



# LATTICE GEOMETRY, GAP FORMATION AND SCALE-INVARIANCE

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# Abstract

In this work, we analyze the gap dynamics of forest in connection with lattice geometry. we also analyze the establishment (birth), growth and mortality (death) of individual trees of Barro Colorado Island (BCI) forest using two slightly different forest disturbance models on three lattices, each with square, triangular, and hexagonal geometry in order to study the effect of geometry. Local nature of ecological interaction can generate complex behaviors such as criticality (scale-invariance). we fit the global and local densities and cluster size distribution from the spatial map of the forest (1983-1984). we also found an effect of geometry on the conditions under which criticality appears in model forests. The scaling exponent of the gap-size distributions, however, was found to be independent of model and geometry.

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I am deeply indebted Dr. Tatak Yirgou , my advisor, for his many suggestions and constant support and friendly approach during this research. His tireless follow up and his consistent support will be in my memory forever.

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# Chapter 1

## Introduction

Biophysicists have been involved in modeling the dynamics of forest the area like gap formation, gap closure, clustering of gaps etc. In this work we follow their lead and try to model the gap dynamics of Barro Colorado Island (BCI) forest.

Our system considered is a BCI forest which is about  $5 \times (10)^4 m^2$  area . It consists so many different trees with different height, age , position etc. The space (distance) between trees is different from point to point. There are areas where trees are densely populated (close to each other) and there are points where tree(s) are scattered (far from each other ) that is points where no tree(s). This shows the distribution of trees on the total area is not uniform. Our aim is taking data from BCI forest to check the effect of changing lattice geometry, to achieve the observed global and local density and gap-size distribution and to check weather this forest follows power law or not.

To study the dynamic properties of any system which is either physical or biological, the total system arrangement as well as the individual arrangement in space and time should be known. Individual-oriented models of ecological systems are based on a detailed description of the behavior of individuals and their interactions with each other [1]. All individuals of a model (trees) have different attribute values like age, height, weight or position in an area that change in time and that influence the behavior of individuals. Individuals can be born or die in a system. So, the behavior of the whole system is described by the aggregation of individuals behaviors and their interaction.

According to this work a cell (site) is the smallest part of the system which has its own defined shape and size; and can give the total system if summed up. The cell considered here is assumed to have neighbors of the same defined shape and size. Any change (or influence) on a particular cell will bring change on the neighbors of a cell according to the rule applied. Since the shape of the cell considered determines the number of neighborhood, it is better to consider different shape of the cell. Here the shape of the cell is simply the space utilized by the particular cell considered.

In real world the space utilized can be regular or irregular. Regular means the total system is considered to be of the same shape and size cells, such as circles, squares, triangles, hexagons etc. Irregular is of two kind; one is a cell is shapeless (means out of circular, square, triangular, hexagonal). The second type of irregularity of the total system consisting of different shape of cell.

It is difficult to predict the number of neighbors for the particular cell. But if the cell and space utilized is considered to be regular, it is possible to predict neighbors of the particular cell. So if the number of the neighbors is known, it is possible to predict and study the behavior of the system in connection with local interaction between cells.

Next to choosing the cell and space utilized to be regular, very critical question is which kind of the lattice geometry should be considered? Most applications of ecological modeling have employed the square lattice [2] because cell (lattice geometry) to be square is logical with the shape of any two dimensional space and it is the most convenient for formalizing ecological systems. However little attention have been paid to the effects of lattice geometry (other than square) on these models. But it is better (realistic) to consider other type of geometry like triangular and hexagonal lattices.

As mentioned above the total system is represented by a regular lattice composed of cells, that means the space utilized by organisms (the interface through which they interact) is often represented by a regular lattice composed of cells. Since the space utilized is regular lattice it is possible to use cellular automata model as it is quite natural

to the study of spatiotemporal dynamics in ecology and have been used extensively in recent years [3].

Cellular automata are generally attributed to Ulam (1952) and Von-Neumann (1966), who introduced the concept in the late forties to provide a realistic model for the behavior of complex systems. So cellular automata are considered to be decentralized, spatially extended systems consisting of large numbers of simple identical components with local connectivity [4]. Such systems have the potential to perform complex computation with a high degree of efficiency, as well as to model the behavior of complex systems in nature [4].

Lattice configuration are not limited to square ones, cellular automata based on other cell geometries, such as triangular and hexagonal lattices, are also possible. Thus, there is no ecological reason for using only the square geometry, and the effect of using alternate geometries should be studied [9]. This models are becoming very popular, as they can even be found in resource dynamics and vegetation process [5]. Due to their easiness of implementation, ability to replicate spatial forms, and capacity to be quickly reaped to reproduce several types of dynamic spatial phenomena, such as forest fire spreading [6] land scape dynamics [7].

Gap dynamics has been postulated to play very important role in maintaining tree diversity and local neighbor effects on long-term survival of individual trees [4]. When a tree dies in a closed-canopy forest, a light-gap is opened. Many ecologists working on forest dynamics have focused their attention on 'gaps', openings created in the forest canopy [8]. According to this paper, from the total system a particular cell is a gap if the cell considered has no tree for a particular time step, this cell may or may not be a gap for the next time step. Consider Barro Colorado Island (BCI) if the chosen cell is a gap in 1983 it may or may not be gap in 1984. Non-gap is a space where there is trees of different heights greater than zero.

Gaps which are areas bounded by forests where trees height is much lower than the average, present opportunities for seedling establishment [9] and can be of different sizes depending on the size of falling trees, the trees surrounding them and the direction the tree falls.

It is reasonable to suppose that trees in a forest interact predominantly with only those trees in their local neighborhood. Thus because local interactions are the key to gap formation in forest dynamics changing the geometry of ecological cells and their neighborhoods should be important [9]. There is one consequence of changing lattice geometry; the number of nearest neighbors available for interaction changes [9]. In real forests, species have been shown to have different aggregation characteristics such that some species interact with a few neighbors and others interact with many [10]. The second consequence of changing lattice geometry is the effect on the rate of propagation of local disturbance [9]. This is because the connectivity of the cells in the lattice is affected by changing the geometry.

## 1.1 Direction of falling of trees

Because a tree can fall to one specified direction at a time the falling trees direction should be considered. According to [9] a tree which falls causes the neighbors to be fallen if its height is greater than the neighbors height. This is to say a tree falls to all the direction at a time, which is not realistic. So considering the direction of falling is important because its effect on the nearest neighbors is restricted to one direction. If a tree which falls is considered to be all the four direction the gap which is formed is going to be above the expected gap and this can influence the total gap formation. Even though we choose the direction of falling to be four for simplicity, it is known that the tree can fall to many direction greater than this numbers. Again this consideration is important to make the system capable with cellular automata model.

## 1.2 Self-organization

Cellular automata modeling of biological system has been used extensively to study the phenomena of self-organization and criticality [11]. Self organization is a process in which pattern formation occurs through interactions internal to a system with out any external influence, and can be observed in many biological and physical systems [12]. If there are not expenditure of external factor (for example lightning, wind-throw, deforestation by human being etc.) forest expansion and closure is a natural phenomena. So a tree in a forest follows nature rule that is, it is born first, then grow to its maximum height, die at the end (biologically). Since the system for the whole forest is assumed to be the same with this tree, it is possible to consider it is in self-organization state. Many models of forest dynamics incorporate a striking level of ecological,biological,and physical details [13]. Some of this models focus on inter-specific interactions and environmental effects (e.g habitat heterogeneity,and light gradients). Some studies suggest that, although these factors are important among seedling and saplings (plantlet), they are not important to gap dynamics at the canopy level.

## 1.3 Criticality

Self-organizing systems are governed by local interactions, but are characterized by long range correlation (and global pattern) not predictable from these local interactions. The critical states are characterized by scale invariance (power-law) [14]. The relevance of self-organized criticality concept to any real system is not universally accepted, but this discussion does emphasize the striking nature of power law [15]. It has also prompted many researchers to try to account for such distributions in terms of (semi) realistic models, and that is one part subject of this model. Self-organization and criticality have been discovered in phenomena such as traffic flow [16], forest fire spreading [6], and gap formation and closure [8]. Although these models represent over simplifications of real

systems, it has been postulated that critical states could be widely present in nature. Again although a critical point is usually a singular state of physical systems, where the system can be held by tuning some external field, it is conjectured that it might be the natural state for a biological system. This property would also make models plausible [11]. Systems that display self-organized criticality may be the most robust and well adapted to external perturbations, and the real evolution under biological constraints would lead unflinchingly to such states.

In gap formation, since local interactions are the key, this could be again the key to criticality. So not only changing the geometry of ecological cells and their neighborhoods that affect the gap formation and criticality but also the direction of falling tree is very important. The connectivity of cells in the lattice is affected by changing the geometry, this causes the nearest neighborhood of the cell to be affected in such a way that the total effect on the whole system is clearly observed. For example cells in the hexagonal lattice geometry have a higher connectivity than those in the triangular lattice geometry [9]. If this is considered as the main case, the direction of falling is connected with its neighborhood (in number) and will bring a little change on the systems dynamics.

Therefore it is important to study this model on different lattice geometry as well as the direction of falling. This may reflect the behavior of different species and the scale-dependent effects of disturbance [11]. In this paper we use alternate geometry (triangular and hexagonal). This was done by [9], and we add the direction of falling of the tree which was not taken into account before. We also propose a modification to FG2 (Forest Game2- model on Lattice Geometry, modification of FG (forest game) which was done by Sole and Manrubia) which gives almost good result compared to the Barro Colorado Island forest data and FG2 result.

# Chapter 2

## Methods

### 2.1 Model structure

We present a simple model of rain-forest dynamics where space is structured as a lattice of  $L \times L$  cells. According to [19] conventional cellular automata model consists of

1. an Euclidean space divided into an array of identical cells;
2. a cell neighborhood of a defined size and shape;
3. a set of discrete cell states;
4. a set of transition rules, which determine the state of a cell as a function of the states of cells in a neighborhood;
5. discrete time steps with all cell states updated simultaneously.

All the above five points are considered in this work. In updating the parameters of a cell, it is necessary to consider the cell's own parameter and the values of the cells in its corresponding neighborhood.

### 2.2 Model data

In modeling any system we need to have data which are realistic and which can fit the observed system. In our system the data used are from real BCI forest which were calculated from the map by researchers, but the data which are used are different from author to author because it is model dependent. The total area of BCI forest is about

$5 \times (10)^4 m^2$ , and this number is changed to suit the cellular automata model. A single cell has a dimension of 5 by 5 meter square, to cover the total land we have 100 by 200 cells.

parameter	square	triangle	hexagon
<b>FG2</b>			
$p_b$	0.56	0.6	0.58
$p_d$	0.06	0.04	0.08
$\gamma$	1.0	1.0	1.0
<b>our model</b>			
$p_b$	0.6	0.69	0.61
$p_d$	0.16	0.12	0.17
$\gamma$	1.0	1.0	1.0

Table 2.1: Parameters used in FG2 and our model to simulate the time evolution of global density ( $\rho_0$ ), local density ( $q_0$ ) and gap size distribution.

parameter	square	triangle	hexagon
<b>FG2</b>			
$p_b$	0.67	0.6	0.71
$p_d$	0.2	0.2	0.2
$\gamma$	1.0	1.0	1.0
<b>our model</b>			
$p_b$	0.7	0.69	0.7
$p_d$	0.22	0.24	0.22
$\gamma$	1.0	1.0	1.0

Table 2.2: Parameters used in FG2 and our model to simulate the time evolution of global density ( $\rho_0$ ), local density ( $q_0$ ) and gap size distribution at criticality.



A tree is born and die every time in the forest but we are not sure which tree is born and die corresponding to its position, so we need to make the system probabilistic and have the probability of birth and death independently. Even though the total system behavior is known exactly the individual cell of the system can not be determined exactly, but we can take random distribution of the individual cell with its age, height and position.

In a real forest the trees growth is dependent on its neighbors because they share nutrients from soil and sun light. A tree which takes more nutrients and sun light will grow be faster than its neighbors. The competition between the trees should be parameterized either in variable form or constant. In our case the competition between them is assumed to be constant in time and space. Its reason was already introduced by some authors. They propose that the BCI forest has the same species of trees, and if the system is homogeneous the competition between trees will be assumed to be constant.

In this model we use a cellular automata with periodic boundary conditions. Each cell might be in a state represented by a real number between zero and maximum value ( $h_{max}$ ) that intends to mimic the height of the tree occupying that site. Each tree is allowed to be born, if the cell (site) is empty, to grow and to fall creating gaps. The empty cell (site) may become non-empty according to the rule applied.

## 2.3 Calculation of densities and gap-size distribution

### Techniques for determining the number of neighbors

#### Von Neumann neighborhood technique

It is a technique which considers all nearest neighbors which are connected adjacent to a cell.

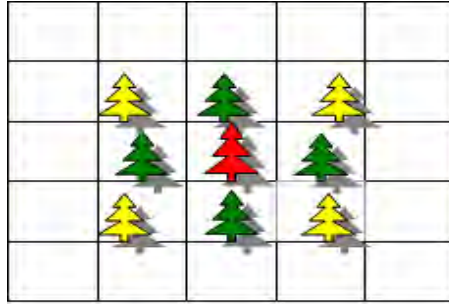


Figure 2.1: A cell (red) with Square lattice geometry and its neighbors. Cells which contain green tree are Von Neumann neighbors. Cells which contain yellow and green tree are Moore neighbors. Total number of Von Neumann neighbors are 4. Total number of Moore neighbors are 8.

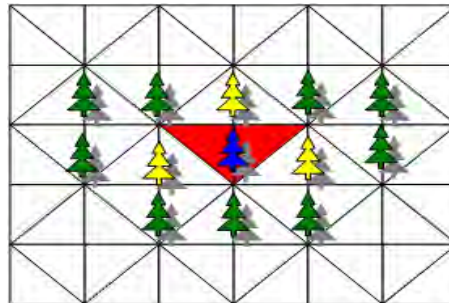


Figure 2.2: A cell (red) with Square lattice geometry and its neighbors. Cells which contain yellow tree are Von Neumann neighbors. Cells which contain yellow and green tree are Moore neighbors. Total number of Von Neumann neighbors are 3. Total number of Moore neighbors are 12.

### Moore Neighborhood technique

It is a technique which considers all nearest neighbors which are connected adjacent to the cell and other neighbors which are connected through the corners (diagonally) with the same cell.

Gap is an area or site that has empty cell, that is the height of the tree on that site is zero meter (or there is no tree on the site). These gaps will proceed us to global and local density which are calculated from global and local gap respectively. From these it is possible to calculate gap size distribution.

#### 2.3.1 Global density( $\rho_0$ )



Figure 2.3: A cell (red) with hexagonal lattice geometry and its neighbors. Cells which contain green tree are both Von Neumann and Moore neighbors. Total number of Von Neumann neighbors are 6. Total number of Moore neighbors are 6.

Global density is defined as total fraction of gaps in the forest.

$$\rho_0 = t_g/t_a \quad (2.3.1)$$

Where  $t_g$  is total number of gaps and  $t_a$  is total area of forest.

### 2.3.2 Local density( $q_0$ )

Local density is defined as the average fraction of gaps among the nearest neighbors of gaps. For each empty cell (cell with no tree) we count the number of empty cells in the Von Neumann neighborhood and divide by the total number of Von Neumann neighbors [9].

$$q_0 = g_{vn}/t_{nv} \quad (2.3.2)$$

Where  $g_{vn}$  is gaps in the Von Neumann neighborhood for gap cell and  $t_{nv}$  is total number of Von Neumann neighborhood.

The local density( $q_0$ ) is taken to be the average over all gaps. Both the results (global and local) are taken as averages over 1000 time steps, after discarding transients

### 2.3.3 Gap size distribution

To calculate the gap size distribution, total gap is considered first, then nearest neighbor gap are clustered together. Two gaps are considered to belong to the same cluster if they are side by side. Points which are connected through the corners belong to different cluster. Other techniques of neighborhood counting do not affect the size distribution. The sum of the total cluster gives total gap. In real forest the gap size distribution displays a power law behavior.

### 2.3.4 Power law

Power law are the mathematical description of self-similarity because they are scale-free laws. Statistical properties of a great quantity of physical systems close to critical points are well described by means of scaling laws. Their relevance in ecology and evolution has been recognized [17]. As can be seen, there is always a range where a power law fits the data quite well. Obviously there is no complete evidence of self-organized criticality (SOC), neither in the simulation nor in real rain-forests. question drives more work. On the other hand our model is still too complex to try any analytical proof. However the results obtained up to now, both in modeling and in analyzing field data, are assumption that rain-forests have evolved towards a critical state characterized by spatial self-similarity [11] and complex fluctuations of self-similar type. This law states that the function  $p(k)$  is directly proportional to  $k$  to the power of negative constant number( $\beta$ ).

$$p(k) \propto k^{-\beta} \tag{2.3.3}$$

Where  $k$  is the gap size of the gap with Von Neumann neighborhood, and  $p(k)$  is the probability that a cell is a member of a gap with size  $k$  [9].

Taking the logarithm of probability and cluster-size of the gap will give the relation between the cluster size distribution of gaps and and number of gaps of the forest. The

scaling exponent ( $\beta$ ) is slope of the line which is determined by fitting the distribution curves ( from the log – log curve) for large gap sizes (gaps larger than neighborhood gap). This constant gives the fractal constant that the two dimensional assumed forest can have. We computed the power law behavior for the three geometries.

We follow the rules given by FG2 and FG which govern birth, growth, death, and gap formation in a forest. The FG2 and our model differs only in their rule in gap formation, that is we add the effect of direction of falling. We use periodic boundary conditions on maps of 40,000 cells for all the three lattice geometries with Moore neighborhood for interaction. Each cell (i,j) in the automaton may contain an individual tree of height  $h(i,j;t)$  at time t. This height is modeled to have height between zero meter and maximum height ( $h_{max}$ ). Trees taller than  $h = 0$  are considered to be non-gaps, and trees which have  $h = 0$  are considered to be gaps.

## 2.4 Algorithm of the model

Model dynamics begins with an initial condition where trees of minimum height ( $h_0$ ) are randomly distributed over half the space. Next to this each tree is located on a cell of  $L \times L$  lattice and defined by a state (the height),  $h(i,j;t)$ , where t is the time step, (i,j) indicates the spatial position of trees.

### Rules of the models

Rules of the model follows the flow chart in figure 2.4.

#### (i)-Birth

An empty lattice cell can be externally colonized with probability of birth  $p_b$ , and this empty cell at time t will have a tree of  $h_0$  at the next time step (t+1). In this case a new tree with size  $h_0$  will grow there. if  $h(i,j;t) = 0.0$  at time t , then  $h(i,j;t) = h_0$  at the next

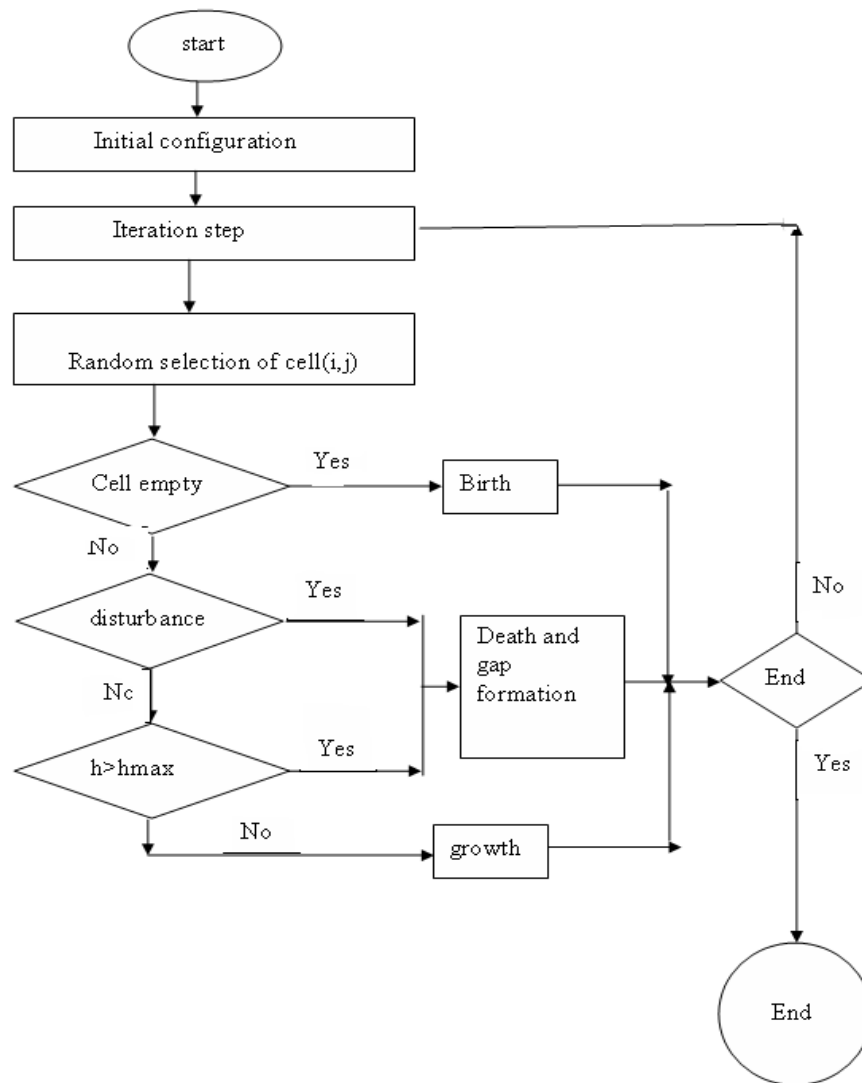


Figure 2.4: Schematic representation of the process taking place in each updating step in the model. Initial configuration is random distribution of trees on the lattice. Distarbunce means the random death of trees.

time step with probability  $p_b$ , where  $h_0$  is the minimum height of tree which is taken to be 0.1m.

### (ii)-Growth

If  $h(i,j;t)$  is in between 0.0 m and ( $h_{max}$ ), the height of the tree at the next time step is:

$$h(i, j; t) = h(i, j; t) + \Theta(1 - \gamma/n \sum_{k,l} (h(k, l; t))) \quad (2.4.1)$$

Where the sum is taken over the n neighboring cells. n is the parameter that depends on the lattice geometry. It is shown in figure 2.1, 2.2 and 2.3. 8 for square, 12 for triangle and 6 for hexagon.

In the real forest a tree can affect its neighbors growth by sharing nutrients, water, and light ingredients. Since it is revealed as an important parameter in the dynamics of the model, we need to model competition between trees( $\gamma$ ). If  $\gamma = 0$  , implies no interaction takes place (trees grow independently), and if  $\gamma = \infty$  , growth becomes impossible. So  $\gamma$  should lie between  $0$  and  $\infty$ , according to Sole and Manrubia and other authors  $\gamma$  is used to be unity ( $\gamma = 1.0$ ). They assumed that, since BCI is homogenous (same type of species) competition between them is the same.  $\Theta(x)$  is zero for  $x < 0.0$  and  $x$  for  $x > 0.0$ .

The value  $h_{max}$  specifies the maximum height that the tree can reach, which is taken to be 30 meters. As can be seen, if the mean height of the neighbors is larger than the height of the tree, then no growth is allowed.

### (iii)-Death

There are two sources of tree mortality, random and mortality and gap propagation. Random mortality is independant of height, gap propagation depends on height. If a tree reaches maximum height, it is removed from the system and die, opening a primary gap.

If the height of the tree is greater than the maximum height ( $h_{max}$ ) at time t, then the height of the tree at the next time step (t+1) is 0.0 m; i.e; if

$$h(i, j; t) > h_{max} \quad (2.4.2)$$

then  $h(i,j;t) = 0.0$  or independent of height, a tree falls down with probability  $p_d$ . Independent of height means a tree may fall by external factor other than achieving  $h_{max}$ .

**(iv)- Gap formation**

When a tree dies, it creates a primary canopy gap. Every tree death is the starting point for local disturbance. Trees at the age of canopy gap (trees less than the average height) have an increased probability of dying. When a tree with height  $h(i,j;t)$  falls down, certain number of trees in its Moore Neighborhood will fall according to the rule:

- a) for FG2 any tree at location  $(k,l)$  in the Moore Neighborhood will fall if

$$h(i, j; t) > h(k, l; t) \quad (2.4.3)$$

is satisfied.

- (b) any tree at location  $(k, l)$  in the Moore Neighborhood will fall if

$$h(i, j; t) > h(k, l; t) \quad (2.4.4)$$

and if the direction of the falling tree which is taken into account is satisfied.

The gap formation rule (iv) reflects the idea that a tree that falls may kill smaller trees surrounding it by simply crushing them. The condition in FG2 requires a falling tree causes any shorter tree in its Moore neighborhood to fall; all trees taller than the tree will not be affected.

To highlight the difference between FG2 and Our model, consider a tree with all shorter neighbors. If this tree falls, local gap formation occurs both in FG2 and our model, the difference lies on the number of gaps formed at a time. Rule (iv) of FG2 allows all the neighboring trees to fall at a time, but in reality a tree can not fall on all the neighbors at a time. To omit this problem, assuming the direction in which the tree falls to be one at time is better approach. So the tree may kill neighbors in the direction it falls.



Since it takes much time to attain  $h_{max}$  for a tree, death and gap formation is less. Probability of death plays a great role in the gap formation. According to [9] choosing larger  $p_d$  will bring large gap formation above the real gap of the forest. But here in this work we choose larger  $p_d$  because the conditions in rule (iv) are added. Even though the probability of a tree to die is taken to be large, the died tree can not make gap simply because it falls. It should fulfil the condition added on equation (2.4.4).

The lattice is updated in a asynchronous fashion. At each time step  $L \times L$  lattice sites are randomly chosen. Depending on the state of the automata, one of the previous rule is applied. The updating of the lattice was chosen in such a way that a time step is approximately equivalent to a year of evolution in a real rain forest [11].

The set of parameters used by FG2 in their simulation are the same with this work. Again we implement in the same way that ensuring each cell undergoes a single state transition at each time step by tagging the cell when its state changes. Otherwise, it would be possible for a new tree to germinate in an empty cell due to probability of birth (rule iv) then immediately die due to probability of death  $p_d$  (rule iii) with in the same time step [9]. Implementing the code such that inhibiting the gap formation rule iv during a maturation phase of the forest (it was chosen 10 time step). This reflects the fact that gap formation can not occur in forests dominated by seedlings (i.e trees of height  $h_0$ ).

# Chapter 3

## Results and discussion

### 3.1 Global and local Densities

The simulations are taken over for the given time steps (250 and 1000) for the area  $L \times L$  by using the best fit parameters  $(p_b, p_d, \gamma)$  for the BCI forest. To validate the simulations, the simulated results were compared with the observed BCI forest global and local densities and critical exponents.

density	square	triangle	hexagon
<b>FG2</b>			
$\rho_0$	$0.332 \pm 0.06$	$0.334 \pm 0.06$	$0.333 \pm 0.06$
$q_0$	$0.515 \pm 0.08$	$0.569 \pm 0.08$	$0.491 \pm 0.08$
<b>our model</b>			
$\rho_0$	$0.332 \pm 0.02$	$0.333 \pm 0.02$	$0.331 \pm 0.02$
$q_0$	$0.565 \pm 0.04$	$0.569 \pm 0.04$	$0.491 \pm 0.04$

Table 3.1: Summary of the results for the global ( $\rho_0$ ) and local ( $q_0$ ) densities of gaps on the three geometries for FG2 and our model using the parameters in table 2.1.

The results obtained by simulating the system using the parameters given in table 2.1 gives good result to the real BCI forest and the FG2. The time evolution of the global density  $\rho_0$  is computed by using the given parameters where the probability of birth  $p_b$  and death  $p_d$  were chosen specifically to reproduce the observed global density of gaps ( $\rho_0$

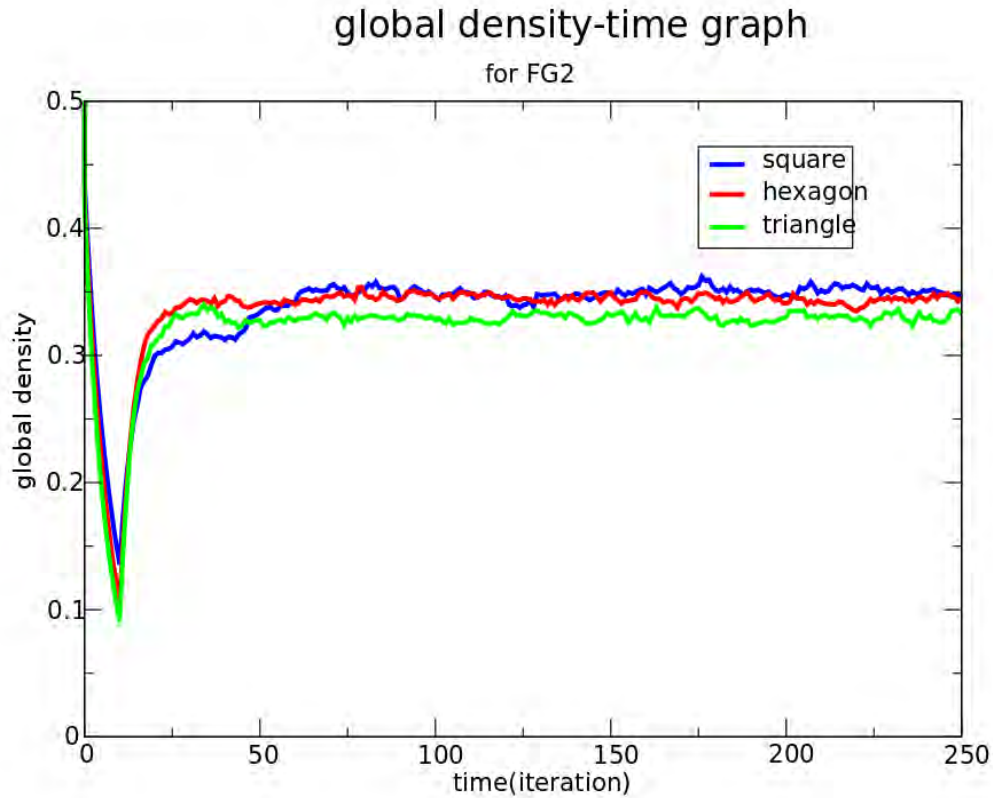


Figure 3.1: The time evolution of the global density ( $\rho_0$ ) for the three lattice geometries using the parameters in table 2.1 for FG2.

= 0.331) in BCI forest.

Figure 4 and Figure 5 displays the time evolution of the global density  $\rho_0$  for FG2 and our model respectively. Since we need to know the change in the gaps of forest we start our simulation with trees of minimum height ( $h_0$ ) randomly distributed on half of the lattice. This means at time  $t=0$ , the global density of the given system is  $\rho_0=0.5$ . The global density  $\rho_0$  decreases to a minimum, then increases and saturates for times typically greater than 50 iterations. The saturation marks the onset of equilibrium. The transient behavior of global density  $\rho_0$  is governed solely by the values of probability of birth  $p_b$  and death  $p_d$ , because no appreciable heights are established for local interaction to take place (rule ii and iv). Since the parameters can vary to suit the results, varying probability

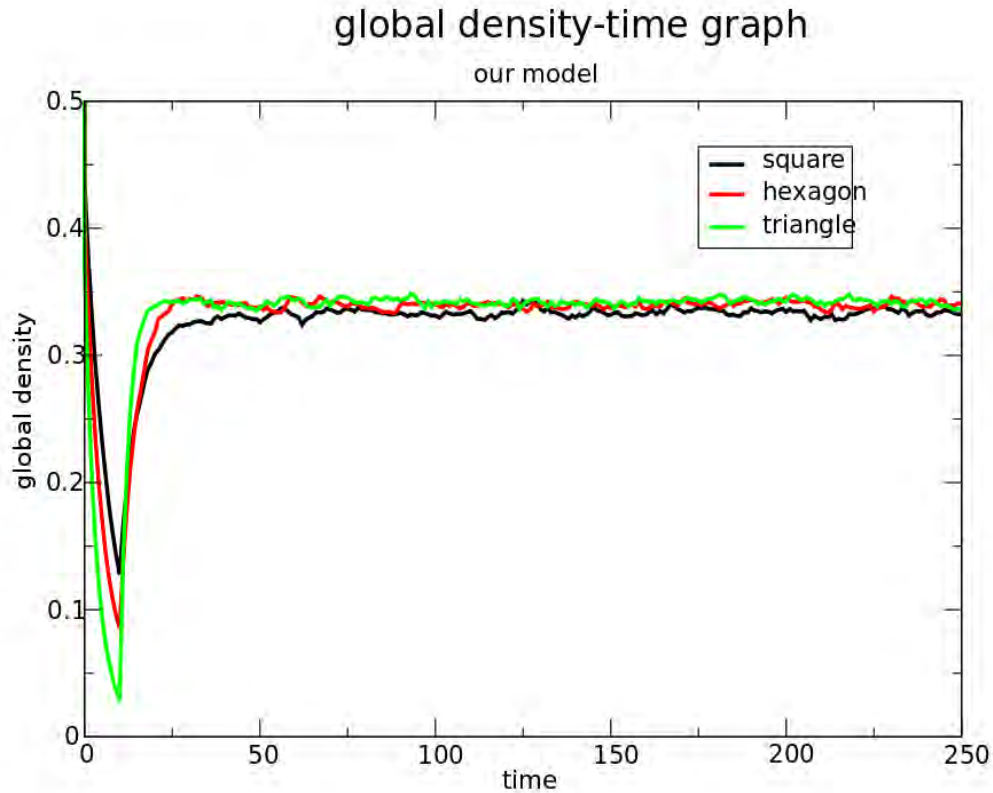


Figure 3.2: The time evolution of the global density ( $\rho_0$ ) for the three lattice geometries using the parameters in table 2.1 for our model.

of birth and death is important, and these values determine the results obtained in the simulation.

The effect of geometry appears for times greater than the minimum time corresponding to the minimum global density. This is because local interactions become relevant only after the forest starts to mature and gap formation becomes a governing process.

The time it takes to reach equilibrium is mostly controlled by the non-random rule for death that means the time it takes for the tree to reach maximum height and becomes greater ( $h(i, j; t) > h_{max}$ ). Because this rule takes the longest to be satisfied, trees in a lattice with a smaller values of total number of Moore neighborhood have less competition with neighbors, and can therefore grow much faster. Since many of these trees can reach

$h_{max}$  more quickly, equilibrium is established more quickly for lattice geometries with smaller number of Moore neighborhood ( $n$ ). Thus the time it takes to reach equilibrium decreases with the Moore neighborhood number ( $n$ ).

Using the parameters given in table 1, we attempt to fit the global and local densities of the BCI data (figure 3.1 and figure 3.2). We obtain an average percentage deviation from the observed local density of BCI forest ( $q_0=0.58$ ) of  $15\%$  for our model and  $22\%$  for FG2. As such our model presents considerable result for all the geometries. For both FG2 and our model, the best fit is given by the triangular lattice. However by adequately choosing probability of birth and death (table 2.2), both models can be made to establish criticality at equilibrium. Interestingly, at criticality the local density is better fit using the square lattice and very well fit using the hexagonal lattice.

## 3.2 Gap-size distributions

When observing a BCI, we are faced with a highly nonlinear dissipative system. The dynamical evolution of such a system is mainly led by the continuous clearings. They leave places for new trees to be grown, and the size of the open canopy is a key element in the selection of which tree becomes established there. This is why a study of the distribution should prove useful in order to understand the underlying dynamical laws. Hence we computed the gap-size distribution from our simulation data.

The gap-size distributions produced by our simulations using the parameters in table 1 to fit the global and local densities are displayed in figure 3.3 for FG2 and our model. The fractal patterns that have been found in BCI may be a finger print of a system in a critical state. The image is quantified by a power law. Fluctuations in the variables of the system change in such a way that they present a wide spectrum of sizes, from very small to the maximum allowed size, and they follow a power law. The appearance of universality in systems close to a critical point is another essential property.

density	square	triangle	hexagon
<b>FG2</b>			
$\rho_0$	$0.561 \pm 0.05$	$0.613 \pm 0.06$	$0.542 \pm 0.04$
$q_0$	$0.632 \pm 0.02$	$0.737 \pm 0.03$	$0.624 \pm 0.03$
<b>our model</b>			
$\rho_0$	$0.541 \pm 0.05$	$0.623 \pm 0.02$	$0.512 \pm 0.02$
$q_0$	$0.642 \pm 0.02$	$0.717 \pm 0.03$	$0.614 \pm 0.02$

Table 3.2: Summary of the results for the global ( $\rho_0$ ) and local ( $q_0$ ) densities of gaps on the three geometries for FG2 and our model using the parameters in table 2.2 at criticality.

Figure 3.4 shows the gap-size distributions at criticality produced using the parameters given in table 2.2. The parameters were chosen to fit the power-law behavior observed in the BCI forest. The slopes of the line in figure 3.4 give the scaling exponent  $p(k)\alpha(k)^{-\beta}$  which is determined by fitting the distribution curves for the larger gap sizes (gaps greater than the Moore neighborhood gap). This way we omit the short range correlation that characterizes criticality. The values of the critical exponent are independent of both model and geometry, and are in very good agreement with the value of  $\beta=1.67$  extracted from the BCI data. Again our value is also in good agreement with FG2 result (with a little deviation),  $\beta=1.66$  at gap-size distribution (from figure 3.3) and  $\beta = 1.71$  at criticality (from figure 3.4).

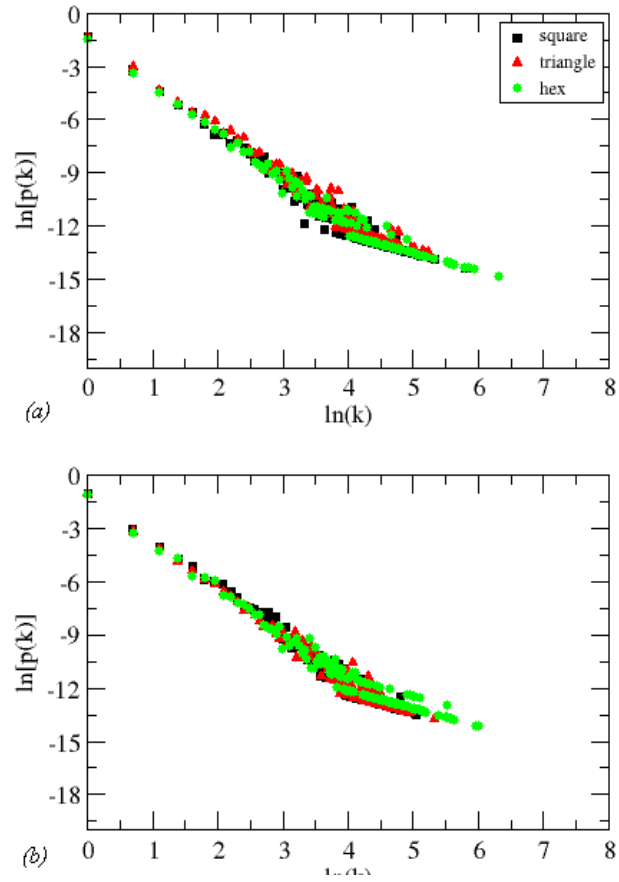


Figure 3.3: Gap-size distribution for the three lattice geometries. In (a) the distributions are shown for FG2 using the parameters in table 2.1. In (b) the distribution are shown for our model using the parameters in table 2.1.

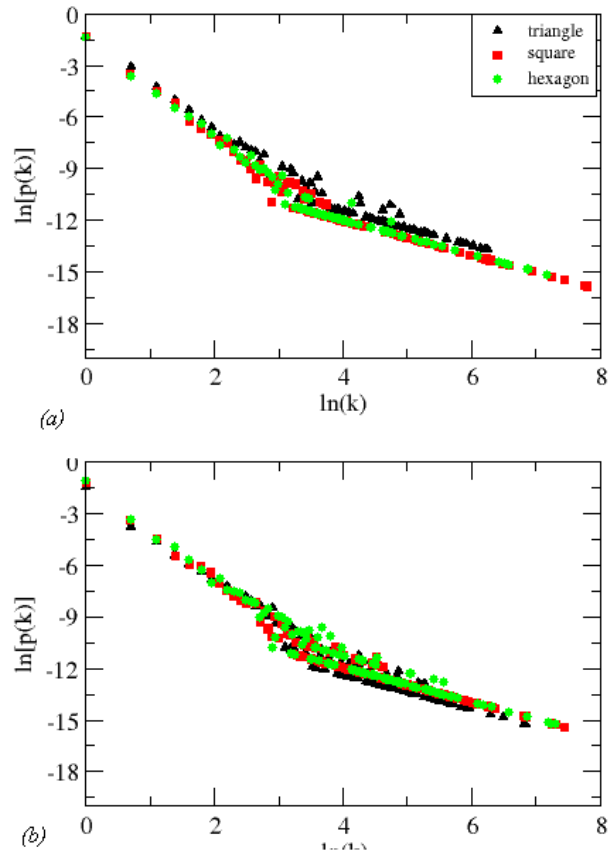


Figure 3.4: Gap-size distribution for the three lattice geometries at criticality. In (a) the distributions are shown for FG2 using the parameters in table 2.2. In (b) the distribution are shown for our model using the parameters in table 2.2. The spread in the data at large sizes is a finite size effect



### 3.3 Discussion

We sought to examine the effect of lattice geometry and direction of falling on gap formation and criticality using simple cellular automata model of forest dynamics and found that while the critical exponent of the gap-size distribution does not depend on the lattice geometry, geometry effects nonetheless exist. In general criticality is realized for each geometry using different set of parameters (birth probability  $p_b$ , death probability  $p_d$  and interaction  $\gamma$ ). In general when equilibrium is reached both FG2 and our model are less dependent on geometry and give better fit to the data because of the production of larger gaps via clustering. Evidence that FG2 and our model is less dependent on geometry is seen, (especially for larger gap sizes and far from criticality).

The question remains as to which parameter set ( $p_b, p_d, \gamma$ ) should be used in the models. When our objective is to fit one aspect of the data, namely the global and local densities, we can find one set of parameters for each geometry that works fairly well. Using the above parameters, however, the critical scaling behavior is not reproduced at all scales. The parameters set which give the best fit to the scaling law are not the same as those which give the best fit to the global and local densities. While the critical scaling exponent does not depend on geometry, the parameter sets that yield criticality are different for each geometry. The most noticeable difference is higher death rate is needed for criticality. While these may appear to be unrealistically high, it is interesting to find that they are in fact biologically realistic for many tree species in the BCI forest.

The fact that we need two different sets of parameters (which are given in table 1 and 2) for the same geometry to fit the densities on one hand and the critical exponent behavior on the other is partly because FG2 and our model do not exhibit self-organizing criticality (can not answer two questions by single parameter). This may also suggest that a mixture of geometries may be needed and or the model of gap formation is still incomplete. Another reason for these discrepancies could be the environmental heterogeneity

due to topography or soil conditions in BCI [9].

When FG2 and our model are at criticality, the scaling exponent converges to 1.7 for all geometries when map sizes are large enough to eliminate statistical fluctuations. Thus we find universality in the scaling exponent as expected for criticality [6]. Our results may be generalizable to other systems which are members of the same universality class. It is interesting to note that ecologists have found a scaling exponent for the height distribution to be 2.0 when they combined bio-mechanical and bioenergetic models of tree growth [20]. Thus, the critical state may be a stable ecological state. It is equally interesting that, at criticality, universality occurs in physical thermodynamic models [21], which depend on a few fundamental parameters analogous to birth rate, death and interaction strength.

We make a few comments about the relationship of the results found here with other models of local interactions in ecology. This is because one of the main effects of our study is to show the importance of local neighborhood size on global properties of the forest. Many recent studies in ecology have highlighted this as an important area of research under the term 'zone of influence' which is defined as the area over which an individual plant utilizes resources or otherwise alters its environment [18]. Our work can help to shed light on this 'zone of influence' by detecting the scale of local interaction as characteristic neighborhood gap size. They concluded that the Moore neighborhood should be used and this is what we have used for the interaction scale.

# Chapter 4

## Conclusion

In this work we studied the effect of lattice geometry in two cellular automata models of forest dynamics. We found an effect of geometry on the specific parameter values required to get scale invariance, and also on the appearance of bumps in the gap-size distribution. However, the critical exponent was robust to changes in geometry. This may be of particular importance to the determination of rates and scales of ecological interaction to guide conservation and restoration efforts. Ecologists may wish to re-examine the results of previously proposed cellular automata models of ecological phenomena on the square lattice, and consider the effects of lattice geometry. Finally, our results may be of general importance for any application of cellular automata models where interactions are local and produce positive feedback within neighborhoods.

In this model we have presented a model ecosystem which is able to imitate rain forest dynamics. This work has proved its usefulness as tool to test some ecological hypothesis. We have shown three examples of good agreement between our model result and present knowledge of real rain forests these are:-

- (1)-global density of gap
- (2)-local density of gap
- (3)-gap-size distribution.

The simulation were performed by using special data that were obtained mainly from the BCI map in 1983 and 1984. in view of this fact, it can be said that the achieved results are encouraging.

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### **Declaration**

This project is my original work, has not been presented for a degree in any other University and that all the sources of material used for the project have been fully acknowledged.

Name: Seid Mohammed

Signature:

**Place and time of submission: Addis Ababa University, June 2010**

This project has been submitted for examination with my approval as University advisor.

Name: Dr. Tatak Yirgou

Signature: