SPIN DEPENDENT 2D ELECTRON SCATTERING BY NANOMAGNETS

By

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Abstract

The fundamental concept of spintronics,the difference between nanoparticle and the corresponding bulk material is presented. Mathematical expression of scattering amplitude is derived for 3D and 2D scattering of electron from small target particle. But emphasis is given for 2D scattering problem of an electron by a magnetized nanoparticle using Born approximation with account of the dipole- dipole interaction of the magnetic moments of electron and nanomagnet. Born approximation doesn't work for nanomagnet gigantic μ ,for slow particles. We can justify it only for fast particle.

The scattering amplitudes in this problem are the two-component spinors. They are obtained as functions of the electron spin orientation, the electron energy and show anisotropy in scattering angle. The initially polarized beam of electrons scattered by the nanomagnet consists of electrons with no spin flipped and spin flipped. The majority of electrons with no spin flipped are scattered by small angles. The majority electrons with spin flipped are scattered in the vicinity of the scattering angles $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.This can be used as one more as one more method of controlling the spin current.

The scattering amplitudes depend on mutual orientation of the magnetic moment of the nanomagnet and the electron,the energy of electron and the scattering angle.

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First of all, I would like to thank the almighty God for letting me to accomplish this study.

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Chapter 1

Introduction to Spintronics and nanomagnets

Introduction

The field of magnetic nanostructures is now an exciting and central area of modern condensed matter science,[1] which has recently led to the development of a major new direction in electronics so called spintronics. This is a new approach in which the electron spin momentum plays an equal role to the electrical charge, and these radical ideas have galvanized the efforts of previously disparate research communities by offering the promise of surpassing the limits of conventional semiconductors.

Clearly the world of magnetism has now entered electronics in a very fundamental manner. This is a very fast growing and exciting field which attracts a steadily increasing number of researchers, bringing a constant stream of new ideas. Both spintronics and magnetic nanostructures are already household names in the broad scientific community and we are now, as a result, at the important stage of beginning to develop entirely new approaches to electronics and information technology. This paper is also part of this new approach because it deals with the effect of electron scattering by nanomagnets.

1.1 Fundamentals of spintronics

Why spintronics is the current issues of new generation? The answer of this question have been given by different researchers. And they assure that spintronics should take an important place in the technology of our century. Generation of spin currents in non-magnetic semiconductors by purely electric means was one of the principal challenges in the emerging field of semiconductor spintronics.

The study of the spin transfer phenomena is one of the most promising new fields of research in spintronics today. In spin transfer experiments, one manipulates the magnetic moment of a ferromagnetic body without applying any magnetic field but only by transfer of spin angular momentum from a spin-polarized current[13,27]. Spintronics today includes a broad class of phenomena exploiting the influence of the spin on electron transport in magnetic nanostructures. Now a days, for spintronics to be useful, three criteria have to be satisfied [33]:

- we have to be able to create spin-polarized populations of charges,
- we have to be able to manipulate the spin-polarization of these populations, and
- we have to be able to detect the ensuing spin polarization, or the difference in spin polarization between the initial and final populations. Much effort has been directed recently on making spintronics work in semiconductor materials.

The main objective of spintronics is finding reliable methods of controlling the spin orientations and arrangement of the so called spin current[2]. One of these methods is the usage of the spin orbit interaction coupling the electron momentum and its spin. It is known that spin-orbit coupling appears as the term proportional to $(v/c)^2$ (v is a velocity of a particle and c is the speed of light) in the non- relativistic limit of the Dirac equation [14] and is relatively small.

Spintronic systems exploit the fact that the electron current is composed of spin-up and spin-down carriers that carry information encoded in their spin state and interact with magnetic materials differently [12,13]. Many research recently conducted in these area use the spin-orbit interaction terms based on [4-11] to gate voltage that makes it convenient in spintronics applications.

1.2 Gigantic nanomagnets

The modern nanotechnology can develop nanomagnets of typical size of $5-100nm$. It has been reported that ferrite particles of $24-27nm$ with Co doping $[12-14]$ exhibit superparamanetism even at room temperatures[2]. These magnetic nanoparticles can be treated as gigantic molecules with anomalous magnetic moments. There are reports about the super magnets with $7-12\mu_B$ (Bohr magnetons) per atom $[15-17]$. Anomalous magnetic moments $21.5\mu_B$ per Fe atom and 22.9 μ_B per Co atom for diluted magnetic semiconductor (Tio) have been reported in [18]. There is also recent report about the values of Bohr magnetons more than these values. For instant, Co_{13} has a value $1-39\mu_B$,and $Co_{13}@Mn_{20}$ has a value $1-117\mu_B$ for the lowest-energy structure [34].

In this paper, we analyze the scattering of electrons by magnetized nanoparticles or nanomagnets. The interesting properties of nanoparticles are not partly due to the aspects of the surface of the material dominating the properties in lieu of the bulk properties. A key reason for the change in the physical and chemical properties of small particles as their size decreases is the increased fraction of the 'surface' atoms, which occur under conditions (coordination number, symmetry of the local environment, etc.) differing from those of the bulk atoms. From the energy stand- point, a decrease in the particle size results in an increase in the fraction of the surface energy in its chemical potential[29,30].

In short, the properties of nanoparticles are different from the properties of the bulk material. This is typical, because nanoparticles have a greater surface per weight than larger particles.

1.3 Gigantic magneto-resistance

In our previous sections, we discussed that electrons have intrinsic magnetic fields, described by the property of spin. That means, a spinning charged object creates a magnetic field. However, in most materials, for every electron that spins in one direction there is an electron that spins in the opposite direction so that their magnetic fields cancel exactly[28]. Therefore, most materials are not magnetic. In a few metals such as iron, cobalt, and nickel are the most common electrons gain energy if most of them spin in the same direction. As a result, a net magnetic field is generated in that direction, making the material magnetic. In the presence of a magnetic field, the laws of quantum mechanics dictate that an electron can spin in one of two directions: either its own magnetic field, as a result of its spin, aligns with the external field, which is the up direction; or its own field aligns opposite to the external field, which is the down direction.

Based on the above discussion,to come across the meaning of Gigantic magneto-resistance, let us begin our discussion with magneto-resistance (MR) which is a term widely used to mean the change in the electric conductivity due to the presence of a magnetic field. Moreover,the magneto-resistance of magnetic thin films and multilayer is one of their most interesting transport properties and results from the different scattering properties of spin up and spin down electrons. The giant magneto-resistance or GMR is another type of MR, existing in heterostructure and discovered in 1988 in iron and chromium multilayers (Fert et al. 1988; Binasch et al. 1989) which is recognized as the birth of spintronics[29]. The GMR of multilayer is induced by the variation of the angle between the magnetizations of consecutive magnetic layers. When a magnetic field aligns all the magnetizations in parallel, the resistance of the multilayer decreases dramatically, this was called giant magneto-resistance or GMR [1,29]. This GMR was associated with a change in the relative alignment of the net spins on two iron layers. Or the term "giant" refers to this larger change. When the net electron spins (or magnetization) on the adjacent iron layers were in opposite directions, the resistance was high; when they were in the same direction, the resistance was low. The origin of the GMR can now be simply understood, as a first step, by only considering the spin-dependent scattering effects.

Several types of theoretical models have been developed for GMR; some of them focus on spin dependent scattering of the electrons which is mainly our interest, others on the dependence of the band structure on the magnetic configuration[29].

In the next three chapters we will see that the effect of nanoparticle on electron for different spin orientations (spin up and spin down) which is called scattering.

Chapter 2

Scattering theory and Born Approximation

2.1 3D scattering theory

Before we focus our attention on 2D scattering theory, it is useful to recapitulate a few basic facts of 3D scattering theory and validity of Born approximation. Let us consider an electron moving to the positive z-axis with an incident plane wave towards a small magnetized target particle placed at the origin. Here we are going to focus on the determination of the the scattering amplitude $f(\varphi)$, it can be obtained from the solutions of Schrodinger equation which can be written as

$$(\bigtriangledown^2 + k^2)\psi(\vec{r}) = \frac{2m}{\hbar^2} V(\vec{r})\psi(\vec{r}), \tag{2.1.1}$$

where $k^2 = \frac{2mE}{\hbar^2}$ and $V(\vec{r})$ is the interaction potential. The general solution of the equation eq.(2.1.1) consists of a sum of two components:

1. a general solution to the homogeneous equation:

$$(\bigtriangledown^2 + k^2)\psi_o(\vec{r}) = 0 \tag{2.1.2}$$

   which is the Schrodinger equation for a free particle (no scattering).

where $k_o^2 = \frac{2mE}{\hbar^2}$, here $\psi_o(\vec{r}) = e^{i\vec{k}_o.\vec{r}}$ is the incident plane wave.

2. and a particular solution of eq.(2.1.1) with the interaction potential.

In the quantum theory of scattering, we imagine an incident plane wave $\psi_o(\vec{r}) = e^{i\vec{k}_o.\vec{r}}$, traveling in the positive z-direction, which encounters a scattering potential, producing an outgoing plane wave. In this case for the three-dimensional system, the wave function well outside the localized target region will involve a superposition of the incident plane wave and the scattered spherical wave.

The asymptotic form of wave function can be straight forwardly written as

$$\psi(\vec{r}) \simeq e^{ik_o r} + f(\varphi)\frac{e^{ikr}}{r} \tag{2.1.3}$$

where the second component of the wave function denotes the change in the outgoing spherical wave due to the potential. It occurs at a distance far from the scatterer ($r \gg 1$). In this case it is useful to replace the differential equation by an integral equation. The transformation to an integral equation is performed most efficiently by regarding $\frac{2m}{\hbar^2}V(\vec{r})\psi(\vec{r})$ on the right side of eq.(2.1.1) temporarily as a given inhomogeneity, even though it contains the unknown function $\psi(\vec{r})$. Formally then, the particular solution of eq.(2.1.1) can be expressed in terms of **Green's function**.

The method of the Green's function by which this may be accomplished is far more general than the immediate problem would suggest. Thus we do have the following form

$$\psi(\vec{r}) = \psi_o(\vec{r}) + \frac{2m}{\hbar^2}\int G(\vec{r},\vec{r'})V(\vec{r'})\psi(\vec{r'})d^3r' \tag{2.1.4}$$

where $G(\vec{r},\vec{r'})$ is the Green's function represents and outgoing spherical wave emitted from r.

The scattering theory is applicable to the case of any arbitrary spherically symmetric potential. The Green's function is obtained by solving the point source equation.

$$(\bigtriangledown^2 + k^2)G(\vec{r}, \vec{r'}) = \delta(\vec{r} - \vec{r'}) \tag{2.1.5}$$

Using Fourier transforms and contour integration the expression of $G(\vec{r}, \vec{r'})$ is given by

$$G(\vec{r}, \vec{r'}) = -\frac{1}{4\pi} \frac{e^{ik|\vec{r}-\vec{r'}|}}{|\vec{r} - \vec{r'}|} \tag{2.1.6}$$

Substituting eq.(2.1.6) into eq.(2.1.4) we obtain for the total scattered wave function.

$$\psi(\vec{r}) = \psi_o(\vec{r}) - \frac{m}{2\pi\hbar^2} \int \frac{e^{ik|\vec{r}-\vec{r'}|}}{|\vec{r} - \vec{r'}|} V(\vec{r'})\psi(\vec{r'}) d^3r' \tag{2.1.7}$$

This is an integral form of the Schrodinger (differential) equation eq.(2.1.1) which is more suitable for scattering theory. Note that eq.(2.1.7) can be solved approximately by means of a series of successive approximations, known as the **Born series**.

The Born approximation is based on the perturbation theory and therefore has the intrinsic assumption that the potential of the scatterer is weak. That means if the potential $V(\vec{r'})$ is weak enough, the first Born approximation is valid whenever the wave function $\psi(\vec{r})$ is only slightly different form the incident plane wave. Keep this in mind we will see the main application of Born approximation for us. To describe the approximation,let us consider the following experimental figure.

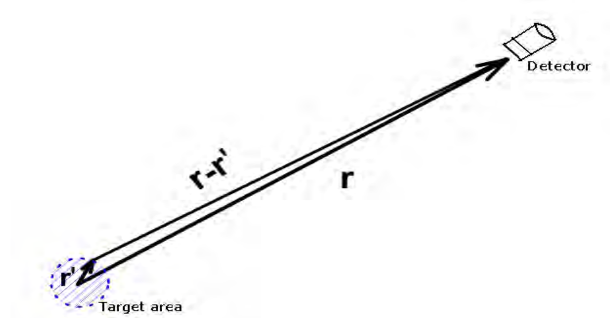


Figure 2.1: Macroscopic and microscopic coordinates.

In a scattering experiment,since the detector is located at distances (away from the target) that are much larger than the size of the target, we have ($r \gg r'$), where r represents the distance from the target to the detector and r' the size of the target. We may have an approximation

$$k|\vec{r} - \vec{r'}| = k\sqrt{\vec{r}^2 - 2\vec{r}.\vec{r'} + r'^2} \simeq kr - k\frac{\vec{r}.\vec{r'}}{r} = kr - \vec{k}.\vec{r'}$$

$$\frac{1}{|\vec{r} - \vec{r'}|} = \frac{1}{r}\frac{1}{|\ 1 - \frac{\vec{r}.\vec{r'}}{r^2}\ |} \simeq \frac{1}{r}(1 + \frac{\vec{r}.\vec{r'}}{r^2}) \simeq \frac{1}{r}$$

Substituting these values into eq.(2.1.7) for $r \gg r'$ gives

$$\psi(\vec{r}) = \psi_o(\vec{r}) - \frac{m}{2\pi\hbar^2}\int \frac{e^{ikr}e^{-i\vec{k}_o.\vec{r}}}{r}V(\vec{r'})\psi(\vec{r'})d^3r' \qquad (2.1.8)$$

This is just the sum of an incident plane wave and a spherical wave emitted from the sample as a whole. We can rewrite the asymptotic form of eq.(2.1.8) as follows

$$\psi(\vec{r}) = e^{i\vec{k}_o.\vec{r}} + f(\varphi)\frac{e^{ikr}}{r} \qquad (2.1.9)$$

where φ is the angle between the z-axis and the direction of the scattered particle (electron) and $f(\varphi)$ is called **the scattering amplitude** in the first Born approximation which is given by

$$f(\varphi) = -\frac{m}{2\pi\hbar^2}\int e^{-i\vec{k}_o.\vec{r}}V(\vec{r'})\psi(\vec{r'})d^3r' \qquad (2.1.10)$$

Moreover, the scattering amplitude is the amplitude of the outgoing spherical wave relative to the incoming plane wave in the stationary-state scattering process.

2.2 2D scattering theory

As we already informed, the intension of this paper is 2D scattering theory. That means, here after (including chapter 3 and 4) the whole problem is to determine the scattering amplitude $f(\varphi)$ in 2D and it tells us the probability of scattering in a given direction φ and hence is related to the differential length.

To derive the expression of scattering amplitude of electron for 2D case and to compare it with eq.(2.1.10) let us consider a particle (electron) in an external electromagnetic field described by the vector potential $\vec{A}(r)$. The electron scattering appears because of the magnetic dipole-dipole interaction between the magnetic dipole moment of the electron and the magnetic field of the magnetized nanoparticle, which has spin and orbital angular momentum contributions. Then the Hamiltonian operator is taken from classical physics to be

$$\hat{H} = \frac{1}{2m}\left(\hat{P} - \frac{q}{c}\vec{A}\right)^2 + \mu_B \hat{\sigma}.\vec{B} \tag{2.2.1}$$

where $\hat{P}$ - momentum operator, q - charge of the particle and c - speed of light

In our case the charge is electron. The interaction of magnetic moments of electrons and nanomagnet appears in the Pauli equation in small parameter $\frac{v}{c}$. So the Hamiltonian for the particle charge electron (charge - e) subjected to an electromagnetic field is given by modifying eq.(2.2.1) as

$$\hat{H} = \frac{1}{2m}\left(\hat{P} + \frac{e}{c}\vec{A}\right)^2 + \mu_B \hat{\sigma}.\vec{B} \tag{2.2.2}$$

where m- mass of the particle, μ_B- Bhor magnetos,

$\hat{\sigma}$- Pauli matrices and $\vec{B}$- magnetic field

For this particular case we use a model of the nanomagnet particle as a point like dipole built in a sphere of radius **a**, which specifies a radius of the nanoparticle. The vector potential and the magnetic field of the dipole $\vec{\mu}$ are given by the expressions.

$$\vec{A} = \frac{\vec{\mu} \times \vec{r}}{r^3} \tag{2.2.3}$$

and

$$\vec{B} = \frac{3(\vec{\mu} \times \vec{n})\vec{n} - \vec{\mu}}{r^3} \tag{2.2.4}$$

where $\vec{n}$ is a unit vector along the radius vector $\vec{r}$,($\vec{n} = \vec{r}/r$)

Upon substituting eq.(2.2.3) and equ.(2.2.4) gives

$$\hat{H} = \frac{1}{2m}\left(\hat{P} + \frac{e}{c}\frac{\vec{\mu}\times\vec{r}}{r^3}\right)^2 + \mu_B\hat{\sigma}.\frac{[3(\vec{\mu}\times\vec{n})\vec{n} - \vec{\mu}]}{r^3}$$

$$\hat{H} = \frac{1}{2m}\left[\hat{P}^2 + 2\hat{P}\frac{e}{c}\frac{\vec{\mu}\times\vec{r}}{r^3} + \left(\frac{e}{c}\frac{\vec{\mu}\times\vec{r}}{r^3}\right)^2\right] + \frac{\mu_B}{r^3}\left[3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}\right]$$

$$\hat{H} = \frac{1}{2m}\hat{P}^2 + \frac{e}{mc}\frac{\vec{\mu}.\hat{L}}{r^3} + \frac{1}{2m}\left(\frac{e}{c}\right)^2\left(\frac{\vec{\mu}\times\vec{r}}{r^3}\right)^2 + \frac{\mu_B}{r^3}\left[3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}\right]$$

where $\hat{P}.(\vec{\mu}\times\vec{r}) = \vec{\mu}.(\vec{r}\times\hat{p})$

Here $\hat{L} = \vec{r}\times\hat{p}$ is the angular momentum operator and since $c \gg e \Rightarrow \left(\frac{e}{c}\right)^2 \to 0$, or we can apply this Hamiltonian only under conditions that allow the A^2 term to be neglected. Then the $\hat{H}$ would have a form

$$\hat{H} = \frac{1}{2m}\hat{P}^2 + \frac{e}{mc}\frac{\vec{\mu}.\hat{L}}{r^3} + \frac{\mu_B}{r^3}\left[3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}\right] \tag{2.2.5}$$

Let the magnetic moment of the nanoparticle $\vec{\mu}$ and the electron momentum P is in the x, z plane. Since our interest is for $2D$ geometry eq.(2.2.5) takes the form

$$\hat{H} = \frac{1}{2m}\hat{P}_2^2 + \frac{\mu_B}{\rho^3}\left[3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}\right] \tag{2.2.6}$$

Here $\hat{P}_2^2$ is the $2D$ momentum operator and ρ is the $2D$ radius vector in the x, z plane and we took into account that the $\vec{\mu}.\hat{L}$ vanishes, because $\vec{\mu}$ is perpendicular to the angular momentum $\hat{L}$.

The Schrodinger equation corresponding to Hamiltonian eq.(2.2.6) takes the form

$$\left(\bigtriangledown_2^2 + k^2\right)\vec{\psi}(\vec{\rho}) = \frac{2m}{\hbar^2}\hat{V}\vec{\psi}(\vec{\rho}) \tag{2.2.7}$$

where the wave number k is given by

$$k^2 = \frac{2mE}{\hbar^2}$$

Here E is the energy of the the scattering electron and $\bigtriangledown_2^2$ is 2D Laplace operator.

The interaction potential $\hat{V}$ can be written as

$$\hat{V} = \frac{\mu_B}{\rho^3}\left[3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}\right] \tag{2.2.8}$$

We know that electron is a spin-$\frac{1}{2}$ particle. In the case of a spin-$\frac{1}{2}$ particle, the wave function is not a complex-valued function but a two-component spinor as shown in the following expression.

$$\vec{\psi}(\vec{\rho}) = \begin{vmatrix} \psi_1(\vec{\rho}) \\ \psi_2(\vec{\rho}) \end{vmatrix}$$

which under a rotation through an angle φ about the y-axis defined by a unit vector **n**. The term **spinor** here stands for doubling of the wave function itself as it is shown in the above equation and characterizing a Pauli spinor

Equation eq.(2.2.7) is a typical equation form the scattering theory with the interaction potential V depending on spin variables. We assume that $V(\rho)$ has a finite range a. Thus the interaction between the electron and the potential occurs only in a limited region of space $\rho \leq a$ which is called **the range of potential** $V(\rho)$, or the scattering region. Outside the range, $\rho > a$ the potential vanishes. If we consider $\psi(\rho)$ is a solution of homogeneous Schrodinger equation, we do have

$$\left(\bigtriangledown_2^2 + k^2\right) \vec{\psi}_o(\vec{\rho}) = 0 \tag{2.2.9}$$

In this case m behaves as a free particle before collision hence described by a plane wave.

$$\psi_o(r) = e^{ik_o r} \tag{2.2.10}$$

where k_o is the wave vector associated with the incident particle.

Consider the case of the presence of the potential $V(\rho)$ the plane wave $\psi(\rho)$ is distorted. The wave function describing the particles is now a solution of the Schrodinger equation of eq.(2.2.7).

The general solution to this equation consists of a sum of two components:

$$\vec{\psi}(\vec{\rho}) = \vec{\psi}_o(\vec{\rho}) + \vec{\psi}_s(\vec{\rho}) \tag{2.2.11}$$

This is a general solution to the homogeneous equation.(i.e eq.(2.2.9)) and a particular solution to eq.(2.2.7). First note that $\psi_o(\rho)$ is nothing but the incident plane wave. As for the particular solution to eq.(2.2.7) is by developing a general expansion of the scattering wave function,$\vec{\psi}_s(\vec{\rho})$, in terms of the Green function of the scattering potential the differential equation of eq.(2.2.7) can be formally presented in the following integral equation.

$$\vec{\psi}_s(\vec{\rho}) = \frac{-2m}{\hbar^2} \int G(\vec{\rho}, \vec{\rho'}) \hat{V}(\rho') \vec{\psi}_o(\vec{\rho'}) d^2\rho' \tag{2.2.12}$$

Now let the incident particle wave be the plane wave propagating along the positive z-axis is represented by

$$\vec{\varphi}_o(\vec{\rho}) = e^{(ikz)} \vec{\chi}_{o(s)} \tag{2.2.13}$$

with the spin function $\vec{\chi}_o(s)$. Since we have $\vec{\varphi}_o(\vec{\rho})$ in eq.(2.2.12) let us modify eq.(2.2.13) for x', z' as

$$\vec{\psi}_o(\rho') = e^{ik_o\rho'} \vec{\chi}_{o(s)} \tag{2.2.14}$$

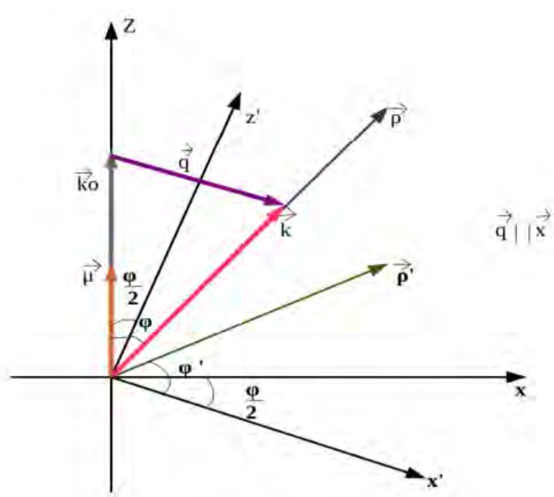


Figure 2.2: The coordinate system:(x,z) and (x', z').The x' axis is parallel to q.

At large distance $\rho' \ll \rho$ from the scattering center, the scattered wave in the 2D geometry is a cylindrical wave

$$\vec{\psi}_s(\rho) = f(\varphi)\frac{e^{ik\rho}}{\sqrt{-i\rho}} \tag{2.2.15}$$

with the scattering "amplitude″ $\vec{f}(\varphi) = \begin{vmatrix} f_1(\varphi) \\ f_2(\varphi) \end{vmatrix}$. Notice that for cylindrical wave the wave function would have the form of eq.(2.2.15). The Green function in eq.(2.2.12) the cylindrical geometry which is the first kind zeroth order Hankel function expressed as[19]:

$$G(\vec{\rho}, \vec{\rho'}) = \frac{i}{4} H_o^{(1)} \left(k|\vec{\rho} - \vec{\rho'}| \right) \tag{2.2.16}$$

At ($\rho' \ll \rho$) the above equation has the asymptotic behavior. The first kind zeroth order Hankel function can be approximated as

$$H_o^{(1)} \left(k|\vec{\rho} - \vec{\rho'}| \right) \simeq \frac{1-i}{\sqrt{\pi\rho k}} e^{ik(\rho - \vec{\rho'}.\vec{n})} \tag{2.2.17}$$

where $\vec{n} = \frac{\vec{\rho}}{\rho} \rightarrow$ unit vector

Substituting equ.(2.2.17) into equ.(2.2.16) gives

$$G(\vec{\rho}, \vec{\rho'}) = \frac{i}{4}\left(\frac{1-i}{\sqrt{\pi\rho k}}\right) e^{ik(\rho - \vec{\rho'}.\vec{n})} = \left(\frac{i+1}{4\sqrt{\pi\rho k}}\right) e^{ik(\rho - \vec{\rho'}.\vec{n})} \tag{2.2.18}$$

Upon substituting eq.(2.2.14) and eq.(2.2.18) into eq.(2.2.12) gives

$$\begin{aligned} \vec{\varphi}_s(\vec{\rho}) \quad &= \frac{-2m}{\hbar^2}\int \left(\frac{i+1}{4\sqrt{\pi\rho k}}\right) e^{ik(\rho - \vec{\rho'}.\vec{n})}\hat{V}(\vec{\rho'})e^{ik_o\vec{\rho'}}\vec{\chi}_{o(s)}d^2\rho' \\ &= \frac{-2m}{\hbar^2}\int \left(\frac{i+1}{4\sqrt{\pi\rho k}}\right) e^{ik(\rho - \vec{\rho'}.\vec{n})}\hat{V}(\vec{\rho'})e^{ik\rho}.e^{-ik\vec{\rho'}.\vec{n})}e^{ik_o\vec{\rho'}}\vec{\chi}_{o(s)}d^2\rho' \end{aligned}$$

$$\vec{\psi}_s(\vec{\rho}) = \frac{-2m}{\hbar^2}\int \left(\frac{i+1}{4\sqrt{\pi k}}\right)\frac{e^{ik\rho}}{\sqrt{\rho}} e^{-i(\vec{k}-\vec{k}_0)\vec{\rho'}.\vec{n})}\hat{V}(\vec{\rho'})\vec{\chi}_{o(s)}d^2\rho' \tag{2.2.19}$$

Substituting eq.(2.2.15) into eq.(2.2.19) gives

$$\vec{f}(\varphi)\frac{e^{ik\rho}}{\sqrt{-i\rho}} = \frac{-m}{\hbar^2}\int \left(\frac{i+1}{2\sqrt{\pi k}}\right)\frac{e^{ik\rho}}{\sqrt{\rho}} e^{-i\vec{q}.\vec{\rho'}}\hat{V}(\vec{\rho'})\vec{\chi}_{o(s)}d^2\rho'$$

$$\vec{f}(\varphi) = \frac{-m}{\hbar^2}\int\sqrt{\frac{2}{4\pi k}}e^{-i\vec{q}.\vec{\rho}'}\hat{V}(\vec{\rho}')\vec{\chi}_{o(s)}d^2\rho'$$

$$\vec{f}(\varphi) = \frac{-m}{\hbar^2\sqrt{2\pi k}}\int e^{-i\vec{q}.\vec{\rho}'}\hat{V}(\vec{\rho}')\vec{\chi}_{o(s)}d^2\rho' \tag{2.2.20}$$

The relationship between $\vec{k}$,$\vec{k}_o$ and φ is shown in the following fig.

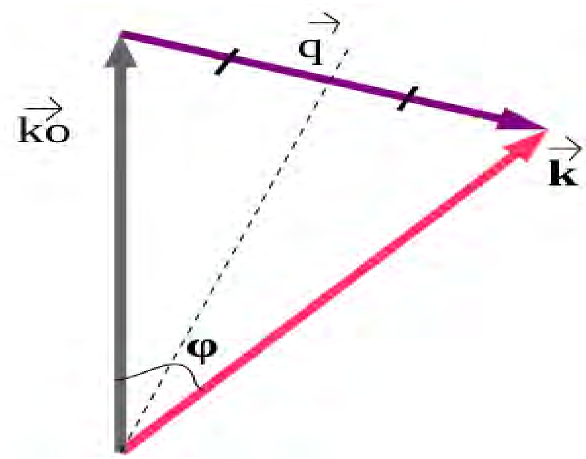


Figure 2.3: Initial wave vector and scattered wave vector

For special case of elastic scattering we have where $k_o^2 = k^2 = \frac{2mE}{\hbar^2}$. Referring the above fig. $\vec{q}$ is the difference between the scattered and initial wave vectors($\vec{q} = \vec{k} - \vec{k}_o$), called **scattering wave vector** and in quantum mechanics is referred to as the momentum transfer. Referring the above fig. the magnitude of $\vec{q}$ can be written as

$$\sin\frac{\varphi}{2} = \frac{q}{2k_o} \Rightarrow q = 2k_o\sin\frac{\varphi}{2}$$

Based on this equation we notice that q will be small for either $\vec{k}_o \to 0$ (the low energy limit) or $\sin\frac{\varphi}{2} \to 0$ (forward scattering). Just to keep our discussion simple, here we focus on elastic collision in which both the energy and the particle number conserved although many of the concepts that we will develop are general[23].

In the next chapter we will see that the effect of magnetic moment of nanoparticle $\vec{\mu}$ for spin orientation(spin up and spin down).

Chapter 3

Scattering Amplitude

3.1 The magnetic moment of nanoparticle($\vec{\mu}$) parallel to the velocity of incident electron

So far we discussed the general form and concepts of 2D scattering theory. In this chapter we are going to see the specific cases of spin orientation. There are four possible cases of spin orientation in which we are interested to see for 2D electron scattering by magnetized nanoparticles.

3.1.1 Electron with spin up

Consider the scattering of electrons moving to the positive z-axis (coming form infinity parallel to the z-axis)by a magnetized nanomagnet placed at the origin. To evaluate the integral of eq.(2.2.20) one has to choose appropriate coordinate system fig.2.2 shows two coordinate systems. In the first one (x,z) the magnetic moment of nanoparticle $\vec{\mu}$ is along the z- axis. But the calculation of the integral in eq.(2.2.20) is convenient to carry out in another coordinate system with x'- axis directed along $\vec{q}$, in which the dot product in the exponent of eq.(2.2.20) has the simplest form $\vec{q}.\vec{\rho'} = q\ \rho' \cos\varphi$ (ρ' and φ are variables of integration). Let us recall, the operator of potential energy of eq.(2.2.8) in this coordinate as

$$\hat{V} = \frac{\mu_B}{\rho'^3}\left[3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}\right] \tag{3.1.1}$$

where $\vec{\mu}' = \vec{\mu}_{x'} + \vec{\mu}_{z'}$

$\vec{n}' = \cos\varphi'\vec{x}' + \sin\varphi'\vec{z}'$ here $\vec{x}'$ and $\vec{z}'$ are unit vectors in x' and z'-axis respectively, and $\hat{\sigma} = \hat{\sigma}_{x'} + \hat{\sigma}_{z'}$

Substituting these expressions into eq.(3.1.1)

$$\begin{aligned}
\hat{V}(\rho) = \quad & \frac{\mu_B}{\rho'^3}\left[3\{(\vec{\mu}_{x'} + \vec{\mu}'_z).(\cos\varphi'\vec{x}' + \sin\varphi'\vec{z}')\}(\cos\varphi'\vec{x}' + \sin\varphi'\vec{z}').(\hat{\sigma}_{x'} + \hat{\sigma}_{z'})\right] \\
& -\frac{\mu_B}{\rho'^3}\left[(\hat{\sigma}_{x'} + \hat{\sigma}_{z'}).(\vec{\mu}_{x'} + \vec{\mu}_{z'})\right] \\
& = \frac{\mu_B}{\rho'^3}\left[(3\mu_{x'}\cos\varphi' + 3\mu'_z\sin\varphi')(\hat{\sigma}_{x'}\cos\varphi' + \hat{\sigma}_{z'}\sin\varphi') - (\mu_{x'}\hat{\sigma}_{x'} + \mu_{z'}\hat{\sigma}_{z'})\right] \\
& = \frac{\mu_B}{\rho'^3}\left[3\mu_{x'}\hat{\sigma}_{x'}\cos^2\varphi' + \frac{3}{2}\mu_{z'}\hat{\sigma}_{z'}\sin 2\varphi' + \frac{3}{2}\mu_{z'}\hat{\sigma}_{x'}\sin 2\varphi' + 3\mu_{z'}\hat{\sigma}_{z'}\sin^2\varphi'\right] \\
& -\left[\frac{\mu_B}{\rho'^3}\{\mu_{x'}\hat{\sigma}_{x'} + \mu_{z'}\hat{\sigma}_{z'}\}\right]
\end{aligned}$$

$$\begin{aligned}
\hat{V}(\rho) = \quad & \frac{\vec{\mu}_B}{\rho'^3}\left[\{\mu_{x'}(3\cos^2\varphi' - 1) + \frac{3}{2}\vec{\mu}_{z'}\sin 2\varphi'\}\hat{\sigma}_{x'}\right. \\
& \left. + \{\vec{\mu}_{z'}(3\sin^2\varphi' - 1) + \frac{3}{2}\vec{\mu}_{x'}\sin 2\varphi'\}\hat{\sigma}_{z'}\right]
\end{aligned} \tag{3.1.2}$$

In this coordinate system the magnetic moment $\vec{\mu}$ has two components:

$\vec{\mu}_{z'} = \mu\cos\frac{\varphi}{2}$, and $\vec{\mu}_{x'} = \mu\sin\frac{\varphi}{2}$. We discussed in section 2.2 that the Hamiltonian of a spin -$\frac{1}{2}$ in an external magnetic field is a Pauli matrix. In this case we know that Pauli matrix has two eigenstates. That is " up state " $\Rightarrow \begin{pmatrix}1\\0\end{pmatrix}$ and " down state"$\Rightarrow \begin{pmatrix}0\\1\end{pmatrix}$

We can treat these states in the x,z coordinate system as

$$\vec{\chi}_{0(s)} = \vec{\chi}_{\uparrow(s)} = \begin{pmatrix}1\\0\end{pmatrix}$$

for the electron with spin parallel to $\vec{\mu}$. But the expression of eq.(2.2.20) is carried out in x', z' the coordinate system. So we have to transform the spinor wave function $\vec{\chi}_{0(s)}$ to this coordinate system. The operator of rotation of a two components spinor through a finite angle θ about the given axis (y-axis) is given by

$$\hat{U}_n = \mathbf{1}\cos\frac{\theta}{2} + i\vec{n}.\hat{\sigma}\sin\frac{\theta}{2} \tag{3.1.3}$$

where $\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is an identity matrix and $\vec{n}$ is a unit vector along the rotation axis. $\vec{n} = \vec{n}_y j$ (unit vector along y-direction). From fig.(2.2) one can see that the required transformation is rotation about the y-axis in the x,z plan by an angle $\theta = \frac{\varphi}{2}$. We can modify equ.(3.1.3) as

$$\hat{U}_y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} cos\frac{\theta}{2} + i\vec{n}_y.\hat{\sigma}_y \sin\frac{\theta}{2}$$

here, $\vec{n}_y.\hat{\sigma}_y = \sigma_y$

$$\hat{U}_y = \begin{pmatrix} \cos\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} \end{pmatrix} + i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\frac{\theta}{2} = \begin{pmatrix} \cos\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\sin\frac{\theta}{2}$$

$$\hat{U}_y = \begin{pmatrix} \cos\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} \end{pmatrix} + \begin{pmatrix} 0 & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & 0 \end{pmatrix}$$

$$\hat{U}_y = \begin{pmatrix} \cos\frac{\varphi}{4} & \sin\frac{\varphi}{4} \\ -\sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix} \tag{3.1.4}$$

Hence, the spinor $\vec{\chi}'_{\uparrow 0(s)}$ in the x', z' coordinate system can be expressed as $\vec{\chi}'_{\uparrow 0(s)} = \hat{U}_y\vec{\chi}_{\uparrow(s)}$

$$\vec{\chi}'_{\uparrow(s)} = \hat{U}_y\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\frac{\varphi}{4} & \sin\frac{\varphi}{4} \\ -\sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{\chi}'_{\uparrow(s)} = \begin{pmatrix} \cos\frac{\varphi}{4} \\ -\sin\frac{\varphi}{4} \end{pmatrix}$$

Substituting eq.(3.1.2) into eq.(2.2.20) with account of results $\hat{\sigma}_x\vec{\chi}'_{\uparrow(s)}$ and $\hat{\sigma}_z\vec{\chi}'_{\uparrow(s)}$

$$\{\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\}$$

$$\begin{aligned} \vec{f}(\varphi) &= \frac{-m}{\hbar^2\sqrt{2\pi k}}\int e^{-iq\rho'\cos\varphi'}\frac{\mu_B}{\rho'^2}\Big[\{\mu_{x'}(3\cos^2\varphi' - 1) + \frac{3}{2}\mu_{z'}\sin 2\varphi'\}\hat{\sigma}_{x'} \\ &\quad +\{\mu_{z'}(3\sin^2\varphi' - 1) + \frac{3}{2}\mu_{x'}\sin 2\varphi'\}\hat{\sigma}_{z'}\Big]\vec{\chi}_{o(s)}d^2\rho' \end{aligned}$$

$$\vec{f}(\varphi) = \frac{-m}{\hbar^2\sqrt{2\pi k}}\int_0^{2\pi}\int_a^{\infty} e^{-i\vec{q}\vec{\rho}'\cos\varphi'}\frac{\mu_B}{\rho'^2}\Big[\big\{\mu_{x'}(3\cos^2\varphi'-1)+\frac{3}{2}\mu_{z'}\sin 2\varphi'\big\}\hat{\sigma}_{x'}\vec{\chi}'_{\uparrow(s)} + \big\{\mu_{z'}(3\sin^2\varphi'-1)+\frac{3}{2}\mu_{x'}\sin 2\varphi'\big\}\hat{\sigma}_{z'}\vec{\chi}'_{\uparrow(s)}\Big]d\rho' d\varphi' \quad (3.1.5)$$

Here $\vec{\mu}_{x'} = -\mu\sin\frac{\varphi}{2}$, $\vec{\mu}_{z'} = \mu\sin\frac{\varphi}{2}$ and $\vec{\chi}'_{0\uparrow(s)} = \vec{\chi}'_{\uparrow(s)}$

$$\hat{\sigma}_{x'}\vec{\chi}'_{\uparrow(s)} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\cos\frac{\varphi}{4}\\ -\sin\frac{\varphi}{4}\end{pmatrix} = \begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} \text{ and}$$

$$\hat{\sigma}_{z'}\vec{\chi}'_{\uparrow(s)} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\cos\frac{\varphi}{4}\\ -\sin\frac{\varphi}{4}\end{pmatrix} = \begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}$$

Substituting all these values into eq.(3.1.5) and let $\gamma = \frac{-m\mu\mu_B}{\hbar^2\sqrt{2\pi k}}$

$$\vec{f}'(\varphi) = \gamma\int_0^{2\pi}\int_a^{\infty}\frac{e^{-iq\rho'\cos\varphi'}}{\rho'^2}\Big[\big\{-\sin\frac{\varphi}{2}(3\cos^2\varphi'-1)+\frac{3}{2}\cos\frac{\varphi}{2}\sin 2\varphi'\big\}\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} + \big\{\cos\frac{\varphi}{2}(3\sin^2\varphi'-1)-\frac{3}{2}\sin\frac{\varphi}{2}\sin 2\varphi'\big\}\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]d\rho' d\varphi' \quad (3.1.6)$$

It is necessary to note that the two component spinor $\vec{f}'(\varphi)$ relates to the x', z' coordinate system that is denoted by the sign $(')$.

Let us arrange the integral of eq.(3.1.6) as follow

$$\vec{f}'(\varphi) = \gamma\int_a^{\infty}\frac{e^{-iq\rho'\cos\varphi'}}{\rho'^2}\Big[\Big\{\int_0^{2\pi}-3\sin\frac{\varphi}{2}\cos^2\varphi' d\varphi' + \int_0^{2\pi}\sin\frac{\varphi}{2}d\varphi' + \int_0^{2\pi}3\cos\frac{\varphi}{2}\sin\varphi'\cos\varphi' d\varphi')\Big\}\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} + \Big\{\int_0^{2\pi}2\cos\frac{\varphi}{2}d\varphi' - \int_0^{2\pi}3\cos\frac{\varphi}{2}\cos^2\varphi' d\varphi' - \int_0^{2\pi}3\sin\frac{\varphi}{2}\sin\varphi'\cos\varphi' d\varphi'\Big\}\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]d\rho' \quad (3.1.7)$$

Now it is convenient for us first to carryout integration over φ' in eq.(3.1.7) with the help of the known presentation of the **Bessel function**.

$$\int_0^{2\pi} e^{-ix\cos\varphi'}d\varphi' = 2\pi J_{0(x)}$$

$$\int_0^{2\pi}\cos^2\varphi' e^{-ix\cos\varphi'}d\varphi' = -\frac{\partial^2}{\partial x^2}\int_0^{2\pi}\cos^2\varphi' e^{-ix\cos\varphi'}d\varphi' = -2\pi\frac{\partial^2}{\partial x^2}J_{0(x)} = \frac{2\pi J_{1(x)}}{x} - 2\pi J_{2(x)}.$$

Here $J_0(x)$, $J_1(x)$ and $J_2(x)$ are the Bessel functions of zeroth, first, and second order, respectively ($x = qp$). The last formula is obtained with the help of recurrence relation $xJ_n(x)$- $xJ_{n+1}(x)$. We also need one more integral result of Bessel function as
$\int_0^{2\pi} \sin\varphi' \cos\varphi' e^{-ix\cos\varphi'} d\varphi' = 0$.
Inserting these results in to eq.(3.1.7) gives

$$\vec{f}'(\varphi) = \gamma \int_a^\infty \frac{1}{\rho'^2}\Big[\big[3\sin\varphi\{\frac{-2\pi J_1(x)}{x} + 2\pi J_2(x)\} + 2\pi J_0(x)\sin\frac{\varphi}{2}\big] \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + \big[2\cos\frac{\varphi}{2} 2\pi J_0(x) - 3\cos\frac{\varphi}{2}\{\frac{2\pi J_1(x)}{x} - 2\pi J_2(x)\}\big] \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}\Big] d\rho'$$

$$\vec{f}'(\varphi) = \gamma \int_a^\infty \frac{1}{\rho'^2}\Big[\sin\frac{\varphi}{2}\{\frac{-6\pi J_{1(x)}}{x} + 6\pi J_{2(x)} + 2\pi J_{0(x)}\} \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + \cos\frac{\varphi}{2}\{4\pi J_{0(x)} - \frac{6\pi J_{1(x)}}{x} + 6\pi J_{2(x)}\} \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}\Big] d\rho'$$

$$\vec{f}'(\varphi) = 6\gamma\pi \int_a^\infty \frac{1}{\rho'^2}\Big[\sin\frac{\varphi}{2}\{\frac{-J_1(x)}{x} + J_{2(x)} + \frac{J_0(x)}{3}\} \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + \cos\frac{\varphi}{2}\{\frac{2J_0(x)}{3} - \frac{J_1(x)}{x} + J_2(x)\} \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}\Big] d\rho' \quad (3.1.8)$$

Using the recurrence relation of Bessel function

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$$

To have the relation for Bessel functions of zeroth, first and second order, let us use $n = 1$

$$J_0(x) + J_2(x) = \frac{2}{x} J_1(x) \Rightarrow$$

$$\frac{J_1(x)}{x} = \frac{J_0(x) + J_2(x)}{2} \quad (3.1.9)$$

Now substituting eq.(3.1.9) into eq.(3.1.8) and simplifying gives us

$$\vec{f}'(\varphi) = 6\gamma\pi \int_a^\infty \frac{1}{\rho'^2}\Big[\sin\frac{\varphi}{2}\{\frac{-J_1(x)}{x} + J_2(x) + \frac{J_0(x)}{3}\} \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + \cos\frac{\varphi}{2}\{\frac{2J_0(x)}{3} - \frac{J_1(x)}{x} + J_2(x)\} \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}\Big] d\rho'$$

$$\vec{f}'(\varphi) = \quad 6\gamma\pi\int_a^\infty \frac{1}{\rho'^2}\Big[\sin\frac{\varphi}{2}\{\frac{-J_o(x)}{2} - \frac{J_2(x)}{2} + J_2(x) + \frac{J_0(x)}{3}\}\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix}$$

$$+\cos\frac{\varphi}{2}\{\frac{2J_o(x)}{3} - \frac{J_o(x)}{2} - \frac{J_2(x)}{3} + J_2(x)\}\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]d\rho'$$

$$\vec{f}'(\varphi) = \quad 6\gamma\pi\int_a^\infty \frac{1}{\rho'^2}\Big[\sin\frac{\varphi}{2}\{\frac{-3J_o(x) - 3J_2(x) + 6J_2(x) + 2J_0(x)}{6}\}\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix}$$

$$+\cos\frac{\varphi}{2}\{\frac{4J_o(x) - 3J_o(x) - 3J_2(x) + 6J_2(x)}{6}\}\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]d\rho'$$

$$\vec{f}'(\varphi) = \quad 6\gamma\pi\int_a^\infty \frac{1}{\rho'^2}\Big[\sin\frac{\varphi}{2}\{\frac{-J_o(x) + 3J_2(x)}{6}\}\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix}$$

$$+\cos\frac{\varphi}{2}\{\frac{J_o(x) + 3J_2(x)}{6}\}\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]d\rho'$$

We know that $x = q\rho' \Rightarrow dx = qd\rho' \Rightarrow d\rho' = \frac{dx}{q}$. Moreover $x^2 = q^2\rho'^2 \Rightarrow \rho'^2 = \frac{x^2}{q^2}$

For the boundary (limit of integral) $\rho' = a \Rightarrow x = qa$

Then the above integral can be rewritten as in the following ways.

$$\vec{f}'(\varphi) = \quad 6\gamma\pi\int_a^\infty \frac{q^2}{6x^2}\Big[-\sin\frac{\varphi}{2}\{J_o(x) + 3J_2(x)\}\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix}$$

$$+\cos\frac{\varphi}{2}\{J_o(x) + 3J_2(x)\}\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]\frac{dx}{q}$$

$$\vec{f}'(\varphi) = \quad \frac{6\gamma\pi}{a}\Big[\int_{aq}^\infty \frac{qa}{6x^2}\{J_o(x) + 3J_2(x)\}dx(-\sin\frac{\varphi}{2})\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix}$$

$$+\int_{aq}^\infty \frac{qa}{6x^2}\{J_o(x) + 3J_2(x)\}dx(\cos\frac{\varphi}{2})\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big]$$

The integral parts of the above equation can be replaced by a certain parameters such as I_1 and I_2 respectively. And then we can write the result as

$$\vec{f}'(\varphi) = \frac{6\gamma\pi}{a}\Big[-I_1\sin\frac{\varphi}{2}\times\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} + I_2\cos\frac{\varphi}{2}\times\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Big] \tag{3.1.10}$$

where

$$I_1 = \frac{qa}{6}\int_{qa}^{\infty}\frac{1}{x^2}\big\{J_0(x) - 3J_2(x)\big\}dx \tag{3.1.11}$$

$$I_2 = \frac{qa}{6}\int_{qa}^{\infty}\frac{1}{x^2}\big\{J_0(x) + 3J_2(x)\big\}dx \tag{3.1.12}$$

The dimensionless factor qa in I_1 and I_2 compensates divergence of these integrals at qa $\rightarrow$ zero. To obtain the spin dependent scattering amplitudes in the original coordinate system (x,z coordinate system). We have to rotate the spinors in eq.(3.1.10) by θ anticlockwise. In our case $\theta = \frac{-\varphi}{2}$. The unitary operator of this rotation can be simplified using eq.(3.1.3) as follows.

$$\hat{U}_n = \mathbf{1}\cos\tfrac{\theta}{2} + i\vec{n}.\hat{\sigma}\sin\tfrac{\theta}{2}\ \text{, here } \vec{n}.\hat{\sigma} = \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

So the unitary operator of this rotation (i.e about y-axis) is given by

$$\hat{U}_y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\cos\frac{(-\varphi)}{4} + i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\frac{(-\varphi)}{4}$$

$$\hat{U}_y = \begin{pmatrix} \cos\frac{(-\varphi)}{4} & 0 \\ 0 & \cos\frac{(-\varphi)}{4} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\sin\frac{(-\varphi)}{4}$$

We know that cosine is an even function and sine is an odd function. As a result we have

$$\hat{U}y = \begin{pmatrix} \cos\frac{\varphi}{4} & -\sin\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix} \tag{3.1.13}$$

In this case we can rewrite eq.(3.1.10) by using eq.(3.1.13) as

$$\vec{f}'(\varphi) = \frac{6\gamma\pi}{a}\Big[- I_1\sin\frac{\varphi}{2}\hat{U}_y\begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + I_2\cos\frac{\varphi}{2}\hat{U}_y\begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}\Big]$$

Using the expression of $\hat{U}_y$ into the above equation gives

$$\vec{f}'(\varphi) = \frac{6\gamma\pi}{a}\Big[-I_1\sin\frac{\varphi}{2}\begin{pmatrix} \cos\frac{\varphi}{4} & -\sin\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix}\begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + I_2\cos\frac{\varphi}{2}\begin{pmatrix} \cos\frac{\varphi}{4} & -\sin\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix}\begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}\Big]$$

$$\vec{f}(\varphi) = \frac{6\gamma\pi}{a}\Big[\big[- I_1\sin^2\frac{\varphi}{2} + I_2\cos^2\frac{\varphi}{2}\big]\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2}\big[I_2 - I_1\big]\sin\varphi\begin{pmatrix} 0 \\ 1 \end{pmatrix}\Big]$$

$$\begin{vmatrix} \vec{f}^{\shortparallel}_{\uparrow\uparrow}(\varphi) \\ \vec{f}^{\shortparallel}_{\uparrow\downarrow}(\varphi) \end{vmatrix} = \frac{6\gamma\pi}{a}\left[\left[-I_1\sin^2\frac{\varphi}{2}+I_2\cos^2\frac{\varphi}{2}\right]\begin{pmatrix}1\\0\end{pmatrix}+\frac{1}{2}\left[I_2-I_1\right]\sin\varphi\begin{pmatrix}0\\1\end{pmatrix}\right] \tag{3.1.14}$$

Here, we introduce the scattering amplitude with no spin flipping $f^{\shortparallel}_{\uparrow\uparrow}$ and $f^{\shortparallel}_{\uparrow\downarrow}$ the scattering amplitude with spin flipping. Rearranging the scattering amplitudes of eq.(3.1.14) take the form

$$f^{\shortparallel}_{\uparrow\uparrow} = -\sqrt{L_o}\frac{[I_1\sin^2\frac{\varphi}{2}+I_2\cos^2\frac{\varphi}{2}]}{\sqrt{ka}} \tag{3.1.15}$$

$$f^{\shortparallel}_{\uparrow\downarrow} = -\sqrt{L_o}\frac{[I_2-I_1]\sin\varphi}{2\sqrt{ka}} \tag{3.1.16}$$

We can see from these two equations the scattering amplitude with no spin flipping $f^{\shortparallel}_{\uparrow\uparrow}$ and the scattering amplitude with spin flipping $f^{\shortparallel}_{\uparrow\downarrow}$ depends on ka and φ. This is the interesting result of this paper. And we will see this dependence with the help of graphs in the next chapter. Clearly one can see that from eq.(3.1.15) and eq.(3.1.16) we can have the same parameter $\frac{\sqrt{L_o}}{\sqrt{ka}}$ and referring equ.(3.1.14) we have $\frac{6\gamma\pi}{a}$ term. Using these two terms we can solve an expression for L_o as follow:

$\frac{\sqrt{L_o}}{\sqrt{ka}} = \frac{6\gamma\pi}{a} \Rightarrow \sqrt{L_o} = \frac{6\gamma\pi\sqrt{ka}}{a}$

$L_o = \frac{(6\gamma\pi)^2 k}{a}$

Let us recall that $\gamma = \frac{-m\mu\mu_B}{\hbar^2\sqrt{2\pi k}}$. Substituting the expression of γ into the above equation and simplifying it gives

$L_o = 18\pi a\left(\frac{m\mu\mu_B}{a\hbar^2}\right)^2.$

where L_o is the typical scattering length of our problem.

Now we discuss the applicability of the Born approximation. It is known that it works for the scattering of particles with arbitrary energy provided that the following inequality holds true[20].

$|V| \ll \frac{\hbar^2}{m\bar{\rho}^2}$

Here, $\bar{\rho}$ is the range of action of the scattering potential $\hat{V}(\rho)$. For evaluations we set $|V| \sim \frac{\mu\mu_B}{\rho^3}$. Due to the fast decay of the dipole - dipole interaction with distance we take $\bar{\rho} = 2a$. We have an assumption $|V| \ll \frac{\hbar^2}{4ma^2}$,

$\frac{\mu\mu_B}{\rho^3} \ll \frac{\hbar^2}{4ma^2}$ and for the magnetic moment of nanoparticle, we can use the formula $\mu = \frac{4}{3}\pi\nu\mu_B n_a a^3$. Keeping this in mind, and rearranging we obtain that the born approximation is applicable when the following inequality holds true:

$$\frac{16\pi\nu\mu_B^2 n_a m a^2}{3\hbar^2} \ll 1 \tag{3.1.17}$$

where ν is a number of Bohr magnetos carried by the ferromagnetic atom, and n_a is the density of atoms of the nanomagnets. In agreement with experimental data [15-18] ν lies in the range 7 -23. Due to the relative weakness of the spin-nanomagnet interaction, eq.(3.1.17) can be satisfied only for nanoparticles of the following size:

$$a^2 \ll \frac{3\hbar^2}{16\pi\nu\mu_B^2 n_a m}$$
$$a^2 \ll \frac{1}{\nu}\frac{3\hbar^2}{16\pi\mu_B^2 n_a m}$$
$$a \ll \sqrt{\frac{1}{\nu}}100nm.$$

This condition is obtained by substituting the numerical values of the parameters into eq.(3.1.17) with $n_a = 10^{22}cm^{-3}$.

3.1.2 Electron with spin dawn

Now, we consider the scattering of electrons moving the z-axis with the spins aligned opposite to the magnetic moment $\vec{\mu}$. The spin wave function for the down state is $\vec{\chi}_{\downarrow 0(s)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in the coordinate system with the z-axis parallel to $\vec{\mu}$ in the coordinate system x', z' is

$$\vec{\chi}'_{\downarrow(s)} = \hat{U}_y \vec{\chi}_{\downarrow 0(s)}$$

$$\vec{\chi}'_{\downarrow(s)} = \begin{pmatrix} \cos\frac{\varphi}{4} & \sin\frac{\varphi}{4} \\ -\sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix}$$

Substituting this spinor and interaction potential eq.(3.1.1) into eq.(2.2.19) and following the same procedure as for the electron spin aligned parallel to magnetic moment $\vec{\mu}$, we get the following result for the scattering amplitudes with no spin and with spin flipping after scattering respectively.

$$f^{\shortparallel}_{\downarrow\downarrow} = -\sqrt{L_o}\frac{[I_1 \sin^2 \frac{\varphi}{2} + I_2 \cos^2 \frac{\varphi}{2}]}{\sqrt{ka}} \tag{3.1.18}$$

$$f^{\shortparallel}_{\downarrow\uparrow} = -\sqrt{L_o}\frac{[I_1 - I_2] \sin \varphi}{2\sqrt{ka}} \tag{3.1.19}$$

Here, the expression of L_o keeps its form as we have for the electron spin aligned parallel to magnetic moment $\vec{\mu}$.

Using eq.(3.1.15), eq.(3.1.16),eq.(3.1.18) and eq.(3.1.19) we will have the following relations

$$f^{\shortparallel}_{\downarrow\downarrow}(ka, \varphi) = f^{\shortparallel}_{\uparrow\uparrow}(ka, \varphi) \tag{3.1.20}$$

$$f^{\shortparallel}_{\downarrow\uparrow}(ka, \varphi) = -f^{\shortparallel}_{\uparrow\downarrow}(ka, \varphi) \tag{3.1.21}$$

This shows us that the probability of spin flipping proportional to the square of the scattering amplitudes for down state is the same as for the up state case. i.e

$$|f^{\shortparallel}_{\downarrow\downarrow}(ka, \varphi)|^2 = |f^{\shortparallel}_{\uparrow\uparrow}(ka, \varphi)|^2 \tag{3.1.22}$$

$$|f^{\shortparallel}_{\downarrow\uparrow}(ka, \varphi)|^2 = |f^{\shortparallel}_{\uparrow\downarrow}(ka, \varphi)|^2 \tag{3.1.23}$$

3.2 The $\vec{\mu}$ perpendicular to the velocity of incident electron.

3.2.1 Electron with spin up

Let us consider the case when the magnetic moment $\vec{\mu}$ is transverse to the incident wave. We keep the previous direction for incident wave along z- axis. But now $\vec{\mu}$ is along positive x-axis. In this case the velocity of incident electron is perpendicular to the magnetic moment of the nanoparticle. The expression of the interaction potential eq.(3.1.1) keeps its form. In this case the change we have is that the components of the magnetic moment of the nanoparticle. That is $\vec{\mu}_{x'} = \mu \cos \frac{\varphi}{2}$, and $\vec{\mu}_{z'} = \mu \sin \frac{\varphi}{2}$. Moreover we know that:

$$\hat{\sigma}_x \vec{\chi}'_{\uparrow(s)} = \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} \text{ and } \hat{\sigma}_z \vec{\chi}'_{\uparrow(s)} = \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix}.$$

Substituting all these into eq.(3.1.5) and keeping the expression of γ gives

$$\begin{aligned} \vec{f}'(\varphi) &= \gamma \int_0^{2\pi} \int_a^{\infty} \frac{e^{-iq\rho'\cos\varphi'}}{\rho'^2} \Big[\{ \cos\frac{\varphi}{2}(3\cos^2\varphi' - 1) + \frac{3}{2}\sin\frac{\varphi}{2}\sin 2\varphi' \} \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} \\ &\quad + \{ \sin\frac{\varphi}{2}(3\sin^2\varphi' - 1) + \frac{3}{2}\cos\frac{\varphi}{2}\sin 2\varphi' \} \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix} \Big] d\rho' d\varphi' \end{aligned}$$

Let us arrange this integral as follow:

$$\begin{aligned} \vec{f}'(\varphi) &= \gamma \int_a^{\infty} \frac{e^{-iq\rho'\cos\varphi'}}{\rho'^2} \Big[\{ \int_0^{2\pi} 3\cos\frac{\varphi}{2}\cos^2\varphi' d\varphi' - \int_0^{2\pi} \cos\frac{\varphi}{2} d\varphi' \\ &\quad + \int_0^{2\pi} 3\sin\frac{\varphi}{2}\sin\varphi'\cos\varphi' d\varphi') \} \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} + \{ \int_0^{2\pi} 2\sin\frac{\varphi}{2} d\varphi' \\ &\quad + \int_0^{2\pi} 3\sin\frac{\varphi}{2}\cos^2\varphi' d\varphi' - \int_0^{2\pi} 3\cos\frac{\varphi}{2}\sin\varphi'\cos\varphi' d\varphi' \} \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix} \Big] d\rho' \end{aligned} \tag{3.2.1}$$

Now let us simplify the integration over φ' in eq.(3.2.1) with the help of the known presentation of the Bessel functions gives

$$\begin{aligned} \vec{f}'(\varphi) &= \gamma \int_a^{\infty} \frac{1}{\rho'^2} \Big[\{ 3\cos\frac{\varphi}{2} \left(\frac{2\pi J_1(x)}{x} - 2\pi J_2(x) \right) - 2\pi J_0(x)\cos\frac{\varphi}{2} \} \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} \\ &\quad + \{ \sin\frac{\varphi}{2} 2\pi J_0(x) - 3\sin\frac{\varphi}{2} \left(\frac{2\pi J_1(x)}{x} - 2\pi J_2(x) \right) \} \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix} \Big] d\rho' \\ &= \gamma \int_a^{\infty} \frac{1}{\rho'^2} \Big[\{ \left(\frac{6\pi J_1(x)}{x} - 6\pi J_2(x) - 6\pi J_0(x) \right) \cos\frac{\varphi}{2} \} \times \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} \\ &\quad + \{ \left(4\pi J_0(x) - \frac{6\pi J_1(x)}{x} + 6\pi J_2(x) \right) \sin\frac{\varphi}{2} \} \times \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix} \Big] d\rho' \end{aligned} \tag{3.2.2}$$

To get more simplified form of this integral substitute eq.(3.1.9) into eq.(3.2.2) gives

$$\begin{aligned} \vec{f}'(\varphi) &= 6\gamma\pi \int_a^{\infty} \frac{1}{\rho'^2} \Big[-\sin\frac{\varphi}{2} \{ J_0(x) - 3J_2(x) \} \cos\frac{\varphi}{2} \begin{pmatrix} -\sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix} \\ &\quad + \{ J_0(x) + 3J_2(x) \} \sin\frac{\varphi}{2} \begin{pmatrix} \cos\frac{\varphi}{4} \\ \sin\frac{\varphi}{4} \end{pmatrix} \Big] d\rho' \end{aligned}$$

As we did so far changing the variable of integration from ρ' to x gives

$$\begin{aligned}\vec{f}'(\varphi) &= \frac{6\gamma\pi}{a}\Bigg[\int_{qa}^{\infty}\frac{qa}{6x^2}\{J_0(x)-3J_2(x)\}dx\cos\frac{\varphi}{2}\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix}\\ &+\int_{qa}^{\infty}\frac{qa}{6x^2}\{(J_0(x)+3J_2(x)\}dx\sin\frac{\varphi}{2}\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Bigg]\end{aligned}$$

$$\vec{f}'(\varphi) = \frac{6\gamma\pi}{a}\Bigg[I_1\cos\frac{\varphi}{2}\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} + I_2\sin\frac{\varphi}{2}\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\Bigg] \tag{3.2.3}$$

Still the expressions of I_1 and I_2 the same as what we have so far (see Appendix). Here is also the dimensionless factor qa in I_1 and I_2 compensates divergence of these integrals at qa $\rightarrow$ zero. To obtain the spin dependent scattering amplitudes in the original coordinate system (x-z coordinate system), we have to rotate the spinors in eq.(3.2.3) by ($\theta = \frac{-\varphi}{2}$). In this case the unitary operator of the rotation is exactly eq.(3.1.13). Thus we can write eq.(3.2.3) the rotation result by using eq.(3.1.13) as

$$\vec{f}(\varphi) = \frac{6\gamma\pi}{a}\left[I_1\cos\frac{\varphi}{2}\hat{U}_y\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} + I_2\sin\frac{\varphi}{2}\hat{U}_y\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\right]$$

Using the expression of $\hat{U}_y$ in this equation gives

$$\vec{f}(\varphi) = \frac{6\gamma\pi}{a}\left[I_1\cos\frac{\varphi}{2}\begin{pmatrix}\cos\frac{\varphi}{4} & -\sin\frac{\varphi}{4}\\ \sin\frac{\varphi}{4} & \cos\frac{\varphi}{4}\end{pmatrix}\begin{pmatrix}-\sin\frac{\varphi}{4}\\ \cos\frac{\varphi}{4}\end{pmatrix} + I_2\sin\frac{\varphi}{2}\begin{pmatrix}\cos\frac{\varphi}{4} & -\sin\frac{\varphi}{4}\\ \sin\frac{\varphi}{4} & \cos\frac{\varphi}{4}\end{pmatrix}\begin{pmatrix}\cos\frac{\varphi}{4}\\ \sin\frac{\varphi}{4}\end{pmatrix}\right]$$

$$\vec{f}(\varphi) = \frac{6\gamma\pi}{a}\Bigg[\Big[-I_1\cos^2\frac{\varphi}{2}+I_2\sin^2\frac{\varphi}{2}\Big]\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2}[I_2-I_1]\sin\varphi\begin{pmatrix}0\\1\end{pmatrix}\Bigg]$$

$$\begin{vmatrix}\vec{f}^{\perp}_{\uparrow\downarrow}(\varphi)\\ \vec{f}^{\perp}_{\uparrow\uparrow}(\varphi)\end{vmatrix} = \frac{6\gamma\pi}{a}\Bigg[\Big[-I_1\cos^2\frac{\varphi}{2}+I_2\sin^2\frac{\varphi}{2}\Big]\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2}[I_2-I_1]\sin\varphi\begin{pmatrix}0\\1\end{pmatrix}\Bigg] \tag{3.2.4}$$

Similarly for this case, we introduce the scattering amplitude with no spin flipping $f^{\perp}_{\uparrow\uparrow}(\varphi)$ and with spin flipping $f^{\perp}_{\uparrow\downarrow}(\varphi)$.

$$f^{\perp}_{\uparrow\downarrow}(ka,\varphi) = \frac{-\sqrt{L_o}[I_1\cos^2\frac{\varphi}{2}+I_2\sin^2\frac{\varphi}{2}]}{\sqrt{ka}} \tag{3.2.5}$$

$$f^{\perp}_{\uparrow\uparrow}(ka,\varphi) = \frac{-\sqrt{L_o}[I_2-I_1]\sin\varphi}{2\sqrt{ka}} \tag{3.2.6}$$

We can observe that; from eq.(3.1.19) and eq.(3.2.6)

$$f^{\shortparallel}_{\downarrow\uparrow}(ka,\varphi) = -f^{\perp}_{\uparrow\uparrow}(ka,\varphi) \tag{3.2.7}$$

3.2.2 Electron with spin dawn

Now let us consider the electrons with spin anti- parallel to direction of propagation. We keep the direction of the magnetic moment of the nanoparticle $\vec{\mu}$ is along positive x-axis. Still the velocity of incident electron is perpendicular to the magnetic moment of the nanoparticle.

The spinor function $\vec{\chi}_{o\downarrow(s)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for down state before scattering and perpendicular to the magnetic moment $\vec{\mu}$. In this case the spin wave function $\vec{\chi}'_{\downarrow(s)}$ in the coordinate x, z is given by

$$\vec{\chi}'_{\downarrow(s)} = \hat{U}_y \vec{\chi}_{o\downarrow(s)} = \begin{pmatrix} \cos\frac{\varphi}{4} & \sin\frac{\varphi}{4} \\ -\sin\frac{\varphi}{4} & \cos\frac{\varphi}{4} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sin\frac{\varphi}{4} \\ \cos\frac{\varphi}{4} \end{pmatrix}$$

Substituting this spinor and interaction potential eq.(3.1.1) into eq.(2.2.19), in this case we can use the components of the magnetic moment of the nanoparticle as $\mu_{x'} = \mu\cos\frac{\varphi}{2}$ and $\mu_{z'} = \mu\sin\frac{\varphi}{2}$. Then following the same procedure as for the electron spin up aligned perpendicular to magnetic moment $\vec{\mu}$. We get the following result for the scattering amplitudes with no spin flipping and with spin flipping after scattering, respectively.

$$f^{\perp}_{\downarrow\downarrow}(ka,\varphi) = \frac{-\sqrt{L_o}[I_1 - I_2]\sin\varphi}{2\sqrt{ka}} = -f^{\perp}_{\uparrow\uparrow}(ka,\varphi) \tag{3.2.8}$$

$$f^{\perp}_{\downarrow\uparrow}(ka,\varphi) = \frac{-\sqrt{L_o}\left[I_1\cos^2\frac{\varphi}{2} + I_2\sin^2\frac{\varphi}{2}\right]}{\sqrt{ka}} = -f^{\perp}_{\uparrow\downarrow}(ka,\varphi) \tag{3.2.9}$$

This shows us that the probability of spin flipping proportional to the square of the scattering amplitudes for dawn state is the same as for the up state case.i.e

$$|f^{\perp}_{\uparrow\uparrow}(ka,\varphi)|^2 = |f^{\perp}_{\downarrow\downarrow}(ka,\varphi)|^2 \tag{3.2.10}$$

$$|f^{\perp}_{\uparrow\downarrow}(ka,\varphi)|^2 = |f^{\perp}_{\downarrow\uparrow}(ka,\varphi)|^2 \tag{3.2.11}$$

For 2D spin independent scattering problem, the solution of Schrodinger equation can be used to calculate the probability per unit length per unit time that the particle m is scattered into a length dL in the direction φ; this probability is the measurable quantity which is the differential length $dL = |f(\varphi)|^2 d\varphi$ (function of the energy of incident particle and the scattering angle φ) and the total scattering length $L = \int |f(\varphi)|^2 d\varphi$.

From the above sections one can generalize that in 2D spin dependent scattering problem, the scattering amplitudes are two component spinors, which describe the processes with different orientation of spins after scattering and in general case depend on a mutual orientation of spin of scattering electrons and the magnetic moment of nanoparticle. We may have other possible spin orientation for the electron even for 2D case. But for simplicity, in this paper, we consider only two spin orientation $\shortparallel$ and $\perp$ to μ.

In the next chapter, we analyze the scattering lengths of the considered process and discuss their peculiarities of typical causes of 2D spin dependent electron scattering by gigantic nanomagnets.

Chapter 4

General properties and numerical analysis of scattering length

4.1 Properties of scattering amplitudes

In the previous chapter we have seen that the way the mathematical expressions of scattering amplitudes of electrons derived for different spin orientation interacting with nanoparticle. Now referring the scattering amplitudes of eq.(3.1.21),eq.(3.2.7) and eq.(3.2.10),for the magnetic moment parallel and perpendicular to the velocity of electron we do have the following equality.

$$|f^{\shortparallel}_{\downarrow\uparrow}(ka,\varphi)|^2 = |f^{\shortparallel}_{\uparrow\downarrow}(ka,\varphi)|^2 = |f^{\perp}_{\uparrow\uparrow}(ka,\varphi)|^2 = |f^{\perp}_{\downarrow\downarrow}(ka,\varphi)|^2 \tag{4.1.1}$$

As in 3D scattering theory [20], we can make general conclusions concerning the scattering amplitudes by considering two limiting cases. The first case is for the fast particle ($ka \gg 1$) corresponds to the small λ, or high energy limit. In this case the wavelength is much smaller than the particle,which means that most of the scattering will be in the forward direction. The second limiting case is that the slow particles ($ka \ll 1$) and as we already considered the property of elastic scattering in the limiting case where the velocities of the particles undergoing scattering are so small that their wavelength is large compared with the radius of action **a**.

The potential field $V(r)$ and energy of the particles are small compared with the field within the radius. For this case we may take an assumption of $I_1 = I_2 = \frac{1}{6}$. This can be checked straight forward using the definitions of I_1 and I_2 (see Appendixes). With this in mind, we get

$$|f^{\shortparallel}_{\uparrow\uparrow}|^2 = |f^{\shortparallel}_{\downarrow\downarrow}|^2 = |f^{\perp}_{\uparrow\downarrow}|^2 = |f^{\perp}_{\downarrow\uparrow}|^2 = \frac{L_o}{36ka} \tag{4.1.2}$$

$$|f^{\shortparallel}_{\uparrow\downarrow}|^2 = |f^{\shortparallel}_{\uparrow\downarrow}|^2 = |f^{\perp}_{\uparrow\uparrow}|^2 = |f^{\perp}_{\downarrow\downarrow}|^2 = 0 \tag{4.1.3}$$

This relation eq.(4.1.3) shows the probability of scattering with spin flipping by the magnetic moment parallel to the velocity of electron and the scattering with no spin flipping by the transversal magnetic moment are close to zero.

The total scattering lengths of slow particles for the process denoted by eq.(4.1.2) is obtained by the integration over all φ as $L(ka) = \int |f(\varphi)|^2 d\varphi = |f(\varphi)|^2 \int d\varphi$

$$L(ka) = \frac{\pi L_o}{18ka} \tag{4.1.4}$$

Form eq.(4.1.2) one can see that the spin dependent electron scattering is isotropic for slow particles ($ka \ll 1$).

For the fast particle ($ka \gg 1$),the exponent in eq.(2.2.20) is fast oscillating and the result of integration is non zero only for small scattering angles ($2ka \sin\frac{\varphi}{2} \sim 1$ or $\frac{1}{ka} \ll 1$). The scattering length is very small in this case because it is suppressed by a large factor $ka \gg 1$. These conclusions are in agreement with the general scattering theory [20] and our numerical calculations will be given in the next section.

4.2 Scattering length

Equalities eq.(3.1.22), eq.(3.2.11) and eq.(4.1.1) allows us to introduce three normalized dimensionless scattering lengths:

$$\tilde{L}_1 = \frac{1}{L_o}\frac{dL_1}{d\varphi} = \frac{1}{ka}\big[I_1 \sin^2\frac{\varphi}{2} + I_2 \cos^2\frac{\varphi}{2}\big]^2 \tag{4.2.1}$$

$$\tilde{L}_2 = \frac{1}{L_o}\frac{dL_2}{d\varphi} = \frac{1}{ka}\big[I_1 \cos^2\frac{\varphi}{2} + I_2 \sin^2\frac{\varphi}{2}\big]^2 \tag{4.2.2}$$

$$\tilde{L}_3 = \frac{1}{L_o}\frac{dL_3}{d\varphi} = \frac{1}{4ka}[I_1 - I_2]^2 \sin^2\varphi \tag{4.2.3}$$

The analytical expressions of I_1 and I_2 as functions of ka and φ in the above equations are given in the appendix. Clearly,these scattering length depends on the orientation of the electron spin relative to the dipole moment. The above three equations shows that the dimensionless scattering length $\tilde{L}_1$ relates to the scattering with no spin flipping of electrons with initial velocity parallel to magnetic moment of a nonoparticle $\vec{\mu}$ and $\tilde{L}_2$ relates to the scattering with spin flipping of electrons with initial velocity of electrons perpendicular to the magnetic moment of nanoparticle $\vec{\mu}$.

The quantity $\tilde{L}_3$ relates to the scattering with spin flipping when initial velocity of electrons is parallel to the magnetic moment of nanoparticle $\vec{\mu}$ and scattering of elections with no spin flipping when the initial electrons velocity is perpendicular to the magnetic moment of nanoparticle $\vec{\mu}$. The formulas (A_3),(A_4) and (A_5) shows all these functions are periodical with a period of 2π. This means that small angles in the vicinity of $\varphi = 0$ and 2π corresponds to the forward scattering. The small angle in the vicinity of $\varphi = \pi$ correspond to the back scattering.

The following graphs clarify more the periodicity of the above functions.

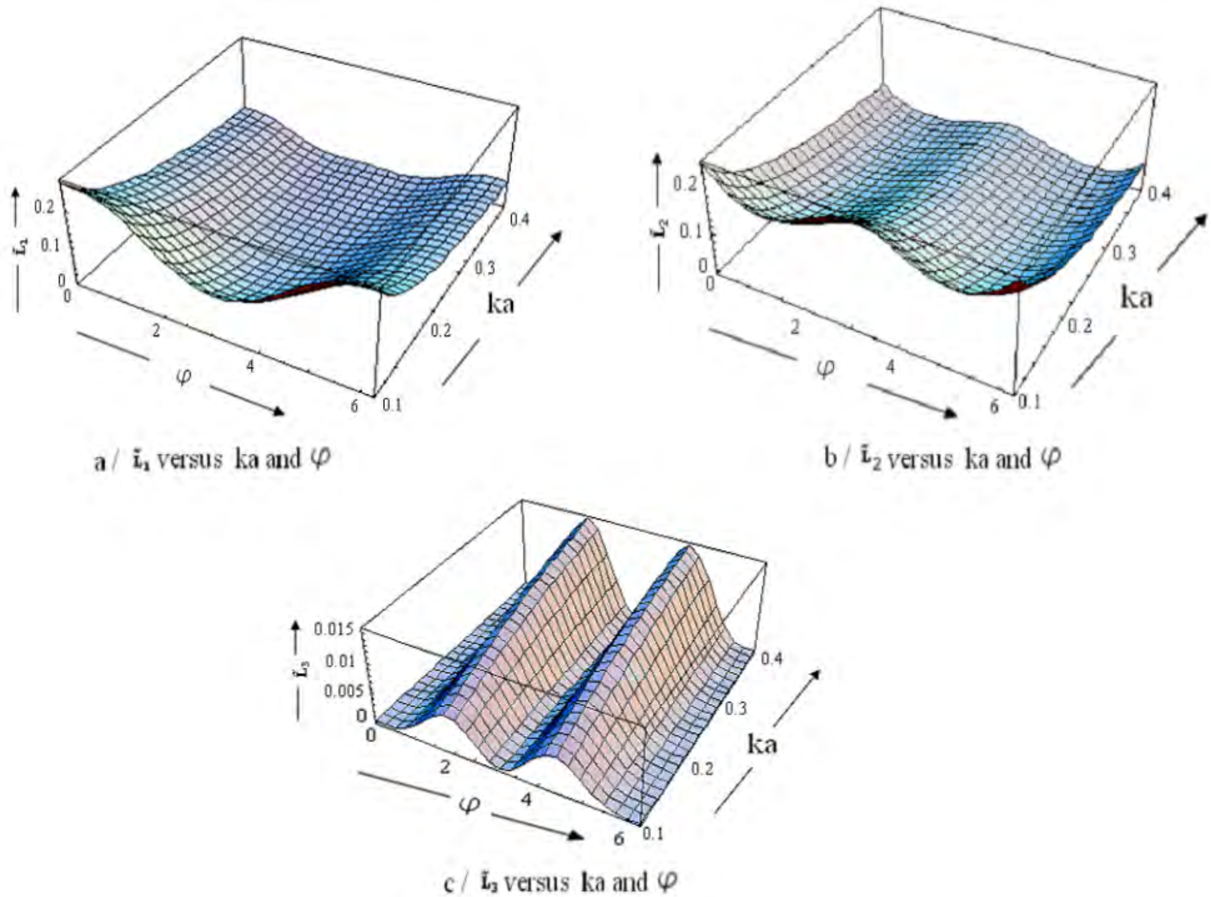


Figure 4.1: The dimensionless differential scattering length $\tilde{L}_{1-3}$ versus ka and φ in rad.(anisotropy)

Fig.(4.1a-c) present 3D graphs of $\tilde{L}_1$,$\tilde{L}_2$ and $\tilde{L}_3$ as functions of ka and φ built using Mathematica 5E with the help of formulas (A_3), (A_4), and (A_5) in the range of ka (0.1 to 0.4), where the spin dependent scattering is anisotropy with respect to the scattering angle φ.

Here the term **anisotropy** refers the property of being directionally dependent, as opposed to isotropy, which implies identical properties in all direction.Anisotropy of the spin dependent scattering exists also for less or extended range of ka . That means, if we consider the range of ka 0.1 to 0.3 which is less than the considered range the graphs show anisotropy. If we take a range of ka 0.1 to 0.5 which is greater than the range we consider, still all the three graphs show anisotropy.

The pattern of the corresponding graphs are the same for the three ranges we consider above. But there exists a slight change in shape for the change we made for the range of ka. Basically anisotropy exists for fast moving particle ($ka \gg 1$).On the other hand,from eq.(4.1.2) one can see that the spin dependent electron scattering is isotropy for slow particles ($ka \ll 1$). We can see this by considering small range for ka. For example,let us consider the following graphs for ka ranges from 0.01 to 0.1.

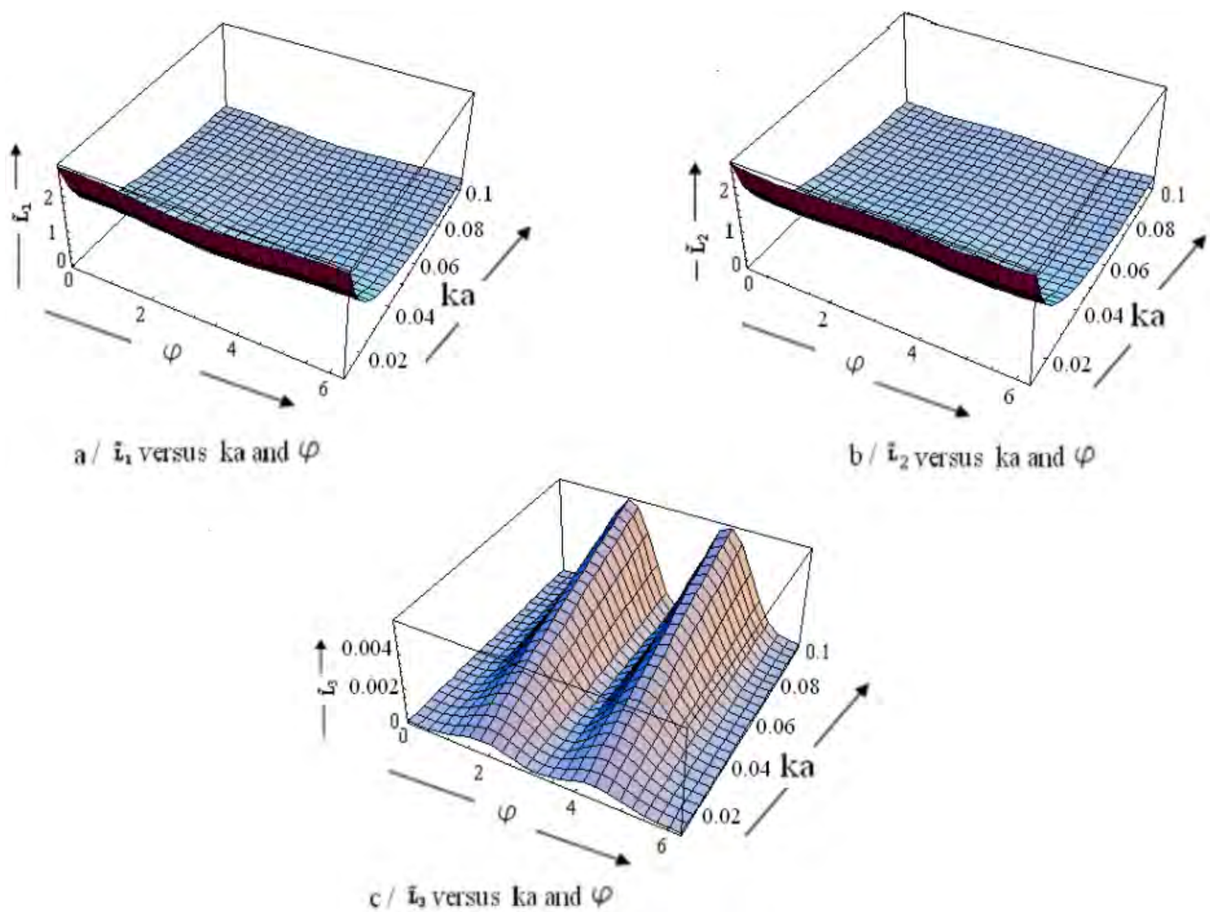
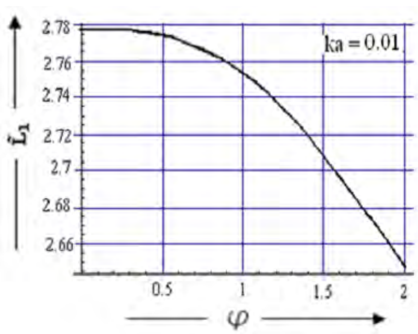


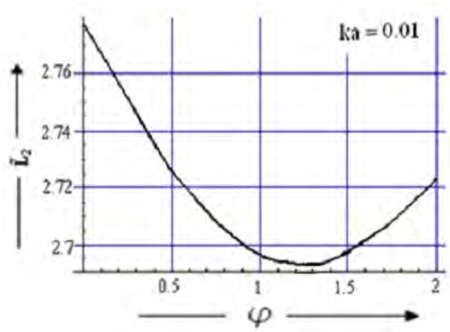
Figure 4.2: The dimensionless differential scattering length $\tilde{L}_{1-3}$ versus ka and φ in rad.(isotropy)

We can easily understand that at ka = 0.01 all the graphs of fig.(4.2a-c) show isotropy for slow particles ($ka \ll 1$).If we have a range ka from 0.01 to 2 the two ends of the graph for the ka side of fig.(4.2c) close to zero. We can see from the graphs of fig.(4.2a and b) for very small values of ka the scattering length has certain constant values as φ increases. Similarly the graphs of fig.(4.2c) shows the scattering length nearly zero for all values of φ for this very small values of ka. Moreover, we can observe from these graphs that for fast particle ($ka \gg 1$) shows anisotropy. Our numerical calculations allow us to specify more precisely for the range of ka, where the scattering becomes anisotropy.

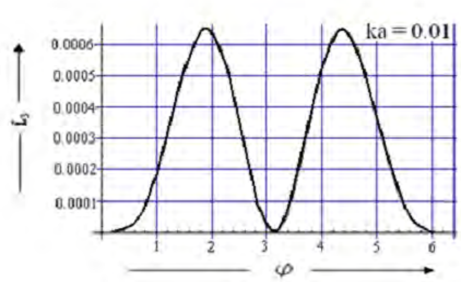
As we already discussed, one can esaly interpenetrates the cross-sections of the above graphs with the help of a plane when ka= constant give dependencies of $\tilde{L}_i \sim (i = 1, 2, 3)$ on φ using the following graphs. The scattering length shows anisotropy and we try to show this with the help of the following graphs(figs.(4.3-4.7) which are built using formulas (A_3), (A_4), and (A_5) for different constant values of ka. For example, as we have seen from the above 3D graphs for $ka \approx 0.01$ the scattering shows isotropy for all values of φ and anisotropy as ka increases. We can see this clearly using the following graphs.



a / $\tilde{L}_1$ versus φ at ka = 0.01



b / $\tilde{L}_2$ versus φ at ka = 0.01



c / $\tilde{L}_3$ versus φ at ka = 0.01

Figure 4.3: Dependence of dimensionless differential scattering length on φ at ka= 0.01

Figs.(4.3a-c) show the differential scattering lengths as functions of φ at ka = 0.01. In this case,the wave length of incident electrons considerably exceeds the size of nanomagnets, $\lambda = 10\pi a \gg a$. With the help of these graphs one can evaluate a certain number of particles scattered with an angle φ in the small interval $\delta\varphi$ in the vicinity of this angle. It is proportional to $\tilde{L}_{i(\varphi)}\delta\varphi$. For example, one can easily visualize using fig.(4.3a) that a number of scattered electrons with no spin flipping is the largest for small scattering angles. Similarly if we look at fig.(4.3b) that the scattering with spin flipping is the largest in the vicinity of this small angles as well. On the other hand, fig.(4.3c) we can generalize that there is no scattered electron or the scattering length is zero for the corresponding small angles. The maximum or the largest scattered electrons of fig.(4.3c) takes place for$\varphi \approx 1.8rad$.

This is the feature of 2D spin dependent electrons scattering by the nanomagnet allows performing experiments on separation of electrons with different orientation of spins. In particular, launching a polarized beam of electrons (spins along the velocity) with ka = 0.01 along the magnetic moment of a nanoparticle ($a \approx 10nm$), we have the electrons with non-flipped spins and electrons with spin flipped. It is possible to compare numbers of these electrons by using the formula $\frac{(\tilde{L}_3(\varphi_1)\delta\varphi)}{(\tilde{L}_1(\varphi_2)\delta\varphi)}$. where φ_1 and φ_2 are the angle in which the first maxima exists for $\tilde{L}_3$ and $\tilde{L}_1$ or $\tilde{L}_2$ respectively. According to graphs of fig.(4.3c) and fig.(4.3a), we can apply the above formula as $\frac{(\tilde{L}_3(1.8rad.)\delta\varphi)}{(\tilde{L}_1(0)\delta\varphi)}$ and we obtain the result close to 2.3×10^{-4} which is the scattered electrons with non-spin flipped.

On the other hand, launching a polarized beam of electrons with the same ka perpendicular to the magnetic moment of the nanoparticle ($a \approx 10nm$), we can see from graphs of fig.(4.3b) and fig.(4.3c) that the ratio $\frac{(\tilde{L}_2(0)\delta\varphi)}{(\tilde{L}_3(1.8rad.)\delta\varphi)} \approx 14.4$ which is the scattered electrons with spin flipped. We can understand from this comparison that the existence of the scattered electrons with spin flipped is much larger than the scattered electrons with no spin flipped for this particular value of ka. These example illustrate anisotropy spin dependent scattering of electrons by nanomagnets.

Let us consider the case where ka = 0.2 as the second example to show that the scattering length is anisotropy.

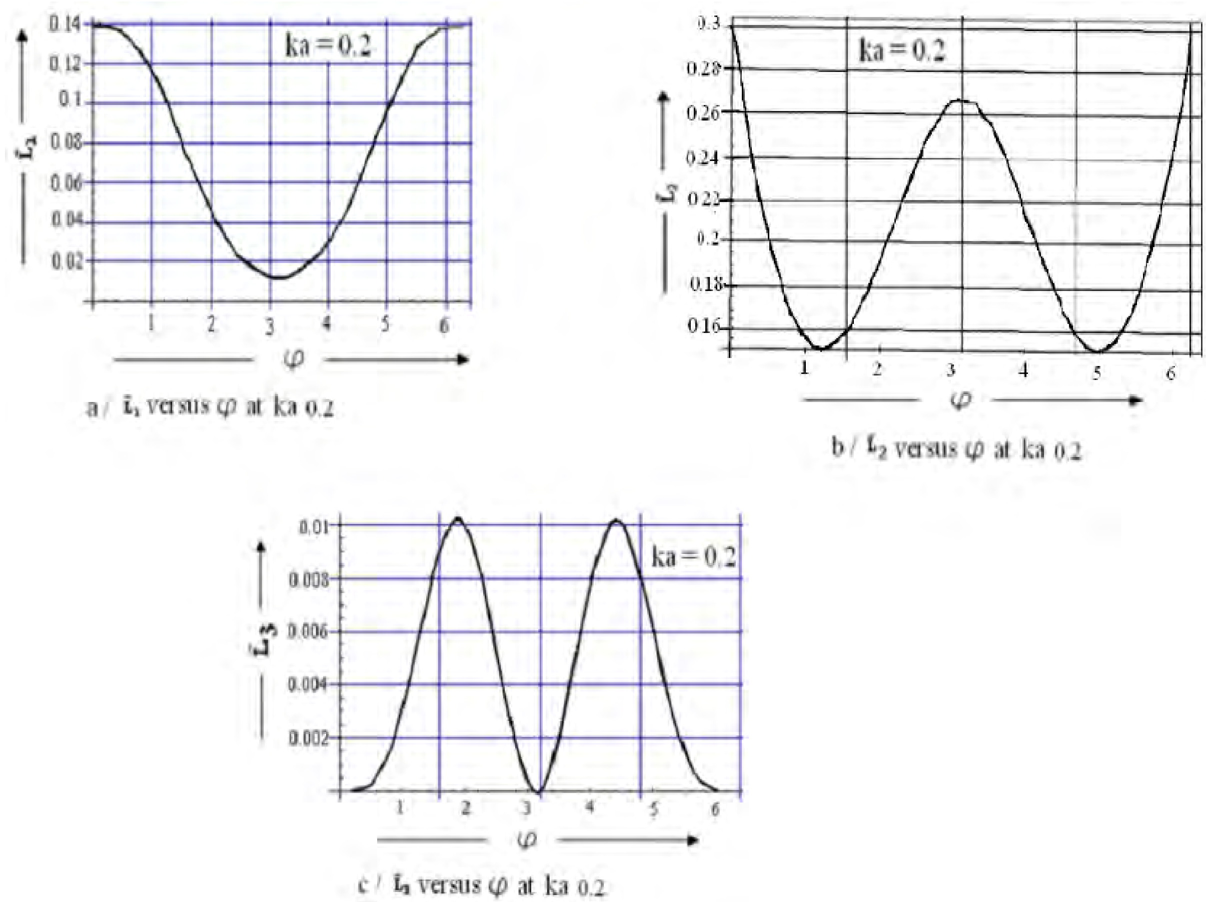


Figure 4.4: Dependence of dimensionless differential scattering length on φ at ka= 0.2

Figs.(4.4a-c) show the differential scattering lengths as functions of φ at ka = 0.2. With the help of these graphs one can evaluate a number of particles scattered with an angle φ in the small interval $\delta\varphi$ in the vicinity of this angle. It is proportional to $\tilde{L}_{i(\varphi)}\delta\varphi$. In this case, we can easily visualize using fig.(4.4a) that a number of scattered electrons with no spin flipping is the largest for small scattering angles. Similarly if we look at fig.(4.4b) that the scattering with spin flipping is the largest in the vicinity of small angles as well. Here also we can see from fig.4.4c that there is no scattered electron or the scattering length is zero for the corresponding small angles. Here the first maximum or the largest scattered electrons of fig.(4.4c) takes place for $\varphi \approx \frac{\pi}{2}$.

Now let us see the nature of the graphs for ka = 1 by using the following figure.

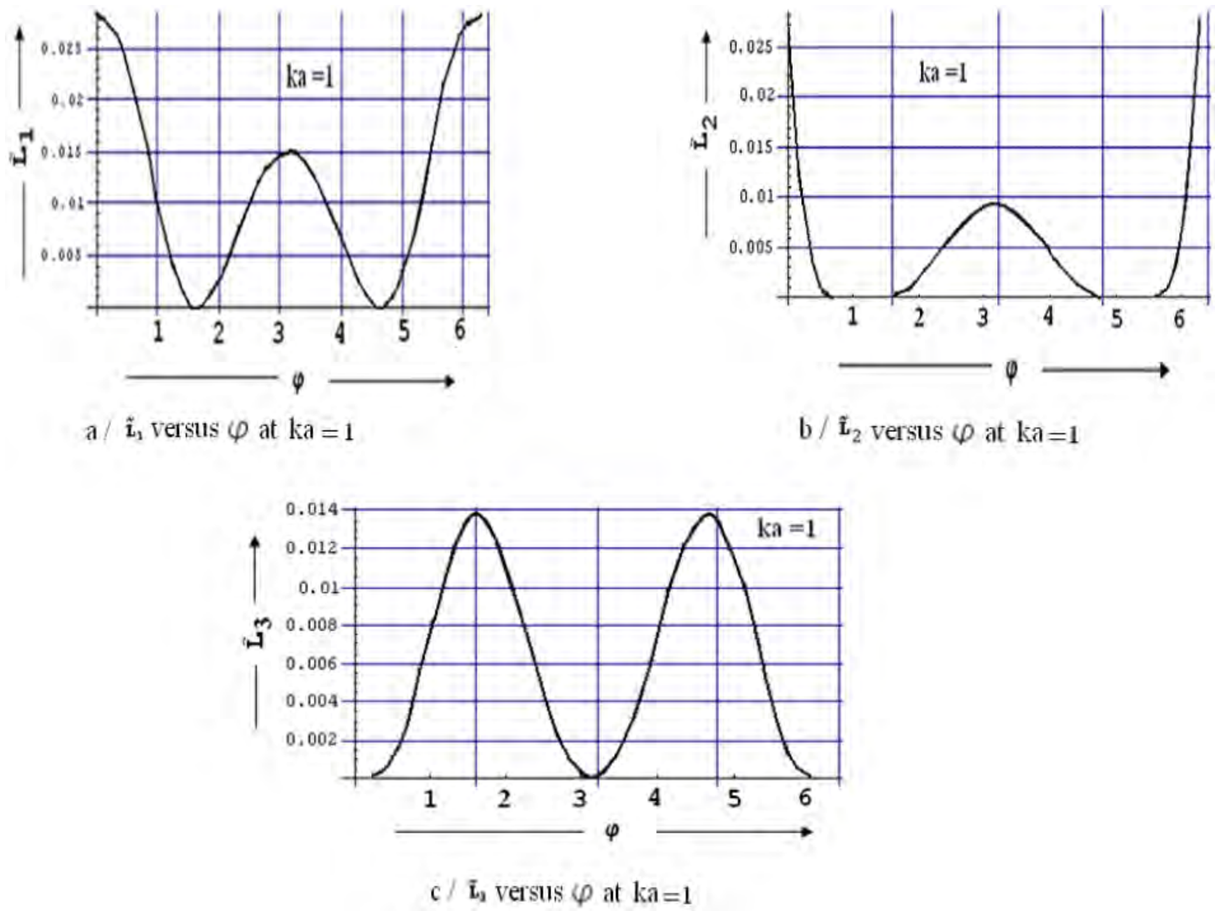


Figure 4.5: Dependence of dimensionless differential scattering length on φ at ka= 1

Still we can observe that,using fig.(4.5a) that a number of scattered electrons with no spin flipping is the largest for small scattering angles. Similarly if we look at fig.(4.5b) that the scattering with spin flipping is the largest in the vicinity of small angles as well. Here also we can see from fig.(4.5c) that there is no scattered electron or the scattering length is zero for the corresponding small angles. The first maximum or the largest scattered electrons of this graph fig.(4.5c) takes place for $\varphi \approx 1.65$rad ($\approx \frac{\pi}{2}$).
On the other hand, the graphs of fig.(4.5a and b) have the second maxima in the vicinity of $\varphi = \pi$. This corresponds to the backward scattering with spin non-flipping and with spin flipping respectively.

The probability of this scattering is smaller than the probability of the forward scattering. Clearly,one can see from the graph of fig.(4.5c) has minima in the vicinity of $\varphi = \pi$.
So far, we have seen that the graphs of scattering length is anisotropy for the small values of ka. Now let us see the nature of the scattering graph for two more little bit larger values of ka than the former values. First let us consider for ka = 5 as follow.

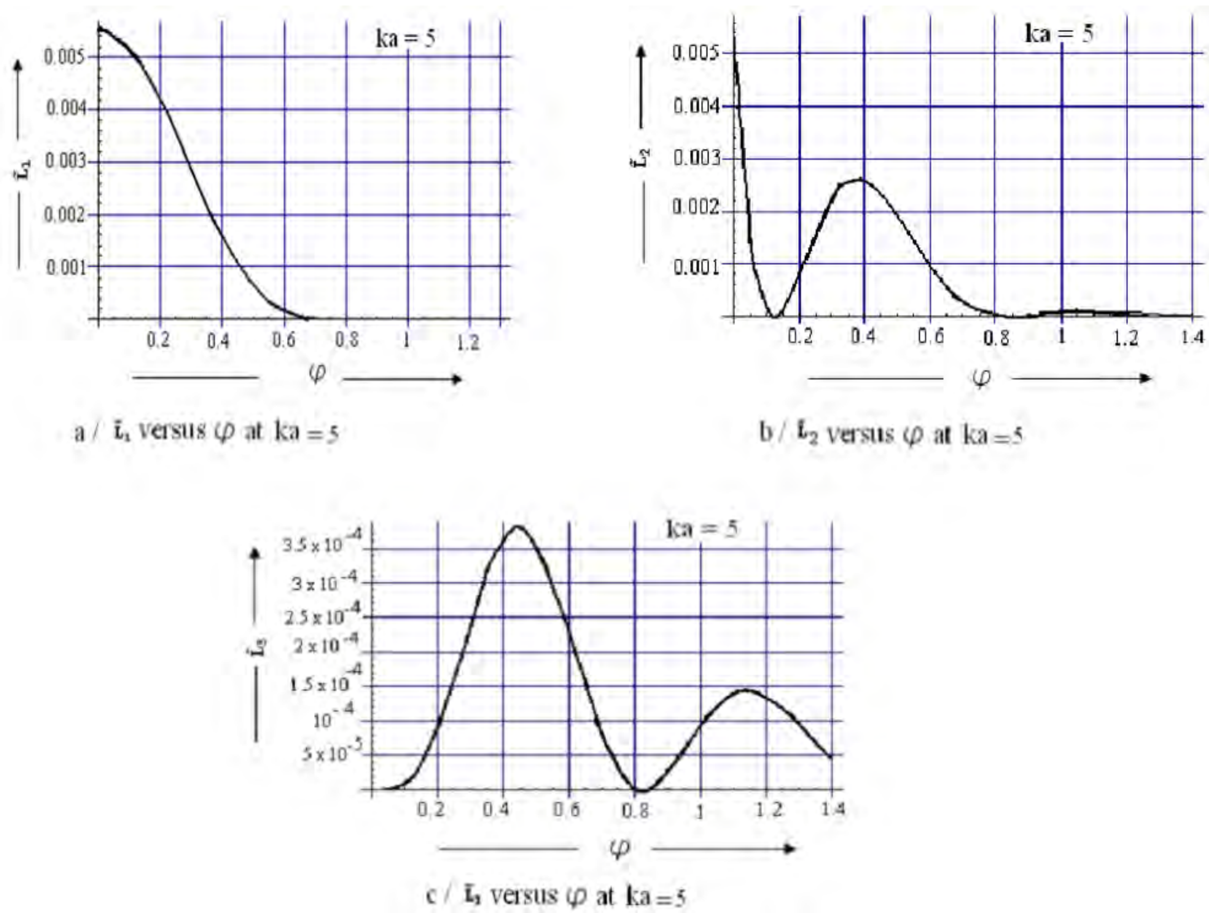


Figure 4.6: Dependence of dimensionless differential scattering length on φ at ka= 5

Here also as we can see from fig.(4.6a) that a number of scattered electrons with no spin flipping is the largest for small scattering angles. Similarly if we look at fig.(4.6b) that the scattering with spin flipping is the largest in the vicinity of small angles too. Moreover, we can see from fig.(4.6c) that there is no scattered electron or the scattering length is zero for the corresponding small angles. The first maximum or the largest scattered electrons of this graph fig.(4.6c) takes place for $\varphi \approx 0.45$rad.

On the other hand, as we observe the graphs of fig.(4.6a and b) the scattering length is zero for $\varphi \approx 0.8$ rad. on wards. Finally, let us consider for ka = 10 and observe the nature of the scattering graph as follow.

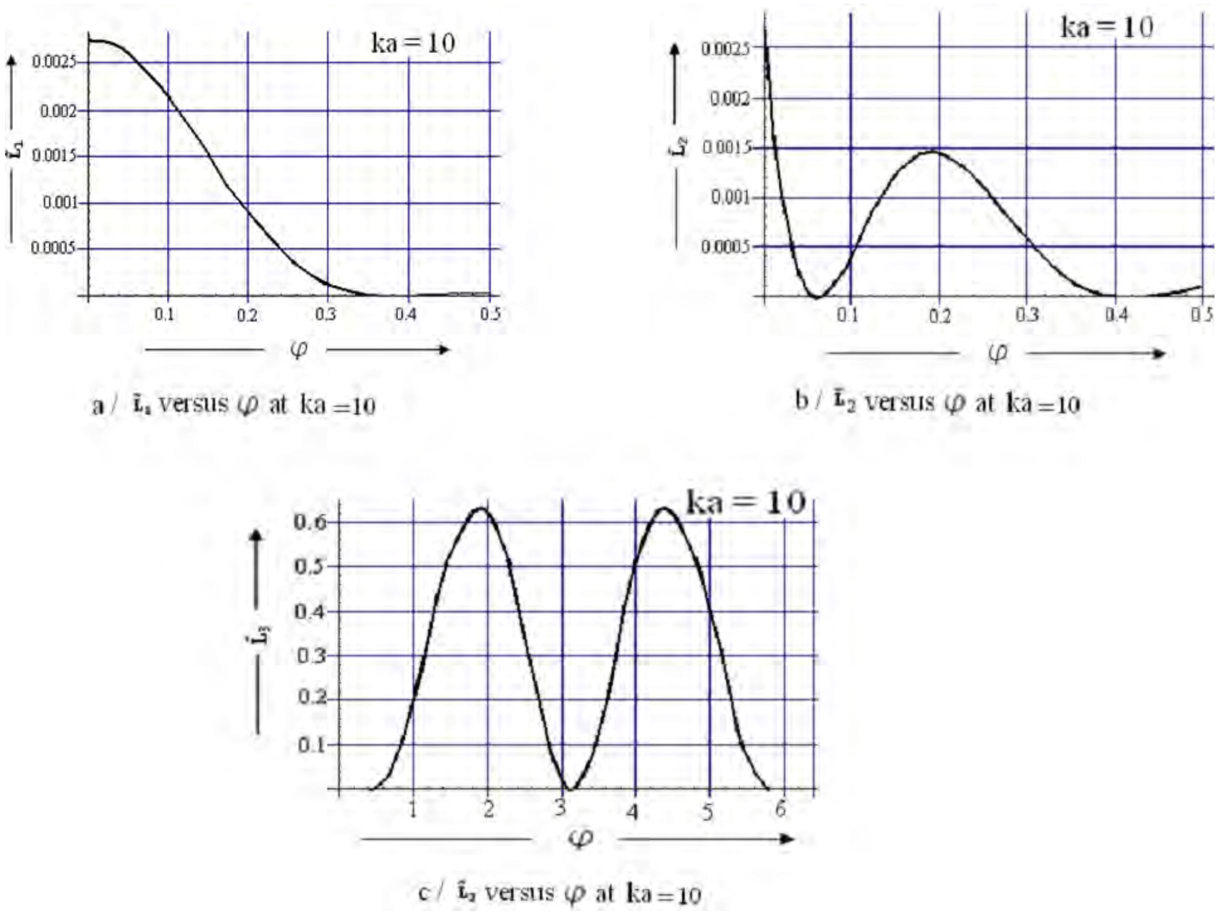


Figure 4.7: Dependence of dimensionless differential scattering length on φ at ka= 10

Still as we see from fig.(4.7a) even if the value is too small that a number of scattered electrons with no spin flipping is the largest for small scattering angles. Similarly if we consider fig.(4.7b) that the scattering with spin flipping is the largest in the vicinity of this small angles as well. Fig.(4.7c) shows that there is no scattered electron or the scattering length is zero for the corresponding small angles. The first maximum or the largest scattered electrons of this graph (fig.(4.7c)) takes place for $\varphi \approx 1.8$rad. Even if the result is too small(close to zero),the scattering result of fig.(4.6a and b) is twice that of the scattering length of fig.(4.7a and b) independently.

As we did for ka = 0.01,it is possible to compare the number of electrons for ka = 10 by using the formula $\frac{(\tilde{L}_3(\varphi_1)\delta\varphi)}{(\tilde{L}_1(\varphi_2)\delta\varphi)}$. Using the graphs of fig.(4.7c) and fig.(4.7a), we can apply the above formula as $\frac{(\tilde{L}_3(1.8rad.)\delta\varphi)}{(\tilde{L}_1(0)\delta\varphi)}$ and we obtain the result close to 237 which is the number of electrons with non-spin flipped.

On the other hand,launching a polarized beam of electrons with the same ka perpendicular to the magnetic moment of the nanoparticle ($a \approx 10nm$), we can see from graphs of fig.(4.3b) and fig.(4.3c) that the ratio $\frac{(\tilde{L}_2(0)\delta\varphi)}{(\tilde{L}_3(1.8rad.)\delta\varphi)} \approx 0.00375$ which is the number of electrons with spin flipped. This implies that the existence of the number of electrons with spin flipped is much larger than the number of electrons with no spin flipped for this particular value of ka.

Now we can generalize from all the above graphs of figures(4.1-4.7)for slow particle or very small values of ka (for example, ka = 0.01) the graphs of the scattering length shows isotropy. But for other values of ka we considered the graphs of scattering length shows anisotropy. More over,as we have seen all the first two graphs (fig.a and b) of figures(4.3-4.7) attains maximum values near small angle for each corresponding values of ka.On the other hand, the scattering result of all fig.c graphs of figures(4.3-4.7) is zero for this small angle.

Another interesting point we have for all graphs of fig.c of figures (4.3-4.7) except fig.(4.6c) the first and the second maxima of $\tilde{L}_3$ attained nearly for $\varphi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ respectively. That mean we can understand from the graph that the majority electrons with spin flipped are scattered for these two especial scattering angles. For example, the scattered electrons with flipped spin initially moving along $\vec{\mu}$ are concentrated in the direction transversal to their initial velocity. The scattered electrons with their spin flipped and initially moving perpendicular to $\vec{\mu}$ are concentrated at small scattering angles. These examples illustrate anisotropy spin dependent scattering of electrons by nanomagnets. It would be interesting to compare the parameter L_0, which specifies the scattering length with a typical radius a of the nanomagnet.

The quantity L_0 (see its definition below eq.(3.1.16)) and substituting the expression of μ, we can rearrange in the following form

$$\frac{L_o}{a} = 18\pi \left[\frac{4\pi}{3} m\nu n_a \left(\frac{\mu_B a}{\hbar} \right)^2 \right]^2$$

$$\frac{L_o}{a} = 32\pi^3\nu^2 \left(\frac{m_e n_a (a\mu_B)^2}{\hbar^2} \right)^2$$

$$\frac{L_o}{a} \simeq 10^3\nu^2 \left(\frac{m\mu_B^2}{\hbar^2} n_a a^2 \right)^2 \tag{4.2.4}$$

The dimensionless combinations $\frac{L_o}{a}$ basically determines the scale of spin dependent scattering. Comparing eq.(4.2.4) with the criterion of applicability of the born approximation eq.(3.1.17), we obtain the following constraint $\frac{L_0}{a} \ll 4$. It shows that $\frac{L_0}{a}$ is always small regardless of the values of the parameters of 2D spin dependent electrons scattering[2]. To our mind it is a manifestation of the short range character of the dipole- dipole interaction in the 2D geometry. According to eq.(4.2.4),taking $\nu = 10$ that is in agreement with reference [16-18], and substituting all numerical values we obtain $\frac{L_0}{a} = 0.1$ for $a \approx 10nm$, which is in agreement with $\frac{L_0}{a} \ll 4$.

As we already discussed from the recent report [34] taking $\nu = 10^2$, using eq.(4.2.4), we obtain $\frac{L_0}{a} = 10$ for $a \approx 10nm$,this result contradict with the above discussion. That means for large value of ν the $\frac{L_0}{a} > 4$. We can generalize from this result that Born approximation doesn't work for nanomagnet gigantic μ ,for slow particles. We can justify it only for fast particle..

Chapter 5

Summary and Conclusion

We discussed that spintronics is the science concerned with the flow of electrons and as it is the spin effect. We extended this as the spin orientation results gigantic magneto-resistance. We tried to discuss the difference between gigantic nanomagnets and the corresponding bulk materials.

We considered at the beginning of chapter 2 that the 3D and 2D scattering theory of electrons by impurity and we derived the mathematical expression of the scattering amplitude. Following this attention is given for 2D scattering of electrons by a nanomagnet when the electron momentum and the magnetic moment of the nanoparticle are in the same plane. Considering the dipole-dipole interaction of the magnetic moments, we solved the problem in the Born approximation. The interaction potential is not spherically symmetric and depends on the electron spin and orientation of $\vec{\mu}$. It considerably complicates the calculations of the scattering lengths that become two-component spinors.

We obtained the scattering lengths for the electrons with energies ka=0.1 - 0.4 and ka=0.01-0.5 with the same orientation of spins scattered by a neutral nanomagnet ($\vec{\mu}$ is parallel or perpendicular to the velocity of incident electrons). It is shown that the scattering lengths for the spin flipping and no spin flipping processes have rather sharp maxima as functions of the scattering angle. The forward and backward scattered electrons preserve the initial direction of spin for the case when the initial velocity of electrons are parallel to magnetic moment of nanoparticle $\vec{\mu}$. This property of the spin dependent scattering allows one to separate electrons with different orientation of spin and compare their numbers.

Now,the following are the conclusions of this thesis, the Born approximation doesn't work for nanomagnet gigantic μ ,for slow particles. We can justify it only for fast particle. The graphs relate the scattering of electrons by a single scattering center. For slow particle or very small values of ka (for example,ka = 0.01) the graphs of the scattering length shows isotropy. The graphs of other value of ka we considered show the anisotropic nature. These effects can be increased if we consider the scattering of electrons by a chain of scattering centers provided that the wave length of the incident electrons is smaller than the average distance between the nanomagnets. It is clear that every subsequent scattering will independently contribute to the relevant scattering length. The electrons scatted by angles $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are oppositely directed beams. This can be used as one more method of controlling of the electron currents with different spin orientation.

To finalize, the paper intended to show how the scattering amplitudes depend on mutual orientation of the magnetic moment of the nanomagnet and the electron, the energy of electron and the scattering angle.

Appendices

Appendix A

Integerals of eq.(3.1.11) and eq.(3.1.12) can be calculated as definite integrals with the help of Mathematica 5 [21]. The result says

$$I_1(qa) = \frac{qa}{6}\Big|\left(\frac{-1}{x}F_1 - \frac{3}{8}xF_2\right)\Big|_{qa}^{\infty} \tag{A.1}$$

$$I_2(qa) = \frac{qa}{6}\Big|\left(\frac{-1}{x}F_1 + \frac{3}{8}xF_2\right)\Big|_{qa}^{\infty} \tag{A.2}$$

Where F_1 and F_2 are $F_1 = HPFQ[\{-\frac{1}{2}\},\{\frac{1}{2},1\},-\frac{x^2}{4}]$ and $F_2 = HPFQ[\{\frac{1}{2}\},\{\frac{3}{2},3\},-\frac{x^2}{4}]$ Here, abbreviation HPFQ stands for the generalized hypergeometric function Mathematica 5 [21]. Unfortunatly; we could not find the reliable expressions of these functions for qa $\rightarrow\infty$ and we get the upper limit of the integerals numerically. As a result,$I_1(qa) \geq 10^3) = -\frac{1}{3}ka$ and $I_2(qa) \geq 10^3) = 0$. This allows us with usage of $eq.$(4.2.1) ,$eq.$(4.2.2),

and $eq.$(4.2.3) to present

$$\tilde{L}_1 = \frac{1}{36ka}\left[-2ka\sin\frac{\varphi}{2} + 2ka\sin\frac{\varphi}{2}\cos\varphi + F_1(qa) - \frac{3}{2}(ka)^2 sin^2\frac{\varphi}{2}\cos\varphi F_2(qa)\right]^2 \tag{A.3}$$

$$\tilde{L}_2 = \frac{1}{36ka}\left[-2ka\sin\frac{\varphi}{2} - 2ka\sin\frac{\varphi}{2}\cos\varphi + F_1(qa) + \frac{3}{2}(ka)^2 sin^2\frac{\varphi}{2}\cos\varphi F_2(qa)\right]^2 \tag{A.4}$$

$$\tilde{L}_3 = ka\left\{\sin\frac{\varphi}{2}\sin\varphi\left[\frac{1}{3} - \frac{ka}{4}\sin\frac{\varphi}{2}F_2(qa)\right]\right\}^2 \tag{A.5}$$

Appendix B

The expressions of Green's function for 1D , 2D and 3D are given below respectively

$$G^{1D}(\vec{r},\vec{r'}) = \tfrac{i}{2\pi}e^{ik|r-r'|} \tag{B.1}$$

$$G^{2D}(\vec{r},\vec{r'}) = \tfrac{i}{4\pi}H_o^1(k|r-r'|) \tag{B.2}$$

$$G^{3D}(\vec{r},\vec{r'}) = \tfrac{e^{ik|r-r'|}}{4\pi|r-r'|} \tag{B.3}$$

where $H_o^1(k|r-r'|)$ is the Hanckle first function of 2D case with the an approximate expression of $H_0^{(1)}\left(k|\vec{\rho}-\vec{\rho'}|\right) \simeq \frac{1-i}{\sqrt{\pi\rho k}}e^{ik(\rho-\vec{\rho'}.\vec{n})}$. We can interpret these free space Green's functions physicaly as follows [22]. The one dimensionnal Green's function is the response doe to a plane source, for which waves go off in both directions. The 2D Green's function is the cylinderical wave from a line source. The 3D Green's function is the spherical wave from a point source.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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