ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

A COMPARATIVE STUDY AND COMPUTER CODING OF LIMIT-EQUILIBRIUM-BASED SLOPE STABILITY ANALYSIS TECHNIQUES

BY

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ADVISOR: Dr. –Ing. Asrat Worku

July 2003, ADDIS ABABA

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LIST OF SYMBOLS

b	=	width of a slice
c'	=	cohesion interms of effective stress;
E	=	the horizontal interslice normal forces;
F	=	safety factor;
FF	=	safety factor obtained by equilibrium of forces;
FM	=	safety factor obtained by equilibrium of moments;
f	=	the perpendicular offset of the normal force (N) from the center of rotation or from the center of moments;
f(x), k	=	scalar interslice force function;
H	=	the height of each interslice surface;
h	=	the height of each slice from the center of the base;
h_t	=	height of line of thrust of interslice force from the base of each slice;
K_h	=	horizontal seismic force coefficient;
K_v	=	vertical seismic force coefficient;
ΔL	=	the length of the base of each slice;
N	=	the total normal force on the base of the slice;
P	=	external vertical point load acting on a slice;
Q	=	external horizontal force acting on the typical slice;
R	=	the radius for a circular slip surface or the moment arm with respect to the center of moments, associated with the mobilized shear force, S, for any shape of slip surface;
r_u	=	pore pressure ratio;
S	=	the shear force mobilized on the base of each slice;
T	=	the vertical interslice shear forces;
u	=	the excess pore pressure acting at the base of each slice;
V	=	equivalent external vertical distributed load (Equivalent Point Load);
VS	=	vertical distributed load;
W	=	the weight of a slice of width b and height h;
$\overline{X}$	=	the horizontal distance from the centerline of each slice to the center of rotation or to the center of moments;

$\overline{X}_P$ = the horizontal distance from the point of action of point load, P, to the center of rotation or to the center of moments;

$\overline{X}_V$ = the horizontal distance from the point of action of equivalent distributed load V to the center of rotation or to the center of moments;

X_c, Y_c = X and Y coordinates of the center of the slip circle;

X_m, Y_m = X and Y coordinates of the center of an arbitrarily chosen center of moments;

X_{LV}, X_{RV} = left and right, X coordinates of distributed load, VS;

X_P, X_V = X coordinates of points of action of loads P and V;

X_Q, Y_Q =X and Y coordinates of point of action of the horizontal external load, Q;

$\overline{Y}$ = moment arm of horizontal seismic load;

$\overline{Y}_Q$ = the vertical distance from the point of action of the external horizontal force on each slice to the center of rotation or to the center of moments;

Z = resultant interslice force;

α = the angle between the tangent to the center of the base of each slice and the horizontal;

δ = inclination angle of resultant interslice force;

λ = a scaling factor;

ϕ' = angle of internal friction of the soil in terms of effective stress;

ϕ'_m = mobilized angle of internal friction

γ = unit weight of soil

σ_n = total normal stress acting at the base of each a slice

A B S T R A C T

General slope stability concepts and the parameters and major considerations associated with slope stability problems are discussed. General features and theories of some of the available methods of analysis such as limit equilibrium methods, limit analysis methods, finite element method, etc. are reviewed.

Special emphasis is made to the limit equilibrium slope stability analysis techniques based on the method of slices. The underlying principles, common features and assumptions employed by these methods are elaborated. The various commonly used methods of slope stability analysis based on the limit equilibrium principles are then thoroughly assessed, compared and contrasted.

These common limit equilibrium based methods of analysis, including Bishop's simplified method, Janbu's simplified method, Bishop's rigorous method, Spencer's/Morgenstern and Price's method, Janbu's rigorous method, are formulated in a unified manner. A computer program is then written for these selected methods, using the Visual Basic language, on the basis of this unified formulation. The computer program has the capacity of analyzing slopes with general-shaped slip surfaces and general loading conditions. It is also capable of incorporating earthquake loading based on the pseudo-static analysis.

Finally, example problems are solved to illustrate the computer program developed and to reflect on the discussions and conclusions in the literature review section. The results obtained for the example problems, using the computer program developed and the different methods of analyses are in close agreement with those obtained by the author's of the original methods.

Key Words: slope stability analysis; limit equilibrium; method of slices; factor of safety

CHAPTER 1: BACKGROUND AND GENERAL SLOPE STABILITY CONCEPTS

1.1 BACKGROUND AND OBJECTIVE

There are various methods of slope stability analysis. Limit equilibrium based analysis techniques are among the most common and widely used methods of analysis.

These analysis techniques have been the subject of extended discussion and research over the years. A lot of work has been done by prominent researchers to assess their theoretical backgrounds, accuracy and reliability of obtained results and their limitations and strong features.

The objective of this work is to review and make comparative study of the common and widely used limit equilibrium based analysis techniques and develop a computer program for each method.

The computer program developed incorporates the basic features and assumptions as employed by the various limit equilibrium based analysis techniques. It can thus help assess reasonableness and consistency of computed safety factors by making comparisons of results as obtained by each method. In addition, sensitivity of the computed safety factors and stresses to variations in the assumptions could be examined by performing comparative studies.

1.2 GENERAL CONCEPTS

Slopes either occur naturally or are engineered by humans. Man made slopes include road way cut and fill slopes, embankments, dams and other similar constructions.

Slope stability problems and associated catastrophes have occurred throughout history when the delicate balance of natural slopes has been disrupted by human beings or by nature. Failure can occur due to faulty designs of engineered slopes or unforeseen natural hazards, which cause the disruption of even engineered slopes because they were not anticipated during the design process.

Increased demands, particularly for engineered cut and fill slopes on various types of construction projects over the years have increased the need to understand failure mechanism. Analytical tools and stabilization methods have been developed to solve and mitigate slope stability problems.

In general, a basic understanding of geology, hydrology and soil properties is required for the proper identification of underlying principles, conditions and applications of slope stability principles to particular or general problems.

Slope stability analyses are carried out with the aim of conducting safe and economical design of excavations, embankments, earth dams, landfills etc. and understanding of nature, magnitude and frequency of potential slope problems.

Topography, geology and material properties often relating to whether the slope was naturally formed or engineered are taken into account in defining and formulating a given slope stability problem.

Through slope stability analysis:

- Stability of slopes under short term, or long term conditions is assessed;
- Failure mechanisms and causes of environmental factors in existing land slides are assessed;
- Effects of seismic loads on slopes and embankments are analyzed;
- Redesign of failed slopes and devising remedial measures, where necessary, are carried out;

Any slope stability analysis, regardless of the method used, demands reasonably accurate modeling of the site subsurface condition, ground behavior and applied loads. Analysis results must be judged based on acceptable risk or safety margins and validity of solution with respect to accepted trends of soil behavior.

1.3 SOIL PARAMETERS AND CRITICAL STABILITY CONDITIONS

Selecting appropriate conditions for analysis of slopes requires consideration of the shear strength of the soils under drained or / and undrained conditions, consideration of pore water pressures and conditions that will control drainage in the field and accurate modeling of geometric and loading conditions.

The methods and procedures of slope stability analysis involve measurement of material properties of the soil and computations of stresses within the soil mass under the influence of actual or anticipated loading conditions.

Computed stresses could vary based on the degree of accuracy in modeling the problem, method of analysis employed (limit equilibrium method, finite element method, etc.) and assumptions associated with the methods.

1.3.1 Soil Strength Parameters – Testing

Measured soil strength parameters are not unique. Their magnitudes depend on a number of factors such as structure of the soil, stress history, drainage condition, method of loading, definition of failure, degree of disturbance, rate of loading, etc.

Generally, quality of laboratory test results depends on type of soil, quality and size of test sample, testing method and skill of testing personnel (technicians).

Size is of particular importance as marked differences have been observed between actual field strengths and performances and laboratory measured parameters. This has been conveniently expressed by Bishop, who pointed out that, 'Field strength values are much closer to that of large scale field tests than to strength measurements on small laboratory samples.' [20].

These discrepancies between observed field behavior and laboratory measured strength parameters could be due to any or all of the following factors:

- Sampling technique/sample disturbance;
- Sample orientation – especially where the soil contains discontinuities or fissures;
- Rate of loading (time to failure) – usually field loading rates are slower than laboratory shearing rates;
- Size of samples – influence of secondary structures like fissures that may not be represented in the small laboratory samples;
- Softening upon removal of load by excavation which is a gradual process;
- Progressive failure conditions in stiff/fissured clays.

Strength testing programs should hence consider the above factors. In order to get reasonably accurate and meaningful results with respect to the particular problem under formulation and analysis, the existing field conditions should be simulated such as:

- natural stress state as an initial condition,
- type of loading,
- pore water conditions,
- drainage conditions (seepage forces or hydrostatic pressures);
- Types of soil involved and existence of any secondary structures, etc.

1.3.2 Drainage Conditions and type of Testing

With respect to drainage, one should consider if the materials under analysis are free draining materials that are able to drain completely within the construction or loading period, or impermeable for which essentially no drainage can take place during construction.

Drained conditions are analyzed in terms of effective stresses, using values of c' and ϕ' determined from drained tests (CD tests) or from undrained tests with pore pressure measurements (CU tests).

Undrained conditions are analyzed in terms of total stress in order to avoid having to rely on estimated values of pore pressure for undrained loading conditions, which one may not be able to predict accurately. Undrained soil parameters could be evaluated using in-situ tests or unconsolidated undrained (UU) tests.

For multistage loading conditions (conditions where a period of consolidation under one set of loads is followed by a change in load under undrained conditions) such as staged construction or rapid draw down, the undrained strength is estimated using CU test results together with values of consolidation pressure estimated by means of consolidation analysis.

1.3.3 Pore Pressure Conditions

Changes in loading condition (excavations or embankment loads) and associated soil volume changes will result in a pore pressure change, Δu, which could be negative or positive.

The change in pore pressure, Δu, may increase or decrease with time depending on the type of soil and the type of stresses involved. Under fully drained conditions, Δu is equal to zero. For partially drained or undrained conditions, evaluation of u depends on the relative rate of loadings as compared to the rate of drainage within the soil.
Evaluation of internal pore pressures is required for effective stress analysis. These are determined by seepage analysis (flow net) or piezometer readings for long-term steady state conditions or by hydrostatic pressure measured from the phreatic surface if there is no flow.

1.3.4 Effective vis-à-vis Total Stress Analysis

Depending on type of slope, rate of loading, foundation condition and material properties, long-term conditions or short-term conditions may be critical. These conditions are reflected by the ability of the soil in the slope to reach equilibrium conditions with respect to volume changes and that may be a reflection of stress changes affecting the slope.

For embankment fills, which are associated with contraction of underlying soils, short-term, end of construction conditions are critical. At this stage, maximum loads will be attained, excess pore pressures will reach maximum value and corresponding strength will be at its lowest. After the end of construction as the excess pore pressures dissipate, strength will increase. Therefore, short-term undrained conditions and total stress analysis should be considered.

For cuts/excavations, long-term drained conditions and effective stress analysis are likely to be critical. This is due to the volume expansion and negative pore pressures that develop as load is removed in cuts, which will give the soil a higher undrained strength at short-term conditions. With time, the slope will absorb water and the negative pore pressure will dissipate, resulting in decrease of effective stress and strength of the soil.

With respect to soil types, normally consolidated (NC) or slightly over consolidated soils tend to decrease in volume when loaded, strength will increase with time as the positive excess pore water pressure dissipates, and the use of undrained strength parameters under short-term conditions should be considered.

For highly overconsolidated soils, the increase in volume during loading (dilatation) will result in decrease of pore water pressure, and high-undrained strength as compared to drained strength. However, the negative excess pore water pressure will tend to draw water into the soil with consequent dissipation of negative pressure and decrease in strength. Hence, long-term drained conditions are critical.

For natural slopes, critical conditions are often associated with high pore pressure and water pressures in cracks, usually during excessively wet periods. These are drained problems and should be analyzed using effective stresses, with water pressures determined from piezometer readings/seepage analysis.

1.4 SLOPE FAILURES

Failure of a slope is commonly called a landslide. Processes that increase shear stresses or decrease shear strength often cause slope failures or landslides.

In general, causes of slope failure could be attributed to one of the following:

- Over steepening of existing slopes;
- Excessive gravity or seismic loading;
- Excess Pore water pressures resulting from high ground water levels, or excessively wet seasons, or from added weight of fill placement;
- Deterioration of shear strength with time;

Landslides are typically classified based on the rate of slope movement and/or the type of movement that characterizes the displacing mass.

According to the type of movement of the displaced mass, landslides can be classified as falling, toppling, sliding, spreading and flowing. Falling and toppling types are associated with the down slope movement of large rock fragments and are applicable for rock slopes. Sliding, spreading and flowing landslides are associated with movements of soil and debris and are applicable for soil slopes.

A slide is a down slope movement of a soil mass occurring predominantly on surfaces of rupture or relatively thin zones of intense shear strain [1].

Movement in slides is usually progressive from an area of local failure. The first overt signs of ground movement are usually cracks in the original ground surface along which the 'main scarp' of the slide will form. [1]

Slides could be translational, rotational or a combination of both. Translational slides involve movement along marked discontinuities or planes of weaknesses, including previously existing failure planes. Whereas rotational slides have a failure surface that is concave upwards and often occur in homogeneous, intact materials, such as those found in constructed fill/embankment.

Slopes can undergo creep movements that are too slow to detect. These creep movements could be continuous or seasonal in nature and may lead to progressive failure.

In addition, post-failure movements may occur on existing slip surfaces at very slow rates. Changes in pore water pressure or other external disturbances may cause these movements. However, one should note that the effect of slides could be to bring the soil mass into a more stable position and no post failure movements may exist.

[1] **Main Scarp is a steep surface on the undisturbed ground at the upper edge of a landslide.**

CHAPTER 2: METHODS OF SLOPE STABILITY ANALYSIS

2.1 GENERAL

The problem of slope stability involves consideration of wide variety of parameters such as body forces, pore water pressures, soil strength parameters, topographic and geologic conditions, etc.

Slope stability problems are statically indeterminate, as the available conditions of static equilibrium are insufficient to determine the stress state within the soil mass.

Generally, the solution to such problems requires application of methods founded on basic continuum mechanics and should employ representative constitutive models of the material involved. To obtain solutions for loading conditions varying from small to sufficiently large to cause collapse of a portion of a soil mass, complete elasto-plastic analysis considering the mechanical behavior of the soil mass until failure should be conducted. However, this leads to very complex computations and is not routinely used.

In continuum mechanics, three types of equations are needed to determine the stresses and strains in a certain body, under the influence of given stresses and displacements on the surface of that body. These include equilibrium equations, constitutive relations and compatibility conditions. This is typically a complex and formidable task even for the simplest types of materials (linear elastic materials).

For soils, which are nonlinear and inelastic, various solution methods have been developed and put to use over the years. Such methods include limit equilibrium methods, limit analysis methods, variational calculus methods, finite element method and more recently discrete element methods of analysis and probabilistic analysis approaches. Some of these methods are briefly discussed in the following sections.

2.2 LIMIT EQUILIBRIUM METHODS

Limit equilibrium methods of slope stability analysis are based on plasticity theories and consider the conditions of equilibrium at impending failure of the soil mass in a slope. The materials involved are assumed to conform to Mohr – Coulomb failure criterion and to have rigid – plastic deformation properties.

Bishop [3] has discussed the validity of considering the existence of plastic equilibrium conditions at least through part of a slope. Based on his previous works, he had shown by a finite difference and relaxation analysis of a typical earth dam that even assuming idealized elastic properties for the soil, local overstress will occur when the factor of safety (by a slip circle method) lies below a value of about 1.8. As the majority of stability problems occur in slopes and dams having lower factors of safety than this, consideration of plastic equilibrium conditions for slope stability analysis is reasonable. Wright et al. [34] have indicated that a minimum average factor of safety of 1.5 is needed to avoid local overstress, based on their studies using finite element method of analysis on embankment sections of varying slopes and strength parameters.

Slope stability problems are generally statically indeterminate as the conditions of the stresses in the soil mass are unknown. If the stresses could be determined, displacements could be predicted using representative stress – strain relationships. Inability to obtain an adequate stress analysis due to different reasons justifies the application of analysis methods based on limit equilibrium principles.

Limit equilibrium method of analysis does not consider the stress – strain relationships and does not hence give a complete picture of the deformations within the soil mass. In addition, compatibility conditions and pre-failure constitutive relations are not considered.

It rather gives a simplified solution by considering the conditions at impending failure and by making assumptions to render the statically indeterminate problem of slope stability analysis determinate. In doing so, some of the equilibrium conditions may be ignored or violated. In line with theories of plasticity, the material properties are conveniently expressed in terms of an equation for the state of limit equilibrium.

Stability condition at limit equilibrium is expressed by the factor of safety. The factor of safety is obtained by comparing the strength necessary to maintain limiting equilibrium with the available strength of the soil.

This implies that the mobilized shear strength required for maintaining equilibrium is given by:

$$S = \frac{1}{F}\{c' + (\sigma - u)\tan\phi'\} \qquad (2.1)$$

Where S is the mobilized shear strength of the soil, c' and ϕ' represent the cohesive strength and the frictional strength of the soil respectively, σ is the total normal stress acting on the soil mass and u is the pore-water pressure.

There are various techniques of analysis available based on the limit equilibrium principle. Most of them are based on the method of slices. In the method of slices, a failure surface is assumed and the soil mass above the failure surface divided into slices. A set of stresses is estimated along the potential failure surface so that conditions of global and local static equilibrium are satisfied for the soil mass overlying the slip surface. This state of mobilized stress, which is not necessarily the true state along this surface, provides an approximate solution that can be used to evaluate the factor of safety. A critical slip surface is then searched, for which the factor of safety is minimized. For the computation of the factor of safety, which is defined in terms of strength, the available strength along the slip surface is computed based on Mohr – Coulomb failure criterion.

There are other techniques of analysis, which are based on limit equilibrium principle, that do not employ the method of slices. These include methods in which the shape and location of failure surface is simplified and dictated by peculiar subsurface conditions, such as infinite slope analysis, block analysis, planar analysis etc. and for which closed from solutions or simpler solutions could be achieved without resorting to the method of slices. Other methods include Log spiral method [13], Friction circle method, etc.

Most of the popular limit equilibrium analysis methods employ the method of slices technique of solution and these will be further discussed in Chapter 3.

2.3 FINITE ELEMENT METHOD (FEM)

The Finite Element method of analysis is a powerful general-purpose method, which can be used for analyzing stresses and deformations, pore pressures, groundwater flows etc. in a soil mass.

With the FEM, it is possible to model complex conditions with a high degree of realism, including non-linear stress-strain behavior, non-homogeneous conditions, and sequences of events such as changes in geometry during construction of embankments or excavations, changes in pore-water pressures, etc.

Unlike limit equilibrium methods, which employ assumptions, Finite Element Method solves the statically indeterminate problem of slope stability analysis by introducing additional equations that consider the stress strain characteristics of the soil and the requirements of compatibility of deformations.

Analysis using FEM requires definition of initial stress state, stress-strain constitutive models and actual construction or loading sequences of the problem being analyzed.

FEM analysis involves dividing the region to be analyzed into a number of elements connected at their common nodal points. The complete state of stress in each element and horizontal and vertical movements of each nodal point at each stage in the analysis is then determined by the finite element method. The analysis thus provides a very detailed picture of stresses, strains, and movements within the region analyzed. Results can be compared with available strength or deformation acceptability criterion.

In addition, finite element analyses conveniently represent nonlinear stress-strain behavior and inelastic properties of soils. To model nonlinear, inelastic behavior of soils, it is necessary to estimate and define:

- The initial stress state in the soil;
- The strength and non-linear behavior of the soil
- The sequence of construction operations and other loading conditions.

Hence, detailed and comprehensive investigations are required to obtain reliable input parameters for the FEM analysis.

Although the FEM provides a powerful technique, it is associated with various complexities that have limited its application to solve practical problems [1, 10, 30].

The major point in this respect is that, it requires accurate information and input data regarding initial stress states involved, correct constitutive models for the slope materials, and correct parameters for the constitutive models. In addition, quality control and sequence of construction should be correctly modeled and be within the tolerance input in the FEM analysis for the results to be meaningful and practical. This requires extensive laboratory and field investigations, involve high costs, and very stringent construction controls.

If uncertainties exist about the input data, the obtained results would be as doubtful and conducting such expensive, rigorous and sophisticated mathematical analysis becomes a futile exercise.

In view of the uncertainties and lack of familiarity associated with FEM, it is not used for routine and typical design problems of highway slopes and embankments. Use of FEM is usually restricted to complex, high cost projects where the consequences of failure are high.

2.4 LIMIT ANALYSIS METHODS

Limit analysis methods are based on the lower and upper bound theorems of plasticity to provide relatively simple bounds to the true solution. The methods assume perfect plasticity behavior of soil and consider the Mohr-Coulomb failure criterion as the yield condition to be satisfied at incipient failure. Solution to the problem of stability is attempted by giving possible upper and lower limits to the stresses and deformations within a soil mass (not the complete stresses and deformations). The two basic theorems of plasticity employed in limit analysis methods are:

- Lower bound theorem – which states that the collapse load corresponding to an equilibrium system or a statically admissible field of stresses, is a lower bound to the actual collapse load. A statically admissible stress field is one that satisfies all stress boundary conditions, is in equilibrium and never violates the Mohr-Coulomb yield criterion.

- Upper bound theorem – which states that the collapse load calculated from a kinematically admissible field of displacements (a mechanism), is an upper bound to the actual collapse load. A kinematically admissible field of displacements is one that gives compatible field of displacements, satisfies the boundary conditions for the displacements and where deformations occur the corresponding stresses satisfy the yield condition.

The mathematical formulations and solutions to the lower and upper bound theorems are obtained in line with plasticity theorems and virtual work principle.

Solution to slope stability problem is obtained in terms of either critical slope height or a collapsing load applied on a portion of the slope boundary, for given soil properties and /or slope geometry.

Most existing methods of analysis of slope stability (limit equilibrium methods), bearing capacity and earth pressure require an assumed failure mechanism and therefore are all potential upper-bound methods.

The range of the true solution or collapse load can be narrowed by finding the highest lower bound solution and the smallest upper-bound solution. This requires that analysis be performed on as many trial statically admissible stress fields and kinematically admissible velocity fields as possible. This is a precarious task and finite element and linear programming techniques are usually employed for limit analysis computations in soils.

For details of the methods, reference can be made to books on Plasticity theories and papers on limit analysis of slope stability problems, [6, 15, 17, 31].

2.5 METHODS BASED ON VARIATIONAL CALCULUS

This method attempts to solve the slope stability problem by making use of the calculus of variations. The factory of safety equation is formulated as a functional, F, of two unknown functions: the potential slip surface function, y(x), and the stress distribution function, σ(x). The factor of safety, F, is obtained by minimizing on the safety functional, F.

Instead of making arbitrary assumptions on the stress functions or the kinematical functions as done by the limit equilibrium methods, it purports to derive the properties of these functions based on the limit equilibrium principles and variational calculus technique.

However, the application of this method has been found questionable both on theoretical basis and from evaluation of practical results. In addition, the method is a mathematically complex method, and is not as commonly used for routine applications [2, 10, 16, 35].

CHAPTER 3: LIMIT EQUILIBRIUM ANALYSIS BY THE METHOD OF SLICES

3.1 GENERAL

Fellenius developed a method of analyzing the stability of embankment slopes by assuming a cylindrical slip surface and dividing the earth mass within this surface into vertical slices, where the conditions of static equilibrium for each slice must be fully satisfied [26]. This is the ordinary method of slices as first developed by Fellenius; it is very flexible and can be applied in different ways.

Failure is defined to consist of simultaneous movement of slices in a downward direction along the slip surface. This simultaneous movement of slices implies transmittance of interslice forces across the boundaries of the slices. Analyses by the method of slices hence incorporate interslice forces and demand making reasonable assumptions regarding the relationships between these forces or their location (point of action).

There are various methods of analysis, which mainly differ in the assumptions employed with regard to location or inclination of the interslice forces. Corresponding results obtained using the various methods depend on the assumptions made.

A cross section of a slope with general shaped failure surface and the forces acting on a typical slice are shown in Fig. 3.1.

If we consider the static equilibrium of the soil mass (of unit thickness) above the slip surface, divided into 'n' number of slices, the available equations and the existing unknowns yet to be determined as part of the solution are as given in Tables 3.1.

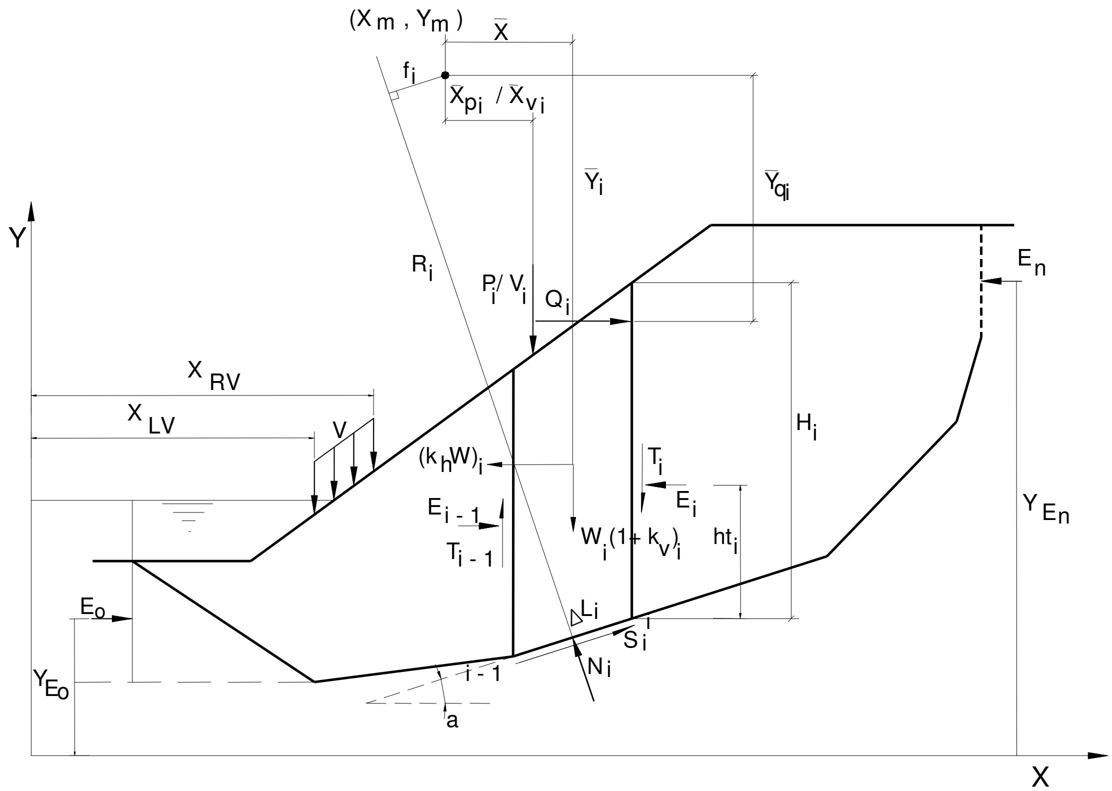

Figure 3.1 Forces acting on a slope and a typical slice.

Table 3.1a: Summary of unknowns

Unknown	Description
1	Factor of safety
n	Normal force at the base of a slice, N
n	Location of normal force at base of slice
n	Shear force at the base of slice, S
n-1	Inter slice horizontal force, E
n-1	Interslice tangential force, T
n-1	Location of inter slice force, h_t
6n-2	**Total unknown**

Table 3.1b: Summary of available equations

Equation	Condition
n	Moment equilibrium for each slice
2n	Force equilibrium in each direction for each slice
n	Mohr – coulomb failure criterion
4n	**Total number of equations available**

Based on static equilibrium conditions and concept of limit equilibrium, 4n equations are available for solution as shown in Table 3.1b.

As there are 6n-2 unknowns, the system is indeterminate and additional 2n-2 assumptions on relationships between the unknowns need to be introduced to render the problem statically determinate.

As discussed in the previous sections, exact and unique solutions can only be obtained if extra equations are developed using the stress-strain relationship of the soils and compatibility conditions. This would require numerical methods that are more advanced.

The implications of making assumptions in limit equilibrium (LE) analysis are:

- There will be a range of possible solutions rather than a unique solution as there are numerous possible sets of assumptions that will satisfy static equilibrium conditions;
- Judgment and intuition will always be required to check the reasonableness of obtained results and to check on the effect of varying the assumptions.

It has been pointed out by Bishop [3], that the range of equally correct values of safety factor (solution) may be quite narrow; i.e. any assumption leading to reasonable stress distribution and magnitudes will give practically the same factor of safety.

From a theoretical point of view, the various methods of slices can be categorized in terms of the conditions of static equilibrium satisfied and the assumption regarding the interslice forces.

Some of the most common methods of analysis, the assumptions they employ and the conditions of equilibrium they satisfy are given in Table 3.2.

Table 3.2: Comparison of various limit equilibrium methods [25]

Method	Force equilibrium*		Moment equilibrium	Assumption
	Vertical	Horizontal		
Ordinary or Fellenius	Yes	No	Yes	Interslice forces are neglected.
Bishop's Simplified	Yes	No	Yes	Resultant interslice forces are horizontal (i.e., there are no interslice shear forces)
Janbu's Simplified	Yes	Yes	No	Resultant interslice forces are horizontal. An empirical correction factor, f_o, is used to account for interslice shear forces.
Janbu's Generalized	Yes	Yes	**	Location of the interslice normal force is defined by an assumed line of thrust.
Bishop's Rigorous	Yes	Yes	Yes	Interslice shear force distribution is assumed.
Spencer	Yes	Yes	Yes	Resultant interslice forces are of constant slope throughout the sliding mass.
Morgenstern-Price	Yes	Yes	Yes	Direction of the resultant interslice forces is determined using an arbitrary function.
US Corps of Engineers	Yes	Yes	No	Direction of the resultant

Method	Force equilibrium*		Moment equilibrium	Assumption
	Vertical	Horizontal		
				interslice force is: i) Equal to the average slope from the beginning to the end of the slip surface or ii) Parallel to the ground surface
Lowe-Karafiath	Yes	Yes	No	Direction of the resultant interslice force is equal to the average of the ground surface and the slope at the base of each slice.

*Any of two orthogonal directions can be selected for the summation of forces.

**Moment equilibrium is used to calculate interslice shear forces.

3.2 CRITICAL SLIP SURFACE

A complete treatment of the problem of slope stability analysis includes locating the slip surface that has the lowest factor of safety - the critical slip surface.

Actual shape and position of the critical slip surface is generally influenced by the distribution of pore pressures, the variation of the shear parameters within the slope, presence of weak layer and physical boundaries.

Usually the simplifying assumption of a circular shape of slip surfaces is made and there are some methods, which were originally developed for the analysis of circular slip surfaces (OMS, Bishop's method [3], Spencer's method [26]).

The assumptions of circular slip surfaces generally make search procedures for critical slip surface simpler, as it will involve only three variables; i.e. coordinates of center of the circle and the radius.

The various studies on location of critical slip surfaces have shown that, unless there are geological and physical constraints, it can be assumed with little loss of accuracy that the critical slip surface is circular [10].

Hence, circular slip surfaces could be assumed in more or less homogenous formations without significant loss in accuracy, where geological and geometrical features are suitable.

However, departure from homogeneity, i.e. heterogeneous nature of soil mass, non uniform distribution of pore pressures etc., which are the cases in most real problems, usually dictate a non circular shape of slip surfaces. Field observations made on landslides and failed dams also confirm the more frequent occurrence of non-circular slip surfaces [18].

Some of the numerous conditions, which definitely warrant consideration of non-circular slip surfaces include:

- Presence of soft (weak) layer in the foundation
- Zoned dams, with different strength parameters and pore pressure distributions
- Presence of drainage blankets in dams, or physical boundaries such as shallow bed-rock
- Rock slopes where slip surfaces are dictated by pattern of joints and fissures in the rock mass, etc.

Some researchers have pointed out that even in homogeneous slopes; the critical slip surface may not be circular in shape. Janbu [14] reported a difference in minimum factor of safety of 3-4% between critical circular and non-circular surfaces (the non-circular slip surfaces being the more critical). Thus, a comprehensive method of slope stability analysis should be able to analyze general shaped slip surfaces.

Determination of a general shape critical slip surface is much more complex. Various methods based on non-linear programming technique, random search techniques, variational calculus technique etc. have been developed and used. Most of these techniques can be used with any of the limit equilibrium analysis methods that are applicable to non-circular surfaces. Such methods include Critical Slip Field (CSF) method, Zhu [35]; Efficient Monte Carlo Method (a random search method), Greco [12]; Simplex minimization method, Nguyen [19] and others. Detailed discussion of methods of locating critical slip surface is beyond the scope of this work and is not considered here.

Numerical difficulties and problems of convergence usually occur with slip surfaces that enter and leave the slope at steep angles. Some researchers have suggested that this could be avoided by specifying maximum inclination angles of the slip surface at the toe (passive zone) and top (active zone) of the slope in line with earth pressure theories at limiting equilibrium. These angles are $45+\phi'/2$ for the active zone (at the top of slope) and $45-\phi'/2$ for the passive zone (at the toe of the slope). Introduction of tension crack near the top of the slope, for cohesive soils, could also alleviate this problem and improve positions of thrust line locations [10, 14, 25, 28].

In addition, when analyzing a particular slip surface, if numerical difficulties are encountered or reasonable stress distributions cannot be achieved for reasonable assumptions of interslice force functions, it could be assumed that the slip surface is not possible and another trial slip surface should be located and assessed [18].

3.3 COMMON FEATURES AND LIMITATIONS

Methods of slope stability analysis based on limit equilibrium principles share some common features and limitations.

3.3.1 Factor of Safety

Engineering design usually incorporates a factor of safety against intolerable deformations and failure. The factor of safety will also compensate for uncertainties associated with measurements of input parameters that enter into design and analysis such as strength parameters, pore-pressure distributions, etc.

The most common definition of factor of safety in limit equilibrium methods of slope stability analysis is one with respect to strength.

$$\text{Factor of Safety, F} = \frac{\textit{Shear strength}}{\textit{Shear stress required for equilbrium}}$$

Factor of safety is hence defined as "the factor by which the shear strength of the soil would have to be reduced to bring the slope to a state of limit equilibrium". Definition of the factor of safety as a factor on shear strength is logical, because shear strength is usually the quantity that involves the greatest degree of uncertainty.

Slope stability analysis is generally conducted with the aim of achieving the minimum value of factor of safety. This value of F is used as an index of the margin of stability for a slope. For the rigorous determination of the minimum F, minimizing a function of two variables, the location and shape of the critical shear surface will be required.

One distinguishing assumption common to all limit equilibrium methods is that the factor of safety is assumed constant along the entire slip surface. In addition, equal proportions of the cohesive and frictional strength are assumed to be mobilized.

Wright et al. [34], Chugh [8] and others have pointed out that the factor of safety actually varies from place to place. The average value of F has, however, been found to be the same for all practical purposes [8].

Criteria for acceptable values of safety factor should normally be established with the following considerations in mind:

1) What is the degree of uncertainty involved in evaluating the conditions and shear strength parameters for analysis?
2) What are the consequences of failure?

When the uncertainties and the consequences of failure are both small, it is acceptable to use small factors of safety, in the order of 1.3 or even smaller in some circumstances. When the uncertainties and consequences of failure increase, large factors of safety are necessary [10].

Typical minimum acceptable values of factor of safety in slope stability analysis are [10]:

- 1.3 for end of construction and multistage loading
- 1.5 for normal long-term loading conditions
- 1.0 to 1.2 for rapid drawdown, in cases where rapid drawdown represents an improbable or infrequent loading condition.

3.3.2 Stress – Strain Behavior

Limit equilibrium methods implicitly assume ductile stress – strain behavior of the soil (i.e. the soil does not have brittle stress – strain curve where the shearing resistance drops off considerably after reaching a peak under continuous deformation).

This limitation results from the fact that the methods provide no information on the magnitudes of strain along the slope, or any indication about how they may vary along the slip surface.

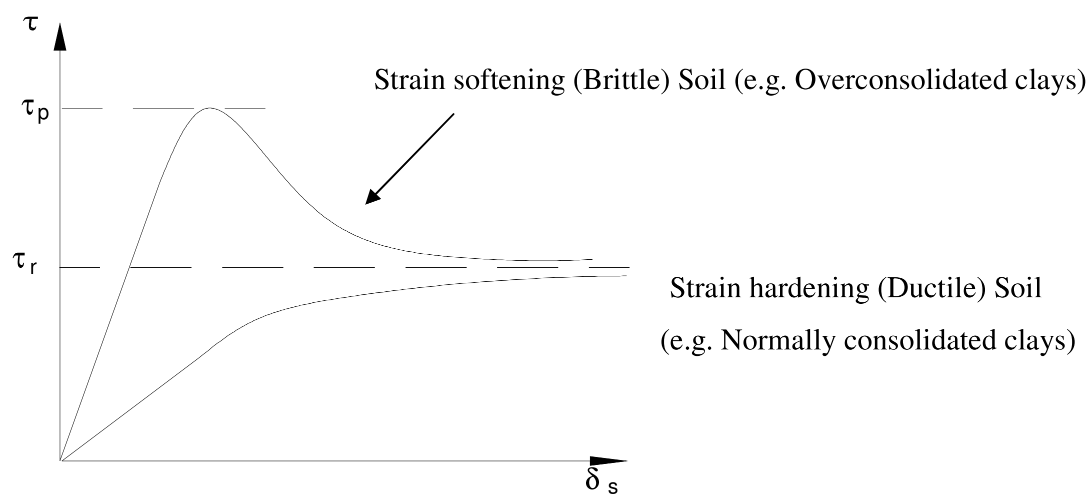

Fig. 3.2 Brittle Vs Ductile soil behavior.

τ = Shear stress

δ_s = Shear displacement

τ_p = Peak shear strength

τ_r = Residual shear strength

Therefore, unless the strengths used in the analysis can be mobilized over a wide range of strains (i.e. ductile stress-strain behavior) there is no guarantee that the peak strength can be mobilized simultaneously along the full length of the slip surface. This also implies that one cannot model brittle stress - strain behavior, as in stiff clays, and associated progressive mode of failure by the limit equilibrium method.

3.3.3 Plain-Strain Analysis

The common limit equilibrium methods of analysis assume plane strain conditions. Analysis results are hence presented in two dimensions (2D).

Over the years, various studies of three dimensional slope stability problems have been conducted. Most of the analysis methods developed are extensions of the common 2D analysis methods [10].

Some of the conditions that warrant three-dimensional analysis include:

- Slopes that are curved in plan or that contain corners;
- Slopes that are subjected to loads of limited extent at the top;
- Slopes in which the potential failure surface is constrained by physical boundaries, such as a dam in a narrow rock-walled valley, etc.

Results of the studies have conclusively indicated that the factor of safety for three-dimensional analysis is greater than or equal to the factor of safety for two-dimensional analysis, provided that the later is calculated for the most critical section.

Based on investigations into the various studies of three dimensional slope stability analyses by various authors, some researchers have come to the conclusion that methods that give F_{3D}/F_{2D} ratios that are smaller than unity either compare inappropriate factors or, more probably, contain simplifying assumptions that neglect important aspects of the problem [10].

Hence, plain-strain analyses of slopes give conservative results of factor of safety and can be utilized for all practical purposes. In addition, use of 2D analysis will avoid the considerable increase in required computational effort of 3D analysis.

3.3.4 Location of Normal Force

It has been pointed out earlier that all the analysis methods based on limit equilibrium employ arbitrary assumption so that the normal stresses on the shear surface may be determined using only static equilibrium conditions.

One common and additional assumption employed by all methods based on the method of slices is that the normal force at the base of each slice acts at the mid point of the base. Espinoza et al [11], pointed out that in the limit, the relationship between internal forces is not affected by the location of the point of application of the base normal force, and assumptions of the normal force distribution should not affect results when the width of slices is small.

Assumptions on locations of base normal forces amount to 'n' assumptions corresponding to ‘n’ number of slices. Thus, the number of unknowns is reduced to '5n-2'. Hence, only 'n-2' assumptions are needed to achieve statical determinacy. It is on the assumptions made to avoid these ‘n-2’ degrees of indeterminacy that the various methods developed so far differ from each other. The most common and widely accepted methods are reviewed in the next chapter.

CHAPTER 4: COMMON LIMIT EQUILIBRIUM BASED SLOPE STABILITY ANALYSIS METHODS

4.1 GENERAL

Various methods of slope stability analysis have been proposed over the years, based on the concept of limit equilibrium and methods of slices.

As discussed in the previous sections, these methods differ in the assumptions they make with regard to the location, distribution or inclination of interslice forces, to attain statical determinacy of the slope stability problem.

Some of the methods do not satisfy all conditions of equilibrium while others rigorously satisfy all force and moment equilibrium conditions of the slices. The force equilibrium equations may be in terms of interslice forces (Bishop [3]; Janbu [28]; Morgenstern and price [18]; Sarma [21]) or in terms of their resultants (Spencer [26], [28]). Moment equilibrium equations can be attained by taking moments about some common axis (Bishop [3]; Spencer [26]; Sarma [21]) or about a different axis for each slice in turn (Morgenstern and price [18]; Janbu [14]; Spencer [28]).

All the methods share common features and assumptions and use the same definition of the factor of safety as pointed out in the previous sections. In summary, these common assumptions are:

- The factor of safety is assumed to be constant along the whole length of the slip surface;
- Equal proportions of the cohesion and friction components of shear strength are mobilized. (i.e. $F_c=F_\phi=F$);
- Plain-strain conditions prevail;
- The width of the slice is sufficiently small so that the normal force and the resultant vertical forces act at the center of the base of each slice.

Following the last assumption, which amounts to 'n' assumptions corresponding to ‘n’ number of slices, the number of unknowns reduce to '5n-2'. Therefore, 'n-2' assumptions will still have to be made to reduce the number of unknowns to ‘4n’ and attain statically determinate condition (refer to section 3.1).

The various methods are thus conveniently classified based on the manner in which these 'n-2' assumptions are made and on whether all conditions of static equilibrium are satisfied (See Table 3.2). Basic features of selected methods will be discussed in the following sections.

4.2 SIMPLIFIED METHODS

In this group, simplified methods such as ordinary method of slices, Janbu's simplified method [14], Bishops simplified method [3] and force equilibrium methods such as Lowe and Karafiath and U.S Army Corps of Engineers Method [1], are categorized.

The above methods make 'n-1' assumptions regarding the inclinations of the interslice forces, which are more than the required 'n-2' assumptions and leave the problem over-determined. Therefore, all the methods have the common feature that they do not rigorously satisfy all the conditions of static equilibrium.

The simplified methods are popular in that a factor of safety could be quickly calculated, even without the use of computers. However, results of F may substantially differ from those obtained using the rigorous methods.

4.2.1 Ordinary Method of Slices (OMS)

This is one of the earliest analytical techniques based on the method of slices and limit equilibrium principles. It assumes interslice force inclinations that are parallel to inclinations of the base of the slices. This amounts to ‘n-1’ assumptions and renders the problem over-determined.

The OMS is applicable to circular slip surfaces only. It only satisfies moment equilibrium conditions. Horizontal and vertical equilibrium conditions are not satisfied. In addition, it fails to satisfy interslice equilibrium where adjacent slices have different base angles. For such cases, the assumption is equivalent to specifying different inclination angles for the same interslice force between adjacent slices. This will generally lead to the calculation of inconsistent effective stresses at the base of the slices.

In deriving the equations of solution, interslice forces are avoided as their net effect cancels out in the moment equilibrium equations, and force equilibrium conditions are considered in the direction perpendicular to the base.

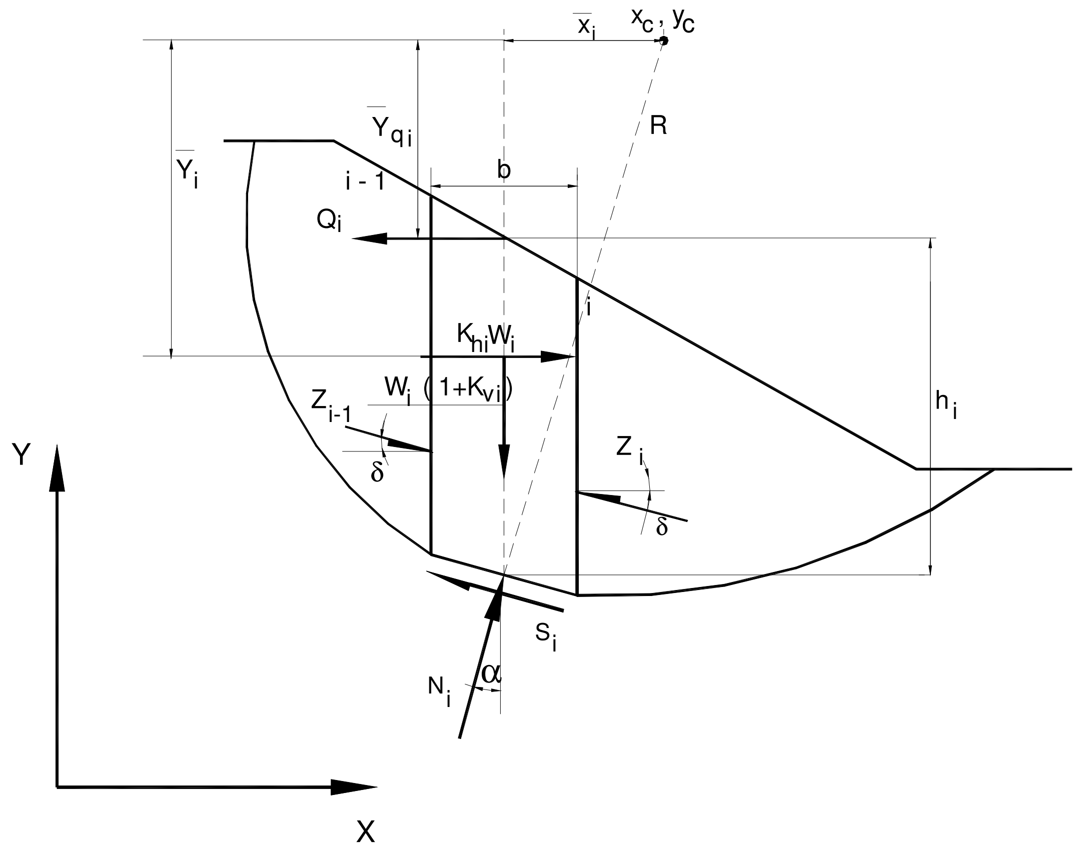

Figure 4.1 Forces on a typical slice in OMS.

N = the normal force at the base of the typical slice of width b and height h;

S = the shear force at the base of the slice;

W = the weight of the soil mass above the base of the slice, including any external vertical load, with moment arm X about the center of the slip circle with radius R and center x_c, y_c;

Q = external horizontal load acting on the slice with moment arm $\overline{Y}_{qi}$ about the center of the slip circle;

K_h, K_v = horizontal and vertical earth quake coefficients;

Z = forces acting on the slice interfaces;

δ = inclination angle of interslice forces, Z.

Consider the slice in Figure 4.1. Resolving forces in the perpendicular direction to the base of the slice:

$$N_i = W_i(1 + K_{vi})\cos\alpha_i - (K_{h_i} \times W_i)\sin\alpha_i + Q_i \sin\alpha_i \quad (4.1)$$

Note that in the OMS, the resultant interslice force is assumed to be parallel to the base of the slice. Hence, the interslice force does not appear in equation (4.1).

Taking summation of moments about the center of the slip circle for all the slices:

$$\sum W_i(1 + K_{vi})R\sin\alpha_i + \sum K_{h_i} W_i \overline{Y_i} - \sum S_i R - \sum Q_i \overline{Y} qi = 0 \quad (4.2)$$

From Mohr Coulomb Failure criterion, we have:

$$S_i = \frac{1}{F}\left[c'\Delta L_i + (N_i - u_i \Delta L_i)\tan\phi'\right] \quad (4.3)$$

Substituting equation (4.1) into equation (4.3) and the resulting expression into equation (4.2), we get:

$$\sum\left[W_i(1 + K_{vi})R\sin\alpha_i + K_{h_i} W_i \overline{Y} i - Q_i \overline{Y} qi\right] =$$
$$\frac{R}{F}\sum\left[c'\Delta L_i + \left(W_i(1 + K_{vi})\cos\alpha_i - u_i \Delta L_i - (K_{h_i} W_i)\sin\alpha_i + Q_i \sin\alpha_i\right)\tan\phi'\right]$$

(4.4)

Solving for F we get:

$$F = \frac{R\sum\left[c'\Delta L_i + \left(W_i(1+K_{vi})\cos\alpha_i - K_{h_i}W_i\sin\alpha_i + Q_i\sin\alpha_i - u_i\Delta L_i\right)\tan\phi'\right]}{\sum\left[W_i(1+K_{vi})R\sin\alpha_i + K_{h_i}W_i\overline{Y}i - Q_i\overline{Yq}_i\right]}$$

(4.5)

Equation (4.5) simplifies to the following when there are no external horizontal loads and seismic loads:

$$F = \frac{\sum\left[c'\Delta L_i + \left(W_i\cos\alpha_i - u_i\Delta L_i\right)\tan\phi'\right]}{\sum W_i\sin\alpha_i} \quad (4.6)$$

Results of Factor of Safety (F) obtained using OMS have been reported to be up to 40-60% in error when compared to rigorous methods such as Morgenstern and Price's. The discrepancy is especially large for cases of effective stress analysis of flat slopes with high pore water pressures. They are reasonably accurate for ϕ=0 analysis and give reasonable results for total stress analysis [10, 32].

4.2.2 Simplified Janbu Method

Janbu et al. have proposed this simplified method for routine calculations, which could be implemented without the need for computers [14]. The expression for the factor of safety is derived based on force equilibrium conditions.

In this simplified method, the interslice shear forces are assumed to be zero. This gives 'n-1' assumptions, which is one more than the required number and leaves the problem over-determined.

This method is a simplified variation of Janbu's rigorous method and is applicable to general shape of slip surfaces. The solution method is derived together with Janbu's rigorous method in Section 4.3.5.

The F is computed based on horizontal and vertical force equilibrium. Moment equilibrium conditions are not considered and not fulfilled by this method of solution.

It has been established by many researchers that factors of safety computed based on force equilibrium conditions are sensitive to the assumptions on inclination of the interslice forces and poor and simplifying assumptions could lead to erroneous results. One should hence consider this when using any force equilibrium method of slope stability analysis [26].

To compensate for the shortcoming of assuming zero interslice shear force, Janbu proposed a correction factor, f_o, that is a function of slope geometry and soil type to be applied to computed factor of safety.

The correction factor, f_o, improves the results of F attained and it is derived based on comparisons of calculated F values using Janbu's simplified and Janbu's rigorous methods, for the same slopes with homogeneous soil conditions.

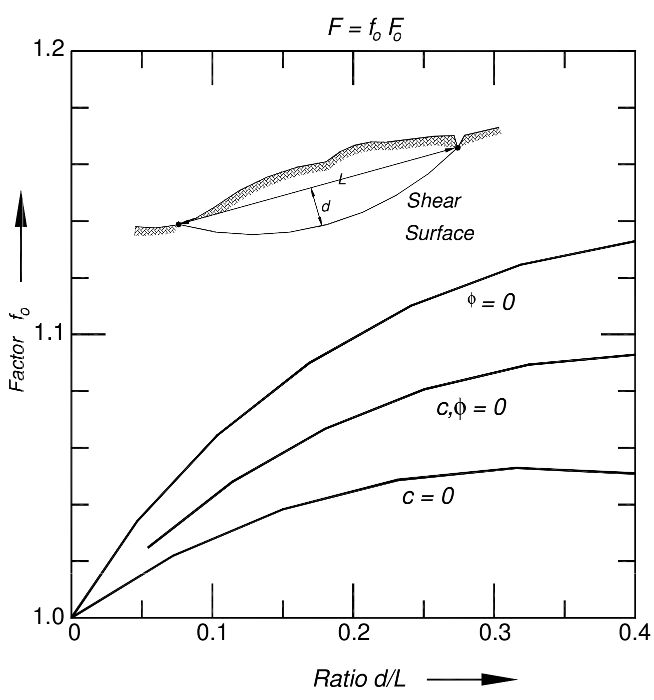

Figure 4.2 Correction factor for Janbu’s simplified method [14]

The formula for f_o, derived from the curve presented by Janbu is as given by equation (4.7) [1]:

$$f_0 = 1 + b_1\left[\frac{d}{L} - 1.4\left(\frac{d}{L}\right)^2\right] \tag{4.7}$$

Where, b_1 varies according to the soil type:

c only soils: $b_1 =$ 0.69

ϕ only soils: $b_1 =$ 0.31

c and ϕ soils: $b_1 =$ 0.5

The computed Factor of safety could thus be multiplied by the appropriate f_0 value to get corrected Factor of Safety.

Janbu's simplified method usually gives conservative values of Factor of Safety. It has been reported that the F could be as much as 30% lower when compared to results obtained using Morgenstern and Price and Spencer's method [1].

4.2.3 Lowe and Karafiath and U.S. Army Corps of Engineers Method [1]

These are force equilibrium methods and do not satisfy moment equilibrium conditions. These methods are not as widely used and discussed in the literature. They will be briefly discussed in this section without going to lengths to derive their equations.

The assumptions employed are:

Lowe and Karafiath (1960): Interslice force inclination angle is equal to the average of the ground surface inclination and slice base angle.

U.S. Army Corps of Engineers Method (1970): Interslice force angles are parallel to the ground surface or the average slope angle between the left and right endpoints of the failure surface.

Remark made earlier regarding the sensitivity of the factor of safety on assumptions of interslice force inclinations for force equilibrium methods should be borne in mind in applying these methods.

4.2.4 Bishop's Simplified Method

Bishop [3] proposed a rigorous method of analysis, which satisfies both force and moment equilibrium conditions and which takes the interslice shear forces into account. In his work, he also suggested a simplified method that allows for the speedy computation of the factor of safety.

The simplified method assumes zero interslice shear forces, similar to Janbu's method, and the problem is over determined. However, the factor of safety (F) expression is developed by considering only moment equilibrium conditions of the soil mass for an assumed circular slip surface.

The developed method of solution does not satisfy all conditions of force equilibrium, horizontal force equilibrium condition in particular. In addition, this method as originally developed is applicable only to circular slip surfaces.

The equations of solution are derived together with the Bishop's Rigorous Method in Section 4.3.1.

Although this method makes the simplifying assumption of zero interslice shear force and does not satisfy all conditions of static equilibrium, it has been shown that it gives reasonably accurate results, which are in good agreement with results attained by the rigorous methods. A factor of safety close to unity was achieved using this method when applied to cases of actual failed slopes [26].

Spencer [26] has investigated the reasons for reported close agreement by comparing results with a rigorous method he developed. His method satisfies all conditions of equilibrium. Accordingly, he reported the following:

- The greatest difference between Bishops simplified F and that obtained using Spencer's rigorous method was about 4%. This was for conditions of relatively high pore pressures (with r_u value of 0.5, where r_u is the pore pressure ratio) and steep slope (3H: 2V). The difference between the factors of safety reduced to less than 0.5% when r_u value of 0.25 and 0 were tried. ($r_u = \frac{u}{\gamma h}$; where u is the pore pressure and γ is the soil unit weight and h is the average slice height).
- The critical slip circle located using Bishop's simplified method was found to be 'practically the same', as that would be obtained using the rigorous method.

Spencer [26] and others have shown that the reason for this agreement in computed F could be the relative insensitivity of F calculated based on moment equilibrium expressions to the inclination angle of interslice forces.

Whitman and Bailey [32] reported that results of F computed using Bishop's simplified method could be from 2% to 7% in error when compared to F obtained using the rigorous Morgenstern and Price method.

Wright et al. [34] used Bishop's simplified method to represent the limit equilibrium methods in assessing their accuracy. Results of F computed were compared with that obtained using Finite Element Method. Bishop's F was found to agree well with the average factor of safety computed using Finite Element Method.

All the above observations have led many to employ Bishop's simplified method as their method of choice in routine applications and for rapid evaluation of stability of design slope sections.

4.3 RIGOROUS METHODS

These are methods based on limit equilibrium principles and which are rigorous in the sense that they satisfy all conditions of equilibrium and within the context of the assumptions they make to attain statical determinacy.

Some argue that these ‘rigorous’ limit equilibrium methods cannot be regarded as rigorous in the strict sense of mechanics, as equilibrium conditions are not actually satisfied at every point in the soil mass and the strain conditions, compatibility conditions and pre-failure constitutive relations are not considered [15].

Limit Equilibrium methods require making assumptions to reduce the degree of indeterminacy of a given problem instead of introducing additional equations (based on compatibility and constitutive relations) to deal with the extra unknowns. Because of this inherent feature of limit equilibrium methods and as there are numerous possibilities in which these assumptions could be made, various methods of analysis have been proposed over the years.

In addition, solutions are not unique and results obtained should be assessed for reasonableness within the context of generally established and accepted soil behavior and slip mechanisms.

It should be noted at this point that all methods of slope stability analysis based on the limit equilibrium principles are approximate methods (including the ‘rigorous’ methods) and as long as all conditions of equilibrium are satisfied and they result in acceptable stress distributions, they should be regarded as equally accurate.

All the rigorous methods recognize the necessity of checking the acceptability of results. As pointed out by Morgenstern and Price [18], for the analysis to be physically acceptable, not only must the equilibrium and boundary conditions and failure criterion along the slip surface be satisfied but the implied state of stress within the soil mass must also be possible.

Accordingly, all methods recommend that the internal stresses should be calculated and the following condition checked:

– The failure criterion within the soil mass at the interslice surface must not be violated; i.e.

$$T_i = \frac{1}{F}\left(\overline{c}_i H_i + \left(E_i - P_{wi}\right)\tan\overline{\phi}_i\right) \tag{4.8}$$

Where, T_i and E_i = Shear and Normal forces at slice interface,

$\overline{c}_i$ and $\overline{\phi}_i$ = Average shear strength parameters along the interface height (H_i)

P_{wi} = Pore water force

F = Local factor of safety (should be greater than 1)

– No state of tension must be implied to exist above the slip surface. This could be checked by computing the line of thrust, (h_{ti}) through summation of moments about the mid point of the base of each slice. The position of line of thrust must be within the potential sliding mass above the slip surface, i.e. $0 < h_{ti} < H_i$.

The different methods will be discussed on the basis of the assumptions they employ. In the following sections, the solution techniques will be derived as presented by the respective authors. In Chapter 5, a common derivation will be presented that incorporates the basic principles employed in deriving the equations by the individual methods so that it serves as a general framework for incorporating the assumptions employed by each method.

4.3.1 Bishop's Rigorous Method

Bishop [3] proposed a method in which an assumption is made on the manner of the distribution of interslice shear force, $\Delta T_i = T_i - T_{i-1}$, so that statical determinacy is achieved. This is equivalent to providing 'n-1' assumptions, which is one more than the required 'n-2' assumption.

However, Bishop argues that among the numerous possible distributions of ΔT_i, there is a set of unique distribution that rigorously satisfies all conditions of equilibrium and gives acceptable results. This unique distribution is yet to be determined and hence an additional unknown.

Bishop considered only circular slip surfaces for analysis, arguing that rigorous determination of the shape of the most critical surface of failure is difficult. The equation for the factor of safety is derived from overall moment equilibrium consideration.

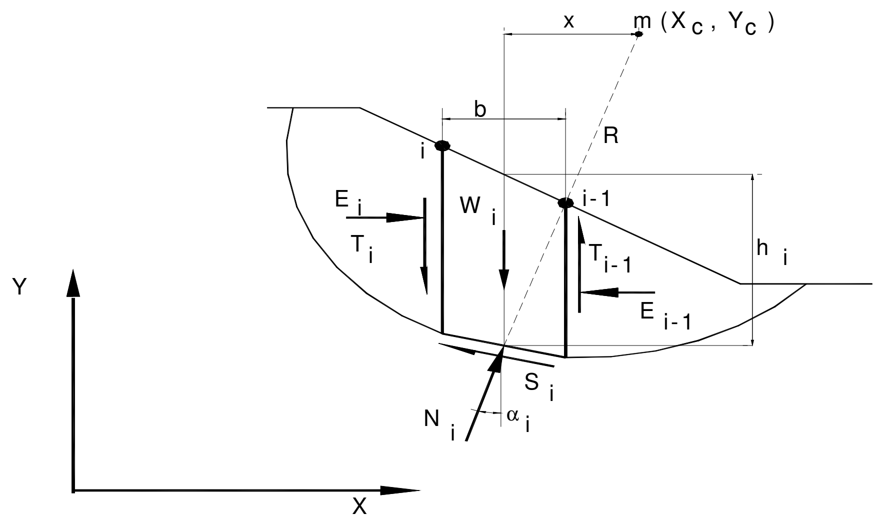

Figure 4.3 A slope section and forces on a typical slice.

With reference to Figure 4.3:

W_i = the weight of soil above the slip surface,

R = the radius of the slip circle,

X_i = the moment arm of W_i about the center of the slip circle, m,

α_i = the inclination angle of the base of the i^{th} slice,

b_i = the width of the i^{th} slice,

h_i = the height of the i^{th} slice at the mid point of the base of the slice, and the other terms are as defined in previous sections.

Mohr-Coulomb failure criterion:

$$S_i = \frac{1}{F}\left[c'\Delta L_i + (N_i - u_i \Delta L_i)\tan\phi'\right] \tag{4.9}$$

Where, ΔL_i is the length of the base of the i^{th} slice. Assuming no external forces and taking summation of moments about the center of the slip circle:

$$\sum W_i X_i = \sum S_i R = \frac{R}{F}\sum\left[c'\Delta L_i + (N_i - u_i \Delta L_i)\tan\phi'\right] \tag{4.10}$$

Solving for F we get:

$$F = \frac{R}{\sum W_i X_i} \sum \left[c' \Delta L_i + (N_i - u_i \Delta L_i) \tan \phi' \right] \quad (4.11)$$

Resolving forces on a slice in the vertical direction and after simplifying and rearranging:

$$N_i - u_i \Delta L_i = m_{\alpha i} \left\{ W_i + (T_i - T_{i-1}) - \Delta L_i u_i \cos \alpha_i - \frac{c' \Delta L_i \sin \alpha_i}{F} \right\} \quad (4.12)$$

Where,

$$m_{\alpha i} = \frac{1}{\cos \alpha_i + \frac{\sin \alpha_i \tan \phi'}{F}} \quad (4.13)$$

Substituting equation (4.12) into equation (4.11) and R sinα for X, after further simplifying and rearranging:

$$F = \frac{1}{\sum W_i \sin \alpha_i} \sum \left[m_{\alpha i} \left(c' \Delta L_i \cos \alpha_i + (W_i - u_i \Delta L_i \cos \alpha_i) \tan \phi' + (T_i - T_{i-1}) \tan \phi' \right) \right]$$

(4.14)

Equation (4.14) is the moment equilibrium expression for the factor of safety.

For Bishop's Simplified Method, setting the (T_i-T_{i-1}) expression to zero one obtains:

$$F = \frac{1}{\sum W_i \sin \alpha_i} \sum \left[m_{\alpha i} \left(c' \Delta L_i \cos \alpha_i + (W_i - u_i \Delta L_i \cos \alpha_i) \tan \phi' \right) \right] \quad (4.15)$$

Resolving forces on a slice in the tangential direction for force equilibrium:

$$E_i - E_{i-1} = S_i \sec \alpha_i - (W_i + (T_i - T_{i-1})) \tan \alpha_i \quad (4.16)$$

From Equation (4.11), substituting R sin α for X, one gets S=W sin α. Hence, if we designate the expression in parenthesis in equation (4.14) as M, i.e.

$$M_i = \lfloor m_{\alpha i} \lfloor c' \Delta L_i \cos\alpha_i + (W_i - u_i \Delta L_i \cos\alpha_i)\tan\phi' + (T_i - T_{i-1})\tan\phi' \rfloor\rfloor \qquad (4.17)$$

Then, S= (M/F) and equation (4.16) becomes, taking summation of forces across the slip surface:

$$\sum\left[E_i - E_{i-1}\right] = \sum\left[\frac{M_i}{F}\sec\alpha_i - (W_i + (T_i - T_{i-1}))\tan\alpha_i\right] \qquad (4.18)$$

Equations (4.14) and (4.18) are the two basic equations in Bishop's Method.

The solution procedure includes the following steps:

1. Assume interslice force distribution $\Delta T_i = T_i - T_{i-1}$, and compute the corresponding factor of safety, F, using equation (4.14), as F appears on both sides of the equation an iterative scheme is employed to solve for F.
2. Check if force equilibrium condition is satisfied by substituting the computed F value and the assumed ΔT_i value from step 1, into Equation (4.18).
3. Repeat steps one and two until the F computed in the first step satisfies the force equilibrium condition in step 2.

For the simplified method, equation (4.15) is iteratively solved for the F.

Bishop has noted after analyzing various slope stability problems using his method that:

- Computed factor of safety values are relatively insensitive to assumed distributions of interslice shear force ΔT_i, provided reasonable distributions are assumed and all conditions of equilibrium are satisfied. (He computed less than 1% variation in a typical case).

- Factor of safety computed by assuming zero interslice shear force (Bishop's simplified method) gives sufficient accuracy for most cases. He computed less than 5% difference between the simplified and rigorous methods. The difference increased with increase in the central angle of the slip circle (deep slip surfaces) and high pore-water pressures.

In general, it is rather difficult to make reasonable assumptions on the distribution of interslice body forces, $\Delta T_i = T_i - T_{i-1}$. This will require sufficient experience and insight into the possible results of the analysis procedure and the problem being studied.

Modified versions of Bishop's simplified and rigorous methods of solution are included in the computer program, which will be discussed in Section 5.4.

4.3.2 Sarma's Method

Sarma [21] presented a different approach of attaining solution within the framework of limit equilibrium principles and method of slices.

In his method, rather than iterating for the factor of safety, the critical horizontal acceleration coefficient, K_c, that is required to bring the mass of soil bounded by the slip surface and the free surface of the slope to limiting equilibrium, is computed. This critical acceleration coefficient (K_c) is supposed to be a measure of the factor of safety of the slope. Sarma's method satisfies all conditions of equilibrium and is applicable to general-shape slip surfaces.

In Sarma's method, it is assumed that under the action of horizontal thrust (K_c multiplied by the weight of the soil above the slip surface, W), the complete shear strength of the soil along the slip surface is mobilized (i.e. F=1 along the slip surface).

Similar to Bishop, Sarma also assumes interslice shear force distribution:

$$\Delta T_i = \lambda \times f_i \tag{4.19}$$

Where, f_i is an assumed distribution function and λ is a scaling factor yet to be determined. This is equivalent to assuming the shape of the interslice shear force distribution but not its magnitude.

However, he proposed to base the assumptions of the interslice shear force distributions (ΔT_i) based on consideration of stress condition inside the soil mass. Hence, his assumptions do not seem to be entirely arbitrary.

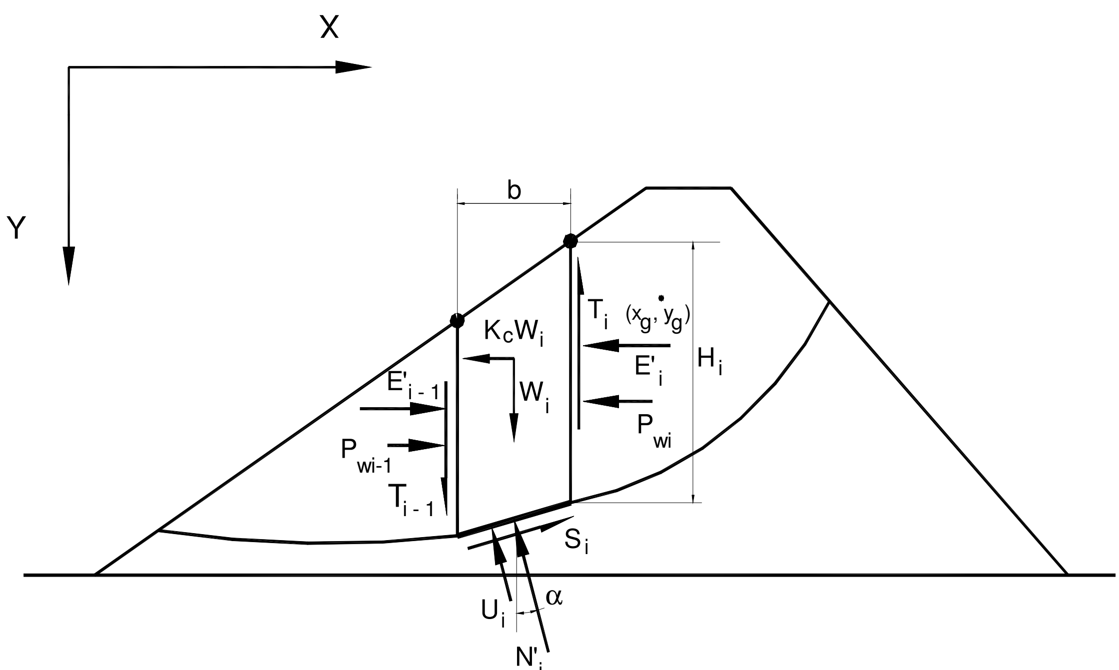

Figure 4.4 Typical embankment section and forces on a slice.

E'_i = Effective interslice normal force,

P_{wi} = Horizontal thrust due to pore water on the interslice surface,

H_i = Height of the interslice surface,

U_i = Pore water force at the bottom of the slice,

W_i = Weight of the soil.

The value of interslice shear force, T_i, that would give acceptable results, is defined as:

$$T_i = \lambda f_i \left[\left(E_i - P_{wi} \right) \tan\phi_i + c_i H_i \right] \tag{4.20}$$

Where, E_i is the total interslice normal force, λ and f_i are as described previously and c_i and ϕ_i are average values of c and ϕ over the height of the interslice surface H_i.

From the Mohr's circle of stresses, considering a plane inclined at an angle α to the horizontal and assuming that within the sliding mass, all planes inclined at this angle are in a state of limiting equilibrium, one can solve for E and P_w over the interslice surface. Substituting the values of E and P_w thus solved into equation (4.20), one gets:

$$T_i = \lambda f_i \left[\left(K_i - R_{ui} \right) \frac{\gamma H_i^2}{2} \tan\phi_i + c_i H_i \right] \tag{4.21}$$

Where,

$$K_i = \frac{1 - \sin\beta_i \left(1 - 2R_{ui}\right)\sin\phi_i + \dfrac{4c_i \cos\phi_i}{\gamma H_i}}{1 + \sin\phi_i \sin\beta_i} \tag{4.22}$$

$\beta_i \quad = \quad (2\alpha_i - \phi_i)$

$$R_{ui} \text{ (Pore pressure ratio)} = \frac{U_i}{W_i \sec\alpha_i} \tag{4.23}$$

See Sarma [21] for detail derivation of the equations.

Referring to Figure 4.3, the Mohr-Coulomb failure criterion, noting that F=1, is:

$$S_i = \left(c'_i \Delta L_i + (N_i - U_i) \tan\phi' \right) \tag{4.24}$$

From the vertical and horizontal equilibrium of the slice, one obtains:

$$N_i \cos\alpha_i + S_i \sin\alpha_i = W_i - \Delta T_i \tag{4.25}$$

$$S_i \cos\alpha_i - N_i \sin\alpha_i = K_c W_i + \Delta E_i \tag{4.26}$$

From equations (4.23), (4.24), (4.25) and (4.26), after simplifying and rearranging, and considering equilibrium of the whole mass one obtains:

$$\sum \Delta T_i \tan(\phi'_i - \alpha_i) + \sum K_c W_i = \sum D_i \tag{4.27}$$

Taking summation of moments about the center of gravity of the sliding mass and noting that the sum of the moments due to W_i and $K_c W_i$ about the center of gravity vanishes, and that the interslice body forces do not produce any net moment, and using equations (4.25), (4.26) and (4.27) one obtains after simplifying and rearranging:

$$\sum \Delta T_i \left[(y_i - y_g)\tan(\phi'_i - \alpha_i) + (x_i - x_g) \right] = \sum W_i (x_i - x_g) + \sum D_i (y_i - y_g) \tag{4.28}$$

Substituting equation (4.20) in to equations (4.27) and (4.28),

$$\lambda \sum f_i \tan(\phi_i - \alpha_i) + K_c \sum W_i = \sum D_i \tag{4.29}$$

$$\left\{ \lambda \sum \left[f_i \left((y_i - y_g)\tan(\phi_i - \alpha_i) + (x_i - x_g) \right) \right] \right\} = \sum \left[W_i (x_i - x_g) \right] + \sum \left[D_i (y_i - y_g) \right] \tag{4.30}$$

Where;

$$D_i = W_i \tan(\phi_i - \alpha_i) + (c'_i b_i \cos\phi_i - R_{ui} W_i \sin\phi_i) \frac{\sec(\alpha_i)}{\cos(\phi_i - \alpha_i)} \tag{4.31}$$

x_i, y_i = the x and y-coordinates of the center of the base of the slice;

x_g, y_g = the x and y coordinates of the center of mass of the slope;

b_i = width of slice i.

Note that in equations (4.29) and (4.30), since f_i is assumed to be known, only two unknowns remain, λ and K_c. Hence, the equations can be solved simultaneously to obtain these two unknowns.

According to Sarma, K_c could be used as a direct measure of the safety of the slope. If the conventional static factor of safety is required, it can be obtained by reducing the strength parameters by a known factor of safety and iterating until zero K_c is achieved.

Slope stability analysis normally involves searching for the critical slip surface, which corresponds to the minimum factor of safety. Using Sarma's approach one can minimize on the basis of K_c, but the critical slip surface corresponding to minimum K_c may not be the same as that for minimum F.

Although his equations can be directly solved for K_c and λ, Sarma has introduced a new parameter outside the conventionally accepted safety index, the factor of safety, and his method is not as widely covered in the literature. In addition, the claimed advantage of carrying out computations without the need for computers is not that much relevant with the current availability of high-speed computing facilities.

4.3.3 Morgenstern and Price Method

Morgenstern and Price [18] presented a comprehensive and rigorous method of solution that satisfies all static equilibrium conditions and that is applicable to slip surfaces of general shape.

To attain statical determinacy of the problem, they recommended making assumptions on interslice force relations, with the intention of avoiding numerical difficulties that may evolve in the solution procedure as well as obtaining reasonable results. The assumption is formulated as follows:

$$T_i = \lambda f(x)_i E_i \tag{4.32}$$

Where, T_i and E_i are interslice shear and normal forces, respectively; $f(x)_i$ is arbitrarily chosen scalar function and λ is a dimensionless scaling factor yet to be determined with the solution.

The equations are presented in a continuous form and are derived by considering force and moment equilibrium conditions, and Mohr-Coulomb failure criterion.

The governing differential equations are derived with reference to Figure 4.5:

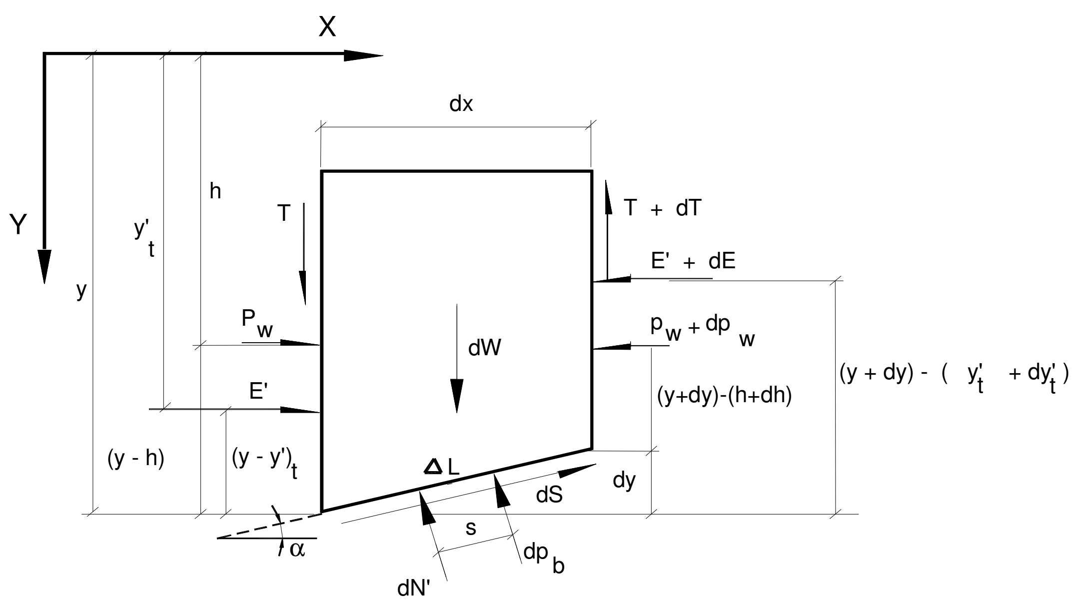

Figure 4.5 Forces acting on a typical slice.

- E' denotes the lateral thrust on the side of the slice in terms of effective stresses,
- T denotes the vertical shear force on the side of the slice,
- dW denotes the weight of the slice,
- P_w denotes the resultant water pressure acting on the side of the slice,
- dP_b denotes the water pressure on the base of the slice,
- dN' and dS denote the effective normal pressure and shear force acting on the base of the slice,
- α denotes the inclination of the base of the slice with respect to the horizontal.

Considering summation of moments about the center point of the base of the slice, and after simplifying, ignoring higher order differentials and proceeding to the limit as dx→0, one obtains the moment equilibrium equation:

$$T = \frac{d}{dx}(E' y_t ') - y\frac{dE'}{dx} + \frac{d}{dx}(P_w h) - y\frac{dP_w}{dx} \tag{4.33}$$

Summation of forces in the normal and tangential direction to the base of each slice and the Mohr-Coulomb failure criterion are considered for force equilibrium.

From equilibrium in the N direction:

$$dN' + dP_b = dW\cos\alpha - dT\cos\alpha - dE'\sin\alpha - dP_w\sin\alpha \tag{4.34}$$

From equilibrium in the S direction:

$$dS = dE'\cos\alpha + dP_w\cos\alpha - dT\sin\alpha + dW\sin\alpha \tag{4.35}$$

Mohr-Coulomb failure criterion in terms of effective stress:

$$dS = \frac{1}{F}\left[c'\,dx\sec\alpha + (dN')\tan\phi'\right] \tag{4.36}$$

By successive substitution to eliminate normal pressure dN and shear force dS from the above equations, noting that $\tan\alpha = -\frac{dy}{dx}$, and after simplifying and rearranging, one obtains:

$$\begin{aligned}&\frac{dE'}{dx}\left[1-\frac{\tan(\phi')}{F}\frac{dy}{dx}\right]+\frac{dT}{dx}\left[\frac{\tan(\phi)}{F}+\frac{dy}{dx}\right]=\frac{c'}{F}\left[1+\left(\frac{dy}{dx}\right)^2\right]+\\&\frac{dP_w}{dx}\left[\frac{\tan(\phi')}{F}\frac{dy}{dx}-1\right]+\frac{dW}{dx}\left\{\frac{\tan(\phi')}{F}+\frac{dy}{dx}-r_u\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{\tan(\phi')}{F}\right\}\end{aligned} \tag{4.37}$$

Where, r_u, the pore pressure ratio is defined as $r_u = \frac{dP_b}{dW\sec\alpha}$.

Using total interslice normal force E and the equivalent base moment:

$$E = E' + P_w \tag{4.38}$$

$$EY_t = E' y_t' + P_w h \tag{4.39}$$

Expressing y (the equation of the slip surface), dW/dx (the rate of change of the soil weight), and the interslice force function f(x) as linear functions of x for each slice, i.e.:

$$y = Ax + B \tag{4.40}$$

$$\frac{dW}{dx} = px + q \tag{4.41}$$

$$f(x) = kx + m \tag{4.42}$$

Substituting equation (4.32) and equations (4.39) to (4.42) in to equation (4.33) and (4.37), after simplifying and rearranging one obtains:

$$T = \frac{d}{dx}\left(Ey_t\right) - y\frac{dE}{dx} \tag{4.43}$$

$$\left(Kx + L\right)\frac{dE}{dx} + KE = Nx + P \tag{4.44}$$

Where;

$$K = \lambda k\left(\frac{\tan(\phi')}{F} + A\right) \tag{4.45}$$

$$L = \lambda m\left(\frac{\tan(\phi')}{F} + A\right) + 1 - A\left(\frac{\tan\phi}{F}\right) \tag{4.46}$$

$$N = p\left[\frac{\tan(\phi')}{F} + A - r_u\left(1 + A^2\right)\frac{\tan(\phi')}{F}\right] \tag{4.47}$$

$$P = \frac{c'}{F}\left(1 + A^2\right) + q\left[\frac{\tan(\phi')}{F} + A - r_u\left(1 + A^2\right)\frac{\tan(\phi')}{F}\right] \tag{4.48}$$

Integrating equation (4.44) across each slice in turn starting from the initial boundary condition of $E=E_o$ at $x=x_o$:

$$E_i = \frac{1}{(L+Kx)}\left[E_{i-1}L + 0.5(Nx^2) + Px\right] \quad (4.49)$$

This equation should satisfy the boundary condition of $E=E_n$ at $x=x_n$, where n refers to the last slice.

Also by integrating equation (4.43):

$$M = E(y_t - y) = \int_{x_o}^{x}\left(T - E\frac{dy}{dx}\right)dx \quad (4.50)$$

Considering the boundary condition at the last slice (the n^{th} slice), $M=M_n$ at $x=x_n$:

$$M_n = \int_{x_o}^{x_n}\left(T - E\frac{dy}{dx}\right)dx \quad (4.51)$$

The following steps are followed in the solution procedure:

1. After choosing a suitable interslice force function, f(x); assume initial values for F and λ and compute E_n and M_n corresponding to the last slice, starting form known boundary conditions at the beginning of the slip surface, x=0 and using equations (4.49) and (4.51).
2. Keeping λ constant, adjust the value of F until E_n equals zero or other appropriate boundary value. Compute all interslice normal and shear forces, E_i and T_i. Use equation (4.49) and (4.32).
3. Using F, E_i and T_i from step 2, adjust the value of λ until M_n equals zero or other boundary value.
4. Use the values of F from step 2 and λ from step 3 and repeat steps 1 to 3 until the changes in F and λ are within tolerable limits.

The authors have pointed out that the F and internal forces are relatively insensitive to variations in assumed interslice force function, f(x), provided that the assumptions are reasonable. Hence, they argue that for an arbitrarily chosen slip surface, if it is not possible to obtain physically admissible internal forces, it must be because the surface itself is unlikely.

The authors recommend the selection of reasonable function f(x) based on elastic theories or field measurements of internal stresses. When this is not possible the functions may be specified based on the following intuitive assumptions.

- For most cross sections, the higher the rate of curvature of the slip surface, the greater the ratio between the shear and horizontal forces at the slice interface.
- In addition, if the slice interface is in a zone of high pore pressure, the amount of shear that could be mobilized would be reduced accordingly and the function should therefore take a lower value in this region.

4.3.4 Spencer's Method

Spencer (1967) presented slope stability analysis method that satisfies all conditions of equilibrium for circular slip surfaces. Later, he generalized and modified his method to adopt it to general slip surfaces [26, 28].

The method presented in 1967 was formulated to investigate the background for the close conformance between results of factor of safety computed using Bishop's simplified method with those computed using rigorous methods.

Equations were conveniently formulated in terms of force and moment equilibrium equation and corresponding force and moment factors of safety. The sensitivity of these factors of safety to variations in the interslice force angle was then investigated.

This method of formulation has formed the basis and background for the current generalized solution methods based on limit equilibrium principles such as the Generalized Limit Equilibrium Method (GLE), which is actually a modification of Spencer's method of solution [1].

Spencer's method considers total interslice force, Z_i, instead of resolved normal and tangential components. Assumption is made on the inclination of the interslice forces, δ_i:

$$\tan \delta_i = k \tan \theta \tag{4.52}$$

Where, δ_i is the inclination angle of forces Z_i, k is an arbitrarily chosen scalar function and θ is unknown and to be determined with the solution. This is equivalent to Morgenstern and Price's assumption; where $\tan \delta_i = \frac{T_i}{E_i}$, $k = f(x)$ and $\lambda = \tan\theta$.

However, Spencer recommends using a constant scalar function, k=f(x)=1, i.e. parallel interslice force. His method of solution is similar to that of Morgenstern and Price's [18], but his equations are presented in a discretized form and he used resultant interslice forces.

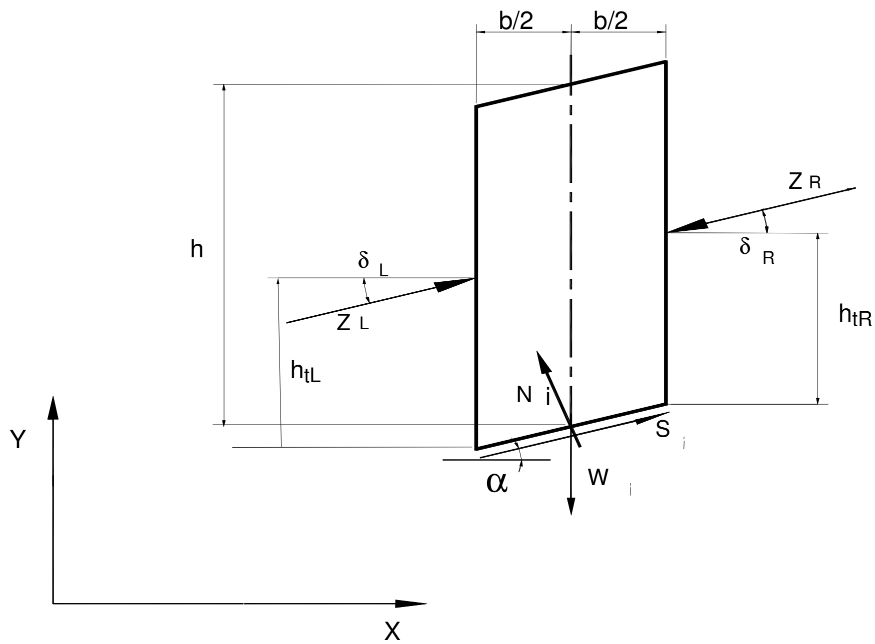

Figure 4.6 Forces acting on a typical slice**.**

- Z_L, Z_R denote the resultant interslice force at the left and right side of the slice,
- δ_L, δ_R denote the inclination angles of the interslice forces Z_L and Z_R respectively,
- h_{tL}, h_{tR} denote height of interslice force above the slip surface
- S denotes total shear force available, and S_m denotes mobilized shear force,

The equations are derived with reference to Figure 4.6.

Mohr-coulomb Failure Criterion:

$$S_m = \frac{S}{F} = \frac{b \sec \alpha}{F}(c' + \sigma' \tan \phi) \tag{4.53}$$

Resolving forces in the normal and tangential directions to the base of the slice, using the expression for S_m in equation (4.53) and eliminating the normal force, N, and rearranging, one obtains:

$$\begin{aligned} &W \sin \alpha - \frac{\tan \phi'}{F}\left[W \cos \alpha - Z_R \sin(\alpha - \delta_R) + Z_L \sin(\alpha - \delta_L) - ub \sec \alpha\right] \\ &- \frac{c'}{F} b \sec \alpha + Z_R \cos(\alpha - \delta_R) - Z_L \cos(\alpha - \delta_L) = 0 \end{aligned} \tag{4.54}$$

Further rearranging and solving for Z_R:

$$\begin{aligned} Z_R = &\frac{\frac{c'}{F} b \sec \alpha - W \sin \alpha + \frac{\tan \phi'}{F}(W \cos \alpha - ub \sec \alpha)}{\cos(\alpha - \delta_R)\left[1 + \frac{\tan \phi'}{F} \tan(\alpha - \delta_R)\right]} + \\ &Z_L \left\{ \frac{\cos(\alpha - \delta_L)\left[1 + \frac{\tan \phi'}{F} \tan(\alpha - \delta_L)\right]}{\cos(\alpha - \delta_R)\left[1 + \frac{\tan \phi'}{F} \tan(\alpha - \delta_R)\right]} \right\} \end{aligned} \tag{4.55}$$

The force on the right hand side of all slices, Z_R can thus be determined using equation (4.55), starting from the known condition of Z_L=0 for the first slice.

To obtain the moment equilibrium equation, the sum of moments about the center of the base of each slice is taken, starting from the first slice. Solving and rearranging the moment equation to get an expression for the height of the line of thrust, h_{ti}, yields:

$$h_{ti} = 0.5 b_i \left(\tan \delta_i - \tan \alpha_i\right) + \frac{1}{Z_i \cos \delta_i} \sum_{i=1}^{i} [J] \tag{4.56}$$

The expression for the external stabilizing moment acting on the last slice (n^{th} slice) is similarly derived:

$$M_n = \gamma_w z^2 \left[\frac{b_n}{2} \tan \alpha_n + \frac{z}{3} \right] - \sum_{i=1}^{n} [J] \tag{4.57}$$

The first expression in equation (4.57) is valid if hydrostatic pressure acts at a tension crack of depth z at the top of the slope, on the last slice. Otherwise, it is equal to zero.

In equations (4.56) and (4.57),

$$J_i = \frac{1}{2} Z_{i-1} \left[\sin \delta_{i-1} \left(b_i + b_{i-1} \right) - \cos \delta_{i-1} (b_i \tan \alpha_i + b_{i-1} \tan \alpha_{i-1}) \right] \tag{4.58}$$

The solution steps are similar to that of Morgenstern and Price:

1. Choose a suitable value for k, and assume trial values of F and $\tilde{\theta}$
2. Compute the slope of each interslice force Z_i using equation (4.52).
3. Use equation (4.55) to compute the value of Z_i at each slice interface and Z_n for the last slice. Adjust F until Z_n becomes zero or other boundary value (Keeping θ constant).
4. Using Z_i and F from step 3, substitute in equations (4.57) and (4.58) and compute M_n. Adjust the value of θ until M_n is zero or other appropriate boundary value.
5. Using the values of F and θ from steps 3 and 4, repeat steps 2 to 4.
6. Repeat steps 2 to 5 until the difference in F and θ between successive cycles are negligible.

Spencer recommends using a constant value for k (or f(x)) and making adjustments on the distribution function k (f(x)) based on thrust line criterion, when found necessary. Accordingly, the locations of the trust lines should be computed for each solution (by considering moment equilibrium of individual slices) and their acceptability checked.

Based on results of his studies using different assumptions of k, he pointed out that unless the line of thrust for total stress remains entirely within the middle third, there may be sections at which the resultant interslice force, Z, due to effective stresses is tensile and thus unacceptable, as soil can take little or no tension.

In addition, introduction of tension crack was found to improve the position of the line of thrust. It should be noted that improvement is with respect to acceptability criterion of computed soil stress state. Otherwise, the assumptions usually do not significantly affect the value of the computed factor of safety. The value of the factor of safety was found to be unaffected either by consideration of and by depth of tension cracks or by the use of different types of interslice force functions, f (x) or k.

From the above considerations, Spencer had concluded that 'Bearing in mind the various uncertainties involved in analyzing the stability of an earth dam, and in particular the determination of the shear strength parameters and the pore water pressure, there seems to be little point so far as the determination of the factor of safety is concerned, in trying many different assumptions regarding the slope of the interslice force' [28].

Modified form of Spencer's and Morgenstern and Price's methods of solution are included in the computer program, which will be discussed in Chapter 5.

4.3.5 Janbu's Generalized Procedure of Slices (GPS)

Janbu's method [14] assumes the location of the thrust line, h_t, of the interslice forces to estimate internal moment distribution.

This amounts to 'n-1' assumptions, but the actual location of the line of thrust that would rigorously satisfy all conditions of equilibrium is unknown (hence 'n-2' unknowns and statically determinate condition).

Janbu's method shares the common assumptions of plain-strain conditions, strength definition of factor of safety, constant factor of safety along the slip surface and location of the normal force at the base of each slice as the other methods.

Making an assumption on the location of the line of thrust is justified, according to Janbu:

- Numerical investigations have shown that the consequence of changing the position of the line of thrust within wide margins has insignificant effects on the factor of safety.
- Knowledge about stress distribution from earth pressure theories makes it a fairly simple matter to select a proper line of thrust.

The equation for the factor of safety is derived based on vertical and horizontal force equilibrium equations and Mohr-coulomb failure criterion.

In his method, rather than assuming interslice force relations, the shear forces are computed directly by considering the moment equilibrium of each slice. Hence, if appropriate line of thrust is chosen, moment equilibrium conditions will be implicitly satisfied.

The computer program, which is to be discussed in chapter 5, uses Janbu's original method of solution. Hence, the method of solution will be discussed in detail and the equations derived in this section will be used in the computer program.

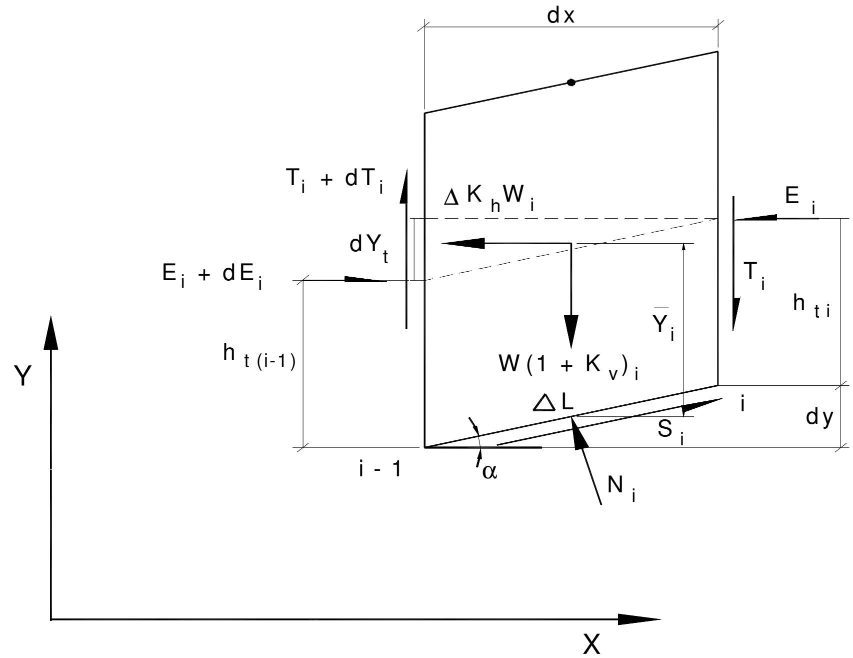

Figure 4.7 Forces acting on a typical slice

Referring to the above figure,

- K_h, K_v denote the horizontal and vertical seismic coefficients,
- dY_t denotes the vertical distance between the points of action of interslice forces at the right and left sides of the i^{th} slice, and the other parameters are as described in previous sections.

Taking summation of Moments about the midpoint of the base of each slice:

$$E_i\left[h_{ti} + \frac{dy}{2}\right] = T_i dx + dT_i \frac{dx}{2} + (E_i + dE_i)\left[h_{ti} + \frac{dy}{2} - dY_{ti}\right] - d(K_{hi}W_i)\overline{Y_i} \quad (4.59)$$

Neglecting higher order differentials and after simplifying and solving for T_i, one obtains:

$$T_i = E_i \frac{dY_{ti}}{dx} - \frac{dE_i}{dx} h_{ti} + \frac{d(K_{hi}W_i)}{dx} \bar{Y}_i \quad (4.60)$$

The values of the differentials $\frac{dE_i}{dx}$ and $\frac{d(K_{hi}W_i)}{dx}$ could be determined by using finite difference techniques. Using assumed values of height of line of thrust, h_{ti}, one can directly solve for the interslice shear, and moment equilibrium is implicitly satisfied.

The force equilibrium equation is derived by considering the Mohr-Coulomb failure criterion and horizontal and vertical equilibrium of forces.

Mohr-Coulomb Failure Criterion:

$$S_i = \frac{1}{F}\left(c'\Delta L_i + (N_i - u_i\Delta L_i)\tan\phi'\right) \tag{4.61}$$

Summation of forces in the vertical direction:

$$N_i \cos\alpha_i = W_i(1+K_{vi}) + \Delta T_i - S_i \sin\alpha_i \tag{4.62}$$

Summation of forces in the horizontal direction:

$$\Delta E_i + \Delta(K_{hi}W_i) = S_i \cos\alpha_i - N_i \sin\alpha_i \tag{4.63}$$

Solving for $N - u\Delta L$ from equations (4.61) and (4.62):

$$N_i - u_i\Delta L_i = \left[W_i(1+K_{vi}) + \Delta T_i - \frac{c'\Delta L_i \sin\alpha_i}{F} - u_i\Delta L_i \cos\alpha_i\right] m_{\alpha i} \tag{4.64}$$

Where, $$m_{\alpha i} = \frac{1}{\left[\cos\alpha_i + \frac{\sin\alpha_i \tan\phi'}{F}\right]} \tag{4.65}$$

Substituting equation (4.62) into (4.63), and simplifying

$$\Delta E_i + \Delta(K_{hi}W_i) = S_i \sec\alpha_i - (W_i(1+K_{vi}) + \Delta T_i)\tan\alpha_i \tag{4.66}$$

From equations (4.61) and (4.64),

$$S_i \sec\alpha_i = \frac{c'\Delta L_i \sec\alpha_i}{F} + \left(W_i(1+K_{vi}) + \Delta T_i\right)\frac{\tan\phi'}{F} m_{\alpha_i} \sec\alpha_i$$
$$- \frac{c'\Delta L_i \sin\alpha_i}{F}\frac{\tan\phi'}{F} m_{\alpha i} \sec\alpha_i - u_i \Delta L_i \cos\alpha_i \frac{\tan\phi'}{F} m_{\alpha i} \sec\alpha_i \quad (4.67)$$

Substituting equation (4.67) into equation (4.66) and after simplifying,

$$\Delta E_i = E_i - E_{i-1} = \frac{c'\Delta L_i}{F} m_{\alpha i} + \left[(W_i(1+K_{vi}))\cos\alpha_i - u_i \Delta L_i\right]\frac{\tan\phi'}{F} m_{\alpha i} \quad (4.68)$$
$$- (W_i(1+K_{vi}))\sin\alpha_i m_{\alpha i} + \Delta T_i \left(\tan(\phi'_m - \alpha_i)\right) - \Delta K_{hi} W_i$$

Where; $\phi_m = \tan^{-1}\frac{\tan\phi'}{F}$.

Taking Summation of forces across the slope and rearranging to solve for the factor of safety, F:

$$F = \frac{\sum\left[\left(c'\Delta L_i + ((W_i(1+K_{vi})\cos\alpha_i - u_i\Delta L_i)\tan\phi'\right) m_{\alpha i}\right]}{\sum\left[W_i(1+K_{vi})\sin\alpha_i m_{\alpha i} + \Delta T_i \tan(\alpha_i - \phi'_m) + \Delta K_{hi} W_i\right] + (E_n - E_o)} \quad (4.69)$$

Where, $\sum \Delta E_i = E_n - E_o$ and E_n and E_o are the boundary horizontal thrust forces at the last and first slices, respectively.

If we set the value of ΔT_i to zero, equation (4.69) then corresponds to Janbu's simplified method.

Equations (4.60), (4.68) and (4.69) are used in an iterative scheme to solve for the factor of safety. The solution to the problem proceeds by following the steps indicated below:

1. The factor of safety is iteratively solved for by setting $\Delta T_i = T_i - T_{i-1} = 0$ in equation (4.69). The F so obtained is equivalent to Janbu's Simplified Factor of Safety.

2. Using the F from step 1 and ΔT_i=0, E_i is computed for each slice using equation (4.68).
3. Using E_i from step 2 and assumed value of height of line of thrust, h_{ti}, T_i is computed for each slice, using equation (4.60).
4. T_i from step 3 is then employed to solve iteratively for the F using equation (4.69).
5. The above steps are repeated until the difference in the F values between two successive iterative loops is sufficiently small.

In Janbu's method of solution, T_i is calculated by considering moment equilibrium conditions, and hence moment equilibrium is implicitly satisfied. However, as pointed out by Sarma [23] and others, Janbu's solution procedure does not check the moment equilibrium of the last slice and this condition is not satisfied for this last slice (upper most slice).

Based on case studies on homogeneous sections, Janbu reported that the change in the factor of safety, F, was found to be quite insignificant for practical purposes even for such a large range of values of h_{ti} as 0.1H to 0.7H, where H is the height of the interslice surface. But, the stress distributions were substantially affected by such large fluctuation in h_{ti}.

Thus, the following recommendations were forwarded so that reasonable stress distributions could be obtained:

- The position of line of thrust, h_{ti}, should be selected in accordance with known plasticity solutions of earth pressure distributions at limit equilibrium. Accordingly:

    - For c'=0, the line of thrust should be selected at or very near to the lower third point,

      $$h_{ti} \cong \frac{1}{3} H_i$$

- For c'>0
  - Line of thrust should be located above the lower third point in the passive zone (near the toe of slopes)
  - Line of thrust should be located some what below the lower third point in the active zone (top of slope)
  - Tension cracks of depth Z_t could be assumed in tension zones.

    Where $Z_t = \frac{2c'}{F\bar{\gamma}}\sqrt{\frac{1+\sin\phi'_m}{1-\sin\phi'_m}}$, F is the factor of safety, $\bar{\gamma}$ is the effective soil unit weight and $\phi'_m = \tan^{-1}\left(\frac{\tan\phi'}{F}\right)$.
  - The failure condition at interslice surface should be checked.

Janbu's method provides a more direct approach to the solution of slope stability problem. By making reasonable assumption of the line of thrust, condition of acceptability of solution, such as implication of no tensile forces within the soil mass are automatically satisfied, and solution can be attained with considerably less computational effort.

In addition, the concepts behind the assumptions are simple and the original method as developed by Janbu is applicable for general loading and geometric conditions as well as being amenable for computer programming [33].

Janbu's method is coded as a solution option in the slope stability analysis computer program.

4.4 CHOICE OF METHOD OF ANALYSIS AND SUMMARY

The various analysis methods discussed in the previous sections share some common fundamental features and limitations.

In general, when interpreting the factor of safety (F) computed using any one of these methods, reliability of the input data that define the conditions under analyses should be considered.

Choice of method of analysis could be regarded with respect to the following:

- Accuracy
- Ease of Application
- Acceptability of Solution.

These are further discussed hereunder.

4.4.1 Accuracy

Various studies have been conducted over the years to evaluate the computational accuracy of limit equilibrium methods. Such studies include Spencer (1967), Wright et al. (1973), Chen and Snitbhan (1975), Huang and Avery (1976), Fredlund and Krhan (1977), Garber and Baker (1979), Sarma (1979), Fredlund et al. (1981), etc. (according to Duncan [10]).

The accuracy of the various methods was evaluated by comparison with what are believed to be correct values for a range of conditions where the slope geometry, water pressures, unit weights and shear strengths are precisely defined.

The accuracy of the limit equilibrium methods has been found to be satisfactory based on studies on failed slopes. The factors of safety and the strength parameters back calculated on existing slide surfaces were found to be reasonably accurate and results were consistent with actually measured and forecasted behavior of the materials involved [34].

Based on review of the various studies, it has been raised that evaluation of computational accuracy of the different methods should not be based on analysis of arbitrarily chosen slip surface; rather the minimum factor of safety for the different methods (corresponding to their own critical slip surfaces, which may vary between the methods) must be compared [10].

Important features and findings of these various studies are summarized as follows [10, 11, 26, 28, 32, 34]:

1. Factors of safety computed using force equilibrium methods are sensitive to the assumed inclination of side force between slices, and poor assumption can lead to serious errors.
2. Ordinary method of slices (OMS) gives quite accurate results for cases of $\phi=0$ because in such cases shear strength does not depend on normal force distribution.
3. Bishop's simplified method is quite accurate and it gives results of F to within 4 – 5% of the rigorous methods. This is attributed to the relative insensitivity of computed F to assumption of interslice force inclinations. Hence, Bishop's simplified method could give accurate results, but insensitivity of the moment equilibrium factor of safety to interslice force relationships must be checked, as this may not hold true in some cases, for example when the slope is subjected to surcharged loads and pore water pressure, [25].
4. Based on analysis of similar slope stability problems, the various studies indicated that factors of safety computed using the limit equilibrium methods, which satisfy all conditions of equilibrium, might differ by as much as 12%. Thus if the average value is considered as accurate, it can be stated that LE methods that satisfy all conditions of equilibrium are accurate to $\pm$ 6%. This is accurate enough for all practical purposes, because slope geometry, water pressures, unit weights and shear strengths can seldom be defined with such degree of accuracy.

4.4.2 Ease of Application

Suitability of a particular method of analysis could be regarded with respect to 'ease of application'. This refers to the relative ease with which one can understand the methods and employ it to solve practical problems.

Limit equilibrium methods of analysis generally involve making assumptions, usually on the inclination or locations of interslice forces and most of the authors of the various limit equilibrium methods recommend employing check procedures to assess acceptability of obtained solutions.

Ease of application could be regarded with respect to the following:

- The ease in making reasonable assumptions
- Amount of computational effort and time required and
- Frequency of numerical problems encountered.

Ease in making assumptions

Assumptions employed by the common limit equilibrium methods can be grouped into three. These are:

Type 1 Assumption - Assumption on interslice shear force distribution (Bishop's Rigorous Method),
Type 2 Assumption - Assumptions on interslice force inclinations (Spencer's Method, Morgenstern and Prices Method),
Type 3 Assumption - Assumptions on the locations of point of action of interslice forces (Janbu's Method).

Making reasonable assumptions on the interslice force distribution (*Type 1*) is rather difficult. This requires experience and familiarity with the method of analysis and the particular problem under study. Considering the differences in the level of expertise of possible users of a general purpose slope stability program, this may limit choice of such methods.

Failure to make reasonable assumption will in turn create numerical problems, very slow convergence or failure to converge to a solution.

For the other groups of assumptions, those regarding interslice force inclination and location (*Type 2* and *Type 3* respectively), one can make reasonable assumptions based on the knowledge of stress distribution within the soil mass and earth pressure theories. General guidelines have been given by Morgenstern and Price [18] and Janbu [14] for making such reasonable assumptions on inclination and location of interslice forces respectively. (See sections 4.3.3 and 4.3.5).

Some of the commonly used interslice force functions are given in Fig. 4.8.

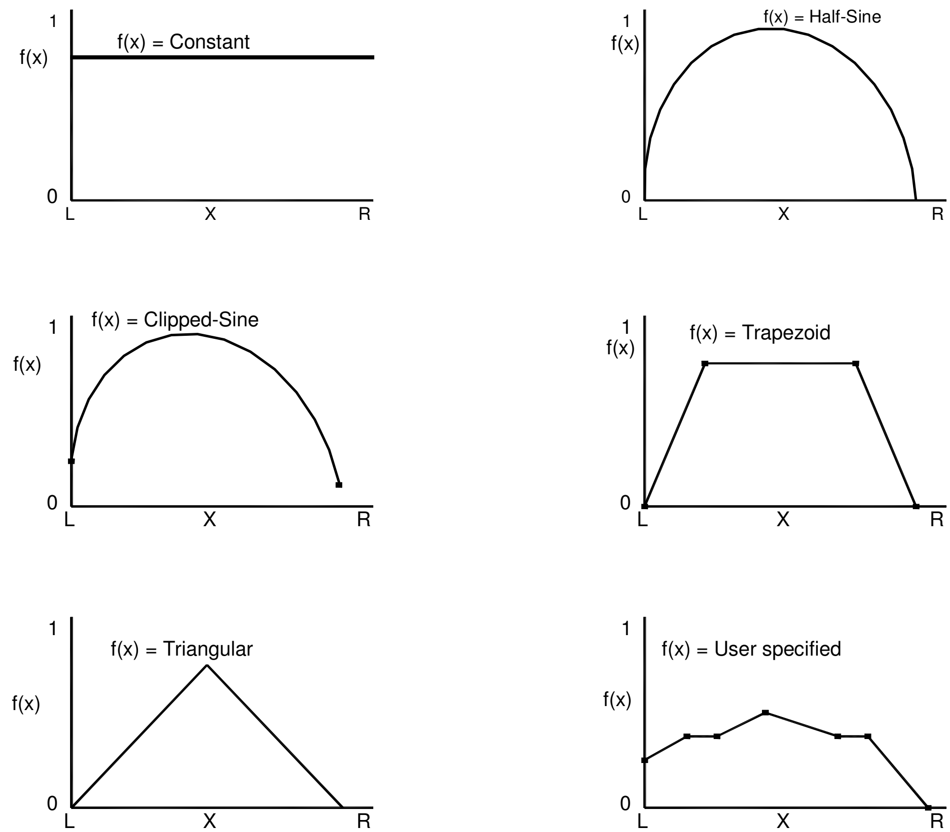

Fig. 4.8 Some commonly used interslice force functions.

Computational effort and time

All the rigorous methods require the use of computers. With the current status in computer technology, computational effort and time required are not issues of major concern.

Frequency of numerical problems encountered

All of the methods that make assumptions of interslice force distributions, relationships or locations can run into numerical problems.

Some authors have noted that Janbu's type of approach, methods that make assumptions categorized as *Type 3*, can run into more frequent numerical problems than the other methods [18]. In addition, some reported that the stresses achieved from this type of analysis are irregular [1]. However, it can still be maintained that, whichever category the type of assumption falls into, failure to make reasonable assumption will lead to numerical problems and unreliable results.

4.4.3 Acceptability of Solution

Acceptability of solution refers to the reasonableness of computed factor of safety and internal stresses. Assessment of acceptability requires detailed calculation of interslice forces and checking that failure criteria is not violated within the sliding mass, no tensile forces are implied and directions of forces are kinematically admissible.

The stresses obtained from a particular analysis depend on the assumption of the distribution, location or relationships of interslice forces. And acceptable solutions could be obtained by considering various interslice force functions and after sufficient trials.

Methods that employ assumptions on location of interslice force (*Type 3* assumption - Janbu's method) have relative advantage as the assumption made is based on direct consideration of equilibrium condition of a slice. Such methods automatically avoid the problem of tensile forces and if reasonable assumptions are made, computational effort required will reduce.

However, failure criterion at interslice boundaries, kinematical admissibility of direction of normal and shear forces at the base of slices, negative and/or excessively large normal stresses at the base of slices, etc., that can have undue effect on the computed factor of safety should still be checked.

Many have argued that the location of the thrust line should be one criterion for acceptability of a solution [26]. Some specify a preferable location of the thrust line to be within the middle third of the height of slice, assuming linear distribution of interslice forces. While others specify that, the location of the thrust line is acceptable if it is within the sliding mass, considering non-linear interslice stress distribution.

Janbu's solution method conforms to the above, and reasonable solution can be obtained if the location of the line of thrust is carefully selected without the need for excessive number of trials, hence reducing the computational effort required.

Another line of thought is often raised by many, who argue that, as computed factors of safety values are often insensitive to assumptions of interslice force function, f(x), irrespective of whether the assumptions result in reasonable or unreasonable internal stress distributions, acceptability should not be based on internal stress distributions.

The principal aim of slope stability analysis should be the determination of the factor of safety and not the stresses. Considering the engineering time that will be required to check the acceptability conditions, it can be concluded that it is not necessary to check the internal force distributions at all, as long as rigorous methods that satisfy all conditions of equilibrium are used. This trend of thought has been concisely summarized by Duncan [10], a prominent researcher in the field of geo-techniques, "If the internal stress distribution implicit in the analysis is unreasonable; the engineer can rely on the fact that there is another solution, with reasonable internal stress distribution, that would give essentially the same factor of safety."

4.4.4 Summary

In general, limit equilibrium methods that satisfy all conditions of equilibrium are equally suitable for general-purpose slope stability analysis, except probably for Bishop's rigorous method (for reasons already discussed).

Choice of a method of analysis should hence depend on subjective factors such as familiarity with a particular method and availability of software.

Various slope stability software are now available on the market, that employ most of the limit equilibrium analysis methods discussed in the previous sections, and that can compute internal stresses, check internal failure criterion, etc. Some soft wares even combine features of Finite Element Method with limit equilibrium principles to get optimal solution. For example, Slope/W, commercial slope stability analysis software, incorporates an option for computing stresses and pore water pressure using finite element method, which together with the soil strength parameters and Mohr-Coulomb failure criterion is used to solve for the factor of safety.

Emphasis should thus be given to the proper formulation of the problem under analysis. This involves the task of accurately defining the soil parameters, pore water pressures and loading conditions, and making reasonable assumption regarding the interrelationship or location of interslice forces or distribution of interslice shear forces, whichever is required by the particular method of analysis. In addition, checking internal stresses and cross checking of computed results, using different methods is recommended as most of the modern soft wares incorporate such features and the additional time or effort required is not considerable.

The above discussions are summarized as follows:

- All limit equilibrium slope stability analysis techniques based on the method of slices considered so far and satisfying all conditions of equilibrium can be regarded as equally accurate.
- Bishop's simplified method is reasonably accurate, for most cases, and could be used to gain quick overview of the stability condition of a slope. However, for complex geometric and loading conditions, results obtained using this method should be properly scrutinized and/or checked by other rigorous methods.
- Force equilibrium methods tend to give conservative results of factor of safety. Results obtained using such methods could significantly differ from that obtained using rigorous methods, and should be regarded with caution.
- Although the factor of safety computed is relatively insensitive to assumptions of interslice force inclination function or variations in the location of the thrust line, some checks should still be run on the solutions obtained. As an alternative or in addition to running checks the analysis should be conducted by different methods and the computed factors of safety checked for consistency, as there is no means of ascertaining the validity of the computed factor of safety otherwise.

CHAPTER 5: COMPUTER PROGRAM FOR SLOPE STABILITY ANALYSIS

5.1 GENERAL FEATURES OF THE COMPUTER PROGRAM

A computer Program has been coded for the most common methods of analysis, using Visual Basic Programming Language. The methods that are incorporated in the program include:

- Bishop Simplified Method
- Janbu Simplified Method
- Bishop Rigorous Method
- Spencer's / Morgenstern and Price's Rigorous Method
- Janbu's General Procedure for Slices (GPS)

The computer program can analyze pre-specified circular and noncircular slip surfaces, homogenous or layered soil profiles and incorporates pore pressure options and earthquake options (pseudo static analysis).

The user inputs the geometric components of the slope, soil parameters and the number of slices into which the sliding mass should be divided. Using these data, the computer program will first divide the slip mass into sections based on the geometry of the slope surface and so that the base of each slice falls on a single soil layer. After that, the program will calculate slices data such as x, y coordinates, and height of the interslice surfaces, the weight, and base inclination angle of the slices, etc. These data will be used in computing the factor of safety, interslice forces and normal and shear stresses at the base of the slice, which are the outputs of the analysis program, according to the method of analysis selected by the user.

The program has been designed to lead the user to input his data systematically. As the user navigates through the program, the appropriate user interfaces for each type of input will appear on the screen, with the label of the user interface frame describing the type of data that need to be entered. In addition, option frames appear at the appropriate time for selecting whether or not pore pressure, external surcharge loads or earthquake conditions are to be considered.

To guide the user to input his data, certain intermediate data that the user will need to input correct data would be displayed. For example, to enter pore pressure head data, the x-coordinates of the interslice surfaces are displayed in the pore pressure head input user interface, so that the user can input the correct data corresponding to the location of the slice.

The program incorporates a database, and allows the user to save his input data. Whenever the user exits the program, he is prompted if he wants to save his data and the user can give his data a file name and save it. Next time he reruns the program, he will have the option to open existing files or create a new file. The program exports the out put data to MS Excel. Hence, the user is able to utilize all the standard features of MS Excel and maneuver his output data.

The computer program is also equipped with error traps. Error messages will appear on the screen if the input data are not numeric, i.e. if the user inputs a text in the data input boxes instead of a number, or if the required input data has not been entered.

The program converges to solution in a relatively short time; it usually takes from few seconds to less than a minute, to converge to a solution. However, as discussed in the previous chapters, it is rather difficult to make reasonable assumption on the interslice shear force distribution, as required by Bishop's rigorous method, and analysis by this method usually takes longer time to converge to a solution. To avoid excessively long duration of processing time, the computer program incorporates a timer function, and if the elapsed time from the start of the iterations is more than 10 minutes, error message will be displayed informing the user to check his input data or his assumed force function.

In the next sections, the equations coded by the computer program for the different methods together with the derivation and the solution procedure employed will be discussed.

5.2 GENERALIZED FORMULATION OF EQUATIONS FOR THE COMPUTER PROGRAM

The theories and assumptions employed by the common Limit Equilibrium Slope Stability Analysis methods, their limitations and strength, their equations and methods of solution have been discussed in the previous sections.

Review of the solution equations and the procedures to solutions of most of the limit equilibrium methods show that basically the same static equilibrium conditions are considered. Their major differences are in the approaches adopted to obtain the equilibrium equations and in the assumptions made on the interslice force relations.

Some methods use resultant interslice forces with their inclination angles (Spencer [26], [28]) while others use the resolved interslice normal and shear forces for analysis (Morgenstern and Price [18], Janbu [14], Bishop [3]). Some methods consider moment equilibrium about a common axis (Bishop [3], Spencer [26], Sarma [21]) while others consider moment equilibrium conditions for each slice (Morgenstern and Price [18], Spencer [28], Janbu [14]).

Spencer [28] has pointed out that, other things being equal, the approaches adopted in obtaining the equilibrium equations do not affect the final solutions to a given problem.

Review of the methods have revealed that Bishop's method, Spencer's method and Morgenstern and Price's method are particularly suited to formulation in a common frame work with a flexible option for accommodating their distinguishing features (the assumption each method employs regarding interslice force relations or distributions).

Hence, in the computer program, instead of preparing a different code for each method, a generalized approach representing the common features of the various methods in a unified manner is coded within a common framework. Appropriate options are also provided to incorporate their distinguishing features. The generalized approach also incorporates the simplified methods.

A separate procedure has been coded for Janbu's GPS, as the method of solution involved is not directly amenable to adapting to the generalized formulation proposed here. Janbu's method directly considers the moment equilibrium conditions of each slice in deriving the relation between interslice forces as opposed to the other rigorous methods, which make simplifying assumptions on these relations.

Chugh (1986) and Fredlund et al. (1981) (according to Abrahamson et al. [1]), and Espinoza et al. [11], have recognized the advantages of such a generalized approach to solve slope stability analysis problems. They have developed a unified solution formulation that can encompass most of the assumptions used by the various methods and which are applicable to slip surfaces of general shape.

The unified formulations presented by the above authors are not independent methods and do not make separate assumptions of their own. Rather the approaches of solution of the various limit equilibrium methods have been unified and presented in a manner such that the different assumptions that distinguish the methods of analyses could be accommodated. Therefore, they could be regarded as modifications of some of the rigorous methods of analysis like Spencer's, Morgenstern and Prices', etc.

An approach similar to the 'Unified formulation of slope stability analysis methods' presented by Espinoza et al. [11] is adopted here. This unified formulation considers vertical and horizontal force equilibrium conditions for all slices and overall moment equilibrium about a common axis. The vertical and horizontal components of the interslice forces are used for the analysis.

Force equilibrium factor of safety, FF, is computed based on force equilibrium conditions, and moment equilibrium factor of safety, FM, is computed based on moment equilibrium conditions.

5.2.1 Derivation of Equations

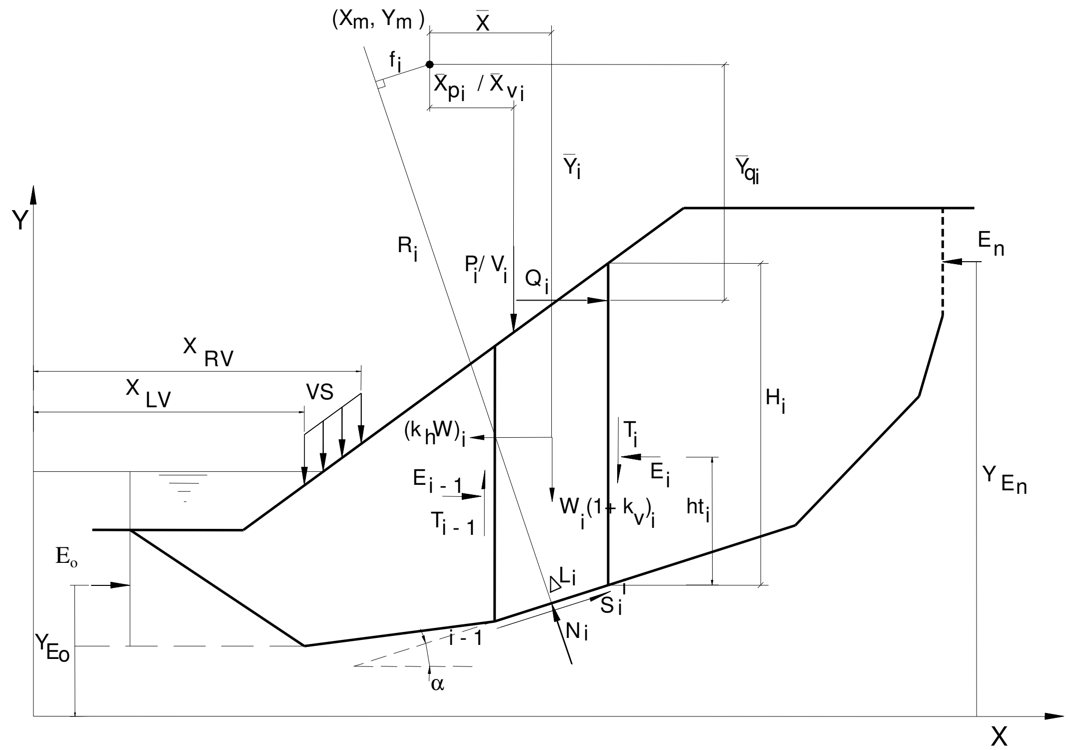

Fig. 5.1 Forces on a general slope section and typical slice.

Consider the typical slice in Figure 5.1, showing all the forces acting on it.

Equations of statics used to solve for the factor of safety

1. The summation of moments about a common axis for all slices. The Moment equilibrium factor of safety, FM, is derived from this.
2. The summation of forces in the horizontal direction for all slices, from which the force equilibrium factor of safety, FF, is derived.
3. The summation of forces in the vertical direction for each slice to get the normal force '*N*' at the base of each slice.
4. The summation of forces in the horizontal direction for each slice. This is used to compute the interslice normal force *E*.

5. Summation of moments about the center of the base of each slice. This is used to derive the interslice shear forces based on assumed thrust line positions for Janbu's GPS procedure, or to compute the line of thrust for the other methods.

Mohr Coulomb failure criterion – effective stress analysis

The shear strength, s, of a soil mass at any point is given by:

$$s = \left[c' + (\sigma_n - u)\tan\phi'\right] \quad (5.1)$$

Where,

c' and ϕ'	=	Effective cohesion and angle of internal friction of the soil
σ_n	=	Total normal stress at the base
u	=	Pore water pressure

Mobilized shear force to satisfy conditions of limiting equilibrium

The mobilized shear force, S, on a segment of a slip surface of length, ΔL, is expressed as:

$$S_i = \frac{\left[c'\Delta L_i + (N_i - u_i \Delta L_i)\tan\phi'\right]}{F} \quad (5.2)$$

Where,	N	=	Normal Force at the base of each slice
	F	=	Factor of safety

Note that the factor of safety is assumed to be constant along the slip surface and equal proportions of the cohesive and frictional components of strength are assumed to be mobilized.

Derivation of Moment Equation

Summation of forces in the vertical direction for a typical slice

$$N_i \cos\alpha_i = W_i(1+K_{vi}) + P_i + V_i + \Delta T_i - S_i \sin\alpha_i \tag{5.3}$$

Solving for $N - u\Delta L$ from Equations (5.2) and (5.3):

$$N_i - u_i\Delta L_i = \left[W_i(1+K_{vi}) + P_i + V_i + \Delta T_i - \frac{c'\Delta L_i \sin\alpha_i}{F} - u_i\Delta L_i \cos\alpha_i \right] m_{\alpha i} \tag{5.4}$$

Where, $m_{\alpha i} = \dfrac{1}{\left[\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi}{F} \right]}$

Substituting the value (N- $u\Delta L$) of equation (5.4) into equation (5.2):

$$S_i = \frac{1}{F}\left[\left(c'\Delta L_i \cos\alpha_i + (W_i(1+K_{vi}) + P_i + V_i + \Delta T_i - u_i\Delta L_i \cos\alpha_i) \tan\phi' \right) \right] m_{\alpha i} \tag{5.5}$$

Taking summation of moments about an arbitrarily selected point for the whole sliding body, and assuming the moment due to the normal force, N, to be positive (clockwise),

$$\begin{aligned} &\Sigma(W_i(1+K_{vi})\overline{X_i}) + \Sigma(P_i \overline{X}pi) + \Sigma(V_i \overline{X}vi) - \Sigma(Q_i \overline{Yq}_i) + \\ &\Sigma(K_{hi} W_i \overline{Y}i) = \Sigma S_i R_i - \Sigma N_i f_i \end{aligned} \tag{5.6}$$

Adding $u_i\Delta L_i f_i$ on both sides of equation 5.6, substituting for S_i and (N_i-$u_i\Delta L_i$) from equations 5.4 and 5.5 into equation 5.6 and on simplifying and rearranging to solve for moment equilibrium factor of safety, FM, one obtains:

$$FM = \frac{\Sigma \left[\left(c'\Delta L_i \cos\alpha_i (R_i + f_i \tan\alpha_i) + (W_i(1+K_{vi}) + P_i + V_i - u_i\Delta L_i \cos\alpha_i) \tan\phi' R_i \right) m_{\alpha i} \right]}{\Sigma \left[\begin{array}{l} W_i(1+K_{vi})\overline{X}i + P_i \overline{Xp}_i + V_i \overline{Xv}i - Q_i \overline{Yq}_i + K_{hi} W_i \overline{Y}i + u_i\Delta L_i f_i + \\ \Delta T_i m_{\alpha i}(f_i - \tan\phi_m R_i) + m_{\alpha i} f_i (W_i(1+K_{vi}) + P_i + V_i - u_i\Delta L_i \cos\alpha_i) \end{array} \right] + \left(E_n(Y_m - Y_{E_n}) - E_o(Y_m - Y_{E_o}) \right)} \tag{5.7}$$

Where, the boundary thrust forces at the first and last slices, E_0 and E_n , respectively, are positive if compressive and negative if tensile.

Derivation of Force Equation

The force equation has already been derived in section 4.3.5, Janbu's GPS method. The equations are:

$$\Delta E_i = E_i - E_{i-1} = \frac{c'\Delta L_i}{F} m_{\alpha_i} + \left[(W_i(1+K_{vi}) + P_i + V_i)\cos\alpha_i - u_i \Delta L_i \right] \frac{\tan\phi}{F} m_{\alpha i}$$
$$- (W_i(1+K_{vi}) + P_i + V_i)\sin\alpha_i m_{\alpha_i} + \Delta T_i \left(\tan(\phi_m - \alpha_i) \right) + Q_i - K_{hi} W_i$$

(5.8)

Where; $\phi_m = \tan^{-1} \frac{\tan\phi'}{F}$

The equation of force equilibrium factor of safety, FF, is:

$$FF = \frac{\sum\left[\left(c'\Delta L_i + ((W_i(1+K_{vi}) + P_i + V_i)\cos\alpha_i - u_i \Delta L_i)\tan\phi' \right) m_{\alpha i} \right]}{\sum\left[\left(W_i(1+K_{vi}) + P_i + V_i \right)\sin\alpha_i m_{\alpha i} + \Delta T_i \left(\tan(\alpha_i - \phi_m) \right) - Q_i + K_{hi} W_i \right] + (E_n - E_o)}$$

(5.9)

Equation for Interslice normal Force, E_i

$$E_i = E_{i-1} + \frac{c'\Delta L_i}{F} m_{\alpha i} + \left[(W_i(1+K_{vi}) + P_i + V_i)\cos\alpha_i - u_i \Delta L_i \right] \frac{\tan\phi}{F} m_{\alpha i}$$
$$- (W_i(1+K_{vi}) + P_i + V_i)\sin\alpha_i m_{\alpha i} + \Delta T_i \left(\tan(\phi_m - \alpha_i) \right) + Q_i - K_{hi} W_i$$

(5.10)

The different assumptions used by the various methods and solution schemes adopted by the computer program for each method based on the unified approach will now be considered.

5.2.2 Simplified Methods

Bishop's Simplified Method

Bishop's simplified method assumes zero interslice shear force and the original method is applicable for circular slip surfaces only.

In Bishop's Method, assumption is made on the distribution of interslice shear forces and the moment equilibrium equation, equation (5.7) is used to compute the factor of safety, FM.

The assumption is formulated as:

$$\Delta T_i = T_i - T_{i-1} = \lambda \left[f(x)_i - f(x)_{i-1} \right] \qquad (5.11)$$

If we set λ equal to zero, the interslice shear force distribution in equation (5.11) and (5.7) will automatically be zero.

Hence, by setting the interslice shear force expression $\Delta T m_{\alpha}(f - \tan\phi_m R)$ in equation (5.7) to zero, one obtains:

$$FM = \frac{\left[\left(c'\Delta L_i \cos\alpha_i (R_i + f_i \tan\alpha) + (W_i(1+K_{vi}) + P_i + V_i - u_i \Delta L_i \cos\alpha_i)\tan\phi' R_i\right) m_{\alpha_i}\right]}{\sum \begin{bmatrix} W_i(1+K_{vi})\overline{X}i + P_i \overline{Xp}_i + V_i \overline{Xv}i - Q_i \overline{Yq}_i + K_{hi} W_i \overline{Y}i + u_i \Delta L_i f_i \\ + m_{\alpha i} f_i (W_i(1+K_{vi}) + P_i + V_i - u_i \Delta L_i \cos\alpha_i) \end{bmatrix} + \left(E_n (Y_m - Y_{En}) - E_o (Y_m - Y_{Eo})\right)}$$

(5.12)

This is Bishop's simplified equation for the general case of non-circular slip surface and external loading. Note that for the case of circular slip surface, f_i (moment arm of the normal force at the base), will be equal to zero and the above equation could be simplified further to yield Bishop's original equation for the simplified version.

Equation (5.12) is then directly solved for the factor of safety, FM, by simple iteration procedure. Note that FM appears on both sides of the equation (m_α is also a function of FM). An initial value of the factor of safety is assumed, and based on that a new factor of safety is calculated. Using the calculated FM, the procedure is repeated until the difference between two successive iterations is acceptable. The computer program uses a difference of 0.001 as the acceptability criterion.

Janbu's Simplified Method

Janbu's simplified method also assumes zero inter slice shear force. However, the factor of safety is computed by using the force equilibrium equation. Setting the value of $\Delta T_i\left(\tan(\alpha_i - \phi_{mi})\right)$ in equation (5.9) to zero, one obtains:

$$FF = \frac{\sum\left[\left(c'\Delta L_i + ((W_i(1+K_{vi}) + P_i + V_i)\cos\alpha_i - u_i\Delta L_i)\tan\phi\right)m_{\alpha i}\right]}{\sum\left[\left(W_i(1+K_{vi}) + P_i + V_i\right)\sin\alpha_i m_{\alpha i} - Q_i + K_{hi}W_i\right] + (E_n - E_o)} \tag{5.13}$$

The factor of safety is computed in a similar manner as Bishop's simplified method by iteration.

5.2.3 Rigorous Methods

Here are included methods that satisfy all conditions of equilibrium. Janbu's rigorous method is also categorized here, but a separate procedure has been incorporated in the computer program using the equations and solution procedures described in Section 4.3.5. Hence, Bishop's rigorous method, Spencer's method and Morgenstern and Price's method are considered.

The force equilibrium formulation of Janbu's method is actually the same as the others. The difference lays in the formulation of the moment equilibrium conditions. Janbu proposes to make assumption on the height of point of action of the interslice force and determining the interslice shear force by directly considering the moment equilibrium of each slice. The other rigorous methods assume the inclination of the line of action of the interslice resultant force or the distribution of interslice shear force and make iterations on this assumed values to satisfy both force and moment equilibrium.

In Bishop's rigorous method, the equations of solution were originally developed by assuming circular slip surface [3]. This simplified the overall moment equilibrium equation because it avoids the normal force, N, as its line of action passes through the center of the slip circle and the resulting moment arm will be zero.

The moment equilibrium factor of safety equation used in the computer program (equation 5.7) has been derived for slip surfaces of general shape by introducing the moment arm, f_i, for the normal force. Thus starting from the basic principles employed in developing the original equations, the method has been modified to apply to general shape slip surfaces.

In Bishop's rigorous method, assumption is made on the shape of the interslice force distribution, ΔT_i, to get statically determinate system. This assumption is formulated as:

$$\Delta T_i = T_i - T_{i-1} = \lambda \left[f(x)_i - f(x)_{i-1} \right] \qquad (5.14)$$

The underlying principles in developing the methods of solution of Spencer [28] and Morgenstern and Price [18], are basically identical. Spencer used the interslice resultant force in his solution schemes while Morgenstern and Price used the resolved forces; and the later presented their equations in a continuous form while Spencer's were presented in a discrete form. In addition, both methods considered moment equilibrium conditions of individual slices in deriving their moment equilibrium expressions.

The basic approach to solution employed by both methods is otherwise similar and the assumptions employed to achieve statical determinacy by both methods is identical. Both methods make assumption on the inclination angle of the interslice force. The fact that the assumptions are identical has been shown in section 4.3.4.

The basic difference lies in the recommended interslice force inclination functions, f(x) or k. Spencer recommends the assumption of parallel interslice forces, i.e. constant k value (taken as 1) while Morgenstern and Price's method recommends certain guide lines based on soil behavior in setting this function (usually half sine or triangular functions are employed). Spencer's assumption is one out of the numerous possibilities in which f(x) could be defined and hence Spencer's method could be regarded as a subset of Morgenstern and Price's method.

For these methods, the interslice force relation is given by:

$$T_i = \lambda f(x)_i E_i \tag{5.15}$$

The computer program uses the same procedure for both methods. The moment equilibrium factor of safety equation, (equation 5.7), the force equilibrium factor of safety equation (equation 5.9), the equation for interslice normal force (equation 5.10) and interslice force relation expressions, (equation 5.14 or 5.15) together with assumed scalar functions, f(x) are used to solve for the factor of safety.

Solution Procedure

1. First, the chosen interslice force functions f(x) values are given for each slice interface.
2. λ is set to zero (which automatically sets the value of T to zero), and corresponding FM and FF values computed using Equations (5-7) and (5-9). These correspond to Bishop Simplified and Janbu Simplified factors of safety respectively.
3. λ is then incremented by a known amount (in the computer program a value of 0.001 is used for Spencer or M-P methods and a value of 0.1 used for Bishop's method). Then using equation (5-10) and starting from boundary condition at the start of the slip surface, interslice normal force values E_i are computed at each slice interface. (Equation 5.14 or 5.15 is substituted for the T term in equation (5-10)).
4. The normal force values are then used to compute interslice shear force values T_i at each slice interface using Equations 5.14 or 5.15.

5. Substitute the T_i values computed in step 4 in to Equations (5-7) and (5-9) to compute a new set of FF and FM corresponding to the λ.
6. Repeat steps 2 to 5 until the difference between FF and FM reduces to an acceptable level (a value of 0.001 is used by the computer program).

5.2.4 General Remarks

The unified approach created a convenient means for preparing a computer program that can solve slope stability problems and compare computed results using the various available methods that are based on limit equilibrium principles and the method of slices. This could help assess the sensitivity of a particular problem to different assumptions as used by the various methods.

The following points should be considered concerning the above formulation:

- The slice density (no. of slices, N) should be sufficient so that the assumption on the location of the normal force at the base of the slice is valid. Janbu [14] recommended that six to ten (6-10) slices are sufficient while Spencer [26] concluded based on his study that the factor of safety is practically insensitive to the number of slices for slice numbers > 16. Espinoza et al. [11] recommended 10 to 20 slices to attain reliable factor of safety.
- Location of the center of moment could be any arbitrary point and has little effect on the final results. Fredlund et al. (1994) have pointed out that location of moment center has practically no effect on computed results for methods that satisfy all conditions of equilibrium (according to Espinoza et al. [11]). However, the position of the moment center could have effect on the computed factor of safety when the slip surface is non-circular and methods of solution do not satisfy all conditions of equilibrium. Hence should numerical instabilities arise in such cases, the position of the center of moments should be considered as a possible cause. It is generally recommended to set the center of moments at the center of an approximate circle that closely encompasses the slip surface.

- Unreasonable values of normal forces are sometimes achieved due to unrealistic values of m_α.

  Where; $\boldsymbol{m}_\alpha = \dfrac{1}{\cos\alpha + \dfrac{\sin\alpha\tan\phi}{\boldsymbol{F}}}$ and the normal force, *N*, is a function of m_α.

  (See equation 5.4).

  These conditions correspond to very high m_α values corresponding to very small value of the expression in the denominator ($\cos\alpha + \dfrac{\sin\alpha\tan\phi}{F}$), which will result in very high normal forces and unduly affect computed factor of safety. In addition, negative m_α values will reduce the shear strength and reduce the factor of safety computed. m_α is one of the outputs of this program, and the user could inspect the computed values for reasonableness, if inconsistent results of stresses and/or factor of safety are observed.

These types of cases usually arise due to in appropriate shape of assumed slip surface. Hence, care should be exercised and the kinematical admissibility of the slip surfaces checked.

Earth pressure theories could be used to establish the limiting conditions for the shape of the slip surfaces.

In the passive zone (near the toe) the maximum obliquity should be:

$$\alpha_p < \frac{\phi'}{2} - 45^o$$

Similarly for the active zone,

$$\alpha_a < \frac{\phi'}{2} + 45^o .$$

In the active zone, if α_a is exceeded tension cracks may be specified.

5.3 INPUT AND OUT PUT DATA OF THE COMPUTER PROGRAM

5.3.1 Input Data

The main input data are the geometry of the slip mass and the soil parameters. In addition to these, depending on the complexity of the problem at hand, external surcharge loads, pore water pressure data and earthquake coefficients data may be required. If the rigorous methods of analysis are selected, the user should also input the selected interslice force function or height of line of thrust.

The input data are typed in to text boxes and when the user clicks on an 'Enter' command button, the data will be transferred and displayed in a list box. The user cannot edit the data once it is in the list box.

The program presupposes that all input data are given in SI units and output data are accordingly in SI units. The user interfaces and the input procedure are described below.

User Interface One (Slope Surface Data)

1. X_g, Y_g; x and y coordinates of the slope surface. The coordinates of the slope surface at the beginning and end of the slip surface and at changes in slope of the surface should be entered. The program stores these data as an array, with the array counter initialized at zero;
2. Radius and x and y coordinates of the center of the slip circle (R, X_c and Y_c), if slip surface is circular;
3. The number of slices, N, into which the slip mass is to be divided for analysis.

This interface also contains option check box for specifying whether shape of the slip surface is circular or not.

User Interface Two (Slip Surface Data)

This interface is enabled if the checkbox for circular slip surface is unchecked (circular slip surface option is not chosen) in user interface one. The data input are:

1. X_s, Y_s; x and y coordinates of the slip surface. For noncircular slip surfaces, the user should divide the slip surface in to a series of straight lines and enter the x and y coordinates of the the points that define the slip surface (at changes in slope). The data will be stored as an array, with array index initialized at zero.

User Interface Three (Soil Layer Data)

This interface appears directly after user interface one if circular slip surface is specified. Otherwise, it will be enabled after user interface two. The data input are:

1. X_{gl}, Y_{gl}; x and y coordinates of the point of intersection of soil layer interface line and the slope surface.
2. X_{sl}, Y_{sl}; x and y coordinates of the point of intersection of soil layer interface line and the slip surface.
3. Soil unit weight (γ), Effective cohesion (c') and Angle of internal friction (ϕ') in degrees, for the soil layer below the layer interface line.

   These data are entered for each layer within the slip mass. If the soil is homogenous, one should enter the coordinates of the start of the slip mass at the intersection of the slip surface and the slope surface at the toe, ($X_g(0)$, $Y_g(0)$) as X_{gl}, Y_{gl} and the end point of the slip mass at the intersection of the slip surface with the ground at the top ($X_g(n)$, $Y_g(n)$) as X_{sl}, Y_{sl}. The soil parameters should also be entered.

   If the layers interface line, (the approximate dividing line between two soil layers) is not a straight line with the same slope, the x and y coordinate at points of change of slope of this line should be entered as X_{sl}, Y_{sl} for the preceding line, and as X_{gl}, Y_{gl} for the continuing line. For each layer interface data entered, the corresponding soil parameters (for the soil below the layer interface) should be entered. The slope surface data beginning from the last X_{gl}, Y_{gl} and ending at the end of the slip surface ($X_g(n)$, $Y_g(n)$) should always be entered as the last layer interface. See figure 5.2 for illustrations of various possible cases.

4. The check boxes by which the user specifies the following options are also in this user interface
   a) If there are external load data (like point loads or other surcharge loads, boundary forces at the first and last slices of the slip mass);
   b) If pore pressure is to be considered;
   c) If earth quake conditions are to be considered;

The user should click on the appropriate check box or check boxes if they are to be considered in the analysis.

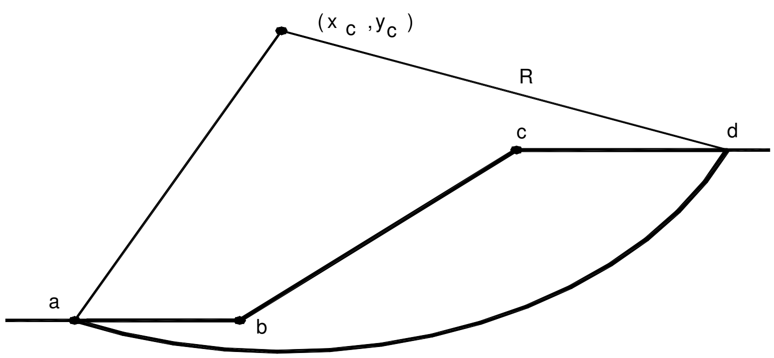

Fig. 5.2 Geometric data input for a homogeneous slope with circular slip surface.

Slope Surface Input	Slip Surface Data	Layer Interface
a=Xg(0), Yg(0)	R, Xc, Yc	a=XGL(0), YGL(0)
b=Xg(1), Yg(1)		d=XSL(0), YSL(0)
c=Xg(2), Yg(2)		c'=c'(0), $\phi=\phi(0)$, $\gamma=\gamma(0)$
d=Xg(3), Yg(3)		

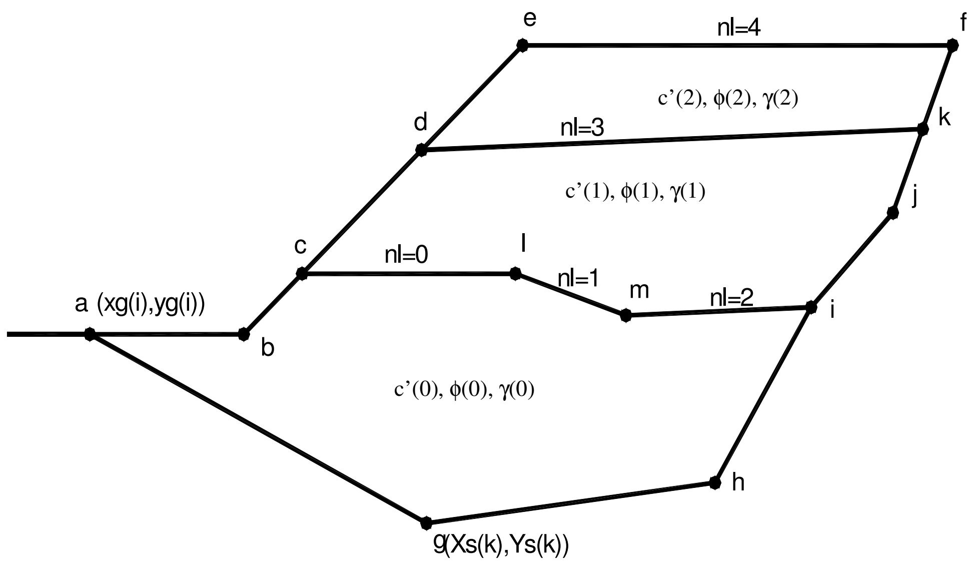

Fig. 5.3 Geometric data input for a multi-layered slope with non-circular slip surface.

Slope Surface Input	**Slip Surface Data**	**Layer Interface**
a=Xg(0), Yg(0)	a=Xs(0), Ys(0)	c=XGL(0), YGL(0)
b=Xg(1), Yg(1)	g=Xs(1), Ys(1)	l=XSL(0), YSL(0)
c=Xg(2), Yg(2)	h=Xs(2), Ys(2)	c'=c'(0), φ=φ(0), γ=γ(0)
d=Xg(3), Yg(3)	i=Xs(3), Ys(3)	l=XGL(1), YGL(1)
	j=Xs(4), Ys(4)	m=XSL(1), YSL(1)
	k=Xs(5), Ys(5)	c'=c'(o), φ=φ(0), γ=γ(0)
		m=XGL(2), YGL(2)
		i=XSL(2), YSL(2)
		c'=c'(o), φ=φ(0), γ=γ(0)
		d=XGL(3), YGL(3)
		k=XSL(3), YSL(3)
		c'=c'(1), φ=φ(1), γ=γ(1)
		d=XGL(4), YGL(4)
		f=XSL(4), YSL(4)
		c'=c'(2), φ=φ(2), γ=γ(2)

User Interface Four (External Loads Data)

If the external load check box in interface three is checked, the external force entry frame will be enabled. The data to be input are:

1. Magnitude of Point Load, P (KN) and x coordinate of its point of action, X_P.
2. Magnitude of distributed load, V (KN/m) and x coordinates of the start and end of the distributed load V, X_{LV} and X_{RV} respectively.
3. Magnitude of Horizontal Load (other than earthquake load, such as one due to inclined loads, etc.), Q (KN) and the x and y coordinates of its point of action, X_q and Y_q.
4. Magnitudes of boundary shear forces (if any) at the sides of the first and last slices, T_0 and T_n in KN.
5. Magnitudes of boundary normal forces, E_0 and E_n in KN, and their point of action, Y_{E0} and Y_{En} (Y-coordinates).

The user then goes back to interface three in which the command button that enable the execution of computation of slices data is situated.

User Interface five (Earth Quake Coefficients Data Input)

If the earthquake option check box in user interface three is checked, after calculating the slices data, the earthquake coefficients input frame will appear on the screen. The interface will display the slice number, x-coordinates of the interslice surfaces and the y coordinates of the interslice surfaces at intersection with the slip surface and the slope surface. This is so that the user can enter the appropriate horizontal and vertical earthquake coefficients, K_h and K_v, based on the given information for each slice. Then the user will be prompted to enter the coefficients. Dynamic soil strength parameters should be used for the pseudo static earthquake analysis.

Horizontal earthquake coefficients should always be entered as positive, while the vertical earthquake coefficients could be either positive or negative. If one does not want to consider effects of earthquake in the vertical direction, zero value should be entered for K_v. The earthquake coefficients should be dimensionless.

User Interface six (Pore Pressure Data Input)

If the pore-pressure option check box in user interface three is checked, the next interface will be the pore pressure data input frame. In this frame, a list box containing the slice index data and the x-coordinates of the slice interfaces will be displayed to assist the user to input the corresponding data. The user should then enter the pore pressure head or pore pressure ratio in the input text box.

The user should also specify if the data corresponds to phreatic surface, piezometric surface or if it is pore pressure ratio, by clicking at the appropriate option in the pore pressure options frame, which is situated within user interface six. The computer program will read the option chosen and compute the pore pressure as follows:

Phreatic surface: $u = \gamma_w h_{pw} \cos^2 \theta$

Piezometric surface: $u = \gamma_w h_{pw}$

Pore pressure ratio: $u = \dfrac{r_u \times W}{\Delta L \cos \alpha}$

Where, γ_w the unit weight of water, h_{pw} is the pore pressure head, r_u is the pore pressure ratio, θ is the inclination angle of the phreatic surface and the other terms have been described in previous sections.

User Interface seven (Method of Analysis)

This interface will enable the user to choose from the methods given in section 5.1. In addition, data input frame for entering the x and y coordinates of the center of moments for the analysis of moment equilibrium conditions, and input box for specifying initial factor of safety are included in this interface. If the user does not specify initial factor of safety, the program will assume a value of 1.5 to start the iteration process.

If the user chooses the simplified methods for analysis, this will be the last user input interface. The user can then click on the 'CalcFS' command button to get the factor of safety and other outputs. But, if the rigorous methods are chosen, either of the following interfaces will appear.

User Interface Eight/A (Interslice Force Function Data Input)

If Bishop Rigorous Method or Spencer/Morgenstern and Price Method are selected, the interslice force function input frame will be enabled. This frame when activated will display the slice indices and x-coordinates of the interslice surfaces.

The user is then prompted to enter numerical values of his chosen interslice force function at each slice interface. Note that if trigonometric functions are employed, data should be calculated in radians. The user then goes back to user interface seven and orders the program to compute the factor of safety.

User Interface Eight/B (Height of Line of Thrust Data Input)

This is the thrust line height (h_t) input interface and it is enabled if Janbu's Rigorous method is selected. The input frame will display the slice indices and the height of the interslice surfaces (H_i) for each slice, so that the user can calculate his h_t values based on the given H_i.

The user should then go back to interface seven to execute the final command of computation of factor of safety.

5.3.2 Data Output

The outputs from the analysis program are exported to MS Excel. These outputs are stored in two sheets in MS Excel.

Slices data, such as x and y coordinates of the interslice surfaces, height of the interslice surfaces (H), m_α, inclination angle of the base of the slices (α), weight (W) and pore pressure (u) for each slice and length (ΔL) of the base of each slice, c' and ϕ' at the slice base are stored in the first sheet named 'Data Sheet'. The data could be then checked to verify the results of geometric computations done by the program. Also, m_α and α values could be checked for reasonableness.

The second sheet, named Stresses, will display the main output of the program, which include:

- Factor of Safety
- Method of Analysis
- Lambda(λ), F, Force Equilibrium Factor of Safety, FF, and Moment Equilibrium Factor of Safety, FM, if Bishop Rigorous method or Spencer/Morgenstern and Price Method were used.
- Interslice Normal Force, E(i) at each interslice surface;
- Interslice Shear Force, T(i) at each interslice surface;
- Normal Force, N(i) at the base of each slice;
- Shear force, S(i) at the base of each slice;
- Location of Line of thrust of the interslice force above the base of each interslice surface.

The output data could be maneuvered as necessary, using the standard MS Excel features. For example, each output could be plotted against the X-axis and the outputs from different methods of analysis compared.

5.4 EXAMPLES

In this section, example problems are solved using the computer program developed and results briefly discussed. The example problems were taken from papers on slope stability analysis methods, and have been solved by other authors using the original methods and/or their own approach.

It should be noted that some discrepancy in the computed safety factors is inevitable as it is difficult to accurately reproduce the geometry of the slopes from the published figures, which are drawn to a very small scale.

Example One

The homogeneous soil profile shown in figure 5.4 was analyzed by Janbu [14] and Asrat W. [33]. Janbu analyzed the most critical slip surface, which is non-circular and represented by curve I in the figure, and the critical slip circle. He obtained a safety factor of 1.48 for the non-circular surface and 1.52 for the circular slip surface. Asrat W. [33] also analyzed the critical circular slip surface, represented by curve II, using a computer program he developed. He obtained a safety factor of 1.58. The same circular slip surface analyzed by Asrat W. is analyzed using the computer program developed and the various analyses approaches.

The safety factor values obtained and the assumptions made by each method are given in Table 5.1.

The input data for the program are:

Slope surface data (X_g, Y_g) :- 0, 0; 31, 12.4; 32.766, 12.4

Slip surface data (R, X_c, Y_c) :- 37.8683, 4.5, 37.6

Number of slices:- 10

Soil layer data (X_{gl}, Y_{gl}, X_{sl}, Y_{sl}, γ, c’, ϕ’) :- 0, 0, 32.766, 12.4, 19.616, 9.81, 33.8045

Pore pressure data (r_u- Pore pressure ratio) :- 0.4

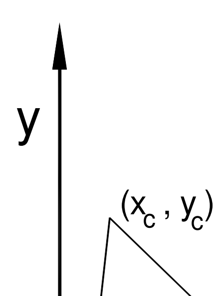

Figure 5.4 *The homogenous slope of Example 1.*

Table 5.1 Summary of safety factor (F) obtained using different methods for Example 1

Method of analysis	Factor of Safety	Lambda (λ)	Assumptions/Remark
Results obtained by Janbu and Asrat W.			
Janbu[14]	1.52	-	-
Asrat W. [33]	1.58	-	-
Results obtained using the computer program developed			
Simplified Bishop	1.522	0	Zero interslice shear force
Simplified Janbu	1.438	0	Same as above
Bishop Rigorous	1.525	66.6 KN	Interslice shear force distribution function : f(x) =sin(k^1 x Π)
Spencer's method	1.525	0.34	Parallel interslice force inclination: f(x)=1
Morgenstern and Price's method	1.525	0.416	Half sine interslice force function: f(x)= =sin(k^1 x Π)

Janbu's Method	1.519	-	Position of line of thrust assumed at one third of slice height.

[1]$k=(x-x_a)/(x_d-x_a)$, where x_a and x_d represent the x coordinates of the beginning and end points of the slip mass.

The results shown in table 5.1, those obtained using the rigorous methods, conform with those obtained by Janbu and Asrat W., and also they are in good agreement with each other. The difference between individual results is less than 0.5%.

Review of the output results reveals that the stress distributions obtained using Janbu's, Spencer's and Morgenstern and Prices's methods appear to be reasonable. However, the position of the line of thrust, for the last (uppermost) slice and for the later two methods, lies outside the slope body. Introduction of tension crack and water pressure in the tension crack could help obtain reasonable position of the line of thrust and avoid the implication of tensile forces in the soil mass.

In addition, although the factor of safety value obtained using Bishop's rigorous approach gave identical result, as that of Spencer's and Morgenstern and Price's, the stress distributions and positions of line of thrust of interslice forces are inadmissible. See appendix 3 for the complete analysis outputs from the computer program.

Example Two

This example depicts a layered slope and non-circular slip surface. The critical slip surface shown in figure 5.5 was identified by Chen and Shao (1988), using an optimization technique together with the Morgenstern-Price's method and employing an interslice force inclination function $\theta(x)$ as suggested by Chen and Morgenstern (1983) (according to Leshchinsky et al. [16]). This same problem was analyzed by Leshchinsky et al. [16], using variational limit equilibrium analysis technique which the authors developed. Also, Greco [12] has analyzed this problem using his 'Efficient Monte Carlo technique for locating critical surfaces' and pattern search method. Results obtained by the above authors and analysis results using the computer program developed are given in table 5.2.

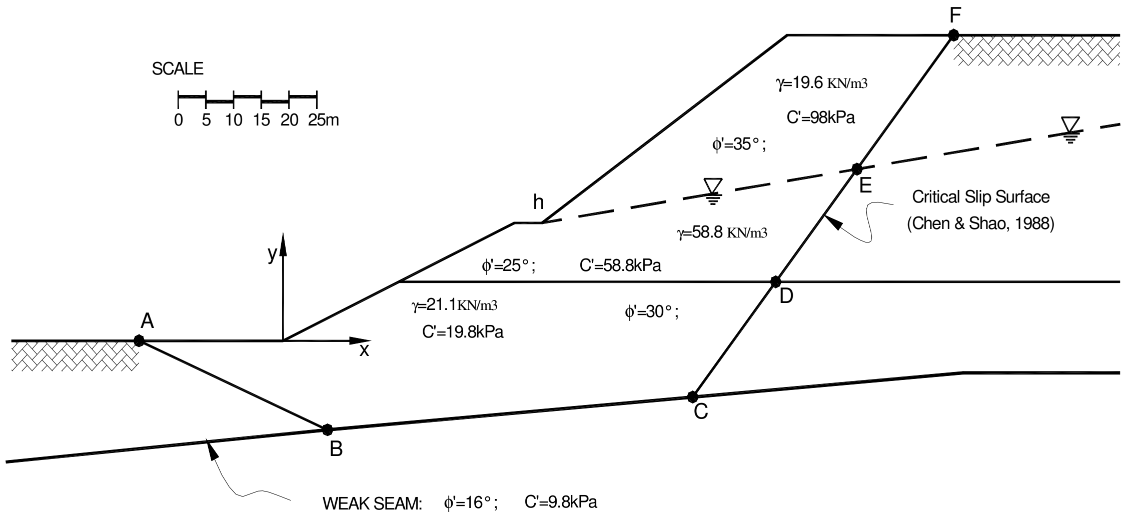

Figure 5.5 *Layered slope of Example 2.*

The input data for the program are:

Slope surface data (X_g, Y_g) :- -25, 0; 0, 0; 42, 21; 47, 21; 92, 52; 124, 52.

Slip surface data (X_s, Y_s):- -25, 0; 6, -14; 75, -7; 124, 52.

Number of slices: - 15

Center of moments (X_m, Y_m) 31.7, 158.7

Soil layer data (X_{gl}, Y_{gl}, X_{sl}, Y_{sl}, γ, c', ϕ') :-

-25, 0, 6, -14, 21.1, 19.8, 30

-25, 0, 75, -7, 21.1, 9.8, 16

22, 11, 89, 9.86, 21.1, 19.8, 30

47, 21, 106, 30.33, 18.6, 58.8, 25

47, 21, 124, 52, 19.6, 9.8, 35

Pore pressure data (Y_{pw}, Phreatic surface):-

Pore pressure head, Y_{pw}, corresponds with the slope surface for $x_a <= x <= x_h$

Pore pressure head, $Y_{pw} = 21+0.158(x-x_g)$ for $x_h <= x <= x_e$.

Table 5.2 Summary of safety factor (F) obtained using different methods for Example 2

Method of analysis	Factor of Safety	Lambda (λ)	Assumptions/Remark
Results obtained by other authors			
Morgenstern and Price [12, 16]	1.01	-	-
Leshchinsky et al. [16]	1.061	-	Variational limit equilibrium method.
Greco [12][2]	0.973 – 0.974	-	Monte Carlo method with 13 vertices.
Results obtained using the computer program developed			
Simplified Bishop	0.947	0	Zero interslice shear force
Simplified Janbu	0.9	0	Same as above
Bishop Rigorous	1.0	2,213.41 KN	Interslice shear force distribution function : f(x) =sin(k^1 x Π)
Spencer's method	1.051	0.239	Parallel interslice force inclination: f(x)=1
Morgenstern and Price's method	1.031	0.31	Half sine interslice force function: f(x)=sin(k^1 x Π)
Janbu's Method	0.977	-	Position of line of thrust assumed at one third of slice height.

[1] $k=(x-x_a)/(x_d-x_a)$, where x_a and x_d represent the x coordinates of the beginning and end points of the slip mass.

[2] Greco's critical surface may not correspond to that of Chen and Shao's.

The individual results of safety factors, for the rigorous methods, computed using the computer program, differ by about 7%. In addition, it can be seen from table 5.2 that they are in good agreement with the results obtained by the other authors.

Review of the stress output revealed that the stress distributions and direction of forces appear to be reasonable, except for Bishop's rigorous method. However, the position of the line of thrust of the interslice force for the upper most slice lie outside the slip mass in both Spencer's and Morgenstern and Prices's methods.

The stresses corresponding to Bishop's rigorous method are, however, inadmissible and the position of the lines of thrust all lie outside the slip mass.

CHAPTER 6: DISCUSSION AND CONCLUSION

A generalized approach for the analysis of slope stability problems based on limit equilibrium principles has been presented. The approach incorporates the basic principles of the major limit equilibrium methods.

The computer program developed presents these major limit equilibrium based slope stability analysis techniques, in a unified formulation. This type of approach is advantageous in that one can assess the sensitivity of the factor of safety and stresses to variations of the assumptions as employed by the different limit equilibrium methods. In addition, one can easily check for consistency of the computed safety factor by crosschecking results obtained by the different methods. The computer program is user friendly and could easily be understood and used by students. Hence, it could also be used for educational purposes.

Analysis results of the example problems using the presented approach and the developed computer program indicate that the factor of safety is relatively insensitive to the assumptions on the distribution or inclination of interslice forces. Very close values of safety factors were obtained, especially for the homogeneous soil profile presented in example 1, using the different approaches. A maximum difference of 0.4% between the rigorous methods was observed for the homogeneous slope. The deviation between the safety factors obtained using the various methods increases for the layered soil profile with non-circular slip surface: about 7% difference between computed factors of safety values (rigorous methods).

This indicates that with increasing departure from homogeneity and for complex problems involving pore water pressure, surcharge loads, etc., the relative sensitivity of computed factors of safety to the particular assumptions employed increases.

The simplified methods also showed the above trend. Deviation of Bishop's simplified method, from safety factor obtained using Morgenstern and Prices' method, increased from 0.2 % for the homogenous soil profile to 6 % for the layered soil. Also, Janbu's simplified method gave safety factors that deviate from Morgenstern and Price's method by 8% and 12% for the homogenous and layered soil profiles, respectively.

In addition, although the safety factor values obtained using the different methods are in close agreement, the difference between the stresses is considerable.

Bishop's rigorous method, in particular, gave inadmissible stress distribution and positions of lines of thrust, although the computed factor of safety was within 3 % of that obtained using Morgenstern and Price's method.

The above observations lead to the following conclusions:

- The factor of safety is relatively insensitive to the assumptions employed regarding the interslice force inclinations, positions or distributions. However, the stresses are highly affected by such variations. This is in conformance with the observations made by many prominent researchers in the field of geotechniques, including the authors whose work has been discussed at length in the previous chapters.
- Relative sensitivity of the factor of safety to the different assumptions increases with departure from homogeneity of the slope geometry and the soil mass.

Acceptability criterion of solution should not be based on the reasonableness of computed stresses but rather on the reasonableness of the safety factors, where slope stability analysis by the limit equilibrium methods is concerned. If safety factor values obtained using the various approaches are consistent, it can be accepted as correct regardless of the stress distributions. The stress distributions, even if they are reasonable, are approximations and if accurate stress analysis is required, one should resort to more advanced methods such as the finite element method.

It should be noted that it is not the purpose of this work to assess the computational accuracy of the limit equilibrium based analysis methods. In order to check that, the minimum factor of safety for the different methods (corresponding to their own critical slip surfaces, which may vary between the methods) must be compared. Rather, in this work, arbitrary critical slip surfaces that correspond to a particular method as established by the respective authors mentioned in the example problems were used.

This is to illustrate the computer program and show the conformance of the computed safety factors using this computer program with already published results.

Finally, it should be mentioned that the computer program developed lacks one basic feature in that it does not incorporate search method for the critical slip surfaces (it can only analyze pre-specified slip surfaces). Various search methods for the critical slip surface have been developed over the years, and incorporating one such method in the computer program and linking it to one or all of the approaches presented is recommended.

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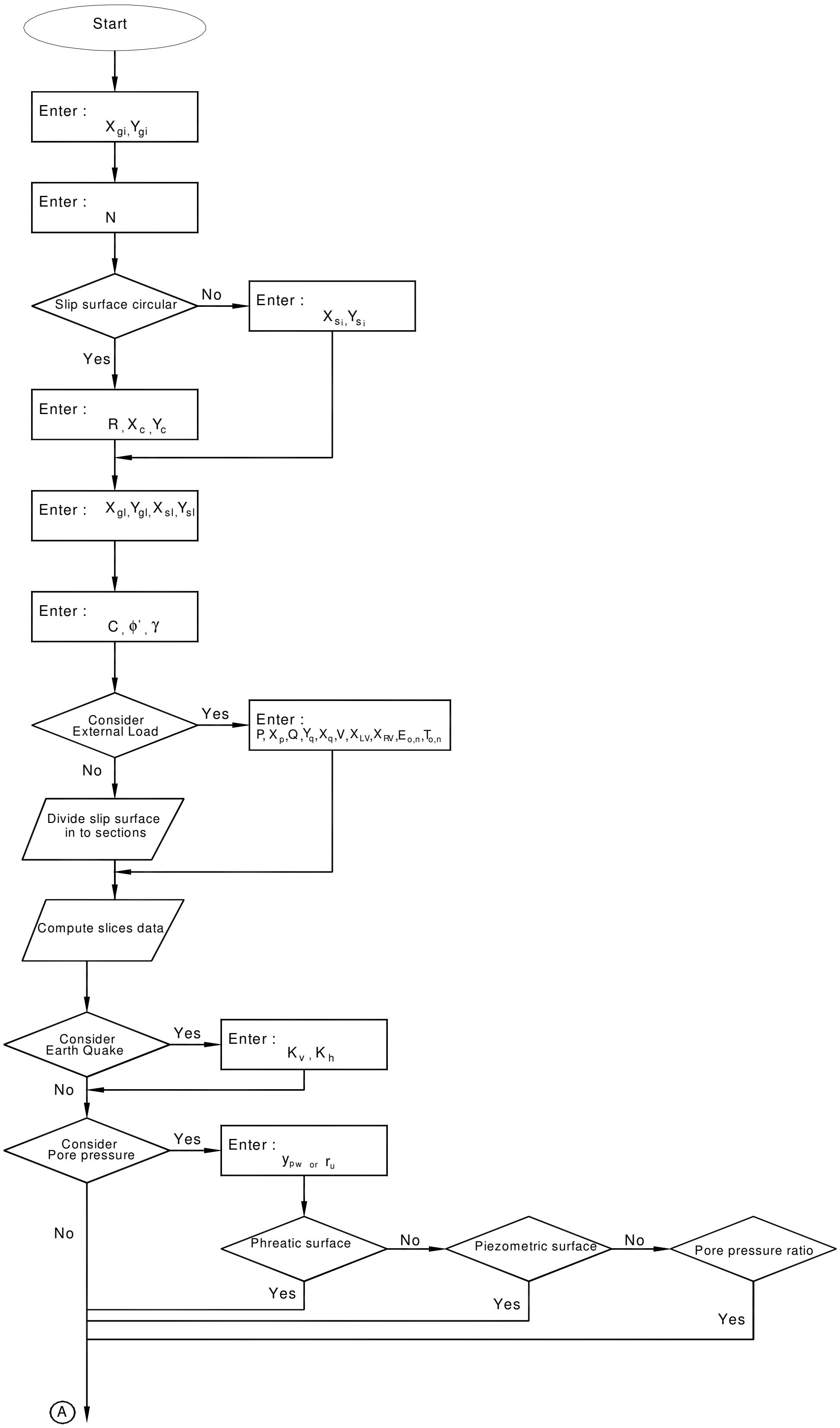
APPENDIX 1 FLOW CHART



FLOW CHART cont'd