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## **Abstract**

This paper is intended to discuss different approaches in proving identities. The paper comprises of two sections. The first one deals with basic concepts in combinatorics that are used in farther discussions and the second Section is the main body that discusses the proofs that illustrate the power and elegance in enumerative/algebraic combinatorial arguments, modern machine assisted techniques of Wilf-Zeilberger and the classical tools of generatingfunctionology.



School of Graduate Studies  
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*A graduate seminar report On*  
**COMBINATORIAL AND AUTOMATED PROOFS OF  
CERTAIN IDENTITIES**

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**January 2012**

**Addis Ababa**

# **DECLARATION**

I declare that this project has been composed by me and that no part of the project has formed the basis of the award of any Degree, Diploma, Associate ship, Fellowship or any other similar title to me.

**Addis Ababa**  
**January 2012**

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**Brhane G/Micheal**

# PERMISSION

This is to certify that this project is compiled by **Mr. Brhane Gebremicheal** in the department of Mathematics, Addis Ababa University, under my supervision. I hereby also confirm that the project has been submitted for evaluation by examiners and eventual defense.

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**January 2012**

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