ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES
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DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Performance Study of Space-Time Coding over Spatially Correlated Rayleigh Fading Channel

A thesis submitted to the school of graduate studies in Partial fulfillment of the requirements of Masters of Science in Electrical and Computer Engineering (Communication)

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Declaration

I, the undersigned declare that this thesis is my original work, and has not been presented for a degree in this or any other university, and all sources of materials used for the thesis have been fully acknowledged.

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Above all I praise Lord who is always with me.

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Acronym	Expansion
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CSI	Channel State Information
EGC	Equal Gain Combining
i.i.d.	Independently identically distributed
LOS	Line-of-Sight
MGF	Moment Generating Function
MIMO	Multiple Input Multiple Output
ML	Maximum-Likelihood
MRC	Maximum Ratio Combining
ODSTBC	Orthogonally Designed Space-Time Block Code
PAM	Pulse Amplitude Modulation
PEP	Pairwise Error Probability
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
RF	Radio Frequency
SC	Selection Combining
SISO	Single Input Single Output

SNR	Signal-to-Noise Ratio
SSC	Switch and Stay Combining
STC	Space-Time Codes
STTC	Space-Time Trellis Codes

List of Symbols

d	Diversity gain
E_s	Transmitted power at each transmit antenna
$h_{j,i}$	Fading coefficient
$\mathbf{H}$	Complex correlated channel matrix
$\mathbf{H}_{\omega}$	i.i.d. complex channel matrix
k	In put symbols
$\mathbf{N}$	Gaussian noise vector
n	Complex Gaussian noise
N_o	Variance
n_R	Number of receive antennas
n_T	Number of transmit antennas
p	Signaling interval
Q(x)	Q-function
$\mathbf{R}$	Correlation matrix of MIMO channel
R	Code rate
$\mathbf{R}_{\mathbf{Rx}}$	Receive correlation matrix
$\mathbf{R}_{\mathbf{Tx}}$	Transmit correlation matrix
$\mathbf{X}$	Transmitted signal vector
$x(t)$	Transmitted signal at time t

Y Received signal vector

y(t) *Received signal at time t*

β SNR per receive antenna

β_t SNR per transmit antenna

$\Phi(s)$ Moment generating function

Abstract

In this thesis we discuss the space-time block coding, a new paradigm for communication over Rayleigh fading channels using multiple transmit antennas with out information of the channel at the transmitter. Data is encoded using a space-time block code and the encoded data is split into n_T transmit antennas. The received signal is linear superposition of the n_T transmitted signals perturbed by noise.

Previous work on space-time coding has been restricted on the idealistic case of uncorrelated spatial fading. In practice, however, insufficient antenna spacing or lack of scattering cause the individual antennas to be correlated. In the second part of this work, we study the impact of spatial fading correlation on the diversity and coding gains.
We derive the exact pairwise error probability (PEP) for space-time coding over qusi-static Rayleigh fading channels. We furthermore show that if a space-time code achieves full diversity in the uncorrelated case, the diversity order achieved in the correlated case is given by the product of rank of the transmit correlation matrix and the rank of receive correlation matrix. Finally, we provide simulation results demonstrating the impact of transmit spatial fading correlation on the performance of space-time block coding for different correlation values.

Chapter 1

Introduction

1.1 Wireless Communications

Wireless systems are developing rapidly to provide voice, data and multimedia messaging services. These services require reliable wireless channels with large capacities. Systems which communicate over a Single Input Single Output (SISO) wireless channel have limited capacity. Also, in some situations, communication over a SISO channel is not reliable due to multipath fading. Information theoretic investigations in the past few years have shown that very high capacities can be obtained by employing multiple antenna elements at both the transmitter and the receiver of a wireless system [1,2]. (MIMO) wireless systems have become an active research area. Compared with the single input single output (SISO) systems, two kinds of gains are provided by the MIMO wireless systems, namely, diversity gain and multiplexing gain [3]. By employing multiple antennas, multiple independent replicas of the information signal are received at the receiver, which means that more reliable reception can be achieved. For example, in a Rayleigh flat fading channel with one transmit antenna and n_R receive antennas, at high signal-to-noise ratio (SNR), the average error probability decays as $1/(\mathrm{SNR})^{n_R}$, instead of 1/SNR for SISO channels [1]. Recently, transmit diversity has been studied extensively because of the feasibility of having multiple antennas at the base station [4]. The idea of spatial multiplexing is that MIMO systems in a rich scattering environment provide multiple data pipes within the same frequency band yielding a linear increase in capacity. It is shown in [5] that for a system with n_T transmit antennas and n_R receive antennas, the capacity is about $\min(n_T, n_R)$ times larger than that of a system with a single transmit and a single receive antenna.

Most of the existing work in this area assumes that the antenna elements at the transmitter and the receiver of the MIMO system are placed far enough (spatially) such that the effect of the channel at a particular antenna element is different from the effect at all other antenna elements. This implies independent or spatially uncorrelated fading.

This holds true only if spacing between transmit antennas or receive antennas is of the order of several wavelengths. However, in reality, the individual antennas could be correlated due to insufficient antenna spacing and lack of scattering.

For orthogonally designed space-time blocks (ODSTBC), we analyze the effects of transmit correlation on the performance in terms of error probability. MIMO antenna system together with space-time coding significantly improve the performance of wireless communication system by exploring the spatial and temporal diversities of the system. In general, the presence of spatial fading correlation between antenna elements will affect the performance of any space-time coding scheme. However, the OSTBC has an inherent protection against information loss due to the spatially correlated fading [7]. This motivates the investigation of fading resistance provided by OSTBC when spatial correlation is present.

1.2 Literature Review

Fading in wireless channels causes loss in performance. To overcome this bad effect, communication engineers came up with techniques to decrease the probability of having an overall weak channel. These techniques are mainly called diversity. In general, diversity means using different dimensions of the channel, e.g. space, time and frequency to improve the equivalent channel seen by the receiver. A space-time code [8] is in general any modulation scheme, which is designed for a multiple transmitter wireless system that tries to achieve antenna (space) diversity. The very first designs of space-time codes were in the form of trellis-coded modulation, and suffered from exponential decoding complexity as the number of transmit antennas increased. After a while, Alamouti [9] proposed a simple transmitter diversity scheme, which benefited from both full diversity of a two-transmit antenna channel as well as simple Maximum-Likelihood (ML) decoding. The good properties of this code inspired Tarokh *et al.* [10] to inspect the existence of similar designs for more numbers of transmit antennas. In the case of complex codes, i.e. modulation schemes using complex constellation members, the authors proposed a structured modulation scheme, called Orthogonal Space-time Block

Code that could send on average one symbol in every two-time slots, and achieved full diversity as well as simple ML decoding. They presented examples for three and four transmit antennas with average rate of ¾, or in case of real constellations they presented rate 1 codes for any number of transmit antennas. It was shown in that paper that for complex constellations there is no square rate-1 code, i.e. a code for which the time length of the block equals the number of transmit antennas like that of Alamouti, for more than two transmit antennas. This means that Alamouti code was the only square full-rate complex orthogonal space- time block code.

Tarokh *et al.* [8] analyzed the multiple input multiple output channel following the lines of [11]. They showed that the diversity gain of the code is characterized by the minimum of the ranks of the code difference matrices and that the coding gain is given by the minimum of the product of the non-zero eigenvalues of the code difference matrices. They also designed several codes following the design criteria of the rank and determinants of the code difference matrices and the concept of space-time coding was introduced.

The rank criterion that determines the diversity gain of the system is relatively easy to satisfy and there are several simple observations (given in [8] and [12], among others), which help to achieve the maximum diversity gain. It is widely believed that the diversity gain is more important than the coding gain, for reliable performance over fading channels.

Recently Bolcskei *et al.* [13] studied the performance of modulation schemes in channels with spatial correlation. Basically they expand the pairwise error probability and average it on the distribution of the channel coefficients. They propose a closed form distance matrix and design criteria that is essentially the same as the one in [8]. When correlation of the antennas is such that it causes rank deficiency in the distance matrix, we do not get full diversity. Direct usage of Space-time codes is not the best we can do at this case. Because space-time codes are full rank codes and they are meant to achieve diversity equal to the number of transmit antennas. They concentrate on the quasi-static channel and find Chernoff bounds that in the high-SNR regime. They also calculate the degradation of average pairwise error probability due to antenna correlation. Usal and Georghiades [14] drive pairwise expressions for transmit antenna correlation. With no conditions on the receive correlation.

1.3 Notations and Mathematical Preliminaries

Throughout this work, the following notations and mathematical formulations are used frequently:

- $\mathbf{I}_n$ denotes the identity matrix of size n.
- $[.]^T$, $[.]^*$ and $[.]^H$ denotes the transpose, complex conjugate and conjugate transpose, respectively.
- The matrix **A** is said to be Hermitian if $\mathbf{A} = \mathbf{A}^H$.
- $E[.]$ denotes the mathematical expectation.
- A circularly symmetric complex Gaussian random variable is a random variable

  $z = (x + jy) \sim CN(0, \sigma^2)$ where x and y are i.i.d $N\left(0, \frac{\sigma^2}{2}\right)$.
- Frobenius norm of matrix **A** is defined to be $\|\mathbf{A}_{p\times Q}\|^2 = \sum_{i=1}^{p}\sum_{j=1}^{Q}|a_{ij}|^2$

  $= \mathrm{tr}(\mathbf{A}^H\mathbf{A})$ where tr (.) is the trace operator.
- Vec (.) denotes the vector operator that concatenates the columns of an n × m matrix respectively into nm × 1 vector.
- The matrix **A** is nonnegative definite if for any 1×n complex vector $x\mathbf{A}x^* \geq 0$.
- The n × m matrix **B** is a square root of an n × n matrix **A** if $\mathbf{B}\mathbf{B}^H = \mathbf{A}$.
- A row eigenvector V corresponds to eigenvalue λ of an n × n matrix **A** is a vector that satisfies $v\mathbf{A} = \lambda v$.
- For two matrices **A** and **B** the Kronecker product $\mathbf{A} \otimes \mathbf{B}$ is defined to be

$$\begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{n1}\mathbf{B} & \cdots & a_{nn}\mathbf{B} \end{bmatrix}.$$

1.4 Objective of the thesis

In this work, our aim is to present an analysis that is general and comprehensive. We assume that the channel conditions are known exactly to the receiver, i.e. we have perfect channel state-information (CSI). We present the analysis of fading channels, receive diversity, transmit diversity and space-time coding which provides full diversity as well as coding gain.

We consider the realistic channel conditions, the individual antennas could be correlated due to insufficient antenna spacing and lack of scattering, and we present generalized analysis for all space-time codes. In particular we propose to

1. Study the performance gain by Space-time Coding.

2. Investigate the effects of correlation on the performance of space-time block code.

3. Compare the bit error probability under uncorrelated and correlated condition for OSTBC as a function of signal-to-noise ratio (SNR).

1.5 Thesis Lay out

The work is organized as follows; chapter 2 begins with a brief description of fading channels and diversity techniques that are being used to characterize the performance of MIMO schemes. Then, methods of receive diversity are introduced. Later, a detailed description of the basic features of Space-Time Block Coding (STBC), a popular transmit diversity, are given. We further extend Tarokh's generalized new scheme to an arbitrary number of transmits antennas.

In chapter 3, we develop a more detailed performance evaluation for Space-Time Codes (STCs) by deriving analytical bounds. It is observed that in fading channels, the code construction criterion depends on the value of the possible diversity gain of the system. In slow fading channels, when the diversity gain is small, the rank and determinant criteria are valid for code design. On the other hand, when the diversity gain is reasonably large, the trace of the codeword distance matrix, or, equivalently, the minimum squared Euclidean distance, will be the dominant parameter for the code performance.

Chapter 4 is organized as follows, we introduce the channel model, and we drive the pairwise error probability (PEP) of space-time codes as a function of the transmit and the receive correlation matrix, we discuss the impact of spatial fading correlation on the performance of Space-Time coding.

In chapter 5 we provide simulations and analysis for both uncorrelated and correlated Channel conditions.

Chapter 6 summarizes the work and presents a brief discussion on possible research direction, which can extend the investigation presented in this work.

Chapter 2

Space-Time Block Coding

2.1 Introduction

In a high data rate wireless communication system, bandwidth limitation and channel fading are two major obstacles to achieve reliable communication. Teletar [15], Foschini and Gans [16] have recently shown that there is a huge potential capacity gain of multiple antenna systems compared to single antenna systems. They showed that the capacity of a multiple antenna system grows at least linearly with the number of transmit antennas; provided that the number of receive antennas is greater than or equal to the number of transmit antennas. To approach the potential by huge capacity of multiple antenna systems, new coding and modulation, which is called space-time coding, has attracted considerable attention.

Space-time block coding was introduced as one of the methods to achieve spatial diversity. In STBC data is encoded using the space-time block code, this data is then split into a number of streams and transmitted at once, using the same number of antennas. Space-Time Codes (STC) have been introduced by Alamouti [9] and Tarokh [10] as a novel means of providing transmit diversity for the multiple-antenna fading channel. The space-time coding scheme is basically a joint design of channel coding, modulation, transmit and optimal receiver diversity. It exploits the utilization of multiple antennas to improve the spectral efficiency and performance over fading channels of communication systems. The space-time trellis-coding scheme was first introduced by Tarokh in 1998 on a Rayleigh's fading channel, providing maximum diversity gain as well as greater coding gain. For a given number of transmit antennas, the decoding complexity increases exponentially with the transmission rate. In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmission using two transmit antennas. These codes were much less complex than trellis codes and promising in terms of simplicity as well as performance. Tarokh then generalized this scheme for any number of transmitter antennas known as the space-time block codes. The space-time block codes provide the full diversity advantage but are not optimized for coding gain. STBC uses fast decoding algorithm and decode all the transmitted symbols linearly, hence the decoding complexity increases linearly and not exponentially.

The space-time block codes can be categorized with respect to the symbols transmitted as real orthogonal designs and complex orthogonal designs. The real orthogonal designs deal with Pulse Amplitude Modulation (PAM) while the complex orthogonal designs deal with complex constellations like such as the Phase Shift Keying (PSK) and the Quadrature Amplitude Modulation (QAM). Tarokh *et al* [10] proposed systematic constructions of real orthogonal designs for any number of transmit antennas with full rate. However, complex orthogonal designs are not well understood. There exist several different types of space-time block codes from complex orthogonal designs. In STBC, the data is encoded using block codes and the encoded data is split into a number of streams which are simultaneously transmitted using the same number of transmit antennas. The signal at each receiver is a linear superposition of all the transmitted signals perturbed by noise.

Decoupling of the signals transmitted from different antennas helps to achieve maximum likelihood decoding in a rather simple way. Decoupling of signals uses the orthogonal structure of the space-time block code and gives a maximum likelihood-decoding algorithm based on linear processing at the receiver. The classical mathematical framework of orthogonal designs is applied to construct the space-time block codes. Space-time block codes are designed to achieve the maximum diversity order for a given number of transmit and receive antennas by means of simple decoding algorithms. Compared with trellis-codes, they have advantages and disadvantages

- complexity: computationally simple (advantage)
- coding gain: limitations in coding gain (disadvantage).

In STTC the decoding complexity increases exponentially with the transmission rate. However, the STBC are much less complex than trellis codes and are promising in terms of simplicity as well as performance. STBC use a fast decoding algorithm and decode all the transmitted symbols linearly, hence the decoding complexity increases linearly and not exponentially.

2.2 Fading Channels

The wireless characteristic of the channel places fundamental limitations on the performance of wireless communication systems. Unlike wired channels that are stationary and predictable, wireless channels are extremely random and are not easily analyzed due to the diverse environment, the motion of the transmitter, the receiver, and the nature of the surrounding objects. In this section, characteristics of wireless channels and Rayleigh flat fading model are discussed.

In a wireless environment, the surrounding objects, such as buildings, trees, and houses act as a reflectors of electromagnetic waves. Due to these reflections, electromagnetic waves travel along different paths of varying lengths and therefore have various amplitude and phases. The interaction between these waves causes multiple fading at the receiver location, and the strength of the waves decreases as the distance between the transmitter and the receiver increases. Thus, fading results from the superposition of transmitted signals, which have experienced differences in attenuation, delay and phase shift while traveling from the source to the receiver. Traditionally, propagation modeling focuses on two aspects. Propagation models that predict the mean signal strength for an arbitrary transmitter-receiver separation distances are called large-scale propagation models since they characterize signal strength over large transmitter-receiver distances. Propagation models that characterize the rapid fluctuations of the received signal strength over short travel distances or short time durations are called small-scale or fading models. Small-scale fading is the result of many factors, such as multiple-path propagation, speed of the transmitter and receiver relative to each other, speed of surrounding objects, and the transmission bandwidth of the signal. In this work, narrowband systems are considered, in which the bandwidth of the transmitted signal is smaller than the channels coherence bandwidth, which is defined as the frequency range over which the channel fading process is correlated. Small-scale fading is usually divided into fading based on multipath time delay spread and based on Doppler spread.

There are two types of fading based on multipath time delay spread:

Flat fading: where the bandwidth of the signal is less than the coherence bandwidth of the channel or the delay spread is less than the symbol period.

Frequency selective fading: where the bandwidth of the signal is greater than the coherence bandwidth of the channel or the delay spread is greater than the symbol period.

There are two types of fading based on Doppler spread:

Fast fading: which has a high Doppler spread, and the coherence time is less than the symbol period. Fast fading describes a condition where time duration in which the channel behaves in a correlated manner is short compared to the time duration of a symbol. Therefore, it can be expected that the fading character of the channel will change several times while a symbol is propagating, leading to distortion of the base band pulse shape.

Slow fading: which has a low Doppler spread. The coherence time is greater than the symbol period. Here, the time duration that the channel behaves in a correlated manner is long compared to the time duration of a transmission symbol. Thus, one can expect the channel state to virtually remain unchanged during the time in which a symbol is transmitted. The propagating symbols will likely not suffer from the pulse distortion as described above. The primary degradation in a slow fading channel, as with flat fading, is loss in SNR [17, 42].

2.2.1 Rayleigh Flat-Fading Channel

The Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal. It is also used to model fading channels in this work. For a typical mobile wireless channel in indoor or urban areas, we may assume that the direct line-of-sight (LOS) wave is obstructed and the receiver obtains only reflected waves from the surrounding objects. When the number of reflected waves is large, according to central limit theory, two quadrature components of the received signal are uncorrelated Gaussian random processes with mean zero and variance σ^2. As a result, the envelope of the received signal at any time instant has a Rayleigh distribution and its phase is uniform between $-\pi$ and π.

The probability density function of the Rayleigh distribution is given by

$$P(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & \text{if} \quad r \geq 0 \\ 0 & \text{if} \quad r < 0 \end{cases} .$$

Another widely used channel model is the Ricean model which is suitable for the case when there is a dominant stationary signal component, such as a line-of-sight propagation path. The small-scale fading envelope is Ricean, with probability density function,

$$P(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} I_0\left(\dfrac{Ar}{\sigma^2}\right) & \text{if} \quad r \geq 0 \\ 0 & \text{if} \quad r < 0 \end{cases} .$$

The parameter A is always positive and denotes the peak amplitude of the dominant signal, and I_o (**.**) is the zeroth -order modified Bessel function of the first kind. The best way to combat fading is to ensure that multiple versions of the same signals are transmitted, received, and coherently combined. This is usually termed as diversity [42], which we discuss in the next section.

2.3 Diversity

Unlike the Gaussian channel, the wireless channel suffers from attenuation due to destructive addition of time varying multi-path fading in the propagation media and due to the interference from other users. Severe attenuation makes it impossible for the receiver to determine the transmitted signal compared to fiber or coaxial cable or line-of-sight (LOS) microwave or even satellite transmissions unless some less-attenuated replica of the transmitted signal is provided to the receiver. This source is called diversity and it is the single most important contributor to reliable wireless communication [8].

Diversity techniques are base on the notion that errors occur in reception when the channel attenuation is large, i.e., when the channel is in a deep fade. If we can supply to the receiver several replicas of the same information signal transmitted over independently fading channels, the probability that all the signal components will fade simultaneously is reduced considerably [43]. The basic idea of diversity is that, if two or more independent samples of a signal are sent and then fade in an uncorrelated manner, the probability that all the samples are simultaneously below a given level is much lower than the probability of any one sample being below that level. For example, suppose we have one transmit and two receive antennas and that one of the channels goes in deep fade and is basically unusable. In this case, the other channel may still be able to recover the data. While both channels might fail simultaneously, this is highly unlikely compared to the event of a single channel failure. This is demonstrated by the following property of independent events.

Pr (“Channel 1 fails” and “Channel 2 fails”)
= Pr (“Channel 1 fails”) Pr (“Channel 2 fails”).

Thus, properly combining various samples greatly reduces the severity of fading and improves reliability of transmission. There are lots of diversity techniques that can be classified according to the domain where diversity is introduced. Commonly used techniques include:

Temporal (Time) Diversity: diversity over time can be obtained via coding and interleaving, information is coded and the coded symbols are dispersed over time in different coherence periods so that different parts of the codewords experience independent fades [19].

Frequency Diversity: The fact that waves transmitted on different frequencies induce different multi-path structure in the propagation media is exploited. Thus, replicas of the transmitted signal are provided to the receiver in the form of redundancy in frequency domain [8].

Polarization Diversity: Orthogonally polarized waves with independent fading characteristics can be used as a source of diversity. Such techniques are called polarization diversity techniques. In urban environments where space is limited, polarization diversity is particularly advantageous as we can place antennas together. It however provides only two diversity branches. In environments with a number of reflections, the polarization may ultimately be lost and this technique may no longer be useful. Line-of-sight communications can use polarization diversity to improve its performance [8].

Space Diversity: In exploiting time diversity, interleaving and coding over several coherence time periods is necessary. When there is a strict delay constraint and / or the fading is slow, this may not be possible. In this case other forms of diversity have to be obtained. Space diversity can be obtained by placing multiple antennas at the transmitter and /or the receiver. If the antennas are placed sufficiently far apart, the channel gains between different antenna pairs more or less independent and independent signal paths are created. The required antenna separation depends on the local scattering environment as well as on the carrier frequency. For a mobile which is near the ground with many scatterers around, the channel decorrelates over shorter spatial distances, and typical antenna separation of half to one carrier wavelength is sufficient. For base stations on high towers, larger antenna separation of several 10's of wavelengths may be required. One should observe that the higher the carrier frequency, the shorter the wavelength and the shorter spatial separation needed. Space diversity can be classified into two categories receive diversity, using multiple receive antennas, and transmit diversity, using multiple transmit antennas [19].

2.3.1 Diversity Advantage of Multiple Antennas

The diversity order of any communication system measures its reliability at high SNR. When a well-designed communication system transfers data across any channel, the probability of error P_e (SNR) is typically a decreasing function of the SNRs. The diversity order d of the communication system is defined as [44]

$$d = -\lim_{SNR\to\infty} \frac{\log p_e(\mathrm{SNR})}{\log(\mathrm{SNR})}. \tag{2.1}$$

The diversity order of the channel is defined as the diversity order of the best possible communication system that can be used for that channel. The diversity order represents the asymptotic slope of a log-log plot of error probability vs. SNR. At high SNR, the error probability goes to zero as SNR^{-d} and hence a high diversity order is desirable.

2.4 Receive Diversity

In receive diversity the independent fading paths associated with multiple receive antennas are combined to obtain a resultant signal that is then passed through a standard demodulator. The combining can be done in several ways which vary in complexity and overall performance. Most combining techniques are linear: the output of the combiner is just a weighted sum of the different fading paths or branches. The four most common space diversity techniques employed at the receiver are selection combining, threshold combining, maximal ratio combining and equal gain combining.

1. Selection Combining: In selection combining (SC), the combiner outputs the signal on the branch with the highest SNR. SC often requires just one receiver that is switched into the active antenna branch. However, a dedicated receiver on each antenna branch may be needed for systems that transmit continuously in order to simultaneously and continuously monitor SNR on each branch. With SC the path output from the combiner has an SNR equal to the maximum SNR of all the branches. Moreover, since only one branch output is used, co-phasing of multiple branches is not required, so this technique can be used with either coherent or differential modulation.

2. Threshold Combining: SC for systems that transmit continuously may require a dedicated receiver on each branch to continuously monitor branch SNR. A simpler type of combining, called threshold combining, avoids the need for a dedicated receiver on each branch by scanning each of the branches in sequential order and outputting the first signal with SNR above a given threshold. As in SC, since only one branch output is used at a time, co-phasing is not required. Thus, this technique can be used with either coherent or differential modulation. Once a branch is chosen, as long as the SNR on that branch remains above the desired threshold, the combiner outputs that signal. If the SNR on the selected branch falls below the threshold, the combiner switches to another branch. There are several criteria the combiner can use to decide which branch to switch to. The simplest criterion is to switch randomly to another branch. With only two-branch diversity this is equivalent to switching to the other branch when the SNR on the active branch falls below threshold. This method is also called switch and stay combining (SSC).

3. Maximal Ratio Combining: In SC and SSC, the output of the combiner equals the signal on one of the branches. In maximum ratio combining (MRC) the output is a weighted sum of all branches. In this classic approach, multiple antennas are used at the receiver and maximum ratio combining (MRC) of the received signals is employed for improving the performance [44]. In Figure 2.1 we show the MRC technique involving one transmitter and two receivers. It is assumed that the receiver has perfect channel side information. If the transmitted signal at time t is $x(t)$, the received signal at the receiver j is given by [9]

$$y_j(t) = x(t)\, h_j(t) + n_j(t) \tag{2.2}$$

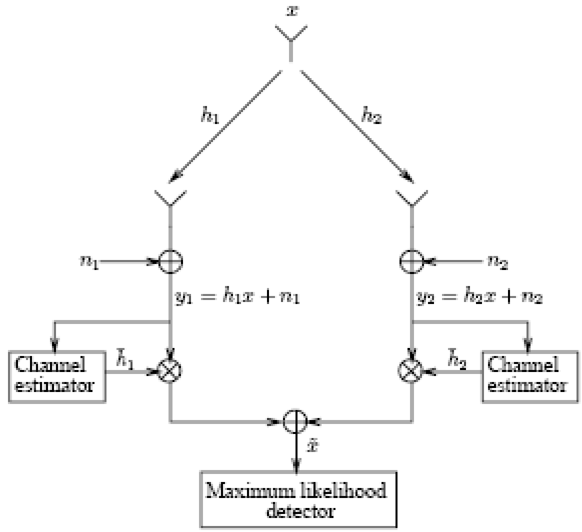

Figure 2.1: Maximum Ratio Combining With 1 Tx and 2 Rx

$$\begin{aligned} h_1 &= \alpha_1 exp\left(j\,\theta_1 \right) \;; \quad h_2 = \alpha_2\, exp\left(j\,\theta_2 \right) \\ y_1 &= h_1\, x + n_1 ; \; y_2 = h_2 x + n_2 \end{aligned} \tag{2.3}$$

$$\begin{aligned} \tilde{x} &= h_1^*\, y_1 + h_2^*\, y_2 = h_1^* \left(h_1 x + n_1 \right) + h_2^* \left(h_2\, x + n_2 \right) \\ &= \left(\alpha_1^2 + \alpha_2^2 \right) x + h_1^* n_1 + h_2^*\, n_2 \end{aligned} \tag{2.4}$$

The maximum likelihood decision rule at the receiver is:

Choose

$$x_j ,\; iff \; \left(\alpha_1^2 + \alpha_2^2 - 1 \right) \left| x_j \right|^2 + d^2 \left(\tilde{x_0}, x_j \right) \leq \left(\alpha_1^2 + \alpha_2^2 - 1 \right) \left| x_k \right|^2 + d^2 \left(\tilde{x_0}, x_k \right) \quad \forall j \neq k \tag{2.5}$$

For equal energy constellation, since $\alpha_1^2 + \alpha_2^2 = 1$ the above decision rule can be simplified to

Choose $x_j ,\; iff \; d^2 \left(\tilde{x_0}, x_j \right) \leq d^2 \left(\tilde{x_0}, x_k \right) \quad \forall j \neq k$ (2.6)

4. Equal-Gain Combining: In equal gain combining (EGC) weights are not dependent on the channel estimates. Signals from each receive antenna are multiplied by the same weight so as to give a lower SNR performance when compared to MRC. Even though the performance for EGC is lower than for MRC, no channel estimation needs to be done in EGC.

2.5 Transmit Diversity

In transmit diversity there are multiple transmit antennas with the transmit power divided among these antennas. Transmit diversity is desirable in systems such as cellular systems where more space, power, and processing capability is available on the transmit side compared to the receive side. Transmit diversity design depends on whether or not the complex channel gain is known at the transmitter or not. When this gain is known, the system is very similar to receiver diversity. However, without this channel knowledge, transmit diversity gain requires a combination of space and time diversity via a novel technique called the Alamouti scheme [44].

We can achieve the same diversity effects with multiple antennas at the transmitter as with multiple antennas at the receiver. Thus, transmit diversity is appealing in systems with multiple information recipients, such as broadcast and cellular schemes. This is because we can increase diversity in all subscriber units by adding just one antenna to the base station, instead of a new antenna to each individual receiving unit.

Theoretically, the most effective technique to mitigate multi-path fading in a wireless channel is transmitter power control [8]. Systems employing transmit diversity fall into three general categories. These are:

- Schemes using feedback,
- Those with feed forward or training information but no feedback, and
- Blind schemes.

The first category uses implicit or explicit feedback of information from the receiver to configure the transmitter [8]. If channel conditions as experienced by the receiver on one side of the link are known at the transmitter on the other side, the transmitter can predistort the signal in order to overcome the effect of the channel at the receiver. But there are two fundamental problems with this approach. The major problem is the required transmitter dynamic range. For the transmitter to overcome a certain level of fading, it must increase its power by that same level, which in most cases is not practical because of radiation power limitations and the size and cost of the amplifier. The second problem is that the transmitter does not have any knowledge of the channel experienced by the receiver except in systems where the uplink (remote to base) and downlink (base to remote) transmissions are carried over the same frequency. Hence the channel information has to be feedback from the receiver to the transmitter [8].

Transmit diversity schemes mentioned in the second category use linear processing at the transmitter to spread the information across the antennas. At the receiver, information is obtained by either linear processing or maximum likelihood decoding techniques. The time interleaving, together with error correction coding, can provide diversity improvement. The same holds for spread spectrum. However, the time interleaving results in large delays when the channel is slowly varying. Equivalently, spread spectrum techniques are larger than the spreading bandwidth or equivalently, where there is relatively small delay spread in the channel [8].

The third category uses multiple transmit antennas combined with channel coding to provide diversity. In most scattering environments, antenna diversity is a practical, effective and commonly used technique for reducing the effect of multi-path fading. The conventional approach is to use multiple antennas and perform combining or selection and switching in order to improve the quality of the received signals at the receiving ends [8].

However, this approach (receiver diversity) causes the problem of size and cost and power of the remote units of the mobile terminals. The use of multiple antennas and radio frequency (RF) chains (or selection and switching circuit) makes the remote units larger and more expensive. As a result, diversity techniques are almost exclusively applied to base station to improve reception quality [9]. Therefore, transmit diversity, which uses multiple antennas at the transmitter, has drawn a lot of interest because of its relative simplicity of implementation and feasibility of having multiple antennas at the base station [20].

An appropriately designed channel code/interleaver pair is also used to provide temporal diversity benefit. Another scheme is to encode information using a channel code and transmit the code symbols using different antennas in an orthogonal manner. But a disadvantage of these schemes over the previous two categories is the loss in bandwidth efficiency due to the use of the channel code. Using appropriate coding, it is possible to relax the orthogonality requirement needed in this scheme and obtain the diversity as well as coding advantage that is offered without sacrificing bandwidth. This is possible when the whole system is viewed as a multiple-input/multiple-output system and suitable code are used [8].

2.6 Space-Time Block Codes

Transmit diversity is applicable when multiple antennas are used at the transmitter and this has become an active area of research in the past few years. Exploiting diversity in such systems does not necessarily require channel knowledge at the transmitter. However, suitable design of the transmitted signal is required to extract diversity. Space-time coding is a powerful transmit diversity technique that relies on coding across space (transmit antennas) and time to extract diversity [21].

The simplest form of space-time blocks code (as shown in Figure 2.2) was introduced by Alamouti [9]. The system has two transmit antennas and one receive antenna. Denoting the signal transmitted from antenna one by x_1 and from antenna two by x_2 in a given symbol period, during the next symbol period signal $-x^*_2$ is transmitted from antenna one and x^*_1 is transmitted from antenna two. Assuming that the fading is constant across two consecutive symbol intervals, the fading coefficients from transmit antenna one to the receive antenna can be designated as h_1 and from antenna two to the receive antenna as h_2

both as complex numbers which remain constant over two consecutive symbol intervals and for quasi-static can be written as [9]

$$h_1(t) = h_1(t+T) = \alpha_1 exp(j\theta_1);$$
$$h_2(t) = h_2(t+T) = \alpha_2 exp(j\theta_2) \tag{2.7}$$

The received signal can then be expressed as

$$y_1(t) = h_1 x_1 + h_2 x_2 + n_1;$$
$$y_2(t+T) = -h_1 x_2^* + h_2 x_1^* + n_2 \tag{2.8}$$

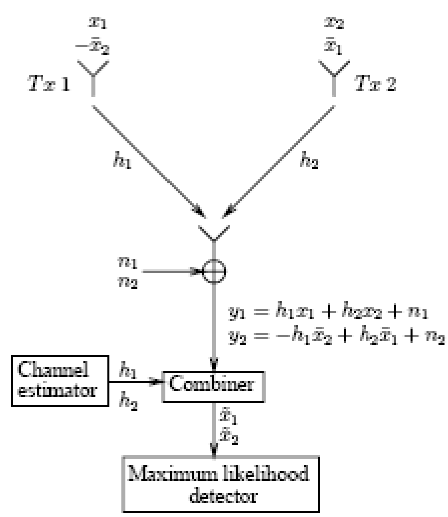

Figure 2.2: Simple twin- transmitter STBC using one receiver

Where y_1 and y_2 are the received signals at time t and $t+T$ respectively, and n_1 and n_2 represent complex Gaussian noise. The signals x_1 and x_2 can be chosen from any complex constellation. After reception of the signal, the combiner scheme builds the following two combined signals

$$\tilde{x}_1 = h_1^* \, y_1 + h_2 \, y_2^* = \left(\alpha_1^2 + \alpha_2^2\right) x_1 + h_1^* n_1 + h_2 \, n_2^*;$$

$$\tilde{x}_2 = h_2^* \, y_1 - h_1 \, y_2^* = \left(\alpha_1^2 + \alpha_2^2\right) x_2 - h_1 n_2^* + h_2^* \, n_1 \tag{2.9}$$

Equation (2.8) can be written in the following matrix form

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

In Alamouti's scheme, the transition matrix is

$$\mathbf{G}_2^c = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}.$$

From Equations (2.9), we can easily see that the signals x_1 and x_2 are separated by simple multiplication and additions. Due to the orthogonality of the STBC $\mathbf{G}_2^c$, the unwanted signal x_2, x_1 is removed from $\tilde{x}_1$, $\tilde{x}_2$, respectively. Then signals $\tilde{x}_1$ and $\tilde{x}_2$ are passed to the maximum likelihood detector for decision rules here are the same with those used earlier in the classic MRC technique.

Choose

$$x_j,\ iff\ \left(\alpha_1^2 + \alpha_2^2 - 1\right) \left|x_j\right|^2 + d^2\left(\tilde{x}_0, x_j\right) \le \left(\alpha_1^2 + \alpha_2^2 - 1\right)\left|x_k\right|^2 + d^2\left(\tilde{x}_0, x_k\right) \quad \forall j \ne k$$

For equal energy constellation, since $\alpha_1^2 + \alpha_2^2 = 1$ the decision rule can be simplified to

Choose

$$x_j,\ iff\ d^2\left(\tilde{x}_0, x_j\right) \le d^2\left(\tilde{x}_0, x_k\right) \quad \forall j \ne k$$

2.7 Alamouti's Scheme Using Multiple Receivers

Alamouti also extended his scheme to the case of two transmit antennas and multiple receive antennas. The encoding and transmission sequence are identical to the case of a single receiver. For i and j representing transmit and receive antenna indices, the scheme can be generalized as the following [46]:

$$y_{j1} = h_{j1} x_1 + h_{j2} x_2 + n_{j1}$$
$$y_{j2} = -h_{j1} x_2^* + h_{j2} x_1^* + n_{j2}$$

Where h_{ji}, $i = 1, 2$; $j = 1, 2, \ldots .n_R$.

$$\tilde{x}_1 = \sum_{j=1}^{nR} \left(h_{j1}^* \, y_{j1} + h_{j2} \, y_{j2}^* \right) = \sum_{j=1}^{nR} \left(\left| h_{j1} \right|^2 + \left| h_{j2} \right|^2 \right) x_1 + h_{j1}^* \, n_{j1} + h_{j2} \, n_{j2}^*$$
$$\tilde{x}_2 = \sum_{j=1}^{nR} \left(h_{j2}^* \, y_{j1} - h_{j1} \, y_{j2}^* \right) = \sum_{j=1}^{nR} \left(\left| h_{j1} \right|^2 + \left| h_{j2} \right|^2 \right) x_2 + h_{j2}^* \, n_{j1} - h_{j1} \, n_{j2}^* \qquad (2.10)$$

Signals $\tilde{x}_1$ and $\tilde{x}_2$ are finally computed and passed to the maximum likelihood detector. From equations (2.10), we can see that the same decision rules as equations (2.9) can be used again to determine the maximum likelihood transmitted symbols.

2.8 Rate of a Space-Time Code

The rate of a space-time code is defined as the number of complex input symbols that it encodes per signaling interval. For K inputs in a block lasting p signaling intervals, we get a rate of $R = K / p$. High rate is desirable, because it indicates that a large fraction of the transmitted symbols carry actual information, not redundancy. Viewed differently, suppose each input symbol to the space-time code is drawn from a QAM (or PSK) constellation of size 2^b. Then, each symbol carries b bits of information. Assuming a pulse shape with zero excess bandwidth, the information rate transmitted by a rate R space-time code with 2^b-QAM input symbols is Rb bits / s / Hz. Thus, for the same input constellation, space-time codes with higher rate transmit at a higher data rate. Conversely, to achieve the same data rate, high rate codes can use a smaller constellation. Another useful view of rate is obtained by considering the effective channel formed by the combination of the space-time code and the MIMO fading channel. The rate is the number of complex inputs multiplexed by the effective channel per signaling interval. Again, high rate implies more multiplexing is desirable.

2.9 Real Orthogonal Designs of STBC

Tarokh *et al.* [10] generalized Alamouti's transmit diversity scheme to an arbitrary number of transmit antennas, and presented more complex space-time block codes akin to Alamouti's. The theory of orthogonal code design was invoked in order to construct STBC having more than two transmit antennas. These codes can achieve maximum-likelihood decoding through linear processing at the receiver, and exhibit maximum diversity. In this section, we will present results of orthogonal designs for real signal constellations (such as PAM), which can provide the maximum possible transmission rate of 1.

2.9.1 Real Orthogonal Designs

A square real orthogonal design of size n_T is an $n_T \times n_T$ orthogonal matrix where each of the columns is orthogonal to all others whose rows are permutations of real numbers $\pm x_1, \ldots, \pm x_{nT}$. Without loss of generality, the first row can be assigned as $(x_1, \ldots, x_{nT})$. For example, real orthogonal matrices for $n_T = 2$ and $n_T = 4$ are given by

$$\mathbf{G}_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix}, \quad \mathbf{G}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix}.$$

The existence of square real orthogonal designs for different values of n_T is known as the Hurwitz-Radon problem in mathematics. It was shown that square real orthogonal matrices exist if and only if $n_T = 2$, 4, or 8 [23].

2.10 Generalized Real Orthogonal Designs

Simple maximum likelihood decoding algorithm can be achieved because of orthogonality of columns of the design matrix, generalized real orthogonal designs are further introduced in [10]. A generalized orthogonal design **G** of size n_T is an p x n_T orthogonal matrix with entries $0, \pm x_1, \ldots, \pm x_k$ such that $\mathbf{G}^T \mathbf{G} = \mathbf{D}$, where **D** is a diagonal matrix with $\mathbf{D}_{i,i}$ of the form $(l_1^i x_1^2 + \ldots + l_k^i x_k^2)$, with the coefficients $l_1^i = \ldots = l_k^i$ all strictly positive integers. The rate of **G** is $R = k/p$. Generalized orthogonal matrix **G** satisfies $\mathbf{G}^T \mathbf{G} = \mathbf{I}\left(x_1^2 + \ldots + x_k^2 \right)$, where the diversity order of generalized orthogonal designs is $n_T n_R$.

Tarokh *et al.* [10] also studied how to construct high-rate linear processing orthogonal designs with low decoding complexity and full diversity order. This means that for a given rate R and transmit antennas n_T, we must attempt to minimize p. For a given R and n_T, define A (R, n_T) to be the minimum number p such that there exists a p x n_T generalized orthogonal design with rate at least R. A generalized orthogonal design attaining the value A (R, n_T) is called delay-optimal. Since the generalized orthogonal designs of full rate are bandwidth efficient, we especially pay attention to the computation of A $(1, n_T)$. It is shown in [10] that the value of $A(1, n_T) = \min(2^{4c+d})$, where the minimization is taken over the set $\{c, d / 0 \le c, 0 \le d < 4 \text{ and } 8c + 2^d \ge n_T \}$.

In particular, $A(1,2) = 2$, $A(1,3) = A(1,4) = 4$, $A(1, n_T) = 8$ for $5 \le n_T \le 8$.
Examples of orthogonal designs of sizes of 3 and 5 are given below [10].

$$\mathbf{G_3} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{bmatrix}, \quad \mathbf{G_5} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 \end{bmatrix}.$$

2.11 Complex Orthogonal Designs

Complex orthogonal designs O_c of size n_T are completely analogous to their real counterparts, except that they now contain entries $\pm x_1,\ldots,\pm x_n$, their conjugates $\pm x_1^*,\ldots,\pm x_n^*$ or multiples. It is shown by simple construction in [10] that a complex orthogonal design of size n_T determines a real orthogonal design of size $2n_T$, and a corollary of this is that complex orthogonal designs only exist for $n_T = 2$ or 4, since their real counterparts only exist for $n_T = 2$, 4 or 8.

A complex linear processing orthogonal design, **G**, is a complex orthogonal design such that $\mathbf{G}^H\mathbf{G} = \mathbf{D}$, where **D** is a diagonal matrix where all diagonal entries are the linear combinations of $|x_1|^2,\ldots,|x_{nT}|^2$ with all strictly positive coefficients. Note that complex linear processing orthogonal designs only exist when $n_T = 2$. Hence, the STBC proposed by Alamouti, which uses the following complex linear orthogonal design where the columns of the matrix representing "space" and the rows representing "time" is given by

$$\mathbf{G}_2^c = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}.$$

2.12 Generalized complex orthogonal designs

From the results obtained for generalized real orthogonal designs, Tarokh *et al.*[10] further proposed generalized complex orthogonal designs. A generalized complex orthogonal design $\mathbf{G}_c$ of size n_T is an $p \times n_T$ orthogonal matrix with entries 0, $\pm x_1$, $\pm x_1^*$,…., $\pm x_k$, $\pm x_k^*$, such that $\mathbf{G}_c^H\mathbf{G}_c = \mathbf{D}_c$, where $\mathbf{D}_c$ is a diagonal matrix with $\mathbf{D}_{i,i}^c$ of the form $\left(l_1^i|x_1|^2 + l_2^i|x_2|^2 + \ldots + l_k^i|x_k|^2\right)$ and the coefficients, $l_1^i,\ l_2^i,\ldots,\ l_k^i$ all strictly positive integers. The rate of $\mathbf{G}_c$ is $R = k/p$. Generalized complex orthogonal matrix $\mathbf{G}_c$ satisfies the condition $\mathbf{G}_c^H\mathbf{G}_c = \left(|x_1|^2 + |x_2|^2 + \ldots + |x_k|^2\right)\mathbf{I}$.

Transmission using a complex generalized orthogonal design is similar to that of a generalized orthogonal design. Maximum-likelihood decoding is also analogous to that of Alamouti's scheme and can be done using linear processing at the receiver.

Tarokh *et al*. [10] also studied how to construct high-rate complex generalized linear processing orthogonal designs with low decoding complexity that achieves full diversity. They showed that a complex orthogonal design of size n_T and full rate exists if and only if $n_T = 2$, which turns out to be the Alamouti's scheme in [9]. For complex constellations, STBCs $\mathbf{G}_c$ can be constructed for any numbers of transmit antennas, which can provide full spatial diversity and half rate.

$$\mathbf{G}_c = \begin{bmatrix} Ln(x_1, x_2, \ldots, x_k) \\ Ln(x_1^*, x_2^*, \ldots, x_k^*) \end{bmatrix}$$

Where, *Ln* () represents corresponding real orthogonal designs.

The half-rate STBC employing three and four transmit antennas are defined as follows.

$$\mathbf{G}_3^c = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}, \quad \mathbf{G}_4^c = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}$$

For 3 and 4 transmit antennas, [10] also proposed rate 3/4 generalized complex linear processing orthogonal designs, which are given by

$$\mathbf{H}_3 = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1 - x_1^* + x_2 - x_2^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2 + x_2^* + x_1 - x_1^*}{2} \end{bmatrix}$$

$$\mathbf{H}_4 = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1 - x_1^* + x_2 - x_2^*}{2} & \frac{-x_2 - x_2^* + x_1 - x_1^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2 + x_2^* + x_1 - x_1^*}{2} & -\frac{x_1 + x_1^* + x_2 - x_2^*}{2} \end{bmatrix}.$$

Chapter 3

Design and Performance of Space-Time Coding

3.1 Introduction

Space –time coding is a powerful technique to improve the error performance of wireless communications systems by using multiple transmit antennas. In the first performance investigation of space-time trellis codes (STTCs) [8], analytical bounds and design criteria were proposed for slow and fast fading channels. It has been pointed out that in slow fading channels, the critical parameters are the rank and determinant of the codeword distance matrix.

In this section, we discuss performance evaluation for space-time codes (STCs) by suggesting analytical bounds. It has been observed that in fading channels, the code construction criterion depends on the value of the possible diversity gain of the system. In slow fading channels, when the diversity gain is small, the rank and determinant criteria are valid for code design. On the other hand, when the diversity gain is reasonably large, the trace of codeword distance matrix, or, equivalently, the minimum squared Euclidean distance, will be the dominant parameter for the code performance.

3.2 System Description

We consider communication systems with n_T transmit and n_R receive antennas and frequency non-selective quasi-static fading channel. The fading coefficient $h_{ji,}$ which forms the channel matrix **H**, is the complex path gain from transmit antenna i to receive antenna j. We assume that the coefficients are independently normally distributed with zero mean and unit variance. Channel matrix **H** is assumed to be known to the receiver, but not to the transmitter.

We also assume that **H** remains constant within a block of p symbols. With these assumptions, we have the following expression for the received vector **Y:**

$$\mathbf{Y} = \sqrt{\beta_t}\,\mathbf{HX} + \mathbf{N} \tag{3.1}$$

Where **N** is an independent zero mean Gaussian noise vector, with variance (N_0) normalized to give out the desired signal to noise ratio and β_t denotes the SNR per transmit antenna. The matrix **X** stands for a $p \times n_T$ block of data

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,nT} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,nT} \\ \vdots & \vdots & \ddots & \vdots \\ x_{p,1} & x_{p,2} & \cdots & x_{p,nT} \end{bmatrix} \tag{3.2}$$

In order to come up with a design criterion, we first need to quantify the effects of mistaking one codeword for each other. In the case of a space-time code, a codeword is a $p \times n_T$ matrix given by Equation (3.2). Let us assume that we transmit a codeword **X** as shown by the above equation. An error occurs if the decoder mistakenly decides that we have transmitted another codeword, for example **E**.

$$\mathbf{E} = \begin{bmatrix} e_{1,1} & e_{1,2} & \cdots & e_{1,nT} \\ e_{2,1} & e_{2,2} & \cdots & e_{2,nT} \\ \vdots & \vdots & \ddots & \vdots \\ e_{p,1} & e_{p,2} & \cdots & e_{p,nT} \end{bmatrix} \tag{3.3}$$

3.3 Upper Bound on Error Probability

We define pairwise error probability (PEP) between any two codeword **X** and **E**, $P(\mathbf{X} \rightarrow \mathbf{E})$ as the probability that the codeword **X** is wrongly decoded as codeword **E** and vice versa, at the receiver. From [12], the pairwise error probability conditioned on the channel realization is given by

$$P\left(\mathbf{X}\rightarrow\mathbf{E}|\mathbf{H}\right) \leq \frac{1}{2}\exp\left(-\frac{d^2(\mathbf{X},\mathbf{E})E_s}{4N_0}\right) \tag{3.4}$$

Where

$$d^2(\mathbf{X},\mathbf{E}) = \sum_{t=1}^{p}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T} h_{j,i}\left(x_{t,i}-e_{t,,i}\right)\right|^2 \tag{3.5}$$

which follows from an approximation to the Q function (Gaussian tail function). The channel coefficients appear in $d^2(\mathbf{X},\mathbf{E})$. In the sequel, the code difference matrix, **X-E** will be denoted by Δ. The average pairwise error probability over a quasi-static fading channel at, high SNR is given by [8].

$$P(\mathbf{X}\rightarrow\mathbf{E}) \leq \left(\prod_{i=1}^{r}\lambda_i\right)^{-n_R}\left(\frac{E_s}{4N_0}\right)^{-rn_R} \tag{3.6}$$

Where λ_i is the i-th non-zero eigenvalue of $\Delta\Delta^H$ and r is the rank of the matrix $\Delta\Delta^H$ where $\Delta\Delta^H$ = **A** (**X**, **E**) = (**X**-**E**) (**X**-**E**)^H. And $\frac{E_s}{N_0}$ is the signal to noise ratio (SNR).

3.3.1 Diversity and Coding Gains

For high SNR diversity advantage is the power of SNR in the expression for the pairwise error probability. Besides diversity, coding gain is another useful parameter used to measure the error performance of a system over fading channels. Originally it was used in AWGN channels as the ratio of the minimum free Euclidean distance of a coded system to the minimum Euclidean distance of an uncoded system. This value asymptotically reflects the SNR reduction of a coded system over an uncoded system for achieving the same amount of error probability. This term is now also applied in coded MIMO systems with fading channels, but may not refer to Euclidean distance.

In [22] coding gain/advantage is defined from the pair-wise error probability as follows. If the PEP is upper bounded as

$$PEP \leq \delta . SNR^{-d}$$

in the region of high SNR, then the system is said to have diversity advantage of d and coding advantage of δ. As can be seen, the coding gain in MIMO systems shifts the upper bound of the PEP up or down and is the approximate measure of the gain over an uncoded system operating with the same diversity order. In a space-time coded system with diversity advantage of $d = n_T n_R$, the coding gain refers to the minimum determinant of the Hermitian square of codeword error matrix [22].

Diversity advantage causes the slope of PEP versus SNR graph to change in the following way: the larger the diversity, the more negative the slope. Coding advantage shifts the graph horizontally: the greater the coding advantage, the larger is the left shift [23].

3.3.2 STC Design Criteria

A. Design Criteria for Slow Rayleigh Fading Channels

As the error performance upper bounds (3.6) indicate, the design criteria for slow Rayleigh fading channels will depend on the value of rn_R. The maximum possible value of rn_R is $n_T n_R$. For small values of $n_T n_R$, corresponding to a small number of independent subchannels, the error probability is dominated by the minimum rank of matrix **A**(**X**, **E**) over all possible codeword pairs. The product of the minimum rank and the number of receive antennas, rn_R, is called the minimum diversity. In addition, in order to minimize the error probability, the minimum product of the nonzero eigenvalues of matrix **A**(**X**, **E**) along the pairs of codewords with the minimum rank should be maximized. Therefore, if the value of $n_T n_R$ is small, the STC design criteria for slow Rayleigh fading channels can be summarized as follows [8].

Design Criteria Set 1:

a) To obtain maximal diversity, we need to maximize the minimum rank r of matrix **A**(**X**, **E**) over all pairs of distinct codewords. A diversity advantage of rn_R is achieved.

b) Let rn_R be the target diversity advantage. Then the design goal is to maximize the minimum determinant $\prod_{i=1}^{r} \lambda_i$ of matrix **A** (**X**, **E**) along the pairs of distinct codewords with the minimum rank. This set of design criteria is referred to as the rank and determinant criteria.

For large values of $n_T n_R$ corresponding to large number of independent subchannels, the PEP is upper bounded as given in [46] is,

$$P\left(\mathbf{X} \rightarrow \mathbf{E}\right) \leq \frac{1}{4}\exp\left(-n_R \frac{E_s}{4N_0}\sum_{i=1}^{r}\lambda_i\right). \tag{3.7}$$

From (3.7), it can be seen that in order to minimize the error probability, the minimum of the sum of all eigenvalues of matrix **A**(**X**, **E**) among all the pairs of distinct codeword should be maximized. For a square matrix, the sum of all the eigenvalues is equal to the sum of all the elements on the main diagonal, or the trace of matrix **A**(**X**, **E**). It can be expressed as [46]

$$\mathrm{tr}\left(\mathbf{A}\left(\mathbf{X},\mathbf{E}\right)\right) = \sum_{i=1}^{r}\lambda_i = \sum_{i=1}^{n_T}\mathbf{A}_{i,i} \tag{3.8}$$

Where $\mathbf{A}_{i,\,i}$ are the elements on the main diagonal of matrix **A** (**X**, **E**). The trace of A(**X**, **E**) can be expressed as [46]

$$\mathrm{tr}\left(\mathbf{A}\left(\mathbf{X},\mathbf{E}\right)\right) = \sum_{i=1}^{n_T}\sum_{t=1}^{p}\left|\left(x_{t,i}-e_{t,,i}\right)\right|^2. \tag{3.9}$$

Equation (3.9) indicates that the trace of matrix A(**X**, **E**) is equivalent to the squared Euclidean distance between the pair of codewords **X** and **E**. In other words, the PEP is minimized if the Euclidean distance is maximized. This design criterion is referred to as the trace criterion. It should be pointed out that (3.7) is valid for a large number of independent subchannels under the condition that the minimum value of rn_R is high. In this case, the STC design criteria for slow fading channels can be summarized as follows.

Design Criteria Set 2:

a) Make sure that the minimum rank r of matrix $\mathbf{A}(\mathbf{X}, \mathbf{E})$ over all pairs of distinct codewords is large enough, such that $rn_R \geq 4$.

b) Maximize the minimum trace $\sum_{i=1}^{r} \lambda_i$ of matrix $\mathbf{A}$ $(\mathbf{X}, \mathbf{E})$ among all pairs of distinct codewords. It is important to note that this design is consistent with the trellis-code design in fading channels with a large number of diversity branches. A large number of diversity branches reduces the effect of fading and consequently, the channel approaches an AWGN model [25]. Therefore, the trellis-code design criteria for AWGN channels apply to fading channels with a large number of diversity. In a similar way, in STC design, when the number of independent subchannels rn_R is large, the channel converges to an AWGN channel. Thus, the code design is the same as that for AWGN channels.

The boundary value rn_R of between the two design criteria sets was chosen to be four. This boundary is determined by the required number of random variables rn_R to satisfy the central limit theorem. In general, for random variables with smooth pdfs, the central limit theorem can be applied if the number of random variables in the sum is larger than four [45]. In the application of the central limit theorem, the choice of four as the boundary has been further justified by the code design, the best codes based on the trace criterion outperform the best codes based on the rank and determinant criteria [26].

Based on the previous discussion, we can conclude that code design in fading channels depends on the possible diversity gain of the STC system. For codes on slow fading channels, the total diversity is the product of the receive diversity, n_R, and the transmit diversity provided by the coding scheme, r.

STC satisfying the criteria for both slow and fast fading channels were called ‘Smart and greedy’ in [8] .The code is called smart and greedy because the encoder dose not know the channel but can exploit the benefits provided both by the transmit and receive antennas as well as by possible rapid changes in the channel.

Chapter 4

Spatially Correlated Channel

4.1 Introduction

Since the introduction of Space-Time Codes, the study of efficient coding schemes for multiple antenna systems has been an active area of research. Previous work in this area has been focused mainly on the idealistic assumption of fading channel with independent identically distributed (i.i.d) gains. In practice, however, insufficient antenna spacing and lack of scattering environment (non-isotropic distribution) cause spatial correlations among antennas. To establish a reliable and spectrally efficient communication system, codes should be analyzed and designed with consideration of spatial correlation of the channel [31]. This problem has been addressed for Rayleigh fading channel in
[12,27, 28]. Spatial correlation changes the distributions of the eigenvalues of the channel realization matrix [29] and reduces channel capacity. It also causes performance degradation by decreasing the diversity order and the coding gain [13].

One can classify four different types of fading correlations: uncorrelated, semi correlated, semi correlated type-2 and fully correlated fading [30].
Table 4.1, is a summery of the types of correlation.

Case	Receiver Side	Transmitter Side	Type of Fading
1	Uncorrelated	Uncorrelated	Uncorrelated
2	Uncorrelated	Correlated	Semi-correlated
3	Correlated	Uncorrelated	Semi-correlated type-2
4	Correlated	Correlated	Fully-correlated

Table- 4.1 summery of the types of correlation.

Gesbert *et al.* [31] have proposed a model for describing spatial correlation in the channel. They show that correlation can be split into two factors, receive and transmit correlation. Based on this model, Bolcskei *et al.* [13] studied the effect of spatial fading correlation on the performance of space-time codes. They show how diversity order of any space-time code, is affected by those two factors in correlated channels. The effect of receive correlation was shown to be multiplicative and independent of the code. On the other hand, the effect of transmit correlation has been shown to be completely dependent on the code structure.

A popular channel model that has been used in MIMO performance analysis is the "Kronecker" model. In this model, the correlation properties of the MIMO channel are modeled at the transmitter and receiver separately, neglecting the statistical interdependency between scattering distributions at the transmitter and receiver antenna apertures [32].

Bolcskei *et.al.* [5] studied the impact of spatial fading correlation on the performance of space-time codes by quantifying the loss of diversity and coding gain. The results in [5], derived, based on the Chernoff bound, on the pairwise error probability (PEP), are therefore relevant for the case of asymptotically high signal-to-noise ratio (SNR). In this section, we derive the exact PEP for space–time coding over quasi-static Rayleigh fading channels in the presence of spatial correlation. In the presence of receive correlation; the PEP degrades for all SNRs. Receive correlation has a negligible effect on the performance of space–time coding at low SNR. Furthermore, by employing the notion of "majorization" [6], we provide a complete study of how receive correlation degrades the performance of space–time codes. Results in [6, 40] show that for all levels of SNR, the more "spread out" the receive correlation matrix, the worse the performance in terms of PEP.

4.2 System Model

Consider a MIMO system consisting of n_T transmit antenna and n_R receive antennas. Assuming frequency non-selective fading channel where the fading coefficient from the i-th transmit antenna to the j-th receive antenna is denoted by $h_{j,i}$ as in Figure 4.1., the space-time encoder modulates the binary data and maps the symbols to the transmit antenna, the signal received by antenna j is given by

$$y_j = \sqrt{\beta_t} \sum_{i=1}^{n_T} h_{j,i} x_i + n_j \qquad (4.1)$$

Where the noise n_j is modeled as independent samples of a zero- mean complex Gaussian random variables with variance 0.5 per dimension, x_i is the signal transmitted from antenna i normalized such that $E[|x_i^2|] = 1$ and $\beta t = \beta / n_T$ denotes the SNR per transmit antenna, where β is SNR per receive antenna, regardless of the number of transmit antenna. The total received SNR is given by $n_R\ \beta$. For quasi-static Rayleigh fading channels, we assume the channel remains constant over N symbol periods and changes in an independent fashion to a new realization from block to block. The following matrix form of the input-output relation of the MIMO channel model is obtained as

$$\mathbf{Y} = \sqrt{\beta_t}\,\mathbf{HX} + \mathbf{N} \qquad (4.2)$$

Where $\mathbf{Y} = \left[y_1 ,\ldots, y_{nR} \right]^{\mathrm{T}}$ is the received signal vector, $\mathbf{X} = \left[x_1 ,\ldots, x_{nT} \right]^{\mathrm{T}}$ is the transmitted signal vector, $\mathbf{N} = \left[n_1 ,\ldots, n_{nR} \right]^{\mathrm{T}}$ is the i.i.d. AWGN noise vector at the receiver with covariance matrix given by $\mathbf{R}_{\mathrm{N}} = E\left[\mathbf{NN}^{\mathrm{H}}\right] = \mathbf{I}_{nR}$ and

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1nT} \\ h_{21} & h_{22} & \cdots & h_{2nT} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nR1} & h_{nR2} & \cdots & h_{nRnT} \end{bmatrix}$$

is the MIMO channel matrix.

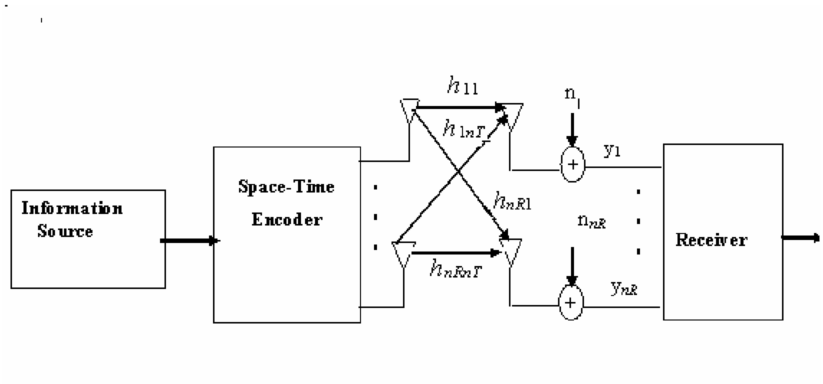

Figure 4.1 Block diagram of the system.

We restrict ourselves to a Rayleigh fading scenario, therefore, the $n_R \times n_T$ elements of **H** are composed of circularly symmetric zero mean complex Gaussian random variables (possibly correlated). We assume that the spatial fading correlation occurs at the transmitter. The spatial fading correlation depends on the physical geometries of the antenna and channel characteristics. We assume the Channel state information (CSI) i.e., **H**, is known at the receiver, but not at the transmitter.

With two transmit antennas (n_T = 2) and two receive antennas (n_R = 2), equation (4.2) becomes;

$$\begin{bmatrix} y_1(t) \\ y_1^*(t) \\ y_2(t) \\ y_2^*(t+T) \end{bmatrix} = \sqrt{\frac{\beta}{2}} \begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \\ h_{12} & h_{22} \\ h_{22}^* & -h_{12}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_1^* \\ n_2 \\ n_2^* \end{bmatrix}. \tag{4.3}$$

4.3 Spatially Correlated model

When designing space-time codes, the main assumption being made is that the channel gains between the transmitter and the receiver antennas undergo independent fading, i.e., the spatial fading is uncorrelated. However, in reality, the individual antennas could be correlated due to lack of scattering and insufficient antenna spacing in either the transmitter or the receiver.

In such conditions the fading coefficients $h_{j,i}$ will be correlated. Assuming a general structure for the correlation of fading coefficients, i.e., the correlation structure is stationary (time-invariant) the most general form of such a correlation model can be constructed using a linear, vectorizing operator Vec (.) We employ the spatial fading correlation model as in [33, 41], namely

$$\mathbf{H} = \mathbf{R}_{\mathrm{Rx}}^{1/2} \, \mathbf{H}_{\omega} \, \mathbf{R}_{\mathrm{Tx}}^{1/2} \tag{4.4}$$

Where $\mathbf{H}_{\omega}$ is an $n_R \times n_T$ matrix composed of i.i.d complex Gaussian entries with zero mean and unit variance, and $\mathbf{R}_{\mathbf{Tx}}$ and $\mathbf{R}_{\mathbf{Rx}}$ are the transmit and receive correlation matrices, and their sizes are $n_T \times n_T$ and $n_R \times n_R$ respectively.

In the case $\mathbf{R}_{\mathbf{Tx}} = \mathbf{I}_{nT}$ and $\mathbf{R}_{\mathbf{Rx}} = \mathbf{I}_{nR}$, the model simplifies to the ideally uncorrelated MIMO channel. If $\mathbf{R}_{\mathbf{Rx}} = \mathbf{I}_{nR}$ and $\mathbf{R}_{\mathbf{Tx}} \neq \mathbf{I}_{nT}$, the model represents transmit correlation only; that is the receive antennas are assumed to be placed in a rich scattering environment, whereas there are not many scatterers around the transmit antennas. This situation is typical in the downlink channel in mobile communication systems. On the other hand the case $\mathbf{R}_{\mathbf{Rx}} \neq \mathbf{I}_{nR}$ and $\mathbf{R}_{\mathbf{Tx}} = \mathbf{I}_{nT}$ represents the receive- correlation only scenario. The joint transmit-receive correlation represents the scenario when both the transmit and receive correlation exist; that is when $\mathbf{R}_{\mathbf{Tx}} \neq \mathbf{I}_{nT}$ and $R_{Rx} \neq \mathbf{I}_{nR}$.

4.3.1 Channel Correlation Coefficients

The complex channel correlation coefficients between receiver antennas $j_1$and j_2 can be defined as

$$\rho_{j_1 j_2}^{\mathrm{Rx}} = E\left[h_{j_1 i} h_{j_2 i}^{*} \right] \quad \text{for all transmitting antennas } i. \tag{4.5}$$

Note that, this definition is subjected to the assumption made below.

- All antenna elements in the receiver and the transmitter antenna arrays have the same polarization and the same radiation pattern identical to each other.
- Correlation between two antenna elements in one antenna array is independent of the antenna element selected from the other antenna array. Using (4.5), the receiver channel correlation matrix can be defined as

$$\mathbf{R}_{\mathbf{Rx}} \triangleq \begin{bmatrix} \rho_{11}^{\mathrm{Rx}} & \cdots & \rho_{1nR}^{\mathrm{Rx}} \\ \vdots & \ddots & \vdots \\ \rho_{nR1}^{\mathrm{Rx}} & \cdots & \rho_{nRnR}^{\mathrm{Rx}} \end{bmatrix} \tag{4.6}$$

A similar definition can be given for the correlation matrix at the transmitter in terms of complex channel correlation coefficients observed at the transmitter, say $\rho_{i_1 i_2}^{\mathrm{Tx}}$. For which,

$$\mathbf{R}_{\mathbf{Tx}} \triangleq \begin{bmatrix} \rho_{11}^{\mathrm{Tx}} & \cdots & \rho_{1nT}^{\mathrm{Tx}} \\ \vdots & \ddots & \vdots \\ \rho_{nT1}^{\mathrm{Tx}} & \cdots & \rho_{nTnT}^{\mathrm{Tx}} \end{bmatrix} \tag{4.7}$$

The correlation matrices $\mathbf{R}_{\mathbf{Tx}}$ and $\mathbf{R}_{\mathbf{Rx}}$ are Hermitian matrices. The correlation coefficient between two arbitrary channel gains connecting two input - output points of antennas is given by

$$\rho_{i_2 j_2}^{i_1 j_1} = E\left[h_{j_1 i_1} h_{j_2 i_2}^{*} \right] = \rho_{i_1 i_2}^{\mathrm{Tx}} \rho_{j_1 j_2}^{\mathrm{Rx}} \tag{4.8}$$

Provided that $\rho_{j_1 j_2}^{\mathrm{Rx}}$ is independent of the transmitting antennas and $\rho_{i_1 i_2}^{\mathrm{Tx}}$ is independent of receiving antennas. This gives that the correlation matrix of the MIMO channel **H** as the krocnecker product of the correlation matrices observed at the transmitter and the receiver, i.e,

$$\mathbf{R} = \mathbf{R}_{\mathbf{Tx}} \otimes \mathbf{R}_{\mathbf{Rx}} . \tag{4.9}$$

The kronecker product model is a special case where correlations among all transmitter – receiver antenna branches are taken into consideration when defining the correlation matrix of the MIMO channel [7].

Namara *et.al*. [34] proposed that the introduction of spatial correlation is made easier if it can be assumed that correlations amongst the elements of any array are independent of the selected antenna element at the other array. This assumption is justified by the reasoning that each element within an array will illuminate the same scatterers in the surrounding environment. The energy arriving at the second array from each of transmit elements will therefore exhibit the same power-angle spectrum and hence the same spatial correlation.

4.3.2 Correlation Channel gains for a 2 × 2MIMO System

We investigate the performance of OSTBC for two transmitter antennas and two receiver antennas. The restriction to two transmitter antennas and two receiver antennas allows us to use the complex – orthogonal block code proposed by Alamouti. Consider a 2 × 2 MIMO system with correlated channel gain matrix **H**. Normalized spatial correlation matrices observed at the transmitter and receiver antenna elements are,

$$\mathbf{R}_{\mathbf{Rx}} = \begin{bmatrix} 1 & \rho \\ \rho^* & 1 \end{bmatrix}, \qquad \mathbf{R}_{\mathbf{Tx}} = \begin{bmatrix} 1 & \vartheta \\ \vartheta^* & 1 \end{bmatrix}$$

Where $\rho = E\left[\, h_{11} h_{21}^* \,\right] = E\left[\, h_{12}\; h_{22}^* \,\right]$ and $\vartheta = E\left[\, h_{11} h_{12}^* \,\right] = E\left[\, h_{21}\; h_{22}^* \,\right]$

Then the correlation matrix **R** of the MIMO channel **H** is found from $\mathbf{R} = \mathbf{R}_{\mathbf{Tx}} \otimes \mathbf{R}_{\mathbf{Rx}}$.
Let $\mathbf{H}_{\omega}$ be 2 × 2 uncorrelated MIMO channel matrix and
h = Vec ($\mathbf{H}_{\omega}$) = [h_{11}, h_{12}, h_{21}, h_{22}], where elements in $\mathbf{H}_{\omega}$ are i.i.d complex zero –mean Gaussian variable with unit variance. Using the method described in [7] we derive the correlated channel matrix **H** as

$$\mathbf{H} = \begin{bmatrix} h_{11} & \rho^* h_{11} + \kappa h_{12} \\ \vartheta^* h_{11} + \xi h_{21} & \vartheta^* \rho^* h_{11} + \vartheta^* \kappa h_{12} + \rho^* \xi h_{21} + \kappa \xi h_{22} \end{bmatrix},$$

Where $\kappa = \sqrt{1 - |\rho|^2}$ and $\xi = \sqrt{1 - |\vartheta|^2}$.

Note that for spatially uncorrelated channels $\mathbf{R}_{\mathbf{Rx}} = \mathbf{R}_{\mathbf{Tx}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

4.4 Exact PEP for Correlated Rayleigh Fading Channel

For maximum-likelihood (ML) detection, the PEP that the receiver decides erroneously in favor of **E** when codeword **X** is transmitted, given the channel realization as given in [8].

$$P(\mathbf{X} \to \mathbf{E}|\gamma) = Q\left(\sqrt{\gamma}\,\right) \tag{4.10}$$

where $\gamma = \frac{d^2(\mathbf{X}, \mathbf{E}) E_s}{2N_0}$, $d(\mathbf{X}, \mathbf{E})$ is the Euclidean distance between code words **X** and **E**, and E_s is the transmitted power at each transmit antenna.

Traditionally, the PEP is given by Chernoff bound on the Q-function. However, the exact simple alternative expression for Q-function due to Crang [35] formula for Gaussian function is,

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta \tag{4.11}$$

Using this expression, and averaging over all channel realizations, one can write the PEP as,

$$P(\mathbf{X} \to \mathbf{E}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp\left(-\frac{\gamma}{2\sin^2\theta}\right) p(\gamma)\, d\gamma\, d\theta$$

We recognize the inner integral as the moment generating function (MGF) of γ,

Φ (s) = E [$e^{-s\gamma}$], evaluated at s = -1/sin $^2\theta$. Hence the PEP is given by

$$P(\mathbf{X} \to \mathbf{E}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Phi_\gamma\left(-\frac{1}{\sin^2\theta}\right) d\theta \tag{4.12}$$

Using techniques made popular by the work of Crang, we first calculate the conditional PEP and then integrate over channel gain coefficients. An error occurs when the noise random variable **N** takes values larger than $\| \mathbf{H}(\mathbf{X}-\mathbf{E}) \|$. Denoting the code word differences in the n-th time interval by $\mathbf{\Delta}_n = \mathbf{X}_n - \mathbf{E}_n$, the PEP for a given channel realization is

$$P(\mathbf{X} \to \mathbf{E}|\mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_0}\sum_{n=1}^{N} \| \mathbf{H}_n \mathbf{\Delta}_n \|^2}\right). \tag{4.13}$$

Using the integral expression for Q-function and subsequently integrating over the randomness of the channel coefficients to obtain the unconditional PEP.

$$P(\mathbf{X} \to \mathbf{E}|\mathbf{H}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{E_s}{4N_0\sin^2\theta}\sum_{n=1}^{N} \| \mathbf{H}_n \mathbf{\Delta}_n \|^2\right) d\theta \tag{4.14}$$

$$P(\mathbf{X} \to \mathbf{E}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Phi\left(-\frac{E_s}{4N_0\sin^2\theta}\right) d\theta , \tag{4.15}$$

Where Φ (.) is the moment generating function of the random variable $\sum_{n=1}^{N} \| \mathbf{H}_n \mathbf{\Delta}_n \|^2$.

Whenever channel coefficients are spatially independent, the over all moment generating function can be decomposed into a product of marginal MGF's, simplifying the calculation of the above integral. In the presence of spatial correlation, however, this product decomposition dose not exists and thus complications arise. This work is mainly devoted to providing exact PEP expression under spatially correlated channel conditions. We consider spatially correlated antennas. We use the spatially correlated channel model. For easy reference, we state the result once again. **H** is $n_R \times n_T$ channel matrix. Transmit side correlation is $\mathbf{R}_{\mathbf{Tx}}$ and receive side correlation is $\mathbf{R}_{\mathbf{Rx}}$. Denoting an innovations channel with i.i.d. Components as $\mathbf{H}_\omega$, the correlated channel can be represented as,

$$\mathbf{H} = \mathbf{R}_{\mathrm{Rx}}^{1/2}\ \mathbf{H}_\omega\ \mathbf{R}_{\mathrm{Tx}}^{1/2}$$

4.4.1 Quasi-static Fading with Spatial Correlation

In quasi-static fading channel the matrix channel is assumed to be constant over the duration of a codeword, hence $\mathbf{H}_n = \mathbf{H}$ for $n = 1,\ldots, N$. Denote the difference of two codeword matrices with $\Delta = [\Delta_1,\ldots,\Delta_N]$. Then, the argument of the MGF in Equation (4.15) can be described in terms of $\mathbf{R}_{\mathbf{Rx}}$ and $\mathbf{R}_{\mathbf{Tx}}$.

$$\sum_{n=1}^{N} \| \mathbf{H}_n \mathbf{\Delta}_n \|^2 = \| \mathbf{H\Delta} \|^2 = tr\left(\mathbf{H\Delta\Delta}^H \mathbf{H}^H\right)$$

$$= vec\left(\mathbf{H}^H\right)^H \left(\mathbf{I}_{nR} \otimes \mathbf{\Delta\Delta}^H\right) vec\left(\mathbf{H}^{\mathrm{H}}\right) \tag{4.16}$$

$$= vec\left(\mathbf{H}_\omega{}^H\right)^H \mathbf{R}_s^{1/2}\left(\mathbf{I}_{nR} \otimes \mathbf{\Delta\Delta}^H\right)\mathbf{R}_s^{H/2}\, vec\left(\mathbf{H}_\omega{}^H\right).$$

Where $\mathbf{R}_s$ is the covariance matrix of vec ($\mathbf{H}^{\mathrm{H}}$), which leads to $\mathbf{R}_s = \mathbf{R}_{\mathbf{Rx}} \otimes \mathbf{R}_{\mathbf{Tx}}$. We wish to calculate the MGF of the above random variable, which consists of a positive semi-definite quadratic form involving Gaussian vectors vec ($\mathbf{H}_\omega{}^{\mathrm{H}}$). Using the result in [37]

$$\Phi(s) = \left| \mathbf{I}_{nRnT} - s\,\mathbf{R}_s^{1/2}\left(\mathbf{I}_{nR} \otimes \mathbf{\Delta\Delta}^{\mathrm{H}}\right)\mathbf{R}_s^{\mathrm{H}/2} \right|^{-1},$$

$$= \left| \mathbf{I}_{nRnT} - s\left(\mathbf{I}_{nR} \otimes \mathbf{\Delta\Delta}^{\mathrm{H}}\right)\mathbf{R}_s \right|^{-1},$$

$$= \left| \mathbf{I}_{nRnT} - s\left(\mathbf{I}_{nR} \otimes \mathbf{\Delta\Delta}^{\mathrm{H}}\right)\left(\mathbf{R}_{\mathrm{Rx}} \otimes \mathbf{R}_{\mathrm{Tx}}\right) \right|^{-1}, \tag{4.17}$$

$$= \left| \mathbf{I}_{nRnT} - s\mathbf{R}_{\mathrm{Rx}} \otimes \left(\mathbf{\Delta\Delta}^{\mathrm{H}}\mathbf{R}_{\mathrm{Tx}}\right) \right|^{-1},$$

$$= \prod_{i=1}^{nT}\prod_{j=1}^{nR}\left(1 - s\,\lambda_j^{(r)}\mu_j\right)^{-1} \tag{4.18}$$

Where $\lambda_j^{(r)}$ are eigenvalues of $\mathbf{R}_{\mathbf{Rx}}$ and μ_i are the eigenvalues of $\mathbf{\Delta\Delta}^{\mathrm{H}}\mathbf{R}_{\mathrm{Tx}}$ therefore the PEP in quasi-static case is

$$P(\mathbf{X} \rightarrow \mathbf{E}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left| \mathbf{I}_{nRnT} + \frac{E_s}{4N_0 \sin^2\theta} \mathbf{R}_{\mathbf{Rx}} \otimes \left(\mathbf{\Delta\Delta}^{\mathrm{H}} \mathbf{R}_{\mathbf{Tx}} \right) \right|^{-1} d\theta \tag{4.19}$$

$$\leq \frac{1}{2}\left| \mathbf{I}_{nRnT} + \frac{E_s}{4N_0} \mathbf{R}_{\mathbf{Rx}} \otimes \left(\mathbf{\Delta\Delta}^{\mathrm{H}} \mathbf{R}_{\mathbf{Tx}} \right) \right|^{-1}, \tag{4.20}$$

Where in Equation (4.20) we have the Chernoff bound for the PEP.

Other attempts have been made to understand and quantify the performance of spatially correlated MIMO channels in quasi-static fading. We briefly mention them and their relationship to our results. For the special case of i.i.d. Channel substitute $\mathbf{R}_{\mathbf{Tx}} = \mathbf{I}_{nT}$ and $\mathbf{R}_{\mathbf{Rx}} = \mathbf{I}_{nR}$ in (4.19) to obtain an expression whose integral kernel is $\left| \mathbf{I}_{nT} - s\mathbf{\Delta\Delta}^{\mathrm{H}} \right|^{-nR}$. This special case has been reported in [38]. Bolcskei and Paulraj [5] offer a different but equivalent expression for the same Chernoff bound. Liu and Sayeed [39] have reported the Chernoff bound corresponding to Equation (4.18).

It is insightful to approximate the Chernoff bound for the high-SNR region as in [5]. Substituting (4.18) in (4.15) gives an alternative Chernoff bound which in high SNR takes the form

$$P(\mathbf{X} \rightarrow \mathbf{E}) \leq \frac{1}{2}\left(\frac{E_s}{4N_0} \right)^{-r\alpha} \prod_{i=1}^{r} \prod_{j=1}^{\alpha} \frac{1}{\lambda_j^{(r)} \mu_i} \tag{4.21}$$

Where r = rank ($\Delta\Delta^{\mathrm{H}}\mathbf{R}_{\mathbf{Tx}}$) and α = rank ($\mathbf{R}_{\mathbf{Rx}}$). Whenever there is no transmit correlation, i.e. $\mathbf{R}_{\mathbf{Tx}} = \mathbf{I}_{nT}$ and $\mu_i = 1$.

In the high-SNR regime, the quality of a code is usually analyzed via the diversity order and the coding gain. The diversity order of a pair of code words is the exponent of SNR, i.e. d = $r\alpha$ = rank ($\Delta\Delta^{\mathrm{H}}\mathbf{R}_{\mathbf{Tx}}$). rank ($\mathbf{R}_{\mathbf{Rx}}$) in Equation (4.21). Thus, the rank of receiver correlation appears directly, while the transmit correlation appears via
$r \leq \min\{\mathrm{rank}(\mathbf{R}_{\mathbf{Tx}}),\mathrm{rank}(\Delta\Delta^{\mathrm{H}})\}$.

Remark: Equation (4.19) is exact PEP of space-time coded system applied to a spatially correlated slow fading MIMO channel following the channel model as proposed in section (4.3).

4.5 Antenna Correlations

Antenna Correlations in many applications arise as the transmit and/or the receive antennas can be correlated. For example, in cellular systems, the base-station antennas are typically unobstructed and have no local scatterers. This induces correlation across the base-station antennas, as a result of which the MIMO channel matrix entries do not fade independently. Antenna correlations tell us about the available spatial diversity in a MIMO channel. If the antennas are highly correlated, very little spatial diversity gain can be extracted from the channel.

For quasi-static Rayleigh fading channel, one can show the following proposition from equation as given in [6].

$$P\left(\mathbf{X} \rightarrow \mathbf{E}\right) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \prod_{i=1}^{nT} \prod_{j=1}^{nR} \left(1 + \frac{\beta_t}{4\sin^2\theta} \lambda_j^{(r)} \mu_i \right)^{-1} d\theta \ , \tag{4.22}$$

Where $\lambda_j^{(r)}$ are the eigenvalues of $\mathbf{R}_{\mathbf{Rx}}$ and μ_i are the eigenvalues of $\mathbf{\Delta\Delta}^{\mathrm{H}}\mathbf{R}_{\mathrm{Tx}}$, and $\Delta = (\mathbf{X}\text{-}\mathbf{E})$.

The above integral form for PEP can be evaluated in a closed form using the techniques of direct partial fraction expansion or partial fraction expansion via eigenvalue decomposition as in [36].

4.5.1 Effects of Receive Correlation

Let us assume that the transmitter is surrounded by many scatterers while the receiver is placed high enough such that there are not many scatterers in its vicinity. Assume also that the antenna at the transmitter spacing is sufficiently large so that fading associated with each antenna is almost independent. In this case the correlations exist only among the sub channels from certain transmit antenna to all receive antennas.

In this subsection, we discuss the effect of receive correlation on the performance of space–time codes in terms of PEP. For simplicity, we focus on the case of receive Correlation only, i.e. $\mathbf{R_{TX}} = \mathbf{I}_{nT}$. However, the results also apply to the case where transmit correlation is included. In this case $\mu_i = \mathbf{\Delta\Delta}^{\mathrm{H}}$. At low SNR i.e., $\beta_t \leq 1$, as in [6] we have

$$P(\mathbf{X} \to \mathbf{E}) \approx \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\beta_t}{4\sin^2\theta} \sum_{j=1}^{nR} \lambda_j^{(r)} \sum_{i=1}^{nT} \mu_i \right)^{-1} d\theta \ ,$$

$$= \frac{1}{2}\left\{1 - \sqrt{\frac{\beta_t nR \left\|\mathbf{X}-\mathbf{E}\right\|^2}{\beta_t nR \left\|\mathbf{X}-\mathbf{E}\right\|^2 + 4}}\right\} \tag{4.23}$$

Therefore, from the above equation at low SNR, receive correlation has negligible effect on PEP performance.

The correlation matrix $(\mathbf{R_{RX}})_1$ is said to be less spread out than another correlation matrix $(\mathbf{R_{RX}})_2$, denoted by, $(\mathbf{R_{RX}})_1 \preceq (\mathbf{R_{RX}})_2$ if the vector of eigenvalues of $(\mathbf{R_{RX}})_1$ majorizes the vector of eigenvalues of $(\mathbf{R_{RX}})_2$ [6].

The notion of spread out can be taken as a measure of the strength of correlation; the more spread out, the stronger the correlation. For MIMO quasi-static Rayleigh fading channels with receive correlation,

i f $(\mathbf{R_{RX}})_1 \preceq (\mathbf{R_{RX}})_2$, then $P_{(\mathbf{R_{RX}})_1}(\mathbf{X} \to \mathbf{E}) \leq P_{(\mathbf{R_{RX}})_2}(\mathbf{X} \to \mathbf{E})$.

The more spread out $\mathbf{R_{RX}}$ is the stronger the receive correlation and the worse is the performance in terms of PEP.

Since i.i.d. fading corresponds to the identity correlation matrix, which is the least spread out among all the correlation matrices, we obtain

$$P(\mathbf{X} \to \mathbf{E}) \geq \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{nT} \left(1 + \frac{\beta_t}{4\sin^2\theta} \mu_i \right)^{-nR} d\theta \ ,$$

$$= P_{i.i.d.} \ (\mathbf{X} \to \mathbf{E}) \ . \tag{4.24}$$

In the above equation the right hand side of the expression represents the PEP of space-time codes over spatially uncorrelated (i.i.d.) Rayleigh fading channels.

Comparing the right hand and left sides of the inequality, one can conclude that for all SNRs, the presence of receive correlation leads to higher PEP. By employing the notion of "majorization," the above results provide us with a complete analysis of how receive correlation degrades the performance of space-time codes.

4.5.2 Effects of Transmit Correlation

Exchanging the roles of the receiver and the transmitter, that is, the receiver is in richly scattering environment while the transmitter is not. In such a set-up, each antenna at the receiver observes correlated fading gains from the transmitter antennas. In the following, for simplicity, we assume there is no receive correlation, i.e., $\mathbf{R_{RX}} = \mathbf{I}_{nR}$. As we have shown in equation (4.22), the PEP performance depends on the eigenvalues of the matrix $\mathbf{\Delta\Delta}^H \mathbf{R}_{Tx}$. We assume a space-time code, which achieves full diversity in the i.i.d. case, its diversity order in the correlated fading case is given by the product of rank of transmit correlation matrix and the rank of the receive correlation matrix i.e.,

r = rank $(\Delta\Delta^H) = n_T$ for any codeword pair **X** and **E**.

For asymptotically high SNR, as in section (4.4.1) the diversity gain is given by

$$d = r\alpha = \text{rank}(\Delta\Delta^H \mathbf{R_{Tx}}).\ \text{rank}(\mathbf{R_{Rx}}) = \text{rank}(\mathbf{R_{Tx}}).\ \text{rank}(\mathbf{R_{Rx}}). \quad (4.25)$$

Under the condition where there is no receive correlation the above equation reduces to

$$d = r\alpha = \text{rank}(\mathbf{R_{Tx}}).\ n_R \quad (4.26)$$

The diversity gain is maximized when rank ($\mathbf{R_{Tx}}$) = n_T.

Chapter 5

Simulation and Results

5.1 Introduction

This section illustrates the performance of space-time block codes through computer simulation. MATLAB is used to construct the computer code of the STBC and the simulations were run in MATLAB environment. For performing the simulations, the chain shown in Figure 5.1 was developed under the MATLAB environment.

The simulation results are divided into two parts. The first part gives the results for uncorrelated conditions for Orthogonal Space-time block codes. In second part we provide the results for correlated channel conditions using the kronecker model where the correlation is from the transmit side.

5.2 Simulation set up

The simulation set up is composed of four distinct parts, namely the bit generator, the STBC-encoder, the channel and the decoder.

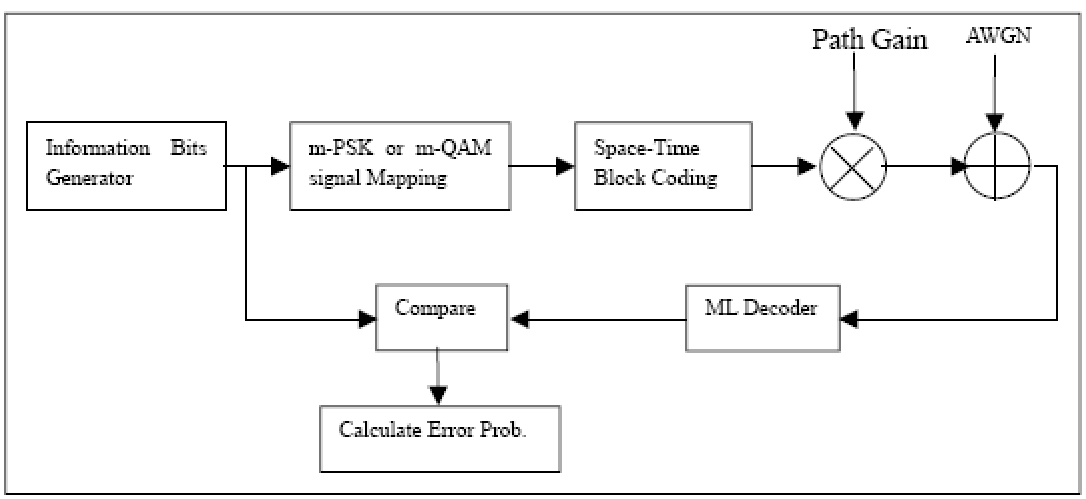

Figure 5. 1. Block Diagram for Simulation

1. Information Bit Generator

This generates the sequence of bits composed of 0 and 1 using uniformly distributed random numbers.

2. M-PSK Mapping

BPSK and 4-PSK modulations are used for this simulation. The bit sequence is divided into symbols which are composed of several bits, i.e., 2 bits is one symbol for Q-PSK, and then each symbol are mapped to the constellation points using Gray-Coded ordering. Finally, the gray coded symbol is changed to the complex output form. In this simulation, the phase offset is 0.

3. Space -Time Block Coding

As explained in section 2.11 and 2.12, the space-time block code is defined by a (p x n_T) transmission matrix, which is a combination of the signals that will be transmitted. In this simulation we use the transmission matrix of $\mathbf{G}_2^c$, $\mathbf{G}_3^c$ and $\mathbf{G}_4^c$.

4. Channel Mode

The channel is considered as Rayleigh flat fading channel. The dominant factor is the path gain from each transmit antenna to each receive antenna. The path gain is the independent complex Gaussian random variables with variance 0.5 per real and imaginary parts. Additionally, the usual additive white Gaussian noise corrupts the signal. The AWGN and the Rayleigh fading channel are generated using

$n_j =$ randn (1, N) + j * randn(1, N));

$h_{j,i} = \sqrt{0.5}$ * ((randn (1,N) + j* randn (1,N)).

In the case where the antenna elements in either the transmitter or receiver are not sufficiently far apart, the fading coefficients $h_{j,i}$ will be correlated. The spatial fading correlation model as given in equation (4.4) is

$$\mathbf{H} = \mathbf{R}_{\text{Rx}}^{1/2}\ \mathbf{H}_{\omega}\ \mathbf{R}_{\text{Tx}}^{1/2}$$

Where $\mathbf{R}_{\mathbf{Rx}}$ and $\mathbf{R}_{\mathbf{Tx}}$ are the correlation matrices at the receiver and transmitter sides.
For (2Tx, 1Rx) and (2Tx, 2Rx) MIMO systems the correlation matrices are respectively

$$\mathbf{R}_{\mathbf{Rx}} = 1, \qquad \mathbf{R}_{\mathbf{Tx}} = \begin{bmatrix} 1 & \vartheta \\ \vartheta^* & 1 \end{bmatrix}$$

$$\mathbf{R}_{\mathbf{Rx}} = \begin{bmatrix} 1 & \rho \\ \rho^* & 1 \end{bmatrix}, \quad \mathbf{R}_{\mathbf{Tx}} = \begin{bmatrix} 1 & \vartheta \\ \vartheta^* & 1 \end{bmatrix}.$$

For conditions where there is no correlation at the receiver side the correlation matrices for (2Tx, 1Rx) and (2Tx, 2Rx) systems are respectively.

$$\mathbf{R}_{\mathbf{Rx}} = 1, \text{ and } \mathbf{R}_{\mathbf{Rx}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

5. Maximum Likelihood (ML) Decoding

The ML decoding algorithm is to find the complex signal, ***s***, which minimizes the following equation.

$$\sum_{t=1}^{l}\sum_{j=1}^{nR}\left| y_t^j - \sum_{i=1}^{n_T} h_{j,i}\, x_t^i \right|^2 .$$

This calculation is performed with all possible $\boldsymbol{s}$ for each modulation. However, we use the simplified form of the above equation in our simulation. For instant, in the case of
2 Transmit Antennas, for detecting s_1,

$$\left| \left[\sum_{j=1}^{nR} \left(y_1^j h_{j,1}^* + \left(y_2^j \right)^* h_{j,2} \right) \right] - s_1 \right|^2 + \left(-1 + \sum_{j=1}^{nR} \sum_{i=1}^{2} \left| h_{j,i} \right|^2 \right) \left| s_1 \right|^2 \text{ is minimized}$$

for detecting s_2

$$\left| \left[\sum_{j=1}^{nR} \left(y_1^j h_{j,2}^* - \left(y_2^j \right)^* h_{j,1} \right) \right] - s_2 \right|^2 + \left(-1 + \sum_{j=1}^{nR} \sum_{i=1}^{2} \left| h_{j,i} \right|^2 \right) \left| s_2 \right|^2 \text{ is minimized}$$

6. Bit Error Rate of the system

BER is computed as the ratio of incorrect data bits divided by the total number of data bits transmitted.

5.3 Simulation Results

The following simulation results provide the performance of space-time block codes. Because of symbol and bit energy symmetry the linear modulation PSK is used. The results for space-time block codes are based on bit error rate (BER) performance over a range of SNR.

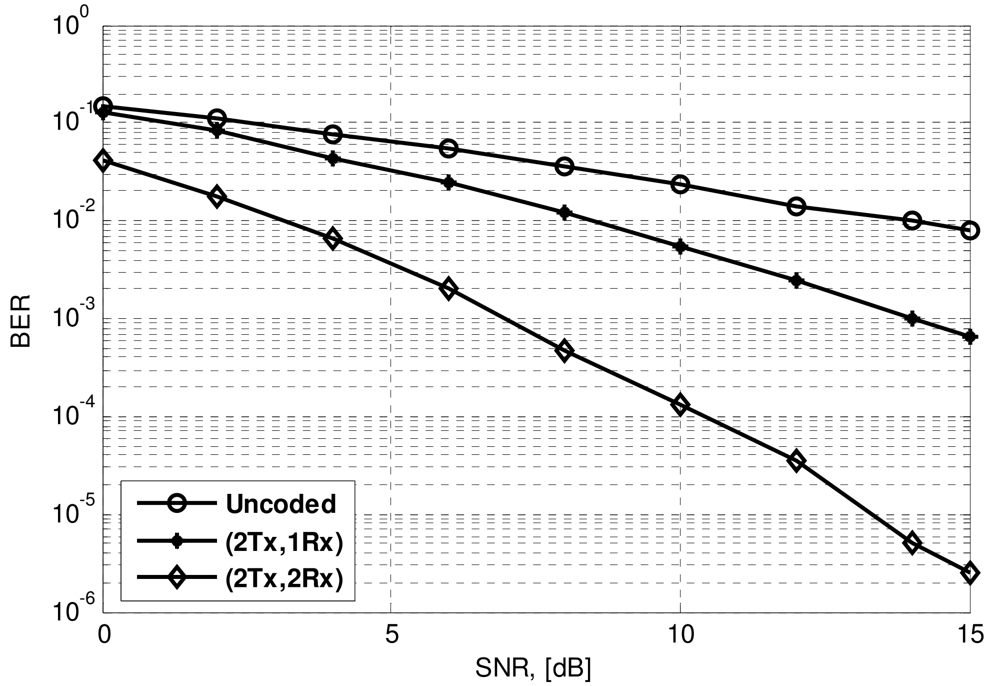

Figure 5.2. Bit error rate vs. SNR at 1 bit/sec/Hz over uncorrelated Rayleigh fading channel.

Figure 5.2 shows the bit error rate for transmission of 1bit/sec/Hz for the following cases: Uncoded case which uses one transmit antenna and one receive antenna, two transmit antennas and one receive antenna, and two transmit antennas and two receive antennas. BPSK constellation is used.

Table 5.1, is a summery of the performance at 10dB. Where the bit error rate improvement over the uncoded case in percentage is computed as

$$\left| \frac{\mathrm{BER}_{\mathrm{uncoded}} - \mathrm{BER}_{(n\mathrm{Tx}\,,\, n\mathrm{Rx})}}{\mathrm{BER}_{\mathrm{uncoded}}} \right| \times 100\,\% \tag{5.1}$$

Antenna Set-up	Uncoded	(2Tx,1Rx)	(2Tx,2Rx)
BER	2.352×10^{-2}	5.5×10^{-3}	$8.421x10^{-5}$
BER improvement over uncoded	-	76.62 %	99.7%

Table-5.1 BER performance for uncoded and Alamouti's Scheme at10 dB.

Figure 5.2 presents the BER performance of the uncoded and the Alamouti's Scheme. It shows that as the number of transmit or receive antennas increases the performance is getting better.

From the plots of figure 5.2 at bit error rate of 10^{-2} the (2Tx, 1Rx) and (2Tx, 2Rx) systems give about 5dB and 10dB gain in SNR respectively over the use of the uncoded system.

From equation (2.9), one can see that there are two fading amplitude terms, i.e., two independent paths associated with transmitting the symbol. Therefore, if either of the paths is in a deep fade, the other path may still provide a high reliability for the transmitted signal. This explains why the performance of a system with 2Tx and 1Rx is better than that of a system with no diversity (uncoded). If the number of receivers is increased to 2, one can see in equation (2.10) that there are now 4 propagation paths as those in equation (2.9). This explains why the performance of a system with 2Tx and 2Rx is better than that of a system with 2Tx and 1Rx thus; its diversity gain is increased from 2 to 4.

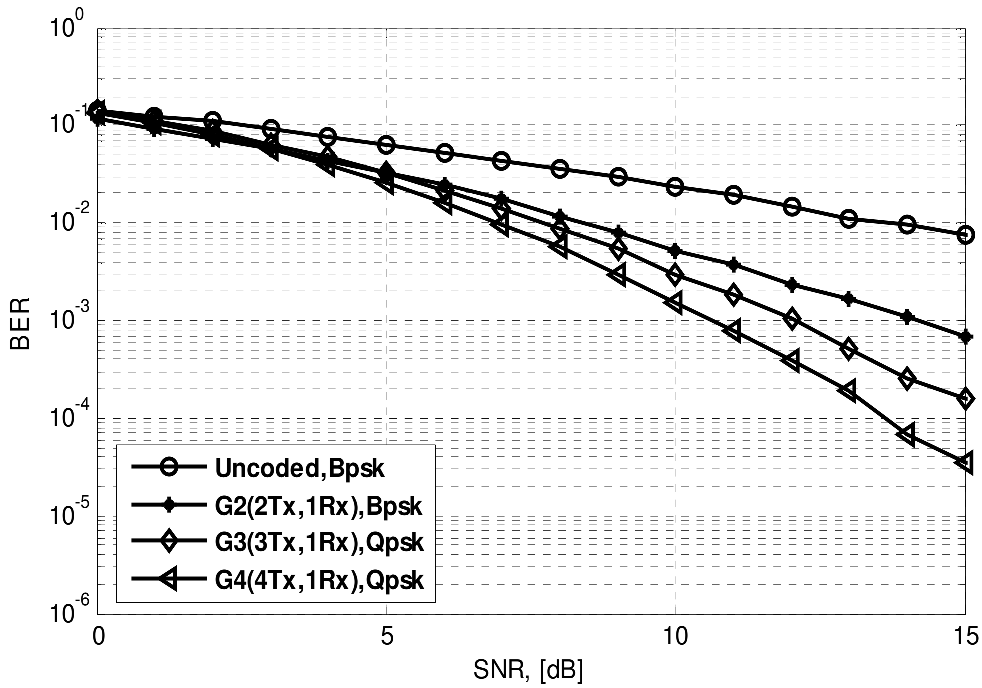

Figure 5.3. Bit error rate vs. SNR at 1 bit/sec/Hz over uncorrelated Rayleigh fading channel of space-time codes G_2, G_3 and G_4; one receive antenna.

Figure 5.3 shows the bit error rate for transmission of 1 bit/sec/Hz for the following cases: Uncoded, two, three and four transmit antennas. The transmission using two transmit antennas employs the BPSK constellation and the code $\mathbf{G}_2$. For three and four transmit antennas, the 4-PSK constellation and $\mathbf{G}_3$ and $\mathbf{G}_4$, are used. Since $\mathbf{G}_3$ and $\mathbf{G}_4$ are rate ½ codes, the total transmission rate in each case is 1 bit /sec/Hz.

Table 5.2, is a summery of the performance at 10dB using equation (5.1).

Antenna Set-up	Uncoded	(2Tx,1Rx)	(3Tx,1Rx)	(4Tx,1Rx)
BER	2.352×10^{-2}	5.5×10^{-3}	2.897×10^{-3}	1.514×10^{-3}
BER improvement over uncoded	-	76.62%	87.6%	93.5%

Table-5.2 BER performance for uncoded, G2, G_3 and G_4 at10 dB.

From the plots of figure 5.3 at bit error rate of 10^{-2} the (2Tx, 1Rx), (3Tx, 1Rx) and (4Tx, 1Rx) systems give about 5dB, 5.5dB and 6dB gain in SNR respectively over the use of uncoded system.

Figure 5.3 shows, the plots of space-time block codes, which improve the diversity order at the receiver. This can be inferred since the slope of the BER curve for 2Tx-1Rx case is steeper than the 1Tx-1Rx case as the former has a second order diversity advantage. Similarly the slope for 3Tx-1Rx case is steeper than 2Tx-1Rx case as its diversity order is greater. Finally the slope 4Tx-1Rx case is steeper than 3Tx-1Rx and its diversity gain is increased from 3 to 4 thus the performance is getting better as the number of transmit antennas increases.

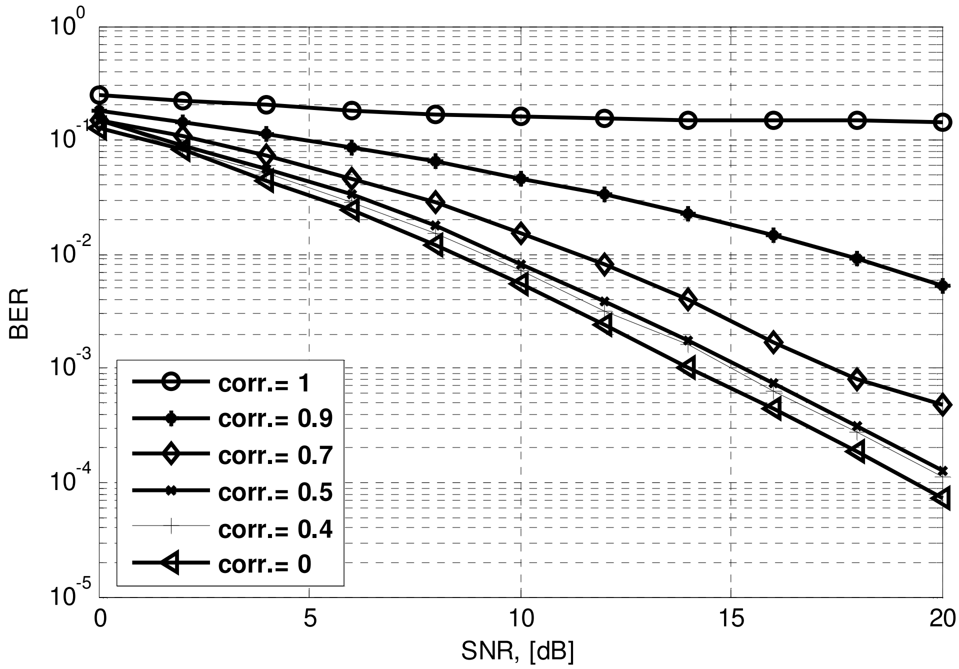

Figure 5.4 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr. values of 0, 0.4, 0.5, 0.7, 0.9 and 1 using BPSK for (2Tx, 1Rx).

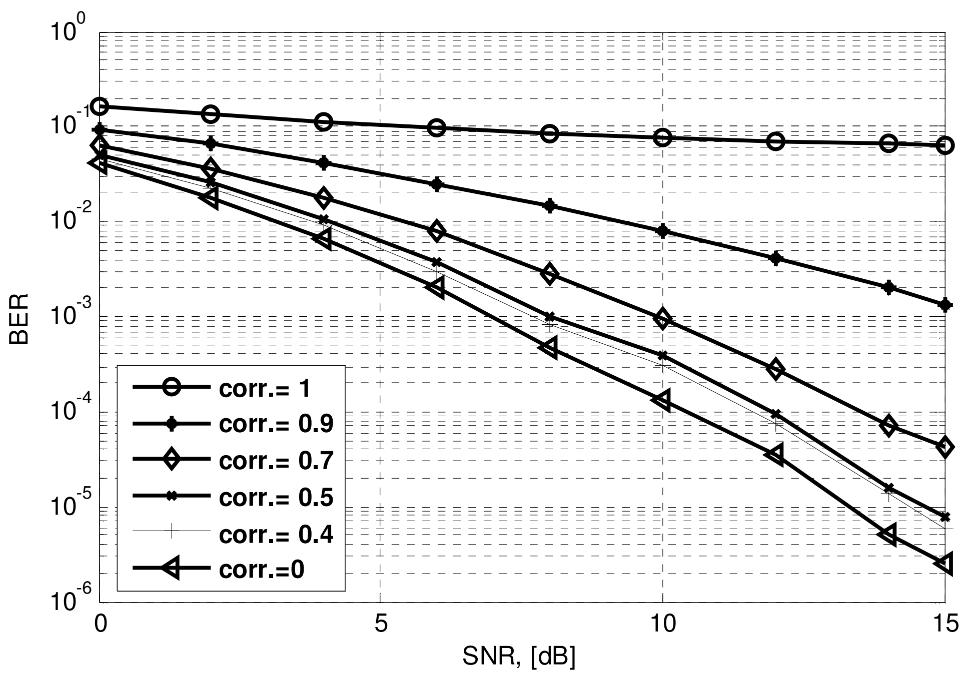

Figure 5.5 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr. values of 0, 0.4, 0.5, 0.7, 0.9 and 1 using BPSK for (2Tx, 2Rx).

From the plots of figure 5.4, the loss in SNR at bit error rate of 10^{-2} for correlation values of 0.4, 0.5, 0.7 and 0.9 are respectively 0.7dB, 0.8dB, 2.5dB and 9.5dB.

From the plots of figure 5.5, the loss in SNR at bit error rate of 10^{-2} for correlation values of 0.4, 0.5, 0.7 and 0.9 are respectively 0.7dB, 0.8dB, 2dB and 6dB.

Since the transmit correlation matrix i.e. $\mathbf{R}_{Tx} = \begin{bmatrix} 1 & \vartheta \\ \vartheta^* & 1 \end{bmatrix}$ is full rank for correlation values of 0.4, 0.5, 0.7 and 0.9 the diversity order of the correlated cases as given by equation (4.26) , d = rank ($\mathbf{R}_{Tx}$). n_R, is the same as the spatially uncorrelated case, thus the performance degradation due to transmit correlation is mainly due to coding loss and the degradation in performance is getting worst for correlation values above 0.5.

For fully correlated systems the transmit correlation matrix $\mathbf{R}_{Tx} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is non-full rank thus the (2Tx,1Rx) and (2Tx,2Rx) systems loss their diversity gains.

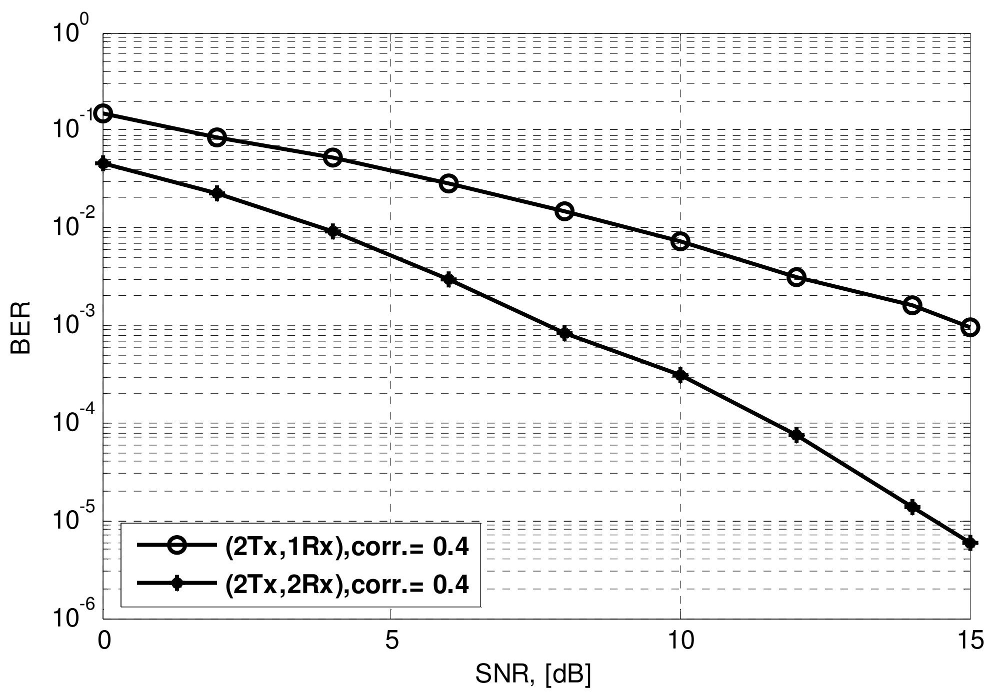

Figure 5.6 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr.of 0.4.

Figure 5.6 shows bit error rate for transmission of 1bit/sec/ Hz with two transmit antennas and one receive antenna, two transmit antennas and two receive antennas BPSK constellation is used. The performance of the Orthogonal Space-time block code for (2Tx, 2Rx) at a correlation of 0.4 is getting better than the (2Tx, 1Rx) at the same correlation of 0.4 as the SNR is increased.

Table 5.3, is a summery of the performance at 10dB. Where the bit error rate improvement over the (2Tx, 1Rx) system in percentage is computed as

$$\left| \frac{\mathrm{BER}_{(2\mathrm{Tx},1\mathrm{Rx})} - \mathrm{BER}_{(2\mathrm{Tx},2\mathrm{Rx})}}{\mathrm{BER}_{(2\mathrm{Tx},1\mathrm{Rx})}} \right| \times 100\,\% \qquad (5.2)$$

Antenna Set-up	(2Tx,1Rx)	(2Tx,2Rx)
BER	7.23×10^{-3}	3.025×10^{-4}
BER improvement over (2Tx,1Rx)	-	95.8%

Table-5.3 BER performance for (2Tx, 1Rx) and (2Tx, 2Rx) at 10 dB with corr. of 0.4.

Using equation (5.2) the bit error improvement of (2Tx, 2Rx) over (2Tx, 1Rx) system in the uncorrelated case is 98.5% and the bit error improvement of (2Tx, 2Rx) over (2Tx,1Rx) system with correlation value of 0.4 is 95.8%, comparing with the uncorrelated and the correlated systems the degradation in BER is
(98.5-95.8) % = 2.7%.

From the plots of figure 5.6 at bit error rate of 10^{-2} the (2Tx, 2Rx) system gives about 5dB gain in SNR over the use of (2Tx, 1Rx) at transmit correlation of 0.4.

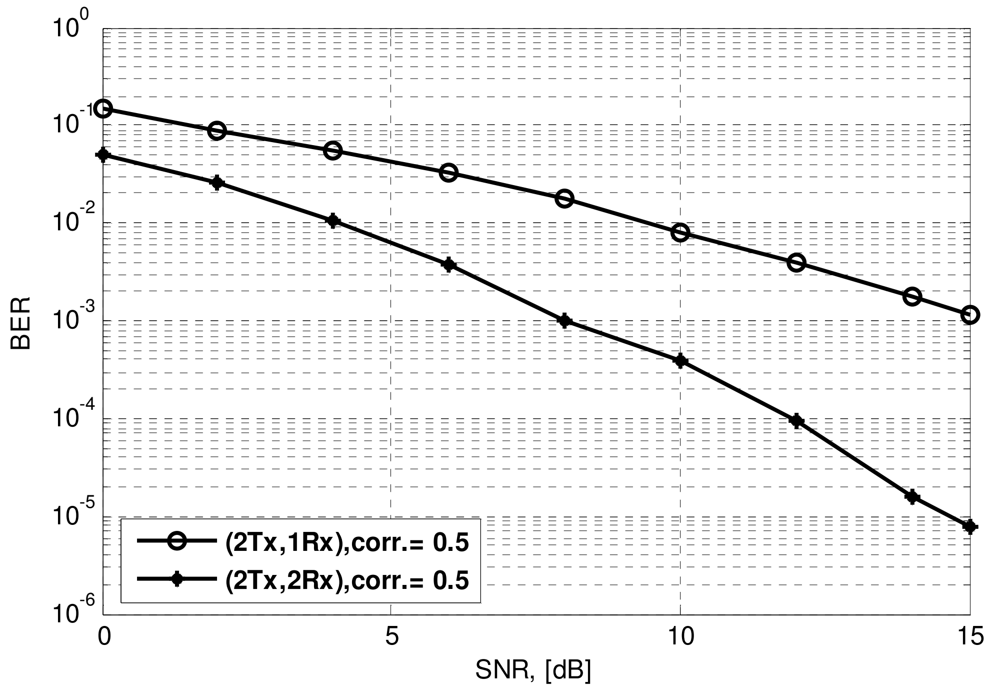

Figure 5.7 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr.of 0.5.

Figure 5.7 shows bit error rate for transmission of 1bit/sec/ Hz with two transmit antennas and one receive antenna, two transmit antennas and two receive antennas Bpsk constellation is used. The performance of the orthogonal space-time block code for (2Tx, 2Rx) at a correlation of 0.5 is getting better than the (2Tx, 1Rx) at the same correlation of 0.5 as the SNR is increased.

Table 5.4, is a summery of the performance at 10dB using equation (5.2).

Antenna Set-up	(2Tx, 1Rx)	(2Tx, 2Rx)
BER	8.707×10^{-3}	4×10^{-4}
BER improvement over (2Tx, 1Rx)	-	95.4%

Table-5.4 BER performance for (2Tx, 1Rx) and (2Tx, 2Rx) at 10 dB with corr. of 0.5.

The bit error improvement of (2Tx, 2Rx) over (2Tx, 1Rx) system at correlation value of 0.5 is 95.4%, comparing with the uncorrelated case the degradation in BER is (98.5-95.4) % = 3.1%.

From the plots of figure 5.7 at bit error rate of 10^{-2} the (2Tx, 2Rx) system gives about 5.5dB gain in SNR over the use of (2Tx, 1Rx) at transmit correlation of 0.5.

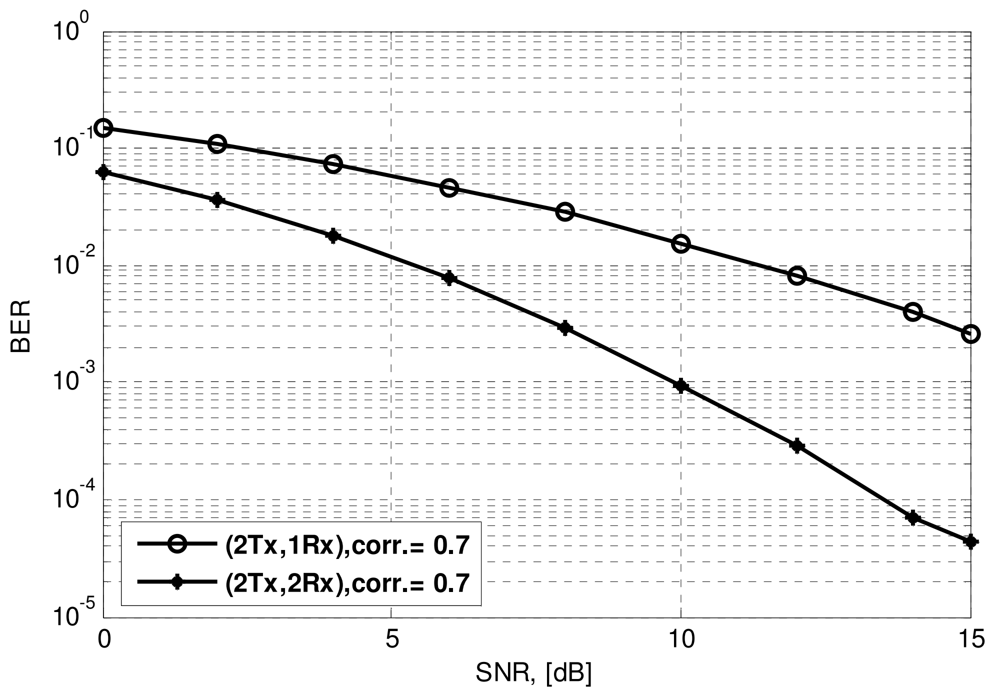

Figure 5.8 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr.of 0.7.

Figure 5.8 shows bit error rate for transmission of 1bit/sec/ Hz with two transmit antennas and one receive antenna, two transmit antennas and two receive antennas Bpsk constellation is used. The performance of the orthogonal space-time block code for (2Tx, 2Rx) at a correlation of 0.7 is getting better than the (2Tx, 1Rx) at the same correlation of 0.7as the SNR is increased.

Table 5.5, is a summery of the performance at 10dB.

Antenna Set-up	(2Tx, 1Rx)	(2Tx, 2Rx)
BER	1.543×10^{-2}	9.99×10^{-4}
BER improvement over (2Tx, 1Rx)	-	93.5 %

Table-5.5 BER performance for (2Tx, 1Rx) and (2Tx, 2Rx) at 10 dB with corr. of 0.7.

The bit error improvement of (2Tx, 2Rx) over (2Tx, 1Rx) system at correlation value of 0.7 is 93.5 %, comparing with the uncorrelated case the degradation in BER is (98.5-93.5) % = 5 %.

From the plots of figure 5.8 at bit error rate of 10^{-2} the (2Tx, 2Rx) system gives about 6.5dB gain in SNR over the use of (2Tx, 1Rx) at transmit correlation of 0.7.

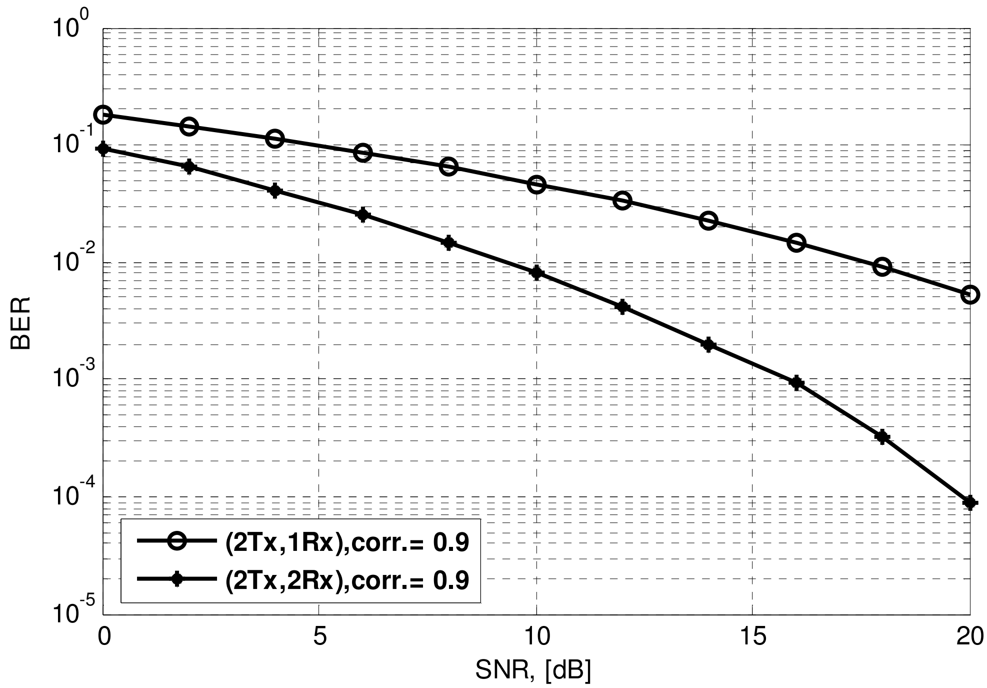

Figure 5.9 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr.of 0.9.

Figure 5.9 shows bit error rate for transmission of 1bit/sec/ Hz with two transmit antennas and one receive antenna, two transmit antennas and two receive antennas Bpsk constellation is used. The performance of the orthogonal space-time block code for (2Tx, 2Rx) at a correlation of 0.9 is getting better than the (2Tx, 1Rx) at the same correlation of 0.9 as the SNR is increased.

Table 5.6, is a summery of the performance at 10dB.

Antenna Set-up	(2Tx, 1Rx)	(2Tx, 2Rx)
BER	4.629×10^{-2}	9.56×10^{-3}
BER improvement over (2Tx, 1Rx)	-	79.3 %

Table-5.6 BER performance for (2Tx, 1Rx) and (2Tx, 2Rx) at 10 dB with corr. of 0.9.

The bit error improvement of (2Tx, 2Rx) over (2Tx, 1Rx) system at correlation value of 0.9 is 79.3 %, comparing with the uncorrelated case the degradation in BER is (98.5-79.3) % = 19.2%.

From the plots of figure 5.9 at bit error rate of 10^{-2} the (2Tx, 2Rx) system gives about 8.5dB gain in SNR over the use of (2Tx, 1Rx) at transmit correlation of 0.9.

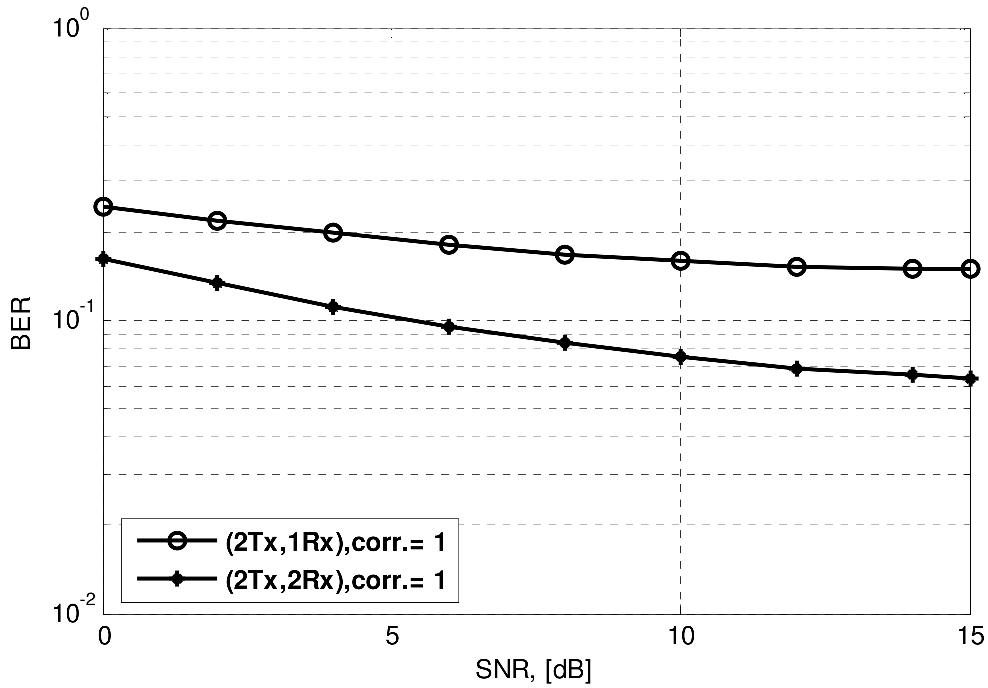

Figure 5.10 Bit error rate vs. SNR at 1 bit/sec/Hz over correlated Rayleigh fading channel with corr.of 1.

Figure 5.10 shows bit error rate for transmission of 1bit/sec/ Hz with two transmit antennas and one receive antenna, two transmit antennas and two receive antennas Bpsk constellation is used. The performance of the orthogonal space-time block code for (2Tx, 2Rx) at a correlation of 1 is getting better than the (2Tx, 1Rx) at the same correlation of 1 as the SNR is increased.

Table 5.7, is a summery of the performance at 10dB.

Antenna Set-up	(2Tx, 1Rx)	(2Tx, 2Rx)
BER	1.591×10^{-1}	7.58×10^{-2}
BER improvement over (2Tx, 1Rx)	-	52.4 %

Table-5.7 BER performance for (2Tx, 1Rx) and (2Tx, 2Rx) at 10 dB with corr. of 1.

The bit error improvement of (2Tx, 2Rx) over (2Tx, 1Rx) system at correlation value of 1(fully correlated) is 52.4 %, comparing with the uncorrelated case the degradation in BER is (98.5- 52.4) % = 46.1%.

It is seen from the plots of figure 5.10 the (2Tx, 1Rx) and (2Tx, 2Rx) systems don't attend the BER of 10^{-2} simultaneously since the two schemes loss their diversity gain.

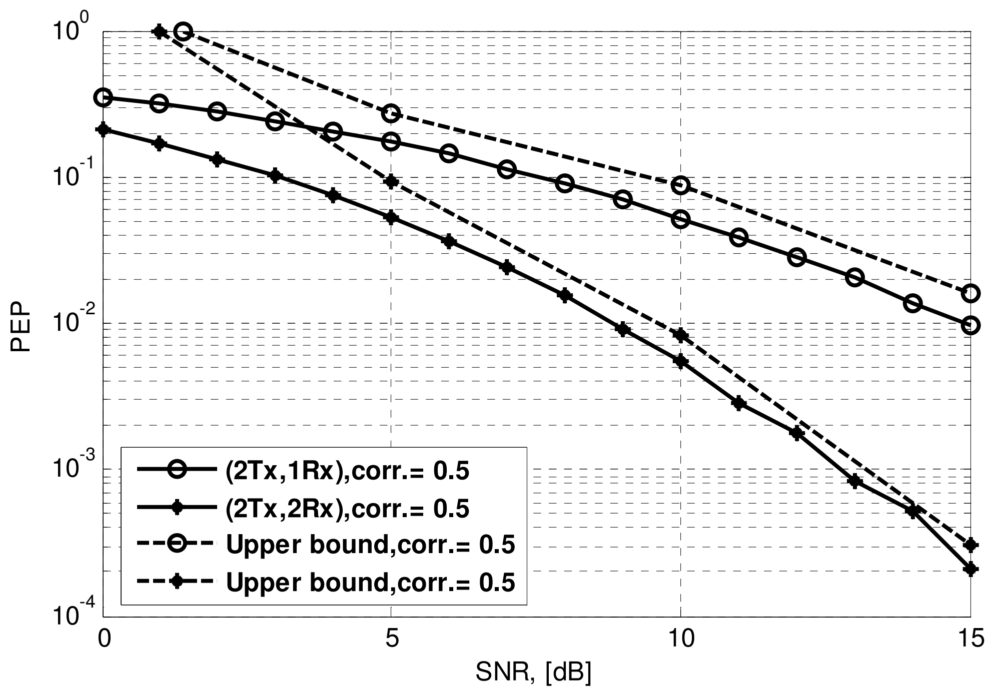

Figure 5.11 PEP vs. SNR for (2Tx, 1Rx) and (2Tx, 2Rx) over correlated Rayleigh fading channel with corr. of 0.5.

Figure 5.11 shows pairwise error probabilities for transmission of 2bits/sec/ Hz with two transmit antennas and one receive antenna, two transmit antennas and two receive antennas and upper bounded using the upper bound of equation (4.20) QPSK constellation is used. The performance of the orthogonal space-time block code for (2Tx, 2Rx) at a correlation of 0.5 is getting better than the (2Tx, 1Rx) at the same correlation of 0.5 as the SNR is increased.

Table 5.8, is a summery of the performance at 10dB.

Antenna Set-up	(2Tx,1Rx)	(2Tx,2Rx)
PEP	5.139×10^{-2}	5.49×10^{-3}
PEP improvement over (2Tx,1Rx)	-	89.3%

Table-5.8 PEP performance for (2Tx, 1Rx) and (2Tx, 2Rx) at 10 dB with corr. of 0.5.

Figure 5.11 presents a comparison of PEP performance for correlation value of 0.5 using (2Tx, 1Rx) and (2Tx, 2Rx) from the Figure, the bounds are tighter for (2Tx,2Rx) than (2Tx,1Rx) and the system is highly bounded at high SNR rather than at low SNR since the diversity is increasing as SNR is increasing.

Chapter 6

Conclusion and Future work

In this work coded modulation schemes designed for wireless channel using multiple antennas without any information about the channel at the transmitter is discussed. The focus has been on space-time block codes benefits and limitations of primary designs are explained. It has been discussed space-time block coding, is a simple and elegant method for transmission using multiple transmit antennas in a wireless Rayleigh fading environment. These codes have a very simple maximum-likelihood decoding algorithm, which is only based on linear processing. Moreover, they exploit the full diversity given by transmit and receive antennas.

It is also studied the performance of space-time code in correlated Rayleigh fading environment using the Kronecker channel model with transmit correlation. Furthermore it is expressed if a space-time code achieves full diversity in the uncorrelated case, the diversity order achieved in the correlated case is given by the product of the rank of transmit correlation matrix and rank of receive correlation matrix.

The simulation results are analyzed both for the case of spatially uncorrelated, as well as spatially correlated fading conditions, some of these important results are listed below.

- ❖ Significant performance gains can be achieved by increasing the number of transmit/ receive antennas with little decoding complexity.

- ❖ Performance plots for STBC indicate a relatively small degradation of performance for low to medium values of correlation values.

- ❖ STBC loss their diversity gains for non-full rank transmit correlation matrices.

- STBC which achieves full diversity in the uncorrelated case, the performance degradation in the correlated case is mainly due to coding loss.

- A relatively high performance degradation is observed for high correlation values, particularly for correlation values of 0.9 and 1, it is difficult to differentiate the two transmitting antennas.

- Antenna systems with high diversity are potentially providing better performance in correlated channel than systems with low diversity thus diversity reduces the effect caused by spatial correlation. Practically the gain in SNR increases as the transmit correlation grows up.

6.1 Future work

Based on the results in this work, a number of interesting research directions can be pursued in the future. These are outlined below.

- To further improve the performance of space-time block in coded system, in both uncorrelated and correlated conditions convolutional and turbo decoding methods involving an inner space-time block code can be investigated.

- To investigate the performance of space-time code over correlated fading channels. In particular, the effect of antenna spacing, spatial geometry of the antenna arrays and the non-isotropic scattering environments on the performance of space-time codes of interest.

- To extend to wideband channel model where the correlation changes over time interval.

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