Addis Ababa University
School of Graduate Studies
Department of Civil Engineering

PERFORMANCE OF PILED RAFT FOUNDATIONS FOR ADDIS ABABA SOILS.

By

Metsihafe Mekbib

B.Sc. in Civil Engineering,
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A Thesis Submitted to School of Graduate Studies in
Partial Fulfillment of the Requirement for Degree of
Master of Science
in
Civil Engineering

Advisor
Dr.–Ing. Girma Boled

Assistant Professor of Civil Engineering
Addis Ababa University

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Approved by Board of Examiners:

Dr.-Ing. Girma boled ________________ ______________
Advisor *Signature* *Date*

Dr.-Ing. Samuel Tadesse ________________ ______________
External Examiner *Signature* *Date*

Dr. Messele Haile ________________ ______________
Internal Examiner *Signature* *Date*

Dr.-Ing. Asrat Worku ________________ ______________
Chairperson *Signature* *Date*

Declaration

I, undersigned declare that this thesis is my original work, has not been presented for a degree in any other universities and that all sources of materials used for this thesis have been duly acknowledge.

Name: Metsihafe Mekbib

Signature: _______________

Place: Faculty of Technology, Addis Ababa University

Date of Submission: March 2004

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Notations:

S_{tot} : total loads of the structure over contact pressure

R_{raft} : load carried by the raft only

$\sum R_{pile,i}$: sum of load carried by piles

K_{pr} : stiffness of piled raft

K_p : stiffness of the pile group

K_r : stiffness of the raft alone

α_{cp} : raft – pile interaction factor

P_r : load carried by the raft

P_t : total applied load

r_c : average radius of pile cap

r_o : radius of pile

ν : Poisson's ratio of soil

L : pile length

E_{sl} : soil Young's modulus at level of pile tip

E_{sb} : soil Young's modulus of bearing stratum below pile tip

E_{sav} : average soil Young's modulus along pile shaft.

P_{up} : ultimate load capacity of the piles in the group

X : proportion of load carried by the piles.

S_{pr} : settlement of piled raft

S_p : settlement of raft without piles subjected to the total applied loading

K_r **:** stiffness of raft

K_{pr} : stiffness of piled raft.

τ : the shear stress

σ_z : the vertical total stress

G : tangent shear modulus at an applied shear stress, τ_o

G_i : initial shear modules at small strains

R_f : stress -strain curve fitting constants (failure ratio)

τ_{max} : shear stress at failure

Δw_b : incremental base settlement

ΔP_b : incremental pile base load

G : initial tangent shear modulus

$\varphi = R_f \; P_b / P_{b\max}$: pile base load level

P_b : current base load

$P_{b\max}$: limiting pile base load

Δw_{si} : incremental soil settlement at node i,

f_{ij} : soil settlement flexibility coefficient denoting the settlement at node i due to unit load at node j,

ΔP_j : incremental load at load

n : total number of nodes in the group.

$\{\Delta w_{si}\}$: vector of incremental load settlement;

[F] : total settlement flexibility matrix; and

$\{\Delta P_{pi}\}$: incremental nodal loads vector of pile interfaces.

$\lfloor K_p \rfloor$: assembled stiffness matrix of the piles

$[K_r]$: global stiffness matrix of the raft element

$\{\Delta Q\}$: applied load on the raft incrementally

$\{\Delta d\}$: the nodal degree of freedom of the plate elements.

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Abstract

This thesis presents the investigation of the performance of the concept of piled raft foundation compared to the conventional pile and raft foundation for Addis Ababa soils. The investigation has been carried out using soil profiles in Addis Ababa where high-rise buildings are going to be constructed and have been already constructed. To support the results of the investigation, soil profile from Wabe Shebele River bridge site has been also considered.

In this work, first, different design philosophies and method of analysis have been discussed. From already available methods of analysis, the hybrid method of analysis has been chosen for its computational efficiency. The program is validated by anaysing examples available in literature and has already been analyzed using 3D finite element code (FLAC 3D).

Then, two case studies have been made to explore the performance of the piled raft for Addis Ababa soils. The first case is the Awash bank head quarter building, which is going to be constructed near national theater. The second case considered is the soil profile on which the former marine transport authority building is constructed. For both cases, comparison has been made among raft, pile, and piled raft foundation based on settlement and bending moment.

Another case study, conducted to supplement the out come of the foregoing two cases, is the abutment foundation of Wabe Shebele River Bridge. This case specifically conducted aiming at reducing the total settlement of the foundation that is above the permissible limit for masonry abutment. For the this case, the results have shown that

two fifth of the number of piles required for conventional piles can be reduced using piled raft design concept. When raft alone is used, the settlement exceeds the permissible value.

Generally, it has been observed that piled raft concept is very advantageous in minimizing the total and differential settlements as compared to the conventional raft foundation. It also reduces number of piles required as compared to the conventional pile foundation. For the cases investigated in this work, the reduction in number of piles ranges from 25% to 45% of that required for conventional pile foundation, though the variation in raft bending moment and other issues are no considered in detail.

Chapter One

Introduction

1.1 Back Ground

The foundations that designers are most likely to consider first for major structures on deep deposits of clay or sand are reinforced concrete rafts. Rafts spread the load from columns and load bearing walls over the widest possible area and the differential settlements can be minimized or controlled by varying the raft stiffness. Generally settlement consideration are the most important determinants of the final design and only in cases of extremely heavy structures must the possibility of bearing capacity failures can be seriously examined. To limit the settlement to the allowable value the practice is to use pile foundations.

Conventional pile foundations are commonly designed by adopting a relatively high factor of safety for the piles. These piles are placed in such a manner that they will sustain the entire design load of the superstructure. Although the connection "cap"(often raft) is in close contact with the soil, its contribution to the total bearing capacity and general pile group behaviour is seldom considered.

Nevertheless, in the past few decades, there has been an increasing recognition that the use of pile groups in conjunction with the raft can lead to considerable economy without compromising the safety and performance of the foundation. Such a foundation makes use of both the raft and the piles, and is referred to here as a pile-enhanced raft or a piled raft. This concept makes clever use of deep foundation elements to selectively supplement and load share with mats and similar foundations.

Very often, the deep foundation elements (piles or shafts) are only placed beneath portions of a foundation and are intended to carry only a portion of the superstructure load. Thus this is fundamentally different from foundation application where the piles or shafts are placed beneath the entire foundation and are assumed to carry all loads. An additional unique aspect of the piled - raft concept is that the deep-foundation elements are sometimes designed to reach their ultimate geotechnical axial compressive capacity under service loads.

The piled-raft concept has also proven to be an economical way to improve the serviceability of foundation performance by reducing settlements to acceptable levels. Although the piled-raft concept has been most notably applied to new construction involving high-rise buildings it is also potentially useful for remedial works and moderate height structures.

1.2 Objectives of The Research

Despite the fact that piled raft foundations are recognized to be able to become a cost-effective alternative to conventional pile foundations, piled rafts represent a lesser known and underutilized geotechnology in many areas, in fact the same also in Ethiopia. Prior to introducing this intriguing foundation concept to the construction industry in the country, it is necessary to investigate the advantages and performance of piled raft for our cases.

The main objectives of this thesis is therefore, to investigate the performance of piled raft foundations compared with conventional pile foundation and raft foundation for Addis Ababa soils.

1.3 Organization of Thesis

This thesis consists of five Chapters. The first Chapter is the introduction part and it discusses briefly on the importance of the piled raft foundations, the objective, the scope and organizational summary of the thesis. The second Chapter is the summarized literature review of different design concepts where some methods of analysis for piled raft are discussed. The third Chapter deals with detailed formulation for the analysis of piled raft foundation using hybrid finite element – load transfer approach. The fourth Chapter presents comparative case studies for Addis Ababa soils showing the contribution of pile cap in carrying the loads from superstructure and the use of piles in reducing settlements. The reduction in numbers of piles utilizing piled raft concept is also investigated. The fifth Chapter includes the summary of the thesis and its findings, and recommendation for future work.

Chapter Two

Literature Review

2.1 Introduction

The piled raft foundation is an effective foundation concept used to minimize total settlement and differential settlement, to improve the bearing capacity of a shallow foundation and to reduce in an economical way the initial stress level and bending moments of a raft. The concept of piled raft combines the bearing elements piles, raft and soil in a composite structure. The understanding of these interaction effects is an indispensable condition for a reliable design of piled rafts.

This chapter reviews the general philosophies of piled raft design, and the design issues, which need to be addressed. Various methods of analysis of piled raft foundations are then reviewed, and their capabilities and limitations are discussed and an efficient method is selected for the present work.

2.2 Design Concepts

2.2.1 Definition and Concept

The piled raft foundation is a foundation concept, which acts as a composite construction consisting of the three bearing elements: piles, raft and sub soil. According to its stiffness the raft distributes the loads of the structure S_{tot} over contact pressure, represented by R_{raft} , as well as over the piles, generally represented by the sum of pile resistance $\sum R_{pile,i}$ in the ground (Figure 2.1). So the total resistance of piled raft is:

$$R_{total} = R_{raft} + \sum R_{pile,i} \geq S_{tot} \qquad (2.1)$$

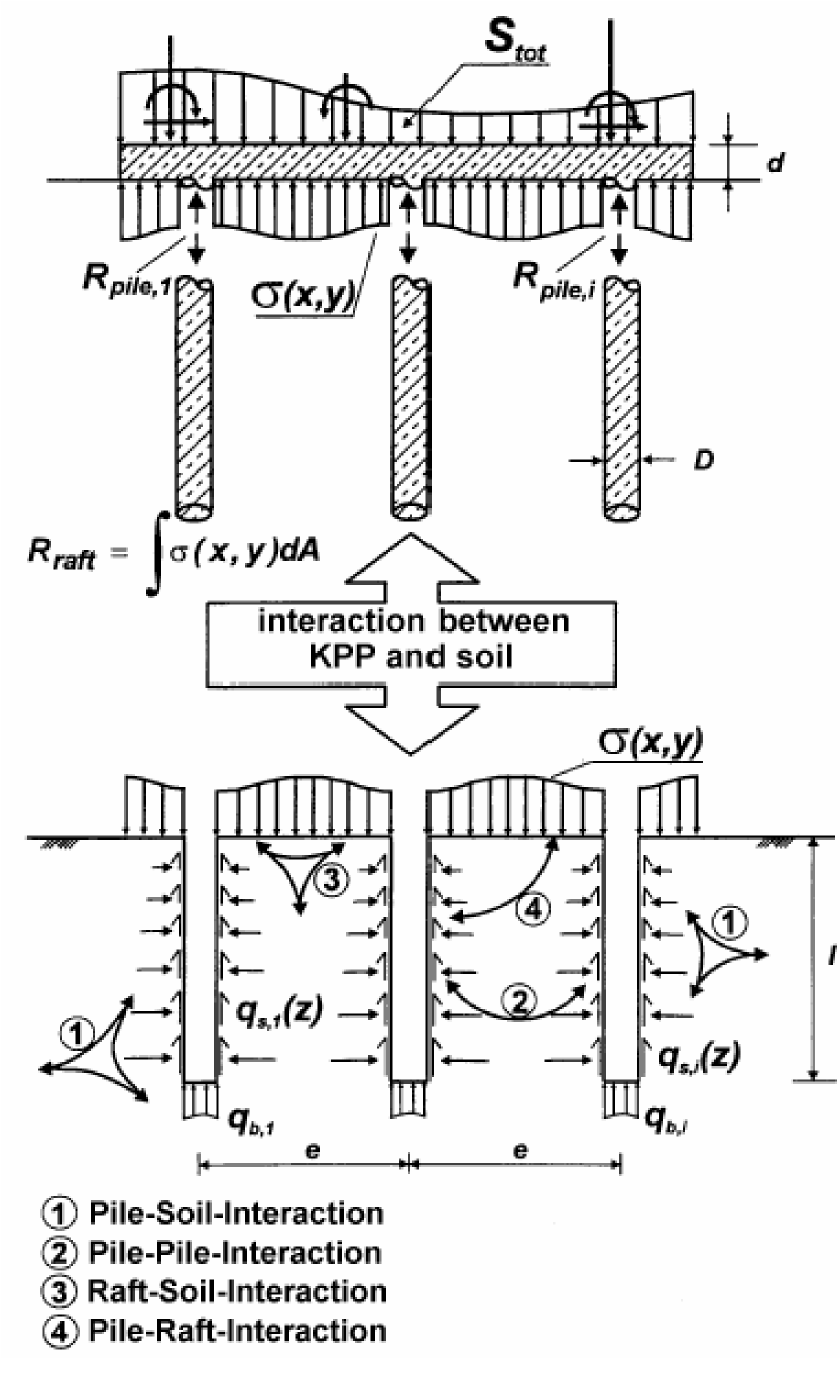


Figure 2.1 Piled raft foundation as composite construction of the bearing elements piles, raft and subsoil (Poulos, 2000)

By conventional foundation design it has to be proved that the building load is transferred either by the raft or the piles in the ground. In each case it has to be proved that either the raft or the piles will support the working load of the building with adequate safety against bearing resistance failure.

The piled raft foundation indicates a new understanding of soil – structure interaction because the contribution of the rafts as well as the piles is taken into consideration to satisfy the proof of the ultimate bearing capacity and the serviceability of a piled raft as an over all system. Besides this, the interaction between raft and piles makes it possible to use the piles up to a load level which can be significantly higher than the permissible design value for the bearing capacity of comparable single standing pile. This leads us to different design philosophies with respect to piled raft foundation.

2.2.2 Alternative Design Philosophies

Randolph (1994) according to Poulos (2001) has defined clearly three different design philosophies with respect to piled rafts:

- The "conventional approach", in which the piles are designed as a group to carry the major part of the load, while making some allowance for the contribution of the raft, primarily to ultimate load capacity.
- "Creep Piling" in which the piles are designed to operate at a working load at which significant creep starts to occur, typically 70-80% of the ultimate load capacity. Sufficient piles are included to reduce the net contact pressure between the raft and the soil to below the preconsolidation pressure of the soil.
- Differential settlement control, in which the piles are located strategically in order to reduce the differential settlements, rather than to substantially reduce the overall average settlement.

In addition, there is a more extreme version of creep piling, in which the full load capacity of the piles is utilized, i.e. some or all of the piles operate at 100% of their ultimate load capacity. This gives rise to the concept of using piles primarily as settlement reducers, while recognizing that they also contribute to increasing the ultimate load capacity of the entire foundation system.

Clearly, the latter three approaches are most conducive to economical foundation design, and will be given special attention herein. However, it should be emphasized that the analysis and design methods to be discussed allow any of the above design philosophies to be implemented.

According to Poulos (2001), De Sanctis et al. (2001) and Viggiani (2001) have distinguished between two classes of piled raft foundations:

1. “Small” piled rafts, where the primary reason for adding the piles is to increase the factor of safety (this typically involves rafts with widths between 5 and 15 m);
2. “Large” piled rafts, whose bearing capacity of the raft is sufficient to carry the applied load with a reasonable safety margin, but piles are required to reduce settlement or differential settlement. In such cases, the width of the raft is large in comparison with the length of the piles (typically, the width of the piles exceeds the length of the piles). These two categories broadly mirror the conventional and creep-piling philosophies considered by Randolph.

Figure 2.2 illustrates, conceptually, the load-settlement behaviour of piled rafts designed according to the first two strategies. Curve O shows the behaviour of the raft alone, which in this case settles excessively at the design load. Curve 1 represents the conventional design philosophy, for which the behaviour of the pile-raft system is governed by the pile group behaviour, and which may be largely linear at the design load. In this case, the piles take the great majority of the load. Curve 2 represents the case of creep piling where the piles operate at a lower factor of safety, but because there are fewer piles, the raft carries more load than for Curve 1. Curve 3 illustrates the strategy of using the piles as settlement reducers, and utilizing the full capacity of the piles at the design load. Consequently, the load-settlement may be nonlinear at the design load, but nevertheless, the overall foundation system has an adequate margin of safety, and the settlement criterion is satisfied. Therefore, the design depicted by Curve 3 is acceptable and is likely to be considerably more economical than the designs depicted by Curves 1 and 2.

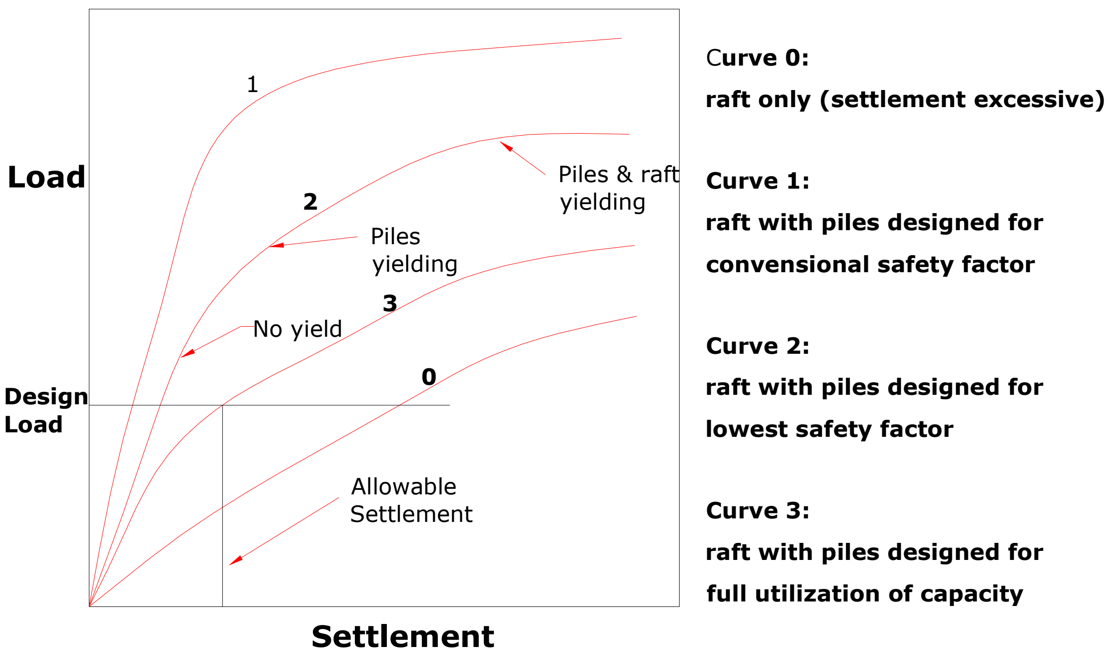


Figure 2.2. Load settlement Curves for piled raft foundation according to varies design philosophies. (Poulos, 2000)

2.2.3 Design Issues

As with any foundation system, design of a piled raft foundation requires the consideration of a number of issues, including:

1. Ultimate load capacity for vertical, lateral and moment loadings
2. Maximum settlement
3. Differential settlement
4. Raft moments and shears for the structural design of the raft
5. Pile loads and moments, for the structural design of the piles.

In much of the available literature, emphasis has been placed on the bearing capacity and settlement under vertical loads. While this is a critical aspect and is considered herein, the other issues must also be addressed. In some cases, the pile requirements may be governed by the overturning moments applied by wind loading, rather than the vertical dead and live loads.

The bearing behaviour of a piled raft is characterized by a complex soil – structure interaction between the elements of the foundation and the subsoil. In detail there are the following interaction effects illustrated in Figure 2.1.

- Soil – pile interaction
- Pile – pile interaction
- Soil – raft interaction
- Pile – raft interaction

The interaction effects between the adjacent piles and between the piles and the raft indicate for example the important fact, that bearing behaviour as known from a comparable single standing pile. The awareness of this interaction effects and the development of an adequate calculation method taking into the most substantial interaction effects is the main condition for a reliable design of piled rafts.

2.3 Methods of Analysis

Several methods of analyzing piled rafts have been developed, and some of these have been summarized by Poulos (2000, 2001). Three broad classes of analysis method have been identified:

- Simplified calculation methods
- Approximate computer-based methods
- More rigorous computer-based methods.

Simplified methods include those of Poulos and Davis (1980), Randolph (1983,1994), van Impe and Clerq (1995), and Burland (1995). All involve a number of simplifications in relation to the modelling of the soil profile and the loading conditions on the raft.

The approximate computer-based methods include the following broad approaches:

- Methods employing a "strip on springs" approach, in which the raft is represented by a series of strip footings, and the piles are represented by springs of appropriate stiffness (e.g. Poulos, 1991). Methods employing a "plate on springs" approach, in which the raft is represented by a plate and the piles as springs (e.g. Clancy and Randolph, 1993; Poulos, 1994).

The more rigorous methods include:

- Simplified finite element analyses, usually involving the representation of the foundation system as a plane strain problem (Desai,1974) or an axi-symmetric problem and corresponding finite difference analyses via the commercial program FLAC.

- Three-dimensional finite element analyses and finite difference analyses via the commercial program FLAC 3D.
- Boundary element methods, in which both the raft and the piles within the system are discretized, and use is made of elastic theory (e.g. Butterfield and Banerjee, 1971; Wiesner and Brown).
- Methods combining boundary element for the piles and finite element analysis for the raft (e.g. Hain and Lee, 1978; Franke et al, 1994)

2.3.1 Simplified Analysis Methods

2.3.1.1 Poulos – Davis - Randolph (PDR) Method

For preliminary estimates of piled – raft behaviour, a convenient method of estimating the load – settlement behaviour may be developed by Poulos and Davis (1980) and Randolph (1994). As a consequence, the method to describe below will be referred to as the Poulos – Davis – Randolph (PDR) method. The method involves two main steps:

1. Estimation of the ultimate load capacity of the foundation.
2. Estimation of the load – settlement behaviour via a simple tri – linear relationship.

For assessing vertical bearing capacity of a piled raft foundation using simple approaches, the ultimate load capacity can generally be taken as the lesser of the following two values:

- The sum of the ultimate capacities of the raft plus all the piles
- The ultimate capacity of a block containing the piles and the raft, plus that of the portion of the raft outside the periphery of the piles.

For estimating the load-settlement behaviour, an approach similar to that described by Poulos and Davis (1980) can be adopted. However, a useful extension to this method can be made by using the simple method of estimating the load sharing between the raft and the piles, as outlined by Randolph (1994). The definition of the pile problem considered by Randolph is shown in Figure 2.3. Using his approach, the stiffness of the piled raft foundation can be estimated as follows:

$$K_{pr} = (K_p + K_r(1-\alpha_{cp}))/(1-\alpha^2{}_{cp} \;.K_r/K_p) \qquad (2.2)$$

where K_{pr} = stiffness of piled raft

K_p = stiffness of the pile group

K_r = stiffness of the raft alone

α_{cp} = raft – pile interaction factor.

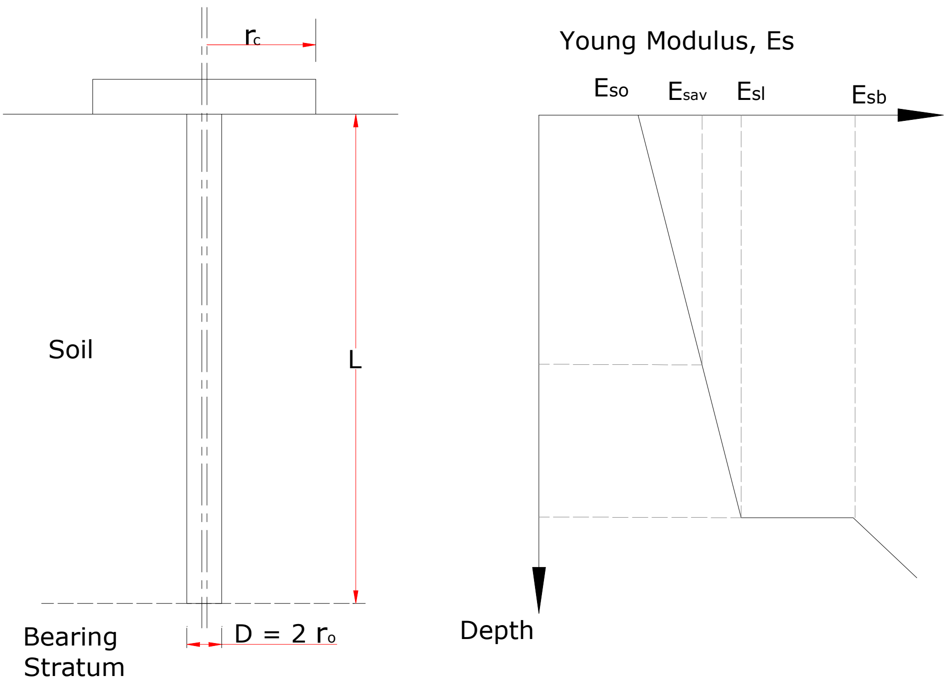


Figure 2.3. Simplified representation of a pile-raft unit (Poulos, 2001)

The raft stiffness K_r can be estimated via elastic theory, for example using the solutions of Fraser and Wardle (1976) or Mayne and Poulos (1999). The pile group stiffness can also be estimated from elastic theory, using approaches such as those described by Poulos and Davis (1980), Fleming et al (1992) or Poulos (1989). In the latter cases, the single pile stiffness is computed from elastic theory, and then multiplied by a group stiffness efficiency factor which is estimated approximately from elastic solutions.

The proportion of the total applied load carried by the raft is:

$$P_r / P_t = K_r (1-\alpha_{cp}) / (K_p + K_r (1-\alpha_{cp})) = X \qquad (2.3)$$

where P_r = load carried by the raft

P_t = total applied load.

The raft – pile interaction factor a_{cp} can be estimated as follows:

$$\alpha_{cp} = 1 - \ln (r_c / r_o) / \zeta \qquad (2.4)$$

where r_c = average radius of pile cap, (corresponding to an area equal to the raft area divided by number of piles)

r_o = radius of pile

$\zeta = \ln (r_m / r_o)$

$r_m = \{0.25 + \xi [2.5 \rho (1-\nu) - 0.25]\} * L$

$\xi = E_{sl}/E_{sb}$

$\rho = E_{sav}/E_{sl}$

ν = Poisson's ratio of soil

L = pile length

E_{sl} = soil Young's modulus at level of pile tip

E_{sb} = soil Young's modulus of bearing stratum below pile tip

E_{sav} = average soil Young's modulus along pile shaft.

The above equations can be used to develop a tri-linear load-settlement curve as shown in Figure 2.4. First, the stiffness of the piled raft is computed from Equation (2.2) for the number of piles being considered. This stiffness will remain operative until the pile capacity is fully mobilized. Making the simplifying assumption that the pile load mobilization occurs simultaneously, the total applied load, P_1, at which the pile capacity is reached is given by:

$$P_l = P_{up}/(1-X) \qquad (2.5)$$

where P_{up} = ultimate load capacity of the piles in the group

X = proportion of load carried by the Rafts (Equation 2.3).

Beyond that point (Point A in Figure 2.4), the stiffness of the foundation system is that of the raft alone (K_r), and this holds until the ultimate load capacity of the piled raft foundation system is reached (Point B in Figure 2.4). At that stage, the load-settlement relationship becomes horizontal.

The load – settlement curves for a raft with various numbers of piles can be computed with the aid of a computer spreadsheet or a mathematical program such as MATLAB.

In this way, it is simple to compute the relationship between the number of piles and the average settlement of the foundation. Such calculations provide a rapid means of assessing whether the design philosophies for creep piling or full pile capacity utilization are likely to be feasible.

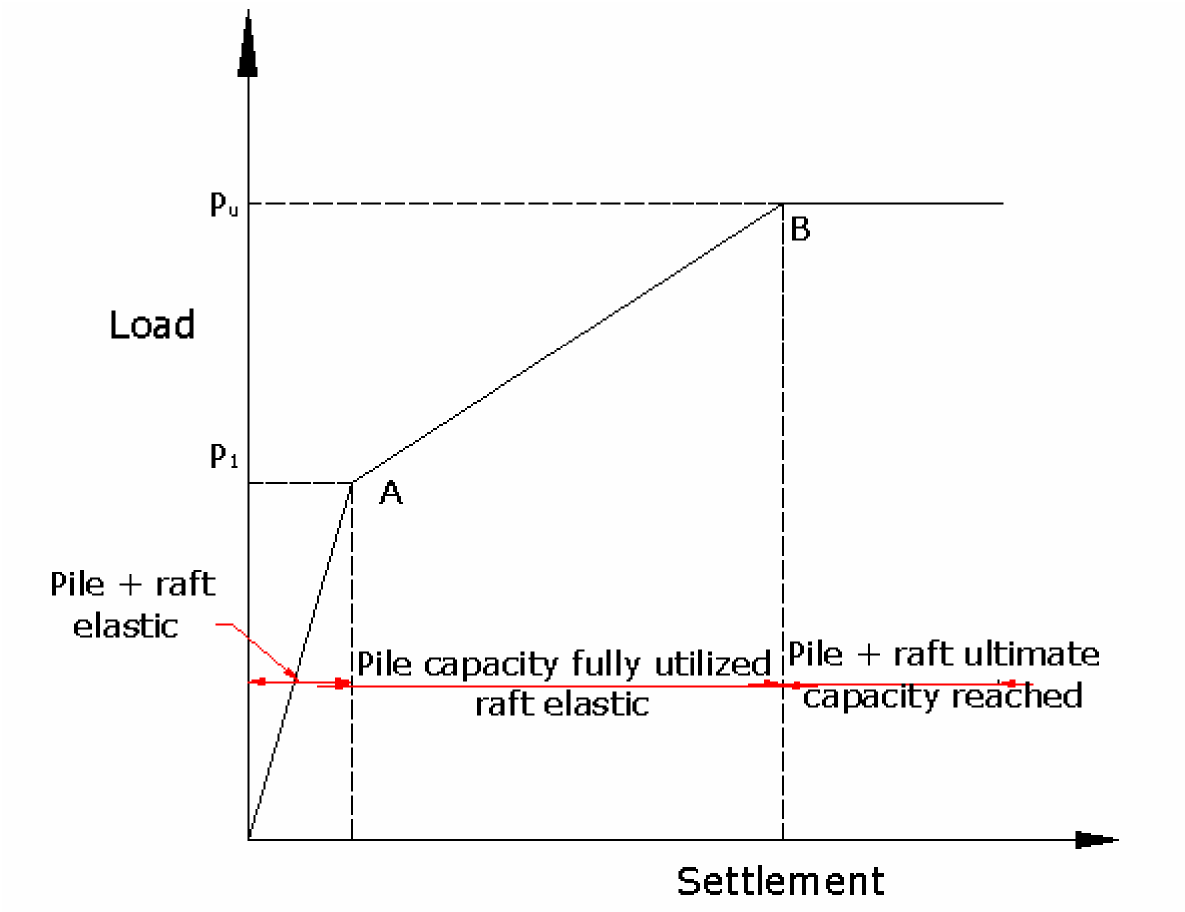


Figure 2.4 Simplified load-settlement curve for preliminary analysis (Poulos, 2001)

2.3.1.2 Burland's Approach

When the piles are designed to act as settlement reducers and to develop their full geotechnical capacity at the design load, Burland (1995) has developed the following simplified process of design:

- Estimate the total long-term load-settlement relationship for the raft without piles (see Figure 2.5). The design load P_0 gives a total settlement S_0.
- Assess an acceptable design settlement S_d, which should include a margin of safety.
- P_1 is the load carried by the raft corresponding to S_d.
- The load excess $P_o - P_1$ is assumed to be carried by settlement-reducing piles. The shaft resistance of these piles will be fully mobilized and therefore no factor of safety is applied. However, Burland suggests that a "mobilization factor" of about 0.9 be applied to the 'conservative best estimate' of ultimate shaft capacity, P_{su}.
- If the piles are located below columns which carry a load in excess of P_{su}, the piled raft may be analyzed as a raft on which reduced column loads act. At such columns, the reduced load Q_r is:

$$Q_r = Q - 0.9\, P_{su} \tag{2.6}$$

- The bending moments in the raft can then be obtained by analyzing the piled raft as a raft subjected to the reduced loads Q_r.
- The process for estimating the settlement of the piled raft is not explicitly set out by Burland, but it would appear reasonable to adopt the approximate approach of Randolph (1994) in which:

$$S_{pr} = S_r * K_r \,/ K_{pr} \qquad (2.7)$$

where S_{pr} = settlement of piled raft

S_r = settlement of raft without piles subjected to the total applied loading

K_r = stiffness of raft

K_{pr} = stiffness of piled raft.

Equation 2.2 can be used to estimate K_{pr}

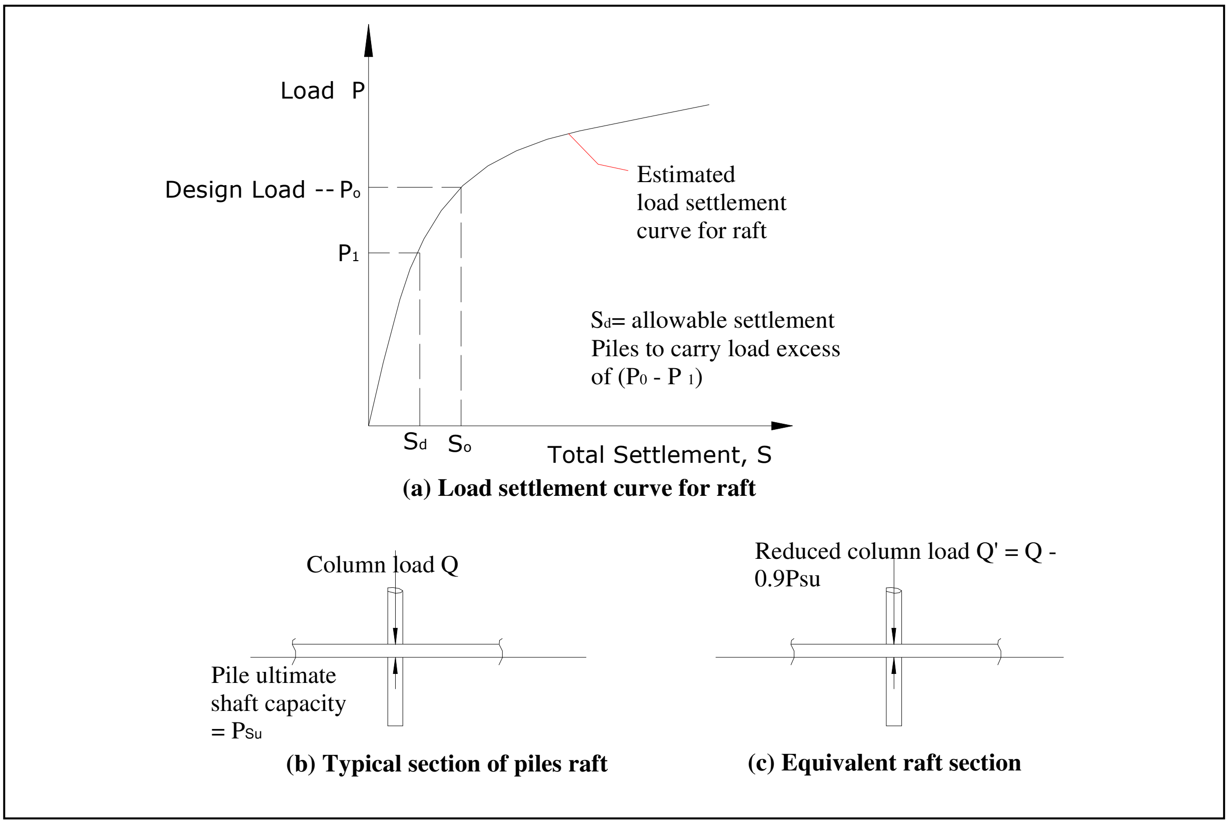


Figure 2.5 Burland's simplified design concept (Poulos, 2001).

2.3.2 Approximate Computer Methods

2.3.2.1 Strip on Springs Approach (GASP)

In this method, a section of the raft is represented by a strip, and the supporting piles by springs (Figure 2.6). Approximate allowance is made for all four components of interaction (raft-raft elements, pile-pile, raft-pile, pile-raft), and the effects of the parts of the raft outside the strip section being analyzed are taken into account by computing the free-field soil settlements due to these parts. These settlements are then incorporated into the analysis, and the strip section is analyzed to obtain the settlements and moments due to the applied loading on that strip section and the soil settlements due to the sections outside the raft.

The method has been implemented via a computer program GASP (Geotechnical Analysis of Strip with Piles) and has been shown to give settlements, which are in reasonable agreement with more complete methods of analysis. However, it does have some significant limitations, especially as it cannot consider torsional moments within the raft, and also because it may not give consistent settlements at a point if strips in two directions through that point are analyzed.

GASP can take account of soil non-linearity in an approximate manner by limiting the strip soil contact pressures not to exceed the bearing capacity (in compression) or the raft uplift capacity in tension. The pile loads are similarly limited to not exceed the compressive and uplift capacities of the piles. However, the ultimate pile load capacities must be predetermined, and are usually assumed to be the same as those for isolated piles. In reality, as shown by Katzenbach et al (1999), the loading transmitted to the soil by the raft can have a beneficial effect on the pile behaviour in the piled raft system. Thus, the assumptions involved in modelling piles in the GASP analysis will tend to be conservative (Poulos, 2001).

In carrying out a nonlinear analysis in which strips in two directions are analyzed, it has been found desirable to only consider nonlinearity in one direction (the longer direction) and to consider the pile and raft behaviour in the other (shorter) direction to be linear.

Such a procedure avoids unrealistic yielding of the soil beneath the strip and hence unrealistic settlement predictions.

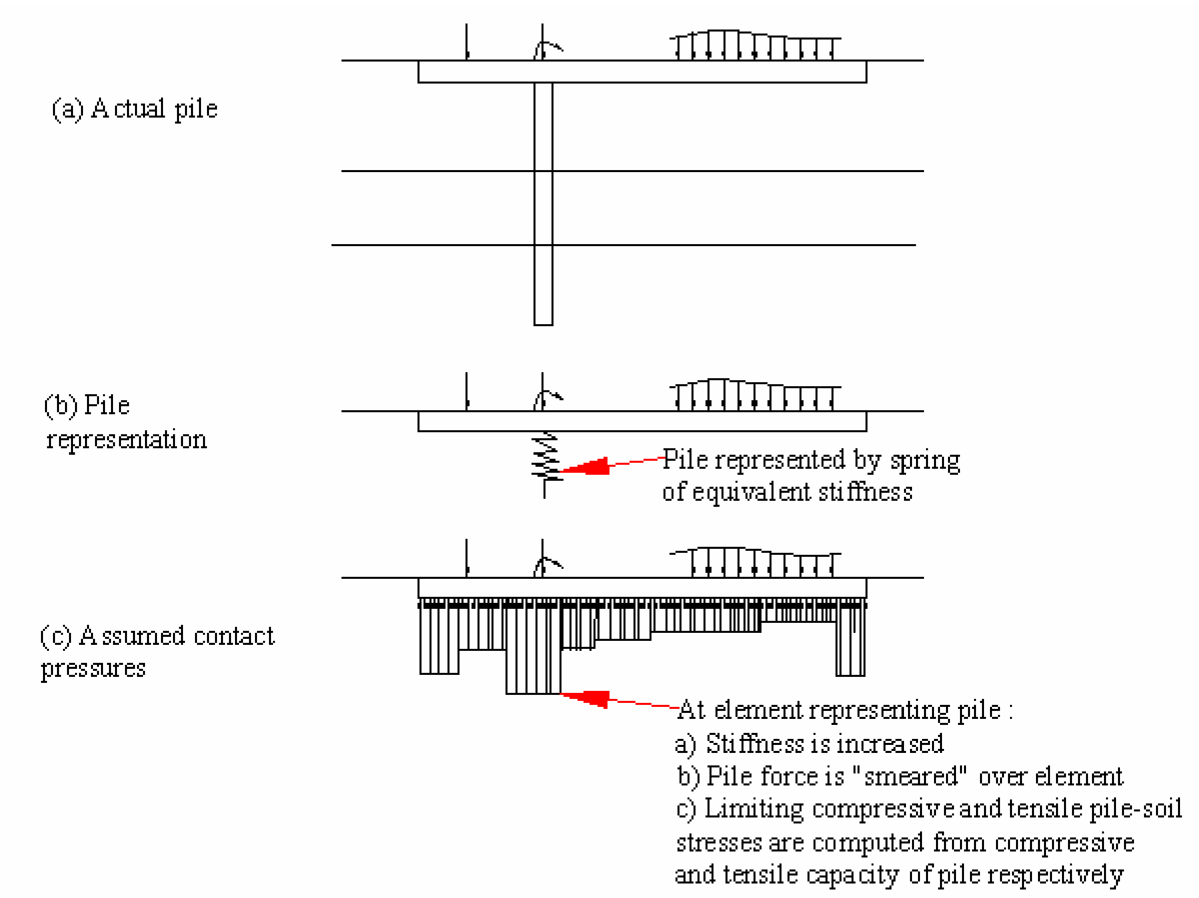


Figure 2.6 Representation of piled strip problem via GASP analysis (Poulos, 2001)

2.3.2.2 Plate on Springs Approach (GARP)

In this type of analysis, the raft is represented by an elastic plate, the soil is represented by an elastic continuum and the piles are modelled as interacting springs. Some of the approaches in this category (e.g. Kim et al 2001) neglected some of the components of interaction and gave pile-raft stiffnesses, which were too large.

Poulos (1994) has employed a finite difference method for the plate and has allowed for the various interactions via approximate elastic solutions. This analysis has been implemented via a program GARP (Geotechnical Analysis of Raft with Piles). Allowance has been made for layering of the soil profile, the effects of piles reaching their ultimate capacity (both in compression and tension), the development of bearing capacity failure below the raft, and the presence of free-field soil settlements acting on the foundation system. The approximations involved are similar to those employed in the program GASP for piled strips.

A later version of GARP (Sales et al, 2000) has replaced the finite difference analysis for the raft with a finite element analysis, and has employed a modified approach to considering the development of the ultimate load capacity in the piles.

Russo (1998) and Russo and Viggiani (1997) have described a similar approach to the above methods, in which the various interactions are obtained from elastic theory, and non-linear behaviour of the piles is considered via the assumption of a hyperbolic load-settlement curve for single piles. Pile-pile interaction is applied only to the elastic component of pile settlement, while the non-linear component of settlement of a pile is assumed to arise only from loading on that particular pile.

Most analyses of piled rafts are based on the raft being treated as a thin plate, and it is of interest to see what the effect of using thick plate theory is on the numerical predictions. Poulos et al (2001) have examined the effect of the method of modelling the raft as a thin plate who analyzed a typical problem using firstly, a three dimensional finite element program where the raft was firstly modelled using thin shell theory, and then secondly, by making the raft 0.3m thick, and assigning the raft modulus to that part of the finite element mesh representing the raft. It was assumed in the analysis that there was no slip between the raft and the soil or between the piles and the soil. It was found that there was not a great deal of difference in the computed deflections for the raft, for both a stiff raft and a flexible raft. It was concluded that the use of thin shell (plate) elements to represent the raft will lead to reasonable estimates of deflections, and therefore moments, as long as the raft is not extremely thick.

2.3.3 More Rigorous Computer Methods

2.3.3.1 Two – Dimensional Numerical Analysis

Methods in this category are exemplified by the analyses described by Desai (1974), Hewitt and Gue (1994) and Pradoso and Kulhawy (2001). In the former case, the commercially available program FLAC has been employed to model the piled raft, assuming the foundation to be a two-dimensional (plane strain) problem, or an axially symmetric three-dimensional problem. In both cases, significant approximations need to be made, especially with respect to the piles, which must be “smeared” to a wall and given an equivalent stiffness equal to the total stiffness of the piles being represented. Problems are also encountered in representing concentrated loadings in such an analysis, since these must also be smeared. Unless the problem involves uniform loading on a symmetrical raft, it may be necessary to carry out analyses for each of the directions in order to obtain estimates of the settlement profile and the raft moments. As with the plate on springs approach, this analysis cannot give torsional moments in the raft.

2.3.3.2 Three – Dimensional Numerical Analysis

2.3.3.2.1 3D Finite Element Method

A complete three-dimensional analysis of a piled raft foundation system can be carried out by finite element analysis (e.g. Katzenbach et al, 1998). In principle, the use of such a program removes the need for the approximate assumptions inherent in all of the above analyses. Some problems still remain, however, in relation to the modelling of the pile-soil interfaces, and whether interface element should be used. If they are, then approximations are usually involved in the assignment of joint stiffness properties. Apart from this difficulty, the main problem is the time involved in obtaining a solution, in that a non-linear analysis of a piled raft foundation can take several days, even on a modern computer running at high frequencies. Such analyses are therefore more suited to obtaining benchmark solutions against which to compare simpler analysis methods, rather than as routine design tools.

2.3.3.2.2 Hybrid FEM – BEM Approach

A FE – BE coupling approach developed by Hain and Lee (1978) considers thin plate finite element for the raft and boundary element method for the pile group. In this method Mindlin's elastic continuum solution is used for interaction of piles, where numerical integration of the equation over the pile surface is required.

Later, Griffith, Clancy and Randolph (1991), presented a hybrid finite element – elastic continuum – load transfer approach, which was developed specifically to minimize the amount of computation. It also takes advantage of quadrant symmetry to allow the complete analysis of piled rafts with more than 200 piles.

This hybrid method provides a relatively rigorous and yet considerably more efficient method of analysis for piled raft foundation than has previously been available. Fewer equations need to be solved than for the finite element method, and the time consuming numerical integration of the boundary element method are not necessary. The hybrid method allows for variable geometry, pile stiffness and raft thickness.

Therefore, this method is chosen and adopted for this study and it is further discussed deeply in next chapter. A program for the analysis of piled raft foundation has been developed using this approach. The results have been validated against more rigorous 3D finite element solution available in literature.

Chapter Three

Hybrid Method of Analysis.

3.1 Introduction

The Method described here is a ‘hybrid’ numerical method, which has been developed specifically to minimize the amount of computation (Griffith et al, 1991). It combines finite elements to model the structural elements of the foundation (the raft and pile) and simplified closed form analytical expressions for the diagonal terms of the soil response stiffness matrices. The three kinds of interactions, namely, pile- soil-pile, pile-soil-raft and raft-soil-raft are accounted for using the elastic theory of Mindlin (1936).

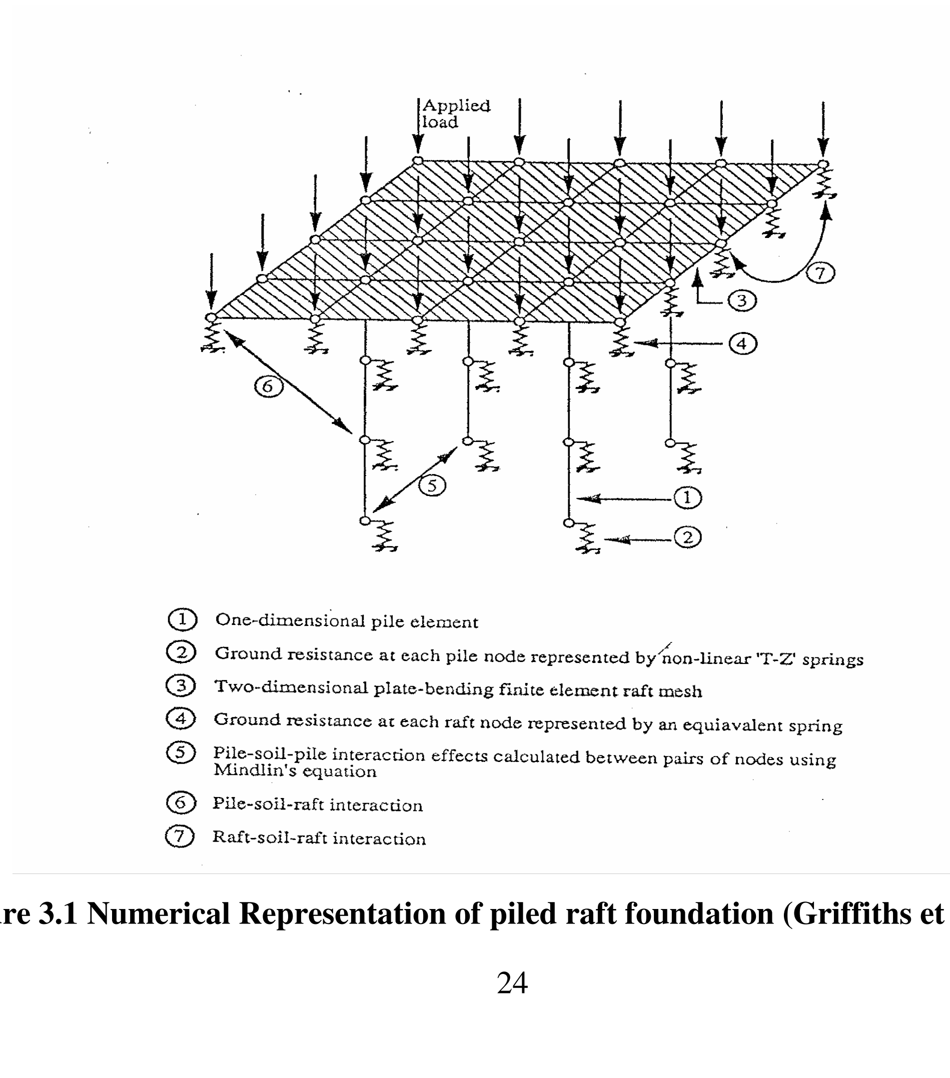


Figure 3.1 Numerical Representation of piled raft foundation (Griffiths et al., 1991)

These simplifications, which have been checked for accuracy against more refined approaches, enable problems of practical proportion to be analyze (Chow, 1986).

3.2 Method of Analysis

A piled raft foundation consists of a raft, a group of piles and underlining soil as illustrated in Figure 3.1. The raft is idealized as a plate because the thickness of the raft is relatively small compared with other dimension of the raft. The thin plate theory, in which the shear deformation of the raft is not taken into account, is adopted in the analysis.

The basic equation representing the piled raft foundation and used to analyze the raft under applied loads and the soil and pile reaction forces is

$$D[K]\{w\} = \{Q\} - \{P\} \tag{3.1}$$

In which, $D = \frac{E\,t^3}{1-v^2}$, flexural rigidity of raft/ unit width; E = Young's modulus of raft material; v = Poisson's ratio of raft material; t = thickness of raft; [K] = finite element stiffness matrix of the raft; $\{w\}$ = nodal degree of freedom for the raft;{Q} = vector of applied nodal loads; and {P} = vector of soil and pile reaction forces.

Since the formulation of the finite element stiffness matrix of the raft element is available in most of standard finite element books, it will not be described here. The flexibility matrix of an equivalent soil 'spring' response is calculated for each raft node using an analytical solution due to Giroud (1968) for the average displacement under a uniformly loaded rectangular area. According to Giroud the elastic settlement of uniformly loaded rectangular flexible area (B x L) at its corner can be calculated as

$$w = \frac{1-v^2}{E_s} B \, I_s \ \mathrm{P} \qquad (3.2)$$

In which w = elastic settlement, v = Poisson's ratio of soil, Es = Young's modulus of soil, P= uniform pressure and I_s = influence factor given by

$$I_s = \frac{1}{\pi}\left[\ln\,(\alpha + \sqrt{1+\alpha^2}\ ) + \alpha \ln\,(\frac{1+\sqrt{1+\alpha^2}}{\varepsilon}) \right] \quad \text{where } \alpha = L/B\,, \quad for \quad L \geq B\,.$$

The area of raft contributing to each node is calculated by summing the area of each raft element to which the node is attached, and by dividing this area by four (since each element has four nodes).

The soil settlement influence matrix must be modified in two ways to take account of the presence of piles. Firstly, since there is no zone pressure above a pile, the term arising from these pressures should be replaced by soil surface settlements due to unit load on a pile. Secondly, all terms contributing to settlement at location of a pile should be replaced by terms for the settlement of pile. However, it has been assumed that the reciprocal theorem applies to a pile in an elastic continuum, so that the settlement of a pile due to a unit surface soil load at a point is equal to the settlement at the point on the soil surface due to a unit load on the pile. The settlement influence matrix obtained in this way describes the combined soil – pile system. The soil flexibility matrix between raft node and pile nodes (raft – soil - pile interaction) is also calculated using Mindlin's equation.

The flexibility matrix for the raft –soil and raft – pile interaction will be

$$\{\Delta w_i\} = [F]\{\Delta P_{pri}\} \qquad (3.3)$$

Where $\{\Delta w_{si}\}$= vector of incremental load settlement; [F]= total settlement flexibility matrix; and $\{\Delta P_{pri}\}$ = incremental nodal loads vector of piles and raft interfaces.

The flexibility matrix due to pile – soil – pile interaction is added to this flexibility matrix. The resulting soil flexibility matrix is inverted to give the load-displacement relationship of the soil for the interfaces of the pile group and raft.

$$\{\Delta P\} = [K_s]\{\Delta w_{si}\} \qquad (3.4)$$

where $[K_s] = [F]^{-1}$ = soil stiffness matrix .

Next, the settlement analysis of piles will be discussed. Various theoretical methods are available for the analysis of vertically loaded pile groups. The most powerful of these methods is the finite element method. In principle, the method is able to deal with soil in-homogeneity and non-linearity in a consistent manner. However, the three-dimensional nature of the problem makes the method expensive, even for the assumption of linear soil behavior.

The pile group problem is more tractable using the integral equation method (also known as the boundary element method). Since the method involves only a discretization of the boundaries, the number of equations to be solved is generally smaller than in the finite element method. Design charts are available for the computation of group response for various pile group configurations. (Poulos and Davis ,1980)

Although the method has been extended to include non-homogeneous soil and soil non-linearity, the discretization of the soil layers and the introduction of volume elements for non-linear calculations rapidly increase the size of the system of equations to be solved, making the method less attractive. It is noted that an approximate procedure has been suggested by Poulos and Davis (1980) to deal with soil inhomogeneity and soil non-linearity, while retaining the use of Mindlin's solution as the kernel.

Randolph and Wroth (1979) have introduced an approximate analytical model for the computation of the vertical deformation of pile groups. The method is based on the superposition of the displacement fields of individual piles within the group. The soil may be homogeneous, or its stiffness may increase linearly with depth. However, the method is confined to the assumption of linear elastic soil behavior.

O'Neill et al. (2001) have proposed a hybrid model in which the response of individual piles is modelled using the load-transfer (t-z) method, and the interaction between the piles, through the soil, is effected using Mindlin’s solution for a point load within a homogeneous, isotropic elastic half-space. Briefly, the approach is as follows:

1. Determine the response of individual piles within the group, ignoring interaction effects.
2. Using soil reactions determined from Step 1, compute additional soil displacements at the nodes of other piles in the group using Mindlin's solution. Interaction effects between nodes within the same pile are ignored. (This is the essence of the load-transfer method.)
3. The load-transfer (t-z) curves for the individual pile nodes are adjusted using the additional soil displacements to account for group effects.

With the modified load-transfer curves, steps 1-3 are repeated, and the solution proceeds in an iterative manner. The main limitation of the method is that true pile-soil-pile interaction is not considered directly, and the convergence of the iterative procedure has not generally been demonstrated.

Later, a refinement of the solution procedure for the hybrid model is described by Chow (1986), in which true pile-soil-pile interaction is considered directly. The accuracy of this approach is examined by comparison with the more rigorous integral equation method and finite element method. This method is adopted and described here.

3.2.1 Load Transfer Mechanism (Single Pile)

A one-dimensional rod finite element is used to represent each axially loaded piles, coupled with a soil response at each pile node modelled by discrete load transfer (or t – z) springs. (Shown figure 3.2). The load transfer between the pile and soil is divided in to two parts:

1. The load transfer along the pile shaft, where the concept of t -z curve developed by Kraft et al, (1981) is used to obtain the shear or wall (frictional) resistance response of the pile, and
2. The load transfer at the pile tip, where the concepts of p - z load transfer is used.

The procedure involves finding of the t - z curves at different depths along the pile. Then the p - z curves is formed at the pile tip. Finally, the forces and displacements at different depths are found using the computational procedure in which the t-z and p-z curves are introduced.

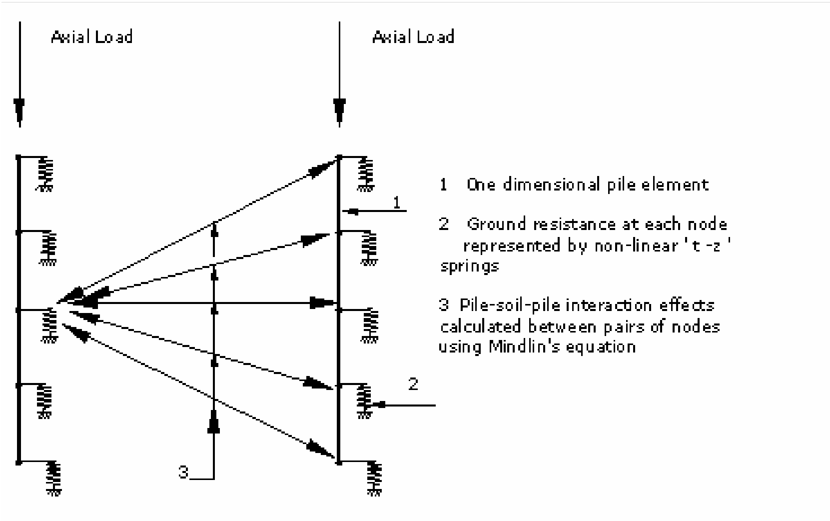


Figure 3.2 One-dimensional pile element (Griffiths et al., 1991)

3.3 Development of t - z Curves

The load-transfer curves used in the analysis describe the relationship between mobilized shaft friction and pile movement, and in the offshore industry they are commonly referred to as t-z curves. The stress and displacement of each point along a pile are influenced by the stress at every other point; the degree of influence decreases with distance from the point of interest. The sub grade reaction approach assumes that the displacement at any point depends only on the stress at that point. Soil - pile represented by springs that relate shear stress at the soil - pile interface and displacement of the pile (Fig 3.2).

Success in developing realistic t - z relationships for pile depends on the accuracy of the ultimate load transfer values of the soil (pile capacity), the distribution of these values along the pile, and the displacement characteristics of the soil during load transfer.

To construct rational t- z curves, pre failure and post failure t - z responses are considered separately. Pre failure t - z relationships were described by a theoretical model based on elasticity. Post failure relationships were modeled considering the residual stress deformation behavior at the soil - pile interface.

3.3.1 Pre failure t - z curves

1) The theoretical formulation discussed below uses a concentric cylinder approach with the following key assumptions and conclusions as described by Randolph and Wroth (1978).
2) The displacement pattern of soil can be modeled as concentric cylinders in shear (Figure 3.3).
3) Radial soil displacements due to pile loads are assumed negligible when compared to vertical deformations. Therefore, simple shear condition pre failure in the soil.
4) Shear stress decreases with distance such that $\tau r = \tau_o r_o$; in which τ = shear stress at distance r; τ_o = the shear stress on the pile soil interface, and r_o = the pile radius.
5) Shear stress are negligible beyond a radial distance r_m (or zone of influence and the soil does not deform beyond that point.

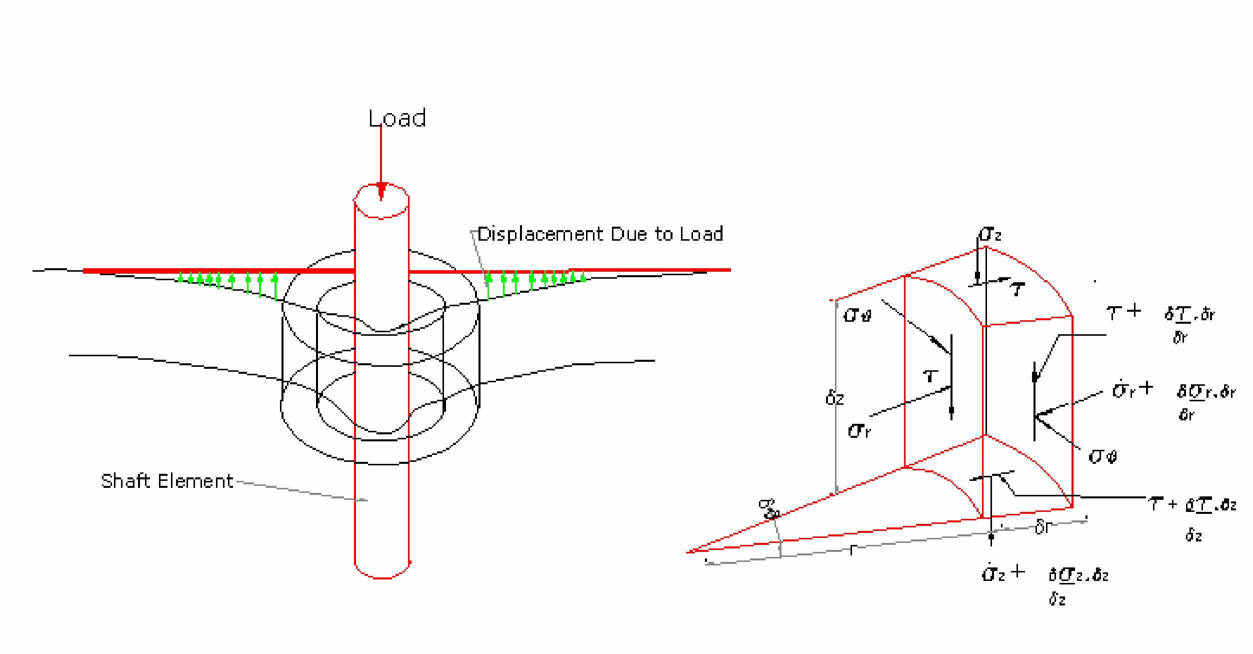


Fig. 3 .3 (a) Settlement molded as shearing concentric cylinders.
(b) Stresses on soil element (Randolph and Wroth, 1978)

Consideration of vertical equilibrium of an element of soil (see figure 3.3.b) yields.

$$\frac{\partial}{\partial r}(r\tau) + r\frac{\partial \sigma_z}{\partial z} = o \qquad (3.5)$$

in which τ = the shear stress increament; and

σ_z = the vertical total stress increment (taking compressive stresses as positive)

When the pile is loaded, the increase in shear stress, τ , in the vicinity of the pile shaft will be much greater than the increase in vertical stress, σ_z, and thus Eqn. 3.5 may be approximated to be come.

$$\frac{\partial}{\partial r}(r\tau) \cong o \qquad (3.6)$$

Writing the shear stress at the pile shaft, $r = r_o$, as τ_o, Eqn. 3.6 may be integrated to give:

$$\tau = \frac{\tau_o r_o}{r} \tag{3.7}$$

The shear strain (reduction in angle taken as positive) is

$$\gamma = \frac{\tau}{G} = \frac{\partial u}{\partial z} + \frac{\partial \omega}{\partial r} \tag{3.8}$$

in which u = the radial, and w = the vertical displacement of the soil.

The primary displacement will be vertical and thus, ignoring $\partial u/\partial z$ integration leads to:

$$w_s \cong \frac{\tau_o r_o}{G} \int_0^{\infty} \frac{dr}{r} \tag{3.9}$$

In which w_s = settlement of the pile in this expression for w_s implies infinite settlement of the pile, which is clearly unacceptable. The normal assumption is that there is some magical radius, r_m, at which the shear stress becomes negligible, and thus

$$w_s = \frac{\tau_o r_o}{G} \ln\left(\frac{r_m}{r_o}\right) = \zeta \frac{\tau_o r_o}{G} \tag{3.10}$$

in which $\zeta = \ln(r_m/r_o)$.

The settlement at a radius, r, may be expressed (from Eqn. 3.9 and Eqn. 3.10) as

$$w = w_s - \frac{\tau_o r_o}{G} \ln\left(\frac{r}{r_o}\right) \tag{3.11}$$

A suitable value for the average magical radius is $r_m = 2.5 * (1 - \nu) * L$.

3.3.1.1 Radial Variation of Shear Modulus

A typical distribution of the soil modulus is shown in figure 3.4 for four conditions

1) Immediately after installation of pile loads that result in small soil strain.

2) Immediately after installation for pile loads near failure that result in large soil strains;

3) After consolidation for small pile loads; and

4) After consolidation for pile loads near failure.

Variation in modulus due to pile installation and soil consolidation is estimated from results of cavity expansion to simulate changes in soil stresses during installation and consolidation theory to simulate the stress changes due to dissipation of pore pressures induced during installation (Kraft et al, 1981).

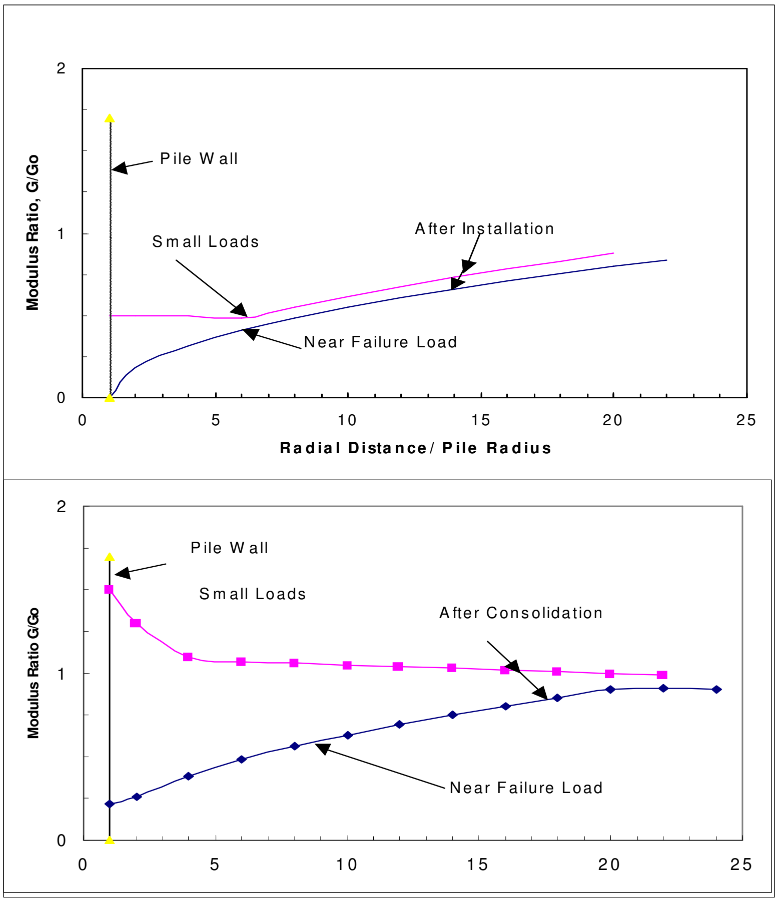


Figure 3.4 Variations in Soil Modulus Ratio (Kraft et al, 1981)

For conditions after consolidation, the non-linear effects are more important than the installation effects. Therefore, further assessment and consideration of the effect due to installation will be neglected in the following word and only the non-linear effect is considered.

3.3.1.2 Non-linear Stress-Strain

The shear stress – shear strain behavior of some soils is described by a hyperbolic expression.

$$G = G_i \left(1 - \frac{\tau_o R_f}{\tau_{\max}}\right)^2 \tag{3.12}$$

in which G = tangent shear modulus at an applied shear stress, τ_o;

G_i = Initial shear modules at small strains

R_f = stress -strain curve fitting constants (failure ratio)

$\tau_{\max}$ = Shear stress at failure

By combining Eqn. 3.6 and Eqn.3.12 and the distribution of τ with r, the equation or t - z response becomes.

$$w(z) = \frac{\tau_o r_o}{G_i}\left[\ln\left(\frac{r_m - \varphi}{r_o - \varphi}\right) + \frac{\varphi(r_m - r_o)}{(r_m - \varphi)(r_o - \varphi)}\right] \tag{3.13}$$

in which $\varphi = \tau_o r_o R_f / \tau_{\max}$

Eqn. 3.13 can be used to generate t - z curves. Although this equation neglects radial heterogeneity caused by the installation processes, after a long setup for driven piles, may be minor compared to uncertainties in the properties of the undisturbed soil and to the approximations used in developing the equations. If a situation arises where both radial heterogeneity and stress-strain non-linearity share equal importance, both effects can be combined as discussed previously.

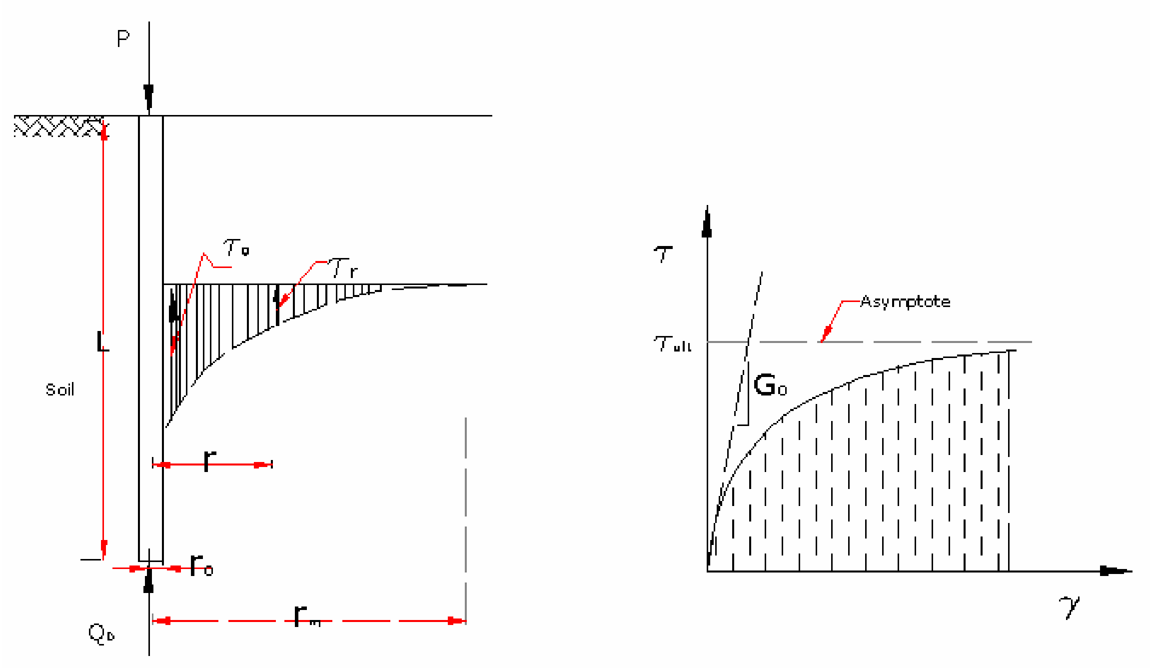


Figure 3.5 (a) Shear Stress distribution in Radial Direction
(b) Nonlinear Hyperbolic Stress –strain relationships (Kraft et al, 1981)

3.3.1.3 Variation of Shear Modulus with Depth

The soil modulus varies along the pile, with radial distance from the pile, and with pile load as a result of natural soil variation, soil disturbance during pile installation, soil consolidation during setup, and non linear stress - strain response. The t - z response at a particular depth, however, is taken to be controlled by soil modulus conditions at that depth. Using this assumption, Randolph and Wroth (1978) approximated a linear increasing variation in soil modulus along the pile shaft. The load - displacement response computed was in good agreement with more rigorous solutions. Therefore, this basic assumption in the t - z approach showed account for vertical variation in soil modulus.

3.3.2 Post Failure t - z curves

To develop an analytical model that includes t - z behavior at and beyond failure, some related problem include

1) Maximum shaft friction
2) Pile displacement (or strain) at which maximum friction is mobilized.
3) Residual shaft friction at large pile displacements
4) Displacement behavior between maximum (peak) and residual stresses.

In this study, the maximum shaft friction was computed by conventional means and the pile displacement, max, at which the maximum friction is mobilized with Eqn. 3.13 Some conventional methods to compute shaft friction provide estimates of average values along the pile shaft. These methods accounts empirically for differences between maximum and residual load transfer and the degree of shear resistance between these values that is mobilized at maximum load on the pile. Also, these conventional methods provide average resistances along the shaft that may not reflect the actual distribution along the shaft. The distribution of shaft friction along the pile is important. Some discretion should be used to establish the distribution of shaft friction along the pile when using conventional methods to compute the maximum friction.

Once the soil reaches a state of incipient failure, additional deformation at the pile wall results from a shear strain in the soil with a rotation of principal planes, movements along slip planes, or a combination of these conditions. Then the slip planes develop, the soil behavior cannot be deformed in terms of stress and strain.

Therefore, equation 3.13 cannot be used to define the t -z response at post failure conditions after slip plane develop. One approach to post failure behavior is to model a section of the pile - soil system in a direct shear test at some laboratory simulation. Replacing one half of steel, concrete, or wood does this. If the failure is in the soil rather than the soil - pile interface, a conventional direct shear test may be more appropriate to define the load transfer displacement response for post failure conditions.

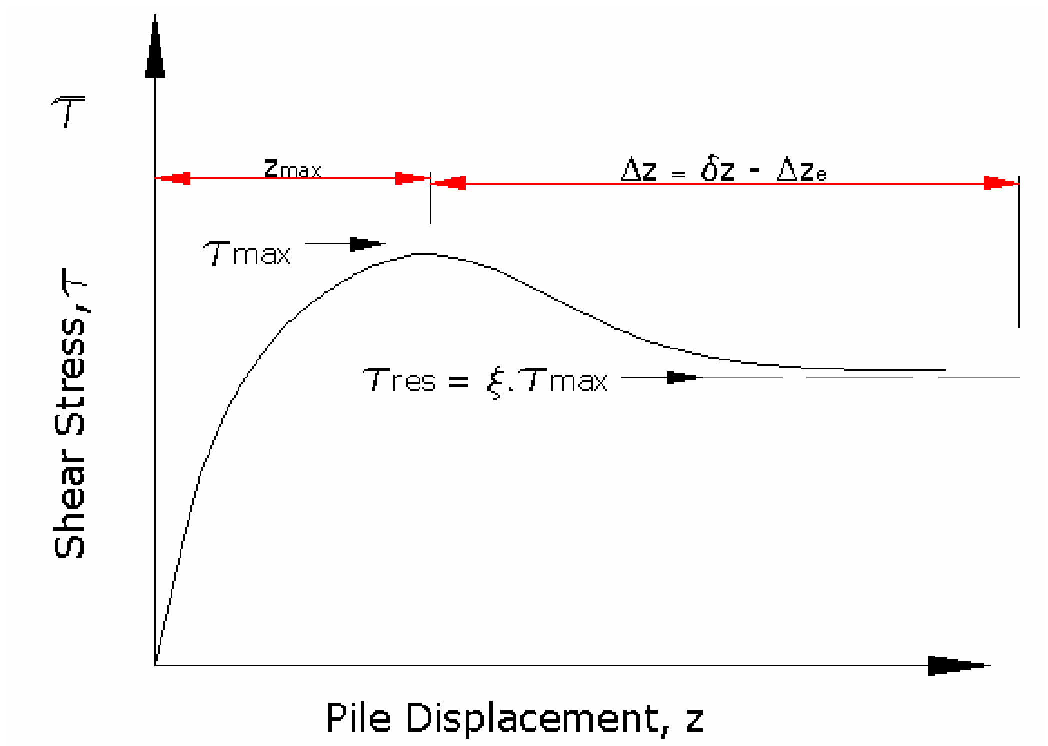


Figure 3.6 Development of t – z Curve (Kraft et al, 1981)

3.3.3 Interpretation of Direct Shear Tests

The direct shear test is not perfect simulation of the load transfer- displacement behaviour of a pile segment. The total normal stress is held constant during direct shear test. This boundary condition is different from the displacement boundary condition (nearly zero radial movement) during pile loading. The stress concentrations caused by the small contact area of the laboratory test and spacing between shear boxes also result in differences between simulation and prototype. Furthermore, the stress displacement curve generated from a direct shear test simulates the condition along failure plane and doesn't account for elastic strains that occur in the soil some distance away from the pile.

3.3.4 Procedure to construct Theoretical t - z curve

The t-z curve is constructed in two parts, pre failure and post failure. The basic concept is summarized in figure 3.6 and pre failure t-z is governed by Eqn. 3.13. This equation is flexible and can be easily altered to simulate changes in shear modulus with shear strain and changes of shear modulus with radial distance. The load displacement relationship requires a maximum unit load transfer value, τ_{max} ; shear strength S_u; soil modulus G_{max} and stress strain constant (failure ratio), R_f. The value of τ_{max} may be less than S_u, for example, for piles in staff, over consolidated clays.

τ_{max} is computed in the same way as it would be for shaft friction in the pile capacity computations, but some discretion should be used to select the distribution of τ_{max} along the pile. For example, a linear increasing distribution of τ_{max} is probably more appropriate for short driven piles in stiff, over consolidated clays than a uniform value, even though the shear strength may be constant over the pile length. The average of the distribution should be consistent with the value obtained from the conventional pile capacity procedures. If laboratory data show that the residual shear resistance is less than $\tau_{\max}$, the pile capacity procedure is used to provide a value of shear resistance between $\tau_{\max}$ and τ_{res}. Laboratory data can be used to determine the non-linear response, as reflected by R_f with the hyperbolic model.

Once the failure stress has been reached, the post - failure behavior can be estimated from results of direct shear tests.

3.3.5 Pile Tip Deformation

The pile base is represented as rigid punch acting at the surface of an infinite soil mass ignoring the pile shaft and the surrounding soil depth (Randolph and Wroth, 1978) and, as such, its load deformation behavior is described by (Timoshenko and Goodier, 1970):

$$\Delta w_b = \frac{P_b\ (1-v)}{4\,G\,r} \tag{3.14}$$

Hence, considering the non-linear variation of shear modulus, the pile base relative incremental settlement is given as follows:

$$\Delta w_b = \frac{P_b\ (1-v)}{4\,G\,r(1-\varphi)^2} \tag{3.15}$$

Where Δw_b =incremental base settlement ; ΔP_b = incremental pile base load ; G = initial tangent shear modulus ; $\varphi = R_f\ P_b / P_{b\max}$ =pile base load level , P_b current base load ; and $P_{b\max}$ = limiting pile base load.

3.4 Pile Group in Homogeneous Soil

The pile –soil-pile interaction problem may be represented schematically in Figure 3.2 for a pile group embedded in a homogeneous, isotropic elastic half-space. The loading of any pile will induce additional displacements of other piles in the group.

The relative incremental settlement of node (i) due to its own loading and additional loadings at nodes of other piles in group may be expressed as follows:

$$\Delta w_{si} = \sum_{j=1}^{j=n} f_{ij} \Delta P_j \qquad (3.16)$$

Where Δw_{si} = incremental soil settlement at node (i), f_{ij} =soil settlement flexibility coefficient denoting the settlement at node (i) due to unit load at node (j), ΔP_j = incremental load at node (j), and n = total number of nodes in the group.

Similar displacement-load relationships may be written for the rest of the nodes, giving the following matrix equation. The overall incremental load-settlement relationship for the pile group is given as follows:

$$\{\Delta w_{si}\} = [F]\{\Delta P_{pi}\} \qquad (3.17)$$

Where $\{\Delta w_{si}\}$= vector of incremental load settlement; [F]= total settlement flexibility matrix; and $\{\Delta P_{pi}\}$ = incremental nodal loads vector of pile interfaces.

The settlement flexibility coefficient for node (i) due to unit load at node (i), f_{ii}, for shaft and base loads are obtained from Eqn. (3.15) and Eqn. (3.13), respectively, with **{P}** equal to unity.

Inherent in the load-transfer method for modeling soil behaviour in the single pile is the assumption that the soil reactions are uncoupled, i.e. the displacement at a particular node will only affect the soil reaction at that node. Thus, the values of the flexibility coefficients, $f_{ii} = 0$, for loadings at nodes j which are associated with the same pile as node (i), and for $i \neq j$. In determining interaction between piles, the continuously distributed loads at the pile shaft and pile base are replaced by point loads acting at the pile nodes; the rationale being that, at some distance, their effects on the soil displacements are indistinguishable. Numerically, it is more convenient to determine the influence coefficients due to point loads (Chow 1986). The flexibility influence coefficients due to interaction between piles may be determined based on Mindlin's solution for a vertical point load in homogeneous, isotropic elastic half-space.

3.4.1 Extension to non-linear soil behaviour

The use of the load-transfer curves advocated by Kraft et al. (1981) for the single pile provides a natural transition from linear elastic to non-linear response of the pile group. At high soil strains at the pile- soil interfaces of the individual piles in the group, slippage begins to take place. This non-linearity is confined only to a narrow zone of soil adjacent to the pile shaft, whereas the bulk of the soil between the piles is subjected to relatively low strain levels, and hence remains essentially elastic. Thus, at this stage, the non-linear response of the group is dominated by the non-linear behaviour of the individual piles whereas interaction effects remain essentially elastic (characterized by the use of G_i in Mindlin's solution). However, when the shear strength of the soil is fully mobilized at a particular node, full slippage takes place at the node. Further increase in load acting on the pile group will not increase the soil reaction at that node. Moreover, further increase in loads at the remaining nodes in the group will not causes further increase in displacement of that particular node because of the discontinuity resulting from full slippage staking place. Thus, there is no further interaction through the soil between that node and the remaining nodes in the pile group. It may be noted that the manner in which the non-linear response of the pile group is obtained is approximate. Before full slippage at a particular node has taken place, the non-linear soil behaviour adjacent to the pile shaft introduces a discontinuity in the material property. Thus, the use of Mindlin's solution to determine the interaction effects is only approximate.

The solution procedure can be outlined as follows:

For each load increment

1. Evaluate the coefficients of the flexibility matrix . The non-linear response of the soil for the single pile is obtained from the tangent values of the flexibility coefficients
2. For nodes at which the shear strength is fully mobilized, set the corresponding rows and columns in the flexibility matrix to zero.
3. Invert the flexibility matrix to give the stiffness matrix for the soil
4. Assemble the pile and soil stiffness matrices.
5. Solve the stiffness equations to give the incremental settlement of the pile group.

3.4.2 Extension to non-homogeneous soil.

The interaction between the piles in the group is effected through the use of Mindlin's solution, which is strictly valid for homogeneous, isotropic elastic half-space.

Interaction effects in non-homogeneous soil may be approximated by using the mean of the soil shear modulus at node (i) (where the displacement is evaluated) and node (j) (where the unit load is applied) in Mindlin's solution. Reasonably accurate solutions have been reported by Poulos (1994) using a similar approach.

3.4.3 Approximate Treatment of Finite Depth and Layered soils.

The elements of flexibility matrix previously calculated approximately (Mindlin's equation) apply only for a soil mass of infinite depth. For soil layers of finite depth or relatively rigid base (shear modulus, G_b) at some depth, the elements of **[Fs]** may be obtained approximately by employing the Stein Brenner's approximation.

The settlement flexibility coefficients may also be evaluated by using more accurate and complex analytical solutions for point load embedded in two layer elastic half space (Davies and Banerjee, 1978). However, Mindlin's solution (SteinBrenner approximation) provides adequate practical approximation for the settlement flexibility coefficients and since it is relatively simple hence the essence of load transfer approach is retained.

3.5 Raft – Soil Flexibility Matrix

The flexibility matrix of an equivalent soil 'spring' response is calculated for each raft node using an analytical solution due to Giroud (1968) for the average displacement under a uniformly loaded rectangular area. According to Giroud the elastic settlement of uniformly loaded rectangular flexible area (BxL) at its corner can be calculated as

$$w = \frac{1 - v^2}{E_s} B \, I_s \, \mathrm{P} \qquad 3.18$$

In which w= elastic settlement , v = Poisson's ratio of soil, Es = Young's modulus of soil , P = uniform pressure and I_s = influence factor given by $I_s = \frac{1}{\pi}\left[\ln(\alpha + \sqrt{1+\alpha^2}) + \alpha \ln(\frac{1+\sqrt{1+\alpha^2}}{\varepsilon}) \right]$ where $\alpha = L/B$, *for* $L \geq B$

The area of raft contributing to each node is calculated by summing the area of each raft element to which the node is attached, and by dividing this area by four (since each element has four nodes). The soil flexibility matrix between raft node and pile nodes (raft – soil - pile interaction) is also calculated using Mindlin's equation.

Forming the global flexibility matrix for the raft soil-interface and considering the raft-soil-raft interaction and pile –soil - raft interaction using Mindlin's equation.

$$\{\Delta w_i\} = [F]\{\Delta P_{pri}\} \qquad 3.19$$

Where $\{\Delta w_{si}\}$= vector of incremental load settlement; [F]= total settlement flexibility matrix; and $\{\Delta P_{pri}\}$ = incremental nodal loads vector of piles and raft interfaces.

This flexibility matrix due to pile – soil – pile interaction is added to this flexibility matrix. The resulting soil flexibility matrix is inverted to give the load-displacement relationship of the soil for the interfaces of the pile group and raft.

$$\{\Delta P\} = [K_s]\{\Delta w_{si}\} \qquad 3.20$$

where $[K_s] = [F]^{-1}$ = soil stiffness matrix.

Considering the compatibility and equilibrium conditions, the analyses are combined by attaching piles to the raft via common nodes at connecting points. The vertical freedoms are linked, resulting only axial load being transmitted to piles. It is assumed that there is no raft-soil contact at the common nodes.

$$[[K_s] + [K_p] + [K_r]]\{\Delta d\} = \{\Delta Q\} \qquad 3.21$$

in which $\lfloor K_p \rfloor$ = assembled stiffness matrix of the piles , $\lfloor K_p \rfloor$ = global stiffness matrix of the raft element, $\{\Delta Q\}$ = applied load on the raft incrementally and $\{\Delta d\}$ = the nodal degree of freedom of the plate elements.

The complete incremental load-displacement relationship for the pile group – raft system is expressed as follows

$$[K_G]\{\Delta\, d\} = \{\Delta Q\} \qquad 3.22$$

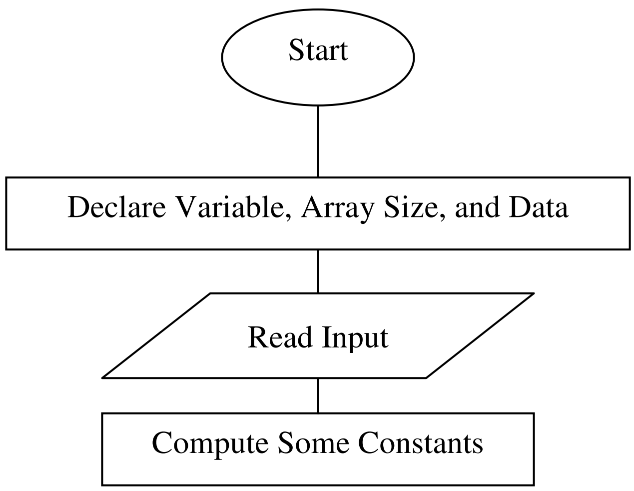
3.6 Stress Computation

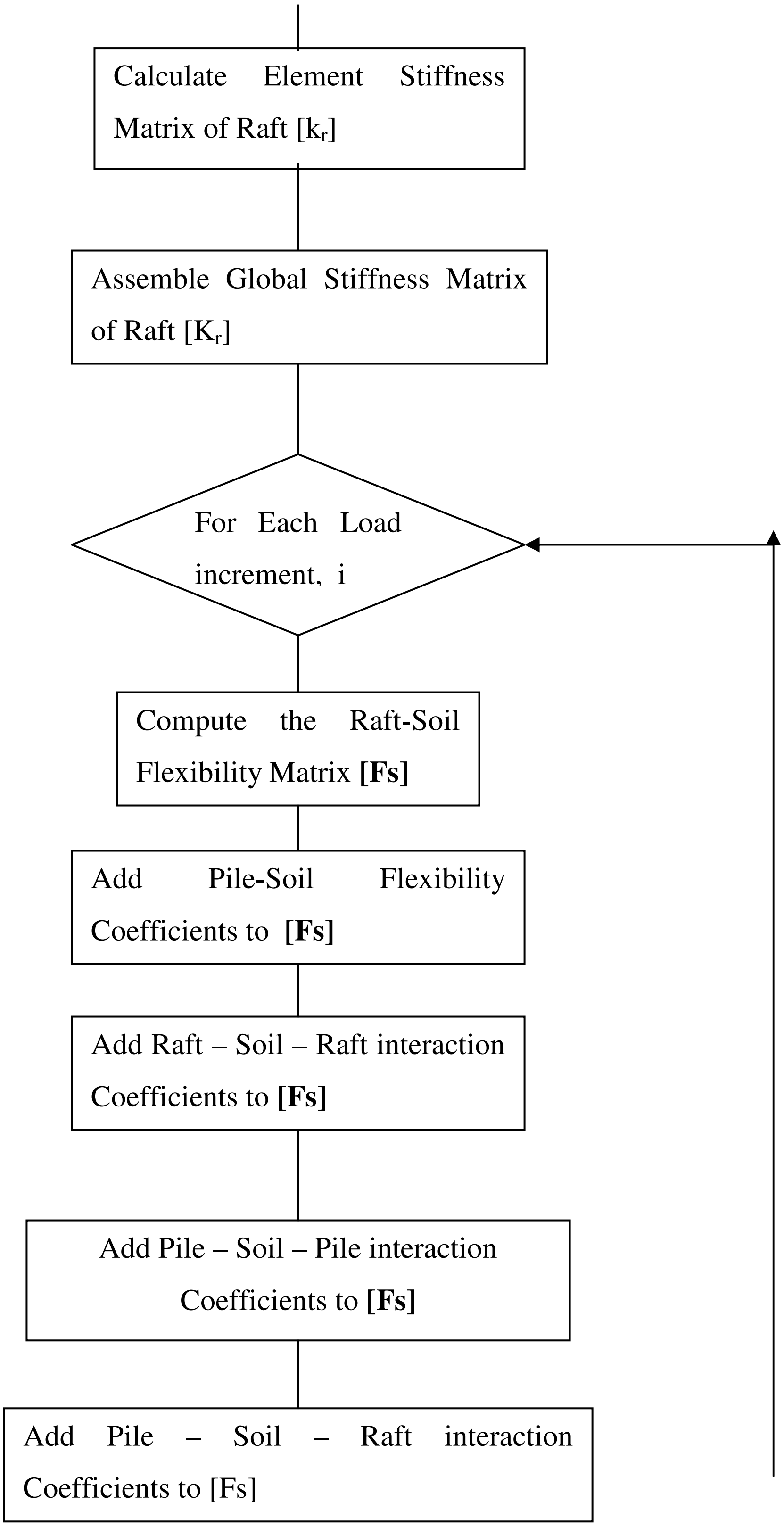
When element nodal d.o.f {d} are known, moments and shears can be evaluated using standard finite element procedure of plate element.

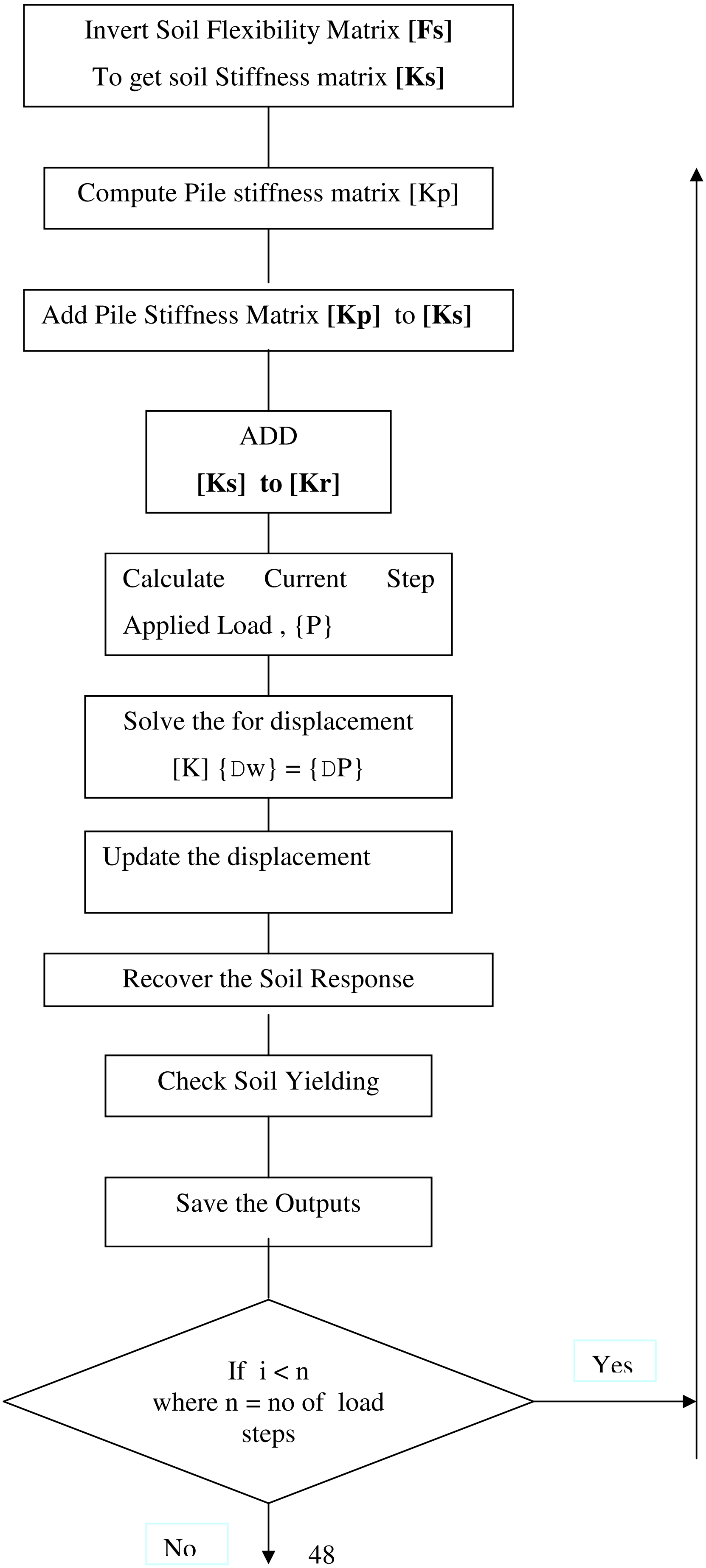
Based on the above formulation a computer program is developed using FORTRAN language and debugged and compiled by the Lahey ED Devleoper. The concise flow chart of the program developed is given below.

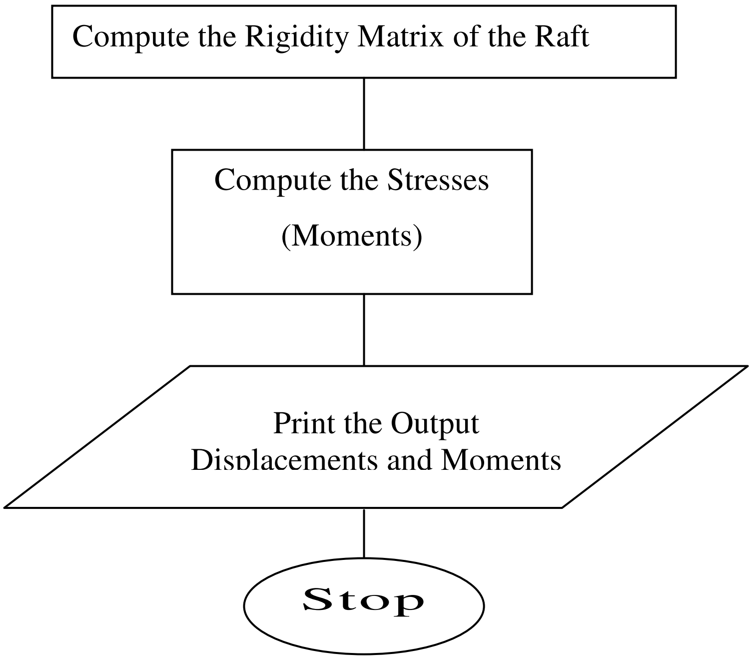
3.7 Concise Flow Chart for the Program Piled Raft Analysis

Concise Flow Chart for the Program Piled Raft Analysis




Calculate Element Stiffness Matrix of Raft [kr]
Assemble Global Stiffness Matrix of Raft [Kr]
For Each Load increment, i
Compute the Raft-Soil Flexibility Matrix [Fs]
Add Pile-Soil Flexibility Coefficients to [Fs]
Add Raft – Soil – Raft interaction Coefficients to [Fs]
Add Pile – Soil – Pile interaction Coefficients to [Fs]
Add Pile – Soil – Raft interaction Coefficients to [Fs]


Invert Soil Flexibility Matrix [Fs]
To get soil Stiffness matrix [Ks]
Compute Pile stiffness matrix [Kp]
Add Pile Stiffness Matrix [Kp] to [Ks]
ADD
[Ks] to [Kr]
Calculate Current Step
Applied Load , {P}
Solve the for displacement
[K] {Dw} = {DP}
Update the displacement
Recover the Soil Response
Check Soil Yielding
Save the Outputs
If i < n
where n = no of load
steps
Yes
No



3.8 Validation of the Program

Prior to conducting numerical analysis on selected case studies, the method used and the program developed are validated using a more rigorous results of 3D finite element analysis available in Poulos (2000). The first problem used to investigate the effectiveness of the present method is illustrated in the following Figure 3.7. The results show that it has a good agreement with the 3D finite element.

Case	P_1	P_2	No. of Piles	Remarks
A	2.0	1.0	15	—
B	2.5	1.25	15	—
C	2.0	1.0	9	Piles A are not present

Bearing capacity of raft = 0.3 MPa
Load capacity of each pile
= 0.786 MN (Tension)
= 0.873 MN (Compression)

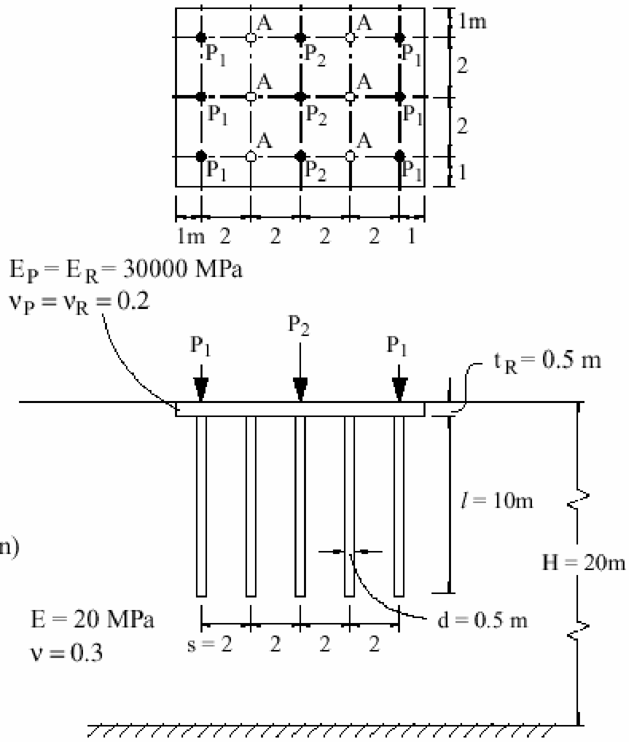


Figure 3.7 Simple problems analyzed for comparison (Poulos, 2000)

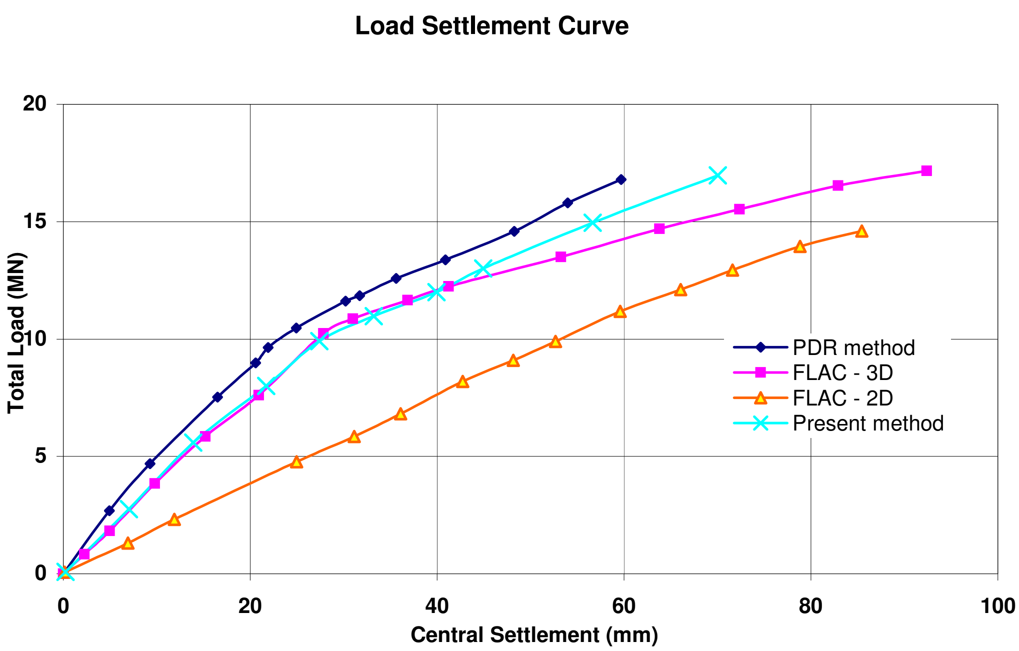


Figure 3.8. Comparison of the present method with other methods for load - settlement s analysis.

Figure 3.8 compares the computed load-settlement relationships (up to a total load of 18 MN) computed from the various methods for the center of the raft with 9 identical piles, one under each column. There is a reasonability good agreement between the computed load –settlement curves from present method and 3D finite element analysis. The FLAC3-D analysis gives a softer response than the other methods for loads in excess of about 12 MN, presumably because of the progressive development of plastic zones beneath the raft, and consequent increase of plastic deformation. However, the FLAC2-D analysis seriously over predicts the settlements because of implicit assumption of plain strain in the analysis.

Table 3.1 summarizes the performance of a piled raft with 9 piles, for a typical design load of 12 MN (equivalent to an overall factor of safety of about 2 against ultimate failure). For the various methods described in chapter two, values are given for the central settlement, the settlement under a corner pile, the maximum moment in the raft, and the proportion of load carried by the raft.

Table 3.1 Summary of Piled raft Behaviour for Total Load = 12 MN

Method	Central Settlement (mm)	Corner pile Settlement (mm)	Maximum raft Moment (MNm/m)	Percentage of load taken by piles
PDR	36.8	-	-	77.0
GARP	34.2	22.6	0.684	65.1
GASP	33.8	22.0	0.563	65.5
Burland	33.8	29.7	0.688	65.5
FLAC2-D	65.9	60.5	0.284	79.5
FLAC3-D	39.9	35.8	0.484	58.2
HyPR (present)	***39.98***	***31.5***	***0.545***	***64.1***

The other problem analyzed for comparison purpose is the ‘ Messe-Torhaus’ - the first piled raft project in Frankfurt Clay and in Germany as well. The detailed profile of the soil and plan of foundation is described in Katzenbath et al. (2000).

The piled raft foundation of the ‘Messe - Torhaus’ exists of two separate rafts, each with 42 bored piles (Figure 4.3). The piles are having a length of 20 m and a diameter of 0.9 m. The 6 x 7 piles are arranged uniformly under each of both rafts with a pile spacing of 3 to 3.5 times the diameter of D of the piles. Both rafts have dimensions of 17.5 m x 24.5 m in plan. The foundation level of the raft lies only 3m below ground surface. Each raft forced by an effective structural load of 200 MN.

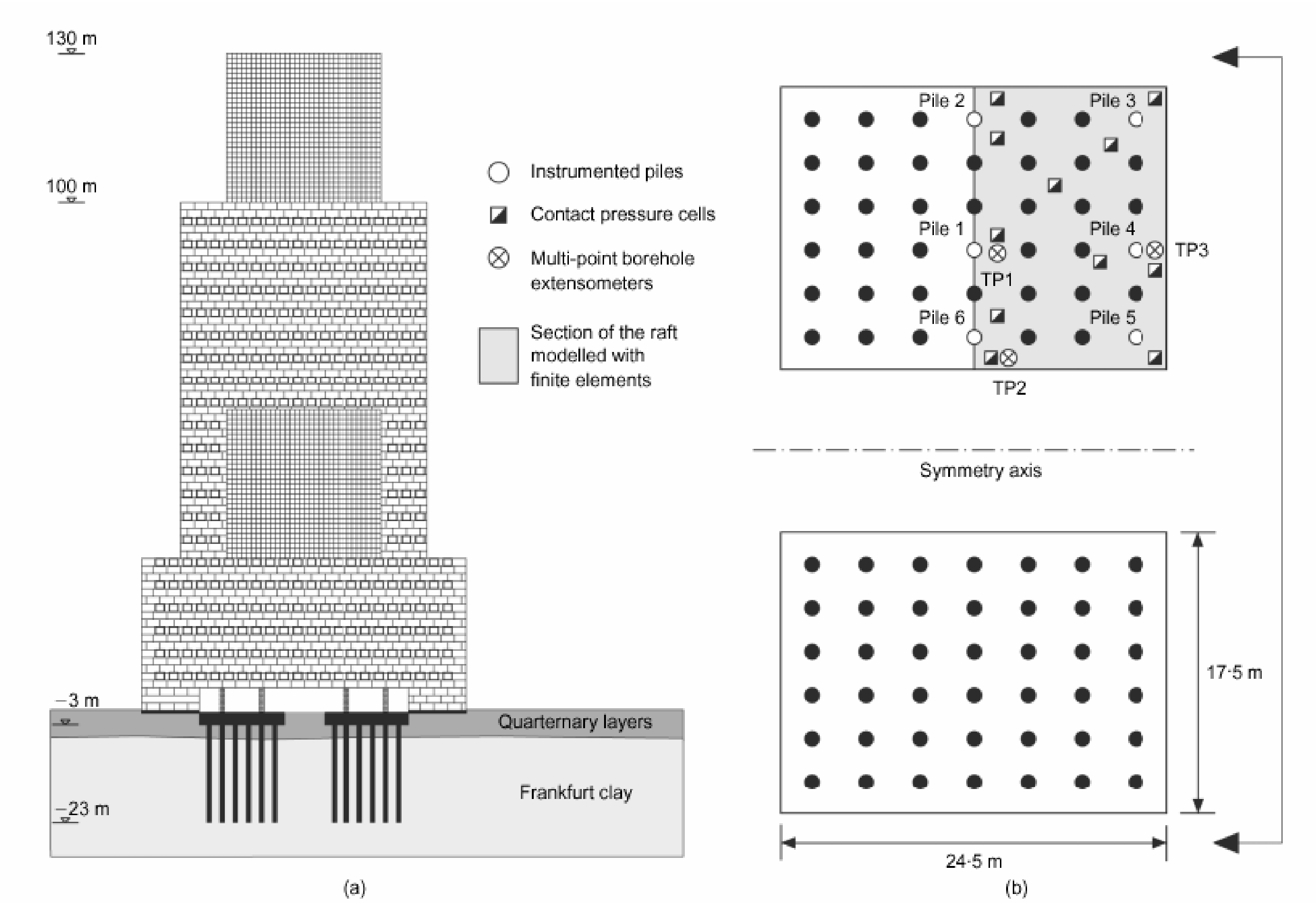


Figure 3.9 Profile view of ‘ Messe-Torhaus’ building (Katzenbath et al. ,2000)

During construction the bearing behaviour of the piled raft of 'Messe-Torhaus' was carefully monitored by geotechnical measurement program. The measured settlements and computed settlements by 3D finite element model and the present method are shown in Figure 3.10.

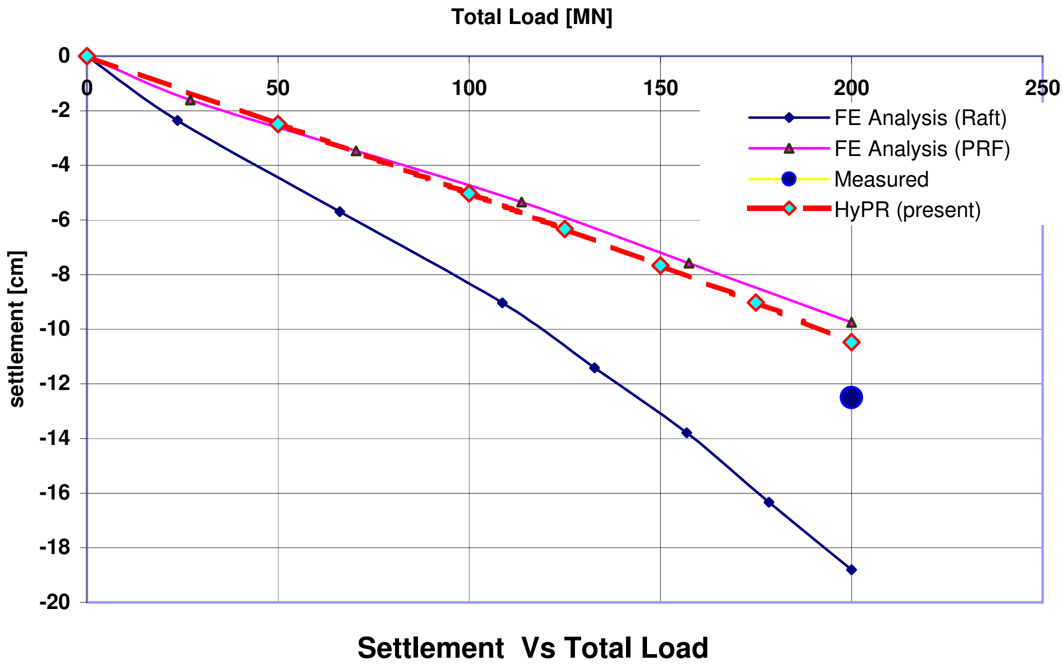


Figure 3.10 Load – Settlement Curves of 'Messe - Torhaus'

The computed settlement by the developed program is in close agreement with settlements of 3D FE model and even with the measured settlements. Other verifications, not presented here are also indicates good agreement with other methods. Therefore the developed program can be used for numerical study part with full confidences.

Chapter Four

Comparative Case Studies.

4.1 Introduction

In this chapter the outcome of three comparative case studies will be presented. These consist of two for Addis Ababa soil condition and one for a soil condition of a bridge site at Wabe Shebele River. The case studies are conducted to investigate the performance of piled raft foundation for Addis Ababa soil condition. The significance of the concept of piled raft has been compared with that of the conventional foundation design. The investigation is based on actual buildings built and ideal cases.

The strategy adopted for the comparative study is as follows.

First the raft alone is analyzed and its behavior (i.e. settlement and bending moment) for the working load was assessed. Then the conventional pile foundation design concept was used to determine the number, length and diameter of piles. Finally, using the concept of piled-raft the change in settlement, number of piles and raft bending moment were investigated and the results were compared with that of the raft and conventional piles.

The cases in Addis selected for comparative purpose are the Awash Bank Head Office Complex and the Marine Transport Head Office Building. In addition to these cases, which are typical for Addis Ababa soils, a bridge project out of Addis is considered to supplement the outcome of other cases.

4.2 Case Studies

4.2.1 AIB/ AIC Headquarter Building

Awash International Bank/ Awash Insurance Company headquarter building (a 20 story twin-tower high-rise building) is to be constructed near the National Theatre. It rests on the total area of about 70m x 40m and exerts a load of about 280.0 MN on the foundation. The RC load bearing walls exerts major loads. The foundation selected for the building is pile foundations and is designed using conventional pile design concept.

4.2.1.1 Site Characteristics

Detailed geotechnical investigation was conducted at the project site. The field and laboratory tests reveal that the soil profile at the project is composed of mostly dark grey, highly to moderately decomposed, weak vesicular basaltic fragments with joints filled by calcite. Typical and averaged soil profile is shown in Figure 4.1. The bearing capacity of piles and other soil parameters are correlated from the SPT-N values and also some laboratory test results.

Table 4.1 Summary of Laboratory Test Results for AIB/ AIC Building Site

Layer	I_p [%]	γ [KN/m^3]	γ_d [KN/m^3]	w [%]	G_s [ ]	e_o [ ]	c [KPa]	ϕ [^{o}]	q_u [kPa]
2	34	18.7	13.70	37	2.46	0.81	65	13	177
3/5	38	17.0	11.4	49	2.51	1.24	50	14	47
4	28	18.0	12.9	40	2.50	0.95	87	26	196
9	44	16.4	10.7	55	2.48	1.35	-	-	106

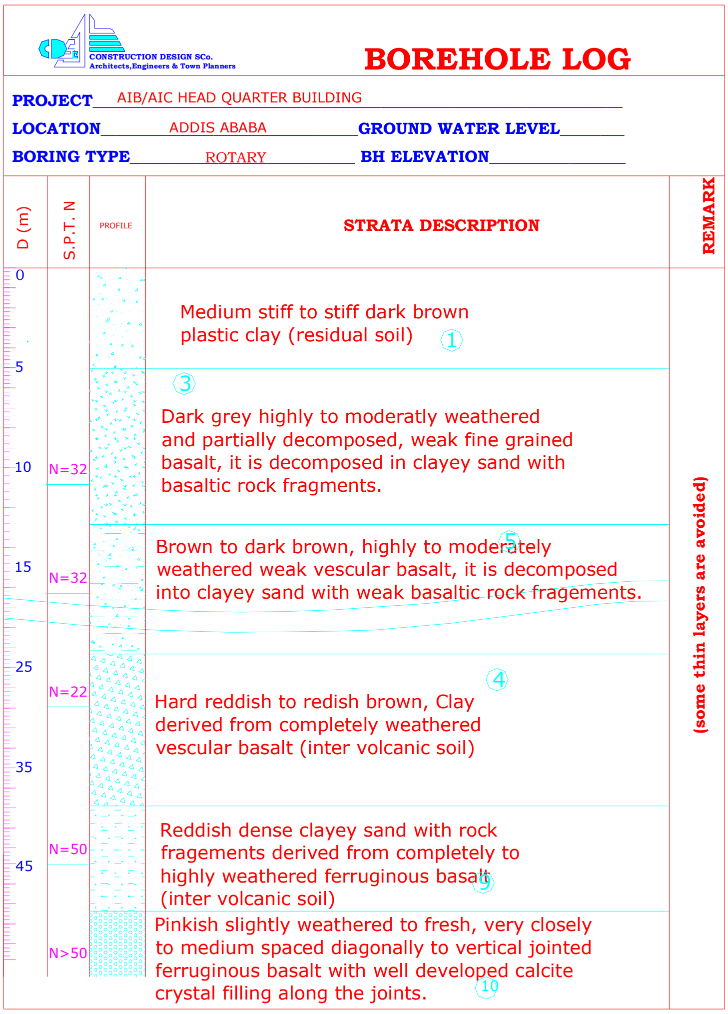


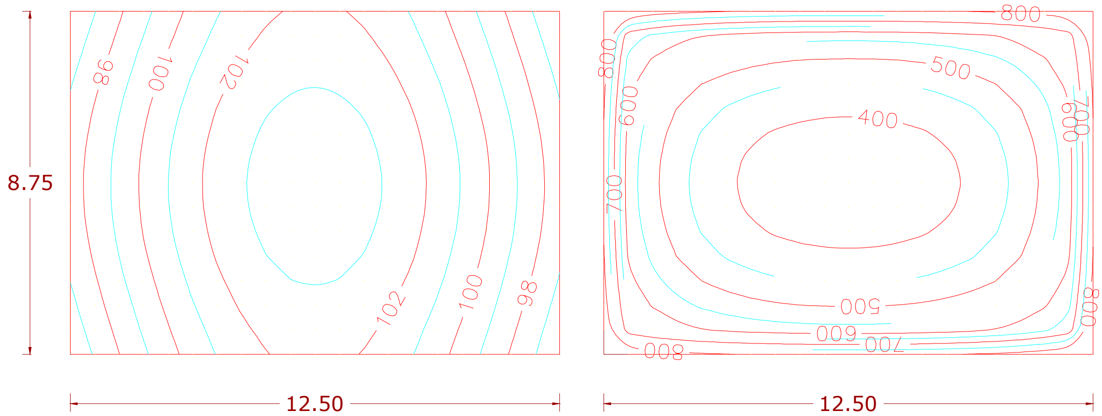
Figure 4.1 Typical Bore hole log Data of AIB site

The building consists of four load bearing walls (the shafts of the sky scrapers) with almost similar loads. Therefore, the behaviour of only the foundation of one shaft is considered here. In actual design, due to heavy loads from four RC shear wall systems of the building, large pile groups (for one shear wall system about 33 pile in a group) are used to carry the load (see figure 4.3). The pile groups are capped with 12.5m x 8.75m wide and 1.0m thick concrete pile cap.

For this case, the performance of the three foundation types (raft, piles, and piled-raft) was investigated as follows.

4.2.1.2 Behaviour of the Raft alone

The raft was analyzed assuming that it can support the entire load. The total applied load is about 58.5 MN. The load is applied distributing over the raft. Figure 4.2 shows the contour of the settlement and the raft node reactions. It undergoes total settlement at the center of raft about 103.5mm and differential settlement along the length about 6.5mm. Maximum moment at the center is 1487.4 kN-m/m (figure 4.5) and the average bearing pressure is also above the allowable bearing capacity of the raft, which is calculated to be 450 kPa.



a) Settlement Contour of the Raft alone **b) Raft node reaction**

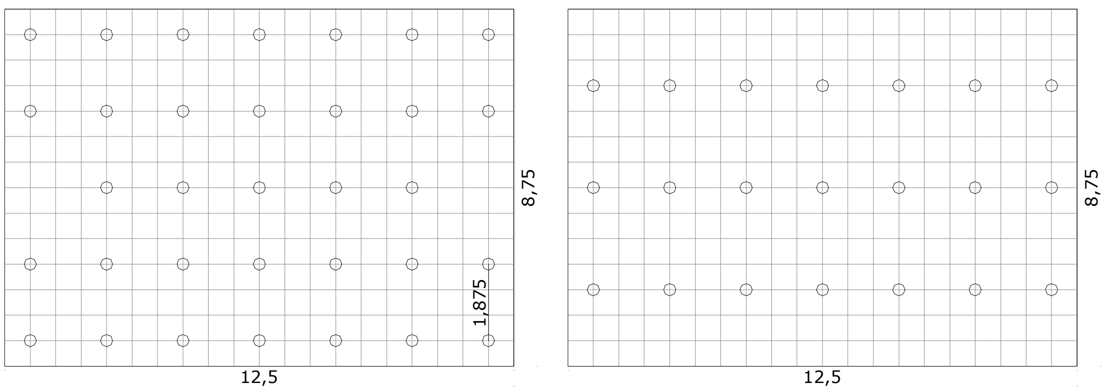
Figure 4. 2 Contours of raft settlement and nodal reaction

The performance of the raft alone is not satisfactory and therefore using the pile to reduce the settlement to its serviceability requirement and improve the bearing behaviour is essential.

4.2.1.3 Free Standing Pile Groups (Conventional Method)

In this case the 33 pile group of 25m length and 0.60m diameter used as foundation for ABI/ AIC building was analyzed using the developed program. The analysis was carried out using the conventional method, which assumes piles carry all loads. The arrangement of the pile is shown in Figure 4.3. The governing criterion was found to be the failures of the piles bearing capacity. The settlement at the center of the raft was 48.6 mm and differential settlement between the center and mid of edge was 4.30mm.

To investigate the load shared by the cap and the effect of raft –soil-pile interaction, the same pile group used for the conventional method was analyzed by allowing the mat (pile cap) to rest on the surface (piled raft concept). This has decreased the total settlement and differential settlement to 48.3 mm and 4.20mm respectively. The load shared by the raft was less than 8.0%. The results are shown in figure 4.4 (a) and (b). One can see that the effect of the raft is negligible due to the large number of piles provided.



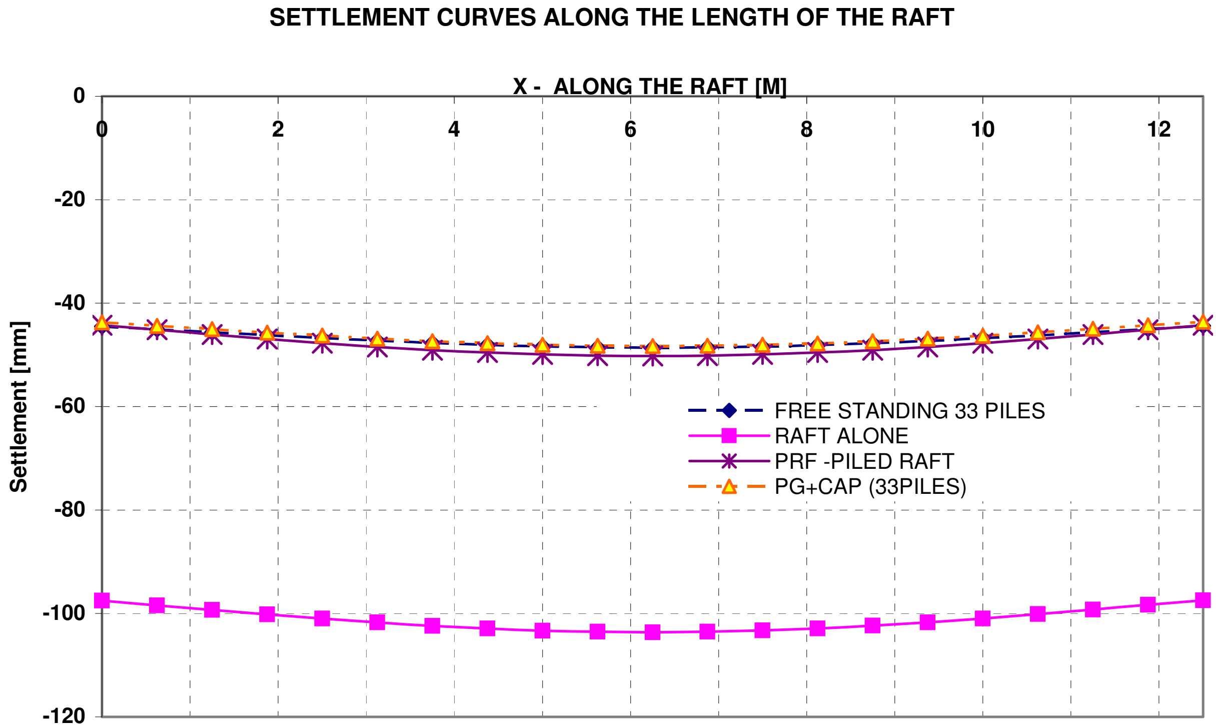
a) Pile Groups alone (33 piles) **b) Piled - Raft (21 piles)**

Figure 4.3 Arrangement of piles used for analysis

The advantage of piled raft can be seen clearly, if the foundation is redesigned based the concept of piled raft

4.2.1.4 Piled -Raft Concept

Using the concept of piled raft, which utilize the piles to their ultimate capacity, the required numbers and length of piles required are determined through some trial and error of different arrangement. Though it is not the most optimal solution, for a settlement of about 50.35 and satisfactory bearing behaviour 21 piles are required for the system. The arrangement of the piles is shown in figure 4.3(b). For this piled raft system, the percentage of the load carried by the raft is about 17.5 %. The maximum bending moment in the raft moment is 825.6 kN-m/m. It can be observed from figure 4.5 that the positive raft bending moment along the width has decreased substantially. This is because the piles are strategically placed at the center of the raft to reduce the differential settlement. During the choice of the pile arrangement it was observed that it is not only the number of the piles but also their arrangement that matters for limiting the differential settlements to the allowable value and to reduce the raft bending moments.



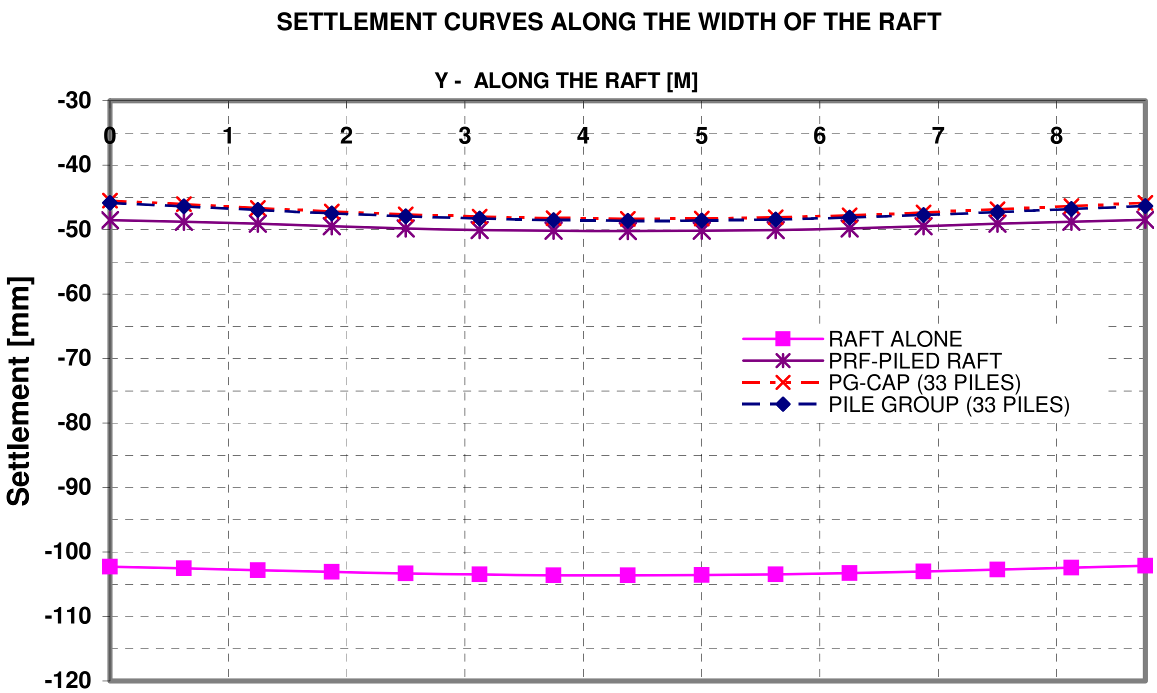
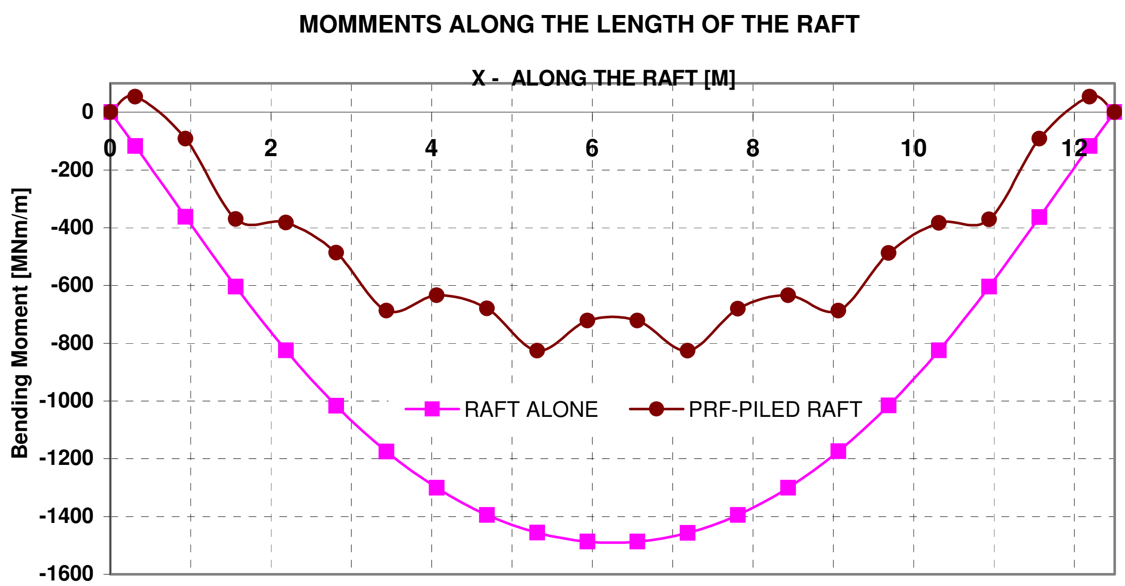


Figure 4.4 Settlement curves of raft along the length and width of the raft



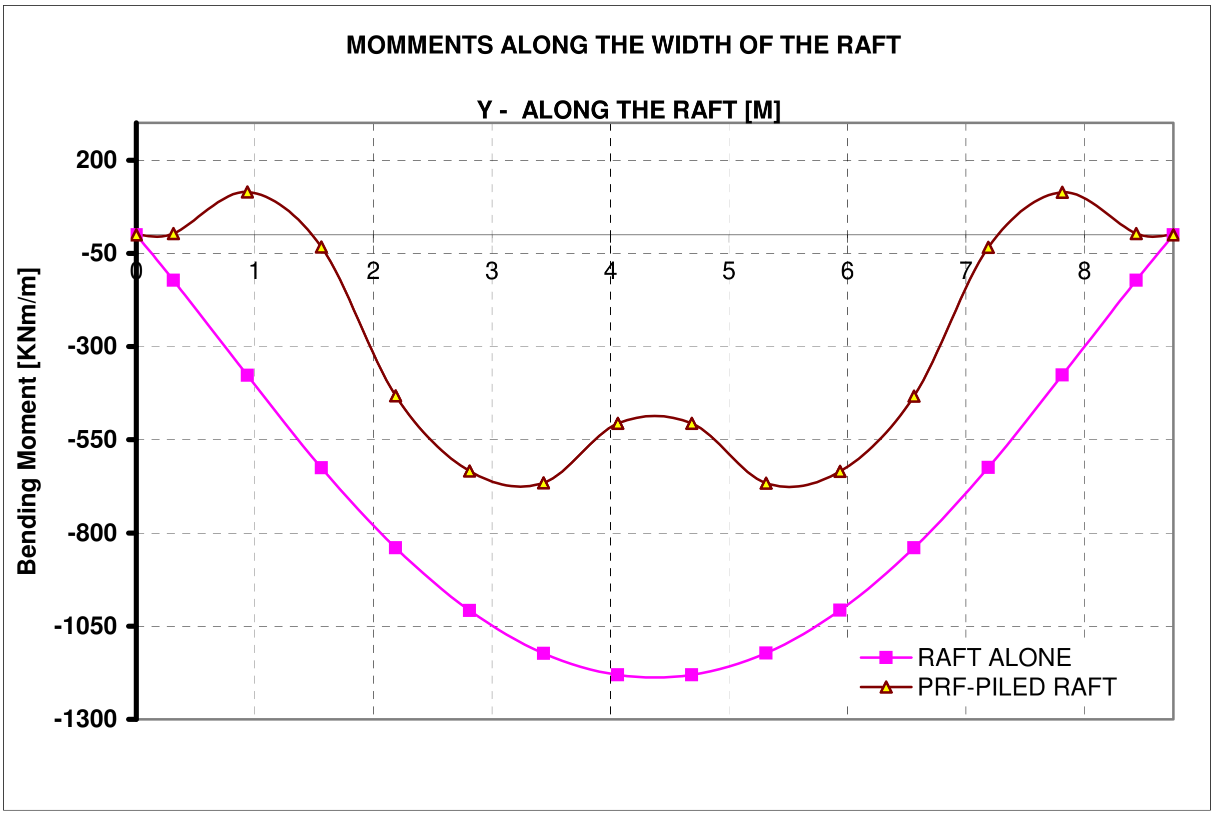
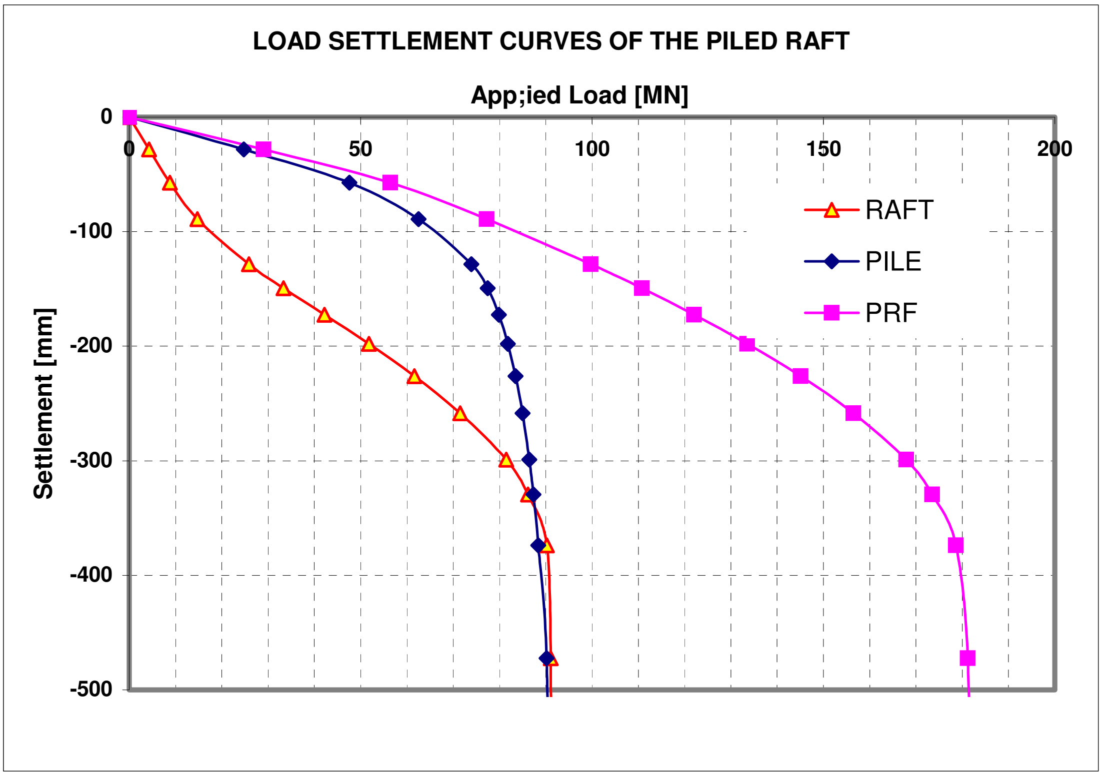


Figure 4.5 Moments along the length and width of the raft



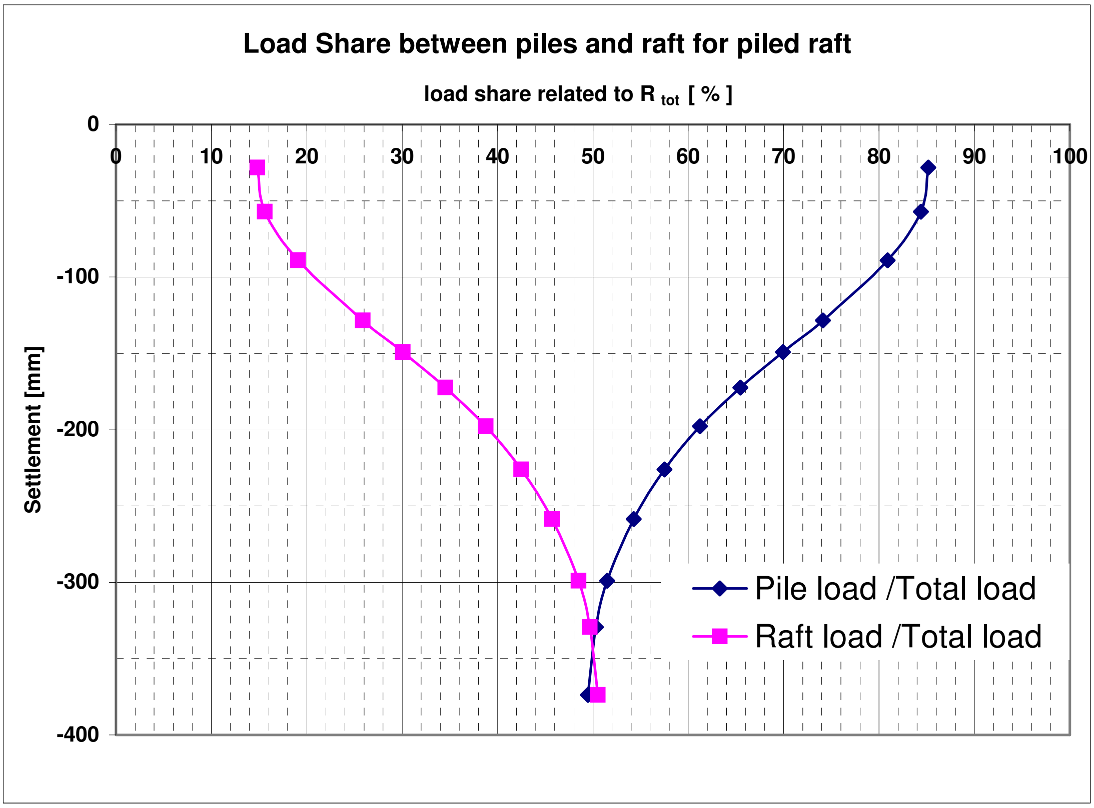


Figure 4. 6 Load - settlement curves of different components PRF system

Table 4.2 Summary of comparison of different design approach

Analysis	Settlement (mm)	Differential Settlement (mm)	Maximum Bending Moments (kN-m/m)	
			M_{xx}	M_{yy}
Raft only	104.5	6.5	1180.9	1487.4
Pile groups(33 piles)	48.60	4.20	729.7	749.8
Pile + Cap (33 piles)	48.30	4.80	756.7	796.0
Piled raft (21 piles)	50.35	5.93	666.6	825.6

From the above table we can see that using the concept of pile raft we can decrease the bending moment of the raft and settlement substantially compared with raft alone and the number of pile by 1/3 compared with the conventional pile.

4.2.2 M.T.A Headquarter Building Site

Knowing that the foundation of the former Marine Transport Authority head office building is pile foundation, it become to the interest of the author consider it for case studies. However, upon searching the data required only the borehole log data were found. Since the main factor, which affects the performance of the piled raft, is supposed to be the soil profile, ideal foundation loads were used for the case study.

4.2.2.1 Site Characteristics

The borehole log data used for the analysis is shown in figure 4.7.

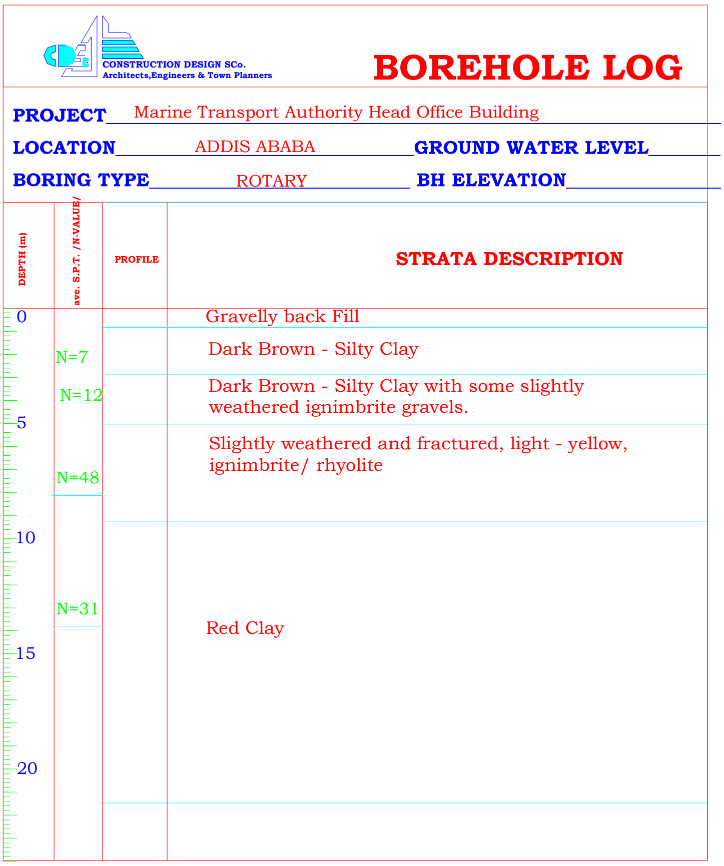


Figure 4.7 Typical Bore hole log data for M.T.A. Building site

The loading considered is concentrated column loads rather than uniform loads. The total applied load on the foundation is 55.0MN. (Figure 4.8)

4.2.2.2 Behaviour of the Raft alone

The size of the raft was determined to be 25mx13m. The thickness of the raft required satisfying punching and wide beam shear failure was 1.3m. The raft was analyzed using the program and the results are shown in figures 4.9 to 4.10. From the figure, one can see that the maximum settlement at the center of raft is about 96.4mm and differential settlement of about 31mm. These values, especially the differential settlement that is about one third of the total settlement, are above the allowable. The allowable bearing pressure is taken to be 300kPa. Due to the excessive differential settlement the raft experiences a maximum moment of about 1974 kN-m/m.

4.2.2.3 Piled -Raft Concept

For the case mentioned above, aspiring decreasing the total settlement, differential settlement and thereof the maximum bending moment of the raft, the concept of piled raft is utilized and the PRF is designed. The number and size of piles required to limit the settlements below allowable is found to be 32 piles with diameter of 1.0m and length of 24.0m using some trials. The plan and profile view of the foundation is shown in the Figure 4.8. The analysis results are shown in figures 4.9 to 4.10

This system decreases the overall total settlement, differential settlement and the bending moments to 57.6mm, 15mm and 1084.4 kN-m/m respectively. The percentage of the load carried by the raft alone is about 25 %. As compared with the raft foundation alone, the decrease in differential settlement and bending moment is about by half. This means, the raft reinforcement decreases proportionally. If piles were placed strategically to minimize the differential settlement, the bending moments would have been decreased more.

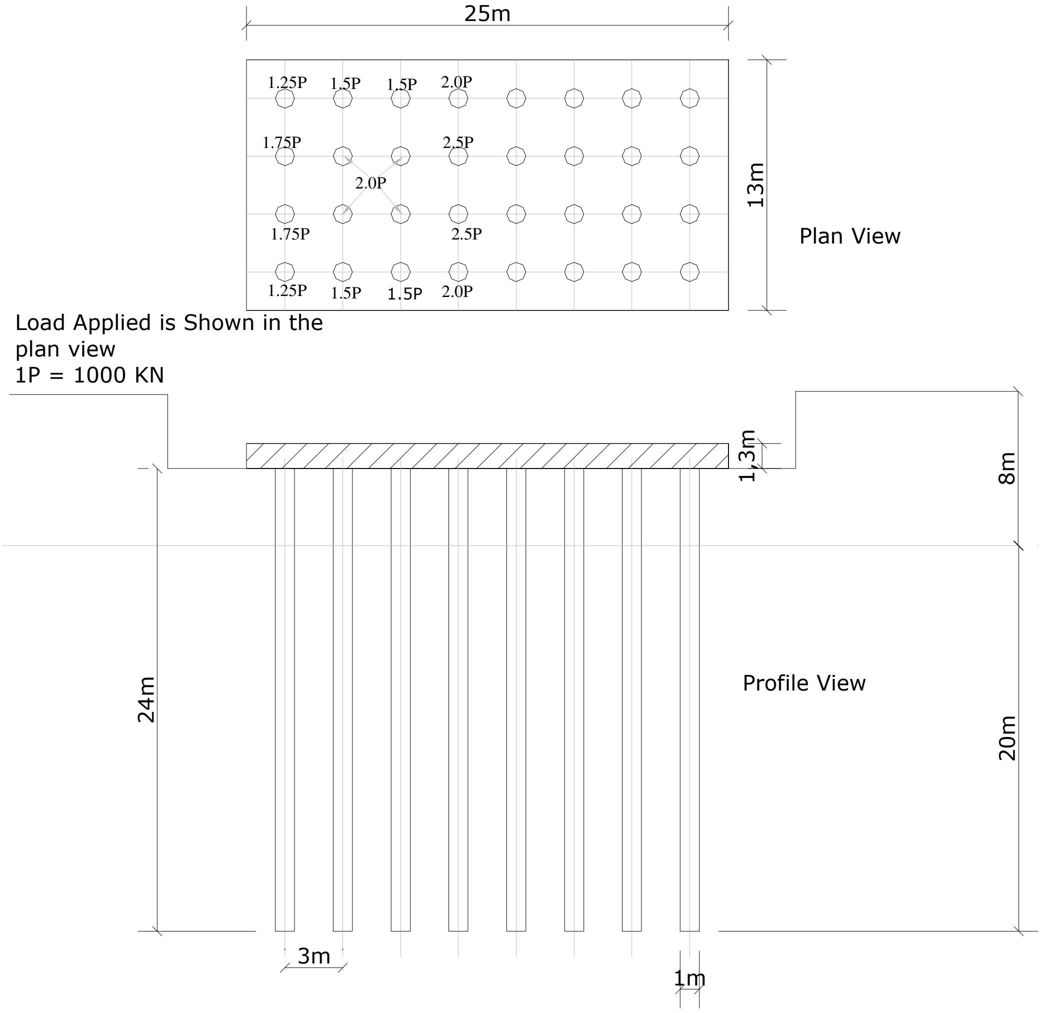


Figure 4.8 The plan and profile view of the foundation used for analysis

4.2.2.4 Free standing piles

The foundation is designed using conventional pile design principle that assumes the entire load to be carried by the piles only. For safety factor of 3 for the pile capacity, about 40 piles of the same size with that of PRF are required.

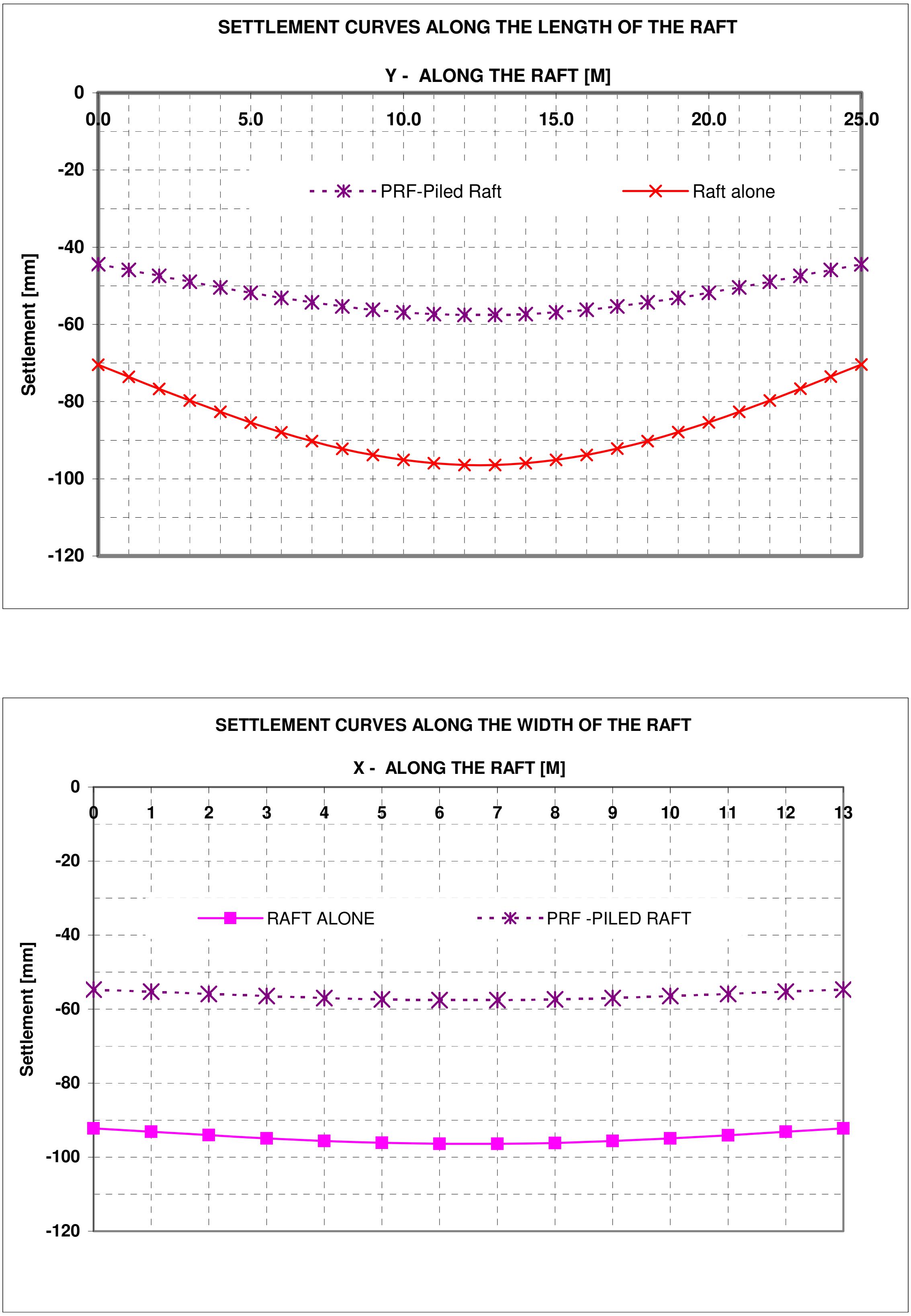
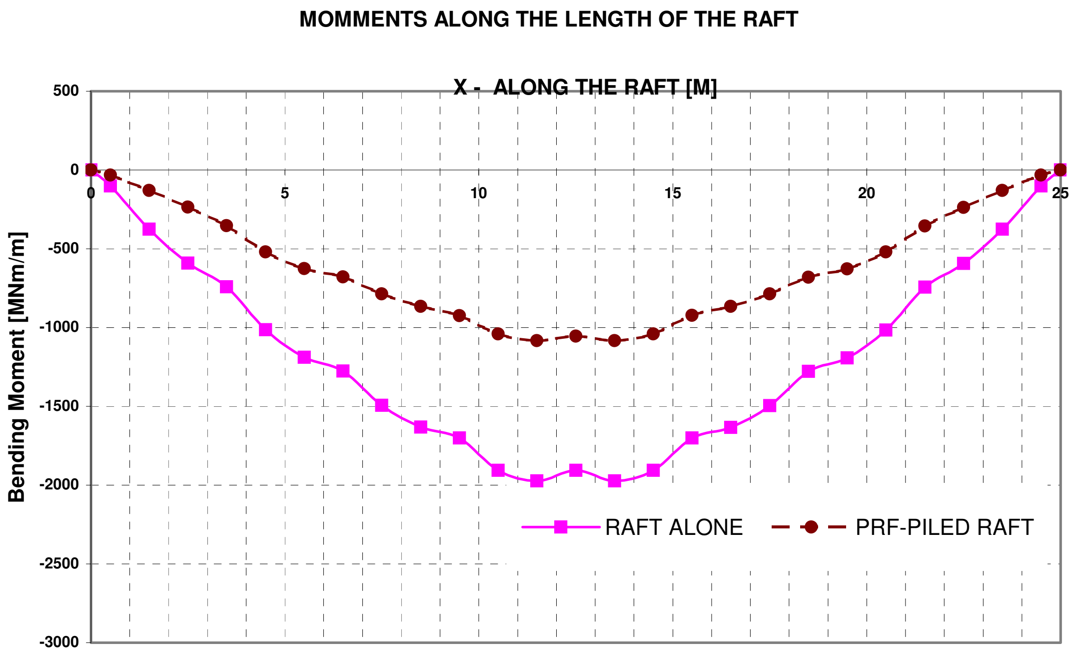


Figure 4.9 Settlement curves of along the length and width of the raft



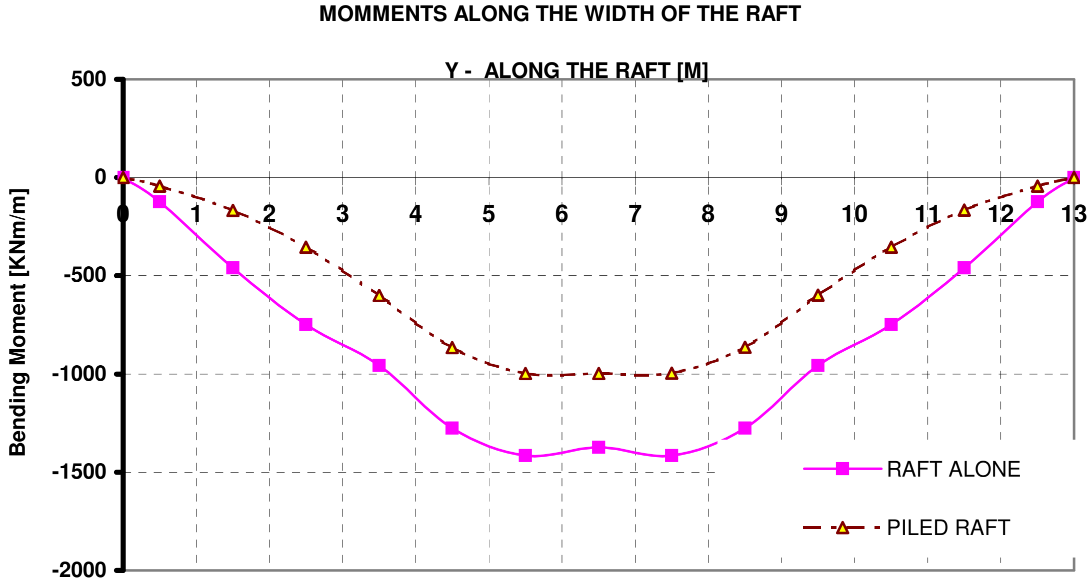


Figure 4.10 Bending moment curves of along the length and width of the raft

Table 4.3 Summary of comparison of raft and piled raft foundations

Analysis	Settlement (mm)	Differential Settlement (mm)	Maximum Bending Moments (kN-m/m)	
			M_{xx}	M_{yy}
Raft only	96.4	31	1974	1415
Pile Group	48.6	12	899	859
Pile group + Cap	48.4	12.8	906	842
Piled raft (32 piles)	57.6	15	1084	997

The above table shows the decrease in differential settlement and raft bending moment of the piled raft foundation as compared to the raft foundation. Using piled raft concept the length of piles has been decreased by 1/3 of the total length required by conventional pile design.

4.2.3 Wabe Shebele River bridge

This case was investigated to supplement the investigation made for Addis Ababa soils. In this case the performance of piled raft for bridge abutment foundation as compared to those raft and conventional pile was investigated.

4.2.3.1 Site Characteristics

As shown in figure 4.11, the geotechnical investigation result reveals that the founding material is loose silty sand with SPT-N value ranges 3 – 15 up to the depth of 20m. The mat of dimension 6.25 m x 10 m was used for the analysis. The foundation is subject to a uniform load of 215 kN/m^2. The thickness of the raft is 0.75m. the depth of the bed rock is assumed to at 50m below ground level.

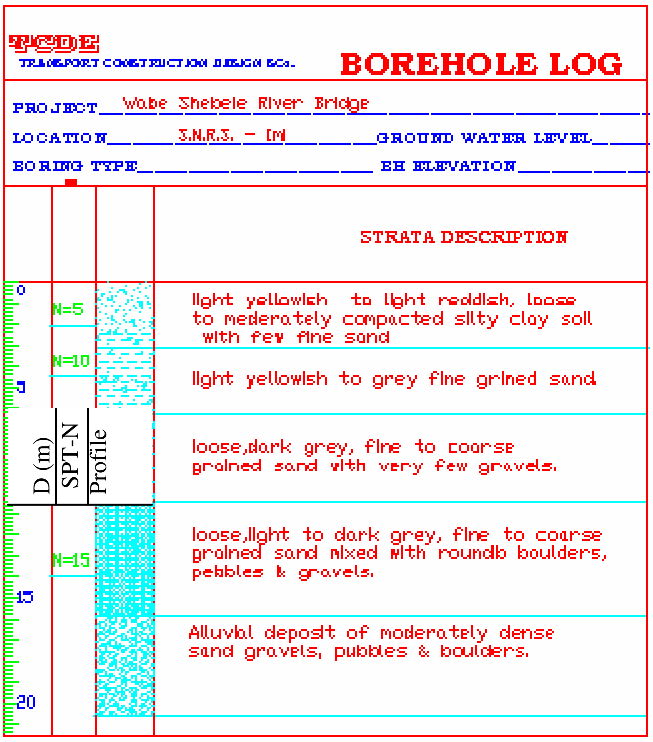


Figure 4.11 Soil Profile for Wabe Shebele River Bridge Site

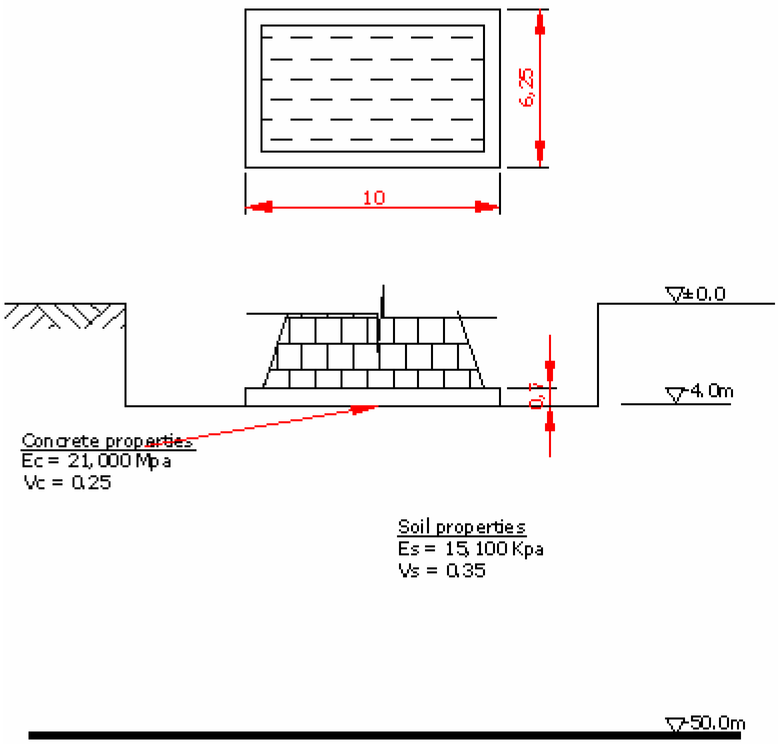


Figure 4.12 Existing raft foundation of the abutment

4.2.3.2 Behaviour of the existing raft

The raft is analyzed assuming it can support the total load. Figure 4.12 shows the distribution of contact pressure underneath the raft. The soil has yielded at the periphery of the raft and the average pressure is also above the allowable which is 150 kPa. The raft undergoes overall total settlement of about 80.5mm. Maximum moment of the raft is 447.4 kN-m/m.

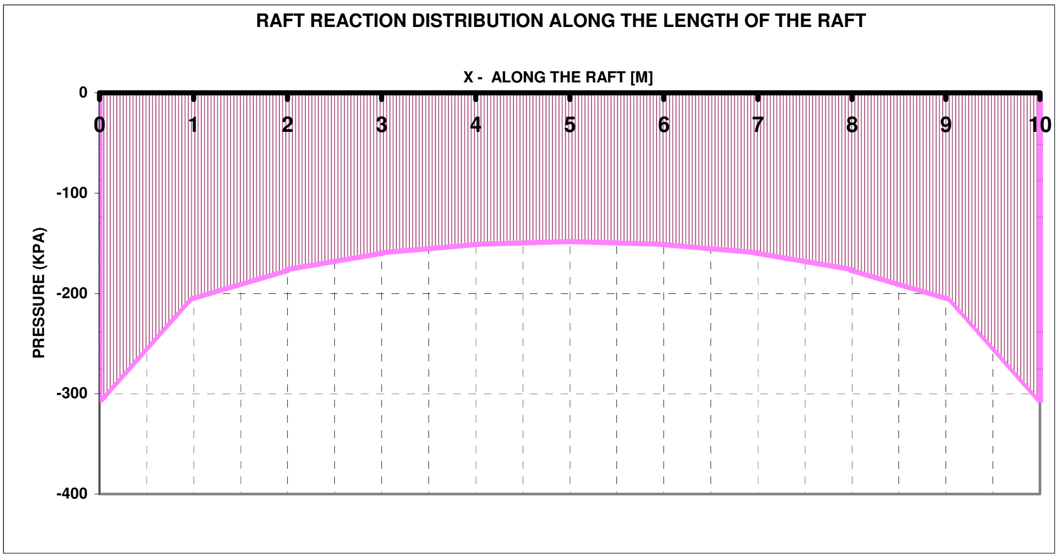


Figure 4.13 Soil pressure under the raft foundation

4.2.3.3 Free standing pile groups (conventional method)

Analysis was carried out using the conventional method which assumes all loads are carried by piles only and a pile group of 15 piles of 20m length and 0.60m diameter was required to limit the total settlement to less than the allowable value and a global safety factor of 2 for ultimate pile failure. The governing criterion was found to be the failures of the pile group as a block - block failure. The settlement at the center of the raft/cap was 45.03 mm. Maximum bending moment in the raft was 210.2kN-m/m. In this analysis, the raft is assumed to be supported at the pile location only. In this case, when the contribution of the cap is considered, the settlement decreases to 31.4mm and the contribution of the cap in carrying the load is about 23%.

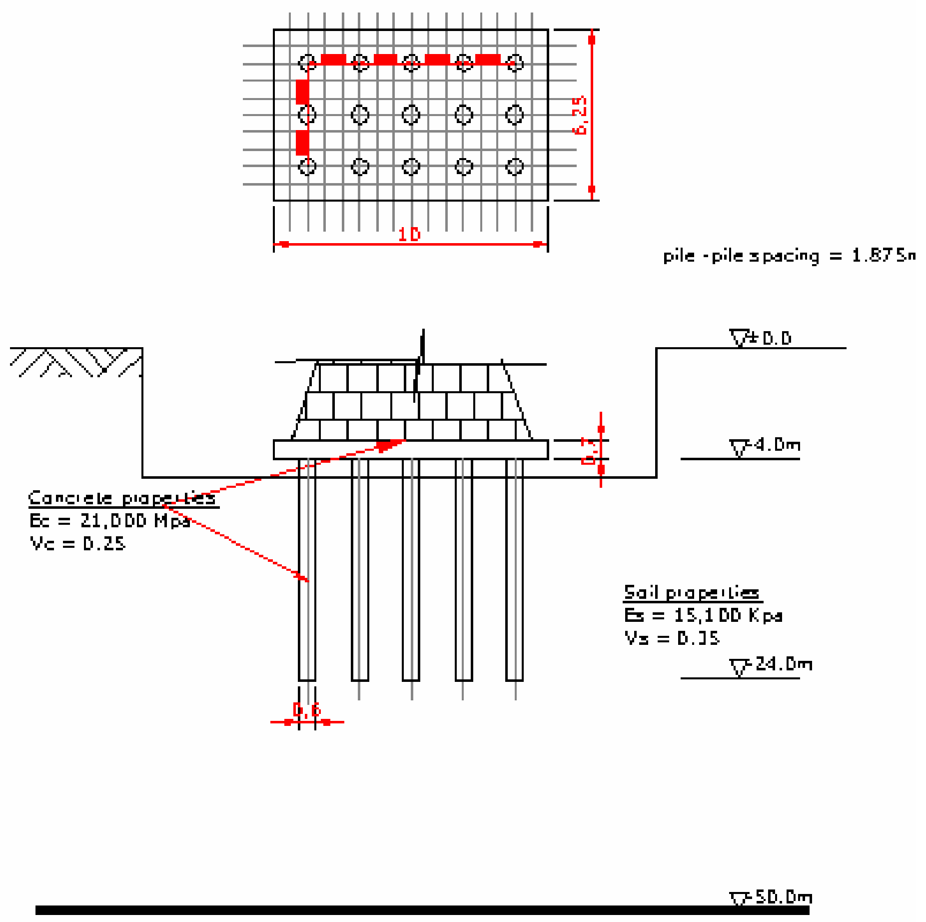
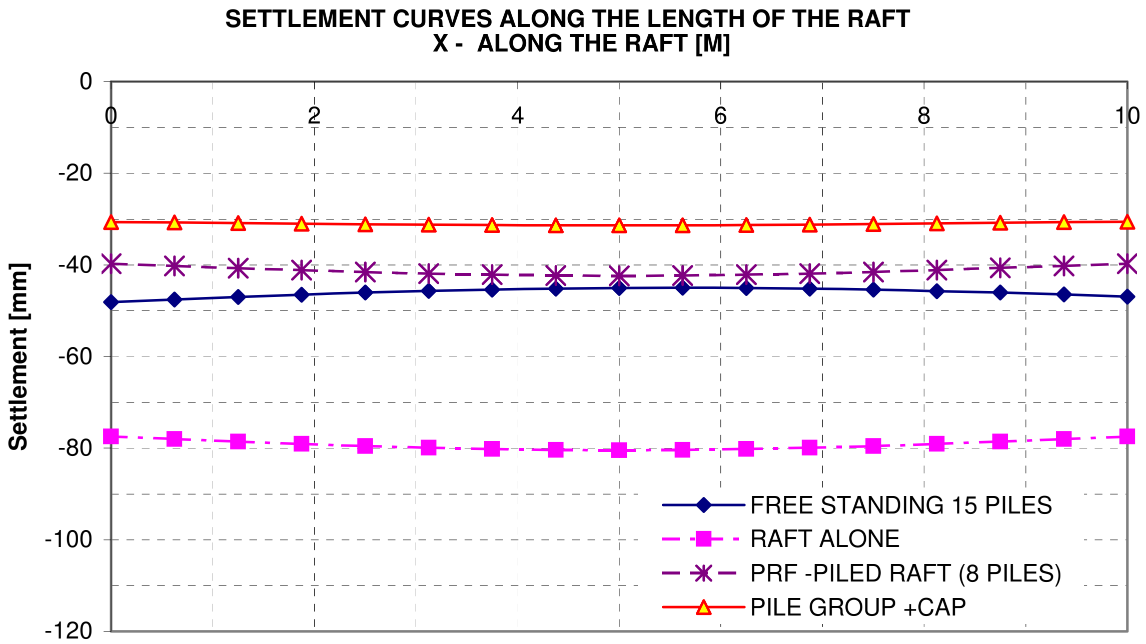


Figure 4.14 Plan and profile views of pile foundation

4.2.3.4 Piled -Raft Foundation

Using the concept of piled raft, the required numbers and length of piles are 8 and 20m respectively. The maximum total settlement of the raft and maximum bending moment is found to be 42.3mm and 173.5kN-m/m respectively (Figure 4.14). The percentage of the load carried by the raft alone is increased to 52 % compared to that of contribution of pile cap of conventionally designed piles.



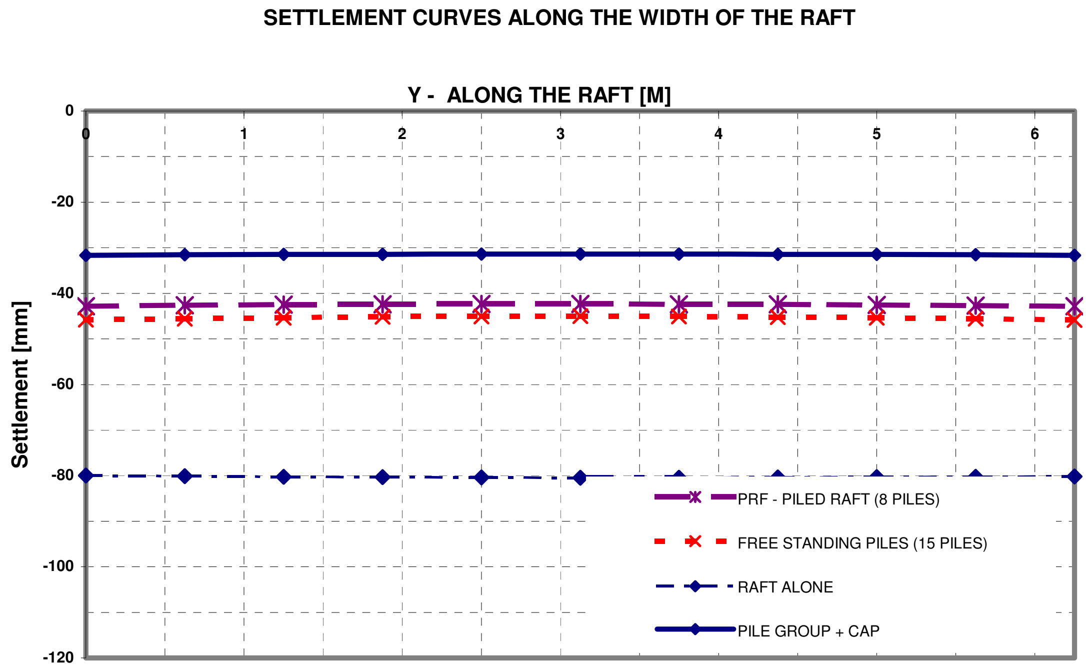
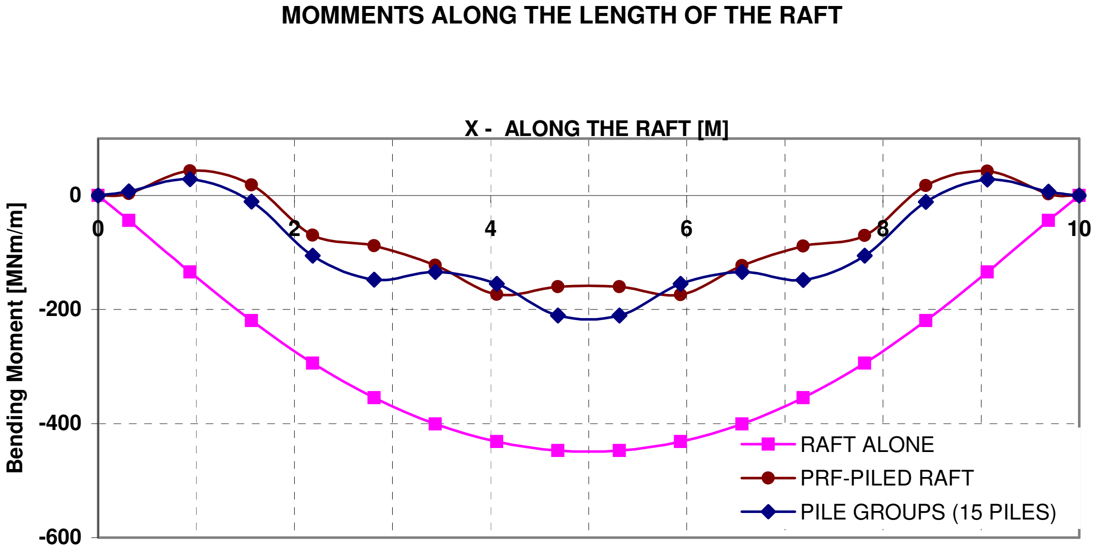


Figure 4.13 Settlement curves of along the width and length of the raft



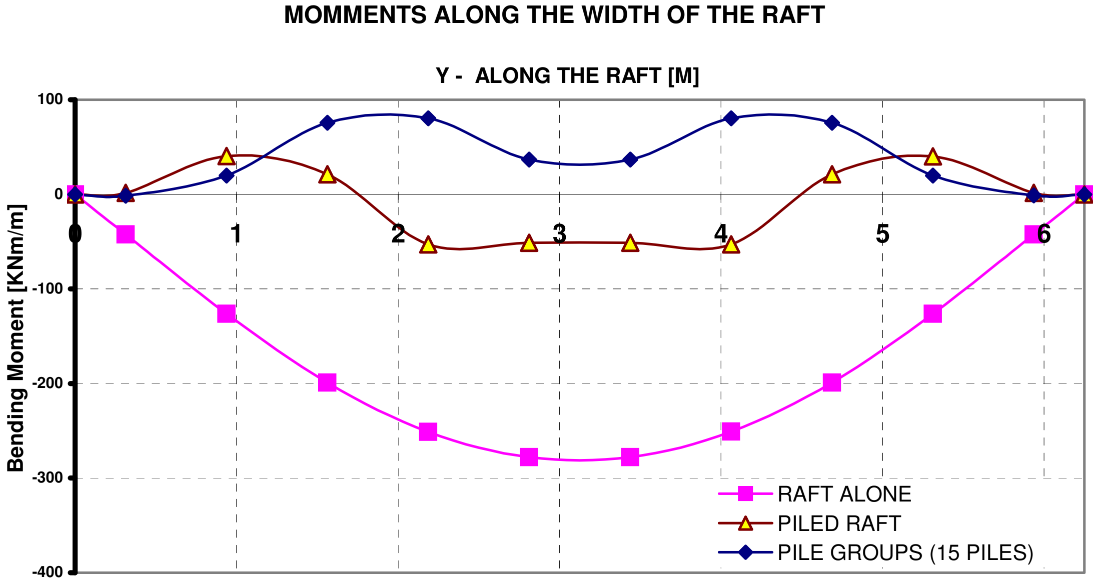


Figure 4.14 Bending moment curves of along the width and length of the raft

Table 4.3 Summary of comparison of different design approach

Analysis	Settlement (mm)	Differential Settlement (mm)	Maximum Bending Moments in the raft (kN-m/m)	
			M_{xx}	M_{yy}
Raft only	80.5	3.2	447.2	277.9
Pile groups(15 piles)	45.03	1.2	210.2	(80.30)
Piled raft (8 piles)	42.3	2.3	173.5	53.0

(Value in the bracket is positive bending moment, i.e. tension on top fiber)

The figures and the table above shows that the behaviour of the raft is highly improved with small number piles of strategically placed under the raft to supplement it. For the case under consideration, the reduction in number of piles is about 46 %.

Chapter Five

Conclusions and Recommendations.

5.1 Conclusions

The case studies indicate that piled raft foundation concept has significant advantages in comparison to conventional foundation for some Addis Ababa soils. From the case studies, the following points have been observed.

- In comparison to shallow (raft) foundations piled rafts reduce effectively the settlements, the differential settlements and the bending moment proportionally.

- In comparison to a conventional pile foundation the application of piled rafts leads to reduction of 30%-45% of the total number and length of piles needed for the foundation, though it needs to conduct detailed economic analysis

- To reduce the differential settlement and moment the piles should be place strategically using some trial and error or using parametric study. Generally, placing the piles beneath the column location reduce the differential settlement considerably rather than placing on the field.

- Upon increased foundation load the contribution of the raft become increased.

5.2 Recommendations

Though this research is limited in its scope, it can be a good starting point for further research in the area. For a further improvement in designing and construction of piled rafts the following are recommended:

(1) This research is based on limited number of data on the soil behaviour. The design of piled rafts requires a detailed and adequate investigation program to collect all relevant data of the subsoil and groundwater conditions. So, further research should be done that incorporate full scale and model tests.

(2) In present work, elastic analysis is considered. Therefore, further researches should be conducted considering 3D-FE geometrical model for the continuum and an elasto-plastic constitutive model for the soils that considers non-linearity and non-homogeneity in consistent manner.

(3) Detailed economic analysis should be conducted considering conventional piles for individual columns and piled raft concept separately. The variation in the amount of reinforcement, foundation excavation and time of construction should also be considered.

(4) The practice of other countries in piled raft foundation should be adopted for our cases and design guidelines for the design of piled rafts should be included in the code of practice of Ethiopia.

```
 PROGRAM PILED RAFT
*******************************************************************
*                                                                 *
*    APPENDIX A. HYBR - FINITE ELEMENT PROGARM CODING              *
*                                                                 *
*******************************************************************
C     PROGRAM  PILED RAFT - FOR THE ANALYSIS OF TRANSVERSLY LOADED
C     RECTANGULAR PILED RAFTS
C
C     ALTER NEXT LINE TO CHANGE PROBLEM SIZE

      PARAMETER(IKVR=2000,INFR=2000,IKV=2000,INCRE=600,
     +ILOADS=2000,INO=650,INX=650,IDEE=3,IBE=3,IDER=16)
C
      REAL KVR(IKVR,IKVR),LOADS(ILOADS),KMR(16,16),DTD(16,16),
     +FUN(16),D1X(16),D2X(16),D1Y(16),D2Y(16),D2XY(16),SAMP(7,2),
     +KV(IKV,IKV),SM(IKV,IKV),FORCE(IKVR),KM(2,2),FMAXRR(INFR),
     +COORD(INO,2),RCOORD(INFR,2),ZCOORD(INX),FMAX(INX),FSOIL(INO,INX),

+FRAFT,FSOILR(INFR),FMAXR(INFR),SUMM(INO),QINC(INCRE),KMM(IKV,IKV),
     +VALL(INO),KVI(IKV,IKV),GRATE,FORCES(IKV),DISP(IDER),DEE(IBE,IBE),
     +MOMM(IBE),KEE(IDEE),DISPL(IKV),KVS(IKV,IKV),SUMR(INCRE),
     +DEFLEC(IKV),BEE(IDEE,IDER),COORDD(4,2),pJAC(2,2),JAC1(2,2),
     +DER(2,4),DERIV(2,4),TOTSUM(INCRE),ALPHA,ELCCOR(INFR,2),
     +SETTLE(INCRE),SUMLOD(INCRE),TOTAL(INCRE),PMAXR(INFR),
     +SMM(IKV,IKV),CMM(IKV,IKV)
C
      INTEGER NFR(INFR,4),GR(16),IKVR,INFR,M,MN,INDX(IKV),
     +G(2),GP(INX),GQ(INX),NO(INO),NOP(INO),INCS
C
      DATA ICOORD,NODOFR/2*4/,ISAMP/7/,IDOFR,IKMR,IDTD/3*16/
      DATA IJAC,IJAC1,IDERIV,IDERR,IT,IKM,IDOF/7*2/,RF/0.9/,IBER/3/
C
      OPEN(10, FILE="INWSpgm.FOR",STATUS="OLD", ACTION="READ")
      OPEN(11, FILE="OTWSpgm.FOR",STATUS="OLD", ACTION="WRITE")
C
C    INPUT SECTION
      READ(10,*)
      READ(10,*)NXER,NYER,NR,NNR,NRR,NGP,ER,VR,TH,AA,BB
      CALL READNF(NFR,INFR,NNR,NODOFR,NRR)
      READ(10,*)NXE,E,PILEN,R0,GSURF,GRATE,VSOIL,RHO,NPILE,ZF,ZL
      READ(10,*)(NOP(I),I=1,NPILE)
      READ(10,*)(FMAX(I),I=1,NXE+1)
      READ(10,*)QULT
      READ(10,*)INCS,(QINC(I),I=1,INCS)
      READ(10,*)PULT
```

```
      READ(10,*)NL,(NO(I),VALL(I),I=1,NL)
C
      D=ER*TH**3/(12.*(1.-VR*VR))

      NN=NXE+1
      M=(NXER+1)*(NYER+1)
      IR=NR+NPILE*NXE
      MN=M+NPILE*NXE
      ELL=PILEN/FLOAT(NXE)
      PI=ACOS(-1.)
      CSA=PI*R0*R0
      RM=2.5*RHO*(1.-VSOIL)*PILEN
C
      CALL NULL(FSOIL,INO,NPILE,NN)
      CALL NULVEC(FSOILR,NNR)
      DO 10 I=1,NN
   10 ZCOORD(I)=FLOAT(I-1)*ELL+ZF
      FACT2=3.-4.*VSOIL
      FACT3=(1.-VSOIL)**2
      CALL NULL(KVR,IKVR,IR,IR)
      CALL NULL(KMR,IKMR,IDOFR,IDOFR)
      CALL NULL(RCOORD,INFR,NNR,2)
      CALL GAUSS(SAMP,ISAMP,NGP)
      CALL FORMCR(RCOORD,INFR,NXER,NYER,NNR,AA,BB)
      DO 12 I=1,NPILE
      DO 21 J=1,2
        COORD(I,J)=RCOORD(NOP(I),J)
   21 CONTINUE
   12 CONTINUE
      CALL RAFTU(FMAXR,NXER,NYER,AA,BB,QULT)
C
C
C     FORM ELEMENT STIFFNESS MATRIX OF THE RAFT BY NUMERICAL
INTEGRATION
C
      DO 100 I=1,NGP
      DO 100 J=1,NGP
      CALL FMPLAT(FUN,D1X,D1Y,D2X,D2Y,D2XY,SAMP,ISAMP,AA,BB,I,J)
      DO 200 K=1,IDOFR
      DO 200 L=K,IDOFR
      DTD(K,L)=4.*AA*BB*D*SAMP(I,2)*SAMP(J,2)*(D2X(K)*D2X(L)/(AA**4)+
     +        D2Y(K)*D2Y(L)/(BB**4)+(VR*D2X(K)*D2Y(L)+
     + VR*D2X(L)*D2Y(K)+2.*(1.-VR)*D2XY(K)*D2XY(L))/(AA**2*BB**2))
      DTD(L,K)=DTD(K,L)
  200 CONTINUE
  100 CALL MATADD(KMR,IKMR,DTD,IDTD,IDOFR,IDOFR)
C
```

```
C    GLOBAL STIFFNESS MATRIX ASSEMBLY FOR THE RAFT
C
     DO 300 IP=1,NXER
     DO 300 IQ=1,NYER
     CALL FORMGP(IP,IQ,NYER,GR,NFR,INFR)
 300 CALL FORMKU(KVR,IKVR,KMR,IKMR,GR,NR,IDOFR)
C
C     LOAD INCREMENT LOOP (LOAD/DISPLACEMENTS APPLIED IN
INCREMENTS)
C
     CALL NULVEC(TOTSUM,INCRE)
     CALL NULVEC(SUMR,INCRE)
     CALL NULVEC(TOTAL,INCRE)
     DO 1010 INC=1,INCS
     CALL NULL(SM,IKV,IR,IR)
     CALL NULL(KV,IKV,MN,MN)
     CALL NULL(KVS,IKV,MN,MN)
C
C     FORM SOIL FLEXIBILITY MATRIX USING MINDLIN'S EQUATION
C      (PILE - SOIL - PILE INTERACTION)
C
     DO 50 IP=1,NPILE
     CALL GEOM(IP,NN,NPILE,GP)
     IF(IP+1.GT.NPILE)GOTO 50
     DO 60 IQ=IP+1,NPILE
     CALL GEOM(IQ,NN,NPILE,GQ)
     RR=SQRT((COORD(IP,1)-COORD(IQ,1))**2+
    +       (COORD(IP,2)-COORD(IQ,2))**2)
     DO 70 I=1,NN
     IF(FSOIL(IP,I)/FMAX(I).GT..9999)GOTO 70
     ZZ=ZCOORD(I)
     GSOILI=GSURF+GRATE*ZZ
     IF(I.EQ.1)GSOILI=GSURF+GRATE*.25*ELL
     IF(I.EQ.NXE)GSOILI=GSURF+GRATE*(ZZ+.25*ELL)
     DO 80 J=1,NN
     IF(FSOIL(IQ,J)/FMAX(J).GT..9999)GOTO 80
     CC=ZCOORD(J)
     GSOILJ=GSURF+GRATE*CC
     IF(J.EQ.1)GSOILJ=GSURF+GRATE*.25*ELL
     IF(J.EQ.NXE)GSOILJ=GSURF+GRATE*(CC+.25*ELL)
     GSOIL=.5*(GSOILI+GSOILJ)
     FACT1=1./(16.*PI*GSOIL*(1.-VSOIL))
     ZMC2=(ZZ-CC)*(ZZ-CC)
     ZPC2=(ZZ+CC)*(ZZ+CC)
     ZMLC2=(ZL-CC)*(ZL-CC)
     ZPLC2=(ZL+CC)*(ZL+CC)
     R1=SQRT(RR*RR+ZMC2)
```

```
      R2=SQRT(RR*RR+ZPC2)
      S1=SQRT(RR*RR+ZMLC2)
      S2=SQRT(RR*RR+ZPLC2)
C     SOIL DISPLACEMENT AT IP DUE TO UNIT LOAD AT IQ
C     OBTAINED USING MINDLIN'S EQUATION
C
      NNI=M+GP(I)-NPILE
      MMI=M+GQ(J)-NPILE
      IF(I.EQ.1)NNI=NOP(GP(1))
      IF(J.EQ.1)MMI=NOP(GQ(1))
         KV(NNI,MMI)=FACT1*(FACT2/R1+(8.*FACT3-FACT2)/R2+ZMC2/R1**3+
     +       (FACT2*ZPC2-2.*CC*ZZ)/R2**3+6.*ZZ*CC*ZPC2/R2**5
     +       -(FACT2/S1+(8.*FACT3-FACT2)/S2+ZMLC2/S1**3
     +       +(FACT2*ZPLC2-2.*CC*ZL)/S2**3+6.*ZL*CC*ZPLC2/S2**5))
      ZMLC2=(ZL-ZZ)*(ZL-ZZ)
      ZPLC2=(ZL+ZZ)*(ZL+ZZ)
      S1=SQRT(RR*RR+ZMLC2)
      S2=SQRT(RR*RR+ZPLC2)
         KV(MMI,NNI)=FACT1*(FACT2/R1+(8.*FACT3-FACT2)/R2+ZMC2/R1**3+
     +       (FACT2*ZPC2-2.*CC*ZZ)/R2**3+6.*ZZ*CC*ZPC2/R2**5
     +       -(FACT2/S1+(8.*FACT3-FACT2)/S2+ZMLC2/S1**3
     +       +(FACT2*ZPLC2-2.*CC*ZL)/S2**3+6.*ZL*CC*ZPLC2/S2**5))
   80 CONTINUE
   70 CONTINUE
   60 CONTINUE
   50 CONTINUE
C
C     ADD IN FLEXIBILITY CONTRIBUTIONS FROM DISCRETE SOIL SPRINGS
ON THE PILES
C     AND THE RAFT
C
      DO 90 IP=1,NPILE
      CALL GEOM(IP,NN,NPILE,GP)
      NNI=NOP(GP(1))
      GSOIL=GSURF+GRATE*.25*ELL
      CALL FORMXI(FSOIL(IP,1),FMAX(1),RF,RM,R0,XI)
      FSHAFT=XI/(PI*GSOIL*ELL)
      IF(FSOIL(IP,1)/FMAX(1).GT.0.9999)FSHAFT=1.E12
      KV(NNI,NNI)=KV(NNI,NNI)+FSHAFT
      GSOIL=GSURF+GRATE*(ZCOORD(NXE)+.25*ELL)
      CALL FORMXI(FSOIL(IP,NXE),FMAX(NXE),RF,RM,R0,XI)
      NNI=M+GP(NXE)-NPILE
      FSHAFT=XI/(PI*GSOIL*3.*ELL)
      IF(FSOIL(IP,NXE)/FMAX(NXE).GT..9999)FSHAFT=1.E12
      KV(NNI,NNI)=KV(NNI,NNI)+FSHAFT
      DO 220 I=2,NN-2
      NNI=M+GP(I)-NPILE
```

```
      GSOIL=GSURF+GRATE*ZCOORD(I)
      CALL FORMXI(FSOIL(IP,I),FMAX(I),RF,RM,R0,XI)
      FSHAFT=XI/(2.*PI*GSOIL*ELL)
      IF(FSOIL(IP,I)/FMAX(I).GT..9999)FSHAFT=1.E12
  220 KV(NNI,NNI)=KV(NNI,NNI)+FSHAFT
C
C     ADD TIP FLEXIBILITY
C
      GSOIL=GSURF+GRATE*ZCOORD(NN)
      NNI=M+GP(NN)-NPILE
      FTIP=(1.-VSOIL)/((4.*GSOIL*R0)*(1.-RF*FSOIL(IP,NN)/FMAX(NN))**2)
      IF(FSOIL(IP,NN)/FMAX(NN).GT..9999)FTIP=1.E12
   90 KV(NNI,NNI)=KV(NNI,NNI)+FTIP
C
C  ADD   THE FLEXIBILTY MATRIX OF THE RAFT - SOIL - PILE INTERACTIONS
C
C       IF (QULT< 0.001)GO TO 5000

      CALL FORMCR(RCOORD,INFR,NXER,NYER,NNR,AA,BB)
      DO 140 II=1,M
      IF(FSOILR(II)/FMAXR(II).GT..9999)GOTO 140
      DO 110 I=1,NPILE
      IF(II.EQ.NOP(I))GOTO 140
  110 CONTINUE
      ZZ=ZF
      GSOILI=GSURF+GRATE*ZZ
      DO 130 IP=1,NPILE
      CALL GEOM(IP,NN,NPILE,GP)
      RR2=SQRT((RCOORD(II,1)-COORD(IP,1))**2+
     +        (RCOORD(II,2)-COORD(IP,2))**2)
      DO 120 I=1,NN
      IF(FSOIL(IP,I)/FMAX(I).GT..9999)GOTO 120
      IF(I.EQ.1)GSOILI=GSURF+GRATE*.25*ELL
      CC=ZCOORD(I)
      GSOILJ=GSURF+GRATE*CC
      IF(I.EQ.NXE)GSOILI=GSURF+GRATE*(CC+.25*ELL)
      GSOIL=.5*(GSOILI+GSOILJ)
      FACT1=1./(16.*PI*GSOIL*(1.-VSOIL))
      ZMC2=(ZZ-CC)*(ZZ-CC)
      ZPC2=(ZZ+CC)*(ZZ+CC)
      ZMLC2=(ZL-CC)*(ZL-CC)
      ZPLC2=(ZL+CC)*(ZL+CC)
      S1=SQRT(RR2*RR2+ZMLC2)
      S2=SQRT(RR2*RR2+ZPLC2)
      R1=SQRT(RR2*RR2+ZMC2)
      R2=SQRT(RR2*RR2+ZPC2)
      MMI=M+GP(I)-NPILE
```

```
      NNI=II
      IF(I.EQ.1)MMI=NOP(GP(1))
C
      KV(NNI,MMI)=FACT1*(FACT2/R1+(8.*FACT3-FACT2)/R2+ZMC2/R1**3+
     +      (FACT2*ZPC2-2.*CC*ZZ)/R2**3+6.*ZZ*CC*ZPC2/R2**5
     +      -(FACT2/S1+(8.*FACT3-FACT2)/S2+ZMLC2/S1**3
     +      +(FACT2*ZPLC2-2.*CC*ZL)/S2**3+6.*ZL*CC*ZPLC2/S2**5))
      ZMLC2=(ZL-ZZ)*(ZL-ZZ)
      ZPLC2=(ZL+ZZ)*(ZL+ZZ)
      S1=SQRT(RR2*RR2+ZMLC2)
      S2=SQRT(RR2*RR2+ZPLC2)
      KV(MMI,NNI)=FACT1*(FACT2/R1+(8.*FACT3-FACT2)/R2+ZMC2/R1**3+
     +      (FACT2*ZPC2-2.*CC*ZZ)/R2**3+6.*ZZ*CC*ZPC2/R2**5
     +      -(FACT2/S1+(8.*FACT3-FACT2)/S2+ZMLC2/S1**3
     +      +(FACT2*ZPLC2-2.*CC*ZL)/S2**3+6.*ZL*CC*ZPLC2/S2**5))
  120 CONTINUE
  130 CONTINUE
  140 CONTINUE
C
C    FLEXIBILITY MATRIX OF RAFT - SOIL- RAFT INTERACTION
C
      CALL FORMCR(RCOORD,INFR,NXER,NYER,NNR,AA,BB)
      DO 170 II=1,M
      IF(FSOILR(II)/FMAXR(II).GT..9999)GOTO 170
      DO 150 I=1,NPILE
      IF(II.EQ.NOP(I))GOTO 170
  150 CONTINUE
      DO 160 JJ=II+1,M
      DO 155 I=1,NPILE
      IF(JJ.EQ.NOP(I))GOTO 160
  155 CONTINUE
      IF(FSOILR(JJ)/FMAXR(JJ).GT..9999)GOTO 160
      RR2=SQRT((RCOORD(II,1)-RCOORD(JJ,1))**2+
     +      (RCOORD(II,2)-RCOORD(JJ,2))**2)
      GSOIL=GSURF
      FACT1=1./(16.*PI*GSOIL*(1.-VSOIL))
      ZZ=ZF
      CC=ZF
      ZMC2=(ZZ-CC)*(ZZ-CC)
      ZPC2=(ZZ+CC)*(ZZ+CC)
      ZMLC2=(ZL-CC)*(ZL-CC)
      ZPLC2=(ZL+CC)*(ZL+CC)
      S1=SQRT(RR2*RR2+ZMLC2)
      S2=SQRT(RR2*RR2+ZPLC2)
      R1=SQRT(RR2*RR2+ZMC2)
      R2=SQRT(RR2*RR2+ZPC2)
      MMI=JJ
```

```
      NNI=II
      KV(NNI,MMI)= FACT1*(FACT2/R1+(8.*FACT3-FACT2)/R2+ZMC2/R1**3+
     +       (FACT2*ZPC2-2.*CC*ZZ)/R2**3+6.*ZZ*CC*ZPC2/R2**5
     +       -(FACT2/S1+(8.*FACT3-FACT2)/S2+ZMLC2/S1**3
     +       +(FACT2*ZPLC2-2.*CC*ZL)/S2**3+6.*ZL*CC*ZPLC2/S2**5))
      KV(MMI,NNI)=KV(NNI,MMI)
  160 CONTINUE
  170 CONTINUE
C
C     ADD DISCRETE SOIL SPRING COEFFICIENT OF RAFT-SOIL INTERFACE
C
      DO 190 II=1,M
      DO 180 I=1,NPILE
      IF(II.EQ.NOP(I))GOTO 190
  180 CONTINUE
      NNI=II
      ALPHA=MAX(AA,BB)/MIN(AA,BB)
      PO=(2./PI)*(LOG(ALPHA +SQRT(1+ALPHA**2))+
     +    ALPHA*LOG((1+SQRT(1+ALPHA**2)/ALPHA)))
      FRAFT=(1.-VSOIL)*MIN(AA,BB)*PO/(2.*AA*BB*GSURF*(1.0
     +     -RF*FSOILR(II)/FMAXR(II))**2)
      IF(FSOILR(II)/FMAXR(II).GT.0.9999)FRAFT=1.E12
      KV(NNI,NNI)=KV(NNI,NNI)+FRAFT
  190 CONTINUE
C
C      INVERT SOIL FLEXIBILITY MATRIX TO GIVE STIFFNESS MATRIX
C
      DO 810 I=1,MN
        DO 800 J=1,MN
          CMM(I,J)=0.0
800   CONTINUE
          CMM(I,I)=1.0
810   CONTINUE
      CALL LUDCMP(KV,IKV,MN,INDX,D)
      DO 820 J=1,MN
      CALL LUBKSB(KV,IKV,MN,INDX,CMM(1,J))
820   CONTINUE
      CALL MATCOP(CMM,IKV,KVS,IKV,MN,MN)
      CALL NULL(KVI,IKV,IR,IR)
      DO 225 KK=1,MN
      DO 210 LL=1,MN
      II=3*M+KK
      JJ=3*M+LL
      IF(KK.LE.M)II=4*(KK-1)+1
      IF(LL.LE.M)JJ=4*(LL-1)+1
      KVI(II,JJ)=KVI(II,JJ)+KVS(KK,LL)
C      KVI(JJ,II)=KV(LI,IL)
```

```
  210 CONTINUE
  225 CONTINUE
      CALL MATCOP(KVI,IKV,SM,IKV,IR,IR)
C
C     FORM AND ASSEMBLE STIFFNESS MATRIX OF THE PILE ELEMENT
C
      CALL NULL(KMM,IKV,IR,IR)
      CALL AXIKM(KM,CSA,E,ELL)
      DO 430 IP=1,NPILE
      CALL GEOM(IP,NN,NPILE,GP)
      DO 440 I=1,NXE
      G(1)=4*M+GP(I)-NPILE
      IF(I.EQ.1)G(1)=4*(NOP(GP(1))-1)+1
      G(2)=4*M+GP(I+1)-NPILE
      CALL FORMKU(KMM,IKV,KM,IKM,G,IR,IDOF)
  440 CONTINUE
  430 CONTINUE
C
C
      CALL MATADD(KVI,IKV,KMM,IKVR,IR,IR)
      CALL MATADD(KVI,IKV,KVR,IKVR,IR,IR)
C
C    EQUATION SOLUTION
C
      CALL NULVEC(LOADS,IR)
      CALL NULVEC(PMAXR,M)
      SUMLOD(INC)=0.0
      DO 550 I=1,NL
 550  LOADS(NO(I))=VALL(I)*QINC(INC)
      CALL RAFTU(PMAXR,NXER,NYER,AA,BB,PULT)
C
      DO 560 I=1,M
      K=4*(I-1)+1
      LOADS(K)=LOADS(K)+PMAXR(I)*QINC(INC)
      SUMLOD(INC)=SUMLOD(INC)+LOADS(K)
  560 CONTINUE
C     TOTAL(INC)=TOTLOD
C
C     SOLVE GLOBAL MATRIX TO GIVE RAFT AND PILE DISPLACEMENTS
C
      CALL MATCOP(KVI,IKV,SMM,IKV,IR,IR)
      CALL LUDCMP(KVI,IKV,IR,INDX,D)
      CALL LUBKSB(KVI,IKV,IR,INDX,LOADS)
      CALL VECCOP(LOADS,FORCES,IR)
      CALL VECADD(DISPL,FORCES,IR)
      NC=NR/2-1
      SETTLE(INC)=DISPL(NC)
```

```
      CALL NULVEC(DEFLEC,IR)
      DO 777 J=1,NR,4
      IF(FORCES(J).GT.0.0)DEFLEC(J)=DEFLEC(J)+FORCES(J)
  777 CONTINUE
      DO 776 J=NR+1,IR
      IF(FORCES(J).GT.0.0)DEFLEC(J)=DEFLEC(J)+FORCES(J)
  776 CONTINUE
C
C     RECOVER SOIL SPRING FORCES
C
      CALL  MVMULT(SM,IKV,FORCES,IR,IR,FORCE)

      WRITE(11,'(/7X,A23/)')'THE FORCE MATRIX'
C     DO 235 K=1,IR
      WRITE (11,'(8X,84F20.12)')(FORCE(L),L=1,NR)
C
      WRITE(11,'(/7X,A29/)')'THE DISPLACEMENT MATRIX OF PILE'
      DO 222 I=1,NPILE
      LL=NR+I
      KK=4*(NOP(I)-1)+1
      WRITE (11,'(8X,16F20.12)')FORCES(KK),(FORCES(L),L=LL,IR,NPILE)
  222 CONTINUE
      DO 490 II=1,M
      DO 445 I=1,NPILE
      IF(II.EQ.NOP(I)) GOTO 490
  445 CONTINUE
      RRAFT=FORCE(4*(II-1)+1)
      IF(RRAFT.GT.0.0)FSOILR(II)=FSOILR(II)+RRAFT
      IF(FSOILR(II).GT.FMAXR(II))FSOILR(II)=FMAXR(II)
      SUMR(INC)=SUMR(INC)+FSOILR(II)
  490 CONTINUE
       DO 460 IP=1,NPILE
      SUMM(IP)=0.0
      CALL GEOM(IP,NN,NPILE,GP)
      RSOIL=FORCE(4*(NOP(GP(1))-1)+1)
      IF(RSOIL.GT.0.0)FSOIL(IP,1)=FSOIL(IP,1)+RSOIL
      IF(FSOIL(IP,1).GT.FMAX(1))FSOIL(IP,1)=FMAX(1)
      SUMM(IP)=SUMM(IP)+FSOIL(IP,1)
      DO 470 I=2,NN
      RSOIL=FORCE(4*M+GP(I)-NPILE)
      IF(RSOIL.GT.0.0)FSOIL(IP,I)=FSOIL(IP,I)+RSOIL
      IF(FSOIL(IP,I).GT.FMAX(I))FSOIL(IP,I)=FMAX(I)
  470 SUMM(IP)=SUMM(IP)+FSOIL(IP,I)
C     WRITE(11,1000)SUMM(IP)
      TOTSUM(INC)=TOTSUM(INC)+SUMM(IP)
  460 CONTINUE
      DO 456 IP=1,NPILE
```

```
      WRITE (11,'(6X,I4,16E20.10))')IP,(FSOIL(IP,I),I=1,NN)
  456 CONTINUE
      TOTAL(INC)=TOTAL(INC)+SUMR(INC)+TOTSUM(INC)
 1010 CONTINUE
      DO 444 LL=1,M
      LLL=4*(LL-1)+1
      SETTMM=1000.*DISPL(LLL)
      WRITE (11,'(6X,3F20.10))')RCOORD(LL,1),RCOORD(LL,2),SETTMM
  444 CONTINUE
      WRITE(11,'(/7X,A23/)') 'THE ACTION RAFT FORCE  VECTOR'
      KK=0
      DO 459 IP=1,NXER+1
      DO 459 IQ=1,NYER+1
      KK=KK+1
      FMAXRR(KK)=FSOILR(KK)/(AA*BB)
      IF(IP.EQ.1.OR.IQ.EQ.1)FMAXRR(KK)=FSOILR(KK)/(AA*BB/2.)
      IF(IP.EQ.NXER+1.OR.IQ.EQ.NYER+1)FMAXRR(KK)=FSOILR(KK)/(AA*BB/2.)
      IF((IP.EQ.1).AND.(IQ.EQ.1))FMAXRR(KK)=FSOILR(KK)/(AA*BB/4.)
      IF(IP.EQ.NXER+1.AND.IQ.EQ.NYER+1)FMAXRR(KK)=FSOILR(KK)/(AA*BB/4.)
      IF((IP.EQ.1).AND.IQ.EQ.NYER+1)FMAXRR(KK)=FSOILR(KK)/(AA*BB/4.)
      IF((IQ.EQ.1).AND.IP.EQ.NXER+1)FMAXRR(KK)=FSOILR(KK)/(AA*BB/4.)
  459 CONTINUE
      DO 454 LL=1,M
      WRITE (11,'(6X,3F20.10))')RCOORD(LL,1),RCOORD(LL,2),FMAXRR(LL)
  454 CONTINUE
      DO 555 IP=1,NPILE
      WRITE (11,'(6X,3E20.10))')(COORD(IP,J),J=1,2),SUMM(IP)
  555 CONTINUE
      WRITE (11,'(/6X,10E20.10/))')(SUMLOD(I),I=1,INCS)
      WRITE (11,'(/8X,A45/)')' TOTAL  SUMMPILE   SUMRAFT  SETTLEMENT'
      DO 168 LM=1,INCS
      WRITE (11,'(6X,I3,2X,4F20.10))')LM,TOTAL(LM),TOTSUM(LM),SUMR(LM),
     +SETTLE(LM)
  168 CONTINUE
C
      CALL FORMD2(DEE,IDEE,ER,TH,VR)
      CALL PRINTA(DEE,IDEE,IBE,IBE)
      NIP=4
      CALL GAUSS(SAMP,ISAMP,NIP)
      CALL FORMECO(ELCCOR,INFR,NXER,NYER,NNR,AA,BB)
C
C     RECOVER STRESSES AT ELEMENT GAUSS-POINTS
C
      IK=0
      DO 303 IP=1,NXER
      DO 303 IQ=1,NYER
      CALL GEOM4Y(IP,IQ,NYER,AA,BB,COORDD,ICOORD,GR,NFR,INFR)
```

```
      CALL FORMGP(IP,IQ,NYER,GR,NFR,INFR)
      IK=IK+1
      DO 333 K=1,IDOFR
      DISP(K)=DISPL(GR(K))
  333 CONTINUE
      CALL NULL(BEE,IBE,IBE,IDOFR)
      DO 302 I=1,NIP
      DO 301 J=1,NIP
      CALL FMPLAT(FUN,D1X,D1Y,D2X,D2Y,D2XY,SAMP,ISAMP,AA,BB,I,J)
      CALL FORMBE(BEE,IBE,D2X,D2Y,D2XY,AA,BB,NODOFR)
  301 CONTINUE
  302 CONTINUE
      CALL MVMULT(BEE,IBE,DISP,IBER,IDOFR,KEE)
      CALL MVMULT(DEE,IBE,KEE,IBER,IBE,MOMM)
      WRITE(11,'(2X,(2X,5F20.10))')(ELCCOR(IK,JK),JK=1,2),
     +(MOMM(JI),JI=1,IBER)
  303 CONTINUE
 1000 FORMAT(E12.4)
 2100 FORMAT(4E20.10)
 2200 FORMAT(3E20.10)
 2000 FORMAT('100(I10,E20.8)')
 1111 STOP
      END
C
C
C--------- ***------THE FOLLOWINGS ARE SUBROUTINES------*** ---------
C
C
      SUBROUTINE FORMB(BEE,IBEE,DERIV,IDERIV,NOD)
C
C     THIS SUBROUTINE FORMS THE STRAIN/DISPLACEMENT
C     FOR PLATE BENDING (B MATRIX)
C
      REAL BEE(IBEE,*),DERIV(IDERIV,*)
      DO 1 M=1,NOD
      K=4*M-1
      L=K-1
      X=DERIV(1,M)
      BEE(1,L)=X
      BEE(3,K)=X
      Y=DERIV(2,M)
      BEE(2,K)=Y
      BEE(3,L)=Y
    1 CONTINUE
      RETURN
      END
C
```

```
      SUBROUTINE FORMLN(DER,IDER,FUN,SAMP,ISAMP,I,J)
C
C     THIS SUBROUTINE FORMS THE SHAPE FUNCTIONS AND
C     THEIR DERIVATIVES FOR 4-NODED QUADRILATERAL ELEMENTS
C
      REAL DER(IDER,*),FUN(*),SAMP(ISAMP,*)
      ETA=SAMP(I,1)
      XI=SAMP(J,1)
      ETAM=.25*(1.-ETA)
      ETAP=.25*(1.+ETA)
      XIM=.25*(1.-XI)
      XIP=.25*(1.+XI)
      FUN(1)=4.*XIM*ETAM
      FUN(2)=4.*XIM*ETAP
      FUN(3)=4.*XIP*ETAP
      FUN(4)=4.*XIP*ETAM
      DER(1,1)=-ETAM
      DER(1,2)=-ETAP
      DER(1,3)=ETAP
      DER(1,4)=ETAM
      DER(2,1)=-XIM
      DER(2,2)=XIM
      DER(2,3)=XIP
      DER(2,4)=-XIP
      RETURN
      END

      SUBROUTINE TWOBY2(JAC,IJAC,JAC1,IJAC1,DET)
C
C     THIS SUBROUTINE FORMS THE INVERSE OF A 2 BY 2 MATRIX
C
      REAL JAC(IJAC,*),JAC1(IJAC1,*)
      DET=JAC(1,1)*JAC(2,2)-JAC(1,2)*JAC(2,1)
      JAC1(1,1)=JAC(2,2)
      JAC1(1,2)=-JAC(1,2)
      JAC1(2,1)=-JAC(2,1)
      JAC1(2,2)=JAC(1,1)
      DO 1 K=1,2
      DO 1 L=1,2
    1 JAC1(K,L)=JAC1(K,L)/DET
      RETURN
      END

      SUBROUTINE FORMCR(RCOORD,INFR,NXER,NYER,NNR,AA,BB)
C
C    THIS SUBROUTINE FORMS COORDINATES OF THE NODES OF PLATE
ELEMENT
```

```
C
      REAL RCOORD(INFR,*)
      INTEGER I
      I=0
      DO 600 IP=1,NXER+1
      DO 500 IQ=1,NYER+1
      I=I+1
      IF(I.GT.NNR)GOTO 1000
      RCOORD(I,2)=BB*(IQ-1)
      RCOORD(I,1)=(IP-1)*AA
  500 CONTINUE
  600 CONTINUE
 1000 RETURN
      END
      SUBROUTINE FORMECO(ELCCOR,INFR,NXER,NYER,NNR,AA,BB)
C
C     THIS SUBROUTINE FORMS COORDINATES OF THE NODES OF PLATE
ELEMENT
C
      REAL ELCCOR(INFR,*)
      INTEGER I
      I=0
      DO 600 IP=1,NXER
      DO 500 IQ=1,NYER
      I=I+1
      IF(I.GT.NNR)GOTO 1000
      ELCCOR(I,2)=BB*(IQ-1)+BB/2.
      ELCCOR(I,1)=(IP-1)*AA +AA/2.
  500 CONTINUE
  600 CONTINUE
 1000 RETURN
      END

C
      SUBROUTINE READNF(NF,INF,NN,NODOF,NR)
C
C      THIS SUBROUTINE READS THE NODAL FREEDOM DATA
C
      INTEGER NF(INF,*)
      DO 1 I=1,NN
      DO 1 J=1,NODOF
    1 NF(I,J)=1
      IF(NR.GT.0)READ(10,*)(K,(NF(K,J),J=1,NODOF),I=1,NR)
      N=0
      DO 2 I=1,NN
      DO 2 J=1,NODOF
      IF(NF(I,J).NE.0)THEN
```

```
      N=N+1
      NF(I,J)=N
      ENDIF
    2 CONTINUE
      RETURN
      END
C
      SUBROUTINE NULL(A,IA,M,N)
C
C     THIS SUBROUTINE NULLS A 2-D ARRAY
C
      REAL A(IA,*)
      DO 1 I=1,M
      DO 1 J=1,N
    1 A(I,J)=0.0
      RETURN
      END
C
      SUBROUTINE SAMPLE(SAMP,ISAMP,NIP)
C
C     THIS SUBROUTINE PROVIDES  SAMPLING POINTS
C     FOR GAUSS-POINTS
C
      REAL SAMP(ISAMP,*)
C
      ROOT3=1./SQRT(3.)
      GO TO(2,4),NIP
    2 SAMP(1,1)=-1./SQRT(3.)
      SAMP(2,1)=-SAMP(1,1)
      GOTO 111
    4 samp(1,1)=-root3; samp(1,2)= root3
      samp(2,1)= root3; samp(2,2)= root3
      samp(3,1)=-root3; samp(3,2)=-root3
      samp(4,1)= root3; samp(4,2)=-root3
  111 CONTINUE
      RETURN
      END
C
      SUBROUTINE GAUSS(SAMP,ISAMP,NGP)
C
C     THIS SUBROUTINE PROVIDES THE WEIGHTS AND SAMPLING POINTS
C     FOR GAUSS-LEGENDRE QUADRATURE
C
      REAL SAMP(ISAMP,*)
      GO TO(1,2,3,4,5,6,7),NGP
    1 SAMP(1,1)=0.
      SAMP(1,2)=2.
```

```
 GOTO 100
2 SAMP(1,1)=1./SQRT(3.)
 SAMP(2,1)=-SAMP(1,1)
 SAMP(1,2)=1.
 SAMP(2,2)=1.
 GO TO 100
3 SAMP(1,1)=.2*SQRT(15.)
 SAMP(2,1)=.0
 SAMP(3,1)=-SAMP(1,1)
 SAMP(1,2)=5./9.
 SAMP(2,2)=8./9.
 SAMP(3,2)=SAMP(1,2)
 GO TO 100
4 SAMP(1,1)=.861136311594053
 SAMP(2,1)=.339981043584856
 SAMP(3,1)=-SAMP(2,1)
 SAMP(4,1)=-SAMP(1,1)
 SAMP(1,2)=.347854845137454
 SAMP(2,2)=.652145154862546
 SAMP(3,2)=SAMP(2,2)
 SAMP(4,2)=SAMP(1,2)
 GO TO 100
5 SAMP(1,1)=.906179845938664
 SAMP(2,1)=.538469310105683
 SAMP(3,1)=.0
 SAMP(4,1)=-SAMP(2,1)
 SAMP(5,1)=-SAMP(1,1)
 SAMP(1,2)=.236926885056189
 SAMP(2,2)=.478628670499366
 SAMP(3,2)=.568888888888889
 SAMP(4,2)=SAMP(2,2)
 SAMP(5,2)=SAMP(1,2)
 GO TO 100
6 SAMP(1,1)=.932469514203152
 SAMP(2,1)=.661209386466265
 SAMP(3,1)=.238619186083197
 SAMP(4,1)=-SAMP(3,1)
 SAMP(5,1)=-SAMP(2,1)
 SAMP(6,1)=-SAMP(1,1)
 SAMP(1,2)=.171324492379170
 SAMP(2,2)=.360761573048139
 SAMP(3,2)=.467913934572691
 SAMP(4,2)=SAMP(3,2)
 SAMP(5,2)=SAMP(2,2)
 SAMP(6,2)=SAMP(1,2)
 GO TO 100
7 SAMP(1,1)=.949107912342759
```

```
      SAMP(2,1)=.741531185599394
      SAMP(3,1)=.405845151377397
      SAMP(4,1)=.0
      SAMP(5,1)=-SAMP(3,1)
      SAMP(6,1)=-SAMP(2,1)
      SAMP(7,1)=-SAMP(1,1)
      SAMP(1,2)=.129484966168870
      SAMP(2,2)=.279705391489277
      SAMP(3,2)=.381830050505119
      SAMP(4,2)=.417959183673469
      SAMP(5,2)=SAMP(3,2)
      SAMP(6,2)=SAMP(2,2)
      SAMP(7,2)=SAMP(1,2)
  100 CONTINUE
      RETURN
      END
C
      SUBROUTINE
FMPLAT(FUN,D1X,D1Y,D2X,D2Y,D2XY,SAMP,ISAMP,AA,BB,I,J)
C
C     THIS SUBROUTINE FORMS THE SHAPE FUNCTIONS AND THEIR 1ST
C     AND 2ND DERIVATIVES FOR RECTANGULAR PLATE BENDING
ELEMENTS
C
      REAL FUN(*),D1X(*),D1Y(*),D2X(*),D2Y(*),D2XY(*),SAMP(ISAMP,*)
      X=SAMP(I,1)
      E=SAMP(J,1)
      XP1=X+1.
      XP12=XP1*XP1
      XP13=XP12*XP1
      EP1=E+1.
      EP12=EP1*EP1
      EP13=EP12*EP1
      P1=1.-.75*XP12+.25*XP13
      Q1=1.-.75*EP12+.25*EP13
      P2=.5*AA*XP1*(1.-XP1+.25*XP12)
      Q2=.5*BB*EP1*(1.-EP1+.25*EP12)
      P3=.25*XP12*(3.-XP1)
      Q3=.25*EP12*(3.-EP1)
      P4=.25*AA*XP12*(.5*XP1-1.)
      Q4=.25*BB*EP12*(.5*EP1-1.)
      FUN(1)=P1*Q1
      FUN(2)=P2*Q1
      FUN(3)=P1*Q2
      FUN(4)=P2*Q2
      FUN(5)=P1*Q3
      FUN(6)=P2*Q3
```

```
FUN(7)=P1*Q4
FUN(8)=P2*Q4
FUN(9)=P3*Q3
FUN(10)=P4*Q3
FUN(11)=P3*Q4
FUN(12)=P4*Q4
FUN(13)=P3*Q1
FUN(14)=P4*Q1
FUN(15)=P3*Q2
FUN(16)=P4*Q2
DP1=1.5*XP1*(.5*XP1-1.)
DQ1=1.5*EP1*(.5*EP1-1.)
DP2=AA*(.5-XP1+.375*XP12)
DQ2=BB*(.5-EP1+.375*EP12)
DP3=1.5*XP1*(1.-.5*XP1)
DQ3=1.5*EP1*(1.-.5*EP1)
DP4=.5*AA*XP1*(.75*XP1-1.)
DQ4=.5*BB*EP1*(.75*EP1-1.)
D2P1=1.5*X
D2P2=.25*AA*(3.*X-1.)
D2P3=-D2P1
D2P4=.25*AA*(3.*X+1.)
D2Q1=1.5*E
D2Q2=.25*BB*(3.*E-1.)
D2Q3=-D2Q1
D2Q4=.25*BB*(3.*E+1.)
D1X(1)=DP1*Q1
D1X(2)=DP2*Q1
D1X(3)=DP1*Q2
D1X(4)=DP2*Q2
D1X(5)=DP1*Q3
D1X(6)=DP2*Q3
D1X(7)=DP1*Q4
D1X(8)=DP2*Q4
D1X(9)=DP3*Q3
D1X(10)=DP4*Q3
D1X(11)=DP3*Q4
D1X(12)=DP4*Q4
D1X(13)=DP3*Q1
D1X(14)=DP4*Q1
D1X(15)=DP3*Q2
D1X(16)=DP4*Q2
D1Y(1)=P1*DQ1
D1Y(2)=P2*DQ1
D1Y(3)=P1*DQ2
D1Y(4)=P2*DQ2
D1Y(5)=P1*DQ3
```

```
D1Y(6)=P2*DQ3
D1Y(7)=P1*DQ4
D1Y(8)=P2*DQ4
D1Y(9)=P3*DQ3
D1Y(10)=P4*DQ3
D1Y(11)=P3*DQ4
D1Y(12)=P4*DQ4
D1Y(13)=P3*DQ1
D1Y(14)=P4*DQ1
D1Y(15)=P3*DQ2
D1Y(16)=P4*DQ2
D2X(1)=D2P1*Q1
D2X(2)=D2P2*Q1
D2X(3)=D2P1*Q2
D2X(4)=D2P2*Q2
D2X(5)=D2P1*Q3
D2X(6)=D2P2*Q3
D2X(7)=D2P1*Q4
D2X(8)=D2P2*Q4
D2X(9)=D2P3*Q3
D2X(10)=D2P4*Q3
D2X(11)=D2P3*Q4
D2X(12)=D2P4*Q4
D2X(13)=D2P3*Q1
D2X(14)=D2P4*Q1
D2X(15)=D2P3*Q2
D2X(16)=D2P4*Q2
D2Y(1)=P1*D2Q1
D2Y(2)=P2*D2Q1
D2Y(3)=P1*D2Q2
D2Y(4)=P2*D2Q2
D2Y(5)=P1*D2Q3
D2Y(6)=P2*D2Q3
D2Y(7)=P1*D2Q4
D2Y(8)=P2*D2Q4
D2Y(9)=P3*D2Q3
D2Y(10)=P4*D2Q3
D2Y(11)=P3*D2Q4
D2Y(12)=P4*D2Q4
D2Y(13)=P3*D2Q1
D2Y(14)=P4*D2Q1
D2Y(15)=P3*D2Q2
D2Y(16)=P4*D2Q2
D2XY(1)=DP1*DQ1
D2XY(2)=DP2*DQ1
D2XY(3)=DP1*DQ2
D2XY(4)=DP2*DQ2
```

```
      D2XY(5)=DP1*DQ3
      D2XY(6)=DP2*DQ3
      D2XY(7)=DP1*DQ4
      D2XY(8)=DP2*DQ4
      D2XY(9)=DP3*DQ3
      D2XY(10)=DP4*DQ3
      D2XY(11)=DP3*DQ4
      D2XY(12)=DP4*DQ4
      D2XY(13)=DP3*DQ1
      D2XY(14)=DP4*DQ1
      D2XY(15)=DP3*DQ2
      D2XY(16)=DP4*DQ2
      RETURN
      END
C
      SUBROUTINE FORMGP(IP,IQ,NYE,G,NF,INF)
C
C     THIS SUBROUTINE FORMS THE 'STEERING' VECTOR FOR A
C     4-NODE RECTANGULAR PLATE BENDING ELEMENT
C
      INTEGER G(*),NF(INF,*)
      I1=(IP-1)*(NYE+1)+IQ
      I2=I1+1
      I3=IP*(NYE+1)+IQ
      I4=I3+1
      DO 1 I=1,4
      G(I)=NF(I1,I)
      G(I+4)=NF(I2,I)
      G(I+8)=NF(I4,I)
      G(I+12)=NF(I3,I)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE FORMGPP(IP,IQ,NYE,G,NF,INF)
C
C     THIS SUBROUTINE FORMS THE 'STEERING' VECTOR FOR A
C     4-NODE RECTANGULAR PLATE BENDING ELEMENT
C
      INTEGER G(*),NF(INF,*)
      I1=(IP-1)*(NYE+1)+IQ+1
      I2=I1-1
      I3=IP*(NYE+1)+IQ+1
      I4=I3-1
      DO 1 I=1,4
      G(I)=NF(I1,I)
      G(I+4)=NF(I2,I)
```

```
      G(I+8)=NF(I4,I)
      G(I+12)=NF(I3,I)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE FRMGRF(IP,IQ,NPILE,GRF,NXER,NYER,M,NOP,NODOFR)
C
C     THIS SUBROUTINE FORMS THE 'STEERING' VECTOR FOR A
C     FOR SOIL-RAFT FLEXIBILITY MARIX
C
      INTEGER M,GRF(*),NOP(*)
      I1=(IP-1)*(NYER+1)+IQ
      I2=I1+1
      I3=IP*(NYER+1)+IQ
      I4=I3+1
      DO 1 I=1,1
      GRF(I)=I1
      GRF(I+1)=I2
      GRF(I+2)=I4
      GRF(I+3)=I3
    1 CONTINUE
C     DO 2 J=1,4
C     DO 2 II=1,NPILE
C   2 IF(GRF(J).EQ.NOP(II))GRF(J)=0
      RETURN
      END
C
      SUBROUTINE FORMKU(KU,IKU,KM,IKM,G,IW,IDOF)
C
C     THIS SUBROUTINE ASSEMBLES ELEMENT MATRICES INTO
SYMMETRICAL
C     GLOBAL MATRIX
C
      REAL KU(IKU,*),KM(IKM,*)
      INTEGER G(*)
      DO 1 I=1,IDOF
      IF(G(I).EQ.0)GO TO 1
      DO 2 J=1,IDOF
      IF(G(J).EQ.0)GO TO 2
      KU(G(I),G(J))=KU(G(I),G(J))+KM(I,J)
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE FORMKV(KV,IKV,KM,IKM,GRF,NODOFR)
```

```
C
C     THIS SUBROUTINE ASSEMBLES ELEMENT MATRICES INTO
SYMMETRICAL
C     GLOBAL MATRIX
C
      REAL KV(IKV,*),KM(IKM,*)
      INTEGER GRF(*)
      DO 1 I=1,NODOFR
C     IF(GRF(I).EQ.0)GO TO 1
      DO 2 J=1,NODOFR
C     IF(GRF(J).EQ.0)GO TO 2
      KV(GRF(I),GRF(J))=KV(GRF(I),GRF(J))+KM(I,J)
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE NULVEC(VEC,N)
C
C     THIS SUBROUTINE NULLS A COLUMN VECTOR
C
      REAL VEC(*)
      DO 1 I=1,N
    1 VEC(I)=0.
      RETURN
      END
C
      SUBROUTINE PRINTV(VEC,N)
C
C     THIS SUBROUTINE WRITES A COLUMN VECTOR TO OUTPUT CHANNEL11
C
      REAL VEC(*)
      WRITE(11,1)(VEC(I),I=1,N)
    1 FORMAT(1X,16E20.10)
      RETURN
      END
C
      SUBROUTINE MATADD(A,IA,B,IB,M,N)
C
C     THIS SUBROUTINE ADDS TWO EQUAL SIZED ARRAYS
C
      REAL A(IA,*),B(IB,*)
      DO 1 I=1,M
      DO 1 J=1,N
    1 A(I,J)=A(I,J)+B(I,J)
      RETURN
      END
```

```
C
      SUBROUTINE VECADD(A,B,M)
C
C     THIS SUBROUTINE ADDS TWO EQUAL SIZED ARRAYS
C
      REAL A(*),B(*)
      DO 1 I=1,M
    1 A(I)=A(I)+B(I)
      RETURN
      END
C
      SUBROUTINE GEOM(IP,NN,NPILE,G)
C
C     THIS SUBROUTINE FORMS THE NODE NUMBERS OF PILE IP
C
      INTEGER G(*)
      DO 1 I=1,NN
    1 G(I)=IP+(I-1)*NPILE
      RETURN
      END
C
      SUBROUTINE FORMXI(FSOIL,FMAX,RF,RM,R0,XI)
C
C     THIS SUBROUTINE FORMS PART OF THE SPRING STIFFNESS TERM
C
C
      PHI=FSOIL*R0*RF/FMAX
      XI=LOG((RM-PHI)/(R0-PHI))+PHI*(RM-R0)/((RM-PHI)*(R0-PHI))
      RETURN
      END
C
      SUBROUTINE VECCOP(A,B,N)
C
C     THIS SUBROUTINE COPIES VECTOR A INTO VECTOR B
C
      REAL A(*),B(*)
      DO 1 I=1,N
    1 B(I)=A(I)
      RETURN
      END
C
      SUBROUTINE AXIKM(KM,CSA,E,ELL)
C
C     THIS SUBROUTINE FORMS THE STIFFNESS MATRIX FOR AN
C     AXIALLY LOADED LINE ELEMENT
C
      REAL KM(2,2)
```

```
      KM(1,1)=CSA*E/ELL
      KM(2,2)=KM(1,1)
      KM(1,2)=-KM(1,1)
      KM(2,1)=KM(1,2)
      RETURN
      END
C
C
      SUBROUTINE GEOM4Y(IP,IQ,NYE,AA,BB,COORD,ICOORD,G,NF,INF)
C
C     THIS SUBROUTINE FORMS THE COORDINATES AND STEERING VECTOR
C     FOR 4-NODE QUADS COUNTING IN Y-DIRECTION
C
      REAL COORD(ICOORD,*)
      INTEGER G(*),NF(INF,*),NUM(4)
      NUM(1)=(IP-1)*(NYE+1)+IQ+1
      NUM(2)=NUM(1)-1
      NUM(3)=IP*(NYE+1)+IQ
      NUM(4)=NUM(3)+1
      INC=0
      DO 1 I=1,4
      DO 1 J=1,4
      INC=INC+1
    1 G(INC)=NF(NUM(I),J)
      COORD(1,1)=AA*(IP-1)
      COORD(2,1)=AA*(IP-1)
      COORD(3,1)=AA*IP
      COORD(4,1)=AA*IP
      COORD(1,2)=-BB*IQ
      COORD(2,2)=-BB*(IQ-1)
      COORD(3,2)=-BB*(IQ-1)
      COORD(4,2)=-BB*IQ
      RETURN
      END
C
      SUBROUTINE PRINTA(A,IA,M,N)
C
C     THIS SUBROUTINE WRITES A 2-D ARRAY TO OUTPUT CHANNEL 11
C
      REAL A(IA,*)
      DO 1 I=1,M
    1 WRITE(11,2)(A(I,J),J=1,N)
    2 FORMAT(1X,100E12.4)
      RETURN
      END
C
      SUBROUTINE MATCOP(A,IA,B,IB,M,N)
```

```
C
C     THIS SUBROUTINE COPIES ARRAY A INTO ARRAY B
C
      REAL A(IA,*),B(IB,*)
      DO 1 I=1,M
      DO 1 J=1,N
    1 B(I,J)=A(I,J)
      RETURN
      END
      SUBROUTINE MATINV(A,IA,N)
C
C     THIS SUBROUTINE FORMS THE INVERSE OF A MATRIX
C     USING GAUSS-JORDAN TRANSFORMATION
C
      REAL A(IA,*)
      DO 1 K=1,N
      CON=A(K,K)
      A(K,K)=1.
      DO 2 J=1,N
    2 A(K,J)=A(K,J)/CON
      DO 1 I=1,N
      IF(I.EQ.K)GOTO 1
      CON=A(I,K)
      A(I,K)=0.
      DO 3 J=1,N
    3 A(I,J)=A(I,J)-A(K,J)*CON
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE MVMULT(M,IM,V,K,L,Y)
C     THIS SUBROUTINE MULTIPLIES A MATRIX BY A VECTOR
C
      REAL M(IM,*),V(*),Y(*)
      DO 1 I=1,K
      X=0.
      DO 2 J=1,L
    2 X=X+M(I,J)*V(J)
      Y(I)=X
    1 CONTINUE
      RETURN
      END
      SUBROUTINE MATMULL(A,IA,B,IB,C,IC,L,M,N)
C
C     THIS SUBROUTINE FORMS THE PRODUCT OF TWO MATRICES
C
      REAL A(IA,*),B(IB,*),C(IC,*)
```

```
      DO 1 I=1,L
      DO 1 J=1,N
      X=0.0
      DO 2 K=1,M
    2 X=X+A(I,K)*B(K,J)
      C(I,J)=X
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE FORMD2(DEE,IDEE,E,T,V)
C
C     THIS SUBROUTINE FORMS THE 2-D STRAIN
C     STRESS/STRAIN MATRIX
C
      REAL DEE(IDEE,*)
      DO 1 I=1,3
      DO 1 J=1,3
    1 DEE(I,J)=0.0
      DEE(1,1)=1.
      DEE(2,2)=1.
      DEE(3,3)=(1.-V)/2
      DEE(1,2)=V
      DEE(2,1)=V
      DO 2 I=1,3
      DO 2 J=1,3
    2 DEE(I,J)=DEE(I,J)*(E*T**3)/(12.*(1.-V**2))
      RETURN
      END
C
C ----------- SUROUTINE FOR LU DECOMPOSITION-----------------------
      SUBROUTINE LUDCMP(A,IKV,N,INDX,D)
      PARAMETER (NMAX=2600,TINY=1.0E-20)
      REAL A(IKV,*),VV(NMAX)
      INTEGER INDX(*)
      D=1.
      DO 12 I=1,N
        AAMAX=0.
        DO 11 J=1,N
          IF (ABS(A(I,J)).GT.AAMAX) AAMAX=ABS(A(I,J))
11      CONTINUE
        IF (AAMAX.EQ.0.)PAUSE 'Singular matrix.'
        VV(I)=1./AAMAX
12    CONTINUE
      DO 19 J=1,N
        IF (J.GT.1) THEN
          DO 14 I=1,J-1
```

```
          SUM=A(I,J)
          IF (I.GT.1)THEN
            DO 13 K=1,I-1
              SUM=SUM-A(I,K)*A(K,J)
13            CONTINUE
            A(I,J)=SUM
          ENDIF
14        CONTINUE
        ENDIF
        AAMAX=0.
        DO 16 I=J,N
          SUM=A(I,J)
          IF (J.GT.1)THEN
            DO 15 K=1,J-1
              SUM=SUM-A(I,K)*A(K,J)
15            CONTINUE
            A(I,J)=SUM
          ENDIF
          DUM=VV(I)*ABS(SUM)
          IF (DUM.GE.AAMAX) THEN
            IMAX=I
            AAMAX=DUM
          ENDIF
16        CONTINUE
        IF (J.NE.IMAX)THEN
          DO 17 K=1,N
            DUM=A(IMAX,K)
            A(IMAX,K)=A(J,K)
            A(J,K)=DUM
17          CONTINUE
          D=-D
          VV(IMAX)=VV(J)
        ENDIF
        INDX(J)=IMAX
        IF(J.NE.N)THEN
          IF(A(J,J).EQ.0.)A(J,J)=TINY
          DUM=1./A(J,J)
          DO 18 I=J+1,N
            A(I,J)=A(I,J)*DUM
18          CONTINUE
        ENDIF
19      CONTINUE
      IF(A(N,N).EQ.0.)A(N,N)=TINY
      RETURN
      END
*************** SUBROUTINE FOR LU BACK
SUBSITITUTION********************
```

```
      SUBROUTINE LUBKSB(A,IKV,N,INDX,B)
      REAL A(IKV,*),B(*)
      INTEGER INDX(*)
      II=0
      DO 22 I=1,N
        LL=INDX(I)
        SUMI=B(LL)
        B(LL)=B(I)
        IF (II.NE.0)THEN
          DO 21 J=II,I-1
            SUMI=SUMI-A(I,J)*B(J)
21          CONTINUE
        ELSE IF (SUMI.NE.0.) THEN
          II=I
        ENDIF
        B(I)=SUMI
22      CONTINUE
      DO 24 I=N,1,-1
        SUMI=B(I)
        IF(I.LT.N)THEN
          DO 23 J=I+1,N
            SUMI=SUMI-A(I,J)*B(J)
23          CONTINUE
        ENDIF
        B(I)=SUMI/A(I,I)
24      CONTINUE
      RETURN
      END
      SUBROUTINE RAFTU(FMAXR,NXER,NYER,AA,BB,QULT)
C
C     COMPUTE SOIL ULTIMATE  REACTION OF RAFT-SOIL INTERFACE
C
      REAL FMAXR(*)
      KK=0
      DO 313 IP=1,NXER+1
      DO 313 IQ=1,NYER+1
      KK=KK+1
      FMAXR(KK)=QULT*AA*BB
      IF(IP.EQ.1.OR.IQ.EQ.1)FMAXR(KK)=QULT*AA*BB/2.
      IF(IP.EQ.NXER+1.OR.IQ.EQ.NYER+1)FMAXR(KK)=QULT*AA*BB/2.
      IF((IP.EQ.1).AND.(IQ.EQ.1))FMAXR(KK)=QULT*AA*BB/4.
      IF(IP.EQ.NXER+1.AND.IQ.EQ.NYER+1)FMAXR(KK)=QULT*AA*BB/4.
      IF((IP.EQ.1).AND.IQ.EQ.NYER+1)FMAXR(KK)=QULT*AA*BB/4.
      IF((IQ.EQ.1).AND.IP.EQ.NXER+1)FMAXR(KK)=QULT*AA*BB/4.
  313 CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE FORMBE(BEE,IBEE,D1X,D1Y,FUN,AA,BB,NODOFR)
C
C     THIS SUBROUTINE FORMS THE STRAIN/DISPLACEMENT MATRIX
C     FOR PLATE BENDING
C
      REAL BEE(IBEE,*),D1X(*),D1Y(*),FUN(*)
      DO 1 I= 1,16
        K =I
C        K= 4*(I-1)+1
         L = K+1-1
         J = L+1-1
         BEE(1,K)=BEE(1,K)+D1X(K)/(4.*AA**2)
         BEE(2,L)=BEE(2,L)+D1Y(L)/(4.*BB**2)
         BEE(3,J)=BEE(3,J)-2.*FUN(J)/AA*BB
    1 CONTINUE
      RETURN
      END
```

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