GENERATION AND COLLIMATION MECHANISMS OF RELATIVISTIC JETS IN AGN

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Abera Shiferaw

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Generation and collimation mechanisms of relativistic jets in AGN

By

Abera Shiferaw

Department of Physics

Faculty of Science

APPROVED BY THE EXAMINATION COMMITEE

Name	**Signature**
Dr. Ramanand Jha, Advisor	______________
Proffesor P.Singh, Examiner	______________
Proffesor A.K.Chaubey, Examiner	______________

Dated: June 2006.

"Thanks and Questions to God"

God ! if you let me to address you some paradoxes of your laboratory outputs of what your creatures, may be only human being, are so ambitious to know, these are my questions: What is your interest to bring the universe into picture? To what extent you plan the universe to expand?, why? Or is that the universe already infinite in size and ever existing? Where does that infinity ends and what is beyond that infinity? If the infinity has no boundary through out, then I must believe that you are the universe of limitless extent and all my questions are wrong.

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This is the least that i can say regarding my mom:

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Abstract

Most of the phenomenon performed by active galaxies are left uncovered among which highly collimated jet-like outflows, that span hounders of Kpc to Mpc via interstellar medium, is the one. Our current research, which is fully analytic, is targeted on finding the plausible mechanism of jets generation and collimation in these Mysterious active galaxies. In this work, Kerr geometry is preferred to describe the dynamics of electromagnetic field and plasma fluid around rotating Kerr black hole. we find out that the force-free region, in the immediate neighborhood of rotating black hole, is the site where jets generation and launching are fabricated. Toroidal magnetic field and centrifugal force, on the accreting plasma fluid, are the key elements for the generation and launching of jets. Collimation of jets is the responsibility of: magnetic hoop stress, radiation pressure, from the accretion disk, and centrifugal force. Also, magnetic field is found to be crucially important in extracting the spin energy of the black hole. Energy and angular momentum are continuously supplied to the jets through differential rotation.

Notation and convention:

The fundamental constants, the vacuum speed of light, c, and the gravitational constant, G, are set to unity. Then, we deal with so-called " geometrized units.

The signature of the metric is (- + + +).

Einstein's summation convention: sum over any repeated index in the product.

Typical mass scale in astrophysics is the solar mass:

$1M_{\odot} = 1.989 \times 10^{30} Kg = 1.989 \times 10^{33} g.$

The usual luminosity scale in astrophysics is the solar luminosity:

$1L_{\odot} = 3.853 \times 10^{33} ergs^{-1}.$

1Parsec = 3.2615ly = $3.0856 \times 10^{13} Km.$

The cgs unit for energy is: $1erg = 10^{-7} J.$

The 4-velocity of , U^{μ}, is normalized by the condition:

$g_{\mu\nu} U^{\mu} U^{\nu} = -1.$

Introduction

It was the early scientists' interest to watch the fascinating objects, like galaxies, in the local universe. In 1610, Galileo Galilei used a telescope to study the bright band on the night sky known as the Milky Way and discovered that it is composed of a huge number of faint stars. In 1755, Immanuel Kant speculated (correctly) that the Galaxy might be a rotating body of a huge number of stars, held together by gravitational forces and then orbiting about a common center of gravity. The first attempt to describe the shape of the Milky-Way(our galaxy) and the position of the Sun within it was carried out by William Herschel in 1785 by carefully counting the number of stars in different regions of the sky. Towards the end of the 18th century, Charles Messier compiled a catalog containing the 109 brightest nebulae which later followed by a larger catalog of 5000 nebulae assembled by William Herschel. Until the 1920s it remained unanswered whether these were gaseous nebulae within our own galaxy, or separate "island universes" (i.e. galaxies like our own). Following this, appreciation of the universe beyond the Milky Way greatly increased early in the twentieth century. For simple understanding, classifying of galaxies had been made by the early astronomers in different ways based on different criterias. However; the most commonly accepted classification is based on the: structures (shapes), stellar content, dust content and gas content of the galaxies. Based on these clues, galaxies have been classified into three main groups as:

Elliptical galaxies : These types of galaxies are elliptical in shape, contain older stars and very little gas and dust.

Spiral galaxies : These types of galaxies(to which our Milk-way galaxy belongs)are spiral in shape, tend to contain more middle-aged stars along with clouds of gas and dust . They are more brighter than elliptical galaxies

Irregular galaxies : An irregular galaxies have undefined shapes and a lot of young stars, dust and gas.

According to currently recorded statistical data, there are about 1.3×10^{11} visible galaxies in the universe out of which 12% are elliptical galaxies, 34% are spiral galaxies and 54% are irregular galaxies. Remember that relative number of galaxies may change with time. Except for irregular galaxies, elliptical and spiral galaxies are rotating galaxies each rotating about its own center. Galaxies are not fixed in space relative to each other as well. Some galaxies are found to move towards each other while some others move apart. For instance, studies show that our host Galaxy (milk-way galaxy) is moving towards the nearby galaxy (Andromeda galaxy) at about 130 km/s, and depending upon the lateral movements, the two may collide in about 5-6 billion years. This leads to galaxies merge and form large cluster. Astronomers had recognized that galaxies differ in luminosity as well. Based on this, galaxies are classified as: **classI** up to **classV**, where **classI** galaxies are the most luminous and **classV** being the least in luminosity. Repeated direct observations towards galactic centers using more advanced astronomical instruments revealed unusual phenomenon in some galaxies. The universe, with its endless secretes, keeps on providing assignments to those who are interested in. The astronomers were able to observe surprising events like: extreme luminosity and unusual jet-like out flows from some galactic centers and then initiated to find out the solution for this assignment. Those galaxies in which these events observed are termed as active galaxies. Extreme luminosity and jet-like outflows from some active galactic centers are mostly inter-related. In the 1940s and

1950s research into active galaxies took off seriously. This started with Seyfert's spectroscopic work on the centers of spiral galaxies, and culminated with the beginning of radio astronomy. After extensive studies of active galactic centers,researchers of the field come to a common consensus that the plausible reason for the extreme luminosity in some galactic centers is the presence of compact object (like black hole) in galactic center which accretes materials close to it. Accreting materials (collection of; stars, dusts and gases), release tremendous amount of gravitational potential energy in the form of radiation. This enormous radiation exceeds the radiation usually observed in normal galaxies. The central gravitating black hole can be either rotating or non-rotating by itself. Accretion onto non-rotating black hole is omni-directional, while accretion onto rotating black is confined to equatorial plane perpendicular to the black hole's rotation axis. Evidences show that almost all black holes are rotating so that we have accreting materials confined in equatorial plane and form disk-like structure called accretion disk. Even if the eagers to know the reasons behind strange outflow in some galactic centers keep on increasing, the puzzle persists till today. A lot of efforts had been made in the past to understand the mysteries behind these unusual phenomenons and still keep on. Black hole, accretion disk and magnetic field are repeatedly proposed as the tools for the generation of outflows in galactic centers and collimation over a large astronomical distance. Understanding possible site of magnetic field generation around black hole is still missing by itself. Our current aim is also focused on looking for the fundamental reasons that meet the observed outflows. The main purpose of this paper is to obtain a qualitative and quantitative pictures of the mechanism by which these outflows are generated and collimated. Locating the possible site of jets production is also the other problem to be discussed along with the above ones'. The effects of disk radiation and magnetic field on the dynamics of materials which accrete onto central rotating black hole is discussed in detail as spine for the generation and then collimation of out flows. There are different mechanisms, proposed by many related field researchers, by which these jet-like out flows are produced and collimated from close to the center of active galaxies. All the mechanisms proposed so far have their own drawbacks of harmonizing with the phenomenon obtained by direct observations. We, here based on analytical way, propose a new model which is more plausible reason for not only their production and collimation but also for their observed large scale propagation strongly related with the central gravitating astrophysical objects. In our mechanisms, the increment of toroidal component of magnetic field strength towards the immediate neighborhood of central rotating black hole and the centrifugal force, on the accreting plasma fluid, take central role for production of relativistic jets. Radiation pressure from the accretion disk rectifies the generated jets to be collimated along the rotation axis of rotating black hole. Collimation of relativistic jets is also supported by the magnetic field induced by the accelerating charged particles in the accretion disk and the external gas pressure from interstellar medium. In first chapter, we review the history of observational discoveries of active galaxies and their distinct features. In the next chapter, we discuss the effects of black holes on the local region of spacetime, the nature of magnetic fields and particle dynamics around black hole. In the third chapter, it is the output of accreting matters which is discussed. The fourth chapter deals with the central theme of our mechanisms for relativistic jets production and collimation and finally, in the last chapter, we discuss the outputs of our research.

Chapter 1

Active galaxies

The highly luminous galaxies are strongly related to the presence of active super-massive (with mass $\geq 10^5 M_{\odot}$) black holes in their centers. Super-massive black hole accretes materials from the surrounding environment thereby, in-falling of these materials release enormous amount of gravitational binding energy in the form of radiation. Therefore, the nucleus of the galaxy increases in luminosity and become visible as so-called active galactic nuclei (AGN hereafter). AGN is best defined from theoretical standpoint as any galactic nucleus that harbors a massive accreting active black hole.

The luminosity of AGNs range from $10^{42} ergs^{-1}$ to $10^{48} ergs^{-1}$

Maia, Machado and Willmer in 2002 reported that about 2.6% of the total galaxies,in local universe,are active galaxies . Active galaxies range from faint compact radio sources like that in M31 to recently discovered quasars like 3C273. These include: Seyfert galaxies, radio galaxies, quasars, and Blazars.

Seyfert Galaxies: They are named after the astronomer Carl Seyfert, who studied them extensively in the 1940s. Seyfert galaxies are the first type of active galaxies to be discovered. These subclasses of active galaxies are identical with normal spiral galaxies in shape except containing an extremely bright nucleus. Only a few percent of Seyfert galaxies might be elliptical in shape. The light from the nucleus varies in less than a year, which implies that the emitting region must be less than one light year across. In addition, most of the radiations produced by a Seyfert galaxies are in the infrared and radio part of the spectrum. The radio emission is believed to be synchrotron emission from the jet. The infrared emission is due to radiation in other bands being reprocessed by dust near the nucleus. Most Seyfert galaxies are very distant; however, a few lie relatively close to the Milky-way galaxy. Approximately 2% of all spiral galaxies are Seyfert galaxies

Radio Galaxies: They are the largest class of active galaxies with strong radio emission. They are generally associated with elliptical galaxies. For instance, M87 is giant elliptical active galaxy with strong radio source and its jet is fired out into space over a length of some 5000 ly with luminosity equal to 10^7solar luminosities. Radio galaxies differ from Seyfert galaxies in that they radiate most of their radiation in the long wavelength, radio part of the spectrum. They are very luminous at radio wavelengths (up to $10^{38}W$ between $10MHz$ and $100GHz$). The radio emission is due to the synchrotron process. Unlike Seyfert galaxies which produce their energy in the nucleus, most radio galaxies produce their energy from an extremely small area in the nucleus called the core. Radio galaxies can be subdivided into compact radio galaxies and extended radio galaxies. Compact radio galaxies often display very small radio sources. Extended radio galaxies have radio emission larger than optical emission of the galaxies. Cygnus A, one of the strongest radio sources in the sky and one of the first discovered, provides an excellent example of the typical double structure of the luminous extended radio galaxy. Its radio output is about 10^{11}solar luminosities, comes from two giant lobes set on opposite sides of the galaxy.

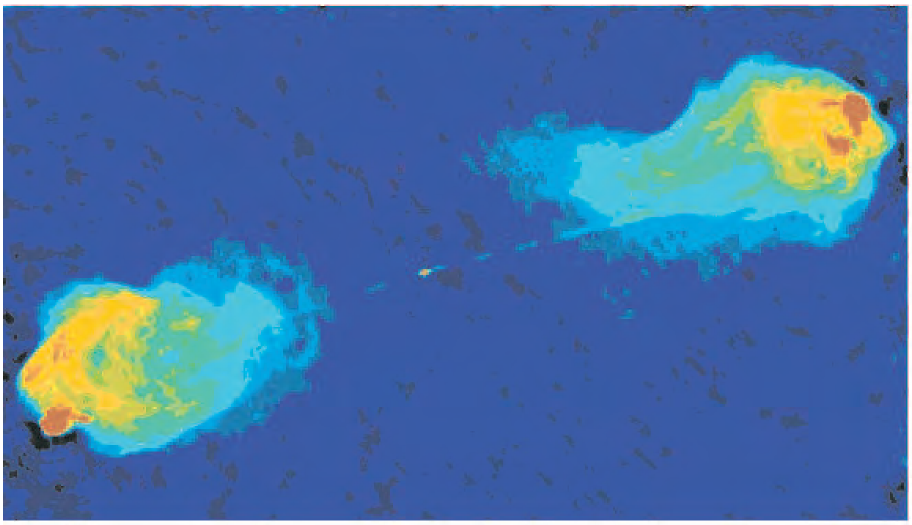

Figure 1.1: jets nourishing the two lobes in Cygnus A

These two lobes each on either sides of the galaxy are continuously nourished by the radio jets that extended out from the nucleus of the galaxy.

Quasars: the quasars were discovered in the 1960s. The name is a contraction of 'quasi-stars' or 'quasi-stellar radio source', which refers to the fact that quasars are generally point sources (like stars). These radio sources correspond to point-like objects on visible light photographs. Currently known that, in fact, most quasars are 'radio quiet', i.e., they have very little radio wave emission. Only 10% of all quasars can be seen with radio telescope while other can be seen in other wavelengths. Quasars are found to vary in luminosity in differing time periods. Some vary in brightness every few months, weeks, days, or hours. This recent evidence has allowed scientists to theorize that quasars exhibit energy in a very small region. Their light is unusually shifted towards redder wavelengths, which indicates that they are very distant the closest being 240 Mpc (780 million ly) away and the farthest being 4 Gpc (13 billion ly) away. Most quasars are known to lie above 1.0 Gpc in distance. Since light takes such a long time to cover these great distances, we are seeing quasars as they existed long ago. The highest redshift currently known for a quasar is 6.4. To be visible to us so far away, they must be very luminous. Hence, quasars are the brightest objects in the known universe. Up to the current, more than 60, 000 quasars are known. It was eventually realized that quasars are active galaxies that appear point-like because the active nucleus outshines all the stars in the galaxy.

Blazars: blazars are believed to be active galaxies having relativistic jets pointed toward the Earth so that what we observe is primarily emission from the jet region. They are thus similar to quasars, but are not observed to be as luminous. The visible and gamma-ray emission from blazars is variable on timescales from days to minutes. They are highly polarized and radiate energetically across the broadband spectrum, from radio to gamma rays.

To date more than 60 blazars have been detected.

From the above studies we can say that active galaxies share the following common characteristic physical features:

I) High luminosity: active galactic nuclei are characterized by high luminosity (comparable or surpassing the luminosity of their host) ranging from $10^{42} ergs^{-1}$ to $10^{48} ergs^{-1}$. This high luminosity confirms the existence of massive black hole such that gravity can combat radiation pressure which would otherwise blow the object apart. For an active nucleus with luminosity of $10^{46} ergs^{-1}$, the black hole mass estimated is 10^8 times as large as the solar mass.

II) variability: Since the light from normal galaxies is dominated by starlight, variability does not exist. In active galaxies, a region near the center produces enormous amounts of emission across the entire electromagnetic spectrum. Highly random motions are observed which accounts for the variability of the luminosity from this region. The emission is often variable on time scales of years and sometimes on time scales of days, hours, or even minutes, from which it can be deduced that the emitting region must be very small.

III) broadband continuum spectra: Most AGN have very broad band spectra compared to normal galaxies. The narrow lines are believed to originate from the outer part of the AGN where velocities are lower, while the broad lines originate closer to the black hole. This is confirmed by the fact that the narrow lines do not vary detectably, which implies that the emitting region is large, contrary to the broad lines which can vary on relatively short timescales. Due to the random motion, particles speeding toward our line of sight will be blue-shifted while those particles speeding away will acquire red-shifted spectrum. Hence all spectrum band is observable

IV)polarization: Most AGN show at least some level of polarization at optical and radio wavelengths. Strong polarization generally indicates non-thermal emission mechanism, most likely synchrotron emission.

V) strong emission lines: strong nebular emission lines characterize the spectra of most AGN. These lines tend to be broader than those found in starbrust galaxies

VI) small angular size: Most of the IR, Optical, or x-ray emission is released very close to the horizon of the black hole

Despite their common characterstic features, AGNs differ in some respects. However; their classification is a very confusing subject. This is because of observational problems in relation with their orientation to our line of sight. It is not possible to obtain full spectral coverage for all objects and so, it is somehow easy to reconcile a classification based on say, X-ray properties with that of optical emission lines of a different sample. For instance, on the bases of their radio properties, AGNs can be classified into two categories as: radio-loud objects and radio-quiet objects.

Radio-loud objects are characterized by showing:

i) highly relativistic outflows of energetic particles along the pole of the rotating black hole.

ii) collimated radio emitting jets that lead to extended radio sources.

iii) strong radio emission having luminosity $L_{5GHZ} > 10^{24}WHZ^{-1}SR^{-1}$.

Radio-quiet objects characterized by showing:

i) sub-relativistic outflows of particles.

ii) jets are not well collimated.

iii) weaker radio emission with luminosity $L_{5GHZ} < 10^{24}WHZ^{-1}SR^{-1}$.

On the other hand, some active radio galaxies, which seem quiet different, are investigated that they are similar unless their orientations make them so different. With this in mind, researchers put a unifying model which makes all AGNs identical. According to unifying model all AGNs have cylindrical structure and that various spectral types in them come from the difference of viewing angle of this cylindrical system. Even though origin of relativistic jets(outflows) needed to power some AGNs remains a difficult astrophysical problem, they have been put to a good use in testing orientation-based unification schemes for radio galaxies and radio quasars in the 3CR sample. If the axis of radio source is observed within an angle of 45^0 with the line of sight, a radio quasar is observed and, if at a greater angle, a radio galaxy is observed. Radio-jets ejected from radio-loud active galactic nuclei sometimes show apparent velocity exceeding speed of light. The widely accepted explanation for this phenomenon, called superluminal motion, is relativistic jet flow along the direction of the line of sight of the observer. One may ask that for how long will active galactic center exist being active. It is believed that when the black hole, in active galactic center, has eaten all of the gas and dust in its neighborhood then the active galactic nucleus ceases to emit large amounts of radiation. This indicates that there is the end for the unusual activities observed in AGNs. The accreting matter onto central supermassive black hole may exhausts resulting in the declining of high luminosity. Under this condition supermassive black hole remain starved once for all. The lifetime of AGNs can then be estimated for given black hole mass(M) and mass accretion rate($\dot{M}$) as: $t_{AGN} \approx \frac{M}{\dot{M}} \approx 10^8 yrs(\frac{M}{10^8 M_\odot})(\frac{\dot{M}}{M_\odot yr^{-1}})^{-1}$

Chapter 2

Black holes

In stars, the inward gravitational pull is held in equilibrium by the radiation pressure of core reaction. If the star exhausts its supply of nuclear fuel the radiation pressure will ultimately disappear and the star will collapse under its own weight. The collapse may be sudden or gradual, but in any case it can be halted only if an alternative mechanism for generating sufficient pressure becomes available at high density. In white dwarf stars and neutron stars such an alternative mechanism is available. These stars are so dense that the quantum mechanical zero-point pressure becomes dominate. The degenerate Fermi gas of electrons supplies the equilibrium pressure in a white dwarf and a Fermi gas of neutrons supplies the equilibrium pressure in a neutron star. The equation of state(pressure as a function of density)based on this model permits equilibrium configuration, provided that the total mass is below critical upper limit. If the mass exceeds this critical limit for white dwarf, then the electron pressure can not support the star. A rough calculation for critical mass limit is as follows:
Assuming that the gas has uniform density throughout the star, the Fermi momentum for a gas of N electrons in a volume V is:

$$P_F = (\frac{3\pi^2\hbar^3 N}{V})^{\frac{1}{3}} \tag{2.1}$$

and the Fermi energy equals,

$$E_F = \frac{P_F}{2m_e} = N\frac{(3\pi^2\hbar^3 N/V)^{\frac{2}{3}}}{2m_e}. \tag{2.2}$$

The Newtonian gravitational binding energy is of the order of,

$$E_N = -\frac{GM^2}{R}, \tag{2.3}$$

where, M is mass of the star and R is its radius. For stable equilibrium to happen, the net effect of equations: [2.2] and [2.3] should be minimum-zero.
This results in,

$$R = N^{\frac{5}{3}}\frac{(9\pi\hbar^3/4)^{\frac{2}{3}}}{GM^2 m_e}, \tag{2.4}$$

where we have taken, $V = \frac{4\pi R^3}{3}$.
The critical mass beyond which the equilibrium disappears may be estimated by asking when the electron gas turns relativistic. This happens when $P_F \simeq m_e c$ Substituting this into equation [2.1], we get,

$$R = \frac{(9\pi\hbar^3/4)^{\frac{1}{3}}}{m_e c}. \tag{2.5}$$

By comparing equations [2.4] and [2.5] and assuming that the nuclei in white dwarf is Helium, we have the upper critical mass limit to be,

$$M_{crit} = \sqrt{\frac{9\pi}{64}}[\frac{\hbar c}{Gm_n}]^{\frac{3}{2}} m_n. \tag{2.6}$$

Numerically, $M_{crit} = 2.4 \times 10^{33} g \approx 1M_{\bigodot}$. Thus, in our rough calculation, the critical mass should be in the order of one solar mass for white dwarf star. Similar calculation holds for neutron star. According to Chandrasekhar, the maximum limit that neutron star can exist is $1.47M_{\bigodot}$. However; recent observation discovered the neutron star with mass of $2M_{\bigodot}$. For mass beyond this critical upper limit, there is nothing to halt the gravitational collapse of the star. The collapse goes on endlessly. But the endless collapsing is only for the distance observer. For an astronaut, who rides along with the collapsing mass, the collapse takes finite time to end. The collapse results in dimensionless singular point in spacetime where something entered can't gate out. Thus, this region of singularity of spacetime is termed as black hole.

Effect of gravitational force :

The escape velocity for a particle fired away from a massive body of mass,M, and radius r is

$$V = \sqrt{\frac{2GM}{r}}, \tag{2.7}$$

where, G, is universal gravitational constant. A particle with velocity less than this, will return while otherwise escape. The dimensionless quantity, $\frac{2GM}{rc^2}$,obtained from eq: [2.7] is may be regarded as a measure of the strength of the gravitational field. Where, c is speed of light. Since eq: [2.7] is independent of the mass of the particle, we may hope that it also applies to the case of light. If the dimensionless quantity given above approaches unity, it is so hard for a particle with non-zero rest mass to escape the gravitating body. Also, light happens to bend around gravitating body under this condition. The effects become spectacular when the dimensionless quantity equals unity. For example, when the radius r = $\frac{2GM}{c^2}$, the gravitational field of the body is so strong that nothing can escape from its grip. This result was obtained by Michell in 1784 and by Laplase in 1795, who speculated that a sufficiently massive and compact star might appear dark and black. A star appears black if it grips light. And due to its enormous gravitational force, it gulps in all the materials around. Since, we don't have information about the last fate of the materials, we designate the region by a hole. The term "black hole" was then coined by Wheeler in 1968, and has since made a lasting impression. Black hole is very compact celestial object with its fascinating complex mechanisms. It is formed when a very massive star runs out of fuel. Without the power to support its mass, the star implodes and the core collapses to a point of infinite density. According to current theory, black hole can be formed in three different ways.

The first: when star of mass $10^9 M_\odot$ exhaust all its nuclear fueling, it will gravitationally collapse to black hole.

The second: when two neutron stars merge together, they will collapse to black hole.

The third: this is proposed by quantum cosmologist Stephen Hawking. He theorized that trillions of black holes where produced in the Big Bang with some existing today. This theory is not as widely accepted as the other two above.

Two types of black holes are found in the Universe: stellar-mass black holes and super-massive black holes. These are characterized by different masses and formation mechanisms.

A stellar-mass black hole forms when a heavy star collapses under its own weight in a supernova explosion. This happens after the nuclear fuel, which makes the star shine for millions of years, is exhausted. The resulting black hole is a little heavier than our Sun and has an event horizon a few miles across. The existence of such black holes has been inferred in cases where the black hole pulls gas of a companion star that orbits around it. The gas heats up as it falls towards the black hole and then produces X-rays that can be observed with Earth-orbiting satellites.

Super-massive black holes are found in the centers of galaxies that contain billions of stars. They are millions or billions times as heavy as our Sun, as determined from the motions of stars and gas surrounding them. Spectacular activity can occur when gas falls onto the black hole. Some of accreting materials onto black hole will finally be swallowed by the black hole. Hence, black hole is believed to be an astrophysical object consuming the energy budget of the universe behind the closed door. Thanks to the distance limited effect of the black hole, the distant universe's products are saved from being swallowed all in all. Researchers of the field are knocking this door to shed light on the mysteries behind it through different techniques.

Following the prediction of general relativity, theoretical physicists were exploring about the properties of black holes. From mathematical point of view, black holes are solutions of Einstein's field equation. Karl Schwarzschild was the first scientist to find out the solution for Einstein's field equation in 1916. Schwarzschild obtained the solution for static and spherically symmetric gravitational field produced by non-rotating neutral black hole. He characterized a black hole by its mass(M) alone. According to Schwarzschild geometry of spacetime, the metric expression of world-line (line element) of an event in this gravitational field takes the form:

$$ds^2 = (1 - \frac{2GM}{rc^2})dt^2 - (1 - \frac{2GM}{rc^2})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (2.8)$$

The surface of radius $r_s = \frac{2GM}{c^2}$ (Schwarzschild radius) is a surface of infinite-redshift. That is, a clock placed at rest near this radius shows a proper time, $d\tau = \sqrt{1 - \frac{2GM}{rc^2}}dt$ that approaches zero as r → r_s; which is to mean that the clock runs infinitely slow compared with a clock at a large distance. The vanishing of $d\tau$ is characteristic of the worldline of a light signal and hence only a light signal, aimed in outward direction, can remain at rest at r_s. This radius is the radius of the surface on which light is infinitely redshifted. The infinite-redshift surface is sometimes called the static limit, because particles with velocity less than that of light will not remain at rest in this surface.

The radius of this surface is obtained from the blowing up of the coefficient of, dr^2, in eq:[2.8],

i.e $\quad \big(1 - \frac{2GM}{rc^2}\big)^{-1} \rightarrow \infty \qquad \Rightarrow \qquad r = \frac{2GM}{c^2},$

Or when $r = \frac{2GM}{c^2}$, g_{rr} becomes infinite. However; $r = \frac{2GM}{c^2}$ is not an intrinsic singularity, because spacetime itself is not singular there. The particles may remain stationary at a certain point (r, θ, ϕ), out side a black hole ($r > \frac{2GM}{c^2}$), but it is impossible for a particle inside the black hole ($r < r_s$) to be at rest, i.e to maintain fixed values of co-ordinates (r, θ, ϕ). The argument runs as follows: We recall that the worldline of any particle, of non zero rest mass, must be time-like, namely the separation $d\tau^2$ along the worldline is always positive. However; for a particle at rest with $dr = d\theta = d\phi = 0$, $d\tau^2 = (1 - \frac{2GM}{rc^2})dt^2$. So that when $r < \frac{2GM}{c^2}$, $d\tau^2$, is negative which means that it is impossible for a particle to have $\triangle r = 0$.

Forty seven years later Roy Kerr obtained axially symmetric, stationary, but not static gravitational field produced by rotating neutral black hole. In his geometry of spacetime, adopted in Boyer-Lindquist coordinates, the worldline(line element) of an event has the form:

$$ds^2 = (\frac{4aM^2r^2sin\theta - \rho^4\Delta}{\Sigma^2\rho^2})dt^2 - \frac{4Mr}{\rho^2}a\sin^2(\theta)\,dt\,d\phi + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2}{\rho^2}sin^2(\theta)d\phi^2, \tag{2.9}$$

where, $\rho^2 \equiv r^2 + a^2cos^2\theta$,
$\Sigma^2 \equiv (r^2 + a^2)^2 - a^2\Delta sin^2\theta$,
$\Delta \equiv r^2 - 2Mr + a^2$,
and 'a' is the angular momentum(J) per unit mass of black hole(M) and a $\epsilon\,(0, M)$. The units used in Kerr metric solution is geometrical units in which r, M and a have units of length.

This solution is different from the solution obtained by Schwarzschild. Hence according to Kerr, black hole is characterized by: its mass (M) and angular momentum (J). Kerr solution degenerates to Schwarzschild solution under zero angular momentum condition.

Two years later another scientist Ezra Newman found the solution for Einstein's field equation around rotating, electrically charged black hole. This solution is called the Kerr-Newman solution.

In general black holes are characterized by all or either of the three parameters: mass(M), angular momentum(J), and charge(Q).

By "characterization" we mean that black holes interact with the external environment only through the effects cause by mass, charge and angular momentum; no further interaction comes into play. Non-rotating neutral black hole (Schwarzschild black hole) is characterized only by its mass (M).

Non-rotating charged black hole (Reissner-Nordstrm black hole) is characterized by its mass(M),and charge (Q).

Rotating neutral black hole (Kerr black hole) is characterized both by its mass (M), and angular momentum (J).

Rotating charged black hole (Kerr-Newman black hole) is characterized by its mass(M), angular mumentum (J), and charge (Q).

Rotating black holes are thought to be formed in the gravitational collapse of a massive rotating star or from the collapse of a collection of stars with an average non-zero angular momentum. Most stars rotate and hence an evidence for most black holes in nature to be rotating black holes. Rotating black hole can release an energy at the expense of its rotation in the presence of strong magnetic field. Black holes were long viewed more as a mathematical curiosity rather than a physical reality. This changed in the 1960s when they became the favored explanation for active galaxies. The extreme luminosity of AGN is explained by an accreting central supermassive black hole. A black hole, by its very nature, is observationally elusive. Progress has, however, been made on several fronts over the past few years. Optical, infrared, and radio studies of the nuclei of several galaxies, including our own, have revealed the presence of large masses within small radii that can only plausibly be black holes Near-infrared observations of the nucleus of our own Galaxy show the proper motion (that is, motion perpendicular to the line of sight) of stars there. This motion increases toward the position of the Galactic Center, which is coincident with the radio source Sgr A*. The stars are presumably in orbit about the Galactic Center that must then have a mass of $2.5 \times 10^6 M_\odot$. The activity in a galaxy ceases when the black hole runs out of "fuel", or when this fuel stops being efficiently transformed into radiation. Hence, a normal galaxy might have had an active phase in the past, and if so would still have a massive black hole lurking in its center. To test this idea, astronomers have used the Hubble telescope not only to study the centers of active galaxies but also of normal galaxies. Indeed, black holes are found in normal galaxies as well and may even exist in the centers of all galaxies. In our own Milky Way galaxy, ground-based observations detected rapidly moving stars. These are believed to move under the influence of the gravitational pull of a black hole with 3 million times the mass of our Sun.

2.1 Kerr Black hole

The dynamics of a particle near a black hole is determined by the curvature of spacetime around the black hole. A spinning black hole drags the surrounding spacetime into a tornado-like circulating motion and allows matters to orbit closer to it than is possible for a non-spinning black hole. Rotation of black hole can not be arbitrarily large as it is limited by the maximum value of the angular momentum per unit mass of black hole $(a = M)$. The angular velocity of the central black hole plays an important role in determining not only the space-time geometry exterior to it but also the dynamics of plasma fluid and the configuration of electromagnetic field. We select Kerr metric to express the designs of the spacetime, electrodynamics and plasma dynamics and their interaction in the vicinity of rotating black hole. As stated above, Kerr geometrized the spacetime, around rotating and neutral black hole, as stationary and axially symmetric. In this spacetime, the metric expression of squared line-element of an event has the form:

$$(ds)^2 = g_{tt}(dt)^2 + g_{t\phi}dtd\phi + g_{\phi\phi}(d\phi)^2 + g_{rr}(dr)^2 + g_{\theta\theta}(d\theta)^2. \tag{2.10}$$

Refereing to the above eq:[2.9], we have the relations:

$g_{tt} = (\frac{4aM^2r^2sin\theta - \rho^4\Delta}{\Sigma^2\rho^2})$, $g_{t\phi} = -(\frac{4Mr}{\rho^2})asin^2\theta$, $g_{\phi\phi} = (\frac{\Sigma^2}{\rho^2}sin^2\theta)$,
$g_{rr} = \frac{\rho^2}{\Delta}$, $g_{\theta\theta} = \rho^2$,

where the off diagonal metric element, $g_{t\phi}$, arises because of the rotation of central black hole.

The gravitational field of a rotating mass differs from that of non-rotating mass by the presence of the off diagonal components. The determinant of the Kerr metric elements, given above, is $-g = \rho^4 sin^2\theta \Rightarrow \sqrt{-g} = \rho^2 sin\theta$.

The field of Kerr black hole, in steady rotation, possesses two evident symmetries: stationarity ($\partial_t g_{\mu\nu} = 0$) and axial symmetricity

($\partial_\phi g_{\mu\nu} = 0$). These axial symmetry and stationarity of gravitational field of the central rotating black hole greatly reduce the complexity of the governing equations. The gravitational field is thus dependent on the poloidal co-ordinates ($\theta \quad and \quad r$).

To understand the dynamics around black hole, we can put fiducial observer, an observer who is at rest with respect to black hole. In the case of rotating black hole, this observer has special name called "zero angular momentum observer (ZAMO)" for he is co-rotating with the black hole having constantly zero velocity along poloidal components. Since ZAMO's trajectory is not geodesic, he feels gravitational field or his clock delays. This effect is quantified by the redshift parameter called lapse function α as:

$$\alpha = (\frac{d\tau}{dt})_{fo}, \tag{2.11}$$

where, 'fo' stands for fiducial observer and the time coordinate 't' is universal time. The lapse function vanishes at the horizon which inturn means that the redshift parameter, Z, defined by:

$$Z \equiv \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}} = \frac{1}{\alpha} - 1 \tag{2.12}$$

becomes infinite. This is the reason for blackness of black hole.

Due to frame dragging effect of Kerr black hole, the spatial co-ordinates, of the inertial frame, have velocity with respect to the ZAMO. This velocity is termed as shift vector

(β), which can be defined as:

$$\beta^i = (\frac{dx^i}{dt})_{fo}. \tag{2.13}$$

This shift vector, or sometimes called frame dragging frequency, falls off very rapidly with radius, i.e ($\beta^i \propto r^{-3}$). It parameterizes the rotation of Kerr space-time. The 4-velocity of the fiducial observer ($\vec{n}$) can be expressed interms of lapse function and shift vector as:

$$\vec{n} = \frac{d}{d\tau} \quad \xrightarrow[(t,\phi,r,\theta)]{} \quad (\frac{1}{\alpha}, -\frac{\beta^i}{\alpha}). \tag{2.14}$$

The lapse function and the shift vector, in Boyer-Lindquist coordinate system, are given by:

$$\alpha = \sqrt{g_{tt} + g_{t\phi}^2/g_{\phi\phi}} = \frac{\rho}{\Sigma}\sqrt{\Delta}, \tag{2.15}$$

and,

$$\beta = \frac{g_{t\phi}}{g_{\phi\phi}} \quad \xrightarrow[(\phi,r,\theta)]{} \quad (\frac{-2Mra}{\Sigma^2}, 0, 0). \tag{2.16}$$

Therefore, the 4-velocity of the ZAMO is rewritten as:

$$\vec{n} \quad \xrightarrow[(t,\phi,r,\theta)]{} \quad \frac{\Sigma}{\rho\sqrt{\Delta}}(1, \frac{-2Mra}{\Sigma^2}, , 0, 0). \tag{2.17}$$

ZAMO's angular velocity is given as:

$$\Omega = \frac{n^\phi}{n^t} = \frac{2Mra}{\Sigma^2}. \tag{2.18}$$

In particular, on the event horizon the angular velocity of ZAMO has the form,

$$\Omega \Rightarrow \Omega_H = (\frac{a}{r_+^2 + a^2}), \tag{2.19}$$

where, Ω_H and r_+ are the angular velocity of black hole and radius of outer event horizon.

Gravitational field, Υ, on fiducial observer is:

$$\Upsilon = -\frac{1}{\alpha}\nabla\alpha. \tag{2.20}$$

That is,

$$\Upsilon = -(\frac{k}{\rho^3\Sigma^2\sqrt{\Delta}})\sqrt{\frac{\Delta}{\rho^2}}\frac{\partial}{\partial r} + (\frac{2Mra^2(r^2+a^2)}{\Sigma^2\rho^3})\frac{cos\theta sin\theta}{\sqrt{\rho^2}}\frac{\partial}{\partial\theta}, \tag{2.21}$$

where, $k = M\rho^2(r^4 - a^4) + 2Mr^2a^2\Delta sin^2\theta$.

With respect to the fiducial observer, the dynamics of a particle around the Kerr black hole will have the following form:

$$d\tau_p = \frac{dt}{\gamma} = \frac{d\tau_{fo}}{\alpha\gamma} = \frac{d\tau_{fo}}{\Gamma}, \tag{2.22}$$

where Γ is Lorentz factor against FIDO.

2.2 Black hole's event horizon

It is mathematically defined, locally undetectable membrane, around black hole, beyond which a distant observer is unable to observe events and hence called "event horizon". Now it is known that this horizon membrane behaves as though it were a membrane with a variety of physical properties. The existence of event horizon around a black hole, is evidenced by the absence of certain type of x-ray explosion, which is observed around neutron star. The idea of using the absence of X-ray bursts to confirm the presence of event horizons around black holes was proposed in 2002 by Harvard's Narayan and Dr. Jeremy Heyl of the University of British Columbia in Vancouver. According to the researchers, the observed x-ray bursts on hard surfaces of neutron stars, could not be observed on black holes. This one way gate horizon rotates rapidly that any object near the horizon, in falling or not, is dragged along with the rotating horizon due to black hole's extremely high gravitational force. Karl Schwarzschild was the first physicist to derive a formula for the radius of event horizon of non-rotating neutral black hole, $r_s = 2M$.

Just below this radius no observer/particle can maintain itself at a constant radius. It is forced to fall inwards. From mathematical point of view, any object of non-zero rest mass can end up with black hole as long as it fulfills the following minimum but physically impossible requirements: The object must, by some magic, collapse to a radius less than r_s or set into motion to acquire a light speed.

Later on, in more advanced way, Kerr calculated out two radii (inner radius and outer radius) of event horizons for rotating neutral black hole. These radii of the event horizons are the radii at which the coefficient of the dr^2 term of the Kerr metric blows up. i.e when,

$$\Delta = r^2 - 2Mr + a^2 = 0 \quad \Rightarrow \quad r_{\pm} = M \pm \sqrt{M^2 - a^2}, \tag{2.23}$$

where, r_- is the radius of inner event horizon and r_+ is the radius of outer event horizon. At r = r_+ and r = r_-, a light signal necessarily has zero velocity in the radially outward direction. We, here, focus on the outer event horizon. The surface of infinite-redshift and event horizon coincide in the case of Schwarzschild solution. In Kerr geometry, this infinite-redshift concept is complex. We encounter two infinite-redshift surfaces, where $g_{tt} = 0$

i.e $g_{tt} = 1 - \frac{2Mr}{\Sigma} = 0$

$\Rightarrow \quad r^2 + a^2cos^2\theta - 2Mr = 0 \quad \Rightarrow \quad r_{\pm} = M \pm \sqrt{M^2 - a^2cos^2\theta}.$

Here, in Kerr geometry, the vanishing of $g_{tt} = 0$, merely tells us that a particle can not be at rest(with $dr = d\theta = d\phi = 0$) at these surfaces.

These surfaces, like Shwarzschild case, do not correspond to any physical singularity. The two infinite-redshift surfaces are distinct from the two event horizons except at the poles.

Consider the figure below:

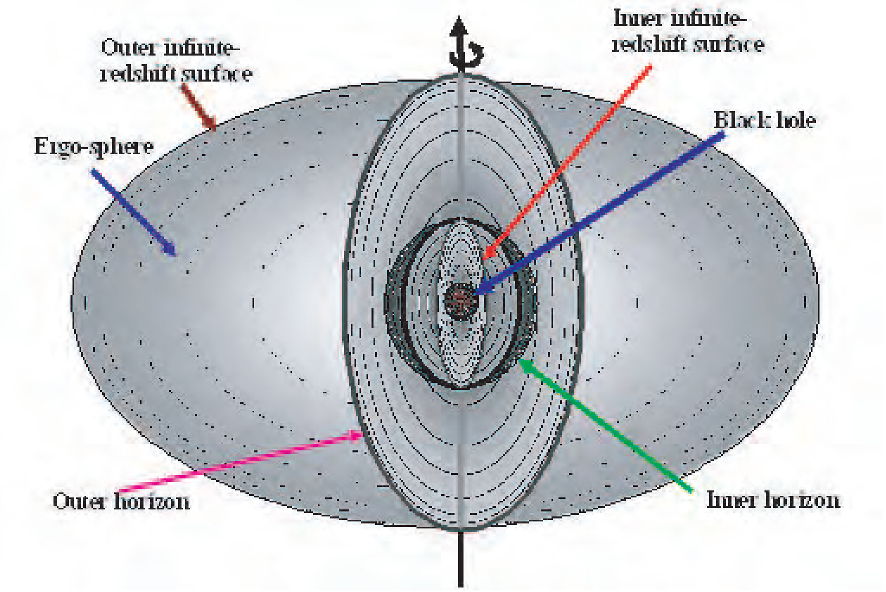

Figure 2.1: The two event horizons, and infinite-redshift surface

The region between the outer event horizon and the outer infinite-redshift surface is termed as an ergo-region. Geometrically, it is stretched out along the equatorial plane and meets the outer event horizon at the poles as shown in the fig: [2.1]. The ergo-region exists only for spinning black holes. Particles falling into the ergo-region are forced to rotate fast(with angular velocity of black hole) and thereby gain energy. Because they are still outside the event horizon, the particles can escape from the black hole. The net process is that the rotating black hole emits energetic particles at the cost of its own total energy.

One thing to be noted is that for an astronaut making a trip to massive black hole, locally, nothing un usual is there till she crosses the event horizon where she starts to feel a tidal force and difficulty to turn back. Rotating black hole is different from Schwarzschild black hole in that the rotation of the black hole will cause the creation of two (inner and outer) event horizons. As the spin of the black hole increases, the inner event horizon moves outward, and the outer ones moves inward. If the spin is great enough (a = M), the two will eventually merge and shrink to form a single singularity. With no event horizons to hide it from the rest of the universe, the black hole ceases to be black and will instead be a naked singularity.

In the region between two event horizons, $g_{rr} > 0$ and hence r is time-like co-ordinate. As in the Schwarzschild case, the r dependance of the metric therefore implies that the metric in this region is necessarily dynamic. Neither at the infinite-redshift surfaces nor at the horizons does the Kerr geometry develop a true singularity. The only true singularity occurs at r = 0. This true singularity can be checked using the behavior of curvature tensor. Curvature is an intrinsic geometric property and its singularity is independent of any coordinate system. Indeed, the curvature tensor diverges at r = 0. The diverging nature of the curvature at this point indicates that the energy and momentum becomes infinitely large in this region of spacetime , which in turn implies that the black hole is confined in small region of spacetime. Thus, black hole may has another alternative definition as: a finite region of spacetime into which signals can enter, but from which no signal can ever emerge. Following this, we can assume that spacetime between the black hole and its event horizon is comparatively empty and hence, the solutions obtained so far, for Einstein field equation, are exterior vacuum solutions.

2.3 Electromagnetic fields in Kerr spacetime

The Maxwell's equations, expressed in tensor form in curved spacetime, acquire the following structure:

the source-free Maxwell equations,

$$F_{[\mu\nu\,:\,\alpha]} = 0, \qquad \mu, \nu, \alpha \quad are\, cyclic, \tag{2.24}$$

and, the inhomogeneous Maxwell's equations:

$$F^{\mu\nu}_{:\nu} = \frac{1}{\sqrt{-g}}(\sqrt{-g}F^{\mu\nu}), \nu = 4\pi J^{\mu}, \tag{2.25}$$

where J^{μ}, is the generalized 4-current density. In Kerr spacetime, the above Maxwell's equations reduce to the simplest form:

$$F_{tr,\theta} + F_{\theta t,r} = F_{\phi r,\theta} + F_{\theta\phi,r} = 0, \tag{2.26}$$

and,

$$-[\frac{\Delta sin\theta}{\alpha^2}(F_{tr} - \beta F_{\phi r})], r - [\frac{sin\theta}{\alpha^2}(F_{t\theta} - \beta F_{\phi\theta})], \theta = 4\pi\sqrt{-g}J^{t}, \tag{2.27}$$

$$[\frac{\Delta sin\theta}{\alpha^2}(\beta F_{tr} - \frac{g_{tt}}{g_{\phi\phi}}F_{\phi r})], r + [\frac{sin\theta}{\alpha^2}(\beta F_{t\theta} - \frac{g_{tt}}{g_{\phi\phi}}F_{\phi\theta})], \theta = 4\pi\sqrt{-g}J^{\phi}, \tag{2.28}$$

$$(\frac{\Delta sin\theta}{\rho^2}F_{r\theta}), \theta = 4\pi\sqrt{-g}J^{r}, \tag{2.29}$$

$$(\frac{\Delta sin\theta}{\rho^2}F_{\theta}r), r = 4\pi\sqrt{-g}J^{\theta}, \tag{2.30}$$

respectively.

Let us define the quantities:

$$\psi_{\theta} \equiv \Sigma\frac{F_{t\theta} - \beta F_{\phi\theta}}{\rho^2\Delta}, \tag{2.31}$$

$$\psi_{r} \equiv \Sigma\frac{F_{tr} - \beta F_{\phi r}}{\rho^2}, \tag{2.32}$$

$$B_{\phi} \equiv (\frac{\Delta sin\theta}{\rho^2})F_{r\theta}, \tag{2.33}$$

$$j^{\phi} \equiv J^{\phi} + \beta J^{t}, \tag{2.34}$$

where, j^ϕ, α, β, are the electric current density measured by ZAMO, lapse function and shift vector respectively. Using the azimuthal co-ordinate killing vector, $\vec{\xi}_{(\phi)}$, we can find that the magnetic vector potential, $A_\phi(r, \theta)$, is constant along each azimuthal flow line. That is: $A_{\mu,\nu}\xi^\nu_{(\phi)} = A_{\mu,\phi} = 0$

Thus, we have four independent variables: $\psi_r, \psi_\theta, B_\phi$, and A_ϕ. In terms of the above defined quantities, the inhomogeneous Maxwell's equations are,

$$\frac{1}{sin\theta}B_{\phi,\theta} = 4\pi\rho^2 J^r \quad , \tag{2.35}$$

$$\frac{1}{sin\theta}B_{\phi,r} = -4\pi\rho^2 J^\theta \quad , \tag{2.36}$$

and,

$$(\frac{\Delta}{g_{\phi\phi}}A_{\phi,r}), r + \frac{1}{sin\theta}(\frac{sin\theta}{g_{\phi\phi}}A_{\phi,\theta}), \theta = -4\pi\rho^2 j^\phi. \tag{2.37}$$

The homogeneous Maxwell's equation becomes,

$$(\frac{\rho^2}{\Sigma}\psi_r), \theta - (\frac{\rho^2\Delta}{\Sigma}\psi_\theta), r = \beta_{,r}A_{\phi,\theta} - \beta_{,\theta}A_{\phi,r} = 0. \tag{2.38}$$

2.4 Magnetohydrodynamics around Kerr black hole

The study of dynamics of magnetized plasma fluid around Kerr black hole is an essential part to break up the puzzle of jet generation and collimation Mechanisms. The accreting plasma fluid around kerr black hole induces electromagnetic fields. This electromagnetic field has an effect to perturb the dynamics of the plasma fluid. However; the effect of both plasma fluid and electromagnetic field is of no consequence on the geometry of spacetime laid by the rotating Kerr black hole. The stress energy-momentum tensor of the plasma fluid and electromagnetic field is given as,

$$T^{\mu\nu} = (\rho_o + \varepsilon + p)U^\mu U^\nu + pg^{\mu\nu} + \frac{1}{4\pi}(F^{\mu\gamma}F^\nu_\gamma - \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}), \tag{2.39}$$

where,

$$T^{\mu\nu}_{MT} = (\rho_o + \varepsilon + p)U^{\mu}U^{\nu} + pg^{\mu\nu} \tag{2.40}$$

is matter stress energy-momentum tensor. Where, $\rho_o \equiv$ rest-mass density, $\varepsilon \equiv$ internal energy, p $\equiv$ pressure, $U^{\mu} \equiv$ the fluid four-velocity.
The stress energy-momentum tensor for the electromagnetic field is,

$$T^{\mu\nu}_{EM} = \frac{1}{4\pi}(F^{\mu\gamma}F^{\nu}_{\gamma} - \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}), \tag{2.41}$$

where, refereing to equation [2.24], $F^{\mu\nu}$ can be expressed in terms of generalized vector potential A_{μ} as:

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{2.42}$$

Close to the Kerr black hole, the magnetic field is so strong that it controls the dynamics of plasma fluid. In this region of spacetime, the inertia of the plasma fluid is neglected. The condition under which the inertia of the plasma fluid is taken to be negligible is known as 'force-free' condition. In reality, accreting plasma fluid is not so free in all directions. The plasma fluid is squeezed radially by the coupled effects of gravitational force and centrifugal force. But the non-radial sites are free of any such external forces whenever we infer the Kerr geometry. Hence, we can talk about the non-radial freeness of the plasma fluid. Traditionally, the 'force-free' equation is expressed as:

$$F_{\mu\nu}J^{\nu} = 0. \tag{2.43}$$

In terms of the defined quantities in section (2.3), the force free condition is expressed as:

$$\psi_r J^r + \Delta\psi_{\theta}J^{\theta} = 0, \tag{2.44}$$

$$A_{\phi,\,r}J^r + A_{\phi,\,\theta}J^{\theta} = 0, \tag{2.45}$$

$$\psi_r J^t - A_{\phi,\,r} j^\phi - \big(\frac{\rho^2}{\Delta sin\theta} B_\phi\big) J^\theta = 0, \tag{2.46}$$

$$\Delta\psi_\theta J^t - A_{\phi,\,\theta} j^\phi + \big(\frac{\rho^2}{\Delta sin\theta} B_\phi\big) J^r = 0. \tag{2.47}$$

Following eq: [2.43], we deduce the degenerate electromagnetic field,

$$^*F^{\mu\nu} F_{\mu\nu} = 0, \tag{2.48}$$

where $^*F^{\mu\nu}$, is dual to $F_{\mu\nu}$ and has the relation: $^*F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ in the case of highly ionized plasma, where this is so near the black hole. $\epsilon^{\mu\nu\alpha\beta} = -(1/\sqrt{-g}\,)[\mu\nu\alpha\beta]$, is covariant form of antisymmetric Levi-Civita tensor. In the force-free limit the governing equations are then,

$$T^{\mu\nu}_{EM:\,\nu} = 0 \qquad and \qquad F_{[\mu\nu:\,\alpha]} = 0. \tag{2.49}$$

Expanding eq:[2.48] we have,

$$A_{\phi,\,\theta} A_{t,\,r} - A_{t,\,\theta} A_{\phi,\,r} = 0. \tag{2.50}$$

This may be re-written as:

$$\frac{A_{t,\,\theta}}{A_{\phi,\,\theta}} = \frac{A_{t,\,r}}{A_{\phi,\,r}} = -\omega(r,\,\theta), \tag{2.51}$$

where, $\omega(r,\theta),$ is usually interpreted as the " rotation frequency " of electromagnetic field. $\omega(r,\theta)$, is constant along each flowline only varying from flowline to flowline. This yields $F_{\mu\nu}$ in terms of the free functions ω, A_ϕ, and B^ϕ- the toroidal magnetic field:

$$F_{tr} = \omega A_{\phi,\,r} \quad F_{t\theta} = \omega A_{\phi,\,\theta} \quad F_{r\theta} = \sqrt{-g} B^\phi \quad F_{r\phi} = A_{\phi,\,r} \quad F_{\theta\phi} = A_{\phi,\,\theta} \tag{2.52}$$

with all other components zero.

Let's define the electromagnetic field components in terms of the conserved quantities in Kerr spacetime.

Using the Maxwell's homogenous equation [2.26] and the expanded 'force-free' condition which has the forms:

$$F_{tr}U^r + F_{t\theta}U^\theta = 0, \tag{2.53}$$

$$F_{rt}U^t + F_{r\theta}U^\theta + F_{r\phi}U^\phi = 0, \tag{2.54}$$

$$F_{\theta t}U^t + F_{\theta r}U^r F_{\theta\phi}U^\phi = 0, \tag{2.55}$$

and,

$$F_{\phi r}U^r + F_{\phi\theta}U^\theta = 0, \tag{2.56}$$

we get:

$$\frac{d}{d\tau}ln(F_{tr}) = -U^\theta(\frac{U^r}{U^\theta}),r\,, \tag{2.57}$$

and,

$$\frac{d}{d\tau}ln(F_{\phi r}) = -U^\theta(\frac{U^r}{U^\theta}),r. \tag{2.58}$$

For detail information see appendix.

Subtracting equation [2.57] form equation [2.58],we get:

$$\frac{d}{d\tau}(ln(\frac{F_{tr}}{F_{\phi r}})) = 0. \tag{2.59}$$

Refering to equation [2.51], we see that:

$$\frac{F_{tr}}{F_{\phi r}} = -\omega \tag{2.60}$$

is proved to be constant. Equation [2.58] yields,

$$\frac{d}{d\tau}ln(F_{\phi r}) = -U^r,r + U^\theta,r\frac{U^r}{U^\theta}, \tag{2.61}$$

where, in Kerr geometry:

$$U^{\mu}_{,\mu} = U^{r}_{,r} + U^{\theta}_{,\theta}\,, \quad and \quad U^{\mu}U^{\theta}_{\mu} = U^{r}U^{\theta}_{,r} + U^{\theta}U^{\theta}_{,\theta}. \tag{2.62}$$

Substituting equation [2.62] into equation [2.61], we get:

$$\frac{d}{d\tau}ln(F\phi r) = -U^{\mu}_{,\mu} + \frac{d}{d\tau}ln(U^{\theta}). \tag{2.63}$$

Mass conservation law is expressed as;

$$(\rho_o U^{\mu}), \mu = -\frac{d}{d\tau}ln(\sqrt{-g}\rho_o). \tag{2.64}$$

Fall derivation can be obtained in the appendix.

Substituting this into equation [2.63] we get another constant(k) as,

$$k = \frac{F_{\phi r}}{\sqrt{-g}\rho_o U^{\theta}} \quad \Rightarrow F_{\phi r} = k\sqrt{-g}\rho_o U^{\theta}. \tag{2.65}$$

Thus, magnetic field components are given as;

$$F_{ta} = \omega F_{a\phi},\ (a = r,\ \theta), \tag{2.66}$$

$$F_{\phi r} = k\sqrt{-g}\rho_o U^{\theta}, \tag{2.67}$$

$$F_{\phi\theta} = k\sqrt{-g}\rho_o U^{r}, \tag{2.68}$$

$$F_{r\theta} = k\sqrt{-g}\rho_o(\omega U^{t} - U^{\phi}), \tag{2.69}$$

2.5 Energy extraction from rotating black

The possibility of extracting spin energy from a rotating black hole was first proposed by the mathematician Roger Penrose in 1969 and is thus called the Penrose process. Rotating black holes in astrophysics are a potential source of large amount of energy and are used to explain energetic phenomena, such as gamma ray bursts. Recent observation of black hole candidates (BHCs) in our galaxy suggests that galactic superluminal sources contain very rapidly rotating black holes (a = 0.9 to 0.95) while the black hole in Cyg x-1 1124-68 spinning less rapidly (a = 0.3 to 0.5) (Cui Zhang and and Chen 1998).

All AGNs will not produce relativistic jets (outflows) even though they harbor massive black holes in their centers. The great majority of AGNs are radio quiet which do not produce powerful relativistic radio jets. Objects with superluminal jets, as GRS 1915+105 and GRO J1655-40, belong to the radio loud class (Mirabel and Rodriguez 1995). Other objects such as Cyg x-1 and GS 1124-68, in which observation shows that these class of objects harbor slowly or non-rotating supermassive black holes, are relatively radio quiet and produce little or no jet. Numerical simulations for Schwarzschild (non-rotating black hole) metric show only subrelativistic jet flow (Koide, Shibata and Kudoh in 1999). The energetic relativistic jets are commonly observed in AGNs that contain rotating black holes in their centers. Hence the rotation of central black hole must have close relation with relativistic jets production. The evidence for the rotation of black hole is particularly proposed in the center of our own galaxy. Recent flare observations in the vicinity of Sgr A reinforce the existence of rotating black hole. Rotation is nature of many cosmic objects like planets, stars, and galaxies. Black holes are the offsprings of these class of objects(stars) and, thus, may retain the rotation nature after birth, though few of there angular momentums radiate away by emission of gravitational waves. For the super-Eddington luminosity observed mostly around AGN that harbor rotating black hole, the free energy that is extractable from this rotating black hole is one fundamental reason. Rotational energy could escape from a spinning black hole when it is in a strong magnetic field and is conveyed into the inner most parts of the accretion disk.

It was calculated by Christodoulou, in 1970 that for the Kerr black hole, about 29 % of the total energy can be extracted.

Using the equations given in previous section, we can evaluate the amount of spin energy and angular momentum that can be extracted from rotating black hole.

Following the geometry of Kerr spacetime and the stress energy-momentum of equation[2.39], the conserved energy and the angular momentum fluxes are defined respectively as:

$$e^{\mu} = T^{\mu\nu}\xi_{(t)\nu}, \tag{2.70}$$

and,

$$l^{\mu} = -T^{\mu\nu}\xi_{(\phi)\nu}, \tag{2.71}$$

where, $\xi_{(t)\nu}$ and $\xi_{(\phi)\nu}$ are temporal and azimuthal co-ordinate killing vectors respectively.

The radial component of energy and momentum fluxes,due to radial differential rotation, are then expressed as,

$$e^{r} = (\rho_o + \varepsilon + p)U^{r}U_{t} + \frac{1}{4\pi}F^{r\theta}F_{\theta t}, \tag{2.72}$$

and,

$$l^{r} = -(\rho_o + \varepsilon + p)U^{r}U_{\phi} - \frac{1}{4\pi}F^{r\theta}F_{\phi\theta}. \tag{2.73}$$

Similarly, vertical differential rotation gives raise to vertical fluxes of energy and angular momentum. These are expressed as,

$$e^{\theta} = (\rho_o + \varepsilon + p)U^{\theta}U_{t} + \frac{1}{4\pi}F^{\theta r}F_{tr}, \tag{2.74}$$

and,

$$l^\theta = -(\rho_o + \varepsilon + p)U^\theta U_\phi - \frac{1}{4\pi}F^{\theta r}F_{\phi r} \tag{2.75}$$

respectively.

Whether the fluxes of energy and angular momentum are inward or away from the central gravitating black hole are determined by the relative velocity of black hole's angular velocity to angular velocity of magnetic field lines. For example, using equation:[2.73] we can determine the direction of radial angular momentum flux as:

$$l^r = -(\rho_o + \varepsilon + p)U^r U_\phi + \frac{1}{4\pi}B_\phi A_{\phi,\theta}, \tag{2.76}$$

where, $B_\phi = F^{r\theta}$, and $F_{\phi\theta} = A_{\phi,\theta}$.

According to Komissarov S.S in 2004, we have the expression:

$$B^\phi = \frac{\alpha H_\phi - B^r sin^2\theta(2r\Omega - a)}{\Delta sin^2\theta}, \tag{2.77}$$

where, H_μ and B^μ are the magnetic field components measured by the ZAMO and the distant observer respectively. From the given expression, for the B^ϕ to remain finite at the horizon (where, $\Delta \to 0$), the numerator has to vanish as well and hence, we obtain:

$$B_\phi = \frac{(2r_+\Omega - a)sin\theta}{r_+^2 + a^2cos^2\theta}A_{\phi,\theta}, \tag{2.78}$$

with the condition, $B_\phi = \alpha H_\phi$, under the assumption that the poloidal component E_θ is negligible compared to magnetic fields.

This leads to:

$$l^r = \frac{(r_+^2 + a^2)(\Omega_H - \omega)}{4\pi(r_+^2 + a^2cos^\theta)^2}. \tag{2.79}$$

Since in ergo-sphere $U_\phi = 0$ (i.e is because of the ZAMO being an observer in a plasma fluid rest frame) in equation: [2.76], the first term on the right side vanishes. Hence, inside the ergo-sphere, the angular momentum flux is not due to differential rotation in plasma fluid. It is angular velocity of magnetic field that determines the flux and its direction. From equation: [2.79], we see that the angular momentum flux is radially out ward if $\Omega_H < \omega$ otherwise the flux is inward, if $\Omega_H > \omega$.

The Kerr black hole is, thus, communicates with the external environment by releasing out its rotation energy and angular momentum which is expressed in equation [2.79] form. Here, when we say, " rotating black hole releases energy and angular momentum", we do not mean that these energy and angular momentum come out of the black hole crossing the event horizon. It is to mean that the particles falling into rotating black hole dissipate their energy and angular momentum, due to differential rotation, in the external environment. The energy and angular momentum left outside of the black hole are the consequence of black hole rotation. And hence, we can say that rotating black hole releases out energy and angular momentum.

Chapter 3

Accretion disk around rotating black hole

Compact(high density) object curves spacetime around itself creating a kind of gravitational crater. Hence a body approaching a compact object(like black hole) will either constantly orbits (accrete) or escape or fail onto the central black hole depending on the velocity of the body and the amount of the curvature of the spacetime around the black hole. The trajectory of accreting matters in the vicinity of compact object shows the behavior of the local spacetime. For non-rotating black hole the accretion of matters has spherical (omni-directional) symmetry while accretion onto rotating black hole has axial symmetry (disk like structure perpendicular to the axis of rotating black hole). This disk-like feature of accreting materials acquire the name "accretion disk". Along with neutron stars and black holes, accretion disks are one of a handful of astrophysical objects whose existence was predicted well in advance of observation. The detailed study of the physics of accretion really did not begin seriously until Lynden-Bell (1969) put forth his disk/black-hole model as the central power source for quasars. Observation evidenced that most if not all systems contain accretion disk around the central source. Accretion disks are observed around: newly formed stars and proto-planets, binary star systems, and close to the center of active galaxies. Here, we focus on the disks in AGNs. There are three force acting on the accreting materials onto compact rotating object. These are inward gravitational force, centrifugal force and and outward radiative force(radiation from the innermost accreting matters). At certain region around gravitating object, the inward gravitational force is balanced by the two outward forces and hence the accreting matters orbit the central gravitating objects without falling in. This orbit,as we will see in the next subsection, is known as stable obit in which materials confined. However; it does not mean that this is the last fate of the accreting materials. The accreting materials at the innermost stable orbit have higher velocity compared to that of the outermost parts. This differential rotation leads to viscosity. Due to the action of viscosity, we expect energy and angular momentum to be transformed from the faster-moving inner regions of the disk to the slower-moving outer regions. As the materials in an inner layer loses angular momentum, it spirals inward. Hence, it is viscosity which determines the rate of radial inflow of matter and therefore the rate at which the gravitational potential energy is converted into other forms, like radiation. If the materials had no viscosity, then they would keep on going in circular orbits and there would be no release of gravitational energy after the formation of the disk. Mathematically, we can explain the gravitational potential energy released by the accreting materials around compact object as: $E_{acc} = \frac{GMm}{R}$, where G, M, m, R represents gravitational constant, mass of central compact object, mass of accreting matter and radius of central compact object respectively. If the central compact object is a rotating black hole then, R is the radius of the outer event horizon. In fact accretion disk of materials around neutron star or black hole is the disk of plasma(electrically conducting)fluid with high temperature. Due to high radiation in the central region, the materials in accretion disk will be ionized and then form plasma. This plasma has the properties of high conductivity and diamagnetism.

3.1 The inner most stable orbit

Accreting plasma fluids do not continuously fall inward. They have to confine themselves in the region where effective(net)potential is found to be minimum. This region can be evaluated from the squared line element of dynamic particle in Kerr spacetime given in eq:[2.9]. Assuming the disk to be thin and the particle is constrained on an equatorial plane where $\theta = \frac{\pi}{2}$. Hence, the line element is re-written as:

$$(ds)^2 = (\frac{4aM^2 - r^2\Delta}{\sigma})(dt)^2 - \frac{4aM}{r}dtd\phi + (\frac{\sigma}{r^2})(d\phi)^2 + \frac{r^2}{\Delta}(dr)^2, \tag{3.1}$$

where, $\sigma = (r^2 + a^2)^2 - \Delta$.

Reminding that,$(ds)^2 = -(d\tau)^2$, and for Lagrangian density($\mathcal{L}$), $2\mathcal{L} = mU^\mu U_\mu$, equation:[3.1] can be written in the form of Lagrangian density as,

$$2\mathcal{L} = -(\frac{4aM^2 - r^2\Delta}{\sigma})(\frac{dt}{d\tau})^2 + \frac{4aM}{r}\frac{dt}{d\tau}\frac{d\phi}{d\tau} - \frac{\sigma}{r^2}(\frac{d\phi}{d\tau})^2 - \frac{r^2}{\Delta}(\frac{dr}{d\tau})^2. \tag{3.2}$$

The geodesic equations of motion result in energy and angular momentum conservations which are expressed as,

$$E = -(\frac{4aM^2 - r^2\Delta}{\sigma})\frac{dt}{d\tau} + \frac{2Ma}{r}\frac{d\phi}{d\tau}, \tag{3.3}$$

and

$$L = (\frac{2Ma}{r})\frac{dt}{d\tau} - (\frac{\sigma}{r^2})\frac{d\phi}{d\tau} \tag{3.4}$$

respectively.

Substituting equations[3.3] and [3.4] into equation[3.2] to eliminate $\frac{dt}{d\tau}$, and $\frac{d\phi}{d\tau}$, we obtain;

$$(\frac{dr}{d\tau})^2 = -(\frac{\sigma}{r^4})E^2 - (\frac{4aM^2 - r^2\Delta}{\sigma})\frac{L^2}{r^2} - \frac{2Ma}{r^3}EL - \frac{m^2\Delta}{r^2}, \tag{3.5}$$

where, $-m^2 = g_{\mu\nu}p^\mu p^\nu$ for the particle non-zero rest mass. The effective potential($V_{(r,E,L)}$), for radial geodesic motion, can be identified as:

$$V^2_{(r,E,L)} = \frac{1}{2}(\frac{dr}{d\tau})^2 + \frac{1}{2}(1 - (\frac{E}{m})^2). \tag{3.6}$$

From equation:[3.6], the conditions for the motion along circular orbit is,
$V_{(r,E,L)} = \frac{\partial V_{(r,E,L)}}{\partial r} = \frac{dr}{d\tau} = 0$.
Following this and the stability condition $\frac{d^2 V_{(r,E,L)}}{dr^2} \leq 0$, E and L have the form:

$$\frac{E}{m} = \frac{r^2 - 2Mr + a\sqrt{Mr}}{r(r^2 - 3Mr + 2a\sqrt{Mr})}, \tag{3.7}$$

and

$$\frac{L}{m} = \frac{\sqrt{Mr}(r^2 - 2a\sqrt{Mr} + a^2)}{r(r^2 - 3Mr + 2a\sqrt{Mr})}. \tag{3.8}$$

The alternative "$<$" sign in $\frac{\partial^2 V_{(r,E,L)}}{(\partial r)^2}$, tells us the orbit of marginal stability that marks the minimum radius in the equatorial plane where stable orbit around the black hole is possible. This minimum radius of stable obit, obtained from the above equation is:

$$r_{ms} = M(3 + Z_2 \mp \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}, \tag{3.9}$$

where, $Z_1 = 1 + (1 - \frac{a}{M})^{1/3}((1 + \frac{a}{M})^{1/3} + (1 - \frac{a}{M})^{1/3})$,
$Z_2\sqrt{3\frac{a^2}{M^2} + Z_1^2}$.
The "$+$" is for co-rotating whereas the "$-$" is used for counter-rotating orbits.
In equation:[3.9], we see that the radius depends only on mass of black hole and the angular momentum per unit mass of black hole.

3.2 Accretion disk luminosity

Accretion disk is well known not only by its use as a fuelling mechanism for central gravitating object (like black hole), but also as principal means of high luminosity observed in active galactic centers. For the most powerful luminosities observed in some astrophysical objects in the universe, nuclear reactions are wholly inadequate to guaranty. The extraction of gravitational potential energy from plasma fluid which accretes inward onto a gravitating body is now known to be the principal source of powerful luminosity and is widely believed to provide the power supply in active galactic nuclei. Hence, large mass accretion rate results in high disk luminosity. The maximum luminosity that could be emitted by spherically accreting materials (plasma fluid)around non-rotating black hole was set by Eddington to be,

$$L_{Edd} \approx 1.3 \times 10^{38} (\frac{M}{M_{\odot}}). \tag{3.10}$$

The luminosity is firmly related to the rate of mass inflow, known as mass accretion rate. Following Eddington, mass accretion rate is evaluated as:

$$\dot{M}_{Edd} \approx 2.2 (\frac{0.1}{\epsilon}) (\frac{M}{10^8 M_{\odot}}) \tag{3.11}$$

where "ϵ" is the ratio of the gravitational energy released to the rest energy. The ratio "ϵ" is 0.057 for Schwarzschild black hole and is 0.423 for maximally rotating black holes. AGN with black hole of mass $10^8 M_{\odot}$ needs the mass supply of $2.2 M_{\odot}$ per year to sustain the luminosity. Sometimes, the mass accretion rate may exceeds the critical rate and thus the disk local luminosity overwhelms the maximum Eddington luminosity limit. These super-Eddington luminosities are observed in some active galactic centers that contains rotating black holes. Studying the characteristic difference in the structure of accreting plasma fluid onto rotating and non-rotating black holes, we can deduce some reasons for the super-Eddington luminosity observed around rotating black holes. Plasma fluid accretes spherically onto non-rotating black hole. The radiation released by in-falling plasma fluid is obscured by the spherically symmetric accretion. On the other hand, plasma fluid accreting onto rotating black hole is confined along the equatorial plane. That is, axial symmetry of the gravitational field confines the plasma fluid along equatorial plane. Then the radiation produced by the inflow of plasma fluid near the innermost stable orbit is not obscured off equatorially by the accreting plasma fluid. Thus the luminosity of the disk may exceeds Eddington luminosity. This reason may account for the observed super-Eddington luminosity in some active galaxies that dock rotating black holes in their centers.

Chapter 4

Relativistic jet

Heber Curtis, in 1918 using optical instrument, observes an elliptical galaxy M87 having strange feature, elongated outflow apparently connected with the nucleus. The name "jet" was given to this outflow by Baade and Minkowski (1954). "relativistic" is added to this jet (outflow) for its escape velocity approaches the light speed. After the development of radio astronomy in the 1960's, jets emanating from the nuclei of certain active galaxies became a major theme of research in astrophysics. Geometrically, relativistic jets are narrow conical or cylindrical protrusion that carry: energy, momentum and magnetic flux. Jets are seen in a number of astrophysical objects like: young stellar objects, proto-planetary nebulae, binary stars(micro-quasars), neutron stars, and active galactic nuclei (mostly radio loud active galaxies). Jets associated with young stellar objects where discovered by Hebig and Hro in 1940's for the first time. The velocities of the jets in these astrophysical objects, like proto-stars with mass equal to the mass of the sun, are observed to be the order of $100 kms^{-1}$ which are close to the escape velocities. Recent space observations of the sun have also revealed that even the solar corona is full of jets and flares whose observed spectra and time variability are quite similar to that of cosmic flares and jets (shibata 2005).

Most of these relativistic jets (outflows) are morphologically very similar suggesting common physical origin. Though all types of jets have common physical origin, they differ in size, velocity and amount of energy transport. For example, in one extreme, AGN jets have typical size greater than 10^6pc, nuclear velocity of the order of light velocity and parent sources (which are massive black holes)with mass $10^{6-9}M_{\odot}$ and luminosity of about $10^{43-48}ergs^{-1}$ while in other extreme , in young stellar objects, jets have typical size less than 1pc, nuclear velocity less $10^{-3}c$, and emerge from low mass proto-stars with mass $1M_{\odot}$ and luminosity, $(0.1 - 2 \times 10^{-4})L_{\odot}$.

Nonetheless, all the jets share common properties as they are:

i) usually collimated to small opening angle and in most cases two-sided.

ii) originated from the vicinity of compact objects.

iii) often terminate in emission lobes (with line emission in the case of young stellar objects and synchrotron continuum emission in case of AGN and Micro-quasar jets).

v) associated with magnetic fields (the currently most accepted mechanism for jet production).

vi) showing evidence of accretion matter onto the central source via an accretion disk.

Consider the feature of relativistic jets in AGN in the figure below.

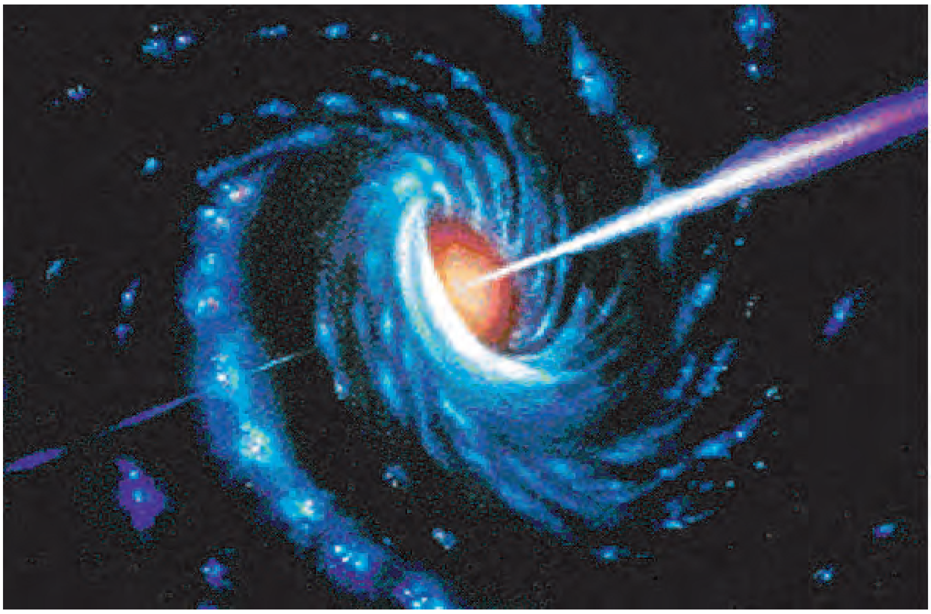

Figure 4.1: Relativistic jets in AGN

Starting from the past many decades, researchers of the field have been trying, observationally, theoretically and experimentally(through simulations) to find out the plausible mechanism for the jets generation and collimation to a large distance, hundreds of KPc to MPc. Through observation, jet sources have been studied in a wide range of electromagnetic wavelength, from radio to x-ray. In radio region, hundreds of jets have been found in AGNs. In infrared and optical region, internal structure of jets from YSOs is studied. In x-ray region, the structure and dynamics of the most inner region of the jet source from which the jet accelerates are investigated. In theories, formation and collimation of jets are investigated in various fields; structure of accretion disks, acceleration and collimation process of jets with numeric simulations, shock waves in accretion disk, particle acceleration mechanisms at shocks, etc. Various kinds of jet acceleration mechanism have been proposed among which: jets are accelerated by; gas pressure, magneto-centrifugal force, magnetic pressure, or radiation pressure are some. Also collimation processes are proposed as: collimation by funnel flow, pinch effect of plasma, or cylindrical environment. Though a lot have been tried, through different techniques, about these astrophysical jets, still now no general consensus have been reached on the mechanism of their formation and collimation. These issues are remained puzzle to the researchers. On the other hand, what mechanism provides the large amount of energy-momentum flux in relativistic jets and restricts this flow to an opening angle of the order of 10^0 is the problem of current research.

4.1 Relativistic jet formation

In the beginning, the formation process of jets was described with pure hydrodynamics. But it turns out that purely gas-pressure driven outflows do not fit the direct observations to relativistic jets. e.g. neither reach high relativistic speeds nor show collimation. Magnetic fields proved to be an efficient mechanism to generate outflows and to drive and collimate jets. Hence, magnetohydrodynamics (MHD) is proved to be the right framework to study jets. According to the current understanding, relativistic jet launching mechanism is by the collaboration of: central compact object, accretion disk and magnetic field. The corner stones of our mechanisms(of relativistic jets generation and collimation) are the twisted magnetic field lines (due to frame dragging effect of rotating black hole), radiation pressure released by the accreting plasma fluid and the centrifugal force on the accreting plasma fluid. As fundamentally accepted, it is the radiation pressure which balances the enormous gravitational collapse of central black hole in AGN while radiation pressure produced by nuclear reaction is additional means to balance gravitational collapse of central object other than black hole. When the radiation pressure exceeds the gravitational collapse of the central object, we don't expect materials to accrete on to the central object. AGN represents a large population of compact extragalactic object characterized by electromagnetic radiation produced in very compact volumes. The presence of magnetic field in jet powering process is repeatedly confirmed by direct observations. Radio astronomers, in 2002, directly observed the dying star called W43A producing twin jets confined by powerful magnetic field.

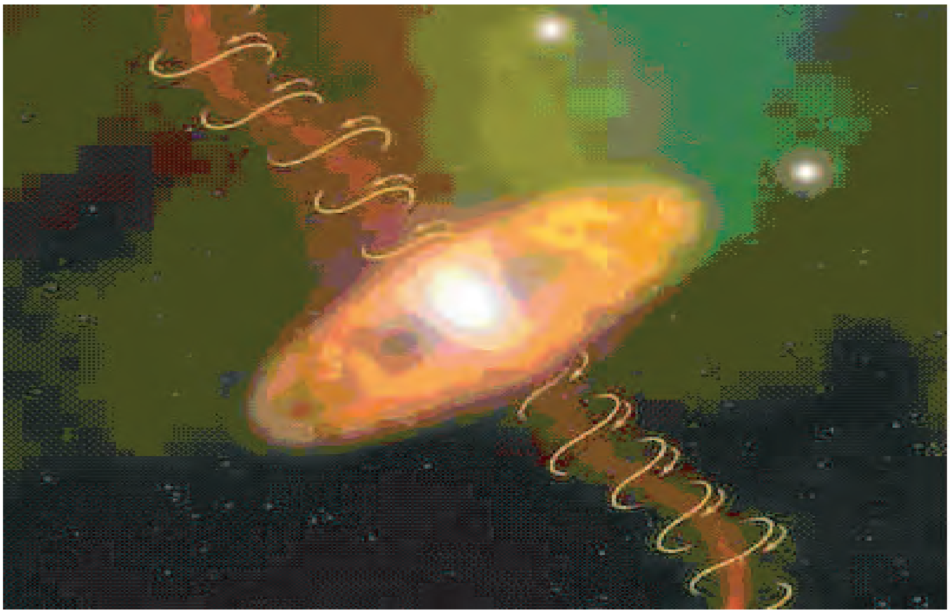

Figure 4.2: Twins jets confined by magnetic field

The location of jet production, by itself, is another unsolved current research problem. Since the ratio of the observed jet velocity to the escape velocity from the central object is the order of unity, we roughly guess that jets must have to originate from the vicinity of the central gravitating object.

Though most of the observed relativistic jets are related to the presence of black holes in the centers of galaxies, it is not pre-request to have black hole for jet production as observations evidence that there are jets produced in the vicinity of: young stellar objects, neutron stars and even our sun. The widely accepted mechanism by which relativistic jets are powered and collimated is through the presence of large scale magnetic field. However; the generation of this magnetic field around the central black hole by itself is a current strong area of debate by the related field researchers. Black hole doesn't have magnetic pole. So the magnetic field should arise by another mechanism. The reasonable theory is that the plasma particles generate magnetic field around black hole. As most researchers of the field support the case, in most case the only tenable explanation for the origin of magnetic field is the motion of electrically conducting fluid (plasma) which rotates onto central compact object. The in-falling plasma fluid has radial and azimuthal velocity components. The radial motion of charged particles generate toroidal component of the magnetic field while the azimuthal motion generates the poloidal components of the magnetic field. However; because of centrifugal force, the plasma fluid dominantly acquires the poloidal magnetic field components as it approaches the rotating central black hole. The frame dragging effect of this rotating black hole twists the poloidal field lines and align them along the azimuthal component. Here, having a close eye to the nature of the trajectory of the charged particles around the rotating black hole is very crucial for better understanding of the mentioned magnetic field components.

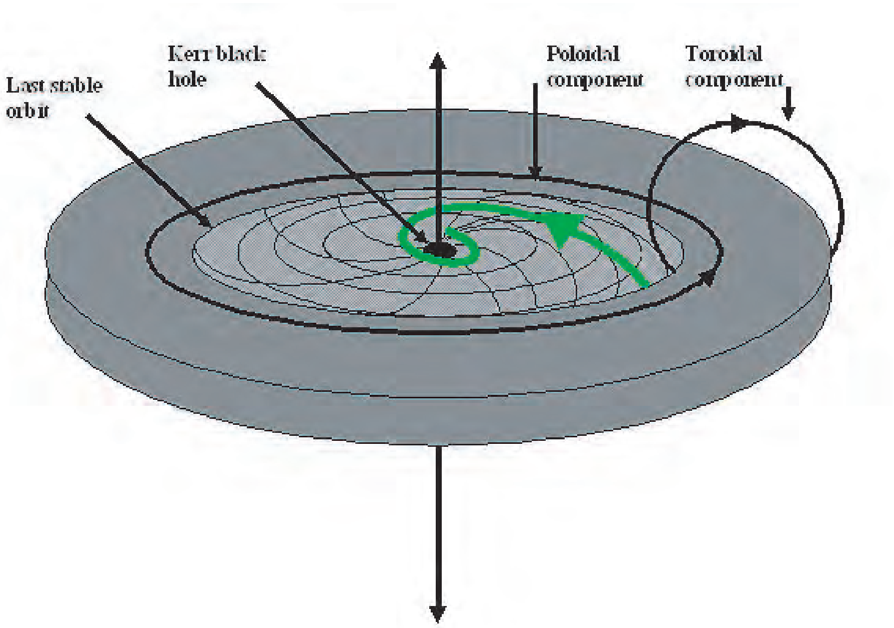

Figure 4.3: Trajectory of particles around Kerr black hole

Close to the central gravitating Kerr black hole, plasma fluid keeps on following circular orbit. The twisted poloidal components frozen into this plasma fluid. Hence in the vicinity of Kerr black hole, the azimuthal (toroidal) magnetic field component is enhanced.

See the figure below:

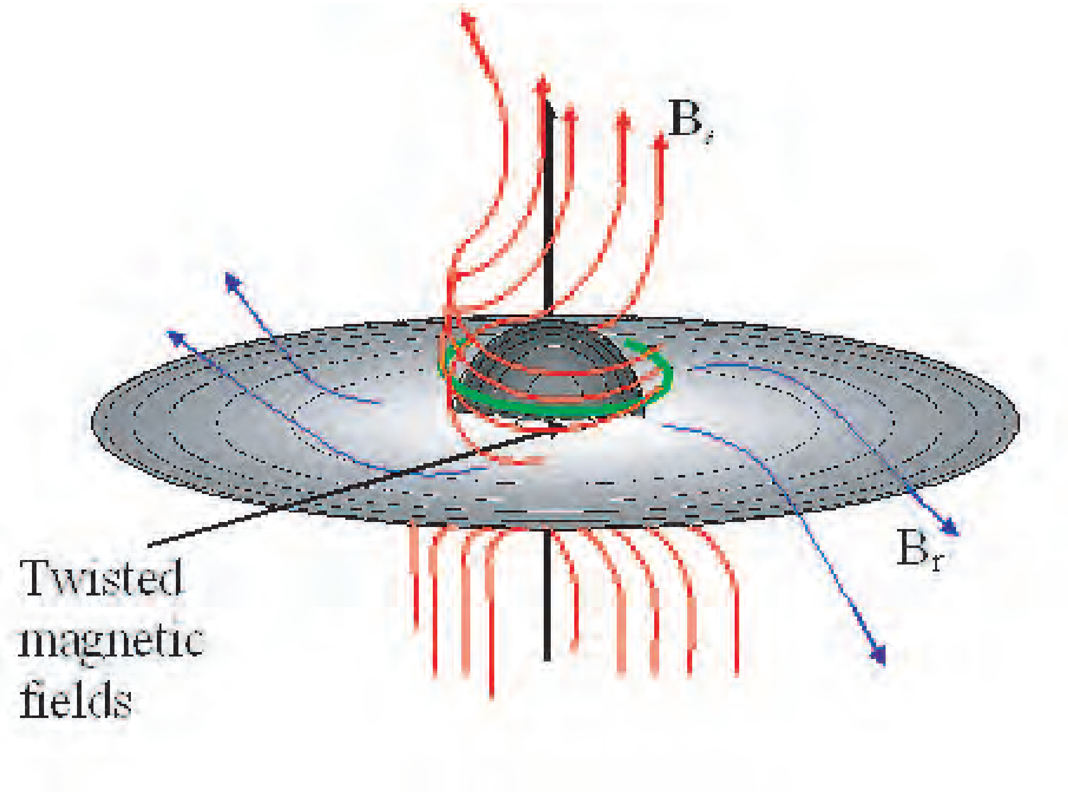

Figure 4.4: Twisted magnetic field lines due to frame dragging effect

This enhanced magnetic field component significantly perturb the dynamics of the plasma fluid. A region where the dynamics of plasma particles is determined by magnetic field is known as magneto-region or magneto-sphere. In this region of spacetime, the inertia of plasma particles is taken to be negligible. We take this condition as force-free condition which was formulated in equation [2.43] as:

$$F_{\mu\nu}J^{\nu} = 0. \tag{4.1}$$

This implies,

$$E_{\mu}\rho_e U^t = -F_{\mu i}J^i \quad , i, \mu \neq t, \tag{4.2}$$

where, ρ_e, E_μ,and U^α are charge density, electric field and four-velocity of the plasma fluid respectively.

The electric field can be related to a force(f) as: $E_\mu = \frac{f_\mu}{q}$, where "q" is a charge. Replacing this in equation [4.2], we have,

$$f_\mu = -q(\frac{U^i}{U^t})F_{\mu i}. \tag{4.3}$$

Remembering that in the vicinity of black hole all magnetic field lines are twisted azimutally and hence, we have net azimuthal magnetic field $(B_{\phi n})$.

Thus, the force drifting the particles off the equatorial plane is expressed as:

$$f_\theta = q(\frac{U^r}{U^t})B_{\phi n}(\frac{\rho^4}{\Delta}). \tag{4.4}$$

Now, let us express the net effective toroidal magnetic field $(B_{\phi n})$ in terms of the conserved quantities.

The net toroidal magnetic field is so strong in the ergo-sphere, where every thing acquires the angular velocity of Kerr black hole. Hence the ZAMO is now can be taken as an observer in the plasma fluid rest frame. The ZAMO observes only the j^t, j^r, j^θ components of the four-electric current density. According to the ZAMO in the plasma fluid rest frame, there is no electric current along the azimuthal co-ordinate (i.e $j^\phi = 0$). Refereing to the equation: [2.47], we have the following equation:

$$\frac{1}{U^t} = \frac{\Sigma\Delta sin\theta(\omega + \beta)}{\rho^4 B_{\phi n}}(k\rho_o\sqrt{-g}). \tag{4.5}$$

Where, $B_\phi \rightarrow B_{\phi n}$. Detail derivation of this is obtainable in the appendix. The shift vector (β) vanishes in the ergo-sphere where spatial grids and ZAMO attained the same angular velocity.

The net toroidal magnetic field is then expressed as:

$$B_{\phi n} = \frac{K\Sigma\Delta sin\theta\rho_o U^t\sqrt{-g}\omega}{\rho^4}. \tag{4.6}$$

Inside the ergo-sphere there is no differential rotation. The absence of differential rotation is so because all; ZAMO, plasma fluid and magnetic field lines subject to the same angular velocity Ω_H. Hence, energy and angular momentum fluxes, due to differential rotation, cease just on the inside ergo-surface. With this feature, the angular velocity of magnetic field lines is equal to the angular velocity of central rotating Kerr black hole, i.e $\omega = \Omega_H$. Thus, the net toroidal magnetic field can be re-written as:

$$B_{\phi n} = \frac{k\rho_o U^t \Sigma \Delta sin^2\theta \Omega_H}{\rho^2}. \tag{4.7}$$

Substituting this equation back to equation:[4.4], we have the final form drifting force:

$$f_\theta = qk\rho_o \Sigma \rho^2 sin^2\theta U^r \Omega_H. \tag{4.8}$$

Drifted charges follow helical path around the black hole being off equatorial plane. The gravitational effect of Kerr black hole, given in equation [2.21], reduces radially outward and off the equatorial plane and hence the centrifugal force, on the drifted particles, starts to dominate the gravitational force. The moment particles leave the equatorial plane, they suffer differential rotation and radiation pressure from the accretion disk in addition to centrifugal force. Differential rotation supplies energy and momentum whereas radiation pressure pushes the drifted particles along the axis of rotation. This is the mechanism by which jets produced.

The drift velocity (V_d), with which particles are launched, is expressed interms of centrifugal force (f_c) and the net toroidal component of magnetic field ($B_{\phi n}$). That is,

$$V_d = \frac{\vec{f_c} \times \vec{B}}{qB^2}, \tag{4.9}$$

where, q is the charge of the electron.
Centrifugal force, on a single electron, in ergo-sphere is given as,

$$f_c = m_e \Omega_H^2 \breve{r}. \tag{4.10}$$

Where, $M + \sqrt{M^2 - a^2} < \breve{r} < M + \sqrt{M^2 - a^2 cos^2\theta}$, m_e is electron mass and $\Omega_H = \frac{a}{a^2 + r_+^2}$, is the angular velocity of the black hole. a and r_+, are angular momentum per unit mass of black hole and radius of the outer event horizon respectively. Substituting equation [4.10] into [4.9], we obtain:

$$V_d = \frac{m_e \breve{r}}{qB_{\phi n}} (\frac{a}{a^2 + r_+^2})^2. \tag{4.11}$$

So, the plasma particles will have the expressed velocity(in opposite direction based on their sign)when ejected off the equatorial plane.

4.2 Relativistic jet collimation

In our jet production mechanism, we have seen that the net toroidal magnetic field is responsible to drift plasma particles outward along the poles of rotating Kerr black hole. We, here, focus on the collimation of these launched outflows of plasma particles. Observations and theoretical arguments indicate that the jets bear higher fraction of energy, momentum and magnetic flux retaining their direction and collimated over a long astronomical distance. Highly collimated, oppositely directed jets are observed in active galaxies and in binaries(Micro-quasars). Furthermore, well-collimated emission-line of jets are seen in young stellar objects. The extent of collimation is measured by the collimation factor defined as the ratio of length to diameter of the jet. Collimation factor for AGNs is several hundreds while it is 3 to 30 for Micro-quasars and young stellar objects. Radio jets from AGNs are extremely collimated. The jet encounters a wide range of external barriers as it propagates out from the galactic nucleus, through the interstellar medium of the host galaxy, and in some cases into intergalactic space. Different mechanisms could well be responsible for collimating jets on different scales. Leaving the question "why relativistic jets are collimated?" for the observational evidences, we here focus on finding the feasible answer for the question "how relativistic jets are collimated?". As discussed in the previous section opposite charges are drifted off the equatorial plane where the gravitational field is relatively weak to bound the charges and hence centrifugal force dominates. Now the drifted charges encounter: Kicks by the disk surface(due to differential rotation), centrifugal force, magnetic hoop stress and radiation force from the accretion disk.

Differential rotation : Differential rotation has a great importance in transporting energy and momentum from region of fast rotation to region of slow rotation. Accretion disk creates two differential rotations: radial differential rotation and vertical differential rotation. In the case of horizontal differential rotation, the energy and angular momentum transfer outward along the equatorial plane. This arises a sort of radially outward force on the exterior accreting matters. However; the gravitational collapse can balance this effect so that matters keep on accreting. In the case of vertical differential rotation, energy and angular momentum are transported vertical upward above the upper surface of the accretion disk and downward below the lower surface of the accretion disk. Close to of Kerr black hole, the effect of differential rotation is pronounced (specially just out side of the ergo-surface). The supply of energy and angular momentum from the plasma fluids in erg-sphere is via the outer boundary of the ergo-surface. This kicks the particles off the equatorial plane. Supply of energy and angular momentum is sustainable as long as the particles are drifted vertically. The centrifugal force pushes the particles radially out and the magnetic hoop stress does the opposite. Meanwhile, radiation pressure forces the particles along the rotation axis. Many researchers have studied a radiatively driven outflows (jets) from the disk surface. For example; Bisnovatyi, kogan, Blinnikov in 1977, Katz, Icke in 1980, Melia, K$\ddot{a}$nigl in 1989, Watarai, Fukue in 2003 extensively studied in the context of models in astrophysical jets. However; they didn't mentioned the question "what mimics the accreting matter(plasma fluid)to leave the equatorial plane. Radiation, released from the disk, forces the drifted charges away parallel to the rotation axis. As the jets are fired into interstellar medium, they suffers another external stress(gas pressure) which make them maintain narrow path over along distance. The launching velocities of the jets and the strength of radiation pressure determine the distance the jets span being collimated. The launching velocity is firmly related to the compactness of the body from which the jets escape. As it is well known, the radiation pressure released form accretion disk should be so strong that it could balance the gravitational force of compact stars. If there is no enormous gravitational force to balance it, We can Contemplate how much will be the effect of radiation pressure on accreting materials.

Here, lets discuss mathematically the effect of the magnetic field and radiation force on the charged particles on the either sides of the disk:

Radiation force on drifted particles : The drifted electrons above the accretion disk are acted on by the radiation(photon) force (F_r), from the accretion disk. The momentum transfered from the photon of energy $E = h\nu$ to the electron in one scattering event is $P = \frac{h\nu}{c}$, where c is the light speed. If there is an emission of N photons per unit area and per unit time from the accretion disk, the number of photons colliding with a given electron per unit time is $N\sigma_T$, where $\sigma_T = 6.65 \times 10^{-29} m^2$, is

the cross-section of the electron for the collision, that is the area it presents to the photons to be hit. This is well known as Thomson's scattering cross-section
Thus, in general the electrons receive a total momentum per unit time given as:

$$\hat{F_r} = (\frac{\sigma_T}{c})Nh\nu. \tag{4.12}$$

This is the radiation force that drifts electrons. The radiation force perpendicular to the accreting disk around rotating black hole, is derived by G.S.Bisnovatyi Kogan in 2002, to be:

$$F_r = \frac{\sigma_T}{c}H(r), \tag{4.13}$$

where, $H(r) = \frac{3}{8\pi}\frac{GM\dot{M}}{r^3}\mathcal{J}$, is the radiative flux emitted per unit area from one side of the disk. $\mathcal{J} = (1-(\frac{r_{in}}{r})^2)$, and r_{in} is radius of the innermost disk where in our case is the radius just outside of the outer event horizon. From equations: [4.12] and [4.13], we have the relation:

$$\hat{F_r} = \eta F_r, \tag{4.14}$$

where, $\eta \,\epsilon\, (0,1)$, is the ratio of radiation force that targets the electrons to the total radiation from the accretion disk.

The radiation force that accelerates electrons, assuming for simplicity that the electrons heating effect, photons scattering by electrons and the force flux via inter-electrons spacing are negligible, has the expression,

$$\hat{F_r} = nm_e < a >, \tag{4.15}$$

where, n and $< a >$ are the number of accelerated electrons and average acceleration of electrons respectively. With the help of equations: [4.13] to [4.15], we get the average acceleration to be:

$$< a >= \eta\frac{3\sigma_T}{8\pi cnm_e}\frac{GM\dot{M}}{r^3}(1-(\frac{r_{in}}{r})^2) \tag{4.16}$$

Effect of magnetic fields on jet collimation : Initially electrons follow the helical path. This manner of electrons' dynamics creates to two components of magnetic field; the poloidal component and weak toroidal component. As radiation pressure rectifies electrons vertically, the toroidal component of magnetic field dominates the poloidal component of the magnetic field.

The rectified electrons induce the toroidal magnetic field which creates hoop stress or magnetic tension on themselves.

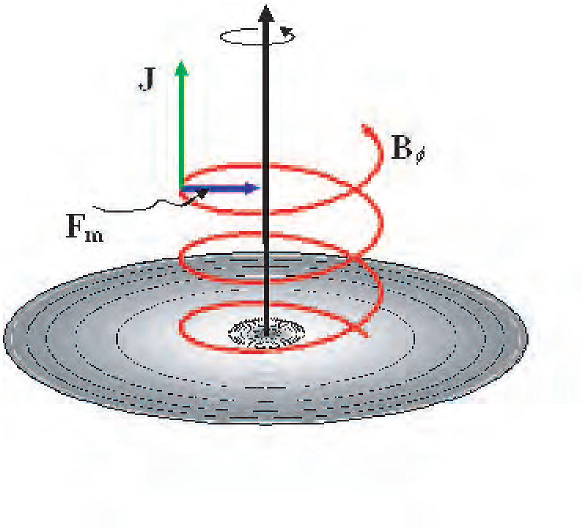

Figure 4.5: Magnetically collimated relativistic jet

This magnetic tension, that ties electrons together, is quantified as:

$$\vec{F_m} = \vec{J_z} \times \vec{B_\phi}. \tag{4.17}$$

Where B_ϕ, $F_m \, and \, J_z$, are induced toroidal magnetic field, magnetic force and electrons current density respectively. Also the gas pressure from the interstellar medium affects the structure and the dynamics of of the jet. The jet has a hallow cylindrical structure so that the external gas pressure pushes it both from inside out and from outside in. In addition to that the jet head suffers a continues collision with the external gas pressure. This continues collision damps the velocity of jet thereby reducing the magnetic tension effect on the jet. This in turn lets the jet to widen and finally form lobe type. The jet terminates in this lobe and continually channel energy and angular to it.

Chapter 5

Discussion

Even though jets are observed in a number of astrophysical objects, our research targets on the relativistic jets in active galaxies enclosing rotating neutral black hole. We have assumed the accretion disk to be thin except near the rotating black hole, where the toroidal magnetic field inflates the disk, and neglected the misalignment of this disk with the equatorial plane of the rotation axis of the central black hole. Also, we assumed steady states wherein all physical quantities are independent of time since analytical treatment acquires its simplest form in this approach. In the mechanism of relativistic jets generation, we focus on the dynamics of charged particles and the configurations of electromagnetic field in the force-free region of spacetime. This force-free region is found to be the immediate neighborhood of rotating black hole. The dynamics of charged particles in this region is governed by the existing magnetic field. Though; not that much significant, the effect of magnetic field on the charged particles exists throughout the accretion disk as long as charged particles are in motion. However; due to frame dragging effect of Kerr black hole, the magnetic field lines tend to be harmonized along the azimuthal component and generate the pronounced perturbation. Hence, the overall effect of azimuthal magnetic field component greatly affect the dynamics of charged particles in a force-free region. Once the charged particles are drifted off the equatorial plane of the accretion disk, centrifugal force tends to widen the orbit of particles and radiation pressure rectifies the particle along the black hole rotation axis. We find out that the drifting force is $f_\theta = q(\frac{U^r}{U^t})B_{\phi n}$ and the drifting velocity is $V_d = \frac{m_e \check{r}}{qB_{\phi n}}\big(\frac{a}{a^2+r_+^2})^2$. The rectified particles induce a magnetic field around themselves which ties them together over a long distance. In addition to this magnetic hoop stress, there is an external gas pressure from the interstellar medium. Since the jet has the a hallow cylindrical structure, the interstellar gas pressure affects the jet from two sides (i.e the gas inside the cylinder, of the jet, forces the jet out while the external gas force the jet inward). The jet head, propagating through galactic medium, encounters continues collision with the galactic medium and results in the damping of the jet velocity. This in turn reduces the magnetic field which ties the charged particles together.

Appendix A

appendix

$$F_{tr,\theta}+F_{\theta t,r}=F_{\phi r,\theta}+F_{\theta\phi,r}=0 \tag{A.1}$$

From force-free condition,

$$F_{tr}U^{r}+F_{t\theta}U^{\theta}=0 \tag{A.2}$$

$$F_{rt}U^{t}+F_{r\theta}U^{\theta}+F_{r\phi}U^{\phi}=0 \tag{A.3}$$

$$F_{\theta t}U^{t}+F_{\theta r}U^{r}F_{\theta\phi}U^{\phi}=0 \tag{A.4}$$

$$F_{\phi r}U^{r}+F_{\phi\theta}U^{\theta}=0 \tag{A.5}$$

From the above equations we have the following relations,

$$F_{tr,\theta}=F_{t\theta,r}=-(F_{tr}\frac{U^{r}}{U^{\theta}}),r \tag{A.6}$$

this implies,

$$F_{tr,r}\frac{U^{r}}{U^{\theta}}-F_{tr}(\frac{U^{r}}{U^{\theta}}),r \tag{A.7}$$

hence,

$$F_{tr,r}U^r + F_{tr,\theta}U^\theta = -F_{tr}U^\theta(\frac{u^r}{U^\theta}),r \tag{A.8}$$

$$\Rightarrow U^\mu F_{tr,\mu} = \frac{d}{d\tau}F_{tr} = -F_{tr}U^\theta(\frac{U^r}{U^\theta}),r \tag{A.9}$$

$$\Rightarrow \frac{d}{d\tau}ln(F_{tr}) = -U^\theta(\frac{U^r}{U^\theta}),r \tag{A.10}$$

By equation [A.1] and [A.6],

$$F_{r\phi,\theta} = (F_{\phi r}\frac{U^r}{u^\theta}),r = F_{\phi r,r}\frac{U^r}{U^\theta} + F_{\phi r}\frac{U^r}{U^\theta},r \tag{A.11}$$

$$\Rightarrow \frac{d}{d\tau}ln(F_{\phi r}) = -U^\theta(\frac{U^r}{U^\theta}),r \tag{A.12}$$

Subtracting [A.12] from [A.12],

$$\frac{d}{d\tau}ln(\frac{F_{tr}}{F_{\phi r}}) = 0 \tag{A.13}$$

$$\frac{F_{tr}}{F_{\phi r}}) = -\omega \tag{A.14}$$

equation [A.10] yields

$$\frac{d}{d\tau}ln(F_{\phi r}) = -U^r_{,r} + U^r(\frac{U^r}{U^\theta}),r \tag{A.15}$$

We derive the mass conservation equation in Kerr geometry as:

$$0 = (\rho_o U^\mu)_{;\mu} = \frac{1}{\sqrt{-g}}(\sqrt{-g}\rho_o U^\mu)_{,\mu}. \tag{A.16}$$

This implies that,

$$0 = (\sqrt{-g}\rho_o)_{,\mu}U^\mu + \sqrt{-g}\rho_o U^\mu_{,\mu}, \tag{A.17}$$

this in turn means,

$$(\rho_o U^\mu),\mu = -\frac{1}{\sqrt{-g}}\frac{d}{d\tau}(\sqrt{-g}\rho_o) = -\frac{d}{d\tau}ln(\sqrt{-g}\rho_o). \tag{A.18}$$

Substituting equation: [A.18] into equation: [A.15], we finally have,

$$\frac{d}{d\tau}ln(F_{\phi r}) = \frac{d}{d\tau}ln(\sqrt{-g}\rho_o) + \frac{d}{d\tau}ln(U^{\theta}) \tag{A.19}$$

$$\Rightarrow \frac{F_{\phi r}}{\sqrt{-g}\rho_o U^{\theta}} = k(constant) \tag{A.20}$$

Equation: [2.47] has the following form in side the ergo-sphere:

$$\Delta\psi_{\theta}J^t = -(\frac{\rho^2}{\Delta sin\theta}B_{\phi})J^r = 0 \tag{A.21}$$

This implies:

$$\frac{U^r}{U^t} = \frac{\Delta^2\psi_{\theta}sin\theta}{\rho^2 B_{\phi}} \tag{A.22}$$

Substituting for ψ_{θ} from equation: [2.31], we have:

$$\frac{U^r}{U^t} = \frac{\Delta\psi_{\theta}sin\theta}{\rho^2 B_{\phi}}(\frac{\Sigma F_{\phi\theta}(\omega+\beta)}{\rho^2}) \tag{A.23}$$

Then, substitute $F_{\phi\theta}$, using equation: [2.68]to obtain;

$$\frac{1}{U^t} = \frac{k\rho_o\Sigma^2\Delta sin^2\theta(\omega+\beta)}{\rho^4 B_{\phi}} \tag{A.24}$$

The net toroidal magnetic field is thus,

$$B_{\phi} = \frac{k\rho_o\Sigma^2\Delta sin^2\theta(\omega+\beta)U^t}{\rho^4} \tag{A.25}$$

Assuming that in the ergo-sphere: $B_{\phi} \rightarrow B_{\phi n}$, and $\omega \rightarrow \Omega_H$, we end up with:

$$B_{\phi n} = \frac{k\rho_o\Sigma^2\Delta sin^2\theta(\Omega_H+\beta)U^t}{\rho^4} \tag{A.26}$$

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DECLARATION

I hereby declare that this thesis is my original work and has not been presented for a degree in any other University. All sources of material used for the thesis have been duly acknowledged.

Name: ***Abera Shiferaw***

Signature:_______________

This thesis has been submitted for examination with my approval as university advisor.

Name: ***Dr. Ramanand Jha***

Signature:_______________

Addis Ababa University
Department of Physics
June, 2006.