Addis Ababa University (Since 1950)

VALIDATION OF THE 2008 EARTH GRAVITY MODEL USING GPS/LEVELING DATA – A CASE STUDY AROUND DEBRE BIRHAN


A Thesis Submitted to the School of Graduate studies of Addis Ababa University in the Partial Fulfillment of the requirements for the Degree of Master of Science in Civil Engineering under Geodesy

Submitted by

Ermias Worku

June, 2013

Addis Ababa University

Institute of Technology

Department of Civil Engineering

This undersigned hear by certify that they have read and recommend to the school of technology for acceptance a thesis entitled " **Validation of the 2008 earth gravity model using GPS/Leveling – a case study around Debre Birhan** " by Ermias Worku in the partial fulfillment of the requirement for the degree of Master of Science.

Date: June, 2013

Advisor:

1. ______________________________ ___________________ ______________.

Name Signature Date

Co-Advisor:

1. ______________________________ ___________________ ______________.

Signature Date

Examiners:

1. ______________________________ ___________________ ______________.

Name Signature Date

2. ______________________________ ___________________ ______________.

Name Signature Date

3. ______________________________ ___________________ ______________.

Name Signature Date

Abstract

The accuracy of the 2008 Earth Gravity Model (EGM08) is evaluated using GPS/ Leveling data around Debre Birhan town. Now, the advents of satellites gravimetery and Global Positioning System have revolutionized the way Geopotential number is computed on the earth's surface. The Space Domain Spherical Harmonic Analysis of the EGM08 Stokes' dimensionless coefficients $C_{n,m}$ and $S_{n,m}$ provides Geopotential height at GPS observation points. This study compared the EGM08 derived orthometric height, normal height, geoid and height anomaly with GPS/Leveling data.

Three first order Blue Nile leveling benchmarks available around Deber Birhan town with a tie to the port of Alexandria mean sea level tide gauge datum were used for evaluating the quality of EGM08. A total length of 1200 m first order geodetic leveling work was done to shift all the three old Blue Nile leveling benchmarks to new locations that have good GPS satellite visibility. The spatial GPS coordinates of these benchmarks through which the Geopotential heights are computed were determined within few millimeter accuracy using GAMIT/GLOBK software. The accuracy of the EGM08 derived Geopotential heights are in general good to few centimeters when compared to GPS/Leveling data. The agreement between the orthometric height derived from EGM08 and level heights has a standard deviation of 2.41cm with a mean of -3.14cm. This error is reduced to 1.09cm when the prey assumption for mean gravity is constrained by surface gravity observation.

Besides, the comparisons of the gravimetric geoid and gravimetric height anomaly (zeta) with the geoid undulation derived from GPS/Leveling, have a standard deviation of 2.24cm & 2.26cm, respectively. In general, the comparison of orthometric height with level height gave the best result with datum offset of -3.73 cm between the gravimetric geoid defined by Wo=Uo and leveling datum. The datum offset may be attributed to the choice of the Wo value and real displacement between the mean sea level and the geoid.

Key words: Geopotential height, leveling height, EGM08, GPS coordinates.

Acknowledgement

First of all I would like to pass my deepest gratitude to my advisor Dr. Tulu Besha. He has a nice potential and knowledge of geopotential height system. He give me professional guidance, full supports and consecutive comments, discussions and suggestions by approaching me as a friend during the draft and final thesis preparation in addition helping me in filed data collection.

I also pass my deepest gratitude to my co-advisor Dr. Ing Elias Lewi for constant support of my study. He facilitates successive class presentations and gives me very good comments and advice. In addition helping me in all filed data collection, especially in gravity data measurement and processing.

Many thanks should also be given to Dr Addisu Hunegnaw and Dr Roger Hipkin for guiding me to do the thesis in height system. I appreciate their nice lecture during study of their course.

I would like to tank Edulink and Addis Ababa University for their financial support.

I would like also to thank the city administrative government and security Bureau of Deber Dirhan town for their cooperatives during collection of data.

Finally, but not least I thank my wife for strengthening me to complete the study.

Table of Contents

List of Tables

List of Figures

List of Acronyms

2D	Two Dimensions
3D	Three Dimensions
CHAMP	CHAllenging Minisatellite Payload
C/A	Code Accusition
DB01	Debre Birhan first station
DB02	Debre Birhan second station
DB03	Debre Birhan third station
DOP	Dilution Of Precision
EGM	Earth Gravity Model
FFT	Fast Fourier transforms
GDOP	Geometrical Dilution Of Precision
GNSS	Global Navigation Satellite System
GOCE	Gravity filed and steady state Ocean Circulation Explorer
GPS	Global Positioning System
GRACE	Gravity Recovery And Climate Experiment
HDOP	Horizontal Dilution Of precision
IAG	International Association of Geodesy

IGS	International Global position system Service
ITRF	International Terrestrial Reference Frame
JPL	Jet Propulsion Laboratory
MSL	Mean See Level
NASA	National Aeronautics and Space Administration
NGS	National Geodetic Survey
NIMA	National Imagery and Mapping Agency
NRMS	Normal Root Mean Square
PPS	Precise Positioning Service
PDOP	Position Dilution Of Precision
RINEX	Receiver Independent Exchange Format
SPS	Standard positioning Service
TCE	Total Electron Content
TDOP	Time Dilution of Precision
VDOP	Vertical Dilution Of Precision
U.S	United State
USA	United States of America
WRMS	Weighted Root Mean Square

List of Symbols

Δg	Gravity anomaly
δg	Gravity disturbance
ψ	Geocentric radius
A	Azimuth angle
C	Geopotential number
W_p	Gravitational potential at point p
W_o	Gravitational potential at reference surface
$\overline{C_m^n}$	Fully normalize cosine spherical harmonic coefficient
$\overline{S_m^n}$	Fully normalize sine spherical harmonic coefficient
$\overline{p_m^n}$	Fully normalized legendary polynomial
n , m	Degree and Order of spherical harmonic coefficients
M	Mass of the earth
G	Newton's Universal gravitational constant
φ	Geodetic latitude
λ	Geodetic longitude
h	Ellipsoidal height
a	Semi-major axis
N	Geoid height

H_p	Orthometric height at point p
H_p^n	Normal height at point p
H_p^{dyn}	Dynamic height at point p
ϴ	Angel between plumb line and ellipsoidal normal
U_o	Normal potential
Z_p	Height anomaly at point p
$\bar{g}_p$	Average gravity along the plumb line
∆n	Change in height increment
ω	Angular velocity
J_2	Second degree zonal harmonics
GM	Geocentric gravitational constant
ϑ	Geocentric co-latitude
θ	Geodetic co- latitude
γ	Normal gravity
γ_{45}	Normal gravity at 45^0 latitude
g_{obs}	Observed gravity
R	Radius of earth

Chapter One

1 Introduction

1.0 Study area

The study area is located in the central part of Ethiopia around Debre Birhan town. It is located at about 135km North East of Addis Ababa. This particular study area extends from 9.6427^0N to 9.7051^0N latitude and 39.5055^0E to 39.5815^0E longitudes. Figure 1.1 shows the geographical location of this study area in relation to Ethiopia.

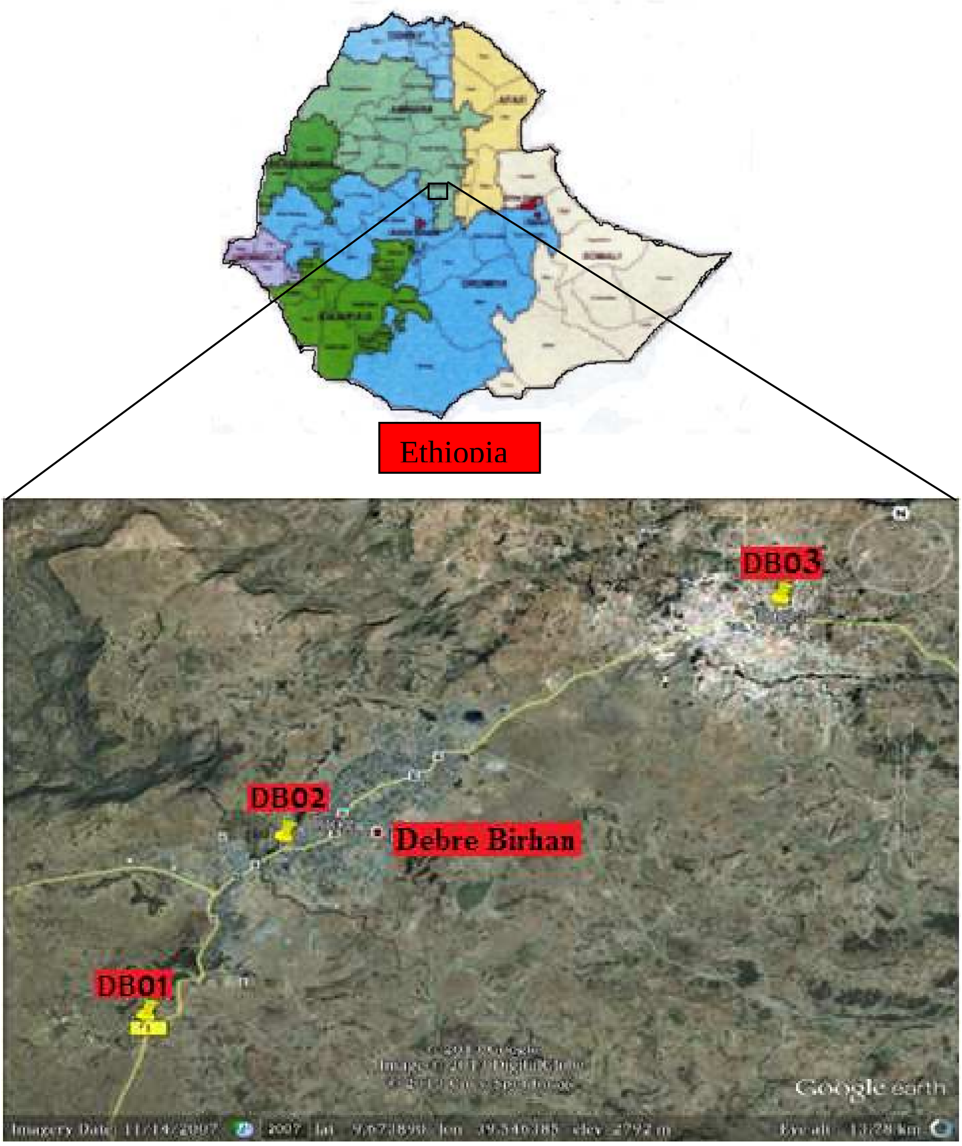


Figure 1.1: Location of the study area (Taken from Google Earth).

1.0.1 Location of station DB01

Station DB01 is located eight kilometer south west direction from Debre Birhan town. DB01 has geographic coordinate of 9.6427391^{0} latitudes, 39.50549467^{0} longitudes, 2814.250m mean see level height and ellipsoidal height of 2808.416m. This new benchmark is located at about 200 m south west of the old benchmark. It is also located at about 100m north east of the old Bahr Hile military training camp. For more detail site descriptions refer to Figure 4.2 below.

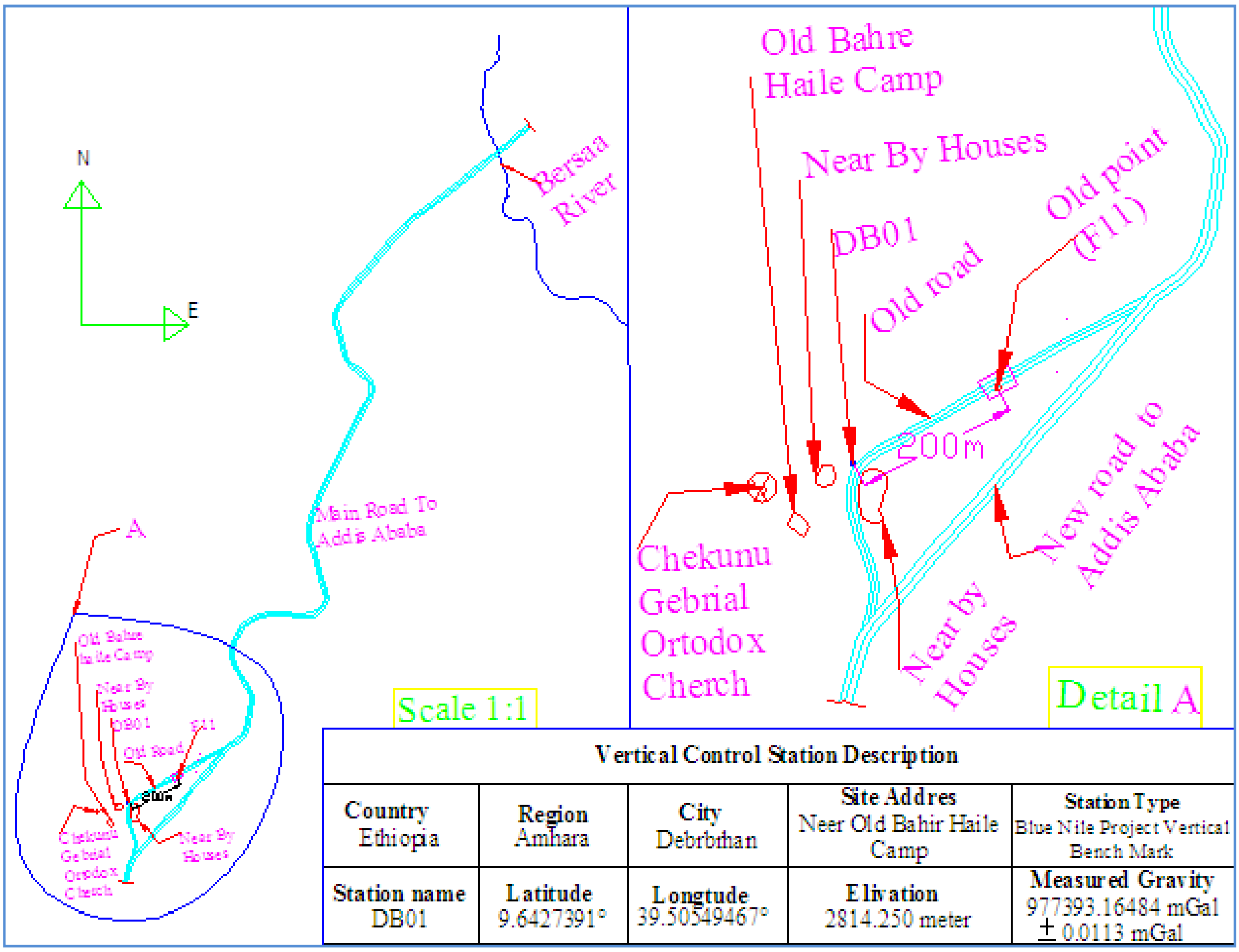


Figure 1.2: Site location and station description of DB01.

1.0.2 Site location of station DB02

Station DB02 is found inside the compound of the Debre Birhan teacher training campus. It is 20 meter perpendicular towards the campus fence from the center of Debre Birhan to Addis Ababa main road and 360 meter far from Berressa river main bridge towards teacher training campus along the main road (see, Figure 4.3). The geographical location of this new benchmark is 9.66945553^0 latitude, 39.52186606^0 longitude, 2767.987 meter and 2762.044 meter above mean sea level and above the ellipsoid respectively.

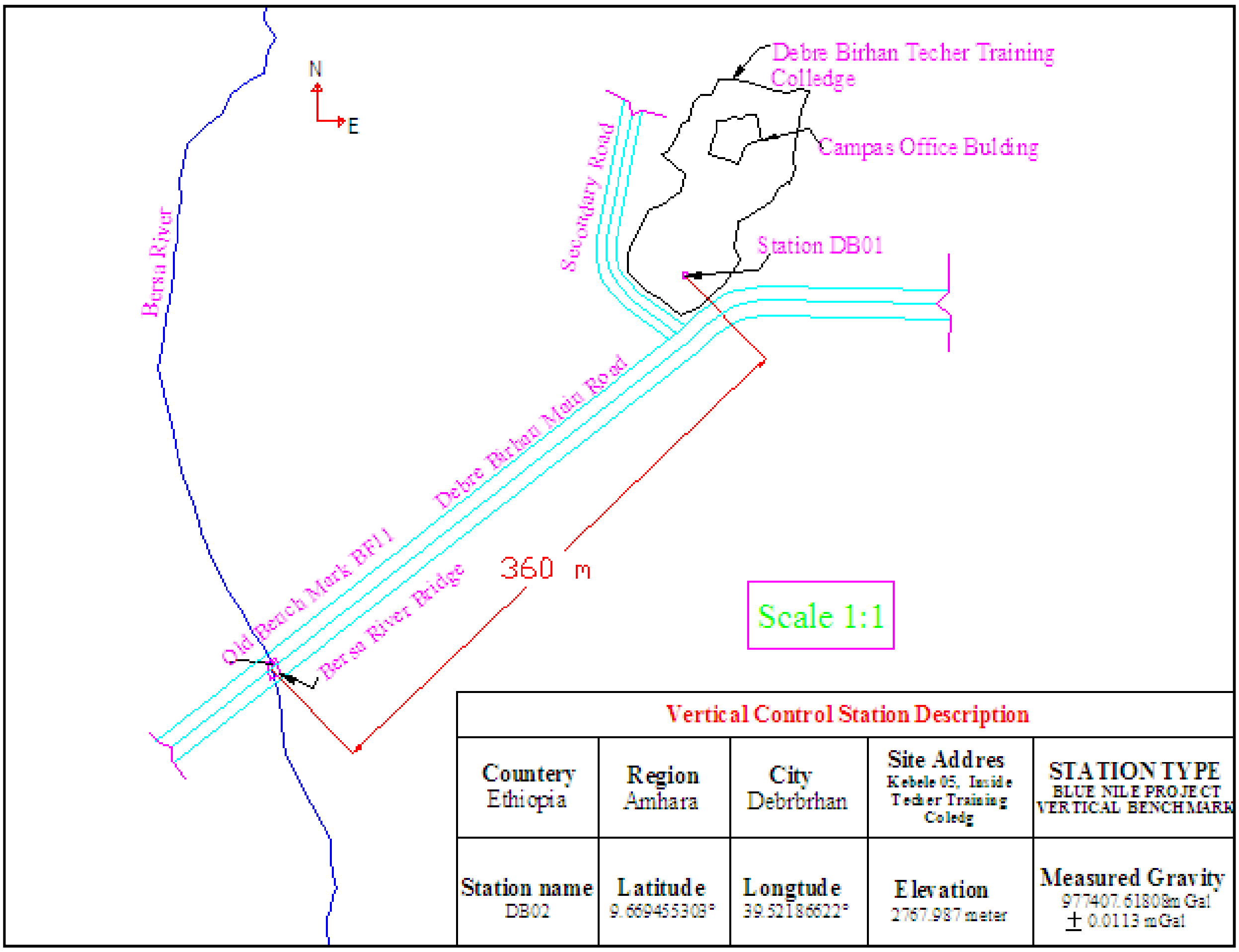
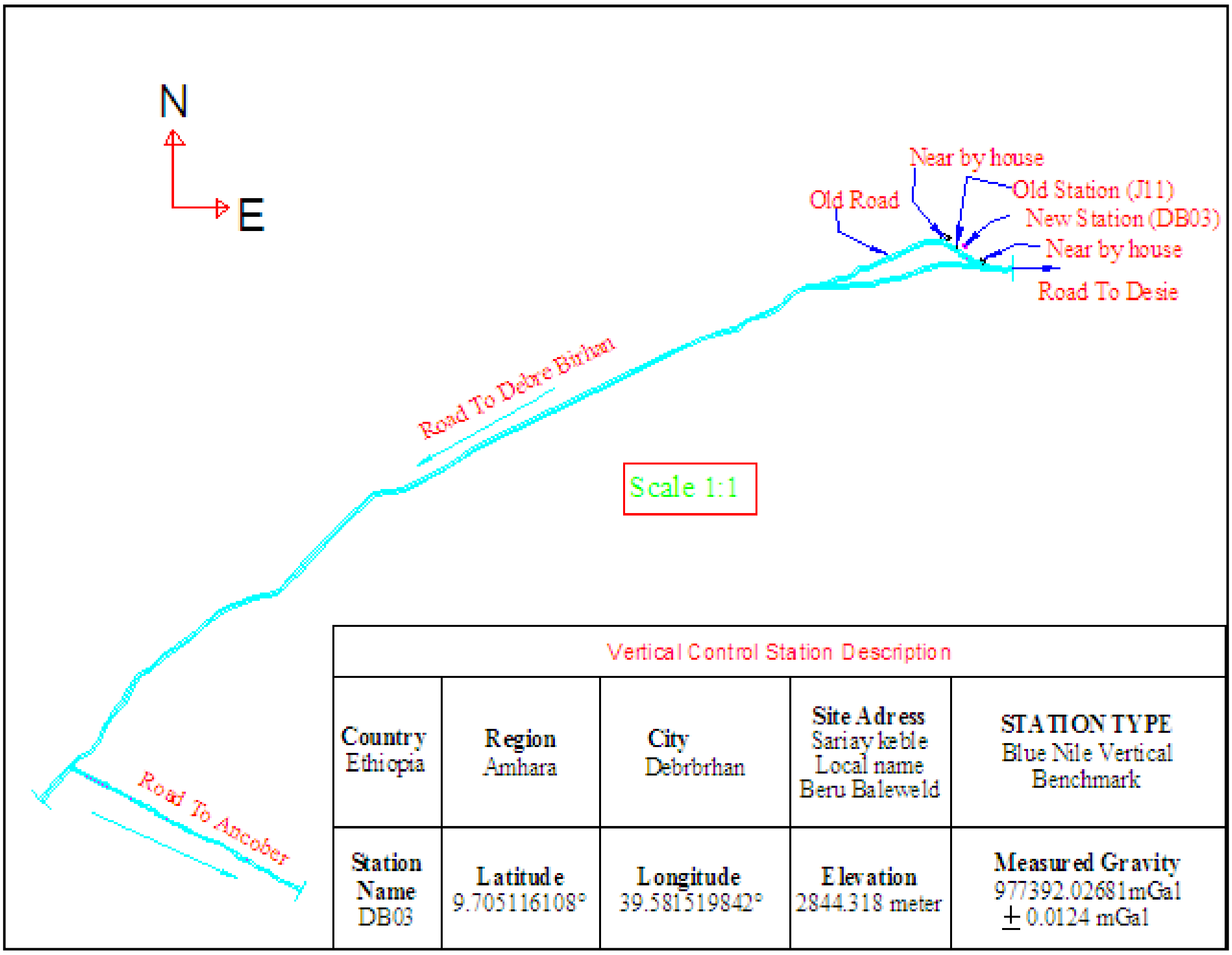


Figure 1.3: Site location and station description of DB02.

1.0.3 Site location of station DB03

Station DB03 is located in Saria Kebele which is known with its local name called Berru Baleweld. It is at about 32m in the north direction from the main Addis Ababa Dessie road (see, Figure 4.4). The geographical location is 9.7051159^0 latitude, 39.58151969^0 longitude, 2844.318 meter above mean sea level and 2838.249 meter of ellipsoidal height.



Vertical Control Station Description				
Country Ethiopia	**Region** Amhara	**City** Debrbrhan	**Site Adress** Sariay keble Local name Beru Baleweld	**STATION TYPE** Blue Nile Vertical Benchmark
Station Name DB03	**Latitude** 9.705116108°	**Longitude** 39.581519842°	**Elevation** 2844.318 meter	**Measured Gravity** 977392.02681mGal ± 0.0124 mGal

Figure 1.4: Site location and station description of DB03.

1.1 Background

A vertical reference system can be defined by directly computing the gravitational potential on or above the Earth's surface from observations of gravity. The gravitational energy difference between the gravitational potential of the mean sea level called 'geoid' and the gravitational potential of any point on or above the earth surface is called Geopotential height (Hofmann-Wellenhof and Helmut Moritz, 2005). In practice, Geopotential height can be determined in two ways. They are the classical and modern approaches. Classically, height is determined by connecting the continental filed point to a predefined mean sea level tide gauge station by a series of first order high quality geodetic spirit leveling fitted with two parallel micro plate optical telescope surveying. This technique measures the geometrical height difference between two points, where the reference surface is the local horizontal defined by the set-up of the leveling instrument. In principle, leveling increment /heights/ must be combined with gravity to determine the geopotential height difference between two points.

However, in most practical application leveling is not connected with gravity data, the geometrical leveling height increment are alone used to establish a classical vertical reference control benchmarks that refers to mean sea level. Mean sea level which serves as a vertical datum of the leveling-based traditional height system varies at regional and global scale due to earth system mass redistribution caused by climatic and geodynamical processes (Hipkin, 2002). Hence, this sort of height system doesn't provide an absolute datum - height cannot be reliably predicted at unknown filed points from the existing network of leveled heights (Hipkin, 2002; Bedada, 2010). Also, it is very time consuming, expensive and prone to accumulation of unmodeled errors. However, it is rigorous and can simulate to the new gravimetric approach of determining height from gravity observation.

Now with the availability of fast and accurate Global navigation Satellite System (GNSS) surveying technology, this sort of height system can be directly predicted from the earth gravity model. Different computational methods such as Stokes Integral (Stokes, 1849), Fast Fourier Transform (Hipkin & Hussain, 1983, Hipkin, 1988, Bedada, 2010) and Space Domain Spherical Harmonic Analysis (Hipkin, 2001; Pavlis et al.,2008) can be used to covert gravity to height measured in meter.

“In 1849, Stokes formulated a method of computing earth gravity potential. He computed a potential for a particular point by integrating surface gravity measurements over the whole globe. To simplify the integration process the Stokes like integrals operate on disturbing part of the Earth gravity filed. Figure 1.1 shows how stokes-like integrals relate the anomalous Geopotential T at point D (the origin) to a global integral of gravity anomalies, Δg or gravity disturbance δg.” (Bedada, 2010).

The Stokes’ integral is given by:

$$\mathrm{T_D} = \frac{R}{4\pi}\int_{\psi=0}^{\pi}\left[\int_{\alpha=0}^{2\pi}\Delta \mathrm{g}(\psi,\alpha)d\alpha\right]S(\psi)\sin\psi\, d\psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.1$$

Hotine Koch Integral written as

$$\mathrm{T_D} = \frac{\mathrm{R}}{4\pi}\int_{\psi=0}^{\pi}\left[\int_{\alpha=0}^{2\pi}\delta \mathrm{g}\,(\psi,\alpha)\mathrm{d}\alpha\right]\mathrm{K}(\psi)\sin\psi \mathrm{d}\psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.2$$

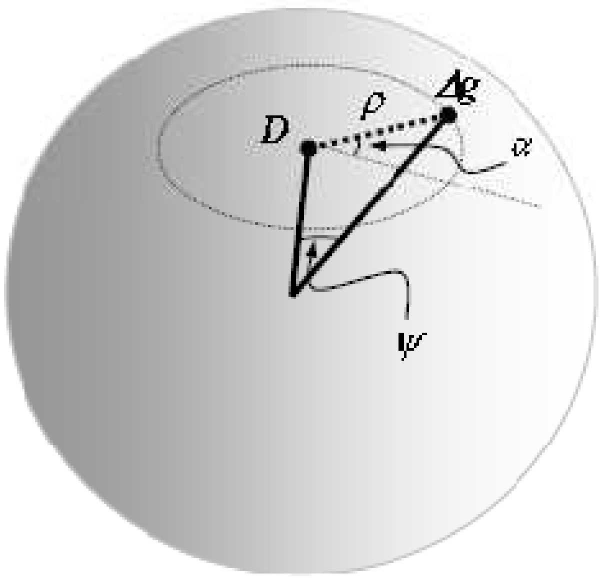


Figure 1.5: Spherical cap (dotted circle) defined by polar coordinates on sphere (ρ is radial distance (chord length) equivalent to ψ) (Bedada, 2010).

Where the gravity disturbance $\delta g = g - \gamma$ is the difference between observed gravity g and normal gravity evaluated at the same point, while the gravity anomaly $\Delta g= g-\gamma$ is the difference between observed gravity g at height 'H' above sea level and normal gravity γ evaluated at a height 'h' above the reference ellipsoid, R is the radius of the Earth, ψ is the geometric spherical angel, $S(\psi)$ is the Stokes' kernel $k(\psi)$ is Hotine-Koch Kernel, α is azimuth angle.

The modern approach of computing height from gravity provides a height that is absolute - the new height system refers to a global geoid defined by Wo=Uo, where Wo is potential of the real earth and Uo is potential of model earth (Hipkin, 2002, Bedada, 2010). This new height is called a geopotential height. The Geopotential height is defined by the gravitational potential difference between the potential Wo of a global datum point and the potential W of any point on or above the earth's surface. This is mathematically expressed by the following equation:

$$c = Wo - W(\varphi, \lambda, h) \quad \text{(1.3)}$$

$$W(\varphi, \lambda, h) = U(\varphi, \lambda, h) + \delta T(\varphi, \lambda, h) \quad \text{(1. 4)}$$

Where c is Geopotential number

Wo is the potential of the geoid

W(φ,λ,h) is the potential of the filed point at which GNSS measurement is taken.

U is the potential of the reference ellipsoid computed at GNSS observation point.

δT is the disturbing potential that can be computed by using Stokes integral (see, equation 1.1 & 1.2) or directly from the Stokes' coefficients $C_{n,m}$ &$S_{n,m}$ of the Geopotential model (EGM08), see section 2.3 for more detail.

φ is geodetic latitude

λ is λ is geodetic longitude

h is ellipsoidal height

This modern Gravimetric method of computing Geopotential height either from observation of local gravity data or directly from the Stokes' dimensionless cosine and sine coefficients of the earth gravity model (e.g. EGM2008) involves a Remove-Compute-Restore technique. Much active research has been conducted to determine the Geopotential height from the knowledge of the earth gravity filed using the Remove- compute - Restore approach. For example, Bedada (2010) has established an absolute Geopotential height system for Ethiopia in a Remove-Compute-Restore approach by combining the airborne gravity data, EGM08, gravity and potential models of the geometrical Shuttle Radar Topographic Mission (SRTM) and gravity and potential models of the spherical harmonic representation of the same topographic mass condensed on to the reference ellipsoid as a single surface density layer.

Using the Stokes-like integral, the remove stage removes the gravity effect of the earth gravity model, normal gravity of the reference ellipsoid and gravity of topographic effect from measured gravity. Compute stage first downward continues the residual gravity anomalies on to the reference ellipsoid. Secondly, it transforms gravity disturbance to disturbing potential. Finally it up ward continuous the disturbing potential at the desired filed point. Restore stage puts back the effect of removed mass distributions in the form of their potential. In essence it restores the potential of the reference ellipsoid (normal potential) to the potential of the earth gravity model and that of topographic contributions.

A space domain spherical harmonic analysis will be used in this study to validate the accuracy of the Geopotential height that is computed directly from the Stokes' dimensionless coefficients $C_{n,m}$ and $S_{n,m}$ of the 2008 Earth gravity model. Specifically, this study will compare the EGM08 derived geopotental height with high quality first order geodetic leveling data around Debere Birhan town. These leveling data to which our gravity based heights will be compared are collected by the Ethiopian Geodetic Survey project, 1957 - 1961.

The task of converting the Geopotential coefficients ($C_{n,m}$, $S_{n,m}$) to Geopotential height also involves the Remove - compute - Restore technique. The remove stage removes the normal potential of the reference ellipsoid from the EGM08 model - the remaining is the disturbing potential generated by density variation and topographic irregularity. The compute stage computes the disturbing potential at the field point. The last Restore stage, adds back the potential of the removed reference ellipsoid to providing the Geopotential, W of the earth gravity model at the desired GPS sites on or above the earth's surface.

This modern gravimetric approach of computing the Earth's potential W or equivalently the Geopotential number 'c' on or above the earth's topography need to know the absolute coordinate of a particular filed point from GNSS observation. In this research conducted GNSS observation co-located with new leveling measurements with a tie to the existing leveling benchmarks of the Blue Nile Investigation Project around Debre Birhan town. This will provide a unique opportunity to evaluate the accuracy of the EGM08 against GPS/Leveling in the study area.

1.2 Previous study-The Blue Nile geodetic leveling height network

The Blue Nile river basin geodetic control project of 1957 to 1961, which was accomplished under the authority of agreement between the imperial Ethiopian government and the government of the United States, specifically the "Third Operational Agreement for the Extension of the Program for the Study of the Water Resource of Ethiopia for Multipurpose Blue Nile River Basin Investigation" dated June 26, 1956. This agreement provided for a joint program under the stake holder of the Ministry of Public Works of the Imperial Ethiopian Government and the United State Operation Mission to Ethiopia of the international cooperation administration. A special service agreement signed on March 5, 1957; the U.S. Department of Commerce agreed to furnish the international cooperation administration and technical assistance in geodesy connection with the cooperative program between the United States and Ethiopia. This agreement established the authority purpose and scope for the conduct of the geodetic control survey on the Blue Nile river basin of

Ethiopia. The technical requirements of the program were comprise aerial photograph, preparation of topographic map and the establishment of precise geodetic control points.

The area covered by Geodetic Control Project include the Ethiopian watershed of the Blue Nile River Basin which was refers from existing maps, it extended to assure inclusion of all tributaries and to provide map boundaries. The other phase in the project was the coverage of the areas required to make the connection to Sudanese control and to existing triangulation in Ethiopia. The triangulation extended from Khashum el Girba in Kassala Province of Sudan, eastward across the Ethiopian-Sudan border in to western Eritrea, thence southward along the border to the vicinity of Metemma in Ethiopia, and thence in to the Blue Nile River Basin area. The leveling survey extended from Gedaraf of Sudanise province to south east ward , and the leveling survey continued to Ethiopian province; Azezo, Addis Zemen, Dessie, Addis Ababa, Nekhempti, Asosa, Metekel, Bahar Dar, with a loop tie-back into Addis Zemen (Ethiopian geodetic survey, 1957-61).

Currently Ethiopia uses the traditional leveling height tide to the Blue Nile benchmarks. These benchmarks are connected to a tide gauge benchmark found at port of Egypt which is 2500 km away (Ethiopian geodetic survey 1957-61). Now, these benchmarks are mostly destroyed due to mainly man made factors like emerging of small village, expansion of road, construction of new building, expansion of cultivated land, erosion and land degradation. The Blue Nile geodetic team have establishes about 40 leveling benchmarks between Addis Ababa and Debre Brihan but at present (2013), only seven benchmarks exist-three of them are located around Debere Birhan and the other four are found in Addis Ababa. The original Blue Nile benchmark between Addis Ababa and Debre Birhan is presented in the figure2.1

below. This traditional way of computing height is tedious, time consuming, take much money and material, it is exposed to accumulation of an unmodelled random error. So there is a real need for a new approach which computes the height of the filed point by referring to a global reference surface called geoid using Earth gravity model (EGM2008) and GPS technology. This research will show how the new gravimetric based height prediction can best simulate to the classical leveling data.

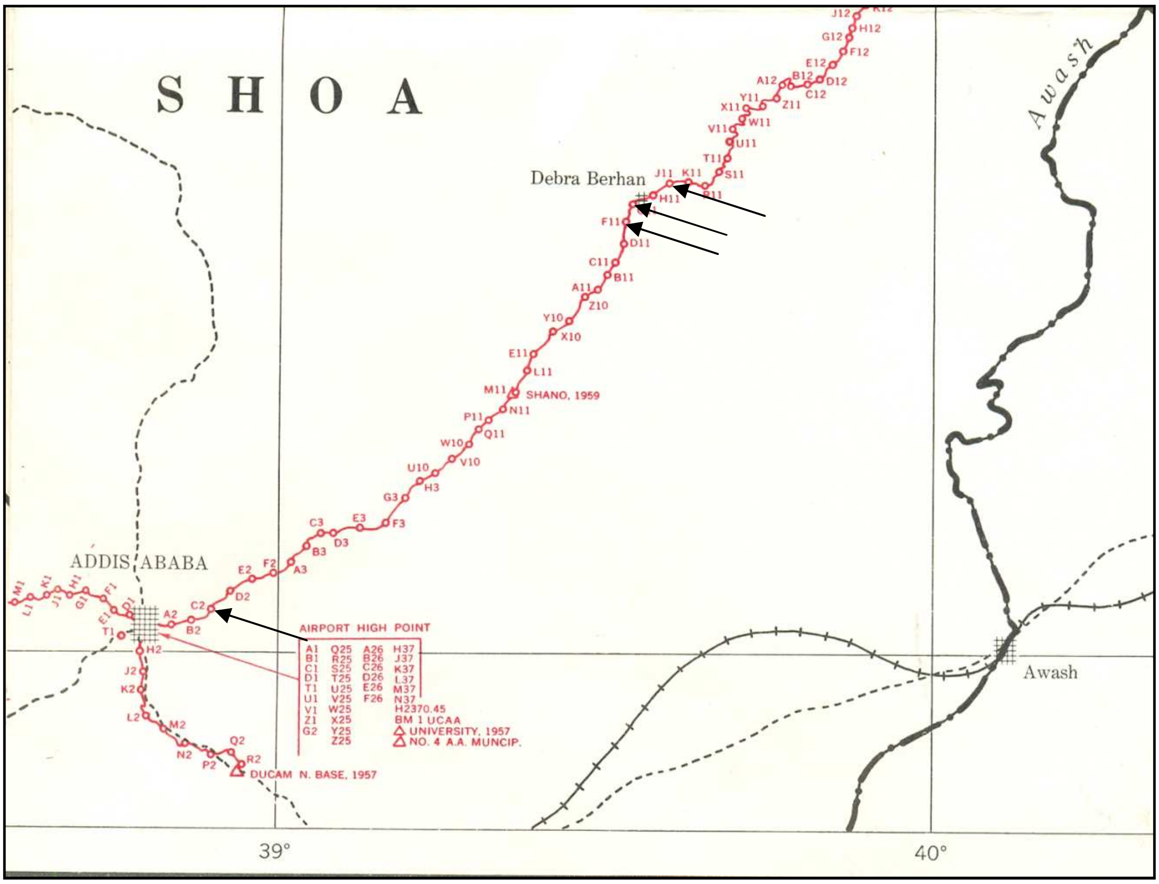


Figure 1.6: Original Blue Nile benchmark between Addis Ababa and Debre Birhan (Blue Nile Geodetic Control Project, 1957 1961).

1.3 The Statement of the research problem

The present practice of height computation in Ethiopia involves the task of connecting the field point to the local reference benchmarks by using geodetic leveling. This classical approach has the following practical problems.

1) Computing height for a single point involves the task of physically connecting the particular point to a local network of leveled heights. This cannot provide an opportunity to interpolate height for an unknown point from the existing network of leveled height.

2) The existing vertical control points are few and poor so that engineering works in an area where there is no local benchmark is expensive and time consuming.

3) It is prone to accumulation of unmodelled random error.

With a very increasing infrastructural development growth in Ethiopia, the old system cannot provide the required fast and economically cheap surveying technology. Therefore, this thesis proposes the fastest, cheapest and reliable approach of predicting height directly from the knowledge of the earth gravity filed and GPS technology. It determine height for any isolated filed point on or above the earth's surface from EGM08 and GPS coordinate without connecting to any local network of vertical control point.

The merits of the new technology are:

1) It is cheap and fast.

2) It has an opportunity to interpolate height for an unknown point from predefined grids of gravitational potential.

3) It can determine the height of the point independently without connecting to any local network of vertical control points.

1.4 Objective

1.4.1 Main objective

To precisely determine the height of a point using earth gravity model and Global Positioning System (GPS) measurement and compare the new height with the classical height and evaluate the quality of the new technology.

1.4.2 Specific Objective

1) To compute Geopotential height from global Geopotential coefficients $C_{n,m}$, $S_{n,m}$.
2) To compare gravity based orthometric height with leveling ortometric height.
3) To compare gravity based normal height with leveling height.
4) To compare gravimetric geoid height with GPS/leveling geometric height.
5) To compare gravity derived height anomaly (Zeta) with GPS/leveling geometric geoid height.
6) To establish an absolute accurate local vertical control benchmarks for civil and water engineering works as well as future geodetic research work.

1.5 Thesis Organization

This thesis comprises five chapters. Chapter two explains different methods of computing potentials, computing different type of Geopotential height based on the Geopotential number and their evaluation, definition, meaning, the relation among them are discussed, in addition the Earth gravity model is discussed.

Chapter three discussed about theory of GPS and answers the question of how the GPS satellite determines the user position. In addition, overviews of GPS working mechanism, GPS system and sources of errors in GPS measurements are presented.

Chapter four comprises the main findings of the research. In this chapter, the site location of the study area, leveling work, GPS and gravity data process outcomes are presented. In addition comparison of EGM08 derived Geopotential heights with GPS/leveling in centimeter accuracy are presented.

Chapter five contains conclusions and recommendations.

Chapter two

2 Computing Geopotential number

2.1 Introduction

Geopotential number is the gravitational energy difference between the potential $W(\varphi,\lambda,h)$ of any point on or above the earth's surface with respect to a particular datum potential Wo (i.e., potential of the geoid). In general, Geopotential number can be computed by using two different techniques. The first technique involves the classical approach of computing Geopotential number using geodetic sprite leveling connected with gravity measurement (see, section 2.4). The second approach computes the Geopotential number from observation of gravity field by using the Stokes'- like integral or equivalently by computing the Geopotential number directly from global gravity model (e.g., EGM08) using the spherical harmonic analysis of the Stokes dimensionless sine and cosine coefficients.

As the aim of this study is to evaluate the accuracy of EGM08 geopotential height around Debre Birhan town using an independent leveling data obtained from the Blue Nile Geodetic survey project (1957-1961) document, in this chapter, the second approach of computing Geopotential number is presented. This chapter first reviews how to compute Geopotential height using Stokes- like integral (section 2.2) and then it presents Space-Domain Spherical Harmonic Analysis that is used in this study to determine Geopotential height directly from the Stokes' coefficients $C_{n,m}$ & $S_{n,m}$ of the 2008 Earth gravity model (EGM08), see section 2.3. Section 2.4, 2.5, 2.6, 2.7 & 2.8 also explores how to measure a true height system based on the Geopotential number or earth gravity field. Section 2.9 presents the global gravity model used in this thesis to compute Geopotential height system.

2.2 Stokes' approach of computing potential

In 1849, Stoke's device methods of computing potentials, he relates the disturbing potential of the computation point to a global integral of gravity anomaly Δg or gravity disturbance δg. The transformation formulas have been shown in equation 1.1 and 1.2, see chapter one. Stokes integral involves the regularization of a global topographic model and an integral transform of residual gravity anomalies over the whole glob to compute anomalous potential (Bedada, 2010). The approach starts with spherical Earth model and subtracts in situ topographic masses, adding back as a coating on the surface of zero height. The gravity filed above this surface then satisfies Laplace's condition.

The remove - compute - restore technique computes the anomalous potential rather than computing the whole components. The remove process subtracts the normal gravity from observed gravity to get the observed gravity disturbances at point $(\varphi,\lambda.h)$. Trend free and smoothly varying residual gravity disturbance is obtained by further subtracting the gravity disturbance derived from the EGM08 model and the topographic gravity model from the observed gravity disturbance and by adding back the long wave length component of the topographic mass condensed as a single layer surface density on to the reference ellipsoid. The task of mathematically removing the gravitational effect of the topographic mass distribution above the reference ellipsoid create a unique opportunity to apply Laplace's equation in transforming gravity anomalies to disturbing potential when computed on the surface of the reference ellipsoid. The compute stage transforms residual gravity to potential on to the ellipsoid. The restore stage computes $W(\varphi,\lambda.h)$, by first continuing the disturbing potential evaluated on to the reference ellipsoid at the desired filed point and then adding back the potential of the EGM08, normal potential and the potential of in situ topography followed by subtracting the potential of the condensed topography.

In this approach, the potential W(φ,λ,h) is obtained from evaluation of the low degree spherical harmonic coefficient of the global gravity model plus contribution of high resolution local gravity anomalies from the Stoke's integral (Bedada, 2010). The equation is written as:

$$T_p = T^{n}_{EGM08} + \frac{R}{4\pi}\int_{\psi=0}^{\psi_o}\left[\int_{\alpha=0}^{2\pi} \Delta g(\psi\,,\alpha)d\alpha\right]S(\psi)\,sin\,\psi\,d\psi$$
$$+\frac{R}{4\pi}\int_{\psi=\psi_o}^{\pi}\left[\int_{\alpha=0}^{2\pi} \Delta g(\psi,\alpha)d\alpha\right]S(\psi)\,sin\,\psi\,d\psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

Where the first term in equation 2.1 represents the contribution from the long wave length component of the global Geopotential model complete to spherical harmonic degree m and order n, the other two integral are contribution from the residual gravity anomalies.

2.3 Space-Domain spherical harmonic analysis

The gravity field of the Earth is represented by a spherical harmonic expansion with harmonic geopotential coefficients (Stokes parameters) complete to degree and order. The data for such a geopotential model comes from satellite observations (long wave length spectrum) and airborne/ terrestrial and shipborne gravity survey recovering medium and short wave length spectrum of the earth's gravity filed.

This research uses space domain spherical harmonic analysis to determine the Geopotential heights directly from Stokes' dimensionless sine and cosine coefficients. First, the regular grid of disturbing potential δT is computed at different layers (0, 500,1000, ……...., 4000km) ellipsoidal heights and the normal potential of the removed reference ellipsoid is restored to compute the gravitational potential W and the Geopotential height c, at the desired field point on or above the earth surface (Hofmann-Wellenhof and Helumt Mortz, 2005). The Potential W(φ,λ,h) of the field point can be computed from the spherical harmonic coefficients using the following formula.

$$W_{(\varphi,\lambda,h)} = \frac{GM}{a}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^{n+1}\sum_{m=o}^{n}\overline{P_n^m}\,(cos\vartheta)\left\{(\overline{C_n^m}\,cos(m\lambda) + \overline{S_n^m}Sin(m\lambda))\right\} \ldots\ldots (2.2)$$

Where

$\overline{P_n^m}$ - is fully normalised Legendre's function

$\overline{C_n^m}$ - is fully normalised cosine coefficient

$\overline{S_n^m}$ - is fully normalised sine coefficient

G is universal gravitational constant

M is mass of the earth

a is semi major axis

n & m are degree and order of spherical harmonic coefficients

r is radius of the reference ellipsoid

ϑ is geocentric colatitudes

φ, is geodetic latitude

λ is geodetic longitude

The disturbing potential is obtained by subtracting the normal potential of the reference ellipsoid from the real potential Wo of the earth gravity field evaluated on the surface of the 1980 Geodetic Reference System (GRS80) or equivalently on the 1984 World Geodetic System (WGS84). Practical computation of the normal potential and therefore disturbing potential involves the use of the geodetic parameters defining a particular reference ellipsoid used. For example, the current values of the four geodetic parameters (GM, a, J_2 and ω) defining the 1980 Geodetic Reference System (GRS80) ellipsoid are:

$GM=3.986005x10^{14}m^3s^{-2}$, a=6378137m, $J_2=1.08263x10^{-3}$, $\omega=7.292115x10^{-5}rads^{-1}$

Where, GM is the geocentric gravitational constant; 'a' is semi-major axis of the reference ellipsoid; J_2 dynamical form factor of the earth; and ω is the earth rotation rate. Values of geodetic earth parameters of the 1984 World Geodetic System (WGS84) are: a = 6378137m, $GM=3.986004418x10^{14}$, $\omega=7.292115x10^{-5}$, f=1/298.257223563, this flattening is derived from the normalized second-degree zonal gravitational coefficient (C2,0), (Hofmann-Wellenhof and Helmut Mortz, 2005).

The spherical harmonic analysis uses a geodetic coordinate system. Therefore a conversion from ellipsoidal coordinate to geocentric coordinate system is necessary. The conversion from one another is done by the following formula:

$$\vartheta = 90 - \varphi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

ϑ is geocentric colatitudes & φ is geodetic latitude.

The recurrence relations of Legendre's function play a key role to synthesis a spherical harmonic analysis fast. Each associated Legendre function is evaluated from one or two already calculated value. The spherical harmonic synthesis program "Schmidt" uses Associated Legendre Functions $\overline{(P_n^m)^*}$ of the Schmidt Semi-normalized form. The fully normalized spherical harmonic coefficient of the EGM08 has to be converted to Schmidt Semi-normalized coefficients. Therefore, the Schmidt semi-normalized Legendre function has the following form:

$$\overline{(P_n^m)^*} = \frac{1}{\sqrt{(2n+1)}}\, x\, \overline{P_n^m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots (2.4)$$

For non-sectorial harmonics (n≠m), the Legendre polynomials are computed with a recurrence relation using equation 2.5, but with m=n-1 second term of equation 2.5 has a zero multiplier, so the non-existent polynomial P_{n-2}^{n-1} is not required. The recurrence relation then has only one term.

$$P_n^m \cos\theta = \frac{(2n-1)\cos\theta}{\sqrt{n^2-m^2}}\, P_{n-1}^m(\cos\theta) - \frac{\sqrt{(n-1)^2-m^2}}{\sqrt{n^2-m^2}} P_{n-2}^m \cos\theta \ \ldots\ldots\ldots (2.5)$$

For sectorial harmonic, the recurrence relation is given by:

$$P_n^n(\cos\theta) = \frac{\sqrt{2n-1}}{\sqrt{2n}} \sin\theta\, P_{n-1}^{n-1}(\cos\theta) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2.6)$$

Spherical harmonic could be geometrically represented by zonal, tesserial and sectorial. Zonal harmonics are defined by the mth order term is zero (m=0), this harmonic are termed zonal since the curves with center at the origin on which $P_n^m \cos\theta$ vanishes are parallel of latitudes which divided the surface in to zones, see, figure 2.1a for representation of zonal harmonics. Tesserial harmonic includes all other terms (m ≠0) and these terms depend on latitude and longitude. They divide the sphere into compartments in which they are alternatively positive and negative and it's representation is shown in figure 2.1b. Sectorial harmonics are occur when the degree and order are equal (n=m). These terms depend only on the longitude and degenerate function that divided the sphere into positive and negative sectors, see figure 2.1c.

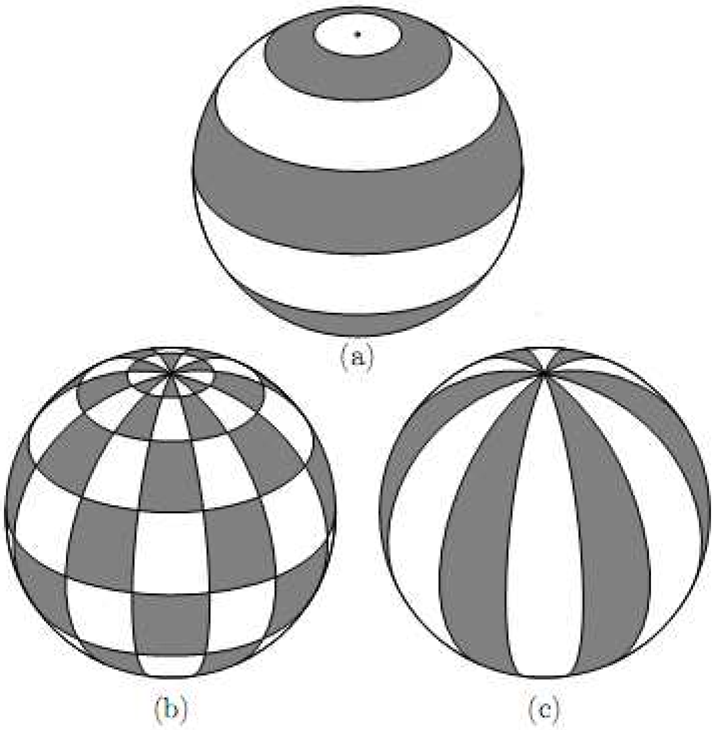


Figure 2.1: Kind of spherical harmonics: (a) zonal, (b) tesserial, (c) Sectorial, Hofmann-Wellenhof and Helmut Mortz, 2005).

2.4 Geopotential number

Geopotential number is defined as energy difference between the potential of the geoid and the potential of particular field point on or above the earth's surface. This energy difference or geopotential number is equal to the work done by the conservative gravitational force along the path length projected in the direction of gravity field. This is given by evaluating the Stokes line integral as below (Heiskanen and Mortiz, 1967). Geopotential number is used to trace a level or equipotential surface on the earth's surface through a GPS coordinate (φ,λ,h). Equipotential surfaces are closed and continuous surfaces with a unique constant gravity potential value, W.

$$C_p = W_o - W_p = \int_0^P \boldsymbol{g.dn} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.7)$$

Where

Wo- is the constant potential value of the geoid.

Wp represents the potential of the equipotential surface through the point p at the earth's surface.

Geopotential number is independent of the particular leveling line used to relate the continental filed point 'p' to sea level (datum). Geopotential number is defined to be always positive (Hofmann-Wellenhaf and Helmut mortiz, 2005). Figure 2.2 shows how leveling connects a point on the continent to the mean sea level tide gauge station through a series of leveling network.

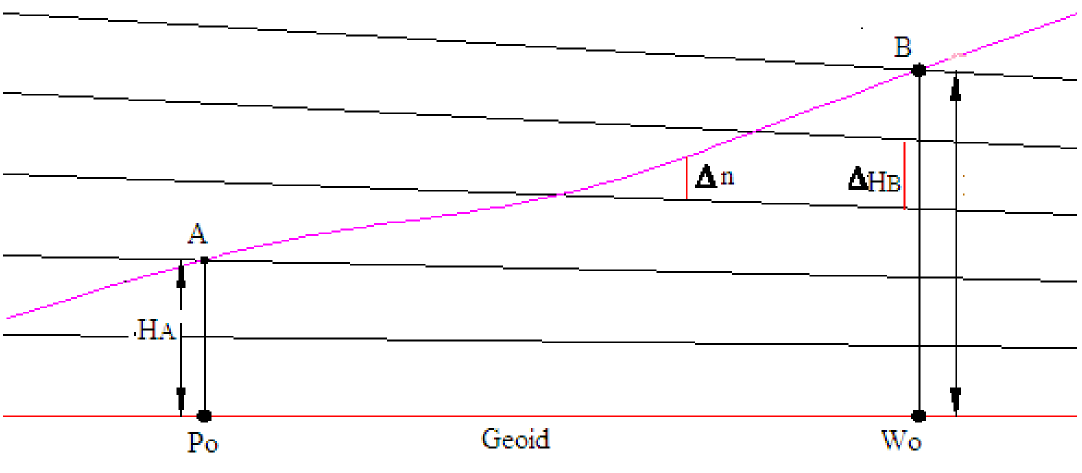


Figure 2.2: The principles of leveling and equipotential surface (Hofmann Wellenhof and Helmut Mortiz, 2005), Po is a point on the geoid defined by a unique potential Wo.

The leveled height differences are path dependant and not the same as physical height differences ($\sum \Delta n \neq H_B - H_A$). The value of the geopotential number depends on the vertical differences, Δn, obtained from leveling between the equipotential surfaces and on the value of the gravity, g, measured at the leveling points. The geopotential number c is independent of any particular leveling line connecting a particular field point to sea-level (geoid). The Geopotential number is measured in geopotential unit (g.p.u) where 1 g.p.u = 1kGal m = 1000Gal m (Hofmann-Wellenhof and Morttiz, 2005). It is usually scaled by gravity to obtain height measured in meter at the required field point. Different height values can be obtained at the same point depending on the gravity value used in its computation. The next sections deal with convenient approaches of converting Geopotential numbers to different form of height in meters.

2.5 Orthometric height

Orthometric height is the distance along the plumb line between the geoid and the point located on the earth's surface (Hofmann-Wellenhof and Helmut Mortiz, 2005). Figure 2.3 shows a representation of the orthometric height- the curvature of the plumb line us exaggerated to show its difference with respect to the direction of ellipsoidal normal. The orthometric height of a point 'P' on the earth's surface is denoted by Hp and it is computed by the following equation:

$$H_p = \frac{c_p}{\bar{g}} \quad \ldots\ldots (2.8)$$

$\bar{g}$, is the averaged gravity value along the plumb line computed between the geoid and the field point 'p' on the earth surface. It is calculated by using the following integral:

$$\bar{g}_p = \frac{1}{\mathrm{H}}\int_0^H g dH \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.9)$$

In principle, $\bar{g}$ cannot be determined exactly due to inadequate knowledge of the surface density variation along the plumb line. Either, it is not practical to measure the gravity along the plumb line inside the earth's surface. Thus, the determination of the orthometric height depends on the approximation used in computing the mean value of gravity evaluated between the geoid and the filed point (Hofmann-Wellenhof and Helmut Mortiz, 2005).

The first approximation uses Helmert's approach that considers the topography above the geoid with an infinite Bouguer plate of constant density and of height 'H'. This approximation assumes that the density gradient along the plumb line is essentially zero while the effect of gravity disturbance is only manifested by free-air gradient of the normal gravity filed. The gradient of the actual gravity filed caused by vertical variations of subsurface density/mass distribution is neglected. The mathematical representation of the Helmert's assumption is given by the following equation.

$$\bar{g} = g_p - 2\pi G\rho H_p - \frac{1}{2}\frac{\partial\gamma}{\partial h}H_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

Where, G is the Newton's gravitational constant, and its value is 66.7x10^{-9} cm^3 kg^{-1} sec^{-2}. The expression is simplified by using a crust mean density of ρ=2.67 g cm^{-3} and a normal gravity gradient $\frac{\partial\gamma}{\partial h}$ = 0.3086 mGal m^{-1}. After substitution of these numerical values, the simplified expression is given by:

$$\bar{g} = g_p + 0.0424H_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

Therefore, the general equation for Helmert orthometric height can be rewritten as:

$$H_p = \frac{c_p}{g_p + 0.0424H_p} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2.12)$$

Where, c is in g.p.u; g is in gal; Hp is in km

This equation is solved by iterations due to the fact that the computation of gravity along the plumb line always requires H_p information (Hofmann-Wallenhof and Helmut Moritz, 2005; Jekeli, 2000).

Prey has also proposed an alternative approach of converting Geopotential number in to orhtometric height. His reduction assumes that it is sufficient to calculate $\bar{g}$ as orthometric mean of gravity g measured at the surface point 'P' and of gravity g_o computed at the corresponding geoid point 'p_o' (Hofmann-Wellenhof and Helmut Mortiz, 2005). This approximation supposed that gravity varies linearly along the plumb line. The approximation of mean gravity is given by:

$$\bar{g} = \frac{1}{2}(g + g_o) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (2.13)$$

The approximation can usually be assumed with sufficient accuracy, even in extreme cases as shown by Mader (1954) and by Ledersteger (1955). The accuracy of this prey approximation will be compared with the Helmert's approach as a test experiment in this research.

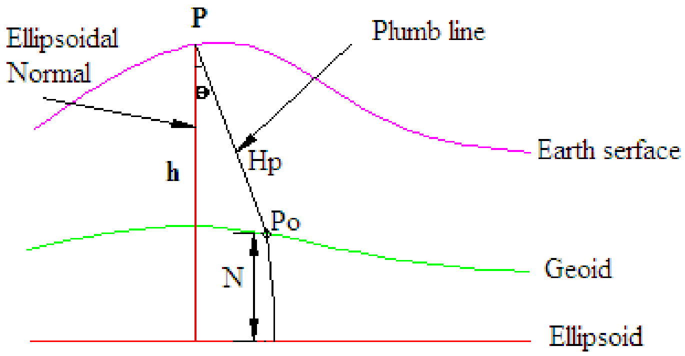


Figure 2.3: Relationship between orthometric, ellipsoidal & geoid height.

2.6 Dynamic height

Dynamic height is the scaled geopotential number by a normal gravity evaluated at 45 degree latitude γ_{45} on to the reference ellipsoid (see, Equation 2.14) below. The conversion of a geopotential number to dynamic height is definitive. Dynamic height can map level surfaces-point with the same dynamic heights are strictly on the same equpotential surface. However, due to historical reasons dynamic height is considered as an obsolete.

$$H_p^{dyn} = \frac{c_p}{\gamma_{45}} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .. (2.14)$$

Where H_p^{dyn} is dynamic height of point p, Geopotential number at point p, γ_{45} is normal gravity at latitude of 45^0, and it is equal to 9.806199203ms^{-2} (Hofmann-Wallenhof and Helmut Moritz, 2005).

2.7 Normal height

Normal height is the result of Geopotential number scaled by normal gravity. It is introduced in order to avoid any hypothesis or modeling of the density distribution of the topographic masses. This is attained by using the normal gravity filed which can exactly be calculated at any point. The normal height is computed as follows:

$$H_p^n = \frac{\mathrm{c_p}}{\bar{\gamma}} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots . \dots (2.15)$$

Where H_p^n normal height at point p, C_p is Geopotential number at point p, $\bar{\gamma}$ is the mean normal gravity along the plumb line, a point ‘P’ on the earth’s surface has a certain normal potential Up, but in general Wp≠Up.

However, there is a certain point ‘o’ on the plumb line of p, such that Uo’ = Wp; that is, the normal potential U at ‘o’ is equal to, the actual potential W at P. the normal height H_P^n of point p is defined as the ellipsoidal height of point 'o' above the ellipsoid. This definition of normal height can be equivalently defined by using the definitions of height anomaly, telluroid and quasi-geoid as shown in the figure 2.4. The distance between the telluroid (defined by Uo=Wo) and the earth's surface is called height anomaly, and it is denoted by Z_p. Height anomaly can be also defined as a vertical height separation between the ellipsoid and the quasi-geoid. The normal height of the point p, is represented by the distance between the point on the earth's surface and the quasi- geoid-the distance H_P^n and Zp are often reversed along the plumb line. The surface obtained by plotting Z_p above the ellipsoid is called quasi-geoid. It is a geoid like surface obtained by Molodensky's solution (Hofmann-Wellenhof and Helmut Moritz, 2005). Unlike the geoid, the quasi-geoid is not an equipotential surface either in the normal or the actual gravity filed and has no physical meaning. There are two advantages of using normal heights:

1) Density information is not required to compute the normal height (Hofmann-Wellenhof and Helmut Moritz, 2005; Jekeli, 2000).

2) The exact value of the normal height can be calculated using normal gravity filed.

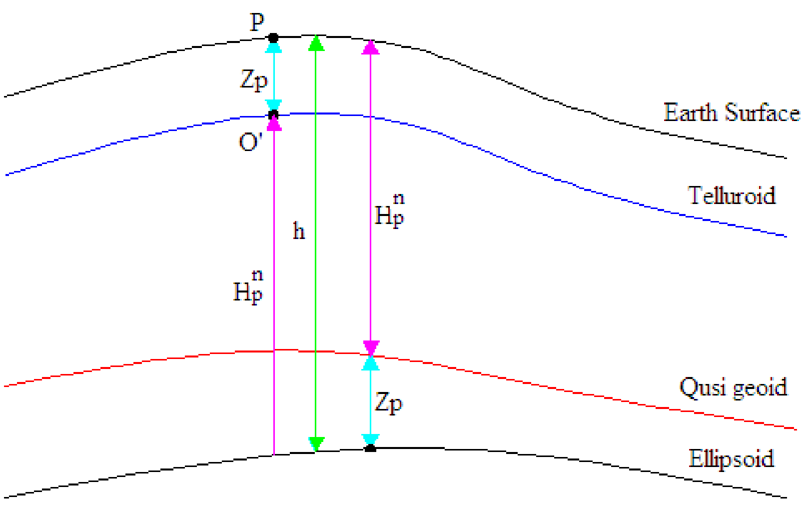


Figure 2.4: Relation between Normal height, height anomaly, telluride & quasi-geoid.

2.8 Relationship between orthometric, Normal and Dynamic heights

In theory, the geoid height and the height anomaly, as well as all other type of heights, can be linked by the Geopotential number. Also we can write the relationship between the geoid height and height anomaly by the following equation:

$$h = H + N = H^{\mathrm{n}} + Z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots.\ldots.\ldots. \quad (2.16)$$

And

$$N - Z_p = H_p^n - H_p = \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H_p \approx \frac{\Delta g_b}{\bar{\gamma}} H_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.17)$$

Where Δg_b is the Bouguer gravity anomaly and $\bar{\gamma}$ is the mean normal gravity along the normal plumb line (Hofmaann-Wellenhof and Mortiz, 2005).

The orthometric and normal heights are defined geometrically. The calculation of the orthometric height requires the knowledge of the mass density of the crust. In the contrary the exact value of the normal height can be determined exactly with no density knowledge. Unlike the orthometric, normal or ellipsoidal heights, the dynamic height are physically meaningful and indicate the direction of the flow of water.

The relationship between orthometric height, normal heights, geoid height and height anomaly (quasi-geoid height) are shown in figure 2.5. The geoid undulation or geoid height N, represents the separation between the ellipsoid and the geoid along the ellipsoidal normal, and the height anomaly, Z_{p} , represents the separation between the ellipsoid and the quasi-geoid along the ellipsoidal normal (see figure 2.5). In flat areas, the height anomaly is close to the geoid height.

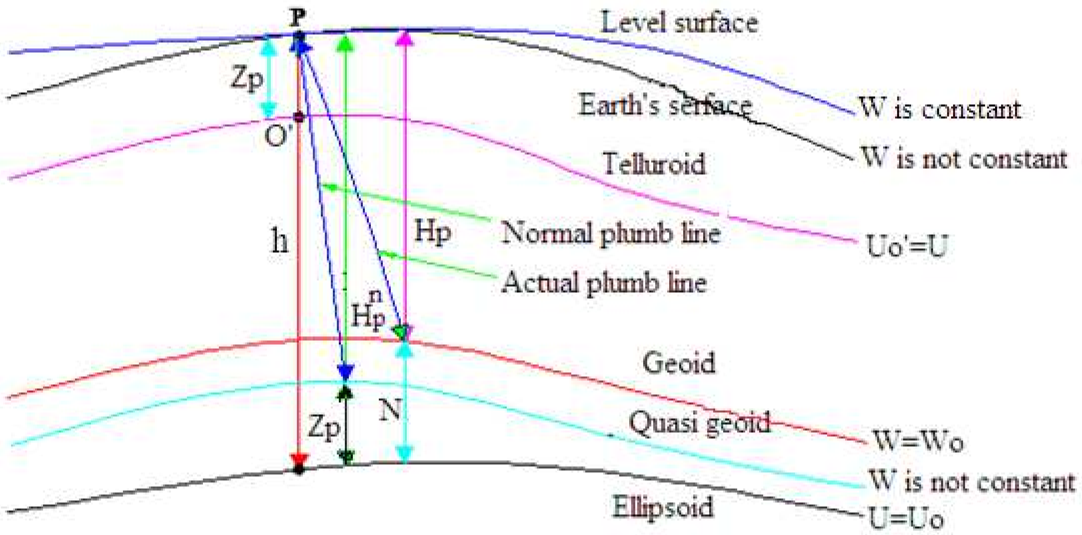


Figure 2.5: Relationships between, geoid, quasi-geoid, ellipsoid and height system.

Table 2.1: Height system and their meaning.

Height Type	**Formula**	**Meaning**	**Implication**
Ellipsoidal height	$h_p = H_p + N$	Geometrically meaningful.	Geometrically defined along the perpendicular to the ellipsoid.
Dynamic height	$H_p^{dyn} = \frac{c_p}{\gamma_{45}}$	Physically meaningful and associates with a value computed at a fixed latitude	Indicates the direction of water flow.
Orthometric height	$\mathrm{H_p} = \frac{\mathrm{c_p}}{\bar{g}_p}$	Geometrically meaningful and cannot be determined exactly.	The distance along the plumb line between the geoid and point on the earth's surface. The calculation needs a complete knowledge of sub surface density variation.
Normal height	$H_p^n = \frac{\mathrm{c_p}}{\bar{\gamma}_\mathrm{p}}$	Geometrically meaningful and can be determined exactly	The distance between the quasi-geoid and the point on earth's surface. There is no need to make approximations for the density of the earth crust
Geoid height	N=hp-Hp	The separation between the geoid and the reference ellipsoid	Used in the conversion of the geometrically defined height in to physical heights.
Height anomaly	$Z_p = h_p - H_p^n$	The separation between the quasi-geoid and the reference ellipsoid	Approximation of the geoid undulation according to the Molodensky's theory.

2.9 Earth Gravity model

Earth gravity model is the representation of the gravity filed of the earth. The theoretical background and practical examples of the Earth gravity models can be found from Schwintzer et al., 1991 or Lemoine et al., 1998. This study uses the 2008 earth gravity model EGM08 (Pavlis et al., 2008) developed by the U.S.A. National Geospatial Intelligence Agency (NGA). It consists of a total of ~4.8 million spherical harmonic coefficients complete to degree and order 2159, with additional spherical harmonic coefficients complete to degree 2190 and order 2159. It resolves a small changes in the gravity filed spectrum up to 18 km wave length – equivalent to 9 km spatial resolution. It provided a significant improvement as compared to EGM96 which recovers up to a minimum resolvable wave length of 110 km, equivalent to 55 km spatial resolution. The first Earth Gravity Model is the Smithsonian standard Earth III which was developed in 1972 with a spherical harmonics coefficient complete to degree and order of 24. The second generation of the Earth Gravity Model is the 1987 Ohio State University gravity model (OSU 1987) complete to degree of 180. The OSU1987 earth gravity model has been upgraded to a new version in 1991 (OSU1991) complete to degree and order of 360. The third Earth gravity model (EGM96) is developed with a spherical harmonic coefficients complete to degree and order of 360. The recent earth gravity model is the 2008 (EGM08) complete to degree of 2160 with an additional spherical harmonic coefficient of 2190. EGM08 is a combined gravitational model of low- frequency and high frequency. The low frequency portion of the model comes from spaceborne data (GRACE , CHAMP), and the high-frequency part comes from terrestrial and airborne gravity and altimetry data. In other words the data for such a geopotential model comes from satellite observations (long wave length spectrum) and airborne/ terrestrial and shipborne gravity survey recovering medium and short wave length spectrum of the earth's gravity filed. The data contribution for such a model is shown in the figure below:

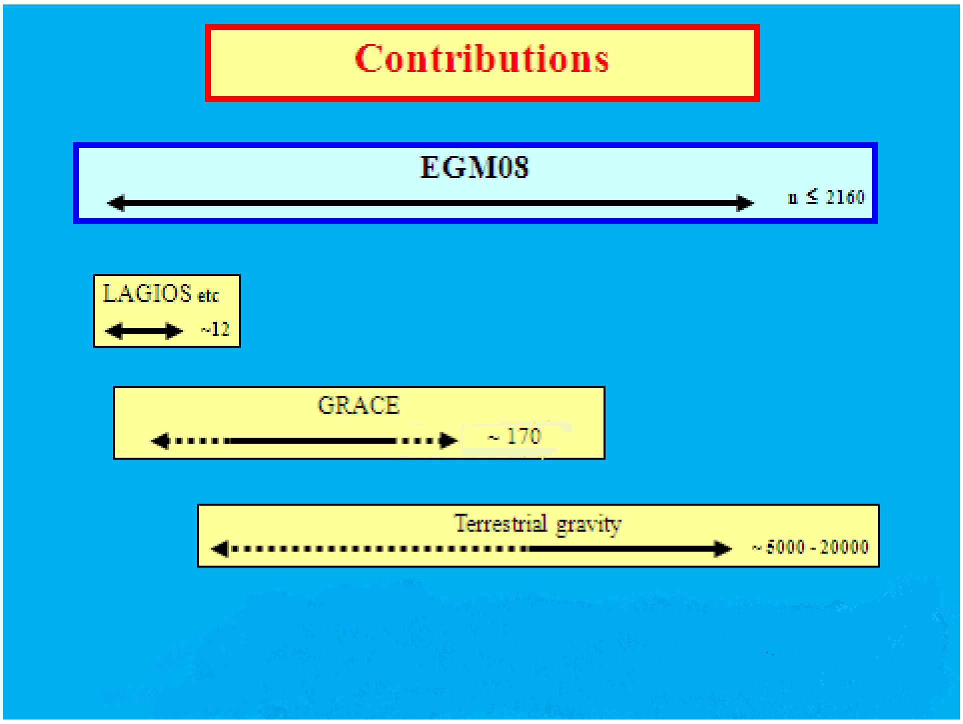


Figure2. 1: Data contribution of satellite and terrestrial gravity.

Chapter Three

3 Theory of Global Positioning System

3.1 Introduction

Global Positioning System (GPS) is space based satellite navigation systems that provides an absolute geocentric coordinates (x,y,z) or (φ,λ,h) for a particular point on or above the earth surface using satellite signals.

This provides a unique opportunity to compute a Geopotential height directly from the Global Gravity model at GPS observation points. In a more practical term, GPS provides a technology that can helps to trace the ellipsoidal height of a level surface through any point 'p' on or above the earth's surface by solving $W(h,\varphi,\lambda) = W_p$ for $h(\varphi,\lambda)$ (Bedada, 2010). With the current GPS satellite constellation and advanced data processing technology spatial position coordinates (x,y,z) or (φ,λ,h) measurement can be accurately predicted by conducting at least three days (72 hours) continuous GPS observations.

This research has conducted three days GPS measurement using Trimble GPS receivers at three leveling benchmark points called DB01, DB02 and DB03, which are found around Debere Birhan town. The GPS data is processed using Gamit Globk software (King & Bock, 2002), which is develop by Massachusetts Institute of technology (MIT) to be used as an input into the EGM08 geopotential model in order to determine a Geopotential height at each leveling benchmark. Detail discussion on GPS data processing will be presented in chapter 4 section 4.4. The next sections discuss the overall principles of GPS positioning system and errors affecting the propagation of GPS radio signal.

3.2 Principles of GPS systems

The GPS system is a composition of space, control and user segment. The space segments consists of twenty four satellites, of which four satellites present in each six orbital plane carrying atomic clock and inclined at 55^0 with respect to earth's equatorial plan(see, Figure 3.1). They are distributed in such a way that from any point on the earth surface four or more satellites are seen above the local horizon. The satellites that make the space segment are orbiting the earth at about 20,000 km altitude above the earth surface or at orbital radius of 26,600 km. They travel at a speed of 11,000 kilometer in an hour. They are moving constantly and making two complete orbits in less than twenty four hours (Keith, 2002).

GPS satellites are powered by solar energy. They have backup batteries onboard to keep them running in the event of a solar eclipse and also has got small rocket boosters on each satellite so as to keep them flying in the correct path.

The control segment is consists of a number of ground-based monitoring stations, which continually gather information from the satellite. These data are sent to Master Control Station in Colorado Springs which analyzes the constellation and projects the satellite ephemerides and clock behavior forward for the next few hours. This information is then uploaded to the satellite for transition to user.

The user segment consists of all geodetic standard GPS receivers that are receiving signals transmitted from the satellites and enabling us to determine position, velocity and time at the desired field points on or above the earth's surface.

GPS satellite transmits its position and other navigational information via the L-band radio signal, L1 (1575.43MHz) and L2 (1227.60MHz). The L- band carrier signal is modulated with data carrying information such as the satellite status, the satellite clock error and the ephemeris (Hofmann-Wellenhof et al., 1997). The L1 carrier signal is modulated with a precision code (p-code) known as Precise Positioning Service (PPS) code and a Coarse Acquisition Code (C/A-code), known as Standard Positioning Service (SPS) code. On the other hand, the L2 carrier signal is modulated with only the precise positioning service (Remondi, 1985).

GPS receiver takes information and use trilateration to calculate the user's exact location (Witte & Wilson, 2004). The datum used for GPS refers to the World Geodetic System (WGS84) ellipsoid (NIMA). Essentially, the GPS receiver compares the time which a signal is transmitted by a satellite with the time it is received; this elapsed time interval is scaled to be a distance by multiplying with the speed of light and tells how far the GPS receiver is away from the satellite. GPS receiver must receive signal at least from four satellites to calculate latitude, longitude and ellipsoidal height of the field point with centimeter and even millimeter accuracy.

However, GPS signal is influenced by many factors such as satellite orbit error, satellite clock error, receiver clock error, antenna phase center variation, tropospheric delay, ionspheric delay, multipath, electromagnetic interface and signal attenuation. In practice, the errors associated with GPS signals are modeled and removed from carrier phase and range observation. The error correction applied in calculating position using pseudo range and carrier phase's measurements are shown in the formula below:

$$P_A^j(t) = \rho_A^j(t) + c\delta t_A - c\delta T^j + \Delta_A^{IONO} + Tropo + MP_{PA} + \varepsilon \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.\ 1)$$

$$\Phi_A^j(t) = \frac{1}{\lambda}\rho_A^j(t) + \frac{1}{\lambda}\left[\delta t_A - \delta T^j\right] + N_A^j - \Delta_A^{iono} + Tropo + MP_{\Phi i} + \varepsilon \ldots\ldots\ldots\ldots\ldots (3.\ 2)$$

Where:

P = pseudo-range observation in meters

ρ = distance between satellite and receiver in meter

c = Speed of light in meter/sec

δT^{j}= Satellite clock error in sec

δt_{A}= Receiver clock error in sec

Iono = Ionospheric range error in meters

Tropo = Tropospheric rang error in meters

N =integer cycle ambiguity

MP_{pi}= pseudo-rang multipath

Mp_{pi}= phase multipath

λ= wave length

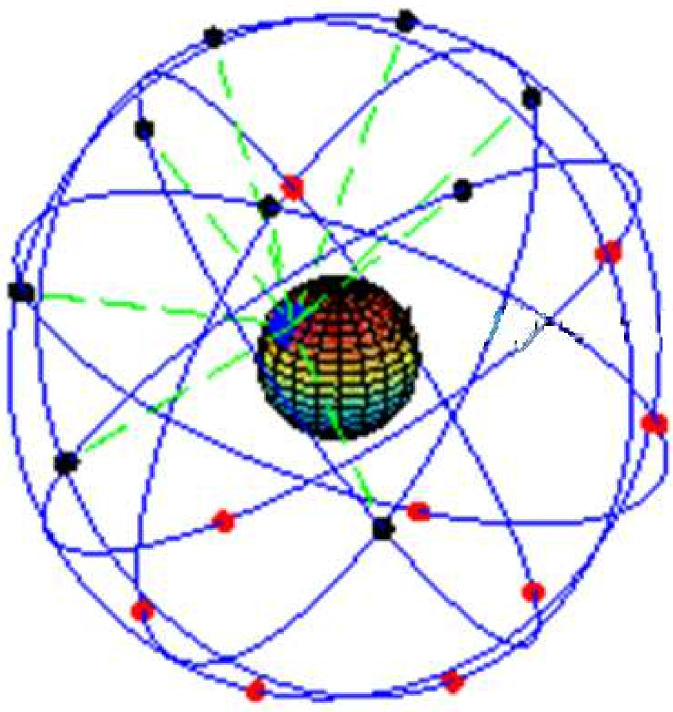


Figure 3.1: Orbital orientation of GPS satellites.

3.3 Error in GPS

Irrespective of the cause, errors can be divided in to three: systematic, random and blonder or gross errors. Systematic errors are errors that have systematic pattern and governed by some physical law, if the law that govern the error is known it is possible to model this error and remove their effect from measurements. Random error is an error that remains after the systematic error is removed; it is caused by the limitation in the capacity of observer or the instrument itself. This type of error does not follow any pattern, they are mostly distributing around the average value and they are not governed by any physical law, it is hard to avoid them. Blender (mistakes) types of errors are mostly caused by the observer and can be easily identified and removed from observations (Elias Lewi Dr .Ing, Lecture note).

There are many possible sources of errors that will degrade the accuracy of positions computed by a GPS receiver. The travel time of the signal can be altered by atmospheric effects; when the signal passes through the ionosphere and troposphere it is refracted, causing the speed of the signal to be different from the speed of a GPS signal in space. Sunspot activity also causes an interference with GPS signals. Another source of error is caused by electrical interference or errors inherent in the GPS receiver itself. Errors in the ephemeris data and variation in the atomic clock (clock drift) will also cause error in computing position. Multipath effect arises when signals transmitted from the satellites bounce off a reflective surface before getting to the receiver antenna. During this circumstance, the receiver gets the signal in straight line path as well as delayed path which results in multipath error.

Satellite geometry can also affect the accuracy of GPS positioning. This effect is called Geometrical Dilution of Precision (GDOP), refers to where the satellites are in relation to one another, and is a measure of the quality of the satellite configuration. In general, the wider the angle between satellite the better the measurement, because the signals comes in different atmospheric condition, that help to get better reading. GPS receivers usually report the quality of satellite geometry in terms of Position Dilution of Precision (PDOP), which refer to the Horizontal Dilution of Precision (HDOP) and Vertical Dilution of precision (VDOP) measurements (latitude, longitude and altitude). A low Dilution of Precision (DOP) value indicates a higher probability of accuracy, and a high DOP indicates a lower probability of accuracy. The PDOP value 4 is excellent, a PDOP between 5and 8 is acceptable and PDOP value of nine or greater is poor (Mohinder S. Grewal; Lawrence R. weill; Angus P. Andrews, 2007).

In general the sources of errors that affects the GPS signals can be grouped in to satellite position and clock errors, signal propagation error (ionosphere, troposphere and multipath) and receiver errors (antenna phase center variation and electromagnetic interference), (Seeber, 2003). These error sources are discussed in depth in the next sections.

3.4 Satellite position and clock error

GPS positioning is accomplished by a process similar to trilateration, there are two key information upon which GPS position depend: signal propagation time and the location of satellite. Signal propagation time is used to determine the range from the satellite to the receiver antenna phase center. Signal propagation time is biased due to an immeasurable time offset between GPS satellite clock and a receiver's internal clock. Receive clock errors are considered systematic errors that can be reduced by differencing the GPS code and phase observable acquired from one satellite using two receivers in different places.

Satellite position information can be obtained from ephemerid data. The ephemeris data are two types which are broadcast and precise ephemerides data. Broadcast ephemerides are the navigation information of satellite position guessed on physics-based and locate the future position of satellite based on the past location and velocity information. Precise ephemerides are produced by observing the satellite position and deducing their position after haw much time elapse they available. Broadcast ephemerides are not as accurate as precise ephemerides. The accuracy of the broadcast ephemerides is currently around one to three meters (Seeber, 2003).

Orbital error, ephemerides and inconsistency of clock on the board of the satellites can make positional error. This error can be removed by using precise ephemerid data obtained from various organizations such as NIMA, JPL, NASA (NIRL) and International GNSS Service (IGS). Most of the precise ephemerides corrections are acquired from IGS - IGS collects records from more than 300 globally distributed stations and distribute the precise broadcast data after a week, (Seeber, 2003).

3.5 GPS signal propagation errors

In general, when the GPS radio signal propagated through the atmosphere, it can be affected by refraction, diffraction or reflection. Refraction is the gradual change of the straight line path of signal direction as it passes through the different layer of atmosphere (troposphere & ionosphere). This is caused by the density variation in the atmospheric layer. GPS signal diffraction occurs when the signal encounters an obstruction-the signal tends to travel around it and arrive at the receiver. During such situation there would be signal loss due to energy absorption. Reflective media may also cause GPS signal delay thereby creative multipath error. The details of how the atmospheric layer (ionosphere, troposphere) and multipath can cause an error in GPS signal propagation will be briefly discussed in the next section.

3.5.1 Ionspheric delay

The ionosphere is a part of the atmosphere that is found at high altitude between 50km to 1000km above the earth's surface. It is composed, of charged particles that have been ionized by solar radiation. The ionosphere has different layers which are known as D, E and F layers. The D layer occur at a height between 50– 90 km, the E-layer is 90 – 150 km height characterize by the ultraviolet and x-rays during the day, and cosmic rays and meteors during night. The F layer is called Appleton layer, and it is divided in to two regions F1 (150 – 200 km height), and F2 (200 -1000 km height). The ionosphere refracts GPS signals in a manner similar to how water in a glass refracts light, such that a pencil appears to have a sharp bend in it (Thomas H Meyer, 2007). The two GPS signals, L1 and L2 frequency delays differently. This difference can be detected by dual frequency receivers (Brunner and Welsch, 1993; Hofmann-Wellenhof et al., 1997; Leick, 1995; Seeber, 2003). Single frequency receiver cannot detect the ionospheric delay.

The ionospheric refraction is modeled as a function of the electron density represented by the Total Electronic Content (TEC). The time delay is directly proportional to the total electron content along the path between the GPS satellite and the GPS receiver. TEC is a function of many variables; local time, season, geographic location, solar activity of sun and magnetic field of the earth. The TEC attains maximum at place located near the equator (-20^{0}S to +20^{0}N latitude).

3.5.2 Tropospheric delay

The lower part of the earth atmosphere is composed of dry gases and water vapor, which lengthen the propagation path due to refraction. Magnitude of the signal delay depends on refractive index of the air along the propagation path. The troposphere is non dispersive at the GPS frequencies, so delay is not frequency dependant-tropospheric path delay is the same for code and carrier signal components.

The refractive index of the troposphere consists of the dry gas component and the water vapor component, which contribute about 90 % and 10% respectively (John Wiley & Sons, 2007). Knowing the temperature, pressure and humidity along the propagation path can help to determine the refractivity. However, using standard atmospheric model of dry delay can permit the determination of the zenith delay within 0.5 m (Wiley & Sons, 2007). These standard atmospheric models are based on the low of ideal gases and assume spherical layers of constant refractivity with no temporal variation and an effective atmospheric height of about 40 km. Estimation of dry delay can be improved considerably if surface pressure and temperature measurements are available.

The component of tropospheric delay due to water vapor is much more difficult to model, because there is considerable spatial and temporal variation of water vapor in the atmosphere. Model of the standard atmosphere at antenna location is used to estimate the combined zenith delay due to both wet and dry components. The delay is modeled as the zenith delay multiplied by a factor which is a function of the satellite elevation angle. This factor is unity at the zenith, and it increases with decreasing elevation angle as the length of the propagation path through the troposphere increases. Typical values of the multiplication factor are 2 at 30^0 elevation angles, 4 at

15^0 elevation angles, 6 at 10^0 elevation angle and 10 at 5^0 elevation angles. The accuracy of the model decreases at low elevation angle (John Wiley & Sons, 2007).

3.5.3 Multipath error

Multipath is the situation where GNNS radio signals arrive at the receiver via more than one path, these happens when the signals reflect from reflective surfaces (e.g. building, cars, trees, reflective ground e.t.c) around the GPS receiver antenna. Multipath propagation affects both code and carrier phase measurements but the effect on code observation is more compared to that of carrier phase observation. According to Seeber (2003), causes phase shift on carrier phase observation introducing a significant periodic bias of several centimeters in to the range observation. Multipath effects introduce an error up to 10 – 20 m for code pseudo ranges (Wells et al, 1987). The multipath effects on carrier phase in a relative positioning with short baselines, generally will not be greater than 1 cm, but even in this case a simple change of the height of the receiver may increase the multipath thereby degrading the accuracy of position coordinates. When performing GPS observations for relatively longer periods, the multi path effect can be easily identified from its repeated nature of patterns. However, it is difficult to identify or model the multipath effect with a limited amount of data acquired over short period of GPS observation.

In general, the following mitigation techniques are commonly applied to reduce the multi path effect on GPS signal propagation.

1) Select the site carefully, avoid nearby reflectors.
2) Deploy radio signal absorbing material on the ground.
3) Select carefully the antenna type, like chock ring antenna.

4) Take longer GPS observations
5) Use bigger antenna heights.

3.6 Receiver errors

Receiver errors are errors that come from the instrument imperfection associated with the receiver hardware. GPS receiver has different component there are antenna, radio frequency and intermediate frequency, signal tracker, microprocessor for receiver control, oscillator, power supply, memory (data storage) and user interface (Seeber, 2003). The error source that comes from receiver can be categorized as antenna phase center variation and electromagnetic interference. These errors will be discussed in the next two sections (section 3.6.1 & 3.6.2).

3.6.1 Antenna phase center variation

The phase center of an antenna can be defined as the point in space where the electrostatic field of the signal exactly matches the signal emerging from the antenna terminal. The phase center can vary with the arrival direction of the signal, usually within the range of one centimeter or less (John Wiley & Sons, 2007). The phase center changes with the zenith and azimuth angle of the incoming signal in addition to its dependence on frequency of the signal. The phase center for L1 frequency is typically different than that for L2 frequency (Mader, 1999). The main cause of phase center variation is use of incorrect serial number for the GPS antenna.

The phase center variation causes an error in the computation of position. There are two basic methods of eliminating the phase center variation errors. One method is to calibrate the phase center as a function of signal arrival direction. The calibration uses a physical point on the antenna as reference point. Another method is to use identical antenna oriented in the same way at the two receivers in a differential GNSS system.

3.6.2 Electromagnetic interference and signal attenuation

The radio signals broadcasted by the GPS satellites have relatively low power, around 50 watts. Although GNSS signals occupy a protected frequency band, nearby sources of broadband electromagnetic noise can overcome them (Johannessen, 1997; Butsch, 2002), thus causing decreased signal to noise ratios and loss of signal tracking (Seeber, 2003). Power transmission lines, television and radio stations, and radar installations are possible examples of such noise sources. To help address this problem, the GPS modernization program includes a third high- power frequency L5 which is expected to reduce this problem (Hatch, Jung, Enge & Pervan, 2000). Unfortunately, new receivers will probably have to be purchased when enough satellites have been placed in orbit to make use of L5 practical and to take advantage of its potential. The obstruction that comes over the receiver can attenuate or block transmissions, this causes decreased in signal to noise ratio.

Chapter four

4 Result and Discussion

4.1 Introduction

Classically Geopotential height is determined by using geodetic leveling. This techniques connects the continental leveling to the mean see level tide gauge datum through a series of first order high accuracy leveling. In principle, level heights has to combined with gravity measurement to determine geopotential number $c = \sum_{i=1}^{n} \delta h_i g_i$, but in most practice, leveling increment are used alone with some correction like temperature, pressure, earth's curvature and refraction. For a country like Ethiopia, where the reference tide gauge is far away, the connection of the practical datum to mean sea level is a large additional source of error (Bedada, 2010).

Therefore with the advent of satellite gravity missions (e.g. CHAMP, GRACE, GOCE) and GPS navigation system, the task of computing a new Geopotential height system has to involve finding the Geopotential $c(\varphi,\lambda,h) = W_o - W(\varphi,\lambda,h)$.

This study used the 2008 Earth gravity model (Pavlis et al, 2008) to compute the Geopotential heights at three first order leveling benchmarks established by the Ethiopian Geodetic Survey project (1957-1961) around Debere Birhan town. The accuracy of the high spectral resolution of EGM08 complete to degree and order 2160 spherical harmonic coefficient will be evaluated against an independent level heights (orthometric height) found around Debre Birhan and tied to mean sea level at port of Alexandria. The task of simulating the new Geopotential height to the classical leveling height involves GPS observation at each leveling benchmarks. However, the Geographical site of each leveling benchmark is not suitable for GPS satellite visibility and has lots of obstruction, a new first order geodetic leveling has to be conducted to the old benchmark points to the new location. In the research, all the three old Blue Nile benchmark points (F11, G11and J11) (Ethiopian Geodetic servay1957-61) has been shifted to the new locations that are suitable for GPS observation (see, section 4.3).

Then GPS data were collected at three leveling benchmarks for three consecutive days using Trimble GPS receivers. The geodetic coordinates needed to compute the Geopotential height at the new leveling benchmarks have been determined with millimeter accuracy by processing the GPS data using GAMIT/GLOBK software (see, section 4.4). Moreover, absolute gravity values have been measured at each leveling benchmark so as to better approximate the mean gravity along the plumb line between the geoid and the benchmark point required to change Geopotential number 'c' to orthometric and Helmert orthometric heights.

Section 4.5 compares the orthometric heights, Helemert orthometric height, geoid and height anomaly (zeta) computed directly from gravity model to the classical leveling height. In addition result of previous research works for Ethiopia as well as from the other countries are presented for comparison purpose.

4.2 Leveling survey

Leveling is used to determine a Geopotential height that is referenced to mean sea level through one or more tide gauge stations. Traditionally, geodetic leveling connects the continent to the predefined mean sea level tide gauge station by carrying out series of level heights using a telescope fitted with sprite level and two parallel plate microscopes.

Three leveling techniques can be used to establish basic network of vertical control points. These are differential, trigonometric and barometric. Deferential leveling is the most accurate of the three methods. With the instrument locked in position, readings are made on two calibrated staffs held in an upright position ahead and behind the instrument. The difference between readings is the difference in elevation between the points. Trigonometric leveling involves measuring a vertical angle from a known distance with a thedolite and computing the elevation of the point. With this method, vertical measurement can be made at the same time horizontal angle are measured for triangulation. It is therefore, a more economical method but less accurate than differential leveling. In barometric leveling, difference in heights is determined by measuring the difference in atmospheric pressure at various elevations. The accuracy is not as great as either of the two; it is used in reconnaissance survey where a high degree of accuracy is not required.

This research conducts differential leveling technique at three places to shift the old benchmarks to the new location so as to get good GPS satellite visibility. The three old benchmarks are called F11, G11, & J11 and they are found along old road culverts, bridges and on the top of an out crop. The elevations of F11, G11and J11 are 2811.135 m, 2747.658 & 2840.080 m above mean see level tide gauge station located at Alexandria, respectively. For more information, the site descriptions of three benchmarks are presented in Annex I. The new benchmarks are established on hard bedrocks by drilling and anchoring stainless still bolts which are durable for many years. The description of these new benchmark sites are shown in figures 1.2, 1.3 & 1.4 in chapter one. A total length of 1220 m leveling survey was done with a closed loop. The elevation of the new benchmark called DB01 with tie to the old 'F11' benchmark has involved the task of surveying 400m leveling distance with ten backsight and foresight readings with offset distance of 20m interval. The height of DB01 benchmark is determined to be 2814.250 m above mean see level with a closer error of 0.00176 m, for more detail see, Table 4.1.

Table 4.1: Surveying result of station DB01.

Level Position	**Back sight (m)**	**Foresight (m)**	**leveling increment (m)**	**Reduced level (m)**	**Remark**
BM			0.00000	2811.13500	F11
1	1.18950	1.22007	-0.03057	2811.10443	
2	1.43040	0.99097	0.43943	2811.54387	
3	1.41337	1.12347	0.28990	2811.83377	
4	1.51250	0.94600	0.56650	2812.40027	
5	2.19843	0.34817	1.85027	2814.25053	DB01
6	0.34817	2.19833	-1.85017	2812.40037	
7	1.06600	1.58567	-0.51967	2811.88070	
8	1.22540	1.54000	-0.31460	2811.56610	
9	1.21783	1.48750	-0.26967	2811.29643	
10	1.07350	1.23317	-0.15967	2811.13677	F11

The second new leveling benchmark called DB02 is established by carrying out a total surveying distance of 720m with tie to the old benchmark called 'G11'. This involved eighteen backsights and eighteen foresights readings with approximately

20m offset distance to get the new elevation for the DB02 to be 2767.987m. This height is predicted with a closure error of 0.0014m (see, Table 4.2).

Table 4.2: Surveying Result of station DB02.

Level Position	Back sight (m)	Foresight (m)	Leveling increment (m)	Reduced level (m)	Remark
BM			0.0000	2747.65800	G11
1	2.2550	0.1653	2.0897	2749.74767	
2	2.4982	0.1323	2.3658	2752.11350	
3	2.4828	0.1282	2.3546	2754.46810	
4	2.4912	0.1180	2.3732	2756.84127	
5	2.4013	0.1637	2.2377	2759.07893	
6	2.3750	0.1775	2.1975	2761.27643	
7	2.2788	0.2015	2.0773	2763.35377	
8	2.3870	0.1145	2.2725	2765.62627	
9	2.6755	0.3148	2.3607	2767.98693	DB02
10	0.3148	2.6657	-2.3508	2765.63610	
11	0.2600	2.5543	-2.2943	2763.34177	
12	0.4720	2.5362	-2.0642	2761.27760	
13	0.3983	2.5960	-2.1977	2759.07993	
14	0.4057	2.6420	-2.2363	2756.84360	
15	0.3075	2.6815	-2.3740	2754.46960	
16	0.2455	2.6003	-2.3548	2752.11477	
17	0.2437	2.6103	-2.3667	2749.74810	
18	0.2503	2.3390	-2.0887	2747.65943	G11

The third station is called DB03 and it is located at a distance of 80m away from the old ‘J11’ benchmark. DB03 was established with a closed loop involving four backsight and four foresight readings. Doing so, the elivation of the new DB03 benchmark is determined to 2844.318m with a closure error of 0.002m above mean see level (see Table 4.3).

Table 4.3: Surveying result of Station DB03.

Level Position	Back sight (m)	Foresight (m)	Leveling increment (m)	Reduced level (m)	Remark
BM			0.00000	2840.08000	J11
1	2.94133	0.16817	2.77317	2842.85317	
2	1.93850	0.47400	1.46450	2844.31767	DB03
3	0.47383	1.93850	-1.46467	2842.85300	
4	0.39033	3.16133	-2.77100	2840.08200	J11

This research has also conducts Gravity measurement at the three newly established benchmark points. The measurement is done using an instrument called LaCoste & Romberg Gravimeter. First the Gravimeter reading is tied to the absolute gravity value of the Addis Ababa university Geophysical observatory. Then the second measurement was taken at Sheno followed by three successive measurements at each new benchmark points with different time interval. Finally, the measurement is made to tie with the Addis Ababa absolute gravity Station. The measured data is processed using gravity data process software, by Dr. Ing Elias Lewi by making temperature, pressure and tidal corrections. The results of the processed gravity data is presented in table 4.4 below.

Table 4.4: Gravity data at three new leveling benchmarks.

Station	Measured gravity mGal	Accuracy m Gal
DB01	977393.16484	$\pm$0.0113
DB02	977407.61808	$\pm$0.0113
DB03	977392.02681	$\pm$0.0124

4.3 GPS data processing

The GPS data are processed using GAMIT/GLOBK software (King and Bock, 2002). This software is a comprehensive GPS analysis package developed by MIT (Massachusetts institute of technology). It is a script for the estimation of three dimensional relative positions of ground stations and satellite orbits. It is designed to run under any UNIX operating system. In the software there is a table folder which contains files such as station.info, sittbl, sestbl, process default, site table and itrf.08, which are necessary for processing. The file station.info contains receiver and antenna information recorded and edited by manually. The antenna type used for this measurement is Trimble and the antenna heights are 0.15 cm, 0.56cm and 0.56cm for the stations DB01, DB02 and DB03 respectively. Sittbl is the input control file, specifying for each site the clock and atmospheric model to be used and the a priori coordinate constrains. Sestbl is an input control file, specifying the type of analysis and the priori measurement errors and satellite constraints. Itrf08 is a file containing the priory information of coordinate of the IGS stations. Sp3 file contains a priori information of the satellite coordinates. The raw GPS data is a binary file this binary data change to rinex file using Translating Editing and Quality Controlling commands (TEQC).

The GPS measurement has conducts at the three new benchmark points. Also the measurement has made for three consecutive days (72 hours) using Trimble GPS receiver on the day of 150, 151, 152 and 153. The geodetic coordinates (φ,λ,h) for each benchmark have been computed good to millimeter accuracy both in the horizontal and vertical components using GAMIT/GLOBK (see, Table 4.5). In the Gamit processing, all the coordinates were determined with a tie to five IGS network stations (ADIS, BJCO, MAL2. RAMO and YIBL), for a graphical presentation see figure 4.4.

Table 4.5: The GPS coordinate of the three points i.e. DB01, DB02 and DB03.

Station Name	Latitude (Degree)	Longitude (Degree)	Ellipsoidal height (m)	Error in Height (m)	Error in latitude (m)	Error in longitude (m)
DB01	9.6427391	39.50549467	2808.4163	0.00672	0.00133	0.00185
DB02	9.6694553	39.52186606	2762.0443	0.00513	0.00111	0.00151
DB03	9.7051159	39.58151969	2838.2490	0.00451	0.00105	0.00139

Table 4.5 shows that, the geodetic coordinates (latitude, longitude and ellipsoidal height) of the three benchmarks (DB01, DB02 and DB03) are determined in millimeter accuracy. For instance, the accuracy of DB01benchmark coordinates are computed 0.00672m, 0.00133m and 0.00185m in the Up, North and East directions, respectively. For station DB02, the accuracy the coordinates are 0.00513m in the Up direction, 0.00111m and 0.00151m in the north and East directions respectively. Whereas the coordinate of DB03 benchmark has 0.00451m, 0.00105m and 0.00139m accuracy in the Up, North and East directions, respectively.

Gamit incorporates a weighted least square algorithm to estimate the relative position of a set of station, orbital and Earth rotation parameters, Zenith delays, and phase ambiguities by fitting to double difference phase observation. Weighted mean square is occurring when the off diagonal elements of the correlation matrix are zero. GLOBK is used to compute residual plots in the North East Up (NEU) computed coordinates showing the Normalized Root Mean Square (NRMS) and Weighted Root Mean Square (WRMS) see, table 4.6 and figure 4.1, 4.2 and 4.3.

Table 4.6: Normal and weighted mean square of the three stations in three directions.

Location	DB01		DB02		DB03	
	Nrms (mm)	Wrms (mm)	Nrms (mm)	Wrms (mm)	Nrms (mm)	Wrms (mm)
North	1.43	4.10	1.42	3.50	1.67	3.90
East	0.81	3.10	1.69	5.40	1.73	5.10
UP	0.21	2.80	0.83	8.90	0.85	8.10

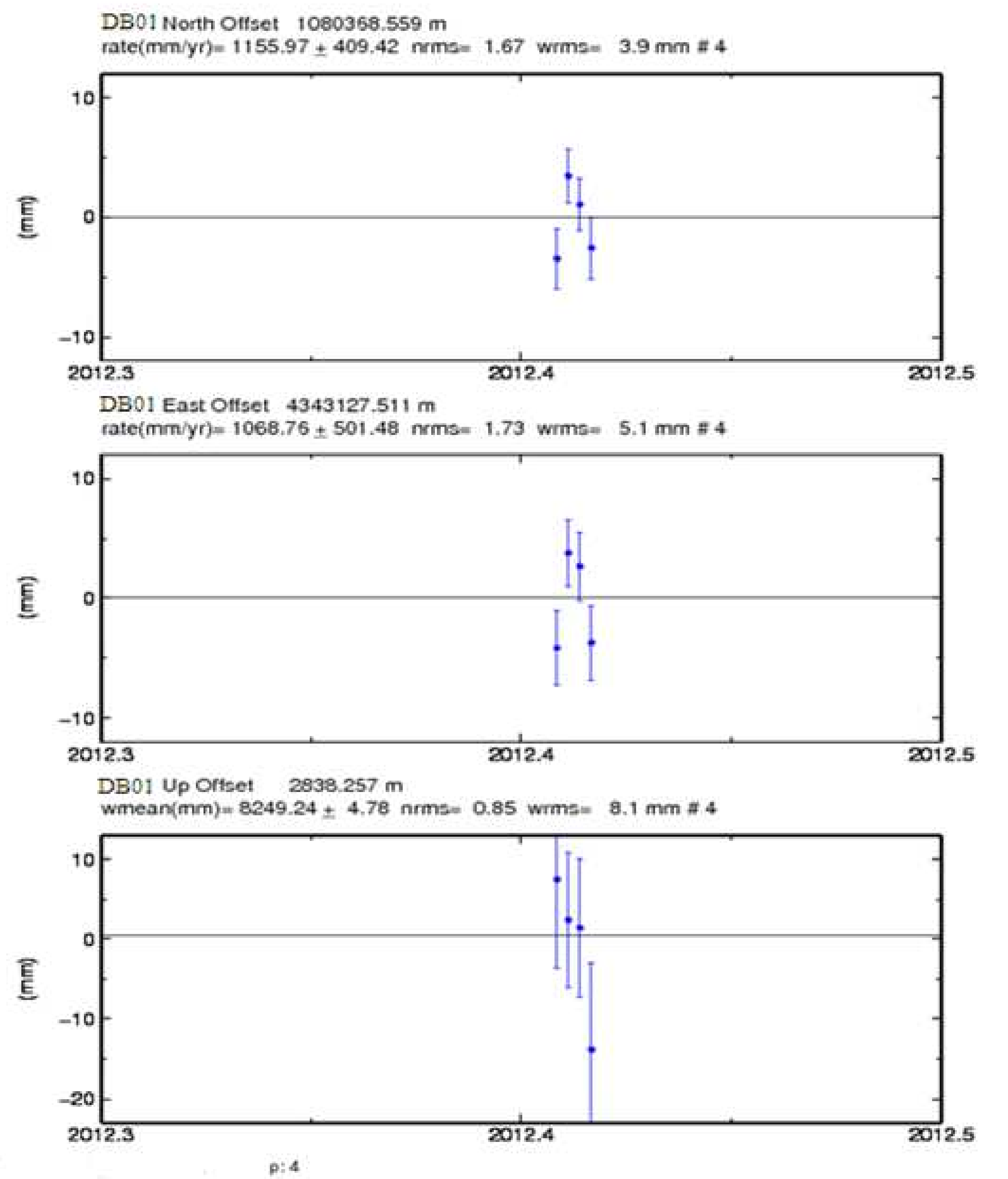


Figure 4.1: Residual plot of station DB01.

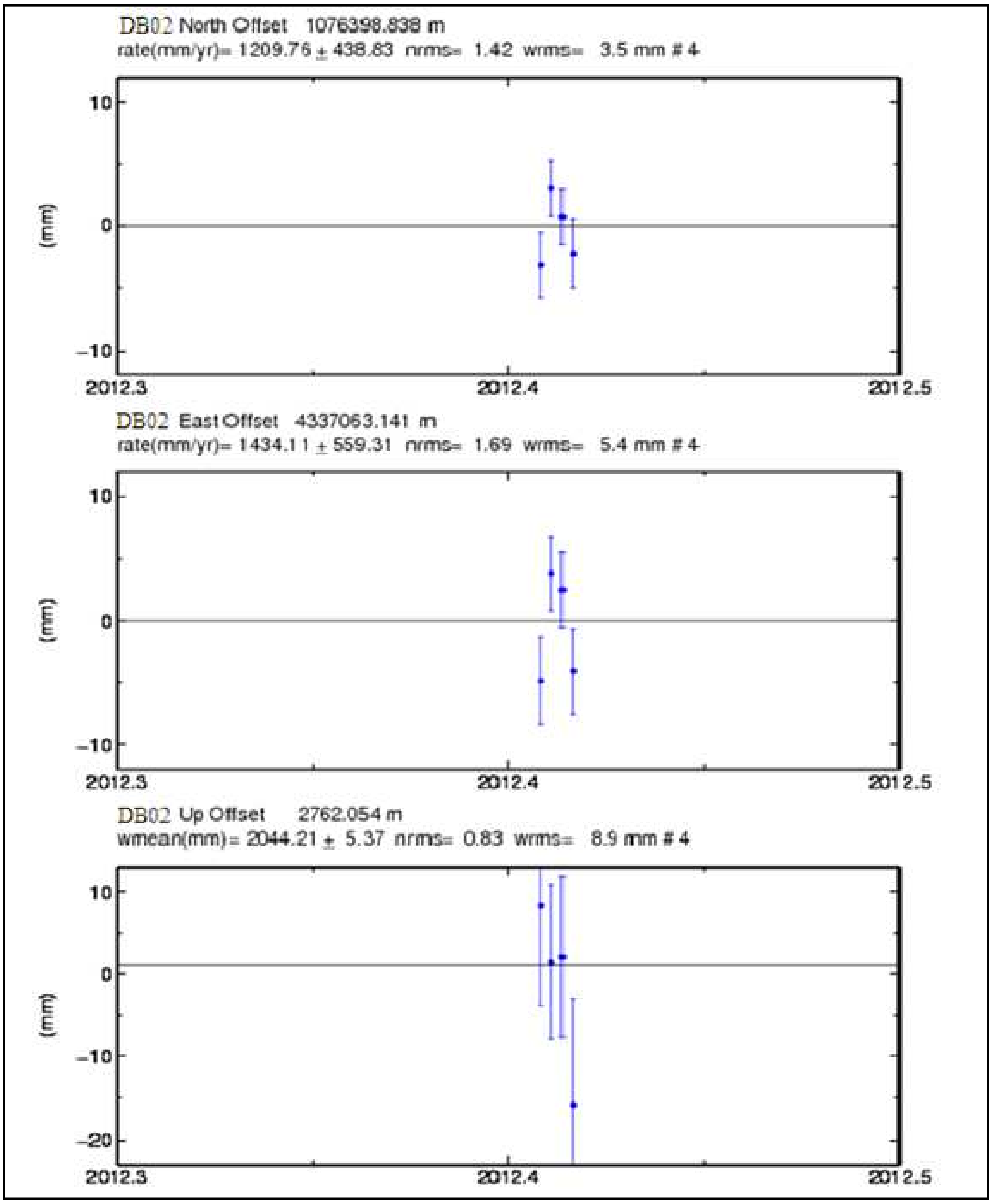


Figure 4.2: Residual plot of station DB02.

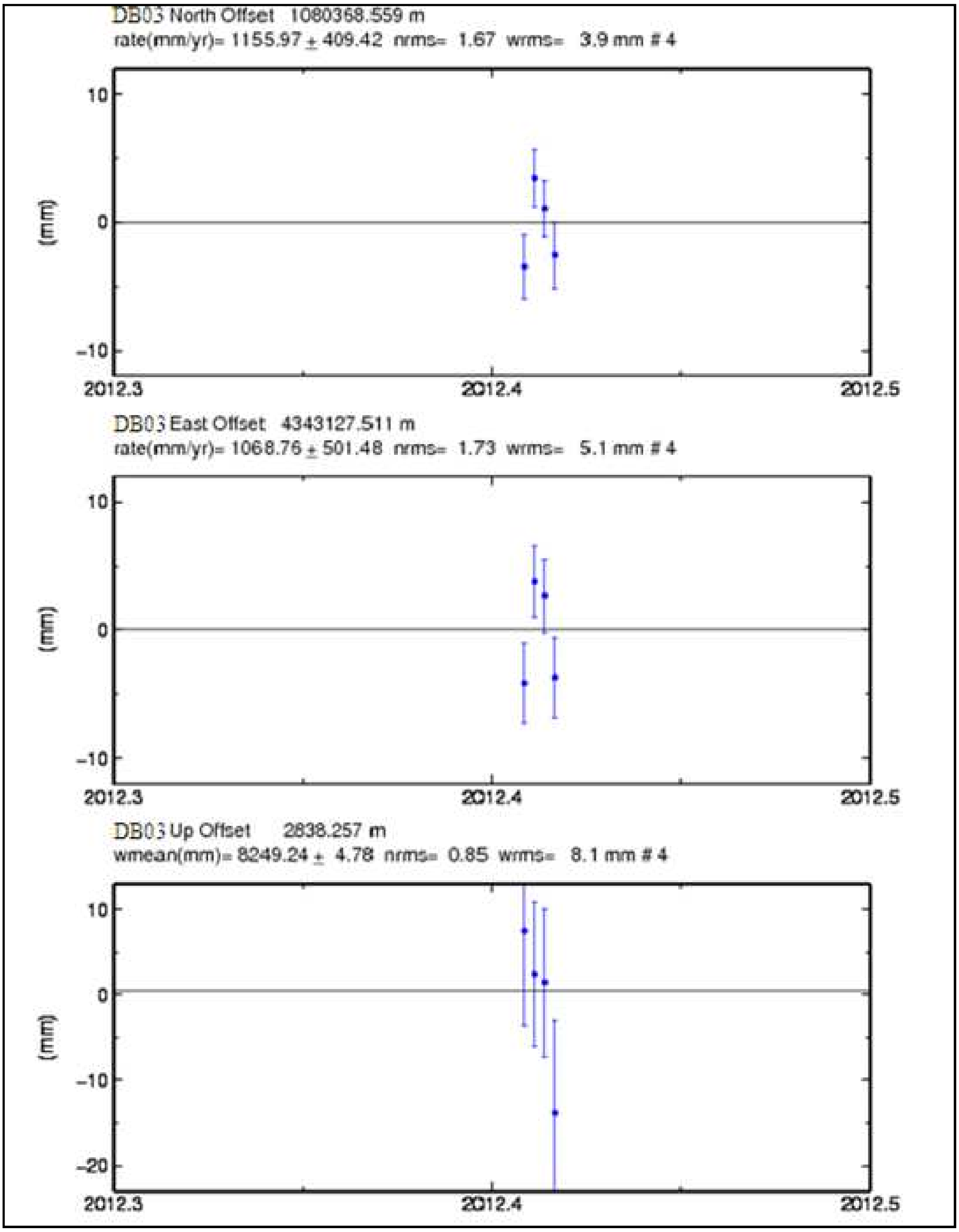


Figure 4.3: Residual plot of station DB03.

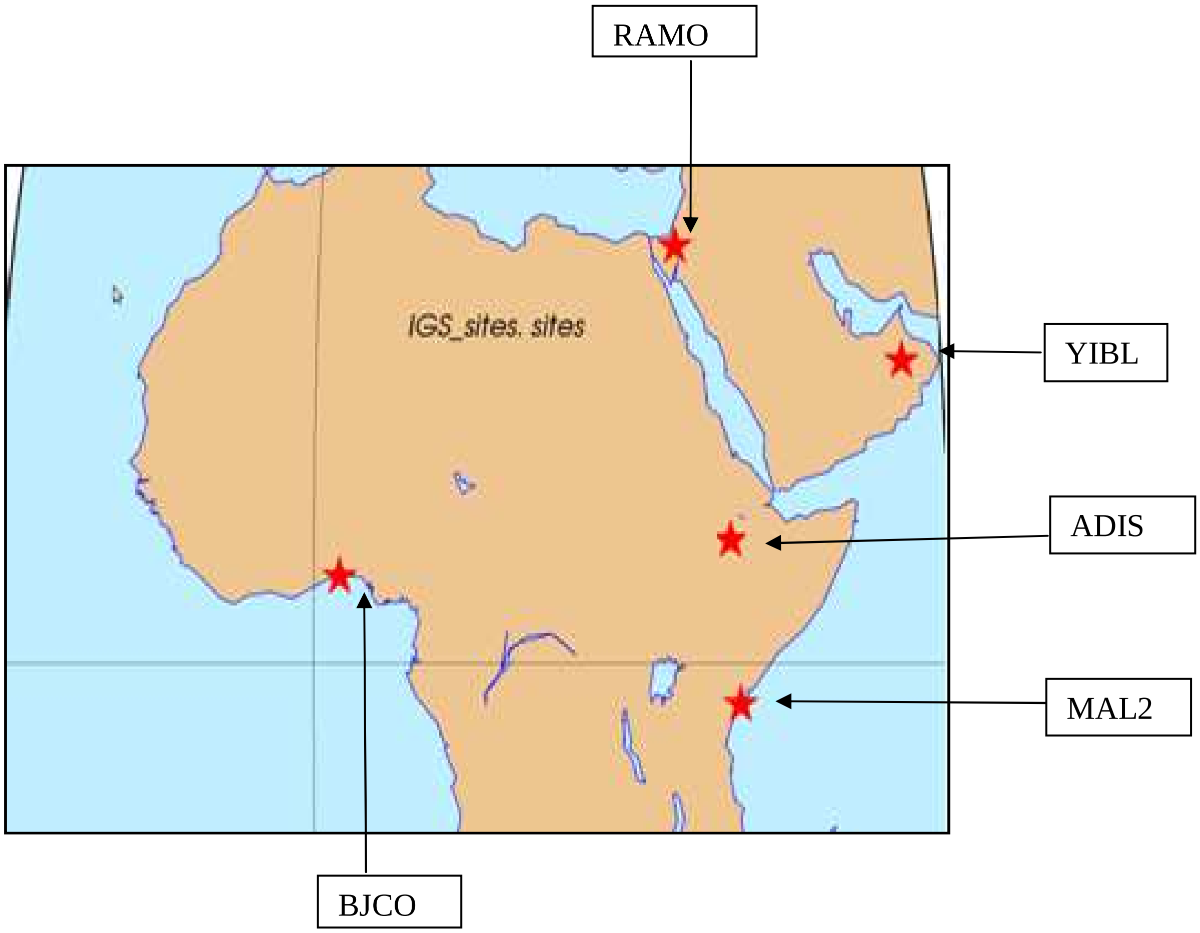


Figure 4.4: Shows Location of five International GPS Station (IGS) used in Gamit processing.

Table 4.7: Shows list of countries and cities that five used IGS station are located.

NO	IGS station Name	Location Country	Cities
1	ADIS	Ethiopia	Addis Ababa
2	BJCO	Benin	Cotonou
3	MAL2	Kenya	Malindi
4	RAMO	Israel	Mitlpe Ramon
5	YIBL	Oman	Yibal

4.4 Comparison of EGM08 derived heights with leveling heights

The evaluation and quality assessment of EGM08 is important for using in various geodetic and other scientific applications at global and regional scales. The evaluation of the EGM08 is based on the comparison with the external data like GPS/leveling observations. The objective of this research is to validate and perform quality assessment of EGM08 model. For example, Bedada (2010) compared leveling heights with the EM08 derived Geopotential heights in Ethiopia and obtained an improved standard deviation from 5.3cm with EGM08 to 4.6 cm and 3.9 cm when airborne gravity and airborne gravity plus topographic models are included, respectively. In addition, various researches have been conducted at different countries to validate the accuracy of EGM08 by using GPS/leveling. For instance, EGM08 has been compared with GPS/Leveling in United States, Germany, Australia, Canada, Turkey and South Korea and the leveling data were observed to simulate to the new earth gravity model on each of the above listed country with a standard deviation of 25cm, 4cm, 22cm, 13cm, 24.2cm and 18.5cm, respectively (Newton's Bulletin, 2009; Ellmann ,2010).

In general, the prediction of Geopotential heights is done in two ways. The first approach compares leveling with EGM08 models alone. The second approach combines EGM08 with surface gravity observation while computing orthometric height, Helmert orthomertic height Geoid heights and height anomaly (zeta). The values of the predicted Geopotential heights from the model are presented in table 4.8 below. The mismatch between the orthometric heights computed from the EGm08 using prey approximation and leveling height has a standard deviation of 2.41 cm. When the prey formula for predicting the mean gravity $\left(\bar{g} = \frac{(g_{obs}+g_o)}{2}\right)$ between the geoid and the leveling benchmark need to convert the Geopotential number to ortohmetric height is modified by actual surface gravity observation, the mismatch between the leveling height and the resulting orthometric heights has small standard deviation of 1.09 cm. More result of the comparison between EGM08 derived Geopotential heights with leveling heights are presented in table 4.9.

Table 4.8: Values of EGM08 predicted geopotential heights of the three stations.

Site Name	Ellipsoidal height (m)	Orthometric height (m)	Normal height (m)	Geoid height (m)	Height anomaly (zeta)(m)	Hlemert ortometric height (m)
DB01	2808.416	2814.2916	2814.5602	-5.8756	-6.1442	2812.1994
DB02	2762.044	2767.9851	2768.2416	-5.9411	-6.1976	2765.9630
DB03	2838.249	2844.3716	2844.5937	-6.1226	-6.3447	2842.2091

Table 4.9: Comparison of EGM08 derived heights with GPS/Leveling data.

	Prediction from EGM08		Prediction from EGM08+ measured gravity	
	Mean(m)	Stdv (m)	Mean(m)	Stdv(m)
Leveling – Ortometric Height prediction	-0.0314	0.0241	-0.0373	0.0109
Leveling – normal height prediction	-0.2804	0.0226	-0.2804	0.0226
Leveling – Ellipsoidal height + Geoid(N) prediction	0.0154	0.0244	0.0154	0.0244
Leveling – Ellipsoidal height + Zeta prediction	-0.2832	0.0226	-0.2832	0.0226
Leveling – Helmert Ortometric Height Prediction	2.0609	0.0348	2.0490	0.0670

Generally, the mean value tells the datum shift between the leveling height datum benchmark (mean sea level) and the geoid used as a datum for Geopotential height derived from EGM08. The minus sign shows the geoid is below mean sea level at the particular point. The standard deviation shows the accuracy of predicting ortometric heights using the EGM2008 model as compared with the classical first order spirit leveling heights.

Table 4.9, showed that, the normal height simulates with leveling heights with an accuracy of 0.0226m. In addition, the standard deviation is not changing when measured gravity is used to predict mean gravity along the plumb line between the geoid and the point of height determination. This shows, normal height has no relation with terrestrial measured gravity rather it is a Geopotential number scaled with normal gravity. Normal gravity has a closed formula and it can be predicted at any point without introducing error.

From Table 4.9, one can observe that, the geoid & zeta are approximated with an accuracy of 0.0244m & 0.02226m standard deviation, respectively. Even though, the formula of zeta is a function of surface gravity, the mean and standard deviation of zeta compared to leveling heights have not been changed when measured gravity is used together with EGM08 as compared to using EGM08 alone. This is due to the fact that, the effect is less than a millimeter level.

Table 4.9, also shows that Helemert-height prediction has an increased standard deviation when measured gravity combined with EGM08. This might be due to the drawback of Helmert's assumption that the density variation of the sub surface is considered to be constant (ρ=2.67g/cm^3), so that the gradient of normal gravity cannot adequately presume the actual gravity gradient that is needed to be cancelled out when combined with actual gravity observation in the process of predicting mean Helmert gravity value.

Table 4.10: Comparison of EGM08 derived heights with GPS/Leveling data using WGS84.

	Prediction from EGM08		Prediction from EGM08+ Observed gravity	
	Mean(m)	Stdv (m)	Mean(m)	Stdv(m)
Leveling – Ortometric Height prediction	0.2014	0.0241	0.1956	0.0109
Leveling – normal height prediction	-0.0478	0.0226	-0.0478	0.0226
Leveling – Ellipsoidal height + Geoid(N)	0.0248	0.0244	0.0248	0.0244
Leveling – Ellipsoidal height + Zeta	-0.0505	0.0226	-0.0505	0.0226
Leveling – Helmert Ortometric Height	2.2931	0.0348	2.2816	0.0670

Table 4.10 shows the comparison of EGM08 derived quantity with GPS/Leveling using World Geodetic System (WGS84). The difference between the leveling and the EGM08 geopotential heights has larger mean values compared with leveling against GSR80 based Geopotential heights as illustrated in Table 4.9 above. This implies that the potential W_0=Uo of the GRS80 ellipsoid is close to the mean sea level tide gauge located at the port of Alexandria to which the leveling heights are tied as compared to the potential W_0 =U_0 of the WGS84 reference ellipsoid. The standard deviation remains almost the same when leveling is compared to gravity based Geopotential heights. This justifies that the EGM08 gravity model is accurate enough to simulate to the classical leveled heights good to few centimeters.

Chapter 5

5 Conclusions and Recommendations

5.1 Conclusions

From the research finding we can conclude that:

1. Geopotential heights derives from EGM08 are determined to be good to 2.26cm accuracy when compared with the leveling Data.

2. The prey orthometric height determined with 1.09 cm accuracy when EGM08 combines with terrestrial gravity observations at each leveling benchmark.

3. The prediction of normal height and height anomaly are determined good to 2.26cm. When EGM08 constrained with surface gravity and EGM08 alone. This shows that the two heights are not affected by surface gravity measurement.

4. As compared with other Geopotential height the prediction of Helmert ortometric height is poor, this might be due to the fact that the assumption of uniform sub surface density variation and constant normal gravity gradient.

5. Since the error in the ellipsoidal height is directly transferable to the Geopotential height, one can conclude that the GPS (GNSS) position coordinates have to be determined with millimeter accuracy so as to get Geopotential heights accurate to few centimeters up to 2.26cm.

6. Compared to results from other counties the result achieved in this work showed that the EGM08 over Ethiopia is high quality due to the usage of the airborne gravity data for its development.

5.2 Recommendations

From the research result one can recommend:

1. The accuracy of the EGM08 based Geopotential heights have been validated around Deber Birhan town with an independent Blue Nile benchmarks. Further validation of this form has to be adequately applied at different part of the country so as to evaluate the quality of EGM08 over the entire region of Ethiopia.

2. Farther Assessment has to be done to recover some of the existing Blue Nile leveling benchmark for EGM08 validation purpose and maintaining the classical leveling control networks.

3. Research should be done on the Blue Nile benchmark points to know the effects of geodynamic movement on the position of benchmark points.

4. A new first order high quality geodetic leveling survey has to be conducted at different part of the country using a nearby standard datum so as to adequately validate the EGM08 and the future new generation of Earth Gravity models.

Annex – I

BLUE NILE RIVER BASIN, ETHIOPIA
VERTICAL CONTROL DATA

SEA LEVEL DATUM

DESCRIPTION OF SUPPLEMENTARY ELEVATION POINT — SP

Extra Foresight No. 5 G State Ethiopia County Shoa
Distance and direction from bench mark 1.0 km east of K 11
Detailed description The surface of the centerline of the Addis Ababa-Dessie road, near the center of a large masonry bridge 7.3 meters above the water in the stream.

DESCRIPTION OF BENCH MARK

Designation K 11 State Ethiopia County Shoa
Nearest town Debra Berhan County Shoa Chief of party Don A. Jones
Distance and direction from nearest town 10.46 kms. northeast Leveling date 9-17-58
Character of mark Chiseled burr Stamping
Established by Blue Nile Geodetic Control Project
Detailed description 10.46 kilometers northeast along the Addis Ababa-Dessie road from the police station at Debra Berhan, 1.1 kilometers east of kilometer post 140, at a large sweeping curve, north and across the road from a rock studded bluff, a chiseled burr surrounded by a 0.05-by 0.05-meter chiseled square on the top of a boulder, 9.2 meters north of the centerline of the road, and about 0.4 meter higher than the road.

DESCRIPTION OF BENCH MARK

Designation J 11 State Ethiopia County Shoa
Nearest town Debra Berhan County Shoa Chief of party Don A. Jones
Distance and direction from nearest town 5.95 kms. northeast Leveling date 9-17-58
Character of mark Chiseled burr Stamping
Established by Blue Nile Geodetic Control Project
Detailed description 5.95 kilometers northeast along the Addis Ababa-Dessie road from the police station at Debra Berhan, a chiseled burr surrounded by a 0.05-by 0.05-meter chiseled square on the top of an outcrop, 16 meters northeast of the centerline of the road, 11.4 meters northeast of the east end of a culvert, 8.2 meters northeast of the center of a drainage ditch, about 1.4 meters higher than the bottom of the ditch, and about 0.7 meter higher than the road.

DESCRIPTION OF SUPPLEMENTARY ELEVATION POINT — SP

Extra Foresight No. 27 State Ethiopia County Shoa
Distance and direction from bench mark Near J 11
Detailed description The surface of the centerline of the Addis Ababa-Dessie road, at the surface of the center of the northwest end of culvert 380.

DESCRIPTION OF BENCH MARK

Designation H 11 State Ethiopia County Shoa
Nearest town Debra Berhan County Shoa Chief of party Don A. Jones
Distance and direction from nearest town 1.28 kms. east Leveling date 9-16-58
Character of mark Monel rivet Stamping H 11
Established by Blue Nile Geodetic Control Project
Detailed description 1.28 kilometers east along the Addis Ababa-Dessie road from the police station at Debra Berhan, at a masonry bridge, in the top of the southwest end of the northwest head wall, 4.2 meters northwest of the centerline of the road, 0.4 meter northeast of the southwest end of the head wall, 0.65 meter above the surface of the bridge, and about 0.5 meter higher than the road.

DESCRIPTION OF SUPPLEMENTARY ELEVATION POINT — SP

Extra Foresight No. 26 State Ethiopia County Shoa
Distance and direction from bench mark 1.28 kilometers south of H11
Detailed description The surface of the centerline of the Addis Ababa-Dessie road (main street of Debra Berhan), opposite the police station and a row of round tukuls.

DESCRIPTION OF BENCH MARK

Designation G 11 State Ethiopia County Shoa
Nearest town Debra Berhan County Shoa Chief of party Don A. Jones
Distance and direction from nearest town 2.32 kms. southwest Leveling date 9-15-58
Character of mark Monel rivet Stamping G 11
Established by Blue Nile Geodetic Control Project
Detailed description 2.32 kilometers southwest along the Addis Ababa-Dessie road from the police station at Debra Berhan, at a large masonry bridge, in the top of the northwest head wall, 8.2 meters southwest of the northeast end of the head wall, 4.2 meters northwest of the centerline of the road, 0.8 meter lower than the top of the head wall, and about level with the road.

DESCRIPTION OF BENCH MARK

Designation F 11 State Ethiopia County Shoa
Nearest town Debra Berhan County Shoa Chief of party Don A. Jones
Distance and direction from nearest town 5.76 kms. southwest Leveling date 9-10-58
Character of mark Monel rivet Stamping F 11
Established by Blue Nile Geodetic Control Project
Detailed description 5.76 kilometers southwest along the Addis Ababa-Dessie road from the police station at Debra Berhan, at a large masonry bridge, in the top of the southwest end of the southeast head wall, 4.5 meters southeast of the centerline of the road, 1.8 meters northeast of the southwest end of the head wall, and 0.15 meter above the road.

DESCRIPTION OF SUPPLEMENTARY ELEVATION POINT — SP

Extra Foresight No. 24 State Ethiopia County Shoa
Distance and direction from bench mark 0.84 km. southwest of F 11
Detailed description The surface of the centerline of the Addis Ababa-Dessie road, at the surface of the "T" junction of a dirt road leading northwest to an army camp.

455

References

Bedada. T, (2010): An Absolute Geopotential Height System for Ethiopia.

Brunner. FK and Welsch. WM., (1993): Effect of the troposphere on GPS measurements. GPS world, 4(1): 42-51.

Butsch, F, (2002): Radiofrequency interference and GPS: A growing concern, GPS world 13(10), 40-49.

Elias Lewi Dr Ing: Lecture note, Chapter one, Measurements and error.

Ellmann. A, (2010): Validation of the New Earth Gravitational Model EGM08 over the Baltic countries. International Association of Geodesy Symposia, 135, 489-496.

Ethiopia Geodetic Survey, (1957 – 1961), Report on Horizontal and Vertical Control Surveys of the Blue Nile River Basin, by U.S Department Of Commence Coast and Geodetic Survey, for Ethiopia- United States Cooperative Program For Water Resource.

Hatch. R., Jung. J, Enge. P and Pervan. B, (2000): Civilian GPS, The Benefits of three Frequencies' GPS Solutions 3(4), 1-9.

Heiskanen WA and Moritz H. (1967): Physical geodesy. Freeman, San Francisco.

Hipkin and Hussian, (1983): Regional gravity anomaly IGS Report.

Hipkin RG. (1988). Bouger anomalies and the geoid: are-assessment of Stokes's method. Geophysical Journal International, 92, 53-66.

Hipkin RG. (2001): The statistics of pink noise on a sphere: applications to mantle density anomalies, Geophysical Journal International.

Hipkin RG.(2002). Defining the geoid by W=Wo=Uo: theory and practice of a modern height system. In: Gravity and Geoid 2002, 3rd Meeting of the International Gravity and Geoid Commission, Thessaloniki, Greece, 367- 377.

Hofmann-Wellenhof B.et al., (1997): GPS, Theory and Practice.

Hofmann-Wellenhof and Helmut Moritz, (2005): Physical Geodesy, Institute of navigation and Satellite geodesy Technical University, Austria.

Jekeli. C, (2000): Heights, the Geopotential, and Vertical Datum's. Ohio State University, Departement of Geodetic Science and Surveying, Report No.459, The Ohio State University, Columbus, USA.

Johannessen, R, (1997): Interference, Source and Symptoms. GPS World, 8 (11): 45-48.

John wiley and Sons, (2007): Global Positining, Inertial navigation & Integration. New York.

Keith DM, (2002): The modernization of GPS: Plans, new capabilities and the future relationship to Galileo.Journal of Global Positioning Systems, 1, No. 1, 1-17.

King RW & Bock Y. (2002): Documentation for the MIT GPS analysis software: GAMIT, Massachusetts Institute of Technology.

Ledersteger. K, (1955): Publication dedicated to Weikko A. Heiskanen. Finnish Geodetic Institute, Helsinki, vol 46 1: 165-173.

Leick. A, (1995): GPS satellite surevaying.2nd. Edition, John Wiley, New York.

Lemoine et al., (1998): The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential model EGM96, NASA Report TP-1998-206861, Goddard Space Flight Center.

Mader. G, (1999): ‘GPS antenna calibration at the National Geodetic Survey’, GPS Solutions 3(1), 1521-1886.

Mobinder S. Grewal; Lawrence R. Weill; Angus P. Andrews, (2007): Global Positioning Systems, Inertial Navigation, and Integration, Second edition.

Newton’s bulletin, External Quality Evaluation Reports of EGM08, Issue no 4, April 2009.

Pavlis et al., (2008): An Earth Gravitational Model to degree 2160: EGM 2008, General Assembly of the European Geosciences Union. Vienna, Australia.

Remondi BW, (1985): Global Positioning System Carrier Phase: Description and use.

Schwintzer et al., (1991): A new global Earth‘s gravity model from the satellite orbit perturbation.

Seeber. G, (2003): Satellite Geodesy.2nd completely revised and extended edition.

Stokes, G.G., (1949): On the variation of gravity at the surface of the Earth. Trans. Cambridge Philos.Soc.VIII, 672-695.

Thomas H. Meyer, (2007): what dose Really Mean?, Department of Natural Resource and the Environment Monographs, University of Connecticut.

Wells et al., (1987): Gide to GPS Positioning, Canadian GPS Associates, Fredericton.

Witte TH & Wilson AM., (2004): Accuracy of non-differential GPS for the determination of speed over ground, Journal of Biomechanics, 37, 1891 -1898.

Declaration

“This thesis is my original work and has not been presented for a degree in any other university, and that all sources of material used for the thesis have been duly acknowledge”

Submitted by:

Ermias worku.	___________________.	_________________.
	Signature	Date

Conformation by:

Dr Tulu Besha.	__________________.	________________.
	Signature	Date