AN INVESTIGATION OF ATTENUATION AND DISPERSION IN OPTICAL FIBER

By

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This is to certify that the project prepared by **Mesfin Gudato**, entitled "**AN INVESTIGATION OF ATTENUATION AND DISPERSION IN OPTICAL FIBER**" and submitted in partial fulfillment of the requirements for the degree of **Master of Science**. compiles with the regulations of the University and meets the accepted standards with respect to originality.

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Abstract

Study of non-linearity in optical fiber material has attracted tremendous attention over last two decades. Despite many experimental and theoretical efforts to find the origin of non-linearity and to quantify the loss due to non-linearity still a debable issue. The aim of this project work is to find a mechanism and to quantify non-linearity that causes attenuation and dispersion in optical fiber. We specifically calculated solution to non-linear envelope equation (non-linear *Schrödinger* equation) from Maxwell is theory choosing appropriate envelope function. Our calculation are in agreement with other researchers as far as loss and non-linearity are con cerned. We found the Soliton solution which is very interesting result that can be further looked at in detail to observe the non-linear effect in fiber material. It is found that the non linearity balances dispersion and a stable pulse is formed which does not alter its shape during propagation. This simple calculation gives understanding for the mechanism of the origin of non-linearity. Moreover, it helps to control non-linearity by tuning some of the componsion of fiber material.

Objectives

• Understand the propagation of pulses in optical fiber and how one can able to choose a physical parameter for the reduction of unwanted effects and comes to final result and conclusion using different equations and solutions of electromaginatic waves with major infulence of non-linearity.

Specific Objectives

• Investigation propagation of electromagnetic pulses through an optical fiber.
• Derive wave equation for a general electromagnetic wave traveling along the fiber.
• Introduce attenuation and dispersion and non-linearity leading non-linear *Schrödinger* equation for the envelope of the electromagnetic pulses.
• De rive the width and intensity of pulses equation as a function of the traveled distance.
• Find the equation that keeps the shape of the pulse during propagation.

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Chapter 1

Introduction

1.1 Fiber Optics

Optical fiber is the backbone component in long-distance and high capacity optical communication systems. The choice of fiber depends on where and how it is applied and what one kind of fiber can offer over the other. Optical fibers are found in a variety of important applications such as sensors, communication systems, telecommunication networks, etc.

A clear understanding of the fundamental properties of fibers is needed not only to support the current optical networks, but to ensure the development of more powerful networks in the future too. An understanding of fiber optics is also important for the development of the fiber components found not only in communication systems but also in the many types of fiber sensors. Modern optical fiber system design typically requires the use of mathematical numerical computer models to estimate system performance.

Optical fibers with modified dispersion characteristics, such as dispersion-shifted, dispersion-flattened, and dispersion compensating fibers, are of considerable interest in broadband fiber-optical communication systems. Dispersion-shifted fibers offer a very small dispersion at the wavelength of $\lambda = 1.55\mu m$. This wavelength corresponds to the minimum attenuation wavelength of silica-based optical fibers. Thus, dispersion-shifted fibers cause much less signal distortion and attenuation than ordinary single-mode fibers [1-3]. Dispersion-flattened fibers provide small dispersion over an extended range of wavelengths. These fibers have application in wavelength-division multiplexed systems in which several optical channels are simultaneously transmitted on the same fibers. With a flattened dispersion characteristic, all channels suffer small signal distortions [4-5]. Both dispersion- shifted and dispersion-flattened fibers are single-mode with multiple cladding geometries. Their propagation properties have been studied extensively in the past [6-8]. Unlike dispersion-shifted and dispersion-flattened fibers which are desired to have very small dispersion at $\lambda = 1.55\mu m$, dispersion compensating fibers are designed to provide very large negative dispersion at this wavelength. These fibers are used to upgrade the $1.3\mu m$ fiber-optic links. Optical fibers designed for use at $1.3\mu m$ wavelength may be operated at $1.55\mu m$ in order to take advantage of lower the fiber attenuation at this wavelength. However, such fibers at $1.55\mu m$ have fairly large positive dispersion which results in signal distortion. To compensate for the accumulated dispersion over the length of the link, the fiber is concatenated with a shorter length of a dispersion compensating fiber with large negative dispersion. Most dispersion compensating fibers reported in the literature have a single cladding layer and operate in LP_{01} or LP_{11} mode [9-11]. Although much work has been done on the analysis and design of fibers with modified dispersion characteristics, little has been reported on the optimization of their designs. For example, dispersion-shifted fibers provide a nominal zero dispersion at $\lambda = 1.55\mu m$. However, it is understood that only the second-order dispersion vanishes at $1.55\mu m$ in these fibers, and third and higher-order dispersions exist and contribute to signal distortion. An optimum design will not only provide zero second-order but also zero third- order dispersion. (Fourth and higher-order dispersions are very small and neglected). For the case of dispersion compensating fibers operating in the LP_{11} mode, there are difficulties with mode conversion, beccause such fibers are dual mode. A more practical design is a single-mode diperssion compensating fiber. Moreover, the amount of negative dispersion should be as large as possible in order to reduce the required length of such fibers. In this project, improved designs for dispersion-shifted and dispersion compensating fibers are presented. A triple-clad cylindrical dielectric structure is used as a common geometry for all fiber designs. The refractive index profile is determined based on design requirements. A new dispersion-shifted fiber is proposed which provides zero second-order as well as third-order dispersion. The advantage of this fiber over the conventional dispersion-shifted fiber is in its zero third-order dispersion. Also, a dispersion compensating fiber is designed which provides a dispersion of about - 400 ps / nm. km at $\lambda = 1.55\mu m$. The important aspect of this design is that this large negative dispersion is associated with the fundamental LP_{01} mode and hence problems of mode coupling encountered in dual-mode dispersion compensating fibers are avoided. In addition to dispersion-shifted and dispersion compensating designs mentioned above, the triple-clad geometry is used to obtain a dispersion-flattened design with less than 1ps / nm. km chromatic dispersion over the $1.38\mu m - 1.59\mu m$ wavelength range.

Another contribution of this work is a generalized field analysis of triple-clad cylindrical dielectric structures. This generalized solution can accommodate all possible triple-clad fibers with step-index profiles. Accordingly, the analysis and design of the proposed dispersion-shifted, dispersion-flattened, and dispersion compensating fibers are performed using a single unified formulation. The formulation can also treat various double-clad configurations and the single-clad fiber as special cases in which the thickness of one or more layers becomes arbitrarily small. This generalized formulation lends itself to straightforward computer algorithms and code development for the calculation of propagation properties of single, double, and triple-clad fibers.

The outline of the remaining chapters of this project is as follows. Chapter two addresses various attenuation and dispersion mechanisms in optical fibers. Dispersion-altered fibers and their applications are discussed. The effect of attenuation on dispersion is also briefly examined. Chapter three is devoted to a generalized analysis of non-linearity fibers. Scalar field solutions, characteristic equation, and cutoff conditions for non-linearity are presented. Numerical results for variations of normalized propagation constant as well as chromatic dispersion versus wavelength and radial field distributions are provided in this chapter.

1.2 An Optical Fiber

A fiber core is fairly thick relative to a wavelength of light and allows the light to enter it at many difference angles. There is a finite number of angles at which the rays reflect and propagate the length of the fiber. Each angle defines a path or a mode. here are basically two modes of a transmission in a fiber. A multimode fiber has a number of paths in which light ray may travel. A single mode has a light ray in one direction only. Fibers are further classified by the refractive index profile of their core. They can be either step index or graded index. Three main types of fibers are multimode step-index fiber , multimode graded index fiber and single mode fiber. The refraction index of multimode step-index fiber is uniformly throughout the core. The refraction index of multimode graded-index fiber is gradually less dense), light travels radically outward it begins to bend back toward the center, eventually reflecting back. Because the material also less dense, the light travels faster. Reducing the core diameter to that of a single wavelength $(3 - 10\mu m)$ will let the light propagates along one mode only. Optical fibres are basically made of Silica. As shown in figure 1.1 the optical fibres have a core and a cladding. The light travels and is actually confined inside the core but the cladding is necessary to keep the light in the inner cylinder, the core. This core/cladding structure is very fragile and must be protected using other materials depending on the application before it is ready to be used as a finished fibre optics cable. The diameter of the core is very small, just a few μm in most of

the optical fibres used for telecommunications, while the cladding is several times bigger. Core and cladding diameters are used to classify different types of optical

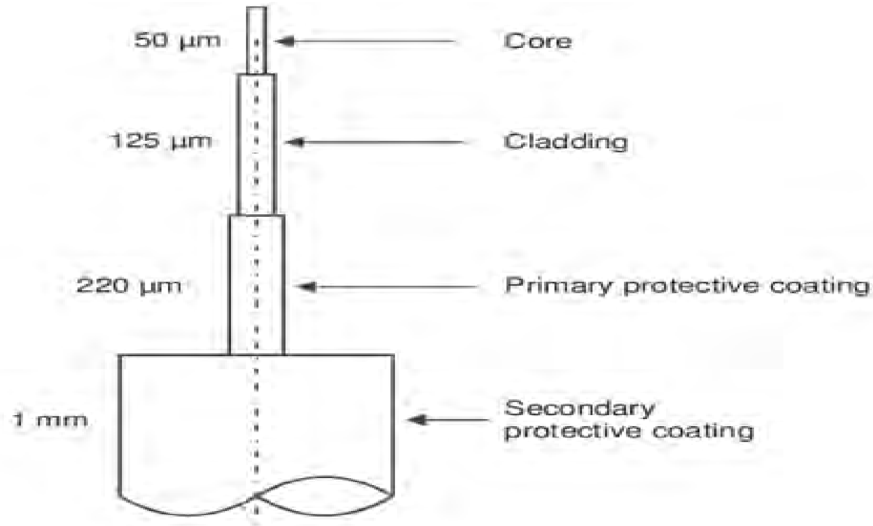

Figure 1.1: optical fiber

fibres. So if we have a $5/125\mu m$ optical fibre it means the core diameter is $5\mu m$ and the cladding diameter is $125\mu m$. The properties of every type of fibre will depend on different factors but most important are the core and cladding composition and diameters. The two most important properties used to classify the optical fibres are attenuation and dispersion. Attenuation indicates how much optical power is lost in the optical fibre and is normally expressed in terms of dB/km. Dispersion describes how the optical fibre deforms the light pulses travelling through the fibre and is one of the most important factors limiting the bandwidth. Attenuation depends on the fibre composition and construction and on the wavelength just in the same way attenuation in a coaxial cable depends on the frequency. If we measure the attenuation of an optical fibre as a function of the light wavelength we obtain a quite interesting graph, figure 1.2, which sets the basics for wavelength selection in all fibre optic communications.

The most common wave lengths are used in Optical fiber communication selected where the attenuation is minimum. These are

850nm   1st attenuation minimum

1310nm   2nd attenuation minimum

1550nm   3rd attenuation minimum

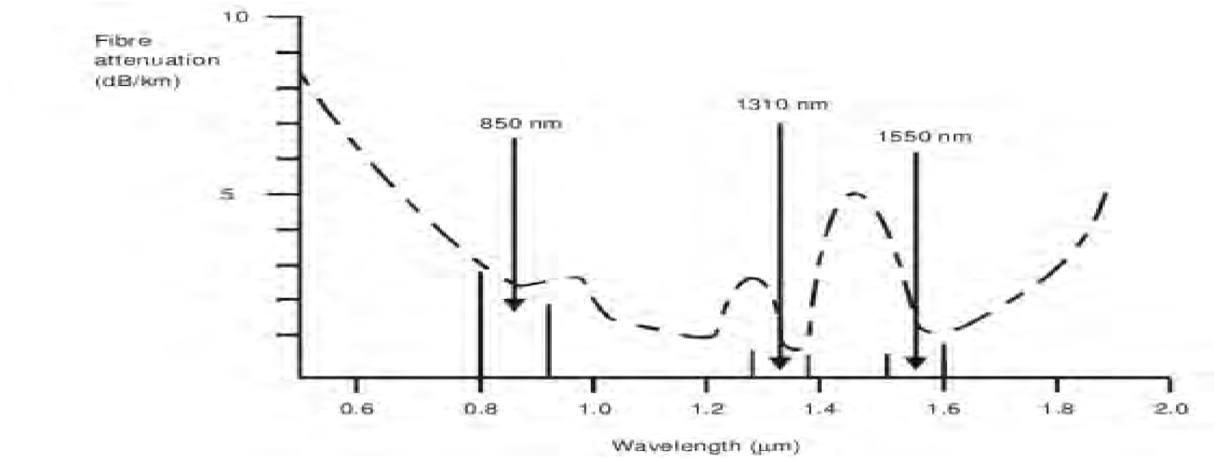

Figure 1.2: Variation of attenuation of Fiber with wavelength.

None of these wavelengths are visible but infrared because the attenuation at lower wavelengths would be unacceptably high for long distance applications.

1.3 Why Use Fibre Optics?

Fibre Optics technology will play a vital part in the future of interconnection systems in the Defence, Aerospace and Transportation markets. Its ability to carry vast amounts of data through very harsh environments makes it suitable for a variety of applications. Typical areas of deployment include: Communications systems Sensor systems (in cameras etc) Structural health monitoring (in strain gauges and systems that monitor for vital changes in temperature and pressure) Defensive-aid sub systems (for example, for detecting and deploying missiles to intercept enemy incoming signals) Databuses and tranmissions systems

The optical fibers are often considered as a perfect transmission medium with almost limitless bandwidth. In practice the propagation through optical fiber is beset with several limitations especially as distance is increased to multi-span amplified systems. As the transmission systems evolved to longer distances and higher bit rates, the linear propagation effect of fibers, which are the attenuation and dispersion, become the important limiting factors . As for wave length division multiplexing (WDM) systems that transmit multiple wavelengths simultaneously at even higher bit rates and increased distances, the non-linear effects in the fiber beginning to set a serious limitation. The major linear effects include group velocity dispersion (GVD) of standard single-mode fiber, fiber loss, polarization mode dispersion (PMD), accumulated amplified spontaneous emission (ASE) noise etc. The non-linear effects on the other hand include self phase modulation (SPM), cross phase modulation (XPM), four-wave mixing (FWM), stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS). These effects are summarized as in Fig. 1.3. The main focus of this project is to investigate the compensation of these linear and non-linear fiber transmission impairments by using digital backward propagation algorithm. This chapter briefly explains the fiber transmission impairments. It also explains the fundamentals of conventional methods, i.e. all-optical signal processing and digital signal processing, in optical communication systems to compensate linear and non-linear transmission impairments.

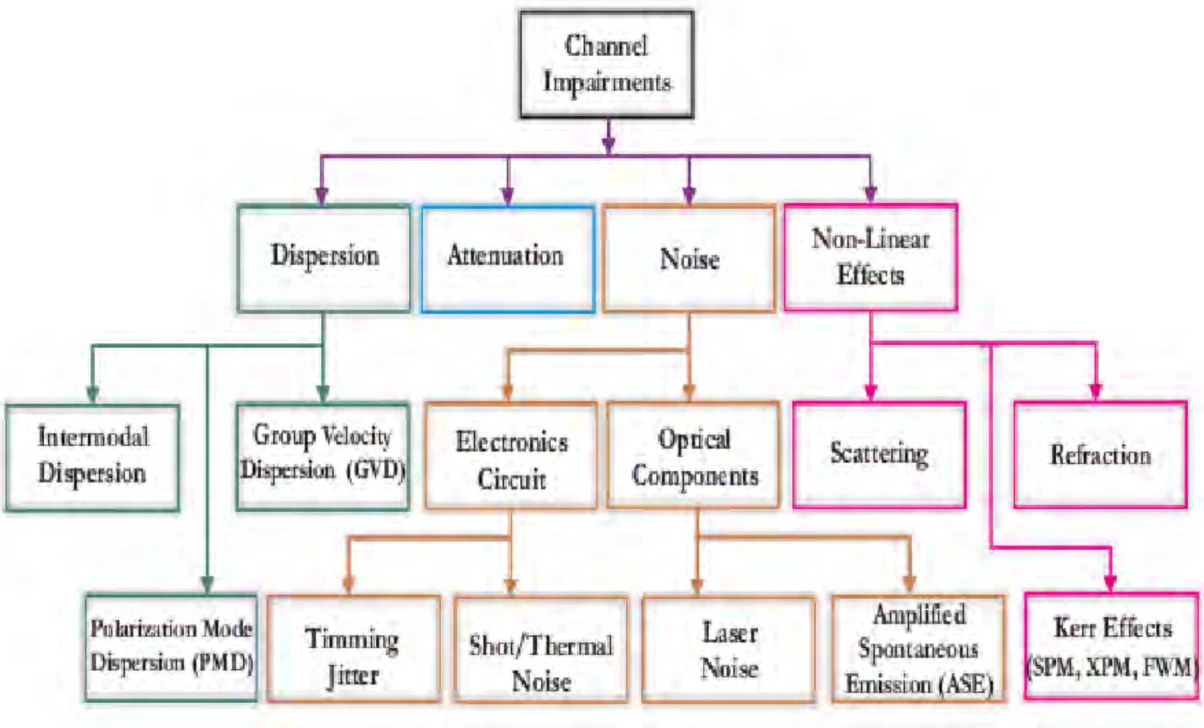

Figure 1.3: Linear and non-Linear optical fiber effects

1.4 Types of Fiber

Optical fibres come in two types: Single-mode fibres and Multi-mode fibres. Single-mode fibres have small cores (about 9 microns in diameter) and transmit infrared laser light. Multi-mode fibres have larger cores (62.5 microns in diameter) and transmit infrared light from light emitting diode (LEDs). Some optical fibres can be made from plastic. These fibres have a large core (1 mm diameter) and transmit visible red light from Light emitting diode (LEDs).

Optical fibres may be classified according to the following three:

1.Multimode

a.Step Index

i.Plastic and liquid ii.Silica high purity iii.Silica Doped iv.Fluoride v.Incoherent and imaging bundles vi.Tapered fibres

b.Gradient Index

2.Single Mode

3.Photonics

a.Silica cavity b.Hollow cavity

Understanding the characteristics of different fiber types aides in understanding the applications for which they are used. Operating a fiber optic system properly relies on knowing what type of fiber is being used and why. Multimode fiber is best designed for short transmission distances, and is suited for use in local area network (LAN) systems and video surveillance. Single-mode fiber is best designed for longer transmission distances, making it suitable for long-distance telephony and multichannel television broadcast systems

1.5 Multimode Fiber

Multimode fiber, the first to be manufactured and commercialized, simply refers to the fact that numerous modes or light rays are carried simultaneously through the optical waveguide. Modes result from the fact that light will only propagate in the fiber core at discrete angles within the cone of acceptance. This fiber type has a much larger core diameter, compared to single-mode fiber, allowing for the larger number of modes, and multimode fiber is easier to couple than single-mode optical fiber. Multimode fiber may be categorized as step-index or graded-index fiber.

Multimode Step-index Fiber

Multimode fiber allows an infinite number of possible paths to be transmitted. These paths are called modes and identify the general characteristic of the light transmission system being used. Fiber that has a core diameter large enough for the light used to find multiple paths is called multimode fiber see figure (1.4).

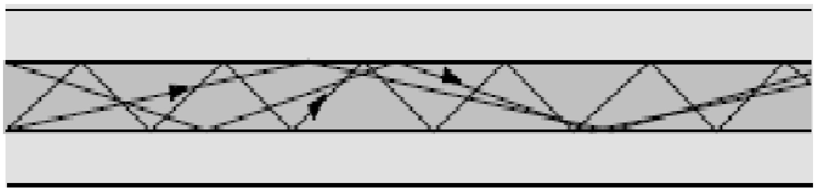

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Figure 1.4: Multi-Mode step-Index Fiber

1.6 Single-Mode Fiber

Single-mode fiber allows for a higher capacity to transmit information because it can retain the fidelity of each light pulse over longer distances, and it exhibits no dispersion caused by multiple modes. Single-mode fiber also enjoys lower fiber attenuation than multimode fiber. Thus, more information can be transmitted per unit of time. Like multimode fiber, early single-mode fiber was generally characterized as step-index fiber meaning the refractive index of the fiber core is a step above that of the cladding rather than graded as it is in graded-index fiber. Modern single-mode fibers have evolved into more complex designs such as matched clad, depressed clad and other exotic structures.

Single-mode fiber has disadvantages. The smaller core diameter makes coupling light into the core more difficult. The tolerances for single-mode connectors and splices are also much more demanding. Single-mode fiber has gone through a continuing evolution for several decades now. As a result, there are three basic classes of single-mode fiber used in modern telecommunications systems. The oldest and most widely deployed type is non dispersion-shifted fiber(NDSF). These fibers were initially intended for use near 1310nm. Later, 1550nm systems made NDSF fiber undesirable due to its very high dispersion at the 1550nm wavelength. To address this shortcoming, fiber manufacturers developed, dispersion-shifted fiber(DSF), that moved the zero-dispersion point to the 1550 nm region. Years later, it was observed that while DSF worked extremely well with a single 1550 nm wavelength, it exhibits serious nonlinearities when multiple, closely-spaced wavelengths in the 1550 nm were transmitted in dense wave length division multiplexing (DWDM) systems. Recently, to address the problem of nonlinearities, a new class of fibers is introduced. These are classified as non zero-dispersion-shifted fibers (NZ-DSF). The fibers are available in both positive and negative dispersion varieties and is rapidly becoming the fiber of choice in new fiber deployment.

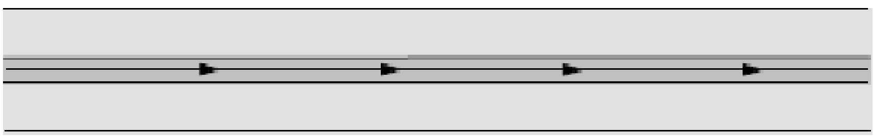

Figure 1.5: Single-Mode Fiber

Optical fibers can be used as sensors to measure strain, temperature, pressure and other quantities by modifying a fiber so that the property to measure modulates the intensity, phase, polarization, wavelength, or transit time of light in the fiber An optical fiber is a cylindrical dielectric waveguide (nonconducting waveguide) that transmits light along its axis, by the process of total internal reflection. The fiber consists of a core surrounded by a cladding layer, both of which are made of dielectric materials. To confine the optical signal in the core, the refractive index of the core must be greater than that of the cladding. The boundary between the core and cladding may either be abrupt, in step-index fiber, or gradual, in graded-index fiber.

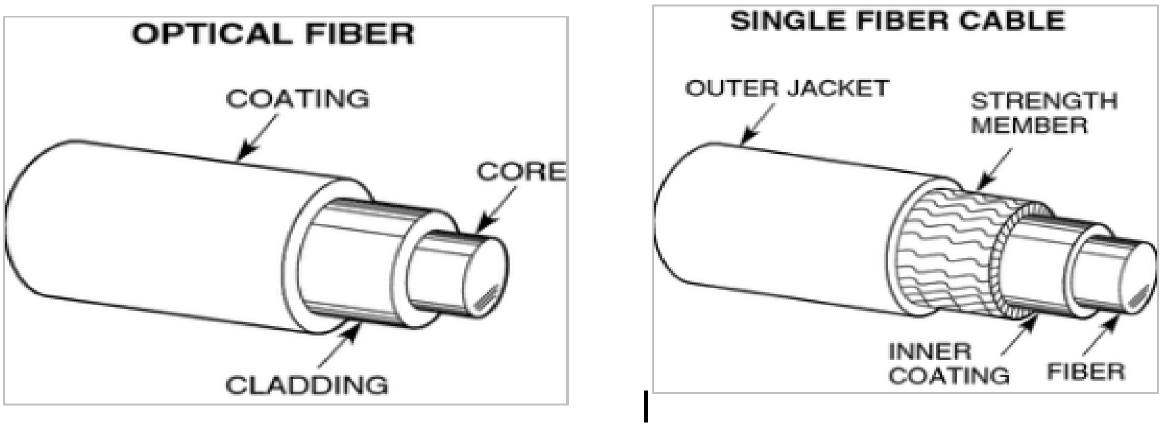

Figure 1.6: Optical fiber consists of a core surrounded by a cladding layer,

Fiber Geometry

An Optical Fiber Is Made of Three Sections

The core carries the light signals

The cladding keeps the light in the core

The coating protects the glass Coating

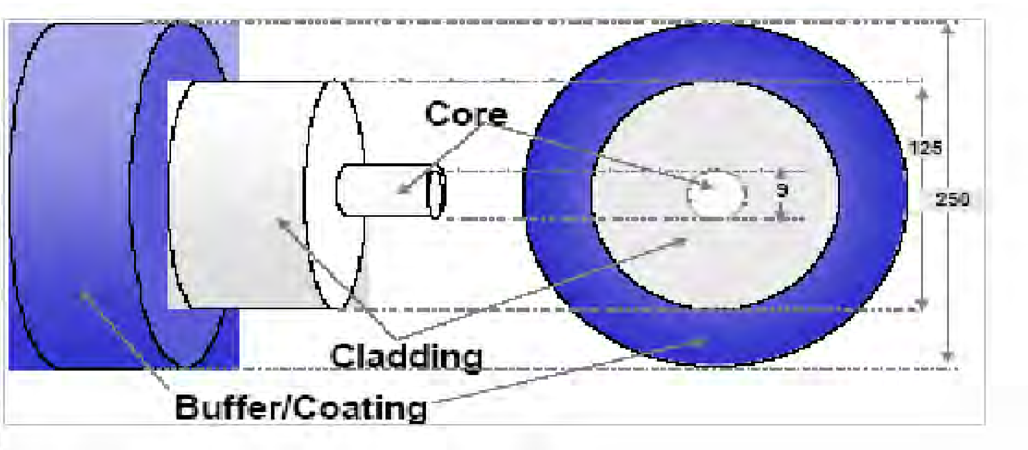

Figure 1.7: Geometrical fiber optics

Chapter 2

Attenuation and Dispersion in Fiber Optics

The aim of this chapter is to describe the system of optical fiber communications and to also describe how light propagates through optical fibers. This chapter presents the characteristics of optical fiber as well as various concepts and techniques which may be of help in the design of optical transmission lines. Sound waves propagating in solid materials are subjected to two phenomena: dispersion and attenuation. While dispersion refers to sound velocity dependency upon frequency, attenuation is described as the wave amplitude decreasing with distance. Since these phenomena are closely related, dispersion measurements can be used to predict attenuation data as a function of frequency, and vice versa.

An information signal becomes distorted due to attenuation and dispersion as it travels in an optical fiber. Attenuation is the loss of signal power and is governed by different mechanisms, including absorption, scattering, and radiation. Since optical fibers were introduced for communication applications three decades ago, great progress has been accomplished in producing optical fibers that exhibit very low signal attenuation. On the other hand, dispersion is the spreading in the time domain of a signal pulse as it travels through the fiber. Spectral components of a pulse propagating down an optical fiber reach their destination at slightly different times. This translates into a wider pulse at the receiving end of the fiber. Both attenuation and dispersion affect repeater spacing in a long distance fiber optic communication system. Dispersion affects the bandwidth of the system, hence maintaining low dispersion is of equal importance for ensuring increased system information capacity, versatility and cost effectiveness.

2.1 Dispersion in Single-Mode Fibers

Dispersion in single-mode fibers is an intramodal effect and is a result of group velocity dependence on wavelength. Because of that, the amount of signal distortion depends on the spectral width of the optical source used. Three mechanisms contribute to intramodal dispersion: material dispersion, waveguide dispersion, and polarization-mode dispersion.

2.1.1 Material Dispersion

Material dispersion is caused by variations of refractive index of the fiber material with respect to wavelength. Since the group velocity is a function of the refractive index, the spectral components of any given signal will travel at different speeds causing deformation of the pulse. Variations of refractive index with respect to wavelength are described by the Sellmeier's equation which is expressed as follows [12]

$$n(\lambda) = [1 + \Sigma_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2}]^{\frac{1}{2}} \tag{2.1.1}$$

where λ is the wavelength of light, and A_i and λ_i are the Sellmeier's coefficients.

2.1.2 Polarization-Mode Dispersion

Single-mode fibers, in reality, support two orthogonally-polarized fundamental modes. In perfectly circular fibers, these two modes have identical propagation constants and pulse spreading due to polarization-mode dispersion does not exist. In practical fibers, however, there is a small difference between the propagation constants of these two modes due to the slight ellipticity of the core. In other words, common single-mode fibers actually support two modes and thus are not truly single-mode. The presence of two fundamental modes contributes to pulse spreading. This phenomenon is known as polarization-mode dispersion.

2.1.3 Dispersion in Multimode Fibers

In applications where two or more modes travel simultaneously though the fiber, intermodal as well as intramodal dispersions exist. Intermodal dispersion does not occur in single-mode fibers, but is a significant effect in multimode fibers. It occurs as a result of different modes having different group velocities at the same frequency. Graded-index fibers with nearly parabolic-index profile were developed mainly to reduce the effect of intermodal dispersion. Here, bound rays deviating from the axis of the fiber travel a longer distance but at larger velocities, reaching the receiving end of the fiber at about the same time with the other rays, thus in graded-index fibers pulse spreading is significantly reduced. Although all forms of dispersion present in single-mode fibers exist in multimode fibers too. Thus, pulse spreading in multimode fibers is largely due to material dispersion and intermodal delay distortion. Polarization-mode dispersion is a much weaker effect than material dispersions and intermodal delay, and is often neglected in the analysis and design of fiber-optic links. Apart from the three dispersion effects described above, there is yet another kind of dispersion referred to as profile dispersion. This effect is attributed to core and cladding materials having slightly different material dispersions. In this project, the profile dispersion is accounted for as part of material dispersion and thus does not require a separate analysis.

2.1.4 Group Velocity, Group Delay, and Dispersion

Let us consider an information signal propagating in a single-mode fiber of length L . Each spectral component of the signal undergoes a time delay t_g . The time delay per unit length, denoted as τ_g , is obtained as

$$\tau_g = \frac{t_g}{L} = \frac{1}{V_g} = \frac{1}{c}\frac{d\beta}{d\kappa_o} = \frac{-\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \tag{2.1.2}$$

where $V_g = (d\beta/d\omega)^{-1} = c(d\beta/d\kappa_0)^{-1}$ is the group velocity, $\kappa_0 = 2\pi/\lambda$ is the free-space wave number, and c is the velocity of light in free space. Spectral components travel at different speeds and experience different time delays. We are interested in the pulse spreading arising from group delay variations. Let the root-mean-square spectral width of the optical source be σ_λ , then the total delay difference is given by

$$\delta_\tau = \frac{d\tau_g}{d\lambda}\sigma_\lambda L \tag{2.1.3}$$

The amount of pulse spread per unit length of fiber and per unit spectral width of light source is defined as dispersion. Thus, the expression of dispersion is written as

$$D = \frac{1}{Ld\lambda}\delta_\tau = \frac{-\lambda}{2\pi c}[2\frac{d\beta}{d\lambda} + \lambda\frac{d^2\beta}{d\lambda^2}] \tag{2.1.4}$$

A more suitable formula for calculation of dispersion is that expressed in terms of the normalized propagation constant $\bar{\beta}$, defined as $\bar{\beta} = \beta/\kappa_0 = 2\pi\beta/\lambda$. Substituting for β , in terms of $\bar{\beta}$, (2.1.4) is reduced to

$$D = -\frac{\lambda}{c}\frac{d^2\beta}{d\lambda^2} \tag{2.1.5}$$

The cumulative effect of waveguide and material dispersion is usually referred to as chromatic dispersion. Chromatic dispersion in ordinary single-mode fibers is approximated by adding the material and waveguide dispersion effects determined separately. This approximation, while satisfactory for ordinary single-mode fibers, may not be adequate for ultra-low dispersion fibers such as dispersion-shifted and dispersion-flattened fibers [6-7]. Therefore, for better accuracy, material and waveguide dispersion effects are calculated simultaneously. In doing so, the wavelength dependence of the refractive index of each material is accounted for when determining the propagation constant β . The propagation constant is calculated by numerically solving the characteristic equation which may be expressed as $f(\lambda, \beta, n_i, i = 1, 2, ..., N) = 0$, for a fiber consisting of N layers. The function f also includes variables such as core and cladding radii, which are independent of wavelength, and the azimuthal mode number. The refractive indices n_i are determined using the Sellmeier equation given in (2.1). Using numerical techniques, $\bar{\beta}, d\bar{\beta}/d\lambda, d^2\bar{\beta}/d\lambda^2$ are determined and put in (2.1.5) to obtain the total dispersion. It is emphasized that using this method of calculation, waveguide, material, and profile dispersion effects are simultaneously accounted for.

2.2 Dispersion-Altered Fibers

The objective in the optimum design of an optical fiber is to achieve the lowest attenuation and dispersion at the wavelength of operation. In ordinary single-mode fibers,total dispersion vanishes at a wavelength of about $1.3\mu m$. However, the lowest attenuation for glass fibers occurs at $1.55\mu m$. Alteration of dispersion in a fiber is attained by manipulating the index profile and geometry of the fiber. Dispersion-altered fibers include dispersion-shifted, dispersion-flattened, and dispersion compensating fibers discussed below.

2.2.1 Dispersion-Shifted Fibers

Dispersion-shifted fibers are the type in which the wavelength of zero dispersion is shifted to the region of lowest attenuation, which for $S_iO_2 - GeO_2$ based fibers lies in the $1.55\mu m$ region [1] and [21-22]. Providing minimal dispersion over a very narrow range of wavelengths, dispersion-shifted fibers are best suited for single channel transmission. The systems efficiency is increased due to longer repeater spacing, one of the most important considerations in designing long-distance optical fiber communication systems. Multiclad fibers with step-index, as well as graded cores can be used to design dispersion-shifted single-mode fibers [2] and [14].

2.2.2 Dispersion-Flattened Fibers

The possibility of low dispersion over an extended range of wavelengths was presented by Kawakami and Nishida in 1974 [15], and studied extensively thereafter [6] and [23]. By manipulating the index profile of a fiber, total dispersion can be made to go to zero at two or three different wavelengths, and remain close to zero in between. Dispersion flattening occurs by partial cancellation of waveguide dispersion by material dispersion in the wavelength range of operation. In some applications such as wavelength division multiplexing, where a number of signals with different wavelengths are carried by one fiber, it is desired to design the fiber optic system such that all optical signals experience relatively the same low distortion. The information capacity of fiber-optic systems using dispersion-flattened fibers and wavelength division multiplexing (WDM) schemes can be increased many folds. Multiclad fibers, including double, triple, and quadruple-clad fibers can be used to design dispersion-flattened fibers. Fiber designs have been reported where dispersion is less than 1 (ps / nm. km) over the entire range $1.31\mu m$ to $1.67\mu m$ [16].

2.2.3 Dispersion Compensation

The performance of a long distance optical fiber communication system is limited by various factors, one of which is dispersion, as mentioned earlier. Pulse distortion reduces maximum spacing between optical transmitters and receivers if the same BER performance for the system is to be maintained.

When commercial single-mode optical fiber links were first introduced and installed, they were designed to offer zero dispersion at $1.3\mu m$, since that was the wavelength of . commercially available light sources. Operated nowadays at $1.55\mu m$, these fibers exhibit . substantial positive dispersion that may be canceled out by using dispersion compensating fibers which provide large negative dispersion at that wavelength [9-10]. A signal traveling through an old-generation single-mode fiber link suffers a total dispersion DL over a distance L , where D is the dispersion per unit length, described earlier in equation (2.1.5), measured at $1.55\mu m$. DL maybe be of considerable magnitude after a long distance is traveled. Dispersion compensation is realized by splicing an optical fiber of length l that exhibits large negative dispersion, $D^{'}$, at the wavelength of operation, such that $D^{'}$ l cancels out DL ; that is $DL + D^{'}l = 0$. The length of the compensating fiber needed l is thus obtained from $l = |D/D^{'}|L$. Dispersion compensating fibers make it possible to upgrade existing $1.3\mu m$ links without unnecessary and expensive replacements.

2.3 Effect of Attenuation on Dispersion

When calculating dispersion in an optical fiber, it is commonly assumed that the fiber is lossless. In other words, the refractive indices of various layers constituting the fiber are all assumed to be real. In reality, these indices are complex with small imaginary parts which account for the losses in the fiber. The effect of losses on dispersion is generally negligible. However, it seems that no investigation has been made to verify how small this effect may be and if it might have to be considered in ultra-low dispersion fibers. Here, the effect of attenuation on dispersion is assessed using a perturbation approach. If the scalar field of a lossless weakly guiding fiber of refractive index profile n is ψ and that of a low-loss fiber with refractive index $\hat{n}$ is $\hat{\psi}$, it can be shown that [17]

$$\beta^2 - \hat{\beta}^2 = \kappa_0^2 \frac{\int_s (n^2 - \hat{n}^2)\psi\hat{\psi} ds}{\int_s \psi\hat{\psi} ds} \tag{2.3.1}$$

where β and $\hat{\beta}$ are the propagation constants of the lossless and the low-loss fibers respectively, S is a z=constant plane (assuming that the z-axis coincides with the fiber axis), and $\kappa_0 = 2\pi/\lambda$ is the free-space wave number. Here, the low-loss fiber is regarded as a perturbation of the lossless fiber. The refractive index n is real, while $\hat{n}$ is complex with a small imaginary part accounting for fiber losses. n and $\hat{n}$ differ only in the imaginary part of n . That is $\hat{n} = nj\delta_n, \delta n << n$. It is clear that $beta$ and $\hat{\beta}$ also differ by a small amount, and we can write

$\hat{\beta} = \beta + \delta_\beta; \delta_\beta << \beta$.

Substituting for $\hat{\beta}$ and $\hat{n}$ in terms of $\hat{\beta}$ and $\hat{n}$, respectively, and also using $\psi\hat{\psi}$, from (2.3.1) we obtain

$$\delta_\beta = -\frac{\kappa_0^2}{2\beta}\frac{\int_s (j2n\delta_n + \delta_n^2)\psi^2 d_s}{\int_s \psi^2 d_s} \tag{2.3.2}$$

The imaginary part of δ_β , denoted as $-\alpha$, is in fact the attenuation coefficient of the fiber,

$$\alpha = \frac{\kappa_0^2}{\beta}\frac{\int_s n\delta_n \psi^2 ds}{\int_s \psi^2 ds} \tag{2.3.3}$$

while the real part of δ_β is

$$\delta\beta_r = \frac{-\kappa_0^2}{\beta}\frac{\int_s \Delta tan^2 \psi^2 ds}{\int_s \psi^2 ds} \tag{2.3.4}$$

For weakly guiding fibers n ,$\hat{n}$, and δ_n vary very slowly over the fiber cross section and thus may be assumed as nearly constant. This approximation essentially amounts to considering the fiber as a homogeneous medium. Here, we are content with this approximation, because our purpose is to assess the order of magnitude of the attenuation effect on dispersion. One can always use (2.3.3) and (2.3.4) for a more accurate evaluation of this effect. Using this approximation and noting that $\kappa_0 n\beta$, we obtain

$$\alpha \cong \frac{\kappa_0 n\delta_n}{\beta} = \kappa_0\delta_n \tag{2.3.5}$$

$$\delta\beta_r = -\frac{\kappa^2}{2\beta}\Delta tan^2 = -\frac{\alpha^2\beta}{2\kappa_0^2 n^2} = -\frac{\alpha^2\hat{\beta}}{2\kappa_0 n^2} \tag{2.3.6}$$

where $bar\beta = \beta/\kappa_0$. A change in the amount of $\delta\beta_r$ in β brings about a change in the dispersion, δD , which can be calculated from (2.1.5) as follows

$$\delta D = \frac{-\lambda}{c}\frac{d^2(\delta\hat{\beta}_r)}{d\lambda^2} \tag{2.3.7}$$

where $\delta\beta_r = \delta\beta_r/\kappa_0$ Combining (2.3.6) and (2.3.7), we obtain

$$\delta D = f(\lambda)D(\lambda) + \frac{\lambda}{c}\frac{[2df(\lambda)}{d\lambda}\frac{d\beta\bar{(\lambda)}}{d\lambda} + \frac{d^2 f(\lambda)}{d\lambda^2}\bar{\beta}(\lambda)] \tag{2.3.8}$$

where $D(\lambda)$ is the dispersion of the lossless fiber given by (2.1.5) and $f(\lambda)$ is defined as

$$f(\lambda) = \frac{\lambda^2 \alpha^2(\lambda)}{8\pi^2 n^2(\lambda)} \tag{2.3.9}$$

At $\lambda = 1.55\mu m$, the fiber attenuation in dB is $20\alpha log_{10}^{e} = 0.21dB/km$, and a typical fiber has n = 1458 . With these data, we obtain $\alpha = 0.024181/km$ and . $f(\lambda) \mid_{\lambda=1.55\mu m} = 7.836$ x 10^{24}. It is Clear that δD is on the order of 10^{20} ps/nm.km and thus can be neglected in all fibers of practical applications.

2.4 Attenuation in the Fiber

Attenuation (in some contexts also called extinction) is the gradual loss in intensity of any kind of flux through a medium. For instance, sunlight is attenuated by dark glasses, X-rays are attenuated by lead, and light and sound are attenuated by water. Fiber attenuation, which necessitates the use of amplification systems, is caused by a combination of material absorption, Rayleigh scattering, Mie scattering, and connection losses. Although material absorption for pure silica is only around 0.03 dB/km (modern fiber has attenuation around 0.3 dB/km), impurities in the original optical fibers caused attenuation of about 1000 dB/km. Other forms of attenuation are caused by physical stresses to the fiber, microscopic fluctuations in density, and imperfect splicing technique.

The light propagating through a medium loses some percentage of its power if the dielectric material is not perfectly transparent. Phenomenologically, this phenomenon can be demonstrated by a complex susceptibility,

$$X = X' + iX'' \tag{2.4.1}$$

corresponding to the complex permittivity $\epsilon = \epsilon_0(1 + x)$. The wavenumber will be complex valued ($\kappa = \beta - i\alpha/$) where the imaginary part of the wavenumber will be responsible for the attenuation of light.

The transmitted power (P_{out}) of a light beam with an initial power of P_{in} can be given by the following expression after the propagation length L

$$P_{out} = P_{in} exp(-\alpha L) \tag{2.4.2}$$

where the attenuation constant α is a measure of total fiber losses from all sources. α in Eq. (2.4.2) has a unit of 1/m in SI. It is customary however to express α in units of dB/km. The conversion between a ratio R in SI and in decibel can be easily done by the general definition

$$R(indB) = 10log10R(inSI) \tag{2.4.3}$$

The most common use of decibel scale occurs for power ratios as it is the case in Eq. (2.4.2)

$$\alpha[dB/m] = -\frac{10}{L} 10log_{10}[\frac{P_{out}}{P_{in}}] = -\frac{10}{L} ln[\frac{P_{out}}{P_{in}}]/ln10 = \frac{10}{ln10}\alpha_{\frac{1}{m}} \tag{2.4.4}$$

where ln stands for the natural logarithm and we used Eq. (2.4.2) to substitute the ratio of output and input power with $\alpha\{\frac{1}{m}\}L$. Subscripts next to the α parameters are intended to show the unit of the parameter. In order to get the loss in dB/km we should simply multiply the above equation by thousand

$$\alpha[dB/km] = \frac{10^4}{ln10}\alpha_{\frac{1}{m}} \tag{2.4.5}$$

This expression is often used in the simulations for obtaining the loss in SI from the loss data provided by the fiber manufacturers. The loss in a fiber is also wavelength dependent. Different frequency components of the propagating light are attenuated with different magnitudes. The factors which contribute to the loss spectrum are the Rayleigh scattering, water (OH^-) absorption and metal-oxide absorption peaks. Silica glass, for example, has electronic resonances in the UV and vibrational resonances in the FIR region. Therefore, this type of glasses can transmit light in the $0.5 - 2.2\mu m$ region. Rayleigh scattering is a fundamental loss mechanism arising from the density fluctuations frozen into the fused silica during manufacturing. Resulting local fluctuations in the refractive index scatter light in all directions. The Rayleigh scattering loss varies with λ^{-4} therefore its effect is dominant at short wavelengths. Another loss factor is the bending loss which may scatter light at the core cladding interface. In communication systems the splicing loss and connector losses may also contribute to the attenuation of the transmitted light.

2.5 Aplications of Attenuation and Dispersion in the Fiber Optics

Correct functioning of an optical data link depends on modulated light reaching the receiver with enough power to be demodulated correctly. Attenuation is the reduction in power of the light signal as it is transmitted. Attenuation is caused by passive media components, such as cables, cable splices, and connectors. While attenuation is significantly lower for optical fiber than for other media, it still occurs in both multimode and single-mode transmission. An efficient optical data link must have enough light available to overcome attenuation. Dispersion is the spreading of the signal in time. The following two types of dispersion can affect an optical data link:

· Chromatic dispersionThe spreading of the signal in time resulting from the different speeds of light rays.

· Modal dispersionThe spreading of the signal in time resulting from the different propagation modes in the fiber.

Optical fiber is used by many telecommunications companies to transmit telephone signals, Internet communication, and cable television signals. Due to much lower attenuation and interference, optical fiber has large advantages over existing copper wire in long-distance and high-demand applications.

Chapter 3

Nonlinearity

The terms linear and nonlinear in optics, mean intensity independent and intensity-dependent phenomena respectively. Nonlinear effects in optical fibers occur due to:-

(1) change in the refractive index of the medium with optical intensity and,

(2) inelastic scattering phenomenon. The power dependence of the refractive index is responsible for the Kerr-effect. Depending upon the type of input signal, the Kerr- effect nonlinearity manifests itself in three different effects such as Self-Phase Modulation (SPM), Cross-Phase Modulation (CPM) and Four-Wave Mixing (FWM).

The origin of nonlinear response is related to anharmonic motion of bound electrons under the influence of an applied field. Mathematically, this phenomena can be expressed by the nonlinear dependence of induced polarization vector (P) from the electric field (E) . Most of the nonlinear effects in optical fibers originate from nonlinear refraction which is a phenomenon referring to the intensity dependence of refractive index

$$n(\omega) = n_0(\omega) + n_2|E|^2 \tag{3.0.1}$$

where $n_0(\omega)$ is the linear part and n_2 is the nonlinear-index relates to the third order susceptibility . The intensity dependents of the refractive index leads to SPM, for example. SPM refers to self-induced phase shift experienced by an optical field during its propagation in optical fibers. Its magnitude can be given by

$$\Phi(\omega) = n(\omega)\kappa_0 L = n_0 + n_2|E|^2\kappa_0 L \tag{3.0.2}$$

where $\kappa_0 = 2\pi/\lambda$, L is the fiber length and $\Phi_{NL} = n_2\kappa_0 L|E|^2$ is the nonlinear phase-shift. An other class of nonlinear effects in optical fibers is the inelastic scatterings. In this case, the optical field transfers energy to the nonlinear medium. Such effects are the stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). The main difference between the two is that optical phonons participate in SRS while acoustic phonons participate in SBS. Both scatterings can be explained the same way. A photon of the incident field is annihilated to create a photon at lower frequency and a phonon with the right energy (and momentum) conserving energy and momentum.

3.1 Non-linear *Schrödinger* Equation (NLSE)

Nonlinear optics (NLO) is the branch of optics that describes the behavior of light in nonlinear media, that is, media in which the dielectric polarization P responds nonlinearly to the electric field E of the light. This nonlinearity is typically only observed at very high light intensities (values of the electric field comparable to inter-atomic electric fields, typically ($10^8 V/m$) such as those provided by pulsed lasers. Above the *Schrödinger* limit, the vacuum itself is expected to become nonlinear. In nonlinear optics, the superposition principle no longer holds.
The nonlinear *Schrödinger* equation is derived in the followings starting from the Maxwells equation. The used approximations are discussed in detail and higher order approximations and additional effects are described too. We note that c is used instead of c_o in further sections because the necessary physical constant we need is the speed of light in vacuum.

3.2 $Maxwell's$ Equations and Wave Equation

The complete equation system that can describe all electromagnetic phenomena are the Maxwells equations whose differential and integral forms are presented here

$$\nabla \times H = J + \frac{\partial D}{\partial t}, \qquad \oint_{(c)} H dr = \int_s (J + \frac{\partial D}{\partial t}) dS, \tag{3.2.1}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}, \qquad \oint_{(c)} E dr = -\frac{\partial}{\partial t} \int_s B dS, \tag{3.2.2}$$

$$\nabla \cdot B = 0, \qquad \oint_s B dS = 0, \tag{3.2.3}$$

$$\nabla \cdot D = \rho, \qquad \oint_s D dS = \int_v \rho dV \tag{3.2.4}$$

where H is the magnetic field vector, E is the electric field vector, B and D are the magnetic and electric flux densities, respectively. The current density vector is J and the charge density is ρ. The notation (c) under the sign of the integral means that the integral is carried out for a closed curve, S is for surface and V is for volume. The corresponding constitutive relations are given by

$$D = \epsilon_0 E + P, \tag{3.2.5}$$

$$B = \mu_0 H + M, \tag{3.2.6}$$

$$J = \sigma E \tag{3.2.7}$$

where $\epsilon_0 = 8.885 \times 10^{12} As/Vm$ is the vacuum permittivity, $\mu_0 = 4\pi \times 10^7 Vs/Am$ is the vacuum permeability, σ is the conductibility, its unit of measure is A/Vm, and P and M are the induced electric and magnetic polarizations. In optical fibers the following quantities are zeros: J, ρ (no free charges) and M (nonmagnetic medium). Therefore if we take the curl of Eq. (3.2.4) and using (3.2.5), (3.2.6) and (3.2.7), yields Eq. (3.2.8) where we also used the relation $\mu_0 \epsilon_0 = 1/c^2$

$$\nabla \times \nabla \times E + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2} \tag{3.2.8}$$

There is a well-known vector-analytical relation for $\nabla \times \nabla \times E$ apply in Eq. (3.2.8):

$$\nabla \times \nabla \times E = \nabla(\nabla E) - \nabla^2 E = -\nabla^2 E, \tag{3.2.9}$$

because the fiber can be considered isotropic and $\rho = 0$, therefore ∇E vanishes $(\nabla D = \epsilon_0 E = 0)$. With this substitution Eq. (3.2.8) becomes:

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2}, \tag{3.2.10}$$

3.3 Induced Polarization Vector and Susceptibility Tensor

In order to solve Eq. (3.2.10) one has to determine the relation between the induced polarization vector P and electric field vector E. In general, a quantum mechanical approach is needed but if the applied optical frequency is far from the medium resonances which means the wavelength of the field is between 0.5 and $2.2\mu m$ ($f_c = 140 - 600THz$) then the electric-dipole approximation is valid. Assuming that the medium response is local, the induced polarization vector can be written as

$$P_{(r,t)} = \epsilon_0 \int_{-\infty}^{\infty} X^{(1)}(t-\tau)E(r,\tau)d\tau + \epsilon_0 \iint_{-\infty}^{\infty} \chi^{(2)}(t-\tau, t-\theta)E(r,\tau)E(r,\theta)d\tau d\theta +$$

$$\epsilon_0 \iiint_{-\infty}^{\infty} X^{(3)}(t-\tau, t-\theta, t-\eta)E(r,\tau)E(r,\theta)E(r,\eta)d\tau d\theta d\eta + ... \qquad (3.3.1)$$

If the medium response is instantaneous compared to the pulse duration ($\tau/T_0 \ll 1$ where T_0 is the pulse width and τ is the nonlinear response time of the medium) then Eq. (3.3.1) may be approximated by

$$P(r,t) \approx \epsilon_0[X^{(1)}E(r,t) + X^{(2)}E(r,t)E(r,t) + X^{(3)}E(r,t)E(r,t)E(r,t)] \qquad (3.3.2)$$

where $X^{(j)}$ is the jth order susceptibility, a tensor of rank j + 1.

✓$X^{(1)}$ is the linear susceptibility. Its effects are included in the linear refractive index n_0 and the attenuation coefficient α .

✓$X^{(2)}$ is the second order susceptibility. The second order susceptibility is responsible for the second-harmonic generation and sum-frequency generation. It is non-zero only for media that has a lack of inversion symmetry at molecular level. SiO_2 is a symmetric molecule, therefore $X^{(2)}$ vanishes for silica glasses.

✓$X^{(3)}$ is the third order susceptibility. It is responsible for the third-harmonic generation, four-wave mixing and nonlinear refraction.

The following notation will be used:

$$P_L = \epsilon_0 X^{(1)} E, \tag{3.3.3}$$

$$P_{NL} = \epsilon_0 X^{(3)} EEE \tag{3.3.4}$$

where

$$P = P_L + P_{NL} \tag{3.3.5}$$

and P_L denotes the linear part while P_{NL} the nonlinear part of the induced polarization vector.

3.4 Deriving the nonlinear *Schrödinger* equation

We can evaluate now a basic propagation equation from Eq. (3.2.10) using the (3.3.3) and (3.3.4) relations between E and P. Here, we can make some simplifying assumptions:

P_{NL} is treated as a small perturbation compared to P_L (nonlinear effects are weak in silica fibers).

The optical field is assumed to maintain its polarization along the fiber length.

The optical field assumed to be quasi-monochromatic ($\Delta\omega/\omega_0 \ll 1$ where ω_0 is the center frequency and $\Delta\omega$ is the spectral width).

According to the slowly-varying-envelope approximation it is useful to separate the rapidly varying part of the electric field by writing it in the form of

$$E(r,t) = \frac{1}{2}\hat{\chi}[E(r,t)exp(-i\omega_0 t) + E(r,t)exp(i\omega_0 t)], \tag{3.4.1}$$

where $\hat{X}$ is the polarization unit vector of the light assumed to be linearly polarized along the x-axis, E(r, t) is a slowly-varying function of time (relative to the optical period) and E means the complex conjugate of E. Eq. (3.4.1) is substituted into Eq. (3.3.4) and (3.3.3) and a similar form is used in the polarization vector as in Eq. (3.4.1):

$$P_L(r,t) = \frac{1}{2}\hat{\chi}[P_L exp(r,t)exp(-i\omega_0 t) + P_L(r,t)exp(i\omega_0 t)], \tag{3.4.2}$$

$$P_{NL}(r,t) = \frac{1}{2}\hat{\chi}[P_{NL}(r,t)exp(-i\omega_0 t) + P_{NL}(r,t)exp(i\omega_0 t)]. \quad (3.4.3)$$

A Fourier-transformation is applied on Eq. (3.2.10) and Eq. (3.4.2) are substituted and (3.4.3) in that where we express $P_L(r,t)$ and $P_{Nl}(r,t)$ with their relation to E(r, t). The obtained wave equation will have a form of

$$\nabla^2 \tilde{E} + \epsilon(\omega)\kappa_0^2 \tilde{E} = 0 \quad (3.4.4)$$

where $\tilde{E}$ denotes the Fourier-transform of $E(r,t), \kappa_0 = \omega_0/c$ and

$$\epsilon_0(\omega) = 1 + \tilde{\chi}^{(1)}_{XX} + \frac{3}{4}\tilde{\chi}^{(3)}_{XXXX}|E(r,t)|^2 \quad (3.4.5)$$

Eq. (3.4.4) is known as Helmholtz equation and can be solved by using the method of separation of variables

$$\tilde{E}(r,\omega-\omega_0) = F(x,y)\tilde{E}(z,\omega-\omega_0)e^{i\beta_0 z} \quad (3.4.6)$$

where $\tilde{E}(z,\omega-\omega_0)$ is a slowly varying function of z and F(x, y) is a function which corresponds to the transverse electric modes in the (x, y) plane if the z-axis is identical to the propagation direction. We note here, that both sids of Eq. (3.4.6) contain the E function but at the left hand side it depends on all spatial coordinates while at the right hand side E is only z dependent. In the following, only E(z, t) will be used in the derivation process therefore the argument of the function will not be noted in all cases.

Writing back Eq. (3.4.6) into Eq. (3.4.4) we obtain

$$\frac{1}{F}(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}) + \epsilon(\omega)\kappa_0^2 = \frac{1}{\tilde{E}e^{i\beta_0 z}}\frac{\partial^2}{\partial z^2}(\tilde{E}e^{i\beta_0 z}) \quad (3.4.7)$$

The two sides of the equation depend on different variables. Therefore the right hand side and the left hand side must be equal with the same constant. Thus we obtain the following differential equations:

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + [\epsilon_0(\omega)\kappa_0^2 - \bar{\beta}^2]F = 0 \quad (3.4.8)$$

$$\frac{\partial^2 \tilde{E}}{\partial z^2} + 2i\beta_0\frac{\partial E}{\partial z} + [\bar{\beta}^2 - \beta_0^2]\tilde{E} = 0 \quad (3.4.9)$$

where $\bar{\beta}$ is the wavenumber and it is determined by solving the eigenvalue equation (3.4.8). In Eq. (3.4.9), the second derivative can be neglected because $\tilde{E}(z,\omega)$ is a slowly varying function of z. The eigenvalue $\bar{\beta}$ can be written in the form of

$$\bar{\beta}(\omega) = \beta(\omega) + \Delta\beta' \tag{3.4.10}$$

where $\Delta\beta$ is a perturbation term and $\beta(\omega)$ is the frequency dependent mode-propagation constant. Thus, from Eq. (3.4.9) we obtain

$$\frac{\partial\tilde{E}}{\partial z} - \frac{i}{2}\{[\beta(\omega)^2 + 2\beta(\omega)\Delta\beta]\frac{1}{\beta_0} - \beta_0\}\tilde{E} = 0 \tag{3.4.11}$$

It is useful to expand $\beta(\omega)$ in a Taylor-series around the carrier frequency of ω_0 and we use the same notation as in Subsection 2.1.1 for the derivatives $(\beta_1, \beta_2, etc.)$. Writing back the Taylor expanded form of $\beta(\omega)$ to Eq. (3.4.11) and neglecting the terms that are higher than second order such as $\beta_1\Delta\beta$ and $\beta_2\Delta\beta$. Thus we obtain the following equation in the Fourier space from (3.4.11):

$$\frac{\partial\tilde{E}}{\partial z} - i\beta_1(\omega-\omega_0)\tilde{E} - \frac{i}{2}\beta_2(\omega-\omega_0)^2\tilde{E} - i\beta_0\Delta\beta\tilde{E} = 0 \tag{3.4.12}$$

Now, performing the inverse Fourier-transformation on Eq. (3.4.12) and taking into consideration the following equations:

$$F^{-1}\{(\omega-\omega_0)\tilde{E}(z,\omega-\omega_0)\} = i\frac{\partial E(z,t)}{\partial t}, \tag{3.4.13}$$

and

$$F^{-1}\{(\omega-\omega_0)^2\tilde{E}(z,\omega-\omega_0)\} = -\frac{\partial^2 E(z,t)}{\partial t^2}. \tag{3.4.14}$$

F^1 means the operation of inverse Fourier-transformation. We obtain the next equation

$$\frac{\partial E}{\partial z} + \beta_1\frac{\partial E}{\partial t} + i\frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} - i\Delta\beta E = 0 \tag{3.4.15}$$

The term with $\Delta\beta$ includes the effect of fiber loss and nonlinearity. It can be evaluated from Eq. (3.4.8) using a first-order perturbation theory.

$$\Delta\beta = -\frac{\alpha}{2} + i\gamma|E|^2 \tag{3.4.16}$$

where γ is the nonlinear coefficient defined by

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}}. \tag{3.4.17}$$

A_{eff} is the effective core area in (3.4.17) which is inversely proportional to the non-linearity. n_2 is the so-called nonlinear refractive index which perturbs the linear index at higher intensities $n = n_0 + n_2 I$. The effective core area is given in the form of

$$A_{eff} = \frac{[\iint_{-\infty}^{\infty} |F(x, y)|^2 dxdy]^2}{\iint_{-\infty}^{\infty} |F(x, y)|^4 dxdy}. \tag{3.4.18}$$

where F(x, y) is the transverse mode field distribution that can be obtained from the eigenvalue equation (3.4.8). Substituting Eq. (3.4.16) into Eq. (3.4.15) and making a variable transformation with

$$T = t \frac{z}{\upsilon_g} = t\beta_1 z, \tag{3.4.19}$$

one can obtain the NLS equation (3.4.20). The variable transformation yields the frame moving with the group velocity of the pulse envelope. This is the reduced time useful for describing pulse propagation in a coordinate system fixed to the pulse.

The differential equation obtained the light propagation in a lossy, dispersive and nonlinear fiber in can be written as [20]

$$\frac{\partial E(z, T)}{\partial z} = -\frac{\alpha}{2} E - \frac{i\beta_2}{2} \frac{\partial^2 E}{\partial T^2} + i\gamma |E|^2 E \tag{3.4.20}$$

which is often referred as NLS equation in the case of $\alpha = 0$. Attenuation is described by the first term at the right-hand side in Eq. (3.4.20), GVD corresponds to the second term and nonlinearity, or SPM is the third term with the intensity dependence.

In the followings, we describe more general forms of the NLS equation using higher order approximations and including an inelastic stimulated scattering effect.

3.5 Higher Order Dispersion and Nonlinearity

Equation (3.4.20) does not include inelastic scattering such as Raman or Brillouin scattering which becomes important above a threshold of pulse peak intensities and below a certain time duration of the pulses. Raman effect is usually important below the 1ps time scale if the Raman threshold is reached which can be approximated as follows

$$P_{er}^{0} \approx 16\frac{A_{eff}}{L_{eff}gR} \tag{3.5.1}$$

where $L_{eff} = (1 - exp(-\alpha L))/\alpha$ is the effective fiber length with the pulse attenuation α and fiber length L. A_{eff} is the effective core area in Eq. (3.5.1) and gR is the Raman gain curve as a function of frequency shift. The maximal value of gR is about $10^{-13}m/W$ for fused silica which is approximately 13.5 THz shift from the reference frequency.

Below 1ps the spectral width can be broad enough that Raman gain transfers energy from the low-frequency components to the higher frequency components. This results in the self-frequency shift of the pulse whose physical origin comes from the delayed nature of Raman response.

In this approximation the nonlinear response of the medium is comparable with the pulse width. Thus Eq. (3.3.1) should be used in the derivation of generalized nonlinear *Schrödinger* equation (GNLSE). Assuming the following functional form of the nonlinear susceptibility

$$X^{(3)}(t-\tau, t-\theta, t-\eta) = X^{(3)}R(t-\tau)\delta(t-\theta)\delta(t-\eta) \tag{3.5.2}$$

where R(t) is the nonlinear response function normalized the same way as the delta function ($\int_{-\infty}^{\infty} R(t)dt = 1$) . Higher order dispersion terms can be easily added including higher order Taylor coefficients from the expansion of $\beta(\omega)$ during the derivation process of (3.4.20) at the step Eq. (3.4.11).

Substituting Eq. (3.5.2) into Eq. (3.3.1) and performing a similar derivation process to the case of Eq. (3.4.20) yields [42]

$$\frac{\partial E(z,t)}{\partial z} = -\frac{\alpha}{2}E - [\sum_{m=1}^{m} \frac{i^{m-1}}{m!}\beta_m \frac{\partial^m}{\partial t^m}] + i\gamma(1 + \frac{i}{\omega_0}\frac{\partial}{\partial t})(E(r,t)\int_{-\infty}^{\infty} R(t^{'}|E(z,t-t^{'})|)^2 dt^{'}) \tag{3.5.3}$$

The response function R(t) includes the electronic (instantaneous) and vibrational (delayed) Raman response

$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t) \tag{3.5.4}$$

where f_R is the fractional contribution of the delayed Raman response to the non-linear polarization and $h_R(t)$ is the Raman response function. Eq. (3.5.3) can be simplified with the assumption $\varDelta\tau \gg 10fs$ to the following expression

$$\frac{\partial E(z,T)}{\partial z} = -\frac{\alpha}{2}E - \underbrace{[\sum_{m=2}^{5}\frac{i^{m-1}}{m!}\beta_m\frac{\partial^m}{\partial T^m}]E}_{} +$$

$$i\gamma[\underbrace{|E|^2E}_{} + \underbrace{\frac{i}{\omega_0}\frac{\partial}{\partial T}(|E|)^2E)}_{} - \underbrace{T_R E\frac{\partial|E|^2}{\partial T}]}_{} \tag{3.5.5}$$

1^{st} is loss , 2^{nd} stands for Dispersion , the 3^{rd} stands for SPM , the 4^{th} is self-steepening and fifth stands for SRS

where

$$T_R \approx f_R\int_0^\infty th_R(t)dt \tag{3.5.6}$$

where h_R is the Raman response function can be given by an approximate formula which has a Lorentz shape in the Fourier space

$$h_R = \frac{\tau_1^2 + \tau_2^2}{T_1T_2^2}sin(\frac{t}{T_1})exp(-\frac{t}{T_2}) \tag{3.5.7}$$

where τ_1 and τ_2 are adjusting parameters with typical values in silica 12.2 fs and 32 fs, respectively. Using this form of the Raman response function, the integration of Eq. (3.5.6) can be performed analytically:

$$T_R \approx f_R\frac{2T_1^2T_2}{T_{T_1^2+T_2^2}} \tag{3.5.8}$$

This can be used to approximate Raman scattering effect in the last term of (3.5.5).

3.6 Numerical Algorithm

3.6.1 Split-Step Fourier Method

The Split-Step Fourier (SSF) Method applies the linear propagation (diffraction) operator and index nonhomogeneity in separate steps (see Fig. 3.1). The linear propagation operator (L) is applied in the Fourier space and simply represents the k - sphere appropriate to the polarization, direction of propagation and material symmetry. The index nonhomogeneity is a result of a waveguiding structure or third-order nonlinearity. The SSF method is commonly used to integrate several types of nonlinear partial differential equations. In simulating NLS systems, SSF is predominantly used, rather than finite difference method (FDM), as SSF is often more efficient [5.6, 5.7].

Considering one of the simplest NLS type system, the equation contains the terms of attenuation, dispersion and nonlinearity (See Eq. (3.4.20)).

In order to solve Eq. (3.4.20) by the SSF method, we write the differential equation in the following functional form

$$\frac{\partial A_{(z,t)}}{\partial z} = [\hat{L} + \hat{N}]A_{(z,t)}, \tag{3.6.1}$$

where $\hat{L}$ and $\hat{N}$ are the linear and nonlinear parts of (3.4.20) , respectively,

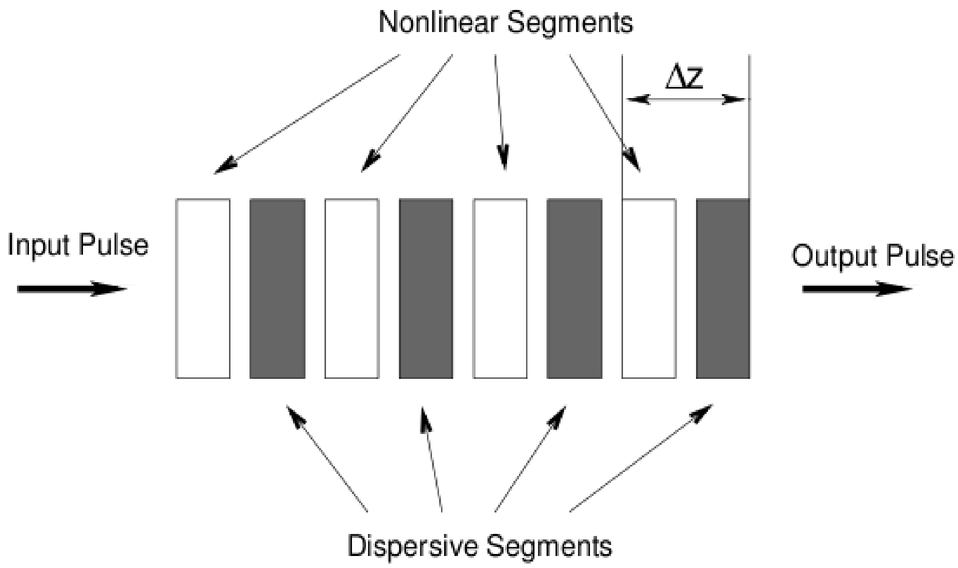

Figure 3.1: Schematic illustration of the calculation. The material is represented by a sequence of thin segments

where

$$\hat{L} = -\frac{\alpha}{2} - \frac{i}{2}\beta_2 \frac{\partial^2}{\partial T^2}, \tag{3.6.2}$$

$$\hat{N} = i\gamma|E|^2. \tag{3.6.3}$$

Integrating (3.6.1) along z using a small space interval Δz, the solution can be written in the form of

$$E_m(z + \Delta z, t) = exp[\Delta z(\hat{L} + \hat{N})]E(z, T), \tag{3.6.4}$$

where the effects of the linear operator (3.6.2) can be easily implemented because the time derivatives become multiplications in the Fourier space by: $(i\omega)^n$ where n is the order of the derivative

$$exp[\Delta z\hat{L}]E(z, T) = \{F^{-1}exp[\Delta z\hat{L}(i\omega)]F\}E(z, T) \tag{3.6.5}$$

where F denotes the Fourier transformation, F^{-1} the inverse Fourier transformation and $\hat{L}(i\omega)$ is the Fourier transform of $\hat{L}$ which is obtained from Eq. (3.6.2).

Chapter 4

Conclusion

The important facts derived from the numerical results and obtained according to the mathematical model can be summarized as follows:

1· A mathematical model is carried out numerical and it is applicable to any optical fiber structure.

2· The dispersion does not predominate by a direct relation. To get the minimum dispersion of any fiber structure, a careful attenuation should be paid to fiber parameters throughout the design process.

3· Large effective area associated with large mode field diameter leads to increase bending loss. Thus, a large effective-area and low bending loss are conflicting requirements.

4· The nonlinearity causes a change in refractive index due to the Kerr effect which is very small according to the low phase modulation nonlinear effects.

Summing up, we can say that for the linear case, the result stays unsatisfactory for a real situation. There are two major effects occurring, attenuation and dispersion. These two effects oppose against the goal to have a fast effective and error-free information flux. The model shows that one physical parameter, the wave length of the carrier can be chosen to reduce the undesired effects. However, the linear case sets limits to the effectiveness of the communication system because attenuation and dispersion can not be optimized simultaneously , low dispersion does not necessarily mean low attenuation. The way out of this dilemma is to increase the amplitude of the signal into the range, where non-linearity has major influence. In this case the non-linearity balances the dispersion and we can choose a carrier wave with a wave length corresponding to minimal loss, without penalty.

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Declaration

This project is my original work, has not been presented for a degree in any other University and that all the sources of material used for the project have been dully acknowledged.

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