STUDY OF FLUCTUATION EFFECT ON ELECTRICAL CONDUCTIVITY AND MAGNETIC SUSCEPTIBILITY OF SPIN GLASS SUPERCONDUCTORS

By

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TO WORDS OF GOD

For I know the plan that I have for you

Table of Contents

List of Tables

List of Figures

Abstract

The fluctuation effect on electrical conductivity and magnetic susceptibility of spin-glass superconductor are studied using the macroscopic generalized Ginzburg-Landau theory. A free energy functional that consists of spin glass (g) and superconducting (ψ) order parameters and their coupling is used to show the influence of spin glass magnetic ordering on superconducting cooper pairs and vice versa. It is found that during coupling of spin glass and superconductor, cooper pairs begin to break by freezing of spins in spin glass and the spins from the pair participate in the process of magnetization so that to increase the magnetic susceptibility of spin glass superconductor until spin glass and superconductor coexist. In the coexistence region the susceptibility is less than pure spin glass system. The electrical conductivity of spin glass superconductor is also affected by the rise of spin glass transition, which leads to the decrease of the superconducting order parameter and the cooper pair density. Consequently the electrical conductivity is decreased.

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Introduction

A major success of low temperature physics was achieved with the introduction by Landau of the notion of quasiparticle. The property of many body interacting systems at low temperatures are determined by the spectrum of some low energy, long living excitations (quasiparticles). Another important many body theory is the mean field approximation (MFA), which permitted achieving considerable progress in the theory of phase transition. Phenomena which can't be described by the quasiparticle method or by MFA are usually called fluctuation. The appearance of superconductivity fluctuation above critical temperature leads to precursor effects of the superconducting phase occurring while the system is still in the normal phase, sometimes far from T_c. The electrical conductivity, the diamagnetic susceptibility and specific heat etc. may increase considerably in the vicinity of the transition temperature. The fluctuation contribution to the heat capacity of a superconductor in the vicinity of T_c even above T_c was done by Ginzburg [1]. In this way the fluctuation change the temperature where, in accordance with the phenomenological -Landau theory of second order phase transition, a jump and discontinuity should take place.

In BCS theory only the Cooper pairs forming a Bose-condensate are considered. Fluctuation theory deals with the Cooper pairs out of the condensate. In some phenomena these fluctuating Cooper pairs behave similarly to the quasi particles but with one important difference. While for the well defined quasi particle the energy has to be much larger than its inverse lifetime, for the fluctuation Cooper pairs the "binding energy" turns out to be of the same order. The Cooper pair life time is determined by its decay into two free electrons. Evidently at the transition temperature the Cooper pair starts to condense and its lifetime goes to infinity.

Above T_c fluctuation causes some vestiges of superconductivity to remain. This was first observed by Glover in 1967 who found that the conductivity of amorphous films of superconductors diverges at $(T - T_c)^{-1}$ as one approaches T_c form above. This Curie-weiss form of temperature dependence with appropriate coefficient was also predicted theoretically at about the same time. Some what later the corresponding effect was also observed in diamagnetic susceptibility of pure bulk samples in this case the basic divergence $(T - T_c)^{-1/2}$.

Thus, there is not only some resistance below T_c but also some superconductivity above T_c. Although the apparently abrupt switch over observed by Kamerling Onnes, the superconducting transition is basically much sharper than other second order phase transitions, because the coherent length ξ is so much larger than the interatomic distances. That is why it is an important difference of fluctuation Cooper pairs from quasi particles lies in their larger size $\xi(T)$. The size of coherent length is determined by the distance on which the electrons forming fluctuation Cooper pair move away during the pair life time.

Using the observation of the coexistence of superconductivity and spin glass was made by Davidov [2]. We will include spin glass order parameter contribution to the effect of fluctuation of spin glass superconductors. These effects can change the superconducting behavior as well known that the effective electron-electron interaction due to the exchange of spin waves is repulsive. Thus, it can lower the superconducting transition temperature.

In Ginzburg theory, T_c is defined as the temperature at which the leading term in the free energy expression changes its sign. Above T_c the free energy takes minimum value, when the superconducting order parameter vanishes. However, thermal fluctuations raising the free energy by an amount KT are common, since the probability falls only as $exp(-F/KT)$. This leads to the existence of fluctuation induced superconducting effect above T_c. These fluctuations are largest in amplitude if confined to small volumes, since the total energy increase must be only KT.

Chapter 1

Literature review

1.1 Overview of superconducting behavior

Superconductivity is mostly characterized by vanishing electrical resistance below a certain temperature T_c called the critical temperature. Below T_c there is no measurable DC resistance in a superconductor and if a current is set up in it, it will flow without dissipation practically for ever. Experiments trying to detect changes in a magnetic field associated with current in a superconductor give estimates that it is constant for $10^6 - 10^9$ years. Thus superconducting state is not a state of merely very low resistance, but one is a truly zero resistance. This is different from the case of very good conductors. Infact, materials which are very good conductors in their normal state typically do not exhibit superconductivity. The reason is that in very good conductor there is little coupling between phonons and electrons, since it is scattering by phonons which gives rise to resistance in a conductor, where as electron-phonon coupling is crucial for superconductivity specially of low T_c superconductors. The drop in DC resistance from its normal value above critical temperature (T_c) to zero takes place over a range of temperature of order $10^{-2} - 10^{-3}T_c$, which is indicated in the figure (1.1), i.e the transition is rather sharp.

For typical superconductor (T_c) is in the range of a few degrees Kelvin, which has made it difficult to take advantage of this extraordinary behavior in practical application, because cooling the specimen to with in a few degree of absolute zero is quite difficult so it requires the use of liquid He, the only non solid substance at such low temperatures, as coolant, which is expensive and cumbersome [3].

In 1986 a new class of superconducting material was discovered by Bendnorz and Muller, dubbed high temperature superconductor, in which the T_c is much higher than in typical superconductors. In general it is in the range of $90K$, but in certain compounds it can exceed $130K$. This is well above the freezing point of N_2 ($77K$), so that this much more abundant and cheap substance can be used as the coolant to bring the superconducting materials below their critical point. The discovery of high temperature superconductors has opened the possibility of many practical applications, but these materials are ceramics and therefore more difficult to utilize than the classical superconductors, which are typically elemental metals or simple metallic alloys. It also re-invigorated theoretical interest in superconductivity, since it seemed doubtful that the microscopic mechanism responsible for low - temperature superconductivity could also explain its occurrence at such high temperatures. J.G.Bednorz and K.A.Muller were awarded the 1987 Nobel prize for physics for their discovery, which has sparked an extraordinary activity both in experiment and in theory, to understand the physics of the high temperature superconductors.

A number of other properties are also characteristics of typical superconductors. Among these characteristics; the AC resistance in the superconducting state is also zero below a certain frequency(ω_g) [4], which is related to the critical temperature by $\hbar\omega_g \approx 3.5k_BT_c$. This is indicative of a gap in the excitation spectrum of the superconductor that can be over come only above a sufficiently high frequency to produce decrease in the superconductivity and consequently an increase in the resistance, which then approaches its normal state value as shown in the figure (1.2)

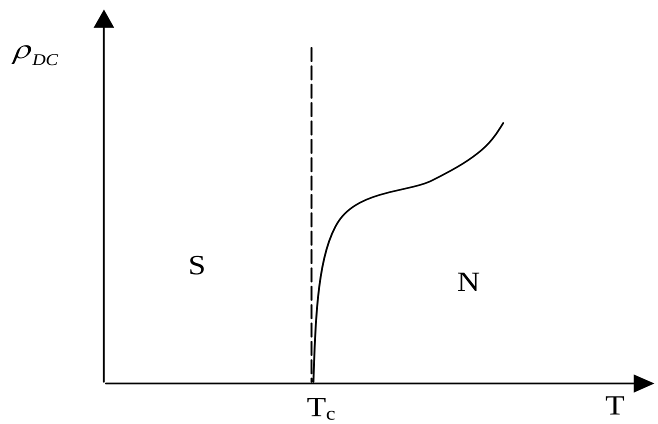

Figure 1.1: *DC Resistivity versus temperature of superconductive transition.*

Superconductors can expel completely an external magnetic field (**H**) imposed on them, which is known as the Meissner effect. The total magnetic field (**B**) inside the superconductor is given by

$$\mathbf{B} = \mathbf{H} + 4\pi m \tag{1.1.1}$$

Where m is the magnetization. B=0 inside the superconductors, that is $m = -\frac{1}{4\pi}\mathbf{H}$. In other words, the superconductor is a perfect diamagnetic a behavior that can be easily interpreted as perfect shielding of the external field by the dissipationless currents that can be set up inside the superconductor. This is true up to a certain critical value of the external field, above which the field is too strong for the

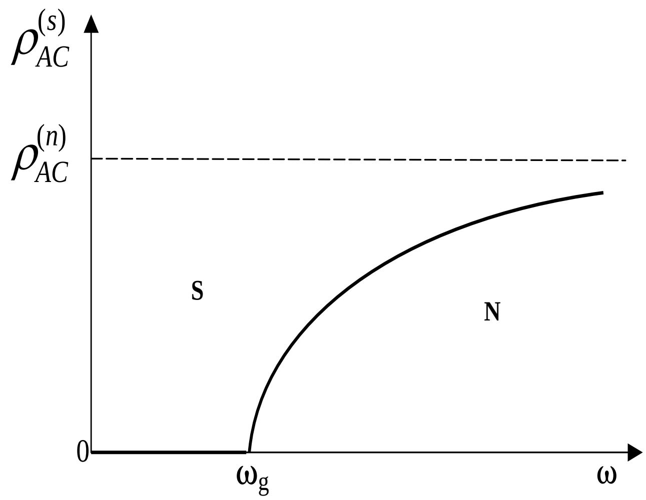

Figure 1.2: *AC resistivity in superconducting state versus frequency.*

superconductor to resist the field abruptly penetrates into the superconductor and the magnetic moment becomes zero, as shown in figure (1.3). Materials that exhibit this behavior are called type I superconductors.

The expulsion of magnetic field costs energy because it bends the magnetic field lines around the superconductors depending on the shape of the specimen, even when the magnetic field is smaller than H_c it may still penetrate some regions which revert to the normal state, while the rest remains in the superconducting state. This is called intermediate state. In type I superconductors, the normal or superconducting regions in the intermediate state are of microscopic dimensions. The behavior of magnetic field B(x) and order parameter $\psi(x)$, which will be discussed in the next chapter in details can determine the penetration length λ and coherence length ξ. The magnetic field B(x) and the magnitude of order parameter $|\psi(x)|$ as a function

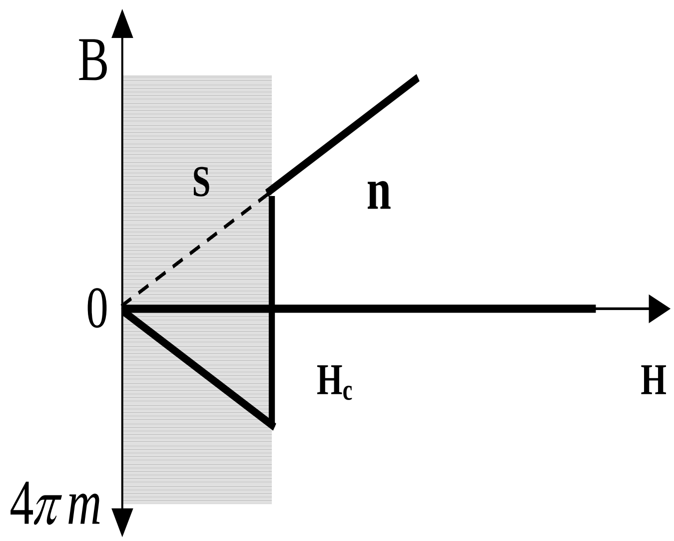

Figure 1.3: *Type I magnetic field versus critical field.*

of the distance x from the normal-superconducting interface, located at $x = 0$ this is shown in figure (1.4).

There exist a different class of superconductors, in which the Meissner effect is also observed up to a critical field H_{c1}, but then a gradual penetration of the field into the superconductor begins that is completed at a higher critical field H_{c2}, beyond which a magnetic moment is again zero as illustrated in figure (1.5). These material are called type II superconductors. The phase in which the external field partially penetrates in the superconductor is called the vortex state and has very interesting physics in it such as vortices can move in response to external force or thermal fluctuation [5].

In type I superconductors the critical field H_c is a function of temperature and vanishes at T_c. The behavior of H_c with temperature is described by the

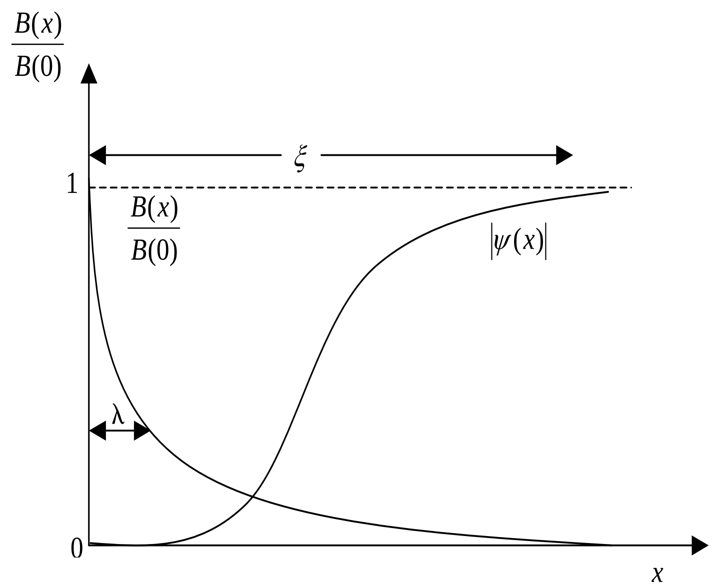

Figure 1.4: *Magnetic field B(x) Versus the magnitude of order parameter* $|\psi(x)|^2$ *as a function of distance* (x)

expression

$$H_c(T) = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right] \tag{1.1.2}$$

Where H_o is a constant field at T=0. An interesting consequences of this behavior is that the slope of the critical field with temperature, $\frac{dH_c}{dT}$, vanishes at $T = 0$ but not at $T = T_c$, where it takes the value $\frac{-2H_0}{T_c}$; this is related to the nature of the phase transition between the superconducting and normal state.

The theoretical explanation of superconductivity remained a big puzzle for almost a half century after its discovery. Although extremely useful phenomenological theories were proposed to account for the various aspects of superconductivity due to London , Pipard, Ginzburg and Landau, Abricosov and others. A theory based on microscopic considerations was lacking until 1957, when it was developed by J.Bardeen,

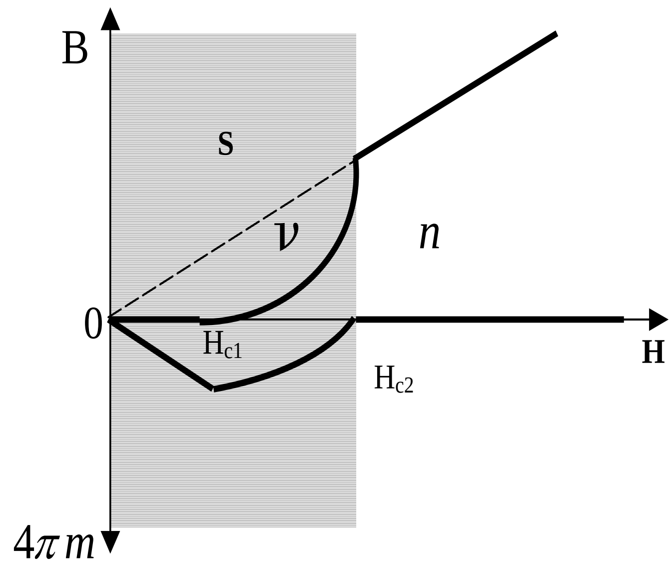

Figure 1.5: *Total magnetic field B and the magnetic moment $4\pi m$ as a function of the external field (H) for type II superconductors*

L.N.Cooper and J.R.Schrieffer, who were awarded the 1972 Nobel prize for physics for their contribution;this is referred to as BCS theory [6].

A common feature of the phenomenological theories is the presence of two important length scales, the so called "penetration length" denoted by λ and the coherence length denoted by ξ which is indicated in figure (1.4). The penetration length gives the scale over which the magnetic field inside the superconductor is shielded by the supercurrents. If (x) measured the distance from the surface of the superconducting sample towards its interior, the total magnetic field behaves like

$$B(x) = B_o e^{\frac{-x}{\lambda}} \tag{1.1.3}$$

That is, it decays exponentially over a distance of order λ. It is natural that such a length scale should exist, because the field cannot drop to zero immediately inside the superconductor as this would imply an infinite surface current. The coherence length determines the scale over which there exist strong correlations which stabilizes the superconducting state. Or, equivalently, it is a measure of the distance over which the superconducting state is affected by fluctuations in the external field or other variables. If the length scale of these fluctuations is much larger than ξ, the superconducting state is not affected (the correlations responsible for the stability are not destroyed); if it is the order ξ, it is affected [7].

In the phenomenological theories the superconducting state is typically described by a quantity called the order parameter ($\psi(x)$), whose absolute value is between unity and zero, at these two extremes the superconducting state is either at full strength or has been completely destroyed. The typical behavior of B(x) and $|\psi(x)|$ in a type I superconductor is shown in figure (1.4)

1.2 Cooper pair formation and superconducting condensate

There are two main ingredients in the microscopic theory of superconductivity developed by Bardeen, Cooper, and Schieffer. The first is an effective interaction between two electrons mediated by phonons that have opposite momenta (larger in magnitude than the fermi momentum) and opposite spins, which leads to the formation of the so called cooper pairs. The second is the condensation of the cooper pairs into a single coherent quantum state. Which is responsible for all the manifestation of superconducting behavior. The fact that creation of cooper pairs is energetically favorable (it has a positive binding energy), the preferred state of the system under conditions where cooper pairs can be formed in a state with the maximum possible number of cooper pairs. However, this cannot be done for all available electrons since then the fermi surface would collapse, removing the basis for the stability of cooper pairs. A sufficiently large number of cooper pairs can be created to take maximum advantage of the benefits of pairing [8].

Each pair of electrons with opposite spins and momenta, retains the antisymmetric nature of two fermions due to the spin components which is singlet state, but as a whole it has zero total momentum and zero total spin. In this sense each pair can be thought of as a composite particle with bosonic character, and the total wave function as a coherent state of all these bosons occupying a zero momentum state. The concept of the macroscopic wave function $\psi(r)$ is central to understanding atomic Bose-Einstein condensate, and superconductivity with in the Ginzburg-Landau theory. The physical meaning of the Ginzburg-Landau order parameter was not at all clear until after 1957 . When Bardeen, cooper and schrieffer (BCS) published the first truly microscopic theory of superconductivity. Soon afterwards the connection was finally established by Gorkov, who was able to show that, at least in the range of temperatures near T_c, the Ginzburg-Landau theory can indeed be derived from the BCS theory [9]. Further more this provides a physical interpretation of the nature of order parameter. Essentially, it is described as a macroscopic wave function, or condensate of cooper pairs.

1.3 Thermodynamics of the superconducting transition

The thermodynamics of the superconducting transition can be expressed as a function of the external magnetic field (H) and temperature (T). The existence of a reversible Meissner effect implies that superconductivity will be destroyed by a critical magnetic field H_c which is related thermodynamically to the free energy difference between normal and superconducting state in zero field, the difference in the free energy is called condensation energy of the superconducting state [10]

$$F_n(T) - F_s(T) = \frac{H_c(T)}{8\pi} \tag{1.3.1}$$

condensation energy is the measure of the gain in free energy per unit volume in the superconducting state compared to the normal state at the same temperature. The entropies of superconducting and normal state also derived from the partial derivatives of the free energies per unit volume to the external field has the form.

$$S_n(T, H_c) - S_s(T, H_c) = \frac{-H_c}{4\pi}\frac{dH_c}{dT} \tag{1.3.2}$$

If we define the latent heat per unit volume $L = T\Delta S$ we can rewrite equation (1.3.2) in the following manner:

$$\frac{dH_c}{dT} = \frac{-4\pi}{H_c}\frac{L}{T} \tag{1.3.3}$$

Which has the familiar form of the Clausis-Clapeyron equation for the first order phase transition. We note that $\frac{dH}{dT} \leq 0$, which means that when we go from the normal state to the superconducing state, the latent heat is a positive quantity; that is; the superconducting state is more ordered (has lower energy) than the normal state. The transition in zero field at (T_c) is of second order because the entropy difference is zero. And the transition in the presence of the field is of first order phase transition.

Differentiating both sides of equation (1.3.2) with respect to temperature, and using the standard definition of the specific heat $C = \frac{dQ}{dT} = T\frac{dS}{dT}$, we find for the difference in specific heats per unit volume.

$$C^n(T) - C^s(T) = -\frac{T}{4\pi}\left[(\frac{dH_c}{dT})^2 + H_c(\frac{d^2H_c}{dT^2})\right] \tag{1.3.4}$$

We will assume that (H_c) as a function of temperature is given by equation (1.1.2), which leads to the following expression for the difference in the specific heats between the two states.

$$C_T^s - C_T^n = \frac{H_o^2}{2\pi}\frac{T}{T_c^2}\left[3(\frac{T}{T_c})^2 - 1\right] \tag{1.3.5}$$

At $T = T_c$ this expression reduces to

$$C^s(T_c) - C^n(T_c) = \frac{H_o^2}{\pi T_c} \tag{1.3.6}$$

That is the specific heat has a discontinuity at the critical temperature (T_c), as the temperature is increased through the transition points. And from equation (1.3.4) the specific heat of the superconducting state is higher than the specific heat of the normal state for $\frac{T_c}{\sqrt{3}} < T < T_c$, which is schematically in figure (1.6).

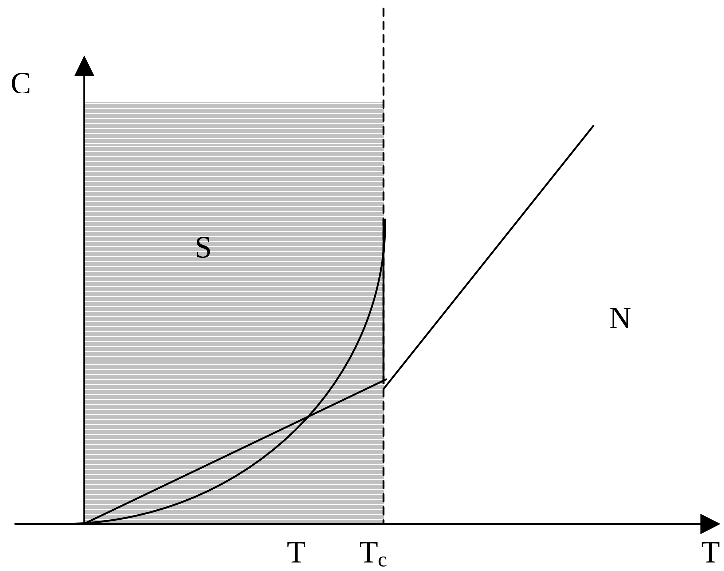

Figure 1.6: *Specific heat of a superconductor near T_c in Ginzburg-Landau model*

1.4 Spin glass superconductor

Historically the first observation of the coexistence of superconductivity and spin glass was made by Davidov. But recent experimental observations of spin glass ordering in the Fe-doped high-T_c superconductors, $Y_{1-x}Fe_xBa_2Cu_3O_{7+y}$ have renewed interest in the study of the coexistence of superconductivity and spin glass ordering with transition temperature $95K$ and $20K$ respectively [11]. The spin glass magnetic ordering is characterized by the freezing of randomly localized magnetic moment due to magnetic impurities into random orientations without any long-range order below a characteristic temperature, known as the spin glass transition temperature. This type of ordering occurs due to competition of ferromagnetic and antiferromagnetic interaction between the randomly distributed magnetic moment. The state of spin glass can be described by the spin glass order parameter, first introduced by Edwards and Anderson [12] in their paper entitled by canonical spin glass (i.e CuMn, AgMn, AuFe etc...).

The macroscopic theory for the spin glass superconductors found that the spin glass ordering did not affect superconductivity due to total compensation of paramagnetic effects and the effect of spin flip scattering, freezing out in a spin glass state. In this theory the dynamics and the quantum nature of impurity spins had been neglected. These effects can change the superconductivity behavior as is well known that the effective electron-electron interaction due to exchange of spin waves is repulsive, thus lowering the superconducting transition temperature. Later on some attempts were made by adding an arbitrary static spectral density due to spin glass ordering in the dynamic spectral density of the Eliashberg theory of superconductivity. Recently Stephan and Carbotte [13] and Nicol and Carbotte [14] have used this theory to calculate the thermodynamics and electromagnetic properties of the spin glass superconductors.

We have spin glass transition temperature (T_{go}) and critical transition temperature of superconductor (T_c) from there normal state. During the coexistence of spin glass and superconducting system, the occurrence of change temperature is visible from T_{co} and T_{go}, which are the transition temperatures of pure superconducting and spin glass system respectively. T_{co} and T_{go} may depend on the magnetic impurity concentration. For example, in the absence of superconducting order T_{go} increases as the impurity concentration increases. At the same time in the absence of spin glass order T_{co} decreases as the impurity concentration increases.

We can also indicate the arbitrary impurity concentration dependence of T_{co} and

T_{go} in phase diagram in (T,X) space,where X is the impurity concentration. A schematic phase diagram as shown in figure (1.7) below

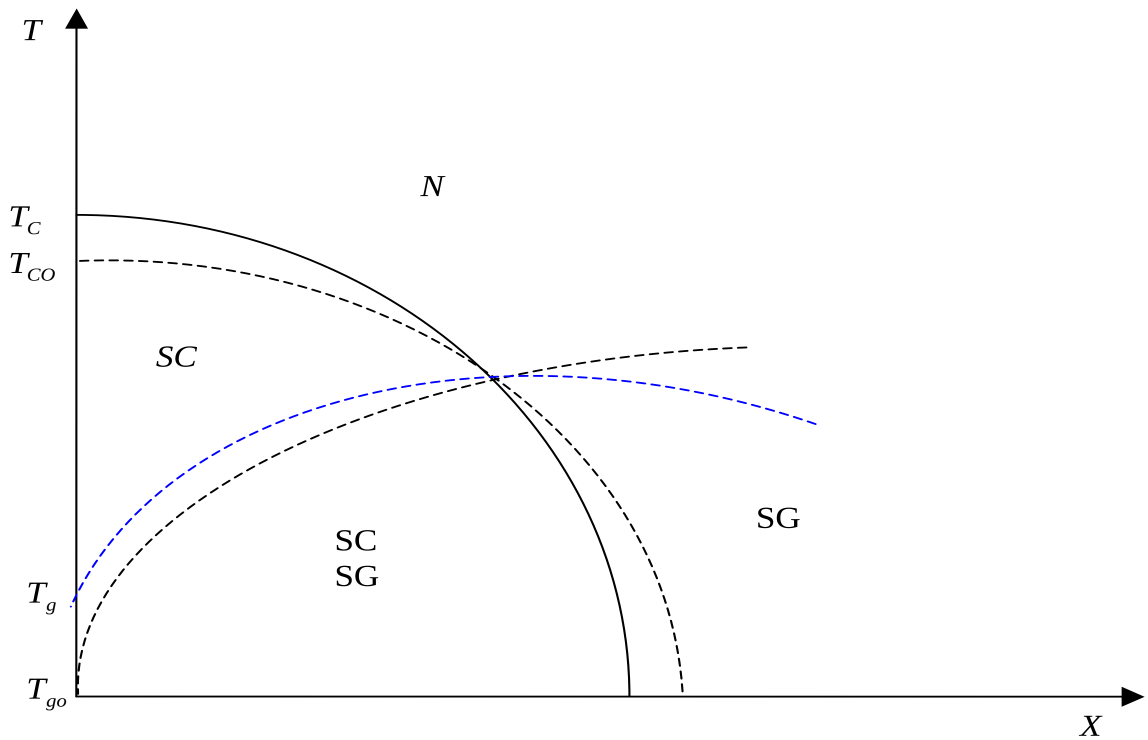

Figure 1.7: *Phase diagram in (T,X) space for superconducting (SC), spin glass (SG), and normal (N) states. T and X are the temperature and the magnetic concentration respectively*

1.5 Spin glass magnetic ordering

Spin glasses are disordered or random solid state magnetic systems in non-magnetic host characterized by a random freezing of moments with out long-range order at rather well defined temperature T_g. In practice, however it is very difficult to investigate the effects of single magnetic impurities experimentally because of measurement sensitivity. To be able to detect the effects, one needs a sufficient number of these impurities. It is also important to note that there exists a long-range interaction between the (localized) impurities via the conduction electrons. The very important property of spin glass is their unique nature is the oscillating character of the exchange interaction. This is so because the randomly located spin then have interactions of essentially random sign, so that no ferromagnetic or long-range ordered antiferromagnetic state is particularly favored energetically. This is the peculiar characteristics of spin glass.

The effective coupling between the magnetic moments can be either ferromagnetic or antiferromagnetic, depending on the separation between two impurities. Now, since the impurities are randomly located with in the crystals. The magnetic interactions are also randomly distributed. Thus, the "term spin glass" is used in analogy with a real glass or an amorphous solid where the atoms are randomly distributed without any order or regular structure.

The electrical resistivity, as well as the specific heat has been measured in the spin glass systems. From this measurement it has been found that a great deal of short-range magnetic clustering is already present at $T \gg T_g$. Such measurements establish beyond doubt that a majority of the spin participate in a local type of correlation.

As temperature is lowered, many of the randomly located, freely rotating spins come together, by means of the correlation, into clusters that can then rotate as a whole. The remaining isolated spins are uncorrelated but serve to transmit interactions between the clusters. At $T = T_g$ these independent isolated spins freeze out in random directions [15].

Spin glass is rapidly developing aspect of soilds which limits the usage of metallic alloys where long-range magnetic interactions are present. It is this long-range interaction (short-range interaction may also be present, through to a much smaller extent) that produces the random freezing of the spin moments a rather well defined temperature T_g. We have also examined a variety of recent experiments performed on spin glass alloys with respect to three ranges of temperature $T > T_g, T = T_g$ and $T < T_g$. In the interpretation of the experimental behavior, a phenomenological model depending on the dynamically growing clusters has been proposed by Mydosh [16]. It has been observed, even at $T \gg T_g$, that some local correlations among the randomly separated spins also exist. The growth of magnetic clusters continues until $T \backsimeq T_g$, when a few of the largest clusters freeze out in random orientations. The spin glass behavior may be theoretically described by the method of critical phenomenon by Edward and Anderson. Apart from small ferromagnetic clusters above T_g, clusters with more or less random spin directions are indicated by specific heat measurements. For a fixed impurity configuration we have, for $T > T_g$,

$$\langle S_i \rangle = 0$$

Since the internal energy felt by a spin averages to zero if the (thermal) average is taken over a sufficiently long time, whereas, for $T < T_g$, we have, for spin glasses,

$$\overline{\langle S_i \rangle . \langle S_i \rangle} \neq 0$$

Hence the spin glass order parameter is related

$$g = \overline{\langle S_i \rangle . \langle S_i \rangle} \neq 0$$

Chapter 2

Ginzburg-Landau theory of fluctuation

While the BSC is a weak-coupling microscopic theory, which is very powerful and provides at least a quantitatively correct description of most aspects of classic superconductors, whose energy gap is constant in space. There is a complimentary theory which is simpler and more physically transparent, although valid only near the transition and provides exact result under certain circumstances such as existence of spatial inhomogeneity. This is the macroscopic Ginzburg-Landau theory. In the present work this phenomenological theory is used in order to discuss the fluctuations effect on electrical conductivity and diamagnetic susceptibility during spin-glass superconducting transition.

2.1 Ginzburg - Landau free energy

The Ginzburg - Landau theory postulated the existence of complex pseudo wave function $\psi(r)$ which was equivalent to an order parameter [17], and proposed that on symmetry ground alone, the free energy density of a superconductor should be expressible in terms of an expansion in this quantity

$$F_s - F_{no} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{m^*}\left[\left(\frac{\hbar}{\imath}\nabla - \frac{e^*}{c}A\right)\psi\right]^2 + \frac{H^2}{8\pi} \tag{2.1.1}$$

Where F_s and F_{no} denote the superconducting and normal state free energy. Evidently if $\psi = 0$, this reduces properly to the free energy of the normal state.

$$F_s = F_{no} + \frac{H^2}{8\pi} \tag{2.1.2}$$

Now consider the remaining three terms describing the superconducting effects. In the absence of fields and gradients, we have

$$F_s - F_{no} = \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 \tag{2.1.3}$$

Ginzburg and Landau assumed that the free energy of the superconductor must depend smoothly on the parameters ψ. Since ψ is complex and the free energy must be real, the energy can only depend on $|\psi|$. Further more since ψ goes to zero at the critical temperature T_c, we can expand the free energy in the powers of ψ, in which only the first two terms are retained. Due to the fact that expansion in powers of ψ itself is excluded, since (F) must be real. The two terms in equation (2.1.3) should be adequate so long as one stays near the second -order phase transition at T_c, where the order parameter $|\psi|^2 \longrightarrow 0$. Looking at equation (2.1.3) shows that β must be positive if the theory is to be useful; otherwise, the lowest free energy would occur for arbitrarily large values of $|\psi|^2$, where the expansion is surely in adequate.

If α is positive, the minimum free energy occur at $|\psi|^2 = 0$ corresponding to the normal state. On the other hand, if α is negative, the minimum occurs when

$$\alpha + \beta|\psi|^2 = 0 \tag{2.1.4}$$

$$|\psi|^2 = |\psi_\infty|^2 = \frac{-\alpha}{\beta} \tag{2.1.5}$$

Where the notation ψ_∞ is conventionally used because ψ approaches this value infinitely deep in the interior of the superconductor, where it is screened from any surface fields or currents. When this value of ψ is substituted back to equation (2.1.3) it comes out:

$$F_s - F_{no} = \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 \tag{2.1.6}$$

$$F_s - F_{no} = \alpha(\frac{-\alpha}{\beta}) + \frac{1}{2}\beta(\frac{-\alpha}{\beta})^2 \tag{2.1.7}$$

$$F_s - F_{no} = \frac{-\alpha^2}{\beta} + \frac{\alpha^2}{2\beta} = \frac{-\alpha^2}{2\beta} \tag{2.1.8}$$

using the definition of thermodynamic critical field (H_c) which was described in equation (1.3.1)

$$F_s - F_{no} = \frac{-\alpha^2}{2\beta} = \frac{-H_c^2}{8\pi} \tag{2.1.9}$$

$$\frac{\alpha^2}{2\beta} = \frac{H_c^2}{8\pi} \tag{2.1.10}$$

Evidently $\alpha(T)$ must change from positive to negative at (T_c), since by definition (T_c) is the highest temperature at which $|\psi|^2 \neq 0$. Making a Taylor's expansion of $\alpha(T)$ about (T_c) and keeping only the leading term, one has

$$\alpha(t) = \alpha^{'}(t-1) \tag{2.1.11}$$

Where $t = \frac{T}{T_c}$ and $\alpha^{'} > 0$

from equation (2.1.10) this assumption is consistent with the linear variation of (H_c) with $(1-t)$, if β is regular at (T_c). Putting these temperature variation of α and β into the equation (2.1.5), we see that :

$$|\psi|^2 \propto 1 - t \tag{2.1.12}$$

Near T_c, this is consistent with correlating $|\psi|^2$ with (n_s), the density of superconducting electrons in the London theory, since $n_s \propto \lambda^{-2} \propto (T_c)$ near T_c. We will see the Ginzburg-Landau approach to London theory and penetration length.

$$n_s = |\psi(x)|^2 \tag{2.1.13}$$

2.2 Ginzburg - Landau supercurrent

Let's now focus our attention on the terms in the Ginzburg-landau free energy which leads to super current [18]. The kinetic energy parts of equation (2.1.1) will discussed in the following calculations.

$$F_{kin} = \int d^3r \tfrac{1}{2m^*} \left| \left(\nabla + \tfrac{\imath e^*}{c} \overrightarrow{A} \right) \psi \right|^2$$

After few steps and keep on substitution $\psi = |\psi| e^{\imath\phi}$ the kinetic part of free energy density of superconductor becomes:

$$F_{kin} = \int d^3r \frac{1}{2m^*} \left[(\nabla|\psi|)^2 + \left(\nabla\phi - \frac{e^*}{c} \overrightarrow{A} \right)^2 \right] \tag{2.2.1}$$

These expressions deserve several remarks. First, note that the free energy is gauge invariant, if we make the transformation

$$\overrightarrow{A} \rightarrow \overrightarrow{A} + \nabla\Lambda \tag{2.2.2}$$

Where Λ is any scalar function of position. While at the same time changing $\psi \rightarrow \psi exp\left(\frac{-\imath e^* A}{c} \right)$. Second, note that in equation (2.2.1), the kinetic part of free energy is splited into a term dependent on the gradient of the order parameter, magnitude of $|\psi|$ and on the gradient of the phase ϕ.

The energy of superconducting state below T_c is lower than that of the normal state by an amount called the condensation energy. From equation (2.1.1) in zero field this is of order $|\psi|^2$ very close to the transition. To make spatial variations of the magnitude of (ψ) must cost a significant fraction of condensation of energy in the region of space in which it occurs. On the other hand the zero field free energy is actually one work with respect to change in ϕ, so fluctuation of ϕ alone costs no energy.

If we apply a weak magnetic field described by (A) to the system. Since it is a small perturbation we don't expect it to couple to $|\psi|$ but rather to the phase ϕ. The kinetic energy density should then reduce to the second term in equation (2.2.1) and further more we expect that it should reduce to the intuitive two - fluid expressions for the kinetic energy due to super currents, $\frac{1}{2}mn_s v_s^2$. Recalling from the super fluid He analogy, we expect that $|\psi|^2 = n_s^*$ to be a kind of density of superconducting electrons, but that we are not certain of the charge or mass of the particle let's put

$$F_{kin} \simeq \frac{1}{2m^*}|(\nabla + \frac{\imath e^*}{c}\overrightarrow{A})\psi|^2 = \int d^3r \frac{1}{2m^*}(\nabla\phi - \frac{e^*}{c}A)^2|\psi|^2 \tag{2.2.3}$$

$$F_{kin} \simeq \frac{1}{2m^*}|(\nabla + \frac{\imath e^*}{c}\overrightarrow{A})\psi|^2 = \frac{1}{2}m^* n_s^* v_s^2 \tag{2.2.4}$$

From equation (2.2.3) and (2.2.4) we can equalize the expression given to the kinetic part of the free energy

$$\frac{1}{2m^*}|(\nabla + \frac{\imath e^*}{c}\overrightarrow{A})\psi|^2 = \frac{1}{2}m^* n_s^* v_s^2 \tag{2.2.5}$$

$$\left|\frac{(\nabla + \frac{\imath e^*}{c}\overrightarrow{A})|\psi|}{m^*}\right| = n_s^* v_s^2 \tag{2.2.6}$$

We find that the super fluid velocity must be

$$\overrightarrow{V_s} = \frac{1}{m^*}(\nabla\phi + \frac{e^*}{c}\overrightarrow{A}) \tag{2.2.7}$$

Thus the gradient of the phase is related to the superfluid velocity, but the vector potential also appears to keep the entire formalism gauge-invariant. The super current density will certainly be just:

$$\overrightarrow{j_s} = -e^* n_s^* \overrightarrow{v_s} = \frac{-e^* n_s^*}{m^*}(\nabla\phi + \frac{e^*}{c}\overrightarrow{A}) \tag{2.2.8}$$

Take the curl of this equation, the phase drops out and we find the magnetic field.

$$\nabla \times j_s = \frac{-e^* n_s^*}{m^* c}\overrightarrow{B} \tag{2.2.9}$$

Now recall the Maxwell equation and use in equation (2.2.8)

$$\overrightarrow{j_s} = \frac{c}{4\pi}\nabla \times \overrightarrow{B} \tag{2.2.10}$$

$$\nabla \times (\frac{c}{4\pi}\nabla \times \overrightarrow{B}) = \frac{-{e^*}^2 n_s^*}{m * c}\overrightarrow{B} \tag{2.2.11}$$

$$\nabla \times (\nabla \times \overrightarrow{B}) = \frac{4\pi}{c}(\frac{-{e^*}^2 n_s^*}{m * c})\overrightarrow{B} \tag{2.2.12}$$

$$\nabla^2 \overrightarrow{B} = \frac{4\pi}{c}(\frac{-{e^*}^2 n_s^*}{m^* c})\overrightarrow{B} \tag{2.2.13}$$

$$\lambda^2 \nabla \overrightarrow{B} = \overrightarrow{B} \tag{2.2.14}$$

Where $\lambda^2 = \frac{m^* c^2}{4\pi {e^*}^2 n_s^*}$

$$\lambda = \sqrt{\frac{m^* c^2}{4\pi {e^*}^2 n_s^*}} \tag{2.2.15}$$

Notice now that if we use, what we know about cooper pairs, this expression reduces to the BCS / London penetration depth. We assume e^* is the charge of the pair, namely $e^* = 2e$ and similarly $m^* = 2m$ and $|\psi|^2 = n_s^* = \frac{n_s}{2}$ since n_s^* is the density of pairs and n_s is the number of single electrons in the condensate. With this convention $\frac{n_s^* {e^*}^2}{m^*} = \frac{n_s e^2}{m}$ so the London penetration depth is unchanged by pairing.

2.3 Ginzburg-Landau coherence length ($\xi(T)$)

The London penetration depth λ is fundamental length that characterizes a superconductor. An independent length is the coherence length ξ. The coherence length is a measure of the distance with in which the superconducting electron concentration cannot change drastically in a spatially-varying magnetic field this is indicated in figure (2.1).

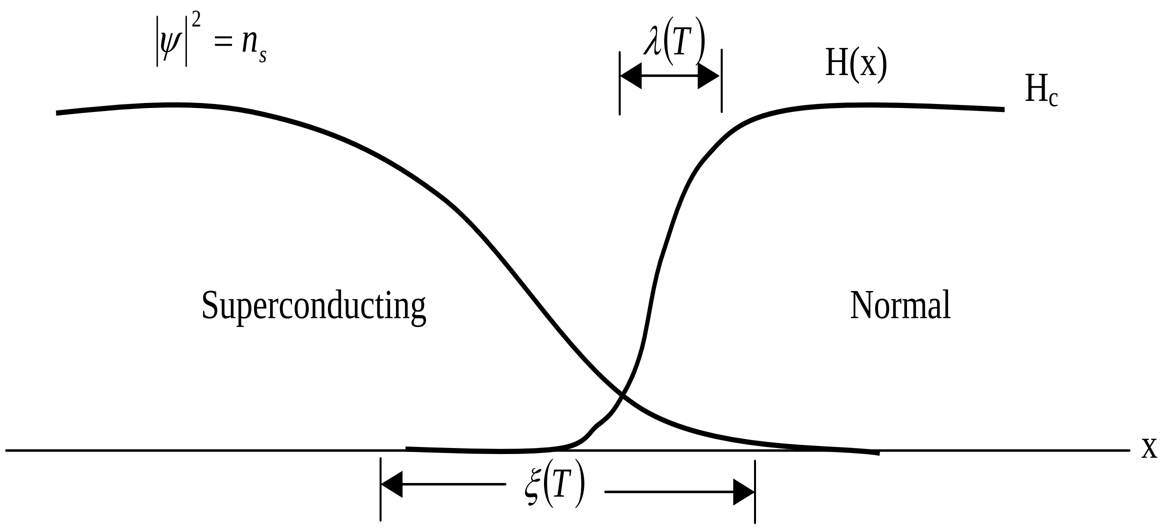

Figure 2.1: *Interface between superconducting and normal domains in the intermediate state.*

Now Ginzburg-landau theory introduces a characteristic length which is temperature dependent from equation (2.1.1). Using variational principle to minimize the free energy functional, and we will have the following expression:

$$\alpha\psi + \beta|\psi|^2 + \frac{1}{2m^*}\left(\frac{\hbar}{\imath}\nabla - \frac{e^*}{c}A\right)^2\psi = 0$$

We first consider a simplified case in which no fields are present. Then A=0, and we can take ψ to be a real, since the differential equation has only real coefficients. If we introduce a normalized wave function

$$f = \frac{\psi}{\psi_\infty} \tag{2.3.1}$$

Where

$$\psi_\infty = \tfrac{-\alpha}{\beta} > 0$$

The equation becomes in one dimension

$$\tfrac{\hbar^2}{2m^*|\alpha(T)|}\tfrac{d^2 f}{dx^2} + f - f^3 = 0$$

This makes it natural to define the characteristic length (ξ(T)) for variation of ψ by

$$\xi^2(T) = \frac{\hbar^2}{2m^*|\alpha(T)|} \propto \frac{1}{1-t} \tag{2.3.2}$$

$\xi(T)$ diverges at T_c

$$\xi^2(T)\frac{d^2 f}{dx^2} + f - f^3 = 0 \tag{2.3.3}$$

The significance of $\xi(T)$ as a characteristic length for variation of ψ (or f) can be made even evident by considering a linearized form of (2.3.3), in which we set f(x)=1+g(x), where $g(x) \lll 1$. Then we have,

$$\xi^2 g^{"}(x) + (1+g) - (1+3g+....) = 0 \tag{2.3.4}$$

or

$$g^{"}(x) = \left(\tfrac{2}{\xi^2}\right) g(x)$$

So that

$$g(x) \sim e^{\pm \frac{\sqrt{2}x}{\xi(T)}} \tag{2.3.5}$$

Which shows that a small disturbance of ψ from ψ_∞ will decay in a characteristics length of order $\xi(T)$.

2.4 Fluctuation induced superconducting effects

Thermal fluctuations raising the free energy by an amount $\sim KT$ are common, since the probability falls only as $e^{\frac{-F}{KT}}$. This leads to the existence of fluctuation-induced superconducting effects above T_c [19]. These fluctuations are largest in amplitude if confined to small volumes, since the total energy increment be only $\sim KT$. We can get a useful orientation on this problem by considering first a particle which is small compared to ξ, so that we can treat ψ as constant over it's volume V. This might be called the zero -dimensional limit. Then the Ginzburg-Landau free energy relative to the normal state (in the absence of any field) is

$$F = V(\alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4) \tag{2.4.1}$$

Where

$$\alpha = \alpha_o\left(\frac{T}{T_c} - 1\right) = \alpha_o\left(\frac{T - T_c}{T_c}\right)$$

Below T_c, this leads to the usual result that the minimum is

$$\left.\frac{\partial F}{\partial|\psi|^2}\right|_{\psi=\psi_o} = V(\alpha + \beta|\psi_o|^2) = 0$$

So $|\psi_o|^2$ is the minimum order parameter so that to get the minimum free energy.

$$|\psi_o|^2 = \tfrac{-\alpha}{\beta}, F_o = F(|\psi_o|^2)$$

$$F_o = F(|\psi_o|^2) = V(\alpha(\tfrac{-\alpha}{\beta}) + \tfrac{1}{2}\beta(\tfrac{-\alpha}{\beta})^2)$$

$$F_o = F(|\psi_o|^2) = V(\frac{-\alpha^2}{\beta} + \frac{-\alpha^2}{2\beta}) = \frac{-\alpha^2}{2\beta}V \tag{2.4.2}$$

$$F_o = F(|\psi_o|^2) = \frac{-\alpha^2}{2\beta}V \tag{2.4.3}$$

$$F_o = F(|\psi_o|^2) = \frac{-\alpha_o^2}{2\beta}\left(\frac{T - T_c}{T_c}\right)^2 V \tag{2.4.4}$$

The fluctuations about this ψ_o can be estimated by computing

$$\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_o} = -4\alpha V = -4\alpha_o\left(\frac{T - T_c}{T_c}\right)V \tag{2.4.5}$$

$$\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_o} = 4\alpha_o\left(\frac{T_c - T}{T_c}\right)V \tag{2.4.6}$$

And setting

$$\langle F - F_o\rangle = \frac{1}{2}\left.\frac{\partial^2 F}{\partial \psi^2}\right|_{\psi_o}(\delta\psi)^2 \approx KT \tag{2.4.7}$$

$$\frac{(\delta\psi)^2}{\psi_o} \approx \frac{KT}{4F_o} = \frac{2\pi KT}{H_c^2 V} \approx \frac{10^{-20}}{(1-t)^2 V} \tag{2.4.8}$$

From this we see that the fluctuations can cause a very small fractional change in ψ except very near T_c or in a very small sample. Therefore we have generally been well justified in using the "mean field" ψ_o. However, by use of very small particles ($d < 1000\mathring{A}$) it has been possible to probe the so called "critical region", where $(\frac{\delta\psi}{\psi_o})^2$ is not necessarily small, and the mean field results becomes inaccurate. In this connection it is important to note that the apparent divergence of equation (2.4.8) at T_c is actually cut off by the anharmonic terms in the free energy expansion, so that even at T_c, $(\delta\psi)^2$ has a finite value.

$$(\delta\psi)^2\Big|_{T_c} \approx \left(\frac{2kT_c}{V\beta}\right)^{\frac{1}{2}} \tag{2.4.9}$$

Now lets us examine the situation above T_c. Here $\alpha > 0$, so that by inspection of equation (2.4.1) we see that the minimum free energy is $F_o = 0$ (relative to the normal state) which occurs for $\psi_o = 0$. The fluctuations are limited by :

$$\frac{\partial^2 F}{\partial\psi^2}\Big|_{\psi=0} = 2\alpha V = 2\alpha_o\left(\frac{T-T_c}{T_c}\right)V \tag{2.4.10}$$

which we see differs only by a factor of two from the value given by equation (2.4.6) for a temperature an equal distance below T_c. The corresponding fluctuation level is:

$$\langle F - F_o\rangle = \frac{1}{2}\frac{\partial^2 F}{\partial\psi^2}\Big|_{\psi=0}(\delta\psi)^2 \approx KT \tag{2.4.11}$$

$$(\delta\psi)^2 \approx \frac{KT}{\alpha V} = \frac{KT}{\alpha_o\left(\frac{T-T_c}{T_c}\right)V} \tag{2.4.12}$$

Again this is the same order as fluctuation below T_c, but since ψ_o is now zero, all the superconducting effect arises from the fluctuations. We see that $(\delta\psi)^2$ tends to diverge as $(T-T_c)^{-1}$, as in the familiar Curie-weiss law in the statistical mechanics of paramagnetism, but this divergence is cut off very near T_c by the quadratic term which leads to equation (2.4.9). Infact, by equating (2.4.1) to KT and solving exactly one obtains:

$$F|_{\delta\psi} = KT \tag{2.4.13}$$

$$V(\alpha(\delta\psi)^2 + \frac{1}{2}\beta(\alpha(\delta\psi)^4) = KT \tag{2.4.14}$$

$$[(\delta\psi)^2]^2 + \tfrac{2\alpha}{\beta}(\delta\psi)^2 - \tfrac{2KT}{\beta V} = 0$$

$$(\delta\psi)^2 = \frac{\alpha}{\beta}\left[\left(1 + \frac{2\beta KT}{\alpha^2 V}\right)^{\frac{1}{2}} - 1\right] \tag{2.4.15}$$

where $T > T_c$

$$(\delta\psi)^2 = \langle\psi\rangle^2 = |\psi|^2$$

Which reduces to equation (2.4.12) and (2.4.9) in appropriate limits.

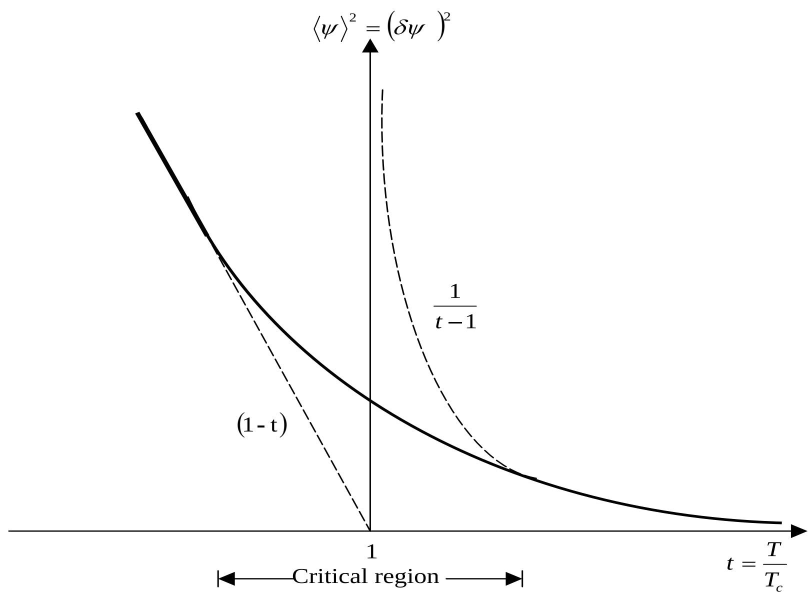

Figure 2.2: *Temperature dependence of Cooper pair density of zero dimensional superconductor near T_C*

The cooper pair density which is found in equation (2.4.15). $\langle\psi\rangle^2$ should rise as $(1-t)^{-1}$ as the temperature is reduced, but then rise more slowly once the critical region is entered; finally well below T_c,$\langle\psi\rangle^2$ should rise as $(1-t)$ after fluctuation effects are swamped by mean-field superconductivity. These dependencies are shown in figure (2.2) exactly this behavior was observed by Buhrman and Halperin [20] in measurements on fine aluminum powders.

2.5 Fluctuation-enhanced conductivity above-T_c

In the absence of superconducting fluctuation, the normal DC-conductivity is given by

$$\sigma_n = \frac{ne^2\tau}{m} \tag{2.5.1}$$

where τ is the mean free scattering time of the normal electrons and (n) is the number of them per unit volume. By analogy, we might expect the fluctuations to contribute an extra term

$$\sigma^{'} \approx \frac{(2e)^2}{m^*}\sum_k \frac{\langle|\psi_k|^2\rangle}{2}\tau_k \tag{2.5.2}$$

$$\tau_k = \frac{\tau_o}{1+k^2\xi^2} \tag{2.5.3}$$

This equation is integrated over k space, this prescription gives results which differ from the exact calculations only by small numerical factors. In particular the temperature dependence of $\sigma^{'}$ is correctly found to be $(T-T_c)^{\frac{-(4-d)}{2}}$ where d=1,2,3 is the dimensionality of the system.

Since the calculation is quite tractable and with some insight, let us now compute $\sigma^{'}$ exactly, within the framework of the Ginzburg-Landau linearized fluctuation theory. This is done most easily using the Kubo formalism, which relates linear response coefficients to the fluctuations in the unperturbed systems, as required by the fluctuation dissipation theorem [21]. We confine our attention to uniform fields and currents. Then, our general starting point is the Kubo result.

$$\sigma_{xx}(\omega) = \frac{1}{KT} = \int_0^\infty \langle J_x(0)\rangle\langle J(t)\rangle cos\omega t dt \tag{2.5.4}$$

We assumed that the normal quasi-particle current fluctuation are unchanged by the superconducting fluctuations. (This is strictly correct very near T_c, where the fluctuations are strong) Thus, to compute (σ'_{xx}), we compute only the fluctuating supercurrent in equation (2.5.4).

For a wave function ψ which can be expressed in the following way

$$\psi = \sum_k \psi_k e^{\imath k.r}$$

The current density is given by

$$J = \frac{e\hbar}{m^*\imath}(\psi^*\nabla\psi - \psi\nabla\psi^*)$$

$$J = \frac{e\hbar}{m^*\imath}(\sum_{k,g}(2k+g)\psi_k^*\psi_{k+g}e^{\imath g.r} \tag{2.5.5}$$

For uniform current in x-direction (g=0) and it reduces to

$$J_x = \frac{2e\hbar}{m^*}\sum_k k_x|\psi_k|^2 \tag{2.5.6}$$

We want to compute a current-current correlation function

$$\langle J_x(0)J_x(t)\rangle = \left(\frac{2e\hbar}{m^*}\right)^2 \left\langle \sum_{k,k'} k_x, k'_x|\psi_k(0)|^2|\psi_{k'}(t)|^2 \right\rangle$$

Since ψ_k and $\psi_{k'}$ are statistically independent, the cross term average out, and this can be written as

$$\langle J_x(0)J_x(t)\rangle = \left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2\langle\psi_k(0)\psi_k(t)\rangle^2 \tag{2.5.7}$$

Taking equation (2.5.4) into account and using the exponential time decay of

$$\langle\psi_k(0)\psi_k(t)\rangle = \langle|\psi_k|^2\rangle e^{\frac{-t}{\tau_k}} \tag{2.5.8}$$

The current-current correlation function becomes

$$\langle\psi_k(0)\psi_k(t)\rangle = \left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2 \langle\psi_k^*(0)\psi_k(t)\rangle^2$$

$$\langle\psi_k(0)\psi_k(t)\rangle = \left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2 \langle|\psi_k|^2\rangle^2 e^{\frac{-2t}{\tau_k}}$$

Insert in equation (2.5.8) and we will have

$$\sigma'_{xx}(\omega) = \frac{1}{KT}\int_0^\infty \left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2 \langle|\psi_k|^2\rangle^2 e^{\frac{-2t}{\tau_k}} \cos\omega t dt$$

$$\sigma'_{xx}(\omega) = \frac{1}{KT}\left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2 \langle|\psi_k|^2\rangle^2 \int_0^\infty e^{\frac{-2t}{\tau_k}} \cos\omega t dt$$

carrying out the integration and we get a result for the expression

$$\int_0^\infty e^{\frac{-2t}{\tau_k}} \cos\omega t dt = \frac{1}{2}\left[\frac{4\tau_k}{4+\omega^2\tau_k^2}\right] \tag{2.5.9}$$

Then we have to use in the equation of

$$\sigma'_{xx}(\omega) = \frac{1}{KT}\left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2 \langle|\psi_k|^2\rangle^2 \int_0^\infty e^{\frac{-2t}{\tau_k}} \cos\omega t dt$$

$$\sigma'_{xx}(\omega) = \frac{1}{KT}\left(\frac{2e\hbar}{m^*}\right)^2 \sum_k k_x^2 \langle|\psi_k|^2\rangle^2 \left[\frac{\frac{\tau_k}{2}}{1+(\frac{\omega\tau_k^2}{2})^2}\right] \tag{2.5.10}$$

Specializing the DC-case ($\omega = 0$) and inserting $\langle|\psi_k|^2\rangle$ and τ_k from equation

$$\langle|\psi_k|^2\rangle = \frac{2m^*}{\hbar}\left(\frac{kT}{k^2+\frac{1}{\xi^2}}\right)$$

$$\tau_k = \frac{\tau_o}{1+k^2\xi^2}$$

With few steps the value of conductivity for three dimension and two dimensions respectively can be obtained as

$$\sigma_{xx}(0)\Big|_{3D} \cong \frac{e^2}{32\hbar}\frac{\xi(T)}{\xi(0)}\left(\frac{T}{T-T_c}\right)^{\frac{1}{2}} \tag{2.5.11}$$

And the conductivity in two dimension can be expressed as:

$$\sigma_{xx}(0)\Big|_{2D} \cong \frac{e^2}{16\hbar}\frac{\xi(T)A}{\xi(0)}\left(\frac{T}{T-T_c}\right) \tag{2.5.12}$$

This relation explains the dependance of electrical conductivity on coherence length and transition temperature of spin glass superconductor.

Chapter 3

Formulation of the problem

3.1 Generalized Ginzburg-Landau theory of fluctuation

The well known phenomenological theory of superconductivity, proposed by Ginzburg and Landau based on second order phase transitions according to which the superconducting order parameter is non-zero in ordered (superconducting) state and zero in disordered (normal) state. In this work a study is made for a superconducting spin glass by expanding the free energy functional in the powers of superconducting and spin glass order parameter ψ and g respectively. The free energy functional F can be expressed as

$$F = F_n + F_{so} + F_{go} + F_{sg} \tag{3.1.1}$$

The first term F_n is the free energy functional of the normal state (ψ=g=0), the next two terms F_{so} and F_{go} are the free energy functionals of the pure superconducting and spin glass order parameters. The last term F_{sg} is the free energy functional due to coupling of superconducting and spin glass order parameter. We consider a homogeneous system with constant order parameter (ψ, g) for simplicity. The term F_{so} for pure superconducting state in equation (1.1.1) can be expressed as

$$F_{so} = \alpha_s|\psi|^2 + \frac{1}{2}\beta_s|\psi|^4 \tag{3.1.2}$$

α_s is assumed that $\alpha_s = a\left(\frac{T-T_c}{T_c}\right)$ where a and β_s are positive constant and T_c is the superconducting transition temprature of pure superconducting system. Due to molecular field theory of spin glass, F_{go} can be approximated as [22]

$$F_{go} = \frac{1}{2}\alpha_g g^2 + \frac{1}{3}\beta_g g^3 \tag{3.1.3}$$

where α_g and β_g are approximated as $\alpha_g = k(T_{go} - T)$ and $\beta_g = -kT_{go}$

Here T_{go} is the spin glass transition temperature of the pure spinglass system. By keeping only the lowest-order terms the coupling term F_{sg} can be assumed to be

$$F_{sg} = \eta|\psi|^2 g^2 \tag{3.1.4}$$

where η is the coupling parameter. The generalized Ginzburg-landau equations (GGLE) for the spin glass superconductor, described by the free energy functional in the equation (3.1.1), can be obtained by using the variational principle

$$\delta \int F dr = 0 \tag{3.1.5}$$

from equation (3.1.1), the free energy functional can be rewritten as:-

$$F = F_n + \alpha_s|\psi|^2 + \tfrac{1}{2}\beta_s|\psi|^4 + \tfrac{1}{2}\alpha_g g^2 + \tfrac{1}{3}\beta_g g^3 + \eta|\psi|^2 g^2$$

Using variational principle we can get the following equations

$$\alpha_s|\psi| + \beta_s|\psi|^3 + \eta|\psi| g^2 = 0 \tag{3.1.6}$$

$$\alpha_g + \beta_g g + 2\eta|\psi|^2 = 0 \tag{3.1.7}$$

collectively we have two equation from arbitrary variations $|\psi|$ and g and for simplicity (η) is very small for weak coupling and it is less than unity. So the term containing η^2 is neglected.

$$\alpha_g^2 = \beta_g^2 g^2 + 4\beta_g g\eta|\psi|^2 \tag{3.1.8}$$

$$\alpha_s = -\beta_s|\psi|^2 - \eta g^2 \tag{3.1.9}$$

Now we have two equations containing the term $|\psi|^2$ and g^2 the two order parameter of superconductor and spin glass respectively. The solution can be obtained by using determinant method with the help of approximation.

$$\begin{pmatrix} \beta_g^2 & 4\beta_g g\eta \\ -\eta & -\beta_s \end{pmatrix} \begin{pmatrix} g^2 \\ |\psi|^2 \end{pmatrix} = \begin{pmatrix} \alpha_g^2 \\ \alpha_s \end{pmatrix}$$

Adding the following two equations, which we get from the matrix equation

$$4\beta_g g\eta|\psi|^2 + \beta_g^2 g^2 = \alpha_g^2 \tag{3.1.10}$$

$$-\beta_s|\psi|^2 - \eta g^2 = \alpha_s \tag{3.1.11}$$

When we add equations (3.1.10) and (3.1.11) we get the following expression:-

$$4\beta_g g\eta^2|\psi|^2 - \beta_s\beta_g^2|\psi|^2 = \eta\alpha_g^2 + \beta_g^2\alpha_s$$

Using the weak coupling approximation $\eta^2 \to 0$. We can neglect the term containing η^2 it becomes

$$-\beta_s\beta_g^2|\psi|^2 = \eta\alpha_g^2 + \beta_g^2\alpha_s$$

$$|\psi|^2 = \frac{-\eta\alpha_g^2}{\beta_s\beta_g^2} - \frac{\alpha_s}{\beta_s} \tag{3.1.12}$$

Where $\alpha_g = k(T_{go} - T)$,$\alpha_s = \frac{a(T-T_{co})}{T_{co}}$ and $\beta_g = -kT_g$

$$|\psi|^2 = \frac{a}{\beta_s}\left(1 - \frac{T}{T_{co}}\right) - \frac{\eta}{\beta_s}\left(1 - \frac{T}{T_{go}}\right)^2 \tag{3.1.13}$$

From the equation (3.1.8) we can calculate the order parameter of spin glass and substitute the magnitude of superconductive order parameter from equation (3.1.13).

$$\alpha_g^2 = \beta_g^2 g^2 + 4\beta_g g\eta|\psi|^2$$

$$\beta_g^2 g^2 = \alpha_g^2 - 4\beta_g g\eta\left[\tfrac{a}{\beta_s}\left(1 - \tfrac{T}{T_{co}}\right) - \tfrac{\eta}{\beta_s}\left(1 - \tfrac{T}{T_{go}}\right)^2\right]$$

$$\beta_g^2 g^2 = \alpha_g^2 - \frac{4\beta_g g\eta a}{\beta_s}\left(1 - \frac{T}{T_{co}}\right) + \frac{4\beta_g g\eta^2}{\beta_s}\left(1 - \frac{T}{T_{go}}\right)^2 \tag{3.1.14}$$

For weak coupling, we can neglect the term containing η^2.

$$\beta_g^2 g^2 = \alpha_g^2 - \tfrac{4\beta_g g\eta a}{\beta_s}\left(1 - \tfrac{T}{T_{co}}\right)$$

$$\beta_g^2 g^2 + \frac{4\beta_g g\eta a}{\beta_s}\left(1 - \frac{T}{T_{co}}\right) - \alpha_g^2 = 0 \tag{3.1.15}$$

$$g^2 + \tfrac{4g\eta a}{\beta_s\beta_g}\left(1 - \tfrac{T}{T_{co}}\right) - \tfrac{\alpha_g^2}{\beta_g^2} = 0$$

From the values given to the constant $\beta_g = -kT_{go}$ and $\alpha_g = k(T_{go} - T)$ in the expression of free energy of spin glass are substituted in the equation we can get the following:

$$g^2 - \tfrac{4\eta a}{kT_{go}\beta_s}\left(1 - \tfrac{T}{T_{co}}\right)g - \tfrac{k^2(T_{go}-T)^2}{k^2T_{go}^2} = 0$$

$$g^2 - \frac{4\eta a}{kT_{go}\beta_s}\left(1 - \frac{T}{T_{co}}\right)g - \left(\frac{T_{go} - T}{T_{go}}\right)^2 = 0 \tag{3.1.16}$$

This expression has a quadratic form of $Ag^2 + Bg + C$, where the coefficients are $A = 1,\ B = -\frac{4\eta a}{kT_{go}\beta_s}\left(1 - \frac{T}{T_{co}}\right),\ C = -\left(\frac{T_{go}-T}{T_{go}}\right)^2$. Hence we can solve for g using a formula known by

$$g = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$g = \frac{1}{2}\left(\frac{4\eta a}{kT_{go}\beta_s}\left(1 - \frac{T}{T_{co}}\right) \pm \sqrt{\frac{16a^2\eta^2}{k^2T_{go}^2\beta_s^2}\left(1 - \frac{T}{T_{co}}\right)^2 + 4\left(\frac{T_{go}-T}{T}\right)^2}\right)$$

Since the value g runs from 0 to 1, we take plus sign. Using weak coupling approximation, we can neglect the term containing η^2. Then the spin glass order parameter can be expressed as the following:

$$g = \left(1 - \frac{T}{T_{go}}\right) + \frac{2\eta a}{k\beta_s T_{go}}\left(1 - \frac{T}{T_{co}}\right) \tag{3.1.17}$$

3.2 Expression for superconducting and spin glass transition temperatures

At T_c, the superconducting order parameter should vanish

$$\lim_{T \to T_c} |\psi|^2 = 0 \text{ or } T = T_c\ ; |\psi|^2 = 0$$

Which gives for T_c

$$T_c = T_{co} - \frac{\eta T_{co}^3}{aT_{go}^2}\left(1 - \frac{T_{go}}{T_{co}}\right)^2 \tag{3.2.1}$$

For $T_{go} > T_{co}$, the value of T_c should be greater than T_{co}

For $T_{go} < T_{co}$, the value of T_c should be greater than T_{co}

For $T_{go} > T_{co}$and $T_{go} < T_{co}$ the value of T_c should be less than T_{co}

At or near T_g, the spin glass order parameter vanishes which is shown in figure (1.7) i.e

$$\lim_{T \to T_g} g = 0 \text{ or } T = T_g c \text{ ; } g = 0$$

we get expression for T_g

$$T_g = T_{go} + \frac{2\eta a}{k\beta_s}\left(1 - \frac{T_{go}}{T_{co}}\right) \tag{3.2.2}$$

These are relations for the transition temperature for spin glass and superconductor during coupling. In the coming chapter we will see the nature of fluctuation in spin glass and superconducting order parameters around their transition temperatures.

Chapter 4

Results and Discussion

4.1 Fluctuation of superconducting order parameter

From the expressions which we derived in previous chapters for the superconducting and spin glass order parameters using Ginzburg Landau equation can be analyzed using heaviside step function.

$$|\psi|^2 = \frac{a}{\beta_s}\left(1 - \frac{T}{T_{co}}\right)\Theta(T_c - T) - \frac{\eta}{\beta_s}\left(1 - \frac{T}{T_{go}}\right)\Theta(T_c - T)\Theta(T_g - T) \qquad (4.1.1)$$

where $\Theta(x)$ is heaviside step function. Heaviside step function $\Theta(T - T')$ is zero for $T < T'$ and one for $T > T'$. For $T_c > T_g$ the behavior of $|\psi|^2$ shows a small discontinuity at a spin glass transition temperature (T_g). Substituting the value of T_g from equation(3.2.2) into equation (4.1.1), we obtain the magnitude of the discontinuity. $T_{go} < T_g$ shows that the presence of superconductivity increases the spin glass transition temperature, thus superconductivity assists the freezing of spins. This may be due to a decrease of scattering of spins by electrons which participate in forming the superconductivity cooper pairs.

$$\Delta|\psi|^2 = \lim_{\delta\to 0}[|\psi|^2(T = T_g + \delta) - |\psi|^2(T = T_g - \delta)]$$

$$\Delta|\psi|^2 = \frac{4\eta^3 a^2}{k^2\beta_s^3 T_{go}}\left(1 - \frac{T_{go}}{T_{co}}\right) \tag{4.1.2}$$

The temperature of which supercondcting order parameter is maximum can be calculated by differentiating $|\psi|^2$ w.r.t temperature (T)

$$\tfrac{d|\psi|^2}{dT} = \tfrac{-a}{\beta_s T_{co}} - \tfrac{2\eta}{\beta_s}\left(1 - \tfrac{T}{T_{go}}\right)\tfrac{-1}{T_{co}} = 0$$

$$\tfrac{d|\psi|^2}{dT} = \tfrac{-a}{\beta_s T_{co}} + \tfrac{2\eta}{\beta_s T_{go}} - \tfrac{2\eta T}{\beta_s T_{go}^2} = 0$$

$$\tfrac{2\eta T}{\beta_s T_{go}^2} = \tfrac{-a}{\beta_s T_{co}} + \tfrac{2\eta}{\beta_s T_{go}}$$

Rearranging terms by multiplying the whole expression by $\frac{\beta_s T_{go}^2}{2\eta}$, the maximum temperature becomes

$$T = T_m = T_{go} - \frac{aT_{go}^2}{2\eta T_{co}} \tag{4.1.3}$$

For $T_c < T_g$, $|\psi|^2$ does not show any discontinuity but can have a maximum at the temperature given by equation (4.1.3). In both cases the superconductive order parameter in the coexistence region (both ψ and g not equal to zero) is always greater than the pure superconducting case ($\eta = 0$). It shows that spin glass ordering acts as a cooper pair breaking effect. Figure (4.1) shows that square of the superconductivity order parameter $|\psi|^2$ as a function of temperature. Broken lines corresponds to the pure superconductive case.

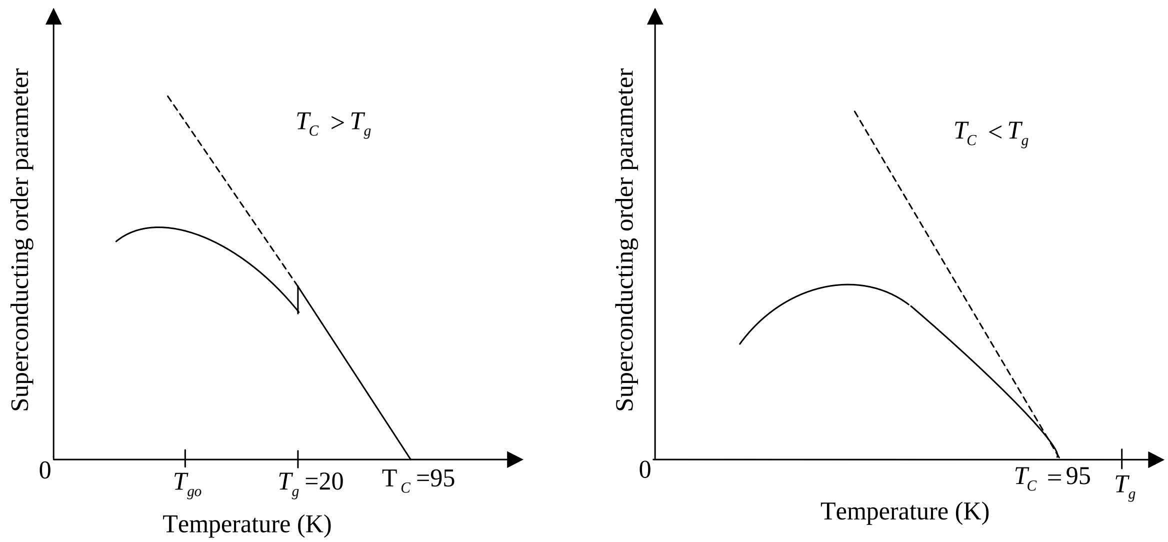

Figure 4.1: *Square of the superconductivity order parameter as a function of temperature*

4.2 Fluctuation of spin glass order parameter

The spin glass order parameter, from the solutions of GGLE can be expressed as

$$g = \left(1 - \frac{T}{T_{go}}\right)\Theta(T_g - T) + \frac{2\eta a}{k\beta_s T_{go}}\left(1 - \frac{T}{T_{co}}\right)\Theta(T_c - T)\Theta(T_g - T) \qquad (4.2.1)$$

As shown in figure (4.2), the spin order parameter shows a small discontinuity at (T_c). For the case $T_c < T_g$ substituting the value of T_c from equation (3.2.1) in equation (4.2.1), the magnitude of this discontinuity is given as:

$$\Delta g = \lim_{\delta \to 0}[g(T = T_c + \delta) - g(T = T_c - \delta]$$

$$\Delta g = \frac{-2\eta^2}{k\beta_s T_{go}} \left(\frac{T_{co}}{T_{go}} - 1 \right)^2 \tag{4.2.2}$$

The cause of discontinuity is spin glass breaking effect in superconductivity which makes a drastic change to spin glass magnetic order. There is no discontinuity for $T_c > T_g$. The magnitude of the spin glass order parameter in the coexistence region is always greater than that in the pure spin glass case. It suggests that the spins of electrons, coming from the broken superconducting pairs, contribute to the spin glass order. In the figure (4.2) spin glass order parameter (g) as a function of temperature. Broken lines correspond to the pure spin glass case.

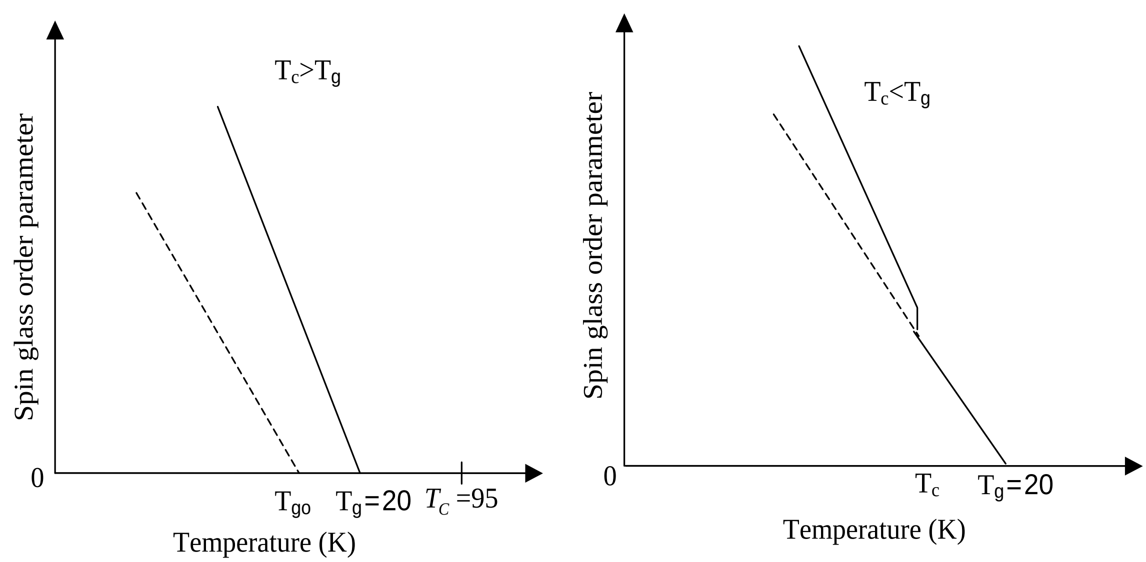

Figure 4.2: *Spin glass order parameter as a function of transition temperatures*

4.3 Ginzburg-Landau spin glass coherence length

From the equation (2.3.2) the inverse of coherence length also behaves as the superconducting order parameter just like the penetration length [23].

$$\xi^2(T) = \frac{\hbar^2}{2m^*|\alpha(T)|} \propto \frac{1}{1-t} \tag{4.3.1}$$

Since ψ approaches infinitely deep in the interior of the superconductor it has the value ψ_∞ or ψ_o

$$|\psi_o|^2 = \frac{-\alpha(T)}{\beta_s} \tag{4.3.2}$$

and

$$|\psi|^2 = \frac{\alpha(T)}{\beta_s}$$

Then the spin glass coherence length can take the following form

$$\xi^2(T) = \frac{\hbar^2}{2m^*\beta_s|\psi|^2}$$

And from the equation (3.1.7) we have

$$-\alpha_s = -\beta_s|\psi|^2 - \eta g^2$$

$$\frac{-\alpha_s}{\beta_s} = |\psi|^2 + \frac{\eta g^2}{\beta_s}$$

$$|\psi_o|^2 = |\psi|^2 + \frac{\eta g^2}{\beta_s}$$

$$|\psi|^2 = |\psi_o|^2 - \frac{\eta g^2}{\beta_s}$$

$$|\psi|^2 = |\psi_o|^2\left(1 - \frac{\eta g^2}{\beta_s|\psi_o|^2}\right) \tag{4.3.3}$$

where $\xi_{sg} = 1 - \frac{\eta g^2}{\beta_s|\psi_o|^2}$

The coherence length of spin glass superconductor taken to be ξ_{sg} then it will be related to

$$|\psi|^2 = |\psi_o|^2\xi_{sg}$$

4.4 Fluctuation induced electrical conductivity

The electrical conductivity obtained by using Kubo formalism from equation (2.5.4):

$$\sigma_{xx}(\omega) = \frac{1}{K_BT}\int_0^\infty \langle J_x(0)J_x(t)\rangle cos\omega t dt$$

where $\langle J_x(0)J_x(t)\rangle$ is current-current correlation. The fluctuation induced electrical conductivity σ_{xx} for spin-glass superconductor can be expressed as

$$\sigma_{xx}|_{3D} \cong \frac{e^2}{32\hbar}\frac{\xi_{sg}}{\xi(0)}\left(\frac{T}{T-T_c}\right)^{\frac{1}{2}} \tag{4.4.1}$$

$$\sigma_{xx}|_{2D} \cong \frac{e^2}{16\hbar}\frac{\xi_{sg}A}{d}\left(\frac{T}{T-T_c}\right) \tag{4.4.2}$$

Where ξ_{sg} is given by using the solution of GGLE

$$|\psi|^2 = |\psi_o|^2\xi_{sg}$$

$$\xi_{sg} = \left(1 - \frac{\eta g^2}{\beta_s|\psi_o|^2}\right) \tag{4.4.3}$$

The coherence length of spin glass superconductor has inverse relation with spin glass order parameter which is shown in figure (4.3). And it can be explained in terms of other parameters. Since the spin glass order parameter in equation (3.1.16) and the

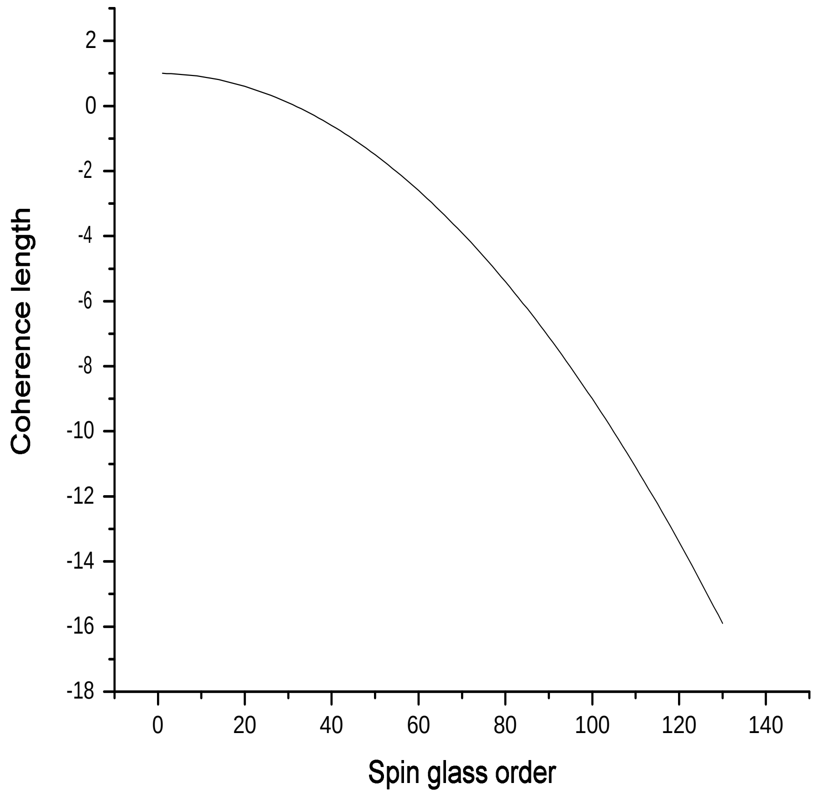

Figure 4.3: *The coherence length of spin glass superconductor versus spin glass order parameter.*

relation stated in coherence length $\alpha(T) = -\beta_s|\psi_o| = k(T_{go} - T)$ can substituted in equation (4.4.3) then we can have a relation as follows

$$\xi_{sg} = \left(1 - \frac{\eta g^2}{\beta_s|\psi_o|^2}\right)$$

$$\xi_{sg} = 1 + \frac{\eta}{\alpha(T)}\left(\frac{4\eta a}{kT_{go}\beta_s}\left(1 - \frac{T}{T_{co}}\right)g + \left(\frac{T_{go}-T}{T_{go}}\right)^2\right)$$

The terms which contain η^2 are neglected using weak coupling approximation:

$$\xi_{sg} = 1 + \frac{\eta}{\alpha(T)}\left(\frac{T_{go}-T}{T_{go}}\right)^2$$

$$\xi_{sg} = 1 + \frac{\eta}{k(T_{go}-T)}\left(\frac{T_{go}-T}{T_{go}}\right)^2$$

then the coherence length of spin glass superconductor has an inverse relation with the spin glass transition temperature

$$\xi_{sg} = 1 + \frac{\eta(T_{go} - T)}{kT_{go}^2} \tag{4.4.4}$$

Making use of the above expression in the conductivity equation σ_{xx} then we get:

$$\sigma_{xx}|_{3D} \cong \frac{e^2}{32\hbar}\frac{\xi_{sg}}{\xi(0)}\left(\frac{T}{T-T_c}\right)^{\frac{1}{2}}$$

using the spin glass coherence length in equation (4.4.3) we can express conductivity in three dimension as:

$$\sigma_{xx}|_{3D} \cong \frac{e^2}{32\hbar\xi(0)}\left(1 - \frac{\eta g^2}{\beta_s|\psi_o|^2}\right)\left(\frac{T}{T-T_c}\right)^{\frac{1}{2}} \tag{4.4.5}$$

These expressions clearly show that the fluctuation enhanced conductivity will be reduced by g. The superconducting order parameter and cooper pair density are lowered by freezing of spins. Hence it will suppress electrical conductivity. The conductivity of spin glass superconductor can be seen in figure (4.4).

4.5 Fluctuation induced magnetic susceptibility

To discuss fluctuation induced magnetic susceptibility χ of spin-glass superconductor. We use the famous spin glass susceptibility equation given by Young [24]

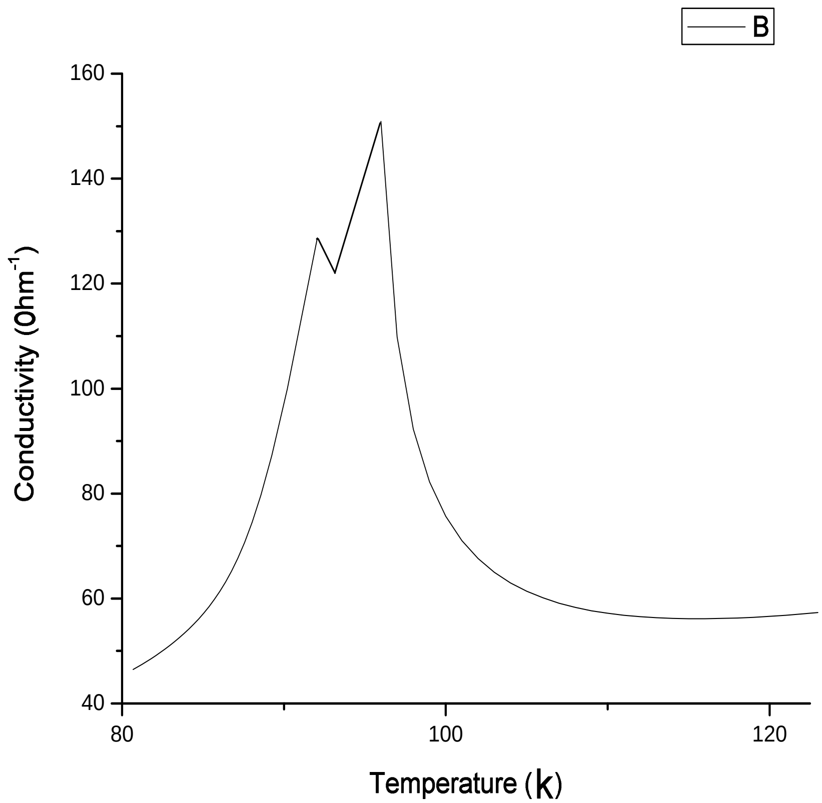

Figure 4.4: *Conductivity versus temperatures of spin-glass superconductors.*

$$\chi = \frac{1-g}{kT} \tag{4.5.1}$$

But from the equation (3.1.17) we found that

$$\chi = \frac{1}{kT} - \frac{1}{kT}\left[\left(1-\frac{T}{T_{go}}\right) + \frac{2\eta a}{k\beta_s T_{go}}\left(1-\frac{T}{T_{co}}\right)\right]$$

$$\chi = \frac{1}{kT_{go}} - \frac{2\eta a}{k^2\beta_s T_{go} T}\left(1-\frac{T}{T_{co}}\right)$$

For $T_c > T_g$ the spin glass susceptibility shows discontinuity (cusps) at T_g, for the temperature (T) such that, $T_g < T < T_c$ the susceptibility increases. Where as $T < T_g < T_c$ in the coexistence region the susceptibility is less than the pure spin glass system. This is indicated in figure (4.5).

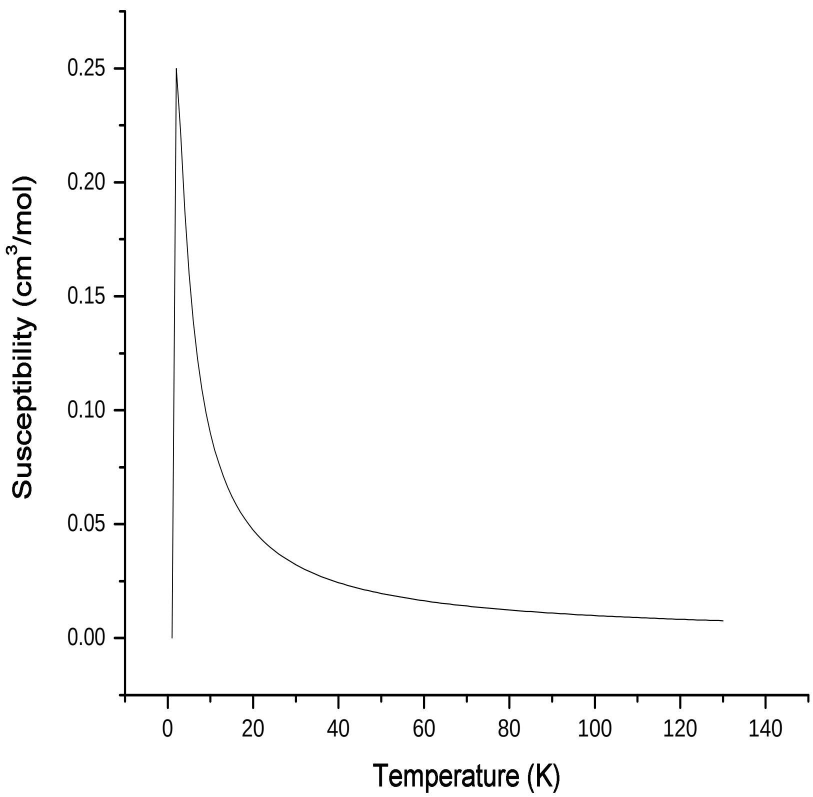

Figure 4.5: *Magnetic susceptibility versus temperature of spin-glass superconductor* $T_c > T_g$.

Chapter 5

Conclusion

A generalized phenomenological theory for a possible coexistence of spin glass order with superconductivity, based on the free energy function has been developed. The free energy function has been is consists the functional of superconducting and spin glass order parameter and their mutual coupling. Due to coupling the transition temperature of superconductor decreases from the pure superconducting transition temperature. On the contrary the spin glass temperature rises from the pure spin glass transition temperature. For the temperature greater than the superconducting and spin glass transition temperature the spin glass and superconducting order is not affected.

When spin glass ordering and superconductivity coexists, the superconducting transition temperature affects the properties of spin glass superconductor. The superconducting order parameter decreases due to spin glass order parameter and the spin glass order parameter increases due to superconducting order parameter. Thus in the coexistence region, the spin glass order acts as a pair breaking effect for superconducting order which helps in freezing of spin and maximize the magnetization by ordering the spins so that to increase the magnetic susceptibility. Also the spins of electrons coming from the broken superconducting pairs may help in increasing the spin glass order parameter. Susceptibility exhibits cusps at spin glass transition temperature.

The electrical conductivity of superconductors is enhanced by fluctuation effect. When spin glass coexist with superconductivity the spin glass order parameter is increased, then the coherence length of spin glass superconductor is also reduced. This tells us superconducting order parameter and cooper pair density are lowered due to breaking of the conducting pair by freezing of spin so it minimize the electrical conductivity of spin glass superconductor.

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