ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

FACULTY OF TECHNOLOGY

ELECTRICAL AND COMPUTER ENGINEERING

DEPARTMENT

Comparative Performance Study on Wavelet Based Orthogonal Frequency Division Multiplexing (OFDM) Using Different Wavelets

By

Yesuf Shiferaw

A thesis submitted to the school of Graduate studies of Addis Ababa University in partial fulfillment of the requirements for the degree of

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Declaration

I, the undersigned, declare that this thesis work is my original work, has not been presented for a degree in this or any other universities, and all sources of materials used for the thesis work have been fully acknowledged.

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$\psi(t)$	Basic or Mother Wavelet
f_d	Doppler Spread of the Channel (Hz.)
$\Psi(w)$	Fourier Transform of $\psi(t)$
$\frac{N_o}{2}$	Power Spectral Density of the Additive Noise
$\alpha(t)$	Rayleigh Distributed Fading Signal
τ	Time Delay (Sec.)
σ^2	Variance of a Random Process
$\psi_{s\tau}(t)$	Wavelet Basis Function (Baby Wavelet)
$\phi(t)$	Wavelet Scaling Function

List of Abbreviations

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CoifN	Coiflets family order of N
CWT	Continuous Wavelet Transform
DbN	Daubechies family order of N
DFT	Discrete Fourier Transform
DFT-OFDM	Discrete Fourier Transform based OFDM
Dmey	Discrete Meyer Wavelet Family
DWT	Discrete Wavelet Transform
DWT-OFDM	Discrete Wavelet Transform based OFDM
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FWT	Fast Wavelet Transform
Haar	Haar wavelet family
ICI	Inter-Carrier Interference
ICWT	Inverse Continuous Wavelet Transform
IDWT	Inverse Discrete Wavelet Transform
IFFT	Inverse Fast Fourier Transform
IFWT	Inverse Fast Wavelet Transform
ISI	Inter-Symbol Interference
MCM	Multi-Carrier Modulation
MRA	Multi-Resolution Analysis
OFDM	Orthogonal Frequency Division Multiplexing
ONBs	Orthonormal Bases
PAM	Pulse Amplitude Modulation
QMF	Quadrature Mirror Filter
SNR	Signal-to-Noise Ratio
STFT	Short Time Fourier Transform
SymN	Symlets family order of N
WM	wavelet Modulation
WT	Wavelet Transform

Abstract

As we move in to the future there is a rising demand for high performance, high capacity and high bit rate wireless communication systems to integrate wide variety of communication services such as high-speed data, video and multimedia traffic as well as voice signals. As proven by the success of orthogonal frequency division multiplexing (OFDM), it provides an efficient means to handle high-speed data streams over a multipath fading environment .Conventional OFDM implementation uses Fourier filters for data modulation and demodulation , via the inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) respectively. Much research suggests wavelet OFDM implementation provides performance gains over Fourier based OFDM, due to superior spectral containment properties of wavelet filters. All the characteristics of OFDM modulated signals are directly dependent on the set of waveforms of which it makes use. Several authors have foreseen wavelet theory as good plat form on which to build OFDM waveform bases.

In this work the performance of five different most widely used Wavelet bases OFDM schemes over wireless channels is studied and wavelet basis that will be suitable for OFDM, for its optimum performance gains will be investigated. The bit error rate (BER) versus signal-to-noise ratio (SNR) and an elapsed time for simulation of wavelet OFDM are used as a measure of the system performance.

The results show that the wavelet based orthogonal frequency division multiplexing (OFDM) system operates at its optimum performance with wavelets of Daubechies family.

Chapter 1

Introduction

1.1 Motivational Overview

The communication systems and communication networks of the future will fundamentally improve the way people communicate. One among the services expected to have major impact in the future include wireless communication that will permit mobile telephony and data transfer anywhere on the planet . Delivering and receiving these services to the large and rapidly growing commercial markets has created new technological challenge in signal design, modulation, detection, and signal processing.

For wireless communication systems, limited bandwidth allocations coupled with a potentially large pool of users restrict the bandwidth availability to the users. The success of wirelesses communication systems thus depends heavily on the development of bandwidth efficient data transmission schemes.

Wireless multicarrier modulation *(MCM-OFDM)* is a technique of transmitting data by dividing the input data stream into parallel substreams that are each modulated and multiplexed onto the channel at different carrier frequencies [7].

Wavelet transform is a tool for studying signals in the joint time-frequency domains. That is, it is capable of providing the time and frequency information simultaneously, hence giving a time-frequency representation of the signal [8]. Wavelets are known to have compact support (localization) both in time and frequency domain, and possess better orthogonality [8].

A promising application of wavelet transform is in the field of digital wireless multicarrier communication where they can be used to generate waveforms that are suitable for transmission over fading channels [1, 2, 3, 4, 30]. With the ever increasing need for enhanced performance, communication systems can be designed for their optimum performance. The major advantage of wavelet based OFDM is its optimal performance over conventional OFDM. Wavelet bases therefore appear to be a more logical choice for building orthogonal waveform sets usable in communication.

In this work we study orthogonal wavelet bases OFDM. Orthogonal wavelets are capable of reducing the power of intersymbol interference (ISI) and intercarrier interference (ICI) which are caused by loss of orthogonality between the carriers as a result of multipath propagation over the wireless fading channels. The work addresses performance of wavelet OFDM using different orthogonal wavelet basis families such as Haar, Daubechies, Symlets, Coiflets and Discrete Meyer over wireless channels and tries to investigate a suitable wavelet basis to Wavelet OFDM for its better performance.

1.2 Review of Related Research

Wornell and Oppenheim outlined the design of the transmitter and receiver for wavelet modulation (WM) [30]. The performance of wavelet modulation in an additive white Gaussian noise (AWGN) channel was also evaluated in Wornell's work [30]. Wornell showed that the bit error rate (BER) performance of wavelet modulation as function of Signal -to-Noise Ratio (SNR) in the channel: the estimate of the received bit becomes more accurate as the number of noisy observations used to calculate it is increased.

Haixia Zhag et al. based on their work titled research of discrete Fourier transform based OFDM (DFT-OFDM) and discrete wavelet transform based OFDM (DWT-OFDM)on Different Transmission Scenarios concluded that DWT-OFDM performs much better than DFT-OFDM .But they observed an error floor in DWT- OFDM systems [2]. They suggested that it may be resulted from the Haar wavelet base, since different wavelet base is of different characteristics. Some other wavelet bases are expected to be employed to improve performance of DWT- OFDM [2].

Akansu et al. [24] emphasize the relation between filter banks and transmultipxer theory and predict that wavelet packet based modulation has a role to play in future communication systems.

Dereje Hailemariam [3] in his thesis work titled Wavelet Based Multicarrier Code Division multiple Access communication (MC-CDMA) for wireless Environment investigated the performance of MC-CDMA in three transmission scenarios and in his direction to future work he predicts designing of wavelet filters which are better suited to OFDM left as an area to be explored.

B.G.Negash and H.Nikookar on their paper Wavelet Based OFDM for wireless channels reached to results wavelet based multicarrier highly reduce ICI and ISI powers and stressed the non-uniform division of the transmission bandwidth by this modulation technique makes it more attractive for future application dependent OFDM services .

Antony Jamin and Petri Mahonen on the article wavelet Packet Modulation for Wireless Communications concluded the performance results of the new modulation scheme: wavelet packet modulation is a viable alternative to conventional OFDM and they stressed the best method to be used in order to select suitable wavelets is a topic to be studied further [11]. The selection of orthogonal wavelet bases, which can match the corresponding channel characteristics and other parameters for wireless communication, was left for future investigation.

1.3 Significance and Objectives of the Thesis

In recent years researches and literatures indicate that discrete wavelet transform based OFDM (DWT-OFDM) has gained popularity due to high performance gains of the system. DWT- OFDM provides performance gains due to superior spectral containment properties of wavelets. So selecting suitable wavelet basis for a better performance OFDM system is hot area to be explored. To this end performance of bit error rate (BER) as a function of Signal-to-Noise Ratio (SNR) and simulation processing time is used as measure of performance and we explore the performance of five wavelet families OFDM in two channel types: Rayleigh flat fading and Frequency selective fading. By comparing the bit error rate performance of wavelets OFDM scheme in fading channels, we attempt to select suitable wavelet basis for optimum performance. Further more, the performance and elapsed simulation time for each wavelet OFDM is studied in the fading channels. Using the results we try to determine the wavelet family with shortest duration which leads to fast transform and provides superior BER performance gains.

The general objectives of the thesis are to explore discrete wavelet transform (DWT), different wavelets, their demanded characteristics for wireless Multicarrier modulation ((MC-OFDM) and investigate the performance of DWT-OFDM using different wavelets over the wireless multipath fading channels.

The specific objectives of this work can be summarized as**:**

- Study the bit error rate performance of discrete wavelet transform based OFDM using five most widely used wavelets.
- Compare the elapsed simulation time of each wavelet OFDM.
- Based on the results we try to select *suitable wavelet basis* to *DWT-OFDM* for its *optimum* performance.

1. 4 Outline of the Thesis

The thesis is organized as follows: In chapter 2 overview of continuous wavelet transform (CWT), discrete wavelets transform (DWT), and the relation of discrete wavelet transform with digital filter banks theory is mentioned. Discrete wavelets transform (DWT) based Orthogonal Frequency Division multiplexing (OFDM) is described in chapter 3. Simulation implementation issues, performance analysis will be discussed and simulation results will be given in chapter 4 followed by concluding the thesis and suggests directions for future work in chapter 5.

Chapter 2

Wavelet Transform

2.1 Introduction

The transform of the signal is just another form of representing the signal. In Fourier theory a signal can be represented as the sum of a possibly infinite series of sinusoids, which is referred to as a Fourier expansion. Fourier expansion works well with time invariant signals. For a time-varying signal, a complete characterization in the frequency domain should include the time aspect, resulting in the time-frequency analysis of a signal. In the past several solutions have been developed which are more or less able to represent a signal in the joint time-frequency domain, which include the short time Fourier transform (STFT) and wavelet transform [3, 30].

The wavelet transform provides the time-frequency representation of the signal. There has been intensive research on wavelets. In particular, Mallat and Meyer discovered a close relationship between wavelet and multiresolution analysis, which leads to a simple way of calculating wavelet functions. Their work also established a connection between wavelet and digital filter-bank. Daubechies developed a systematic technique for generating finite-duration orthogonal wavelets with finite impulse response (FIR) filter banks [26].

The major outcomes of recent studies on wavelet transform indicate that development of several new applications. One of a promising application of wavelet transforms is in the field of digital wireless communications where they can be used to generate waveforms that are suitable for transmission over wireless fading channels .Construction of filters in wavelet based orthogonal frequency division (OFDM) transceiver is one of the applications. In this chapter, we provide a comprehensive review of wavelets and wavelet transforms since this signal processing technique will be used throughout the thesis.

2.2 Wavelet Transform

2.2.1 The continuous Wavelet Transform

The Continuous Wavelet Transform (CWT) is used to represent a continuous time signal in terms of shifted and scaled versions of a prototype wavelet function $\psi(t)$. The function $\psi(t)$, known as the basic (or mother) wavelet function, plays a vital role in CWT similar to that played by sinusoidal wave in Fourier Transform. The CWT of a continuous time signal x (t) is the set of coefficients $X_{CWT}(s,\tau)$ for $s \in (-\infty,\infty)$, $\tau \in (-\infty,\infty)$. The coefficients are obtained using the inner product operation between x (t) and $\psi(t)$ as follows:

$$X_{CWT}(s,\tau) = \int x(t)\psi_{s,\tau}(t)dt \qquad (2.1a)$$

The wavelet *basis function* $\psi_{s\tau}(t)$ is defined in terms of the mother wavelet function as:

$$\psi_{s\tau}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right) \tag{2.1b}$$

and is sometimes called *baby wavelet* [27]. As seen in the above equations, the transformed signal is a function of two variables: "s" called the *scale*, which scales a function by compressing or stretching it and " τ ", the *translation* parameter along the time-axis. The scale parameter "s"**,** is defined as $|1/\,frequency|$ and corresponds to frequency information of the signal. The translation parameter " τ ", relates to the location of the wavelet as it is shifted through the signal and it corresponds to the time information of the signal in the wavelet transform. The factor $1/\sqrt{|s|}$ is for energy normalization across the different scales.

2.2.2 Inverse Continuous Wavelet Transform (ICWT)

The invertibility of the CWT is an important feature of wavelet transform. When reconstruction of the original signal based upon the transformation is required, the selection of the mother wavelet ψ(t*)* should satisfy the admissibility condition (explained under wavelet properties section) given by [27]:

$$C_\psi = \frac{1}{2\pi}\int \frac{|\Psi(\omega)|^2}{|\omega|}\, d\omega < +\infty \tag{2.2}$$

Where $\Psi(\omega)$ is the Fourier transform of ψ(t). The admissibility condition implies that a wavelet ψ(t) is a zero-mean continuous band pass function. Once ψ(t) meet the admissibility condition, the Inverse Continuous Wavelet Transform (ICWT) can be applied to the above set of coefficients to reconstruct the signal x (t) as follows:

$$x_{ICWT}(t) = \frac{1}{C_\psi} \iint \frac{1}{s^2} X_{CWT}(s,\tau)\psi_{s,\tau}(t) ds d\tau \qquad (2.3)$$

2.3 Multi-resolution Analysis (MRA)

An interesting and key factor leading to the proliferation of wavelet based signal processing was the discovery and construction of orthonormal wavelet bases (ONBs). Developed by Mallat, the main mathematical frame work and mechanism by which the construction of orthonormal wavelet bases is facilitated is called multi-resolution analysis (MRA). MRA is the concept wherein a signal is analyzed using different scales, each of which provides a different resolution of the signal under consideration.

Wavelet transform is used for multi-resolution analysis (MRA) of signals. Wavelet analysis involves decomposition of the signal of interest, x (t), into subspaces on $L^2(R)$ that is vector space of *finite energy* signals, denoted by $L^2(R)$. This space is such that

$$x(t) \in L^2(R) \Rightarrow \int |x(t)|^2 dt < \infty \qquad (2.4)$$

In the discussion on MRA, we restrict our attention to the vector space $V = L^2(R)$ [27, 30].

Scaling Function, Wavelet Functions and Vector Spaces

Multi-resolution analysis is a decomposition of $L^2(R)$ in to a chain of closed subspaces. This is represented as

$$\ldots \subset V_m \subset V_{m+1} \subset V_{m+2} \subset V_{m+3} \subset V_{m+4} \subset \ldots \subset L^2(R) \qquad (2.5)$$

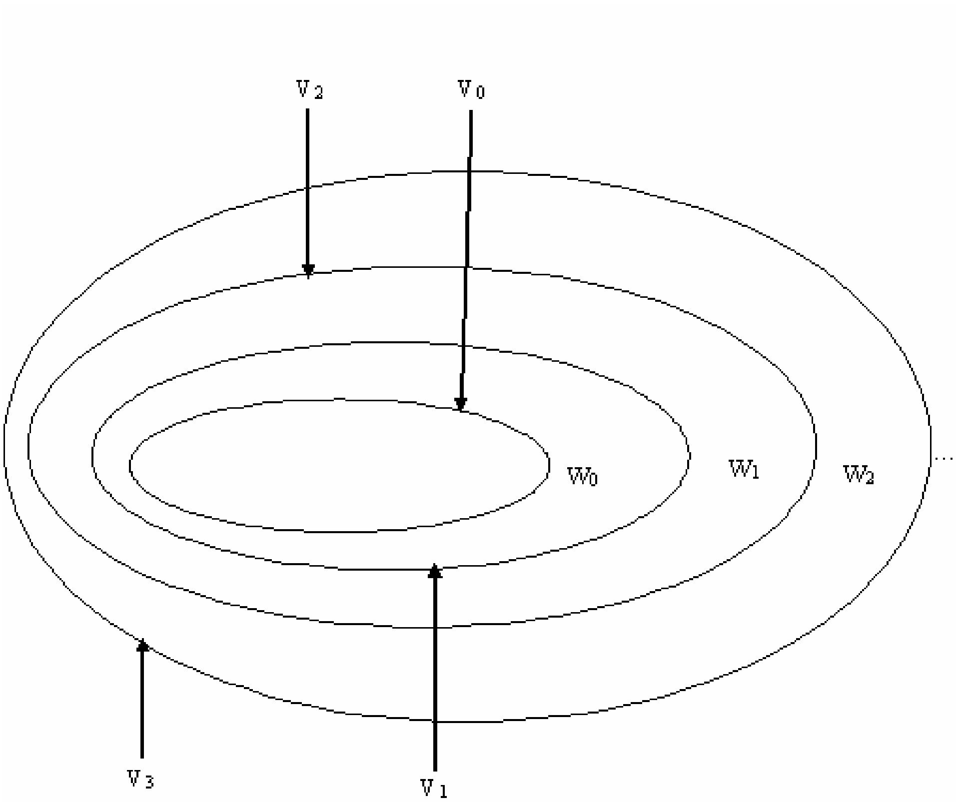
Equation (2.5) implies that a function in one subspace also present in all finer subspaces. In general, the nested subspace V_m , has a basis of $\{ 2^{m/2}\Phi(2^m t-k), k \in Z \}$ where $\Phi(t)$ is the scaling function. The scaling function $\Phi(t)$ is orthogonal to its translations $\{ \Phi(t-k); k \in Z \}$; however, $\Phi(t)$ is not orthogonal across dilations $\{ \Phi(2^m t); m \in Z \}$.

Wavelet analysis involves decomposition of the signal of interest **x (t)** into subspaces. The signal is projected onto these nested subspaces V_m . The subspace V_m is contained in the next finer subspace V_{m+1}. The wavelet space W_m is defined as the difference between V_m and V_{m+1}. It is the orthogonal complement of V_m in V_{m+1} and can be represented as

$$V_{m+1} = V_m \oplus W_m \qquad (2.6)$$

Where $\oplus$ is space addition.

The subspace W_m contains the *detail information* of the x (t) where as V_m contains the average or approximate information of the signal. If $x_m(t)$ is the projection of x(t) onto the subspace V_m, and $x_{m+1}(t)$ is the projection of x(t) onto the subspace V_{m+1}, then the information $\Delta x_m(t) = x_{m+1}(t) - x_m(t)$ is the detail information of x (t) contained in W_m. There exists a wavelet function ψ (t) such that all of its translations at scale **m**, $\{ 2^{m/2}\Psi(2^m t-k); m,k \in Z \}$ forms an orthonormal basis for the subspace W_m. The relationship between the V_m and W_m , is shown in the figure [3, 30].



$$V_0 \subset V_1 \subset V_2 \subset ... \subset V_\infty$$
$$V_m \perp W_m \perp W_{m+1}$$

Figure 2.1: Multi-resolution analysis showing the relation between and nesting of spaces spanned by scaling and wavelet functions

Ψ (t) is orthogonal (in the sense of the usual L^2 inner product) to its scaling Ψ (2^mt) (dilations for **m** < 0; compressions for **m** > 0) and to the translations Ψ (2^mt − k) of its scalings. The set of all scaling and translations $\{\Psi_k^m(t) = 2^{m/2}\Psi(2^m t - k); m, k \in Z\}$ forms a complete orthonormal basis for L^2(R).

In MRA, a low pass type function known as *Scaling function* $\Phi(t)$ plays an important role like *wavelet function* $\psi(t)$ in wavelet transforms.

We will try to show that the scaling functions and the wavelet functions are used to extract the low frequency and high frequency components in the signal x(t) respectively [3,27,30]. Let V_o denote the space spanned by the shifted versions of the scaling function, i.e.

$$Vo = Span\ \{\Phi(t-k)\}, \quad k \in Z, \quad \Phi(t) \in L^2(R) \tag{2.7}$$

Where Z is the set of integers. From basic linear system concept we know that

$$x(t) \in V_0 \Rightarrow x(t) = \sum_k h_k \Phi(t-k) \tag{2.8a}$$

Where the constant h_k is given by the inner product operation

$$h_k = \int x(t)\Phi^*(t-k)dt \tag{2.8b}$$

Let the scaled and shifted versions of the scaling function be defined as

$$\Phi_{m,k}(t) = 2^{m/2}\Phi(2^m t - k), \quad m,k \in Z, \tag{2.9}$$

and the space spanned by these functions is denoted by V_m i.e.

$$V_m = Span\{\Phi_{m,k}(t)\}, \quad k \in Z, \tag{2.10}$$

From the above set of equations, we see that the nesting of the subspaces $V_m, m \in Z$ as it is shown in (2.5) is satisfied. With

$$\begin{aligned} V_{-\infty} &= \{0\} \\ V_{\infty} &= L^2(R) \end{aligned} \tag{2.11}$$

Combining equations (2.5) (2.9) and (2.10), it is easy to deduce that

$$x(t) \in V_m, x(2t) \in V_{m+1} \tag{2.12}$$

Hence the scaling function $\Phi(t)$ is the *basis vector (or basis function*) for the vector space V_o, the vector $x(t) = \Phi(t) \in V_0$.

From (2.12), we see that $\Phi(t)$ must be a vector in the space V_1 whose space is spanned by the basis vectors $\Phi_{1,k}(t)$. Hence the vector $\Phi(t)$ can be expressed in terms of these basis vectors (for the space V_1) as:

$$\Phi(t) = \sum_k h(k)\Phi_{1,k}(t)$$

$$= \sum_k h(k) 2^{1/2} \Phi(2t-k) \tag{2.13}$$

The above recursive equation is referred to as the *dilation equation* or the *multiresolution analysis (MRA)* equation. The coefficients $h(k)$ are called the scaling function coefficients.

For orthogonal wavelet system, the scaling function and wavelet functions are orthogonal to each other i.e.

$$<\Phi_{m,k}(t), \Psi_{m,k}(t)> = \int \Phi_{m,k}(t)\Psi_{m,k}(t)dt = 0 \qquad m,k \in Z \tag{2.14}$$

Since orthogonality is one of the demanded features for communication signals, we restrict our attention to orthogonal wavelet systems. In orthogonal wavelet system, the wavelet functions $\Psi_{m,k}(t), k \in Z$ spans the difference of the sub spaces V_{m+1} and V_m. The complementary space of subspace V_m in space V_{m+1}, denoted by W_m, as it is shown in equation (2.6).

The space V_1 spanned by the scaling functions $\Phi_{1,k}(t), k \in Z$ can be obtained by summing the space V_0 spanned by the scaling functions $\Phi_{0,k}(t), k \in Z$ and the space W_0 spanned by the wavelet functions $\Psi_{0,k}(t) = \Psi(t-k), k \in Z$.

That is in mathematical terms,

$$V_1 = V_0 \oplus W_0 \tag{2.15}$$

The space V_2 is obtained as

$$V_2 = V_1 \oplus W_1 \tag{2.16}$$

$$= V_0 \oplus W_0 \oplus W_1$$

By doing the above step recursively, we reach the space V_∞,

$$V_\infty = L^2(R) = V_0 \oplus W_0 \oplus W_1 \oplus \ldots \tag{2.17}$$

The vector space $L^2(R)$ is also the sum of the spaces $W_m, m \in Z$, i.e.

$$L^2(R) = W_{-\infty} \oplus \ldots \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus \ldots W_\infty \tag{2.18}$$

Comparing (2.17) and (2.18),

$$V_0 = W_{-\infty} \oplus \ldots \oplus W_{-2} \oplus W_{-1} \tag{2.19}$$

Since the wavelet function $\Psi(t) \in W_0$, it must also exist in the space V_1. Hence the vector $\Psi(t)$ is expressed in terms of the basis vectors for V_1 as

$$\Psi(t) = \sum_k g(k)\Phi_{1,k}(t)$$

$$= \sum_k g(k)\sqrt{2}\Phi(2t-k) \tag{2.20}$$

The coefficients $g(k), k \in Z$ are called the *Wavelet Function Coefficients*. The designing of a wavelet system involves choosing the coefficients $g(k)$ and $h(k)$. The functions $\Phi_{0,k}(t),\ k \in Z$ and $\Psi_{m,k}(t),\ m \in (0,\infty), k \in Z$ span the space $L^2(R)$.

Therefore from the discussion above any vector x (t) in $L^2(R)$ can be represented as:

$$x(t) = \sum_k a_{0,k} \Phi_{0,k}(t) + \sum_{m=0}^{\infty} \sum_k w_{m,k} \Psi_{m.k}(t) \tag{2.21}$$

Where $a_{m,k}$ and $w_{m,k}$ are the scaling (averaging) coefficients and wavelet coefficients, respectively, at scale index **m** and translation index **k**. Since the wavelet system under consideration is orthogonal, these coefficients are obtained by projecting the vector x (t) onto the basis vectors $\Phi_{m,k}(t)$ and $\Psi_{m,k}(t)$.

$$a_{m,k} =< x(t), \Phi_{m,k}(t) >= \int x(t) \Phi_{m,k}(t) dt \tag{2.22}$$

$$w_{m,k} =< x(t), \Psi_{m,k}(t) >= \int x(t) \Psi_{m,k}(t) dt \tag{2.23}$$

2.4 Discrete Wavelet Transform (DWT)

Wavelets are wave forms with desirable characteristics of localization both in time and frequency. They also possess the property of orthogonality across scale and translation. The discrete wavelet transform (DWT) provides a means of decomposing sequences of real numbers in a basis of compactly supported orthonormal sequences each of which is related by being a scaled and shifted version of a single function. As such it provides the possibility of efficiently representing those features of a class of sequences localized in both position and scale and possess the property of orthogonality across scale and translation. In (2.1) and (2.3), both (s, τ) are continuous variables and there is a redundancy in the CWT representation of x (t). To overcome this problem, s and τ can be restricted to take discrete values.

That is

$$s = 2^{-m}, m \in Z$$
$$\tau = k2^{-m}, k \in Z \qquad (2.24)$$

Where Z is the set of integers in (-∞,∞). The DWT of a signal $x(t)$ is the set of coefficients $X_{DWT}(m,k)$ for m and k obtained as the inner product of the signal $x(t)$ and the wavelet function, $\Psi_{m,k}$ .The discrete wavelet and inverse discrete representation of a signal $x(t)$ is given by (2.25) and (2.26) respectively.

$$X_{DWT}{}_{k}^{m} = \int_{-\infty}^{\infty} x(t) 2^{m/2} \Psi(2^m t - k) dt \qquad (2.25)$$

$$x_{IDWT}(t) = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} X_k^m 2^{m/2} \psi(2^m t - k) dt \qquad (2.26)$$

Where is $\Psi_{m,k}$ the wavelet function [30].

Mallat's fast wavelet transform (FWT) provides a computationally efficient, practical, discrete time *algorithm* for computing the DWT.

2.4.1 Filter Banks, DWT and IDWT

In this work interest is focused on discrete wavelets developed in MRA algorithm. Multiresolution analysis (MRA) approach is used to explain Mallat's FWT [27, 30].

Let A_{m+1} be the projection operator which approximates the signal x(t) at 2^{m+1} and $\Phi_k^{m+1} = \{2^{m+1}\Phi(2^{m+1}t-k); k \in Z\}$ is an orthonormal basis for V_{m+1}.

The resolution limited approximation of the signal $x(t)$ is given by

$$A_{m+1}x(t) = \sum_k a_{m+1,k}\Phi_{m+1,k}(t) \tag{2.27}$$

Where $a_{m+1,k}$ are approximation coefficients obtained by projections of x (t) on to basis functions.

$$a_{m+1,k} = \int_{-\infty}^{\infty} x(t)\Phi_{m+1,k}\,dt \tag{2.28}$$

The approximation coefficients and wavelet coefficients at any coarser scale can be computed by

$$a_k^m = \sum_p h(p-2k)a_{m+1,k} \tag{2.29}$$

$$w_k^m = \sum_p g(p-2k)w_{m+1,k} \tag{2.30}$$

Where $h(-k)$ and $g(-k)$ are the low pass and the high pass filters in the associated filter bank structure corresponding to DWT shown in figure 2.3.Equations (2.29) and (2.30) represent the FWT to compute the DWT in equation (2.25).Conversely, it is possible to construct the approximation coefficients using

$$a_{m+1,k} = \sum_p h(2p-k)a_{m,p} + g(2p-k)w_{m,p} \tag{2.31}$$

The synthesis filter bank operation is shown in figure 2.5.This is the IFWT for computing the IDWT in equation (2.26) .In the figure 2.5 $h(k)$ and $g(k)$ are the low pass and the high pass filters .Together Equations (2.29), (2.30) and (2.31) form the DWT and the IDWT. These discrete time algorithms with filter bank implementations are used in wavelet modulation.

Analysis

The analysis (*Finer to Coarser Scales*) of a signal $x(t)$ using DWT involves obtaining the scaling coefficients and wavelet coefficients at the desired coarsest scale and at higher scales. For the orthogonal wavelet system, the scaling coefficients at scale m are obtained as:

$$\begin{aligned} a_{m,k} &= \int x(t)\Phi_{m,k}(t)dt \\ &= \int x(t)2^{m/2}\Phi(2^m t - k)dt \\ &= 2^{-1/2}\int x(t)2^{(m+1)/2}\Phi(2^m t - k)dt \end{aligned} \tag{2.32}$$

From (2.13) we see that

$$\Phi(2^m t - k) = \sum_u h(u)\sqrt{2}\Phi(2^{m+1} t - 2k - u) \tag{2.33}$$

Substituting (2.33) expression for $\Phi(2^m t - k)$ in (2.32) and rearranging,

$$\begin{aligned} a_{m,k} &= \sum_u h(u)\int x(t)2^{(m+1)/2}\Phi(2^{m+1} t - 2k - u)dt \\ &= \sum_u h(u-2k)\int x(t)2^{(m+1)/2}\Phi(2^{m+1} t - u)dt \\ &= \sum_p h(p-2k)a_{m+1,p} \end{aligned} \tag{2.34}$$

Similarly, the wavelet coefficients at scale m also obtained as

$$w_{m,k} = \sum_p g(p-2k)w_{m+1,p} \tag{2.35}$$

Synthesis

The synthesis (*Coarser to Finer Scales*) involves reconstructing the original signal $x(t)$ from the scaling coefficients at the coarsest scale and the wavelet coefficients at higher scales. It has been shown that the scaling coefficients at finer scale are recursively computed using [3, 30].

$$a_{m+1,k} = \sum_p a_{m,p} h(k-2p) + \sum_p w_{m,p} g(k-2p) \tag{2.36}$$

DWT as Multi-Rate Signal Processing System

Multirate Signal Processing systems are systems that use multiple sampling rates in the processing of discrete time signals. DWT is one such system.

Decimation in DWT

Decimation and interpolation form the backbone of multirate signal processing systems. In decimation, the sampling rate of discrete time signals $x(k)$ is reduced. Usually, this is achieved by first passing $x(k)$ through a low pass filter $h(k)$ and down sampling the output of the filter as shown in Figure 2.2. The output of the filter is

$$z(k) = h(k) * x(k) = \sum_p h(p)x(k-p) \tag{2.37}$$

The output of the filter $z(k)$ is then down sampled by two to get

$$y(k) = z(2k) = \sum_p h(p)x(2k-p) \tag{2.38}$$

If we represent $a_{m,k}$ by $a_m(k)$, then equations (2.30) and (2.31) becomes

$$a_m(k) = \sum_p h(-(2k-p))a_{m+1}(p)$$

$$w_m(k) = \sum_p g(-(2k-p))w_{m+1}(p) \qquad (2.39)$$

Comparing the above equations with (2.34), we see that the scaling and wavelet coefficients at coarser scales can be obtained from scaling coefficients at finer scales using the recursive filter bank structure shown in Figure 2.3.

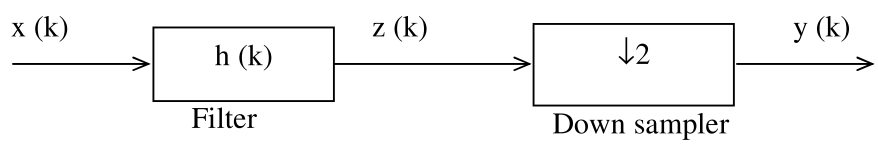


Figure 2.2 Decimation

Interpolation in DWT

In Interpolation, the sampling rate of a discrete time signal $x(k)$ is increased. Usually, this is achieved by first upsampling $x(k)$ and then filtering it with a filter $g(k)$ as shown in Figure 2.4. The upsampling is defined mathematically as:

$$z(2k) = x(k) \quad and \quad z(2k+1) = 0 \qquad (2.40)$$

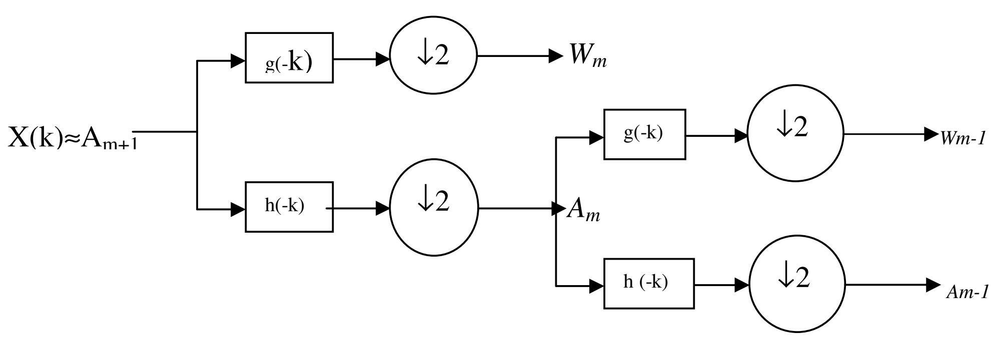


Figure 2.3: Filter bank structure of DWT to obtain coarser (lower) scale scaling coefficients from finer (higher) scale scaling coefficients

The upsampled signal $z(k)$ is then filtered with $g(k)$ to get

$$y(k) = g(k) * z(k)$$

$$= \sum_p g(k-p)z(p) = \sum_p g(k-2p)z(2p)$$

$$= \sum_p g(k-2p)x(p) \qquad (2.41)$$

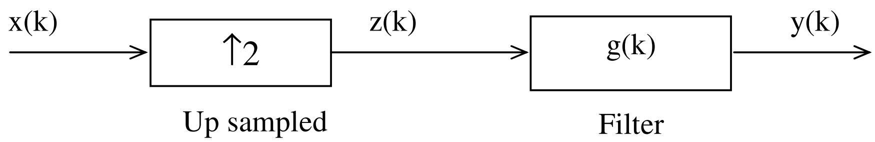


Figure 2.4 Interpolation

Rewriting Equation (2.36) as before, we obtain:

$$a_{m+1}(k) = \sum_p a_m(p)h(k-2p) + \sum_p w_m(p)g(k-2p) \qquad (2.42)$$

Comparing equation (2.42) with (2.41), we see that the scaling coefficients at the finer scale can be obtained from scaling and wavelet coefficients at coarser scales using the recursive filter bank structure in Figure 2.5

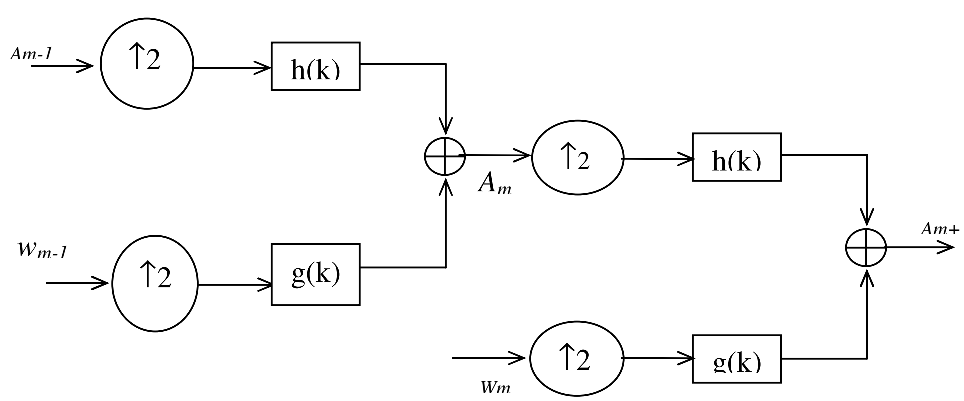


Figure 2.5: Filter bank structure of DWT to obtain finer (higher) scale scaling coefficients from coarser (lower) scale scaling coefficients.

Viewing the above discussions in the frequency domain, it has been shown in the literatures that the dilation equation (2.13) is completely determined by [3, 24, 27, 30].

$$\Phi(w) = \prod_{m=1}^{\infty} H\left(\frac{\omega}{2^m}\right) \tag{2.43}$$

Where H(w) is the Fourier transform of $h(k)$, the scaling function coefficients, and it is low pass type filter i.e. H(0)=1 and H(π)=0. The mother wavelet is also computed using:

$$\psi(w) = G\left(\frac{\omega}{2}\right)\prod_{m=2}^{\infty} H\left(\frac{\omega}{2^m}\right) = G\left(\frac{\omega}{2}\right)\Phi\left(\frac{\omega}{2}\right) \tag{2.44}$$

Where $G(w)$ is the Fourier transform of $g(k)$, wavelet function coefficients. It has the characteristics of a high pass filter (i.e. G (0) =0 and G (π) =1).The high pass filter is obtained from the low pass filter by $g(k) = (-1)^K h(L_0 - 1 - k)$ where L_0 is the filter length. The filters $H(w)$ and $G(w)$ are called Quadrature Mirror Filters (QMF).

To sum up the above discussion, for an orthogonal wavelet in the Multiresolution framework, we start with the low pass type filter $H(w)$; compute the scaling function $\Phi(t)$ and then the wavelet function $\Psi(t)$.

Subband Coding

So far we have seen that the scaling functions and wavelet functions are associated with low pass and high pass filters, respectively. Figure 2.3 shows the decomposition of a discrete time signal $x(k)$ into different frequency bands i.e. scaling and wavelet functions by successive high pass and low pass filtering $x(k)$. The original signal $x(k)$ is first passed through a high pass filter $g(-k)$ and a low pass filter $h(-k)$ and decimated by 2.

This procedure, which is also known as the *Subband Coding*, can be repeated for further decomposition. At every level, the filtering and decimation will result in half the number of samples and hence half the time resolution and half the frequency bands spanned and hence double the frequency resolution [3, 27].Note that this method of decomposition serves as Fast Wavelet Transform (FWT) algorithm.

To reconstruct the original signal the above procedure is followed in reverse order. The signals at every level are interpolated by two, passed through the synthesis filters $g(k)$), and $h(k)$ and then added. The procedure is shown in Figure 2.5. The analysis and synthesis filters are identical to each other, except for a *time reversal*.

Five widely used wavelets such as: Haar, Daubechies with vanishing order N, Symlets with vanishing order N, Coiflets with vanishing order N, and Discrete Meyer are generated using MRA algorithm and are employed in the simulation.

Figure 2.7 depicts the wavelet and scaling functions for the above wavelets.

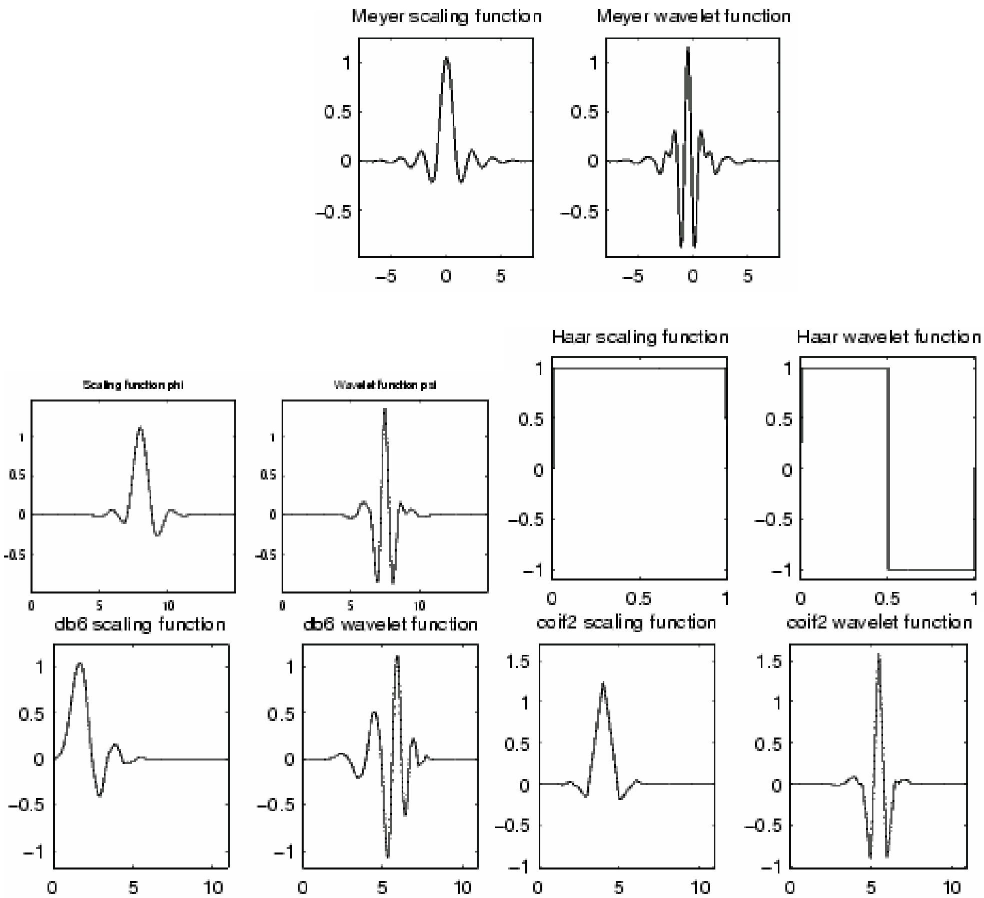


Figure 2.7 Wavelet families Scaling and Wavelet functions (Meyer, Symlets N=3, Haar, Daubechies N=6 and Coiflets N=2) respectively).

Properties of wavelets

The most important properties of wavelets are the *admissibility* and the *regularity* conditions and these are the properties which gave wavelets their name. The square integrable function $\Psi(t)$ satisfying the *admissibility condition*,

$$\int \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < +\infty \qquad (2.45)$$

Can be used to first analyze and then reconstruct the signal without loss of information. In the above equation $\Psi(w)$ is the Fourier Transform of $\Psi(t)$ .The admissibility condition implies that the Fourier Transform of $\Psi(t)$ vanishes at zero frequency, i.e.

$$|\Psi(\varpi)|^2 \;\Big|_{w=0} = 0 \qquad (2.46)$$

This means that wavelets must have a band-pass like spectrum. A zero at the zero frequency also means that the average value of the wavelet in time domain must be zero, i.e.

$$\int \Psi(t)dt = 0 \qquad (2.47)$$

Therefore it must be oscillatory. That is $\Psi(t)$ must be a wave.

Regularity

As can be seen from equation (**2.1**) the wavelet transform of a one-dimensional function is two-dimensional. The time-bandwidth product of the wavelet transform is the square of the input signal and for most practical applications this is not a desirable property. Therefore one imposes some additional conditions on the wavelet functions in order to make the wavelet transform decrease quickly with decreasing scale **s**. These are the *regularity* conditions and they state that the wavelet function should have some smoothness and concentration in both time and frequency domains. Regularity is a quite complex concept and we will try to explain it a little using the concept of vanishing moments (approximation order).

Vanishing moments

If we expand the wavelet transform (2.1) into the Taylor series at t=0 until order **n** (let **τ = 0** for simplicity) we get

$$X(s,o) = \frac{1}{\sqrt{s}}\left[\sum_{p=0}^{N} x^{(p)}(0)\frac{t^p}{p!}\psi(\frac{t}{s})dt + R(n+1)\right] \qquad (2.48)$$

Here $x^{(p)}$ stands for the p^{th} derivative of x and $R(n+1)$ means the rest of the expression. Now if we define the moments of the wavelet by

$$N_p = \int t^p \Psi(t)dt \qquad (2.49)$$

Then we can write the equation (2.49) into the finite development

$$X(s,0) = \frac{1}{\sqrt{s}}\left[x(0)N_o s + \frac{x^{(1)}(0)}{1!}N_1 s^2 + \frac{x^{(2)}(0)}{2!}N_2 s^3 + \ldots \frac{x^{(n)}(0)}{n!}N_n s^{n+1} + R(s^{n+2})\right] \qquad (2.50)$$

From the admissibility condition we already have that the 0^{th} moment $N_0 = 0$ so that the first term in the right hand side of (2.50) is zero. If we now manage to make the other moments up zero, then the wavelet transform coefficients $x(s,\tau)$ will decay as fast as s^{n+2} for a smooth signal $x(t)$. This is known in as the vanishing moments or approximation order. If a wavelet has **N** vanishing moments, then the approximation order of the wavelet transform is also ***N.*** For large classes of Wavelets, more regularity implies more vanishing moments and with increasing number of vanishing moments the wavelet becomes smoother or more regular.

Summarizing, the *admissibility* condition gave us the wave, *regularity* and *vanishing moments* gave us the fast decay or the let, and put together they give us the wavelet.

Compact Support

Compactly supported wavelets (that is, be zero off for some finite intervals) were first developed by Daubechies. The scaling function is compactly supported if and only if the filter constructed by the wavelet has a finite support and their supports are the same. Once a scaling function is found that constitutes a valid MRA, an appropriate analyzing wavelet is readily determined.

Symmetry

Symmetric scaling functions and wavelet functions are important because they are used to build basis of *regular wavelets* over an interval. The following table shows the summary of five wavelet families and their associated properties.

Table 2.1 The Five wavelet families and their associated properties

Property	Haar (haar)	Daubechies (dbN)	Symlets (symN)	Coiflets (coifN)	Discrete Meyer (dmey)
Arbitrarily regular		*	*	*	
Compactly supported orthogonal	*	*	*	*	
Symmetry	*				*
Asymmetry		*			
Near symmetry			*	*	
Arbitrary number of vanishing moments		*	*	*	
Vanishing moments for Φ				*	
Existence of Φ	*	*	*	*	*
Orthogonal analysis	*	*	*	*	
Exact reconstruction	*	*	*	*	*
FIR filters	*	*	*	*	*
Continues transform	*	*	*	*	
Discrete transform	*	*	*	*	*
Fast transform	*	*	*	*	*

Wavelet filter length

The waveforms length can be derived from a detailed analysis of the tree algorithm. Explicitly, the wavelet filter of length L_0 generates **M** waveforms of length:

$$\mathbf{L = (M - 1)\,(L_0 - 1) + 1}$$

In Daubechies wavelet family for instance, the length L_o is equal to twice the wavelet vanishing order N. For the order 2 Daubechies wavelet, L is equal to 4, and thus a 32 subcarrier wavelet transform is composed of waveforms of length 94.Daubechies has proved to generate an orthogonal wavelet with **N** vanishing moment a filter with minimum length **2N** had to be used. *Daubechies* filters, which generate *Daubechies* wavelets, have a length of **2N**.

Table 2.2 Summary of Wavelet Families characteristics and their associated wavelet filter lengths 26]

Wavelet Family Name	Abbreviated Name	Vanishing Order	Wavelet filter Length L_o
Haar	haar	1	2
Daubechies	dbN	N	2N
Symlets	symN	N	2N
Coiflets	coifN	N	6N
Discrete Meyer	dmey	-	62

2.5 Wavelet Filter Design for OFDM

Since the focus of this work is on wavelet filter design that is suitable for OFDM for its high bit rate or high throughput and improved bit error performance, we mention a few key points to design wavelet filters for wavelet filter bank implementations. Desirable time-domain properties of wavelet filters include the following: *compact support, orthogonality, overlapped orthogonality* to preserve signaling rate and fast algorithms. Desirable frequency domain properties of wavelet filters include control of: main lobe width, *asymptotic rate of decay, maximum side lobe height and overlap of adjacent main lobes*. The construction of a wavelet basis is entirely defined by the wavelet scaling filter. Hence its selection is critical. This filter solely determines the specific characteristics of the transform. We limit our performance analysis to wavelet OFDM based on widely used wavelet families such as *Haar*, *Daubechies*, *Symlets*, *Coiflets* and *Discrete Meyer*. The primary wavelet family we have been focusing is the one that satisfies the characteristics which are demanded features for representing the signal in wireless transmission over fading channels. So we are essentially interested in wavelets that provide demanded features for representing signals in communication with properties: shortest duration which leads to fast transform, excellent orthogonality between subcarriers, wonderful spectral containment and adequately adjustable localization in the frequency domain by selecting their vanishing order.

Daubechies has proved the *asymmetric* wavelet family satisfies these characteristics which are demanded feature for representing the signal in communication. The primary wavelet family that leads to fast transform is the one from Daubechies [11], since it presents wavelets with the shortest duration. Furthermore, Daubechies basis localization in the frequency domain can be adequately adjusted by selecting their vanishing order. Further work carried out after her initial research led to a *near symmetric* extension of the Daubechies family, since it was a demanded feature in applications such as *image compression*. We will refer to this family as *Symlets*. The other *near symmetric* wavelet we will make reference to has been constructed at R. Coiffman demand for *image processing* applications and will be referring to Coiflets [26].

Chapter 3

Orthogonal Frequency Division Multiplexing (OFDM)

3.1 Introduction

Orthogonal frequency division multiplexing (OFDM) is based on the multicarrier communications technique. Multicarrier modulation, (MCM) is commonly employed to combat channel distortion and improve the spectral efficiency .Multicarrier Modulation schemes divide the input data into bands upon which modulation is performed and multiplexed into the channel at different carrier frequencies so that information is transmitted on each of the sub carriers, such that the sub channels are nearly distortion less. For transmission in wireless environment, MCM is refereed to as *Orthogonal Frequency Division Multiplexing,* (OFDM) [24].

Unlike the conventional multicarrier communication scheme in which spectrum of each subcarrier is non-overlapping and band pass filtering is used to extract the frequency of interest, in OFDM the frequency spacing between sub-carriers is selected such that the subcarriers are mathematically orthogonal to each other so that the spectra of subcarriers overlap each other. This overlapping property makes OFDM more spectrally efficient than the conventional multicarrier communication scheme. A large benefit of OFDM is that ISI can be avoided since the time duration of the signal waveform representing one symbol is increased compared to a single carrier system.

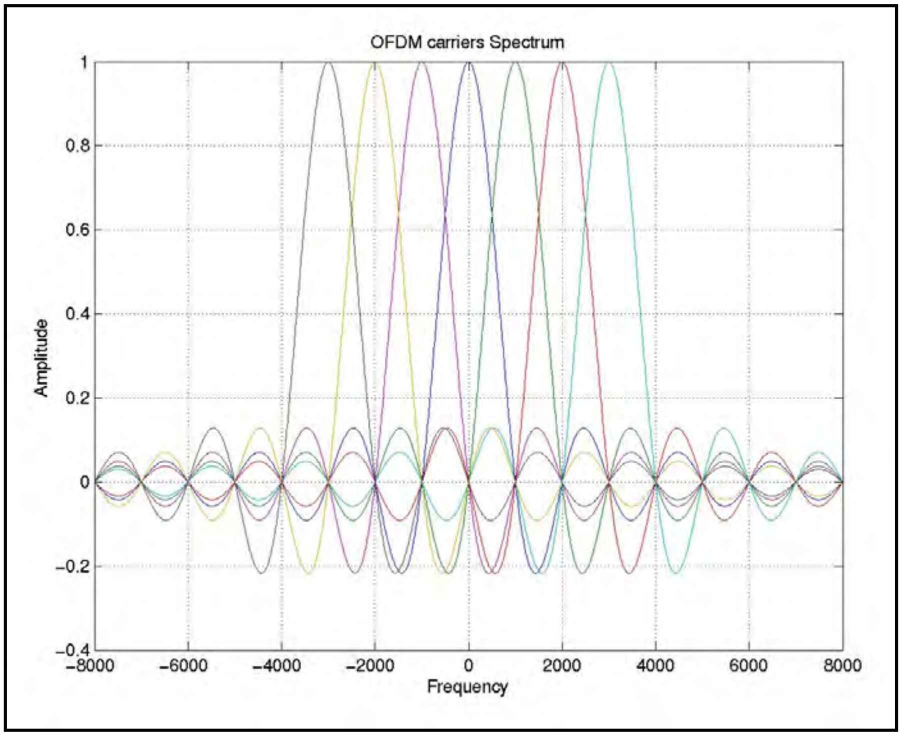


Figure 3.1 Orthogonal subchannels in the frequency Domain

Instead of having N sequential single-carrier symbols where each one is occupying the whole bandwidth (BW) it is possible to transmit N parallel multicarrier symbols where each one is occupying a bandwidth of BW/N for a time period of N/BW. The OFDM signals spectrum is shown in Figure 3.1.

It is of interesting to note that before equalizers were developed, the parallel transmission method was the means of achieving high data rates over a dispersive channel, in spite of its high cost and relative bandwidth inefficiency. In 1971, S. B. Weinstein and P.M. Ebert [9] proposed the use of discrete Fourier transform, (DFT) for Multicarrier modulation. This removed the need to use banks of oscillators to produce all of required sub carriers.

The DFT exhibits the desired orthogonality and can be implemented efficiently through the fast Fourier transform, (FFT) algorithm. But their innovation had a drawback in that their system could not guarantee orthogonality between sub carriers over a dispersive channel. In 1980, A. Peled and A. Ruiz solved the orthogonality problem with their introduction of the cyclic prefix (guard band) (CP). The CP is a copy of the last part of the OFDM symbol attached to the front of the transmitted symbol. When the CP length is longer than channel delay response, ISI and inter-channel interference (ICI) can be avoided. In practical OFDM systems, equalization techniques must also be used in addition to CP.

But in recent years Wavelet-based OFDM has gained popularity. Wavelet OFDM is implemented via overlapped waveforms to preserve data rate. Due to its very high spectral containment properties of wavelet filters, wavelet OFDM can better combat narrowband interference and is inherently more robust with respect to ICI than traditional FFT filters. The classic notion of a guard band does not apply for wavelets; hence data rates can surpass those of FFT implementations, one of its key motivating factors [3].

3.2 Discrete Wavelet Transform Based OFDM

Fourier based OFDM (DFT-OFDM) implementations have used conventional Fourier filters, and have accomplished data modulation and demodulation via the IFFT and FFT operations respectively. Wavelet based OFDM provides performance gains due to superior spectral containment properties of wavelet filters and in wavelet-based OFDM (DWT-OFDM) the IFFT and FFT blocks are simply replaced by an inverse discrete wavelet transform (IDWT) and discrete wavelet transform (DWT) wavelet filter blocks respectively. In DWT-OFDM data modulation and demodulation are accomplished via IDWT and DWT operations respectively.

Block Diagram of DWT-OFDM

The basic block diagram for wavelet based OFDM transceiver is shown in Figure 3.2 [1, 20]. At the transmitter, the user information bit sequence is first subjected to channel encoding and then the bits are mapped to symbols. The symbol sequence is converted to parallel format and IDWT (DWT-OFDM modulation) is applied and then the sequence is once again converted to the serial format. The resulting sequence is converted to an analog signal using a DAC and passed on to the radio frequency (RF) modulation and then to transmission.

At the receiver, first RF demodulation is performed. Then, the signal is digitized using an ADC and timing and frequency synchronization are performed. The sequence is converted to parallel format and then DWT (DWT-OFDM demodulation) operation is applied. The output is then serialized and symbol de-mapping is done to get back the coded bit sequence. Channel decoding is, then, done to get the user bit sequence.

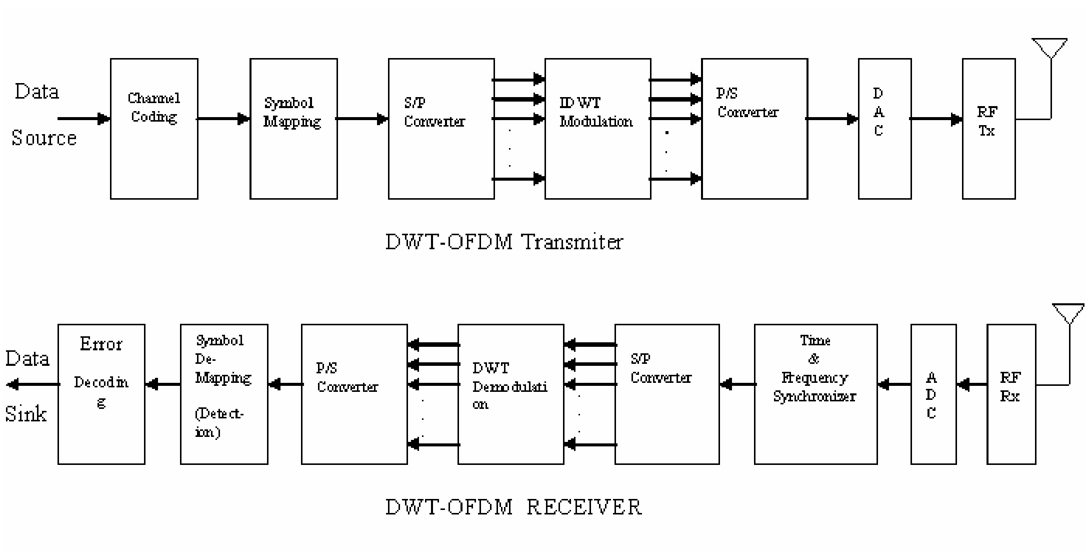


Figure 3.2 Wavelet based OFDM system Block Diagram

DWT-OFDM Modulation Basics

The real wavelet transform, converts real numbers to real numbers, and hence binary signaling must be used in each sub channels. Since the most commonly encountered WT are defined in the real domain, it has naturally leaded us to use pulse amplitude modulation (PAM) for each subcarriers. The wavelet modulated signal to be transmitted $x(t)$ can be generated via:

$$x[t] = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k) 2^{m/2} \Psi(2^m t - K) \tag{3.1}$$

Where $x(k)$ is the data that is modulated onto the wavelets at different scales. $x(t)$ is completely specified by $x(k)$ and vice versa; $x(k)$ is the generating sequence for the transmitted signal $x(t)$. In practical system $x(k)$ is modulated onto a finite number of frequency bands (i.e. m has finite limits: there are a finite number of scales available). Consequently, the transmitted signal $x(t)$) is given by:

$$x[t] = \sum_{m \in M} \sum_{k=-\infty}^{\infty} x(k) 2^{m/2} \Psi(2^m t - K) \tag{3.2}$$

Where M is a finite set of integers.

For the data to be recovered at the rate 2^m, the smallest base band bandwidth is 2^{m+1} Hz.

In wavelet based modulation the subcarrier waveforms are obtained through the wavelet transform. The inverse wavelet transform is used to build the transmitted symbol [11].The processing of a signal through inverse wavelet transform is usually referred to as synthesizing into wavelet coefficients, while the reverse operation is called analyzing from wavelet coefficients.

A particularity of the waveforms constructed through the wavelet transform (WT) is that they are longer than the transform size. Hence, wavelet modulation (WM) belongs to the family of overlapped transforms. That is beginning of a new symbol is being transmitted before the previous ones ends. The waveforms being **K-**shift orthogonal, the inter-symbol orthogonality is maintained despite this overlap of consecutive symbols. The waveforms length can be derived from a detailed analysis of the tree algorithm. The wavelet filter of length $\mathbf{L_0}$ generates **M** waveforms of length

$$L = (M - 1)\,(L_0 - 1) + 1 \qquad (3.3)$$

In Daubechies wavelet family for instance, the length L_o is equal to twice the wavelet vanishing order N. The length of the waveform is there for greater than the corresponding OFDM symbol.

In multicarrier systems, the primary characteristic of the waveform composing the multiplex signal is out-of-band energy. This is the most important source of interference when propagation through the fading channels causes the orthogonality of the transmitted signal to be lost. It is interesting to use waveforms of short duration to ensure that the symbol duration is far shorter than the channel coherence time. Further more, short waveforms require less memory, limit the modulation-demodulation delay and require less computation. For the simulation work we limit our comparative performance study on wavelet OFDM based on five widely used wavelets as it is given in chapter 2.

Chapter 4

Simulation and Results

4.1 Simulation Implementation Issues

The wavelet OFDM system BER performance as a function of SNR is examined. Simulation have been carried out to compare the performance of the five orthogonal wavelet families based OFDM system over Rayleigh fading and frequency selective wireless channels . Both the flat and frequency selective channels are slow fading channels.

The study and comparisons are based on simulation done using MATLAB. The BER performance as a function of SNR is examined for Rayleigh fading and frequency selective fading with Doppler frequencies (f_d = 60Hz). The elapsed processing time is also considered as another measure of performance for each wavelet family based OFDM. Five wavelet families (Haar, Daubechies, Symlets, Coiflets and Discrete Meyer) were considered for comparison. To compare the computational transform complexity using the elapsed time took for simulation similar vanishing order is used for Dubechies, Symlets and Coiflets family wavelets (as vanishing order directly relates with filter length). All BER results are the average of five independent trials.

4.1.1 Simulation Parameters and System Model for DWT-OFDM

Simulation Parameters

In an attempt to use realistic and typical values the following parameters were used:

- Wireless Channel Environments
  - Flat Rayleigh Fading + AWGN
  - Frequency Selective + AWGN
- The five wavelets:
  - Haar (haar)
  - Daubechies with vanishing order N=2 (dbN=2)
  - Symlets with vanishing order N =2 (symN=2)
  - Coiflets with vanishing order N =2 (CoifN=2)
  - Discrete Meyer (dmey)
- Channel Bandwidth: 20 MHz,32MHz
- Bite Rate : 20 Mbps,32Mbps,
- Data Size: 256Kbits,20Mbits
- Doppler Frequency : 60 Hz
- DWT & IDWT Size (Sub carriers): 64,256
- Modulation & Demodulation: BPSK
- Subcarriers frequency spacing $\Delta f = \frac{Bandwidth}{numberofsubcarriers}$ =0.3125 MHz for BW=20 MHz and for 64 number of subcarriers.

The simulation flow block model for the DWT-OFDM

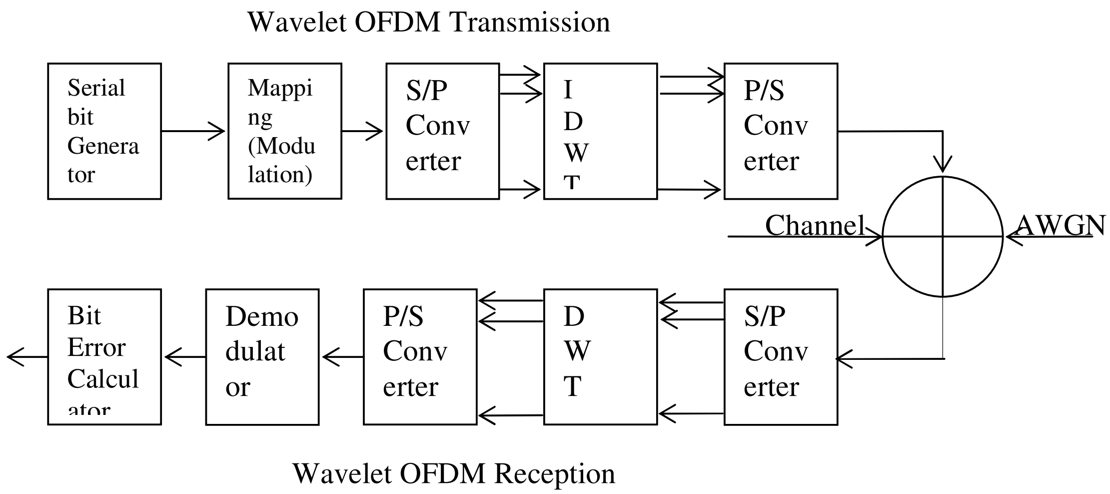


Figure 4.1 Simulation model for DWT-OFDM

Simulation Using MATLAB

For performing the simulations, the flow chain shown in the Figure 4.1 is developed under a MATLAB 7 environment. The blocks are implemented by MATLAB functions. The function ofdmchain provides the interface to the system. The following is an overview of procedures to simulate the DWT-OFDM scheme shown above.

✓ *Transmitter Side*

- A binary data is generated using the standard random bit generator.
- The data are BPSK modulated.
- Perform the serial-to-parallel (S/P) conversion using the MATLAB function.
- Wavelets are generated and signal synthesis are performed using IDWT blocks (see DWT and IDWT Section below)
- P/S is conversion is performed to make the signals ready for transmission

- ✓ *Channels*

- ➢ The transmitted signal is passed through Rayleigh fading and frequency selective channels added with additive white Gaussian noise (The channel models and the channel signal generation for the model are discussed under wireless channel models section).

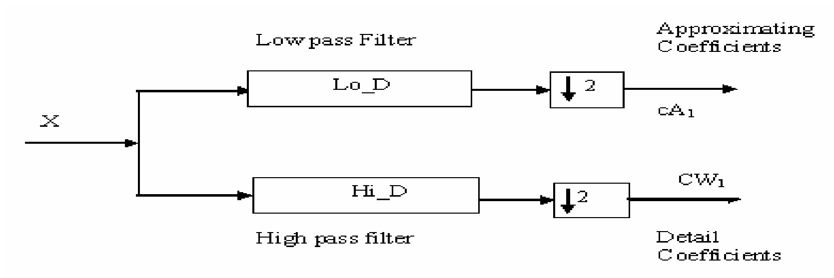
- ✓ *The receiver Side*

- ➢ Perform S/P conversion
- ➢ Decomposing signal in to wavelet filter coefficients is performed using DWT block (see DWT and IDWT Section below)
- ➢ Perform the P/s conversion
- ➢ Demodulate the BPSK symbols
- ➢ The probability of bit error rate of the system is calculated by comparing the transmitted and the received data.

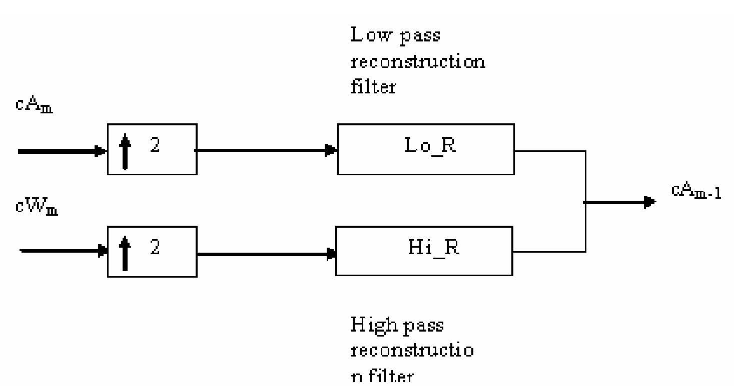
The DWT and IDWT

Mallat produced a fast wavelet decomposition and reconstruction algorithms for an orthogonal wavelet, in the *Multiresolution (MRA) framework*. The decomposition (DWT) algorithm starts with signal x, and then calculates the approximating coefficients cA_1 and detail coefficients cW_1, and then those of cA_2 and cW_2, and so on. These vectors are obtained by convolving x with the low-pass filter Lo_D for approximation coefficients, and with the high-pass filter Hi_D for detail coefficients, followed decimation by two.

More precisely the first step is:



The reconstruction (synthesizing) algorithm called inverse discrete wavelet transform (IDWT) starts from the coefficients of cA_m and cW_m next calculating the coefficients cA_{m-1} ,inverting the decomposition by inserting zeros and convolving the results with reconstruction filters, and then using the coordinates of cA_{m-1} and cD_{m-1} calculates these of cA_{m-2} and so on. More precisely the first synthesizing step is (it is done using MATLAB built in functions in the Wavelet tool box)



4.2 Wireless Channel

In this work, the performance of wavelet OFDM using five wavelet family functions has been examined in two channel models: the Rayleigh and frequency selective fading channels. Each of these channel models are described below.

Additive White Gaussian Noise (AWGN) Channel

Noise and interference are common transmission nuisances in wireless communication. Noise is an inevitable problem that is usually modeled as Additive White Gaussian Noise (AWGN) .The Gaussian distribution model is due to the central limit theorem and the fact that noise is the cumulative result of contributions from a number of independent sources. In this channel, zero-mean white Gaussian noise is added to the transmitted signal $x(t)$, so that the received signal $r(t)$ can be represented as:

$$r(t) = x(t) + n(t) \tag{4.1}$$

Where n (t) is a zero-mean Additive White Gaussian Noise process, with power spectral density $\frac{N_0}{2}$.

A Gaussian random number generator generates the AWGN added at the front end of the receiver.

Multipath Fading

Fading occurs due to multipath components in a channel. In the wireless propagation channel there is no single direct link between the transmitter and the receiver; rather the transmitted signal undergoes multiple reflections, refractions, diffractions and scattering. The effect can cause time spreading of the signal and results in fluctuations in the received signal's amplitude, phase, and angle of arrival, giving rise to the terminology multipath fading. It is possible that either or both the transmitter and receiver are moving. These moments cause Doppler shifts in the received signal and hence result in intercarrier interference. The constructive (or destructive) interference results in random fluctuation in the received signal. For the simulation purpose the multipath channel can be modeled as:

Rayleigh fading channel

The time dispersion in a multipath environment causes the signal to undergo either flat with Rayleigh distribution or frequency selective fading. Small-scale fading is also called Rayleigh fading because if the multiple reflective paths are large in number and there is no line-of-sight signal component, the envelope of the received signal is statistically described by a Rayleigh probability density function (pdf) given as:

$$p(x) = \begin{cases} (x/\sigma^2)\exp\left(-\dfrac{x^2}{2\sigma^2}\right) & 0 \le x < \infty \\ 0 & \textit{otherwise} \end{cases} \tag{4.2}$$

Where x is amplitude of the received signal, and $2\sigma^2$ is the mean power of the multipath signal envelop.

Simulation of Rayleigh Fading Signals

A Rayleigh fading waveform can be simulated by using Clarke and Gans's fading model [30].The generation is done by generating independent inphase and quadrature complex Gaussian noise samples and filtering them by $H(f)$ as shown in Figure 4.2. $S_{Ez}(f)$ is the spectrum of the Doppler filter. The sum of the inphase and quadrature terms at the output of the IFFT forms the complex Rayleigh fading waveform. The waveform has a Rayleigh distributed amplitude and a uniformly distributed phase in the interval (-π,π).

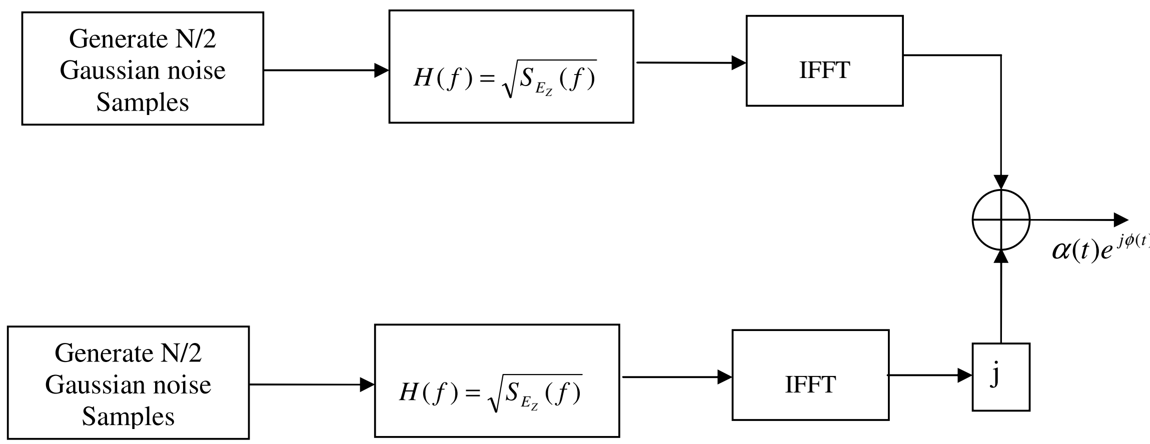


Figure 4.2: Generation of a Rayleigh Fading waveform

Frequency Selective Channel

If the bandwidth of the signal of interest exceeds the coherence bandwidth of the channel, the signal undergoes *frequency selective fading*. Viewed in the frequency domain, the channel causes different levels of attenuation for different frequency components of the signal.

Frequency selective fading is caused by multipath delays which approach or exceeds the symbol period of the transmitted symbol i.e. $T_s \subset \sigma_\tau$, where σ_τ is delay spread of the channel.

Simulation of Frequency Selective Fading Signals

Simulating a multipath frequency selective fading often requires the generation of multiplex independent Rayleigh faders. One possible model for a frequency selective radio channel is a transversal filter with tap spacing τ_n and random complex gain $\alpha_n(t)$ as in Figure 4.3 [30]. This is an approximation of the actual multipath component that is continuous in time into discrete paths at different delays. Each ray $\alpha_n(t)$ will then be modeled as a flat fading channel, with Rayleigh distribution.

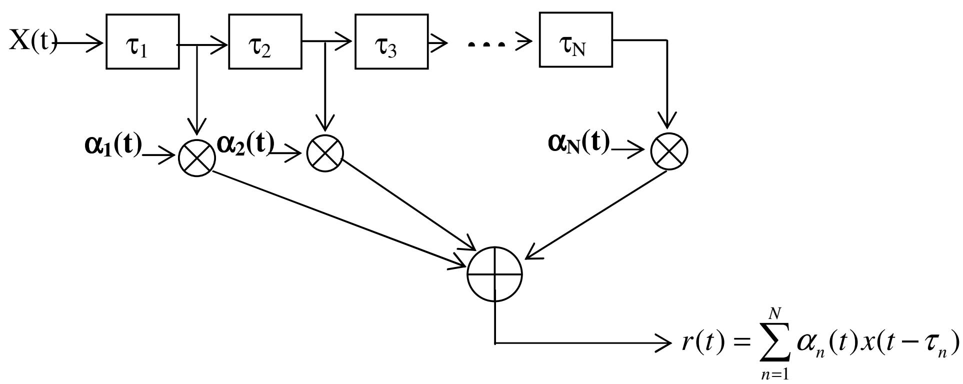


Figure 4.3 Tapped delay line for frequency selective channels wave form generation

In this work we used two-ray channel model with a channel. For the frequency selective fading channel, the received signal is given by

$$r(t) = \alpha_0 ray_0(t)\, x(t) + \alpha_1 ray_1(t)\, x(t - \tau_1) + n(t) \quad (4.3)$$

Where α_0 and α_1 are the amplitudes of the main ray and the secondary ray respectively. The signal energy in the main component along with the power of the noise term determines the SNR of the signal. The secondary component with a factor of α_1 and delayed by τ_1 corresponds to the first multipath bin with significant amplitude.

4.3 Performance Measures of DWT-OFDM

To investigate suitable wavelet base family for the wavelet based OFDM scheme, the superiority of one particular wavelet filter OFDM scheme over another is, of course determined by how well each fulfills the objectives of high throughputs (fast transform processing which leads in less computational transform complexity) and reduced bit error communication .

Fast Transform Wavelet

Since computational transform processing complexity and throughputs (which results from fast transform in this particular case) is another key issue, we are essentially interested in wavelets leading to fast transform. It is interesting to use waveforms of shorter duration to ensure that the symbol duration is far shorter than the channel coherence time That is short waveforms require less memory, limit the modulation-demodulation delay and require less transform computational time . We limit our performance analysis to DWT based OFDM using widely used different wavelets families such as Haar, Daubechies, Symlets, Coiflets, and Discrete Meyer and select the one with short computational simulation time. To this end considering the same computer speed, MATLAB 7 environment and keep all other constraints the same, the simulation time is compared for each wavelet families OFDM.

BER performance of DWT-OFDM

Probability of reduced bit error rate (increased performance) is another key issue in wireless communication. To measure the noise robustness of DWT-OFDM communication scheme the relationship of the BER as a function of SNR performance for different levels of noise is a useful performance tool. Then wavelet family that results in high performance gains is selected for optimum performance of wavelet OFDM

4.4 Performance Analysis

We analyze in this section the performance of wavelet based OPFDM in the propagation channels. The results have been obtained by simulations only, due to the fact that no analytical expressions are available for wavelet and wavelet filters.

4.4.1 Performance in Rayleigh Fading Channel

To measure processing speed of decomposition and synthesis of the signal by the wavelet filter banks the five wavelets family based OFDM is simulated independently on the flat fading channel environment.

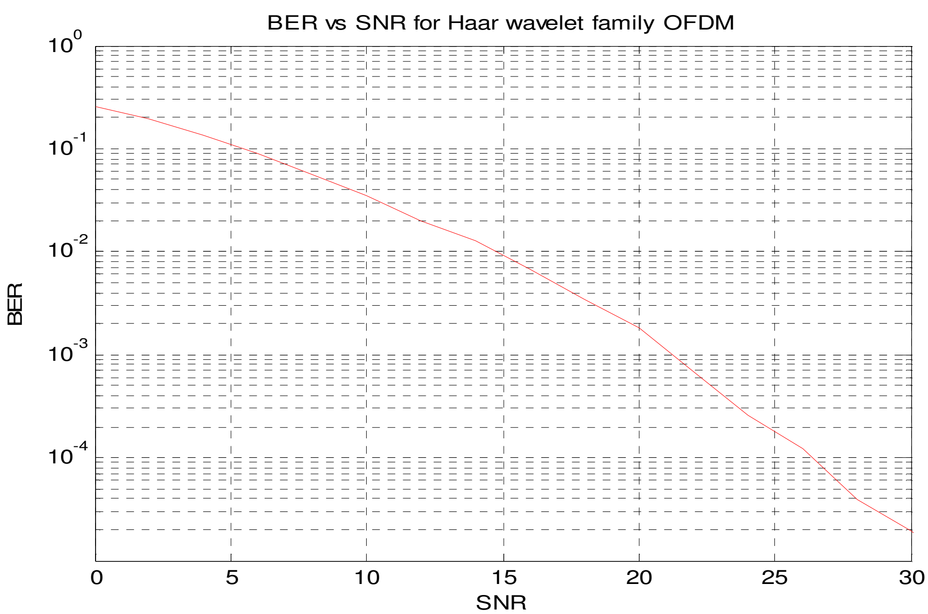


Figure 4.4 BER performance in Rayleigh fading channel with Doppler spread 60 Hz, Haar wavelet based OFDM

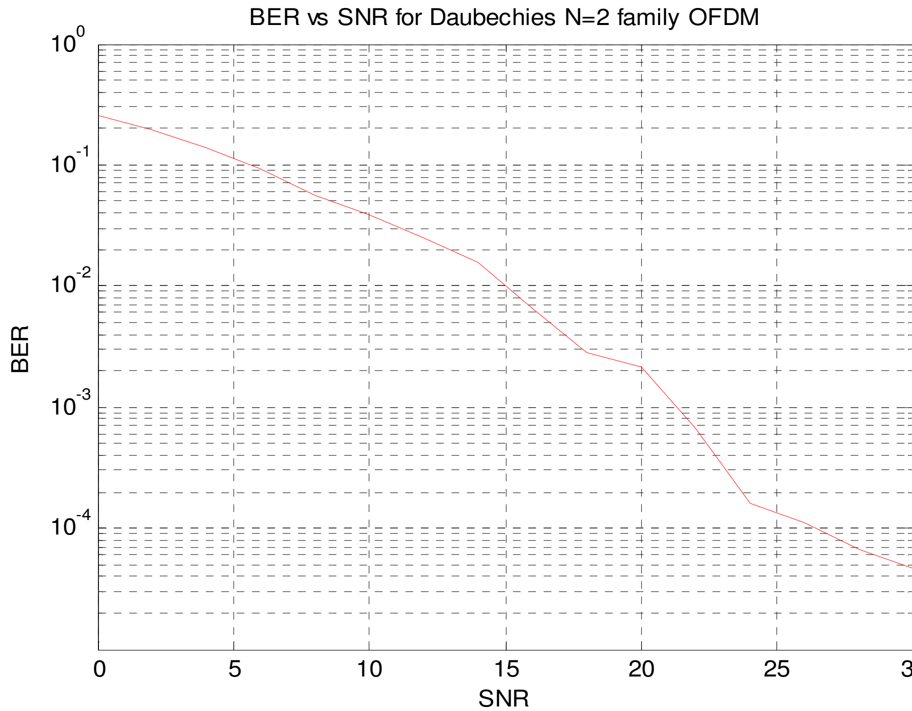


Figure 4.5 BER performance in Rayleigh fading channel with Doppler spread 60 Hz,

Daubechies N=2 wavelet Filter based OFDM

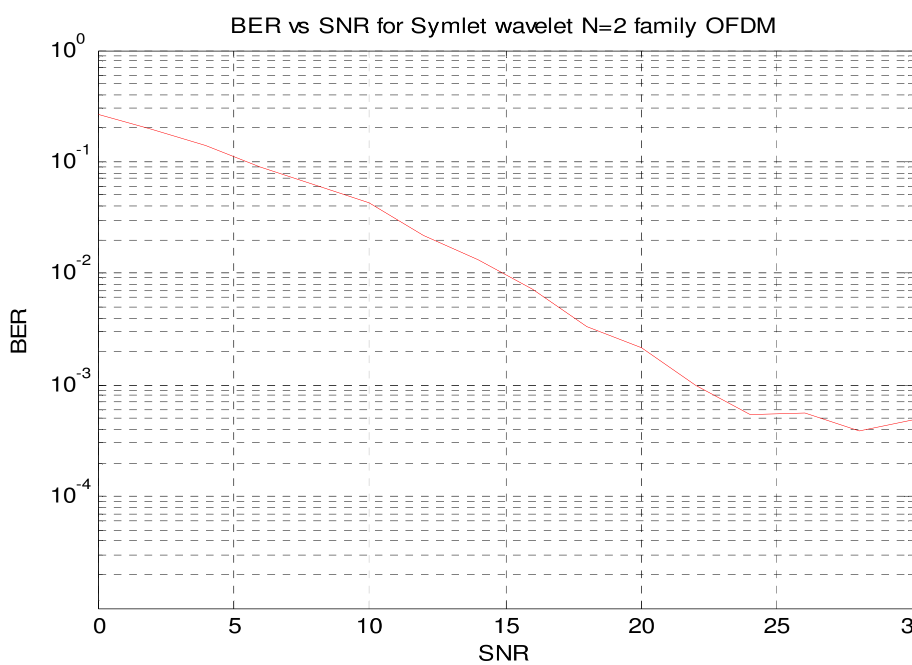


Figure 4.6 BER performance in Rayleigh fading channel with Doppler spread 60 Hz,

Symlets N=2 wavelet based OFDM

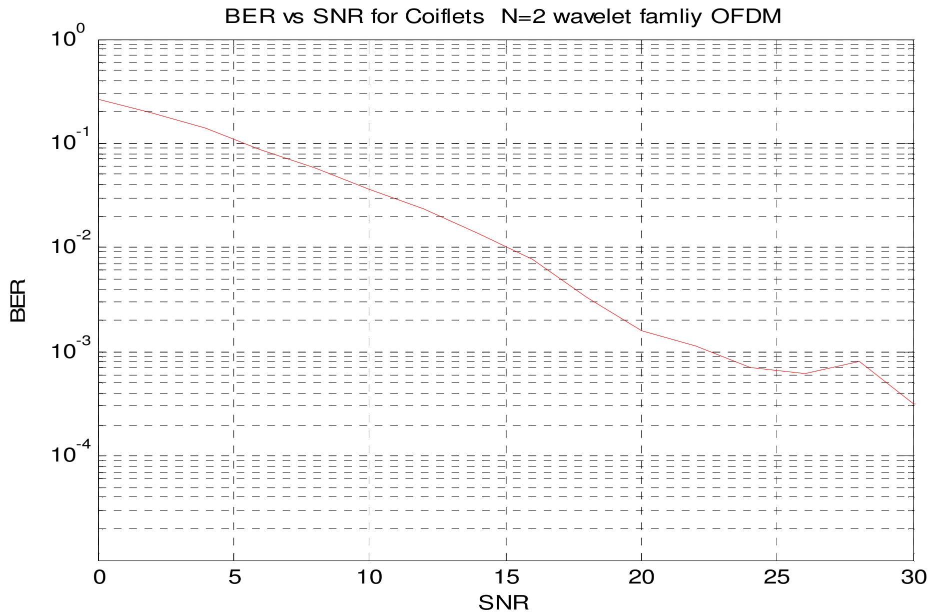


Figure 4.7 BER performance in Raleigh fading channel with Doppler spread 60 Hz,

Coiflets N=2 wavelet based OFDM

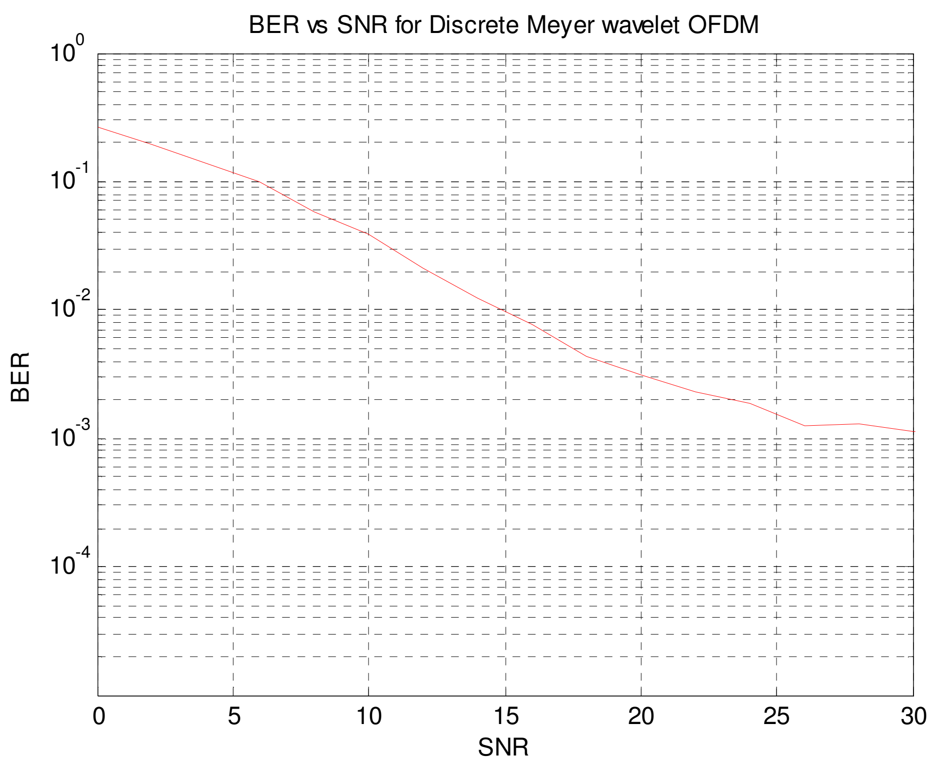


Figure 4.8 BER performance in Rayleigh fading channel with Doppler spread 60 Hz,

Discrete Meyer wavelet based OFDM

Table 4.1 Summary of elapsed simulation time of the five wavelet families based OFDM on Rayleigh Fading channel

DWT based OFDM	Elapsed time for simulation in seconds
Haar -OFDM	95.531000
Daubechies N=2-OFDM	90.203000
Symlets N=2-OFDM	98.693000
Coiflets N=2-OFDM	99.646000
Discrete Meyer-OFDM	121.643000

As it is shown in the table 4.1 Daubechies N=2 wavelet has taken less elapsed processing time for simulation.

For the sake of comparison, the performance of DWT-OFDM over Rayleigh fading, channel is plotted in Figure 4.9.

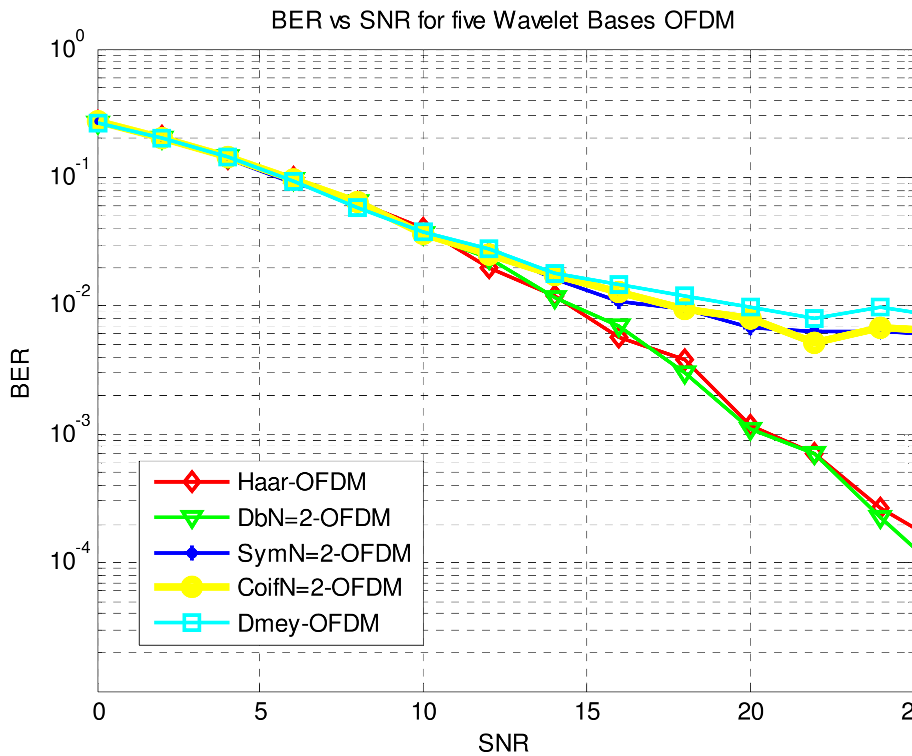


Figure 4.9 BER performances in Rayleigh fading channel with Doppler spread 60 Hz, Haar, Daubechies N=2, Symlets N=2, Coiflets N=2, and Discrete Meyer wavelet Filter Banks based OFDM

Low bit error communication is another key concept in wireless multicarrier communication system. As simulation results have demonstrated the BER performance of Haar and Daubechies wavelet filters based OFDM at high SNR reduce bit error rate at the receiver.

4.4.2 Performance in Frequency Selective Fading Channels

To measure processing speed of decomposition and synthesis of the signal by the wavelet filter banks the five wavelets family based OFDM is simulated independently on the frequency selective channel environment.

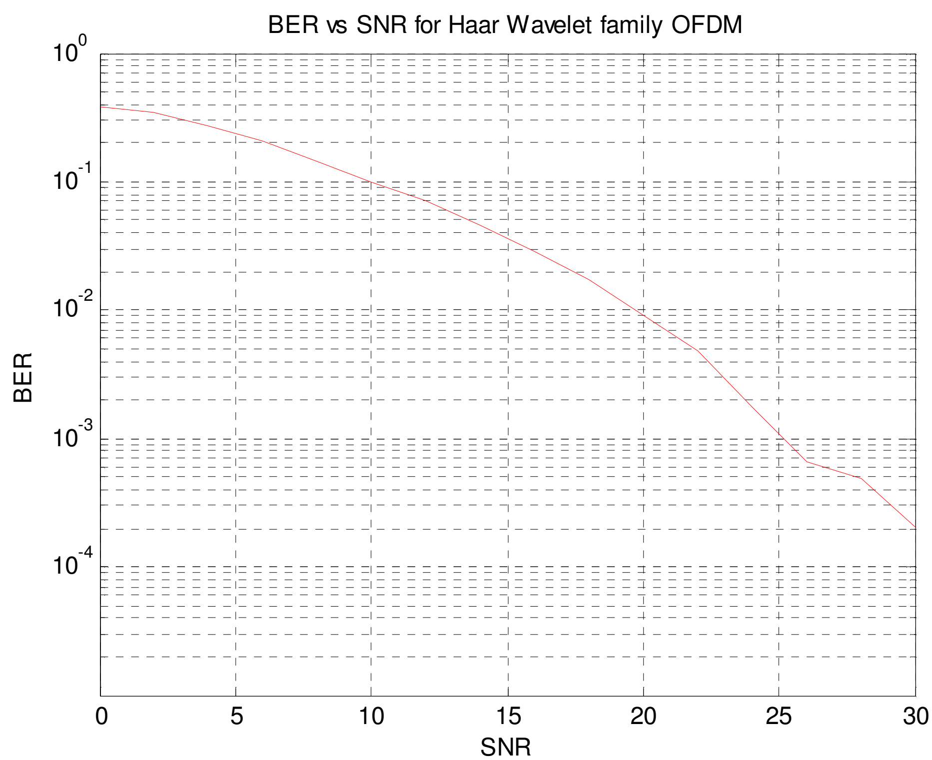


Figure 4.10 BER performance in Frequency Selective fading channels with Doppler spread 60 Hz, Haar wavelet based OFDM

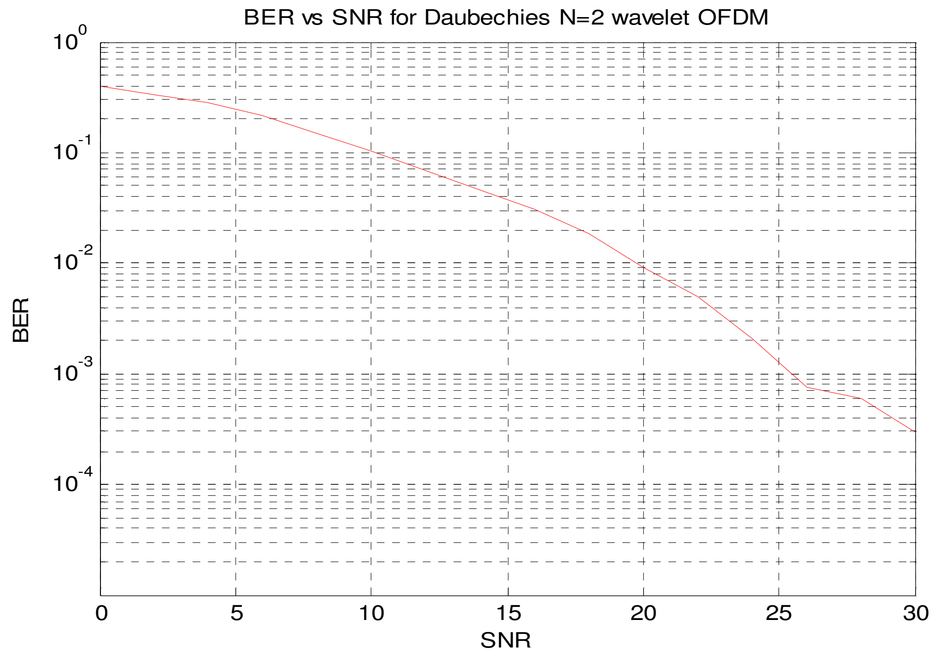


Figure 4.11 BER performance in Frequency Selective fading channels with Doppler spread 60 Hz, Daubechies N=2 wavelet based OFDM

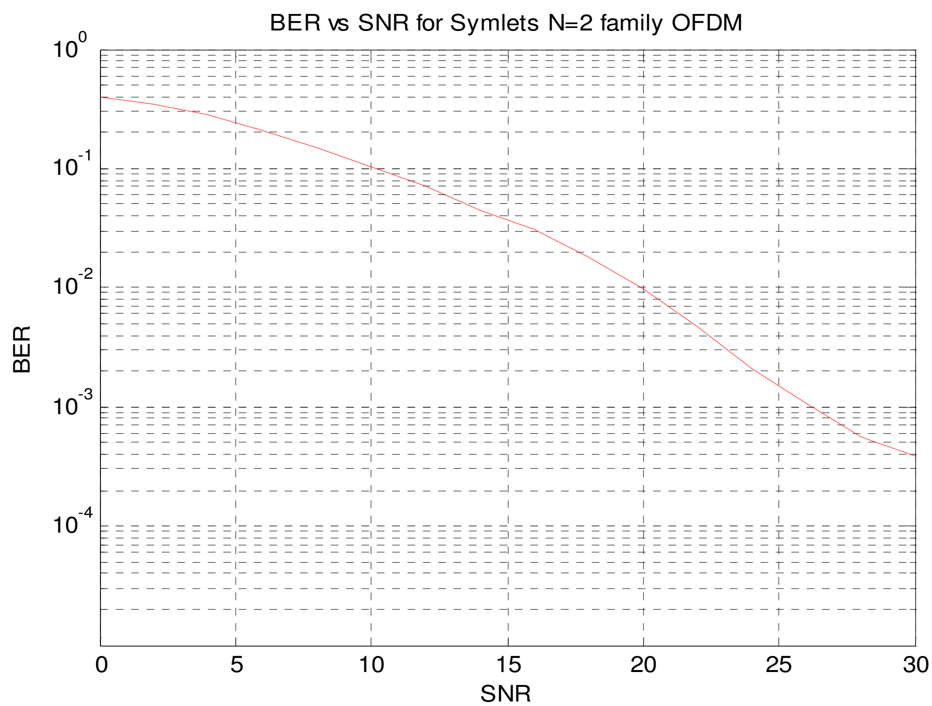


Figure 4.12 BER performance in Frequency Selective fading channel with Doppler spread 60 Hz, Symlets N=2 wavelet based OFDM

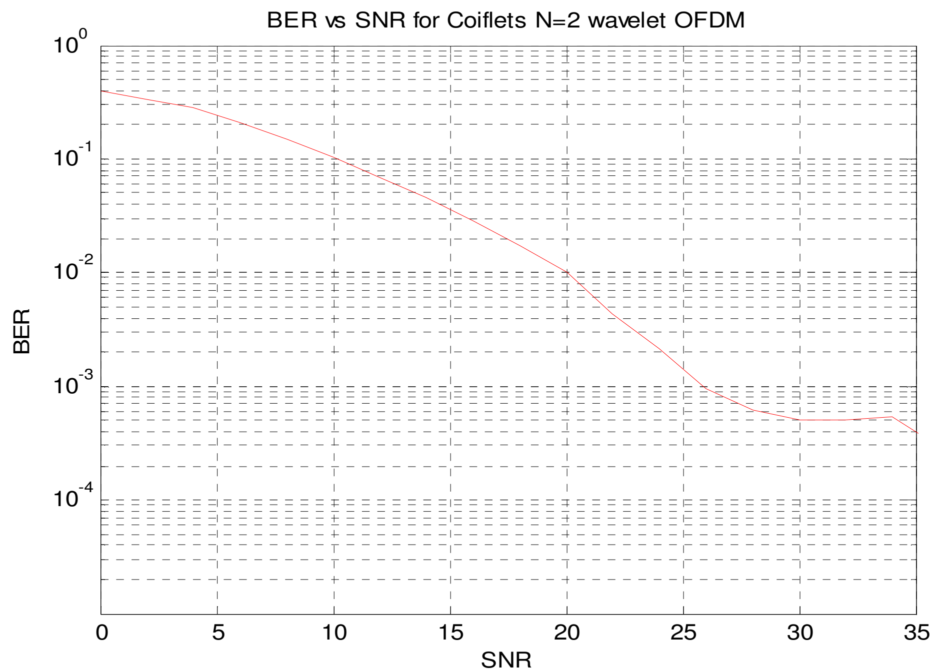


Figure 4.13 BER performance in Frequency Selective fading channel with Doppler spread 60 Hz, Coiflets N=2 wavelet based OFDM

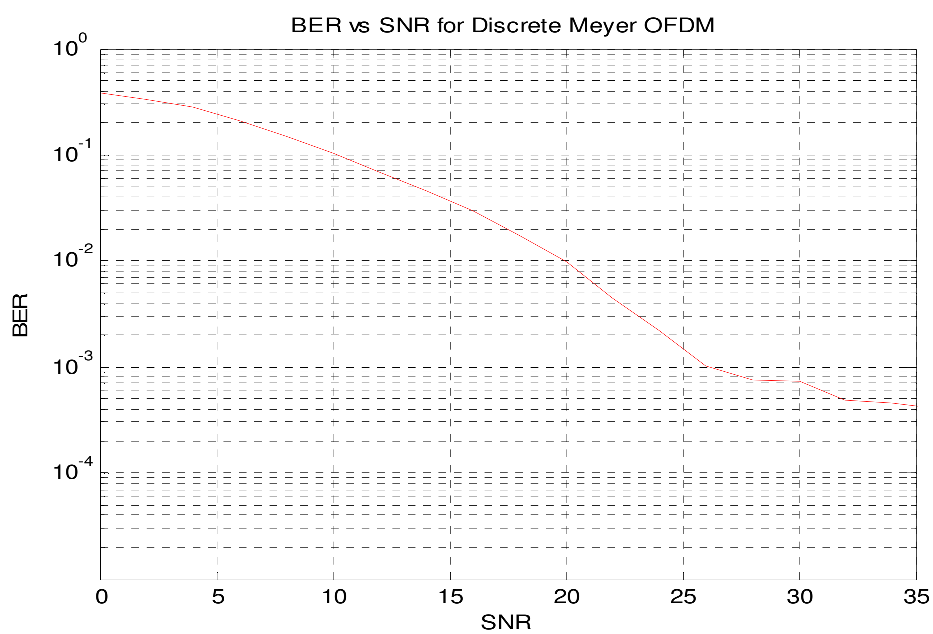


Figure 4.14 BER performance in Frequency Selective fading channels with Doppler spread 60 Hz, Discrete Meyer wavelet based OFDM

Table 4.2 Summary of elapsed simulation time of the five wavelet families based OFDM over Frequency selective fading channel

DWT Based OFDM	Elapsed time for simulation in seconds
Haar-OFDM	31.969000
Daubechies N=2-OFDM	31.641000
Symlets N=2-OFDM	38.470000
Coiflets N=2-OFDM	42.406000
Discrete Meyer- OFDM	57.891000

As it is also shown in the table 4.2 Daubechies N=2 wavelet has taken less simulation processing time.

It shows Daubechies N=2 wavelets generate waveforms of shortest duration. Computational transform complexity is another key issue in the wireless communication systems. Due to the high data rates required in modern applications, low signal synthesis and decomposition complexity is imperative. It is interesting to use waveforms of short duration to ensure that the symbol duration is far shorter than the channel coherence time to reduce interference. Further more short waveforms require less memory, limit the modulation-demodulation delay and require less computational transform time which helps to build less computational transform complex OFDM system.

For the sake of comparison, the performance of DWT-OFDM over frequency selective channel is plotted in Figure 4.14.

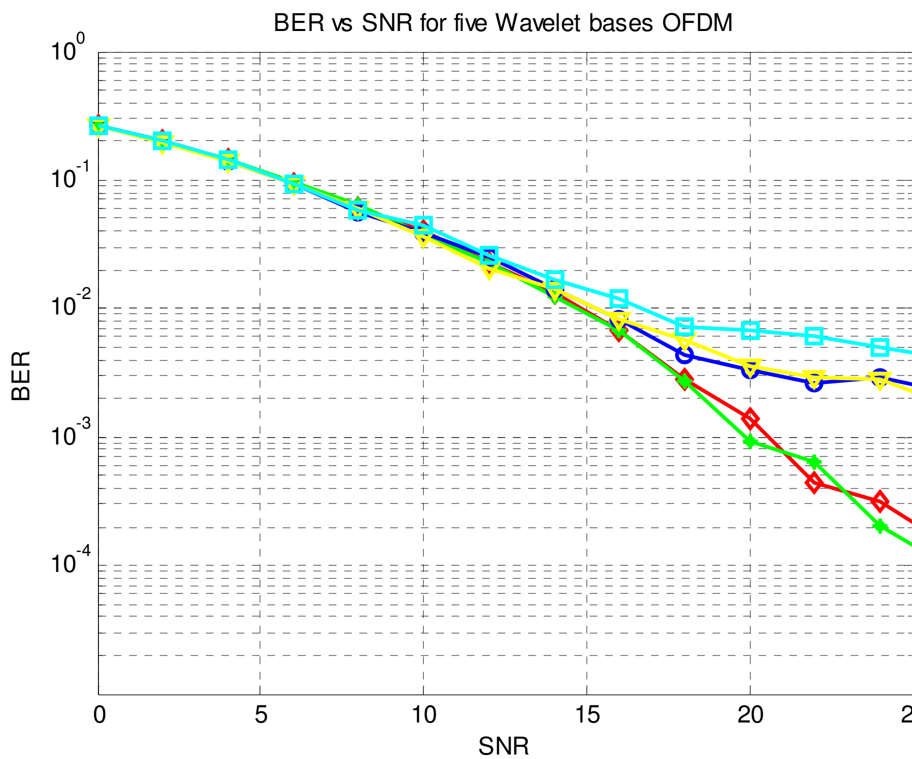


Figure 4.15 BER performances in Frequency selective channels with Doppler spread 60 Hz, Haar, Daubechies N=2, Symlets N=2, Coiflets N=2, and Discrete Meyer wavelet Filter Banks based OFDM

As the simulation results have demonstrated the BER performance of Haar and Daubechies wavelet filters based OFDM at high SNR provide high performance gains. This is also another key objective to be achieved for high performance gains multicarrier wireless communication.

Chapter 5

Conclusions and Future works

In this section we summarize and present conclusions based on the results in Chapter 4. Then we suggest some future areas for wavelet based OFDM systems.

5.1 Conclusions

Over all the performance results of wavelet based OFDM and its ability to fulfill the wide range of requirements of tomorrow's ubiquitous wireless communications lead us to conclude that this mew modulation is a viable alternative to conventional OFDM to be considered in future wireless communication systems.

- As it is shown from the tables of the elapsed time for BER performance simulation results fast transform results of short duration Daubechies N=2 wavelet gives fast processing time. Daubechies wavelet generates short wave forms. Short waveforms require less memory, limit the modulation-demodulation delay and require less computation which helps to implement fast wavelet transform less computational complex wavelet based OFDM scheme. This leads to conclude that Daubechies wavelet family can be a viable alternative suitable basis for OFDM to be considered for future OFDM communication scheme.

- On the Rayleigh fading channel as the simulation results of the BER performance of the five wavelet families based OFDM have indicated that OFDM with wavelet filter bases Haar and Daubechies N=2 at high SNR reduced the probability of bit error rates at the receiver.
- On the Frequency selective fading channel as the simulation results of the BER performance of the five wavelet families based OFDM have indicated that OFDM with wavelet filter bases Haar and Daubechies N=2 at high SNR reduced the probability of bit error rates at the receiver.
- The BER performance measure of the two channel simulation results have indicated that Daubechies (as Haar =Daubechies N=1) based OFDM scheme highly reduced probability of bit error rates, that is provides better performance gains.

Over all the two performance measure results of Daubechies based OFDM lead us to conclude that *Daubechies wavelet* family is suitable basis for OFDM to its optimum performance.

5.2 Future Works

Finally it is important to underline that wavelet theory is still developing .It is expected that more is still to be pointed out as the knowledge of this recently proposed scheme gains more interest .There are many possibilities for future work in this area, and are summarized as follows:

- *Diversity Scheme* on *Wavelet* based *OFDM*: improved transmission integrity may be achieved with aid of diversity. Space, time, frequency diversities are the most physical diversities to be exploited.
- *Equalization techniques*: equalization techniques and channel estimation on wavelet based realization could also be an area to be addressed.
- The initiative could be extended to address Orthogonal *Wavelet* based *Codes* for *Multiple access schemes.*

To sum up researches done in wavelets and their application for communication engineering is still at its infant stage and there are growing number of areas that upcoming researchers are invited to explore.

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