MAGNETIZED PLASMA WITH FERROMAGNETIC GRAINS AS A TUNABLE LEFT HANDED MEDIUM

By

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The undersigned hereby certify that they have read and recommend to the College of Graduate Studies for acceptance a graduate project entitled "**Magnetized Plasma with Ferromagnetic Grains as a Tunable Left Handed Medium**" by **Osman Taha** in partial fulfillment of the requirements for the degree of **Master of Science**.

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I, the undersigned, declare that this M.Sc. project is my original work and has not been presented for a degree in any other university, and that all sources of materials used for the project have been duly acknowledged.

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Abstract

The propagation of electromagnetic waves in a cold magnetized plasma with ferromagnetic grains (MPFG) in the high frequency domain is studied. The MPFG medium that consists of magnetized electron-ion plasma and ferromagnetic grains is assumed to be a homogeneous anisotropic medium. Its dispersion properties which is controlled by the simultaneous characterization of the permittivity and permeability tensors is investigated theoretically and numerically near the resonance frequency. It is found that MPFG becomes transparent for the waves that cannot propagate in conventional magnetized electron-ion plasma. The refractive index of the waves propagating parallel to the applied magnetic field is found to be negative for the extraordinary wave in certain frequency domain. Moreover, by varying the parameters of the electric subsystem (electron-ion plasma), the nontransparent region for electromagnetic wave propagation can be eliminated.

The results obtained show that in a narrow band of the super-high-frequency range near the electron cyclotron frequency, the MPFG medium possesses all the known characteristics of negative refractive index media, which would make it as a viable alternative medium to demonstrate the known and predicted peculiar properties of media having negative index of refraction.

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Chapter 1

Introduction

For a long time all optically transparent crystals were mainly characterized by the refractive index. The refractive index was considered to a quantification of the resistance offered by a medium to the passage of light. Based on this idea, Fermat, known by his famous and extremely useful studies of light propagation in medium with spatially varying refractive index, Erasmus Bartholinus (1669) discovered the realization of polarization by light, Malus in 1808, discovered polarization of light in an anisotropic medium were the first cases when optical material could not be characterized by a single number, but depended on propagation direction and the relative orientation of the crystal [1].

In 1968, Veselago [2] first formally considered media with simultaneous negative ϵ and μ and his analysis showed that for Maxwell's equations to be valid in such media, the corresponding index of refraction $n = \sqrt{\epsilon\mu}$ must be negative [3]. His results, however, did not spark much interest for about 30 years, primarily because there were no media available at the time which had both ϵ and μ negative at a given frequency.

Practical suggestions of how left handed media could be realized experimentally were first given in a series of works by Pendry et al., who also predicted theoretically that these media could be used for the creation of a "perfect lens". After these insights, the physical construction of a composite left handed medium structure has been demonstrated by Shelby et al., and the possibility of achieving subwavelength resolution of an object with the same structures has been shown with further experiments [4].

In the 1990's, following the proposal made by Pendry [5, 6], negative refractive index media (NRIM) and the associated peculiar properties are predicted through the application and the fabrication of "artificial" metamaterials composed of an array of conducting nonmagnetic split-ring resonators and continuous thin wires to make both the effective permittivity and permeability simultaneously negative so that electromagnetic wave (EMW) propagation is made possible. Consequently, the first experimental demonstration of a negative index material was reported in the microwave frequency range using an array of metal lines and split ring resonators [7] and in optical frequencies using an array of paired nanorods, an array of elliptic voids in a multi-layered structure and a fishnet structure.

After this initial demonstration of the first metamaterial at microwave frequencies, various theoretical and experimental methods and materials that can be suitable to realize negative refractive index at higher microwaves and optical frequencies have been reported [8]. More recently, the interest in studying EMWs in anisotropic and bianisotropic media has been renewed by the man-made realization of chiral media and a new type of metamaterials exhibiting left-handed properties [9].

An alternative route toward negative refraction by utilizing material chirality has been theoretically proposed by Tretyakovin in 2003, which is based on chiral nihility. Later, in 2004, Pendry also reported on the possibility to achieve negative refraction in chiral metamaterials [10]. Recently, a medium that consists of magnetized electron-ion plasma and ferromagnetic grains (MPFG) was theoretically shown to have both the permittivity and permeability to be negative for certain frequency band in the microwave frequency region so that the system behaves as left handed medium [11, 12].

In our work, we showed that the MPFG medium behaves as left-handed medium in certain frequency domain. This graduate project is arranged as follows: In Chapter 2, Maxwell's equations, permittivity, permeability, the general dispersion relation, and the phase and group velocities are introduced. Left-handed metamaterials, the concept of negative refractive index and its consequences are discussed in Chapter 3. In Chapter 4, electromagnetic wave propagation in MPFG medium is investigated theoretically and numerically. In particular, the permittivity, permeability and the corresponding refractive index of MPFG system is analyzed numerically for different parameter values of the electric subsystem while keeping the parameter of the magnetic subsystem constant. Finally, a brief summary is presented in the Conclusion.

Chapter 2

Propagation of Electromagnetic Waves in a Medium

The Maxwell's equations completely describes the propagation of electromagnetic waves in matter as well as in vacuum. The equations provide relationships between the electromagnetic fields, that is, electric and magnetic fields, and sources (charges and currents) in a medium. In particular, the Maxwell's curl equations for the electric $\vec{E}$ and magnetic fields $\vec{B}$ can be thought of as equations giving the fields everywhere in space, provided all the sources, the charge density ρ and current density $\vec{J}$, are specified.

In electromagnetism, the electric permittivity ϵ and magnetic permeability μ are the two fundamental parameters characterizing the electromagnetic properties of a medium. Physically the permittivity and permeability describe how an electric and magnetic fields affects, and are affected by a medium, which is determined by the ability of a material to polarize in response to the electric and magnetic fields.

In this Chapter, we presented the Maxwell's equations in a medium both in the presence and absence of sources together with the electromagnetic constitutive relations. The permittivity, permeability, group and phase velocities, and the general dispersion relation are discussed.

2.1 Maxwell's equations in matter

The Maxwell's equations in a medium, in the presence of sources, are given by [13]

$$\nabla \cdot \vec{E} = \rho, \tag{2.1.1}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2.1.2}$$

$$\nabla \cdot \vec{B} = 0, \tag{2.1.3}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \tag{2.1.4}$$

where $\vec{E}$ and $\vec{B}$ are the electric field and magnetic induction, ρ and $\vec{J}$ are the net electric and current densities, respectively, c is the speed of light in vacuum. In a material medium, (2.1.1-2.1.4) may be rewritten as

$$\nabla \cdot \vec{D} = \rho_f, \tag{2.1.5}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2.1.6}$$

$$\nabla \cdot \vec{B} = 0, \tag{2.1.7}$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{J}_f + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}. \tag{2.1.8}$$

Here $\vec{D}$ is the electric displacement vector, $\vec{H}$ is the magnetic field, ρ_f is the 'free' charge density, and $\vec{J}_f$ is the 'free' current density.

In the absence of sources, (2.1.5-2.1.8) takes the form

$$\nabla \cdot \vec{E} = 0, \tag{2.1.9}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2.1.10}$$

$$\nabla \cdot \vec{B} = 0, \tag{2.1.11}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}. \tag{2.1.12}$$

Hence, the set of equation given by (2.1.9-2.1.12) describes the propagation of EMWs in a material medium in the absence of external sources. These equations are supplemented by the electromagnetic constitutive relations which are shown below.

2.2 Permittivity and permeability

2.2.1 Permittivity

When an electric field $\vec{E}$ is applied on a dielectric media, the charges are polarized, a phenomena known as polarization. The electric displacement vector ($\vec{D}$) is, then, given by

$$\vec{D} = \vec{E} + 4\pi\vec{P}, \tag{2.2.1}$$

where $\vec{P}$ is the polarization. For a linear and isotropic medium, the relation between the polarization $\vec{P}$ and the electric field $\vec{E}$ can be expressed as

$$\vec{P} = \chi_e\vec{E}, \tag{2.2.2}$$

where the constant χ_e is the electric susceptibility. In view of Eq. (2.2.2), Eq. (2.2.1) becomes

$$\vec{D} = (1 + 4\pi\chi_e)\vec{E}. \tag{2.2.3}$$

The quantity in bracket, in Eq. (2.2.3), is called the permittivity, ϵ, of the medium. That is,

$$\epsilon = 1 + 4\pi\chi_e. \tag{2.2.4}$$

Thus, Eq. (2.2.3) can be written as

$$\vec{D} = \epsilon\vec{E}. \tag{2.2.5}$$

Equation (2.2.5) is referred to as the electric constitutive relation.

2.2.2 Permeability

The magnetic induction vector of a magnetic material placed in a region where the magnetic field is $\vec{H}$ is given by

$$\vec{B} = \vec{H} + 4\pi\vec{M}, \tag{2.2.6}$$

where $\vec{M}$ is the magnetization of the medium. In linear and isotropic medium, the magnetization is linearly related to the magnetic field by $\vec{M} = \chi_m \vec{H}$, where the proportionality constant χ_m is the magnetic susceptibility of the medium. Using this equation, Eq. (2.2.6) can be rewritten as

$$\vec{B} = (1 + 4\pi\chi_m)\vec{H}. \tag{2.2.7}$$

Moreover, the permeability μ of the medium is defined by

$$\mu = 1 + 4\pi\chi_m, \tag{2.2.8}$$

and Eq. (2.2.8) takes the form

$$\vec{B} = \mu\vec{H}. \tag{2.2.9}$$

Equation (2.2.9) is the magnetic constitutive relation.

2.3 Plane waves in isotropic homogenous media

Next, consider a monochromatic plane wave propagating in an isotropic homogenous medium. The electric and magnetic fields components of the plane wave have the form:

$$\vec{E}(\omega, t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

and

$$\vec{H}(\omega, t) = \vec{H}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)},$$

respectively, where $\vec{E}_0$ and $\vec{B}_0$ are the amplitudes of the electric and magnetic fields, ω is the angular frequency and $\vec{k}$ is the wave vector. The Maxwell's curl equations (in the absence of sources)

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2.3.1}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}, \tag{2.3.2}$$

and using the constitutive equations

$$\vec{D} = \epsilon\vec{E}, \tag{2.3.3}$$

$$\vec{B} = \mu\vec{H}, \tag{2.3.4}$$

can be simplified into

$$\vec{k}\times\vec{E} = \frac{\omega}{c}\mu\vec{H}, \tag{2.3.5}$$

$$\vec{k}\times\vec{H} = -\frac{\omega}{c}\epsilon\vec{E}. \tag{2.3.6}$$

Here ϵ is the permittivity and μ is the permeability of the medium. From the above

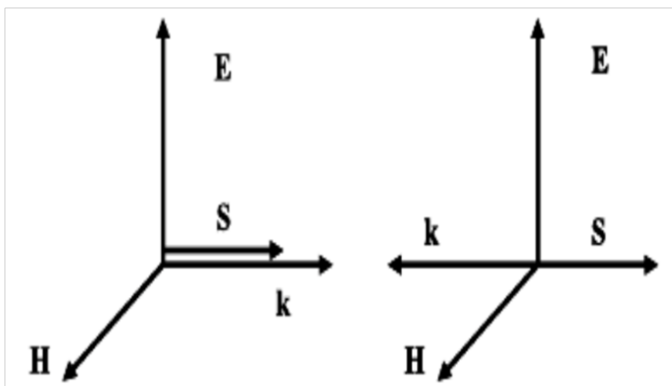

Figure 2.1: Illustration of system of vectors $\vec{E}$, $\vec{H}$, $\vec{k}$, and $\vec{S}$ in left handed media (right) and right handed media (left).

equations, it can be readily seen that $\vec{K}$, $\vec{E}$, and $\vec{H}$ form a right handed triplet of vectors, as a plane wave propagates in normal dielectric materials with $\epsilon > 0$ and $\mu > 0$, as shown in Fig. 2.1. In contrast, these vectors form a left handed triplets in materials with $\epsilon < 0$ and $\mu < 0$, and hence such materials are referred as left handed media (LHM). Moreover, the Poynting vector,

$$\vec{S} = \vec{E}\times\vec{H},$$

is antiparallel to the wave vector $\vec{k}$ in such materials.

2.4 Phase and group velocities

Let us first take the case of a monochromatic plane wave propagating in the z direction. In the time domain, the field is written as $E_y = E_0 \cos(kz - \omega t)$, where E_0 is the amplitude of the wave and $(kz - \omega t)$ is the phase of the wave. For a propagating wave, we can track a point of constant phase and realize that it is traveling at a velocity

$$v_p = \frac{dz}{dt} = \frac{\omega}{k}, \tag{2.4.1}$$

where $z = \omega t/k$. Because of this definition, v_p is called the phase velocity.

1. In the case of free space, $k = \omega/c$ so that the phase front propagates at the velocity of light.

2. In the case of a more general lossless and non-dispersive medium, $k = \omega\sqrt{\epsilon\mu}/c$, which is a linear function of frequency: the phase velocity is constant, typically the velocity of light in the medium. For yet more general dispersive media, the phase velocity is not a constant with frequency and the phase velocity can be typically larger than the speed of light in the medium.

The signal is typically composed of a slowly varying envelope confining a rapidly oscillating wave. The simplest multi-frequency signal is composed of two closely separated frequencies $\omega_0 \pm \triangle\omega$, where $\triangle\omega \ll \omega_0$, to which correspond the wavenumbers $k \pm \triangle k$. The superposition of the two waves is simply written as

$$\begin{aligned} E_y &= \cos[(k + \triangle k)z - (\omega + \triangle\omega)t] + \cos[(k - \triangle k)z - (\omega - \triangle\omega)t] \\ &= 2\cos(\triangle kz - \triangle\omega t)\cos(kz - \omega t). \end{aligned}$$

Tracking the constant fronts of the two terms, yields two velocities.That is,

1. $kz - \omega t =$ constant yields the velocity of the rapidly oscillating wave, which is similar to the monochromatic case discussed previously:

$$v_p = \frac{dz}{dt} = \frac{\omega}{k}.$$

2. $\triangle kz - \triangle\omega t=$ constant yields the velocity of the envelope, called the group velocity:

$$v_g = \frac{dz}{dt} = \frac{\triangle\omega}{\triangle k}.$$

The group velocity is seen to correspond to the velocity of the envelope or the packet, and corresponds to the velocity of propagation of the energy. Also the group velocity may be written as

$$v_g = \frac{1}{dk/d\omega}.$$

Expressing the group velocity in terms of the phase velocity

$$\frac{1}{v_g} = \frac{1}{v_p} + \omega\frac{d}{d\omega}\left(\frac{1}{v_p}\right), \tag{2.4.2}$$

which indicates that if there is no frequency dispersion, $v_g = v_p$. In the case of normal dispersion, $\partial(1/v_p)/\partial\omega > 0$ so that $v_g < v_p$. In the case of anomalous dispersion relation, $\partial(1/v_p)/\partial\omega < 0$ so that $v_g > v_p$: the group velocity can be even larger than the speed of light in vacuum. The group velocity can be generalized to a vectorial relation as $\vec{v}_g = \nabla_k\omega$. This gradient relationship indicates that the direction of the group velocity is normal to the iso-frequency contour in the spectral domain.

2.5 Dispersion relation

The propagation of electromagnetic waves (EMWs) in a medium is described by the Maxwell's curl equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2.5.1}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}. \tag{2.5.2}$$

For a plane EMWs

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)},$$

and

$$\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)},$$

Eqs. (2.5.1) and (2.5.2 can be expressed as

$$\vec{B} = \frac{c(\vec{k} \times \vec{E})}{\omega},$$

and

$$\vec{D} = -\frac{c(\vec{k} \times \vec{H})}{\omega}.$$

Taking the curl of Eq. (2.5.1) and with the constitutive relations

$$\vec{D} = \bar{\epsilon}\vec{E},$$

and

$$\vec{B} = \bar{\mu}\vec{H},$$

where the bar denotes a second-rank tensor, we get

$$\vec{k} \times (\vec{k} \times \vec{E}) = \frac{\omega}{c}\bar{\mu} \cdot \vec{k} \times \vec{H}. \tag{2.5.3}$$

Eliminating $\vec{H}$ from Eq. (2.5.3) using Eq. (2.5.2), we get

$$\vec{k} \times \vec{k} \times \vec{E} = \frac{\omega^2}{c^2}\bar{\epsilon}\bar{\mu} \cdot \vec{E},$$

$$k^2\vec{E} - \vec{k}(\vec{k} \cdot \vec{E}) = \frac{\omega^2}{c^2}\bar{\epsilon}\bar{\mu} \cdot \vec{E}. \tag{2.5.4}$$

Using the definition

$$n = \sqrt{\epsilon\mu} = \frac{c}{\omega}k,$$

where n is the magnitude of the refractive index. Also, in general, $\epsilon = \epsilon_1 \pm \epsilon_2$ and $\mu = \mu_1 \pm \mu_2$. Thus, Eq. (2.5.4) can be expressed in terms the index of refraction, n, as

$$n^2\vec{E} - n(n \cdot \vec{E}) = \bar{\epsilon}\bar{\mu} \cdot \vec{E}. \tag{2.5.5}$$

Thus the dispersion relation takes the form

$$\left(n^2 I - nn - \bar{\epsilon}\bar{\mu}\right) \cdot \vec{E} = 0, \tag{2.5.6}$$

where I is a unit dyad. Equation (2.5.6) in component form can be written as

$$(n^2\delta_{ij} - n_i n_j - \mu_{il}\epsilon_{lj})\vec{E} = 0,$$

$$|n^2\delta_{ij} - n_i n_j - \mu_{il}\epsilon_{lj}| = 0. \tag{2.5.7}$$

This is known as the general dispersion relation.

Chapter 3

Metamaterials as Left Handed Media

Metamaterials[1] have been the most recent development in this quest for control over light via material parameters, with the recognition that engineered materials, structured in specific manners, can exhibit resonances unique to the structure at certain frequencies. The word "meta" means "beyond" and these metamaterials are composite materials consisting of structural units much smaller that the wavelength of the incident radiation and displaying properties not usually found in natural materials. The structures are engineered such that at these frequencies, the wavelength of the electromagnetic radiation is much larger than the structural unit sizes, and thus can excite these resonances. Consequently, an array of these structural units can be well described by effective medium parameters such as a dielectric permittivity ϵ and a magnetic permeability μ.

A photograph of one of the original metamaterial structures possessing a negative index of refraction is shown in Fig. 3.1 and illustrates how the proposals for a negative permittivity and a negative permeability were put together in a single configuration. The system depicted in Fig. 3.1 has negative refractive index for wave propagating in the horizontal plane with the electric field along the vertical direction.

Figure 3.1: The world's first negative refractive index medium at microwave frequencies reported by Shelby, et al (2001). [4]

The ring-like metallic structures printed on a circuit board provide the negative magnetic permeability while metal wires make the composite acquire a negative dielectric permittivity.

In this Chapter, we discussed metamaterials that are meant for left-hand media applications. Negative refractive index and the peculiar characteristics that are predicted to be associated with LHM are discussed.

3.1 Negative Refractive Index

In optics, the refractive index of a material is taken to be a measure of the 'optical density' and from Maxwell's equations the refractive index is given by $n = \sqrt{\epsilon\mu}$, where ϵ is the relative dielectric permittivity and μ is the relative magnetic permeability of the medium. Usual ordinary optical materials have a positive ϵ and μ, and n could easily be taken as $\sqrt{\epsilon\mu}$. Although it was realized that the refractive index would have to be a complex quantity to account for absorption and even a tensor to describe anisotropic materials. In 1968, Veselago [2] first considered the case of a medium that had both negative dielectric permittivity ϵ and negative magnetic permeability μ at a given frequency and concluded that the medium should then be considered to have a negative refractive index (i.e., the negative square root, $n = -\sqrt{\epsilon\mu}$). But finding a material that has these properties naturally is a more difficult task.

3.2 Materials with Negative ϵ and μ

In electromagnetism, electric permittivity ϵ and magnetic permeability μ are the two fundamental parameters characterizing the EM property of a medium. Physically, permittivity (permeability) describes how an electric (magnetic) field affects, and is affected by a medium, which is determined by the ability of a material to polarize in response to the electric (magnetic) field.

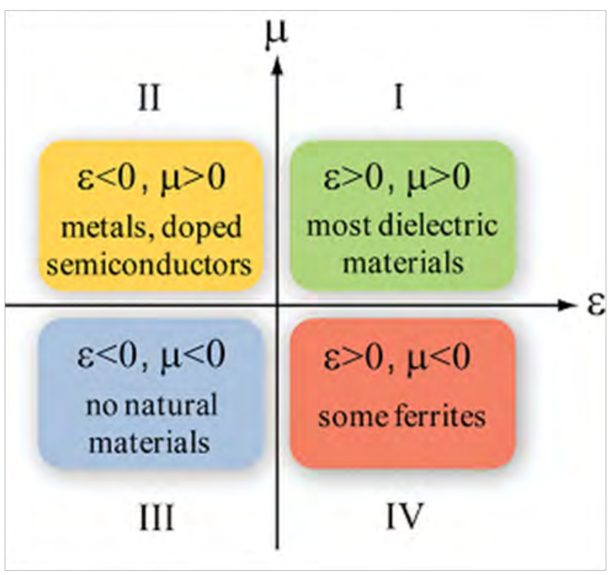

Figure 3.2: Material parameter space characterized by electric permittivity (ϵ) and magnetic permeability (μ). [8]

In order to better understand the effect of negative material parameters, consider an isotropic medium where the $\Im[\epsilon(\omega)] \sim \Im[\mu(\omega)] = 0$, i.e., dissipation is assumed negligibly small at some frequencies (away from resonant frequency). We use the material parameter space as shown in Fig. 3.2 to represent all materials, as far as EM properties are concerned [14].

- Region I: In the upper right quadrant covers materials with simultaneously positive permittivity and permeability, which include most dielectric materials. Electromagnetic radiation can propagate through these media and the vectors $\vec{E}$, $\vec{H}$, and $\vec{k}$ form a right-handed triad.

- Region II: Embraces metals, ferroelectric materials, and doped semiconductors that could exhibit negative permittivity at certain frequencies (below the plasma frequency), a plasma of electric charges. It is well known that a plasma screens the interior of a region from electromagnetic radiation. Indeed, all electromagnetic waves are evanescent inside a plasma and no propagating modes are allowed.

- Region IV: Is comprised of some ferrite materials with negative permeability, the magnetic responses of which, however, quickly fade away above microwave frequencies. Here, too, a wave incident on a medium of this family decays evanescently within the medium and no propagating modes are sustained.

- Region III: The most interesting region in the material parameter space, in which permittivity and permeability are simultaneously negative. In nature there is no such material. Yet, the material with simultaneously negative permittivity and permeability possesses many remarkable properties, which were theoretically investigated in detail by Veselago more than forty years ago. The properties $\Re(\epsilon) < 0$ and $\Re(\mu) < 0$ yield a dispersion condition that allows a real wave vector in the medium, i.e., waves are propagating.

3.2.1 Isotropic Metamaterials

The constitutive equations for homogeneous isotropic materials are

$$n^2 = \epsilon\mu$$

and

$$k^2 = \frac{\omega^2}{c^2} n^2.$$

An isotropic negative index condition has the important property that it exactly reverses the propagation paths of rays within it. The composite medium used to realize the first LHM (Fig. 3.1) made use of an array of metal posts to create a frequency region with $\epsilon_{eff} < 0$, interspersed with an array of split ring resonators (SRRs) having a frequency region with $\mu_{eff} < 0$ [15]. Thus, in the region where both μ_{eff} and ϵ_{eff} are simultaneously negative; the refractive index has a negative real value and hence propagating modes exist in the medium.

3.2.2 Anisotropic metamaterials

The medium is called anisotropic when the electrical and/or magnetic properties of a medium depend upon the directions of field vectors. The relationships between fields can be written in the following form:

$$\vec{D} = \bar{\epsilon} \cdot \vec{E}, \tag{3.2.1}$$

$$\vec{B} = \bar{\mu} \cdot \vec{H}, \tag{3.2.2}$$

where $\bar{\epsilon}$ and $\bar{\mu}$ are relative permittivity and permeability tensors, respectively. Anisotropic materials may be divided into two classes,

1. For those where the natural modes of propagation are linearly polarized, the permittivity and permeability components are symmetric; that is, $\epsilon_{ij} = \epsilon_{ji}$ and $\mu_{ij} = \mu_{ji}$.

2. For those where the natural modes of propagation are circularly polarized, the permittivity and permeability components are antisymmetric (the media is called gyrotropic): that is, $\epsilon_{ij} = -\epsilon_{ji}$ and $\mu_{ij} = -\mu_{ji}$.

Generally, the tensors $\bar{\epsilon}$ and $\bar{\mu}$ have the form:

$$\bar{\epsilon} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix}, \qquad \bar{\mu} = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{pmatrix}. \tag{3.2.3}$$

3.2.3 Chiral metamaterials

A material is defined to be chiral if it lacks any planes of mirror symmetry. In terms of electromagnetic responses, chiral material is characterized by a cross coupling between the electric and the magnetic dipoles along the same direction. This results in the breaking of degeneracy between the two circularly polarized waves; i.e., the refractive index is increased for one circular polarization and reduced for the other. Given the chirality is strong enough, negative refraction may occur for one circularly polarized wave, while for the other circular polarization the refractive index remains positive. This gives rise to interesting phenomena that conventional NIMs do not exhibit.

The development of chiral metamaterials came with the realization that one of the circularly polarized states of light could experience a negative refractive index under suitable conditions (Tretyakov et al. 2003, Pendry 2004, Monzon and Forester 2005). In general a chiral medium is often described by the following bi-isotropic constitutive relations [9]:

$$\vec{D} = \epsilon\vec{E} - i\xi\vec{H}, \tag{3.2.4}$$

$$\vec{B} = i\xi\vec{E} + \mu\vec{H}. \tag{3.2.5}$$

where ξ is the chiral parameter. ξ in general is a complex function of the frequency that also satisfies the Kramer-Kronig's relations. The dispersion relation for a plane wave in this medium is given by

$$k_{\pm} = (\sqrt{\epsilon\mu} \pm \xi)\omega. \tag{3.2.6}$$

3.3 Consequences of Negative Refractive Index

3.3.1 Negative refraction and Snell's law

Wave refraction at an interface between two media is one of the most fundamental phenomena of optics and electromagnetic, and is quantified by Snell's refraction law which stipulates that the transmitted angle θ_2 is related to the incident angle θ_1 by the relation [16]

$$n_1 \sin\theta_1 = n_2 \sin\theta_2,$$

where n_1 and n_2 are the refractive indices of the incident and transmitted media, respectively.

Consider the refraction of an incident ray of light at the interface between ordinary ($\epsilon > 0$ and $\mu > 0$) material and left handed media. Boundary conditions impose continuity of the tangential components of the wave vector along the interface. Thus, from the aforesaid backward propagation in the left-handed region, it immediately follows that, unlike ordinary refraction, the angles of incidence and refraction must have opposite sign. This effect is illustrated in Fig. 3.3.

From the tangential components of the wave vectors of the incident and refracted rays it follows that

$$\frac{\sin\theta_i}{\sin\theta_r} = -\frac{|\vec{k_2}|}{|\vec{k_1}|} = \frac{n_2}{n_1},$$

which is the well-known Snell's law. In this expression, $n_1 > 0$ and $n_2 < 0$ are the refractive indices of the ordinary and left handed media, respectively. Assuming $n_1 > 0$, from the above equation it follows that $n_2 < 0$. That is, the sign of the square root in the refractive index definition must be chosen to be negative.

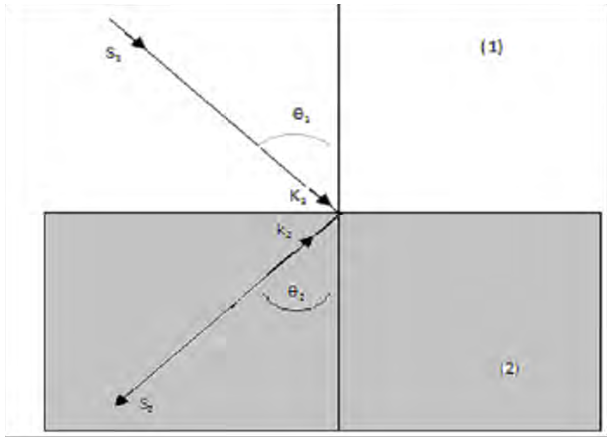

Figure 3.3: Graphic demonstration of the negative refraction between ordinary (1) and left handed media (2).

3.3.2 Inverse Doppler effect

When a moving receiver detects the radiation coming from a source at rest in a uniform medium, the detected frequency of the radiation depends on the relative velocity of the emitter and the receiver. This is the well-known Doppler effect [17]. If the receiver moves towards the source wave fronts and receiver move in opposite directions. Therefore, the frequency seen by the receiver will be higher than the frequency measured by an observer at rest. However, if the medium is a left-handed material, wave propagation is backward, and wavefronts move towards the source. Therefore, both the receiver and the wavefronts move in the same direction, and the frequency measured at the receiver is smaller than the frequency measured by an observer at rest. A straight forward calculation shows that the aforementioned frequency shifts are given by [17]

$$\triangle\omega = \pm\frac{v}{v_p}\omega_0, \tag{3.3.1}$$

where ω_0 is the frequency of the radiation emitted by the source, v is the velocity at which the receiver moves towards the source, v_p the phase velocity of light in the medium, and the +/- sign applies to ordinary/left-handed media. The equation can be written in a more compact form as

$$\triangle\omega = \frac{nv}{c}\omega_0, \tag{3.3.2}$$

where n is the refractive index of the medium and c the velocity of light in free space. In the above equation the $\triangle\omega$ is the difference between the frequency detected at the receiver and the frequency of the oscillation of the source. For $n < 0$ the frequency shift becomes negative for positive v (receiver moving towards the source), the group velocity $d\omega/dk$ in left-handed media, a negative frequency shift results in an increase of k. Therefore, a shift towards shorter wavelengths is seen when the receiver approaches the source, both in ordinary and left-handed media.

3.3.3 Backward Cerenkov radiation

The Cerenkov effect is a relativistic effect whereby a charged particle emits electromagnetic radiation when it travels at a velocity v larger than the velocity of light c in the surrounding medium

$$|\vec{v}| = \frac{c}{|n|}, \tag{3.3.3}$$

where $n = \sqrt{\epsilon\mu}$ is the refractive index of the medium. In addition, this radiation exhibits a cylindrical symmetry and creates the well-known Cerenkov cone whose angle θ is given by

$$\cos\theta = \frac{c}{nv}, \tag{3.3.4}$$

where θ is the angle between the particle velocity and the radiated EM wave front. The realization of this effect greatly depends on simultaneous change of the signs of ϵ and μ, and is known as the Cerenkov effect. It is clear from the logic of the above derivation that the conclusion on the direction of the emission tacitly assumed that the group velocity v_g corresponding to the wave vector $\vec{k}$ was positive, that is directed along $\vec{S}$. But if the phase velocity is negative, i.e., directed opposite to $\vec{S}$, radiation will be directed backward rather than forward as in the case of ordinary material (RHM). Therefore if the index of refractions become negative, it is obvious that angle lies in the second quadrant, and here under, the cone of the Cerenkov radiation is directed back [17].

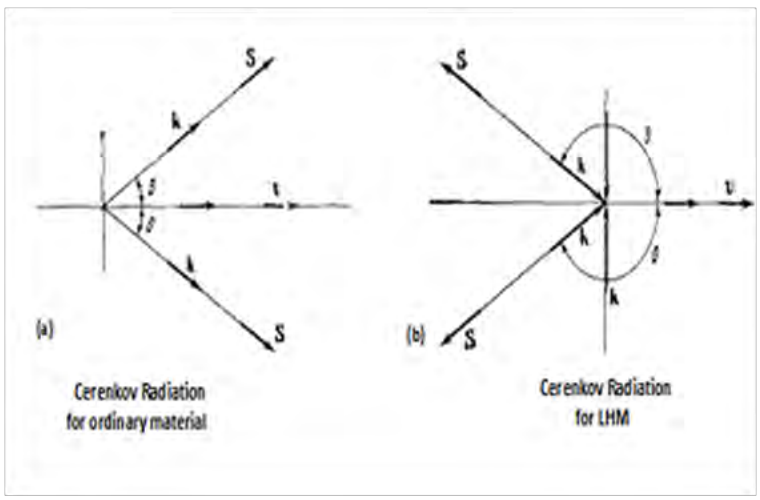

Figure 3.4: Schematic of backward Cerenkov radiation in a left-handed medium, showing the reverse cone.

3.3.4 Perfect lens

A slab of negative refractive index material with $n = -1$ can have a lens-like action, this slab can form the image of a source located on one its side at two locations, one within itself and another on the opposite side of the source. This flat lens structure is often known as a Veselago lens. Its imaging action arises as a direct consequence of the negative refraction of a ray across a planar interface between positive and negative index media. An additional condition for a real image to be formed is that the sum of the distances from the source to the slab (d_1) and the slab to the external image plane (d_2) in the positive medium equals the thickness of the negative index slab ($d = d_1 + d_2$) as shown in Fig. 3.5. All this can be deduced with a simple ray analysis. The Veselago lens is a remarkable device: it maps each point on the object plane onto a point in the image plane and thus suffers from no geometrical aberrations.

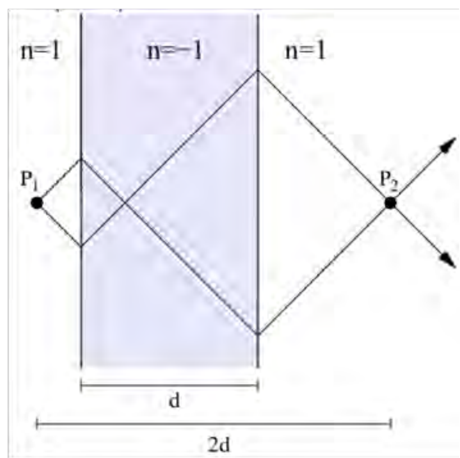

Figure 3.5: The perfect lens.

The Veselago lens is, however, much more than just a flat lens. In 2000, a full wave analysis of the flat lens revealed that, in principle, the image produced by the slab with $\epsilon = -1$ and $\mu = -1$ had an infinite spatial resolution (Pendry 2000). That is to say, the lens can resolve geometrical details in the source that are much smaller than the imaging wavelength, theoretically without any limit. This capability of the Veselago lens to give image resolution beyond the so-called diffraction limit in conventional optics actually derives from the capacity of negative index materials to support surface states. These surface states interact with and involve the non-radiative near-field modes of the source in the image formation process.

Chapter 4

Magnetized Plasma with Ferromagnetic Grains

A medium that consists of conventional magnetized electron-ion plasma with the addition of the ferrite grains that are embedded uniformly in the plasma is known as magnetized plasma with ferromagnetic grains (MPFG). The MPFG medium is assumed to be a homogeneous medium which may be artificially fabricated from magnetized electron-ion plasma with the addition of ferrite grains. Hence the MPFG is essentially a anisotropic medium in a sense that both the permittivity and permeability are second-rank tensors. In certain frequency range ϵ and μ may be simultaneously negative while the refractive index $n = \pm\sqrt{\epsilon\mu}$ is still real so that EMWs can propagate in the medium in the given frequency range, that is, in the vicinity of resonance frequency. By properly choosing the density number of the electrons and the magnetization that depends on the size of the ferrite grains the medium will have a negative refractive index.

For anisotropic materials, the quantities ϵ and μ are tensors. For a magnetized plasma where the direction of the external magnetic field acting on the plasma is in the z-direction, the permittivity tensor, $\bar{\epsilon}$, is given by [3]

$$\bar{\epsilon} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \tag{4.0.1}$$

Similarly, for ferromagnetic grains in which the orientation of the magnetization of the grains is assumed to be directed in the z-direction, the permeability tensor, $\bar{\mu}$, will have the form [3]

$$\bar{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} \tag{4.0.2}$$

Being a homogeneous and anisotropic medium, the effective permittivity and permeability of the MPFG system are taken to be that of the permittivity of the electron subsystem (magnetized cold electron-ion plasma) and the permeability of the magnetic subsystem (ferrite grains).

4.1 Permittivity of MPFG

The permittivity tensor of the plasma is well known and we take it according to the model of cold magnetized plasma. Choosing the z-axis along the external magnetic field, the nonzero components of the high frequency permittivity tensor components can be shown to have the form [8]:

$$\epsilon_1 = 1 - \frac{\Omega_e^2}{\omega^2 - \omega_c^2}, \tag{4.1.1}$$

$$\epsilon_2 = \frac{\omega_c}{\omega}\frac{\Omega_e^2}{\omega^2 - \omega_c^2}, \tag{4.1.2}$$

and

$$\epsilon_3 = 1 - \frac{\Omega_e^2}{\omega^2}, \tag{4.1.3}$$

where $\Omega_e = \sqrt{4\pi e^2 n_e/m}$ is the plasma frequency, $\omega_c = e\vec{H}_0/(mc)$ is the electron cyclotron frequency, n_e is the density number of the electrons, c is the speed of light in vacuum, and m is the mass of an electron.

4.2 Permeability of MPFG

In the presence of the external constant magnetic field H_0 and the relatively weak time-varying field $H(t)$, magnetic dipole moments are induced in the MPFG and form a magnetic subsystem. As a result, the magnetic subsystem possesses variable magnetization that arises from the interaction of the magnetic dipoles with the variable electromagnetic field. The permeability of the system can be obtained by analyzing the motion of the magnetization vector of the ferrite grains in those fields. In particular, for spherical and homogeneously magnetized ferromagnetic grains, the nonzero components of the permeability tensor of the grain's subsystem can be shown to have the form [11]:

$$\mu_1 = 1 - \xi \frac{\omega_M \omega_H}{\omega^2 - \omega_H^2}, \tag{4.2.1}$$

$$\mu_2 = \xi \frac{\omega \omega_M}{\omega^2 - \omega_c^2}, \tag{4.2.2}$$

and

$$\mu_3 = 1, \tag{4.2.3}$$

where $\omega_M = (16\pi^2 a^3 N_g \chi \omega_H)/3$, a is the grain's radius, N_g is the density of the grains, χ is the static magnetic susceptibility, and for the MPFG system we assumed $\omega_H = \omega_c$.

4.3 Refractive Index of MPFG

Consider a monochromatic plane EMW propagating along the applied magnetic field $\vec{H}_0$ with the wave vector oriented along the z-axis, that is, $\vec{k} = k\hat{z}$. Employing the dispersion relation, Eqn. (2.5.7), the refractive index of the MPFG takes the form

$$n_{\pm}^2 = (\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2), \tag{4.3.1}$$

where $\pm$ represent ordinary and extraordinary waves, respectively. Note that setting $\mu_1 = 1$ and $\mu_2 = 0$ in (4.3.1), we find that $n_{\pm}^2 = \epsilon_1 \pm \epsilon_2$; which coincides with the known result for the refractive index of magnetized electron-ion plasma. Furthermore, it is clear that EMWs can propagate in a medium when $n^2 = \epsilon\mu > 0$, whereas it becomes nontransparent when $n^2 = \epsilon\mu < 0$; where $\epsilon \equiv \epsilon_{eff}$ and $\mu \equiv \mu_{eff}$ are the effective values.

Rewriting (4.3.1) separately for the *ordinary* and the *extraordinary* waves, respectively, we find the index of refractions to be

$$n_{+}^2 = \left[1 - \frac{\Omega_e^2}{\omega(\omega + \omega_c)}\right]\left[1 + \frac{\xi\omega_M}{\omega + \omega_c}\right], \tag{4.3.2}$$

and

$$n_{-}^2 = \left[1 - \frac{\Omega_e^2}{\omega(\omega - \omega_c)}\right]\left[1 - \frac{\xi\omega_M}{\omega - \omega_c}\right]. \tag{4.3.3}$$

From (4.3.2), we observe that the ordinary wave propagates for frequencies $\omega > (-\omega_c + \sqrt{\omega_c^2 + 4\Omega_e^2})/2$. For frequencies less than this value, the effective permittivity $\epsilon_{eff} = \epsilon_1 + \epsilon_2$ of the MPFG is negative, while the effective permeability $\mu_{eff} = \mu_1 + \mu_2$ is still positive, so that the MPFG is nontransparent to the ordinary wave. For this wave the medium behaves only as right handed medium (RHM) medium over all frequencies.

However, the situation for the extraordinary wave is different, that is, it may behave as RHM as well as left handed medium (LHM) in various frequency domain.

In particular, from (4.3.3), we observe that the extraordinary wave propagates for frequencies $\omega > (\omega_c + \sqrt{\omega_c^2 + 4\Omega_e^2})/2$ and $\omega > (\omega_c + \xi\omega_M)$, simultaneously. In this case the MPFG behaves as RHM. In the frequency band where $\omega < (\omega_c + \sqrt{\omega_c^2 + 4\Omega_e^2})/2$ and $\omega < (\omega_c + \xi\omega_M)$, simultaneously, EMWs propagates in the medium possessing negative refractive index. That is, in this frequency domain, the MPFG behaves as LHM.

Let us introduce the dimensionless parameters

$$x = \frac{\omega}{\omega_c}, \qquad \alpha_e = \frac{\Omega_e}{\omega_c}, \qquad \text{and} \qquad \alpha_M = \frac{\omega_M}{\omega_c}.$$

Setting $n_- \equiv n$, for convenience, Eq. (4.3.3) becomes

$$n^2 = \epsilon\mu, \tag{4.3.4}$$

where the effective permittivity and the effective permeability are given by

$$\epsilon = \epsilon_1 - \epsilon_2 = 1 - \frac{\alpha_e^2}{x(x-1)}, \tag{4.3.5}$$

$$\mu = \mu_1 - \mu_2 = 1 - \frac{\alpha_M}{x-1}. \tag{4.3.6}$$

Inserting Eqs. (4.3.5) and (4.3.6) into Eq. (4.3.3), we obtain the refractive index to be

$$n(x) = \pm\sqrt{\left[1 - \frac{\alpha_e^2}{x(x-1)}\right]\left[1 - \frac{\alpha_M}{x-1}\right]}, \tag{4.3.7}$$

where the "+" sign is taken when ϵ and μ are simultaneously positive and the medium behaves as RHM; whereas the "-" sign is taken when ϵ and μ are simultaneously negative and the medium behaves as NRIM. From the index of refraction equation of the MPFG, Eq. 4.3.7, the permittivity is positive in the range $0 < x < 1$ and $x > (1 + \sqrt{1 + 4\alpha_e^2})/2$; and negative in the range $1 < x < (1 + \sqrt{1 + 4\alpha_e^2})/2$; and the permeability is positive in the range $0 < x < 1$ and $x > 1 + \alpha_M$; and negative in the range $1 < x < 1 + \alpha_M$. Note that the condition $n^2 > 0$ may be attained when either for $\epsilon > 0$ and $\mu > 0$ or $\epsilon < 0$ and $\mu < 0$, simultaneously.

4.4 Results and Discussions

For cold plasma, typical values of the plasma and the cyclotron frequencies are of the order of $\Omega_e \sim 2\pi(10\ GHz)$ and $\omega_c \sim 2\pi(1\ GHz)$. For instance, the parameter $\alpha_M = (16\pi^2/3)a^3 N_g \chi_0 = 0.85$, for spherical grains of radius $a \simeq 4 \times 10^{-4}\ cm$, density number $N_g \simeq 2.5 \times 10^5\ cm^{-3}$, the magnetic susceptibility $\chi_0 \simeq 5 \times 10^3$.

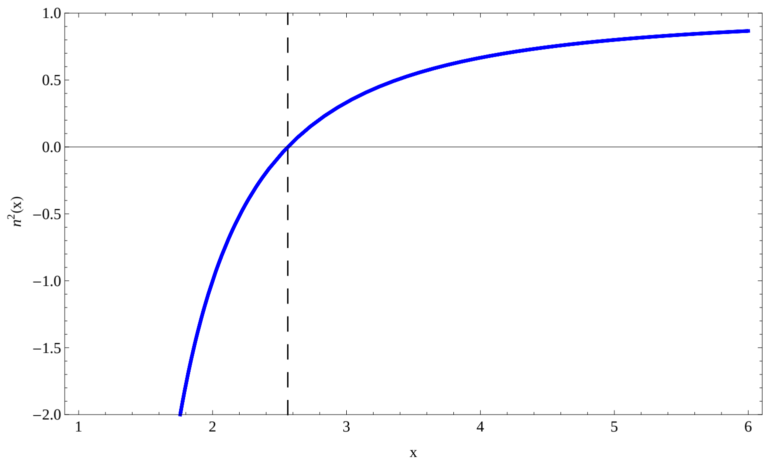

Figure 4.1: The refractive index squared $n^2(x)$ versus the dimensionless frequency (x) of the conventional magnetized plasma (i.e., $\alpha_M = 0$) for $\alpha_e = 2.00$. Note that this system is transparent to EMWs for $x > 2.56$ and opaque below it.

Consequently, for the sake of numerical evaluations we choose: a constant value of $\alpha_M = 0.85$ for the magnetic subsystem, and three different values for the electric subsystem of the MPFG, i.e., $\alpha_e = 2.00,\ 1.60,\ 1.25$. Using these values, we seek to investigate the RHM, LHM, and the nontransparent frequency domains of the MPFG system as a function of α_e while keeping α_M constant.

4.4.1 Case 1: $\alpha_e = 2.00$ and $\alpha_M = 0.85$

For values $\alpha_e = 2.00$ and $\alpha_M = 0.85$, the graphs of ϵ and μ as a function of the reduced frequency are depicted in Fig. 4.2. It shows that $\epsilon > 0$ for $0 < x < 1$ and $x > 2.56$, while $\epsilon < 0$ in the range $1 < x < 2.56$. In the interval $1 < x < 1.85$ the permeability is also negative. That is, the permeability of the ferrite grains that are incorporated in the plasma is found to be negative in certain frequency range as shown in Fig. 4.2. This frequency domain completely lies the region where $\epsilon < 0$. It is the presence of this overlapping frequency domain where ϵ and μ are simultaneously negative which makes MPFG an interesting, potentially viable alternative LHM that can be employed to demonstrate the peculiar characteristics predicted in such media. Note that in the interval $1 < x < 2.56$ the permittivity is negative (Fig. 4.2). In this case, the refractive index becomes purely imaginary and the MPFG is nontransparent to EMWs.

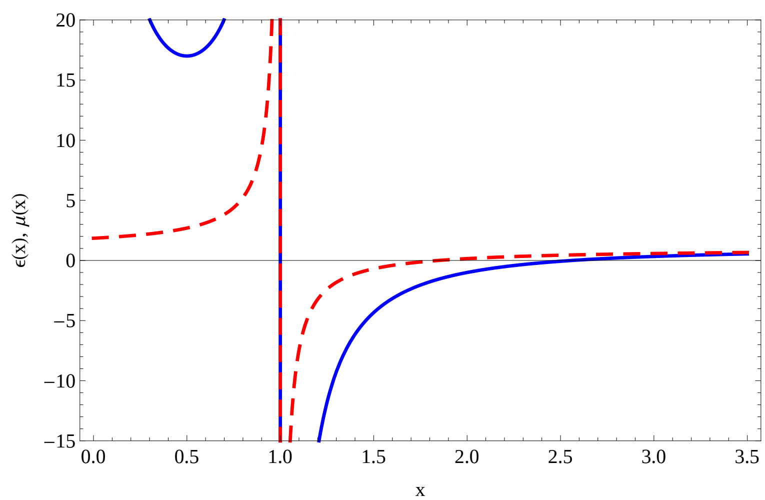

Figure 4.2: The effective permittivity (solid line) and permeability (dashed line) of MPFG versus the dimensionless frequency (x) for $\alpha_e = 2.00$ and $\alpha_M = 0.85$. [18]

Figure 4.3 depicts the graph of the index of refraction, n, and its square n^2 versus the dimensionless frequency, x. It is seen that in the frequency domains $0 < x < 1.85$ and $x > 2.56$, $n^2(x)$ is positive and the MPFG is transparent to EMWs, whereas in the domain $1.85 < x < 2.56$, $n^2(x)$ is negative and the medium is nontransparent to EMWs. The graph of $n(x)$ versus x shows that in the frequency interval $1 < x < 1.85$, the refractive index is negative and consequently the medium behaves as a LHM, in this frequency domain. Note that in the region $1 < x < 1.85$, ϵ and μ are simultaneously negative as shown in Fig. 4.2, so that the refractive index $n(x) = \pm\sqrt{\epsilon\mu}$ is negative but real. Note that the conventional magnetized plasma is nontransparent in this frequency domain as shown in Fig. 4.1. However, the addition of the ferrite grains to the plasma makes it possible for EMWs to propagate in the combined system - MPFG. In the other frequency domains $0 < x < 1$ and $x > 2.56$, the refractive index is positive and the medium behaves as RHM, since both ϵ and μ are positive. On the other hand, for $1.85 < x < 2.56$, $\epsilon < 0$ while $\mu > 0$ and the corresponding refractive index $n(x) = \pm\sqrt{\epsilon\mu}$ is purely imaginary - the MPFG is nontransparent to EMWs.

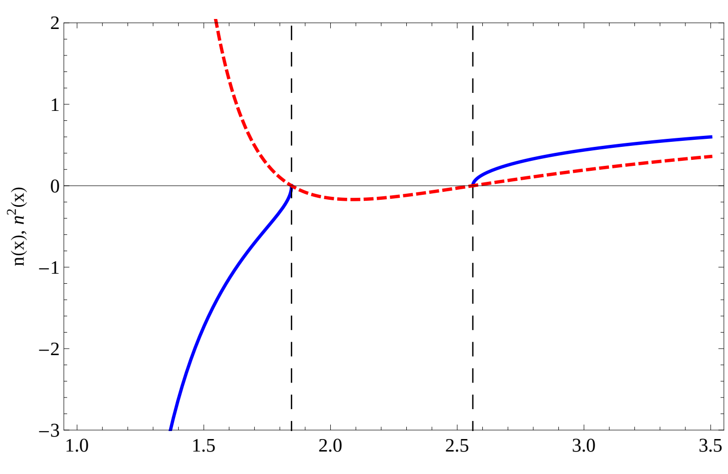

Figure 4.3: The refractive index (solid line) and its square (dashed line) versus the dimensionless frequency (x) for $\alpha_e = 2.00$ and $\alpha_M = 0.85$. [18]

4.4.2 Case 2: $\alpha_e = 1.60$ and $\alpha_M = 0.85$

The nontransparent region, which ranges from $1.85 < x < 2.56$, in the previous case, can be reduced by varying the value of α_e. We can deduce this fact from the graphs of the reduced frequency x versus ϵ and μ, x versus $n(x)$ and $n^2(x)$ drawn for $\alpha_e = 1.60$ and constant $\alpha_M = 0.85$.

The graphs of ϵ and μ as a function of the reduced frequency, for $\alpha_e = 1.60$ and $\alpha_M = 0.85$, are depicted in Fig. 4.4. It shows that $\epsilon > 0$ for $0 < x < 1$ and $x > 2.18$, while $\epsilon < 0$ in the range $1 < x < 2.18$. Similar to case 1 discussed above, $\mu > 0$ in the frequency domain $0 < x < 1$ and $x > 1.85$, whereas $\mu < 0$ in the range $1 < x < 1.85$. Accordingly, it is expected that the MPFG system behaves as LHM in the range $1 < x < 1.85$, where both ϵ and μ are simultaneously negative, and it behaves as RHM in the range $0 < x < 1$ and $x > 2.18$.

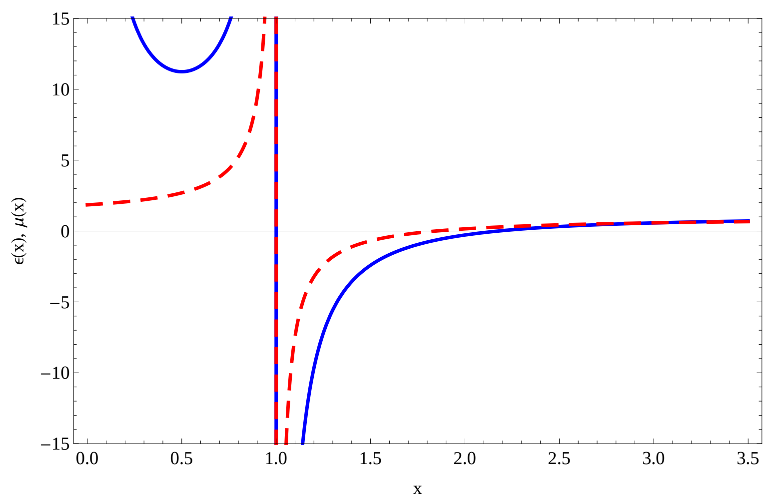

Figure 4.4: The permittivity (solid line) and permeability (dashed line) versus the dimensionless frequency (x) for $\alpha_e = 1.60$ and $\alpha_M = 0.85$.

This can be be clearly shown to be a valid argument from the graph of the index of refraction, n, and its square n^2 versus the dimensionless frequency, x, as depicted in Fig. 4.5. We observe that n is positive for $x > 2.18$ and the system behaves as RHM, and it is negative in the range $1 < x < 1.85$ so that the MPFG behaves as LHM. In the range of frequency $1.85 < x < 2.18$, n becomes imaginary which implies that the MPFG is nontransparent to EMW. Moreover, it is obvious that if $n^2 > 0$ the EMWs propagate in the system and in our case this frequency domain corresponds to $1 < x < 1.85$ and $x > 2.18$. In the region $1.85 < x < 2.18$, $n^2 < 0$ indicating that EMWs do not propagate in the system.

Note that the nontransparent frequency domain is reduced from $\triangle x = 0.71$ for $\alpha_e = 2.00$ to $\triangle x = 0.33$ for $\alpha_e = 1.60$, with the parameter of the magnetic subsystem being kept at a constant value of $\alpha_M = 0.85$. Furthermore, it will be shown in the next case that there is a critical value of α_M below which the nontransparent frequency domain is completely eliminated thereby enabling the system to be transparent for EMWs propagation over the entire frequency domain.

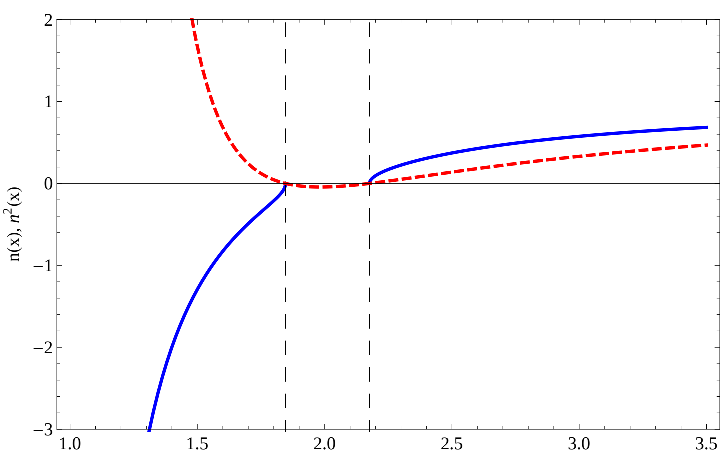

Figure 4.5: The refractive index (solid line) and its square (dashed line) versus the dimensionless frequency (x) for $\alpha_e = 1.60$ and $\alpha_M = 0.85$.

The extent of the nontransparent region is a function of the parameters of the electric and magnetic subsystem. It may be varied (tuned) by appropriate choice of the values of the temperature of the MPFG (the density number of the plasma), the radius, or/and the density number of the grains. Below, we show the case where the nontransparent frequency domain observed in Figs. 4.3 and 4.4 are completely eliminated.

4.4.3 Case 3: $\alpha_e = 1.25$ and $\alpha_M = 0.85$

The nontransparent regions observed in the previous two cases can be completely eliminated by tuning the parameters of the electric subsystem. The critical parameter value of α_e is obtained by equating equations (4.3.5) and (4.3.6) and evaluating the result at zero of $\mu(x)$, i.e., $x = 1 + \alpha_M$. Thus, the result becomes

$$\alpha_e = \sqrt{\alpha_M(1 + \alpha_M)},$$

which for $\alpha_M = 0.85$, we obtain $\alpha_e = 1.25$.

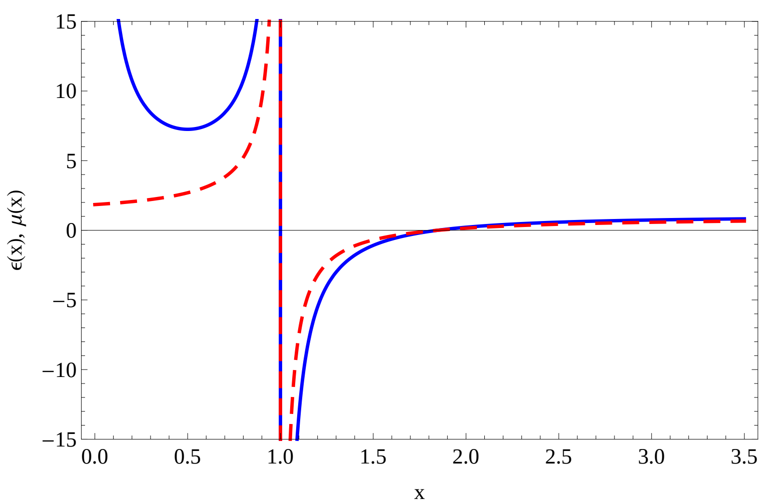

Figure 4.6: The permittivity (solid line) and permeability (dashed line) versus the dimensionless frequency (x) for $\alpha_e = 1.25$ and $\alpha_M = 0.85$. [18]

Figure 4.6 is the plot of the effective permittivity ϵ and the effective permeability

μ of the MPFG versus the dimensionless frequency x, for the tuned parameters $\alpha_e = 1.25$ and $\alpha_M = 0.85$ in the range $0 < x < 3.5$. It is seen that for $0 < x < 1$ and $x > 1.85$, the permittivity is positive; whereas in the interval $1 < x < 1.85$ the permittivity is negative. Similarly, Fig. 4.6 shows that in the frequency interval $0 < x < 1$ and $x > 1.85$, the permeability is positive whereas in the interval $1 < x < 1.85$ the permeability is negative. It is observed that the frequency domain where ϵ is negative completely coincides (overlaps) with that where $\mu < 0$. This overlapping frequency domain where ϵ and μ are simultaneously negative enables the medium to be transparent to EMWs over the whole frequency domain. However, the mode of propagation is different in different frequency domains with the medium behaving as RHM as well as NRIM.

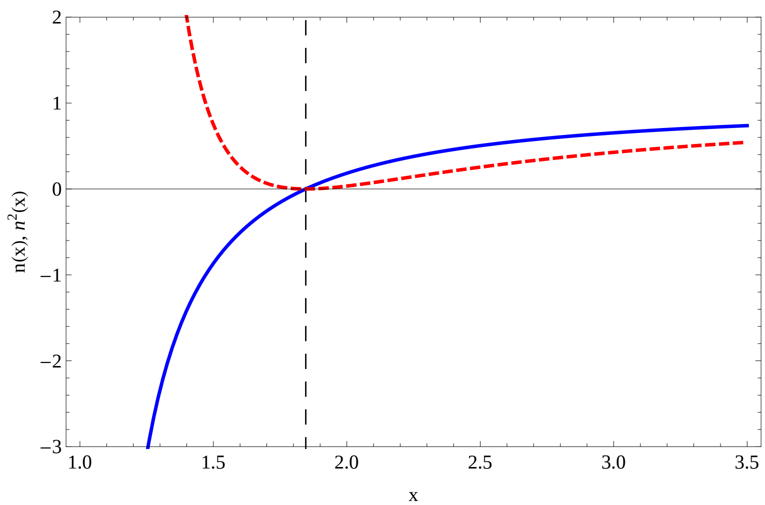

Figure 4.7: The refractive index (solid line) and its square (dashed line) versus the dimensionless frequency (x) for $\alpha_e = 1.25$ and $\alpha_M = 0.85$. [18]

The corresponding graph of the refractive index $n(x)$ and the square of the refractive index $n^2(x)$ of the extraordinary wave versus the relative frequency for $\alpha_e = 1.25$ and $\alpha_M = 0.85$ is shown in Fig. 4.7. The graph (solid line) shows that $n^2(x)$ is positive over all frequencies. It means that the MPFG is transparent to EMWs over the entire frequency range. Also, the plot of n versus x depicts that in the frequency domain $1 < x < 1.85$, where the effective permittivity and permeability are simultaneously negative, the refractive index is negative and consequently the medium behaves as LHM. In the other hand, for the frequency domain $x > 1.85$, both ϵ and μ are simultaneously positive and hence the refractive index is positive so that the medium behaves as RHM. Unlike the previous cases, i.e., Figs. 4.3 and 4.4, here the nontransparent frequency domain is completely eliminated enabling the MPFG to be transparent to EMWs over the entire frequency.

Conclusions

A left-handed medium is a composite medium in which both the permittivity ϵ and the permeability μ are simultaneously negative in a certain frequency band. The condition of negative refractive index can be attained in an electrically tunable magnetized plasma with ferromagnetic grains (MPFG) medium. The conventional magnetized plasma will have a negative permeability in certain frequency ranges depending upon the direction of the electromagnetic wave relative to the magnetic field direction. For EMW propagating parallel to the applied magnetic field the magnetized plasma is nontransparent to electromagnetic waves for frequencies below the electron's plasma frequency, that is, $\epsilon < 0$. Thus, for the magnetized plasma to be transparent in these ranges the permeability in these ranges should be negative. This, negative permeability, is achieved by introducing magnetically active materials such as ferromagnetic materials in the magnetized plasma.

In this work we showed that MPFG behaves as LHM in certain frequency range. In the MPFG, the plasma provides negative permittivity whereas the ferrite grains provide negative permeability in the range $\omega_c < \omega < 1.85\ \omega_c$. In particular, for $\alpha_M = 0.85$ and $\alpha_e = 2.00$, $\epsilon < 0$ in the frequency range $\omega_c < \omega < 2.56\ \omega_c$ and $\mu < 0$ in the range $\omega_c < \omega < 1.85\ \omega_c$. It means that ϵ and μ are simultaneously negative in the range $\omega_c < \omega < 1.85\ \omega_c$ giving rise to negative refractive index in the given region. In the range $1.85\ \omega_c < \omega < 2.56\ \omega_c$, $\epsilon < 0$, $\mu > 0$, and $n(\omega)$ becomes purely imaginary so that the medium is nontransparent to EMWs. For large frequencies ($\omega > 2.56\,\omega_c$), $n(\omega) > 0$, and the MPFG behaves as RHM.

The nontransparent region is a function of the parameters of the electric and magnetic subsystems. By appropriately selecting (tuning) the parameter α_e, while keeping α_M constant the nontransparent domain can be completely eliminated. With $\alpha_M = 1.25$ and $\alpha_e = 0.85$, the MPFG is made transparent to EMWs over the entire frequency ranges. Thus, we conclude that in the MPFG negative refractive index can be attained in a certain frequency range and as a result the MPFG behaves as left-handed medium in the SHF (i.e., in the microwave) region.

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