FILTERING CHIRP WAVEFORMS WITHOUT PRODUCING UNWANTED SIDELOBES

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FILTERING CHIRP WAVEFORMS WITHOUT PRODUCING UNWANTED SIDELOBES

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Abstract

Regardless of the type of waveforms implemented by a radar, using a matched filter at the receiver yields the maximum signal to noise ratio (SNR). Unfortunately, when one applies matched filtering technique in filtering chirp waveforms, it produces unwanted sidelobes at the output of the receiver. In this case it is difficult to pinpoint whether the radar signal is coming from the main peaks or from the sidelobes. This can be a major problem especially when main scatterer lies at the sidelobes. In other words, there will be ambiguity in the measurements. Here we present a new method of filtering chirp waveforms that suppresses the unwanted sidelobes. The method is developed based on an earlier work that has been applied to solve a similar problem created in filtering binary phase coded waveforms by means of matched filter [2]. *Abiyu Zerfu*: June, 2005, Addis Ababa

Preface

The performance of modern radar systems are largely dependant on the type of waveform implemented and also on the signal processing techniques used in the analysis. As a result many scientists and engineers around the world are conducting research on how to find the optimal radar waveform for a specific application.

In this thesis we present how to filter chirp waveforms, which are used by some radar systems, without producing unwanted sidelobes. The thesis is organized as follows.

In Chapter one we present a general background and radar fundamentals and then we go on to discuss radar waveform analysis in Chapter two. In Chapter three we discuss how to model chirp waveforms and also the traditional method of filtering these waveforms. In this chapter we also present how to filter chirp waveforms without producing unwanted sidelobes, which is essentially the main aim of this thesis. In last two chapters we present conclusions and future directions respectively.

Acknowledgment

I would like to acknowledge the following for help, encouragement, and support during the preparation of this thesis. First I thank God for giving me the endurance and perseverance to complete this work. My special thanks must go to Dr. Baylie Damtie for inviting me to write this thesis since I came to be interested in space physics. I am also very much grateful for his being devoted almost totally in checking me where I started to simulate the problem and commenced to start writing the thesis at Bahir Dar university. Besides his valuable comments, I am very much grateful to his assistance in making me familiar with LaTeX, in introducing me how to incorporate many of the diagrams and making me friendly with simulation.

I want to express my appreciation to Sintayehu Tesfa at large who first introduced me the course electrodynamics very well and later with whom I enjoyed my undergraduate senior project, relativistic approximation to point charge radiation; which let me to develop enthusiasm to the present study.

I am specially grateful to Dr. Mulugeta Bekele, head of the department of physics at A.A.U., who from the very start arranged conditions for me to work with my supervisor and later helped me a lot in supplying me personal computer access, with out which this work would have been very difficult.

And I finally want to thank my family and friends for cheerfully letting me devote so much time to this work.

It is my wish to dedicate this thesis to my mother Yeshi Gebrie and my father Zerfu Nedie, who have been remarkably patient of my single minded application of my time to this thesis.

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Chapter 1

General introduction

Physics is one of the fundamental sciences that deals with nature. Nature may reveal itself in many ways. Human beings have different discipline to study the different aspects of nature. Perhaps one of the most interesting subject is an electromagnetism. Several people have contributed to the development of electromagnetism. The basis for the theoretical understanding of electromagnetism was laid down by James Clerk Maxwell, whose equations predicted the existence of electromagnetic waves. After some years, Heinrich Hertz verified this theory by his famous experiment. Hertz showed that it is possible to transmit electromagnetic waves without a wire and he has also showed that these waves are reflected from some objects. He suggested that the reflection of electromagnetic waves can be used to identify the presence and also the location of objects. This may be described as essentially the invention of a radar.

However, the development of radar for practical applications has been done mainly in the military. The idea of developing a device that can identify the hostile aircraft has got significant attentions during world war II. Detailed descriptions on the history of radar can be found from [14], [15] and [16].

The performance of modern radars largely depend on the type of waveforms used. Choosing a particular waveform depends heavily on the specific mission and role of the radar. Certain types of waveforms are better suited for measuring how fast an object is travelling, while others have a very good range resolution.

Here we shall assume that the radar employs chirp waveforms. Once we have made such an assumption, one of the possible ways to improve the performance of such a radar is to apply an optimal filtering technique. If one wants to have an optimal signal-to-noise ratio, then matched filtering technique is the obvious choice. Unfortunately, matched filtering technique yields unwanted sidelobes. In this thesis we present a new method of filtering chirp waveforms. The method eliminates the unwanted sidelobes. Other methods of sidelobe suppression of Barker codes can be found in literature [9] and [21].

In this thesis, we start with a broad topic of electromagnetic waves and then go on to present radar fundamentals and finally we narrow down to discuss method of filtering chirp waveforms without producing unwanted sidelobes.

1.1 The discovery of electromagnetic waves

The science of electricity has its roots in the observation, known to Thales of Melitus in 600 B. C., that a rubbed piece of amber attracts bits of straw. On the other hand, the science of magnetism goes back to the observation that naturally occurring stones (magnetite) attract iron. Electricity and magnetism developed quite separately until 1820. However, Hans Christian Oersted in 1820 observed that an electric current in a wire can affect a magnetic compass needle. This observation showed that electric current has magnetic property. Michael Faraday in 1856 showed a relative motion between a bar magnet and a coil induces an electric current in the coil. The work of Christian Oersted, Michael Faraday, Clerk Maxwell and others led to the birth of a subject called electromagnetism.

Especially, the theory of electromagnetism established by James Clerk Maxwell in 1865 is the basis for development of the subject. The theory is expressed in the form of equations, often called Maxwell's equations, and can be described in the absence of magnetic or polarizable media in the following way:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_o}, \tag{1.1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.3}$$

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} + \varepsilon_o \mu_o \frac{\partial \mathbf{E}}{\partial t}, \tag{1.4}$$

where $\mathbf{E}$ is the electric field vector , $\mathbf{B}$ is magnetic induction, $\mathbf{j}$ is current density, ρ is charge density, $\varepsilon_o \mu_o = 1/c^2$. Here c is the speed of light. These laws play the same role in electromagnetism that Newton's laws of motion and of gravitation do in mechanics.

Although Maxwell's synthesis of electromagnetism rests heavily on the work of his predecessors, his own contribution is central and vital. Maxwell deduced that light is electromagnetic in nature and that it's speed can be found by making purely electric and magnetic measurements. Thus the science of optics was intimately connected with those of electricity and of magnetism. Maxwell's equations together with other related scientific works by others led to the development of many modern devices such as radio, television, radar and so on.

1.2 Radar in a nutshell

As we mentioned earlier, a German physicist Heinrich Hertz first has showed that the electromagnetic waves are reflected from objects. Some years later a German engineer Christian Huelsmeyer proposed the use of radio echoes in detecting devices designed to avoid collisions in marine navigation. A successful radio range finding experiment occurred in 1924, when the British physicist Sir Edward Victor Appleton used radio echoes to determine the height of the ionosphere, an ionized layer of the upper atmosphere that reflects radio waves.

Practical radar system was made in 1935 by the British physicist Sir Robert Watson-Watt, and by 1939 England had established a chain of radar stations to detect aggressors in the air or on the sea. In addition to the development carried out in Britain, radar was developed independently in United States, Germany, France, Russia, Italy, and Japan during the middle and late thirties of the 19^{th} century. In other words radar was developed independently and simultaneously in several countries prior to World War II. The development of the fighter airplane and thereby the need to defend against it has contributed significantly for the development of radar.

The invention of radar just prior to World War II changed forever the nature of the warfare. Radar has eliminated many of the limitations that were imposed by the human ability to see. The other interesting development was that during the war radar operators continually observed precipitation, like rain and snow appearing in their radar fields. Scientists had not known that radar would be sensitive enough to detect precipitation. Only during the war did the use of radar to study weather become obvious. Today, radar is an essential tool for analyzing and predicting the weather. Meteorology today relies on radar as an important tool for analyzing and forecasting weather events. For these and many other reasons, radar has grown in prominence, so that today radar is becoming an absolute necessity.

1.3 Fundamentals of radar

Radar has several components. Detailed descriptions about parts of radar can be found from [1]. A simple radar consists of a transmitter and a receiver. In the transmitter the carrier signal and the coding waveform are created and the modulated waveform goes to the antenna, which transmits these waveforms into the region under investigation and a portion of the transmitted signal is intercepted by a target and is reradiated in all directions. It is the signal that is reflected to the direction of the receiver that is of prime interest to the radar.

The receiving antenna collects the the returned signal and delivers it to the receiver, where it is processed. In the receiver the most serious issue is noise. Reflected signals from relatively distant targets have very small amounts of energy which can be swamped by the noise generated with in the receiver and the noise from external background. The power available to the transmitter is a key factor, particularly in reducing noise problems. In addition to noise, a radar system receives reflections from objects which are not important for the operational purpose of the radar. For example, ship born radars, typically receive much of the reflections from the sea surface. This can mask the reflection from target (it renders the radar system ineffective). Such reflections are collectively called clutter.

Hence, much work has to be done on extraction of signals of importance from the clutter. This is usually done by employing an appropriate signal processing techniques.

Filters can be used to remove noise. Noise is a broadband signal. This means that its spectrum is wide. By using filter that allows only the spectrum of the signal to pass through, many regions of the spectrum of noise is removed. Detection of radar reflections in the presence of noise are discussed in many textbooks, for example see [17], [18], and [19].

The amplifier amplifies the radar reflected signal to make it less susceptible to noise in the later parts of the receiver and prepares it for signal processing. Extraction of target information is performed by the signal processor. The target's range is computed by measuring the time delay it takes a pulse to travel the two way-path between the radar and the target. Since electromagnetic waves travel at the speed of light, $c = 3 \times 10^8 m/sec$, then the range R can be calculated by $R = \frac{c\Delta t}{2}$, where Δt is the time it takes a pulse to travel the two way-path between the radar and the target.

It is important to mention that the receiver must be protected from damage caused by the high power of the transmitter. This protection is provided by duplexer. The duplexer also serves to channel the reflected signal to the receiver and not to the transmitter. The duplexer might consist of a TR and an ATR. The TR protects the the reciever during transmission and the ATR directs the reflected signal to the receiver during reception. A mixer is used in the receiver to extract the modulating waveform. This is possible by mixing the received signal and then passing it through a low pass filter. The basic block diagram of a pulsed radar system is shown in Figure 1.1. The basic qualitative relationship between transmitted power, received power and nature of the target can be established by using the radar equation.

1.4 Radar frequencies

Table 1.1. shows the operating frequencies of radar.

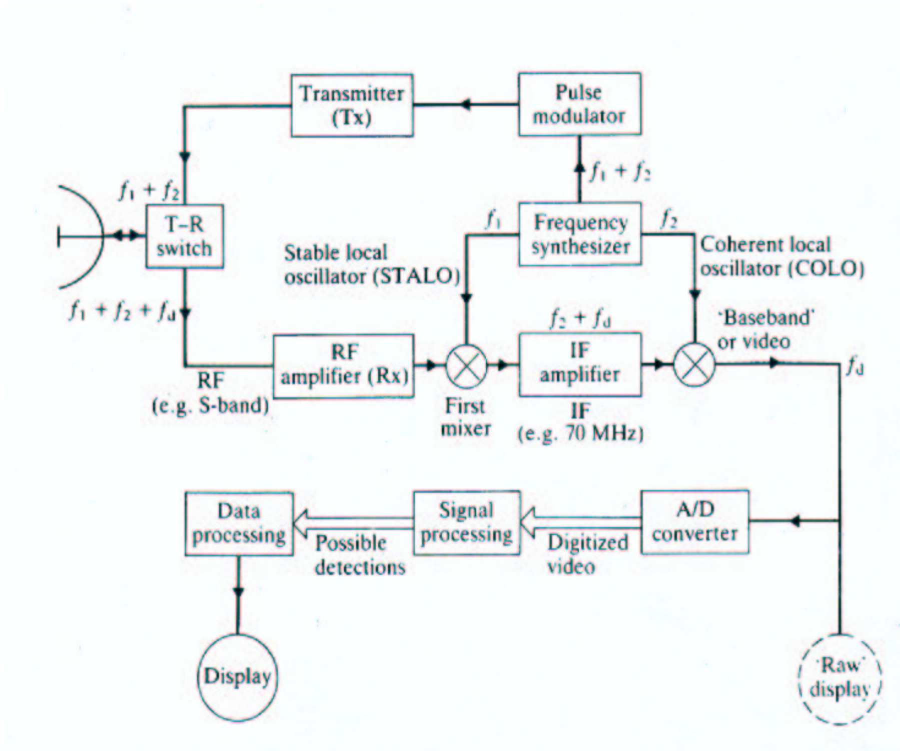

Figure 1.1: A simplified pulsed radar block diagram.

Letter designation	Frequency (GHz)	New band designation (GHz)
HF	0.003 - 0.03	A
VHF	0.03 - 0.3	A$<$ 0.25; B$>$ 0.25
UHF	0.3 - 1.0	B$<$ 0.5; C$>$ 0.5
L- band	1.0 - 2.0	D
S- band	2.0 - 4.0	E$<$ 3.0; F$>$ 3.0
C- band	4.0 - 8.0	$G < 6.0$; H$>$ 6.0
X- band	8.0 - 12.5	I$<$ 10.0; J$>$10.0
Ku- band	12.5 - 18.0	J
K- band	18.0 - 26.5	J$<$ 20.0; K$>$ 20.0
Ka- band	26.5 - 40.0	K
MMW- band	Normally $>$ 34.0	L$<$ 60.0; M$>$ 60.0

Table 1.1. Radar frequencies.

1.4.1 Radar equation

The radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, and target. It is useful not just as a means for determining the distance from the radar to the target, but it can serve both as a tool for understanding radar operation and as a basis for radar design.

The antenna channels or directs the waveform sent with power P_t into some specific direction. The gain G of the antenna is a measure of the increase power of the waveform that is sent in the direction of the target as compared with the signal power that would have been sent from an isotropic antenna. It is given by

$$G = \frac{4\pi A_e}{\lambda^2}, \tag{1.5}$$

where λ is the wavelength of the transmitted radio wave and A_e is the effective area of the antenna. The power per unit area (P_{di}) at the target which is located at a distance R from an antenna with a transmitting gain G is given by

$$P_{di} = \frac{P_t G}{4\pi R^2}. \tag{1.6}$$

The ratio of the amount of the power reradiated back in the direction of the radar to the incident power is denoted as radar cross section, σ. It has the units of area. It is a characteristics of a particular target and is a measure of its size as seen by the radar.

The radar cross section of complex targets such as ships , aircraft, and terrain are complicated functions of the viewing aspect. For example, since a sphere is a sphere from whatever aspect it is viewed, its cross section will not be aspect sensitive. However, the cross section of other objects will depend up on the direction as viewed by the radar. This is to say that the shape of a target greatly determines the radar cross section. For most types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relation ship of the physical area, except that the larger the target size, the larger the cross section is likely to be.

By using the radar cross section in eq.(1.6), we obtain

$$P_{df} = \frac{P_t G \sigma}{(4\pi R^2)^2}, \tag{1.7}$$

where, P_{df} is the power per unit area of the reflected signal. The radar antenna captures a portion of the reflected signal at the location of the receiver. In terms of the effective area of the receiving antenna A_e, the power P_r received by the radar (using eq.(1.7)) is

$$P_r = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4}. \tag{1.8}$$

The maximum radar range R_{max} is the distance beyond which the target can not be detected. It occurs when the received signal power P_r just equals the minimum detectable signal S_{min}. Therefore

$$R_{max} = \left(\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}} \right)^{1/4}. \tag{1.9}$$

This is the basic form of radar equation. Substituting eq.(1.5) in (1.9), we get another form of the radar equation

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right)^{1/4}. \tag{1.10}$$

Note that the radar equation described here is based on a point target. However one can follow an argument similar to this to arrive at the radar equation for an extended target.

1.4.2 Radar ambiguity function

The radar ambiguity function is normally used by radar designers as a means of studying the performance of different waveforms. It can provide insight about how different radar waveforms may be suitable for various applications. This is done by analyzing the radar ambiguity function. A practical application on range ambiguity measurement can be found from [7].

Let us assume that $S(t)$ represents the waveform employed by the radar. The two dimensional (2-D) correlation function for $S(t)$ is given by

$$X(\tau_r; f_d) = \int_{-\infty}^{\infty} S(t')S^*(t' + \tau_r)e^{j2\pi f_d t'}\ dt'. \tag{1.11}$$

The radar ambiguity function for a waveform S(t) is defined as the modulus squared of its 2-D correlation function. More precisely

$$|X(\tau_r; f_d)|^2 = \left| \int_{-\infty}^{\infty} S(t)S^*(t + \tau_r)e^{j2\pi f_d t}\ dt \right|^2, \tag{1.12}$$

where τ_r is the time delay and f_d is the shift in Doppler frequency due to the target motion.

The ambiguity function evaluated at $(\tau_r, f_d) = (0, 0)$ is equal to the filter output that is matched perfectly to the signal reflected from the nominal target of interest. Here the nominal target is assumed to be situated at the origin of the ambiguity function. Thus, the ambiguity function at non zero τ_r and f_d represents the reflected signals from some range and Doppler different from those for the nominal target. The three dimensional(3-D) plot of the ambiguity function versus frequency and time delay is called the radar ambiguity diagram.

Let us now demonstrate how one may visualize the radar ambiguity function by using a rectangular pulse of width τ' and this means that $S(t)$ is given by

$$S(t) = \frac{1}{\sqrt{\tau'}} Rect(\frac{t}{\tau'}), \tag{1.13}$$

where

$$Rect(\frac{t}{\tau'}) = \begin{cases} 1 & \text{if } -\tau'/2 \leq t \leq \tau'/2 \\ 0 & \text{otherwise.} \end{cases} \tag{1.14}$$

Substituting eq.(1.14) into (1.12) and performing the integration yields

$$|X(\tau_r; f_d)|^2 = \left| (1 - \frac{|\tau_r|}{\tau'}) \text{sin} \frac{(\pi f_d(\tau' - |\tau_r|))}{\pi f_d(\tau' - |\tau_r|)} \right|^2, \qquad |\ \tau_r\ | \leq \tau'. \tag{1.15}$$

The plot of eq.(1.15) is shown in Figure 1.2. It is also possible to have a simplified view of the same thing using contour plot (see the bottom panel of Figure 1.2). The ambiguity function cut along the time axis τ_r is obtained by setting $f_d = 0$. More precisely

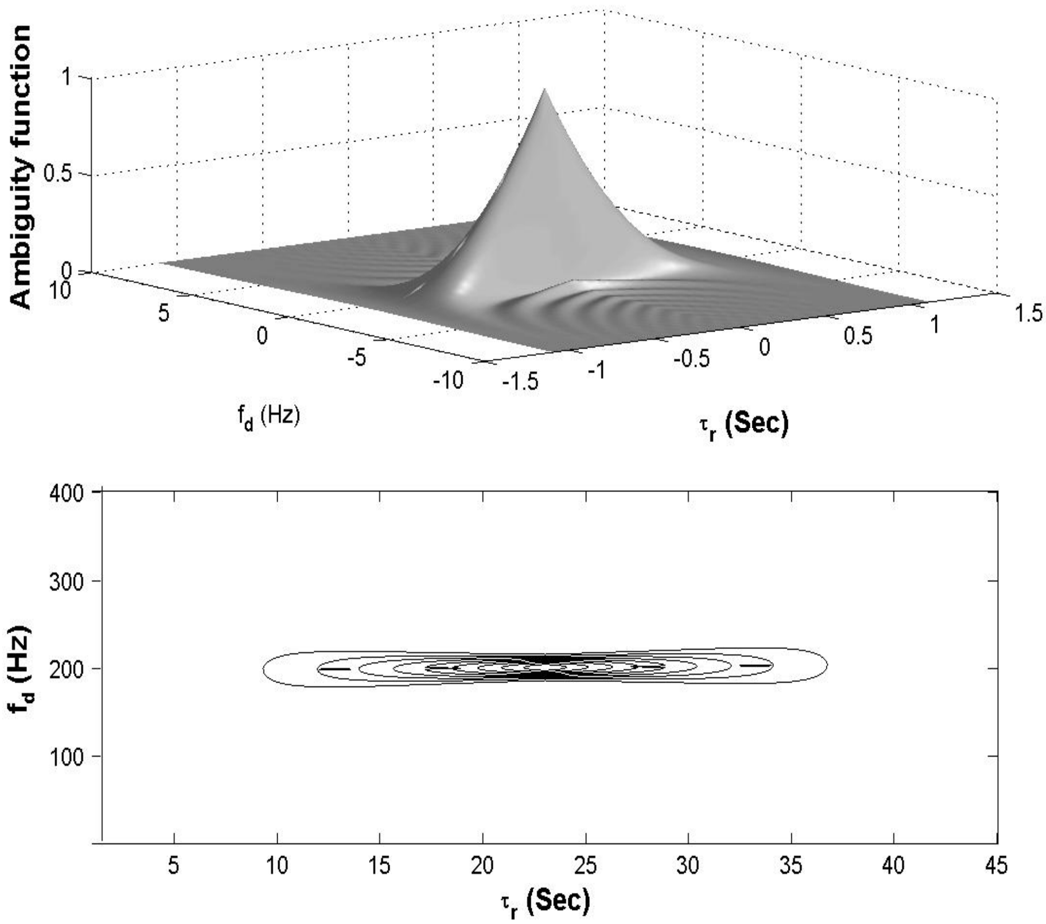

Figure 1.2: Top panel: 3-D ambiguity function plot for a single pulse. Bottom panel: Contour plot corresponding to the 3-D ambiguity plot.

$$|X(\tau_r;0)|^2 = \left|(1-\frac{|\tau_r|}{\tau'})\right|^2, \qquad |\tau_r| \leq \tau'. \tag{1.16}$$

Note that the time autocorrelation function of the signal S(t) is equal to $X(\tau_r;0)$. Similarly the cut along the Doppler axis is

$$|X(0;f_d)|^2 = \left|\frac{sin(\pi\tau' f_d)}{\pi\tau' f_d}\right|^2. \tag{1.17}$$

Figure 1.3 shows that the plots of the ambiguity function cuts defined by eqs.(1.16) and (1.17). From this figure one can visualize that the zero Doppler cut along the time

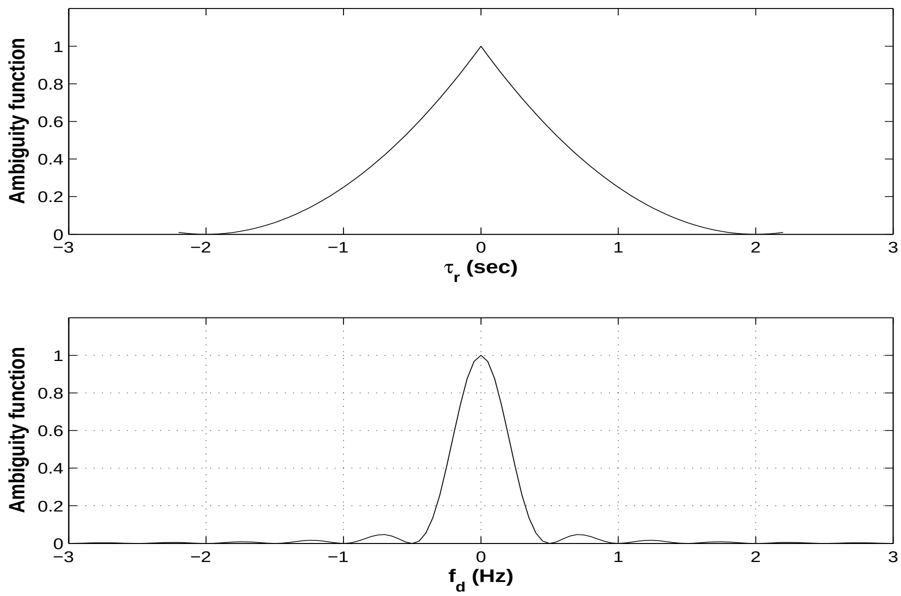

Figure 1.3: Top panel: Zero doppler ambiguity function cut along the time delay axis. Bottom panel: The cut along the doppler axis.

delay axis extends between $-\tau'$ and τ', where here we assumed τ'= 2 sec. This implies that close targets would be unambiguous if they are at least τ' seconds apart. From this figure it can also be inferred that the zero time cut along the Doppler frequency axis has a $(\frac{sin\ x}{x})^2$ shape. It extends from $-\infty$ to ∞. The first null occurs at $f_d = \pm 1/\tau'$. Hence it is possible to detect two targets that are shifted by $1/\tau'$, without any ambiguity.

We conclude that for a single pulse range and Doppler resolutions are limited by the pulse width τ'. Fine range resolutions require that a very short pulse be used. Unfortunately, using very short pulses requires very large operating bandwidths, and may limit the radar average transmitted power to impractical values. The way to get out of this problem is to use a pulse compression technique.

1.4.3 Pulse compression

Utilizing a very short pulse decreases the average transmitted power, which can hinder the radar's normal modes of operation. Since the average transmitted power is directly linked to the receiver SNR, it is often desirable to increase the pulse width (i.e. increase the

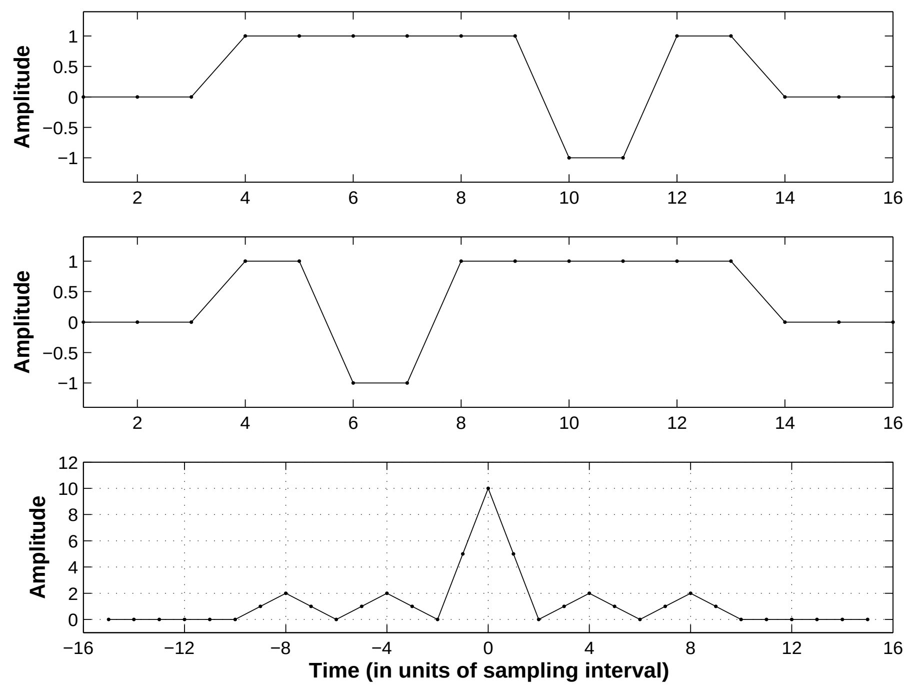

Figure 1.4: Top panel: 5-bit Barker code with 2-samples per bit. Middle panel: Impulse response of the corresponding matched filter. Bottom panel: Output of the matched filtering.

average transmitted power) while simultaneously maintaining adequate range resolution. This can be made possible by using pulse compression technique.

Pulse compression technique allows for the transmission of a low peak power, long duration coded pulse and attain fine range resolution. This is accomplished by widening the band width of the long transmitted pulse by coding it in either phase or frequency, which yields a finer range resolution than can be achieved with a conventional radar system using an uncoded pulse. The range resolution ΔR is given by

$$\Delta R = \frac{c}{2\beta}, \tag{1.18}$$

where β is the bandwidth of the transmitted waveform. The received waveform is processed using a filter that compresses the long pulse to a duration of $\frac{1}{\beta}$. Detailed descriptions about pulse compression are available in [4], [5], and [9]. Let us illustrate pulse compression using a 5-bit Barker coded signal [2]. In the top panel of Figure 1.4 we depict the 5-bit barker code and the middle panel we display the impulse response the filter used for matched filtering. The filter out put is shown at bottom panel of Figure 1.4. The convolution result shows that the 5-bit Barker code in the top panel has been compressed to a pulse of one-bit length.

Chapter 2

Radar waveform analysis

Radar measurement and resolution performance, as well as target detection in clutter, depend largely on the transmitted waveform. For example, see [7] on how a very high range resolution observations of sporadic E-layer can be made using binary phase coded waveform. The choice of a suitable waveform for a particular application is an important problem in radar science. This is so because the waveform controls resolution and clutter performance and also bears heavily on system cost. This explains the sizable efforts that have gone into the studies of radar waveforms, including attempts at the synthesis of optimum waveforms.

How does then one select the waveform that fits a particular application best? A practical approach would be simply to select a signal based on the available results of the waveform theory and design experience, where as at the other extreme one might try to develop a formal theory of waveform synthesis. Radar waveform selection techniques have been discussed by [4].

It is very difficult, if not impossible, to find the best waveform suitable for all radar applications. However, it is possible to find optimum waveforms for specific radar tasks in a given environment. This makes it very important to critically analyze the type of the radar waveform that one uses in a given area of interest. Radar systems can use continuous waveforms or pulsed waveforms with or without modulation. Modulation techniques can be either analogue or digital.

2.1 Continuous waveform radar

Continuous wave (CW) radars utilize continuous waveforms. The transmitter generates a continuous waveform of frequency f_1, which is radiated by the antenna. A portion of the radiated signal is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.

If the target is in motion with a velocity v relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_1 by an amount $\pm f_d$. The plus sign associated with the Doppler frequency applies if the distance between the target and the radar is decreasing (closing target), that is when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target). The received echo signal at a frequency $f_1 \pm f_d$ enters the radar via the antenna and would be analyzed. Thus by measuring the frequency difference CW radars can very accurately extract target radial velocity.

Because of the continuous nature of CW emission, range measurement is not possible without some modifications to the radar operations and waveforms. The modification could be done by interrupting the continuous waveform emission into a relatively short burst of electromagnetic waves, so as to produce pulses of a given duration.

2.2 Pulsed radar

Pulsed radars transmit and receive a train of modulated pulses. Range is extracted from the two-way time delay between transmission and reception. If accurate range measurements are available between the consecutive pulses, then Doppler frequency can be extracted from range rate. This approach works fine as long as the range is not changing drastically over a small time interval. Pulsed radar waveforms may be characterized based on carrier frequency, pulse width, the type of modulation implemented, pulse repetition frequency and so on.

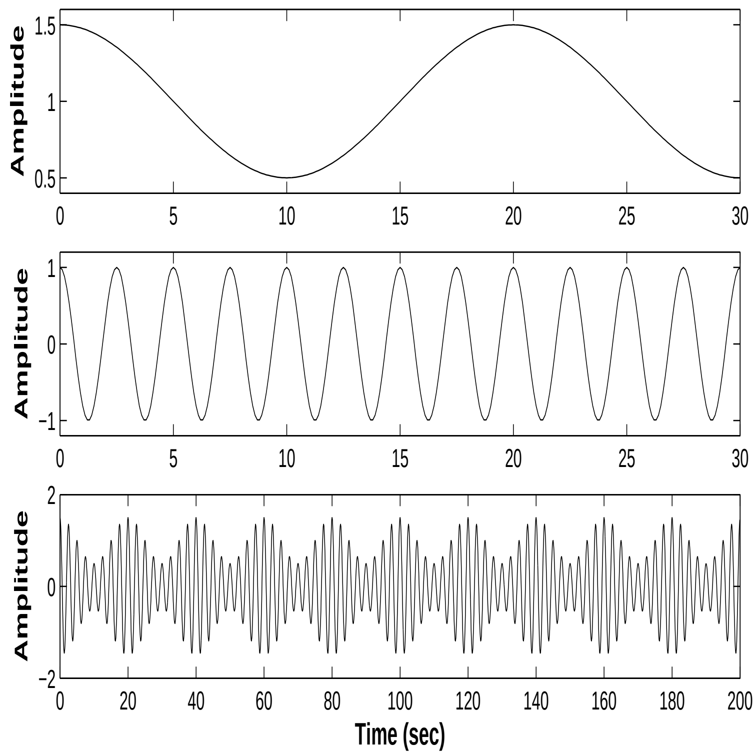

Figure 2.1: Top panel: modulating signal. Middle panel: carrier signal. Bottom panel: part of the modulated signal.

Normally different modulation techniques are used in pulsed radar systems to enhance the radar performance. It is possible to demonstrate how modulation is done. For example let us assume that we want to use the waveform depicted in the top panel of Figure 2.1 as a modulator. When we use the waveform shown in the middle panel as a carrier, we obtain the modulated signal displayed in the bottom panel. In this demonstration, the modulating signal is generated by using a simple signal model $A_o\cos(\omega_o t)$, where $A_o = 1.5$ and $\omega_o = 0.1\pi$. Similarly, the carrier signal is generated from $A_c\cos(\omega_c t)$, where $\omega_c = 0.8\pi$ and $A_c = 1$.

2.2.1 Radar Pulse repetition frequency (PRF)

Pulsed radar systems may employ low, medium, and high PRF schemes. Low PRF waveforms can provide accurate, long, unambiguous range measurements, but exert severe

Doppler ambiguities. Medium PRF measurements resolve both range and Doppler ambiguities; however, they provide adequate transmitted average power as compared to low PRFs. High PRF waveforms can provide superior average transmitted power and excellent clutter rejection capabilities. Alternatively, high PRF waveforms are extremely ambiguous in range. Radar systems utilizing high PRFs are often called Pulsed Doppler Radars (PDR).

2.3 Phase modulation in pulsed radar system

Phase modulation(PM) is usually done by dividing the radar pulse into a set of subpulses of equal duration and the phase of each subpulse is chosen. When two phase values (0^o and 180^o) are used, the modulated waveform is called binary code. Barker codes and alternating codes are examples of binary phase codes, which are widely used by incoherent scatter radar systems.

Barker codes have the property that their autocorrelation functions have a triangular peak around zero time lag, surrounded symmetrically on both sides by a set of smaller triangular peaks of equal height. These codes are frequently used in practice in incoherent scatter radar measurements, for example see [12] and [13]. If a Barker coded signal is filtered by an impulse response which is its mirror image, the resulting waveform contains a sharp peak with a width corresponding to the length of a single modulation element (see Figure 1.4). Hence modulation works approximately as if the transmitted pulse were compressed to the length of a single modulation element conserving the total energy and thus increasing the transmitted power, for more information see [6].

2.4 Frequency modulation

Frequency modulation(FM) is a method of impressing data on to an alternating current(AC) wave by varying the instantaneous frequency of the wave. This scheme can be used with analog or digital data. In analog FM, the frequency of the AC signal wave, also called, carrier, varies in a continuous manner. Thus, there are many possible carrier frequencies. Where as, in digital FM, the carrier frequency shifts abruptly, rather than varying continuously. In [20] one can find more about frequency modulation and the corresponding pulse compression.

Frequency modulation is similar in practice to phase modulation. When the instantaneous frequency of a carrier is varied, the instantaneous phase changes as well. The converse also holds: when the instantaneous phase is varied, the instantaneous frequency changes. But FM and PM are not exactly equivalent. When an FM receiver is used to demodulate a PM signal, or when an FM signal is intercepted by a receiver designed for PM, for example the audio is distorted in the case of audio receivers. This is because the relationship between frequency and phase variations is not linear; that is, frequency and phase does not vary in a direct proportion.

Chapter 3

Chirp waveform

3.1 Introduction

Linear frequency modulated waveforms(LFM) are commonly used to achieve much wider operating bandwidths. In this case the frequency is swept linearly across the pulse width, either upward (up-chirp) or downward (down-chirp). The matched filter band width is proportional to the sweep band width, and is independent of the pulse width.
The LFM up-chirp instantaneous phase can be expressed by

$$\varphi(t) = 2\pi(f_o t + \frac{\mu}{2}t^2), \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}, \tag{3.1}$$

where f_o is the initial frequency, and $\mu = \frac{2\pi B}{\tau'}$ is the LFM coefficient for band width B. Thus, the instantaneous frequency is

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\varphi(t) = f_o + \mu t, \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}. \tag{3.2}$$

Similarly, the down-chirp instantaneous phase and frequency are given, respectively, by

$$\varphi(t) = 2\pi(f_o t - \frac{\mu}{2}t^2), \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}, \tag{3.3}$$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\varphi(t) = f_o - \mu t, \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}. \tag{3.4}$$

Figure 3.1 shows a typical example of a chirp waveform that one finds by setting $\mu = 3$ Hz/s and zero initial frequency. It shows that frequency increases linearly between zero

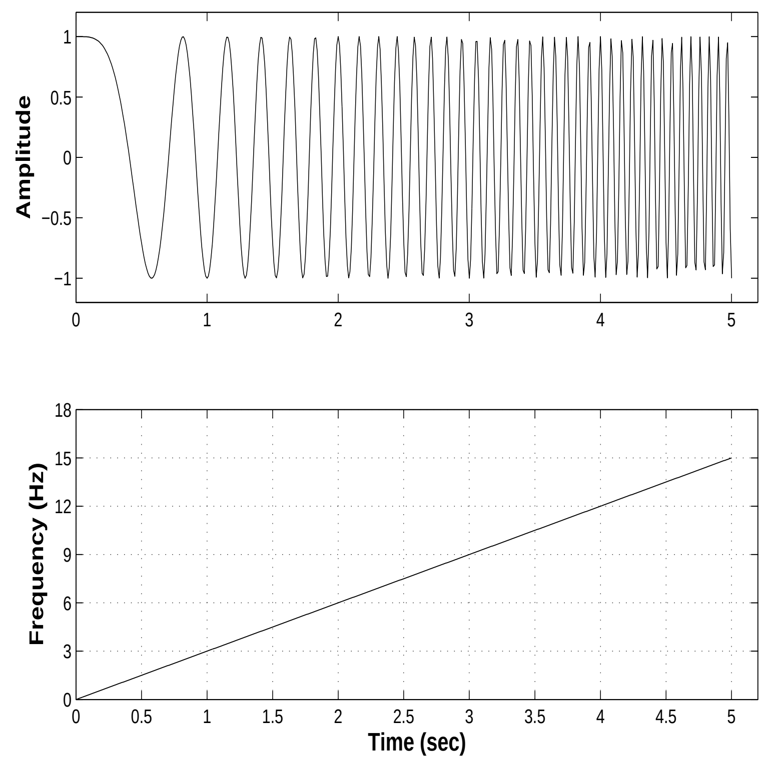

Figure 3.1: Top panel: Time-amplitude behavior of a chirp waveform. Bottom panel: Time-frequency behavior of a chirp waveform.

and 15 Hz. Assuming that the initial frequency is zero, let us take a LFM waveform, $S_1(t)$, which is given by

$$S_1(t) = \frac{1}{\sqrt{\tau'}} Rect(\frac{t}{\tau'}) e^{j\pi\mu t^2}. \tag{3.5}$$

Combining eq.(1.12) and eq.(3.5), the radar ambiguity function of a chirp waveform becomes

$$|X(\tau_r; f_d)|^2 = \left| (1 - \frac{|\tau_r|}{\tau'}) \frac{sin(\pi\tau'(\mu\tau_r + f_d)(1 - \frac{|\tau_r|}{\tau'}))}{\pi\tau'(\mu\tau_r + f_d)(1 - \frac{|\tau_r|}{\tau'})} \right|^2, \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}, \tag{3.6}$$

equation (3.6) can be understood from the 3-dimensional frequency-time plot of the ambiguity function shown in the top panel of Figure 3.2. It is also possible to have a simplified view of the same thing using contour plot (see the bottom panel of Figure 3.2).

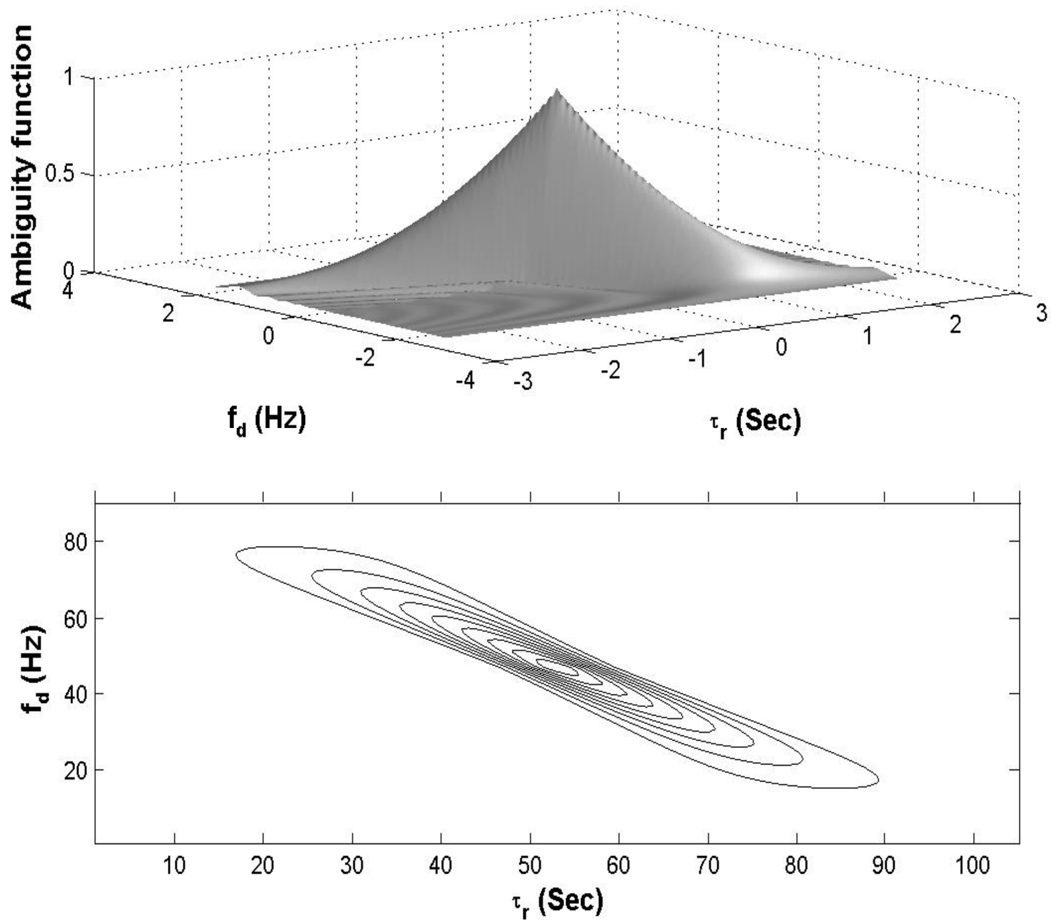

Figure 3.2: Top panel: Up-chirp LFM 3-D ambiguity plot. Bottom panel: Contour plot corresponding to the LFM 3-D ambiguity plot.

The transmitted up-chirp waveform $\varphi(t)$ can be represented by

$$\varphi(t) = cos[\omega_o t + \frac{\mu}{2}t^2], \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}, \tag{3.7}$$

where $\omega_0 = 2\pi f_o$. If matched filter is used in the receiver, then it will have an impulse response $h_t(t)$ that is the time inverse of the signal at the receiver input. Thus

$$h_t(t) = \sqrt{\frac{2\mu}{\pi}}\; cos[\omega_o t - \frac{\mu}{2}t^2], \qquad \frac{-\tau'}{2} \leq t \leq \frac{\tau'}{2}, \tag{3.8}$$

where $\sqrt{\frac{2\mu}{\pi}}$ is the factor that gives the filter unity gain.

The output of the matched filter is then obtained by convolving $\varphi(t)$ and $h_t(t)$ yielding

$$g(\tau_r) = \int_{-\infty}^{\infty} \varphi(t) h_t(t - \tau_r) dt. \quad (3.9)$$

When $\varphi(t)$ and $h_t(t)$ are matched, $g(\tau_r)$ represents the autocorrelation function of input signal.

If the signal and the filter are not matched, then $g(\tau_r)$ represents the cross correlation of the two functions. In the case of general interest that is when the target is moving, by virtue of the Doppler signal shift(ω_d) the received signal mismatches to the filter. For the case at hand we may take the Doppler shifted signal $\varphi_s(t)$ as

$$\varphi_s(t) = cos[(\omega_o + \omega_d)t + \frac{\mu}{2}t^2]. \quad (3.10)$$

Hence the most general output of the matched filter is obtained by evaluating

$$g(\tau_r, \omega_d) = \sqrt{\frac{2\mu}{\pi}} \int_{-\frac{\tau'}{2}}^{\frac{\tau'}{2}} cos[(\omega_o + \omega_d)t + \frac{\mu}{2}t^2] cos[\omega_o(\tau_r - t) - \frac{\mu}{2}(\tau_r - t)^2] dt, \qquad \tau_r > 0. \quad (3.11)$$

3.2 Traditional method of filtering chirp waveforms

Matched filtering technique is used traditionally in filtering chirp waveforms. This means that the impulse response of the filter is a replica of the time inverse of the chirp. Hence, the impulse response $h_t(t)$ is obtained from

$$h_t(t) = \varphi(-t), \quad (3.12)$$

where $\varphi(-t)$ is the replica of the time inverse of the chirp. The convolution of $\varphi(t)$ and $h_t(t)$ is given by

$$w_t(t) = h_t(t) * \varphi(t), \quad (3.13)$$

where $w_t(t)$ denotes the matched filter output.

In Figure 3.3, we demonstrate how matched filtering of a chirp waveform is carried out. The chirp waveform, the form of matched filter implemented, and the filter output (convolution result) are shown in the top, middle and bottom panel, respectively. From this figure one can see that matched filtering technique produces unwanted sidelobes at

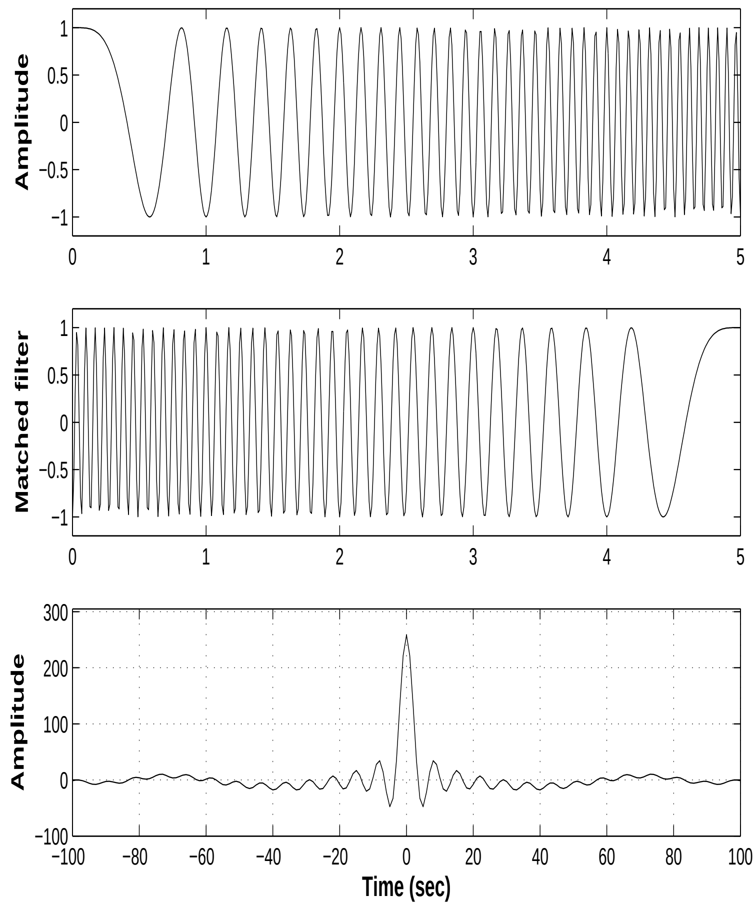

Figure 3.3: Top panel: Time- amplitude behavior of chirp waveform. Middle panel: The impulse response of the matched filter. Bottom panel: Convolution result.

the output of the receiver. In some cases, this makes it difficult to determine whether the scattering is coming from the main peaks or from the sidelobes. In other words there will be ambiguity in the measurement. In this thesis we present a method of suppressing these unwanted sidelobes. The method is demonstrated theoretically and it is based on an earlier work that has been used in filtering binary phase coded waveforms without producing unwanted sidelobes [2].

3.3 Method of suppressing sidelobes

In the traditional method of filtering chirp waveform, one employs matched filtering techniques. This means that the impulse response of the filter is just the inverse replica of the chirp waveform. As shown earlier, this method produces unwanted sidelobes at the output of the receiver. These sidelobes may create problems in some applications. Here we present a method of suppressing these sidelobes.

Let us denote the discrete chirp waveform by $\varphi(n)$. In the traditional filtering method, the impulse response of the filter is given by

$$h_t(n) = \varphi(-n) \tag{3.14}$$

Hence, the convolution result is obtained from

$$w_t(n) = \varphi(n) * h_t(n) \tag{3.15}$$

Here $w_t(n)$ has one peak with other unwanted sidelobes (see Figure 3.3) for example.

The main aim of the present thesis is to find out a method of suppressing these unwanted sidelobes in $w_t(n)$. Let us denote the impulse response of the filter that suppresses these sidelobes by $h(n)$. Then, we can have

$$h(n) * \varphi(n) = w(n) \tag{3.16}$$

where $w(n)$ is the sidelobe free output and it just has one peak with no sidelobes. In other words $w(n)$ is obtained form $w_t(n)$ by removing the sidelobes.

Equation (3.16) can be calculated conveniently by means of Discrete Fourier Transform (DFT). Therefore, it can be rewritten as

$$H(k)\boldsymbol{\varphi}(k) = W(k), \tag{3.17}$$

where

$$H(k) = \sum_{n=0}^{N-1} h(n)e^{-jnk\frac{2\pi}{N}}, \tag{3.18}$$

$$\boldsymbol{\varphi}(k) = \sum_{n=0}^{N-1} \varphi(n)e^{-jnk\frac{2\pi}{N}}, \tag{3.19}$$

and

$$W(k) = \sum_{n=0}^{N-1} w(n) e^{-jnk\frac{2\pi}{N}}. \tag{3.20}$$

Here $k = 0, 1, ..., N-1$, that is we choose N number of samples equally spaced in a desired range from zero to the sampling frequency, namely 0, $\frac{2\pi}{N}$, $2(\frac{2\pi}{N})$,...,$(N-1)(\frac{2\pi}{N})$ radians per sample. This means that the transfer function $H(k)$ is obtained from

$$H(k) = \frac{W(k)}{\boldsymbol{\varphi}(k)}. \tag{3.21}$$

Filters calculated from eq.(3.21) can suppress the sidelobes. The drawback for this kind of filter is that the DFT of the chirp waveform should not have zeroes. Chirp waveforms that satisfy the requirements of eq.(3.21) can be found by searching algorithm.

Let us illustrate the present method with examples. In the late side of Figure 3.4, the top panel shows $\varphi(n)$, the middle panel displays $h_t(n)$ and bottom panel shows the convolution result $w_t(n)$. In the bottom panel one can see that there are sidelobes that may create problems in certain application.

In the right side of Figure 3.4, the top panel shows $\varphi(n)$, the middle panel portrays $h(n)$ and at the bottom panel we show the convolution result $w(n)$ (i.e. $\varphi(n) * h(n)$). One can see clearly that the unwanted sidelobes have been suppressed.

3.4 Discussions

It is a common knowledge that using matched filter yields the maximum signal to noise ratio. This means that the traditional method of filtering chirp waveform is the optimal filtering technique. However, the unwanted sidelobes that appear in the output may create a major problem in some applications. In such circumstances, the method presented here can be employed.

The drawbacks associated to the sidelobe free filtering method presented here are distortion in the SNR performance and the specific requirement about the type of chirp waveform, which can be filtered by the method. The SNR performance of the filter designed by the present method is less than the traditional method. A comparison has

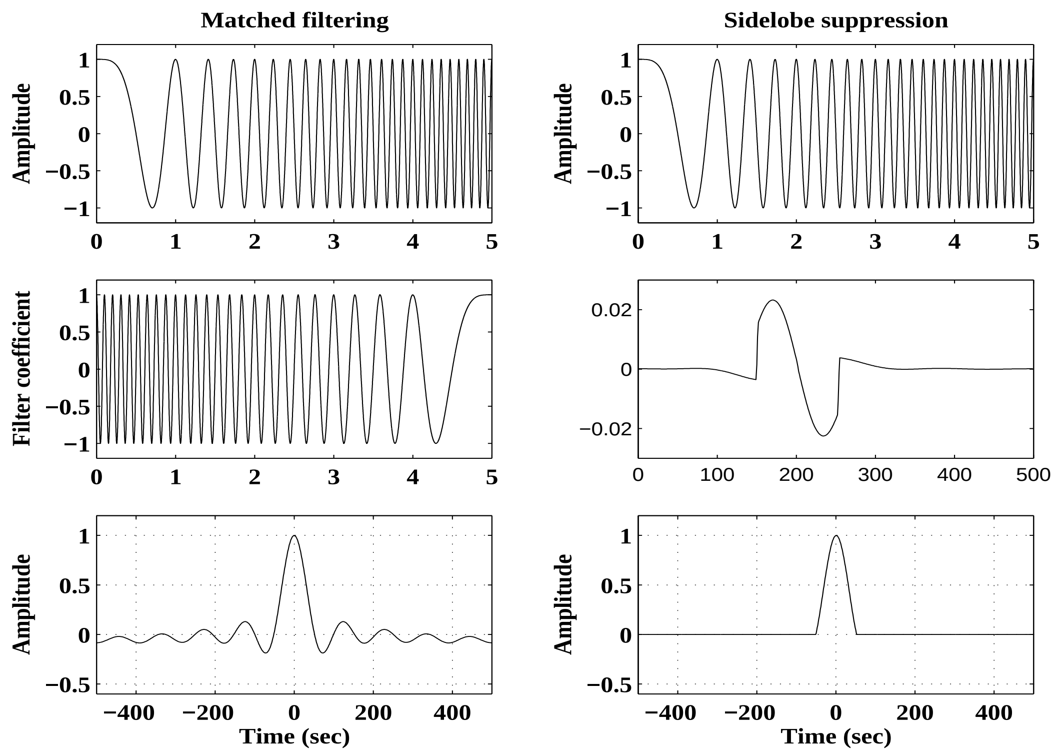

Figure 3.4: A chirp waveform of $\mu = 2$ and zero initial frequency (top:left and right panels). Left: The impulse response of the corresponding matched filter (middle panel) and the matched filter convolution output (bottom panel). Right: The impulse response of the sidelobe free filter (middle panel) and the sidelobe free convolution output (bottom panel).

been made for binary phase codes by [2]. The result shows that the method works better in some codes than others. A similar investigation can be carried out for chirp waveform in terms of SNR.

The other drawback is that the Discrete Fourier Transform (DFT) of the chirp waveform should be non zero. This means that one should search for chirp waveforms that have non zero Discrete Fourier Transform. The search can be made by developing a fast searching algorithm. However, in some cases, the search may be expensive in terms of computer time.

Chapter 4

Conclusions

Unwanted sidelobes can create major problems in some radar applications. For example, when we have good scatter of radio waves immersed in the ionosphere, ionospheric observations by means of radar waveforms that produce side lobes at the filter out may not be optimal. Contributions from the sidelobes may obscure the nominal observation. In this thesis we have presented method of filtering chirp waveforms without creating unwanted sidelobes. This is possible by formulating the transfer function for the filter in terms of the Fourier transform of chirp waveform and the desired shape of the signal at the filter output. The method demands that the Fourier transform of the chirp waveforms should be nonzero. It is worth mention in these conclusions that the present method has poor SNR performance when compared to the matched filtering method.

Chapter 5

Future directions

Chirp waveforms that can be filtered by the method presented in this thesis should have nonzero Fourier transform values. This means that one needs to search for chirp waveforms that satisfy this requirements. Developing an optimal searching algorithm and implementing it in a computer can be carried out as a continuation of the present thesis. The search will be made by investigating many types of chirp waveforms.

One can also carry out research on what types of radar applications the present method is superior than other methods. This will lead to investigation on drawbacks and advantages of filtering chirp waveforms by the present method. The deterioration in SNR performance in the case of sidelobe free filtering method should be weighted against the advantages obtained by removing ambiguity in the measurements. The results will depend on the specific applications. In some applications the SNR may be a major concern and in some other application it may be the ambiguity.

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Declaration

I declare that the thesis is my original work and has not been presented for a degree in any other university, and that all sources of material have been duly acknowledged.

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This thesis has been submitted for examination with my approval as a university advisor.

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