



***A STUDY OF MAGNETIZATION AND  
SPECIFIC HEAT IN DILUTED MAGNETIC  
SEMICONDUCTORS QUANTUM WELL***

By

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*Dedicated To  
My Family.*

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# Abstract

Semiconductor devices that exploit spin as well as charge would operate faster than conventional microelectronic devices and would offer new functionality. The main challenge in building such device is transporting spin-polarized electrons efficiently into and out of the semiconductor region of the device. In the first parts of this thesis, we reviewed the possible magnetic interaction in the bulk diluted magnetic semiconductors. At the last part of the thesis the effect of the small wave vector gapless Goldstone mode of spin excitation on the temperature dependence of the magnetization is discussed for 3D and 2D Mn spin configuration for modulation doped DMSs quantum well. In the 3D case the magnetization decreases as power of three by four ( $T^{\frac{3}{4}}$ ) with temperature and for the 2D configuration it decreases as a power of half ( $T^{\frac{1}{2}}$ ) with temperature. The specific heat of the Mn-spin system increases as a power of three by four and by half for 3D and 2D configurations respectively.

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# Thesis Outline

In the emerging field of spin electronics the role of the spin degree of freedom in the properties of electronic system is exploited in the design of new functional devices. The candidate materials are diluted magnetic semiconductors(DMSs). These materials possess both magnetic and semiconductor properties. In this thesis we consider especially magnetic properties of these systems.

In device application it is mostly used electronic and magnetic thin films. To understand the properties of DMS thin films, it is better to study their properties in lower dimensions such as quantum dots, quantum wires and quantum wells. Understanding the properties of magnetic interaction in DMS quantum well is walking a lot of steps forward to touch the application of DMS in the required spintronics devices. Considering this, DMS quantum well is the basic issue of this thesis giving emphasis on the effect of collective spin excitation on the magnetization and Curie temperature of DMS quantum well.

In the first chapter we review general background of the area briefly.

A brief review of DMSs is given in the second chapter. We shall define DMSs and discuss their properties. We will consider Curie temperature and specially useful DMSs GaMnAs.

The third chapter of this thesis concentrates on the possible magnetic interactions in

DMSs and the dependence of this interaction on the concentration of the magnetic dopant and the carriers.

In chapter four we will formulate our problem, the DMS quantum well, by introducing the form of the Hamiltonian. We shall evaluate the magnetization and specific heat for 3D and 2D Mn configuration.

Finally, the fifth chapter summarizes the results obtained and gives concluding points of our work.

# Chapter 1

## Introduction

### 1.1 Overview

The electron lies at the heart of the microelectronics revolution, where it is shuttled around in semiconductors (usually silicon) to allow transistors and other such devices to operate. Yet these devices only exploit the charge of the electron. For the last 40 years the number of transistors per unit area that can be etched onto a silicon chip - which, for example, governs the processing power of a computer has increased very rapidly. But we are now rapidly approaching the limit of how small and closely packed these transistors can become before the heat that they generate cannot be dissipated fast enough, or unwanted quantum-mechanical effects prevent them from functioning properly.

In addition to their charge, electrons have an intrinsic angular momentum or "spin" that has only two possible orientations in an external field[1]. The remarkable developments currently being made in the field of electronics and information technologies have been made possible by exploiting the properties of electron charge and spin. Integrating circuits used for data processing use the charge of electron in semiconductors, while data storage media such as hard disk use the spin of electrons in a

magnetic materials[2].

Spintronics or Magnetoelectronics is the study of electron or nuclear spin in solid state physics and possibly devices that specifically exploit spin properties instead or in addition to charge degree of freedom [3].

## 1.2 Advantages of spintronics over electronics

The semiconductors currently used in integrated circuits, transistors and lasers, such as silicon and gallium arsenide are non magnetic in which the carriers (holes or electrons) energy is almost independent of the spin direction[2]. Advances in material sciences have led to the development of ferromagnetic semiconductors based on the III-V compound semiconductors already used for electronic devices, it is becoming possible to clearly express and control the degrees of freedom in charge and spin[2]. Because the spin of an electron can be switched from one state to another much faster than charge can be moved around a circuit, spintronic devices are expected to operate faster and produce less heat than conventional microelectronic components. One of the ultimate goals is to build a spin-based transistor that would replace conventional transistors in integrated logic circuits and memory devices, thus allowing the miniaturization trend to continue.

Looking further into the future, spintronic devices could even be used as quantum bits, the units of information processed by quantum computers[1]. Within the next few years, it is expected that magnetoelectronic chips will be used in quantum computers. The other an inherent advantage of magnetoelectronics over electronics is the fact that magnet tends to stay magnetized for long. Hence, this arises interest

in industries to replace the semiconductor-based components of computer with magnetic ones, starting from RAM. The new magnetic RAM will retain data even when the computer is turned off. Most important advantage will be eliminating the time consuming process of booting up information from hard drive to processor like a TV set, all the information would be there [4].

The realization of materials that combine semiconducting behavior with robust magnetism has long been a dream of material physics[5]. Semiconductors are very useful than metals for device application because of the following reasons . Firstly, semiconductors electronic properties can be significantly modified(controlled) by a small amount of impurities ,i.e.by doping. Secondly ,the existing metal-based devices do not amplify signals (although they are successful switches or valves), whereas semiconductor based spintronic devices could in principle provide amplification and serve, in general, as multi-functional devices. Perhaps even more importantly, it would be much easier for semiconductor-based devices to be integrated with traditional semiconductor technology[3].

### **1.3 Current activities in the area**

Semiconductor physics and magnetism are newly established subfields of condensed-matter physics that continue to reveal a reach variety of unusual phenomena , often in new types of solid-state materials. The properties of semiconductors are extraordinarily sensitive to impurity atoms ,defects and charges on external gates[5]. For the spintronics revolution to happen, however, researchers need to find a way to inject, manipulate and detect the spin of electrons in semiconductors, since these materials are likely to remain central to device physics for the foreseeable future. Spin

manipulation should in theory be relatively straightforward, but injecting and detecting spin under practical conditions are huge challenges. So the challenge is to find spinpolarized materials - i.e. materials in which most of the electron spins are aligned in a particular direction - that can be combined with them. Promising candidates are "dilute magnetic semiconductors" (DMS) - semiconductors that, when doped with impurity atoms, display ferromagnetism[1]. Rather than using naturally magnetized semiconductors such as Eu-chalcogenides or  $ZnCr_2Se_4$  spinel, it is better to use artificially prepared diluted magnetic semiconductors (DMS). The DMS are semiconductor compounds in which a fraction of the constituent ions is replaced by magnetic transition metal ions such as Mn, Fe, Co, Ni ...etc. The great interest in DMS system is due to the fact that, in contrast to magnetic semiconductors, DMS offer a possibility of studying the magnetic phenomena in crystal with a simple band structure and excellent magneto-optical and transport properties. Moreover, the well developed technology of growing these DMS allows for tuning their magnetic properties not only by an external magnetic field but by varying the band structure and /or carrier, impurity and magnetic ion concentration[6]. The rapid progress achieved in DMS technologies which use non-equilibrium growth methods, such as molecular beam epitaxy (MBE) the ferromagnetic transition was observed in II-VI, III-V and IV-VI semiconductor compounds[6]. In most studied DMS manganese is used as a magnetic impurity. Manganese doped II-V and IV-VI DMS mostly exhibit antiferromagnetism, spin glasses and /or paramagnetism, but they also show ferromagnetic transition at low temperatures [2]. It has been established that several (III,V) compound semiconductors become ferromagnetic when heavily doped with Mn, and that the highest ferromagnetic temperature reported so far is 110K for (Ga,Mn)As and



60k for (In,Mn)As in bulk form. [5,15].

## 1.4 Difficulties

The most spectacular properties of DMS, example the giant Zeeman splitting of the free carriers spectrum in the magnetic field, result from the spin dependent interactions of band carriers with the localized spins of magnetic ions[5]. In these materials ferromagnetism is expected due to primarily to coupling between magnetic element moments that is mediated by conduction band electrons or valence band holes . Efforts to increase their critical temperature further run into incompletely understood fundamental limits on the ratio of the magnetic transition temperature to the fermi temperature of the free carrier systems. Finding a material that exhibits spin polarization well above room temperature, however, is not the only challenge in developing a practical spin injector. First, it must have a large polarization in order to be able to inject enough spin-polarized electrons into a semiconductor. Second, it must be possible to control the properties of the interface that forms when the injector material is deposited on the semiconductor. While developing magnetic tunnel junctions in the 1990s, researchers learned that the properties of the few atomic layers close to the interface have a critical effect on spin-injection efficiency. This is because very small amounts of chemical intermixing between the layers can scatter the electrons into new states and therefore substantially lower the amount of electrons that make it across the interface while remaining polarized. It is difficult to control the properties of DMS materials in bulk form, and even more so when the material is deposited in a thin film, as is required when fabricating a device. Achieving clean interfaces between

DMS materials and semiconductors, therefore, poses a considerable challenge for researchers trying to build DMS-based spintronic devices. Theoretical predictions show that the Curie temperature of certain DMS materials should increase significantly with ferromagnetic doping. Yet still no-one has found suitable ferromagnetic semiconductor materials that operate at room temperature and can be used in practical semiconductor spintronic devices. While the effort to develop DMS-based spintronics continues, however, the remarkable development of magnetic tunnel junction technology has given great impetus to using ferromagnetic metals in combination with semiconductors. While ferromagnetic transition metals do not offer 100 percent spin polarization, this may not be necessary for practical devices: theoretical predictions suggest that by controlling the interface structure and composition, and using appropriate barriers, future ferromagnetic-metal systems could yield dramatic increases in spin transmission over the injector/detector materials tried so far. The properties of the few atomic layers close to the interface have a critical effect on spin transmission. In the future it will be important to precisely control the structure of the materials used in semiconductor spintronic devices by matching the crystal orientation of the interface with that of the spin injector/detector material, and it is clear that there are many promising new routes to investigate.

# Chapter 2

## DILUTED MAGNETIC SEMICONDUCTORS

### 2.1 Introduction

Diluted Magnetic Semiconductors (DMSs) are alloys between a non-magnetic semiconductor (e. g. GaAs) and a magnetic element, usually manganese (Mn) [5]. They are composed of an inert host semiconductor doped with both localized spins and carriers (electrons or holes) that are either itinerant, or localized in a much longer length scale. In that sense they belong to the general family of the correlated electron systems. While the name diluted magnetic semiconductors implies that the system has only a small percentage of localized spins. The carrier density in DMS is significantly lower than the (low) localized moment density, so the spins become an integral part of the description of the system and its magnetic phase [7]. Diluted magnetic semiconductors (DMS) are expected to play an important role in interdisciplinary materials science and future electronics because charge and spin degrees of freedom accommodated into a single material exhibits interesting magnetic, magneto-optical, magnetoelectronic and other properties [4]. Figure 1.1 shows schematic diagram of DMSs.

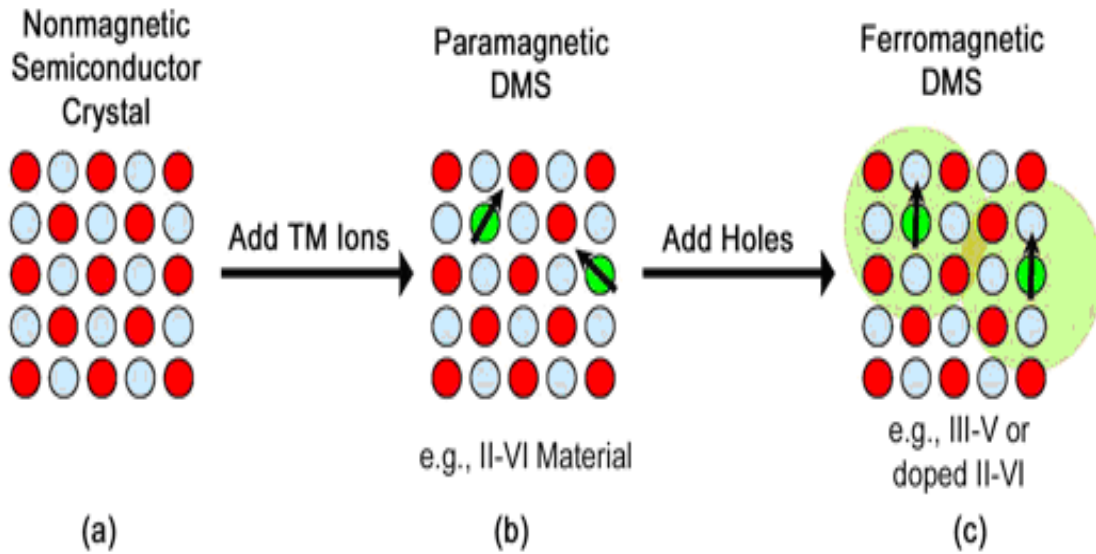


Figure 2.1: Three types of semiconductors: (a) a magnetic semiconductor, which has a periodic array of a magnetic element, (b) a diluted magnetic semiconductor (DMS), an alloy between a nonmagnetic semiconductor and a magnetic element and (c) a non-magnetic semiconductor, which contains no magnetic ions [3].

## 2.2 II-VI DMS

The II-VI DMS are based on semiconductors AB, where A is a group -II element and B is a group -VI element (such as CdTe or ZnSe). In the II-VI DMS, some of the divalent sites (Cd/Zn) are substituted by a magnetic element, typically Mn. This is because such +2 Mn ions are easily incorporated into the host II-VI crystals by replacing group II cations [4]. Mn is also a group -II element, but in addition it has a half-filled 3d shell, with a total spin  $S = \frac{5}{2}$ . In the absence of other types of dopants, the  $A_{1-x}Mn_xB$  (where x is the fractional Mn concentration) is an insulator which exhibits dominantly antiferromagnetic (AFM) tendencies at low temperature[7] which results in the paramagnetic, antiferromagnetic, or spins glass behaviour of the material[4]. The origin of this AMF tendency is expected antiferromagnetic exchange between the Mn spins. However, for low doping concentration x, the average distance between Mn spins is larger and this AFM direct exchange is rather small. When a low density of dopants is introduced in the system, each of them binds a hole (or electrons). Exchange interaction arises between the spin of these charge carriers and the Mn spins which is described by a Heisenberg Hamiltonian

$$H = \sum_{ij} J(r_i, R_j) \vec{s}_i \cdot \vec{S}_j \quad (2.2.1)$$

where,  $S_j$  is the spin of the Mn ion at position  $R_j$  and  $s_i$  is the spin of the electron or hole centered at  $r_i$ . The exchange interaction  $J(r_i, R_j)$  is dependent on the overlap between the orbital  $\phi(r - r_i)$  of the charge carrier and the orbitals  $\psi_d(r - R)$  of the 3d electrons responsible for the Mn spin. The exchange is proportional to the carrier

charge density at the Mn site:

$$J(r_i, R_j) = J_o[\phi(R_j - r_i)]^2 \quad (2.2.2)$$

where  $J_o$  characterizes the strength of the exchange. For Mn spins which are very close to one another the direct antiferromagnetic (AFM) exchange is the dominant interaction. For fairly far apart Mn spins, the dominant magnetic interaction is the exchange between the charge carrier spins and the Mn spins. If the fraction  $x$  of the Mn becomes too large, both types of interactions will be of comparable size for all the Mn spins, and therefore this separation is no longer possible.

Although it is relatively easy to prepare in bulk form as well as thin epitaxial layers, II-VI based DMS are difficult to dope to create p- and n-type magnetic semiconductors, which makes the material less attractive for applications [4]. Simple thermodynamic considerations show [7] that, qualitatively, at a temperature  $k_B T < J(r)$  all Mn spins within a certain distance of a dopant order, their spins align antiferromagnetically with respect to the dopant holes. As a result, a region with a large magnetization (from all the polarized Mn spins) appears near the dopant. This is known as Bound Magnetic Polaron(BMP) . As a result, one expects that long range ferromagnetic ordering appears in the system for the temperature low enough that a continuous percolating network of BMPs is formed. But the Curie temperature below which long range ferromagnetism is observed in these system is very low, only below 5K for all II-VI DMS studied so far.

## 2.3 III-V DMSs

An approach compatible with the semiconductors used in present day electronics is to make nonmagnetic III-V semiconductors magnetic, and even ferromagnetic by introducing a high concentration of magnetic ions. The III-V semiconductors such as GaAs are already in use in a wide variety of electronic equipment in the form of electronic and optoelectronics devices, including cellular phones (microwave transistors), compact disks (semiconductor lasers), and in many other applications. Therefore, the introduction of magnetic III-V semiconductors opens up the possibility of using a variety of magnetic phenomena not present in conventional nonmagnetic III-V semiconductors used in the optical and electronics devices already established [4].

The low solubility of the Mn ion in III-V semiconductors is solved by molecular beam epitaxy (MBE). When Mn is grown in a III-V semiconductor, such as GaAs, the major difference with respect to the II-VI DMS is that the Mn atom provides both the  $S = \frac{5}{2}$  spin and the dopant charge carrier (a hole since divalent Mn substitutes for trivalent Ga). As in the doped(II,Mn)VI systems, the main magnetic interactions in the (III,Mn)V DMS is the exchange between the Mn spins and the hole spins, which is known to be AFM. This interaction is again proportional to the probability of finding the charge carrier at the Mn site [7]. The equilibrium solubility of magnetic impurities in III-V semiconductors is low, and under ordinary crystal growth conditions it is impossible to introduce a high density of magnetic atoms. But using low temperature molecular beam epitaxy (LT-MBE), it is possible to achieve non-equilibrium crystal growth, and thereby succeeded in suppressing the surface segregation of Mn . Thus, using this method ferromagnetic p-type (In,Mn)As and p-type (Ga,Mn)As are grown. (Ga,Mn)As has become indispensable material for the study of semiconductor

spin electronics; and is being actively researched in recent years [2]. III-V materials are among the most widely used semiconductors. There is little doubt about that ferromagnetism in these materials would enable a host of new microelectronic device application if the following criteria were met:

- i. The ferromagnetic temperature should safely exceed room temperature
- ii. The mobile charge carriers should respond strongly to changes in the ordered magnetic state and
- iii. The material should retain fundamental semiconductor characteristics, including sensitivity to doping and light, and electric field produced by gate charges.

## 2.4 Search For High Transition Temperature in DMS

LT-MBE led to the first successful growth of (In,Mn)As and (Ga,Mn)As DMS ternary alloys with more than 1 percent Mn. Since the first report in 1992 of ferromagnetic transition in p-type (In,Mn)As at critical temperature  $T_c = 7.5K$  the study of critical temperature limits in (III,Mn)V DMSs has unfolded in different stages. Initial experiments in (In,Mn)As suggested an intimate relation between ferromagnetic transition and carrier localization. Theoretical critical temperature calculations based on the kinetic exchange model predicts room temperature ferromagnetism in (Ga,Mn)As with 10 percent Mn content. In spite of this optimistic predictions, the goal of breaking 110K record in (Ga,Mn)As remained elusive for nearly four years. Only recently has progress in MBE growth and in the development of post growth annealing techniques made it possible to suppress extrinsic effects, pushing  $T_c$  in (Ga,Mn)As up to 173K. The current  $T_c$  record should be broken if DMS materials with higher concentration



of substitutional Mn ions can be grown [5].

TABLE 2.1. Ferromagnetic transition temperatures in III-V host semiconductors doped with 5 percent of Mn and with itinerant hole densities  $p=0.1$  and  $p=0.5 \text{ nm}^{-3}$  using Mean-field ( $T_c^{MF}$ ), and collective spin wave ( $T_c^{Sw}$ ) approach (taken from [8]).

<i>Host</i>	$p(\text{nm}^{-3})$	$T_c^{MF}(\text{K})$	$T_c^{SW}(\text{K})$
<i>AlAs</i>	0.1	45	41
	0.5	134	105
<i>GaAs</i>	0.1	40	38
	0.5	124	106
<i>InAs</i>	0.1	14	14
	0.5	41	40
<i>AlSb</i>	0.1	19	18
	0.5	58	48
<i>GaSb</i>	0.1	18	18
	0.5	85	82
<i>InSb</i>	0.1	11	11
	0.5	37	35
<i>AlP</i>	0.1	94	73
	0.5	173	105
<i>GaP</i>	0.1	57	50
	0.5	101	43
<i>InP</i>	0.1	66	60
	0.5	136	103
<i>GaN</i>	0.1	379	81
	0.5	657	270
<i>InN</i>	0.1	308	89
	0.5	531	303

In the mean field approximation the Curie temperature is given by the following equation.

$$T_c = \frac{xzJ_{ex}}{3k_B} \quad (2.4.1)$$

where  $x$  is the Mn ion concentration,  $z$  number of nearest neighbors,  $J_{ex}$  is the total exchange interaction between nearest neighbors and  $k_B$  is Boltzmann constant.

Mean-field theory overestimates the transition temperature because it neglects the suppression of the transition temperature due to spin waves (see Table 2.1).

In (II,Mn)P and (III,Mn)N which has smaller lattice constant and wide-gap, the kinetic exchange model predicts  $T_c$ 's far above room temperature , especially in (Ga,Mn)N . Also the solubility limit of Mn is much larger than in arsenides, making it possible in principle to grow highly doped DMS, under or close to equilibrium conditions. However, the nature of magnetic interactions in Mn-doped phosphides and nitrides is not completely understood either theoretically or experimentally [5]. Experimental critical temperature close to 1000K have been reported in some (Ga,Mn)N samples. It is still unclear, however, whether the high temperature ferromagnetism phase should be attributed to (Ga,Mn)N ternary alloy or to the presence of ferromagnetic metal precipitate embedded in the host GaN lattice. However, a comparative study of (Ga,Mn)P and (Ga,Mn)As prepared by the post-MBE ion implantation and pulse laser melting annealing suggests carrier mediated origin of ferromagnetism in the (Ga,Mn)P material[5]. Current understanding of the material physics of (III,Mn)V DMS epilayers suggests that synthesis of a room temperature ferromagnetism will require a level of doping and defect control comparable to what has now been achieved in high-quality (Ga,Mn)As samples, Mn density of order of 10

percent and may require the use of wide-gap *III – V* alloys [5].

## 2.5 Mn impurity in GaAs

Among all (III,V) hosts, Mn impurity has been studied most extensively in GaAs [5]. With the discovery of ferromagnetism in (Ga,Mn)As the attention strongly shifted to *III – V* DMSs. Up to few years, the highest Curie-temperature was 100K for  $Ga_{0.95}Mn_{0.05}As$ . Due to the low temperature non equilibrium growth technique (LT-MBE) and the improvement of annealing procedure which leads to a complete removal of interstitial Mn atoms, the temperature was increased to 170K [5,9]. Large number of experimental and theoretical efforts have been devoted to (Ga,Mn)As and this is now the best understood DMS system. Nevertheless, for (Ga,Mn)As many things remain to be solved. For example, why is it not possible to further increase  $T_c$ , despite the fact that samples with Mn concentration up to 20 percent can be produced.

### 2.5.1 Substitutional Mn impurity ( $Mn_{Ga}$ ) in GaAs

The elements in the (Ga,Mn)As compound have nominal atomic structure  $[Ar]3d^{10}4s^2p^1$  for Ga,  $[Ar]3d^54s^2$  for Mn, and  $[Ar]3d^{10}4s^2p^3$  for As. This suggests that the most stable and therefore most common position of Mn in GaAs host lattice is on the Ga site where its two 4s electrons can participate in crystal bonding in much the same way as the two Ga 4s electrons. Because of the missing valence 4p electron, the substitutional Mn ( $Mn_{Ga}$ ) acts as an acceptor with a character of local moment. The local moment is formed by the three occupied sp-d hybridized bonding states with dominant  $t_{2g}$  ( $3d_{xy}, 3d_{xz}, 3d_{yz}$ ) character and by two occupied non bonding  $e_g$  state ( $3d_{x^2-y^2}, 3d_{z^2}$ ) orbitals that are split from the  $t_{2g}$  states by tetrahedral crystal field

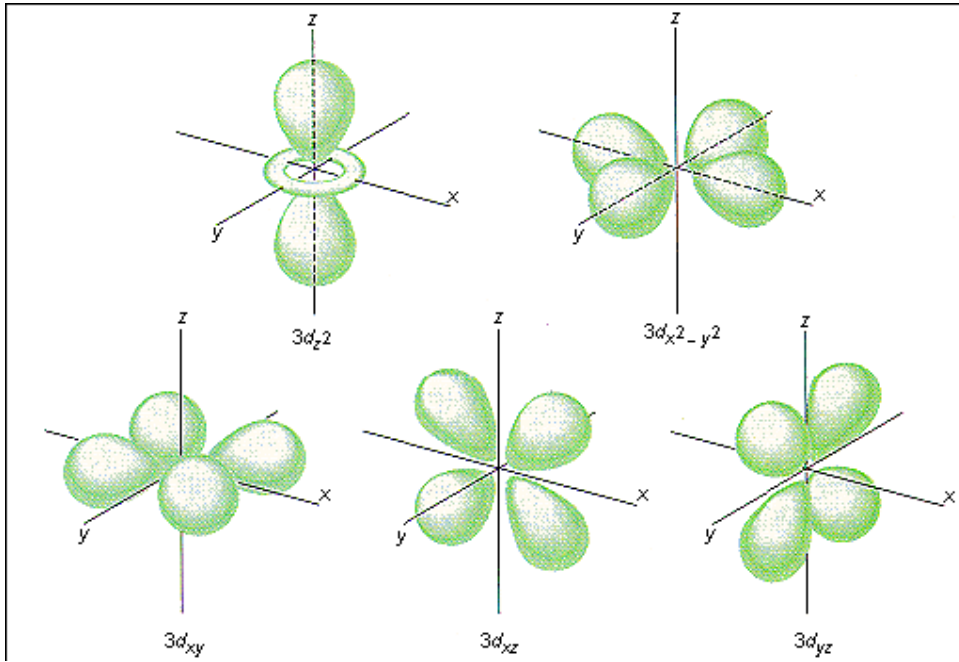


Figure 2.2: Two  $e_g$  3d orbitals and three  $t_{2g}$  3d orbitals of Mn [9].

and do not hybridize with sp orbitals (see Figure 2.2). All the occupied orbitals are assumed to have the same spin orientation and together comprise the  $S = \frac{5}{2}$  local moment.

The weakly bound hole occupies one of the three antibonding sp-d levels with dominant As 4p character. The top of the GaAs valence band is dominated by 4p levels which are more heavily weighted on As than on Ga site. Direct exchange between holes near the top of the band and localized Mn d electrons is weak since  $Mn_{Ga}$  and As belong to different sublattices in the zinc blende structure of the GaAs. This fact allows p-d hybridization to dominate, explaining the antiferromagnetic sign of this interaction seen in the experiment [5]. Substitutional Mn impurity in GaAs is mostly occurs in samples with doping level  $x < 0.1$  percent for which  $Ga_{1-x}Mn_xAs$

random alloy can be grown under equilibrium conditions. Ferromagnetism, however, is observed [5] only for  $x > 1$  percent which is well above the equilibrium Mn solubility limit in GaAs and therefore require non-equilibrium growth technique (LT-MBE) to avoid Mn precipitate. The high concentration leads to occurrence of a large number of metastable impurity states. The most important additional defects are interstitial Mn ions and As atoms on cation (Ga) sites which are antisite defects. Both acts as donors and can have a sever impact on the electric and magnetic properties of DMS which leads to unintended self compensation effect.

### 2.5.2 Interstitial Mn Impurity ( $Mn_I$ )

In highly doped as-grown samples ,experiment shows that 20 percent of Mn residing on interstitial positions. These are metastable that their density can be decreased substantially upon post growth annealing with temperature very close to the growth temperature. Total energy calculation showed that Mn can occupy two metastable interstitial positions, both with a comparable energy, one surrounded by four Ga atoms ( as shown in Figure 2.3) and the other surrounded by four As atoms. The two  $Mn_I$  states have similar local magnetic moments and electronegativity. Since a divalent Mn occupies interstitial position, it acts as a double donor and therefore compensate two substitutional Mn acceptors. It seems likely that because of the strong Coulombic attraction between positively charged  $Mn_I$  and negatively charged  $Mn_{Ga}$  defects, mobile interstitial pair up with substitutional Mn during growth. The total spin of  $Mn_{Ga} - Mn_I$  pair infered from ab-initio calculation and experiment is much smaller than the local spin  $S = \frac{5}{2}$  of isolated  $Mn_{Ga}$  acceptor. Ab-initio calculation of the valence-band spin splitting indicate that the exchange interaction coupling constants of interstitial and substitutional Mn are comparable. This would suggest a negligible

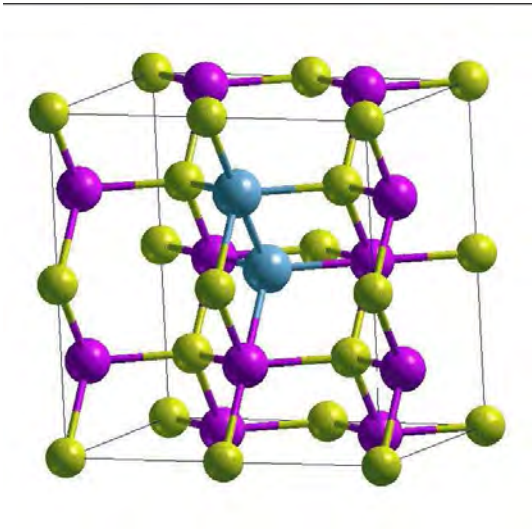


Figure 2.3: Substitutional  $Mn_{Ga}$  and interstitial  $Mn_I$  in the zinc blende structure of GaAs [5].

net p-d coupling between the antiferromagnetically coupled  $Mn_{Ga} - Mn_I$  pair and the valence band holes.

### 2.5.3 Arsenide antisite defect

Low temperature and over pressure on arsenide (As) often used in LT-MBE growth of (Ga,Mn)As is known to lead to the incorporation of high As antisite defects. These double donor defects may contribute to hole compensation. Unlike  $Mn_I$  impurities, As antisites are stable up to temperature of the order of  $450^\circ C$ . This is well above the temperature at which Mn precipitate starts to dominate the properties of (Ga,Mn)As and therefore As antisites can not be removed from epilayer by a post growth annealing treatment.

## Chapter 3

# THEORY OF FERROMAGNETISM IN DMS

Although magnetism is a well known phenomena, it can have very different origins which are difficult to understand, especially in DMSs [9]. The magnetic dipole-dipole interaction strength between two discrete moments separated by a lattice constant in a typical solid is only in the order of 1K, relegating direct magnetic interactions to a minor role in the physics of condensed-matter magnetic order. Although these terms are critical for specific properties like magnetic anisotropy, they are rarely, if ever, crucial for the onset of the magnetic order itself. Instead the universal ultimate origin of magnetic phenomenon is almost always the interplay between electronic spin degree of freedom, the repulsive coulomb interactions between electrons, and the fermionic quantum statistics of electrons. The pauli exclusion principle correlates the spin and orbital parts of the the many-electron wave function by requiring that the total wave function to be antisymmetric under particle interchange. Since magnetism is a strong phenomena, there is no especially useful universal theory of magnetism [2].

Based on the way the local atomic moments couple to each other, i.e., parallel, antiparallel or not at all materials can be classified as follows.

- (a). Diamagnetic: A system is called diamagnetic if it has no inherent magnetization, but when subject to an external field it develops magnetization that is opposite to the field.
- (b). Paramagnetic: A system is said to be paramagnetic if it has no inherent magnetization, but when subjected to an external field it develops magnetization which is aligned with the field.
- (c). Ferromagnetism : A system may exhibit magnetic order even in the absence of a external field below the ordering temperature called Curie temperature. If the magnetic moments tend to be oriented in the same direction (due to the positive exchange interaction ), the system is described as ferromagnetic.
- (d).Antiferromagnetic : If the exchange interaction between nearest neighbor is negative then the antiparallel alignment of their spin is preferred. In such case there is a tendency for ordering to occur in to a state in which up and down spins alternate within the structure and there is no macroscopic magnetization at zero external field.

The existence of ferromagnetic phase in DMSs is experimentally well established [10], but the physical origin of this phenomena is far from being well understood. The RKKY (Ruderman Kittel Kasua Yosida) approach, which successfully explains the ferromagnetism observed in magnetic metals, cannot easily be applied to the case of magnetic semiconductors, which are typically composed of a dilute subsystem of localized magnetic spins and an even more dilute gas of free carriers. On the other hand , the early Zener model of ferromagnetism driven by exchange interaction between free carriers and localized magnetic moments provides a rough estimate of the observed critical ferromagnetic temperatures in DMS materials [10]. Although there



is broad agreement that ferromagnetism in these systems is due to carrier mediated interactions between Mn (true for other transition metals also) local moments, consensus on the details of this picture is still building as the body of experimental studies on well characterized samples growth [14].

The understanding of the origin of the remarkable properties of DMS started with a model of their band structure, in which two electronic sub systems were distinguished: one containing the delocalized band electrons (which are described in terms of the virtual crystal approximation by extended states built from Bloch tight-binding functions) and the other consisting of the magnetic impurity electrons with magnetic moment localized in the ionic open 3d or 4f shells [6].

The low energy dynamics of the mobile carriers electrons in the conduction band (mostly composed of cation s orbitals) and holes in the valence bands (mostly consist of anion p orbitals) is described by effective mass theory. Primarily these effective mass carriers determine the electric and optical properties of DMS crystals. On the other hand, the localized magnetic moments are responsible for the fact that DMS exhibit magnetic properties. The most fascinating DMS features result, however from the strong spin dependent s(p)-d(f) exchange interaction between these systems [6]. The dependence of energy of a system on the relative orientation of the local moments of a system is called exchange interaction[9]. The study of the exchange interaction therefore is crucial for attempting to tailor new combinations of magnetic ions and semiconducting hosts with the desired magnetic properties.

## 3.1 s(p)-d(f) exchange interaction

Strong s(p)-d(f) exchange in DMS leads to the indirect exchange between the local moments of the magnetic ions (d-d exchange), which rules the magnetic properties of these materials. The direct coulomb exchange and the hybridization-mediated kinetic exchange are responsible for spin dependent interactions between band carriers and localized impurity magnetic moments [6].

### 3.1.1 Direct coulomb exchange

For undoped DMS the density of free charge carriers is much smaller than the concentration of the magnetic ion. This is due to some unwanted defect during growth. By doping the system the interaction can be made significant. For n-doped (or s-like conduction band) and for transition metal ions with open d-shell, Liu has shown that the direct coulomb exchange always leads to a ferromagnetic s-d coupling [6]. For p-doped DMS the exchange is antiferromagnetic p-d coupling. In both cases, the free carriers are believed to mediate an effective ferromagnetic coupling between the magnetic moments (spins) of the magnetic ion [11]. Fig.3.1 schematically shows these direct ferromagnetic exchange coupling between carriers and magnetic impurity spins.

### 3.1.2 Kinetic exchange

Schriffer and Wolf proved that the hybridization of orbitals of the magnetic atoms and the band carriers induces an exchange interaction between the two subsystems [6]. The p-d hybridization can be thought about in terms of virtual transitions of an electron between p bands and the ionic d shell. A singly occupied orbital (i.e. any

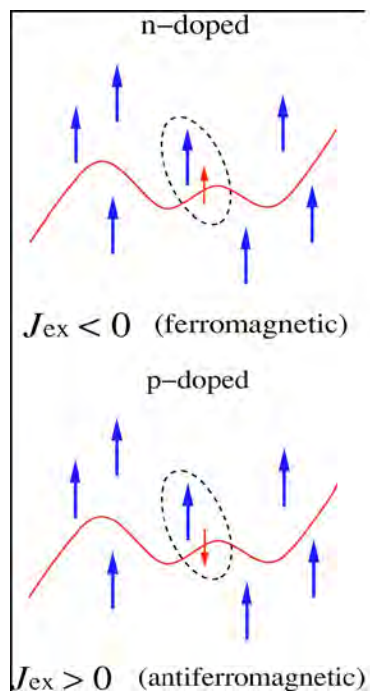


Figure 3.1: Schematic representation of the exchange coupling between itinerant-carrier and localized magnetic impurity spins in n-doped and p-doped DMS's [11]

of the  $t_{2g}$  d orbitals of the  $Mn^{2+}$  ion shown schematically in Figure 2.2 ) can participate in the virtual transitions involving both the creation and the annihilation of a band electron (See Figure 3.2) . The spin dependence of the hybridization-induced interactions results from the Pauli principle, which allows only for virtual transitions decreasing the total spin of the ion (either by removing the electron spin from the d orbital or by adding one with opposite spin). This leads to an antiferromagnetic interaction. Such exchange mechanism occurs when the majority d-states of the magnetic impurity are localized below the center of the valence p-bands [9]. The hybridization mediated kinetic exchange interactions depend not on the filling of all one electron d-orbitals but only the  $t_{2g}$  orbitals of the magnetic ions [6].

For the ion with all  $t_{2g}$  orbitals empty (like  $Sc^{2+}$  and  $Ti^{2+}$ ) only the annihilation, i.e. the transfer of the band electron (whatever its spin) into the ionic d shell, is possible (see Figure 3.3). For an empty (or doubly occupied) orbital the Pauli principle does not favor any direction of electron spin . Still, the interaction remains spin dependent as the ion prefers to increase its total spin, according to Hund's rule. The type of exchange depends on the difference of the energies required to transfer electrons with opposite spins. For very small number of electron transferred it is ferromagnetic. Finally, when some  $t_{2g}$  orbitals are singly occupied and others are empty (or doubly occupied), one gets different exchange interactions [6].

## 3.2 Spin interaction between magnetic ions

When looking for DMS with desired properties ,eg. ferromagnetism, one has to study the interactions that couple the spins of magnetic ions. There are several microscopic

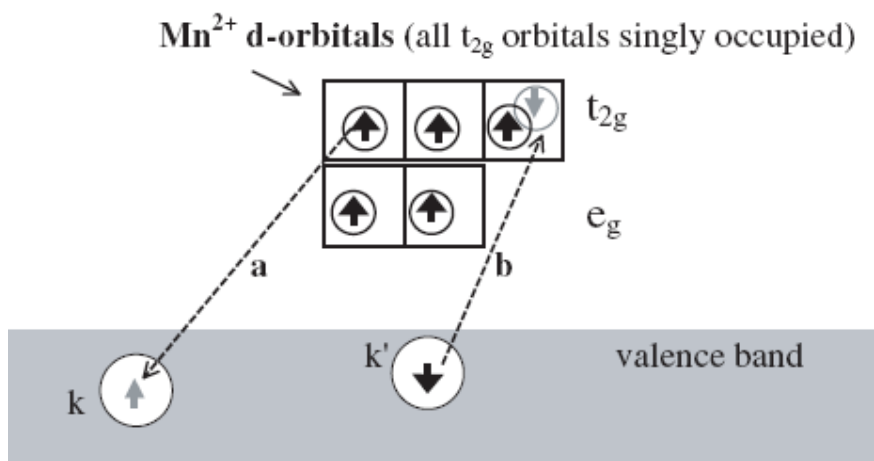


Figure 3.2: For a singly occupied d orbital (like all the  $t_{2g}$  orbitals in  $Mn^{2+}$  or  $Fe^{2+}$  ions) both kinds of virtual transitions, i.e. the transfer of an electron from the ion to the valence band (a) and from the band onto the ionic d shell (b), decrease the total spin of the ion, due to the Pauli principle [6].

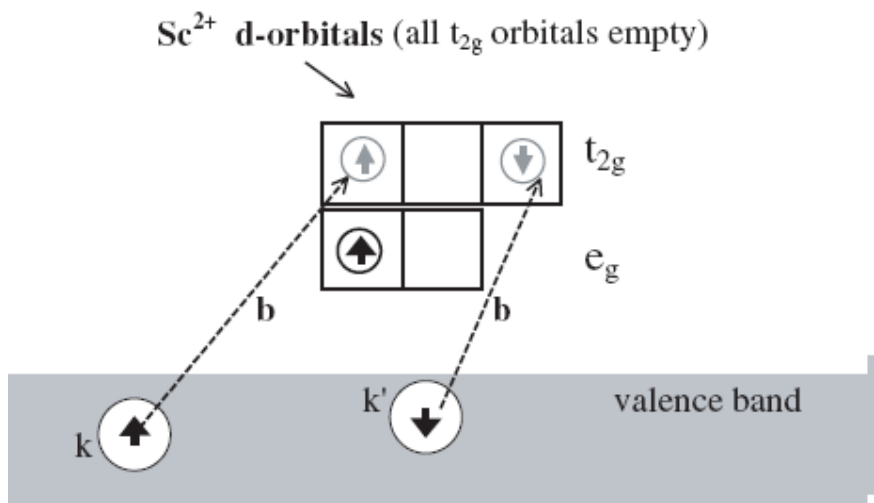


Figure 3.3: For an empty d orbital (like all  $t_{2g}$  orbitals in  $Sc^{2+}$ ) or  $Ti^{2+}$  only the virtual transitions (b) from the valence band into the ion are possible; a band electron with spin up as well as an electron with spin down can be transferred. According to the Hund's rule, the transition which increases the total spin of the ion leads to lower energy [6].

mechanisms that lead to the direct spin-spin (d-d) interactions between two magnetic ions. Ignoring the spin-orbit interaction (relativistic term), the most important of such interactions can be described by an effective Heisenberg Hamiltonian :

$$H = -2 \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad (3.2.1)$$

Where  $\vec{S}_i$  and  $\vec{S}_j$  are spin operators at  $i^{th}$  and  $j^{th}$  positions.  $J_{ij}$  is the exchange interaction constant. The sign and the value of the nearest neighbor (NN) and the next nearest neighbor (NNN) interaction constants  $J_{NN}$  and  $J_{NNN}$  depend on the mechanism involved.

### 3.2.1 Superexchange

Superexchange is an indirect mechanism where the interaction between the magnetic impurities is mediated through non-magnetic atoms . In a crystal environment, an electron can be transferred from a non-magnetic atom to an empty shell of the magnetic atom and interact, via direct exchange, with electrons forming its local moment (Figure 3.4(a)). The non-magnetic atom is polarized and is coupled via direct-exchange with all its magnetic neighbors . The sign of the interaction depends on the direct interaction sign[9]

For II-VI DMSs the superexchange, resulting from the sp-d hybridization is by far the dominant spin-spin interaction because modulatedly undoped such DMSs have no free carriers to mediate the interaction. For doubly occupied or empty d orbitals hybridization-mediated p-d kinetic exchange results both ferromagnetism and antiferromagnetism . When all hybridizing d orbitals ( $t_{2g}$ ) are single occupied (as for  $Mn^{2+}$ ,  $Fe^{2+}$ ,  $Co^{2+}$  ions ) the superexchange for both NN and NNN is antiferromagnetic [6].

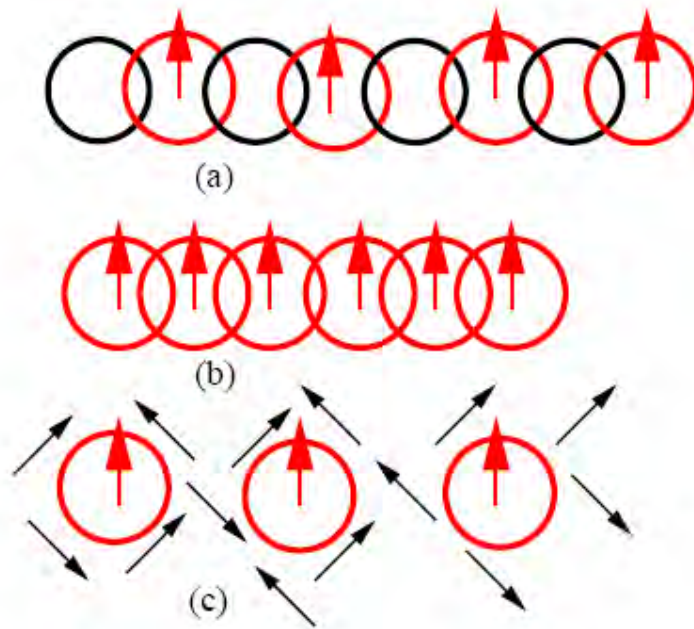


Figure 3.4: Schematic illustration of (a) superexchange where the magnetic ions are interact via the same non-magnetic ion; (b) direct exchange in which magnetic ions overlapping their charges;(c) indirect exchange where the interaction between the magnetic ions is mediated by charge carriers [9].

### 3.2.2 Blomberg-Rowland interaction

This mechanism differs from the superexchange only by selection of the intermediate states and it also includes the interaction of the magnetic ions and the conduction s-band. This can lead to a ferromagnetic d-d interaction even for  $Mn^{2+}$  ions, but was shown[5] to be an order of magnitude less effective than the superexchange.

### 3.2.3 RKKY Interaction

The other mechanism of interactions between magnetic atoms involve mediation through charge carriers. The local moments can have either a ferromagnetic direct

exchange interaction with band electrons on the same site/or an antiferromagnetic interaction due to hybridization between the local moments and band electrons on neighboring sites. Polarization of band electrons due to the interaction at one site is propagated to neighboring sites(Figure 3.4(c)). When the coupling is weak, the effect is described by RKKY theory [9]. What makes RKKY difficult and often not suitable for describing the ion-ion interaction in DMS is that, in contrast to the other mechanisms, it becomes efficient only when a high concentration of free carriers present. Ruderman and Kittel showed that RKKY interaction is ferromagnetic at short distance and at large distances, oscillates with decreasing amplitude [6].

### 3.2.4 Double exchange

The double exchange couples magnetic ions in different charge states (mixed-valence) by virtual hopping of the ‘extra’ electron from one ion to the other. Double exchange favors the ferromagnetic alignment of the ionic spins. For the superexchange constant  $J_s > 0$ , the superexchange also favors ferromagnetism. But for  $J_s < 0$ , there is a competition between the ferromagnetic double exchange and the antiferromagnetic superexchange.

### 3.2.5 Zener double exchange model

It was suggested by Akay [6] that in doped  $III - V$  DMS the double exchange, rather than the RKKY, is responsible for the observed carrier induced ferromagnetism. Dietl et al, showed, however, that the double exchange cannot be the mechanism leading to ferromagnetic correlation between distance Mn spins, as the magnetic electrons remain localized at the magnetic ion and do not contribute to charge transport. Instead, these authors proposed the Zener model which states that the spin polarization



of the localized spins results in a spin splitting of the band carriers and that in this situation the exchange coupling between the free carriers and the localized spins leads to ferromagnetism. The authors showed that partially localized holes in DMS supplied by the Mn ions also transmit magnetic information effectively between the Mn ion spins due to the large density of states in the valence band and strong spin dependent p-d hybridization.

# Chapter 4

## Formulation of the problem

DMSs are very important materials because of their potential use in spintronics as information storage, information processing, communication and quantum computing. But before their possible use for all these applications their properties have to be understood; specially their magnetic and transport properties .

Electrical and optical manipulation of electron spin is becoming increasingly possible. Inview of this ,the study of DMSs in quantum dots, quantum wires and quantum wells is being undertaken.

Recently magnetic and transport properties of DMSs are being made in their quantum forms mentioned above. In this chapter, it is given a special attention on the problem area, i.e., the diluted magnetic semiconductors(DMSs) quantum well.

### 4.1 DMSs Quantum Wells

This system consists a modulation doped diluted magnetic semiconductor quantum well where magnetic Mn impurities have been inserted in the well with a fraction  $x$  on the cation site of the host semiconductor. In such quantum well, two types of systems

have to be considered for the physics we are interested in. The first sub-system is composed by two dimensional carriers populating on a confined level of the well. These carriers originate from the n-type or p-type dopant impurities located in the barrier. They form an itinerant spin sub-system which is coupled to the second sub-system formed by spins of electrons localized on the manganese impurities introduced in the well. These electrons occupy the d-shell of Mn atoms and each Mn atom will behave like a unique  $\frac{5}{2}$  spin.

Assuming the carriers move freely in x-y plane occupying one or several transverse mode, then the three dimensional(3D) spacial coordinate can be splitted into  $(\mathbf{r}, z)$ , where  $\mathbf{r}$  is the two dimensional(2D) x-y projection and  $z$  is along the length  $d$ , of the quantum well. In the mean field theory the local Mn-ions are treated as independent but subjected to an effective magnetic field which originate from their exchange interactions with spin-polarized free carriers confined in the well. Similarly, the itinerant carrier system sees an effective field proportional to the Mn density and polarization.

Then in the absence of applied magnetic field, the Hamiltonian of the system is given by

$$\hat{H} = \hat{H}_{kin} + \hat{H}_{ex} \quad (4.1.1)$$

where  $H_{kin}$  is the kinetic energy of the carrier.  $\hat{H}_{ex}$  is the coupling energy between the carriers and the local Mn spins. It is written as

$$\hat{H}_{ex} = J_{ex} \int d^2r \int_0^d dz \vec{S}(\vec{r}, z) \cdot s(\vec{r}, z) \quad (4.1.2)$$

$J_{ex}$  is the coupling constant,  $\vec{S}(\vec{r}, z)$  is the Mn ion spin density, and  $s(\vec{r}, z)$  is the quantum well carriers spin density.

This model system has given a new access to spin excitations and spectrum of the spin

polarized 2D carriers and the magnetic impurity spin system. Recently, high mobility spin-polarized two dimensional electron gas (SP2DEG) have been obtained in dilute magnetic semiconductor (DMS) heterostructures like  $Cd_{1-x}Mn_xTe/Cd_{1-y}Mg_yTe$  n-type modulation doped quantum wells [10,17].

It is mentioned in the previous chapters that for undoped III-V DMSs the concentration of Mn ion is far far greater than that of the carriers concentration though each Mn-ion supplies a hole to the system. This is due to disorder effect while growing Mn ion in the semiconductors crystal. Theoretical and experimental studies show that the enhancement of the concentration of the carriers enhances the exchange interaction between the two subsystems. This interaction leads to relatively a strong polarization in both systems. Thus, it can solve the problem of the lack of polarized carriers to be used in the spintronics devices. The quantum well with modulation doped barrier can lead to such useful phenomena.

Most recently Diego Frustaglia et al [11] have studied the carrier mediated interaction in quantum well in searching such phenomena. They found anomalous dispersion of spin waves ( magnons) for the Mn system in quantum wells. For small wave vectors, the authors reported the quartic dispersion for gapless Goldstone mode of the form :

$$\Omega = O(\varepsilon_k^2) \tag{4.1.3}$$

where  $\varepsilon_k = \frac{\hbar^2 k^2}{2m^*}$  is the kinetic energy of the an electron,  $m^*$  is the effective mass of electron. We have undertaken the study of magnetization and specific heat of the Mn system taking this anomalous dispersion. The reason is that the magnetization and the specific heat will show completely different temperature dependence which could

shed some interesting light on these quantum systems.

## 4.2 Magnetization, Curie temperature and Specific heat for 3D Mn-spin configuration

### 4.2.1 Magnetization and Curie temperature

The low temperature magnetic properties of a ferromagnets can be understood by spin waves (magnetic excitations) approach .The magnetization of the system at a temperature T is given by:

$$M(T) = M(0) - g\mu_B \sum_k n_k \quad (4.2.1)$$

where  $M(0) = N_{Mn}g\mu_B S$  is the saturated magnetization at  $T = 0$ ,  $N_{Mn}$  is the total number of magnetic atoms per unit volume,  $g$  is the Lande' g factor,  $\mu_B$  is Bohr magneton,  $S$  is total spin per atom and  $n_k$  is the number of spin wave modes with the same wave vector  $k$ . Since magnons carry a unit spin, they are bosons and their distribution function is given by Bose-Einstein statics as :

$$n_k = \frac{1}{\exp(\beta E_k) - 1} \quad (4.2.2)$$

where  $\beta = \frac{1}{k_B T}$ ,  $k_B$  is Boltzmann constant,  $E_k$  is the energy (dispersion) of a magnon and we take the value of  $\hbar = 1$  in the magnon energy  $E_k = \hbar\Omega$ . At low temperature the magnon with large excitation energy (i.e.  $k \gg 1$ ) are very small, the summation over  $k$  can be converted to integration up to infinity. To convert into integration we use the number of states between  $k$  and  $k+dk$  in the three dimensional DMS sample inside the quantum well as :

$$g(\vec{k})d\vec{k} = \frac{v}{(2\pi)^2} k^2 dk \quad (4.2.3)$$

The total number of magnon  $n$  substituting the value of  $n_k$  is given by

$$n = \sum_k n_k = \int_0^\infty n_k g(\vec{k}) d\vec{k} = \frac{v}{(2\Pi)^2} \int_0^\infty n_k k^2 dk$$

$$n = \frac{v}{(2\Pi)^2} \int_0^\infty \frac{k^2 dk}{\exp(\beta E_k) - 1} \quad (4.2.4)$$

Here  $v = ad$  is the volume of the DMS in the quantum well, where (a) is area of quantum well and d is its thickness. Following the work of Diego Frustaglia et al [11] as mentioned in the previous section, the quartic dispersion for a proportionality constant  $p$ , frequency  $\Omega$  and  $\hbar = 1$  can be written as

$$E_k = \Omega = pk^4 \quad (4.2.5)$$

Then  $n$  become  $n = \frac{v}{(2\Pi)^2} \int_0^\infty \frac{k^2 dk}{\exp(\beta pk^4) - 1}$

let  $y = \beta pk^4$ , this leads the form of the integration(4.2.4) to:

$$n = \frac{v}{4(2\Pi)^2 (p\beta)^{\frac{3}{4}}} \int_0^\infty \frac{y^{-\frac{1}{4}} dy}{\exp(y) - 1}$$

Using the expansion  $\frac{1}{\exp(y) - 1} = \sum_{r=1}^\infty \exp(-ry)$ , the value of  $n$  become

$$n = \frac{v}{4(2\Pi)^2 (p\beta)^{\frac{3}{4}}} \zeta\left(\frac{3}{4}\right) \Gamma\left(\frac{3}{4}\right) = h T^{\frac{3}{4}} \quad (4.2.6)$$

where  $\zeta\left(\frac{3}{4}\right)$  and  $\Gamma\left(\frac{3}{4}\right)$  are Riemann Zeta and Gamma functions respectively. Using the approximate values of these functions the constant  $h$  is given by

$$h = \frac{v}{4(2\Pi)^2} \left(\frac{k_b}{p}\right)^{\frac{3}{4}} \zeta\left(\frac{3}{4}\right) \Gamma\left(\frac{3}{4}\right) = 0.891 v \left(\frac{k_b}{p}\right)^{\frac{3}{4}} \quad (4.2.7)$$

Then the magnetization becomes :

$$M(T) = M(0) - g\mu_B h T^{\frac{3}{4}} \quad (4.2.8)$$

The reduced magnetization is given by

$$\frac{M(T)}{M(0)} = 1 - AT^{\frac{3}{4}} \quad (4.2.9)$$

where

$$A = \frac{h}{N_{Mn}S} = \frac{v}{4(2\Pi)^2 N_{Mn}S} \left(\frac{k_b}{p}\right)^{\frac{3}{4}} \zeta\left(\frac{3}{4}\right) \Gamma\left(\frac{3}{4}\right) = 0.356 \frac{v}{N_{Mn}} \left(\frac{k_b}{p}\right)^{\frac{3}{4}} \quad (4.2.10)$$

The temperature dependence of the reduced magnetization is plotted in Figure 4.1.

The curie temperature at which the magnetization vanishes is

$$T_c = \left(\frac{1}{A}\right)^{\frac{4}{3}} = 3.957 \left(\frac{N_{Mn}}{v}\right)^{\frac{4}{3}} \frac{p}{k_b} \quad (4.2.11)$$

## 4.2.2 Specific Heat

The contribution of spin waves to the internal energy of the magnetic quantum well is

$$U_{mag} = \frac{v}{(2\Pi)^2} \int_0^\infty E_k n_k k^2 dk \quad (4.2.12)$$

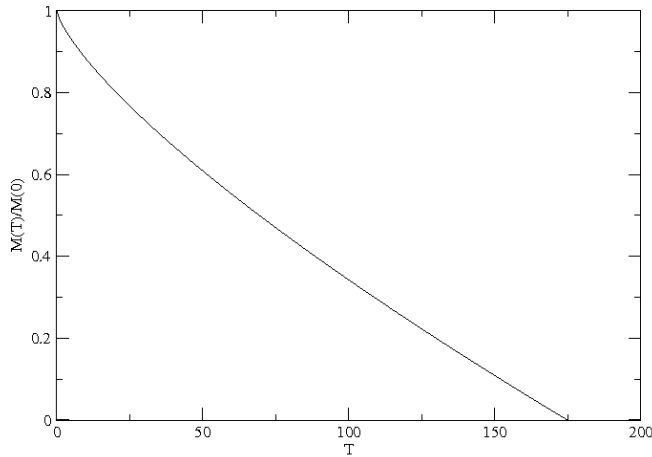


Figure 4.1: Magnetization versus temperature graph in the 3D Mn-spin configuration from eq(4.2.9)

substituting value of  $E_k$  and  $n_k$

$$U_{mag} = \frac{v}{(2\Pi)^2} \int_0^\infty \frac{pk^6 dk}{\exp(\beta pk^4) - 1} \quad (4.2.13)$$

Let  $y = \beta pk^4$ , then

$$U_{mag} = \frac{vp}{4(2\Pi)^2(\beta p)^{\frac{7}{4}}} \int_0^\infty \frac{y^{\frac{3}{4}} dy}{\exp(y) - 1} \quad (4.2.14)$$

Following the same procedure as the previous subsection



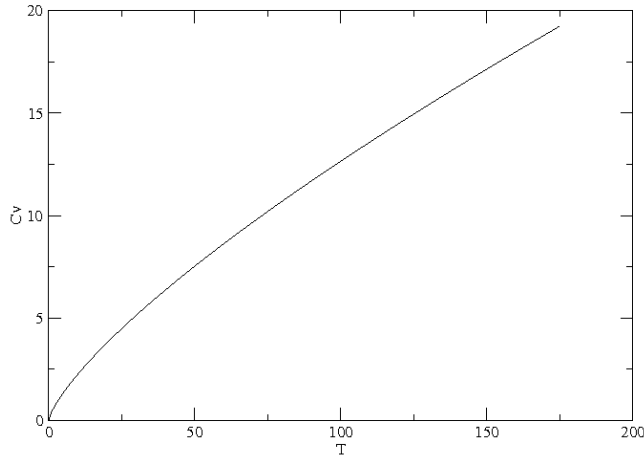


Figure 4.2: Specific heat versus temperature in 3D Mn spin configuration for eq(4.2.16)

$$U_{mag} = \left[ \frac{vk_b^{\frac{7}{4}} \zeta(\frac{7}{4}) \Gamma(\frac{7}{4})}{4(2\Pi)^2 p^{\frac{3}{4}}} \right] T^{\frac{7}{4}} \quad (4.2.15)$$

The specific heat,  $c_v$  is given as

$$c_v = \frac{\partial U_{mag}}{\partial T} = h' T^{\frac{3}{4}} \quad (4.2.16)$$

where

$$h' = \left[ \frac{3vk_b^{\frac{7}{4}} \zeta(\frac{7}{4}) \Gamma(\frac{7}{4})}{16(2\Pi)^2 p^{\frac{3}{4}}} \right] = 0.021 \left[ \frac{vk_b^{\frac{7}{4}}}{p^{\frac{3}{4}}} \right] \quad (4.2.17)$$

The specific heat versus temperature graph is plotted in Figure 4.2

### 4.3 Magnetization, Curie temperature and Specific heat in 2D Mn-spin configuration

If the width of the quantum well is minimized, the three dimensional nature of the magnetic ions dissolves into two dimension. Then we consider the problem in a wave vector plane. Following similar steps like the previous section, the thermodynamic properties of the two dimensional Mn spins in the quantum well is obtained as follows. The number of states between  $k$  and  $k+dk$  in the two dimensional DMS quantum well is given by:

$$g(\vec{k})d\vec{k} = \frac{a}{2\Pi}kdk \quad (4.3.1)$$

where  $a$  is the area of the quantum well. The total number of magnon  $n$  is

$$n = \frac{a}{2\Pi} \int_0^\infty \frac{kdk}{\exp(\beta E_k) - 1} \quad (4.3.2)$$

$$n = \frac{a}{8\Pi(\beta p)^{\frac{1}{2}}} \zeta\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) = \lambda T^{\frac{1}{2}} \quad (4.3.3)$$

where  $\zeta\left(\frac{1}{2}\right)$  and  $\Gamma\left(\frac{1}{2}\right)$  are Rieman Zeta and Gamma functions respectively. The constant  $\lambda$  is given by

$$\lambda = \frac{a}{8\Pi} \left(\frac{k_b}{p}\right)^{\frac{1}{2}} \zeta\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) \quad (4.3.4)$$

The magnetization is obtained as

$$M(T) = M(0) - g\mu_B n = M(0) - g\mu_B \lambda T^{\frac{1}{2}} \quad (4.3.5)$$

The reduced magnetization is given by

$$\frac{M(T)}{M(0)} = 1 - BT^{\frac{1}{2}} \quad (4.3.6)$$

where

$$B = \frac{\lambda}{N_{Mn}S} = \frac{a}{8\Pi N_{Mn}S} \left(\frac{k_b}{p}\right)^{\frac{1}{2}} \zeta\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) = 56.704 \frac{a}{N_{Mn}} \left(\frac{k_b}{p}\right)^{\frac{1}{2}} \quad (4.3.7)$$

This reduced magnetization is plotted in Figure (4.3).

The curie temperature at which the magnetization vanishes is

$$T_c = \left(\frac{1}{B}\right)^2 = 3.11 \times 10^{-4} \left(\frac{N_{Mn}}{a}\right)^2 \frac{p}{k_b} \quad (4.3.8)$$

$N_{Mn}$  is the number of Mn atoms per unit area.

The contribution of spin waves to the internal energy of the 2D magnetic quantum well is obtained as

$$U_{mag} = \frac{a}{2\Pi} \int_0^{\infty} E_k n_k k dk \quad (4.3.9)$$

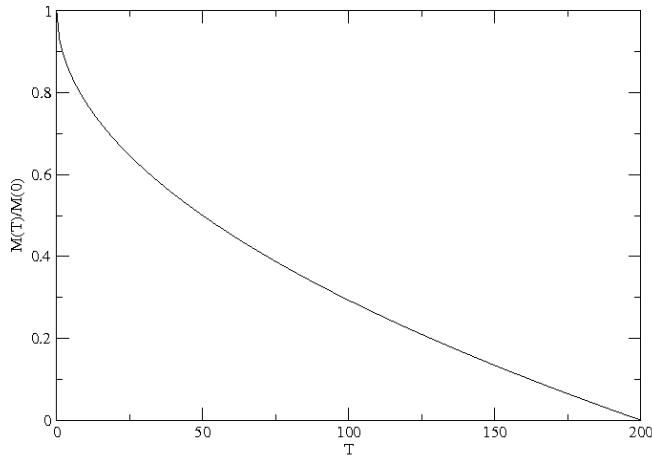


Figure 4.3: Magnetization versus temperature graph in the 2D Mn-spin configuration for eq(4.3.5)

substituting values of  $E_k$  and  $n_k$

$$U_{mag} = \frac{a}{2\Pi} \int_0^{\infty} \frac{pk^5 dk}{exp(\beta pk^4) - 1} \quad (4.3.10)$$

$$U_{mag} = \left[ \frac{ak_b^{\frac{3}{2}} \zeta(\frac{3}{2}) \Gamma(\frac{3}{2})}{8\Pi p^{\frac{1}{2}}} \right] T^{\frac{3}{2}} \quad (4.3.11)$$

The specific heat,  $c_v$  is given by

$$c_v = \frac{\partial U_{mag}}{\partial T} = \lambda T^{\frac{1}{2}} \quad (4.3.12)$$

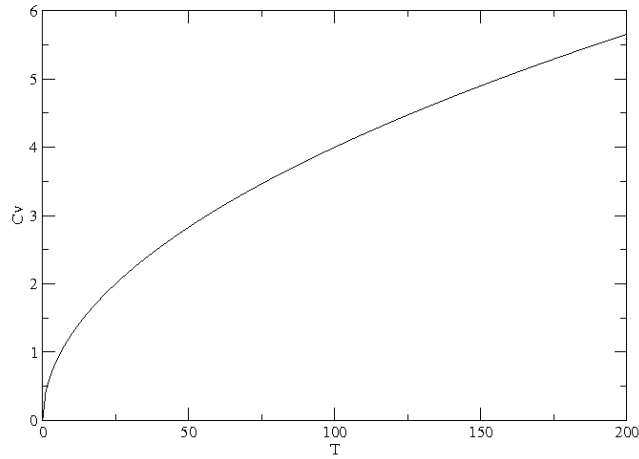


Figure 4.4: Specific heat versus temperature graph in 2D Mn-spin configuration for eq(4.3.12)

where

$$\lambda' = \frac{3ak_b^{\frac{3}{2}}\zeta(\frac{3}{2})\Gamma(\frac{3}{2})}{16\Pi p^{\frac{1}{2}}} = 0.138 \frac{ak_b^{\frac{3}{2}}}{p^{\frac{1}{2}}} \quad (4.3.13)$$

It is plotted in Figure (4.4).

# Chapter 5

## Results, Discussion And Conclusion

The effort to generate and manipulate spin-polarized carriers in a controllable environment in semiconductors has triggered the discovery of carrier induced ferromagnetism in DMS's [12]. Although it was possible to incorporate high Mn concentration in III-V semiconductors, the hole to Mn concentration ratio was very low and was not enough to realize ferromagnetic ordering. In order to obtain high hole concentration, it requires selective p-doped barrier to overlying Mn-doped III-V semiconductors. This successfully maximize the ferromagnetic order among the Mn spins in the III-V semiconductors by overlapping the wave function of the two dimensional hole gas (2DHG) with the Mn doping profile [13]. Extracting various magnetic parameters influencing the spin excitations is essential for gaining complete control of the magnetic properties of DMS films with an eye on successful future applications, such as spin injection and manipulation [15]. We have considered the spin wave approach to extract some magnetic parameters at low temperature.

## 5.1 Results and Discussion

Using Diego Frestaglia [11] report of the form of the dispersion relation for DMS quantum well, we discussed the contribution of the small wavevector gapless Goldstone mode of spin excitation on temperature dependence of the magnetization and the specific heat for 3D and 2D Mn spins configuration.

As it is shown in eq.(4.2.8) and Figure 4.1 the magnetization of the 3D Mn spin configuration in the quantum well decreases as a power of three by four ( $T^{\frac{3}{4}}$ ) with temperature. To obtain the value of A and draw the graph we use  $T_c = 175K$  which is the highest as of now.

For the 2D Mn spins configuration, the magnetization decreases as a power of half with temperature ( $T^{\frac{1}{2}}$ ) as shown in eqs.(4.3.5 and 4.3.6) and Figure 4.3. This shows that the minimization of the quantum well thickness further reduces the rate at which the magnetization decreases with temperature. To obtain the value of B and draw the graph we use  $T_c = 200K$  assuming that the 2D Mn-spin configuration has higher  $T_c$  value than the 3D.

In both cases the magnetization in the ordered region of the quantum well decreases more slowly than the rate it decreases in the bulk DMS whose rate is proportional to the power of three by two ( $T^{\frac{3}{2}}$ ) with temperature[11]. These lead to a wide temperature range magnetic ordering in the DMS quantum well. Qualitatively, the result is in a good agreement with the comparative report of highest temperatures at which ferromagnetic ordering can be observed in bulk and hetrostructures (i.e. thin films) DMS in [13].

As shown in the eqs.(4.2.16) and (4.3.12) and Figures 4.2 and 4.4 the specific heat of the DMS quantum well increases as a power of  $\frac{3}{4}$  and  $\frac{1}{2}$  with temperature for 3D

and 2D Mn-spin configuration respectively. These rates are smaller than the one in the bulk DMS. This slow increase in specific heat helps a small energy dissipation in the spintronics devices and hence they consume small energy. This is one of the advantage of spintronics over electronics.

## 5.2 Conclusion

From the previous chapter and the previous section it is possible to conclude that the quantum confinement of the system, and the increase in the number of the carriers drastically affects the magnetic properties of the DMS. In another words the retarded free-carrier mediated interaction between the Mn-ions is responsible for unusual aspect of the collective spectrum and the magnetization. We can see that  $T_c$  can be enhanced in case of nano sized DMS quantum well. When we compare the 3D and the 2D results we can conclude that the minimization of the well leads to a strong interaction and hence a wide temperature range magnetization.

In the absence of exact parametric values for the system a realistic calculation for the  $T_c$  and other quantities of interest can not be made. However, we have taken some current appropriate values of  $T_c$  in the thin films to obtain the values of the constants A and B. Since DMS are going to be of immense interest for spintronics because of their potential applications in the newly emerging magnetoelectronics, some approximations have been made. It is clear that the parametric dependence of A and B will decide  $M(T)$ ,  $T_c$ , and  $C_v$  for which experimental value has to be accomplished in the precise determination of the parametric dependence of the constants.

Even though there is no universally accepted best model (or theory) of magnetic interaction (especially in DMS), it is believed that a careful and detailed study on



the two dimensional carriers especially their degree of polarization due to their interaction to the localized spin systems is so much important. Improving the degree of polarization of the carriers solves one of the problems of spintronics which is lack of polarized carriers. In addition, further study on the localized spin system to determine the explicit dependence of the  $M(T)$  ,  $T_c$  and  $c_v$  on the concentration and polarization of the two sub-systems, and on the quantum well size also reveal a lot of interesting physics which can leads the application of spintronics instead of the current electronics devices.

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# Decleration

I hereby declare that this thesis is my original work and has not been presented for any acadamic purposes in any other universities. All source of materials used for the thesis have been duly acknowledged.

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Signature :.....

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