REVIEW OF CLASSICAL AND QUANTUM HALL EFFECT

By

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The undersigned hereby certify that they have read and recommend to the School of Graduate Studies of Addis Ababa University for acceptance a project entitled "**REVIEW of CLASSICAL and QUANTUM HALL EFFECT** " by **Lakech Alem** in partial fulfillment of the requirements for the degree of **Master of Science in physics**.

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Abstract

In this project we review the Classical and Quantum Hall Effects. We discuss these two effects theoretically based on the available literature. In classical Hall Effect when a strong magnetic field is applied perpendicular to the electrons plane of movement, the electrons execute tiny cyclotron orbits around the flux lines. If in addition, an electric field applied in transverse direction to the induced electric field, the electrons will tend to drift in a direction perpendicular to both fields, generating the Hall Effect. For the case of Quantum Hall Effect, energy associated with the cyclotron motion is quantized giving rise to the Landau levels and at low temperatures all the electrons are in the lowest Landau level. The filling factor ν *can be changed by varying the magnetic field* B *for a fixed carrier density. This leads to the sitation that the Hall conductivity takes the values equal to* $\nu e^2/h$ *as discussed in Chapter three. The Quantum Hall Effect can be Integral or Fractional. The Integer Quantum Hall Effect can be understood in an independent-particle model, without taking into account the electron-electron interactions, in the Fractional Quantum Hall Effect, where the filling factors take fractional values (1/2,1/3,1/5,...) the electron-electron interactions play an essential role. The Coulomb interaction produces incompressible states of highly correlated carrier motion in high magnetic fields at specific fractional filling levels. At such magnetic fields the electrons can be treated as quasi particles called composite fermions.*

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Lakech Alem
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Chapter 1

INTRODUCTION

Hall Effect is an electrical effect that occurs when a current carrying conductor or semiconductor is subject to a magnetic field or a region of space influenced by a magnet or other magnetizing object. If an electric current is passed along the x-direction through a conductor in which there is a magnetic field along the z-direction then an electric field is set up along the y-axis. This electric field is at right angles to both the applied electric field and the magnetic field. In this case a voltage called the Hall voltage is created across the conductor or semiconductor perpendicular to both the current and the magnetic field [1]. The Hall voltage arises because the magnetic field distorts the flow of electrons or other charge carriers that constitute the current, pushing the charged particles to one side of the conductor. The result of accumulation of electrons on the side boundary builds up a static electric field and balances the magnetic force. Electrons then drift in their original intended direction. The resulting electric field gives rise to a potential difference along the y-direction.

Studies of the Hall Effect have led to a better understanding of the electronic properties of solids, such as conduction in metals and semiconductors.

For instance, the Hall voltage across a metal is much smaller than across a semiconductor carrying the same current in the same magnetic field because the metal contains more charged particles than the semiconductors.

Figure 1.1 depicts Hall experiment. An electric field E_x is applied to a wire extending

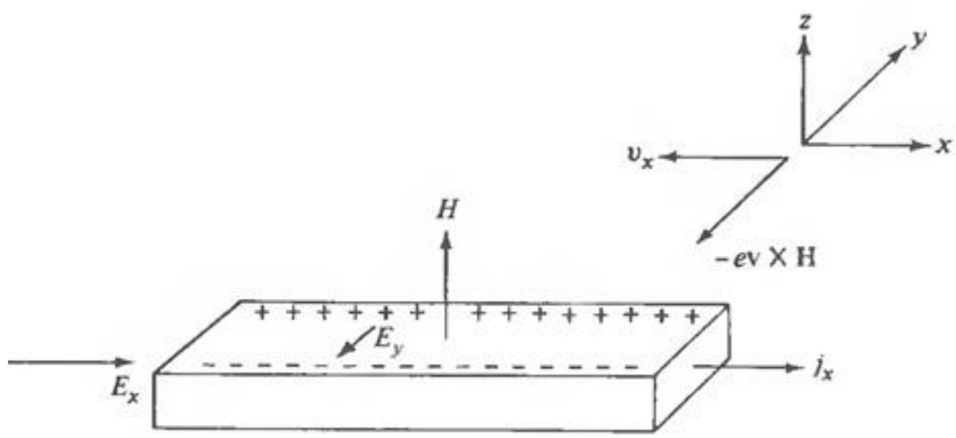

Figure 1.1: Schematic view of Hall's experiment.

in the x-direction and a current density

$$j_x = (-e)nv_x \tag{1.0.1}$$

flows in the wire. Here v_x is the velocity, n is the density of electrons and e is the charge of an electron.

In addition, the applied magnetic field H points in the positive z-direction. As a result the magnitude of the magnetic force in CGS in the y-direction is given by

$$F_y = \frac{(-e)}{c} v_x \times H. \tag{1.0.2}$$

Where c is the speed of light.

It acts to deflect electrons in the negative y-direction (an electron's drift velocity is opposite to the current flow). As the electrons accumulate there, an electric field builds up in the y-direction that opposes their motion and their further accumulation. In equilibrium the transverse field E_y will balance the magnetic force, and current will flow only in the x-direction.

The ratio of the field along the wire E_x to the current density j_x,

$$\rho_H = \frac{E_x}{j_x}, \tag{1.0.3}$$

is called the magneto resistance, or the transverse magneto resistance. The transverse field E_y proportional both to the applied field H and to the current along the wire j_x is

$$E_y = \frac{F_y}{(-e)} \tag{1.0.4}$$

Using equation (1.0.1) and (1.0.2)

$$E_y = \frac{j_x}{(-e)nc}H, \tag{1.0.5}$$

$$\frac{j_x}{E_y} = \frac{(-e)nc}{H}. \tag{1.0.6}$$

This expression is defined as the Hall conductivity (σ_H): it is the ratio of the current in the x-direction to the effective electric field in the y-direction, as usually defined in the classical electrodynamics.

From equation (1.0.6) $\frac{E_y}{j_x H}$ is the Hall coefficient (R_H).

$$R_H = \frac{E_y}{j_x H} = \frac{1}{(-e)nc}. \tag{1.0.7}$$

The Hall field would be opposite to the direction it has for negatively charged carriers. The Hall coefficient is used to determine the sign of the charge carriers. Hall's original data agreed with the sign of the electronic charge later determined by Thomson. One of the remarkable aspects of the Hall effect, however, is that in some metals the Hall coefficient is positive, suggesting that the carriers have a charge opposite to that of the electron [2].

The effective conductance depends on gate voltage. There are plateaus of the Hall voltage near the same values of gate voltage, and the values of the Hall resistance $\frac{V_H}{I_X}$ at these plateaus are accurately equal to (25,813/integer) Ohms, where 25,813 is the value of $\frac{h}{e^2}$ expressed in Ohms [3]. This has led to advances in metrology. Since the ratio $\frac{e^2}{h}$ happens in several fundamental units, the Hall resistivity and conductivity are given by

$$\rho_H = \frac{h}{e^2}. \tag{1.0.8}$$

$$\sigma_H = \frac{e^2}{h}. \tag{1.0.9}$$

Quantization of the Hall conductance has been established to an accuracy of one part in 10^9 and has been used as a standard for electrical resistance since early 1990. Another practical application was the standardization of the fine-structure constant [4,5]. It is a fundamental physical constant characterizing the strength of the electromagnetic interaction between elementary charged particles and is used to determine the electromagnetic coupling constant (the elementary charge constant) $e = \sqrt{4\pi\varepsilon_0\hbar c\alpha}$ [6].

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = \frac{\mu_0}{4\pi}\frac{e^2 c}{\hbar} = \frac{k_e e^2}{\hbar c} = \frac{c\mu_0}{2R_k} = 1/137, \tag{1.0.10}$$

where e the electron charge, μ_0 permeability of free space (magnetic constant), ε_0 is the permitivity of free space (electric constant), k_e is coloumb constant, R_k is Von klitzing constant, $\hbar = \frac{h}{2\pi}$ where h is planks constant and c is the speed of light ($c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$). α (the fine structure) is the separation of light of particular wavelength produced by atoms or molecules into two or more very similar wavelengths, caused by interaction of particular quantum mechanical properties. It is a dimensionless quantity.

This effect was discovered by German physicist Klaus-Olaf Von Klitzing [7], who won the 1985 Nobel Prize in physics for his discovery of the quantum Hall effect, which helps to explain electrical movements in atoms and the constant

$$R_k = \frac{h}{e^2} = 25,812.807499(89)\ \Omega. \tag{1.0.11}$$

Stormer and Tsui [8] discovered that electrons in very strong magnetic fields can act together to form new types of particle-like units called quasi particles. When electrons form these quasi particles, they appear to have only a fraction of their normal electric charge as Laughlin explained this phenomenon [9].

Stormer and Tsui studied conductors at very low temperatures. The electric characteristics, including the Hall Effect, of materials change at low temperatures. In 1980 Klaus-Olaf Von Klitzing examined the behaviour of conductors at temperatures near absolute zero (-273.15^0_c). Von Klitzing also used very strong magnetic fields. He discovered that, in those conditions, the voltage created by the Hall Effect varies in discrete steps.

Quantum theory states that electrons in a magnetic field move in circular orbits perpendicular to the magnetic field. These orbits are also called energy levels. The number of electrons that each orbit, or energy level, can hold depends on the strength of the magnetic field-the stronger the magnetic field, the more electrons can share a single energy level.

As Von Klitzing gradually increased the magnetic field, the electrons began to collect at one side of the conductor. When the first energy level was full, the voltage leveled off, then dropped suddenly as electrons began filling up another energy level. Each new energy level created a new step in voltage. Each step was made up of a whole (integer) number of electrons, so the voltage difference between steps was always greater than or equal to the charge of an electron.

In 1982 Stormer and Tsui tested the quantum Hall effect but used even stronger magnetic fields at even colder temperatures than those used by Von Klitzing. Stormer and Tsui expected to find steps of charge in increments equal to the charge of an electron, as von Klitzing had discovered. Instead, they found many more steps that were smaller than they expected. These steps were only a fraction of the charge of an electron. Electrons cannot split into smaller particles, so Stormer and Tsui could not explain how the steps could be less than the charge of a whole electron.

Their colleague at Bell Laboratories, Robert Laughlin [9], saw that they had found a fractional charge and set out to explain the result using quantum mechanical equations. A year later, he came up with a new theory, involving groups of electrons acting as a unit. He called these units quasi particles.

Individual electrons can couple to more than one quasi particle, dividing their charge between the quasi particles and giving the appearance that the electron has a fraction of a whole charge. In 1997 two groups of physicists saw direct evidence of quasiparticles, supporting Laughlin's explanation.

The effect that Stormer and Tsui discovered is known as the fractional quantum Hall effect. It does not have any immediate practical applications but may allow physicists to understand how electrons behaved at the beginning of the universe. It could play a role in the development of computer memories by using the fractional charges of electrons while they are parts of quasi particle. The fractional quantum Hall effect (FQHE) is a collective behavior in a two-dimensional system of electrons. As in the integer quantum Hall effect, the Hall resistance undergoes certain quantum Hall transitions to form a series of plateaus. Each particular value of the magnetic field corresponds to a filling factor (the ratio of electrons to magnetic flux quanta) [9]. Filling factor is the number of occupied Landau levels.

The first successful theory of the FQHE was developed by Laughlin, who proposed a variation wave function to describe a correlated, incompressible electron liquid at filling factors ($\nu = \frac{1}{2n+1}$), where n is an integer. It was later shown that this class of wave functions are exact for a certain type of short ranged interactions. The incompressibility of the Laughlin state leads to plateaus of Hall resistivity at filling factors, while the absence of gap less excitations leads to vanishing longitudinal resistivity at zero temperature. The Laughlin state also supports novel quasi particle excitations which carry fractional charge and obey fractional statistics. Haldane [10], Halperene [11] and recently Jain [12] have constructed hierarchical wave functions which explain the FQHE at other filling factors as well.

The Hall Effect is useful in the study of plasmas and magneto-hydro dynamics. The function of a variety of electric meters and measuring instruments, such as ammeteres, watt meters, magnetic compasses, and position sensing devices, as well as power-transforming solid-state devices known as transducers, are based on the Hall effect [13].

Chapter 2

CLASSICAL HALL EFFECT

2.1 Introduction

The effect is the basis of many practical applications of devices such as magnetic field measurements, position and motion detectors. With the measurements he made, Hall was able to determine for the first time the sign of charge carriers in a conductor. Even today, Hall Effect measurements continue to be a useful technique for characterizing the electrical transport properties of metals and semiconductors.

The current density along the drift velocity for weak fields is [4]

$$j_x = -n_{2D} e \frac{E_y}{B}, \tag{2.1.1}$$

where n_{2D} is the density of electron in two dimension and $\frac{E_y}{B}$ is the drift velocity of an electron.

2.2 Hall effect in terms of Newton's second law of motion

To understand more about Hall effect [1,14], we use the equation of motion of the electron in an electric field $\vec{E}$ and magnetic field $\vec{H}$, the force $\vec{F}$ on an electron of charge $-e$ in SI unit is

$$\vec{F} = -e\big(\vec{E} + \vec{v} \times \vec{B}\big). \tag{2.2.1}$$

Through the movement of electrons, there is a wave vector $\vec{k}$.

$$-e\vec{E} = \frac{h}{2\pi}\frac{d\vec{k}}{dt}. \tag{2.2.2}$$

which can be solved as follows

$$\vec{k}(t) - \vec{k}(0) = -\frac{2\pi e\vec{E}t}{h}. \tag{2.2.3}$$

Since the right hand side is independent of k we see that all states, k, in the system evolve in time in precisely the same way. This means that in k-space the Fermi sphere moves rigidly in the direction of the electric field $\vec{E}$. Drude recognized this and introduced the average time, τ between collisions which he thought were collisions between the electrons and the ion cores (the cores of charged particles). In that case the average change in wave vectors, $\delta\vec{k}$, between successive collisions is obtained by setting $t = \tau$, The average drift velocity of the electron is just

$$\vec{v} = \frac{h}{2\pi}\frac{\delta\vec{k}}{m} = -\frac{e\vec{E}\tau}{m}. \tag{2.2.4}$$

If there are n electrons per unit area then the current density is

$$\vec{j} = -ne\vec{v} = \frac{ne^2\tau}{m}\vec{E}. \tag{2.2.5}$$

The proportionality between the current density and the electric field is Ohm's law. That is the electrical conductivity defined by $\vec{j} = \sigma\vec{E}$ is

$$\sigma = \frac{ne^2\tau}{m}. \tag{2.2.6}$$

In the presence of a steady magnetic field, the conductivity and resistivity becomes tensors

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \tag{2.2.7}$$

$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}. \tag{2.2.8}$$

Then the current density becomes

$$\vec{j}_x = \sigma_{xx}\vec{E}_x + \sigma_{xy}\vec{E}_y, \tag{2.2.9}$$

$$\vec{j}_y = -\sigma_{xy}\vec{E}_x + \sigma_{yy}\vec{E}_y. \tag{2.2.10}$$

In the absence of electric field, for electrons moving in a uniform magnetic field and the resistance of perfect, static crystal with at least one partially filled electronic band vanishes. The electrons obey the semiclassical equations of motion [20]

$$\vec{v}_k = \frac{2\pi}{h}\vec{\nabla}_k E(k), \tag{2.2.11}$$

$$\frac{h}{2\pi}\frac{d\vec{k}}{dt} = -e\vec{v}_k \times \vec{B} = -\frac{2\pi e}{h}\vec{\nabla}_k E(k) \times \vec{B}, \tag{2.2.12}$$

where E is the electron's energy.

The electrons moves on a constant energy surface in k-space under the action of an applied magnetic field. Therefore, the equations of motion, when dissipation taken into account, is given by

$$m(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}) = -e(\vec{E} + \vec{v} \times \vec{B}). \tag{2.2.13}$$

The term $m(\frac{\vec{v}}{\tau})$ is a 'frictional force' due to the collisions. With $\vec{B}$ along the z-axis, and in the steady state where $\frac{d\vec{v}}{dt} = 0$, equation (2.2.13) becomes the following set of three equations

$$m\frac{\vec{v}_x}{\tau} = -e\vec{E}_x + e\vec{v}_y\vec{B}, \tag{2.2.14}$$

$$m\frac{\vec{v}_y}{\tau} = -e\vec{E}_y + e\vec{v}_x\vec{B}, \tag{2.2.15}$$

$$m\frac{\vec{v}_z}{\tau} = -e\vec{E}_z. \tag{2.2.16}$$

If no current flow then there must be an electric field component E_y such that $v_y = 0$. There is the Hall field and it is given by equation (2.2.15)

$$E_y = v_x B. \tag{2.2.17}$$

But from equation (2.2.14) and $v_y = 0$, we have,

$$E_x = -\frac{v_x}{(e\tau/m)}. \tag{2.2.18}$$

And therefore $\frac{E_y}{E_x} = -\frac{eB\tau}{m}$, then

$$E_y = -(\frac{eB\tau}{m})E_x. \tag{2.2.19}$$

Where (eB/m) = ω_c is the cyclotron frequency of an electron in SI unit.

One of the great successes of Drude's free electron model was the derivation of Ohm's law for a metal. A current I flows in the 2DEG (2 Dimensional Electron Gas) of width W and a longitudinal voltage V_x is measured between two contacts separated by a distance L. At the same time, there is a transverse voltage V_y. The voltages and currents are related by

$$V_x = R_{xx} I_x + R_{xy} I_y, \tag{2.2.20}$$

$$V_y = -R_{xy} I_x + R_{xx} I_y. \tag{2.2.21}$$

where R_{xx} is the longitudinal resistance and R_{xy} is the Hall resistance. In figure 2.1 broad steps can be observed in the Hall resistance. Simultaneously, the longitudinal resistance vanishes.

In a two-dimensional system, the Hall resistance is equal to the Hall resistivity $\rho_{xy} = R_{xy}$. The longitudinal resistance is related to the longitudinal resistivity by $\rho_{xx} = (W/L)R_{xx}$. However, in the quantum Hall regime, $R_{xx} = \rho_{xx} = 0$. Therefore, the resistances are as fundamental as the resistivities in contrast to the three-dimensional case, where geometrical corrections are required. On a plateau, the Hall sample is a perfect conductor with $\sigma_{xx} = 0$. However, due to the tensorial nature of the resistance in two dimensions, it is a perfect insulator as well, $\rho_{xx} = 0$. This can be seen from the relation between the resistivities and the conductivities [7,15].

The diagonal components of the resistivity tensor

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \cdot \begin{pmatrix} I_x \\ I_y \end{pmatrix} \tag{2.2.22}$$

Which gives

$$V_x = \rho_{xx} I_x + \rho_{xy} I_y, \tag{2.2.23}$$

$$V_y = -\rho_{xy} I_x + \rho_{yy} I_y. \tag{2.2.24}$$

In the Hall geometry $V_y = 0$, that is $I_y = (\frac{\rho_{xy}}{\rho_{yy}}) I_x$

$$V_x = \rho_{xx} I_x + \rho_{xy} (\frac{\rho_{xy}}{\rho_{yy}}) I_x, \tag{2.2.25}$$

$$V_x = \big(\rho_{xx} + \frac{\rho_{xy}^2}{\rho_{yy}}\big) I_x. \tag{2.2.26}$$

Assuming that $\rho_{yx} = -\rho_{xy}$ and $\rho_{xx} = \rho_{yy}$

the relation between the two tensors will be;

$$\sigma_{xx} = \frac{\rho_{xx}}{(\rho_{xx}^2 + \rho_{xy}^2)}, \tag{2.2.27}$$

and

$$\sigma_{xy} = \frac{-\rho_{xy}}{(\rho_{xx}^2 + \rho_{xy}^2)}. \tag{2.2.28}$$

Similarly, the diagonal components of the conductivity tensor

$$\begin{pmatrix} I_x \\ I_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} . \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{2.2.29}$$

Which gives

$$I_x = \sigma_{xx} V_x + \sigma_{xy} V_y, \tag{2.2.30}$$

$$I_y = -\sigma_{xy} V_x + \sigma_{yy} V_y. \tag{2.2.31}$$

Using $I_y = 0$, that is $V_y = \big(\frac{\sigma_{xy}}{\sigma_{yy}}\big) V_x$

$$I_x = \sigma_{xx} V_x + \sigma_{xy} \big(\frac{\sigma_{xy}}{\sigma_{yy}}\big) V_x, \tag{2.2.32}$$

$$I_x = \big(\sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{yy}}\big) V_x. \tag{2.2.33}$$

Assuming that $\sigma_{yx} = -\sigma_{xy}$ and $\sigma_{xx} = \sigma_{yy}$

The relation between the two tensors will be;

$$\rho_{xx} = \frac{\sigma_{xx}}{(\sigma_{xx}^2 + \sigma_{xy}^2)}, \tag{2.2.34}$$

And

$$\rho_{xy} = \frac{-\sigma_{xy}}{(\sigma_{xx}^2 + \sigma_{xy}^2)}. \tag{2.2.35}$$

Introducing the Hall coefficient, $R_H = \frac{E_y}{(j_x B)}$, by using equation (2.2.19) and the x-component of equation (2.2.5), in SI unit we obtain

$$R_H = -\frac{1}{en} \tag{2.2.36}$$

Where n equals number of particles per occupied volume.

Chapter 3

QUANTUM HALL EFFECT (QHE)

3.1 Laughlin's Thought Experiment

The explanation of Quantum Hall Effect is based on the Classical Hall Effect. In Laughlin's thought-experiment the $2D$ electron system is bent to form a cylinder Figure 3.1 [3] whose surface is pierced everywhere by a strong magnetic field B to the surface. The current I (former I_x) circles the loop, the magnetic field B acts on the charge carriers to produce a Hall voltage V_H (former V_y) perpendicular to the current and to the magnetic field; that is, V_H is developed between one edge of the cylinder and the other.

The circulating current I is accompanied by a small magnetic flux φ that threads the current loop. The aim of the thought-experiment is to find the relation between I and V_H . We start with the electromagnetic relation that relates I to the total energy U of a resistance less (superconductivity) system:

$$\frac{\partial U}{\partial t} = V_x I_x = \frac{I}{c}\frac{\partial \varphi}{\partial t}; \quad and \quad I = c\frac{\delta U}{\delta \varphi} \tag{3.1.1}$$

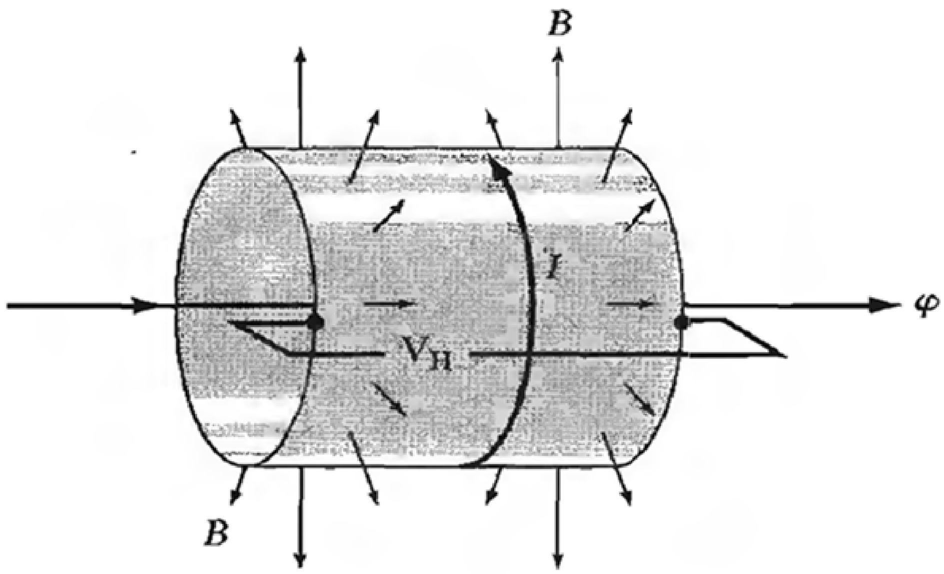

Figure 3.1: Geometry for Laughlin's thought -experiment. The 2D electron system is wrapped around to form a cylinder. A strong magnetic field B pierce the cylinder everywhere normal to its surface. A current I circles the loop, giving rise to the Hall voltage V_H and a small magnetic flux φ through the loop.

In real system $V_x = \frac{1}{c}\frac{\partial\varphi}{\partial t}$.

The value of I can now be found from the variation δU of the electronic energy that accompanies a small variation $\delta\varphi$ of the flux. The carrier states divide into two classes:

1. Localized states, which are not continuous around the 1oop.
2. Extended states, continuous around the loop.

The localized and extended states cannot coexist at the same energy. The two classes of states respond differently to the application of the flux φ. The localized states are unaffected to first order because they do not enclose any significant part of φ . For a localized state a change in φ looks like a gauge transformation, which do not affect the energy. The extended states enclose φ , and their energy may be changed. However, if the magnetic flux is varied by a flux quantum $\delta\varphi = \frac{hc}{e}$ all extended orbits are identical to those before the flux quantum was added. The argument here is identical to that for the flux quantization in the super conducting ring with 2 electron of the cooper pair replaced by e ($\varphi_0 = \frac{hc}{2e} = 2.0678 \times 10^{-7} Gauss\ cm^2$) $\quad in\ CGS\ \ or$ ($\varphi_0 = \frac{h}{2e} = 2.0678 \times 10^{-15} Tm^2$) $\quad in\ SI$ [3].

3.2 QHE in Metallic Oxide Semiconductor Field Effect Transistor (MOSFET)

Von Klitzing's discovery of the quantum Hall Effect in 1980 is a remarkable achievement which led to a new and important area of research in condensed matter physics and his experiment was a pioneering contribution.

The experiment sample used was a MOSFET. It is a semiconductor device that is a metal oxide silicon field effect transistor as shown in Figure 3.1a. For a range of gate voltage, a thin layer of electrons are attracted to the interface between the Si and SiO_2. This is the inversion layer which acts as a very good insulator in order to increase the resistance strongly. Because the inversion layer is so thin ($\sim 100\mathring{A}$) , electrons behave as a 2D gas. The 2D density can be changed by varying gate voltage (V_g).

The transistor used most commonly in the microelectronics industry. It contains two n-type regions, called the source and the drain, with a p-type region in between them, called the channel. Over the channel is a thin layer of nonconductive silicon dioxide topped by another layer, called the gate. For electrons to flow from the source to the drain, a voltage (forward bias) must be applied to the gate. This causes the gate to act like a control switch, turning the MOSFET on and off and creating a logic gate that transmits digital 1s and 0s throughout the microprocessor.

In a quantum Hall effect experiment, the device is placed in a strong magnetic field whose direction is perpendicular to the 2D electron layer. A DC current is applied from the source to the drain and passes through the 2D electron layer. As shown in Figure 3.1b a voltage drop V_{ab} is measured between a and b, from which the Hall resistance is obtained as

$$R_{xy} = \frac{V_{ab}}{I}, \tag{3.2.1}$$

where $V_{ab} = V_H = E_y L_y$ and $I = j_x A = \sigma_H E_y A$.

In general, the Hall resistance R_{xy} is dependent upon the material's temperature T, magnetic field B, electron density, and other physical properties.

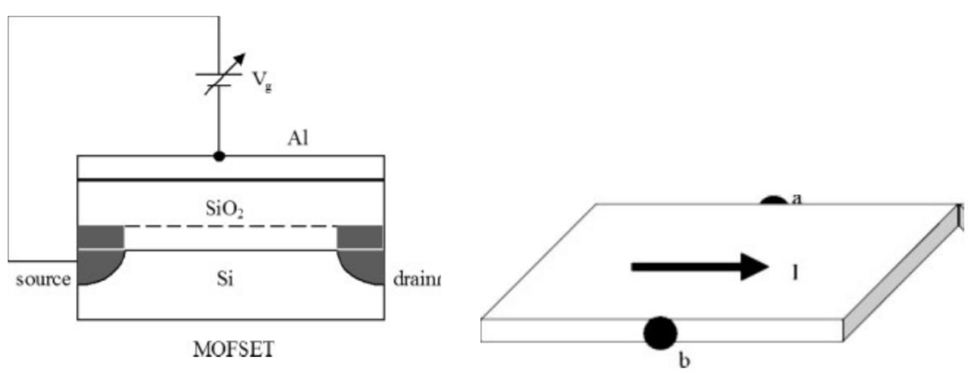

Figure 3.2: (a) MOSFET used in the Quantum Hall Effect experiments (b)Diagram showing the direction of the current flow and the applied voltage in the Quantum Hall Effect experiments.

Von Klitzing's remarkable discovery was performed at very low temperatures and a very high magnetic field. Under such extreme conditions, the Hall resistivity can be expressed as

From equation (3.2.1) $R_{xy} = \frac{V_H}{I} = \rho_H(\frac{L_y}{A})$.

Therefore, $\rho_H = R_{xy}(\frac{A}{L_y})$ is a staircase function of V_g with extremely flat plateaus and L_Y is the sample width in the y-direction and A is the cross sectional area for the current flow in the x-direction. Thus

$$\rho_H = \frac{h}{ne^2}. \tag{3.2.2}$$

where n is an integer that changes from plateau to plateau . Thus far, the accuracy of the plateaus have been one part in 10^9. This phenomenon is the Quantum Hall Effect (QHE)[4].

3.3 An Idealized Model of Quantum Hall Effect

To explain the QHE, we use the following idealized model [4]. Consider a non-interacting 2D electron gas in the presence of an impurity free environment that also has a uniform, perpendicular magnetic field B. Compared to the free case with no external magnetic field, we will find a quite different energy spectrum. This problem was solved by Landau many years ago, but we will review the problem to establish a physical illustration and mathematical language for later discussions. The magnetic field enters the equation through the vector potential $\vec{A}$, where $\vec{B} = \vec{\nabla} \times \vec{A}$. A specific gauge must be chosen and we choose the so-called Landau gauge

$$\vec{A} = \begin{pmatrix} 0 \\ Bx \\ 0 \end{pmatrix}, \tag{3.3.1}$$

where $\hat{x}$ is the position operator. Thus, the Schrödinger equation is

$$\frac{1}{2m^*}\big[\big(-i\hbar\frac{\partial}{\partial x}\big)^2 + \big(-i\hbar\frac{\partial}{\partial y} + eBx\big)^2\big]\psi = E\psi. \tag{3.3.2}$$

where m^* is the effective mass of electrons.

Because of the translational invariance is in the y-direction, we write $\psi = e^{iky}\varphi(x)$. Thus,

$$\frac{1}{2m^*}\big[\big(-i\hbar\frac{\partial}{\partial x}\big)^2 + (\hbar k + eBx)^2\big]\varphi(x) = E\varphi(x). \tag{3.3.3}$$

To find the energies, note that translating the harmonic oscillator potential does not affect the energies. The energies of this system are thus identical to those of the standard quantum harmonic oscillator.

The Hamiltonian of the particle(an electron) is:

$$\hat{H} = \frac{\hat{p}^2}{2m^*} + \frac{1}{2}m^*\omega^2\hat{x}^2, \tag{3.3.4}$$

$$\hat{H} = \frac{1}{2m^*}[\hat{p}^2 + (eB\hat{x})^2]. \tag{3.3.5}$$

given by$\hat{p}$ is the momentum operator, $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, $\hat{x}$ is the position operator and $\omega = eB/m^*$.

The first term in the Hamiltonian represents the possible kinetic energy states of the particle, and the second term represents its respectively corresponding possible potential energy states. We may write the time independent shrodinger equation

$$\hat{H}|\psi\rangle = E|\psi\rangle, \tag{3.3.6}$$

where E denotes the energy level and $|\psi\rangle$ denotes that level's eigen state. By using *ladderoperator* method we can extract the energy eigen values. Initially we define the operators a and its adjoint $a^\dagger$ Thus

$$a = \sqrt{\frac{m^*\omega}{2\hbar}}\big(\hat{x} + \frac{i}{m^*\omega}\hat{p}\big) \tag{3.3.7}$$

And

$$a^\dagger = \sqrt{\frac{m^*\omega}{2\hbar}}\big(\hat{x} - \frac{i}{m^*\omega}\hat{p}\big). \tag{3.3.8}$$

This leads to the use full representation of $\hat{x}$ and $\hat{p}$

$$\hat{x} = \sqrt{\frac{\hbar}{2m^*\omega}}(a + a^\dagger), \tag{3.3.9}$$

$$\hat{p} = i\sqrt{\frac{m^*\omega\hbar}{2}}(a^\dagger - a). \tag{3.3.10}$$

The operator a is not Hermitian, since itself and its adjoint $a^\dagger$ are not equal. Yet the energy eigen states $|n\rangle$, when operated on by these ladder operators give

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \tag{3.3.11}$$

$$a|n\rangle = \sqrt{n}|n-1\rangle. \tag{3.3.12}$$

It is then evident that $a^\dagger$, add single atom of energy to the oscillator, while $\hat{a}$ removes a quantum. From the relation above, we can also define a number operator N, which has the following property:

$$N = a^\dagger a, \tag{3.3.13}$$

$$N|n\rangle = n|n\rangle. \tag{3.3.14}$$

The following commutators can be easily obtained by substituting the canonical commutation relation,

$$[a, a^\dagger] = 1, \tag{3.3.15}$$

$$[N, a^\dagger] = a^\dagger, \tag{3.3.16}$$

$$[N, a] = -a. \tag{3.3.17}$$

By substituting equation (3.3.9) and equation (3.3.10) into equation (3.3.4) and using the relation ships from equation (3.3.13) and equation (3.3.15), the Hamiltonian can be expressed as

$$\hat{H} = (n + \frac{1}{2})\hbar\omega_c. \tag{3.3.18}$$

Therefore, the energy eigen value will be:

$$E = E_n = (n + \frac{1}{2})\hbar\omega_c, \quad n \ \geq \ 0 \tag{3.3.19}$$

where $\omega_c = \frac{eB}{m^*}$ is the angular frequency which is known as the 'cyclotron frequency'. In the presence of a perpendicular magnetic field $H = H_z$ the electron orbits are quantized in units of the cyclotron frequency

$$\omega_c = \frac{eH}{m^* c}, \tag{3.3.20}$$

The corresponding energy levels are equally spaced, separated by intervals $\hbar\omega_c$, these are called Landau levels. This is intuitively understood as the quantization of circular motion due to a harmonic oscillator potential, and from equation (3.3.3) $\varphi(x)$ is the flux quanta. The energy spectrum consists of a series of equally spaced along the x-direction is at $\frac{\hbar k}{eB}$ a restriction provided by the width L_x of the sample in the x-direction must be provided. Thus, the number of k allowed values are Landau levels. Each level is highly degenerate and has many states with the same energy.

The number of states per level is proportional to the area of the 2D gas and is demonstrated in the following arguments. If there are periodic boundary conditions in the y-direction, with $\psi(y + L_y) = \psi(y)$, k is then quantized as $k = \frac{2\pi j}{L_y}$ where j is an integer. Thus,

$$\frac{eBx}{\hbar k} = \Big(\frac{eBL_x}{\hbar}\Big)\Big(\frac{2\pi}{L_y}\Big)^{-1} = (L_x L_y)\frac{eB}{h} = n. \tag{3.3.21}$$

where $\hbar = \frac{h}{2\pi}$ and n is the number of states. This leads to the conclusion that each Landau level has a total number of states $\frac{eB}{h}$ per unit area and also applies in the thermodynamic limit, that is, $L_x, L_y \to \infty$.

If an additional electric field E_y is applied in the y-direction, the center of each wave function moves with a velocity of $\frac{E_y}{B}$ in the x- direction. When the lowest Landau level is full and the others are empty, the current density for this area is

$$j_x = env = e\frac{eB}{h}\frac{E_y}{B} = \frac{e^2}{h}E_y. \tag{3.3.22}$$

For which the Hall conductance is

$$\sigma_H = \frac{j_x}{E_y} = \frac{e^2}{h}. \tag{3.3.23}$$

When the lowest n Landau levels are full and all others are empty, the current density increases by a factor of n and

$$\sigma_H = \frac{ne^2}{h}. \tag{3.3.24}$$

The Hall resistivity is

$$\rho_H = \frac{1}{\sigma_H} = \frac{h}{ne^2}. \tag{3.3.25}$$

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3.4 Effects of Finite Temperature on QHE

In the above idealized model, the Hall conductance at finite temperature is

$$\sigma_H = \frac{e^2}{h}\sum_{n=0}^{\infty}\frac{1}{e^{(E_n-\mu)/K_BT}+1}. \tag{3.4.1}$$

Here μ is the chemical potential7

At T = 0, the Hall conductance is an initial function of the magnetic field and chemical potential, and, results shows that [4]

$$\frac{\partial\sigma_H}{\partial\mu} = \frac{e^2}{h}\sum_{n=0}^{\infty}\delta(E_n-\mu), \tag{3.4.2}$$

At low temperature ($k_BT \ll \mu$),

$$\frac{\partial\sigma_H}{\partial\mu} = \frac{e^2}{h}\frac{1}{k_BT}\sum_{n=0}^{\infty}e^{-(E_n-\mu)/K_BT}, \tag{3.4.3}$$

At the center of a plateau,

$$\mu = (\frac{E_n + E_{n-1}}{2}), \quad (3.4.4)$$

Then

$$E_n - \mu = E_n - \frac{E_n + E_{n-1}}{2} = \frac{E_n - E_{n-1}}{2} = \frac{\hbar\omega_c}{2}, \quad (3.4.5)$$

Equation (3.4.3) will be

$$\frac{\partial\sigma_H}{\partial\mu} \approx \frac{e^2}{h}\frac{1}{k_B T}(e^{-\hbar\omega_c/2k_B T}). \quad (3.4.6)$$

For quantization, we have

$$\frac{\partial\sigma_H}{\partial\mu} = \tan\theta = \frac{e^2}{h}\frac{f}{\hbar\omega_c} \quad with \quad f \ll 1. \quad (3.4.7)$$

That is, θ is the slope angle of the plateau and f is the distribution function $f = \frac{1}{e^{(E_n-\mu)/K_B T}+1}$. Comparing equations (3.4.6) and (3.4.7), we obtain

$$\frac{\hbar\omega_c}{k_B T} = -2\ln\left(\frac{f}{\hbar\omega_c/k_B T}\right). \quad (3.4.8)$$

It has been found within experimental accuracy that $f = 10^{-7}$, for which we need $\frac{\hbar\omega_c}{k_B T} > 40$.

Since $\omega_c = \frac{eB}{m^*}$, in the experiment, the electron effective mass is $m^* = 0.19m_e$.

Where $\hbar = \frac{h}{2\pi} = 1.05459(10^{-34})JS$, and $k_B(Boltzman's\,constant) = 1.38062(10^{-23})JK^{-1}$.

For a magnetic field $B = 10T$, it requires $T < 2K$, as found experimentally.

And Since $\omega_c \propto \frac{1}{m^*}$, the quantizing condition for the Hall conductance is more easily achieved if the electron effective mass is reduced. This observation led Daniel Tsui, et al.[8] to use GaAs/AIGaAs hetero junctions to study the QHE, where $m^* \approx 0.068m_e$.

Chapter 4

TYPES AND APPLICATIONS OF QUANTUM HALL EFFECT

4.1 Integer Quantum Hall Effect (IQHE)

The integer quantum Hall Effect observed in two dimensional electron systems subjected to low temperatures and strong magnetic fields 10T , in which the Hall conductance under goes certain quantum Hall transitions to take on the quantized values $\sigma = \frac{I_{channel}}{V_{Hall}} = \nu \frac{e^2}{h}$.

The pre factor ν is known as the "filling factor" and can take on integer ($\nu = 1, 2, 3, ...$), values without considering the geometry of the sample .

Several workers observed the effect in experiments carried out on the inversion layer of MOSFETs [16]. The integer Quantum Hall Effect can be simply explained in terms of single-particle orbitals of an electron in a magnetic field, the Landau quantization explain this because it is the quantization of the cyclotron orbits of charged particles in magnetic fields. As a result, the charged particles can only occupy orbits with discrete energy values, called Landau levels. The Landau levels are degenerate with the number of electrons per level directly proportional to the strength of the applied magnetic field. Landau quantization is directly responsible for oscillations in electronic properties of materials as a function of the applied magnetic field.

The link between exact quantization and gauge invariance was subsequently elucidated by Robert Laughlin. Currently, most integer quantum Hall experiments are performed on gallium arsenide hetero structures, though many other semiconductor materials can be used. The integer quantum Hall effect has also been found in graphene at room temperature, which is considered high.

4.2 Fractional Quantum Hall Effect (FQHE)

The fractional quantum Hall effect (FQHE) occurs when electrons are confined to two dimensions, cooled to near absolute zero temperature, and exposed to a strong magnetic field. Robert Laughlin, Daniel Tsui and Horst Stormer collaborated on the discovery that electrons can act together to form particle-like units called quasi particles [17]. When electrons form these quasi particles, they appear to have only a fraction of their normal electric charge. Laughlin provided the theoretical analysis to explain Stormer and T sui's experimental discovery of this phenomenon is called the FQHE. And it is a property of a collective state in which electrons bind magnetic flux lines to make new quasi particles, and excitations have a fractional elementary charge and possibly also fractional statistics.

The discovery of the IQHE by von Klitzing [7] in 1980 for a MOSFET, repeated the experiment on GaAs/AlGaAs hetero-structures by Tsui et al. The consequence of the small effective mass of the electrons in the 2DEG was that the QHE observed at higher temperatures and lower magnetic fields. To their surprise, they found, as reported in their 1982 paper, that the Hall conductance exhibited a plateau at $1/3(e^2/h)$ to an accuracy of one part in 10^5 .

Further experiments revealed quantization of the conductance at other simple fractions such as (2/3,1/5,2/5,...). These results help physicists to believe that the 2DEG in a strong perpendicular magnetic field must behave extraordinarily at densities corresponding to fractions of the Landau level capacity.

4.3 The Quantum Hall Resistance

In Figure 4.1, [18], Hall resistance curves versus magnetic flux density are shown as they are measured on two dimensional electron systems of different quality. At low magnetic field, the Hall resistance linearly increases with the magnetic flux density B_z.

$$|R_{xy}| = |V_H/I_x| = |B_z/(-en_s)|, \tag{4.3.1}$$

and allows one to determine the respective sheet charge carrier density n_s of the two-dimensional electron system. At higher magnetic field, the Hall Effect on the two-dimensional charge system shows plateau values which are independent of the magnetic field and described by

$$|R_{xy}| = \frac{h}{\nu e^2} \tag{4.3.2}$$

where $\nu = 1, 2, 3, ...$ or a certain fractional number f which seems to follow $f = p/q$ with $p = 1, 2, 3, ...$ and $q = 3, 5, 7, ...$, with exceptions like $f = 5/2$. The same plateau values are found by keeping the magnetic flux densityB_z constant and varying the sheet electron concentration n_s. Therefore, varying the ratio $\frac{n_s}{B_z}$ in certain ranges allows one to observe Hall resistance plateaus described by Eqn(4.3.2).

Such Hall resistance values are expected due to the comparison between Eqns (4.3.1)

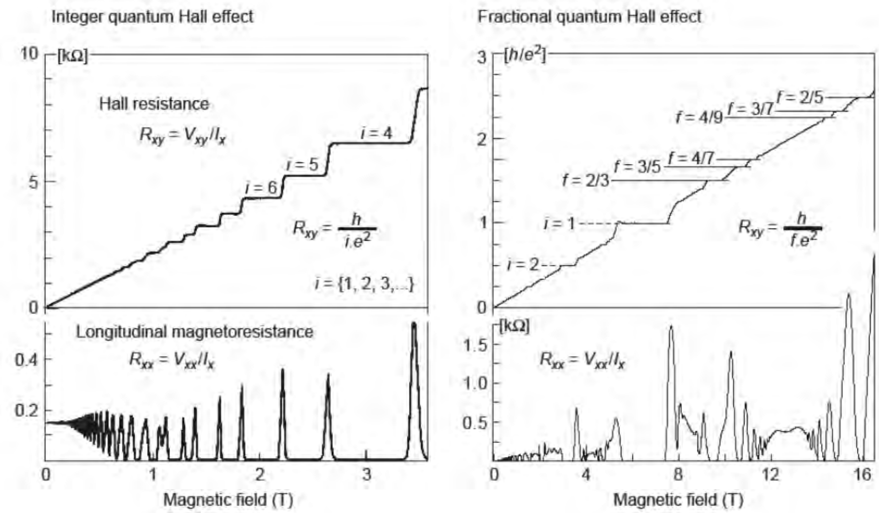

Figure 4.1: Quantum Hall effect: Hall resistance R_{xy} and longitudinal resistance R_{xx} measured on a 2DES as a function of the applied magnetic field . In certain magnetic field regions, plateaus are observed in R_{xy} and at the same time R_{xx} vanishes.

and (4.3.2) only for certain well-defined ratios are important. The meaning of the ratio $\frac{n_s}{B_z} = \frac{\nu e}{h}$ can be expressed slightly differently: taking a certain area out of a homogeneous two-dimensional electron system enclosing the integer number $N = n_s A$ of electrons, the magnetic flux penetrating this area is given by $\phi = AB_z$. The ratio of electron number N and magnetic flux ϕ in units of the magnetic flux quantum $\phi_0 = \frac{h}{e}$ is then $\nu = \frac{N}{(\phi/\phi_0)}$ that is, for each electron, ν^{-1} magnetic flux quanta are present. Whenever a plateau is found in the Hall resistance, the longitudinal resistance $R_{xx} = V_x/I_x$ vanishes that is, $R_{xx} = 0$.

The Hall plateau values with $\nu = 2$ and $\nu = 4$ are reproducible to the standard uncertainty of 10^{-9}, independent of the sheet charge carrier density, the sample geometry, and further properties of the material in which the two-dimensional charge carrier system is embedded.

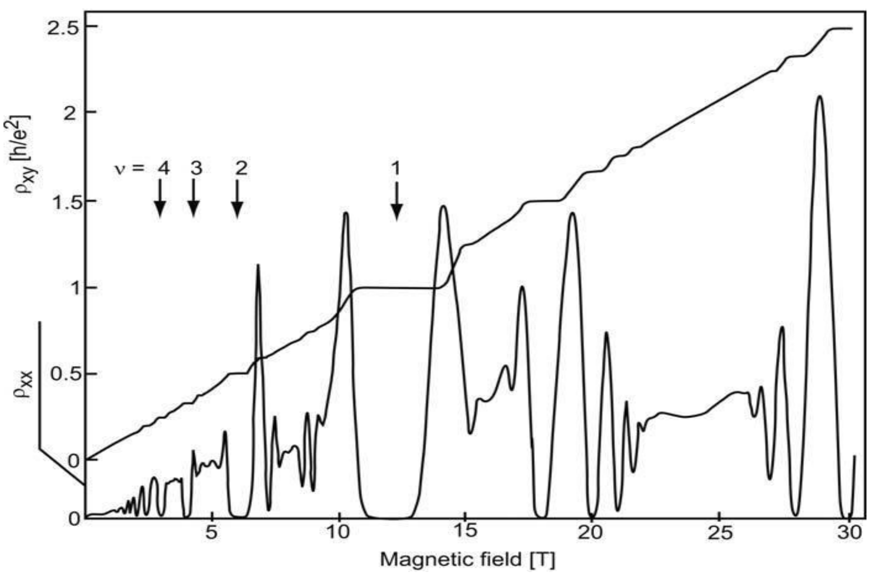

Figure 4.2: The quantum Hall effect and Shubnikov-de Haas oscillations in a Ga[Al]As HEMT, measured in a dilution refrigerator at a temperature of 100mK. A filling factor of ν=1 is reached at $B \approx 12T$.

That is why the quantum Hall resistance has been used since 1990 as a resistance standard. Si-MOSFET or GaAs-AlGaAs-HEMT (High-Electrons-Mobility-Transistor) devices are used for this purpose. To trust in the plateau values obtained on a certain sample for metrological application, the flatness of the Hall plateau, the vanishing of the longitudinal resistance $R_{xx} = 0$, and the invariance of the Hall resistance value with changing the temperature are checked.

Increasing the quality of the samples, the Hall plateaus get smaller. In consequence, a certain amount of disorder is required to obtain well-defined Hall resistance plateaus and, therefore, accuracy in the quantized Hall resistance value.

For metrological application, the sheet density n_s of the two-dimensional electron system in a GaAs/AlxGa1-x As hetero structure is typically in the range of (3 −

6) $\times 10^{15} m^{-2}$ with a mobility m of $(40-80)T^{-1}$, and the measurements are done at the temperature $T = 1.5k$. For high-precision measurements, it is desirable to have the current level as high as possible. However, the current level is limited for the respective sample, since the QHE breaks down suddenly with, current density I_x/W (where W is the sample width) is $\sim 0.5 - 1.5Am^{-1}$. Also, a significant increase of the working temperature degrades the Hall plateau, and the plateaus finally disappear.

In the theoretical Models of the Quantum Hall Effect, a complete microscopic theory, allowing one to derive the Hall plateau values and widths, still does not exist. However, it is commonly accepted that the IQHE can be described within a model of independent electrons whereas for the FQHE, many-particle correlations due to the electron-electron interaction are of important [19].

Furthermore, another quantization of ρ_H has been discovered. In extremely high-quality samples, additional resistance plateaus are observed at $\rho_H = \frac{h}{ke^2}$, with k being a rational number.

Shubnikov-de Haas oscillation is often used to map the Fermi surface of metals. For strong magnetic fields, each Landau level is highly degenerate

Consider the data of figure (4.2), around ν=1/2 , i.e. $B \approx 24T$. The data resemble the IQHE around B = 0. This observation can be substantiated by theory One picture of the FQHE is that so-called composite fermions, which are quasi particles composed of one electron and two magnetic flux quanta, form in strong magnetic fields. These quasi particles (with an effective mass that differs from m*) then undergo Landau quantization in an effective magnetic field which remains after the flux quanta used to form the composite fermions are subtracted [19].

Interestingly, the accurate determination of $\frac{e^2}{h}$ is highly relevant for quantum electrodynamics, since this ratio is contained in the fine structure constant, which describes the coupling of elementary particles to electromagnetic fields.

4.4 Applications of Quantum Hall Effect

Quantum Hall Effect can be used to realize very reproducible resistance values which depend only on natural constants. To be used as a practical standard, the value of the QHR(Quantum Hall Resistance) has to be known in SI units. In the SI, the electrical units are defined in terms of the mechanical basic units metre, kilogram and second (MKS) through the definition of the ampere and the assumption that electrical power and mechanical power are equivalent.

At present, the ohm and the watt are the two chosen units, since they are the most accurately determined. The discovery of the QHE has opened another progression for the realization of the ohm [20].

Most important, however, test measurements can reveal whether the device is in a proper state or not. This means that the value of the QHR can be made as reproducible as today's measurement techniques allow without making reference to an external standard. These are the criteria a standard has to full fill to be accepted as primary standard [15].

Through the realization of the SI unit of voltage, the units of resistance and current are realized in terms of the non-electrical SI units which is needed to make the other electrical units measurable in the SI. With the QHE and the Josephson effect, two fundamentally stable standards are available and thus it was realized the world-wide consistency of electrical measurements could be improved by defining conventional values for klitzing resistance ($R_K = h/e^2$) and for the Josephson frequency to voltage coefficient ($K_J = 2e/h$). All the values for R_K and K_J available by June 1988 in units of the SI were analysed and the following conventional values were proposed.

$$R_{K-90} = 25812.807\Omega \qquad (4.4.1)$$

,

$$K_{J-90} = 453597.9GH_Z/V \qquad (4.4.2)$$

. Relative uncertainties with respect to the international system of 2×10^{-7} and 4×10^{-7} were assigned to the two values respectively. The conventional values were accepted by all member states of the Metre Convention and came into effect as of January 1, 1990 [21].

The von klitzing constant is related with the fine structure which is used to determine R_K and test possible corrections to the Quantum Hall resistance.

Chapter 5

CONCLUSION

In this project work, we have studied the Classical and Quantum Hall Effect and the application of Quantum Hall Effect. The Hall effect is the production of a potential difference (the Hall voltage) across an electrical conductor, transverse to an electric current and a magnetic field .

The quantum Hall Effect is a quantum mechanical version of the Hall effect, observed in two-dimensional electron systems subject to low temperatures and magnetic fields, in which the Hall conductance undergoes certain quantum Hall transitions to take on the quantized values. It is referred to as the integer or fractional Quantum Hall Effect depending on whether the filling factor is an integer or fraction, respectively. The integer Quantum Hall Effect is very well understood, and can be simply explained in terms of single-particle orbitals of an electron in a magnetic field, the Landau quantization explain this because it is the quantization of the cyclotron orbits of charged particles in magnetic fields . As a result, the charged particles can only occupy orbits with discrete energy values, called Landau levels. The Landau levels are degenerate with the number of electrons per level directly proportional to the strength of the applied magnetic field. Landau quantization is directly responsible for oscillations in

electronic properties of materials as a function of the applied magnetic field. It is named after Soviet physicist Lev Landau. The fractional quantum Hall effect is more complicated, as its existence relies fundamentally on electron- electron interactions. Although the microscopic origins of the fractional quantum Hall effect are unknown, there are several phenomenological approaches that provide accurate approximations. For example the effect can be thought of as an integer quantum Hall effect, not of electrons but of charge -flux composites known as composite fermions. The quantization of the Hall conductance has the important property of incredibly precise. Actual measurements of the Hall conductance have been found to be integer or fractional multiples of e^2/h to nearly one part in billion. It has allowed for the definition of a new practical standard for electrical resistance, based on the resistance quantum given by the Von Klitzing constant $R_K = h/e^2 = 25812.807557(18)\Omega$.
This is named after Klaus von Klitzing, the discoverer of exact quantization. Since 1990, a fixed conventional value R_{K-90} is used in resistance calibrations worldwide. The quantum Hall effect also provides an extremely precise independent determination of the fine structure constant, a quantity of fundamental importance in quantum electrodynamics. The observed quantization of the resistance is primarily used for the reproduction of the SI unit ohm, but is also important for high precision measurements of both the fine structure constant and the plank constant. Some current QHE research areas include the analysis of new electron-electron correlation phenomena and the development of a more complete microscopic picture of this quantum Hall effect.

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DECLARATION

I hereby declare that this Project is my original work and has not been presented for a degree in any other universities, and that all sources of material used for the Project have been duly acknowledged.

Name: Lakech Alem

Signature: _____________

This Thesis has been submitted to for examination with my approval as University advisor.

Name: Dr. Teshome Senbata

Signature: _______________

Place and date of submission:

Department of Physics
Addis Ababa University
February 2016