INVERSE BETA-DECAY OF POLARIZED NEUTRON IN CURVED SPACETIME IN THE PRESENCE OF STRONG MAGNETIC FIELD

By

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This is to certify that the thesis prepared by **Tahir Tibesso** , entitled " **Inverse Beta-Decay of Polarized Neutron in Curved Spacetime in the Presence of Strong Magnetic Field**" and submitted in partial fulfillment of the requirements for the degree of **Master of Science**. compiles with the regulations of the University and meets the accepted standards with respect to originality.

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Abstract

We calculate the cross section of the inverse beta decay process,$\nu_e + n \rightarrow p + e^-$ in curved spacetime in the presence of strong magnetic field and.Using exact solutions of the Dirac equation in a curved space-time and Spinors of the Dirac Equation and its solution In Robertson-Walker Spacetime, we find the cross section for arbitrary polarization of the initial neutrons.The magnetic field might provide a net polarization of the neutrons, which we take into account.

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Chapter 1

Introduction

Our universe consists of elementary particles (electron, proton, neutron, positron, neutrino, and photon, etc.) and massive bodies (stars, galaxies, and so on). The components of interstellar medium usually consisting of molecular hydrogen and some dust are the springboardsfor the formation of stars. The interactions of elementary particles show novel features when they occur in non-trivial backgrounds. Study of particle propagation in matter has proved pivotal in the understanding of the solar neutrino problem. Similar studies of particle processes in curved space-time in the background magnetic fields are also important since stellar objects like neutron stars are expected to possess very high magnetic fields. Analysis of these processes might be crucial for obtaining a proper understanding of the properties of these stars. It is by now widely recognized that strong magnetic fields can be a significant factor relevant to diverse astrophysical and cosmological environments. The presence of strong magnetic fields in proto-neutron stars and pulsars is well established. The surface magnetic fields of many radio-pulsars, that can be estimated by the observed synchrotron radiation, are of the order of $B \sim 10^{12} - 10^{14} G$. There are also the so- called magnetars [1,2] whose surface magnetic elds are two or three orders of magnitude higher. Very strong magnetic fields are also supposed to exist in the early Universe [3]. Such fields can influence the primordial nucleosynthesis [4-6] and affect the rate of ^4He production.

Under the influence of strong magnetic fields the direct URCA processes like

$$n \to p + e + \bar{\nu_e} \tag{1.0.1}$$

$$\nu_e + n \rightleftharpoons e + p \tag{1.0.2}$$

$$p + \bar{\nu_e} \rightleftharpoons n + e^+ \tag{1.0.3}$$

can be modified. the first of these reaction is by for the slowest and it does not contribute much to the equilibrium. (proceeding in the forward direction, this reaction is neutron decay, with a half-life of about 10min, which in much longer than the typical time scale or hubble age, of the early universe, proceeding in the backward direction, this reaction requires a simultaneous collision among three particles which is much less likely than a collision between two particles). These reactions play important roles in the neutron star evolution so that the presence of strong magnetic fields significantly change the star cooling rate [7-11]. It is worth mentioning here a recent study of neutrino processes (2) and (3) in strong magnetic fields of the order 1016 G and implication for supernova dynamics [12]. The direct URCA processes have gained a lot of attention because of the asymmetry in the neutrino emission, which can arise in the presence of strong magnetic fields. Various authors have argued that asymmetric neutrino emission during the first few seconds after the massive star collapse could provide explanations for the observed pulsar velocities. Different mechanisms for the asymmetry in the neutrino emission from a pulsar has been studied previously [13-21]. For more complete references on the neutrino mechanisms of the pulsar kicks, see the review papers [22,23].

It is worth mentioning here that the angular dependence of the neutrino emission in URCA processes was first considered for the neutron beta-decay neutrinos in [26,27]. In these papers the probability of the polarized neutron beta-decay in the presence of a magnetic field was derived, as well as the asymmetry in the neutrino emission was studied for the first time. In the two well-known papers, [28,29], the results of [26,27] for the neutron decay rate in a magnetic field were re-derived. However, there was no discussion on the asymmetry in neutrino emission in refs [28,29].

The neutron beta-decay have been studied in different electromagnetic field configurations. we have considered the probability of the polarized neutron beta-decay in the superposition of a magnetic field and a field of an electromagnetic wave (the so- called Redmond field configuration) and have confirmed the results of [26,27] for the decay probability in the magnetic field and also got the probability in the presence of an electromagnetic wave field. The relativistic theory of the beta-decay of the neutron (accounting for the proton recoil motion) in the strong magnetic field has been developed. Many important technical details of the calculations, also useful for the studies performed. The rates of the two inverse processes in eqs (3) and (4) in the presence of a magnetic field have been derived in [16]. The present paper is devoted to a detailed study of the inverse beta-decay of polarized of neutron in curved spacetime in strong magnetic fields

$$\nu_e + n \rightleftharpoons e + p \tag{1.0.4}$$

The process $\nu_n \to pe$ in a magnetic field has been discussed previously by several authors. The contribution of this process to the conditions for beta-equilibrium in the presence of magnetic fields has been considered in [27]. The dependence of the cross-section on the magnetic field has also been discussed [19] in the context of the pulsar kick in the case when the asymmetric magnetic field arises just after the star collapse.

1.1 properties of the Dirac Equation

in the Dirac equation the 4×4 matrices γ^μ, $\mu = 0, ..., 3$, satisfying the anticommutation relations

$$[\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu} \tag{1.1.1}$$

and the Hermiticity condition

$$\gamma^{\mu+} = \gamma^0\gamma^\mu\gamma^0 \tag{1.1.2}$$

the properties are consequences of above eqn. is only and do not depend on choosing a particular representation for the γ-matrices

A fifth anticommuting γ-matrix is defined by

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{1.1.3}$$

and γ^5 has the properties

$$[\gamma^\mu, \gamma^5]_+ = 0, (\gamma^5)^2 = 1, \gamma^{5+} = \gamma^5 \tag{1.1.4}$$

Note that Greek indices will always stand for the value $0 - --, 3$, only, and not for 5

the 4×4 spin matrices

$$\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu] \tag{1.1.5}$$

1.2 Trace,Energy and Spin projection Operators

We next list some rules and relations which are extremely useful in evaluating the trace of a product of γ-matrices.

a) For any Two $n \times n$ matrices U and V

$$Tr(UV) = Tr(Vu) \tag{1.2.1}$$

b) if$(\gamma^\alpha\gamma^\beta....\gamma^\mu\gamma^\nu)$ contains an odd and even number of γ matrices, then

odd number γ matrices

$$Tr(\gamma^\alpha\gamma^\beta....\gamma^\mu\gamma^\nu) = 0 \tag{1.2.2}$$

even number γ matrices

$$Tr(\gamma^\alpha\gamma^\beta) = 4g^{\alpha\beta}, Tr\sigma^{\alpha\beta} = 0 \tag{1.2.3}$$

$$Tr(\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\sigma) = 4(g^{\alpha\beta}g^{\gamma\sigma} - g^{\alpha\gamma}g^{\beta\sigma} + g^{\alpha\sigma}g^{\beta\sigma}) \tag{1.2.4}$$

c) The above results can be extended to products involving the γ^5 matrix, the most important relation being

$$Tr\gamma^5 = Tr(\gamma^5\gamma^\alpha) = Tr(\gamma^5\gamma^\alpha\gamma^\beta) = Tr(\gamma^5\gamma^\alpha\gamma^\beta\gamma^\gamma) = 0$$

and

$$Tr(\gamma^5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\sigma) = -4i\epsilon^{\alpha\beta\gamma\sigma} \tag{1.2.5}$$

The Energy-projection operator are define by

$$\lambda^{\pm}(p) = \frac{\pm\not{p} + m}{2m} \tag{1.2.6}$$

they have the operators, which follow the Dirac Equation of projection out the positive/negative energy solutions.

The Spin projection operator, we choose the plane wave solutions U and V of the Dirac Equation in curved spacetime. as eigenstates of the 4×4 spin matrix

$$\sigma_p = \frac{\sigma.P}{|P|} \tag{1.2.7}$$

corresponding, we now define the operator

$$\Pi^{\pm}(p) = \frac{1}{2}(1 \pm \sigma_p) \tag{1.2.8}$$

the Natural constant $\hbar = c = 1$ except c.g.s

A reasonable interest in the inverse beta-decay of polarized neutron in curved spacetime in strong magnetic fields has been stimulated by a belief that it can be relevant for the neutrino opacity in the proto-neutron star stage after supernova collapse. The first detailed evaluation of the magnetic field effect on the neutrino opacity can be found in [22]. In [22], as well as in [23], the calculations for the cross-section have been performed under the assumption that the magnetic field gives contribution to the phase space integrals only, whereas the process matrix elements have been considered unaffected by the magnetic field.

In this paper, we consider the problem in full detail i.e., we calculate the matrix element using spinor solutions of the electron in a magnetic field, take all possible final Landau levels into account, include the possiblity of neutron polarization, and perform the calculations to all orders in the background field in the 4-fermi interaction theory. The paper is organized as follows. In chapter 2 we provide Solution of Dirac Equation in Curved space-time in the presence of strong magnetic fields.Most of this chapter contains no original material, but we provide it for the sake of completeness, as well as for setting up the notation that will be used in the later sections. In chapter 3, Inverse Beta-Decay of polarized Nuetron presence of strong magnetic field contains we calculate spin sum is curved spacetime in the presence of strong magnetic field, we define the fermion field operator and show how it acts on the states in curved spacetime in the presence of strong magnetic field, the S-matrices of inverse beta-decay and contains the calculation of the scattering cross section for a monochromatic neutrino beam, which contains the main results of our paper. In the last chapter. chapter 4, conclusions and summary.

Chapter 2

Solution of Dirac Equation in Curved space-time in the presence of strong magnetic field

The study of the structure of the Cosmic Microwave Radiation leads us to conclude that the ratio of the total density to the critical density of the universe Ω_0 is likely to be close to one, favoring a spatially flat Robertson-Walker metric over other topologies. It is well known that general relativity is a local metrical theory and therefore the corresponding Einstein field equations do not fix the global topology of spacetime and consequently the universe may have compact spatial sections with a nontrivial topology [4,5], then the observational data does not rule out the possibility that our universe possesses a hyperbolic topology.The study of cosmological models with nonstandard topologies is not new and goes back to the works by Zelmanov [9,10], showing that upon different coordinate transformations, spatially closed or flat sections can be transformed into hyperbolic sections and vice versa. The line element associated with an spatially open Friedman universe has the form

$$ds^2 = a(\eta)^2[d\eta^2 - dr^2 - sinh^2(d\theta^2 + sin^2\theta)d\Phi^2] \qquad (2.0.1)$$

making the coordinate transformation the metric in eqn.(2.1) becomes

$$ds^2 = a^2(\eta)[d\eta^2 - dz^2 - e^{-2z}(dx^2 + dy^2)] \qquad (2.0.2)$$

In order to solve quantum processes in curved spacetime one has to fulfill a preliminary step which constists in having a description of the single-mode solution of the relativistic particles or perturbation in those background field, i.e., exact solution of the relativistic scalar and spinor wave equations. we have different methodes of solving relativistic wave equation in curved space; among them, the methode of separation of varirables is one most widely used.

2.1 Solution of the Dirac equation

The Dirac equation is a system of coupled partial differential equations which is separable in a very restricted set of metrics. Among the spacetimes where the separability of the Klein-Gordon and Dirac equations has been studied one can mention the Stäckel spaces [28], which are those metrics where the Hamilton-Jacobi equation is separable. Nevertheless recently it has been shown that this condition is neither necessary nor sufficient in order to guarantee a complete separability of variables in the Dirac equation. A systematic classification of the gravitational backgrounds where the Dirac equation is separable with the help of the algebraic method is presented in [27]. The line element (2.1.1) belongs to this family and consequently one can apply the algebraic method of separation.
The covariant generalization of the Dirac equation in curved space-time is

$$\bar{\gamma}^{\mu}(\partial_{\mu} - \Gamma_{\mu} - ieA_{\mu})\bar{\Psi} + M\bar{\Psi} = 0 \tag{2.1.1}$$

where the curved Dirac matrices $\bar{\gamma}^{\mu}$ satisfy the anticommutation relation

$$\{\bar{\gamma}^{\mu}, \bar{\gamma}^{\nu}\} = 2g^{\mu\nu} \tag{2.1.2}$$

and the spinor connection Γ_{μ} are

$$\Gamma_{\mu} = \frac{1}{4} g_{\lambda\alpha}[(\frac{\partial b^{\beta}_{\nu}}{\partial x^{\mu}})a^{\alpha}_{\beta} - \Gamma^{\alpha}_{\nu\mu}]S^{\lambda\nu} \tag{2.1.3}$$

where

$$S^{\lambda\nu} = \frac{1}{2}(\bar{\gamma}^{\lambda}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{\lambda}) \tag{2.1.4}$$

and $\Gamma^{\alpha}_{\nu\mu}$ is the affine connection can be computed from the usual

$$\Gamma^{\alpha}_{\nu\mu} = \frac{1}{2} g^{\delta\alpha}\{\frac{\partial g_{\mu\delta}}{\partial x^{\nu}} + \frac{\partial g_{\nu\delta}}{\partial x^{\mu}} + \frac{\partial g_{\mu\nu}}{\partial x^{\delta}}\} \tag{2.1.5}$$

the matrices b^{β}_{ν}, a^{α}_{β} establish the connection between the Dirac matrices $\bar{\gamma}^{\mu}$ on a curved space-time and the flat Dirac matrices γ^{μ} as follows:

$$\bar{\gamma}_{\mu} = b^{\beta}_{\nu}\gamma_{\alpha}, \bar{\gamma}^{\mu} = a^{\mu}_{\beta}\gamma^{\beta}. \tag{2.1.6}$$

from $g_{\mu\nu} = e^{a}_{\mu}(x)e^{b}_{\nu}(x)\eta_{ab}$ it easy to see that we can also write

$$\eta_{ab} = e^{\mu}_{a}e^{\nu}_{b}g_{\mu\nu} \tag{2.1.7}$$

since the line element in eqn.(2.1.2) is associated with a diagonal metric, we can workin the tetrad gauge for $\bar{\gamma}^{\mu}$

$$\bar{\gamma}^{0} = \frac{\gamma^{0}}{a(\eta)}, \bar{\gamma}^{1} = \frac{\gamma^{1}}{a(\eta)e^{-z}},$$
$$\bar{\gamma}^{2} = \frac{\gamma^{2}}{a(\eta)e^{-z}}, \bar{\gamma}^{3} = \frac{\gamma^{3}}{a(\eta)} \tag{2.1.8}$$

substituting eqn.(2.2.4) into eqn.(2.2.3) and solve for the spinor connection we have

$$\Gamma_{\mu} = \frac{1}{4}g_{\lambda\alpha}[(\frac{\partial b^{\beta}_{\nu}}{\partial x^{\mu}})a^{\alpha}_{\beta} - \Gamma^{\alpha}_{\nu\mu}]\frac{1}{2}(\bar{\gamma}^{\lambda}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{\lambda}) \tag{2.1.9}$$
$$\Gamma_{1} = \frac{1}{4}g_{\lambda\alpha}[(\frac{\partial b^{\beta}_{\nu}}{\partial x^{1}})a^{\alpha}_{\beta} - \Gamma^{\alpha}_{\nu1}]\frac{1}{2}(\bar{\gamma}^{\lambda}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{\lambda})$$

$$\Gamma_{1} = \frac{1}{4}g_{00}[(\frac{\partial b^{\beta}_{\nu}}{\partial x^{1}})a^{0}_{\beta} - \Gamma^{0}_{\nu1}]\frac{1}{2}(\bar{\gamma}^{0}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{0}) + \frac{1}{4}g_{11}[(\frac{\partial b^{\beta}_{\nu}}{\partial x^{1}})a^{1}_{\beta} - \Gamma^{1}_{\nu1}]\frac{1}{2}(\bar{\gamma}^{1}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{1})$$
$$+\frac{1}{4}g_{22}(\frac{\partial b^{\beta}_{\nu}}{\partial x^{1}})a^{2}_{\beta} - \Gamma^{2}_{\nu1}]\frac{1}{2}(\bar{\gamma}^{2}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{2}) + \frac{1}{4}g_{33}[(\frac{\partial b^{\beta}_{\nu}}{\partial x^{1}})a^{3}_{\beta} - \Gamma^{3}_{\nu1}]\frac{1}{2}(\bar{\gamma}^{3}\bar{\gamma}^{\nu} - \bar{\gamma}^{\nu}\bar{\gamma}^{3})$$
$$\Longrightarrow \Gamma_{1} = \frac{1}{4}g_{00}\{(\frac{\partial b^{0}_{0}}{\partial x^{1}})a^{0}_{0} - \Gamma^{0}_{01})\frac{1}{2}(\bar{\gamma}^{0}\bar{\gamma}^{0} - \bar{\gamma}^{0}\bar{\gamma}^{0}) + (\frac{\partial b^{1}_{1}}{\partial x^{1}})a^{0}_{1} - \Gamma^{0}_{11})\frac{1}{2}(\bar{\gamma}^{0}\bar{\gamma}^{1} - \bar{\gamma}^{1}\bar{\gamma}^{0})$$
$$+(\frac{\partial b^{2}_{2}}{\partial x^{1}})a^{0}_{2} - \Gamma^{0}_{21})\frac{1}{2}(\bar{\gamma}^{0}\bar{\gamma}^{2} - \bar{\gamma}^{2}\bar{\gamma}^{0}) + (\frac{\partial b^{3}_{3}}{\partial x^{1}})a^{0}_{3} - \Gamma^{0}_{31})\frac{1}{2}(\bar{\gamma}^{0}\bar{\gamma}^{3} - \bar{\gamma}^{3}\bar{\gamma}^{0})\}$$
$$+\frac{1}{4}g_{11}\{(\frac{\partial b^{0}_{0}}{\partial x^{1}})a^{0}_{1} - \Gamma^{1}_{01}]\frac{1}{2}(\bar{\gamma}^{1}\bar{\gamma}^{0} - \bar{\gamma}^{0}\bar{\gamma}^{1}) + (\frac{\partial b^{1}_{1}}{\partial x^{1}})a^{1}_{1} - \Gamma^{1}_{11})\frac{1}{2}(\bar{\gamma}^{1}\bar{\gamma}^{1} - \bar{\gamma}^{1}\bar{\gamma}^{1})$$
$$+(\frac{\partial b^{2}_{2}}{\partial x^{1}})a^{1}_{2} - \Gamma^{1}_{21})\frac{1}{2}(\bar{\gamma}^{1}\bar{\gamma}^{2} - \bar{\gamma}^{2}\bar{\gamma}^{1}) + (\frac{\partial b^{3}_{3}}{\partial x^{1}})a^{1}_{3} - \Gamma^{1}_{31})\frac{1}{2}(\bar{\gamma}^{1}\bar{\gamma}^{3} - \bar{\gamma}^{3}\bar{\gamma}^{1})\}$$
$$+\frac{1}{4}g_{22}\{(\frac{\partial b^{0}_{0}}{\partial x^{1}})a^{2}_{0} - \Gamma^{2}_{01}]\frac{1}{2}(\bar{\gamma}^{2}\bar{\gamma}^{0} - \bar{\gamma}^{0}\bar{\gamma}^{2}) + (\frac{\partial b^{1}_{1}}{\partial x^{1}})a^{2}_{1} - \Gamma^{2}_{11})\frac{1}{2}(\bar{\gamma}^{2}\bar{\gamma}^{1} - \bar{\gamma}^{1}\bar{\gamma}^{2})$$
$$+(\frac{\partial b^{2}_{2}}{\partial x^{1}})a^{2}_{2} - \Gamma^{2}_{21})\frac{1}{2}(\bar{\gamma}^{2}\bar{\gamma}^{2} - \bar{\gamma}^{2}\bar{\gamma}^{2}) + (\frac{\partial b^{3}_{3}}{\partial x^{1}})a^{2}_{3} - \Gamma^{2}_{31})\frac{1}{2}(\bar{\gamma}^{2}\bar{\gamma}^{3} - \bar{\gamma}^{3}\bar{\gamma}^{2})\}$$
$$+\frac{1}{4}g_{33}\{(\frac{\partial b^{0}_{0}}{\partial x^{1}})a^{3}_{0} - \Gamma^{3}_{01})\frac{1}{2}(\bar{\gamma}^{3}\bar{\gamma}^{0} - \bar{\gamma}^{0}\bar{\gamma}^{3}) + (\frac{\partial b^{1}_{1}}{\partial x^{1}})a^{3}_{1} - \Gamma^{3}_{11})\frac{1}{2}(\bar{\gamma}^{3}\bar{\gamma}^{1} - \bar{\gamma}^{1}\bar{\gamma}^{3})$$

$$+(\frac{\partial b_2^2}{\partial x^1})a_2^3-\Gamma_{21}^3)\frac{1}{2}(\bar{\gamma}^3\bar{\gamma}^2-\bar{\gamma}^2\bar{\gamma}^3)+(\frac{\partial b_3^3}{\partial x^1})a_3^3-\Gamma_{31}^3)\frac{1}{2}(\bar{\gamma}^3\bar{\gamma}^3-\bar{\gamma}^3\bar{\gamma}^3)\}$$

$$\Longrightarrow \Gamma_1=\frac{1}{4}g_{00}\{-\frac{1}{2}\Gamma_{11}^0(\bar{\gamma}^0\bar{\gamma}^1-\bar{\gamma}^1\bar{\gamma}^0)-\frac{1}{2}\Gamma_{21}^0(\bar{\gamma}^0\bar{\gamma}^2-\bar{\gamma}^2\bar{\gamma}^0)-\frac{1}{2}\Gamma_{31}^0(\bar{\gamma}^0\bar{\gamma}^3-\bar{\gamma}^3\bar{\gamma}^0)\}$$

$$+\frac{1}{4}g_{11}\{-\frac{1}{2}\Gamma_{01}^1(\bar{\gamma}^1\bar{\gamma}^0-\bar{\gamma}^0\bar{\gamma}^1)-\frac{1}{2}\Gamma_{21}^1(\bar{\gamma}^1\bar{\gamma}^2-\bar{\gamma}^2\bar{\gamma}^1)-\frac{1}{2}\Gamma_{31}^1(\bar{\gamma}^1\bar{\gamma}^3-\bar{\gamma}^3\bar{\gamma}^1)\}$$

$$+\frac{1}{4}g_{22}\{-\frac{1}{2}\Gamma_{01}^2(\bar{\gamma}^2\bar{\gamma}^0-\bar{\gamma}^0\bar{\gamma}^2)-\frac{1}{2}\Gamma_{11}^2(\bar{\gamma}^2\bar{\gamma}^1-\bar{\gamma}^1\bar{\gamma}^2)-\frac{1}{2}\Gamma_{31}^2(\bar{\gamma}^2\bar{\gamma}^3-\bar{\gamma}^3\bar{\gamma}^2)\}$$

$$+\frac{1}{4}g_{33}\{-\frac{1}{2}\Gamma_{01}^3(\bar{\gamma}^3\bar{\gamma}^0-\bar{\gamma}^0\bar{\gamma}^3)-\frac{1}{2}\Gamma_{11}^3(\bar{\gamma}^3\bar{\gamma}^1-\bar{\gamma}^1\bar{\gamma}^3)-\frac{1}{2}\Gamma_{21}^3(\bar{\gamma}^3\bar{\gamma}^2-\bar{\gamma}^2\bar{\gamma}^3)\}$$

but we have to the affine connections from formula

$$\Gamma_{\nu\mu}^\alpha=\frac{1}{2}g^{\delta\alpha}\{\frac{\partial g_{\mu\delta}}{\partial x^\nu}+\frac{\partial g_{\nu\delta}}{\partial x^\mu}+\frac{\partial g_{\mu\nu}}{\partial x^\delta}\}$$

$$\Gamma_{11}^0=\frac{1}{2}g^{\delta 0}\{\frac{\partial g_{1\delta}}{\partial x^1}+\frac{\partial g_{1\delta}}{\partial x^1}+\frac{\partial g_{11}}{\partial x^\delta}\}$$

$$=\frac{e^{-2z}}{a(\eta)}\frac{da(\eta)}{d\eta}$$

similary, $\Gamma_{01}^1=\frac{1}{a(\eta)}\frac{d(\eta)}{d\eta}$, $\Gamma_{11}^3=e^{-2z}$, $\Gamma_{31}^1=-1$ and $\Gamma_{21}^0=0$ by substituting all value of christofel symbol into the spinor connection we get,

$$\Gamma_1=-\frac{1}{8}g_{00}\{\frac{e^{2z}}{a(\eta)}\frac{da(\eta)}{d\eta}(\bar{\gamma}^0\bar{\gamma}^1-\bar{\gamma}^1\bar{\gamma}^0)\}-\frac{1}{8}g_{11}\{\frac{1}{a(\eta)}\frac{da(\eta)}{d\eta}(\bar{\gamma}^1\bar{\gamma}^0-\bar{\gamma}^0\bar{\gamma}^1)\}+\frac{1}{2}(\bar{\gamma}^1\bar{\gamma}^3-\bar{\gamma}^3\bar{\gamma}^1)$$

$$-\frac{1}{8}g_{33}e^{-2z}((\bar{\gamma}^3\bar{\gamma}^1-\bar{\gamma}^1\bar{\gamma}^3)$$

$$=-\frac{1}{8}g_{00}\frac{e^{-2z}}{a(\eta)}\frac{da(\eta)}{d\eta}(2\bar{\gamma}^0\bar{\gamma}^1+\frac{1}{8}g_{11}\{\frac{1}{a(\eta)}\frac{da(\eta)}{d\eta}2\bar{\gamma}^0\bar{\gamma}^1)\}+\frac{1}{2}2(\bar{\gamma}^1\bar{\gamma}^3)+\frac{1}{8}g_{33}e^{-2z}2(\bar{\gamma}^1\bar{\gamma}^3)$$

by substituting the value of g_{00} , g_{11}, g_{22} and g_{33} from the line element diagonal matrix we get:

$$\Gamma_1=-\frac{1}{2}\frac{e^{-z}}{a(\eta)}\{a(\eta)\gamma^1\gamma^3+\frac{da(\eta)}{\eta}\gamma^0\gamma^1\} \tag{2.1.10}$$

In the same way, we find for Γ_2, Γ_3 and Γ_0 their values are given by:

$$\Gamma_2=-\frac{1}{2}\frac{e^{-z}}{a(\eta)}\{a(\eta)\gamma^2\gamma^3+\frac{da(\eta)}{\eta}\gamma^2\gamma^0\}, \tag{2.1.11}$$

$$\Gamma_3=-\frac{1}{2}\frac{da(\eta)}{\eta}\frac{1}{a(\eta)}\gamma^3\gamma^0 \tag{2.1.12}$$

$$\Gamma_0=0 \tag{2.1.13}$$

substituting eqn(2.2.10)-eqn.(2.2.13) into eqn.(2.2.1)

$$\Longrightarrow [\bar{\gamma}^0(\frac{\partial}{\partial\eta}-0)+\bar{\gamma}^1(\frac{\partial}{\partial x}-\{\frac{1}{2}\frac{e-z}{a(\eta)}a(\eta)\gamma^1\gamma^3-\frac{1}{2}\frac{e-z}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^1\gamma^0\}-ieA_1(y))$$

$$+\bar{\gamma}^2(\frac{\partial}{\partial y}-\{\frac{1}{2}\frac{e-z}{a(\eta)}a(\eta)\gamma^2\gamma^3-\frac{1}{2}\frac{e-z}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^2\gamma^0\})+\bar{\gamma}^3(\frac{\partial}{\partial z}-\{\frac{1}{2}\frac{da(\eta)}{d\eta}\frac{1}{a(\eta)}\gamma^3\gamma^0]\bar{\Psi}+M\bar{\Psi})=0$$

$$\Rightarrow[\gamma^0(\frac{\partial}{\partial\eta})+\gamma^1e^z(\frac{\partial}{\partial x}-\{\frac{1}{2}e^{-z}\gamma^1\gamma^3-\frac{1}{2}\frac{e^{-z}}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^1\gamma^0\}-ieA_1(y)+\gamma^2e^z(\frac{\partial}{\partial y}\{\frac{1}{2}e^{-z}\gamma^2\gamma^3-$$
$$\frac{1}{2}\frac{e^{-z}}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^2\gamma^0\})+\gamma^0(\frac{\partial}{\partial z}+\frac{1}{2}\frac{1}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^2\gamma^0)]\bar{\Psi}+Ma(\eta)\bar{\psi}=0$$

we find that the Dirac equation takes the simple form

$$\{\gamma^0\frac{\partial}{\partial\eta}+\gamma^1e^z(\frac{\partial}{\partial x}-eA_1(y))+\gamma^2e^z\frac{\partial}{\partial y}+\gamma^3\frac{\partial}{\partial z}+Ma(\eta)\}\mathit{\Psi}=0 \qquad (2.1.14)$$

where we have introduced the spinor $\bar{\Psi}$

$$\bar{\Psi}=a(\eta)^{\frac{-3}{2}}e^z\mathit{\Psi} \qquad (2.1.15)$$

the factor $a(\eta)^{\frac{-3}{2}}$ was introduced in order to cancel the contribution due to the spinor connections. Regarding Eq. (2.2.15) we should mention that it does exhibits a nonfactorizable structure. we apply the algebraic method of separation of variables The method consists in rewriting the Dirac equation (2.2.15) as a sum of two first order differential operators $\hat{K}_1$,$\hat{K}_2$ satisfying the relation

$$[\hat{K}_1,\hat{K}_2]=0,\{\hat{K}_1,\hat{K}_2\}\Phi=0, \qquad (2.1.16)$$

where the spinor Φ is related to Ψ via the expression $\gamma^3\gamma^0\Psi=\Phi$ and k is a separation constant. the operators $\hat{K}_1$ and $\hat{K}_2$ read

$$\hat{K}_1(x,y)\Phi=\{\gamma^2\frac{\partial}{\partial y}+\gamma^1(\frac{\partial}{\partial x}-eA_1(y))\}\gamma^3\gamma^0\Phi=ik\Phi \qquad (2.1.17)$$

$$\hat{K}_2(z,\eta)\Phi=e^z\{\gamma^0\frac{\partial}{\partial y}+\gamma^3\frac{\partial}{\partial\eta}+Ma(\eta)\}\gamma^3\gamma^0\Phi=-ik\Phi \qquad (2.1.18)$$

It should be noticed that using the pairwise scheme of separation one has been able to reduce the problem of solving the Dirac equation to finding solutions of the decoupled system of Eqs. (2.2.17) and (2.2.18). A further problem arises when we try to separate variables in Eq. (2.2.18). Here it is not possible to reduce the problem to a set of two commuting first order differential operators. In order to separate variables in Eq. (2.2.18) we re-write it in the following form:

$$(\hat{L}_1\gamma^3\gamma^0+\hat{L}_2)\Phi=0 \qquad (2.1.19)$$

where $\hat{L_1}$,$\hat{L_2}$ are two commuting differential operators given by the expressions

$$\hat{L_1} = \gamma^0 \frac{\partial}{\partial \eta} + Ma(\eta), \tag{2.1.20}$$

$$\hat{L_2} = \gamma^0 \frac{\partial}{\partial \eta} + ike^z. \tag{2.1.21}$$

In order to separate variables in Eq. (2.2.19) we introduce the auxiliary spinor Y

$$(\hat{L_1}\gamma^3\gamma^0 + \tilde{L_2})Y = \Phi, \tag{2.1.22}$$

where the differential operator $\tilde{L_2}$ is given by the expression

$$\tilde{L_2} = \gamma^0 \frac{\partial}{\partial z} - ike^z \tag{2.1.23}$$

substituting eqn.(2.2.22) into eqn.(2.2.19) we obtain that Y satisfies the following equation

$$\{\hat{M_1}, \hat{M_2}\} = 0 \tag{2.1.24}$$

with $[\hat{M_1}, \hat{M_2}] = 0$, and

$$(\hat{M_1} + \tilde{\lambda})Y = (-\frac{\partial^2}{\partial z^2} - i\gamma^0 ke^{2z} + \tilde{\lambda})Y = 0 \tag{2.1.25}$$

$$(\hat{M_2} - \tilde{\lambda})Y = (-\frac{\partial^2}{\partial \eta^2} - i\gamma^0 M\frac{da(\eta)}{d\eta} + M^2a^2(\eta) - \tilde{\lambda})Y = 0 \tag{2.1.26}$$

where $\tilde{\lambda}$ is separation constant. Introducing the new variable $u = 2ke^z$, we have that Eq.(2.2.25) can be written as

$$(\frac{\partial^2}{\partial u^2} + \frac{i}{2u}\gamma^0 - \frac{1}{4} + (\frac{1}{4} - \lambda)\frac{1}{u^2})S = 0 \tag{2.1.27}$$

where

$$u^{\frac{-1}{2}}S = y \tag{2.1.28}$$

Choosing the following representation of the Dirac matrices,

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix} \tag{2.1.29}$$

we readily obtain that the spinor Φ has the following structure

$$[\sigma_1\frac{\partial}{\partial y} - i\sigma_2(k_x - A_1(y))]\Phi_1 = ik\Phi_2, \tag{2.1.30}$$

$$[-\sigma_1\frac{\partial}{\partial y}+i\sigma_2(k_x-A_1(y))]\Phi_2=ik\Phi_1, \tag{2.1.31}$$

$$\Phi=\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix}=\Phi=\begin{pmatrix}\phi(y)\\ F\sigma^3\phi(y)\end{pmatrix}exp^{(ik_x x+k_y y+k_z z)} \tag{2.1.32}$$

where

$$\phi(y)=\begin{pmatrix}A(y)\\ B(y)\end{pmatrix} \tag{2.1.33}$$

Using the representation (2.2.29) we obtain that the solution of Eq. (2.2.27) can be written in terms of Whittaker functions[27]

$$S_{1,2}=D_1W_{-1/2,\sqrt{\lambda}(u)}+D_2M_{-1/2,\sqrt{\lambda}(u)},S_{3,4}=D_3W_{1/2,\sqrt{\lambda}(u)}+D_4M_{1/2,\sqrt{\lambda}(u)} \tag{2.1.34}$$

where D_1 , D_2 , D_3 , D_4 do not depend on the variable u. Looking at (2.2.26) and (2.2.27) we have that, for regular solutions at $u=0$, the spinor Y has the following structure:

$$Y=\begin{pmatrix}a(y)c_1(\eta)u^{-1/2}M_{+\frac{1}{2},\sqrt{\lambda}(u)}\\ b(y)c_1(\eta)u^{-1/2}M_{+\frac{1}{2},\sqrt{\lambda}(u)}\\ c(y)c_2(\eta)u^{-1/2}M_{-\frac{1}{2},\sqrt{\lambda}(u)}\\ d(y)c_2(\eta)u^{-1/2}M_{-\frac{1}{2},\sqrt{\lambda}(u)}\end{pmatrix}exp^{i(k_x x+k_y y+k_z z)} \tag{2.1.35}$$

Substituting (2.2.35) into eqn.(2.2.22) and noticing that Eq. (2.2.26) is equivalent to the following system of equations

$$(\frac{\partial}{\partial\eta}-iMa(\eta)c_1(\eta))=\sqrt{\tilde{\lambda}}c_2(\eta) \tag{2.1.36}$$

$$(\frac{\partial}{\partial\eta}+iMa(\eta)c_2(\eta))=\sqrt{\tilde{\lambda}}c_1(\eta) \tag{2.1.37}$$

we obtain that the spinor Φ has the following structure

$$\Phi=\begin{pmatrix}A(\nu)c_1(\eta)u^{-z/2}M_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)}\\ B(\nu)c_1(\eta)u^{-z/2}M_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)}\\ iA(\nu)c_2(\eta)u^{-z/2}M_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)}\\ -iB(\nu)c_2(\eta)u^{-z/2}M_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)}\end{pmatrix}exp^{ik_x x+k_y y+k_z z} \tag{2.1.38}$$

where $A(\nu)$ and $B(\nu)$ satisfy the system coupled system of equations

$$(\frac{d}{dy}-(k_x-A_1(y)))B=ikA \tag{2.1.39}$$

$$(\frac{d}{dy} + (k_x - A_1(y)))A = ikB \tag{2.1.40}$$

where

$$\nu = \frac{A_1 y - k_x}{\sqrt{A_1}} \tag{2.1.41}$$

The corresponding solution of Eq. (2.2.16) in terms of the Whittaker functions $W_{k,\mu}(z)$ has the form

$$\Phi = \begin{pmatrix} i\sqrt{\tilde{\lambda}}A(\nu)c_1(\eta)u^{-z/2}W_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ -i\sqrt{\tilde{\lambda}}B(\nu)c_1(\eta)u^{-z/2}W_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ A(\nu)c_2(\eta)u^{-z/2}W_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ B(\nu)c_2(\eta)u^{-z/2}W_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \end{pmatrix} exp^{i(k_x x + k_y y + k_z z)} \tag{2.1.42}$$

Let us look for solutions of the system (2.2.39) and (2.2.40) when the electromagnetic potential has the simple functional dependence $A_1(y) = A_1 y$. In this case one can obtain exact solutions for $A(\nu)$ and $B(\nu)$ in terms of hypergeometric functions. After making the change of variable (2.2.41) and using the recurrence relations

$$(b-1)M(a, b-1, z) = (b-1)M(a, b, z) + z\frac{dM(a, b, z)}{dz}, \tag{2.1.43}$$

$$\frac{1}{a}\frac{dM(a, b, z)}{dz} + M(a, b, z) = M(a+1, b, z) \tag{2.1.44}$$

$$\frac{dU(a, b, z)}{dz} - U(a, b, z) = -U(a, b+1, z) \tag{2.1.45}$$

we find that the general solution of the system of equations (2.2.39) and (2.2.40) reads

$$A = \sqrt{\frac{2A_1}{ik}}e^{-1/2}\nu^2(c_1 M(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{1}{2}, \nu^2)) + c_2 U(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{1}{2}, \nu^2)) \tag{2.1.46}$$

$$B = e^{-\frac{1}{2}\nu^2}\nu(c_1 M(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{3}{2}, \nu^2)) + c_2 U(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{3}{2}, \nu^2)) \tag{2.1.47}$$

2.2 Spinors of the Dirac Equation and it's solution In Robertson-Walker Spacetime

cosider the following spatially flat isotropically changing RW metrics is given by

$$ds^2 = dt^2 - a^2(t)dx^i dx_j \tag{2.2.1}$$

where a is the scale factor of the expanding Universe. we suppose that the cosmological scale factor has an arbitrary time dependence that asymptotically approaches constant values at early and late values of the cosmic time to this cosmic time t is the proper time of a set of clocks on a geometric worldlines that remains at constant values of spatial coordinates (x,y,z) we take

$$a(t) \sim \begin{cases} a_1, & t \to -\infty \\ a_2, & t \to +\infty \end{cases} \tag{2.2.2}$$

interms of the conformal time parameter given by

$$\eta = \int_t \frac{dt}{a(t)} \tag{2.2.3}$$

we assume that a(t) is sufficiently smooth and approaches constant values sufficiently fast that the statements we make below are well define. we will suppose that a(t) approaches the constant value a_1 and a_2 sufficiently rapidly that the asymptotic forms we write below are actually approached(we can, in fact suppose that $a(t) = a_1$ and $a(t) = a_2$ for arbitrarily long initial and final time intervals, respectively; but such a strong condition is not necessary)

In the initial Minkowski spacetime the metric is that of above the line element equation with $a = a_1$. The coordinate can be rescaled with $x^i \to x'^i = a_1 x^i$,so that we have the usual Minkowski metric and x'^i is the physical or measured distance. The appropiate rescaled physical momentum is then $k'^i = \frac{k^i}{a_1}$, and the physical Energy of the particle is $|\vec{k'}| = \frac{k}{a_1} = \omega k$. It is the physical Volume Va_1^3 and Energy $\omega_1 k$ which must appear. the line element (2.3.1) to be

$$ds^2 = a^2(\eta)(d\eta^2 - dx^i dx_i) \tag{2.2.4}$$

the covariant generalization of the Dirac equation in curved spacetime is

$$\bar{\gamma}^\mu(\partial_\mu - \Gamma_\mu - ieA_\mu)\bar{\Psi} + M\bar{\Psi} = 0 \tag{2.2.5}$$

where $\bar{\gamma}^\mu$ is the curved Dirac matrices and Γ_μ is the spinor connection. As an approximation, we will ignore the effect of curvature on the fermions. so For a particle of charge eQ, the Dirac equation in presence of a magnetic field is given by

$$(i\gamma^{\mu}\partial_{\mu} - ma)\psi = 0$$

$$(i\gamma^{0}\vec{\alpha}.(\vec{\nabla} - ieQ\vec{A}) - \beta ma)\psi = i\gamma^{0}\partial_{0}\psi \tag{2.2.6}$$

where $\vec{\alpha}$, γ^0 and β are the Dirac matrices,a is expantion and $\vec{A}$ is the vector potential. In our convention, e is the positive unit of charge, taken as usual to be equal to the proton charge. for stationary states. we cna write

$$\psi = e^{iE_c t}\begin{pmatrix} \Phi \\ \chi \end{pmatrix} \tag{2.2.7}$$

where ϕ and χ are 2-component objects. We use the Pauli-Dirac representation of the Dirac matrices, in which

$$\alpha = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.2.8}$$

where each block represents a 2 x 2 matrix, and $\vec{\sigma}$ are the Pauli matrices. With this notation, we can write eqn.(2.3.5) as

$$(E_c - m)\phi = \frac{1}{a}\vec{\sigma}.(-i\vec{\nabla} - eQ\vec{A})\chi, \tag{2.2.9}$$

$$(E_c + m)\chi = \frac{1}{a}\vec{\sigma}.(-i\vec{\nabla} - eQ\vec{A})\Phi. \tag{2.2.10}$$

Eliminating χ, we obtain

$$(E_c^2 - m^2)^2 a^2\phi = [\vec{\sigma}.(-i\vec{\nabla} - eQ\vec{A})]^2\Phi \tag{2.2.11}$$

We will work with a constant magnetic field $\vec{B}$. Without loss of generality, it can be taken along the z-direction. The vector potential can be chosen in many equivalent ways. We take

$$A_0 = A_y = A_z = 0, A_x = -yB. \tag{2.2.12}$$

With this choice, Eq. (2.3.10) reduces to the form

$$(E_c^2 a^2 - m^2 a^2)^2\phi = [-\vec{\nabla}^2 + (eQB)^2 y^2 - eQB(2iy\frac{\partial}{\partial x} + \sigma_z)]\phi \tag{2.2.13}$$

Noticing that the co-ordinates x and z do not appear in the equation except through the deriva tives, we can write the solutions as

$$\phi = e^{i\vec{P}.\vec{X_y}} f(y) \tag{2.2.14}$$

where $f(y)$ is a 2-component matrix which depends only on the y-coordinate, and possibly some momentum components, as we will see shortly. We have also introduced the notation $\vec{X}$ for the spatial co-ordinates (in order to distinguish it from x, which is one of the components of $\vec{X}$), and $\vec{X_y}$ for the vector X with its y-component set equal to zero. In other words, $\vec{p}.\vec{X_y} = p_x x + p_z z$, where p_x and p_z denote the eigenvalues of momentum in the x and z directions. There will be two independent solutions for $f(y)$, which can be taken, without any loss of generality, to be the eigenstates of σ_z with eigenvalues $s = \pm 1$. This means that we choose the two independent solutions in the form

$$f_+(y) = \begin{pmatrix} F_+(y) \\ 0 \end{pmatrix}, f_-(y) = \begin{pmatrix} 0 \\ F_-(y) \end{pmatrix} \tag{2.2.15}$$

Since $\sigma_z F_s = sF_s$, the differential equations satisfied by F_s is

$$\frac{d^2 F_s}{dy^2} - (eQBy + p_z)^2 F_s + (E_c^2 a^2 - m^2 a^2 - p_z^2 + eQBs)F_s = 0 \tag{2.2.16}$$

which is obtained from Eq. (2.3.12). The solution is obtained by using the dimensionless variable

$$\xi = \sqrt{eQB}(y + \frac{p_x}{eQB}) \tag{2.2.17}$$

which transforms Eq. (2.) to the form

$$\frac{d^2 F_s(\xi)}{d\xi^2} - \xi^2 F_s(\xi) + a_s f_s(\xi) = 0 \tag{2.2.18}$$

where

$$a_s = \frac{(E_c^2 a^2 - m^2 a^2 - p_z^2 + eQBs)}{eQB} \tag{2.2.19}$$

This is a special form of Hermites equation, and the solutions exist provided $a_s = 2\nu + 1$ for $\nu = 0, 1, 2, .$This provides the energy eigenvalues

$$E_c^2 = \frac{1}{a^2}(m^2 a^2 + p_z^2 - eBQs + eB(2\nu + 1)) \tag{2.2.20}$$

and the solutions for F_s are

$$F_s(\xi) = (\frac{\sqrt{eQB}}{2^\nu \nu! \pi^{1/2}})^{1/2} e^{-\xi^2/2} H_\nu(\xi) = I_\nu(\xi) \tag{2.2.21}$$

where H_ν are Hermite polynomials of order ν, and N_ν are normalizations which we take to be

$$N_\nu = (\frac{\sqrt{e|Q|B}}{2^\nu \nu! \pi^{1/2}})^{1/2} \tag{2.2.22}$$

We stress that the choice of normalization can be arbitrarily made, as will be clarified later. With our choice, the functions I_ν satisfy the completeness relation

$$\sum_\nu I_\nu(\xi) I_\nu(\xi_*) = \sqrt{e|Q|B}\delta(\xi - \xi_*) = \delta(y - y_*), \tag{2.2.23}$$

where ξ_* is obtained by replacing y by y_* in Eq. (2.3.16). So far, Q was arbitrary. We now specialize to the case of electrons, for which $Q = 1$. The solutions are then conveniently classified by the energy eigenvalues

$$E_{cn}^2 = \frac{1}{a^2}(m^2 a^2 + p_z^2 + 2neB), \tag{2.2.24}$$

which is the relativistic form of Landau energy levels. The solutions are two fold degenerate in general: for $s = 1$, $\nu = n1$ and for $s = 1$, $\nu = n$. In the case of $n = 0$, only the second solution is available since ν cannot be negative. The solutions can have positive or negative energies. We will denote the positive square root of the right side by E_{cn} . Representing the solution corresponding to this n^{th} Landau level by a superscript n, we can then write for the positive energy solutions,

$$f_+^n = \begin{pmatrix} f_+(\xi) \\ 0 \end{pmatrix} = \begin{pmatrix} I_\nu(\xi) \\ 0 \end{pmatrix} = \begin{pmatrix} I_{n-1}(\xi) \\ 0 \end{pmatrix},$$

$$f_-^n = \begin{pmatrix} 0 \\ f_-(\xi) \end{pmatrix} = \begin{pmatrix} 0 \\ I_\nu(\xi) \end{pmatrix} = \begin{pmatrix} 0 \\ I_n(\xi) \end{pmatrix} \tag{2.2.25}$$

For $n = 0$, the solution f_+ does not exist. We will consistently incorporate this fact by defining

$$I_{-1}(y) = 0 \tag{2.2.26}$$

in addition to the definition of In in Eq. (2.3.20) for non-negative integers n. The solutions in Eq. (2.3.24) determine the upper components of the spinors through

Eq. (2.3.13). The lower components, denoted by χ earlier, can be solved using Eq. (2.3.9), and finally the positive energy solutions of the Dirac equation can be written as

$$e^{-ip.X_y}U_s(y,n,\vec{p_y}), \tag{2.2.27}$$

where X^ν denotes the space-time coordinate. And U_s are given by

$$U_+(y,n,\vec{p_y}) = \begin{pmatrix} I_{n-1}(\xi) \\ 0 \\ \frac{p_z}{a(E_{cn}+m)}I_{n-1}(\xi) \\ -\frac{\sqrt{2neB}}{a(E_{cn}+m)}I_n(\xi) \end{pmatrix}, U_-(y,n,\vec{p_y}) = \begin{pmatrix} 0 \\ I_n(\xi) \\ -\frac{\sqrt{2neB}}{a(E_{cn}+m)}I_{n-1}(\xi) \\ -\frac{p_z}{a(E_{cn}+m)}I_n(\xi) \end{pmatrix} \tag{2.2.28}$$

A similar procedure can be adopted for negative energy spinors which have energy eigenvalues $E = E_{cn}$. In this case, it is easier to start with the two lower components first and then find the upper components from Eq. (2.3.8). The solutions are

$$e^{-ip.X_y}V_s(y,n,\vec{p_y}), \tag{2.2.29}$$

Where

$$V_+(y,n,\vec{p_y}) = \begin{pmatrix} \frac{p_z}{a(E_{cn}+m)}I_{n-1}(\tilde{\xi}) \\ \frac{\sqrt{2neB}}{a(E_{cn}+m)}I_n(\tilde{\xi}) \\ I_{n-1}(\tilde{\xi}) \\ 0 \end{pmatrix}, V_-(y,n,\vec{p_y}) = \begin{pmatrix} \frac{\sqrt{2neB}}{a(E_{cn}+m)}I_{n-1}(\tilde{\xi}) \\ -\frac{p_z}{a(E_{cn}+m)}I_n(\tilde{\xi}) \\ 0 \\ I_n(\tilde{\xi}) \end{pmatrix} \tag{2.2.30}$$

where $\tilde{\xi}$ is obtained from ξ by changing the sign of the p_x-term.

2.3 Relativistic Landau Energy Level in 4 Dimensions

we now solve the Dirac equation in curved spacetime to find the relativistic energy level of a charged fermion in the presence of strong magnetic field in the 4 dimensional space time as an approximation, we will ignore the effect of curvature on the energy levels of fermions. starting from the Dirac equation in flat spacetime

$$\gamma^\mu \partial_\mu \Psi + ma\Psi = 0 \tag{2.3.1}$$

the energy solution $\Psi(x) = u(p)e^{-iEt+iPx}$ satisfies the equation $(\gamma^{\mu}+ma)u(p) = 0$ Let us consider a particle in an external magnetic field, the effect of magnetic field can be take into account by adding the momentum $p_{\mu} \to p_{\mu} - eA_{\nu}$ we will choose the magnetic field to point in the Z-direction and uniformly distributed over the entive x,y,z space the equation of motion of the fermion in 4 dimensional space becomes

$$p_x + p_y - 2eB_xp_y + e^2B^2x^2 - eB\sigma_z\Phi = (a^2E^2 - a^2m^2)\Phi \tag{2.3.2}$$

the energy condition from the equation of motion is given by

$$a^2E_{cn}^2 = a^2m^2 + P_Z^2 + (2n - v + 1)2am\mu_{\beta}B \tag{2.3.3}$$

if we let $j = n - 1/2$ and $P_n^2 = P_z^2$ then we have

$$E_{cn}^2 = \frac{1}{a^2}(a^2m^2 + P_n^2 + (j + 1/2)4am\mu_{\beta}B) \tag{2.3.4}$$

from the above equation energy is quantized in the x-y plane and contians certain degeneracy of states i.e, there are several states with the same one-particle energy. the number of states g_j of discrete energy level j is

$$g_j = g_s\int dp_xdp_ydxdy = g_sL_xL_y2\Pi\int_{p_j}^{p_{j+1}} PdP = g_sL_xL_y2\Pi(P_{j+1}^2 - P_j^2) = g_sL_xL_y2(4am\mu_{\beta}B) \tag{2.3.5}$$

the degeneracy is proportional to the field and vanishes for $B \to 0$. the discrete energies from the degree of freedom of the plane perpendicural to the magnetic field is called the landau levels. characterized the statistical properties of the fermion system. solution of the Dirac equation in spatially flat in curved space time in the presence of strong magnetic

Chapter 3

Inverse Beta-Decay of polarized Nuetron in Curved Spacetime in the Presence of Strong Magnetic field

Previously, some attempts have been made to find the change in the decay rate of the inverse beta decay whenever a strong magnetic field. In this Chapter we calculate the inverse beta decay polarized neutron in curved spacetime in the presence of strong magnetic fields.

3.1 Spin Sum in Curved Spacetime in the Presence Strong Magnetic field

the spin sum can be conveniently writteen by introducing the following rotation given any vector a^{μ}, we will define the following 4 vectors whole components aare given by

$$a^{\mu}_{||} = (a_0, 0, 0, a_z)$$

$$a^{\mu}_{||} = (a_z, 0, 0, a_0)$$

$$a^{\mu}_{\perp} = (0, a_x, a_y, 0) \tag{3.1.1}$$

in the frame in which the in the strong (background) magnetic field in purely magnetic. then, for any two 4 vectors a and b,

we will write

$$a.b_{||} = a_{\alpha} b^{\alpha}_{||}$$

$$a.b_\perp = a_\alpha b^\alpha_\perp \tag{3.1.2}$$

In notation we derive the spin-sum $\sum_s U_s(y,n,p)\bar{U}_s(y_*,n,p)$ the solutions of the Dirac equation in curved spacetime in presence of strong magnetic field. The two spinors in the above sum can have two different position coordinates in general and so their spatial dependence is explicitly shown to be different.

$$P_{U(y,y_*,n,P_y)} \equiv \sum_s U_s(y,n,p_y)\bar{U}_s(y_*,n,p_y) = \frac{1}{a(E_{cn}+m)} \sum_{i,j=n-1} I_i(\xi)I_j(\xi)T_{i,j} \tag{3.1.3}$$

The spin-sum of the product of the spinors, $U_s(y,n,p_y)\bar{U}_s(y_*,n,p_y)$ will give rise to a 4×4 matrix whose elements will be contain $I_i(\xi)I_j(\xi)$, where i,j runs from $n-1$, n. If these terms as $I_i(\xi)I_j(\xi)$ are taken as common factors then the whole 4×4 spin-sum matrix can be represented as a sum of terms containing the products of $I_i(\xi)I_j(\xi)$ times the corresponding 4×4 matrices called $T_{i,j}$. Using the dispersion relation $E^2_{cn} = \frac{1}{a^2}(m^2a^2 + p_z^2 + 2neB)$,$T_{n,n}$ can be written as,

$$T_{n,n} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a(E_{cn}+m) & 0 & p_z \\ 0 & 0 & 0 & 0 \\ 0 & -p_z & 0 & -a(E_{cn}+m) \end{pmatrix} \tag{3.1.4}$$

In the 2×2 notation the above matrix can be written as,

$$T_{n,n} = aE_{cn}\begin{pmatrix} \frac{1}{2}(1-\sigma_3) & 0 \\ 0 & -\frac{1}{2}(1-\sigma_3) \end{pmatrix} + p_z \begin{pmatrix} 0 & \frac{1}{2}(1-\sigma_3) \\ -\frac{1}{2}(1-\sigma_3) & 0 \end{pmatrix}$$

$$+am\begin{pmatrix} \frac{1}{2}(1-\sigma_3) & 0 \\ 0 & \frac{1}{2}(1-\sigma_3) \end{pmatrix} \tag{3.1.5}$$

where σ_3 is the third Pauli matrix. In the 4×4 notation Eq. (3.1.5) can be written as,

$$T_{n,n} = \frac{1}{2}[am(1-\sigma_z) + aE_{cn}(\gamma^0+\gamma^5\gamma^3) - p_z(\gamma^5\gamma^0+\gamma^3)] = \frac{1}{2}[am(1-\sigma_z) + \not{p}_{||} + \tilde{\not{p}}_{||}] \tag{3.1.6}$$

where $\sigma_z = i\gamma^1\gamma^2$. In the last equation $\not{p}_{||} = p^0\gamma_0 + p^3\gamma_3$ and $\tilde{\not{p}}_{||} = p^0\gamma_3 + p^3\gamma_0$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in our cases $p^0 = aE_{cn}$

In a similar way $T_{n1,n1}$ can be written as:

$$T_{n-1,n-1} = \begin{pmatrix} a(E_{cn}+m) & 0 & -p_z & 0 \\ 0 & 0 & 0 & 0 \\ p_z & 0 & -a(E_{cn}-m) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.1.7}$$

In the 4×4 notation the above equation becomes,

$$T_{n-1,n-1} = \frac{1}{2}[am(1+\sigma_z)+aE_{cn}(\gamma^0-\gamma^5\gamma^3)+p_z(\gamma^5\gamma^0-\gamma^3)] = \frac{1}{2}[am(1+\sigma_z)+\not{p}_{||}-\tilde{\not{p}}_{||}] \tag{3.1.8}$$

From the matrix multiplication in the left hand side of Eq. (3.1.3) it can be seen that $T_{n1,n}$ is given as,

$$T_{n-1,n} = \sqrt{2neB}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.1.9}$$

In the 2×2 notation the above equation looks like,

$$T_{n-1,n} = \sqrt{2neB}\begin{pmatrix} 0 & \frac{1}{2}(\sigma_1+i\sigma_2) \\ -\frac{1}{2}(\sigma_1+i\sigma_2) & 0 \end{pmatrix} \tag{3.1.10}$$

Here σ_1 and σ_2 are the first two Pauli matrices. When converted back to the 4×4 notation the above equation becomes,

$$T_{n-1,n} = -\frac{1}{2}\sqrt{2neB}(\gamma_1+i\gamma_2) \tag{3.1.11}$$

Similarly $T_{n,n1}$ is given by,

$$T_{n,n-1} = \sqrt{2neB}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \tag{3.1.12}$$

when we converted back to the 4×4 notation becomes,

$$T_{n,n-1} = -\frac{1}{2}\sqrt{2neB}(\gamma_1-i\gamma_2) \tag{3.1.13}$$

Supplying the values of $T_{i,j}s$ from Eq.3.1.6,Eq.3.1.8,Eq.3.1.11,Eq.3.1.13 into Eq.3.1.3 we get The spin sum for the U -spinors is

$$P_{U(y,y_*,n,P_y)} \equiv \sum_s U_s(y,n,p_y)\bar{U}_s(y_*,n,p_y)$$

$$= \frac{1}{2a(E_{cn}+m)} \times [am(1+\sigma_z) + \not{p}_{||} - \tilde{\not{p}}_{||} I_{n-1}(\xi) I_{n-1}(\xi_*)$$
$$+am(1-\sigma_z) + \not{p}_{||} + \tilde{\not{p}}_{||} I_n(\xi) I_n(\xi_*)$$
$$-\sqrt{2neB}(\gamma_1 - i\gamma_2) I_{n-1}(\xi_*)$$
$$-\sqrt{2neB}(\gamma_1 + i\gamma_2) I_n(\xi_*)] \tag{3.1.14}$$

Similarly, the spin sum for the V -spinors can also be calculated, and we obtain

$$P_{U(y,y_*,n,P_y)} \equiv \sum_s V_s(y,n,p_y)\bar{V}_s(y_*,n,p_y)$$

$$= \frac{1}{2a(E_{cn}+m)} \times [-am(1+\sigma_z) + \not{p}_{||} - \tilde{\not{p}}_{||} I_{n-1}(\tilde{\xi}) I_{n-1}(\tilde{\xi_*})$$
$$+-am(1-\sigma_z) + \not{p}_{||} + \tilde{\not{p}}_{||} I_n(\tilde{\xi}) I_n(\tilde{\xi_*})$$
$$+\sqrt{2neB}(\gamma_1 - i\gamma_2) I_{n-1}(\tilde{\xi_*})$$
$$+\sqrt{2neB}(\gamma_1 + i\gamma_2) I_n(\tilde{\xi_*})] \tag{3.1.15}$$

3.2 The Fermion field operator

Since we have found the solutions to the Dirac equation, we can now use them to construct the fermion field operator in the second quantized version. For this, we write

$$\psi(X) = \sum_{s=\pm}\sum_{n=0} \int \frac{dp_x dp_z}{D} [f_s(n,\vec{p_y}) e^{-ip.X_y} U_s(y,n,\vec{p_y}) + \hat{f}_s^\dagger(n,\vec{p_y}) e^{ip.X_y} V_s(y,n,\vec{p_y})] \tag{3.2.1}$$

Here, $f_s(n,\vec{p_y})$ is the annihilation operator for the fermion, and $\hat{f_s}^\dagger(n,\vec{p_y})$ is the creation operator for the antifermion in the n^{th} Landau level with given values of p_x and p_z . The creation and annihilation operators satisfy the anticommutation relations

$$[f_s(n,\vec{p_y}), \hat{f}_s'(n',\vec{p_y})]_+ = \delta_{ss'}\delta_{nn'}\delta(p_x - p_x')\delta(p_z - p_z') \tag{3.2.2}$$

and a similar one with the operators $\hat{f}$ and $\hat{f}^{\dagger}$, all other anticommutators being zero. The quantity D appearing in Eq. (3.2.1) depends on the normalization of the spinor solutions, and this is why of the spinors could have been chosen arbitrarily, as remarked after Eq. (2.17). Once we have chosen the spinor normalization, the factor D appearing in Eq. (3.2.1) is however fixed, and it can be determined from the equal time anticommutation relation

$$[\psi(X), \psi^{+}(X_{*})]_{+} = \delta^{3}(\vec{X} - \vec{X}_{*}) \tag{3.2.3}$$

Plugging in the expression given in Eq.(3.1) to the left side of this equation and using the anticommutation relations of Eq.(3.2), we obtain

$$[\psi(X), \psi^{+}(X_{*})]_{+} = \sum_{s}\sum_{n}\int \frac{dp_x dp_z}{D^2}(e^{-ip_x(x-x_*)-ip_z(z-z_*)}U_s(y, n, \vec{p_y})U_s^{\dagger}(y_*, n, \vec{p_y}))$$
$$+e^{ip_x(x-x_*)ip_z(z-z_*)}V_s(y, n, \vec{p_y})V_s^{\dagger}(y_*, n, \vec{p_y}) \tag{3.2.4}$$

Changing the signs of the dummy integration variables p_x and p_z in the second term, we can rewrite it as

$$[\psi(X), \psi^{+}(X_{*})]_{+} = \sum_{s}\sum_{n}\int \frac{dp_x dp_z}{D^2}(e^{-ip_x(x-x_*)-ip_z(z-z_*)}U_s(y, n, \vec{p_y})U_s^{\dagger}(y_*, n, \vec{p_y}))$$
$$+V_s(y, n, -\vec{p_y})V_s^{\dagger}(y_*, n, -\vec{p_y}) \tag{3.2.5}$$

Using now the solutions for the U and the V spinors from Eqs. (2.3.28) and (2.3.30), it is straight forward to verify that

$$\sum_{s}(U_s(y, n, \vec{p_y})U_s^{\dagger}(y_*, n, \vec{p_y}) + V_s(y, n, -\vec{p_y})V_s^{\dagger}(y_*, n, -\vec{p_y}))$$

$$= 2(1+\frac{p_z^2 + 2neB}{a^2(E_{cn} + m)^2})\times diag[I_{n-1}(\xi)I_{n-1}(\xi_*), I_n(\xi)I_n(\xi_*), I_{n-1}(\tilde{\xi})I_{n-1}(\tilde{\xi_*}), I_n(\tilde{\xi})I_n(\tilde{\xi_*})] \tag{3.2.6}$$

where diag indicates a diagonal matrix with the specified entries, and ξ and ξ_* involve the same value of p_x . At this stage, we can perform the sum over n in Eq. (3.2.7) using the completeness relation of Eq. (2.3.23), which gives the δ-function of the y-coordinate that should appear in the anticommutator. Finally, performing the integrations over p_x and p_z , we can recover the δ-functions for the other two coordinates as well, provided

$$\frac{2E_{cn}}{E_{cn}+m}\frac{1}{D^2}=\frac{1}{(2\pi)^2} \tag{3.2.7}$$

using the expression for the energy eigenvalues from Eq. (2.3.24) to rewrite the prefactor appearing on the right side of Eq. (3.2.8). Putting the solution for D, we can rewrite Eq. (3.2.3) as

$$\psi(X)=\sum_{s=\pm}\sum_{n=0}\int\frac{dp_x dp_z}{2\pi}\sqrt{\frac{E_{cn}+m}{2E_{cn}}}\times[f_s(n,\vec{p_y})e^{-ip.X_y}U_s(y,n,\vec{p_y})+\hat{f}_s^{\dagger}(n,\vec{p_y})e^{ip.X_y}V_s(y,n,\vec{p_y})] \tag{3.2.8}$$

The one-fermion states are defined as

$$|n,\vec{p_y}\rangle = Cf^{\dagger}(n,\vec{p_y})|0\rangle \tag{3.2.9}$$

The normalization constant C is determined by the condition that the one-particle states should be orthonormal. For this, we need to define the theory in a finite but large region whose dimensions are L_x , L_y and L_z along the three spatial axes. This gives

$$C=\frac{2\pi}{\sqrt{L_x L_z}} \tag{3.2.10}$$

then

$$\psi U(X)|n,\vec{p_y}\rangle = \sqrt{\frac{E_{cn}+m}{2E_{cn}L_x L_z}}e^{-ip.X_y}U_s(y,n,\vec{p_y})|0\rangle, \tag{3.2.11}$$

where ψU denotes the term in Eq. (3.2.10) that contains the U-spinors. Similarly,

$$\langle n,\vec{p_y}|\bar{\psi}U(X) = \sqrt{\frac{E_{cn}+m}{2E_{cn}L_x L_z}}e^{-ip.X_y}\bar{U}_s(y,n,\vec{p_y})\langle 0| \tag{3.2.12}$$

3.3 The S-matrix Element of Inverse Beta-decay

In this section, we calculate the S-matrix Element for the inverse beta-decay process $\nu_e+n\rightarrow p+e^-$ in the curved spacetime as background magnetic field. The magnetic field might provide a net polarization of the neutrons, which we take into account. so we ignore its effects on the proton and neutron spinors. The electron spinors, on the other hand,are the ones appropriate for the Landau levels.Thus,we can write the process as

$$\nu_e(\overrightarrow{K}) + n(\overrightarrow{P}) \to (\overrightarrow{P'}) + e(\overrightarrow{p_y}, n') \tag{3.3.1}$$

the scattering of particle and antiparticles is closly related to the process of particle-particle scattering.Therefore, the interaction Lagrangian for this process of inverse beta-decay is represented by,

$$L = \frac{G}{\sqrt{2}}[\bar{\Psi}_p\gamma_\mu(1+\alpha\gamma_5)\Psi_n][\bar{\Psi}_e\gamma^\mu(1+\gamma_5)\Psi_\nu \tag{3.3.2}$$

we can rewrite the above equation well-known four-fermion lagrangian in curved spacetime

$$L_{int} = \sqrt{-2g}G_\beta[\bar{\Psi}_{(e)}\gamma_\mu(L)\Psi_{(\nu_e)}][\bar{\Psi}_p\gamma^\mu(g_v - g_A\gamma_5)\Psi_n] \tag{3.3.3}$$

where $L = \frac{1}{2}(1-\gamma_5)$ and $G_\beta = G_F cos\theta_c$, θ_c being the Cabibbo angle and g denote the determinant of the metric $g_{\mu\nu}$ The total cross-section of the process can be written as

$$\sigma = \frac{L^3}{T}\sum_{phasespace}|S_{fi}|^2 \tag{3.3.4}$$

where summation is performed over the phase space of the final particles. We account for the influence of the background magnetic field on the matrix element (3.3.5). The corresponding calculations are performed using the exact solutions of the Dirac equation in the magnetic field for the relativistic electron and proton. Without loss of generality, a constant magnetic field B is taken along the z-direction. We use of our previous the notations of the beta-decay of the polarized neutron in a magnetic field with the proton recoil effects have been accounted for. We also choose the longitudinal in respect to the magnetic field vector B component of the polarization tensor.

The matrix element of the process is given by

$$S_{fi} = \sqrt{2}G_\beta\int d^4X\sqrt{-g}\langle e(\vec{p_y}, n')|\bar{\Psi}_{(e)}\gamma_\mu L\Psi_{(\nu_e)}|\nu_e(\vec{k})\rangle$$

$$\times\langle p(P')|\bar{\Psi}_p\gamma^\mu(g_v - g_A\gamma_5)\Psi_n|n(P)\rangle \tag{3.3.5}$$

For the hadronic part, we should use the usual solutions of the Dirac field which are normalized within a box of volume V , and this gives

$$\langle p(P')|\bar{\Psi}_p\gamma^\mu(g_v - g_A\gamma_5)\Psi_n|n(P)\rangle =$$
$$\frac{e^{i(P'-P).X}}{\sqrt{2\varepsilon V}\sqrt{2\varepsilon' V}}\bar{u_p}(\vec{P'})\gamma^\mu(g_v - g_A\gamma_5)u_n(\vec{P}) \tag{3.3.6}$$

using the notations $\varepsilon = \frac{P_0}{a}$ and $\varepsilon' = \frac{P'_0}{a}$ For the leptonic part, we need to take into account the magnetic spinors for the electron. Using Eq. (3.2.14), we obtain

$$\langle e(\vec{p_y}, n')|\bar{\Psi}_{(e)}\gamma_\mu L\Psi_{(\nu_e)}|\nu_e(\vec{k})\rangle$$
$$= \frac{e^{-ik.x+ip'.X_y}}{\sqrt{2\omega V}}\sqrt{\frac{E_{cn'}+m}{2E_{cn'}L_xL_z}}(\bar{U}_e(y, n', \vec{p_y})\gamma^\mu LU_{\nu_e}(\vec{k})) \tag{3.3.7}$$

Putting these back into Eq. (3.3.5) and performing the integrations over all co-ordinates except y, we obtain

$$S_{fi} = \sqrt{2}G_\beta\int d^4X\sqrt{-g}[\frac{e^{i(P'-P).X}}{\sqrt{2\varepsilon V}\sqrt{2\varepsilon' V}}\bar{U}_p\gamma^\mu(g_v - g_A\gamma_5)U_n(P)]$$
$$\times[\frac{e^{-ik.x+ip'.X_y}}{\sqrt{2\omega V}}\sqrt{\frac{E_{cn'}+m}{2E_{cn'}L_xL_z}}(\bar{U}_e(y, n', \vec{p_y})\gamma^\mu LU_{\nu_e}(\vec{k}))]$$
$$= \sqrt{2}G_\beta\int\sqrt{-g}e^{i(P'-P).X-ik.x+ip'.X_y}d^4X(\frac{E_{cn'}+m}{2E_{cn'}L_xL_z})^1/2$$
$$\times\bar{U}_p\gamma^\mu(g_v - g_A\gamma_5)U_n(P)(\bar{U}_e(y, n', \vec{p_y})\gamma^\mu LU_{\nu_e}(\vec{k}) \tag{3.3.8}$$

using propertie of Delta function and the Diagonal matrix $\sqrt{-g} = a^3$ the above equation give us

$$S_{fi} = (2\pi)^3\delta^3_y(P + k - P' - p')(\frac{E_{cn'}+m}{2E_{cn'}L_xL_z})^{1/2}\times\sqrt{2}G_\beta\bar{U}_p\gamma^\mu(g_v - g_A\gamma_5)U_n(P)$$
$$\times a^3\int dye^{i(P_y+k_y+p'_y)}(\bar{U}_e(y, n', \vec{p_y})\gamma^\mu LU_{\nu_e}(\vec{k}) \tag{3.3.9}$$

$$S_{fi} = (2\pi)^3\delta^3_y(P + k - P' - p')(\frac{E_{cn'}+m}{2\omega V 2\varepsilon V 2\varepsilon' V 2E_{cn'}L_xL_z})^{1/2}M_{fi} \tag{3.3.10}$$

Here, δ^3_y implies, in accordance with the notation introduced earlier, the δ-function for all space- time co-ordinates except y. Contrary to the field-free case, we do not get 4-momentum conservation because the y-component of momentum is not a good quantum number in this problem. The quantity M_{fi} is the Feynman amplitude, given by

$$M_{fi} = \sqrt{2}a^3 G_\beta \bar{U}_p \gamma^\mu (g_v - g_A\gamma_5) U_n(P) \int dy e^{i(P_y+k_y+p'_y)} (\bar{U}_e(y, n', \vec{p_y}) \gamma^\mu L U_{\nu_e}(\vec{k}) \tag{3.3.11}$$

the transition rate in a large time T is given by $\frac{|S_{fi}|^2}{T}$

$$\begin{aligned} |\delta(\varepsilon+\omega-\varepsilon'-E_{cn'})|^2 &= \frac{T}{2\pi}\delta(\varepsilon+\omega-\varepsilon'-E_{cn'}) \\ |\delta(P_x+k_x-P'_x-p'_x)|^2 &= \frac{L_x}{2\pi}\delta(P_x+k_x-P'_x-p'_x) \\ |\delta(P_z+k_z-P'_z-p'_z)|^2 &= \frac{L_x}{2\pi}\delta(P_z+k_z-P'_z-p'_z) \end{aligned} \tag{3.3.12}$$

so

$$\begin{aligned} |S_{fi}|^2 = \frac{T}{2\pi}&\delta(\varepsilon+\omega-\varepsilon'-E_{cn'})\frac{L_x}{2\pi}\delta(P_x+k_x-P'_x-p'_x) \\ &\times\frac{L_x}{2\pi}\delta(P_z+k_z-P'_z-p'_z) \\ &\times(2\pi)^2\left(\frac{E_{cn'}+m}{2\omega V 2\varepsilon V 2\varepsilon' V 2E_{cn'} L_x L_z}\right)|M_{fi}|^2 \end{aligned} \tag{3.3.13}$$

therefore,

$$\frac{|S_{fi}|^2}{T} = \frac{1}{16}(2\pi)^2 \delta_y^3(P+k-P'-p')\frac{E_{cn'}+m}{V^3\omega\varepsilon\varepsilon' E_{cn'}} \tag{3.3.14}$$

3.4 The Scattering Cross Section

collide beam of particles; sometimes the particle will hit and bounce off each; sometimes they will pass right through; The fraction of the time that they collide is cross-section. If the incoming flux F is defined to be the number of incoming particles per area per unit time, then the total number of scattering events N per unit time is given by $N = F\sigma$

We would like to calculate σ from QFT. In fact, we can calculate a more sensitive quantity $d\sigma$ known as the differential cross-section which is the probability for a given scattering process to occur in the solid angle more precisely

$$d\sigma = \frac{differential probability}{unittime.unitFlux} \tag{3.4.1}$$

Therefore, In this section, we calculate the Scattering cross section of Inverse Beta-decay. Now we have all the tools required for calculating the scattering cross-section of the inverse beta- decay process in curved spacetime in the presence of strong magnetic field. The calculations involve evaluating the matrix element using spinor solutions of the electron in a magnetic field, taking all possible final Landau levels into account, including the possibility of neutron polarization, and performing the calculations to all orders in the background field in the 4-fermi interaction theory. The magnetic field might provide a net polarization of the neutrons, which is taken into account. However, the magnitude of the field is assumed to be much smaller than m_p^2/e, so its effects on the proton spinors are ignored. The electron spinors, on the other hand, are the ones appropriate for the Landau levels.

We consider the possibility that the neutrons may be totally or partially polarized in the magnetic field, and find the cross-section as a function of this polarization. The neutrinos are assumed to be strictly standard model neutrinos, without any mass and consequent properties. The presence of the magnetic field breaks the isotropy of the background, and a careful calculation in this background reveals a dependence of the cross-section on the incident neutrino direction with respect to the magnetic field. Using unit flux $\frac{1}{V}$ for the incident particle as usual, we can write the differential cross-section as

$$d\sigma = V\frac{|S_{fi}|^2}{T}d\rho \tag{3.4.2}$$

where $d\rho$, the differential phase space for final particles, is given in our case by

$$d\rho = \frac{L_x}{2\pi}dp'_x\frac{L_z}{2\pi}dp_z\frac{V}{(2\pi)^3}d^3P' \tag{3.4.3}$$

therefore,

$$d\sigma = \frac{1}{64\pi^2}\delta_y^3(P+k-P'-p')\frac{E_{cn'}+m}{\omega\varepsilon\varepsilon' E_{cn'}}|M_{fi}|^2\frac{L_xL_z}{V}dp'_xdp'_zd^3P' \tag{3.4.4}$$

the square of the matrix element is

$$|M_{fi}|^2 = 2a^6G_\beta^2\ell^{\mu\nu}H_{\mu\nu} \tag{3.4.5}$$

where $H_{\mu\nu}$ is the hadronic part and $\ell^{\mu\nu}$ the leptonic part, whose calculation we outline now. For the hadronic part, we can use the usual Dirac spinors because of our assumption thatthe magnetic field is much smaller than m_p^2/e. We will work in the rest frame of the neutron. Due to the presence of the background magnetic field, the neutrons may be totally or partially polarized. We define the quantity

$$S \equiv \frac{N_n^{(+)} - N_n^{(-)}}{N_n^{(+)} + N_n^{(-)}} \tag{3.4.6}$$

where $N_n^{(\pm)}$ denote the number of neutrons parallel and antiparallel to the magnetic field. Then

$$H_{\mu\nu} = \frac{1}{2}(1+S)H_{\mu\nu}^{(+)} + \frac{1}{2}(1-S)H_{\mu\nu}^{(-)} \tag{3.4.7}$$

where $H_{\mu\nu}^{(\pm)}$ denotes the contribution calculated with spin-up and spin-down neutrons respectively. Either of these contributions can be calculated by using the spin projection operator, which is $\frac{1}{2}(1 \pm \gamma^5\gamma^3)$ for up and down spins. A straight forward calculation then yields

$$\begin{aligned} H_{\mu\nu} &= 2(g_v^2 + g_A^2)(P_\mu P'_\nu + P_\nu P'_\mu - g_{\mu\nu} P.P') \\ &+2(g_v^2 - g_A^2) m_n m_p g_{\mu\nu} + 4i g_v g_A \epsilon_{\mu\nu\lambda\rho P^\lambda P'^\rho} \\ &-S[4 g_v g_A m_n (P'_\mu g_{3\nu} + P'_\nu g_{3\mu} - P'_3 g_{\mu\nu}) + 2i\epsilon_{\mu\nu 3\alpha R^\alpha}] \end{aligned} \tag{3.4.8}$$

where we have introduced the shorthand

$$R^\alpha = (g_v^2 + g_A^2) m_n P'^\alpha - (g_v^2 - g_A^2) m_p P^\alpha \tag{3.4.9}$$

We have omitted some terms in the expression for $H_{\mu\nu}$ that involve spatial components of the neutron momentum, with the anticipation that we will perform the calculation in the neutron rest frame.

In the leptonic part $\ell_{\mu\nu}$, we should use the magnetic spinors given in Sec. 2. This gives

$$\ell_{\mu\nu} = \int dy \int dy_* e^{i(P_y + k_y + p'_y)(y_* - y)} Tr(U_p(y, y_*, n', \vec{p_y})\gamma^\mu \not{k} \gamma^\nu L) \tag{3.4.10}$$

where P_U denotes the spinor sum for the electrons, given in Eq. (3.1.14). We now have to perform the integrations over y and y_* . Each of these variables should be integrated in the range $\frac{-1}{2}L_y$ to $\frac{+1}{2}L_y$. However, since we will take the infinite volume limit at the end as usual, we let $L_y \to \infty$ and use the result [24]

$$\int_{-\infty}^{+\infty} dy e^{ixy} I_n(y) = i^n \sqrt{2\pi} I_n(x) \tag{3.4.11}$$

This gives

$$\ell_{\mu\nu} = \frac{2\pi}{eB} \frac{2}{a(E_{cn'} + m)} (\lambda^\mu k^\nu + \lambda^\nu k^\mu - k.\lambda g^{\mu\nu} - i\epsilon^{\mu\nu\alpha\beta}) \tag{3.4.12}$$

where

$$\lambda^\alpha = (I_{n'-1}(\frac{(P_y + k_y + p'_y)}{\sqrt{eB}})^2 (p^\alpha_{||} - \tilde{p^\alpha_{||}}) + [I_{n'}(\frac{(P_y + k_y + p'_y}{\sqrt{eB}})]^2 (p^\alpha_{||} + \tilde{p^\alpha_{||}})$$

$$-2\sqrt{2n'eB} g^\alpha_2 I_{n'}(\frac{(P_y + k_y + p'_y)}{\sqrt{eB}}) I_{n'-1} \frac{(P_y + k_y + p'_y)}{\sqrt{eB}} \tag{3.4.13}$$

thus the Feynman amplitude is by

$$|M_{fi}|^2 = 8a^5 G^2_\beta \times \frac{2\pi}{eB} \frac{1}{E_{cn'} + m} [(g^2_V + g^2_A)(P.\lambda P'.k + P'.\lambda P.k)$$

$$-(g^2_V - g^2_A) m_n m_p k.\lambda - 2g_V g_A (P.\lambda P'.k + P'.\lambda P.k)$$

$$+S(2g_V g_A m_n (P'.\lambda k_z + P'.k\lambda_z) - \lambda_z k.R + k_z \lambda.R) \tag{3.4.14}$$

We now choose the axes such that the 3-momentum of the incoming neutrino is in the x-z plane. We will also assume that $|\vec{P'}| \ll m_p$ for the range of energies of interest to us. In that case, it is easy to see that the terms involving $\sqrt{2n'eB}$ drop out, and we obtain

$$|M_{fi}|^2 = 8a^4 G^2_\beta \times \frac{2\pi}{eB} \frac{1}{E_{cn'} + m} \times [(g^2_V + 3g^2_A)\omega\lambda_0 + (g^2_V - g^2_A) k_z \lambda_z$$

$$+2g_A S(g_V - g_A)\omega\lambda_z + (g_V + g_A) k_z \lambda_0] \tag{3.4.15}$$

We now put this expression into Eq. (3.4.4) and calculate the total cross section by performing the integrations over different final state momenta appearing in that formula. First we integrate over P'_xandP'_z . These appear only in the momentum conserving δ-function. Integration over them therefore just gets rid of the corresponding δ-functions. For the integration over p'_x we refer to Eq. (2.3.17).

Since the center of the oscillator has to lie between $\frac{-1}{2}L_y$ and $\frac{1}{2}L_y$, we conclude that $\frac{-1}{2}L_y eB \leq p'_x \leq \frac{1}{2}L_y eB$ Thus the integration over p_x gives a factor $L_y eB$. Putting back into Eq. (3.4.4) and using $V = a^3 L_x L_y L_z$, we obtain

$$d\sigma = \frac{a^5 G_\beta^2}{4\pi} \frac{\delta(Q + \omega - E_{cn'})}{\omega E_{cn'}} \times [(g_V^2 + 3g_A^2)\omega\lambda_0 + (g_V^2 - g_A^2)k_z\lambda_z$$
$$+2g_A S(g_V - g_A)\omega\lambda_z + (g_V + g_A)k_z\lambda_0] dP'_y dp'_z \qquad (3.4.16)$$

where Q is the neutron-proton mass difference, $m_n - m_p$.

We next perform the integration over P'_y. In the integrand, it occurs only as the argument of the functions I_n and I_{n1} . The functions I_n are orthogonal in the sense that

$$\int_{-\infty}^{+\infty} da I_n(a) I'_n(a) = \sqrt{eB}\delta_{nn'} \qquad (3.4.17)$$

This property can be used to perform the integration over P_y. We have already remarked that the term proportional to $\sqrt{2neB}$ in Eq. (3.4.13) does not contribute. From other two terms, we obtain

$$\int dP'_y \lambda^\alpha = eB[(p_{||}^{'\alpha} - \tilde{p}_{||}^{'\alpha})(1 - \delta_{n',0} + (p_{||}^{'\alpha} + \tilde{p}_{||}^{'\alpha})]$$
$$= [eB(g'_n p_{||}^{'\alpha} + \delta_{n',0}\tilde{p}_{||}^{'\alpha})], \qquad (3.4.18)$$

where

$$g'_n = 2 - \delta_{n',0} \qquad (3.4.19)$$

gives the degeneracy of the Landau level. Notice the appearance of the Kronecker delta, $\delta_{n',0}$, in the expression of Eq. (3.4.18). The reason for this is that, while two terms of Eq. (3.4.13) contribute in the integral for $n = 0$, only one of them contributes for $n = 0$ since $I_1 = 0$.

The final integration is over p'_z . Writing the argument of the remaining δ-function in terms of p_z, we find that the zeros occur when

$$p'_z = p'_+ = \pm\sqrt{(Q + \omega)^2 - a^2m^2 - 2n'eB} \qquad (3.4.20)$$

Therefore,

$$\delta(Q + \omega - E_{cn}) = \frac{Q + \omega}{\sqrt{(Q + \omega)^2 - a^2m^2 - 2n'eB}}(\delta(p'_z - p'_+) + \delta(p'_z - p'_-)) \quad (3.4.21)$$

In the integration, the terms proportional to p'_z in the integrand receive equal and opposite contributions from the two δ functions and cancel. For the other terms, independent of p'_z , both the contributions are equal. So we obtain

$$\sigma'_n = \frac{eBa^5G_\beta^2}{2\pi}[g'_n(g_V^2 + 3g_A^2) + 2g_AS(g_V + g_A)\cos\theta$$

$$+\delta_{n',0}(g_V^2 - g_A^2)\cos\theta + 2g_AS(g_V - g_A)]\frac{Q+\omega}{\sqrt{(Q+\omega)^2 - a^2m^2 - 2n'eB}} \quad (3.4.22)$$

where we have defined the direction of the incoming neutrino by the angle θ, with

$$k_z = \omega\cos\theta \quad (3.4.23)$$

In Eq. (3.4.22), we have denoted the cross section by σ'_n because the electron ends up in a specific Landau level n' . The total cross section is then given as a sum over all possible values of n' ,i.e.,

$$\sigma = \sum_{n'=0}^{n'_{max}} \sigma'_n = \frac{eBa^5G_\beta^2}{2\pi}\sum_{n'=0}^{n'_{max}}[g'_n(g_V^2 + 3g_A^2) + 2g_AS(g_V + g_A)\cos\theta$$

$$+\delta_{n',0}(g_V^2 - g_A^2)\cos\theta + 2g_AS(g_V - g_A)]\frac{Q+\omega}{\sqrt{(Q+\omega)^2 - a^2m^2 - 2n'eB}} \quad (3.4.24)$$

we can, in fact, suppose that $a = a_1$ and $a = a_2$ for arbitrarily long initial and final scale factor constant.

Note that the vector coupling g_V does not contribute to the cross section in this limit. This can be understood easily. The neutrino spin is along the +z direction whereas the electron spin in the lowest Landau level must be in the z direction. Thus there is a spin-flip in the leptonic sector. Conservation of angular momentum then implies that there must be a spin-flip in the hadronic sector as well. In the non-relativistic limit for hadrons that we have employed, this can occur only through the axial coupling.

The possible allowed Landau level has a maximum, n'_{max} , which is given by the fact that the quantity under the square root sign in the denominator of Eq. (3.4.24) must be non-negative, i.e.,

$$n'_{max} = int\frac{1}{2eB}[(Q+\omega)^2 - a^2m^2] \quad (3.4.25)$$

Eq.(3.4.25) gives our result for the cross section of the inverse beta decay process. Some properties of this formula are worth noting. For unpolarized neutrons, $S = 0$, the cross section for $n' = 0$ does not depend on the direction of the incoming neutrino. The same is not true if the electron ends up in the lowest Landau level.The cross section will be asymmetric in this case. All terms in the cross section which depend on S have a common factor g_A . The reason is that,if g_A were equal to zero, the interaction in the hadronic sector would have been spin-independent. If the final electron is in the lowest Landau level and the initial neutrino momentum is antiparallel to the magnetic field, Eq. (3.4.25) shows that

$$\sigma_0 = \frac{a^5 eBG_\beta^2}{2\pi}[4g_A^2(1 - S)]$$

Chapter 4

Summary and Conclusion

In this paper we have developed the cross section of the inverse Beta decay of the polarized neutron in curved spacetime in the presence of strong magnetic field and effects of the physical momentum, energy of the particle and volume. the calculation of the cross section for inverse beta decay process in the strong magnetic field has been performed earlier by several Authors[22][25][26], but no one can calculate the cross section for inverse beta decay process in curved spacetime in the presence of strong magnetic field.

We assumed that the matrix element and cross section is affected by constant 'a' the scale factor the expanding Universe and we have also shown that in the case of total neutron polarization ($S = \pm 1$) the cross section is exactly zero. Developed in curved spacetime treatment of cross-section can be applied to other URCA processes in magnetic field with two particles in initial and final states

$$\nu_e + n \rightleftharpoons e + p$$

$$p + \bar{\nu}_e \rightleftharpoons n + e^+$$

that are important for star cooling and also producing asymmetry in neutrino distribution. Proton momentum quantization and recoil motion are accounted for exactly.

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DECLARATION

I hereby declare that this thesis is my original work, has not been presented for a degree in an other university and that all the sources of material used for the thesis have been dully acknowledged.

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