ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOGY

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

OPTIMIZED DESIGN OF SPHERICAL DOME WITH SKY LIGHT

(Using compatibility method of analysis)

A thesis submitted to the school of Graduate Studies in Partial fulfillment of the Requirements for the Degree of Masters of Science in Civil Engineering (Structures)

By

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Advisor: Dr.-Ing BEDILU HABTIE

April, 2019

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DECLARATION

I, the undersigned, declare that this thesis is the original work and has not been presented a degree in any other university and that all sources of material used for the thesis have been dually acknowledged.

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A area covered by the spherical dome roof structure

A_c section area of concrete

A_{ct} area of concrete with tensile zone (area of concrete in tension just before the initiation of the first crack)

A_{fw} total formwork area

A_r section area of ring beam

A_s area of tension steel

A_s^T area of steel reinforcement for pure tension force

A'_s area of compression steel

A_{s1} area of tensile reinforcement, which extends $\geq l_{bd} + d$ beyond the section considered.

A_{sc} circumferential tension reinforcement of dome

A_{sci} circumferential reinforcement area at spherical dome meridional angle ϕ_i

A_{smi} meridional tensile side reinforcement area at spherical dome meridional angle of ϕ_i .

A'_{smi} meridional compressive side reinforcement area at spherical dome meridional angle of ϕ_i .

$A_{sm,\min}$ minimum meridional reinforcement

$A_{sc,\min}$ minimum circumferential reinforcement

A_{sm} meridional tension side reinforcement of dome

$A_{sm,drs}$ meridional tension side reinforcement for doubly reinforced section

A'_{sm} meridional compression side reinforcement for doubly reinforced section

$A_{sm,srs}$ meridional tension side reinforcement for singly reinforced section

$A_{sc,drs}$ circumferential tension reinforcement for doubly reinforced section

$A_{sc,srs}$ circumferential tension reinforcement for singly reinforced section

A'_{sm} meridional compression side reinforcement of dome

$A_{s,\max}$ maximum area reinforcement

$A_{s,\min}$ minimum area of reinforcement

A_{sv} shear reinforcement area of ring beam in s_l

A_{sc}^{dlb} lower boundary for circumferential tension reinforcement area of dome

A_{sc}^{dub} upper boundary for circumferential tension reinforcement area of dome

$A_{sc,\max}^{d}$ maximum area of circumferential tension reinforcement of dome

$A_{sc,\min}^{d}$ minimum area of circumferential tension reinforcement of dome

$A_{smc,\max}^{d}$ maximum area of meridional compression side reinforcement of dome

$A_{smc,\min}^{d}$ minimum area of meridional compression side reinforcement of dome

A_{smc}^{dlb} lower boundary for meridional compression side reinforcement area of dome

A_{smc}^{dub} upper boundary for meridional compression side reinforcement area of dome

A_{sm}^{dlb} lower boundary for meridional tension side reinforcement area of dome

A_{sm}^{dub} upper boundary for meridional tension side reinforcement area of dome

A_{st}^{r} tension reinforcement area of ring beam.

A_{st}^{rlb} lower boundary for circumferential tension reinforcement area of ring beam

A_{st}^{ulb} upper boundary for circumferential tension reinforcement area of ring beam

$A_{st,\max}^{r}$ maximum area of circumferential tension reinforcement of ring beam

$A_{st,\min}^{r}$ minimum area of circumferential tension reinforcement of ring beam

b width of the rectangular reinforced concrete section.

b_r width of ring beam.

b_r^{lb} lower bound of ring beam width

b_r^{ub} upper bound of ring beam width

b_t the mean width of the tension zone

C concrete resultant compressive force

cc concrete clear cover to the longitudinal reinforcement

C_c^d concrete cost of the spherical dome.

C_c^r concrete cost of the ring beam.

C_f formwork cost of per m^2

C_{fw}^d formwork cost of the spherical dome.

C_{fw}^r formwork cost of the ring beam.

$c_{\min}$ minimum cover

$c_{\min,b}$ minimum cover due to bond requirement

$c_{\min,dur}$ minimum cover due to environmental conditions

C_m^d the total material cost of the spherical dome

C_m^r the total material cost of the ring beam.

c_{nom} nominal concrete cover

C_s steel compression force

C_{su} cost of steel per Kg

C_s^d reinforced steel cost of the spherical dome.

C_s^r reinforced steel cost of the ring beam.

C_s^s the cost of concrete per unit volume.

C_s^s the cost of concrete per unit volume.

C_{st}^r cost of tension reinforcement ring beam.

C_{ss}^r cost of shear reinforcement ring beam.

d effective depth of the reinforced concrete section

d' distance of compression reinforcement from edge of compression side

$d_{balance}$ balanced effective depth

d_ϕ differential meridional angle

d_θ differential circumferential angle

E young's modulus of elasticity

E_{cm} concrete modules of elasticity

E_s steel modules of elasticity

f_{ck} characteristic cylinder strength of concrete

$f_{ct,eff}$ mean tensile strength of concrete effective at the time when the cracks may first be expected to occur

f_{ctm} mean tensile strength of concrete

f_{yd} the design yield strength of the reinforcement

f_{yk} characteristic yield strength of steel

$\mathrm{f}(\mathrm{X})$ objective function

g dead load due to gravity

G_K characteristic dead load

$\mathrm{g}_\mathrm{j}(\mathrm{x})$ inequality constraints

H radial force acting on the ring beam

h over all depth of the rectangular reinforced concrete section.

$h_{c,eff}$ depth of a concrete section for tensile zone

h_r overall depth of ring beam.

h_r^{lb} lower bound of beam depth

h_r^{ub} upper bound of beam depth

k non-linear stress distribution coefficient- leading to a reduction in restraint force

K Gaussian curvature

k_c stress distribution coefficient

k_t factor that accounts for the duration of loading

k_1 the mean concrete stress

k_b coefficient accounting for the bond properties of the reinforcement

k_2 a coefficient accounting for the nature of strain distribution

$k_2 x$ depth to the centroid of the stress block

L_{si} length of a single shear reinforcement.

l_v meridional length of spherical dome for a meridional step angle (ϕ_s)

$l_j(x)$ equality constraints

M_d applied ultimate design moment

M_0 uniformly distributed dome base edge meridional moment

M_R^D doubly reinforced section moment resistance capacity

M_R^S singly reinforced section moment resistance capacity

M_u ultimate moment carrying capacity of the section

M_ϕ bending moment in meridional direction

M_θ bending moment in circumferential direction

$M_{\phi\theta}, M_{\theta\phi}$ twisting moments

n_s number of shear reinforcement provided for ring beam

N_{Ed} design axial force in the cross-section due to loading

N_{ϕ} internal membrane force in meridional direction

$N_{\phi all}$ allowable compression force of concrete

N_{θ} internal membrane force in tangential direction

$N_{\theta c}$ tangential compressive force

$N_{\theta\phi}, N_{\phi\theta}$ membrane shear forces

P external applied loads per unit area

$p^{(d)}$ dead load of spherical dome

$p^{(rd)}$ top ring load of spherical dome

p_i parallel circle perimeter of the spherical dome at spherical dome meridional angle of ϕ_i .

P_Z out ward load projection of external applied load

P_{θ} external applied load projection along circumferential direction

P_{ϕ} external applied load projection along meridional direction

q live load of spherical dome

Q_K characteristic imposed load

Q_0 uniformly distributed dome base edge lateral shear force

Q_{0b} shear force at fixed base

Q_{0s} lateral shear force of spherical dome

Q_{ϕ}, Q_{θ} transverse shear forces

R principal radius of any shell structure

r^r radius of ring beam middle surface.

R_D parallel circle radius of spherical dome base

R_b principal radius of spherical dome

r_i	parallel circle radius of the spherical dome at spherical dome meridional angle of ϕ_i .
r_0	pitch radius of spherical dome
r_1	parallel circle radius of spherical dome element in undeformed state
r_f	parallel circle radius of spherical dome element in deformed state
r_θ	principal radius of any shell structure in tangential direction
r_ϕ	principal radius of any shell structure in meridional direction
s_l	shear reinforcement spacing of ring beam
$s_{l,\max}$	allowable maximum shear reinforcement spacing in beam
$s_{r,\max}$	maximum crack spacing
$S_{\max m}$	maximum bar spacing for meridional reinforcement
$S_{\max c}$	maximum bar spacing for circumferential reinforcement
S_c^{dlb}	lower bound of bar spacing for circumferential reinforcement
S_c^{dub}	upper bound of bar spacing for circumferential reinforcement
S_m^{dlb}	lower bound of bar spacing for meridional reinforcement
S_m^{dub}	upper bound of bar spacing for meridional reinforcement
SORs	shell of revolution
t	thickness of any shell structure
T	steal tension force
t_D	thickness of spherical dome
t_D^{lb}	lower bound of thickness of spherical dome
t_D^{ub}	upper bound of thickness of spherical dome
T_0	horizontal component of membrane meridional force (N_ϕ) at the bottom of dome
T_r	ring beam axial force

T_P pure tension force

u_ϕ displacement along tangent to meridian curve

v poisons ratio

V_θ displacement in circumferential direction

V_c^d total volume of concrete for spherical dome.

V_c^r total volume of concrete for ring beam.

V_{cst}^d total volume of reinforcement in circumferential direction of dome

V_{Ed} design shear force

$V_{\min}$ minimum shear resistance

V_{mst}^d total volume of reinforcement in meridional direction of dome

$V_{Rd,c}$ the design shear resistance

V_{St}^d total volume of steel of reinforcement for dome structure.

w the distance from the neutral axis to the strain ε_{c2}

w_z displacement normal to meridian curve

w_k cracks width

w_{k1} design crack width

x depth of the neutral axis

x_u ultimate neutral axis depth

X vector of design variables

z lever arm

ϕ meridional angle starting from the axis of symmetry

ϕ_b longitudinal bar diameter

ϕ_i the ith spherical dome meridional angle of

ϕ_s	step angle in meridional direction of spherical dome
ϕ_0	pitch angle of spherical dome
ϕ_1	base angle of spherical dome
θ	circumferential angle
ε_{cm}	mean strain in the concrete between cracks
ε_{c1}	ultimate compressive strain of concrete
ε_{c2}	concrete strain at the end of the parabolic section
ε_{sm}	mean strain in the reinforcement
ε_{cc}	concrete strain for triangular stress block
ε_{sc}	compression steel strain for triangular stress block
ε_{st}	tension steel strain for triangular stress block
ε_{ϕ}	meridional strain
ε_{θ}	tangential strain
$\varepsilon^{r}_{\theta H}$	hoop strain of ring beam
γ_{CONC}	unit weight of concrete
γ_C	partial safety factors for concrete
γ_f	factor of safety
γ_S	partial safety factors for steel
$\gamma_{\phi\theta}, \gamma_{\theta\phi}$	membrane shear strain
σ_{ft}	steel flexural tension stress
σ_c	design compressive stress

σ_{call} permissible compressive stress

σ_s stress in tension reinforcement assuming a cracked section.

σ_{tallw} allowable steel direct or flexural tension stresses.

$\rho_{p,eff}$ effective reinforcement ratio

$\rho_{w,\min}$ minimum shear reinforcement ratio

α the inclination of the shear reinforcement to the longitudinal axis of the beam

α_e modular ratio

α_{fb} rotation of fixed support

$\alpha_s^h\big|_{\phi=\phi_1}$ homogeneous spherical dome base rotation tangent to meridional curve

$\alpha_s^p\big|_{\phi=\phi_1}$ particular spherical dome base rotation tangent to meridional curve

β_ϕ spherical dome rotation tangent to meridional curve

$\beta_\phi\big|_{\phi=\phi_1}$ spherical dome base rotation tangent to meridional curve

δ_{fb} displacement of fixed support

δ_{rh} radial displacement of ring beam

δ_r^{lb} lower bound of beam deflection

δ_r^{ub} upper bound of beam deflection

δ_{rh}^h homogeneous ring beam radial displacement

${\delta_{rh}}^p$ particular ring beam radial displacement

δ_{sh} horizontal displacement of spherical dome

$\delta_{sh}\big|_{\phi=\phi_1}$ spherical dome base horizontal displacement

δ_{sh}^{lb} lower bound of dome horizontal deflection

δ_{sh}^{ub} upper bound of dome horizontal deflection

$\delta_{sh}^{h}\big|_{\phi=\phi_1}$ homogeneous spherical dome base horizontal displacement

$\delta_{sh}^{p}\big|_{\phi=\phi_1}$ particular spherical dome base horizontal displacement

ψ change of coordinate angle

Δc_{dev} an allowance in design for deviation

Abstract

Reinforced concrete thin Spherical dome shell roof structure construction for top covering of containment structures, memorial buildings, meeting halls, mosques and cathedral churches is in continuous demand in developing countries, like Ethiopia. Due to its strength, economy, containment of large space and aesthetical value than other shape of roofs, spherical dome shell is largely constructed as a reinforced concrete roof structures.

This structure is classified as shell of revolution having greater than zero Gaussian curvature which shows the greatest stiffness of it. This roof structure can be subjected to axisymmetric and asymmetrical loadings, but in this study the axisymmetric loadings such as loads from top ring beam, live load and self-weights are considered.

In this study, it has been investigated the cost optimized design of reinforced truncated spherical dome shell roof structure which is stiffened by rectangular cross section circular ring beam at base of it. The cost optimized is done for spherical dome having base area from 78.539 m^2 to 530.929 m^2 using Scilab programing language. The developed program follows compatibility method of analysis to get the effects of axisymmetrical external loadings, European codes for designing of the sections and Brute-force optimization method to get the best minimum total material costs of the roofs.

From this program the optimized total minimum material costs are obtained in addition to optimized values of cross sectional and geometrical dimensions, and reinforcement areas. From the result most of the optimized design variables are obtained around the lower bounds of side constraints. The optimized values of meridional reinforcements are obtained from singly reinforced section design, and they have the same value per unit meter width at every meridional angle. The circumferential reinforcements like the meridional have the same values per unit meter at every meridional angle of spherical dome. The total cost spherical dome component of the roof has significant influence on the total cost of the whole roof structure than ring beam structural component and the material cost of the concrete has also significant influence on the total cost of the roof.

CHAPTER ONE

1. INTRODUCTION

Cost optimization design in civil engineering profession generally means maximizing benefit or minimizing cost of civil structures in order to make them constructible as much as possible in minimum cost. In thin shell structures cost of construction is the basic issue. Therefore, cost optimized design of thin shell structure is essential to construct it in optimal minimum cost. Major life-cycle costs of thin shell structure construction are cost of materials (concrete, reinforcements and form work), labor power, transportation, construction machineries, maintenance and disassembling the structure at the end of its life cycle.

One example of thin shell structure is spherical dome structure. Nowadays, these structures are widely constructed as top covering of containment structures, memorial buildings, meeting halls, mosques and cathedral churches. They have been constructed of steel, concrete, composite and even precast and/or prestressed concrete in combined with ring beam located axisymmetricaly at the base of it. Among these, reinforced concrete truncated spherical dome shell stiffened by rectangular cross section ring beam at the base is the most common form used as roof structure. The main superstructure components of this structure are truncated spherical dome shell and rectangular cross section circular ring beam.

This study is all about cost optimized design of truncated dome (dome with skylight) roof stiffened by rectangular cross section ring beam at the base of spherical dome shell structure which is made up of reinforced concrete construction material. The optimization method used to solve this problem is Brute force optimization technique. To optimize the cost of this roof structure with minimizing cost as an objective function using Brute force method needs repeated action of structural analysis, structural designing and cost calculation for each possible values of design variables. For this study the analysis program has been formulated based on compatibility method (force method/flexibility method) of analysis by assuming that the structure has fixed support at its base, the structural designing has followed the Eurocode and the cost includes only the construction materials cost (cost of concrete, reinforcement and form work). And for simplicity they have been coded as one computer program with the help of Scilab programing language.

The steps that has been followed to achieve the objective of this study are ; first, gathering and arranging relevant information like structural analysis, structural design and selection of optimization strategy and optimization method used for cost optimization of spherical dome roof structure has been made; second, developing appropriate cost optimal design computer program using Scilab has been developed by combining the collected relevant information for the selected combined form of structure; third, the cost optimization has been investigated.

At the end of the study, the final outcome of the investigated result of cost optimization structural design value of spherical dome shell roof structure stiffened by rectangular cross section ring beam at the base has been displayed, discussed and concluded.

1.1. Objectives

1.1.1. General objectives

- The general objective of this study is:-

- Investigating cost optimization of spherical dome shell roof structure.

1.1.2. Specific objectives

- The specific objectives of this study are:-

- To obtain cost optimized geometrical and cross sectional dimensions, reinforcement areas and formwork surface areas for construction of the structural components of spherical dome shell roof structure.
- Preparing well organized and easily understandable design charts and design tables for axisymmetricaly loaded spherical dome shell roof structure which can covers areas from 78.539 m^2 to 530.929 m^2.

1.2. Statement of the problem

Nowadays, in developing countries, the rapid growth of population is demanding a large-scale containment structures, memorial buildings, meeting halls, mosques and cathedral churches. Since Ethiopia is a developing country, it is unquestionable that these structures are essential for the fulfillment of the societies' need. Most of the time containment structures, memorial buildings, meeting halls, mosques and cathedral churches are covered by strong and structurally stable spherical dome shell roof structure. To fulfill this need designing and constructing of efficient, durable, structurally stiff and low cost spherical dome shell roof structure should be expected. In order to construct this structure in least cost manner and in short period of time cost optimized design chart and design table are helpful for the designers.

1.3. Limitations of the study

- This study has the following limitations.

- This study only focuses on spherical dome shell roof structure having base areas between 78.539 m^2 to 530.929 m^2.
- This study only deals with super structure part reinforced concrete thin shell (i.e. $r/t \geq 20$) cast in place spherical dome shell roof structure that exists in the combined form of truncated spherical dome shell roof structure supported rigidly and stiffened by a rectangular cross section ring beam at its base.
- This study considers only for spherical dome having design working life of 50 years.
- In this study spherical dome roof with uniform thickness is considered.
- This study only studied for direct actions acts axisymmetricaly on the members of the selected form of structure such as self-weight, live load and load transferred from the connected other members of the structure.
- This study did not consider analysis of second order effects.

CHAPTER TWO

2. LITERATURE REVIEW

2.1. Optimization general

Optimization is the act of obtaining the best result under given circumstances [15]. Optimization is everywhere, from engineering design to computer sciences and from scheduling to economics [17]. In design, construction, and maintenance of any engineering system; engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function [15].The applications of optimization in engineering are diverse. Almost all areas in engineering can use optimization for problem solving. Topics includes pressure vessel design, shape optimization, structural design optimization, building design, energy-efficient design, heat management of electronics, planning, scheduling, and many others[17].

2.2. Statement of an optimization problem

An optimization problem can be stated as follows.

$$\text{Find } X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \text{ which minimizes } f(X) \qquad (1)$$

Subject to the constraints

$$g_j(x) \le 0 \qquad j = 1,2,\ldots\ldots\ldots, m \qquad (2a)$$

$$l_j(x) = 0 \qquad j = 1,2,\ldots\ldots\ldots, p \qquad (2b)$$

Where X is an n-dimensional vector called the design vector, $f(X)$ is termed the objective Function, and $g_j(x)$ and $l_j(x)$ are known as inequality and equality constraints, respectively [15].

Design Vector

Any engineering system or component is defined by a set of quantities some of which are viewed as variables during the design process. In general, certain quantities are usually fixed at the outset and these are called *preassigned parameters.* All the other quantities are treated as variables in the design process and are called *design* or *decision variables* x_i . The design variables are collectively represented as a design vector $X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$ [16].

Design Constraints

In many practical problems, the design variables cannot be chosen arbitrarily; rather, they have to satisfy certain specified functional and other requirements. The restrictions that must be satisfied to produce an acceptable design are collectively called *design constraints.* Constraints that represent limitations on the behavior or performance of the system are termed *behavior* or *functional constraints.* Constraints that represent physical limitations on design variables such as availability, fabricability (constructability), and transportability are known as *geometric* or *side constraints* [15]. The constraints divide the design space into a non-feasible and a feasible region out of which the optimum solution is to be found [5].

Objective Function

A criterion has to be chosen for comparing the different alternative acceptable designs and for selecting the best one. The criterion, with respect to which the design is optimized, when expressed as a function of the design variables, is known as the *criterion or merit* or *objective function* [15].

2.3. Application of optimization in structural design

Civil Structures should satisfy the following four major criteria when they are considered to be constructed.

1. **Appropriateness**. The arrangement of the spaces, spans ceiling height, access and traffic flow must compliment the intended use. The structure should fit its environment and be aesthetically pleasing.
2. **Maintainability**. A structure should be designed to require a minimum of maintenance and to be able to be maintained in a simple fashion.
3. **Economy**. The overall cost of the structure should not exceed the client's budget.
4. **Structural adequacy**. Structural adequacy involves two major aspects.
    a. A structure must be strong enough to safely support all anticipated loadings.
    b. A structure must not deflect, tilt, vibrate, or crack in a manner that impairs its usefulness [9].

Once the architectural preliminary structural layout has been selected specially based on the first three criteria listed above, structural system can be designed normally (conventionally) using the following main procedures. Based on preliminary layout selected structural analysis is carried out to determine the moment, shears, torques, and axial forces in the structure. The individual members are then proportioned to resist these load effects. The proportioning (member design) must also consider the economy, the overall aesthetics, the constructability, coordination with mechanical and electrical systems, and the sustainability of the final structure [9].This conventional design procedures aim at finding an acceptable or adequate design which merely satisfies the functional and other requirements of the problem. In general, in this kind of design practice there will be more than one acceptable design.

To get the best structure which is least cost, structurally stiff and aesthetically striking needs rational or systematic way of designing i.e. structural optimization. Structural optimization can be seen as a rational method of finding a structural design that is the best of all possible designs for a chosen objective and a given set of technological or geometrical and behavioural constraints. Structural optimization combines mathematics and mechanics with engineering and has become a multidisciplinary field with applications in areas such as aeronautical, mechanical, civil, nuclear and marine engineering. In its present state of maturity, it is regarded as a practical, automated and integrated numerical tool for research and design [5].

2.4. Structural optimization classes

Depending on the geometric feature, we divide structural optimization problems into three classes: Sizing optimization, Shape optimization and Topology optimization. In sizing optimization, sizing variables are used to define the thickness distributions or cross-sectional properties of the structural components. In shape optimization, geometrical or shape variables are used to define the structural geometry [5]. Shape optimization is very common in engineering optimization, including design of aerodynamically efficient shape, structural design with minimum materials, and design of special tools [17]. And topological optimization is the most general form of structural optimization. In topological optimization; topological variables define the pattern of connection of elements, regions or components or the number and spatial sequence of elements, joints and supports or the material distribution. In topological design, material is redistributed or elements are added or deleted during the design process and the analysis model, as well as the set of design variables, changes [5].

2.5. Brute-force optimization method

Brute force optimization method is the simplest metaheuristic method of optimization technique. It is also known as generate and test optimization method. It is a very general problem-solving technique that consists of systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the problem's statement. And it is also distinguished by approaching the solution of a problem in the most natural, obvious, simple, or direct method in contrast to a more clever or sophisticated way.

The basic algorithm in order to apply brute force search to a specific class of problems, one must implement four procedures, first, next, valid and output. These procedures should take as a parameter the data p for the particular instance of the problem that is to be solved, and should do the following.

1. First (p): generate a first candidate solution for p.
2. Next (p, c): generate the next candidate for p after the current one c.

3. Valid (p, c): cheek weather candidate c is a solution for p.

4. Output (p, c): use the solution c of p as appropriate to the application.

This method of optimization has its own merits (advantages) and demerits (disadvantages). The main advantages are it is obvious, simple to implement and gives global solution in most natural way. The disadvantage is due to its primitiveness, exhaustiveness and slowness behavior it is inappropriate for large problem and its workability depends on computer's processing power [8].

2.6. Thin shell structures

The term shell is applied to bodies bounded by two curved surfaces, where the distance between the surfaces is small in comparison with other body dimensions. The locus of points that lie at equal distance between these two curved surfaces defines the middle surface of the shell. The length of the segment, which is perpendicular to the curved surfaces, is called the thickness (t) of the shell. The geometry of a shell is entirely defined by specifying the form of the middle surface and thickness of the shell at each point.

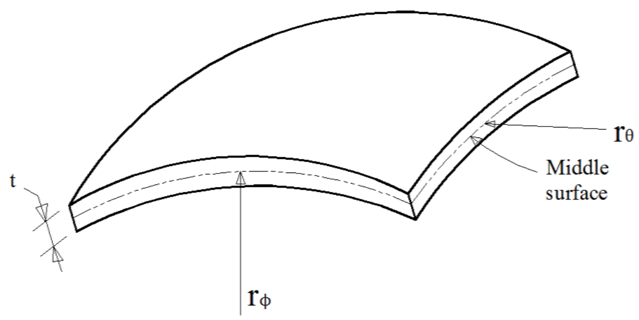


Figure 1: Shell general geometric definition.

The wide application of shell structures in engineering is conditioned by their following advantages:

1. Efficiency of load-carrying behavior.
2. High degree of reserved strength and structural integrity.
3. High strength to weight ratio. This criterion is commonly used to estimate a structural component efficiency: the larger this ratio, the more optimal is a structure. According

to this criterion, shell structures are much superior to other structural systems having the same span and overall dimensions.

4. Very high stiffness.
5. Containment of space.

Based on radius to thickness ratio shell structure can be classified as thin and thick shell structures. If the maximum value of thickness to radius (t/R) ratio of the shell is less than $1/20$ (i.e. max) $t/R \leq 1/20$ it can be regarded as thin shell structure. Shell structures which violets this inequality are referred to as thick shells [6].

2.6.1. Classification of thin shell structure

Thin shell structures can be classified based on different point of views, but the two general way of shell classification may be based on Gaussian curvature and their generated surface. Based on Gaussian curvature thin shell structure can be classified as synclastic surface, anticlastic surface and zero Gaussian curvature (developable surface). Gaussian curvature can be defined as $K = 1/r_{\phi} r_{\theta}$. Where $r_{\emptyset}$ and r_{θ} are principal radii of curvature [11]. Synclastic surfaces, like domes, have a Gaussian curvature greater than zero. They typically show the greatest stiffness and they do not need stiffenings like anticlastic surface structures [1].

Anticlastic surfaces (saddle surfaces) have a negative Gaussian curvature. This type of shell structure surfaces are less stiff than synclastic surface structures and require more stiffening. Curves with $K \equiv 0$ are called developable surfaces; a conspicuous feature being that they can be flattened onto a plane without experiencing any strains or stresses. Examples are cylinders and cones [1].

Based on generated surface, shell structures can be classified as surfaces of revolution, surfaces of translation, Ruled Surfaces and combined surface. Cone, Spherical dome and circular profile cylindrical shells are the three most known examples of shell of revolution (SORs) [10]. SORs appear in many practical forms, such as water towers, tanks and other liquid-containing structures, roof structures (domes), silos and pressure vessels etc. [5].

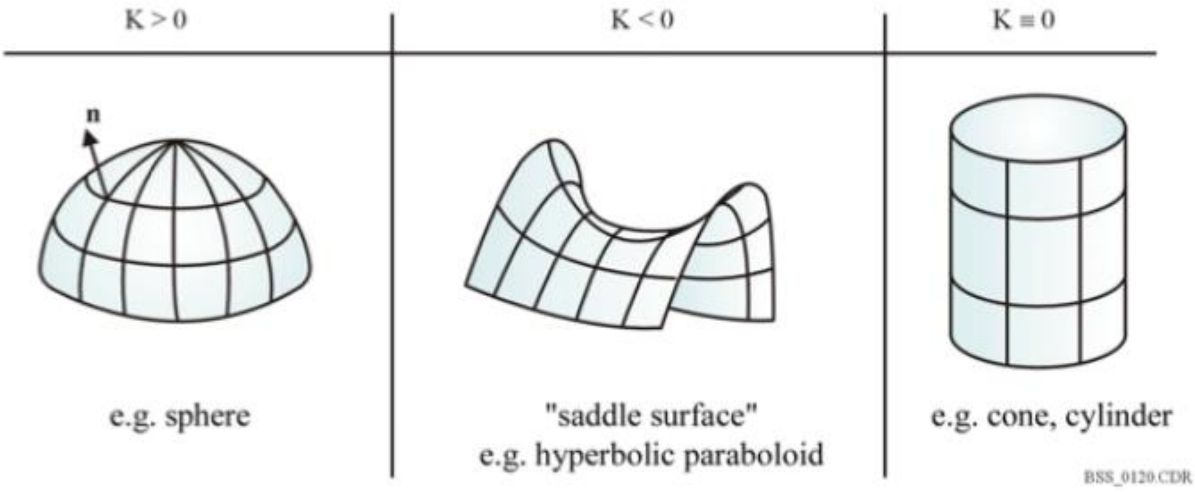


Figure 2: shell with different Gaussian curvature [6].

2.6.2. Spherical dome shell roof structure

As stated by M. Farshad (1992), Domes have synclastic shell surfaces with positive Gaussian curvature. They are strong and structurally stable. Dome roofs can be constructed from steel, various fiber reinforced composites, and reinforced concrete materials. Precast shells made of these materials have also been constructed successfully. Reinforced concrete domes are used to cover large spans of stadiums, memorial buildings, meeting halls, and other large assembly halls. They are also used to cover the roofs of liquid retaining structures, silos, as well as the roofs of containment shells of nuclear power plants. The end caps of concrete containment structures and pressure vessels are also made of these axisymmetric shells. Parts of shells of revolution and /or various combinations of these shell types can also be conceived and designed.

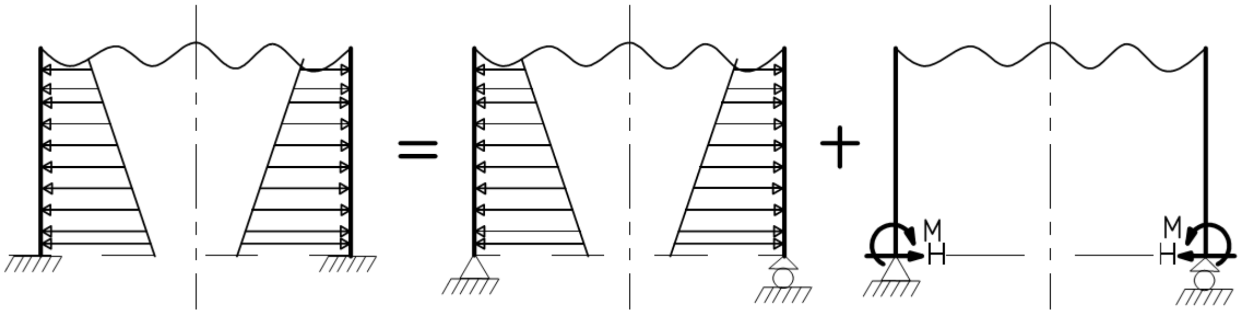
Nowadays, in developing countries, the rapid growth of population is demanding a large-scale buildings for different purposes. Since Ethiopia is a developing country, it is unquestionable that these structures are essential for the fulfillment of the societies' need. Most of the time spherical dome shell roof structure is used to cover different kinds of large spanned buildings which are constructed for different usage. To fulfill this need designing and constructing of efficient, durable, structurally stiff and low cost spherical dome shell roof structure should be expected. In order to construct this structure in least cost manner and in short period of time cost optimized design chart and design table are helpful for the designers.

In this study it has been tried to investigate cost optimized design of reinforced concrete spherical dome shell structures used for roofs. This structure is combined form of dome with skylight as roof structure which is stiffened by the help of ring beam on its base. The amount of areas covered by the selected fixed topology shell structures ranges from 78.539 m^2 to 530.929 m^2.

2.6.3. Shell structure analysis

According to euro code 2 part 1-1 section 5 clause 5.1.1(1), structural analysis is to establish the distribution of either internal forces and moments, or stresses, strains and displacements, over the whole or part of a structure. And carried out using idealizations of both the geometry and the behavior of the structure. Shell Structural analysis can be performed with force (compatibility) method or displacement (stiffness) method. According to M. Farshad (1992), for manual calculation the force (compatibility) method offers certain advantage over the stiffness method. Also it can be automatized as computer software. For this study the analysis of thin shell structure has been done using force (compatibility) method of analysis and using linear elastic behavior of materials. There are four major stapes to analyze shell structure using force method of analysis. These are:-

1. Determining internal membrane forces, displacements and rotations for a given shell structure subjected to loadings using membrane theory by assuming the shells is statically determinate and the boundary condition is compatible with the membrane action of the shell.
2. Using appropriate bending theory obtain internal force, displacements and rotations in terms of unknown corrective redundant bending forces which is applied instead of the applied external loadings. This yields the correction redundant forces due to the bending field which exist at the shell boundary.
3. By combining the results of analysis in stage (1) and (2) Express the compatibility equation in terms of known edge displacements and unknown edge forces that yields a set of simultaneous algebraic equations from which the redundant edge forces can be determined.
4. Finally complete force and deformation field in the shell is determined by superimposing the membrane and the bending fields [11].



a) The shell b) Membrane shell c) Edge effect

Figure 3: An example of shell analysis procedure according to the force method.

2.6.4. Sign convention

- The general sign conventions used in this study are as follows

1. For applied loading
   a. Positive (+ve) sign is used for the applied force as their natural acting direction.
2. For internal membrane stress
   a. Tension force is taken as positive (+ve).
   b. Compression force is taken as negative (–ve).
3. For shear stress
   a. Shear stress which acts outside of the shell considered as positive (+ve)
4. For bending moment
   a. Moment which produce tension in the inner fibers of shell is considered positive (+ve)
5. For displacements
   a. Down ward displacement along tangent to meridian curve (u_{ϕ}) taken as Positive(+ve)
   b. In ward displacement normal to meridian curve (w_z)) taken as Positive(+ve)
   c. Out ward rotation taken as positive (+ve)
   d. Out ward horizontal displacement taken as positive (+ve)

2.6.5. Membrane analysis of shell structure

One of the major reason that made shell structures strong and economical is their curved middle plane. This property of shell enhances their ability to carry out-of-plane load by in-plane membrane forces. Membrane theory of shell structure enables us to study shell structures membrane behavior in momentless state or in membrane state of stress.

- The necessary conditions to have momentless state are:-

1. At the boundary region of the shell are free from transverse shear and moments (i.e. Loads applied to the shell boundaries must lie in planes that are tangent to the middle surface of the shell.).
2. Rotation and normal displacement at the shell edge should be free.
3. Surface of shell must be smooth and continuous.
4. Surface components and edge loads must be also smooth and continuous function of the coordinates.

The governing equation of membrane theory can be obtained directly from the equation of the general shell theory by neglecting the effects of the bending and twisting moments, as well as the transverse shear forces, on the state of stress and strain of thin shells [6]. Thus, for the membrane theory of thin shells, we can assume that: $M_{\phi} = M_{\theta} = M_{\phi\theta} = M_{\theta\phi} = Q_{\phi} = Q_{\theta} = 0$.

Furthermore the governing equation of membrane theory for axisymmetricaly loaded shell of revolution can be simplified by assuming the derivative of membrane forces and displacements with respect to θ is zero, because a given load does not change in the circumferential direction [6].

2.6.5.1. Membrane analysis of axisymmetricaly loaded dome with skylight

The axisymmetric external applied loads per unit area (P) of the middle surface are represented by the components P_{ϕ} and P_Z acting tangent along the meridian and normal direction to the middle surface of the dome, respectively, as depicted in figure 4. Axisymmetric loading component P_θ acting in circumferential direction of the dome is zero. This implies that in the case of axisymmetrical loading membrane shear forces are zero (i.e. $N_{\theta\phi} = N_{\phi\theta} = 0$).

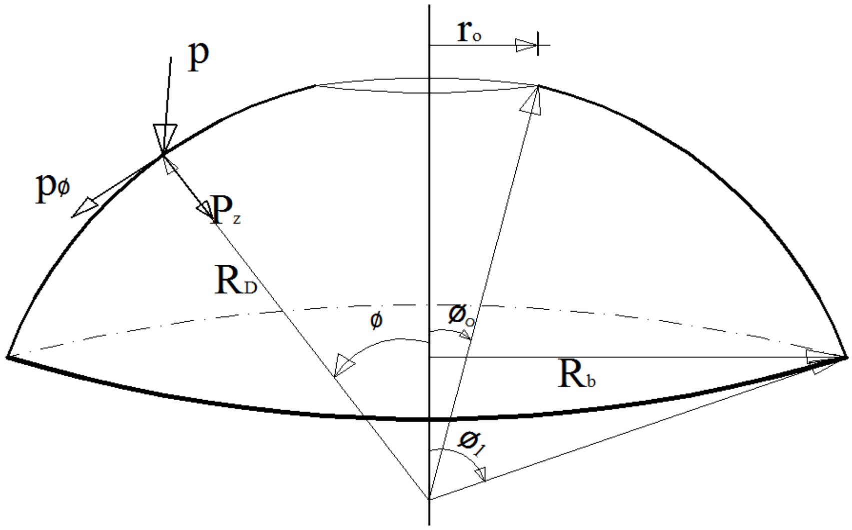


Figure 4: Spherical dome with skylight geometry and positive external load projection.

Spherical coordinate is used to drive the governing equation for membrane analysis of axisymmetric loaded dome with skylight shell. Consider an infinitesimal shell element of size $R_D d_\phi$ in meridian direction and rd_θ in circumferential direction as shown in figure 5. The unknown two internal membrane force and the given two components of applied load make the system statically determinate. Therefore, after some simplification the two equilibrium equations of the element that relate internal membrane forces and loading components in meridional and normal direction can be formulated as follows.

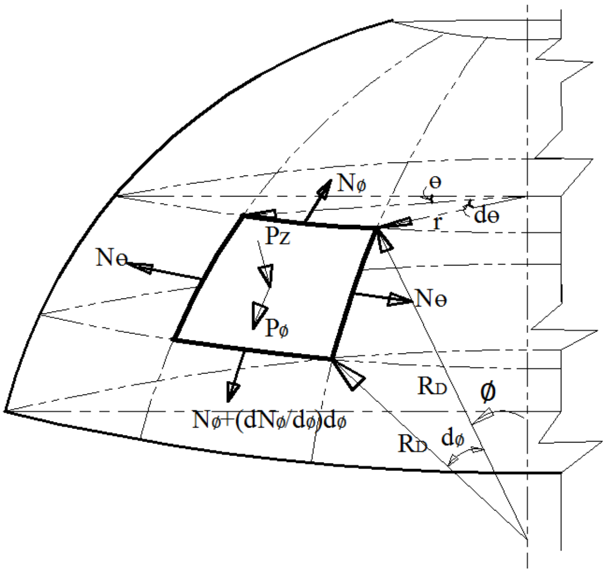


Figure 5: Load component and stress resultant on an infinitesimal element of dome with skylight.

The equilibrium of force along tangent to the meridian direction

$$\frac{dN_{\phi}}{R_D d\phi} + \frac{(N_{\phi} - N_{\theta})}{R_D} \cot\phi + p_{\phi} = 0 \tag{3}$$

The equilibrium of force normal direction to the middle surface

$$\frac{N_{\phi}}{R_D} + \frac{N_{\theta}}{R_D} + p_z = 0 \tag{4}$$

From equation-4 the expression for N_{θ} is

$$N_{\theta} = -R_D\left(p_z + \frac{N_{\phi}}{R_D}\right) \tag{5}$$

Inserting the N_{θ} expression from equation-5 in to equation-3, after some algebra transformation and geometric consideration equation-3 may be reduced to the following equation.

$$\frac{d}{d\phi}(N_{\phi} r \sin\phi) + rR_D(p_{\phi} \sin\phi + p_z \cos\phi) = 0 \tag{6}$$

Integrating equation-6 from ϕ_0 to ϕ and solving for N_{ϕ} we can get

$$N_\phi = -\frac{1}{R_D \sin^2 \phi} \int_{\phi_0}^{\phi} R_D^2 \sin\phi \left(p_\phi \sin\phi + p_z \cos\phi \right) d\phi + \frac{N_\phi^0 \sin^2 \phi_0}{\sin^2 \phi} \tag{7}$$

Axisymmetrical loadings that may be act on dome with skylight structure are top ring loading ($p^{(rd)}$), self-weight (p) and live load (q). Their Projection in positive direction are given in table-1 below and their membrane force effects are summarized in table-2 below.

Table 1: Axisymmetrical loadings and their load arrangement (projection) on dome with skylight.

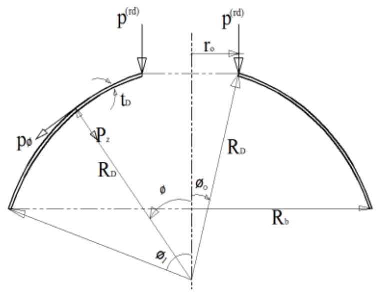
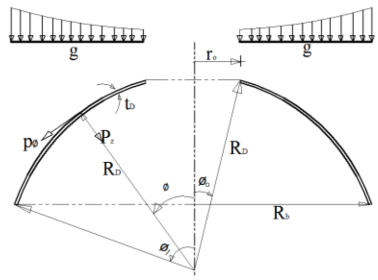
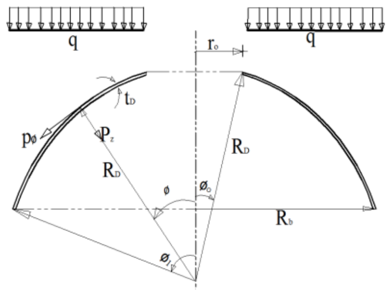
Top ring loading	Dead loading	Live loading
		
$p^{(rd)} = 3640 N/m$, $p_\phi = p^{(rd)} \sin\phi_0$ $p_z = p^{(rd)} \cos\phi_0$	$p^{(d)} = g$, $p_\phi = p^{(d)} \sin\phi$ $p_z = p^{(d)} \cos\phi$	$p_\phi = q \sin\phi \cos\phi$ $p_z = q \cos^2 \phi$

Table 2: membrane analysis result of membrane forces (N_ϕ and N_θ) for axisymmetrically loaded dome with skylight shell structure.

Axisymmetric loads applied on dome with skylight	Projection of loads in Meridional (φ) and normal(z) direction	Membrane forces in meridional ϕ and circumferential direction θ	
		Meridional force (N_ϕ)	Circumferential force (N_θ)
Top ring load ($p^{(rd)}$)	$p_\phi = p^{(rd)} \sin\phi_0$	$-\dfrac{p^{(rd)} \sin\phi_0}{\sin^2\phi}$	$R_D\, p^{(rd)}\left(\dfrac{\sin\phi_0}{R_D \sin^2\phi} - \cos\phi_0\right)$
	$p_z = p^{(rd)} \cos\phi_0$		
Dead load ($p^{(d)}$)	$p_\phi = p^{(d)} \sin\phi$	$R_D P^{(d)}\left(\dfrac{\cos\phi - \cos\phi_0}{\sin^2\phi}\right)$	$-\dfrac{R_D\, p^{(d)}}{\sin^2\phi}\left(\cos\phi\left(\sin^2\phi + 1\right) - \cos\phi_0\right)$
	$p_z = p^{(d)} \cos\phi$		
Live load (q)	$p_\phi = q \sin\phi \cos\phi$	$\dfrac{R_D q}{2}\left(\sin^2\phi_0 \csc^2\phi - 1\right)$	$\dfrac{R_D q}{2\sin^2\phi}\left(\sin^2\phi - 2\cos^2\phi \sin^2\phi - \sin^2\phi_0\right)$
	$p_z = q\cos^2\phi$		

Where

ϕ	is meridional angle starting from the axis of symmetry
ϕ_0	is pitch angle of spherical dome
r_0	is pitch radius of spherical dome
R_D	is principal radius of spherical dome
t_D	is thickness of spherical dome
$p^{(d)}$	is dead load of spherical dome
q	is live load of spherical dome
$p^{(rd)}$	is top ring load of spherical dome
P	is external applied loads per unit area

P_ϕ is external applied load projection along meridional direction

P_Z is out ward load projection of external applied load

g is dead load due to gravity

$= t_D \gamma_{CONC}$

t_D is thickness of spherical dome

γ_{CONC} is unit weight of concrete

N_ϕ is internal membrane force in meridional direction

N_θ is internal membrane force in tangential direction

By assuming the shell is made from linearly elastic material and obeys Hooke's low, the constitutive relations that relate stress with strain for axisymmetricaly loaded dome with skylight are as follows.

$$\varepsilon_\phi = \frac{1}{Et_D}\left(N_\phi - vN_\theta\right) \quad , \quad \varepsilon_\theta = \frac{1}{Et_D}\left(N_\theta - vN_\phi\right) \qquad (8\ (a, b))$$

The kinematic relationship that relates strain and displacement derived in spherical coordinate as follows. For axisymetricaly loaded shell of revolution it can be assumed that displacement in circumferential direction (V_θ) and membrane shear strain ($\gamma_{\phi\theta}$ and $\cdot\gamma_{\theta\phi}$) equal to zero (0). Let us take an infinitesimal dome element AB length $R_D d_\phi$ which curves in the meridional direction as show in the figure 6. AB and A'B' in the figure refers to the undeformed and deformed configuration of the element. u_ϕ and w_z represents the two displacement parameter of deformation in the tangent to the meridional and normal direction, respectively. Positive meridional displacement (u_ϕ) is taken in increasing ϕ direction and positive normal displacement (w_z) is taken in wards.

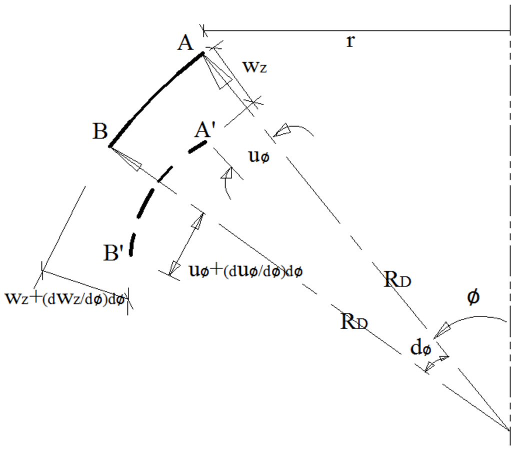


Figure 6: Deformation of infinitesimal element in the meridian plane.

Therefore, the meridional strain (ε_ϕ) equation is

$$\varepsilon_\phi = \frac{A'B' - AB}{AB} \tag{9}$$

Since

$$AB = R_D d\phi \tag{10}$$

$$A'B' = (R_D - w_z)d\phi - u_\phi + (u_\phi + du_\phi) = (R_D - w_z)d\phi + du_\phi \tag{11}$$

Then

$$\varepsilon_\phi = \frac{(R_D - w_z)d\phi + du_\phi - R_D d\phi}{R_D d\phi} = \frac{1}{R_D}\frac{du_\phi}{d\phi} - \frac{w_z}{R_D} \tag{12}$$

The strain in circumferential (hoop) direction is calculated in the same manner by taking the hoop element of the shell. Before deformation the parallel circle radius $r_1 = R_D \sin\phi$. At the deformation state of the element the radius becomes $r_f = r_1 - w_z \sin\phi + u_\phi \cos\phi$.

Therefore, the meridional strain (ε_θ) equation is

$$\varepsilon_\theta = \frac{r_f - r_1}{r_1} = \frac{u_\phi}{R_D}\cot\phi - \frac{w_z}{R_D} \tag{13}$$

From equation -13 Displacement normal to meridian curve (w_z) can be expressed as

$$w_z = u_\phi \cot\phi - R_D \varepsilon_\theta \tag{14}$$

Inserting equation-14 in to equation-12 gives

$$\frac{1}{R_D}\left(\frac{du_\phi}{d\phi} - u_\phi \cot\phi\right) = \varepsilon_\phi - \varepsilon_\theta \tag{15}$$

Multiplying equation-15 both side by $R_D \csc\phi$ gives

$$\frac{d(u_\phi \csc\phi)}{d\phi} = f_1(\phi) = \frac{R_D}{\sin\phi}(\varepsilon_\phi - \varepsilon_\theta) \tag{16}$$

Both sides Integration of equation-16 gives the expression of displacement along tangent to meridian curve (u_ϕ).

$$u_\phi = \sin\phi \int_{\phi_0}^{\phi} f_1(\phi) d\phi - C_1 \sin\phi \tag{17}$$

Where C_1 is a constant of integration. Substituting for u_ϕ from equation-17 in to eqution-14, one finds the following expression for displacement normal to meridian curve (w_z).

$$w_z = \cos\phi \int_{\phi_0}^{\phi} f_1(\phi) d\phi + C_1 \cos\phi - \varepsilon_\theta R_D \tag{18}$$

The meridional rotation (β_ϕ) can be obtained by the following expression

$$\beta_\phi = \frac{1}{Et_D}\left(\cot\phi(1+v)(N_\phi - N_\theta) - \frac{dN_\theta}{d\phi} + v\frac{dN_\phi}{d\phi}\right) \tag{19}$$

And finally, the horizontal expansion or contraction of the membrane shell at angle ϕ can be calculated by the following expression

$$\delta_{sh} = R_D \sin\phi \varepsilon_\theta = \frac{R_D \sin\phi}{Et_D}(N_\theta - vN_\phi) \tag{20}$$

The effect axisymmetrical loadings on Horizontal displacement (δ_{sh}) and meridional rotation (β_{ϕ}) for dome with skylight shell structure are summarized in table-3 as follows.

Table 3: Horizontal displacement (δ_{sh}) and meridional rotation (β_{ϕ}) for dome with skylight.

Axisymmetric loads applied on dome with skylight	Horizontal displacement (δ_{sh})	meridional rotation (β_{ϕ})
Top ring load ($p^{(rd)}$)	$\frac{R_D P^{(rd)}}{\sin\phi E t_D}\left(\sin\phi_0(1+v)+R_D\cos\phi_0\sin^2\phi\right)$	$\frac{(1+v)}{Et_D}\begin{pmatrix}\cot\phi\left(\left(\frac{P^{(rd)}}{\sin^2\phi}\right)\left(R_D\cos\phi_0\sin^2\phi-2\sin\phi_0\right)\right)\\ +2P^{(rd)}\sin\phi_0\left(\frac{\cos\phi}{\sin^3\phi}\right)\end{pmatrix}$
Dead load (P)	$\frac{R_D^2 P^{(d)}}{\sin\phi E t_D}\left((1+v)(\cos\phi_0-\cos\phi)-\cos\phi\sin^2\phi\right)$	$\frac{R_D P^{(d)}}{Et_D}\begin{pmatrix}\cot\phi(1+v)\left(\frac{\cos\phi(\sin^2\phi+2)-2\cos\phi_0}{\sin^2\phi}\right)\\ -\left(\sin\phi+\csc\phi\left(\cot^2\phi+\csc\phi(\csc\phi-2\cos\phi_0\cot\phi)\right)\right)\\ +v\csc\phi\left[\csc\phi(2\cos\phi_0\cot\phi-\csc\phi)-\cot^2\phi\right]\end{pmatrix}$
Live load (q)	$\frac{R_D^2 q}{2\sin\phi E t_D}\begin{pmatrix}(1+v)\left(\sin^2\phi-\sin^2\phi_0\right)\\ -2\cos^2\phi\sin^2\phi\end{pmatrix}$	$-\frac{R_D q}{Et_D}\begin{pmatrix}\cot\phi(1+v)\left(\sin^2\phi\right)\\ +\left(\sin^2\phi_0\csc^2\phi\cot\phi-2\sin\phi\cos\phi\right)-v\sin^2\phi_0\csc^2\phi\cot\phi\end{pmatrix}$

Where

ϕ is meridional angle starting from the axis of symmetry

ϕ_0 is pitch angle of spherical dome

R_D is principal radius of spherical dome

t_D is thickness of spherical dome

E is young’s modulus of elasticity

v is poisons ratio

ε_{θ} is tangential strain

ε_{ϕ} is meridional strain

w_z is displacement normal to meridian curve

u_{ϕ} is displacement along tangent to meridian curve

β_{ϕ} is meridional rotation

δ_{sh} is horizontal displacement of spherical dome

r_1 is parallel circle radius of spherical dome element in undeformed state

r_f is parallel circle radius of spherical dome element in deformed state

2.6.6. Bending analysis of shell structure

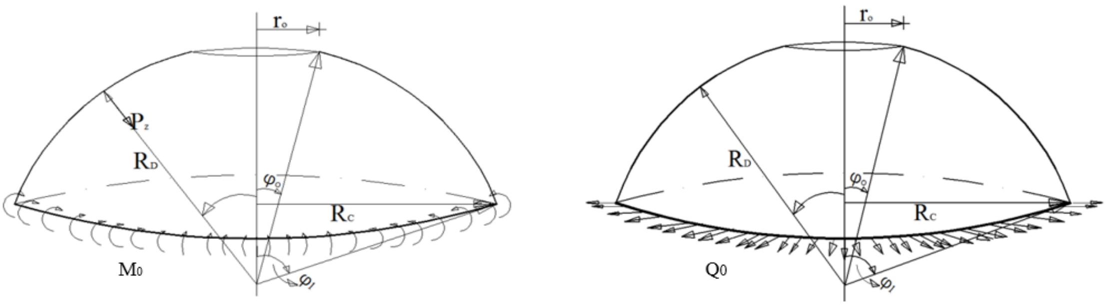
Membrane theory is insufficient to maintain deflection and rotation compatibility at the edge of thin shell structures. Discontinuity of forces and moments are developed at these locations that results the bending and shear stresses in the shell which are localized over a small area and dissipate rapidly along the shell. Edge effect or bending theory is used to correct these incompatibilities [11]. Complete analysis result at the edge of shell is obtained by superimposing the membrane analysis results and bending analysis results.

The edge effect bending moment (M_0) and shear force (Q_0) of shells are found from the compatibility of deformation conditions. If the geometry and loading condition are axially symmetrical, all variable involved are going to be independent of the hoop parameter, P_{θ}.

2.6.6.1. Bending analysis for edge effect of axisymmetric spherical shell structure

Edge effects in axisymmetric spherical shell, which are calculated from compatibility of deformation at the bottom edge, include uniformly distributed moment (M_0) and shear force (

Q_0) as shown in the figure 7. Each of these bottom edge forces produces a bending field in the shell. The stresses and deformations of the bending field due to these edge forces must be superposed on the membrane field to obtain the total field of internal forces and deformations in the shell.



a) b)

Figure 7: Edge loads on dome with skylight, a) distributed edge moment, b) distributed edge shear

Calculations based on more exact theories and experimental results show that the influence of boundary conditions on thin shell structures dies out rapidly away from the edges [11]. The spatial variation of internal force and shell deformations due to distributed edge shear and bending moment could be quantified by bending analysis or actual measurement of shell behavior.

Geckeler’s approximation, based on the physical argument that the boundary effects are localized and thus the higher order gradients are of greatest importance, gives an approximation bending analysis result for axisymmetric shells. The approximated Geckeler’s governing equation for edge effects of spherical shell can be presented as follows.

$$\frac{d^2 Q_{0s}}{d\phi^2} = Et_D \beta_\phi \tag{21}$$

$$\frac{d^2 \beta_\phi}{d\phi^2} = -\frac{R_D^2 Q_{0s}}{k} \tag{22}$$

Where

β_ϕ is spherical dome rotation tangent to meridional curve

$$= \frac{1}{R}\left(u_\phi + \frac{dw_z}{d\phi} \right)$$

u_ϕ is displacement along tangent to meridian curve

w_z is displacement normal to meridian curve

Q_{0s} is lateral shear force of spherical dome

$$K = \frac{Et_D^3}{12(1 - v^2)}$$

By eliminating the variable function β_ϕ we obtain the forth order differential equation on the function Q_{0s}.

$$\frac{d^4 Q_{0s}}{d\phi^4} = 4\lambda^4 Q_{0s} \tag{23}$$

Where

$$\lambda^4 = 3(1 - v^2)\left(\frac{R_D}{t_D} \right)^2 ,$$

The general solution of this equation is

$$Q_{0s} = C_1 e^{\lambda\phi} \cos\lambda\phi + C_2 e^{\lambda\phi} \sin\lambda\phi + C_3 e^{-\lambda\phi} \cos\lambda\phi + C_4 e^{-\lambda\phi} \sin\lambda\phi \tag{24}$$

The last two term in the general solution increase as ϕ decreases, i.e. as we move away from the edge. But we argued that edge effects must decrease as we move away from the edge. Therefore, we set $C_3 = C_4 = 0$, and obtain

$$Q_{0s} = C_1 e^{\lambda\phi} \cos\lambda\phi + C_2 e^{\lambda\phi} \sin\lambda\phi \tag{25}$$

In discussing the edge condition it is advantageous to use a change of coordinate angle ψ, $\psi = \phi_1 - \phi$, representing the angular coordinate and counting off from the shell edge as shown in the figure-8 below.

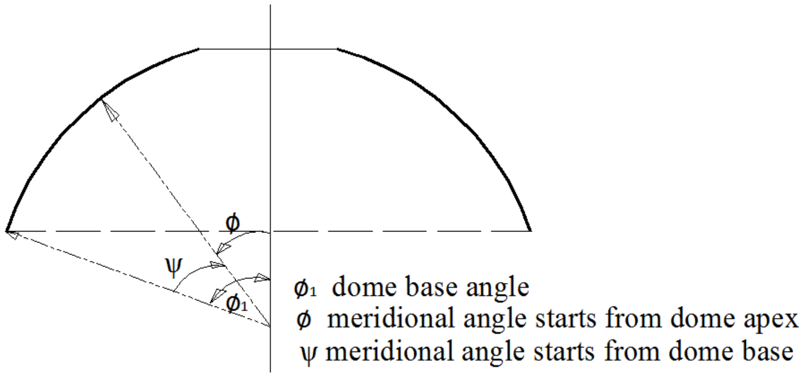


Figure 8: Different coordinate angles of an axisymmetric dome with skylight shell.

Substituting $\phi = \phi_1 - \psi$ for ϕ and replace the integration constants C_1 and C_2 by two new constants C and γ, the above solution takes the form

$$Q_{0s} = Ce^{-\lambda\psi} \sin(\lambda\psi + \gamma) \tag{26}$$

We can now write the bending field forces and deformation as follows.

$$\beta_\phi = \frac{1}{Et_D}\frac{d^2 Q_{0s}}{d\phi^2} = \frac{2\lambda^2}{Et_D} Ce^{-\lambda\psi} \cos(\lambda\psi + \gamma) \tag{27}$$

$$N_\theta = -\frac{dQ_{0s}}{d\phi} = -\lambda\sqrt{2}Ce^{-\lambda\psi} \sin\left(\lambda\psi + \gamma - \frac{\pi}{4}\right) \tag{28}$$

$$N_\phi = -Q_{0s} \cot\phi = -\cot(\phi_1 - \psi)Ce^{-\lambda\psi} \sin(\lambda\psi + \gamma) \tag{29}$$

$$N_\phi = -Q_{0s} \cos\phi = -\cos(\phi_1 - \psi)Ce^{-\lambda\psi} \sin(\lambda\psi + \gamma)$$

$$M_\phi = -\frac{k}{R_D}\frac{d\beta_\phi}{d\phi} = \frac{R_D}{\lambda\sqrt{2}} Ce^{-\lambda\psi} \sin\left(\lambda\psi + \gamma + \frac{\pi}{4}\right) \tag{30}$$

The horizontal displacement, i.e. the change in radius of parallel circle is

$$\delta_{sh} = R_D \sin\phi \varepsilon_\theta = \frac{R_D \sin\phi}{Et_D}\left(N_\theta - vN_\phi\right) \tag{31}$$

Let us drive the expression for the integration constants (i.e. C and γ) by using the boundary condition below for the shell of dome subjected to edge force M_0 and Q_0 shown in the figure 10 above.

Boundary conditions for dome subjected to uniformly distributed edge moment M_0

$$M_\phi\big|_{\phi=\phi_1} = M_0 \tag{32}$$

$$N_\phi\big|_{\phi=\phi_1} = 0 \tag{33}$$

From condition of equation-33 we get $\gamma = 0$ and from condition of equation-32 and γ value we get $C = \dfrac{2M_0\lambda}{R_D}$

Boundary conditions for dome subjected to uniformly distributed edge shear force Q_0

$$M_\phi\big|_{\phi=\phi_1} = 0 \tag{34}$$

$$N_\phi\big|_{\phi=\phi_1} = -Q_0 \cos\phi_1 \tag{35}$$

From condition of equation-34 we get $\gamma = \dfrac{\pi}{4}$ and from condition of equation-35 and γ value we get $C = \dfrac{2Q_0 \sin\phi_1}{\sqrt{2}}$.

The final sum bending field effects of uniformly distributed moment and horizontal shear force are presented as follows.

$$N_\phi = -e^{-\lambda\psi}\left[Q_0\left(\sqrt{2}\cot(\phi_1-\psi)\sin\phi_1 \sin\left(\lambda\psi - \frac{\pi}{4}\right)\right) + M_0\left(\frac{2\lambda}{R_D}\cot(\phi_1-\psi)\sin(\lambda\psi)\right)\right] \tag{36}$$

$$N_{\theta} = -2e^{-\lambda\psi}\left[Q_0\left(\lambda\sin\phi_1\sin\left(\lambda\psi-\frac{\pi}{2}\right)\right)+M_0\left(\frac{\sqrt{2}\lambda^2}{R_D}\sin\left(\lambda\psi-\frac{\pi}{4}\right)\right)\right] \tag{37}$$

$$Q_{0s} = 2e^{-\lambda\psi}\left[Q_0\left(\frac{\sin\phi_1}{\sqrt{2}}\sin\left(\lambda\psi-\frac{\pi}{4}\right)\right)+M_0\left(\frac{\lambda}{R_D}\sin\lambda\psi\right)\right] \tag{38}$$

$$M_{\phi} = e^{-\lambda\psi}\left[Q_0\left(\frac{R_D}{\lambda}\sin\phi_1\sin\lambda\psi\right)+M_0\left(\sqrt{2}\lambda^2\sin\left(\lambda\psi-\frac{\pi}{4}\right)\right)\right] \tag{39}$$

$$\delta_{sh}\big|_{\phi=\phi_1} = \frac{2\lambda\sin\phi_1}{Et_D}\left[Q_0\left(R_D\sin\phi_1\right)+M_0\lambda\right] \tag{40}$$

$$\beta_{\phi}\big|_{\phi=\phi_1} = \frac{2\lambda^2{}_1}{Et_D}\left[Q_0\sin\phi_1+\frac{2M_0\lambda}{R_D}\right] \tag{41}$$

Where

ϕ_1 is base angle of spherical dome

R_D is principal radius of spherical dome

N_{ϕ} is internal membrane force in meridional direction

N_{θ} is internal membrane force in tangential direction

Q_{0s} is lateral shear force of spherical dome

M_{ϕ} is bending moment in meridional direction

δ_{sh} is horizontal displacement of spherical dome

$\delta_{sh}\big|_{\phi=\phi_1}$ is spherical dome base horizontal displacement

β_{ϕ} is spherical dome rotation tangent to meridional curve

$\beta_\phi\big|_{\phi=\phi_1}$ is spherical dome base rotation tangent to meridional curve

The equations listed above gives the flexibility influence coefficient for uniformly distributed unit shear force at the edge of the shell ($Q_0 = 1$) by considering edge moment equals to zero ($M_0 = 0$).

And proceeding along similar line, the flexibility influence coefficient for uniformly distributed unit moment at the edge of the shell ($M_0 = 1$) by considering edge shear force equals to zero ($Q_0 = 0$). These flexibility influence coefficient are use full for deriving for the compatibility relation and for determining the unknown redundant edge forces in force method of axisymmetric shell analysis.

2.6.7. Ring beam analysis

Edge rings stiffen the shell and / or provide lateral support for the shell structure. The lateral support action of the rings is specially needed in cases where there are only vertical supports and thus the lateral thrusts are to be absorbed by the structure itself [11]. Providing Ring beam at the junction can solve an even distribution of shear force and prevent ovalaization of the shell [1]. Atypical connection between sphere and ring beam is shown in figure 9 below.

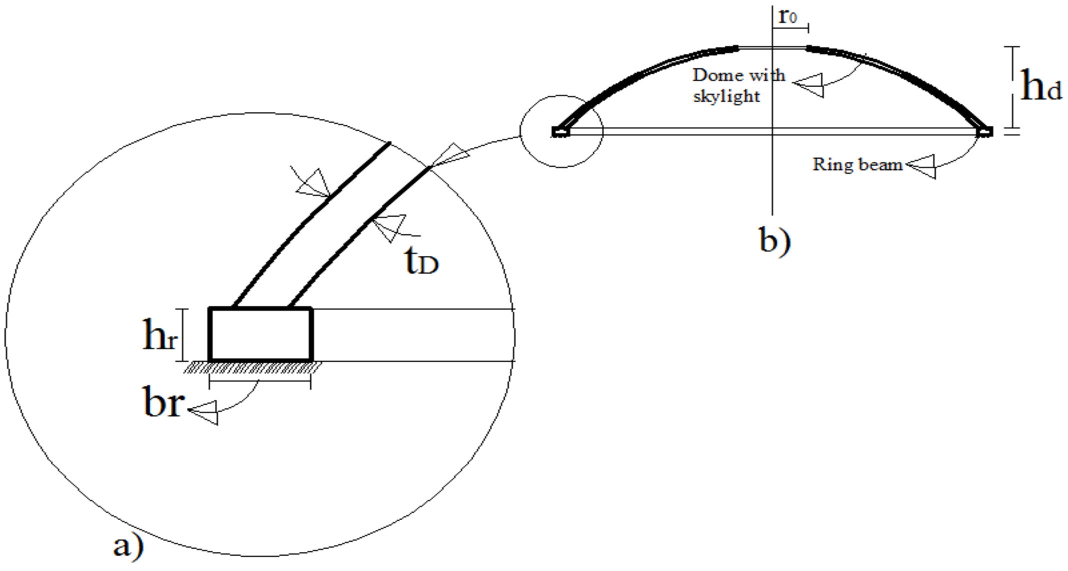


Figure 9: Section view of elevation of the spherical dome a) Junction detail b) section elevation view.

The connection is assumed to be rigid and a vertical deflection of the ring beam is negligible. In the analysis of such shell structures, one can assume that the bending stiffness of the ring in the direction perpendicular to its plane is negligible. Thus, the ring resists only the radial load in its own plane [6]. The free-body diagrams of the spherical dome shell and ring is shown in Figure 10 which shows horizontal force (H) from spherical dome shell acting on the ring beam.

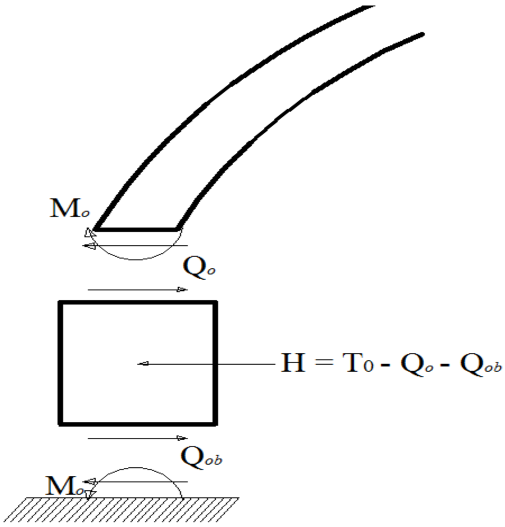


Figure 10: Free body diagram at junction.

The axial force T_r in the ring beam is found from equilibrium of forces:

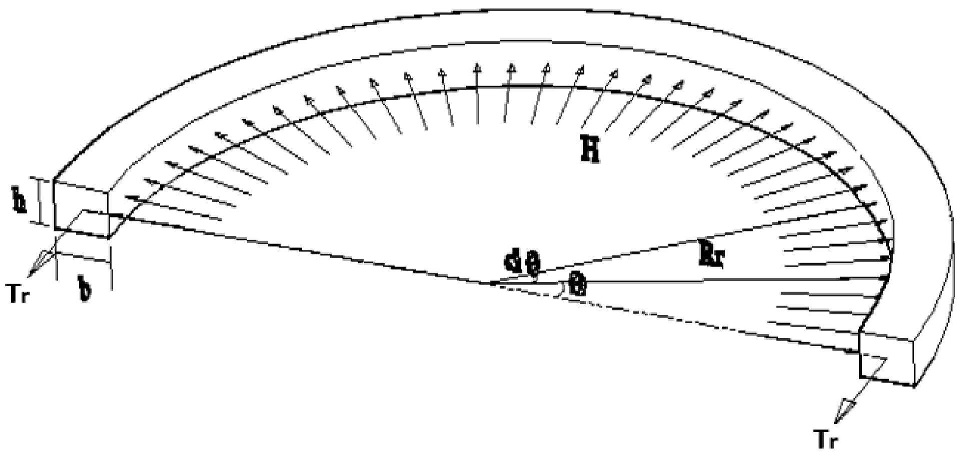


Figure 11: Applied radial load (H) on Ring beam.

$$T_r = \frac{1}{2}\int_0^{\pi} Hr^r \sin\theta d\theta = Hr^r \quad (42)$$

From Hook's low the hoop strain of ring beam becomes

$$\varepsilon_{\theta H}^{r} = \frac{T_r}{EA_r} = H\,\frac{r^r}{EA_r} \tag{43}$$

The radial displacement of ring beam can be expressed as

$$\delta_{rh} = r^r \varepsilon_{\theta H}^{r} = H\,\frac{\left(r^r\right)^2}{EA_r} \tag{44}$$

Where

δ_{rh}	is radial displacement of ring beam
$\varepsilon_{\theta H}^{r}$	is hoop strain of ring beam
θ	is circumferential angle
T_r	is ring beam axial force
H	is radial force acting on the ring beam
r^r	is radius of ring beam middle surface.
A_r	is section area of ring beam
E	is young's modulus of elasticity

2.6.8. Compatibility equation

To get the edge shear forces and bending moments of the shell it is essential to consider compatibility of displacement for dome with ring beam, ring beam with base fixed support and dome with base fixed support. Hence the deformations of the different components must be expressed in the junction. Figure 12 shows the relevant forces and deformations in the junction of the base of spherical dome roof structure, as well as the horizontal force acting on the ring beam. From the figure it is clear that equivalent hoop force H can be expressed as:

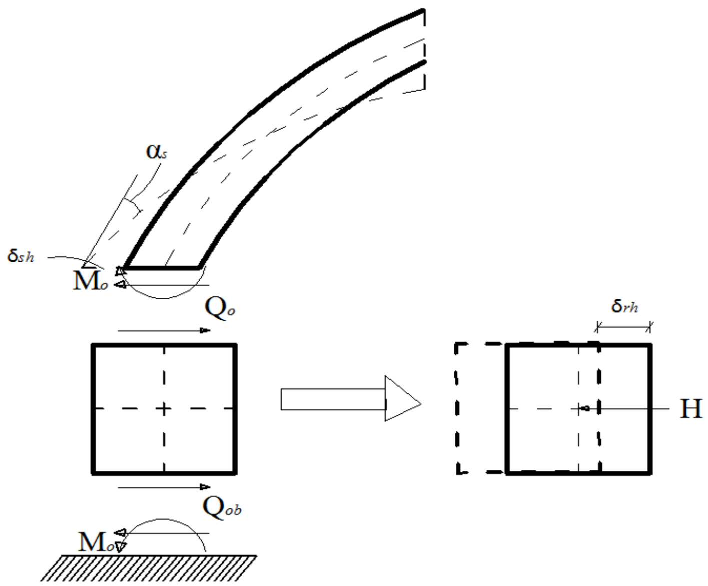


Figure 12: Deformations of spherical dome roof components at junction.

$$H = T_0 - (Q_0 + Q_{0b}) \tag{45}$$

Where

H is radial force acting on the ring beam

T_0 is horizontal component of membrane meridional force (N_ϕ) at the bottom of dome

$= N_\phi \cos\phi_1$

Q_0 is edge lateral shear force at dome base

Q_{ob} is shear force at fixed base

2.6.8.1. Compatibility equation at spherical dome, ring beam and fixed base support

A. Displacement compatibility for dome and ring beam can be expressed as:

$$\delta_{sh}\big|_{\phi=\phi_1} = \delta_{rh} \Rightarrow \delta_{sh}^{h}\big|_{\phi=\phi_1} + \delta_{sh}^{p}\big|_{\phi=\phi_1} = \delta_{rh}^{h} + \delta_{rh}^{p} \tag{46}$$

By inserting and rewriting, we can get the following equation.

$$\left(\frac{2\lambda R_D \sin^2\phi_1}{Et_D}+\frac{(r^r)^2}{EA_r}\right)Q_0+\left(\frac{(r^r)^2}{EA_r}\right)Q_{0b}+\left(\frac{2\lambda^2\sin\phi_1}{Et_D}\right)M_0=\delta_{sh}^p\Big|_{\phi=\phi_1}+\delta_{rh}{}^p \tag{47}$$

B. Displacement compatibility for ring beam and base fixed support can be expressed as:

$$\delta_{rh}=\delta_{fb}\Rightarrow\delta_{rh}^p+\delta_{rh}^p=0 \tag{48}$$

$$\left(\frac{(r^r)^2}{EA_r}\right)Q_0+\left(\frac{(r^r)^2}{EA_r}\right)Q_{0b}=\delta_{rh}{}^p \tag{49}$$

C. Rotation compatibility for dome and base fixed support can be expressed as:

$$\alpha_s\Big|_{\phi=\phi_1}=\alpha_{fb}\Rightarrow\alpha_s^h\Big|_{\phi=\phi_1}+\alpha_s^p\Big|_{\phi=\phi_1}=0 \tag{50}$$

$$\left(\frac{2\lambda^2\sin\phi_1}{Et_D}\right)Q_0+\left(\frac{4\lambda^3}{R_D Et_D}\right)M_0=\alpha_s^p\Big|_{\phi=\phi_1} \tag{51}$$

Now we have three unknowns from junction 1 (i.e. Q_0, Q_{0b} and M_0) and three compatibility equations. Therefore the unknowns can be determined from the following matrix.

$$\begin{vmatrix}\frac{2\lambda R_D\sin^2\phi_1}{Et_D}+\frac{(r^r)^2}{EA_r} & \frac{(r^r)^2}{EA_r} & \frac{2\lambda^2\sin\phi_1}{Et_D}\\ \frac{(r^r)^2}{EA_r} & \frac{(r^r)^2}{EA_r} & 0\\ \frac{2\lambda^2\sin\phi_1}{Et_D} & 0 & \frac{4\lambda^3}{R_D Et_D}\end{vmatrix}\begin{vmatrix}Q_0\\ Q_{0b}\\ M_0\end{vmatrix}=\begin{vmatrix}\delta_{sh}^p\Big|_{\phi=\phi_1}+\delta_{rh}{}^p\\ \delta_{rh}{}^p\\ \alpha_s^p\Big|_{\phi=\phi_1}\end{vmatrix} \tag{52}$$

Where

R_D is principal radius of spherical dome

r^r is radius of ring beam middle surface.

A_r is section area of ring beam

E is young's modulus of elasticity

t_D is thickness of spherical dome

ϕ_1 is base angle of spherical dome

M_0 is uniformly distributed dome base edge meridional moment

Q_0 is uniformly distributed dome base edge lateral shear force

Q_{0b} is shear force at fixed base

$\delta_{sh}\big|_{\phi=\phi_1}$ is spherical dome base horizontal displacement

$\delta_{sh}^{p}\big|_{\phi=\phi_1}$ is particular spherical dome base horizontal displacement

$\delta_{sh}^{h}\big|_{\phi=\phi_1}$ is homogeneous spherical dome base horizontal displacement

δ_{rh} is radial displacement of ring beam

$\delta_{rh}{}^{p}$ is particular ring beam radial displacement

δ_{rh}^{h} is homogeneous ring beam radial displacement

$\alpha_s^{p}\big|_{\phi=\phi_1}$ is particular spherical dome base rotation tangent to meridional curve

$\alpha_s^{h}\big|_{\phi=\phi_1}$ is homogeneous spherical dome base rotation tangent to meridional curve

α_{fb} is rotation of fixed support

δ_{fb} is displacement of fixed support

2.6.9. Design of spherical dome shell structure

The basic requirement of designing of a structure according to Eurocode 0 section 2 clause 2.1(1) is, proportioning of a structure in order to make them capable of sustain all actions and influences likely to occur during execution and use, and meet the specified serviceability requirements for a structure or its element with appropriate degrees of reliability and in an economical way.

A structure that is designed as roof must fulfill all the requirements for normal structures in having adequate strength, durability and freedom from excessive cracking and deflection.

Structural design is often governed by a code of practice. For this study basically Eurocodes are used. Eurocode is concerned with the requirements for resistance, serviceability, durability and fire resistance of concrete structures. And it is written based on the limit state concept used in conjunction with a partial factor method and it encourages using of limit state design principle to design concrete structures. This study is done for persistent (anticipated normal usage) design situation of unfavorable load (action) case of roof structure.

2.6.9.1. Limit state design of spherical dome roof structure

The aim of limit state design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended. When structure or structural member becomes deficient for its planed use, it is said to have reached a limit state. The limit state of concrete structure most of the time can be divided in to two. These are:-

1. Ultimate limit state (states associated with collapse or with other similar forms of structural failure)
2. Serviceability limit state (states that correspond to conditions beyond which specified service requirements for a structure or structural member are no longer met)

2.6.9.2. Ultimate limit state

2.6.9.2.1. Combinations of actions

For this study the fundamental load combination (combination value of variable action) is used for verification of ultimate limit state design. The selected load combination and values of γ_f

for ultimate limit state design of persistent (anticipated normal usage) design situation and unfavorable load case of roof according to Eurocode 0 :2002 is as follows:

$$1.35G_K + 1.5Q_K \tag{53}$$

Where

G_K is characteristic dead load

Q_K is characteristic imposed load

According to Eurocode 1 part1-1 (2002) of section 6 clause 6.3.4.1(1) table 6.10 characteristic imposed load (Q_k) for roof category H (i.e. Roofs not accessible except for normal maintenance and repair.) is equal to 0.4 KN/m^2.

2.6.9.2.2. Material and design stress used for ultimate limit state

In this study the plain concrete with quartzite aggregates having properties of characteristic 28 day compressive cylinder strength (f_{ck}) of $C35/45$, normal weight (γ_{CONC}) value of $25KN/m^3$ and design value of modulus of elasticity (E_{cm}) $34GPa$ is used. Poisson's ratio(ν) may be taken equal to 0.2 for uncracked concrete and 0 for cracked concrete as stated in Eurocode 2 part 1-1 section 3 clause 3.1.3(4). And also ribbed surface bar diameters of $8mm$ for longitudinal is used having of characteristic yield strength (f_{yk}) of $S460$, normal weight value of $78.5\,KN/m^3$, design value of modulus of elasticity (E_s) $200GPa$ and high bond steel properties. For ultimate limit state of fundamental load combination, the partial factors for concrete (γ_c) is 1.5 and for steel (γ_s) is 1.15 are used.

2.6.9.2.3. Concrete cover

According to Eurocode 2 part1-1 of section 4 clause 4.4.1.1(1), the nominal concrete cover is the distance between the surface of the reinforcement closest to the nearest concrete surface (including links and stirrups and surface reinforcement where relevant) and the nearest concrete surface and can be calculated using the following expression.

$$c_{nom} = c_{\min} + \Delta c_{dev} \tag{54}$$

Where

c_{nom} is nominal concrete cover

$c_{\min}$ is minimum cover

$= \max\{c_{\min,b}; c_{\min,dur}, 10mm\}$

$c_{\min,b}$ is minimum cover due to bond requirement,

=diameter of bar(ϕ_b) (for separated arrangement of bars)

$c_{\min,dur}$ is minimum cover due to environmental conditions

=30mm (selected according to Table 4.4N of Eurocode 2 part1-1 for Structural Class of S4 (design working life of 50 years), for exposure condition class XC4 and for strength class of concrete C35/45)

Δc_{dev} is an allowance in design for deviation (for this study the value of Δc_{dev} is taken as zero (0) by assuming that a very accurate measurement device is used for monitoring and non-conforming members during construction.)

Therefore, for durability of the roof structure the nominal value of the concrete cover to the reinforcement (c_{nom}) for this study has been taken as $30mm$ according to the above expression.

2.6.9.2.4. Ultimate limit state design formula for rectangular parabolic section of shell structure subjected to design moment

In this study the rectangular-parabolic stress block is used as shown in Figure 13 for nominal flexural strength calculations. For the rectangular-parabolic stress block, a value of $0.567f_{ck}$ is used for the width of stress block at the ultimate strain of 0.0035.

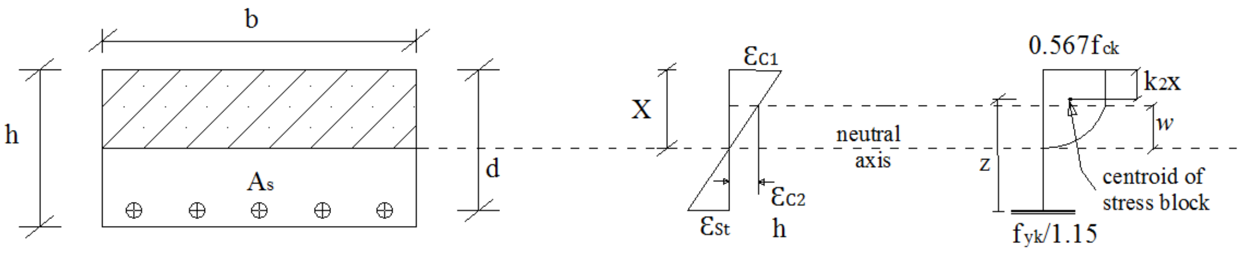


Figure 13: rectangular-parabolic stress block for Section equilibrium.

Where

ε_{c1} is the ultimate compressive strain of concrete

$= 0.0035$

ε_{c2} is the concrete strain at the end of the parabolic section

$= 0.002$

χ is depth of the neutral axis

w is the distance from the neutral axis to the strain ε_{c2}

$= 0.571x$

k_1 is the mean concrete stress

$= 0.459 f_{ck}$ (Calculated using area properties of a parabola)

$k_2 x$ is depth to the centroid of the stress block (where $k_2 = 0.416$)

z is the lever arm

$= d - k_2 x = 0.416x$

The following formulas (equations), which are based on the rectangular-parabolic stress block of a rectangular reinforced concrete section of the shell structure having width (b) and thickness (h) and subjected to a design moment M_d as shown in the figure 13 above.

The first step to design a section is to calculate the balanced effective depth based on ultimate concrete moment carrying capacity of a section (M_u) as follows:

$$M_u = 0.168 f_{ck} b d^2 \quad , \quad d_{balance} = \sqrt{\frac{M_u}{0.168 f_{ck} b}} \qquad (55)$$

Where

$d_{balance}$ is balanced effective depth

M_u is ultimate concrete moment capacity

f_{ck} is characteristic cylindrical strength of concrete

b is width of the rectangular reinforced concrete section. (unit width used in this case)

The second step is to select using of singly reinforced or doubly reinforced concrete section design principle by comparing the value $d_{balance}$ and effective depth (d) as follows:

A. If $d \geq d_{balance}$ use singly reinforced section design procedure.

B. If $d < d_{balance}$ use doubly reinforced section design procedure.

A. Singly reinforced section of shell design procedure ($d \geq d_{balance}$)

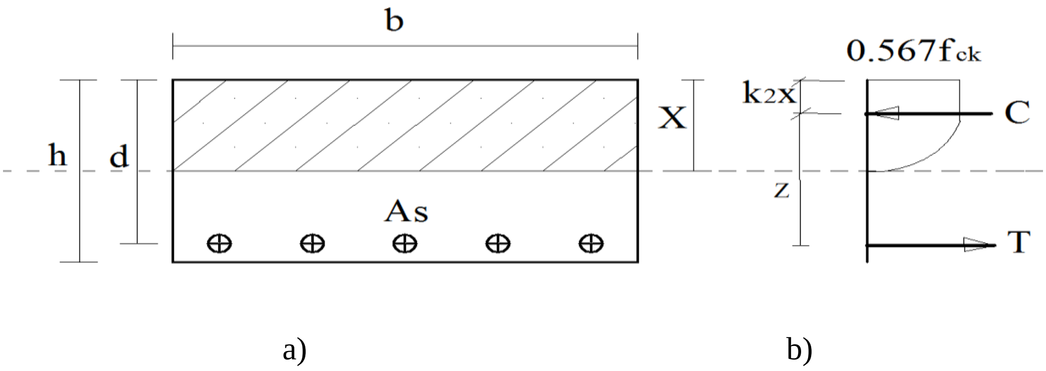


Figure 14: Singly reinforced section of shell, a) section, b) forces.

The depth of the neutral axis in terms of the applied ultimate design moment can be calculated from the following formula.

$$x = \left(1.202 - \sqrt{1.4462 - 5.2371K}\right)d \quad (56)$$

The lever arm z can be calculated as follows.

$$z = \left(0.5 + \sqrt{0.25 - 0.9063K}\right)d \leq 0.95d \quad (57)$$

Where

$K = M_d / f_{ck} bd^2$

x is depth of the neutral axis

z is lever arm

d is effective depth of the reinforced concrete section

According to literatures, it is a good practice to take lever arm (z) is not to exceed $0.95d$ in order to give a reasonable concrete area in compression and to avoid failure by premature crushing of weak concrete near the top of the section due to bending stress.

For a rectangular reinforced concrete section the Eurocode 2 part 1-1 of section 5 clause 5.6.3(2) limits the ultimate neutral axis depth (x_u) to $0.45d$ for concrete strength classes less than or equal to $C50/60$ to ensure that the design is for the under-reinforced case where failure is gradual [4]. And also it should be greater than $0.12d$ (calculated at lever arm (z) $0.95d$).

The reinforcement required to resist a given applied load can be calculated from the following equation.

$$A_s = \frac{M_d}{0.87 f_{yk} z} \geq A_{s,\min} \tag{58}$$

Where

A_s area of tension steel

$A_{s,\min}$ minimum area of reinforcement

M_d is applied ultimate design moment

f_{yk} is characteristic yield strength of concrete

z is lever arm

✓ **Singly reinforced section capacity checking**

After providing reinforcement a check on an existing section can be carried out by calculating the ultimate moment carrying capacity of concrete section and comparing this with the applied ultimate moment. The problem may be solved by considering the equilibrium of the internal forces.

After providing reinforcement a check on an existing section can be carried out by calculating the ultimate moment carrying capacity of the reinforced concrete section and comparing this with the applied ultimate moment. The problem may be solved by considering the equilibrium of the internal forces.

$$M_R^S = Tz = Cz \geq M_d \tag{59}$$

Where

M_R^S is singly reinforced section moment resistance capacity

T is steal tension force

$= 0.87 f_{yk} A_s$

C is concrete resultant compressive force

$= 0.459 f_{ck} b x$

z is lever arm

M_d is design moment

To say the section is safe the concrete section moment carrying capacity (M_R^S) should be greater than the applied design moment (M_d) [16].

B. Doubly reinforced section of shell design procedure ($d < d_{balance}$).

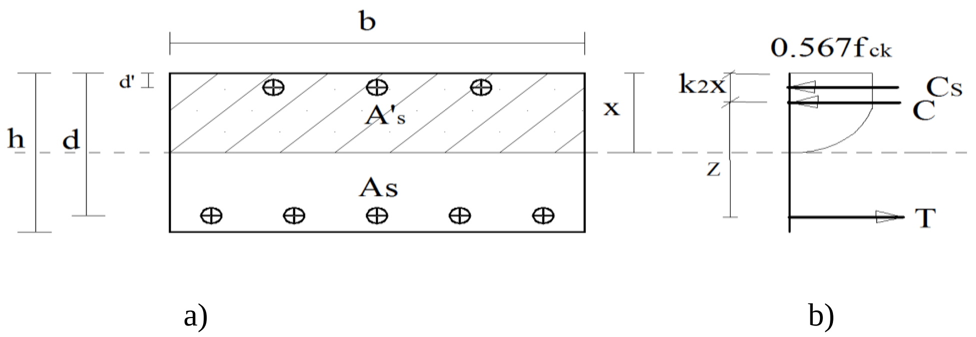


Figure 15: Doubly reinforced section of shell, a) section, b) forces.

If the concrete alone cannot resist the applied moment in compression, reinforcement can be provided in the compression zone. The design formulae for a doubly reinforced beam are derived using the rectangular-parabolic stress block.

If $d < d_{balance}$ (i.e. if ultimate moment carrying capacity of concrete (Mu) of a section is less than the applied ultimate design moment M_d) compression reinforcement should be provided in order to increase the moment carrying capacity of the section compression zone, the compression steel resists a moment $M_d - M_u$. The force in the compression steel is then

$$C_s = \frac{M_d - M_u}{d - d'} \qquad (60)$$

Where

C_s	is steel compression force
M_u	is ultimate moment carrying capacity of the section
	$= 0.168 f_{ck} bd^2$
M_d	is applied ultimate design moment
d	is effective depth of the reinforced concrete section
d'	is distance of compression reinforcement from edge of compression side

The area of compression steel is

$$A'_s = \frac{C_s}{0.87 f_{yk}} \le 0.04 A_c \qquad (61)$$

Where

A'_s	is area of compression steel
A_c	is section area of concrete

For internal equilibrium the section shown in the figure 15 above

$$T = C + C_s \qquad (62)$$

Where

T is steel tension force

C_s is steel compression force

$= 0.87 f_{yk} A'_s$

C is concrete resultant compressive force

$= 0.459 f_{ck} b * 0.45d = 0.20655 f_{ck} bd$

The area of tension steel is

$$A_s = \frac{T}{0.87 f_{yk}} \geq A_{s,\min} \tag{63}$$

- ✓ **Doubly reinforced section capacity checking**

$$M_R^D = Tz = Cz + C_s (d - d') \geq M_d \tag{64}$$

Where

M_R^D is doubly reinforced section moment resistance capacity

$T = 0.87 f_{yk} A_s$

$C_s = 0.87 f_{yk} A'_s$

$C = 0.20655 f_{ck} bd$

z is lever arm at $x = 0.45d$

$= 0.8128d$

d' is distance of compression reinforcement from edge of compression side

$cc + \phi_b / 2$

cc	is concrete clear cover to the longitudinal reinforcement ($30mm$)
ϕ_b	is longitudinal bar diameter

2.6.9.2.5. Ultimate limit state design formula for rectangular section of shell structure subjected to pure tension force (T_p)

$$A_s^T = \frac{1.15 T_P}{f_{yk}} \tag{65}$$

Where

A_s^T	is area of steel reinforcement for pure tension force
T_P	is pure tension force
f_{yk}	is characteristic yield strength of steel

✓ **Checking section capacity for tension strength**

$$\frac{A_s^T f_{yk}}{1.15} \geq T_P \tag{66}$$

2.6.9.2.6. Shear carrying capacity checking of a reinforced concrete section

It is inconvenient to use sheer reinforcement in the spherical dome roof because it is difficult to fix, impedes placing of concrete, and is inefficient in the use of steel. If no shear reinforcement is to be used, the ultimate shear stress must be limited to that permissible on the concrete alone. The thickness therefore should be at least sufficient to allow the ultimate shear force to be resisted by the concrete in combination with longitudinal reinforcement. According to Eurocode 2: section 6 clause 6.2.1 (3) in regions of the member where design shear force resulting from external loading (V_{Ed}) is less than or equal to the design shear resistance ($V_{Rd,c}$) of the reinforced concrete section resistance no calculated shear reinforcement is necessary. Therefore verification is needed for the shear capacity of the section that can varies not only with concrete grade but also with the effective depth of the section and the reinforcement ratio in the section with design shear force resulting from external loading (V_{Ed}) by using the following expression.

$$V_{Rd,c} = \left[C_{Rd,c} k (100 \rho_1 f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \right] bd \geq V_{Ed} \tag{67}$$

Where

$V_{Rd,c}$ is the design shear resistance in N with a minimum of $(V_{min} + k_1 \sigma_{cp}) b_w d$

$$V_{min} = 0.035 k^{\frac{3}{2}} f_{ck}^{\frac{1}{2}}$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c}$$

$$k_1 = 0.15$$

f_{ck} characteristic cylinder strength of concrete in MPa

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$ with d in mm

$$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$$

A_{s1} is the area of the tensile reinforcement, which extends $\geq l_{bd} + d$ beyond the section considered.

b_w is width of the rectangular reinforced concrete section [mm]

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd}$$

N_{Ed} is is design axial force in the cross-section due to loading [in N] ($N_{Ed} > 0$ for compression).

A_c is section area of concrete [mm2]

V_{Ed} is design shear force

2.6.9.3. Serviceability limit state

In addition to requiring structural strength and stability, roofing structures should be designed and constructed for serviceability. This may be achieved by considering the serviceability limit states. the requirement of "serviceability" would include the structural as well as functional criteria. Serviceability limit states considered in this study are limitation on stress, limitation on crack width and limitation on deflection.

2.6.9.3.1. Combinations of actions

- For this study the rare load combination is used for verification of serviceability limit state design. The load combination and values of factor of safety (γ_f) selected for the serviceability limit state design are formulated as follows:-

$$1.0G_K + 1.0Q_K \tag{68}$$

Where

G_K is characteristic dead load

Q_K is characteristic imposed load

2.6.9.3.2. Material and design stress used for serviceability limit state

In this serviceability limit state the same characteristic strength of reinforced concrete materials are used as used in ultimate limit state but the only difference is, for serviceability limit state partial factors for concrete (γ_c) and steel (γ_s) are 1.0 .

2.6.9.3.3. Minimum thickness of the spherical dome roof

For this study 100mm thickness of spherical dome (t_D) roof has been selected as a minimum thickness for practical or constructability case [13].

2.6.9.3.4. Crack

According to Eurocode 2 part 1-1 of section 7 clause 7.3.1(1), Cracking shall be limited to an extent that will not impair the proper functioning or durability of the structure or cause its appearance to be unacceptable. Therefore for Exposure Class XC4, the maximum allowable crack width is limited to 0.3mm.

The cracks width (w_k) which are developed in reinforced concrete members due to loadings should not be greater than the maximum allowable crack width (w_{k1}). The expression for crack width calculation given in Eurocode 2 part 1-1 section 7 clause 7.3.4(1) is:

$$w_{k1} = s_{r,\max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right) \tag{69}$$

Where

w_{k1} is design crack width

$s_{r,\max}$ is maximum crack spacing

ε_{sm} is mean strain in the reinforcement

$$= \frac{\sigma_s}{E_s}$$

σ_s is stress in tension reinforcement assuming a cracked section.

$= \dfrac{M_d}{zA_s}$ or $\dfrac{M_d}{(d - x/3)A_s}$ (Tension stress of steel due to design moment (M))

$= \dfrac{T}{A_s}$ (Tension stress of steel due to pure design tension force (T))

χ is neutral axis depth calculated for elastic condition using triangular stress block of doubly and singly reinforced cross section as shown in the figure 16 and 17, respectively by considering the cross-section of the concrete is cracked in the tension zone and taking the area moments of transformed section about the upper edge ($x = \frac{\sum(Ax)}{\sum A}$) .

- For singly reinforced section

$$= \frac{-\alpha_e A_s + \sqrt{\left[(\alpha_e A_s)^2 + 2b\alpha_e A_s d\right]}}{b}$$

- For doubly reinforced section

$$= \frac{-((\alpha_e - 1)A'_s + \alpha_e A_s) + \sqrt{\left[((\alpha_e - 1)A'_s + (\alpha_e A_s))^2 + 2b(0.5(\alpha_e - 1)A'_s d' + \alpha_e A_s d)\right]}}{b}$$

ε_{cm} is mean strain in the concrete between cracks

$$= \frac{k_t \left(\frac{f_{ct,eff}}{\rho_{p,eff}}\right)(1 + \alpha_e \rho_{p,eff})}{E_s}$$

b is width of the rectangular reinforced concrete section.

α_e is modular ratio

$$= E_s / E_{cm}$$

E_s is steel modules of elasticity (200GPa)

E_{cm} is concrete modules of elasticity (34GPa for $f_{ck} = 35MPa$)

$\rho_{p,eff}$ is effective reinforcement ratio

$$= A_s / A_{c,eff}$$

A_s is the reinforcement area within an effective tension area of concrete of $A_{c,eff}$ as shown in figure 18

k_t is factor that accounts for the duration of loading

= 0.4 (for long-term load)

Therefore, $\varepsilon_{sm} - \varepsilon_{cm}$ is can be calculated by the expression below

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t\left(\frac{f_{ct,eff}}{\rho_{p,eff}}\right)\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6\frac{\sigma_s}{E_s} \tag{70}$$

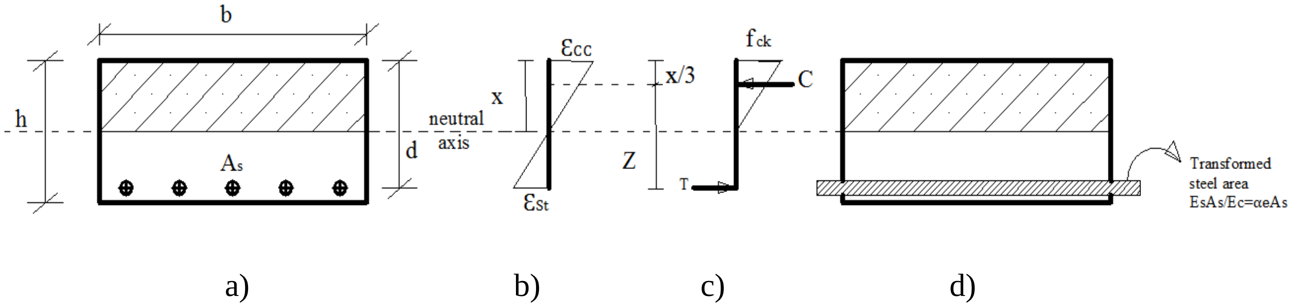


Figure 16: Singly reinforced section a) cross section b) strain c) stress d) transformed Area

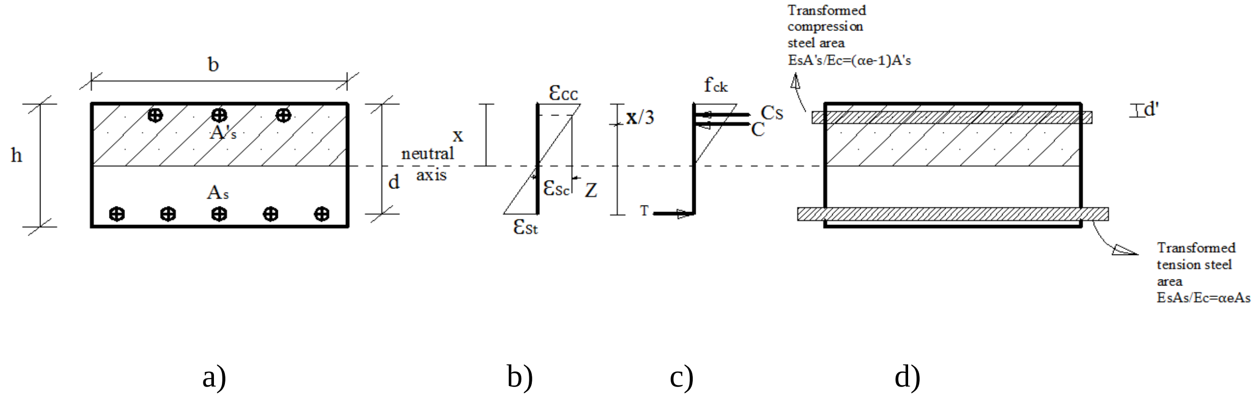


Figure 17: Doubly reinforced section a) cross section b) strain c) stress d) transformed Area

The maximum crack spacing ($s_{r,max}$) is given by the empirical formula:

$$s_{r,\max} = 3.4cc + 0.425k_b k_2 \phi / \rho_{p,eff} \tag{71}$$

Where

ϕ_b is longitudinal bar diameter in mm

cc is concrete clear cover to the longitudinal reinforcement

k_b is coefficient accounting for the bond properties of the reinforcement

$= 0.8$ (for high bond bars)

k_2 is a coefficient accounting for the nature of strain distribution

$= 0.5$ (for bending)

$= 1$ (for pure tension)

A. Allowable concrete compressive stress for serviceability limit state

According to Eurocode 2 part 1-1 of section 7 clause 7.2(2), to avoid longitudinal crack due to compressive design stress, allowable compressive stress is limited to $0.6 f_{ck}$.

B. Allowable steel tensile stress for serviceability limit state

According to Eurocode 2 part 1-1 of section 7 clause 7.2(5), to avoid inelastic strain, unacceptable cracking or deformation due to the design tensile stress allowable tensile stress should be less than $0.8 f_{yk}$.

C. Minimum area reinforcement

According to Eurocode 2 part 1-1 of section 7 clause 7.3.2(1), a minimum amount of bonded reinforcement ($A_{s,\min}$) is required to control cracking in areas where tension is expected. The recommended minimum reinforcement required for controlling thermal and shrinkage cracking with in the acceptable limit. This minimum reinforcement area can be calculated by the following equation

$$A_{s,\min} = k_c k f_{ct,eff} A_{ct} / f_{yk} \quad (72)$$

Where

$A_{s,\min}$ is minimum area of reinforcement

A_{ct} is area of concrete with tensile zone (area of concrete in tension just before the initiation of the first crack)

$= bh_{c,eff}$

b is width of the rectangular reinforced concrete section

$h_{c,eff}$ is depth of a concrete section for tensile zone as shown in the figure 18

(Depth of concrete in tension just before the initiation of the first crack)

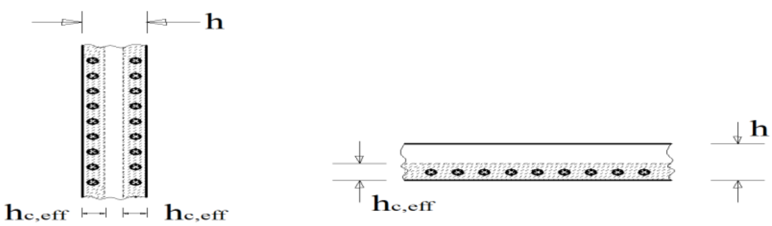
=Lesser of $\{h/2,\ 2.5(h-d),\ (h-x)/3\}$ (for bending moment)

=Lesser of $\{h/2,\ 2.5(h-d)\}$ (for pure tension)

$f_{ct,eff}$ is mean tensile strength of concrete effective at the time when the cracks may first be expected to occur: $f_{ct,eff} = f_{ctm} = 0.3 f_{ck}^{\frac{2}{3}}$.

k_c is stress distribution coefficient (1.0 for pure tension, 0.4 for flexure)

k is non-linear stress distribution coefficient- leading to a reduction in restraint force (1.0 for web less than 300mm deep, 0.65 for web greater than 800mm deep and interpolate for intermediate value).



a) b)

Figure 18: Effective concrete tension area a) member in pure tension b) member in compression

Eurocode 2 part 1-1 of section 9 gives additional rules for minimum and maximum area of reinforcement in order to prevent a brittle failure, wide cracks and also to resist forces arising from restrained actions. According to this section the minimum reinforcement for beam and wall can be expressed as follows:

- For beam and for wall in vertical direction subjected predominantly to out-of-plane bending, according to Eurocode 2 part 1-1 of section 9 clause 9.2.1.1(1), the area of longitudinal tension reinforcement should not be taken as less than $A_{s,\min}$.

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \text{, but not less than } 0.0013bd. \tag{73}$$

Where

b is width of the rectangular reinforced concrete section.

f_{ctm} is mean tensile strength of concrete

d is effective depth of the reinforced concrete section

.

- For wall according to Eurocode 2 part 1-1 of section 9 clause 9.5.2(2), The total amount of longitudinal reinforcement should not be less than $A_{sm,\min}$, if the wall is considered as compression member or column. The minimum longitudinal or vertical reinforcement ($A_{sm,\min}$) can be calculate according to equation 9.12N of Eurocode 2 part 1-1 as follows.

$$A_{sm,\min} = \frac{0.1N_{Ed}}{f_{yd}} \quad , \quad \text{or } 0.002A_c \text{ whichever is greater} \tag{74}$$

Where

$A_{sm,\min}$ is minimum meridional reinforcement

f_{yd} is the design yield strength of the reinforcement

N_{Ed} is the design axial compression force.

A_c is area of concrete section

- For wall according to Eurocode 2 part 1-1 of section 9 clause 9.6.3(1), The total amount of horizontal reinforcement should not be less than $A_{sc,\min}$. The recommended value is either 25% of the vertical reinforcement ($25\% A_{sm}$) or $0.001A_c$, whichever is greater.

D. Maximum area reinforcement

According to Eurocode 2 section 9.2.1.1(3) and 9.6.2(1), largely for practical need to achieve adequate compaction of the concrete around the reinforcement outside lap location, the maximum amount reinforcement is limited in a concrete section of beam and wall, respectively, to $A_{s,max}$ .The value of the maximum limit of vertical reinforcement for beam and wall tension or compression reinforcement is.

$$A_{s,\max} = 4\% A_c = 0.04 A_c \tag{75}$$

Where

$A_{s,\max}$ is maximum area reinforcement

A_c is section area of concrete

E. Minimum spacing of reinforcement

According to Eurocode 2 part 1-1 section 8 clause 8.2(1), the spacing of bars shall be such that the concrete can be placed and compacted satisfactorily for the development of adequate bond. The recommended spacing of bars for free concrete flow around reinforcement during construction should not be less than the maximum of (i) the bar diameter (ii) 20mm, or (iii) the maximum aggregate size plus 5mm.

F. Maximum spacing of reinforcement

In vertical direction of wall according to Eurocode 2 part 1-1 in section 9 clause 9.6.2(3) the spacing of reinforcement is limited to the maximum spacing ($S_{\max m}$) of 400mm or 3 times the thickness of the wall whichever is the lesser.

In horizontal direction of wall according to Eurocode 2 part 1-1 in section 9 clause 9.6.3(2) the spacing of reinforcement is limited to the maximum spacing ($S_{\max}$) of 400mm.

G. Minimum shear reinforcement area

According to Eurocode 2 part 1-1 section 9 clause 9.2.2, the recommended minimum shear reinforcement ratio of a beam can be calculated from the following equation.

$$\rho_{w,\min} = \left(0.08\sqrt{f_{ck}}\right) / f_{yk} \tag{76}$$

Where

$\rho_{w,\min}$ is minimum shear reinforcement ratio

f_{ck} is characteristic cylinder strength of concrete

f_{yk} is characteristic yield strength of steel

According to Eurocode 2 part 1-1 section 9 clause 9.2.2, the shear reinforcement spacing is recommended not to be taken greater than the maximum allowable shear reinforcement spacing ($s_{l,max}$). The maximum allowable shear reinforcement spacing ($s_{l,max}$) can be calculated as follows.

$$s_{l,\max} = 0.75d(1 + \cot\alpha) \tag{77}$$

Where

$s_{l,\max}$ is allowable maximum shear reinforcement spacing in beam

d is effective depth of the reinforced concrete section

α is the inclination of the shear reinforcement to the longitudinal axis of the beam (in this study the value of α has been taken as 90^0).

Therefore the minimum shear reinforcement area can be calculated based on the ratio of shear reinforcement equation given in equation 9.4 of Eurocode 2 part 1-1 section 9 clause 9.2.2(5) as follows:

$$A_{sv,\min} = \rho_{w,\min} s_{l,\max} b \sin\alpha \tag{78}$$

Where

$A_{sv,\min}$ is minimum shear reinforcement area with in length $s_{l,\max}$

$s_{l,\max}$ is allowable maximum shear reinforcement spacing in beam.

b is width of the rectangular reinforced concrete section

$\rho_{w,\min}$ is minimum shear reinforcement ratio

α is the inclination of the shear reinforcement to the longitudinal axis of the beam

2.6.9.3.5. Deflection

To insure an adequate degree of safety and serviceability of roof structure relevant limitation on development of excessive deflection should be considered. The deformation of a member or structure shall not be such that it adversely affects its proper functioning or appearance. According to Eurocode 2 section 7 clause 7.4.1(4), the appearance and general utility of the structure could be impaired when the calculated sag of a beam, slab or cantilever subjected to quasi-permanent loads exceeds span/250. Therefore the maximum horizontal deflection of the roof structural components should not be greater than span/250.

2.6.9.3.6. Buckling strength of spherical shells

Maximum compression stress should be allowed in spherical shells so that buckling does not take place. The theoretical value of permissible compressive stress can be calculated by using the equation-79 [13].

$$\sigma_{call} = \frac{E_c t_D}{R_D \sqrt{3}} \tag{79}$$

Where

σ_{call} is permissible compressive stress

Ec is young's modulus of elasticity of concrete

R_D is principal radius of spherical dome

t_D is thickness of spherical dome

CHAPTER THREE

3. METHODOLOGY

In this study it has been tried to investigate cost optimization of shell structure which has fixed topology (i.e. spherical dome shell roof structure stiffened by rectangular cross section ring beam and fixed supported at its base.) The materials and procedures that have been used to tackle the cost optimization problem of spherical dome shell roof structure with stiffening ring beam have been listed and discussed as follows:-

- Collecting and investigating relevant information from different materials.
- Developing computer program for structural shape optimization of spherical dome shell roof structure with stiffening ring beam.
- Investigation of the cost optimized design of the selected fixed topology roof structure by extracting and showing the cost optimized results from the developed program through design tables and design charts.
- Giving discussion and conclusion to the investigated results.
- Finally, this study has been completed by giving recommendation and developing further research idea based on this study.

3.1. Collecting and investigating relevant information

The most important stage to achieve objectives of any study is doing literature survey and in this research work much more time has been spent on this stage. This has been done through collecting and investigating useful data from data sources. The materials have been used as sources of data are well prepared and applicable analysis and design of shell structure books, structural optimization materials, related thesis works, dissertations, journals, design codes, computer programing books and programing tutorials. The relevant information that has been collected and investigated during literature survey includes theoretically available methods and formulas that are used to analyze the effect of axisymmetric loadings, structural design formulas and procedures used for detailing truncated spherical dome shell roof structure for the analysis results of loading effect, theories of structural optimization techniques used for shape optimization spherical dome shell roof structure and principles and methods behind Scilab computer programing software and language.

3.2. Developing a computer program for optimization of spherical dome roof with stiffening ring beam.

It is useful to develop computer program in order to create a program that exhibits the behavior of brute force optimization method to design spherical dome shell roof structure in least cost manner. This simplify the computation process, save time and energy, reduce errors, accessible, simple to handle. The program integrates the optimization method with the analysis and design of spherical dome shell roof structure. Computer programming (often shortened to programming or coding) is the process of designing, writing, testing, debugging/troubleshooting, and maintaining the source code of computer programs. This source code is written in a programming language.

A programming language is an artificial language designed to express computations that can be performed by a machine, particularly a computer. Programming languages can be used to create programs that control the behavior of a machine, to express algorithms precisely, or as a mode of human communication. A programming language is a notation for writing programs, which are specifications of a computation or algorithm. A programming language's surface form is known as its syntax. The syntax of a language describes the possible combinations of symbols that form a syntactically correct program. There are many developed softwares that can be used to develop programs. These are widely used to develop a program because these softwares have built-in functions and reduce additional effort to define some functions. For example, Scilab or Matlab has functions that can transpose and inverse a matrix. This reduces additional effort to define the transpose and inverse of a matrix.

In this study, it is intended to develop computer program for optimal design of spherical dome shell roof structure by using Scilab Programming language on Scilab software. Scilab is an open scientific software package for numerical computations providing a powerful computing environment for engineering and scientific applications. Scilab Programming language can be used in the fields of scientific, numerical and engineering fields to automate a series of commands. It has sophisticated data structures (including lists, polynomials, rational functions, linear systems...), an interpreter and a high level programming language. It is capable of interactive calculations as well as automation of computations through programming. Its ability to plot 2D and 3D graphs helps in visualizing the data we work with. All these make Scilab an excellent tool for teaching, especially those subjects that involve matrix operations. Further, the numerous toolboxes that are available for various specialized applications make it an important tool for research [9]. It is also used as atool of solving optimization problems. Scilab built-in optimization features mainly include optim function (nonlinear optimization), qpsolve function (quadratic optimization), lsqrsolve function (nonlinear least-square optimization), semidef function (semidefinite programming), optim_ga function (genetic algorithms), optim_sa function (simulated annealing) and lmisolver function (linear matrix inequalities) [12].

Problem solving with computer involves clearly define the problems, analyze the problem and formulate a method to solve it, describe the solution in the form of an algorithm, draw a flow chart of the algorithm, write the computer program (debugging), test the program and interpretation of the results.

In this study it is tried to investigate cost optimized design of spherical dome shell roof structure which covers areas starting from 78.5398 m^2 to 530.929 m^2. This roof structure has two structural components. These are ring beam and spherical dome with skylight. One of the success of roof structure designer is to design these components and construct them in least construction costs. So, to construct by combining these structural components as one huge roof structure in least cost manner needs to select the appropriate optimization method and those design variables which describes the shapes of each component and which influences the construction cost.

To challenge this problem, brute force optimization method has been selected. This method works by trying every possible combination values of design variables before deciding on the best solutions. Every possible combination of design variable can be obtained by dividing each design variable which exist between upper and lower limits in to n numbers and then trying to combine them as a solutions of the problem and calculate the cost of the structure after checking the constraints and then saving the design variables and the corresponding cost values. Finally design variables values selection has been done which gives the least cost of the structure.

For this study the selected design variables are dome thickness (t_D), dome principal radius (R_D) area of reinforcement at each selected points on the structure in both meridional (compressive (A'_{sm}) and tension side (A_{sm})) and tangential direction (A_{sc}), ring beam depth (h_r) and ring beam width (b_r). Based on constructability and codes these design variables have their own upper and lower limits. According to brute force method of optimization, if these design variables trail values combined each other and passes the behavioral and design constraints of the roof structure, calculation should be done in order to know the corresponding cost of the structure which is obtained from the cost formula which is formulated as a function of those design variables. After calculating roof construction costs again and again for all possible combination of design variables value, comparison for minimum cost should be done to select the best design variable values of the roof structure.

For a given base area (A_b) the optimization is started by assigning trial value of spherical dome radius (R_D) from its upper and lower bound, then trial thickness of the dome (t_D) is selected from upper and lower bounds of thickness limit of thin shell structure. Once the initial values of radius and thickness of spherical dome roof structure is assigned, the membrane and bending effect of the loads are calculated for the spherical dome roof structure by assigning width and depth of ring beam based on beam width and depth upper and lower limit values. At this stage the upper limit and lower limit value of reinforcement ratio for pure tension force, bending moment (compression side and tension side) and shear force based on the obtained initial geometries can be determined. Then assigning reinforcements to the necessary points in the structures based on the upper and lower limit and checking the side, behavioral and design constraints of the structures can be done. If they satisfies those constraints the costs of the roof structure is determined and the result is saved for comparing the value with other trials. After doing this for all possible trials comparing the costs and selecting the design variables which gives the list cost is done. Finally, after obtaining the best cost optimized spherical dome roof structures geometry for base areas from 78.5398 m^2 to 530.929 m^2, displaying the result with design charts and design tables is done.

3.2.1. Brute force optimization algorithm for optimized design of spherical dome shell roof structure

1. Input the area of the space to be covered by the spherical dome.
2. Generate trial values of dome and ring beam geometric variables.
3. Calculate maximum, and minimum reinforcement area for meridional bending moment (in both compression and tension side) and tangential pure tension force.
4. Input trail values of these reinforcement area at each selected point on the structure based on those upper and lower limit of reinforcement area.
5. Calculate behavioral constraints and check wither the trial value satisfies the allowable behavioral constraints or not. If they satisfies the constraints go to number 6 otherwise go to number 2.
6. Calculate design constraints and check wither they satisfy the design constraints or not. If they satisfies the constraint go to number 7 otherwise go to number 2.
7. Calculate the construction cost of the roof structure including cost of the concrete, reinforcement steel and formwork.
8. Store each cost value with the corresponding value of design variables.
9. Finally select the design variable which gives the least cost of the roof structure and display them.

3.2.2. Flow chart

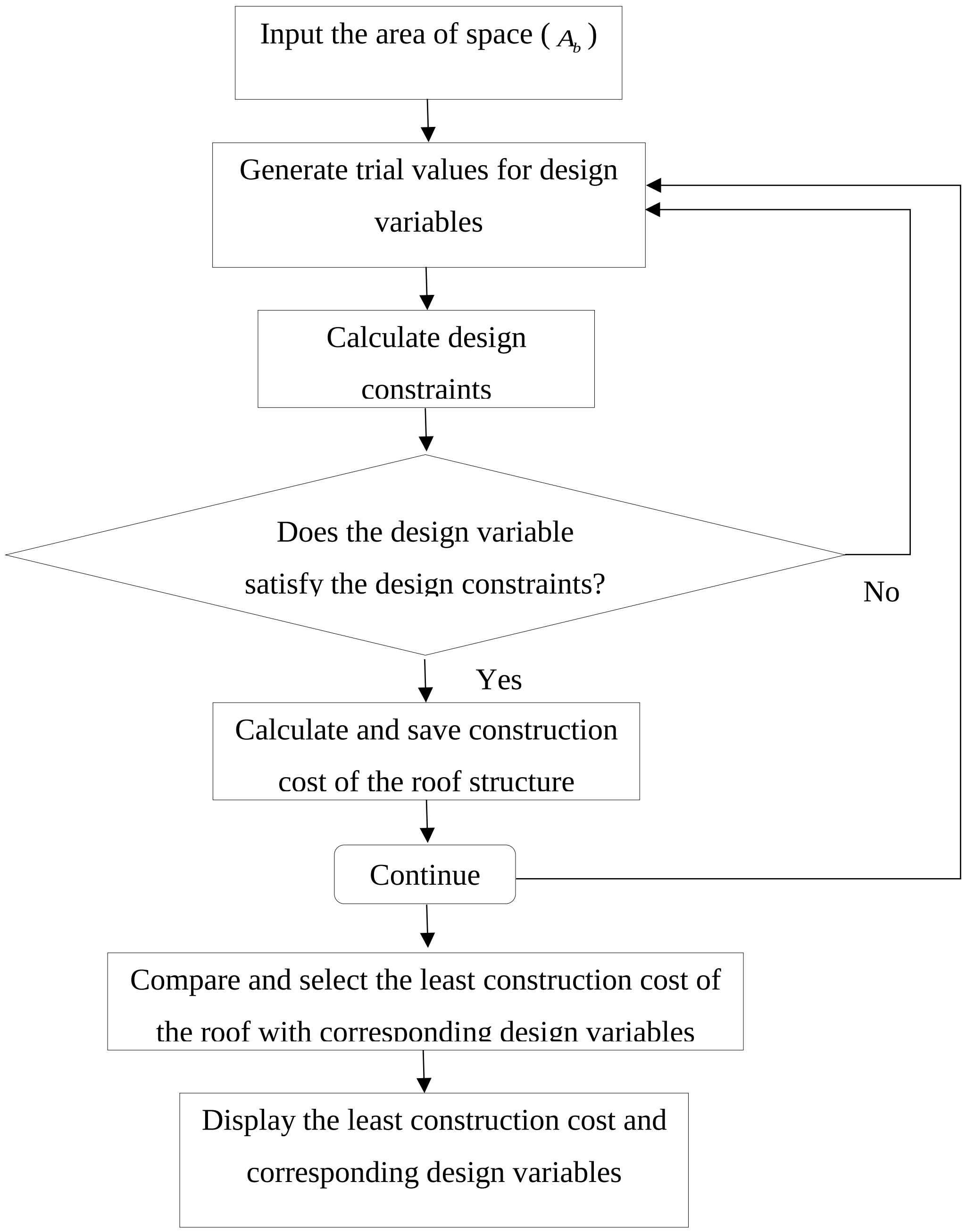


Figure 19: Flow chart for optimized design of roof structure

3.3. Investigation of the cost optimized design of the selected fixed topology roof structure.

After the optimization program has been developed the result is displayed in the form of design chart and design table. This enable us to investigate the design variable behavior in order to give the optimum design. Under this the followings items are studied in order to draw respectable conclusions.

1. The concrete cost contribution to the total cost.
2. The reinforcement steel cost contribution to the total cost.
3. The form work cost contribution to the total cost.
4. The spherical dome cost contribution to the total cost.
5. The ring beam cost contribution to the total cost. and,
6. The effect of doubly and singly reinforced section design on the total cost of the structure.

3.4. Giving discussion and conclusion for investigated results.

Based on the result obtained from the optimization process clear and detail discussion have been given and conclusions have been drawn about the study.

3.5. Giving recommendation and developing further research idea based on the study.

Finally, this study has been completed by giving recommendation and developing further research idea based on the study results.

CHAPTER FOUR

4. OPTIMIZATION FORMULATION

Ideally, Cost optimization of civil engineering structure problem should be formulated in terms of the life-cycle cost, which includes the cost of materials, erection, maintenance, and disassembling the structure at the end of its life cycle. But in cost optimization of concrete structure at least three different cost item should be considered in the objective function i.e. cost of concrete, steel and the form work [18]. This objective function can be affected by different design variables i.e. topology of the structure, types of materials property, cross sectional and geometrical shape design variables. The values of these design variables can be chosen as best design variables if they give the minimum value of objective function of cost by satisfying structural analysis, design, constructability and maintainability of structure constraints.

4.1. Optimized design formulation of spherical dome shell roof structure for minimum cost

For this study a fixed structural topology of reinforced concrete spherical dome shell structure stiffened by rectangular cross-section ring beam at its base which can be designed as roof structure has been selected as shown in the figure 20. The topology that has been selected as a roof structure has been designed for areas starting from 78.5398 m^2 to 530.929 m^2 (i.e. for spherical dome base radius (R_b) between 5m and 13m). And the reinforced concrete materials property used are also fixed i.e. Concrete grade C35 and Steel grade S460. The design variables are the cross section and geometric shape variables of the structural components of the selected roof structure topology. The constraints of this optimization problem includes behavioral, design, constructability and maintainability of the roof structure. The analysis constraints have been defined according to force method of thin shell structure analysis and the design constraints are defined according to Eurocode of standards.

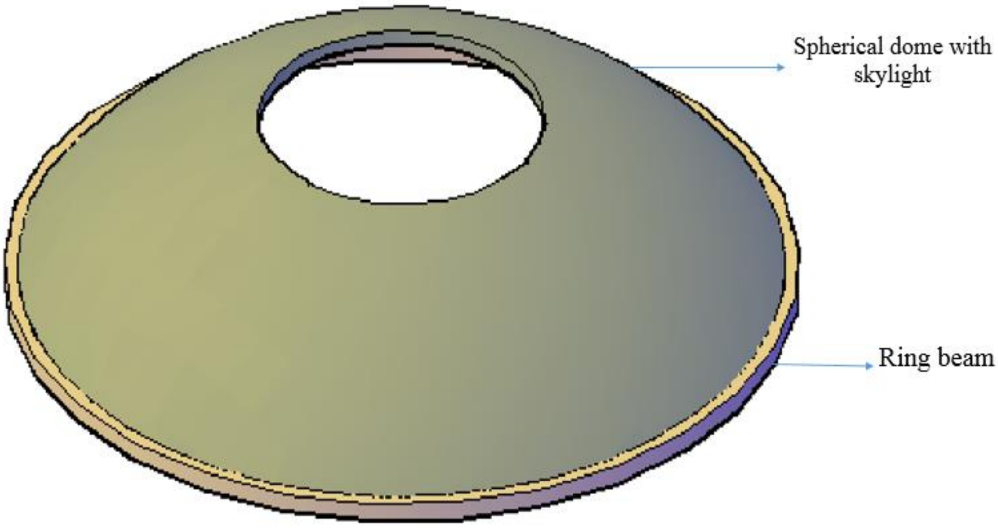


Figure 20: Structural topology of spherical dome shell roof structure.

Cost of concrete, steel and form work are available per unit volume (m^3), per unit weight (kg) and per unit area (m^2), respectively. Current cost of concrete per unit volume (C_s^s), steel per Kg (C_{su}) and formwork (C_f^d) are 2200.00 birr/m^3, 27.00 birr/Kg and 200.00 birr/m^2, respectively [7]. Cost optimization formulation for each member of the roof can be formulated as follows.

4.1.1. Cost objective function formulation for dome

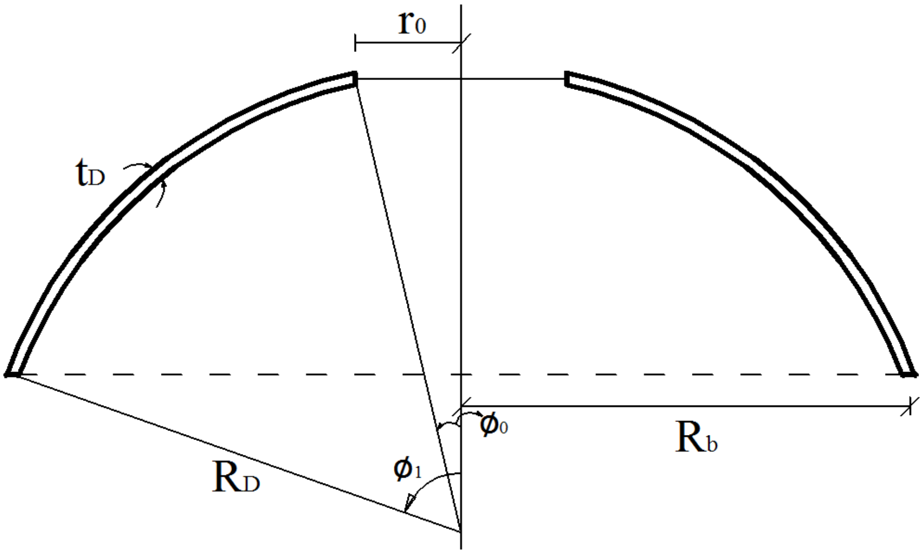


Figure 21: Dome geometric shape and cross-sectional design variables.

The cost of the spherical dome roof can be formulated as follows.

$$C_m^d = C_c^d + C_s^d + C_{fw}^d \quad (80)$$

Where

C_m^d is the total material cost of the spherical dome.

C_c^d is concrete cost of the spherical dome.

C_s^d is reinforced steel cost of the spherical dome.

C_{fw}^d is formwork cost of the spherical dome.

4.1.1.1. Concrete cost of the spherical dome (C_c^d)

Concrete cost of the spherical dome (C_c^d) can be calculated as follows

$$C_c^d = C_s^s V_c^d \quad (81)$$

Where

C_s^s is the cost of concrete per unit volume.

V_c^d is total volume of concrete for spherical dome.

Total volume of spherical dome (V_c^d) can be calculated as follows:

$$V_c^d = 2\pi\left(R_D^2\right)\left(\cos\phi_0 - \cos\phi_1\right)t_D \quad (82)$$

Where

R_D is principal radius of spherical dome

ϕ_1 is base angle of spherical dome

$$= \sin^{-1}\left(\frac{R_b}{R_D}\right)$$

ϕ_0 is pitch angle of spherical dome

$$= \sin^{-1}\left(\frac{r_0}{R_D}\right)$$

r_0 is pitch radius of spherical dome

t_D is thickness of spherical dome.

4.1.1.2. Reinforcement steel cost of the spherical dome (C_S^d)

$$C_S^d = 7850\frac{Kg}{m^3}V_{St}^d C_{su} \tag{83}$$

Where

V_{St}^d is total volume of steel of reinforcement for dome structure.

C_{su} is cost of steel per Kg .

Total volume of steel of reinforcement for dome structure can be calculated as the summation of Total volume of reinforcement in meridional direction of dome (V_{mst}^d) and Total volume of reinforcement in circumferential direction of dome (V_{cst}^d) as follows.

$$V_{st}^d = V_{mst}^d + V_{cst}^d \tag{84}$$

Where

V_{mst}^d is total volume of reinforcement in meridional direction of dome

V_{cst}^d is total volume of reinforcement in circumferential direction of dome

Meridian reinforcements in meridional direction of the dome are provided at step angle (ϕ_s) difference starting from skylight to bottom of spherical dome. It includes the tensile and the compressive side reinforcement areas. The volume of the reinforcement for that spherical dome meridional angle increased by step angle can be calculated first by summing the tensile and compressive side area of reinforcement (i.e. calculated for unit circumferential width and for spherical dome meridional angle ϕ_i moment) then multiplied by spherical dome step angle meridional length and parallel circle perimeter (p_i) of the spherical dome at spherical dome meridional angle ϕ_i. Then total volume of reinforcement in meridional direction of dome (V_{mst}^d) becomes the summation of volumes of reinforcement that are calculated at each point of spherical dome meridional angle (ϕ_i) increased by step angle (ϕ_s).

$$V_{mst}^d = \sum_{i=1}^{n}\left(\left(A_{smi} + A'_{smi}\right) * l_v * p_i\right) \qquad i = 1,2\ldots\ \ldots\ \ldots, n \tag{85}$$

Where

A_{smi} is meridional tensile side reinforcement area at spherical dome meridional angle of ϕ_i.

A'_{smi} is meridional compressive side reinforcement area at spherical dome meridional angle of ϕ_i.

l_v is meridional length of spherical dome for a meridional step angle (ϕ_s)

p_i is parallel circle perimeter of the spherical dome at spherical dome meridional angle of ϕ_i.

Meridional length (l_v) for meridional step angle (ϕ_s) is equal to

$$l_v = \frac{\phi_s \pi R_D}{180} \tag{86}$$

Where

ϕ_s is step angle in meridional direction of spherical dome

R_D is principal radius of spherical dome

l_v is meridional length of spherical dome for a meridional step angle (ϕ_s)

Parallel circle perimeter (p_i) of the spherical dome at spherical dome meridional angle of ϕ_i can be calculated as follows.

$$p_i = 2\pi r_i = 2\pi R_D \sin\phi_i$$

(87)

Where

p_i is parallel circle perimeter of the spherical dome at spherical dome meridional angle of ϕ_i .

r_i is parallel circle radius of the spherical dome at spherical dome meridional angle of ϕ_i .

R_D is spherical dome principal radius.

ϕ_i is the i^{th} spherical dome meridional angle of

Circumferential reinforcements in circumferential direction of the dome are also provided at step angle (ϕ_s) difference starting from skylight to bottom of spherical dome. The circumferential volume of reinforcement (i.e. calculated for meridional length (l_v) of spherical dome for a meridional step angle (ϕ_s) as width and for spherical dome meridional angle of ϕ_i tension force) can be calculated as circumferential reinforcement area that is calculated for each spherical dome meridional angle ϕ_i multiplied by meridional length (l_v) of spherical dome for a meridional step angle (ϕ_s) and parallel circle perimeter (p_i) of the spherical dome at spherical dome meridional angle ϕ_i. Then total volume of reinforcement in Circumferential direction of dome (V_{cst}^d) becomes the summation of volumes of circumferential reinforcement that are calculated at each point of spherical dome meridional angle (ϕ_i) increased by step angle (ϕ_s).

$$V_{cst}^d = \sum_{i=1}^{n} \left(A_{sci} * l_v * pi\right) \qquad i = 1,2, \dots \dots \dots, n \tag{88}$$

Where

A_{sci} is circumferential reinforcement area at spherical dome meridional angle ϕ_i

l_v is meridional length of spherical dome for a meridional step angle (ϕ_s)

pi is parallel circle perimeter of the spherical dome at spherical dome meridional angle of ϕ_i.

4.1.1.3. Formwork cost of the spherical dome (C_{fw}^d)

Formwork cost of the spherical dome (C_{fw}^d) can be calculated as follows

C_{fw}^d =Surface area of spherical dome × cost of double face of form work

$$= 2\pi R_D^2 \left(\cos\phi_0 - \cos\phi_1\right) C_f^d \tag{89}$$

Where

C_{fw}^d is formwork cost of the spherical dome

ϕ_1 is base angle of spherical dome

ϕ_0 is pitch angle of spherical dome

R_D is principal radius of spherical dome

C_f is formwork cost of per m^2

4.1.2. Design variables for dome

The variables used for obtaining the optimum design of spherical dome are the followings:

1. Spherical dome principal radius (R_D).
2. Spherical dome thickness (t_D).
3. Spherical dome reinforcement (A_{st}^d)

4.1.3. Constraints for dome optimization

4.1.3.1. Side constraints

1. Thickness of dome cover

Thin shell structure analysis principle is workable only for shell structures that are fulfills thin shell structure criteria. One of the criteria is the limit of thickness to radius ratio for thin shells.

- Based on thin shell structure thickness to one of its shortest radius ratio (t_D/R_D), the thickness should be between $R_D/1000$ and $R_D/20$.
- For this study for constructability case $100mm$ minimum thickness has been selected.

Therefore: -

Lower bound of thickness (t_D^{lb}) = $R_D/1000$ or 100mm, whichever is the greater one.

Upper bound of thickness (t_D^{ub}) = $R_D/20$

2. Edge angle of dome cover (ϕ_1)

Practically, the Geckeler's approximate edge effect of bending moment analysis can be used for shell of revolution of spherical dome shell if the angle of the shell edge $\phi_1 > 35^0$. And for this thesis the maximum dome edge angle (ϕ_1) has been taken as 90^0.

Therefore: -

Lower bound of dome radius (R_D^{lb}) = R_b

Upper bound of dome edge radius (R_D^{ub}) = $1.75R_b$

4.1.3.2. Behavioral constraints

1. For stability the buckling stress should be below the critical buckling stress.

   i.e. $\sigma_c = N_\phi / t_D \leq \sigma_{call}$ (90)

   $$\sigma_{call} = 0.25\left(\frac{E_c t_D}{R_D \sqrt{3}}\right) \quad (91)$$

   Where

   N_ϕ is internal membrane force in meridional direction

   σ_{call} is permissible compressive stress

   σ_c is design compressive stress

   E is young's modulus of elasticity

   R_D is principal radius of spherical dome

   t_D is thickness of spherical dome

2. The design compressive force should not be beyond allowable compression force of concrete ($N_{\phi all}$).

   i.e. $N_\phi \leq N_{\phi all} = 0.6 f_{ck} t_D$ *and* $N_{\theta c} \leq N_{\theta call} = 0.6 f_{ck} t_D$ (92)

Where

N_ϕ is internal membrane force in meridional direction

$N_{\theta c}$ is tangential compressive force

f_{ck} is characteristic cylinder strength of concrete(the 28 day cylinder strength of concrete in N/mm^2).

3. Steel direct or flexural tension stresses should be less than allowable steel direct or flexural tension stresses $\sigma_{tallw} = 0.8 f_{yk}$.

i.e. $N_\theta \leq 0.8 f_{yk} t_D \quad and \;\; \sigma_{ft} \leq 0.8 f_{yk}$ (93)

Where

f_{yk} is characteristic yield strength of steel

σ_{ft} is steel flexural tension stresses.

4. The actual applied design moment M_d will generally be less than ultimate moment carrying capacity of the section Mu. The maximum permissible value of ultimate moment which can be applied to the section without using compression reinforcement (M_u) is.

i.e. $M_u = 0.168 f_{ck} b d^2 \geq M_d$ (94)

Where

f_{ck} is characteristics concrete strength(the 28 day cylinder strength of concrete in N/mm^2)

b is width of the rectangular reinforced concrete section.

d is effective depth of the reinforced concrete section

M_u is ultimate moment carrying capacity of the section

M_d is applied ultimate design moment.

5. If no shear reinforcement is to be used, the ultimate shear stress must be limited to that permissible on the concrete alone. Therefore the shear capacity of the concrete should be greater than the design shear.

$$V_{Rd,c} = \left[C_{Rd,c} k (100 \rho_1 f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \right] bd \geq V_{Ed} \tag{95}$$

Where

$V_{Rd,c}$ is the design shear resistance in N

$C_{Rd,c} = \dfrac{0.18}{\gamma_c} = 0.12$

f_{ck} in MPa

$k_1 = 0.15$

$k = 1 + \sqrt{\dfrac{200}{d}} \leq 2.0$ with d in mm

$\rho_1 = \dfrac{A_{s1}}{b_w d} \leq 0.02$

A_{s1} is the area of the tensile reinforcement, which extends $\geq l_{bd} + d$ beyond the section considered.

b is width of the rectangular reinforced concrete section.

$\sigma_{cp} = \dfrac{N_{Ed}}{A_c} < 0.2 f_{cd}$

N_{Ed} is the axial force in the cross-section due to loading [in N] ($N_{Ed} > 0$ for compression).

A_c is the area of concrete cross section [mm2]

6. According to RD. Anchor the lateral deflection is likely to be no more than 30 mm and According to Eurocode 2 part 1-1 section 7 clause 7.4.1(4) of 2004, the deflection value should be limited to $span/250$.

Therefore:-

Lower bound of dome deflection (δ_{sh}^{lb}) $= -span/250 \text{ or } -30mm$, whichever is the greater one.

Upper bound of dome deflection (δ_{sh}^{ub}) $= span/250 \text{ or } 30mm$, whichever is the lesser one.

- ♦ According to Eurocode 2 part 1-1 of section 7 clause 7.3(1), Cracking shall be limited to an extent that will not impair the proper functioning or durability of the structure or cause its appearance to be unacceptable. Therefore for Exposure Class XC4, the maximum allowable crack width is limited to 0.3mm.

7. Area of reinforcement.
   - ➢ For meridional direction

- ♦ Tension side (A_{sm})

Lower boundary area of meridional tension side reinforcement (A_{sm}^{dlb}) = $A_{sm,\min}^{d}$

Upper boundary area of meridional tension side reinforcement (A_{sm}^{dub}) = $A_{sm,\max}^{d}$

Where

$A_{sm,\min}^{d}$ is minimum area of meridional tension side reinforcement

$$= \max\left\{k_c k f_{ct,eff} A_{ct} / f_{yk}, 0.26\frac{f_{ctm}}{f_{yk}} b_t d, 0.0013 b_t d, \frac{0.1 N_{Ed}}{f_{yd}}\right\}$$

$A^{d}_{sm,\max}$ is maximum area of meridional tension side reinforcement

$= 0.04A_c$

- Compression side (A'_{sm})

Lower boundary area of meridional compression side reinforcement (A^{dlb}_{smc}) = $A^{d}_{smc,\min}$

Upper boundary area of meridional compression side reinforcement (A^{dub}_{smc}) = $A^{d}_{smc,\max}$

Where

$A^{d}_{smc,\min}$ is minimum area of meridional compression side reinforcement

$= 0$

$A^{d}_{smc,\max}$ is maximum area of meridional compression side reinforcement

$= 0.04A_c - A_{sm}$

- For circumferential (tangential) direction (A_{sc})

Lower boundary area of circumferential reinforcement (A^{dlb}_{sc}) = $A^{d}_{sc,\min}$

Upper boundary area of circumferential reinforcement (A^{dub}_{sc}) = $A^{d}_{sc,\max}$

Where

$A^{d}_{sc,\min}$ is maximum area of circumferential tension reinforcement of dome

$$= \max\left\{ k_c k f_{ct,eff} A_{ct} / f_{yk} \,, 0.26\frac{f_{ctm}}{f_{yk}} b_t d \,, 25\% A_{s,v} \,, 0.001 A_c \right\}$$

$A^{d}_{sc,\max}$ is maximum area of circumferential tension reinforcement of dome

$= 0.04A_c$

8. Reinforcement bar spacing

➢ For meridional direction reinforcement

Lower bound of bar spacing for meridional reinforcement (S_m^{dlb})

$= \max\{\phi, 20mm, aggegatessize + 5mm\}$

Upper bound of bar spacing for meridional reinforcement (S_m^{dub}) $= \min\{3t_D, 400mm\}$

➢ For circumferential direction

Lower bound of bar spacing for circumferential reinforcement (S_c^{dlb})

$= \max\{\phi, 20mm, aggegatessize + 5mm\}$

Upper bound of bar spacing for circumferential reinforcement (S_c^{dub}) $= 400mm$

9. For this study, for durability of the structure a nominal minimum cover of $30mm$ has been selected According to Eurocode 2 part1-1 of section 4 clause 4.4.1.1.

4.1.4. Cost objective function formulation for ring beam

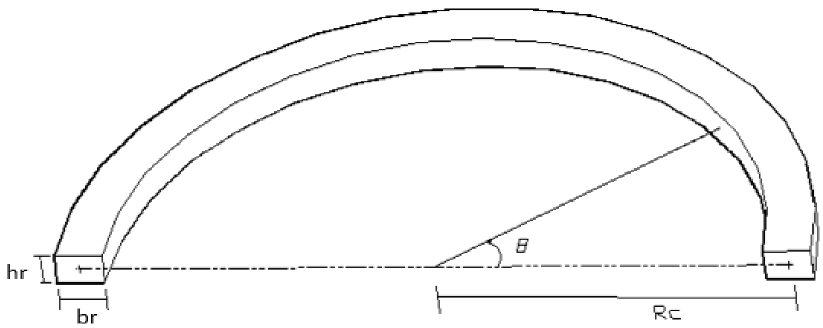


Figure 22: Ring beam geometric shape and cross-sectional design variables.

The cost of the rectangular cross section ring beam can be formulated as follows.

$$C_m^r = C_c^r + C_s^r + C_{fw}^r \quad (96)$$

Where

C_m^r is the total material cost of the ring beam.

C_c^r is concrete cost of the ring beam.

C_s^r is reinforced steel cost of the ring beam.

C_{fw}^r is formwork cost of the ring beam.

4.1.4.1. Concrete cost of the ring beam (C_c^r)

Concrete cost of the ring beam (C_c^r) can be calculated as follows

$$C_c^r = C_s^s V_c^r \tag{97}$$

Where

C_s^s is the cost of concrete per unit volume.

V_c^r is total volume of concrete for ring beam.

Total volume of ring beam (V_c^r) can be calculated as follows:

$$V_c^r = 2\pi r^r b_r h_r \tag{98}$$

Where

r^r is radius of ring beam middle surface.

h_r is overall depth of ring beam.

b_r is width of ring beam.

4.1.4.2. Reinforcement steel cost of the ring beam (C_s^r)

Reinforcement steel cost of the ring beam (C_s^r) can be calculated as follows

$$C_s^r = C_{st}^r + C_{ss}^r \tag{99}$$

Where

C_{st}^r is cost of tension reinforcement ring beam.

C_{ss}^r is cost of shear reinforcement ring beam.

Tension reinforcement (C_{st}^r) of ring beam can be calculated as follows

$$C_{st}^r = 7850 \frac{Kg}{m^3} \left(2\pi r^r A_{st}^r\right) C_{su} \tag{100}$$

Where

r^r is radius of ring beam middle surface.

A_{st}^r is tension reinforcement area of ring beam.

C_{su} is cost of steel per Kg

Shear reinforcement cost (C_{ss}^r) of ring beam can be calculated as follows

$$C_{ss}^r = 7850 \frac{Kg}{m^3} C_{su} \left(A_{sv,\min} L_{si} n_s\right) \tag{101}$$

Where

$A_{sv,\min}$ is area of minimum shear reinforcement within length $s_{l,\max}$

$= \rho_{w,\min} b_r \sin\alpha$

$\rho_{w,\min}$ is minimum shear reinforcement ratio

$$= \frac{0.08\sqrt{f_{ck}}}{f_{yk}}$$

$s_{l,\max}$ is allowable maximum shear reinforcement spacing in beam

$$= 0.75d(1+\cot\alpha)$$

L_{si} is length of a single shear reinforcement.

$$= 2(hr + br - 2cc)$$

cc is concrete clear cover to the longitudinal reinforcement

n_s is number of shear reinforcement provided for ring beam

$$= (2\pi R_b)/s_{l,\max}$$

α is the inclination of the shear reinforcement to the longitudinal axis of the beam

$$= 90^0$$

C_{su} is cost of steel per Kg

4.1.4.3. Formwork cost of the ring beam (C^r_{fw})

Formwork cost of the ring beam (C^r_{fw}) can be calculated as follows

$$C^r_{fw} = 4\pi r^r hr C_f \quad (102)$$

Where

C_f is formwork cost of per m^2

r^r is radius of ring beam middle surface.

h_r is overall depth of ring beam.

4.1.5. Design variables for ring beam

The variables used for obtaining the optimum design of ring beam are the followings:

1. Ring beam width (b_r).
2. Ring beam depth (h_r).
3. Ring beam reinforcement (A_{st}^r)

4.1.6. Constraints for ring beam optimization

4.1.6.1. Side constraints

1. Width and height of ring beam

- According to Eurocode 2 section 5 clause 5.3.1(3), a beam is a member for which the span is not less than 3 times the overall section depth. Width dimension should less than 5 times the overall beam depth.
- According to P.C. Varghese (2010), all dimensions of the ring beam should be not less than twice the thickness of the shell.

Therefore: -

Lower bound of beam depth (h_r^{lb}) $= 2t_D$

Upper bound of beam depth (h_r^{ub}) $= 2r^r/3$

Lower bound of beam width (b_r^{lb}) $= 2t_D$

Upper bound of beam width (b_r^{ub}) $= 5h_r$

Where

r^r is radius of ring beam middle surface.

$= R_b$ (Parallel circle radius of spherical dome base).

t_D is thickness of spherical dome.

4.1.6.2. Behavioral constraints

1. Outward deflection of ring beam generally should be limited to $span/250$ or $30mm$.

Therefore: -

Lower bound of beam deflection (δ_r^{lb}) $= -span/250\ or\ -30mm$ whichever is the greater one.

Upper bound of beam deflection (δ_r^{ub}) $= span/250\ or\ 30mm$, whichever is the greater one.

2. Steel direct stresses should be less than allowable steel direct or flexural tension stresses ($\sigma_{tallw} = 0.8 f_{yk}$)
3. The design compressive force should not be beyond allowable compression force of concrete ($N_{\varphi_{all}}$).

   $$i.e. \quad N_\phi \le N_{\phi all} = 0.6 f_{ck} b_r \quad and \quad N_{\theta c} \le N_{\theta call} = 0.6 f_{ck} b_r \tag{103}$$

   Where

   $N_{\phi all}$ is allowable compression force of concrete

   N_ϕ is internal membrane force in meridional direction

   b_r is width of ring beam.

   f_{ck} is characteristic strength of concrete (the 28 day cylinder strength of concrete in N/mm^2).

4. According to Eurocode 2 part 1-1 of section 7 clause 7.3(1), Cracking shall be limited to an extent that will not impair the proper functioning or durability of the structure or cause its appearance to be unacceptable. Therefore for Exposure Class XC4, the maximum allowable crack width is limited to 0.3mm.

5. Tension reinforcement Area (A_{st}^{r}).

- For ring beam circumferential (tangential) tension reinforcement area

Lower boundary area of circumferential tension reinforcement (A_{st}^{rlb}) = $A_{st,\min}^{r}$

Upper boundary area of circumferential tension reinforcement (A_{st}^{ulb}) = $A_{st,\max}^{r}$

Where

$A_{st,\min}^{r}$ is minimum area of circumferential tension reinforcement of ring beam

$$= \max\left\{ k_c k f_{ct,eff} A_{ct} / f_{yk} \text{ },0.26 \frac{f_{ctm}}{f_{yk}} h_r d_h^r ,0.0013 h_r d_h^r \right\}$$

$A_{st,\max}^{r}$ is maximum area of circumferential tension reinforcement of ring beam

$$= 4\% A_c$$

6. Shear reinforcement Area (A_{sv}) of ring beam.

Minimum shear reinforcement area should be provided for the ring beam.

Therefore:-

$$A_{sv} = A_{sv,\min} = \rho_{w,\min} s_{l,\max} b_w \sin\alpha \tag{104}$$

7. Shear reinforcement spacing for ring beam.

Shear reinforcement spacing of ring beam should not be greater than maximum shear reinforcement spacing.

Therefore:-

$$s_l = s_{l,\max} = 0.75 d(1 + \cot\alpha) \tag{105}$$

8. For this study, for durability of the structure a nominal minimum cover of $30mm$ has been selected According to Eurocode 2 part1-1 of section 4 clause 4.4.1.1.

CHAPTER FIVE

5. RESULTS

5.1. Design Tables

The necessary outputs obtained from the written program for spherical dome shell roof base radius between 5m and 13m are provided in Table 4 and Table 5 and design chart 1 up to design chart 9 to show the whole process of computation involved in the program.

Table 4:The optimized geometrical and cross sectional dimensions of the spherical dome shell roof structure components for base radius from 5m up to 13 m.

Base area (A_b in m^2)	Base radius (R_b in m)	Principal radius (R_D in m)	Thickness (t_D in m)	Ring beam depth (h_r in m)	Ring beam width (b_r in m)	Total Area of formwork (A_{fw} inm^2)
78.5398	5	6.75	0.1350	0.2700	0.2700	101.1350
113.0973	6	8.5	0.1310	0.2619	0.2619	142.4288
153.9380	7	10.25	0.1464	0.2929	0.2929	203.6740
201.0619	8	12	0.1500	0.3000	0.3000	260.5558
254.4690	9	13.75	0.1528	0.3056	0.3056	324.3818
314.1593	10	15.5	0.1550	0.3100	0.3100	395.1360
380.1327	11	15.25	0.1386	0.2773	0.2773	487.4947
452.3893	12	17	0.1417	0.2833	0.2833	572.3535
530.9291	13	20.75	0.1596	0.3192	0.3192	648.8957

Table 5: The optimized reinforcement areas of spherical dome roofs structure components for base radius from 5m up to 13 m.

Base radius (R_D in m)	Meridional tension side reinforcement for singly reinforced section ($A_{sm,srs}$ in m²)	Circumferential tension reinforcement for singly reinforced section ($A_{sc,srs}$ in m²)	Ring beam longitudinal reinforcements (A_{st}^{r} in m²)	Ring beam minimum shear reinforcement area in length $s_{l,\max}$ (A_{sv} in m²)
5	0.0002700	0.0002513	0.0020828	0.0000003
6	0.0002619	0.0002513	0.0000776	0.0000003
7	0.0002929	0.0002513	0.0030986	0.0000003
8	0.0003000	0.0002513	0.0031037	0.0000003
9	0.0003056	0.0002513	0.0031079	0.0000003
10	0.0003100	0.0002513	0.0001112	0.0000003
11	0.0002773	0.0002513	0.0020877	0.0000003
12	0.0002833	0.0002513	0.0030918	0.0000003
13	0.0003192	0.0002513	0.0001184	0.0000003

5.2. Design Charts

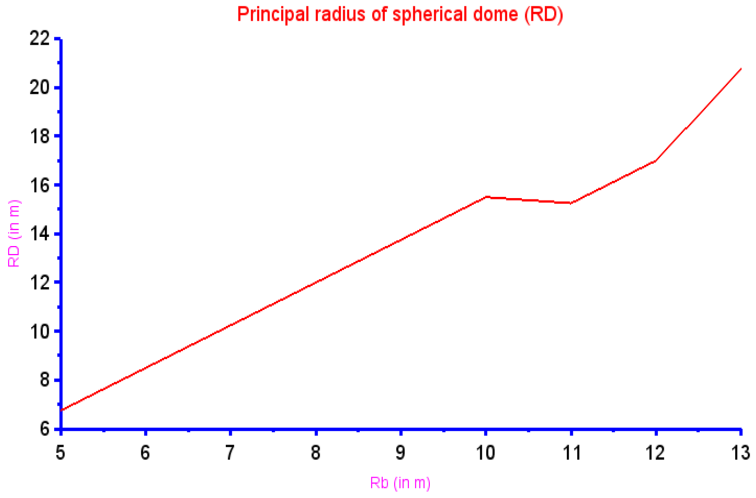


Chart 1: Base radius (R_b) versus principal radius of spherical dome (R_D)

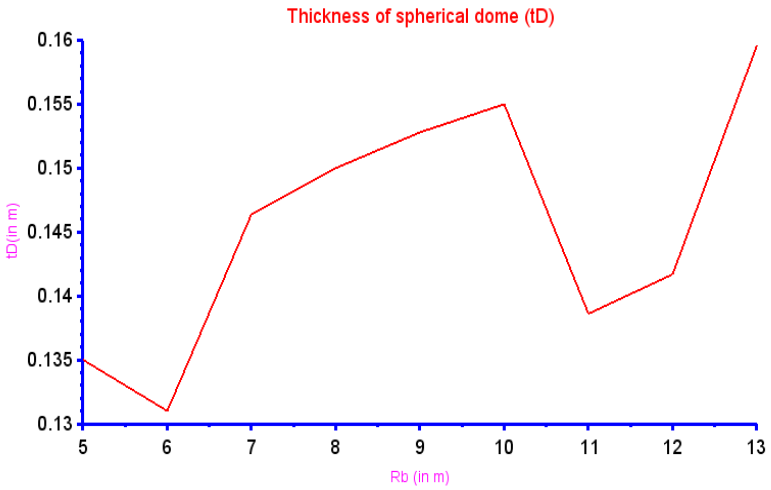


Chart 2: Base radius (R_b) versus thickness of spherical dome (t_D)

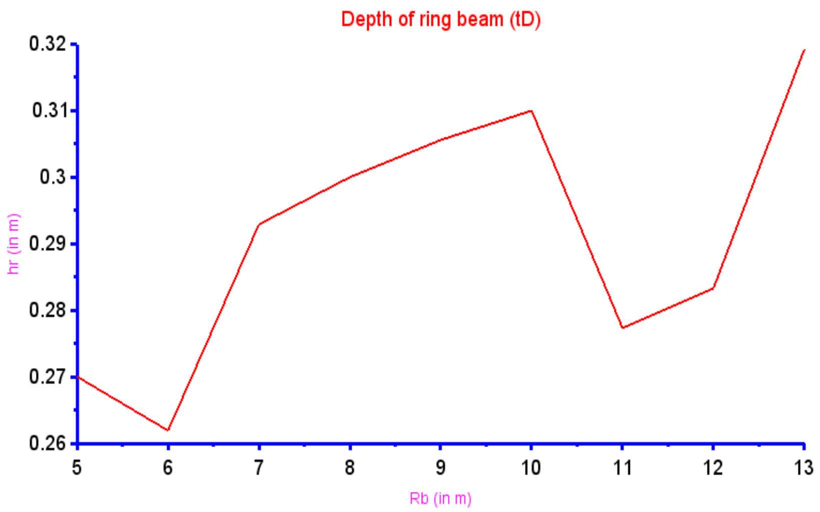


Chart 3: Base radius (R_b) versus depth of ring beam (h_r)

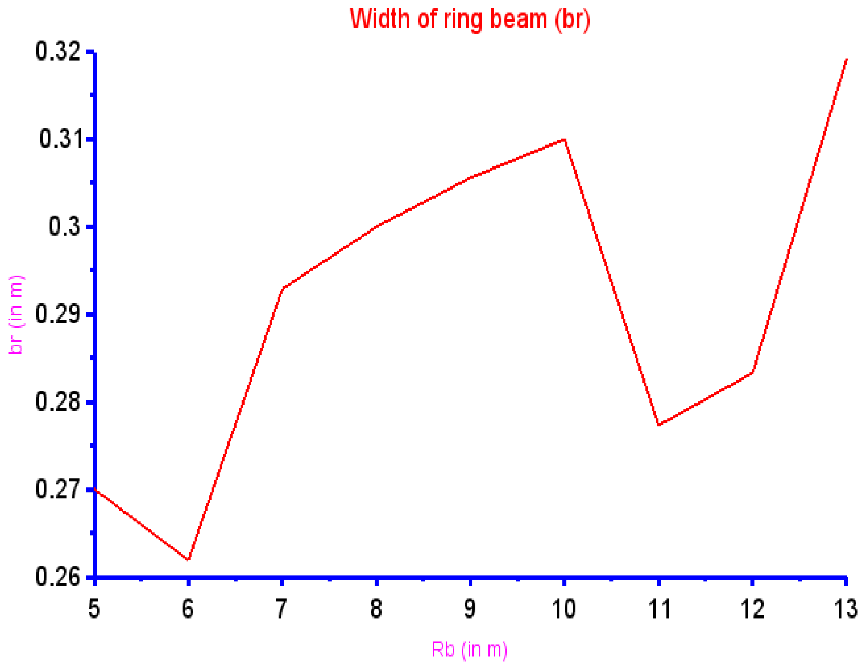


Chart 4: Base radius (R_b) versus width of ring beam width (b_r)

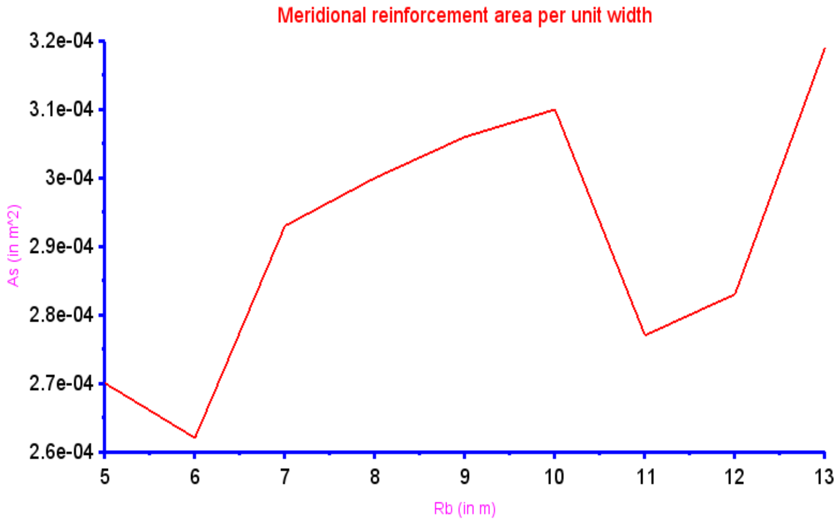


Chart 5: Base radius (R_b) versus meridional reinforcements per unit width (A_s)

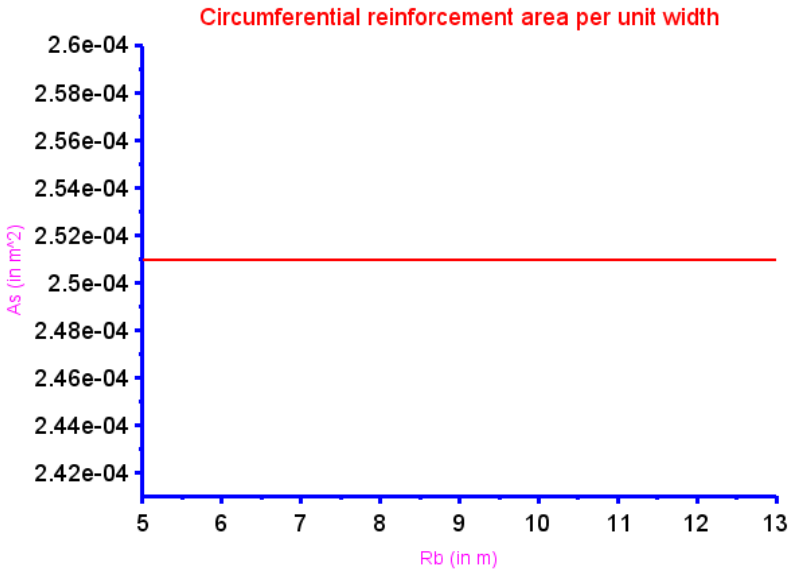


Chart 6: Base radius (R_b) versus circumferential reinforcements per unit width (A_s)

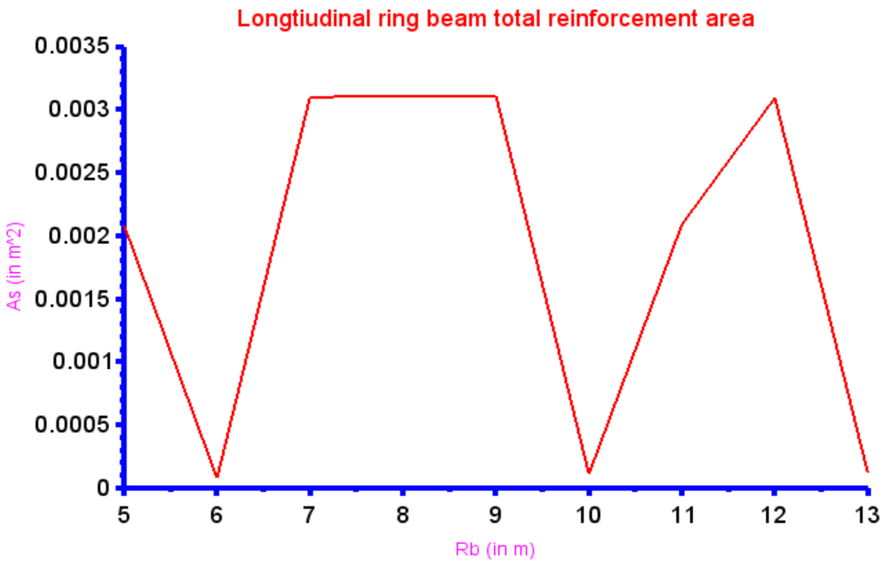


Chart 7: Base radius (R_b) versus longitudinal ring beam reinforcements (A_s)

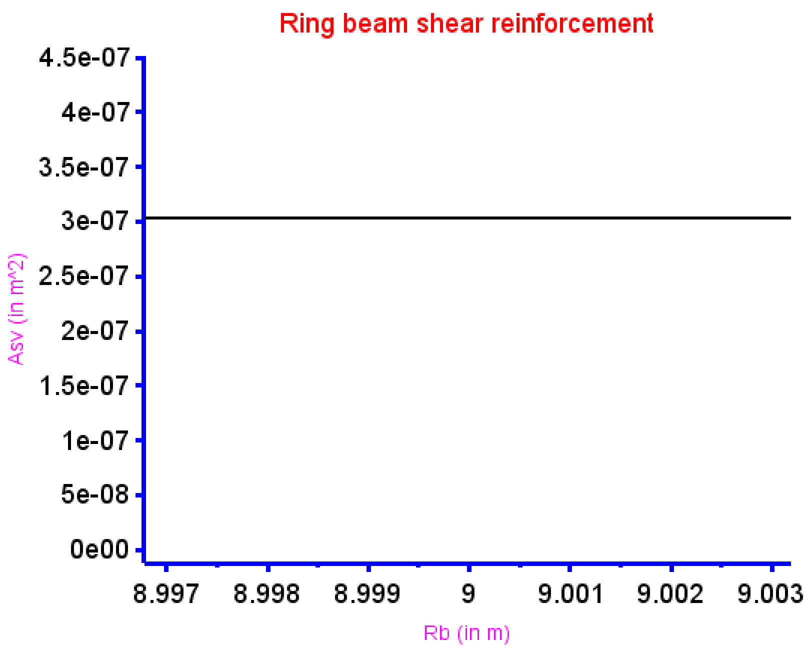


Chart 8: Base radius (R_b) versus ring beam shear reinforcements (A_{sv})

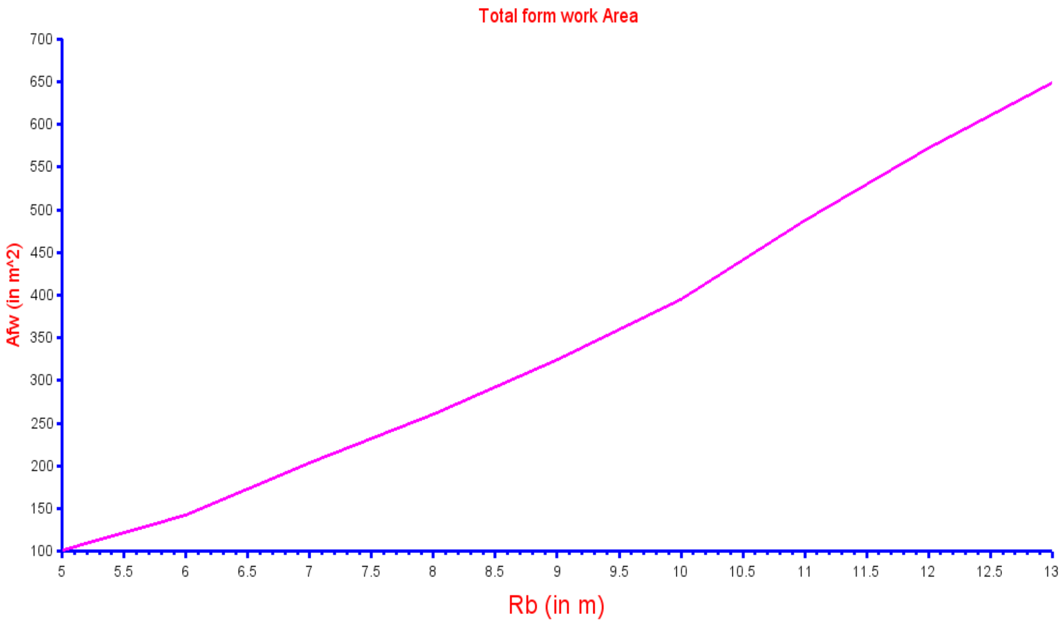


Chart 9: Base radius (R_b) versus total formwork area (A_{fw})

CHAPTER SIX

6. DESCUSION AND CONCLUSIONS

6.1. DESCUSION

From the result obtained in chapter five it is clear that form table 4 and chart 1 if the base radius is between 5 m and 10 m the principal radiuses of spherical dome increases linearly. But, for base radius greater than 10 m the principal radiuses of spherical dome become constant up to base radius 11 m and again increase linearly up to base radius of 13 m. From the values of base radius and principal radius of the spherical dome the optimized results are obtained in base angles (ϕ_1) between 38^0 and 47^0.

The thickness of the spherical dome as shown in table 4 and chart 2 decreases from base radius 5m to 6m and then increases linearly up to base radius 10m. After base radius 10m the thickness of the dome decrease up to base radius 11m then it increases linearly up to base radius 13m.

Ring beam depth, ring beam width and spherical dome meridional reinforcement goes up and down like that of spherical dome thickness as shown in the design table 4 and 5 and design chart 3, 4 and 5.

As shown in the design table 4 and design chart 3 and 4 the optimized depth and width value of ring beam cross section dimensions at cost optimized spherical dome roof structure varies equally as the dome base radius increases that results square section of ring beam.

The spherical dome circumferential reinforcement per unit meter width does not vary with the variation of spherical dome base radius as shown in the design table 5 and design chart 6. From table 5 and design chart 7 it is observable that the longitudinal ring beam reinforcement become maximum at base radius 5, 7, 8,9,11,and 12 m. but at base radius of spherical dome 6m,10m and 13 m has low value. In addition to this the longitudinal reinforcement inversely proportional with the ring beam depth and width at base radius 11, 10, and 7 m. the ring beam shear reinforcement has constant area in maximum spacing of shear reinforcement for all base radius values of spherical dome as shown in design table 5 and design chart 8. As shown in design table 4 and design chart 9 the total formwork area increases from base radius 6m up to 10m and decreases from 5m to 6m and from 10m to 12m.

As we can observe from the design charts and design tables the optimized design variables of spherical dome roof structures are resulted around the lower bounds of side constraints.

When we come to the optimized results of reinforcements at each meridional angles of spherical domes are come from singly reinforced section design.

When we see the materials cost influence on the total optimized costs of the spherical dome roof structures as shown in appendix A of table A-1,A-2 and figure A-4, A-5 and A-6 from 50% to 55% of the total cost is comes from concrete cost. From 28% to 35% cost of total cost is come from formwork cost. Only 15% to 17% of total cost is comes from reinforcement steel cost.

When we see the cost influence at the spherical dome shell roof structure component level as shown in the appendix A of table A-3 and figure A-8 and A-9 the influence of total materials cost of spherical dome is too significant than the ring beam. About 87% to 95% of total cost is raised from spherical dome materials cost. Only 6% to 13% of the total cost is come from ring beam construction materials cost.

6.2. CONCLUSIONS

Most of the optimized cross sectional and geometrical dimensions are obtained around the side constraints lower bounds. Therefore minimum costs are obtained when the spherical dome roof structure are designed conventionally for possible feasible values of cross sectional and geometrical dimensions and reinforcements of spherical dome starting from lower bounds of side constraints.

Optimized minimum cost of spherical dome shell roof structure is obtained when they are designed for base radius and principal radius values which can gives base angle between 38^0 and 47^0

Cost optimized spherical dome shell roof structure is obtained by designing the structure for singly reinforced section than doubly reinforced section.

Using square section ring beam are possible as base stiffening structure and can give cost optimized spherical dome shell roof structure.

In total construction material optimized cost of spherical dome shell roof structure, primarily concrete material cost has significant influence than reinforcement steel and form work costs. And reinforcement steel cost has moderate influence on the total optimized cost of spherical dome shell roof structure. The form work cost on the total optimized cost of spherical dome shell roof structure is relatively small when it is compared with reinforcement and concrete costs. Therefore when we design spherical dome shell roof structure for minimum cost special attention should be given for the cross section of concrete and reinforcement area proportioning.

Also the spherical dome structural component construction material cost has significant influence on total minimum construction cost of spherical dome shell roof structure when it is compared with ring beam structural component construction material cost.

RECOMENDATIONS

It is recommended that further minimum cost can be obtained by refining the step sizes of design variable in to small value.

It is best to design spherical dome shell roof structures for base angle between 38^0 and 47^0 in order to get cost optimized spherical dome roof structures.

It is better to follow the singly reinforced design section as well in order to get cost optimized spherical dome roof structures.

The prepared program will become more applicable when it is modified for asymmetrical loadings.

From the study that has been carried out, the followings are the recommended research ideas drawn for researchers to upgrade this study as full document for reference.

1. Cost optimized design of reinforced concrete spherical dome including material properties as design variable
2. Cost optimized design of reinforced concrete spherical dome for asymmetrical loadings
3. Cost optimized design of reinforced concrete spherical dome using different optimization methods
4. Optimized design of spherical dome based on finite element analysis
5. Stress minimization of reinforced concrete and steel spherical dome roof cover structure
6. Weight minimization of reinforced concrete and steel spherical dome roof cover structure
7. Cost comparison between reinforced concrete spherical dome roof and steel spherical dome roof structure for the same space area covering
8. Cost comparison between spherical dome roof and cylindrical shell roof structure for the same space area covering
9. Sensitivity analysis for optimized design of reinforced concrete spherical dome
10. Cost optimized design of reinforced concrete spherical dome including life-cycle costs

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APPENDIX A

OPTIMIZED MINIMUM COSTS OF SPHERICAL DOME SHELL ROOF STRUCTURAL COMPONENTS AND CONSTRUCTION MATERIALS

Table A- 1: Spherical dome materials cost

Rb(m)	Total Spherical Dome and Ring Beam cost (Birr)	Concrete		Reinforcement		Form Work	
		CCOSDD C_c^d (Birr)	%	RSCOSDD C_s^d (Birr)	%	FWCOSDD C_{fw}^d (Birr)	%
5	58390.66398	24998.615	42.813	9261.414	15.861	16834.084	28.830
6	81293.38716	35344.008	43.477	13117.564	16.136	24536.333	30.182
7	119391.7254	54191.110	45.389	19747.723	16.540	33644.142	28.180
8	157090.9565	72838.800	46.367	26015.911	16.561	44144.727	28.101
9	199684.9802	94166.192	47.157	33101.853	16.577	56032.776	28.061
10	247160.1344	118166.146	47.810	41004.617	16.590	69305.658	28.041
11	288033.438	134051.855	46.540	49248.916	17.098	87902.856	30.518
12	342998.6793	162060.779	47.248	57860.078	16.869	103996.22	30.320
13	418802.8896	206164.850	49.227	69608.758	16.621	117421.38	28.037

Table A- 2: Ring beam materials cost

Rb (m)	Total Spherical Dome and Ring Beam cost (Birr)	Concrete		Reinforcement		Form Work	
		CCORBB C_c^r (Birr)	%	RSCORBB C_s^r (Birr)	%	FWCORBB C_{fw}^r (Birr)	%
5	58390.664	5038.486	8.629	561.605	0.962	1696.460	2.905
6	81293.387	5689.063	6.998	631.704	0.777	1974.716	2.429
7	119391.725	8298.739	6.951	933.905	0.782	2576.106	2.158
8	157090.957	9952.566	6.336	1123.024	0.715	3015.929	1.920
9	199684.980	11615.166	5.817	1313.242	0.658	3455.752	1.731
10	247160.134	13283.910	5.375	1504.229	0.609	3895.575	1.576
11	288033.438	11689.866	4.059	1307.202	0.454	3832.743	1.331
12	342998.679	13316.164	3.882	1492.872	0.435	4272.566	1.246
13	418802.890	18312.827	4.373	2080.031	0.497	5215.044	1.245

Table A- 3: Spherical dome shell roof structural components cost

Rb(m)	Total Spherical Dome and Ring Beam cost (Birr)	Spherical Dome		Ring Beam	
		Total cost (C_m^d) (Birr)	%	Total cost (C_m^r) (Birr)	%
5	58390.6640	51094.1129	87.5039	7296.5511	12.4961
6	81293.3872	72997.9051	89.7956	8295.4821	10.2044
7	119391.7254	107582.9754	90.1092	11808.7500	9.8908
8	157090.9565	142999.4380	91.0297	14091.5185	8.9703
9	199684.9802	183300.8206	91.7950	16384.1596	8.2050
10	247160.1344	228476.4207	92.4406	18683.7137	7.5594
11	288033.4380	271203.6274	94.1570	16829.8106	5.8430
12	342998.6793	323917.0770	94.4368	19081.6023	5.5632
13	418802.8896	393194.9876	93.8855	25607.9020	6.1145

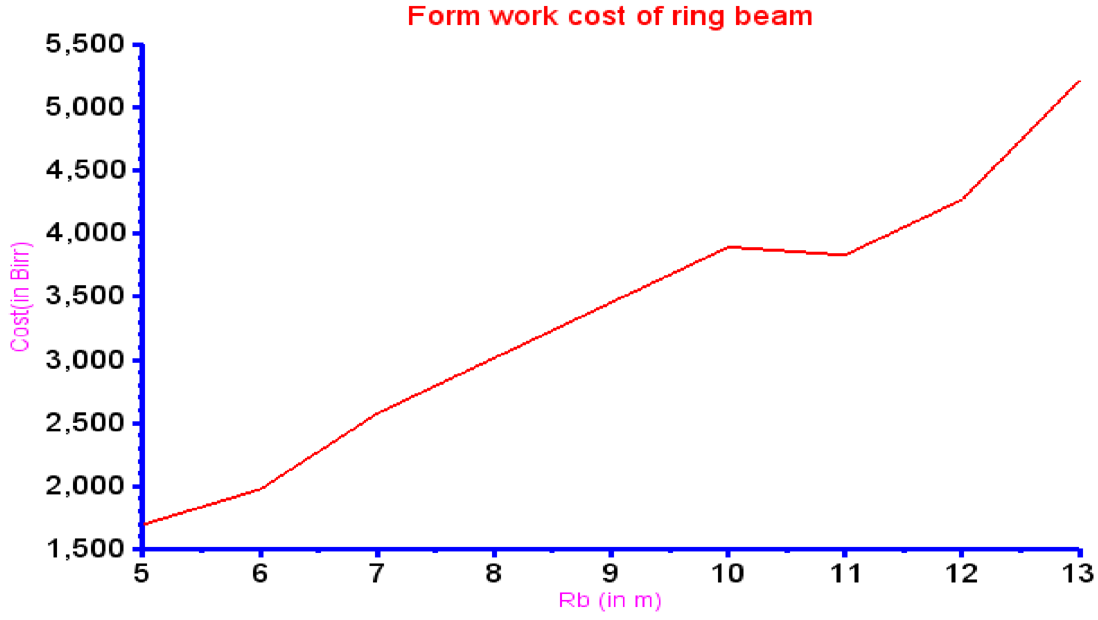

Figure A- 1: Form work cost of ring beam versus base radius

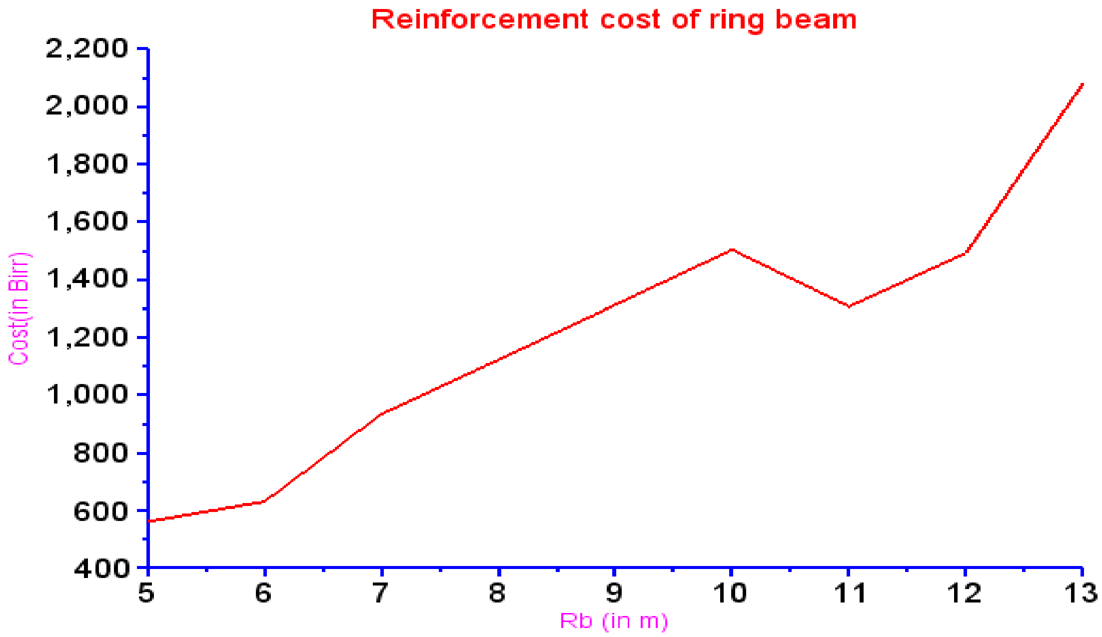

Figure A- 2: Reinforcement cost of ring beam versus base radius

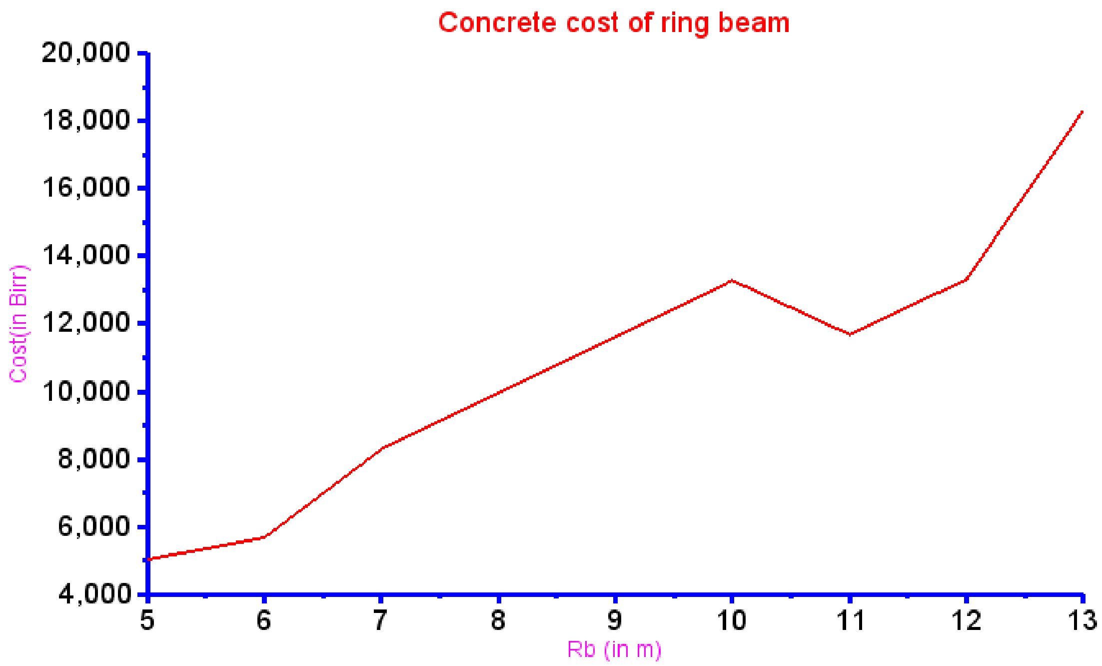

Figure A- 3: Concrete cost of ring beam versus base radius

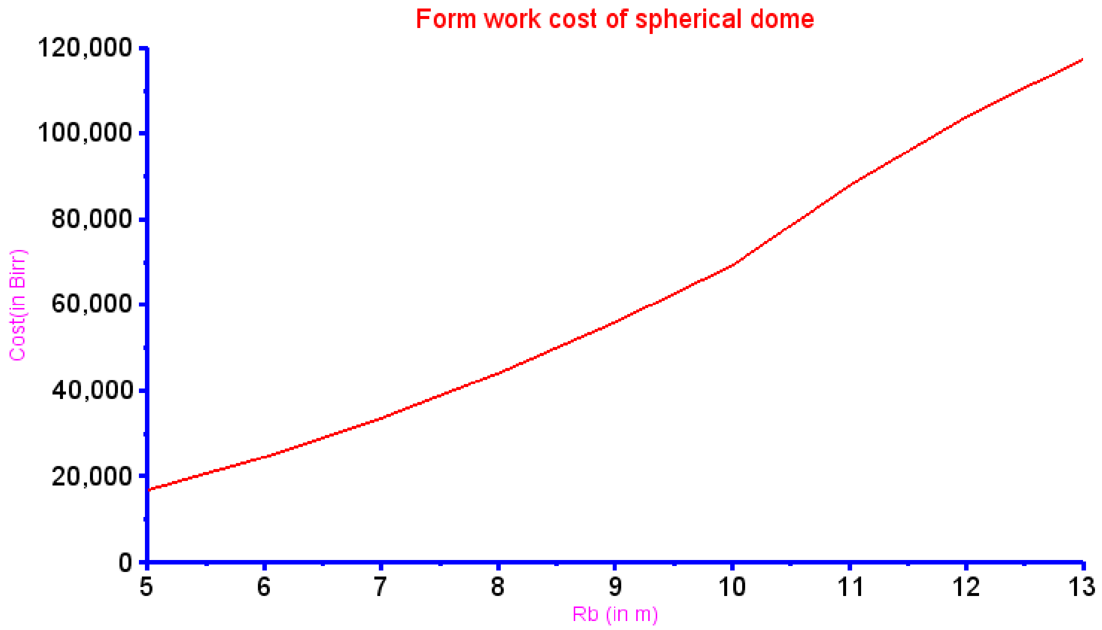

Figure A- 4: Form work cost of spherical dome versus base radius

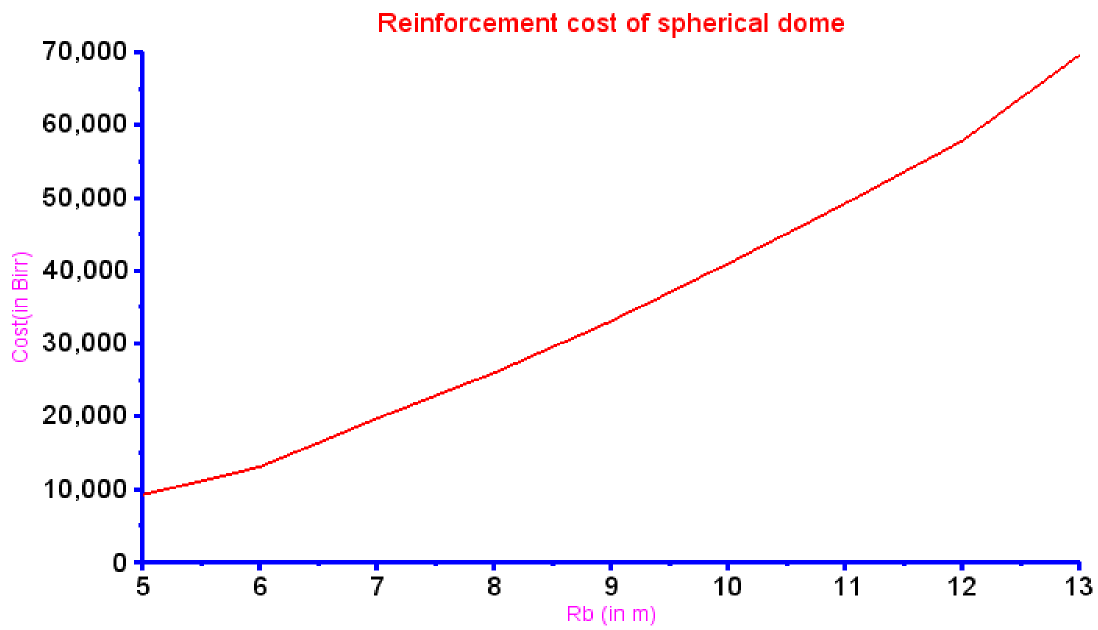

Figure A- 5: Reinforcement costs of spherical dome versus base radius

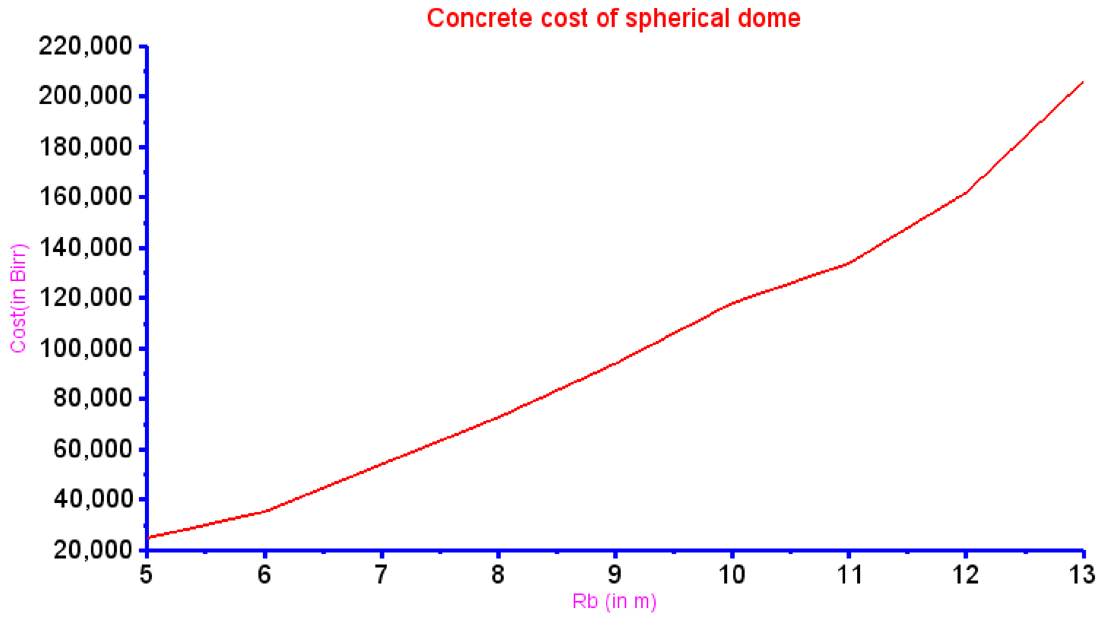

Figure A- 6: Concrete cost of spherical dome versus base radius

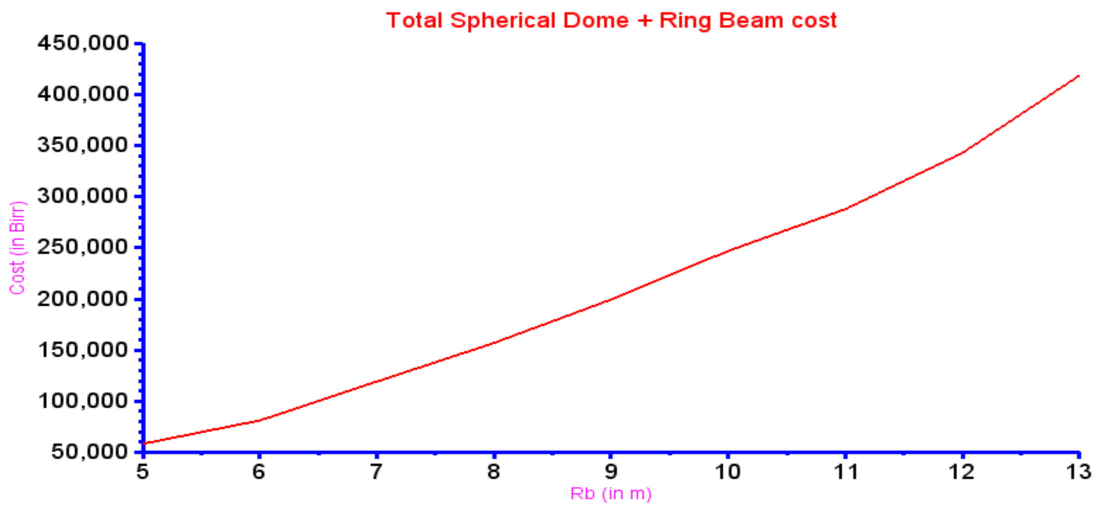

Figure A- 7: Total spherical dome plus ring beam cost versus base radius

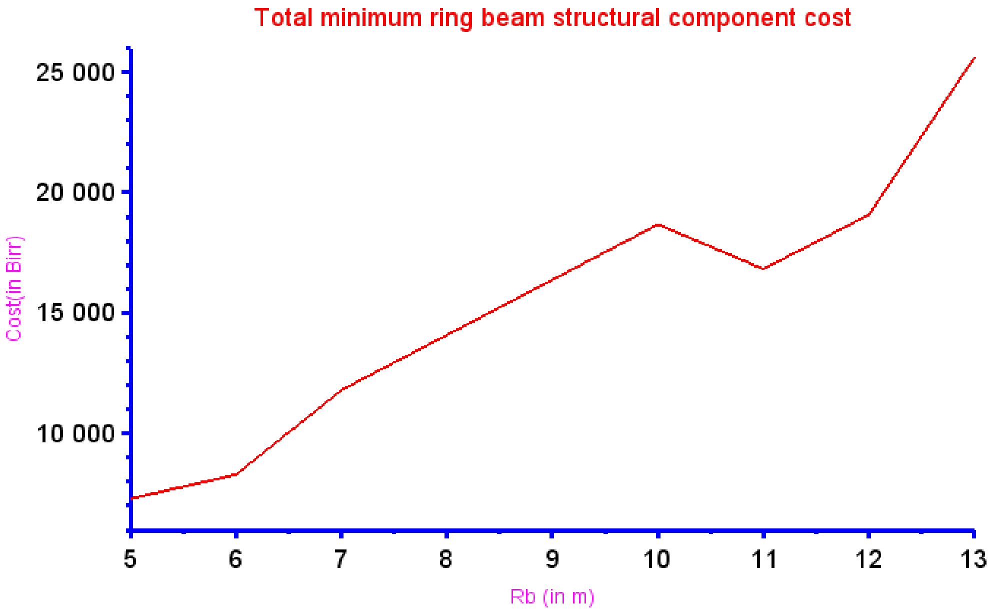

Figure A- 8: Total minimum ring beam structural components cost

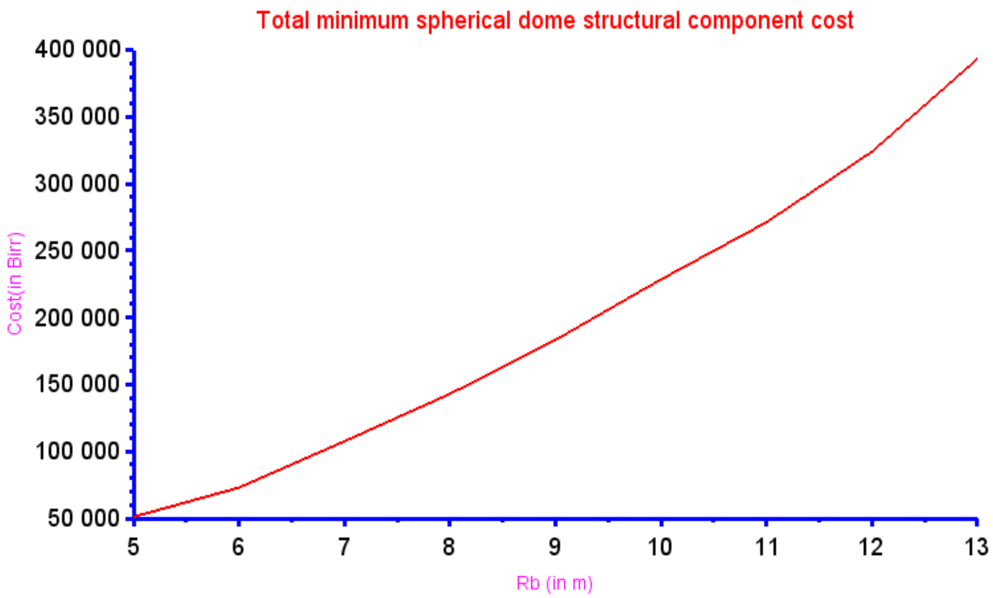

Figure A- 9: Total minimum spherical dome structural components cost

APPENDIX B

SCILAB OPTIMIZATION PROGRAM

```
clear,clc,clf();
funcprot(0);
//DOME PICH AND BASE ANGLES

//pich angle

function A0=DPAngle(r0, RD)
   //r0 is parallel circle radius of the dome at the pich angle A0
   //RD is principal radius of sphere
   //A0 is pich angle
      funcprot(0)
      A0=(asin(r0 / RD))*(180/%pi);
endfunction

//Dome base angle

function A1=DBAngle(RC, RD)
   //RD is principal radius of sphere
   //RC is radius of the cylinder
   //A1 is dome base angle
      funcprot(0)
      A1 = (asin(RC / RD))*(180/%pi);
endfunction

//STRESSES

//MEMBRANE ANALYSIS OF DOME

//Membrane meridional force(NPhi)

function NPhi=DMNPhi(prd, PDD, qD, Phio, RD, Phii)
      NPhi = (-(prd * sin(Phio * (%pi / 180))) / ((sin(Phii * (%pi / 180))) ^ 2)) + ((RD * PDD) * (((cos(Phii *
(%pi / 180))) - (cos(Phio * (%pi / 180)))) / ((sin(Phii * (%pi / 180))) ^ 2))) - ((0.5 * RD * qD) * (1 - (((sin(Phio
* (%pi / 180))) ^ 2) * ((sin(Phii * (%pi / 180))) ^ -2))));
endfunction

//Membrane tangential force(NTeta)

function NTeta=DMNTeta(prd, PDD, qD, Phio, RD, Phii)
      NTeta = ((RD * prd) * (((sin(Phio * (%pi / 180))) / (RD * ((sin(Phii * (%pi  / 180))) ^ 2))) - ((cos(Phio *
(%pi  / 180)))))) - (((RD * PDD) / ((sin(Phii * (%pi  / 180))) ^ 2)) * (((cos(Phii * (%pi  / 180))) * (((sin(Phii *
(%pi  / 180))) ^ 2) + 1)) - (cos(Phio * (%pi  / 180))))) + (((RD * qD)/(2*(((sin(Phii * (%pi  / 180))) ^ 2)))) *
(((sin(Phii * (%pi  / 180))) ^ 2)-(2*((cos(Phii * (%pi  / 180))) ^ 2)*((sin(Phii * (%pi  / 180))) ^ 2))-((sin(Phio *
(%pi / 180))) ^ 2)));
endfunction

//BENDING ANALYSIS OF DOME

//Bending meridional force(NPhi)
```

```
function NPhi=BDBNPhi(v, tD, RD, A1, Phii, M0, Q0)
    si = A1 - Phii;
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2));
    NPhi = -((%e) ^ (-si * Lmd)) * ((Q0 * (Lmd * (sin(A1 * (%pi / 180))) * (sin(((si * Lmd) - 45) * (%pi / 180))))) + (M0 * (((2 * Lmd) / RD) * ((tan((A1 - si) * (%pi / 180))) ^ -1) * (sin((si * Lmd) * (%pi / 180))))));
endfunction

//Bending tangential force(NPhi)

function NTeta=BDBNTeta(v, tD, RD, A1, Phii, M0, Q0)
    si = A1 - Phii
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2))
    NTeta = -2 * ((%e) ^ (-si * Lmd)) * ((Q0 * (Lmd * (sin(A1 * (%pi / 180))) * (sin(((si * Lmd) - 90) * (%pi / 180))))) + (M0 * (((2 ^ (1 / 2)) / RD) * (Lmd ^ 2) * (sin(((si * Lmd) - 45) * (%pi / 180))))))
endfunction

//Bending meridional moment(MPhi)

function MPhi=BDBMPhi(v, tD, RD, A1, Phii, M0, Q0)
    si = A1 - Phii;
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2)) ^ (1 / 4);
    MPhi = ((%e) ^ (-si * Lmd)) * ((M0 * ((2 ^ (1 / 2)) * (Lmd ^ 2) * (sin(((si * Lmd) - 45) * (%pi / 180))))) + (Q0 * ((RD / Lmd) * (sin(A1 * (%pi / 180))) * (sin((si * Lmd) * (%pi / 180))))));
endfunction

//Bending shear force(Q)

function Q=BDBQ(v, tD, RD, A1, Phii, M0, Q0)
    si = A1 - Phii;
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2)) ^ (1 / 4);
    Q = (2 * ((%e) ^ (-si * Lmd))) * ((M0 * (Lmd / RD) * (sin((si * Lmd) * (%pi / 180)))) + (Q0 * (((sin(A1 * (%pi / 180))) / (2 ^ (1 / 2))) * (sin(((si * Lmd) - 45) * (%pi / 180))))));
endfunction

//RING BEAM ANALYSIS

//Ring radial load

function QR=RBQR(NphiA1Hf, Q0D, Q0TC)
    QR = NphiA1Hf-(Q0TC + Q0D );
endfunction

//Ring axial force

function Tr=RBTS(RC, QR)
    Tr = QR * RC;
endfunction

//RING BEAM AXIAL STRESS

function RBAS=RBAST(Tr, Ar)
    RBAS = Tr / Ar;
endfunction

//DOME BOTTOME DISPLAMENTS
```

//Dome bottome displamnt due to actual loadings

```
function TDDRL10=DBDDAL10(v, Ec, tD, RD, A1, A0, PDD, qD, prd)
   //PDD is dome dead load
   //prd is ring loading comes from the top ring of the dome
   //qD live load on dome
   //RD is principal radius of dome
   //tD is dome thichness
   //A1 is dome base angle
   //A0 is pich angle

   //Dome base displacement due to dead load
      DDD10 =  ((((RD^2) * PDD) / (Ec * tD * (sin(A1 * (%pi / 180))))) * (((1 + v) * ((cos(A0 * (%pi / 180))) - (cos(A1 * (%pi / 180))))) - ((cos(A1 * (%pi / 180))) * ((sin(A1 * (%pi / 180))) ^ 2))));
   //Dome base displacement due to live load
      DDL10 =((((RD^2) * qD) / (2*Ec * tD*(sin(A1 * (%pi / 180))))) * (((1 + v) * (((sin(A1 * (%pi / 180))) ^ 2) - (sin(A0 * (%pi / 180))^2))) - (2*((cos(A1 * (%pi / 180))) ^ 2)*((sin(A1 * (%pi / 180))) ^ 2))));
   //Dome base displacement due to ring loading
      DBDDRL =  (((RD*prd) / (Ec * tD * (sin(A1 * (%pi / 180))))) * (((sin(A0 * (%pi / 180))) * (1 + v)) + (RD * (cos(A0 * (%pi / 180))) * ((sin(A1 * (%pi / 180))) ^ 2))));
   //Total dome base displacement due to applied loading
      TDDRL10 = DDD10 + DDL10 + DBDDRL;
endfunction
```

//Dome bottome rotation due to actual loadings

```
function TDRAL20=DBRDAL20(v, Ec, tD, RD, A1, A0, PDD, qD, prd)
   //Dome base rotation due to dead load
      DRD20 = ((RD * PDD) / (Ec * tD)) * (((1 + v) * ((tan(A1 * (%pi / 180))) ^ -1) * ((1 / ((sin(A1 * (%pi / 180))) ^ 2)) * (((cos(A1 * (%pi / 180))) * (((sin(A1 * (%pi / 180))) ^ 2) + 2)) - (2 * (cos(A0 * (%pi / 180))))))) - ((sin(A1 * (%pi / 180))) + (((sin(A1 * (%pi / 180))) ^ -1) * (((tan(A1 * (%pi / 180))) ^ -2) + (((sin(A1 * (%pi / 180))) ^ -1) * (((sin(A1 * (%pi / 180))) ^ -1) - (2 * (cos(A0 * (%pi / 180))) * ((tan(A1 * (%pi / 180))) ^ -1)))))))) + (v * ((sin(A1 * (%pi / 180))) ^ -1) * ((((sin(A1 * (%pi / 180))) ^ -1)*(2 * (cos(A0 * (%pi / 180))) * ((tan(A1 * (%pi / 180))) ^ -1)))-((tan(A1 * (%pi / 180))) ^ -2))));
   //Dome base rotation due to live load
      DRL20 = -((RD * qD) / (Ec * tD)) * (((1 + v) * ((tan(A1 * (%pi / 180))) ^ -1) *((sin(A0 * (%pi / 180))) ^ 2))+((((sin(A0 * (%pi / 180))) ^ 2) * ((sin(A1 * (%pi / 180))) ^ -2) * ((tan(A1 * (%pi / 180))) ^ -1))-(2 * (sin(A1 * (%pi / 180))) * (cos(A1 * (%pi / 180))))) - (v * ((sin(A0 * (%pi / 180))) ^ 2) * ((sin(A1 * (%pi / 180))) ^ -2) * ((tan(A1 * (%pi / 180))) ^ -1)));
   //Dome base rotation due to RING LOAD
      DRR20 = ((1 + v) / (Ec * tD)) * ((((tan(A1 * (%pi / 180))) ^ -1) * ((prd / ((sin(A1 * (%pi / 180))) ^ 2)) * ((RD * (cos(A0 * (%pi / 180))) * ((sin(A1 * (%pi / 180))) ^ 2)) - (2 * (sin(A0 * (%pi / 180))))))) + (2 * prd * (sin(A0 * (%pi / 180))) * ((cos(A1 * (%pi / 180))) / ((sin(A1 * (%pi / 180))) ^ 3))));
   //Total dome base rotation due to applied loading
      TDRAL20 = DRD20 + DRL20 + DRR20;
endfunction
```

//Dome bottome displacement due to unit shear

```
function DDUS11=DBDDUS11(v, Ec, tD, RD, A1)
      Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2)) ^ (1 / 4);
      DDUS11 = ((2 * RD * Lmd * ((sin(A1 * (%pi / 180))) ^ 2)) / (Ec * tD));
endfunction
```

```
//Dome bottome rotation due to unit shear

function DRUS21=DBRDUS21(v, Ec, tD, RD, A1)
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2)) ^ (1 / 4);
    DRUS21 = ((2 * (Lmd ^ 2) * (sin(A1 * (%pi / 180)))) / (Ec * tD));
endfunction

//Dome bottome displacement due to unit moment

function DDUM12=DBDDUM12(v, Ec, tD, RD, A1)
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2)) ^ (1 / 4);
    DDUM12 = ((2 * (Lmd ^ 2) * (sin(A1 * (%pi / 180)))) / (Ec * tD));
endfunction

//Dome bottome rotation due to unit moment

function DRUM22=DBRDUM22(v, Ec, tD, RD)
    Lmd = ((3 * (1 - (v ^ 2)) * (RD ^ 2)) / (tD ^ 2)) ^ (1 / 4);
    DRUM22 = ((4 * (Lmd ^ 3)) / (Ec * RD * tD));
endfunction

//RING DISPLACEMENTS

//Ring beam displacement due to actual loadings

function RDDUS10=RBDDUUS10(NphiA1, Ec, Rr, Ar)
    RDDUS10 = -(((Rr ^ 2) / (Ec * Ar)) * NphiA1);
endfunction

//Ring beam displacement due to unit radial force

function RDDUS11=RBDDUUS11(Ec, Rr, Ar)
    RDDUS11 = -(Rr ^ 2) / (Ec * Ar);
endfunction

//Ring beam rotation due to unit moment

function RBDDUM12=RBRDUUM22(Ec, hr, b, Rr)
    Ir = (b * (hr ^ 3)) / 12;
    RBDDUM12 = (Rr ^ 2) / (Ec * Ir);
endfunction

//DESIGN

//EFFECTIVE DEPTHS

//Balanced effective depth

function dbal=Effdblnc(b, fck, Mu)
  //Muc is ultimmate moment carring capacity of concrete
  //dphi is effective depth in in for meridonal direction
    dbal = ((Mu / (0.168 * fck * b)) ^ (1 / 2));
endfunction

//effective depth for meridional direction
```

```
function dphi=Effdphi(tD, Cc, BDPhi)
   //tD is thickness of dome
   //Cc is clear cover
   //BDPhi bar diameter in meridional direction
   //BDTeta bar diameter in tangential direction
   //dphi is effective depth in in for meridonal direction
      dphi = tD - Cc - (BDPhi / 2);
endfunction

//effective depth for meridional direction

function dTeta=EffdTeta(tD, Cc, BDPhi, BDTeta)
      dTeta = tD - Cc - BDPhi - (BDTeta / 2);
endfunction

//NUETRAL AXIS DEPTH AND LIVER ARM FOR SINGLY AND DOUBLY REINFORCED SECTION

//SINGLY REINFORCED SECTION

//Nuetral axis depth for singly reinforced section

function nads=nadsrs(b, dphi, fck, Md)
      Ks = Md / (fck * b * (dphi ^ 2));
      if (5.2371 * Ks) > 1.4462 then
      nads=0
      else
      nads = (1.202 - ((1.4462 - (5.2371 * Ks)) ^ (1 / 2))) * dphi;
      end

endfunction

//Liver arm for singly reinforced section

function las=lasrs(b, dphi, fck, Md)
      Ks = Md / (fck * b * (dphi ^ 2));
      las = (0.5 + ((0.25 - (0.9063 * Ks)) ^ (1 / 2))) * dphi;
endfunction

//DOUBLY REINFORCED SECTION

//Nuetral axis depth for doubly reinforced section

function nad=naddrs(dphi)
      nad = 0.45 * dphi;
endfunction

//Liver arm for doubly reinforced section

function lad=ladrs(dphi)
      lad = 0.8128 * dphi;
endfunction

//AREA OF REINFORCEMENTS
```

//Minimum area of reinforcement for flexural memeber to minimize crack in tension zone

```
function As02=MARFM1(tD, b, dprime, nadphi, fck, fyk)
       fcteff = 0.3 * ((fck) ^ (2 / 3))
       if tD < 0.3 then
          k = 1;
       elseif tD > 0.8 then
          k = 0.65;
       else
          k = 1.21 - (0.7 * tD);
       end
//Depth of concrete section for tensile zone just before the intiation of the first crack (hceff)
       hceff = tD / 2;
       if 2.5 * (tD - dprime) < hceff then
         hceff = 2.5 * (tD - dprime);
       end
       if ((tD - nadphi) / 3) < hceff then
          hceff = ((tD - nadphi) / 3);
       end
       Act = b * hceff;
//Moment stress distribution coefficient (kc=0.4)
       kc = 0.4;
       As02 = (kc * k * fcteff * Act) / fyk;
endfunction
```

//Minimum area of reinforcement for pure tensioned member

```
function As03=MARPT2(tD, b, dprime, nadphi, fck, fyk)
        if tD < 300 then
          k = 1;
       elseif tD > 800 then
          k = 0.65;
       else
          k = (605 - (0.35 * tD)) / 500;
       end
       fcteff = 0.3 * ((fck) ^ (2 / 3));
       hceff = tD / 2;

       if 2.5 * (tD - dprime) < hceff then
           hceff = 2.5 * (tD - dprime);
       end
       kc = 1;
       Act = b * hceff;
       As03 = (kc * k * fcteff * Act) / fyk;
endfunction
```

//Minimum area of reinforcement for member subjected to out-of-planebending (beams and walls) to prevent abrittle failure, wide cracks and also to resist forces arising from restarained actions

```
function As04=OPAR1(bt, dphi, fck, fyk)
       fctm = 0.3 * ((fck) ^ (2 / 3));
       As0401 = 0.26 * (fctm / fyk) * bt * dphi;
       As0402 = 0.0013 * bt * dphi;
       if As0401 < As0402 then
          As04 = As0402;
```

```
    else
        As04 = As0401;
    end
endfunction
```

//Minimum area of reinforcement for compression member subjected predominantly to compression force (columns and walls) to prevent abrittle failure, wide cracks and also to resist forces arising from restarained actions

```
function As05=CMAR1(tD, b, NED, fyk)
    AC = b * tD;
    fyd = fyk / 1.15;
    As0501 = 0.1 * (NED / fyd);
    As0502 = 0.002 * AC;

    if As0501 < As0502 then
        As05 = As0502;
    else
        As05 = As0501;
    end
endfunction
```

//Minimum area of reinforcement in vertical direction

```
function VDMAR=VDMAR1(MAR, OPAR1, CMAR1)
    VDMAR = MAR;
    if VDMAR < OPAR1 then
        VDMAR = OPAR1;
    end
    if VDMAR < CMAR1 then
        VDMAR = CMAR1;
    end
endfunction
```

//Minimum area of reinforcement in horizontal direction to prevent abrittle failure, wide cracks and also to resist forces arising from restarained actions

```
function As06=HDMAR1(MARPT, ASV, b, tD)
    AC = b * tD;
    As0601 = 0.25 * ASV;
    As0602 = 0.001 * AC;

    As06 = MARPT;

    if As06 < As0601 then
        As06 = As0601;
    end
    if As06 < As0602 then
        As06 = As0602;
    end
endfunction
```

//Minimum shear reinforcement area for ring beam

```
function RBMSR=RBMINSRA(fck, fyk, bw)
```

```
    alpha=90
    Rhowmin = (0.08 * ((fck) ^ (1 / 2))) / fyk;
    RBMSR = Rhowmin *  bw * (sin(alpha * (%pi / 180)));
endfunction

//Sectional area of ring beam from equivalent composit section

function AC=RBSAECS(Es, Ec, RBAFf, RBAPTRF, ALLTS)
    Alphae = Es / Ec;
    AC = ((RBAFf / ALLTS) - ((Alphae - 1) * RBAPTRF));
endfunction

//Maximum area of reinforcement

function As07=MAXAR1(b, tD)
    AC = b * tD;
    As07 = 0.04 * AC;
endfunction

//SPACING OF REINFORMENT

//Minimum reinforcement spacing

function MRSPA=MSRMFF(brd, aggs)
    MRSPA = 0.02;
    if MRSPA < brd then
        MRSPA = brd;
    end
    if MRSPA < (aggs + 0.005) then
        MRSPA = (aggs + 0.005);
    end
endfunction

//Maximum reinforcement spacing in tangential direction

function MAXRSTD=MAXSRMTD()
    MAXRSTD = 0.4;
endfunction

//Maximum reinforcement spacing in meridional direction

function MAXRSMD=MAXSRMD(tD)
    MAXRSMD = 0.4;

    if MAXRSMD > 3 * tD then
        MAXRSMD = 3 * tD;
    end
endfunction

//Length of a single shear reinforcement

function SRL=LOSSR(hr, br, cc)

    SRL = 2*(hr+br-(2*cc));
endfunction
```

//Maximum shear reinforcement spacing for ring beam

function **MAXSRS**=MAXSRSRB(**d**)
    alpha = 90;
    MAXSRS = 0.75 * **d** * (1 + ((tan(alpha * (%pi / 180))) ^ -1));
endfunction

//Number of reinforcement per unit width
function **NR**=NRMFF(**Ass**, **bd**)
    aas = %pi * ((**bd** / 2) ^ 2);
    NR = **Ass** / aas;
endfunction

//spacing of reinforcement per unit width

function **RSPA**=SRMFF(**b**, **NR**)
    RSPA = **b / NR**;
endfunction

//Area of reinforcement based on a given spacing

function **AORF**=ARBOGS(**bd**, **b**, **RSPA**)
    aas = %pi * ((**bd** / 2) ^ 2);
    AORF =(aas* **b**) / **RSPA**;
endfunction

//ALLOWABLE TENSION AND COMPRESSIVE STRESSES

//allowabele compression stress for concrete

function **ALLCS**=AllCSTRS(**fck**)
    ALLCS = 0.6 * **fck**;
endfunction

//allowabele pure tension stress for reinforced concrete

function **ALLTS**=AllTSTRS(**fyk**)
    ALLTS = 0.8 * **fyk**;
endfunction

//allowabele compression force for concrete

function **FCDD**=AllCFC(**tD**, **fcd**)
  //FCDD allowable compressive force for concrete
  //fcd design sterngth of concrete
FCDD = tD * **fcd**;
endfunction

//ULTIMATE MOMENT CARRYING CAPACITY OF TENSION AND COMPRESSION SIDE OF SINGLY AND DOUBLY REINFORCED SHELL SECTION

//SINGLY REINFORCED SECTION

```
//Ultimate moment carring capacity of concrete in compression zone for singly reinforced section based on rectangular - parabolic stress block

function UMCCC=UMCCOSRCS(b, dphi, fck)
   //fck is caracterstic concrete strength
   //ultimate moment carring capacity of singly rinforced concrete section based on rectangular-parbolic stress block of concrete section
      UMCCC = 0.168 * fck * b * (dphi ^ 2);
endfunction

//Moment carring capacity of concrete in compression zone for singly reinforced section based on rectangular - parabolic stress block

function MCCC=MCCOSRCS(b, nadsr, lasr, fck)
   //fck is caracterstic concrete strength
   //moment carring capacity of concrete in compression zone for singly rinforced concrete section based on rectangular-parbolic stress block of concrete section
      MCCC = 0.459 * fck * b * nadsr * lasr;
endfunction

//Moment carring capacity of steel in tension zone for singly reinforced section based on rectangular - parabolic stress block

function MCCS=MCCOSRTS(Assrs, lasrs, fyk)
      MCCS = 0.87 * fyk * Assrs * lasrs;
endfunction

//DOUBLY REINFORCED SECTION

//Moment carring capacity of concrete in compression zone for doubly reinforced section based on rectangular - parabolic stress block

function MCDRC=MCCODRCS(b, dphi, dprime, Asprime, ladrs, fck, fyk)
      MCDRC = ((0.20655 * fck * b * dphi) * ladrs) + ((0.87 * fyk * Asprime) * (dphi - dprime));
endfunction

//Moment carring capacity of steel in tension zone for doubly reinforced section based on rectangular - parabolic stress block

function MCCSD=MCCODRTS(Asdrs, ladrs, fyk)
      MCCSD = 0.87 * fyk * Asdrs * ladrs;
endfunction

//Pure tension capacity of steel

function PTCAS=PTCCOAS(fyk, Asten)
      PTCAS = (fyk / 1.15) * Asten;
endfunction

//Shear carring capacity of concrete

function VC=SCC(fck, Asbend, b, tD, dphi, Nphii)
      fcd = fck / (1.5 * (10 ^ 6));
      bw = b * 1000;
      Ac = (1000 * b) * (1000 * tD);
```

```
    Crdc = 0.18 / Yc;
    k = 1 + ((200 / (dphi * 1000)) ^ (1 / 2));
    if k > 2 then
      k = 2;
    end
    cmpstrs = (-Nphii) / Ac;
    if cmpstrs > 0.2 * fcd then
      cmpstrs = 0.2 * fcd;
    end
    Rho1 = (Asbend * (10 ^ 6)) / (bw * dphi);
    if Rho1 > 0.02 then
    Rho1 = 0.02;
    end
    VC = ((Crdc * k * ((100 * Rho1 * (fck / (10 ^ 6))) ^ (1 / 3))) + (k1 * cmpstrs)) * (bw * 1000) * (dphi * 1000);
    VMIN = 0.035 * ((k) ^ (3 / 2)) * ((fck / (10 ^ 6)) ^ (1 / 2));
    VCMIN = (VMIN + (k1 * cmpstrs)) * (bw * 1000) * (dphi * 1000);
    if VC < VCMIN then
      VC = VCMIN;
    end
endfunction

//Shear carring capacity of concrete

function VC=SCC(fck, Asbend, b, tD, dphi, Nphii)
    k1 = 0.15;
    Yc = 1.5;
    fcd = fck / (1.5 * (10 ^ 6));
    bw = b * 1000;
    Ac = (1000 * b) * (1000 * tD);
    Crdc = 0.18 / Yc;
    k = 1 + ((200 / (dphi * 1000)) ^ (1 / 2));
    if k > 2 then
      k = 2;
    end
    cmpstrs = (-Nphii) / Ac;
    if cmpstrs > 0.2 * fcd then
      cmpstrs = 0.2 * fcd;
    end
    Rho1 = (Asbend * (10 ^ 6)) / (bw * dphi);
    if Rho1 > 0.02 then
      Rho1 = 0.02;
    end
    VC = ((Crdc * k * ((100 * Rho1 * (fck / (10 ^ 6))) ^ (1 / 3))) + (k1 * cmpstrs)) * (bw * 1000) * (dphi * 1000);
    VMIN = 0.035 * ((k) ^ (3 / 2)) * ((fck / (10 ^ 6)) ^ (1 / 2));
    VCMIN = (VMIN + (k1 * cmpstrs)) * (bw * 1000) * (dphi * 1000);
    if VC < VCMIN then
      VC = VCMIN;
    end
endfunction

//CRACK WIDTH CALCULATION

//Design crack width calculation for bending moment singly reinforced section
```

```
function DCRWMDS=DCWFMDSRS(b, tC, dphi, dprime, Cc, bdiam, ASS, Md, Es, Ec, fck)
    Kt   = 0.4;
    K1   = 0.8;
    K2  = 0.5;
    Alphae = Es / Ec;
    fcteff = 0.3 * (fck ^ (2 / 3));
    X = (((-Alphae * ASS) + ((((Alphae * ASS) ^ 2) + (2 * b * Alphae * ASS * dphi)) ^ (1 / 2))) / b);
    hceff = tC / 2;
    if 2.5 * (tC - dprime) < hceff then
       hceff = 2.5 * (tC - dprime);
    end
    if ((tC - X) / 3) < hceff then
        hceff = ((tC - X) / 3);
    end
    Aceff = b * hceff;
    Rhopeff = ASS / Aceff;
    TStres = Md / ((dphi - (X / 3)) * ASS);
    Esm = TStres / Es;
    Eccm = (Kt * (fcteff / Rhopeff) * (1 + (Alphae * Rhopeff))) / Es;
    Ediff = Esm - Eccm;
    if Ediff < (0.6 * (TStres / Es)) then
       Ediff = (0.6 * (TStres / Es));
    end
    MAXCS = (3.4 * Cc) + ((0.425 * K1 * K2 * bdiam) / Rhopeff);
    DCRWMDS = MAXCS * Ediff;
endfunction

//Design crack width calculation for bending moment doubly reinforced section

function DCRWMDD=DCWFMDDRS(b, tC, dphi, dprime, Cc, bdiam, ASS, ASprime, Md, Es, Ec, fck)
    Kt   = 0.4;
    K1  = 0.8;
    K2   = 0.5;
    Alphae = Es / Ec;
    fcteff = 0.3 * ((fck) ^ (2 / 3));
    X = (((-(((Alphae - 1) * ASprime) + (Alphae * ASS))) + ((((((Alphae - 1) * ASprime) + (Alphae * ASS)) ^
2) + (2 * b * ((dprime * ASprime * (Alphae - 1)) + (Alphae * ASS * dphi)))) ^ (1 / 2))) / b);
    hceff = tC / 2;
    if 2.5 * (tC - dprime) < hceff then
       hceff = 2.5 * (tC - dprime);
    end
    if ((tC - X) / 3) < hceff then
       hceff = ((tC - X) / 3);
    end
    Aceff = b * hceff;
    Rhopeff = ASS / Aceff;
    TStres = Md / ((dphi - (X / 3)) * ASS);
    Esm = TStres / Es;
    Eccm = (Kt * (fcteff / Rhopeff) * (1 + (Alphae * Rhopeff))) / Es;
    Ediff = Esm - Eccm;
    if Ediff < (0.6 * (TStres / Es)) then
       Ediff = (0.6 * (TStres / Es));
    end
    MAXCS = (3.4 * Cc) + ((0.425 * K1 * K2 * bdiam) / Rhopeff);
```

```
        DCRWMDD = MAXCS * Ediff;
endfunction

//Design crack width calculation for pure tension

function DCRWTF=DCWFTF(b, tC, dprime, Cc, bdiam, ASS, Td, Es, Ec, fck)
        Kt = 0.4;
         K1 = 0.8;
         K2  = 1;
        Alphae = Es / Ec;
        fcteff = 0.3 * ((fck) ^ (2 / 3));
        hceff = tC / 2;
        if 2.5 * (tC - dprime) < hceff then
            hceff = 2.5 * (tC - dprime);
        end
        Aceff = b * hceff;
        Rhopeff = (0.5 * ASS) / Aceff;
        TStres = Td / ASS;
        Esm = TStres / Es;
        Eccm = (Kt * (fcteff / Rhopeff) * (1 + (Alphae * Rhopeff))) / Es;
        Ediff = Esm - Eccm;
        if Ediff < (0.6 * (TStres / Es)) then
            Ediff = (0.6 * (TStres / Es));
        end
        MAXCS = (3.4 * Cc) + ((0.425 * K1 * K2 * bdiam) / Rhopeff);
        DCRWTF = MAXCS * Ediff;
endfunction
//Unit cost of materials
//Cost of concrete per unit volume(in birr/m^3)
CCS=2200.00;
//Cost of steel per Kg(in birr/Kg)
CSU=27.00;
//Cost of form work per m^2(in birr/m^2)
CFD=200;

//Counter
        l=0;
//MATERIAL PROPERTIES

    //CHARACTERSTIC CYLINDERICAL STRENGTH OF CONCRETE
        fck = 35000000;
    //YIELD STRENGTH OF STEEL
        fyk = 460000000;
    //YOUNG'S MODULUS OF STEEL
        Es  = 200000000000;

    //YOUNG'S MODULUS OF CONCRETE
        Ecm = 22 * (0.8 + (0.1 * (fck / (10 ^ 6)))) * (10 ^ 9);
    //ASSUMING CREEP COEFIINT=1.5
    //LONG TERM YOUNG MOULUS OF CONCRETE (Ec)
        Ec = Ecm / 2.5;
    //Unit weight of concrete (N/m3)
        Yc = 24000;
        //Unit weight of steel
        Ys = 78500;
```

```
    //Poisson's ratio
       v = 0.2;

//DOME GEOMETRY

    //Radius of Dome pich
       r0= 1.75;
    //Dome clear cover
       DCC = 0.03;
       dprime=DCC;

    //Members unit width

       b = 1;

    //BAR GEOMETRIES AND AGGS

    //BAR DIAMETER

    //BAR DIAMETER IN MERIDIONAL DIRECTION
       BDPhi = 0.008;
    //BAR DIAMETER IN TANGENTIAL DIRECTION
       BDTeta = 0.008;
    //aggrigate size
       aggs = 0.02;

//LOADINGS

    //Loads on dome

    //Dome live load in N/m^2
       qD = 400;
    //Dome ring load in N/m(vergashi pg.40)
       prd = 3640;

    //SAFETY FACTOR FOR MATERIALS

    //SAFETY FACTOR OF CONCRETE
       sfc = 1.5;
    //SAFETY FACTOR OF STEEL
       sfs = 1.15;

    //SAFETY FACTOR FOR LOADINGS OF ULTIMATE LIMIT STATE

    //SAFETY FACTOR FOR DEAD LOAD
       sfDL = 1.3;
    //SAFETY FACTOR FOR LIVE LOAD
       sfsLL = 1.5;
    //Factored loads
    //Factored Dome live load
       qDf = sfsLL * qD;
    //Factored Dome ring load
       prdf = sfDL * prd;

          disp( '..........................................................................................')
```

```
disp( '---//---            SPHERICAL DOME SHELL ROOF STRUCTURE OPTIMAL DESIGN PARAMETERS           ----//---')
disp('..........................................................................................')

//Material properties
//USED MATERIALS
disp( ''..........................................................................'')
disp('---//---            USED MATERIALS            ----//---')
disp('..........................................................................')
disp(fck,'Characterstic cylinderical strength of concrete (fck in N/m^2 )=')
disp(fyk,'Yield strength of steel (fyk in N/m^2)=')
disp(Ecm,''Youngs modulus of concrete (Ecm in N/m^2)='')
disp(Es,''Youngs modulus of steel (Es in N/m^2)='')
disp(Ec,'Long term youngs modulus of concrete(Ec in N/m^2)=')
disp(Yc,'Unit weight of concrete(Yc in N/m^3)=')
disp(Ys,'Unit weight of steel(Ys in N/m^3)=')
disp(v,''Poisons ratio (v)='')
//DESIGN STRENGTH OF MATERIALS

//DESIGN STRENGTH OF CONCRETE
fcd = ((0.85 * fck) / (sfc));
//DESIGN STRENGTH OF STEEL
fyd = fyk / sfs;
disp( '..........................................................................')
disp('---//---       DESIGN STRENGTH OF MATERIALS         ----//---')
disp('..........................................................................')
disp(fcd,'Design strength of concrete (fcd in N/m^2)=')
disp(fyd,'Design strength of steel (fyd in N/m^2)=')
//BAR GEOMETRIES
disp('..........................................................................')
disp( '---//---        BAR GEOMETRIES          ----//---')
disp('..........................................................................')
disp(BDPhi,'BAR DIAMETER FOR MERIDIONAL DIRECTION (BDPhi in m)=')
disp(BDTeta,'BAR DIAMETER FOR TANGENTIAL DIRECTION (BDPhi in m)=')
//BASE RADIUS OF THE SPHERICAL DOME ROOF
for Rb=6
  //Counter
    j=0;
  for RD =  (1.75 * Rb):-2:Rb
    //According to thin shell defnition thickness of shell shuld be between RD / 1000 and RD / 20 (RD / 1000 <= tD <= RD / 10)
    for tD = (RD / 60) :0.05: (RD / 20)
      if tD > 0.1 then
        //for hr = 2 * tD :1: (2 *  Rb)
        for hr = 2 * tD :0.2: 0.6
          // for br = 2 * tD :1: (5 * hr)
          for br = 2 * tD :0.2: 0.6
//Essential spherical dome geometric parametrs(principal radius of spherical dome(RD),pich angle(A0) and dome base angle(A1))  that can be calculated from the user inputs
  //Dome base angle(A1)
    A1 = DBAngle(Rb, RD);
  //pich angle(A0)
    A0 =DPAngle(r0, RD);
  //Dome dead load
    PDD = Yc * tD;
```

```
    PDDf = sfDL * PDD;
  //Meridional stress at dome base
    NphiA1 = DMNPhi(prd, PDD, qD, A0, RD, A1);
    NphiA1f = DMNPhi(prdf, PDDf, qDf, A0, RD, A1);
  //Vertical load on ring beam
    prc = NphiA1 * (sin(A1 * (%pi / 180)));
    prcf = NphiA1f * (sin(A1 * (%pi / 180)));
  //Horizontal load on ring beam
    NphiA1H = -NphiA1 * (cos(A1 * (%pi / 180)));
    NphiA1Hf = -NphiA1f * (cos(A1 * (%pi / 180)));
  //ring beam essential values
    Ar = hr * br;
  //DISPLACEMENTS ON THE RESERVIOR
  //DOME BASE DIPLACEBENT AND FLEXIBILITY  ASSIGNMENT AND DISPLAYING THE RESULT TO
THE USER
    DBD10 = DBDDAL10(v, Ec, tD, RD, A1, A0, PDD, qD, prd);
    DBD10f = DBDDAL10(v, Ec, tD, RD, A1, A0, PDDf, qDf, prdf);
    DBR20 = DBRDAL20(v, Ec, tD, RD, A1, A0, PDD, qD, prd);
    DBR20f = DBRDAL20(v, Ec, tD, RD, A1, A0, PDDf, qDf, prdf);
    fcdb11 = DBDDUS11(v, Ec, tD, RD, A1);
    fcdb21 = DBRDUS21(v, Ec, tD, RD, A1);
    fcdb12 = DBDDUM12(v, Ec, tD, RD, A1);
    fcdb22 = DBRDUM22(v, Ec, tD, RD);
  //RING BEAM DIPLACEBENT AND FLEXIBILITY  ASSIGNMENT AND DISPLAYING THE RESULT TO
THE USER
    RBD10 = RBDDUUS10(NphiA1H, Ec, Rb, Ar);
    RBD10f = RBDDUUS10(NphiA1Hf, Ec, Rb, Ar);
    fcrb11 = RBDDUUS11(Ec, Rb, Ar);
  //COMPATIBILITY EQUATIONS AT THE JUNCTION OF DOME-RING BEAM
  //SIMULTANEOUS EQUATION VARIABLE AND CONSTANT DECALARATION
  //A11 (1st raw, 2nd column)
    A11 = fcdb11 + fcrb11;
    A12 = fcrb11;
    A13 =fcdb12;
    A21 =  fcrb11;
    A22 = fcrb11;
    A23 = 0;
    A31 = fcdb21;
    A32 = 0;
    A33 = fcdb22;

    B11 = RBD10 + DBD10;
    B21 = RBD10;
    B31 = DBR20;

    B11f = RBD10f + DBD10f;
    B21f = RBD10f;
    B31f = DBR20f;

    AAAA11=[A11,A12,A13;A21,A22,A23;A31,A32,A33];
    BBBB11=[B11;B21;B31];
    BBBB11f=[B11f;B21f;B31f];

    EAGFRC = inv(AAAA11)*BBBB11;
    EAGFRCf = inv(AAAA11)*BBBB11f;
```

```
//EDGE REACTIONS
      Q0D=EAGFRC(1,1);
      Q0Df=EAGFRCf(1,1);
      Q0b=EAGFRC(2,1);
      Q0bf=EAGFRCf(2,1);
      M0=EAGFRC(3,1);
      M0f=EAGFRCf(3,1);
//counters
   i=0;
   //Spherical dome step angle
   sss=1
for Phii=A0:sss: (A1+sss)

//ALLOWABLE FORCES
   //ALLOWABLE COMPRESSION FORCE FOR CONCRETE OF SPHERICAL DOME
      FCDD = AllCFC(tD, fcd);
   //ALLOWABLE COMPRESSION FORCE FOR CONCRETE OF RING BEAM
      FCDRB= AllCFC(br, fcd);
   //ALLOWABLE COMPRESSION STRESS FOR CONCRETE
      ALLCSA =AllCSTRS(fck);
   //ALLOWABLE PURE TENSION STRESS FOR REINFORCED CONCRETE
      ALLTS = AllTSTRS(fyk);

//SPHRICAL DOME CONSTRAINT CHEAKING
   if Phii<=A1 then

      DNPhi = DMNPhi(prd, PDD, qD, A0, RD, Phii) + BDBNPhi(v, tD, RD, A1, Phii, M0, Q0D);
      DNPhif = DMNPhi(prdf, PDDf, qDf, A0, RD, Phii) + BDBNPhi(v, tD, RD, A1, Phii, M0f, Q0Df);
      ABSDNPhif = abs(DNPhif);
      DNTeta = DMNTeta(prd, PDD, qD, A0, RD, Phii) + BDBNTeta(v, tD, RD, A1, Phii, M0, Q0D);
      DNTetaf = DMNTeta(prdf, PDDf, qDf, A0, RD, Phii) + BDBNTeta(v, tD, RD, A1, Phii, M0f, Q0Df);
      DMPhi = BDBMPhi(v, tD, RD, A1, Phii, M0, Q0D);
      DMPhif = BDBMPhi(v, tD, RD, A1, Phii, M0f, Q0Df);
      DMPhiAbs = abs(DMPhi);          //Unfactored absolute meridional moment
      DMPhiAbsf = abs(DMPhif);        //Factored absolute meridional moment
      DMTeta = v * DMPhi;
      DMTetaf = v * DMPhif;
      DQ = BDBQ(v, tD, RD, A1, Phii, M0, Q0D);
      DQf = BDBQ(v, tD, RD, A1, Phii, M0f, Q0Df);
      DHD = abs((1 / (Ec * tD)) * (DNTeta - (v * DNPhi)));

if ABSDNPhif < FCDD & (ABSDNPhif / tD) < ALLCSA & (DNTetaf/ tD) < ALLTS & (ABSDNPhif / tD) <=
(((Ecm * tD) / (sqrt(3)*RD))/4) & DHD < 0.03 & DHD < ((2*RD * (sin(Phii * (%pi / 180)))) / 250) then
//Spherical dome
   //EFFECTIVE DEPTH
   //Effective depth of spherical dome in meridional direction
      EFFDOSDIMD = Effdphi(tD, DCC, BDPhi);
   //Effective depth of spherical dome in tangential direction
      EFFDOSDITD = EffdTeta(tD, DCC, BDPhi, BDTeta);
   //NUTRAL AXIS DEPTH
   //Nuetral axis depth for singly reinforced section of spherical dome
      NADFSRSOSD = nadsrs(b, EFFDOSDIMD, fck, DMPhiAbsf);
        if NADFSRSOSD < 0.12 * EFFDOSDIMD then
            NADFSRSOSD = 0.12 * EFFDOSDIMD
```

```
        end
    //Nuetral axis depth for doubly reinforced section of spherical dome
        NADFDRSOSD = naddrs(EFFDOSDIMD);
    //LIVER ARM
    //Liver arm for singly reinforced section of spherical dome
       Ks0 = DMPhiAbsf / (fck * b * (EFFDOSDIMD ^ 2));
       qwa=0.9063*Ks0
        if qwa > 0.25 then
           continue
        end
        LAFSRSOSD = lasrs(b, EFFDOSDIMD, fck, DMPhiAbsf);
    //Liver arm for doubly reinforced section of spherical dome
        LAFDRSOSD = ladrs(EFFDOSDIMD)
//ULTIMATE MOMENT CARRYING CAPACITY OF CONCRETE FOR SPHERICAL DOME
        UMCCOCFSD = UMCCOSRCS(b, EFFDOSDIMD, fck);
//MINIMUM AND MAXIMUM AREA OF REINFORCEMENT FOR VERTICAL AND HORIZONTAL
DIRECTION OF SPHERICAL DOME
    //MINIMUM AREA OF REINFORCEMENT FOR COMPRESSION  MEMBER SUBJECTED
PREDOMINANTLY TO COMPRESSION FORCE (COLUMNS AND WALLS)
        CMARFSD = CMAR1(tD, b, DNPhif, fyk);
    //MINIMUM AREA OF REINFORCEMENT FOR SPHERICAL DOME SUBJECTED PREDOMINANTLY TO
OUT- OF PLANE BENDING (BEAMS AND WALLS)
        OPARFSD = OPAR1(b, EFFDOSDIMD, fck, fyk);
    //MINIMUM AREA OF REINFORCEMENT FOR FLEXURAL MEMBER SINGLY REINFORCED SECTION
SPHERICAL DOME TO MINIMIZE CRACK IN TENSION ZONE
        MARFMSRSOSD = MARFM1(tD, b, dprime, NADFSRSOSD, fck, fyk);
    //MINIMUM AREA OF REINFORCEMENT FOR FLEXURAL MEMBER DOUBLY REINFORCED SECTION
SPHERICAL DOME TO MINIMIZE CRACK IN TENSION ZONE
        MARFMDRSOSD = MARFM1(tD, b, dprime, NADFDRSOSD, fck, fyk);
    //MINIMUM AREA OF REINFORCEMENT FOR PURE TENSION FORCE OF SINGLY REINFORCED
SPHERICAL DOME (MAX OF AS MINIMUMS)TO MINIMIZE CRACK IN TENSION ZONE
        MARSRPTOSD = MARPT2(tD, b, dprime, NADFSRSOSD, fck, fyk);
    //MINIMUM AREA OF REINFORCEMENT FOR PURE TENSION FORCE OF DOUBLY REINFORCED
SPHERICAL DOME (MAX OF AS MINIMUMS)TO MINIMIZE CRACK IN TENSION ZONE
        MARDRPTOSD = MARPT2(tD, b, dprime, NADFDRSOSD, fck, fyk);
//FOR VERTICAL DIRECTION
    //MINIMUM AREA OF REINFORCEMENT IN VERTICAL DIRECTION FOR SINGLY REIFORCED
SECTION OF SPHERICAL DOME
        VDMARSRS = VDMAR1(MARFMSRSOSD, OPARFSD, CMARFSD);
    //MINIMUM AREA OF REINFORCEMENT IN VERTICAL DIRECTION FOR DOUBLY REIFORCED
SECTION OF SPHERICAL DOME
        VDMARDRS = VDMAR1(MARFMDRSOSD, OPARFSD, CMARFSD);
//FOR HORIZONTAL DIRECTION
    //MINIMUM AREA OF REINFORCEMENT IN HORIZONTAL DIRECTION FOR SINGLY REIFORCED
SECTION OF SPHERICAL DOME
        HDMARSRS = HDMAR1(MARSRPTOSD, VDMARSRS, b, tD);
    //MINIMUM AREA OF REINFORCEMENT IN HORIZONTAL DIRECTION FOR DOUBLY REIFORCED
SECTION OF SPHERICAL DOME
        HDMARDRS = HDMAR1(MARDRPTOSD, VDMARDRS, b, tD);
    //MAXIMUM AREA OF REINFORCEMENT FOR SPHERICAL DOME
        MAXAR = MAXAR1(b, tD);
    //counter
    k=0;
//Spherical dome reinforcement
    //Tension side reinforemnt steel in meridional direction of doubly reinforced spherical dome.
```

```
  for Astmdr = VDMARDRS:0.001:MAXAR
      //Compression side area of reinforemnt steel in meridional direction of doubly spherical dome.
      for Ascmdr = 0:0.001:(MAXAR -Astmdr)
        //Tension side reinforemnt steel in meridional direction of singly reinforced spherical dome.
        for Astmsr = VDMARSRS:0.001:MAXAR
          //Area of reinforcement for pure tension force of meridionaly doubly reinforced spherical dome.
          for Asttdr = HDMARDRS:0.001:MAXAR
            //Area of reinforcement for pure tension force of meridionaly singly reinforced spherical dome.
            for Asttsr = HDMARSRS:0.001:MAXAR
              if Phii<=A1 then
                if DNTetaf < 0 then
                  if DNTetaf < FCDD & (DNTetaf / tD) < ALLCSA then
                    Asttdrsd = HDMARDRS;
                    Asttsrsd=HDMARSRS
                  else
                    continue
                  end
                end
              end
    if Ascmdr > 0 then
  //DOUBLY REINFORCED SECTION
    //AREA OF REINFORCEMENT FOR PURE TENSION FORCE IN ONE SIDE(HORIZONTAL
DIRECTION)
      Asttdr1 = 0.5 * Asttdr;
    //MINIMUM REINFORCEMENT SPACING
      MRSPA = MSRMFF(BDPhi, aggs);
    //MAXIMUM REINFORCEMENT SPACING IN TANGNTIAL DIRECTION
      MAXRSTD = MAXSRMTD();
    //MAXIMUM REINFORCEMENT SPACING IN MERIDIONAL DIRECTION
      MAXRSMD = MAXSRMD(tD);
    //NUMBER  OF PURE TENSION REINFORCEMENT PER UNIT WIDTH OF ONE SIDE
      NPTR = NRMFF(Asttdr1, BDPhi);
    //NUMBER  OF TENSION REINFORCEMENT PER UNIT WIDTH FOR DOUBLY REINFORCED
SECTION
      NTRDRS = NRMFF(Astmdr, BDPhi);
    //NUMBER  OF COMPRESSION REINFORCEMENT PER UNIT WIDTH FOR DOUBLY REINFORCED
SECTION
      NCRDRS = NRMFF(Ascmdr, BDPhi);
    //SPACING  OF REINFORCEMENT PER UNIT WIDTH
    //ONE SIDE SPACING  OF PURE TENSION REINFORCEMENT PER UNIT WIDTH
      TRSPAPT = SRMFF(b, NPTR);
        if TRSPAPT > MAXRSTD then
          TRSPAPT = MAXRSTD;
          Asttdr=2*ARBOGS( BDPhi , b,TRSPAPT )
          if Asttdr < HDMARDRS then
            Asttdr = HDMARDRS;
          end
        end
        if TRSPAPT < MRSPA then
          continue
        end
    //SPACING  OF TENSION REINFORCEMENT PER UNIT WIDTH FOR DOUBLY REINFORCED
SECTION
      TSRSPAB = SRMFF(b, NTRDRS);
        if TSRSPAB > MAXRSMD then
```

```
          TSRSPAB = MAXRSMD;
          Astmdr=ARBOGS( BDPhi , b,TSRSPAB )
          if Astmdr < VDMARDRS then
             Astmdr = VDMARDRS;
          end
       end
       if TRSPAPT < MRSPA then
          continue
       end
 //SPACING  OF COMPRESSION REINFORCEMENT PER UNIT WIDTH FOR DOUBLY REINFORCED SECTION
    CRSPA = SRMFF(b, NCRDRS);
       if CRSPA < MRSPA then
          continue
       end
 //MOMENT CARRYING CAPACITY OF TENSION AND COMPRESSION SIDE OF  DOUBLY REINFORCED  SHELL SECTION
 //MOMENT CARRING CAPACITY OF  COMPRESSION ZONE
    MCDRC = MCCODRCS(b, EFFDOSDIMD, dprime, Ascmdr, LAFDRSOSD, fck, fyk);
    if MCDRC < DMPhif then
       continue
    end
 //MOMENT CARRING CAPACITY OF STEEL
    MCCSD = MCCODRTS(Astmdr, LAFDRSOSD, fyk);
    if MCCSD < DMPhif then
       continue
    end
 //PURE TENSION CARRYING CAPACITY OF STEEL
    PTCAS = PTCCOAS(fyk, Asttdr);
    if PTCAS < DNTetaf then
       continue
    end
 //MAXIMUM SHEAR CARRYING CAPACITY OF REINFORCED CONCRETE FOR DOUBLY REINFORCED BENDING MOMENT OF DOME
    VCDRBM = SCC(fck, Astmdr, b, tD, EFFDOSDIMD, DNPhif);
    if VCDRBM < DQf then
       continue
    end
//CRACK WIDTH CALCULATION
 //MAXIMUM ALLOWABLE CRACK WIDTH FOR EXPOSURE CLASS XC4 IN m
    MAXCRW = 0.0003;
 //DESIGN CRACK WIDTH CALCULATION FOR BENDING MOMENT DOUBLY REINFORCED SECTION
    DCRWMDD = DCWFMDDRS(b, tD, EFFDOSDIMD, dprime, DCC, BDPhi, Astmdr, Ascmdr, DMPhiAbs, Es, Ec, fck);
       if DCRWMDD > MAXCRW then
          continue
       end
 //DESIGN CRACK WIDTH CALCULATION FOR PURE TENSION
    DCRWTF = DCWFTF(b, tD, dprime, DCC, BDPhi, Asttdr, DNTeta, Es, Ec, fck);
       if DCRWTF > MAXCRW then
          continue
       end
    Astmsr=0;
    Asttsr=0;
```

```
    else
   //SINGLY REINFORCED SECTION
          //AREA OF REINFORCEMENT FOR PURE TENSION FORCE IN ONE SIDE(HORIZONTAL
DIRECTION)
            Asttsr1 = 0.5 * Asttsr;
         //MINIMUM REINFORCEMENT SPACING
            MRSPA = MSRMFF(BDPhi, aggs);
         //MAXIMUM REINFORCEMENT SPACING IN TANGNTIAL DIRECTION
            MAXRSTD = MAXSRMTD();
         //MAXIMUM REINFORCEMENT SPACING IN MERIDIONAL DIRECTION
            MAXRSMD = MAXSRMD(tD);
         //NUMBER  OF PURE TENSION REINFORCEMENT PER UNIT WIDTH OF ONE SIDE
            NPTR = NRMFF(Asttsr1, BDPhi);
         //NUMBER  OF TENSION REINFORCEMENT PER UNIT WIDTH FOR SINGLY REINFORCED
SECTION
            NTRSRS = NRMFF(Astmsr, BDPhi);
         //SPACING  OF REINFORCEMENT PER UNIT WIDTH
         //SPACING  OF PURE TENSION REINFORCEMENT PER UNIT WIDTH
            RSPAPT = SRMFF(b, NPTR);
            if RSPAPT > MAXRSTD then
               RSPAPT = MAXRSTD;
               Asttsr=2*ARBOGS( BDPhi , b,RSPAPT )
               if Asttsr < HDMARSRS then
                  Asttdr = HDMARSRS;
               end
            end
            if RSPAPT < MRSPA then
               continue
            end
         //SPACING  OF FLXURE REINFORCEMENT FOR SINGLY REINFORCED SECTION FOR  PER
UNIT WIDTH
            RSPATFF =SRMFF(b, NTRSRS);
            if RSPATFF > MAXRSMD then
               RSPATFF = MAXRSMD;
               Astmsr=ARBOGS( BDPhi , b,RSPATFF )
               if Astmsr < VDMARSRS then
                  Astmsr = VDMARSRS;
               end
            end
            if RSPATFF < MRSPA then
               continue
            end

      //MOMENT CARRYING CAPACITY OF TENSION AND COMPRESSION SIDE OF  SINGLY
REINFORCED SHELL SECTION
         //MOMENT CARRING CAPACITY OF CONCRETE IN COMPRESSION ZONE
            MCCC = MCCOSRCS(b, NADFSRSOSD, LAFSRSOSD, fck);
            if MCCC >UMCCOCFSD then
               continue
            end
            if MCCC < DMPhif then
               continue
            end
         //MOMENT CARRING CAPACITY OF STEEL IN TENSION ZONE
            MCCS = MCCOSRTS(Astmsr, LAFSRSOSD, fyk);
```

```
            if MCCS < DMPhif then
               continue
            end
         //PURE TENSION CARRYING CAPACITY OF STEEL
            PTCAS = PTCCOAS(fyk, Asttsr);
            if PTCAS < DNTetaf then
               continue
            end
         //MAXIMUM SHEAR CARRYING CAPACITY OF REINFORCED CONCRETE OF DOME
            VCSRBM = SCC(fck, Astmsr, b, tD, EFFDOSDIMD, DNPhif);
            if VCSRBM < DQ then
               continue
            end
      //CRACK WIDTH CALCULATION
         //MAXIMUM ALLOWABLE CRACK WIDTH FOR EXPOSURE CLASS XC4 IN m
            MAXCRW =0.0003;
         //DESIGN CRACK WIDTH CALCULATION FOR BENDING MOMENT SINGLY REINFORCED
SECTION
            DCRWMDS = DCWFMDSRS(b, tD, EFFDOSDIMD, dprime, DCC, BDPhi, Astmsr, DMPhiAbs, Es,
Ec, fck);
            if DCRWMDS > MAXCRW then
               continue
            end
         //DESIGN CRACK WIDTH CALCULATION FOR PURE TENSION
            DCRWTF = DCWFTF(b, tD, dprime, DCC, BDPhi, Asttsr, DNTeta, Es, Ec, fck);
            if DCRWTF > MAXCRW then
               continue
            end
      Astmdr=0;
      Ascmdr=0;
      Asttdr=0;
     end
      k=k+1;
   //total value of dome reinforcement at each meridinal angle
   tvodr(k)=Astmdr+Ascmdr+Asttdr+Asttsr+Astmsr;
//SPHERICAL DOME REINFORCEMENT
               //Asttsrsd
               end
            //Asttdrsd
            end
         //Astmsrsd
         end
      //Ascmdrsd
      end
   //Astmdrsd
   end

   else
     continue
   end
         i=i+1;
      //minimum (mtvodr) and maximum(mxtvodr) reinforcment area sum of dome at each angle
         mtvodr=min(tvodr);
         mxtvodr=max(tvodr);
      //minimum cost of steel at each angle of dome(y1)
```

```
            phiii(i)=Phii;
            mcsaehd(i)=(7850 *CSU)*(mtvodr*(2*(%pi)*RD)*sin(Phii *(%pi/180)))*(sss*(%pi/180)*RD);
        //maximum cost of steel at each angle of dome(Ai)
            MXcsaehd(i)=(7850 *CSU)*(mxtvodr*(2*(%pi)*RD)*sin( Phii  * (%pi/180)))*(sss*(%pi/180)*RD);
else
    //RING BEAM ANALYSIS RESULT
    //RING BEAM RADIAL FORCE
        HQRB = RBQR(NphiA1Hf, Q0D, Q0bf);
        HQRBf = RBQR(NphiA1Hf, Q0Df, Q0bf);
    //RING BEAM AXIAL FORCE
        RBAF = RBTS(Rb, HQRB);
        RBAFf = RBTS(Rb, HQRBf);
    //RING BEAM AXIAL STRESS
        RBAS = RBAST(RBAFf, Ar);
    //absolute value of ring beam stress
        abRBAS=abs(RBAS);
        if RBAS < 0 then
          if abRBAS > FCDD  then
            continue
          end
        end
    //RING BEAM COMPRESSIVE STRESS
        RBCS = prcf;
    //Effective depth of ring beam
        EFFDORB = Effdphi(hr, DCC, BDPhi);
//MINIMUM AND MAXIMUM AREA OF REINFORCEMENT FOR HORIZONTAL DIRECTION OF RING
BEAM
    //MINIMUM AREA OF REINFORCEMENT FOR RING BEAM
        HRBMARSRS = OPAR1(br, EFFDORB, fck, fyk);
    //MAXIMUM AREA OF REINFORCEMENT
        MAXARRB = MAXAR1(br, hr);
//RING BEAM CONSTRAINT CHEAKING
    if (RBCS / br) < AllCSTRS(fck) & abRBAS < ALLTS then
        //counter
        p=0;
    //Ring beam reinforcement
    //Area of reinforcement for pure tension force of singly reinforced ring beam.
         for Asttsrrb = HRBMARSRS:0.001:MAXARRB
           if RBAFf < 0 then
              abRBAFf=abs(RBAFf)
             if abRBAFf < FCDRB & (abRBAFf / br) < ALLCSA then
               Asttsrrb=HRBMARSRS;
             else
               continue
             end
           end
    //SECTIONAL AREA OF RING BEAM SECTION FROM EQUIVALENT COMPOSITE SECTION
        RBSAC = RBSAECS(Es, Ec, RBAFf, Asttsrrb, ALLTS);
        if RBSAC > Ar then
           continue
        end
        //NUMBER  OF PURE TENSION REINFORCEMENT PER UNIT WIDTH FOR SINGLY REINFORCED
SECTION
           NCRDRS =NRMFF(Asttsrrb, BDPhi);
        //PURE TENSION CARRYING CAPACITY OF STEEL
```

```
            PTCAS = PTCCOAS(fyk,Asttsrrb);
            if PTCAS < RBAFf then
               continue
            end

   //CRACK WIDTH CALCULATION
      //MAXIMUM ALLOWABLE CRACK WIDTH FOR TIGHTNESS CLASS-1 STRUCTURE
            MAXCRW = 0.0003;
      //DESIGN CRACK WIDTH CALCULATION FOR PURE TENSION
            DCRWTF = DCWFTF(br, hr, dprime, DCC, BDPhi, Asttsrrb, RBAFf, Es, Ec, fck);
            if DCRWTF > MAXCRW then
               continue
            end
            //total value of ring reinforcement at each change of dome base area
            p=p+1;
            tvorr(p)=Asttsrrb
      //Asttsrrb
   end
//PROVIDED RING BEAM SHEAR REINFORCEMENT
      slmax = MAXSRSRB(EFFDORB);
      RBMSRA = RBMINSRA(fck, fyk, br);
      srl=LOSSR(hr,br,DCC)
      mtvorr=min(tvorr);
      mxtvorr=max(tvorr);
      //minimum cost of steel for ring beam at each base radius of dome(RC)
      mcsabrrb=(7850 *CSU)*((mtvorr+(RBMSRA*srl*(1/slmax)))*(2*(%pi)*Rb));
      //maximum cost of steel for ring beam at each base radius of dome(RC)
      MXcsabrrd=(7850 *CSU)*(mxtvorr+(RBMSRA*srl*(1/slmax)))*(2*(%pi)*Rb);
   else
      continue
   end
end
      //Phii
      end
   //sum of minimum (mtvodr) and maximum(mxtvodr)  reinforcment cost of dome at each radius cange of dome
      smcsaehd=sum(mcsaehd);
      sMXcsaehd=sum(MXcsaehd);
   //sum of minimum (mtvodr) and maximum(mxtvodr) reinforcment area sum of ring beam at each radius cange
of dome
      smtvorr=sum(mcsabrrb);
      smxtvorr=sum(MXcsabrrd);
   //Cost of spherical dome concrete
      CCOSD=CCS*(2*(%pi)*(RD^2)*tD*((cos(A0*(%pi/180)) )-(cos(A1*(%pi/180)))));
   //Cost of ring beam concrete
      CCORB=CCS *(2*(%pi)*Rb*br*hr );
   //Cost of spherical dome reinforcement steel (min)
      RSCOSD=smcsaehd;
   //Cost of spherical dome reinforcement steel (max)
      MARSCOSD=sMXcsaehd;
   //Cost of ring beam reinforcement steel (min)
      RSCORB=smtvorr;
   //Cost of ring beam reinforcement steel (max)
      MARSCORB=smxtvorr;
   //Cost of spherical dome form work
      FWCOSD=CFD*(2*(%pi)*(RD^2)* ((cos(A0*(%pi/180)) )-(cos(A1*(%pi/180)) )));
```

```
    //Cost of ring beam form work
        FWCORB=CFD* (2*(%pi)*Rb*hr);
//OBJECTIVE FUNCTION
    //COST CALCULATION FOR SPHERICAL DOME
    //minimum cost of spherical dome
        fx1min=CCS*(2*(%pi)*(RD^2)*tD*((cos(A0*(%pi/180)) )-(cos(A1*(%pi/180)))))+smcsaehd+(CFD*
(2*(%pi)*(RD^2)* ((cos(A0*(%pi/180)) )-(cos(A1*(%pi/180)) ))));
    //maximum cost of spherical dome
        fx1max=CCS*(2*(%pi)*(RD^2)*tD*((cos(A0*(%pi/180)) )-(cos(A1*(%pi/180)))))+sMXcsaehd+(CFD*
(2*(%pi)*(RD^2)* ((cos(A0*(%pi/180)) )-(cos(A1*(%pi/180)) ))));
    //COST CALCULATION FOR RING BEAM
    //minimum cost of ring beam
        fx2min=(CCS *(2*(%pi)*Rb*br*hr ))+smtvorr+(CFD* (2*(%pi)*Rb*hr ));
    //maximum cost of ring beam
        fx2max=(CCS *(2*(%pi)*Rb*br*hr ))+smxtvorr+(CFD* (2*(%pi)*Rb*hr ));

        j=j+1;
        brr(j)=br;
        hrr(j)=hr;
        tDD(j)=tD;
        RDD(j)=RD;
        RBB(j)=Rb;
    //Cost of spherical dome concrete
        CCOSDD(j)=CCOSD;
    //Cost of ring beam concrete
        CCORBB(j)=CCORB;
    //Cost of spherical dome reinforcement steel (min)
        RSCOSDD(j)=RSCOSD;
    //Cost of spherical dome reinforcement steel (max)
        MARSCOSDD(j)=MARSCOSD
    //Cost of ring beam reinforcement steel (min)
        RSCORBB(j)=RSCORB;
    //Cost of ring beam reinforcement steel (max)
        MARSCORBB(j)=MARSCORB
    //Cost of spherical dome form work
        FWCOSDD(j)=FWCOSD;
    //Cost of ring beam form work
        FWCORBB(j)=FWCORB;

        fx11min(j)=fx1min;
        fx11max(j)=fx1max;
        fx22min(j)=fx2min;
        fx22max(j)=fx2max;
    //TOTAL COST ( SPHERICALDOME COST + RING BEAM COST)
        fx12min(j)=fx1min+fx2min;
        fx12max(j)=fx1max+fx2max;

                    //br
                end
                //hr
                end
            else
                continue
            end
```

```
    //tD
    end
  //RD
end
  l=l+1;
  DODO=cat(2,RBB,CCOSDD,CCORBB,RSCOSDD,RSCORBB,FWCOSDD,FWCORBB);
  lolo=cat(2,RBB,RDD,tDD,hrr,brr,fx11min,fx11max,fx22min,fx22max,fx12min,fx12max);
  [min_val min_pos] = min(fx12min);
  J1=min_pos(1,1);
  k11=lolo(J1,1:11);
  k111(l,1:11)=k11
  K21=DODO(J1,1:7);
  K211(l,1:7)=K21
//Rb
end
//program outputs
    disp(' RBB CCOSDD CCORBB RSCOSDD RSCORBB FWCOSDD FWCORBB')
    disp(K211,'K211')
    disp(' RBB RDD tDD hrr brr fx11min fx11max fx22min fx22max fx12min
fx12max')
    disp(k111,'k111')
```