# MEDIA WITH NEGATIVE REFRACTIVE INDEX AND THEIR ELECTROMAGNETIC PROPERTIES


**By**

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## APPROVED BY THE EXAMINATION COMMITTEE

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# DECLARATION

I declare that the project is my original work, and has not been presented for a degree in any other university and that all sources of materials used for this project have been duly acknowledged.

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June, 2010

This project has been submitted for examination with my approval as university advisor.

**Name: Prof.V.N. MAL'NEV**

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**Date** ___________________

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# Abstract

In this project it was intended to reveal that the main electrodynamical properties of media with negative refractive index. The negative sign of the refractive index was firstly predicted in the theoretical description of the EM properties of materials with simultaneously negative values of permittivity $\varepsilon$ and permeability $\mu$. Medium with negative $\varepsilon$ and $\mu$ is termed as left-handed medium (LHM). At the time being, there are known experimental realization of the systems, which demonstrates the properties of LHM. The properties of LHM are theoretically described in the plasma with ferromagnetic grains. Reversal of wave propagation contains important implications for nearly all electromagnetic phenomena. Electromagnetic properties following a negative refractive index was also explored. It has been also shown that metamaterials can be constructed for which the index of refraction is negative over a finite band of frequency.

We show that the refractive index of the extraordinary wave propagating along the external magnetic field with frequencies close to $\omega_c$ can be negative as well as positive. For the typical parameters of Magnetized Plasma with Ferrite Grains, the group velocity of this wave is much smaller than the speed of light.

# CHAPTER ONE

## Introduction

When an unexplored area is explored in any field of science, names and terms have to be invented for new phenomena and quantities that are being studied. Logic and rationality of naming things may be absent in such a process [1], especially when the field is progressing fast, and reports, articles, and other texts are conceived under pressure. A Russian physicist Victor Veselago in 1967 published his article [2] about the theoretical properties of media that can effectively be said to possess both negative permittivity and negative permeability within a certain frequency range.

The dielectric constant ε and the magnetic permeability μ are the fundamental characteristic quantities which determine completely the peculiarities of propagation of electromagnetic waves in matter. Particularly the dispersion law or dependence of a frequency of wave $\omega$ on the wave vector $\vec{k}$. With the help of the dispersion law one can obtain the phase velocity $\vec{v}_{ph} = \frac{\omega}{k}$ and group velocity $\vec{v}_{gr} = \frac{d\omega}{dk}$. In the case of isotropic substances the dispersion law takes the form:

$$\omega(k) = k\frac{c}{n} = kv \quad , \qquad (1.1)$$

here $v = \frac{c}{n}$ is the speed of light in a medium, c is speed of light in vacuum and $n$ is a refractive index which is given by.

$$n^2 = \varepsilon\mu \quad , \qquad (1.2)$$

It is clear that for the propagation of the wave $n^2 > 0$ other way cannot propagate in the media. But from (1.2) it follows that $n = \pm\sqrt{\varepsilon\mu}$ and the refractive index can be positive for ($\varepsilon > 0$ and $\mu > 0$) as well as negative for ($\varepsilon < 0$ and $\mu < 0$). The negative sign of the refractive index naturally arises in the theoretical description of the electromagnetic properties of materials with simultaneously negative values of dielectric permittivity ε and magnetic permeability μ. A negative refractive index means that the phase velocity $\vec{v}_{ph}$ of a propagating wave, which describes the propagation of individual wave fronts in a wave group, is opposite to the movement of the energy flux of the wave, represented by the Poynting vector $\vec{s}$ [2].

A wave with the phase velocity opposite to the direction of energy flow is termed as "negative phase velocity wave or backward wave" [3]. In an isotropic medium with a negative dielectric permittivity $\varepsilon(\omega) < 0$ and negative magnetic permeability $\mu(\omega) < 0$ over a common band of frequencies $\omega$, an electromagnetic wave propagates with the wave vector **k**, the electric field **E**, and the magnetic field **H**, that form a left-handed orthogonal set. Such a medium is labeled as left-handed (LHM) [4]. The antiparallel orientation of the vectors of the phase and group velocities is an essential condition for negative refraction. Such antiparallel orientation corresponds to so called “backward waves,” or negative group velocity. In our opinion, the term “negative phase velocity” would be more appropriate, bearing in mind that the group velocity, directed from the source to the receiver, is always positive.

In analyzing plane waves propagating in left-handed materials, Veselago noted that the phase and group velocities were reversed, giving rise to a significant modification of numerous electromagnetic properties, including the change in sign of the refractive index. The prospect of a negative refractive index might at first be considered surprising, since the form of the refractive index, given by eq(1.2) would appear to be unchanged whether ε and μ are both positive or both negative, however, analytical arguments based on causality show that, in fact, the negative square root of the index must be taken for a Left-handed material [5]. Specifically, the simultaneously negative dielectric permittivity, $\varepsilon < 0$, and magnetic permeability $\mu < 0$, lead to a negative refraction index, with the left-handed triplet of $E$, $H$, $K$. It is necessary to note that for $n > 0$ poynting vector $\vec{S}$ and $\vec{k}$ are parallel ,and for $n < 0$ they are anti-parallel. It was also generally accepted in the physical optics that the magnetization at optical frequencies is negligible and, hence, did not play any essential role. In accordance with this, the magnetic permeability μ was normally set to be equal to one in the basic Maxwell’s equations, describing the linear and nonlinear optical processes [6]. The concept of the optical length is connected with the total phase wind of the wave that depends on the index of refraction $n$, which defines the phase velocity of light, rather than the group velocity. As it is well known, group and phase velocities are different in dispersive media.

The use of materials with negative values of $\varepsilon$, $\mu$ and $n$ is not the only way to obtain backward waves and hereby negative refraction. Moreover, backward waves exist in many systems that cannot be described by negative permeability and negative index of refraction. The well known backward waves in vacuum electronic devices are a typical example. The discovery of NIMs with their surprising electrodynamic properties attracted much interest and also provoked statements of questionable value. Metamaterials, i.e., artificially designed and engineered materials may have properties unattainable in nature, including a negative refractive index.

## 1.1. Negative refraction

In a NIM, the negative phase velocity has direct implications on the refraction of a light beam at the boundary of two media. Whereas in the ordinary case, the refracted ray follows the regular path and propagates on the other side of the normal perpendicular to the surface (Fig-1.1A), in the case where one of the media is a NIM, the refracted ray undergoes 'negative refraction' and both beams stay on the same side of the normal (Fig-1.1B).


**Fig-1.1.** ***A straw in a glass of water seems disjointed because of refraction (A). But in this rough mock-up of what would happen if water had a negative refractive index (B), the effect is startling. The underside of the water's surface can be seen but not the bottom of the glass.***

The physical picture described above was introduced by Victor Veselago in his 1967 work, along with the concept of 'left -handed materials' and related phenomena. However, some of its elements find their origins in the beginning of the twentieth century. Note that regardless of the signs of $\varepsilon$ and $\mu$, the fundamental Maxwell's equations, and their corresponding boundary conditions for refraction of light, can be formally satisfied by both positive and negative refraction solutions. The appropriate solution is only selected by the additional requirement that in the refracted beam the energy flows away from the interface. In the regular material, this corresponds to the normal refraction, whereas for the negative index media the solution of Maxwell's equations must be chosen such that it leads to negative refraction.

Maxwell's equations determine how electromagnetic waves propagate within a medium and can be solved to arrive at a wave equation of the form, $\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial^2 \vec{r}} = \varepsilon\mu \frac{\partial^2 \vec{E}(\vec{r},t)}{\partial^2 t}$. In this equation $\varepsilon$ and $\mu$ enter as a product and it would not appear to matter whether the signs of $\varepsilon$ and $\mu$ were both positive or were both negative. Indeed, solutions of the wave equation have the form $E(z,t) \approx e^{\left[i\omega\left(n\frac{z}{c}-t\right)\right]}$ where $n=\sqrt{\varepsilon\mu}$ is the refractive index. Propagating solutions exist in the material whether ε and μ are both positive or are both negative. So what, if anything, is the difference between positive and negative materials? It turns out that we need to be more careful in taking the square root, as $\varepsilon$ and $\mu$ are analytic functions that are generally complex valued. There is an ambiguity in the sign of the square root that is resolved by a proper analysis. For example, if instead of writing $\varepsilon=-1$ and $\mu=-1$ we write $\varepsilon=\exp(i\pi)$ and $\mu=\exp(i\pi)$, then

$$n=\sqrt{\varepsilon\mu} = \exp\left(i\frac{\pi}{2}\right)\exp\left(i\frac{\pi}{2}\right)=\exp(i\pi)=-1.$$

The important step is that the square root of either $\varepsilon$ or $\mu$ alone must have a positive imaginary part. This is necessary for a passive material. This briefly stated argument shows why the material Veselago pondered years ago is so unique: the index of refraction is negative. A negative refractive index implies that the phase of a wave advancing through the medium will be negative rather than positive. As Veselago pointed out, this fundamental reversal of wave propagation contains important implications for nearly all electromagnetic phenomena. Many of the exotic effects of negative index have been or are currently being pursued by researchers.

The concept of negative refraction has also been generalized to transmission line structures, common in electrical engineering applications. An experiments and applications have shown that the material Veselago hypothesized more than thirty-five years ago can now be realized using artificially constructed metamaterials, making discussion of negative refractive index more than a theoretical curiosity. The question of whether such a material can exist has been answered, turning the development of negative index structures into a topic of materials or metamaterials physics. As metamaterials are being designed and improved, we are now free to consider the ramifications associated with a negative index of refraction. This material property, perhaps because it is so simply stated, has enabled the rapid design of new electromagnetic structures some of them with very unusual and exotic properties [19].

# CHAPTER TWO

# ELECTROMAGNETIC PROERTIES OF MATERIALS WITH NEGATIVE ε AND μ

The main question now is how the electrodynamics of the materials with negative ε and μ differ from the electrodynamics of materials with positive ε and μ.

There are three possible answers to this question:-

(1) There are no differences, i.e., electrodynamics is invariant with regard to the simultaneous change the signs of ε and μ.

(2) Simultaneously negative values of ε and μ are in principle impossible because this conflicts with some basic principles.

(3) Simultaneously negative values of ε and μ are possible, but the electrodynamics of such materials differ from electrodynamics for the case of positive ε and μ.

In most equations of electrodynamics the values of ε and μ are present as a product, εμ for which a simultaneous change of the signs of both multiplicands is not essential. This completely pertains also to the value of the refractions index *n*. However, from general considerations it is obvious that a simultaneous change of the signs of ε and μ must bring about some essential changes to electrodynamics characteristics of material. From the above possible answers to the question it is easy to show that the answer **3** is correct. Let us consider the Maxwell curl equations:

$$\nabla \times E(\vec{r},t) = -\frac{1}{c}\frac{\partial B(\vec{r},t)}{\partial t} \qquad (2.1)$$

$$\nabla \times H(\vec{r},t) = \frac{1}{c}\frac{\partial D(\vec{r},t)}{\partial t} \qquad (2.2)$$

Where $E(\vec{r},t)$ is a variable electric field, $D(\vec{r},t)$ is the electric induction, $H(\vec{r},t)$ is the magnetic field, and $B(\vec{r},t)$ is the magnetic induction.

after taking Fourier transform for uniform plane waves along with material equations i.e.

$$E(\vec{r},t) = \frac{1}{(2\pi)^4}\int E(\vec{k},\omega)e^{i\vec{k}.\vec{r}-i\omega t}d^3kd\,\omega \qquad (2.3)$$

$$H(\vec{r},t) = \frac{1}{(2\pi)^4}\int H(\vec{k},\omega)e^{i\vec{k}.\vec{r}-i\omega t}d^3kd\omega \quad (2.4)$$

and the general case of material equations;

$$D = \hat{\varepsilon} E \quad (2.5)$$

$$B = \hat{\mu} H \quad (2.6)$$

Where $\hat{\varepsilon}$ and $\hat{\mu}$ are the tensors of electric permittivity and magnetic permeability.

But for isotropic media $\varepsilon(\vec{k},\omega)$ and $\mu(\vec{k},\omega)$ are scalars, which depends on the frequency $\omega$ and wave vector $\vec{k}$. Here we note that dependence on frequency $\omega$ is called the time dispersion and dependence on wave vector *k* is called the space dispersion.

Now we consider isotropic medium that is the case of scalar ε and μ by taking the above Fourier transform of the Maxwell equations we obtain:

$$\vec{K} \times \vec{E} = \mu \frac{\omega}{C} \vec{H} \quad (2.7)$$

$$\vec{K} \times \vec{H} = -\varepsilon \frac{\omega}{C} \vec{E} \quad (2.8)$$

One can immediately see from the above (2.7) and (2.8) equations that the vectors $E, H$ and $K$ form a right-handed triple of vectors for positive ε and μ, for ε < *0* and μ < 0 these vectors form a left-handed triple of vectors . For this reason, NIMs (Negative index materials) are frequently called Left-Handed Materials (LHMs). It is known that the Poynting vector:

$$S = \frac{C}{4\pi}[E \times H], \quad (2.9)$$

always forms a right-handed triple of vectors together with the vectors $E, H$. The direction of the phase velocity $v_{ph}$ of the wave coincides with the direction of the wave vector $K$, whereas the direction of the group velocity $v_{gr}$ complies with the direction of the vector $S$. Thus, it is obvious that the phase and the group velocities are antiparallel, when ε and μ are simultaneously negative. The inverse statement holds: When the phase and the group velocities of an isotropic medium are antiparallel, the medium is characterized by negative values of ε and μ [7].

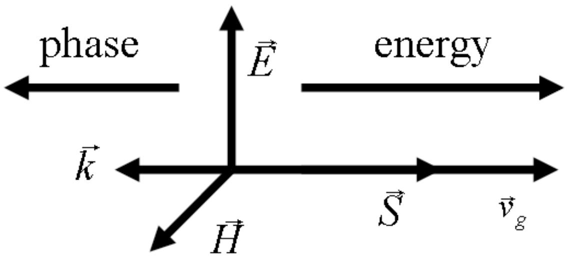


**Fig.2.1.*****Field vectors, wave vector and Poynting vector in left-handed medium***

The fact that for negative ε and μ the phase and group velocities are oppositely directed, leads to a conclusion that some fundamental laws of electrodynamics and optics are expressed in an unusual way, among these laws we will discuss the Doppler effect, Cherenkov effects, and Snell's law [27].

## 2.1. Snell's Law

From the study of refraction in geometrical optics, we know that light rays incident on an interface between two transparent materials are refracted at an angle determined by the incident angle and by the phase velocities of light in the two materials. The ratio of the phase velocity of light in vacuum to that in a material is the refractive index ***n***:

$n = \frac{C}{v_\phi}$ , where c = $3 \text{x} 10^8 \frac{m}{s}$ is the velocity of light in vacuum and the "phase velocity" $v_\phi$ in the medium is the ratio of the angular frequency ω and the "wave number" k, which is equal to the product of the wavelength λ and the temporal frequency $f$:

$$v_\phi = \frac{\omega}{k} = \lambda f$$

By combining the two equations, we obtain an expression for the refractive index in terms of the vacuum velocity c, the angular frequency ω, and the wave number k.

$$n = C\left(\frac{k}{\omega}\right) \qquad (2.10)$$

Snell's law relates the indices of refraction to the angles of light rays in two media. If the angles of the incident and refracted rays are $\theta_1$ and $\theta_2$, respectively (measured from the vector normal to the surface), then Snell's law is:

$$n_1 \sin[\theta_1] = n_2 \sin[\theta_2]$$

$$\Rightarrow \quad \frac{\sin[\theta_1]}{\sin[\theta_2]} = \frac{n_2}{n_1} \qquad (2.11)$$

Snell's law is one of the oldest and most well known of electromagnetic phenomena, quantitatively it describes the bending of a wave as it enters a medium. It is also the basis for a direct measurement of a material's refractive index [8]. A plot of a wave which is incident on the flat side of a wedge shaped sample is transmitted through the transparent sample, striking the second interface at an angle is shown below:

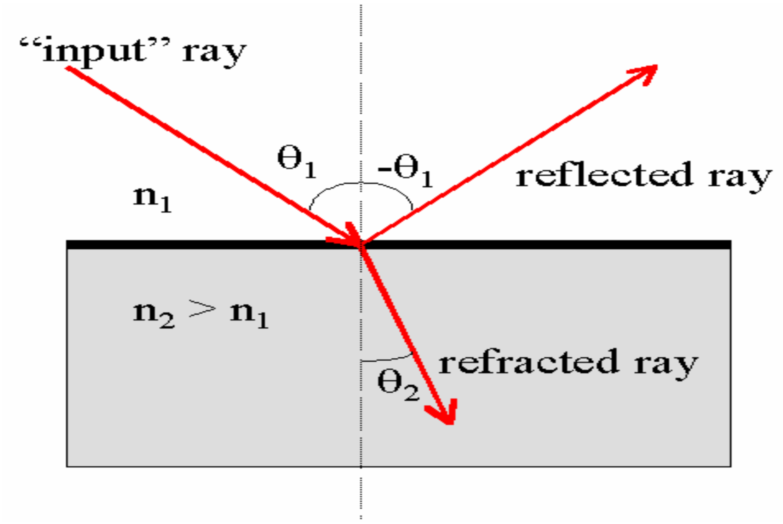


**Fig -2.2.** ***The above fig shows Illustration of Snell's Law***.

In the passage of a ray of light from one medium into another the boundary conditions

$$E_{t_1} = E_{t_2} \qquad H_{t_1} = H_{t_2}\,, \qquad (2.12)$$

$$\varepsilon_1 E_{n_1} = \varepsilon_2 E_{n_2} \qquad \mu_1 H_{n_1} = \mu_2 H_{n_2} \qquad (2.13)$$

must be satisfied, independently of whether or not the media have the same rightness. It follows from (2.12) that the *x* and *y* components of the fields E and H in the refracted ray maintain their directions, independently of the rightnesses of the two media. As for the *Z* component, it keeps the same directions only if the two media are of the same rightness. If the rightness are different, the *Z* component change sign. This corresponds to the fact that in passage in to a medium of different rightness the vector E and H not only change in magnitude owing to the difference in $\varepsilon$ and $\mu$ but also under go a reflection relative to the interface of the two media .the same thing happens to the vector K also.

The simultaneous reflection of all three vectors corresponds precisely to a change of sign of the rightness of the given medium [2]. Because of the difference in refractive index between the material and free space, the beam exits the wedge deflected by some angle from the direction of incidence. One might imagine that an experimental determination of Snell's law should be a simple matter; however, the peculiarities of Metamaterials add a layer of complexity that renders the experimental confirmation somewhat more difficult. Antiparallel phase and group velocities immediately affect Snell's law as illustrated below;

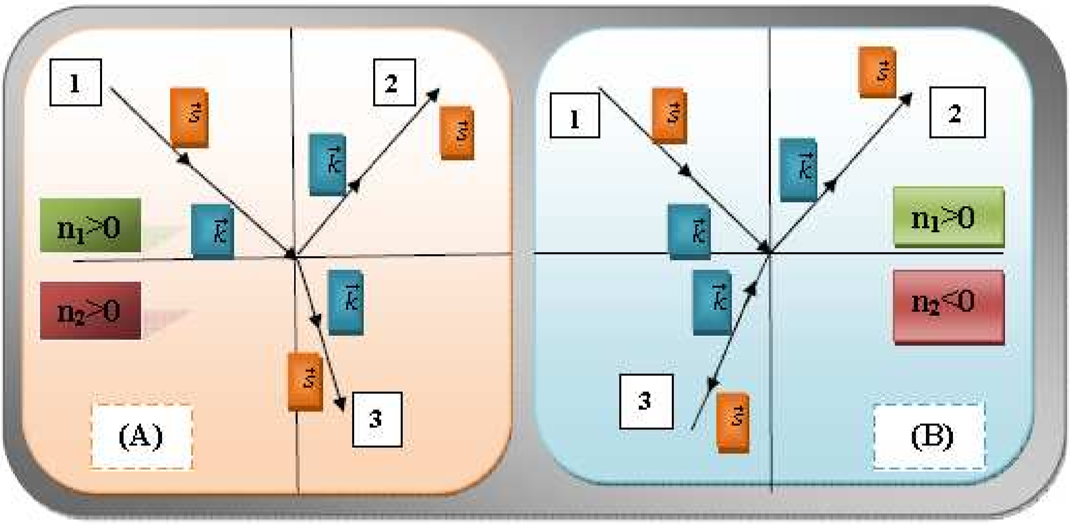


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**Fig-2.3.** ***The beam path at refraction on the boundary of vacuum (n) and material with positive refraction index (n>0) (A) and negative one (n<0) (B).1-incident beams, 2-reflected beams, 3 –refracting beams. S- Poynting vector, and wave vector K.***

The path of the refracted ray produced as the result of such reflections is shown in figure above. When the second medium is left-handed the refracted ray lies on the opposite side of the *z*- axis from its position in the case of a right handed second medium. It must be noted that the direction of the reflected ray is always the same, independent of the rightnesses of the two media.

For positive ε and μ the ray propagates along the way *1–3 in part (A)* through the interface between two media. If one of the media has negative ε and *μ*, the ray propagates along the way *1–3 in part (B)***.** This unusual propagation of the ray is a consequence of the opposite direction of the vectors $\vec{v}_{ph}$ and $\vec{v}_{gr}$ and of the continuity of the tangential components of the wave vector on the interface between the two media. If we want to keep the usual notation of Snell's law:

$$\frac{\sin\ \varphi}{\sin\ \phi} = \frac{n_2}{n_1},$$

also for negative ε and μ, we must accept that the index of refraction is negative when ε and μ are simultaneously negative or when the vectors of the phase and group velocities are anti-parallel [9].

## 2.2. Traveling Waves and the Doppler Effect

The frequency measured by an observer may be different from that emitted from the wave source due to the relative motion between them. This phenomenon is called Doppler Effect. For the electromagnetic waves (we simply call them light in the following text) we have different Doppler Effect equation from the classical (sound wave) because**;**

1) Propagation of light doesn't need medium so the light velocity relative to the observer is always the same;
2) The period of the light may change for the observers at different initial frame of reference (and time dilation).

We start the discussion from the simplest case where the velocities of observer and source are along the wave vector between source and observer.

### 2.2.1. Moving observer

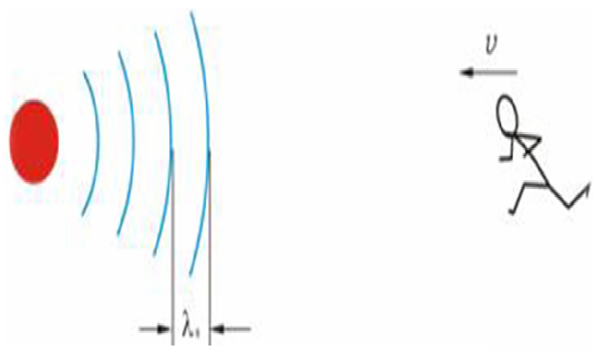


**Fig-2.4**. ***Stationary light sources and moving observer***.

As shown in **Fig. 3**, the observer moves with velocity *v* *toward* the light source. In the inertial frame of reference of light source, **K**, the time for the observer needs to pass two crests (one wavelength) is (the period in *K*)

$$T = \frac{\lambda_s}{c+v} = \frac{1}{\left(1+\frac{v}{c}\right)f_s} \qquad (2.14),$$

where we use $\frac{1}{f_s} = \frac{\lambda_s}{c}$ and

$\lambda_s$ is the wave length of the source

$v$ is the velocity of an observer

$c$ is the speed of light in vacuum

$f_s$ is the frequency of the source .

If this period, $T$, is the same as $T_0$ that measured in the frame of the observer **K'**, then this is exactly the same Doppler effect equation as in sound (classical) waves. However, because of time dilation, the time measured by the observer is:

$$T_0 = \frac{T}{\gamma} = \frac{1}{\gamma\left(1+\frac{v}{c}\right)f_s}, \qquad (2.15)$$

Where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ from this $T_0 = \frac{1}{f_s}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$ and the corresponding frequency is

$$f_0 = f_s\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}, \qquad (2.16)$$

$f_0$ is the frequency of the observer.

### 2.2.2. Moving light source

In the case that the light source moves with velocity *v* toward the observer, then in the frame of reference of the observer, *K'*, the time interval between emission of successive wave crests, *T*, is different from the time interval between the arrival of successive crests because the crests are emitted at different locations. Therefore, the frequency measured by the observer $f_0$, is not $1/T$. During the time *T* the crests ahead of the source move a distance $x_c = cT$, and the source moves a shorter distance $x_s = vT$ in the same direction. So the wavelength becomes $\lambda = x_c - x_s = (c-v)T$, as measured in the observer's frame of reference. The frequency that the observer measures is $f_0 = c/\lambda$. That is,

$$f_0 = \frac{c}{\lambda} = \frac{c}{(c-v)T} = \frac{1}{(1-v/c)T} \qquad (2.17)$$

where $x_c$ is the distance between two successive emitted crests, $x_s$ is the shorter distance moved by the source $f_0$ is the frequency of the observer. Because of the time dilation the transformation between $T_s$ and T *is* given by:

$$T = \gamma T_s = \frac{T_s}{\sqrt{1 - v^2/c^2}} = \frac{1}{f_s\sqrt{1 - v^2/c^2}} \qquad (2.18)$$

Substituting eq(2.18) to Eq. (2.17) yields eq(2.16) again. Therefore, with the light, unlike the sound (classical), there is no distinction between motion of source and motion of observer, only the relative velocity of the two matters. Usually in none relativistic case $v<<c$, so eq(2.16) can be expanded to Taylor series and to the first order, i.e.

$f_0 = f_s\sqrt{\frac{1+\beta}{1-\beta}}$ where $\beta = \frac{v}{c}$ , Therefore $f_0 \approx f_s\sqrt{1+2\beta} = f_s(1+\beta)$

$$f_0 = f_s(1+\frac{v}{c}) \ , \qquad (2.19)$$

Obviously, eq(2.19) shows that when the speed is low compared to $c$, measured frequency is proportional to the relative speed [10]. Similarly from the law of transformation of the 4-vector wave we can consider the Doppler effect, let $\omega_0$ is a frequency of light (Electromagnetic radiation) in the stationary coordinate system $k_0$. Let $v$ be a speed of the source or the velocity of frame of reference $k_0$ with respect to the moving system k. The velocity of the rest system with respect to the moving one is $-v$.

Consider the 4-wave vector of $\vec{k}\,(k_x = k_1, k_y = k_2, k_z = k_3, \frac{\omega}{c} = k_4)$

The transformation law of 4-vector for wave vector is given by:

$$k_1 = \frac{k'_1 - \frac{v}{c}k'_4}{\sqrt{1-(\frac{v}{c})^2}}$$ and

$$k_4(stationary\ \ system) = \frac{k'_4 - \frac{v}{c}k'_1}{\sqrt{1-(\frac{v}{c})^2}} \qquad (2.20)$$

If $v$ is along x-axis The $k = \frac{\omega}{c}$ Dispersion for electromagnetic wave is

$k'_x = \frac{\omega'}{c}\cos\alpha$ .Thus the stationary state transformation ($k_4$) becomes:

$$\frac{\omega}{c} = \frac{\frac{\omega'}{c} - \frac{v}{c} k'_x}{\sqrt{1 - (\frac{v}{c})^2}} \quad , \quad \omega = \frac{\omega' - v \frac{\omega'}{c} \cos \alpha}{\sqrt{1 - (\frac{v}{c})^2}}$$

$$\omega \sqrt{1 - (\frac{v}{c})^2} = \omega' (1 - \frac{v}{c} \cos \alpha)$$

at the stationary system $\omega = \omega_0$ and for $\beta = \frac{v}{c}$

$$\omega' = \omega_0 \frac{\sqrt{1 - (\frac{v}{c})^2}}{(1 - \frac{v}{c} \cos \alpha)} = \omega_0 \frac{\sqrt{1 - \beta^2}}{(1 - \beta \cos \alpha)} ,$$

this is the general result for the frequency change at moving and stationary systems. To make it consistence with previous result it is possible to consider particular cases.

Particular cases:

If α =0 light is emitted the same direction with $v$ along x-axis

$$\omega' = \omega_0 \frac{\sqrt{(1+\beta)(1-\beta)}}{1-\beta} = \omega_0 \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\frac{1+\beta}{1-\beta} \approx (1+\beta)(1+\beta) = 1 + 2\beta + \beta^2 = 1 + 2\beta \qquad \text{Since } \beta^2 << 1 .$$

$$\omega' = \omega_0 \sqrt{\frac{1+\beta}{1-\beta}} \approx \omega_0 \sqrt{1+2\beta} = \omega_0 (1+\beta)$$

$$\omega' = \omega_0 (1 + \frac{v}{c}) \quad , \qquad (2.21)$$

this is the classical limit.

If $\alpha = \frac{\pi}{2}$ $\omega' = \omega_0 \sqrt{1-\beta^2} \approx \omega_0 (1 - \frac{1}{2}\beta^2)$ this is the transverse effect.

For usual materials with positive ε and μ, a traveling wave, radiated from an energy source, can be written in form, which is proportional to the value:

$$e^{i\vec{k}.\vec{r} - i\omega t} = e^{i\omega(\frac{\vec{k}c}{\omega}\frac{\vec{r}}{c} - t)} = e^{i\omega[n\frac{\vec{r}}{c} - t]} ,$$

Where $n$ is index of refraction given by: $n = \frac{c}{v_\phi} = \frac{c}{\omega / \vec{k}} = \frac{\vec{k}\, c}{\omega}$ , and

$$v_\phi = v_{ph} = \omega / k \ .$$

The refractive index is a complex number i.e. $n = n' + in''$, where the imaginary part $n''$ characterizes light extinction (losses). The real part of the refractive index $n'$ gives the factor by which the phase velocity of light is decreased in a material as compared with vacuum. The wave vector in this expression is $K = \frac{\omega}{C} n = \frac{\omega}{C}\sqrt{\varepsilon\mu}$ , and the value of the refraction index $n$ is taken to be positive. This means that the phase velocity $v_{ph} = \frac{C}{\sqrt{\varepsilon\mu}}$ is also positive. If both ε and μ are negative, the index of refraction $n$ also must be taken negative, and this brings about negative phase velocity. The appearance of a negative factor of refraction requires rewriting of (1.2) in more general way,

$$n = \pm\sqrt{\varepsilon\mu} \qquad (2.22).$$

Here the positive sign is used for the usual case, i.e. ($\varepsilon > 0$ and $\mu > 0$), whereas the negative sign is used when ($\varepsilon < 0$ and $\mu < 0$) or both negative. The inverse motion of the sinusoidal wave for materials with negative refraction changes the sign of the Doppler effect, as it can be seen on Fig-5. Suppose a detector of radiation which is in a left-handed medium moves relative to the source which emits a frequency $\omega_0$. In its motion the detector will pursue points of the wave which correspond to some definite phase, as it is shown in fig below. The frequency received by the detector will be smaller than $\omega_0$, not larger as it would be in ordinary (right handed) medium. This becomes clear from the formula for the classical Doppler effect eq(2.21):

$$\omega = \omega_0 (1 + \frac{v}{c/n}) \ ,$$

Where $c/n$ is the velocity of light in a given medium.

$$\omega = \omega_0 (1 - |n| \frac{v}{c}) \ , \qquad (2.23)$$

for $n$ has negative value and , $\omega_0$ is the frequency of source at rest. Here the velocity $v$ of the detector is regarded as positive when it receding from the source. The velocity of the energy flux is regarded as always positive.

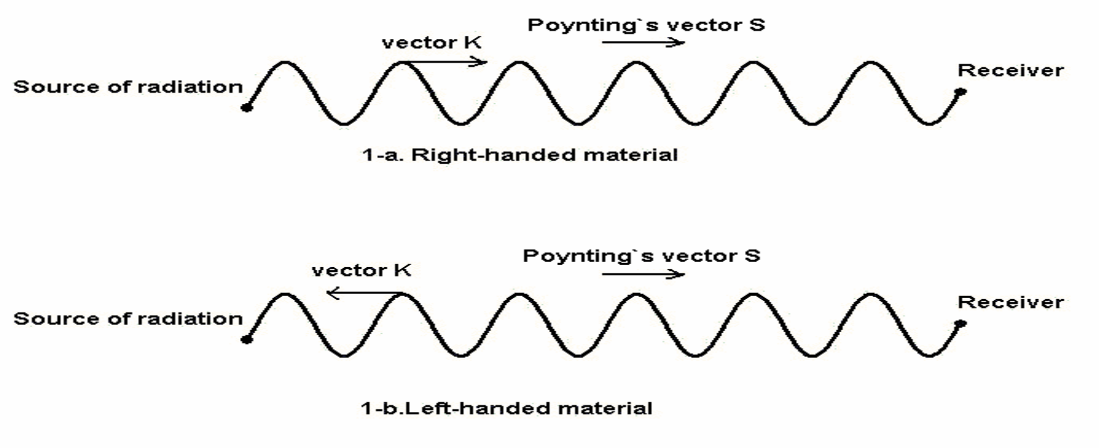


**Fig-2.5.** ***Poynting's vector $\vec{s}$ and the wave vector $\vec{k}$ for wave propagations in right-handed and left- handed materials.***

For the usual materials when the receiver moves to the source, the observed frequency increases, but for NIM (negative index) materials it decreases [9].

## 2.3. Cherenkov Effect

Cherenkov radiation is electromagnetic radiation emitted when a charged particle (such as an electron) passes through an insulator at a constant speed greater than the speed of light in that medium. As a charged particle travels, it disturbs the local electromagnetic field (EM) in its medium. Electrons in the atoms of the medium will be displaced, and the atoms become polarized by the passing EM field of a charged particle. Photons are emitted as an insulator's electrons restore themselves to equilibrium after the disruption has passed. (In a conductor, the EM disruption can be restored without emitting a photon.) In normal circumstances, these photons destructively interfere with each other and no radiation is detected. However, when a disruption which travels faster than light is propagating through the medium, the photons constructively interfere and intensify the observed radiation.

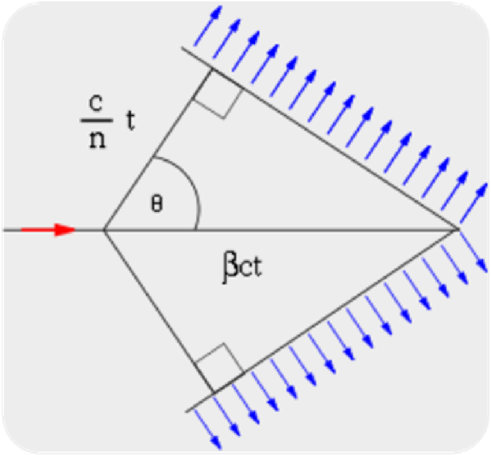


**Fig-2.6** *.The geometry of the Cherenkov radiation (shown for the ideal case of no dispersion).*

In the figure, the particle (red arrow) travels in a medium with speed $v_p$ and we define the ratio between the speed of the particle and the speed of light as $\beta = \frac{v_p}{c}$ where $c$ is speed of light. If $n$ is the refractive index of the medium, then the emitted light waves (blue arrows) travel at speed $v_{em} = \frac{c}{n}$. The left corner of the triangle represents the location of the superluminal particle at some initial moment ($t$=0). The right corner of the triangle is the location of the particle at some later time t. In the given time $t$, the particle travels the distance;

$$x_p = v_p t = \beta ct \quad ,$$

whereas the emitted electromagnetic waves are constricted to travel the distance

$$x_{em} = v_{em} t = \frac{c}{n} t \qquad \text{so,} \ \cos\theta = \frac{1}{n\beta} = \frac{c}{nv_p} \ .$$

Note that since this ratio is independent of time, one can take arbitrary times and achieve similar triangles. The angle stays the same, meaning that subsequent waves generated between the initial time $t$=0 and final time $t$ will form similar triangles with coinciding right end points to the one shown [11].

### 2.3.1. Backward-Wave radiation

One more effect, whose realization greatly depends on simultaneous change of the signs of ε and μ, is the Cherenkov effect. As described above the angle of the radiated conical wave front is given by the velocity of the particle $v_p$ with respect to the phase velocity of EM waves $v_{ph}$ within the medium in the following manner:

$$\cos\theta = \frac{v_{ph}}{v_p} = \frac{c/n}{v_p} \qquad (2.24)$$

Where $n$ *is* the refractive index of the surrounding medium, $c$ is the speed of light in a vacuum, and θ is the angle between the particle velocity and the radiated EM wave front. It is clear from the logic of the above derivation that the conclusion on the direction of the emission tacitly assumed that the group velocity $v_{gr}$ corresponding to the wave vector k was positive, that is, directed along k (this situation is depicted in Fig. 2.7a). If, instead, the group velocity was negative, i.e., $v_{gr}$ was directed opposite to k, the direction of the emission (the energy flow S) would in fact be opposite. The radiation in the latter case forms an obtuse angle with the direction of the particle motion, as was first discussed by Pafomov [23]. The Cherenkov radiation emitted backwards is shown in Fig. 2.7b. It is distributed over the surface of the cone with the same vertical angle. This implies that radiation is directed backward rather than forward as is the case in a usual material (RHM). If the index of refractions becomes negative, it is obvious that angle lies in the second quadrant, and, hereunder, the cone of the Cherenkov radiation is directed back [9].

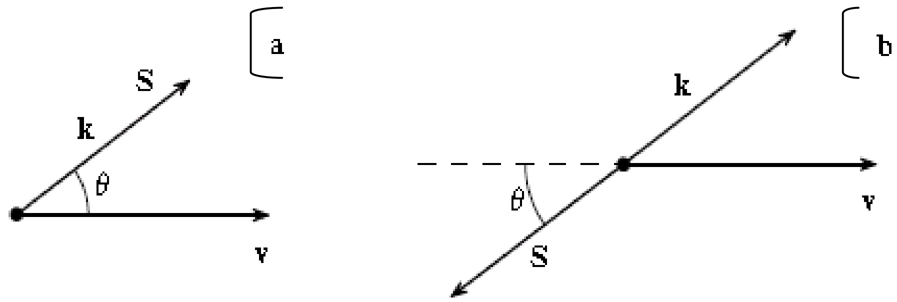


**Fig-2.7.** ***Illustration on the direction of the Cherenkov radiation in a medium with (a) positive and (b) negative group velocity.***

Here, $v$ denotes the direction of the particle velocity, k the direction of the wave vector of the emitted radiation, and S the direction of the Poynting vector. The vector S is directed along the group velocity $v_{gr}$ and determines the actual direction of the emission. As It is already explained in the above the choice of the sign of the $n$ will assure that the energy moves away from the radiating particle to infinity. Then it is clear that for left handed media the vector k will be directed toward the trajectory of the particle and the con of the radiation will be directed backward relative to the motion of the particle. This corresponds to an obtuse angle θ between $v$ and $s$ or $v_{gr}$ [15].

# CHAPTER THREE

# MODERN ARTIFICIAL MATERIALS WITH NEGATIVE $\varepsilon$ AND $\mu$

## 3.1. Electromagnetic Metamaterials

The absence of natural materials with the properties $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ led to neglect of the subject until 1999 when it was shown how to make artificial materials, known as "**Metamaterials**", with negative $\mu(\omega)$. The word 'meta' means 'beyond' in Greek and in this sense the name **"Metamaterials"** refers to 'beyond conventional materials. Metamaterials are typically man-made and have properties that are not found in nature. To understand Metamaterials, it is necessary to understand response in homogeneous materials governed by two parameters to EM waves in general. One of these parameters, is $\varepsilon(\omega)$, describes the response of a material to the electric component of light (EM wave) and the other, $\mu(\omega)$, to the magnetic component at a frequency $\omega$. Both of these parameters are typically frequency-dependent complex quantities, that completely describe the response of an isotropic material to EM radiation at a given frequency,

$$\varepsilon(\omega) = \varepsilon_r(\omega) + i\varepsilon_i(\omega) \qquad (3.1)$$

$$\mu(\omega) = \mu_r(\omega) + i\mu_i(\omega) \qquad (3.2)$$

The imaginary parts of $\varepsilon$ and $\mu$ are responsible for the decay of EM waves. A commonly used EM parameter is that of the index of refraction, which is defined as $n(\omega)^2 = \varepsilon(\omega)\mu(\omega)$. The index of refraction provides a measure of the speed of an EM wave as it propagates within a material. In addition, the refractive index also provides a measure of the deflection of a beam of light as it crosses the interface between two materials having different values for their refractive indices. Also a material's refractive index depends on its response to the electric and magnetic components of an electromagnetic wave. Veselago realized that if a material were found that had negative values for both the electric and magnetic response functions , (i.e. $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$), then its index of refraction would also be negative , n(ω) < 0. Because of artificially structured MMs can have controlled magnetic and electric responses over a broad frequency range, it is possible to achieve the condition ε < 0 and μ < 0 in artificial composites and Veselago's hypothesized material can, indeed, be realized.

Permittivity and permeability of conventional materials derive from the response of constituent atoms to applied fields and ε, μ represent an average response of the system. On a length scale much greater than the separation between atoms all we need to know about the system is given by ε, μ. Metamaterials carry this idea one step further; the constituent material is structured into subunits, and on a length scale much greater than that of the sub-units properties are again determined by an effective permeability and permittivity valid on a length scale greater than the size of the constituent units. In the case of electromagnetic radiation this usually means that the subunits must be much smaller than the wavelength of radiation [13].

The idea behind making a left-handed Metamaterials (LHMMs) is to treat electric and magnetic properties separately. Essentially, a LHM is an assembly of two kinds of cell elements. Split ring resonators (SRR) produce negative μ and a wire array (capacitively loaded strips, CLS) produce negative ε [27] . As a matter of fact, although it is known that the SRR itself does respond to the electric field, the frequencies associated with this response usually does not overlap with the frequency response due to the magnetic field. The wire medium and the SRR medium represent the two basic building blocks, one electric the other magnetic, for a large range of metamaterial response. The addition of the wires to the SRR medium does not affect the permeability properties of the SRR medium. However, the combination of the SRRs and wires results in a permittivity that is less negative than that for wires alone. A metamaterial formed by combining a wire medium with a SRR medium was used to demonstrate negative refraction, and the angle of refraction was utilized to recover *n* as a function of frequency. While a refraction experiment is a useful method to measure the index.

## 3.2. Magnetic response of MMs

When studying the interaction of an EM wave with a material it is impossible to take the response of each atom or electron into account. Instead, we rely on macroscopic parameters such as the index of refraction ($n$), the permittivity ($\varepsilon$) and the permeability ($\mu$), to replace the details of structure in the long wavelength limit. The split ring resonator (SRRs) have attracted a great interest in recent years as key constituent particles for the design of effective media with negative magnetic permeability or left-handed metamaterials (LHMs) [26]. It has been the element typically used for response to the magnetic component of the EM field. This *split ring resonator* (SRR), in its various forms, can be viewed as the Metamaterials equivalent of a magnetic atom. This ‘magnetic atom’ was proposed by Pendry in 1999. The (SRRs) are subwavelength resonators (i.e., electrically very small) that are able to inhibit signal propagation in a narrow band in the vicinity of their resonant frequency [26]. The SRRs consists of a planar set of concentric rings, each ring with a gap. When a wave propagates through a SRR artificial media, its magnetic field induces currents in the rings, creating the magnetic response, while, the dielectric response is negligible ($\varepsilon$=1). From a circuit point of view, a time varying magnetic field induces an electromotive force in the plane of the element, driving currents within the conductor. A gap in the plane of the structure introduces capacitance into the planar circuit, giving rise to a resonance at a frequency set by the geometry of the element.

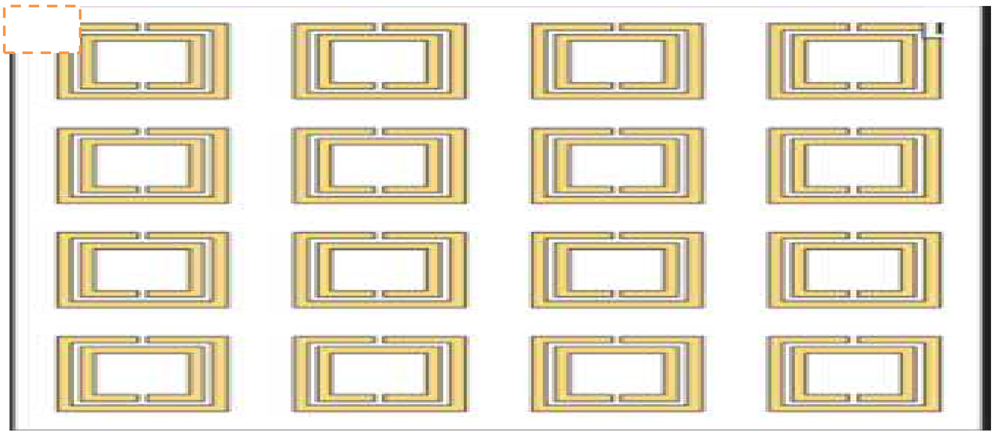

**Fig -3.1.** ***A SRR elements used for construction of MMs arranged in to an array behaves as an effective Material and described by a magnetic response***.

There are two important issues to consider when designing magnetic structures. First is the ‘activity’ of the structure, broadly speaking the range of frequencies over which a negative response is measured, and the amount of loss present as indicated by $\mu_i$ . The activity is a function of the cross section presented to magnetic lines of force. If a large number of the lines of force passing through the material are intersected by the rings, then the activity will be high. Therefore it is important to engineer a high filling fraction. The advantage of large activity is that the region of negative $\mu_r$ may then extend to high frequencies where losses are small. Loss is always an issue in negative materials because it attenuates the effects we seek to create.

Curiously the main source of loss encountered appears to be the surrounding dielectric material in the structure rather than the resistivity of the metal. High quality dielectric substrates are vital to good performance. With a highly active structure and low loss components the critical value of $\mu_r = -1$ can be associated with very small values of loss as measured by $\mu_i$ [13].

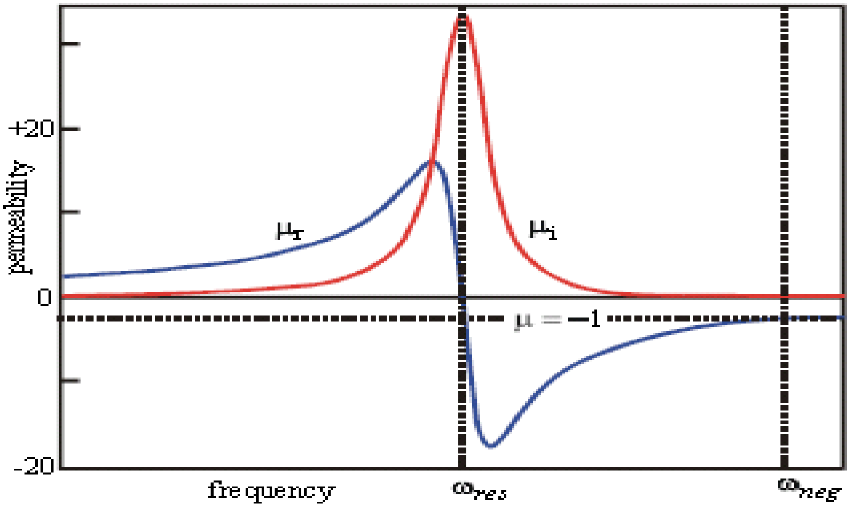


**Fig-3.2**.***Schematic permeability of the magnetic metamaterial shown in Fig-3.1showing the resonant response of the structure at $\omega_{res}$ . Note that the frequency at which $\mu_r = -1$ is far removed from the resonant frequency and in this instance is in a region where $\mu_i$ is small.***

The split ring structure has the philosophy that first we design a resonant element with a magnetic response, and then we assemble these elements into a 2D or 3D structure. However a critical aspect of magnetic activity is the ability of the element to capture a large cross section of incident magnetic flux and this may be limited by the design. For example cylinders of circular cross section cannot be closer packed than the hexagonal close packed structure. This limits the activity of the structure.

A time varying magnetic field polarized perpendicular to the plane of the SRR will induce circulating currents according to Faraday's law. Because of the split gap in the SRR, this circulating current will result in a buildup of charge across the gap with the energy stored as a capacitance. The SRR can thus be viewed as a simple LC circuit, with a resonance frequency of $\omega_0 \sim \sqrt{\frac{1}{LC}}$, where the inductance results from the current path of the SRR . For frequencies below $\omega_0$, currents in the SRR can keep up with the driving force produced by the externally varying magnetic field and a positive response is achieved. However, as the rate of change (frequency) of the external magnetic B-field is increased, the currents can no longer keep up and eventually begin to lag, resulting in an out-of-phase or negative response [12]. The electromagnetic response of SRRs excited by a time-varying magnetic field parallel to the ring axis results from a resonant exchange of energy between the electrostatic fields in the capacitive gaps (splits) and the inductive currents in the rings [27].

### 3.3. Electric response of MMs

Naturally occurring materials that yield a negative response to the electric component of light have been known for several decades. Any metal below its plasma frequency (the frequency at which it becomes transparent) yields negative values of the permittivity. This $\varepsilon_r < 0$ response results from the free electrons in the metal that screen external EM radiation. Many decades ago researchers fabricated structures having $\varepsilon < 0$ using arrays of conducting wires and other unique shapes. This technology was recently reintroduced with a more physics-oriented understanding. Currently, variations of the wire lattice being used to create $\varepsilon_r < 0$ media include straight wires, cut-wire segments, and loop wires. In the context of NIMMs, the wire lattice and its variants are a convenient means of achieving a medium for which $\varepsilon_r < 0$ [12].

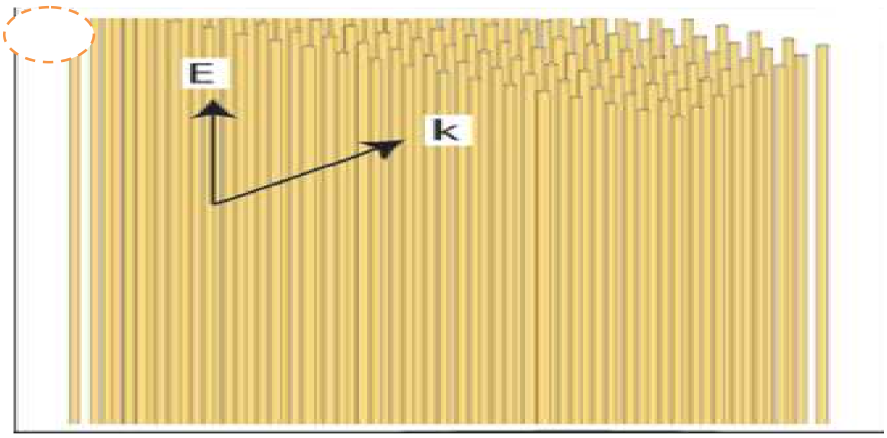


**Fig-3.3.** ***The straight wire medium shown in the fig above is a medium used for electric response***

In addition to a straight wire medium depicted in Fig-3.3 above, there have been further advances in the development of electric MMs with new designs analogous to the SRR. Particular flexibility is given to us in the design of these materials by the very large difference in conductivity between insulators and metals. Therefore all the metamaterials discussed here will owe their activity to the metallic content.

A wire arrays are well-known high-pass filters for EM waves polarized with *E* parallel to the wires. It does not allow propagation in a certain frequency range because of negative ε**.** The required negative permittivity comes from the presence of the wires, which behave as microwave plasma up to a frequency (plasma frequency) that depends on the wires' diameter and separation. By simply tailoring the wire medium to exhibit the plasma frequency above the resonant frequency of SRRs, a left-handed behavior is expected in a narrow band just above the resonant frequency of SRRs.

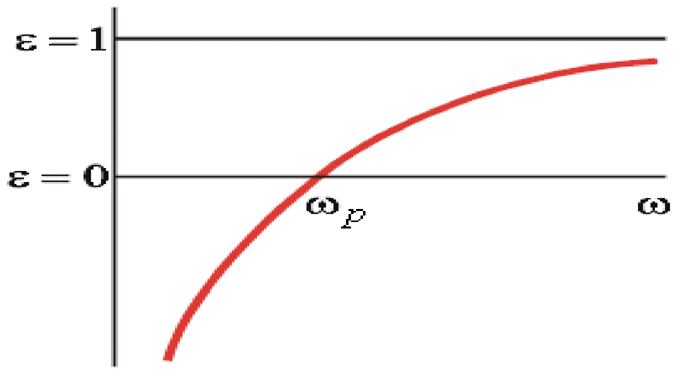


**Fig-3.4.** ***The schematic permittivity, $\varepsilon$ , of a plasma; below the plasma frequency, $\omega_p$ , $\varepsilon$ is negative.***

The first structure designed by the Marconi collaboration and that simulates the properties of low density plasma having a dielectric response of the form,

$$\varepsilon_{metal} = 1 - \omega_p^2 / \omega^2$$

Sketched in Fig-3.4 as shown above. Why this should be? Can be understood as follows; plasma comprises a gas of charged particles such as a metal where the particles are free electrons, or an ionized gas. The plasma frequency is dictated by the density of charges and by their mass.

$$\omega_p^2 = 4\pi n e^2 / m_e$$

Where $n$ is the electron density in the wires, $e$ is the charge of an electron, and $m_e$ is the effective mass of electron. In a wire MM, $n$ and $m$ are related to the geometry of the lattice rather than the fundamental charge carriers, giving MMs much greater flexibility than conventional materials. Because the effective density can be reduced substantially by making the wires thin, which has the added effect of increasing the effective mass of the charge carriers, the effective plasma frequency can be reduced by many orders of magnitude. In the context of NI MMs, the wire lattice and its variants are a convenient means of achieving a medium for which $\varepsilon_r < 0$.

## 3.4. A better focus with negative index

As Veselago showed that at an interface between a negative index materials light would be bent in the wrong way relative to normal. Theory predicted that the energy flow as dictated by the Poynting vector would be in the opposite direction to the wave vector, the implication being that rays travel in the opposite direction to waves. See Fig-3.5. The reversal of energy flow can easily be understood: flipping the sign of both ε and μ is equivalent in Maxwell's equations to flipping the sign of the magnetic field but keeping the same wave vector. Solutions are exactly the same as solutions for a conventional positive system except for this inversion. Since the Poynting vector is given by **E**×**H**, Veselago's result follows immediately. The following fig illustrates that as the waves enter the negative medium; their phase is wound backwards as they progress.

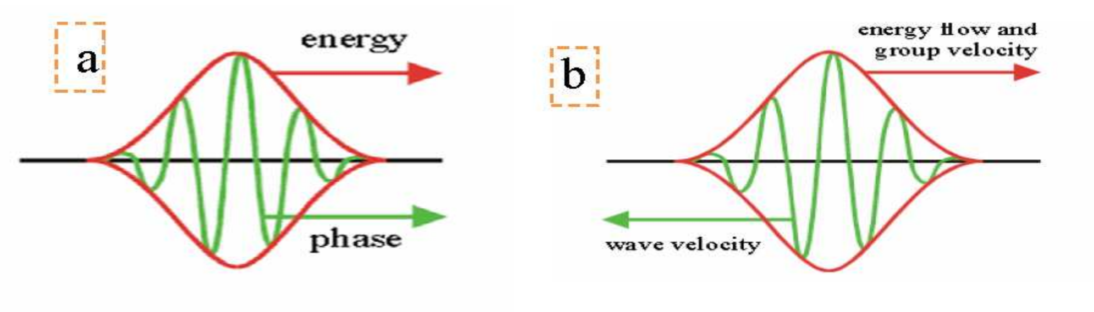


**Fig -3.5.** ***Shows how the focus is achieved by the negative slab 'unwinding' or negating the phase acquired in passing through free space.***

The use of left-handed substances would in principle allow the design of very unusual refracting system. An example of such a system is a simple plate of thickness d made of a left-handed substance with *n*=-1(slab of negative material with ε= μ= -1 acts like a lens), and situated in vacuum. Material with negative refractive index will focus light even when in the form of a parallel-sided slab of material, So that, objects on one side are brought to a focus on the other side. The strangest property follows directly from negative refraction and is the ability of the material to focus light. Fig-3.6 illustrates this effect for the case $n = -1$ when focussing is free from aberration, and all rays radiating from a point source are brought to a double focus: once inside the slab and again outside.

Refraction is the phenomenon responsible for lenses and similar devices that focus or shape radiation. While usually thought of in the context of visible light, lenses are utilized throughout the electromagnetic spectrum, and represent a good starting point to implement negative index materials. It is very interesting to trace the beam path through lenses prepared by materials with $n < 0$. In his early paper, Veselago noted that a negative index focusing lens would need to be concave rather than convex. This would seem to be a trivial matter, but there is, in fact, more to the story. Negative refraction can be used to focus light. In fact a flat slab of material will produce two foci, one inside the medium, the other outside.

A new lens made from a slab of negative material not only brings rays to a focus but has the capacity to amplify the near field so that it contributes to the image thus removing the wavelength limitation. However the resonant natures of the amplification places sever demands on materials; they must be very low loss. As it can be seen in the fig below such a plate can focus at a point, the radiation from a point source located at a distance $l$ < d from the plate. This is not a lens in the usual sense of the word, however, since it will not focus at a point a bundle of rays coming from infinity. As for actual lenses, the paths of rays through lenses made of left-handed substances for the convex and concave lenses have changed places, since the convex lens has a diverging effect and the concave lens has a converging effect.

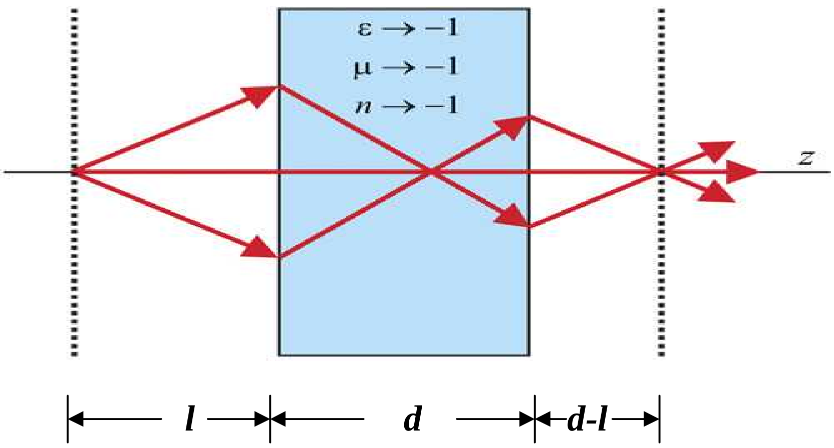


**Fig -3.6.** ***A negative refractive index medium bends light to a negative angle relative the surface normal. Light formerly diverging from a point source in the object plane is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time in the image plane.***

Our new lens is even more efficient. As the wave packets (see figure 3.5 above) enter the negative slab , the phase 'clock' is set in reverse and winds backwards in angle so that propagation through a thickness *d* of this medium unwinds the phase acquired passing through the same thickness of vacuum. Wave packets arrive at the image plane with exactly the same phase as in the object plane. This is another clue to the superior nature of this lens. Overall the slab undoes the effect of an equal thickness of vacuum. Similarly decaying waves have their amplitude restored by passing through the slab. This suggests another view of the focusing action, that of the slab annihilating an equal thickness of vacuum.

## 3.5-A perfect lens

The focussing properties of a negative slab are rather unusual, especially for the case where $\varepsilon \to -1$, $\mu \to -1$. In this instance rays are drawn to an aberration-free focus, and suffer no reflection from the surfaces of the slab. Yet the focussing properties are even more remarkable than this, further investigation shows that the slab is free from the wavelength restriction on resolution. In the ideal limit $\varepsilon \to -1$, $\mu \to -1$ resolution increases without bounds.

First a few words on the wavelength limit to resolution. In figure 3.6 an object emits electromagnetic waves of frequency $\omega$ . Note that because of dispersion we can only define $\varepsilon \to -1$, $\mu \to -1$ at a single frequency. Each wave has by a wave vector, **k**, where, $k_z = \sqrt{k_0{}^2 - k_x{}^2 - k_y{}^2}$ is responsible for driving the wave from object to image, and $k_x$ , $k_y$ define the Fourier components of the image. The larger the magnitude of $k_x$ , $k_y$ we can propagate to the image plane the better the resolution. The problem is that making these transverse wave vectors too large gives $k_z$ an imaginary value and the wave decays exponentially along the $z$-axis. These decaying components of the object field are often referred to as the 'near field'. They are confined to the vicinity of the object and serve to lock away high-resolution information. Hence the biggest Fourier component that we can capture has magnitude $k_0 = \dfrac{\omega}{c_0}$ and the wavelength restriction on resolution follows.

How does our negative slab avoid this limit? Light cannot get into a metal, or at least it cannot penetrate very far, but metals are not inert to light. It is possible for light to be trapped at the surface of a metal and propagate around in a state known as a *surface Plasmon*. These surface states have intriguing properties which are just beginning to be exploited in applications. The secret it deploys is a surface resonance which is used to amplify evanescent waves and restore them to the values taken in the object plane. At a given time, a resonance can build a substantial amplitude using energy drawn from the source. Absorption is the great enemy of resonances so low loss materials are essential if we are to approach the resolution offered by the new lens. The resonances are related to the surface Plasmon excitation familiar on the surfaces of metals, the condition for which is, in the short wavelength limit, $\varepsilon \to -1$. In our case $\varepsilon \to -1$, $\mu \to -1$ , and there are two surface excitations, one of electric the other of magnetic character. This is a remarkable result because, in order to 'focus' the near field, amplification has to take place to compensate for the natural decay and furthermore the amplification has to be exactly tuned to each Fourier component of the field. Surfaces of negatively refracting materials are heavily decorated with resonant states.

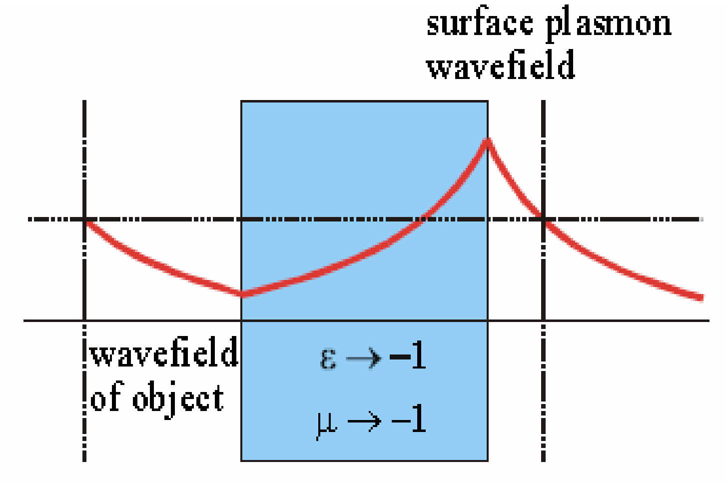


**Fig-3.7.** ***The new lens works by excitation of surface plasmons. Matching the fields at the boundaries selectively excites a surface plasmon on the far surface thus reproducing the same amplitude in the image plane as in the object plane.***

The surface supports an infinite number of resonances degenerate at frequency $\omega$ and in the limit of no losses a system stimulated at its resonant frequency gives an infinite response. In a finite slab degenerate resonances on opposite surfaces interact giving rise to a splitting and the response is no longer infinite. Detuning of the surface resonances results in exactly the degree of excitation required to reproduce the object fields in the image plane. See Fig-3.7 [13].

Up to now we have considered the scalar LHM. But as it was noted by Veselago [2], the dispersion of magnetic permeability requires the external magnetic field, because there are no magnetic monopoles. This magnetic field makes the medium anisotropic which is described by the tensors permittivity and permeability.

# CHAPTER FOUR

# ELECTROMAGNETIC WAVES IN ANISOTROPIC MEDIA

In chapter 2, we consider isotropic media, in which ε and μ are scalars. Let us now go on to anisotropic media. In this case the quantities $\hat{\varepsilon}$ and $\hat{\mu}$ are tensors. Quite recently there have begun to be intensive studies substances in which both $\hat{\varepsilon}$ and $\hat{\mu}$ are tensors. Examples of such substances are pure ferromagnetic metals and semiconductors. The electron-ion plasma with impurity of ferromagnetic grains is one more system with tensor $\hat{\varepsilon}$ and $\hat{\mu}$.

Now we consider anisotropic media that is the case of tensor $\hat{\varepsilon}$ and $\hat{\mu}$ by making use of equations in chapter 2 from eq (2.1) to eq (2.8) of the Maxwell equations, Fourier transform and the material equations we obtain:

$$\vec{k} \times \vec{E}(\vec{k},\omega) = \hat{\mu}\frac{\omega}{c}\vec{H}(\vec{k},\omega) \qquad (4.1)$$

$$\vec{k} \times \vec{H}(\vec{k},\omega) = -\hat{\varepsilon}\frac{\omega}{c}\vec{E}(\vec{k},\omega) \qquad (4.2)$$

$\vec{E}$ and $\vec{H}$ are the Fourier components of electric and magnetic fields, $\hat{\varepsilon}(\vec{k},\omega)$ and $\hat{\mu}(\vec{k},\omega)$ are the Fourier transform of the dielectric permittivity and magnetic permeability tensors respectively. These tensors of substances are given by the form of:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \quad , \quad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}.$$

In this work, we consider the wave propagating along the *z-axis* that is along the direction of the field. The above system of equations (4.1) and (4.2) written in the vector form presents a system of six linear homogeneous algebraic equations for six Fourier components of the electromagnetic field as follows:

$$\begin{vmatrix} i & j & k \\ k_x & o & k_z \\ E_x & E_y & E_z \end{vmatrix} = \frac{\omega}{c}\begin{vmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{vmatrix}\begin{vmatrix} H_x \\ H_y \\ H_z \end{vmatrix},$$

$$\left.\begin{aligned} -k_z E_y &= \frac{\omega}{c}\left(\mu_1 H_x + i\mu_2 H_y\right) \\ k_z E_x - k_x E_z &= \frac{\omega}{c}\left(-i\mu_2 H_x + \mu_1 H_y\right) \\ k_z E_y &= \frac{\omega}{c}\mu_3 H_z \end{aligned}\right\}, \qquad (4.3)$$

and from

$$\begin{vmatrix} i & j & k \\ k_x & 0 & k_z \\ H_x & H_y & H_z \end{vmatrix} = -\frac{\omega}{c}\begin{vmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{vmatrix}\begin{vmatrix} E_x \\ E_y \\ E_z \end{vmatrix},$$

Again we will get three more equations that is given as below

$$\left.\begin{aligned} -k_z H_y &= -\frac{\omega}{c}\left(\varepsilon_1 E_x + i\varepsilon_2 E_y\right) \\ k_z H_x - k_x H_y &= -\frac{\omega}{c}\left(-i\varepsilon_2 + \varepsilon_1 E_y\right) \\ k_x H_y &= -\frac{\omega}{c}\varepsilon_3 E_z \end{aligned}\right\}. \qquad (4.4)$$

After eliminating the magnetic field from the systems eq (4.3) and eq (4.4), we obtain the system of linear algebraic equation for $E_x$, $E_y$, $E_z$ . The nontrivial solution of this system is obtained from the condition of vanishing of its determinant. It is convenient to introduce the index of refraction according to relation $n = \frac{kc}{\omega}$. This quality in general case depends on frequency and direction of the wave propagation i.e. $n(\omega,\theta)$ where $\theta$ is the angle between the wave vector $\vec{k}$ and $\vec{H}_0$. For the electromagnetic waves propagating parallel and transverse to the magnetic field, one can obtain the simple analytic expressions of the refraction index by setting $\theta = 0$. Then an index of refraction of traveling wave along the field is given by:

$$n^2_{\pm} = \left(\frac{kc}{\omega}\right)^2 = (\varepsilon_1 \pm \varepsilon_2)(\mu_1 \pm \mu_2), \qquad (4.5)$$

the refractive indexes of the ordinary and extraordinary circularly polarized electromagnetic waves propagating along the *z-axis*.

Where $\varepsilon_1 = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_c^2}$ , $\varepsilon_2 = \frac{\omega_c}{\omega}\frac{\omega_{pe}^2}{\omega^2 - \omega_c^2}$,

$$\mu_1 = 1 - \frac{\omega_c \omega_M}{\omega^2 - \omega_c^2}, \qquad \mu_2 = \frac{\omega \omega_M}{\omega^2 - \omega_c^2},$$

and $\omega_{pe} = \sqrt{\frac{4\pi e^2 n_e}{m_e}}$ is the electron plasma frequency, $\omega_c = \frac{eH_0}{m_e c}$ is the electron cyclotron frequency, and $\omega_M$ is the parameter characterizing the magnetic subsystem. The refractive index $n_+$ relates to the ordinary wave with the electric vector perpendicular to $H_0$ and $n_-$ is the refractive index of the extraordinary linearly polarized wave with $E_z$ parallel to $H_0$. These waves propagate along the *x*-axis.

It is easy to see that due to the frequency dispersions $\varepsilon_1, \varepsilon_2, \mu_1, \mu_2$ can change their signs depending on relation between $\omega$ and $\omega_c$. Here we ignore the losses in the media that requires $\omega$ should not be close to $\omega_c$. The electromagnetic wave can propagate in the media only if $n^2 > 0$, therefore $n^2$ remains positive and the wave will be propagated. The media with magnetic permeability $\mu_1 = 1\ (\mu_2 = 0)$ is transparent for the EM wave. It means that,

$$n_\pm^2 = \varepsilon_1 \pm \varepsilon_2 > 0 \qquad (4.6)$$

In the frequency range where $\varepsilon_1 \pm \varepsilon_2 < 0$, the waves with the refractive indexes (4.5) do not exist, i.e. the wave cannot propagate in the media. But Magnetized Plasma with Ferrite Grains can be transparent for these waves in the same frequency range provided that $\mu_1 \pm \mu_2 < 0$. The magnetic dispersion permeability completely changes the situation. In the media under consideration, the EM wave can exist if,

$$(\varepsilon_1 \pm \varepsilon_2)(\mu_1 \pm \mu_2) > 0 \qquad (4.7)$$

## 4.1-Negative Refractive Index of Waves Propagating along $H_0$

The dispersion of the components of permittivity and permeability tensors is important at , $\omega \to \omega_c$, we specify this frequency range introducing a dimensionless frequency $x = (\omega - \omega_c)/\omega_c << 1$. With this in mind, we obtain:

$$\left.\begin{array}{c} \mu_1 = 1 - \dfrac{\alpha_M}{x(x+2)},\ \mu_2 = \dfrac{\alpha_M (x+1)}{x(x+2)}, \\ \varepsilon_1 = 1 - \dfrac{\alpha_e^2}{x(x+2)},\ \varepsilon_2 = \dfrac{\alpha_e^2}{x(x+1)(x+2)}, \\ \varepsilon_3 = 1 - \dfrac{\alpha_e^2}{(x+1)^2}. \end{array}\right\} \qquad (4.8)$$

Where $\alpha_e = \omega_p / \omega_c$, $\alpha_M = \omega_M / \omega_c$

Now we consider the refractive index of the waves propagating along the magnetic field in more details. From (4.5) with the help of (4.8), we obtain

$$n_+^{\ 2}(x,0) = \left(1 - \frac{\alpha_e^2}{(x+1)(x+2)}\right)\left(1 + \frac{\alpha_M}{x+2}\right), \qquad (4.9)$$

$$n_-^{\ 2}(x,0) = \left(1 - \frac{\alpha_e^2}{x(x+1)}\right)\left(1 - \frac{\alpha_M}{x}\right). \qquad (4.10)$$

Below we study Magnetized Plasma with Ferrite Grains (MPFG) with $\alpha_e = \omega_p / \omega_c << 1$ and $\alpha_M = \omega_M / \omega_c << 1$. The plasma with the dispersion properties controlled by the permittivity $\hat{\varepsilon}$ and permeability $\hat{\mu}$ tensors simultaneously can be obtained in laboratory by adding the ferrite (ferromagnetic) grains to the magnetized electron-ion plasma. We call it magnetized plasma with ferrite grains (MPFG). In a narrow frequency band in the vicinity of the electron cyclotron frequency, the dispersion of permittivity $\hat{\varepsilon}$ and permeability $\hat{\mu}$ tensors may considerably change the refractive index, phase and group velocities of these waves comparing to the conventional magnetized electron-ion plasma (CMP).

The existence of the waves with negative refraction index makes MPFG interesting as the medium related to the left handed materials (LHM) or materials with negative refraction. The dispersion properties of MPFG are controlled by permittivity and permeability tensors simultaneously. The magnetic subsystem makes MPFG transparent for the waves that cannot propagate in the conventional magnetized electron-ion plasma near the frequency of the ferromagnetic resonance of a grain that coincides with the electron cyclotron frequency.

Because of $\alpha_M << 1$, the simultaneous dispersion of the permeability and permittivity tensors (4.8) is important for $|x| \sim \alpha_M << 1$ remembering that $\omega$ should not be very close to $\omega_c$, where the cyclotron waves strongly decay in the magnetized plasma.

With account $|x| \sim \alpha_M << 1$ and $\alpha_e << 1$, (4.9) yields $n_+^2(x,0) \approx 1 > 0$. The ordinary wave propagates along the magnetic field in MPFG with the above-specified parameters practically as in the free space. The magnetic subsystem cannot change the sign of $n_+^2(x,0) > 0$. It is controlled by the electrons as in Conventional Magnetized Plasma. The extraordinary wave is more interesting. In conventional magnetized plasma (CMP) $(\alpha_M = 0)$, the fast extraordinary wave propagates only if $x > \alpha_e^2$.

Fig-4.1 shows a typical dependence of $n_-^2(x,0)$ of CMP versus dimensionless frequency $x$.

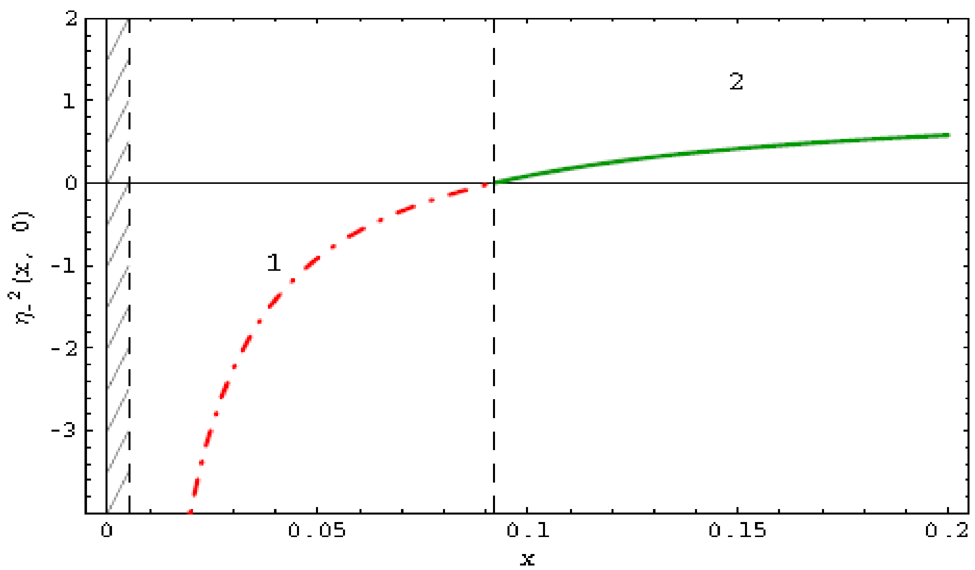


**Fig-4.1.** ***the refractive index squared ($n_-^2$ (x, 0)) of CMP versus the dimensionless frequency (x) of CMP ($\alpha_M$ = 0) for $\alpha^2{}_e$ = 0.1. CMP is transparent in region 2 ($n_-^2$ (x, 0) > 0), and nontransparent in region 1 ($n_-^2$ (x, 0) < 0). In the shaded region 0 ≤ x < $10^{-4}$ the waves strongly decay.***

The shaded region separated by dashed line includes small $x$ where our theory does not work and requires account of the wave damping. It ends at $x = 10^{-4}$. The nontrasparency region 1(dashed red line), where $n^2 < 0$, $10^{-4} < x < (\sqrt{1+4\alpha_e^2} - 1)/2 \approx \alpha_e^2$; the transparency region 2 starts with $x = \alpha_e^2$. The graph is plotted for $\alpha_e^2 = 0.1$.

If the ferromagnetic component is added to such plasma, a window of transparency appears for $x > \alpha_e^2$ and $x < \alpha_M$, simultaneously. But the frequency range $\alpha_M < x < \alpha_e^2$ remains nontransparent $n_-^{\,2} < 0$. Here we assume that $\alpha_M < \alpha_e^2$.

The figures (4.2) and (4.3) show dependencies of $n_-^{\,2}(x,0)$ and $n_-(x,0)$ of MPG as functions of frequency *x*. The sign of the refraction index of the extraordinary wave is chosen to be negative in the frequency range where $\mu_1 - \mu_2 < 0$, $\varepsilon_1 - \varepsilon_2 < 0$ simultaneously and positive in the opposite case. It was also emphasize that the graph $n_-(x, 0)$ in the transparency domains (blue line) shows a normal dispersion for a positive and negative refraction index as it must be in an equilibrium media (see Fig-4.2).

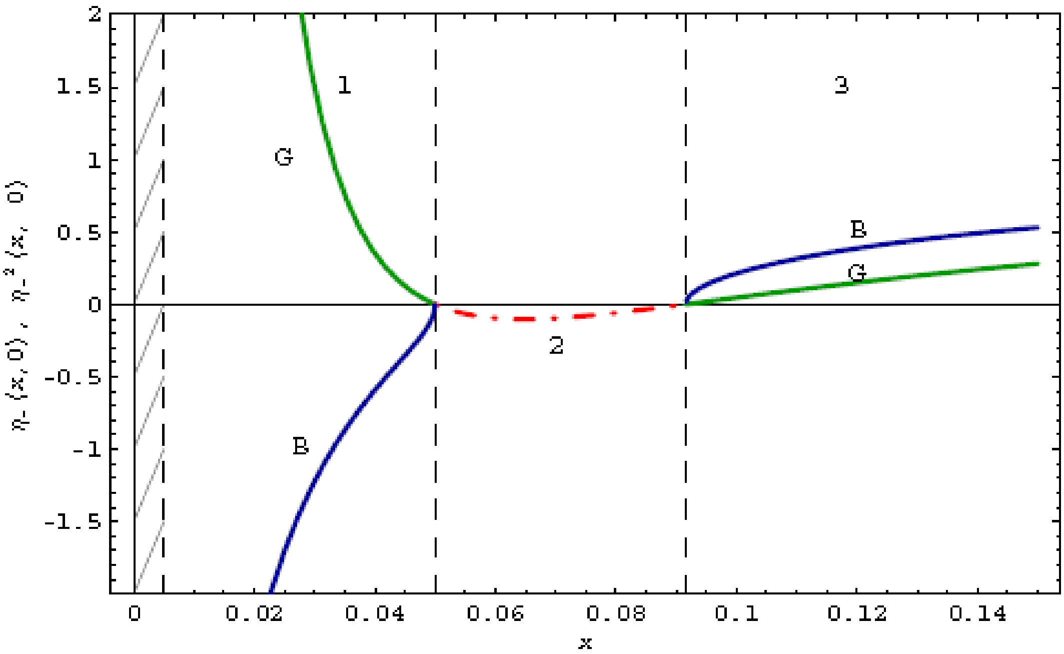


**Fig-4.2.** ***The graphs of $n_-^{\,2}$ (x,0) (green line G) and $n_-$ (x, 0) (blue line B) of MPFG versus x, for $\alpha_e^{\,2}$ = 0.1 and $\alpha_M$ =0.05. MPFG is transparent in regions 1 and 3 ( $n_-^{\,2}$ > 0) and nontransparent in region 2, $n_-^{\,2}$ < 0 (dashed red line)***

In Fig-4.2, above we depict $n_-^{\,2}(x,0)$ versus $x$ of MPFG , again the dashed line excludes frequencies where dissipation must be taken into account , region 1 is the transparency domain, which appears in MPFG , region 2 is nontransparent ($\alpha_M = 0.05 < x < \alpha_e^2 = 0.1$). We see that in region 3 the refraction index is slightly affected by the ferromagnetic grains.

With $\alpha_M \to \alpha_e^2$, the nontransparent domain disappears. Fig-4.3. shows $n_-^{\,2}(x,0)$ versus x of MPFG; the dashed line excludes frequencies where dissipation must be taken into account; $n_-^{\,2}(x,0) \geq 0$ in region 1 and 2. The solid line separating regions 1 and 2 corresponds to $n_-^{\,2}(x,0) = 0$.

The refractive index of the fast extraordinary wave in this case with the great accuracy is obtained from the relation,

$$n_-^{\,2}(x,0) \cong \left[1 - \frac{\alpha_M}{x}\right]^2 \qquad (4.11).$$

Here in CMP the refractive index is always positive. But in MPFG it can be positive as well as negative.

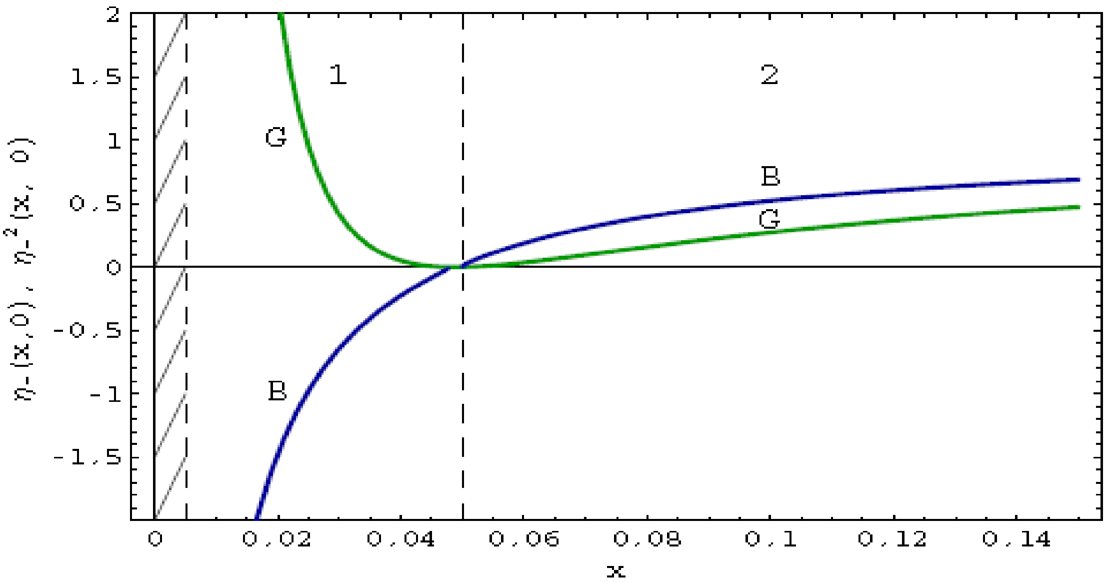


**Fig-4.3.** ***The graphs of $n_-(x, 0)^2$ (green line G) and $n_-(x, 0)$ (blue line B) versus x, for $\alpha_e^{\,2} = \alpha_M = 0.05$. MPFG is transparent in the entire frequency range ($n_-(x, 0)^2 > 0$) with the exception of the shaded area. In region 1, $n_-(x, 0) < 0$ and in region 2, $n_-(x, 0) > 0$***

The refractive index is obtained as the square root of (4.9 -4.11). It can be shown that it is negative if $\mu_1 - \mu_2 < 0$, $\varepsilon_1 - \varepsilon_2 < 0$ simultaneously and positive in the opposite case [30].

For the ordinary wave $\mu_1 + \mu_2 = 1 + \alpha_M / 2 > 0$. For the extraordinary wave in the frequency range where $\mu_1 - \mu_2 < 0$ and $\varepsilon_1 - \varepsilon_2 < 0$ simultaneously, the refractive index must be negative. In particular, for MPFG with $\alpha_e^2 = \alpha_M$ where the nontransparency window disappears, the refractive index with account of (4.11) can be expressed by a simple formula

$$n_{-}(\omega,0) = 1 - \frac{\alpha_M}{x} \equiv 1 - \frac{\alpha_M \omega_c}{\omega - \omega_c}. \qquad (4.12)$$

It shows that $n_{-}(\omega,0)$ for frequencies $\omega < \omega_c(1+\alpha_M)$ is negative, and for $\omega > \omega_c(1+\alpha_M)$, it is positive. For $\omega = \omega_c(1+\alpha_M)$, *n* is very close to zero.

The phase velocity of this wave $V_{ph} = c / n(\omega,0)$ *i*s negative for $\omega < \omega_c(1+\alpha_M)$ and positive for $\omega > \omega_c(1+\alpha_M)$. The group velocity can be found with usage of the definition $n_{-}(\omega,0) = kc/\omega$. Using the relation $v_g = \frac{c}{[n + \omega(dn/d\omega)]}$, substituting (4.12) in this equation the result says:

$$V_g = \frac{cx^2}{\alpha_M + x^2}. \qquad (4.13)$$

It is necessary to remind that in the formulas (4.12) and (4.13) can be applied if the frequency $\omega$ not very close to $\omega_c$. For $\omega_c \sim 10GHz$, we chose $x = (\omega - \omega_c)/\omega_c = 10^{-5}$. For MPFG with $\alpha_M \approx 10^{-3} - 10^{-2}$, $x \sim 10^{-3}$ provides enough margin of safety.

From (4.13) follows that always $V_g > 0$. A direction of group velocity of the extraordinary wave propagating along the magnetic field in MPFG coincides with the direction of the Poynting vector. But in the frequency range where $n_{-}(\omega,0) < 0$, the phase velocity of this wave is negative. It means that the wave propagates in the negative direction of the *z*-axis i.e. opposite to the direction of energy flow. These waves are known as the backward waves. In the frequency range where $n_{-}(\omega,0) < 0$ and $x \leq \alpha_M << 1$, we see that

$$V_g \approx \alpha_M c << c. \qquad (4.14)$$

Therefore, the extraordinary wave propagating in MPFG along the magnetic field in the frequency range $x < \alpha_M$, where $n_{-}(\omega,0) < 0$, and the ordinary wave in the frequency range $x > \alpha_M$, where $n_{+}(\omega, 0) > 0$ are slow waves, Since their group velocity is much smaller than velocity of light [30].

# CHAPTER FIVE

# SUMMARY AND CONCLUSION

## 5.1-Summary

It has been thought that for many years only media with positive refractive index was exist, but in this project we have outlined the most important theoretical, historical and, experimental aspects of negative index materials (NIMs), including a proper mathematical description of fundamental physical laws and equations that may be applied when NIMs are present. There has recently been a huge interest in the physics of electromagnetic wave propagation in media where the permeability and the permittivity in the harmonic regime both have a negative real part. In medium with negative refractive index (LHM) an EM wave propagates with wave vector $\vec{k}, \vec{E},$ and $\vec{H}$ form a left-handed orthogonal set. The fact that for negative ε and μ the phase and group velocities are oppositely directed, leads to a conclusion that some fundamental laws of electrodynamics and optics are expressed in an unusual way, among these laws; Doppler effect, Cherenkov effects, and Snell's law are some of the laws discussed in this project.

The negative sign of the refractive index naturally arises in the theoretical description of the electromagnetic properties of materials with simultaneously negative values of dielectric permittivity $\varepsilon$ and magnetic permeability $\mu$, therefore, regardless of the signs of $\varepsilon$ and $\mu$, the fundamental Maxwell’s equations, and their corresponding boundary conditions for refraction of light, can be formally satisfied by both positive and negative refraction solutions. The appropriate solution is only selected by the additional requirement that in the refracted beam the energy flows away from the interface. In the regular material, this corresponds to the normal refraction, whereas for the negative index media the solution of Maxwell’s equations must be chosen such that it leads to negative refraction.

Since there was no naturally occurring material exists having the property of negative refractive index, the project reveal that experiments to date have been performed on artificially structured metamaterials, for which the negative permeability response results from an array of conducting (nonmagnetic) split ring resonators (SRRs) and the negative permittivity response results from an array of conducting wires.

It was also discussed a slab of negative material with ε= μ= -1 that acts like a lens have many strange properties follow directly from negative refraction and is the ability of the material to focus light.

Finally by considering propagation of EM waves in anisotropic media it was shown that in the anisotropy plasma like media, the account of the dispersion of the permittivity and permeability in the vicinity of the electron cyclotron frequency strongly affects the high frequency dependence of the refractive index of the extraordinary wave which allow us to claim that the magnetized electron-ion plasma with ferromagnetic grains behaves as LHM with respect to the extraordinary wave.

## 5.2-Conclusion

The Project shows provided that at least for a certain frequency due dispersion of the electric permittivity and magnetic permeability an isotropic homogeneous Veselago material can exist, then the Maxwell system leads naturally to negative refraction. The experimental realization of the Veselago materials become possible only recently with the help of modern technology. The opposite direction of EM plane wave's propagation to the flow of energy in a medium with negative refractive index is due to the individual Maxwell curl equations. It is shown that a medium with simultaneously isotropic and negative ε and μ supports propagating solutions whose phase and group velocities are antiparallel. Certain classes of anisotropic media such as magnetized plasma with ferromagnetic grains (MPFG) have identical refractive properties as isotropic negative index materials. Furthermore, Combining the spilt ring resonator (SRR) Metamaterial with a thin wire structure can produce a material that has frequency regions where both the permittivity and the permeability are negative. Negative refractive materials are necessarily frequency dispersive, so that the various frequency components of a modulated beam are refracted at different angles within the medium. It is important that the magnetic permittivity $\mu \to 1$ at high frequencies (far from optical).This means that creation of materials with negative refraction index at optical frequencies requires new approaches.

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