GIANT QUANTUM OSCILLATIONS OF ULTRASONIC ABSORPTION BY METALS IN THE PHASE TRANSITION OF ORDER 2½

by

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C O N T E N T S

LIST OF FIGURES

Page

ABSTRACT

The absorption coefficient Γ of a normal metal like any one of the thermodynamic and kinematic properties of the metal greatly depends upon the state in which the metal is found. At low enough temperatures and the metal going through electronic phase transition, the absorption coefficient under the action of an external magnetic field is shown to assume different values in different regions of Z, the parameter of PT-2½. In PT-2½ assoiated with appearance of a new spheroidal cavity, it is found to be low and monotonous below a certain threshold value Z^* while shows Giant Quantum Oscillations (GQO) with exponentially rising amplitude for values $Z \gg Z^*$, the conditions of observation of GQO being $k_B T \ll h\Omega \ll |Z|$.† In PT-2½ associated with breaking of the Fermi neck the absorption coefficient shows strict dependence on the angle of orientation of the magnetic field (θ). Near $\theta \approx 0$, it is found to behave in the same way as in the appearance of a spheroidal cavity whereas near a critical value of the angle θ_{cr} it shows a sharp anisotropy of the root - type $(\theta_{cr}-\theta)^{-\frac{1}{2}}$.

† Wherever h appears as a symbol here after in this manuscript it must be read as $\hbar$.

INTRODUCTION

The energy spectrum of the electrons of crystalline metal has band character which results from its periodic nature in momentum space. At absolute zero, metals consist of partially filled bands, called conduction bands, which are responsible for the metallic properties of the metal; and in this ground state (T=0) the electrons occupy all states less than the Fermi energy, the surface of this constant energy being the Fermi surface. Within a given band of energy spectrum there are several singular points with critical values of energy ε_k at which the topology of the Fermi surface suffers changes and this occurs whenever under the action of external uniform pressure or of impurity the Fermi level approaches the critical values of energy ε_k (i.e. whenever $\varepsilon_F - {}_k$ passes through zero) [1,2]. A change in the topology of the Fermi surface may be that a new split - off Fermi sheet appears (disappears) or a connecting neck is broken; and this commonly results in singularity of the density of states $\nu(\varepsilon)$ given by

$$\nu(\varepsilon) = \nu_0 \ \varepsilon + \delta\nu(\varepsilon) ,$$

where $\nu_0(\varepsilon)$ is a smooth function of the energy and $\delta\nu(\varepsilon)$ is singular. The singularity of density of states is of the root-type, usually termed as a van Hove - type singularity [3] and is generally given by:

$$\delta\nu(\varepsilon) = \begin{cases} 0 & (\varepsilon < \varepsilon_k) \\ \frac{\sqrt{2}}{\pi^2}\,\frac{V}{h^3}\,(\varepsilon - \varepsilon_k)^{\frac{1}{2}}\,\sqrt{(m_1 m_2 m_3)} & (\varepsilon \geq \varepsilon_k) \end{cases}$$

written under the assumption that the number of sheets increases with increasing energy. V is the volume of the specimen. The singularities of the spectral density $\nu(\varepsilon)$ and the dynamics of the electrons near the critical surface lead to peculiar anomalies in the thermodynamic and kinetic characterstics of the electron gas in a metal, the absorption coefficient of ultrasonic wave being one. Take, for instance, this other case of the thermodynamic potential (Ω) of which many physical properties are derivatives. It can easily be shown that at absolute zero the anomaly of Ω is given by

$$\delta\Omega = \begin{cases} 0 & (\varepsilon < \varepsilon_k) \\ -4/15\,\alpha\,|Z|^{5/2} & (\varepsilon \geq \varepsilon_k) \end{cases}$$

where, $\alpha = \frac{\sqrt{2}}{\pi^2}\,\frac{V}{h^3}\,\sqrt{(m_1 m_2 m_3)}$

and $Z = \varepsilon_F - \varepsilon_k$

Note that, the second derivative of the thermodynamic potential has a singularity proportional to $Z^{\frac{1}{2}}$ while the third derivative has a singularity proportional to $Z^{-\frac{1}{2}}$.

Thus the anomalies of the thermodynamic potential at $Z = o$ carry the name "electronic phase transitions" or "phase transitions of order 2½".

At low enough temperatures absorption of short wave-length sound ($k\ell \gg 1$), where k is the wave - vector of the incident sound and ℓ is the mean - free path of electrons, by a metal can be considered as purely the direct absorption of phonons by the electrons [4] of the metal and the coefficient of absorption for such a case is quite sensitive [5-8] to a phase transition of order 2½ or simply to the parameter Z. The present work is devoted to the investigation of the behaviour of the absorption coefficient by a metal in a magnetic field in the case of electronic phase transition which results from a change in the topology of the Fermi surface for the cases of appearance (disappearance) of a spheroidal cavity and the rupture of a connecting neck.

CHAPTER I

ATTENUATION OF SOUND BY METALS

1.1. Ultrasonic absorption in metals

The most essential mechanism of abosrption of ultrasonic wave in normal metals comes from the interaction of conduction electrons with elastic oscillations of the lattice (phonons). An ultrasonic wave being an intense flow of coherent phonons with the same frequencies and wave vectors, when incident on a metal creates a sequence of compressed and rarefied regions in the lattice moving with the velocity of sound. This sequence of compressed and rarefied regions on the other hand results in a periodic variation of the density of space charge moving along with the wave, which is explained by displacement of valence electrons from regions of compression (i.e. regions of higher density of ion cores) into those rarefied. The resulting positive charge in the compressed regions and negative one in the rarefied regions establishes a periodic distribution of the electric field which remains practically fixed at each instant of time, since, of course, $V_e \gg s$. Here V_e and s are the values of the electron and sound velocities respectively. Being accelerated in this electric field, electrons are then scattered on phonons and thus transfer to the lattice their energy gained from the ultrasonic wave. Energy is also partly lost as Joule's heating $\vec{J}_e \cdot \vec{E}$. This in essence is the electron mechanism of absorption of the energy of the incident ultrasound.

The attenuation of sound in metals depends on the magnitude of the ratio λ/ℓ of the wave length of sound and the electronic mean free path as well as on $\omega\tau$, τ being the relaxation time of electrons and ω the frequency of the incident sound. In case of when $\lambda \ll \ell$ or equivalently $\omega\tau \gg 1$ the attenuation may be thought of to be carried out by absorption and emission of single quanta of sound energy. Thus in this limit of high frequency (i.e. $k\ell \gg 1$) sound attenuation is a pure quantum process; and the process of absorption is subject to the conservation of energy and momentum principles as given by:

$$\vec{p}\,' = \vec{p} + h\vec{k} = \vec{p}+\vec{q}$$

and $$\varepsilon(\vec{p}\,') = \varepsilon(\vec{p}) + h\omega \qquad (1.1.1)$$

where, $\vec{k}$ is the phonon wave - vector and $\vec{q}$ the phonon momentum. From (1.1.1) follows

$$\varepsilon(\vec{p}+h\vec{k}) = \varepsilon(\vec{p}) + h\omega$$

which after expansion in powers of hk (assuming $|h\vec{k}| \ll |\vec{p}|$) delivers

$$\varepsilon(\vec{p}+h\vec{k}) \simeq \varepsilon(\vec{p}) + h\vec{k}.\frac{\partial\varepsilon}{\partial\vec{p}}$$

$$= \varepsilon(\vec{p}) + h\vec{k}.\vec{V}_e$$

Comparison of this last equation with (1.1.1) shows that $\omega = \vec{k}.\vec{V}_e$ which using the identities $\omega = 2\pi f$ and $s = f\lambda$ may be dissolved into

$$s = V_e \cos\theta \qquad (1.1.2)$$

When seen in light of the fact that at temperatures near absolute zero much of the interaction of a normal metal with its surroundings comes due to the activity of its conduction electrons on the Fermi surface, Eq. (1.1.2) suggests that resonant absorption of phonons is only due to such electrons on the Fermi surface which move in phase with the sound wave. Since $V_e = V_F \gg s$, V_F being the Fermi velocity, these electrons move practically in the plane of constant phase of the wave at an angle close to 90 degrees to the vector $\vec{k}$ (the direction of motion of these electrons making an angle of the order of s/V_F with the constant phase plane) (Fig. 1) .

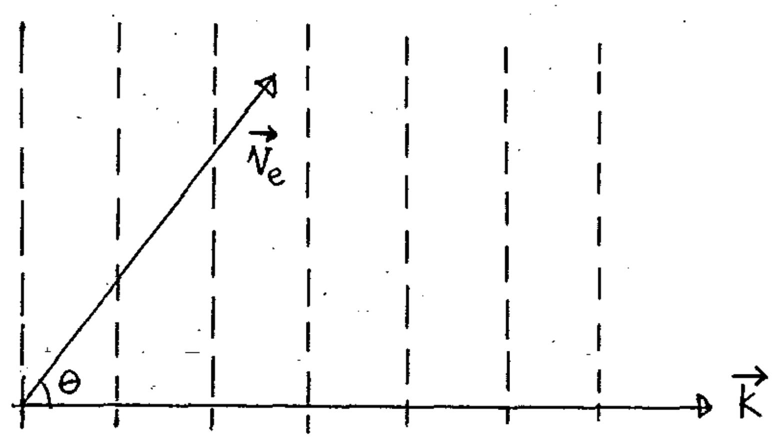


Fig. 1. Relation between direction of incident sound and Fermi velocity of electrons involved in absorption.

For incident phonon energy which normally is quite insignificant and may be taken as only a small periodic perturbation the absorption coefficient under the case discussed above can be determined using time - dependent perturbation theory. The problem actually may be put as that, being given the complete set of solutions to the unperturbed Schrödinger equation

$$H_o\phi_n = E_n^o\phi_n \quad , \qquad (1.1.3)$$

where ϕ_n are Bloch - waves in the crystal, we try to find $\psi(t)$ satisfying the equation

$$ih \frac{\partial\psi\ (t)}{\partial t} = |H_o + \lambda V(t)|\psi(t). \qquad (1.1.4)$$

Here V(t) is the periodic perturbing incident sound energy. The standard procedure of solving this equation begins by expanding $\psi(t)$ using a complete set of functions as

$$\psi(t) = \sum_n C_n\ (t)\quad \phi_n\ \exp(-iE_n^o\ t/h)\ . \qquad (1.1.5)$$

Substitution of (1.1.5) into (1.1.4) together with (1.1.3) delivers

$$\sum_n \frac{\partial C_n}{\partial t}\phi_n\ \exp(-iE_n^o\ t/h) = \lambda\sum_n\ V(t)C_n\ \phi_n\ \exp(-iE_n^o t/h)$$

Multiplying this by ϕ_m^*, integrating the result over all space and using the orthonormality of Bloch - functions

$$< \phi_m | \phi_n > = \delta_{mn}$$

we finally arrive at

$$ih \frac{\partial c_m}{\partial t} = \lambda \sum_n C_n < \phi_m | V(t) | \phi_n > \exp i(E_m^0 - E_n^0)t/h \quad (1.1.6)$$

where, $\int \phi_m^* V(t) \phi_n \, d^3r = < \phi_m | V(t) | \phi_n >$

We shall solve (1.1.6) to first order in the parameter λ .
As an initial condition at t=0 we take the system to be in a particular state ϕ_k, as that $\psi(0) = \phi_k$ which when referred to (1.1.5) gives:

$$C_{nk}(0) = \delta_{nk}$$

where δ_{nk} is the delta Kronecker symbol.
Since departures from these values at latter times will depend on λ, we may, for first - order calculation, substitute this into RHS of (1.1.6). This yields the differential equation (for $m \neq k$)

$$ih \frac{\partial C_m}{\partial t} = \lambda < \phi_m | V(t) | \phi_k > \exp i(E_m^0 - E_k^0)t/h$$

which when solved yields

$$C_m(t) = \frac{\lambda}{ih}\int_0^t dt'\langle\phi_m|V(t')|\phi_k\rangle \exp i(E_m^0 - E_k^0)t/h$$

The probability that at a later time t, the state $\psi(t)$ is an eigen state of H_0 with energy E_n^0, that is, that it is ϕ_n is, according to the expasion postulate

$$P_n(t) = |\langle\phi_n|\psi(t)\rangle|^2 = |C_n(t)|^2$$

So, the probability of transition from the initial state k to the final state m is given by

$$P_{k\to m} = \frac{1}{h^2}\left|\int_0^t dt'\langle\phi_m|U|\phi_k\rangle \exp i(E_m^0 - E_k^0)t/h\right|^2 \quad (1.1.7)$$

where $\lambda V(t')$ is replaced by U.

The energy operator U describing an electron in an external periodic time - dependent field is of the form

$$U = \tfrac{1}{2}(ue^{-i\omega t} + \overset{+}{u}e^{i\omega t}) \qquad (1.1.8)$$

where, $u = \lambda_{ij}(\vec{p})\, u_{ij}(\vec{r},t)\, e^{i\vec{k}.\vec{r}}$

Here the deformation tensor u_{ij} is given by

$$u_{ij} = \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \text{ which is periodic,}$$

u_i being the displacement field of the medium. λ_{ij} is the tensor of deformation potential. The first term in (1.1.8) refers to the absorption of phonon energy while the second term refers to emission. That means (1.1.7), the probability of transition after absorption of a phonon of energy from the incident sound wave, can be rewritten as:

$$P_{k\to m} = \frac{1}{2h^2} \left|\langle\phi_m|u|\phi_k\rangle\right|^2 \left|\int_0^t dt' \; e^{i\Delta t'}\right|^2$$

where, $$\Delta = \frac{E_m^O - E_k^O - h\omega}{h} \, .$$

It can be shown that

$$\left|\int_0^t dt' \; e^{i\Delta t'}\right|^2 = \frac{4}{\Delta^2} \sin^2 \left(\frac{\Delta t}{2}\right) \qquad (1.1.9)$$

Note that $\lim_{t\to\infty} \frac{\sin^2 \alpha t}{\pi t \alpha^2}$ is the Dirac delta function $\delta(\alpha)$

for, when $\alpha \neq o$ this limit is zero while when $\alpha = o$ it is infinite; and moreover when integrated over α in the region $(-\infty,\infty)$ it delivers unity. Thus for large enough t (1.9) yields

$$\frac{4}{\Delta^2} \sin^2 \left(\frac{\Delta t}{2}\right) = 2\pi t \; \delta(\Delta) \, .$$

The probability of transition per unit time may then be rewritten as

$$P_{k\to m} = \frac{\pi}{h} \left|\langle\phi_m|u|\phi_k\rangle\right|^2 \delta(E_m^O - h\omega - E_k^O)$$

or even as

$$P_{\vec{p}\to\vec{p}'} = \frac{\pi}{h} \left|U_{\vec{p}\vec{p}'}\right|^2 \delta(h\omega_{\vec{p}\vec{p}'} - h\omega)$$

where, $\vec{p}$ and $\vec{p}\,'$ refer to initial and final momenta of the electron.

This probability has to be multiplied by $(f_{(\vec{p})} - f_{(\vec{p}\,')})$, where $f_{(\vec{p})}$ is the Fermi-Dirac distribution, to accomodate the case of transitions in inverse directions. Hence, the rate of ultrasonic energy absorption per unit volume and per unit time will be given by:

$$\Omega = \frac{1}{V_O} \sum_{\vec{p}\vec{p}'} \frac{\pi}{h} h\omega_{\vec{p}\vec{p}'} \left|U_{\vec{p}\vec{p}'}\right|^2 \delta(h\omega_{\vec{p}\vec{p}'} - h\omega) \; (f_{(\vec{p})} - f_{(\vec{p}')})$$

from which the coefficient of ultrasonic absorption (Γ) follows by dividing this by the flux $= \frac{1}{2} \rho A^2 \omega^2 s$

$$\Gamma = \frac{2\pi\omega\left|U_O\right|^2}{V_O \rho A^2 \omega^2 s} \sum_{\vec{p}\vec{p}'} \delta(h\omega_{\vec{p}\vec{p}'} - h\omega)\,(f_{(\vec{p})} - f_{(\vec{p}\,')}) \qquad (1.1.10)$$

where, ρ is the density of the crystal, A is the amplitude of oscillations in the sound wave and s is the group velocity of sound.

In (1.1.10), $|U_o|^2$ stands for $|U_{\vec{p}\vec{p}'}|^2$ which can be considered as a constant matrix due to the limitation in the region of interest, namely, the region near and around the critical value of momentum. The summation of (1.1.10) has to be carried out over all quantum states satisfying (1.1.1); and since without a quantizing field in action the summation may be replaced by integration using the relation

$$dN_p = \frac{2V_o}{(2\pi h)^3} d^3p$$

from ,(1.1.10) we get

$$\Gamma = \int |M_o|^2 \, \delta(\varepsilon_{(\vec{p})} + h\omega - \varepsilon_{(\vec{p}+\vec{q})}) (f\varepsilon_{\vec{p})} - f(\varepsilon_{\vec{p}} + h\omega)) d^3p \quad (1.1.11)$$

where, $|M_o|^2$ is a constant matrix given by:

$$|M_o|^2 = \frac{4\pi |U_o|^2 \omega}{\rho A^2 \omega^2 V_s (2\pi h)^3}$$

Eq. (1.1.11) is the quantum formula for the coefficient of sound absorption.

Under the legitmate assumption that $h\omega \ll \varepsilon_{\vec{p}}$ we can write

$$f(\varepsilon_{\vec{p}}) - f(\varepsilon_{\vec{p}} + h\omega) = f(\varepsilon_{\vec{p}}) - \left| f(\varepsilon_{\vec{p}}) + h\omega \frac{\partial f(\varepsilon_{\vec{p}})}{\partial \varepsilon_{\vec{p}}} \right|$$

$$= - h\omega \frac{\partial f(\varepsilon_{\vec{p}})}{\partial \varepsilon_{\vec{p}}}$$

But, $$\frac{\partial f(\varepsilon_{\vec{p}})}{\partial \varepsilon_{\vec{p}}} = -\frac{1}{k_B T} \frac{e^{\Delta/k_B T}}{(e^{\Delta/k_B T}+1)^2} \qquad (1.1.12)$$

where, $\Delta = \varepsilon - \mu$ and μ is the chemical potential.

In the limit as T approaches zero the function

$\frac{e^{\Delta/k_B T}}{(e^{\Delta/k_B T}+1)^2}$ for $\Delta \neq 0$ is identically zero whereas in this same limit for $\Delta=0$ the function reduces to unity. Moreover, it can easily be shown that

$$\int_{-\infty}^{\infty} \frac{e^{\Delta/k_B T}}{(e^{\Delta/k_B T}+1)^2} d\Delta = \frac{k_B T}{2} \int_{-\infty}^{\infty} \frac{1}{\cosh^2 x} dx = k_B T$$

where, $\Delta|_{k_B T} = 2x$

Thus we can conclude that (1.1.12) gives

$\frac{\partial f(\varepsilon_{\vec{p}})}{\partial \varepsilon_{\vec{p}}} = -\delta(\Delta)$ and hence in this case (1.1.11) takes on the form

$$\Gamma = h\omega \int |M_0|^2 \; \delta(\varepsilon_{\vec{p}}-\mu) \; \delta(\varepsilon_{\vec{p}}+h\omega-\varepsilon_{(\vec{p}+\vec{q})}) d^3p \qquad (1.1.13)$$

As compared to (1.1.11) Eq. (1.1.13) is (due to the assumption $h\omega \ll \varepsilon_{\vec{p}}$) a quasiclassical formula for the absorption coefficient of ultrasonic wave; and for the case of $T \simeq 0$, it may be rewritten as

$$\Gamma = h\omega \int |M_o|^2 \; \delta(\varepsilon_{\vec{p}} - \varepsilon_F)\, \delta(\varepsilon_{\vec{p}} + h\omega - \varepsilon_{(\vec{p}+\vec{q})})\, d^3p \qquad (1.1.14)$$

1.2 Absorption of sound in the phase transition of order 2½

As is mentioned in the introduction, a phase transition of order 2½ is accompanied by appearance (disappearance) of a new Fermi sheet or breaking of a connecting neck. Let us pick this case of appearance (disappearance) of a spheroidal cavity, just for the sake of mathematical simplicity. Near such a point of critical energy, the dispersion may be approximated by:

$$p^2/_{2m} = \varepsilon_F - \varepsilon_k = Z$$

where, of course, the energy is measured relative to ε_k. Z is the measure of the size of the new cavity.

The absorption coefficient (1.1.14) when made to accomodate this phase transition then appears as:

$$\Gamma = h\omega |M_o|^2 \int d^3p\, \delta(\varepsilon_k + p^2/2m - \varepsilon_F)\, \delta(\varepsilon_k + \frac{p^2}{2m} + h\omega - \varepsilon_k - \frac{(\vec{p}+\vec{q})^2}{2m})$$

For $|\vec{q}| \ll |\vec{p}|$ this takes on the form

$$\Gamma = h\omega |M_o|^2 \int d^3p \;\; \delta(\varepsilon_k + p^2/2m - \varepsilon_F) \;\; \delta(h\omega - \frac{\vec{p}\cdot\vec{q}}{m}) \qquad (1.2.1)$$

Employing now spherical polar coordiantes in which case

$$d^3p = p^2 \; \sin\theta \; dp \; d\theta \; d\phi$$

Eq. (1.1.15) can be rewritten as

$$\Gamma = h\omega |M_O|^2 \int p^2 \sin\theta\, dp\, d\theta\, d\phi \left(\frac{p^2}{2m} - Z\right) \delta\left(h\omega - \frac{pq\cos\theta}{m}\right)$$

which itself, after making the change of variable

$$x = -\cos\theta \quad \text{gives}$$

$$\Gamma = 2\pi h\omega |M_O|^2 \int p^2\, dp \int_{-1}^{1} dx\, \delta(p^2/2m - Z)\, \delta\left(h\omega + \frac{pqx}{m}\right) \qquad (1.2.2)$$

The integration over x after making a further change of variable

$$Y = x + \frac{h\omega m}{pq} \quad \text{reduces to}$$

$$\frac{m}{pq} \int_{-1+\frac{h\omega m}{pq}}^{1+\frac{h\omega m}{pq}} \delta(y)\, dy = \frac{m}{pq} \quad \text{for} \quad -1 + \frac{h\omega m}{pq} < 0 < 1 + \frac{h\omega m}{pq} \qquad (1.2.3)$$

so that (1.1.16) becomes (since $\varepsilon = p^2/2m$)

$$\Gamma = \frac{2\pi h\omega m^2}{q} \quad |M_O|^2 \int d\varepsilon\, \delta(\varepsilon - Z) \qquad (1.2.4)$$

Notice that the requirement in (1.2.3) may be restated as

$$\varepsilon = \frac{mV_e^2}{2} = p^2/2m > \frac{h^2\omega^2 m}{2q^2} = \frac{ms^2}{2} = Z^* \quad \text{which,}$$

of course,is in agreement with (1.1.2).

Thus (1.2.4) delivers the absorption coefficient given by:

$$\Gamma = \begin{cases} \frac{2\pi m^2 h\omega}{q} \quad |M_0|^2 = \Gamma_0 & \text{for } Z>Z^* \\ 0 & \text{for } Z<Z^* \end{cases} \qquad (1.2.5)$$

The above set of equations indicate that under phase transition of order 2½, the absorption coefficient shows a jump $\Delta\Gamma$, which is comparable to Γ_0 itself and this jump is independent of the parameter Z. Refer to Figure 2.

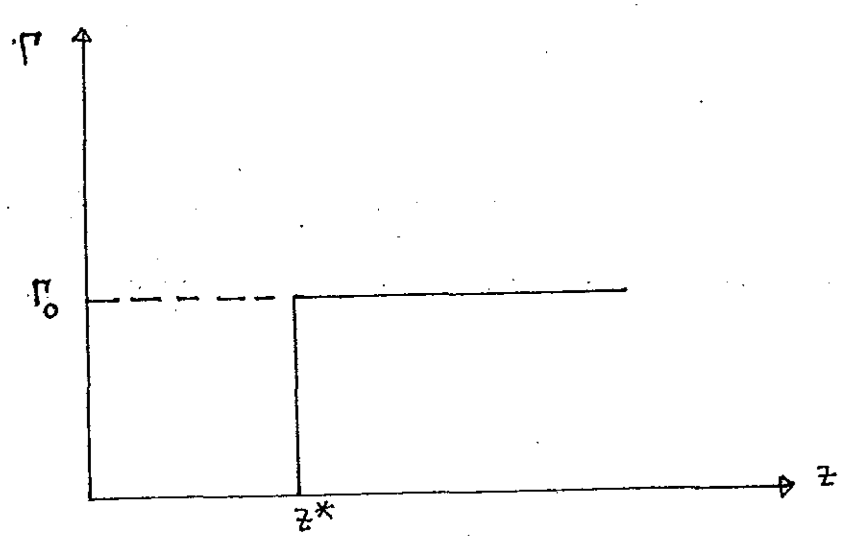


Fig. 2 Absorption coefficient in the case of appearance (dis-appearance) of a spheroidal cavity at $T \simeq 0$.

The above result is calculated specifically for the case of negligible phonon energy $h\omega$ and $T \simeq 0$ so that smoothing of the jump due to temperature may be ignored.

Let us now pick this other case of phase transition of order 2½ accompanied by the breaking of a connecting neck. In the rupture of a connecting neck, let us assume that the constant - energy surface $\varepsilon_{(\vec{p})} = \varepsilon_k$ contains a conical singular point $\vec{p} = \vec{p}_k$. Near such a singular point the dispersion is well approximated by (when energy is measured relative to the point $\vec{p} = \vec{p}_k$):

$$Z = \varepsilon_F - \varepsilon_k = \frac{p_1^2 + p_2^2}{2m_\perp} - \frac{p_3^2}{2m_{/\!/}} \ ; \ (m_\perp, m_{/\!/}) > 0 \qquad (1.2.6)$$

where axis - 3 coincides with the axis of the neck. The above equation stands for a hyperboloid of one sheet if $Z>0$; a cone if $Z=0$; and a hyperboloid of two sheets if $Z<0$ (Figure 3).

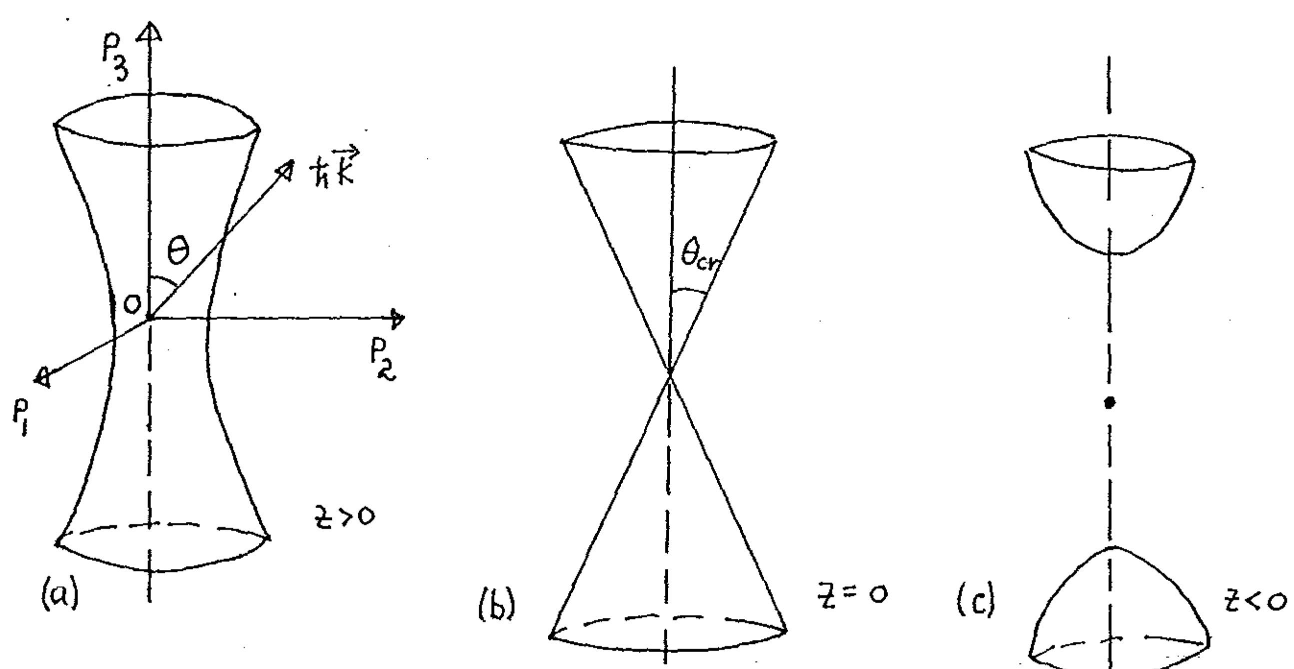


Fig.3. Equal energy surfaces before and after breaking of neck.

For this case of phase transition accompanied by breaking of a connecting neck (1.1.14) must be rewritten in a new form. This can be done by noting that:

$$\varepsilon_{\vec{p}}+h\omega - \varepsilon_{(\vec{p}+\vec{q})} = \left[\frac{p_1^2+p_2^2}{2m_\perp} - \frac{p_3^2}{2m_{//}} + h\omega\right] - \left[\frac{p_1^2}{2m_\perp} + \frac{(\vec{p}_2+\vec{q}\sin\theta)^2}{2m_\perp} - \frac{(\vec{p}_3+\vec{q}\cos\theta)^2}{2m_{//}}\right]$$

Assuming $|\vec{q}| \ll |\vec{p}|$ this may be shown to reduce into:

$$hk\left(s - \frac{p_2\sin\theta}{m_\perp} + \frac{p_3\cos\theta}{m_{//}}\right)$$

where, $s = \omega/k$

Thus: $$\delta(\varepsilon_{\vec{p}}+h\omega - \varepsilon_{(\vec{p}+\vec{q})}) = \frac{1}{hk}\,\delta\left(s - \frac{p_2\sin\theta}{m_\perp} + \frac{p_3\cos\theta}{m_{//}}\right)$$

Using this and the dispersion (1.2.6) the absorption coefficient in (1.1.14) can be restated as (for $T \simeq 0$):

$$\Gamma = \frac{h\omega|M_0|^2}{q} \int \delta\left(\frac{p_1^2+p_2^2}{2m_\perp} - \frac{p_3^2}{2m_{//}} - Z\right)\delta\left(s - \frac{p^2\sin\theta}{m_\perp} + \frac{p_3\cos\theta}{m_{//}}\right)d^3p \qquad (1.2.7)$$

θ defines the angle of inclination between $\vec{p}_3$ and $h\vec{k}$ which is arbitrarily made to lie on the (p_2p_3) plane.

The integration of (1.2.7) is performed along the curve of intersection of the plane

$$s - \frac{p_2 \sin\theta}{m_\perp} + \frac{p_3 \cos\theta}{m_{/\!/}} = 0 \quad \text{and the surface of (1.2.6).}$$

This curve of integration remains closed and elliptic, which can be shown using the method displayed in pp <u>73-77</u>, only for $\theta < \theta_{cr}$ where θ_{cr}, as evident, from Fig.3 (b), being the angle of cone safisfies the equation

$$\tan\theta_{cr} = \frac{p_\perp}{p_{/\!/}} = \frac{\sqrt{(p_1^2 + p_2^2)}}{p_3} \quad .$$

This using (1.2.6) for the case $\varepsilon_F = \varepsilon_k$

$$\frac{p_1^2+p_2^2}{2m_\perp} - \frac{p_3^2}{2m_{/\!/}} = 0$$

may be rewritten as

$$\tan\theta_{cr} = \left(\frac{m_\perp}{m_{/\!/}}\right)^{\frac{1}{2}}$$

The calculation of Γ from (1.2.7) will be simplified if first integration over p_3 is made; and later on simple transformation techniques developed in pp <u>77</u> may be followed to deliver

$$\Gamma = \frac{\Gamma_0}{2\pi m_\perp m_{/\!/}} \left(\frac{m_{/\!/}}{R^{\frac{1}{2}}\cos\theta}\right) \int\int \delta \left| \frac{p^2+q^2}{2m_\perp} - \left(Z + \frac{m_{/\!/} s^2}{2R\cos^2\theta}\right) \right| dp\,dq \qquad (1.2.8)$$

where $R = 1 - \frac{m_{//}}{m_{\perp}} \tan^2\theta$

and $\Gamma_0 = \frac{2\pi m_{//} m_{\perp} h}{q} |M_0|^2$

It can be seen from this that the integral is different from zero when $Z+(m_{//}s^2)/2R\cos^2\theta > 0$, and, consequently, the anomally in the absorption coefficient should be abservable at

$$Z = Z_{cr} = - m_{//}s^2 / 2R\cos^2\theta \qquad (1.2.9)$$

So, as $Z \to Z_{cr}$, the absorption coefficient undergoes a jump equal to

$$\Delta\Gamma = \frac{\Gamma_0}{R^{\frac{1}{2}}\cos\theta} \qquad (1.2.10)$$

which coincides in order of magnitude with Γ_0.
Note that for $Z<Z_{cr}$, the absorption coefficient is equal to zero. Moreover, from (1.2.9) & (1.2.10) we abserve that as θ increases toward θ_{cr} the quantities $|Z|$ and $\Delta\Gamma$ show increments; and it can easily be calculated from (1.2.10) that in the limit $\theta \tilde{=} \theta_{cr}$, the jump of the absorption coefficient shows a root-law increase as given by

$$\Delta\Gamma \tilde{=} \frac{\Gamma_0}{\sqrt{2}\left(\frac{m_{//}}{m_{\perp}}\right)^{\frac{1}{2}}\sqrt{\Delta\theta}} \quad (\Delta\theta \ll \theta_{cr}) \qquad (1.2.11)$$

where, $\Delta\theta = \theta_{cr} - \theta$

In the case of $\theta > \theta_{cr}$ during which the curve of integration of course is open and infinite, the absorption coefficient $\Gamma \neq 0$ for any value of Z. A simple analysis shows that for any $\theta > \theta_{cr}$ there exists a value of the parameter Z at which the absorption coefficient logarithmically diverges according to:

$$\Gamma_{Z \to Z'_{cr}} = \frac{2\Gamma_0}{\pi \sin_\theta (ctn^2\theta_{cr} - ctn^2\theta)^{\frac{1}{2}}} \; \ell n \left| \frac{p_0^2}{m\,(Z - Z'_{cr})} \right|$$

where, $Z'_{cr} = \frac{m_\perp s^2}{2\sin^2\theta} \left(1 - \frac{m_\perp}{m_{/\!/}} ctn^2\theta\right)^{-1}$

the maximum value of the absorption coefficient is attained at $Z = Z'_{cr}$ and is equal to

$$\Gamma_{max} = \frac{\Gamma_0 \ln (4 p_0^2 m_{/\!/} h\omega)}{\pi \sin\theta\, (ctn^2\theta_{cr} - ctn^2\theta)^{\frac{1}{2}}} \qquad (1.2.12)$$

Divergence is removed by using (1.1.11) instead of (1.1.14). Notice that as the angle θ approaches the critical value, the quantity Γ_{max} increases and in the limit (1.2.12) gives

$$\Gamma_{max} \propto (\theta - \theta_{cr})^{-\frac{1}{2}} \qquad (1.2.13)$$

1.3. Absorption of sound in the magnetic field and giant quantum oscillations

The energy of the conduction electron in a magnetic field $\vec{H}$, which may be assumed to be directed along the Z - axis, is quantized (Refer to section 2.1, [9]) being dependent only on the principal quantum number n and p_z the component of the electron momentum along the magnetic field. Ignoring the spin splitting of the energy levels (Landau levels) and assuming for simplicity quadratic dispersion law of the form:

$$\varepsilon = p^2/2m \qquad (1.3.1)$$

the energy of the conduction electron under such condition may be given as:

$$\varepsilon_{\vec{p}_{z,n}} = h\Omega(n+\tfrac{1}{2}) + \frac{p_z^2}{2m} \qquad (1.3.2)$$

where, $\Omega = \frac{eH}{mc}$ (Larmor frequency).

For such a case the absorption coefficient (1.1.10) takes on the form

$$\Gamma_H = |M|^2 \sum_{\vec{p}_z,\vec{p}_z'} \sum_{n,n'} \left[f(\varepsilon_{n,\vec{p}_z}) - f(\varepsilon_{n',\vec{p}_z'})\right] \delta(\varepsilon_{n',\vec{p}_z'} - \varepsilon_{n,\vec{p}_z} - h\omega) \qquad (1.3.3)$$

where,

$$|M|^2 = \frac{2\pi\omega |U_O|^2}{V_O \rho A^2 \omega^2 s} .$$

Not only that, the conservation principles stated in (1.1.1) must now be rewritten as:

$$\left.\begin{aligned} \vec{p}_z{}' &= \vec{p}_z + h\vec{k}_z \\ \text{and} \quad h\Omega(n'+\tfrac{1}{2}) + \frac{p_z'^2}{2m} &= h\Omega(n+\tfrac{1}{2}) + \frac{p_z^2}{2m} + h\omega \end{aligned}\right\} \qquad (1.3.4)$$

But, (1.3.4) under the assumption $|h\vec{k}_z| \ll |\vec{p}_z|$ requires that

$$h\Omega(n'-n) + \frac{p_z h k_z}{m} = h\omega \qquad (1.3.5)$$

which since $\Omega \gg \omega$ delivers the condition $n' = n$. This specifically means that the rise to a higher Landau level (by an electron) after absorption of a phonon of energy is improbable; or equivalently it means that the variation in the state of an electron after absorption will be noted only as an increase of the momentum p_z. Thus under this newly discovered condition (1.3.5) becomes

$$h\omega = h\vec{k}, \frac{\partial \varepsilon(\vec{p}_z)}{\partial \vec{p}_z}$$

$$\text{or} \qquad s = v_z \cos\theta \qquad (1.3.6)$$

This is the condition of absoption by a metal subject to a magnetic field, and it implies that only a small belt of electrons on the Fermi surface are involved in ultrasonic absorption (Figure 4).

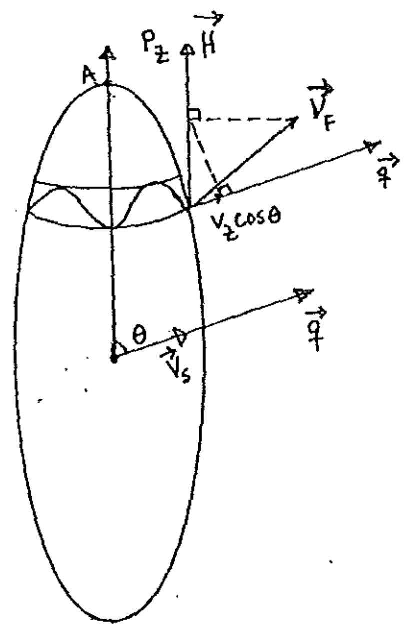


Fig. 4. The belt of electrons effective in absorption.

The value of θ is limited by $\theta_{lim} < \theta \leq 90^{\circ}$ where θ_{lim} satisfies the relation

$$\cos\theta_{lim} = \frac{s}{V_F(A)} \qquad (1.3.7)$$

According to (1.1.12), $[f(\epsilon_{n,\vec{p}_z}) - f(\epsilon_{n'},\vec{p}_z')]$ may be

replaced by $\frac{h\omega}{4k_BT} \quad \frac{1}{\cosh^2[(\epsilon_{n,\vec{p}_z} - \mu)/2k_BT]}$ so that this together with the conditions of absorption:

$$n' = n$$

$$\vec{p}_z{}' = \vec{p}_z + h\vec{k}_z$$

let (1.3.3) have the form

$$\Gamma_H = |M|^2 \sum_{\vec{p}_z} \sum_n \frac{h\omega}{4k_BT} \frac{\delta(\varepsilon_{n,\vec{p}_z+h\vec{k}_z} - \varepsilon_{n,\vec{p}_z} - h\omega)}{\cosh^2|(\varepsilon_{n,\vec{p}_z} - \mu)/2k_BT|}$$

Using the assumption $|h\vec{k}_z| \ll |\vec{p}_z|$ which can be used to expand $\varepsilon_{n,\vec{p}_z+h\vec{k}_z}$ in powers of hk_z, the above expression may be rewritten as:

$$\Gamma_H = \frac{h\omega}{4k_BT} |M|^2 (\frac{m}{hk_z}) \sum_{\vec{p}_z} \sum_n \frac{\delta(p_z - \frac{m\omega}{k_z})}{\cosh^2|(\varepsilon_{n,\vec{p}_z} - \mu)/2k_BT|} \quad (1.3.8)$$

Employing now (1.3.5) under the condition $n' = n$ and noting the fact that p_z is not quantized so that the density of electron states along p_z is given by

$$\nu_n(p_z) = \frac{2VeH}{(2\pi h)^2 c},$$ (1.3.8) can be changed into

$$\Gamma_H = (\frac{h\omega}{4k_BT}) \frac{2VeH}{(2\pi h)^2 c} |M|^2 \frac{m}{hk_z} \sum_n \int dp_z \frac{\delta(p_z - \frac{m\omega}{k_z})}{\cosh^2\left[(\varepsilon_{n,\vec{p}_z} - \mu)/2k_BT\right]} \quad (1.3.9)$$

from which follows

$$\Gamma_H = \left(\frac{h\omega}{4k_BT}\right) |M|^2 \frac{2VeH}{(2\pi h)^2 c} \left(\frac{m}{hk_z}\right) \sum_n \frac{1}{\cosh^2 \left[\frac{h\Omega(n+\frac{1}{2}) - \mu}{2k_BT}\right]}$$

Notice that in the above expression the quantity $\frac{m\omega^2}{2k_z^2}$ is ignored from the argument of the hyperbolic cosine. Since, $\Omega = \frac{eH}{mc}$ and $|M|^2 = |M_O|^2 \frac{(2\pi h)3}{2V}$ this same expression can further be reduced into

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \sum_n \frac{1}{\cosh^2 \left[\frac{h\Omega(n+\frac{1}{2}) - \mu}{2k_BT}\right]} \quad (1.3.10)$$

For $\Omega \to 0$ or equivalently for $h\Omega \ll k_BT$ the summation over n in (1.3.10) may be replaced by an integral so that we get

$\Gamma = (\Gamma_O/2) \int_0^\infty \frac{dy}{\cosh^2 (y - \mu/2k_BT)} \cong \Gamma_O$ which is in perfect agreement with the discussion of section 1.2. Moreover, from (1.3.10) it follows that for $\vec{H}$ large enough so that $k_BT \ll h\Omega$, Γ_H oscillates between approximate maximum value $\Gamma_O \frac{h\Omega}{4k_BT} \gg \Gamma_O$ and approximate minimum value

$$\Gamma_O \frac{h\Omega}{4k_BT} \exp(-\alpha h\Omega/k_BT) \ll \Gamma_O \quad (\alpha \cong 1)$$

which occur whenever the value $h\Omega$ $(n+\frac{1}{2})$ is close to μ and far from μ respectively. Therefore, Γ exhibits very strong oscillations (Figure 11) termed as Giant Quantum oscillations [10-14] which are uniformly spaced in H^{-1}. The period of oscillations may be easily determined using the techniques delivered in pp 64-65 as

$$\Delta(1/H) = eh/_{mc\mu} \qquad (1.3.11)$$

A sketch of the Landau levels as given by (1.3.2) is drawn below (Fig. 5).

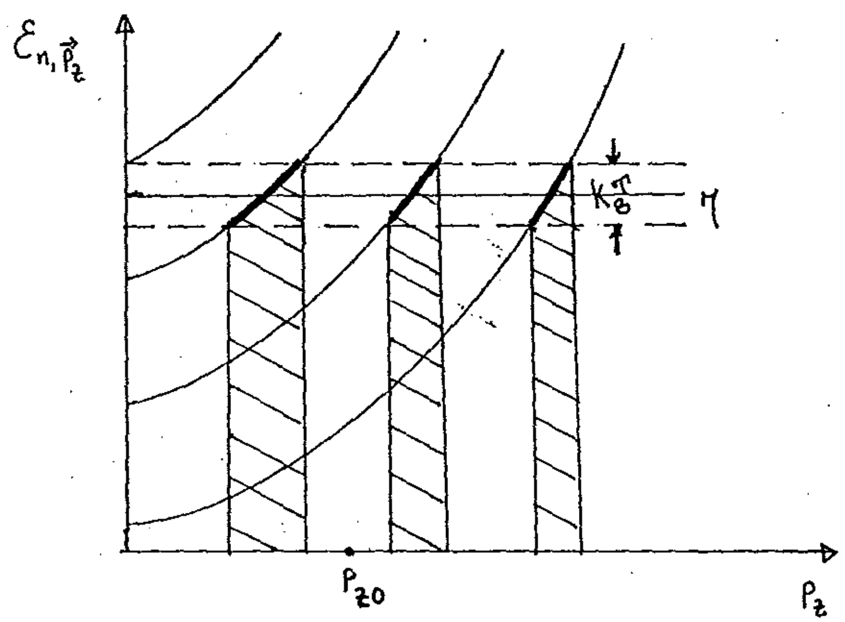


Fig. 5. Origin of Giant Quantum Oscillations.

As has been tried to explain earlier, it is only those electrons with momentum P_{zO} satisfying the condition (1.3.6) which can participate in ultrasonic absorption; and for low enough temperatures their energy is close to the Fermi level found in the narrow strip of energy indicated in Figure 5.

Then whenever P_{zO} merges into one of the allowed segments on the Landau levels marked in block the absorption coefficient Γ_H rises to a large value and drops to a real low value otherwise. That means as the magnetic field is continously increased broadning the gap between the Landau levels, the absorption coefficient passes through peak values periodically which in effect produces the Giant Quantum Oscillations discussed earlier. Notice then the fact that for observation of GQO, μ has to be large enough so that the condition

$$k_B T \ll h\Omega \ll \mu \qquad (1.3.12)$$

is satisfied.

CHAPTER II

GIANT QUANTUM OSCILLATIONS IN THE PHASE TRANSITION OF ORDER 2½

2.1. Quantization of the energy levels of the conduction electron in the magnetic field

It was mentioned at the beginning of section 1.3 that the energy of the conduction electron in the magnetic field is quantized. The quantized energy levels given by (1.3.1) for the simplest case of quadratic dispersion law applied in the free electron model may be derived quite simply using the Schrödinger equation

$$\hat{H}\psi = \varepsilon\psi \qquad (2.1.1)$$

where the Hamiltonian Operator in this case is

$\hat{H} = \hat{k} = \hat{P}^2/_{2m}$, $\hat{P}$ being the generalized momentum introduced as

$$\hat{P} = \hat{p} - \frac{e}{c}\hat{A} \quad (\hat{p} = -i\hbar\nabla = \text{kinematic momentum})$$

Thus:

$\frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}\hat{A}\right)^2 \psi = \varepsilon\psi$, which after expansion delivers

$$-\frac{h^2}{2m}\nabla^2\psi + \frac{ihe}{mc}(\nabla.\vec{A})\psi + \frac{e^2}{2mc^2}A^2\psi = \varepsilon\psi$$

For the arbitrary choice of $\vec{A}$ = (O, Hx,O) this again may be rewritten as:

$$-\frac{h^2}{2m}\nabla^2\psi + \frac{ihe}{mc}\frac{\partial}{\partial_y}(H\,x)\psi + \frac{e^2}{2mc^2}(H^2x^2)\psi = \varepsilon\psi$$

which, since $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$ simplifies into

$$-\frac{h^2}{2m}\frac{\partial^2\psi}{\partial_x{}^2} + \frac{1}{2m}(-\,ih\frac{\partial}{\partial y} - \frac{e}{c}Hx)^2\psi - \frac{h^2}{2m}\frac{\partial^2\psi}{\partial z^2} = \varepsilon\psi \qquad (2.1.2)$$

We now look for the solution of this equation in the form:

$$\psi(\vec{r}) = \phi(x)\,\exp i(p_y y/h + p_z z/h)$$

Substitution of this into (2.1.2) produces

$$-\frac{h^2}{2m}\frac{\partial^2\phi(x)}{\partial x^2} + \frac{e^2H^2}{2mc^2}(x - \frac{cp_y}{eH})^2\phi(x) = (\varepsilon - \frac{p_z^2}{2m})\phi(x) \qquad (2.1.3)$$

Note that (2.1.3) is very similar to the Schrödinger equation for a one dimensional Harmonic - oscillator:

$$-\frac{h^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + \frac{kx^2}{2}\psi(x) = \varepsilon\psi(x) \qquad (2.1.4)$$

whose eigenvalues are given by

$$\varepsilon_n = h\omega \quad (n+\tfrac{1}{2})$$

where, $\omega = (k/_m)^{\frac{1}{2}}$; and notice that there is a shift of origin by $\frac{cp_y}{eH}$.

The quantized energy levels of the electron in the magnetic field is then easily observed from (2.1.3) by comparison with (2.1.4) to be:

$$\varepsilon_{n,\vec{p}z} = h\Omega(n+\tfrac{1}{2}) + \frac{p_z^2}{2m} \qquad (2.1.5)$$

where, $\Omega = \frac{|e|H}{mc}$ (cyclotron frequency)

As pointed out at the beginning, the above derivation is made for the simplest case of free electron model; but the appearance of discrete energy levels for a fixed value of p_z is a property to be expected even for an arbitrary electronic spectrum which might be complicated if the trajectory of the conduction electron in momentum space is a closed curve. Since $\mu \gg h\Omega$ is the legitmate condition for ultrasonic absorption and observation of GQO, the relevant states have high quantum numbers n. Hence the quantized energy levels of an electron with an arbitrary dispersion can be determined using the Semiclassical Bohr - Sommerffeld quantization rule stated as:

$$\oint p_i \, dQ_i = 2\pi h \ (n_i+\gamma) \qquad (2.1.6)$$

where Q_i and p_i are conjugate coordinates and momenta of the electron; and the n_i are integers. Making no distinction between quasimomentum and momentum, which aviods the consideration of quantum - mechanical transitions, (2.1.6) may be rewritten as:

$$\oint p_y dy = 2\pi h \; (n+\gamma) \qquad (2.1.7)$$

the integral being taken along the classical trajectory of the electron. But a glance look at the form of Lorentz' Force

$$\frac{d\vec{p}}{dt} = \frac{|e|}{c}\frac{d\vec{r}}{dt} \times \vec{H}$$

which may be transformed into

$$d\vec{p} = \frac{|e|}{c} d\vec{r} \times \vec{H} \qquad (2.1.8)$$

shows that trajectories in momentum space and real space are rotated with respect to each other by angle of 90^o; and for the magnetic field directed along the Z - axis, which is according to our interest, we from (2.1.8) find the relation

$$dy = \frac{c}{|e|H} dp_x \; .$$

Substituting this into (2.1.7), we finally arrive at

$$\oint p_y dp_x = \frac{2\pi|e|hH}{c} \; (n+\gamma) \qquad (2.1.9)$$

where now the integral is taken along the electron trajectory in momentum space. The expression on the left side is simply the area enclosed by the orbit in momentum space S. The conditions defining the closed orbit may be found straight away from Lorentz' force:

$$\frac{d\vec{p}}{dt} = \frac{|e|}{c} \vec{V}x\vec{H}$$

Dotting this expression with $V = \frac{\partial \varepsilon}{\partial \vec{p}}$ we get that $\frac{d\varepsilon}{dt} = 0$ which indicates the idea that in the magnetic field the electron moves along a constant energy surface $\varepsilon_{(\vec{p})}$ = constant. Moreover dotting this same equation with $\vec{H}$ gives (since $H \neq 0$) $|(\frac{d\vec{p}}{dt})_z| = 0$ where $(\frac{d\vec{p}}{dt})_z$ is a component parallel to $\vec{H}$. The last result shows that the projection of the momentum of the electron along the magnetic field is retained, i.e.

$$p_z = \text{constant}$$

Thus, the intersection of the surface $\varepsilon_{(\vec{p})}$ = constant and the secant plane p_z = constant defines the curve of the path of the electron in momentum space under the action of the magnetic field. Depending on the topology of the constant energy surface and the direction of the magnetic field, this trajectory may be either closed (determining a finite motion) or open, which passes continously through the whole $\vec{p}$ - space (and determines an infinite motion). From (2.1.9) we then can write

$$S(\varepsilon_{n,p_z}) = \frac{2\pi|e|hH}{c} \quad (n+\gamma) \tag{2.1.10}$$

which can be employed to find the quantized energy levels of the conduction electron with an arbitrary dispersion. Eq. (2.1.10) is known as the Lifshitz - Onsager quantization rule and is quasi-classical in nature as the following steps demonstrate. From (2.19) follows:

$$\Delta\varepsilon = \frac{2\pi|e|Hh}{c(\frac{\partial s}{\partial \varepsilon})} \tag{2.1.11}$$

But also from the perpendicular component (perpendicular to the secant plane) of Lorentz force:

$$\frac{d\vec{p}}{dt} = \frac{|e|}{c} \quad [\vec{V} \quad x\vec{H}] \tag{2.1.12}$$

follows (after integration):

$$T_H = \frac{c}{eH} \oint_L \frac{dp}{V} \tag{2.1.13}$$

where T_H is time of rotation

The integral should be taken over the closed contour L of the path on the secant plane (Figure 6).

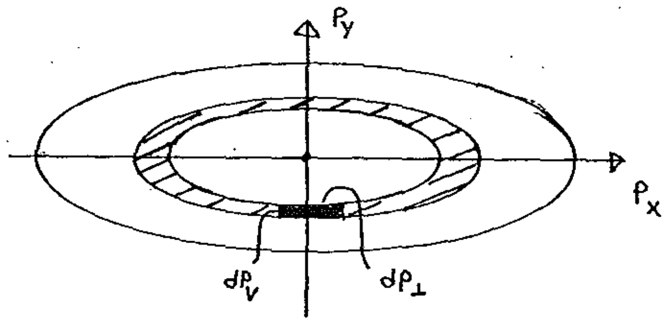


Fig.6. The secant plane.

Notice that a strip of area in the secant plane is given by $\Delta S_{ring} = \oint_L dp_\perp \, dp_v$ which using

$$\oint dp_v = \frac{\partial \varepsilon}{v_\perp} \text{ reduces into } \Delta S_{ring} = \oint_L \frac{\partial \varepsilon}{v_\perp} dp_\perp$$

Thus the total area of the secant plane is given by

$$S = \int_0^{\varepsilon'} \left[\oint_L \frac{dp_\perp}{V} \right] d\varepsilon$$

It implies that

$$\frac{\partial S}{\partial \varepsilon} = \oint_L \frac{dp_\perp}{V_\perp}$$

This when substituted into (2.1.13) gives

$$T_H = \frac{c}{|e|H} \left(\frac{\partial S}{\partial \varepsilon}\right)$$

from which we discover that

$$\Omega = \frac{2\pi}{T_H} = \frac{2\pi |e| H}{c\left(\frac{\partial S}{\partial \varepsilon}\right)}$$

Comparing this with the cyclotron frequency of a free particle $\omega = \frac{|e|H}{mc}$ we identify:

$$\frac{1}{2\pi}\left(\frac{\partial S}{\partial \varepsilon}\right) = m^* \qquad (2.1.14)$$

as the effective cyclotron mass. Thus the expression (2.1.11) takes on the form

$$\Delta\varepsilon = \frac{eHh}{m^* c} = h\Omega$$

But, quantization is possible for $\Delta\varepsilon \ll \varepsilon \cong \varepsilon_F$ so that $h\Omega \ll \varepsilon_F$. This last inequality establishes the fact that Lifshitz - Onsager quantization rule is classical.

As a demonstration of how (2.1.10) may be used to find classical energy levels, let us pick the case of the quadratic dispersion law given by

$\varepsilon_{(\vec{p})} = \frac{p^2}{2m^*}$. The constant energy surface $\varepsilon_{(\vec{p})} = \text{const.}$ in this case is spherical (Figure 7).

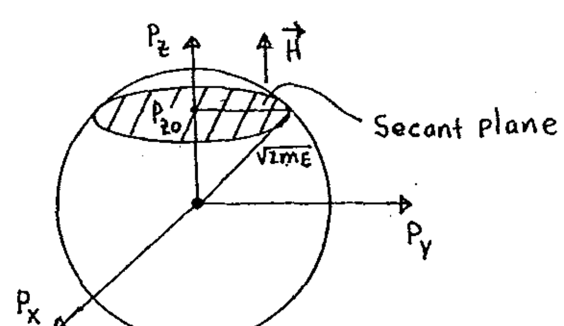


Fig.7. The constant energy surface of the type $\frac{p^2}{2m^*} = \text{const.}$

For the magnetic field directed along the Z- axis the curve of the orbit of the conduction electron is as shown above, in Figure 7.

From, $\frac{p_x^2}{2m} + \frac{p_y^2}{2m} - \frac{p_{z0}^2}{2m} = \varepsilon =$ constant, where p_{z0} is itself a constant shows that the orbit is circular with radius $\sqrt{(2m\varepsilon - p_{z0}^2)}$ so that the area of orbit is:

$$\pi(2m\varepsilon - p_{z0}^2) = S(\varepsilon_{n,p_z})$$

Thus according to (2.1.10) one can write

$$\pi(2m^*\varepsilon - p_{z0}^2) = \frac{2\pi|e|hH}{c}(n+\tfrac{1}{2}).$$

($\gamma = \frac{1}{2}$ for quadratic dispersion law)

From this follows the expression for classical energy levels

$$\varepsilon_n = \frac{p_{z0}^2}{2m^*} + h\Omega\ (n+\tfrac{1}{2})$$

which is in accordance with (2.1.5).

2.2. Conditions for experimental observations of GQO in PT-2½

The conditions of GQO without PT-2½ are indicated in (1.3.10) which for the case of T≃0 may be rewritten as:

$$k_B T \ll h\Omega \ll \varepsilon_F. \qquad (2.2.1)$$

The second inequality imposes the condition that GQO is observable only and only when the characterstic size of the path of the electron in the magnetic field exceeds substantially its de Broglie wavelength λ_B .

$$\text{i.e. } \lambda_B \ll r_\perp \qquad (2.2.2)$$

Actually it can easily be shown that (2.2.2) dissolves into $h\Omega \ll \epsilon_F$.

Integration of (2.1.12) for the magnitudes of $p_\perp$ and $r_\perp$ gives $r_\perp = \frac{cp_\perp}{eH}$. On the otherhand the de Broglie wavelength is given by: $\lambda_B = \frac{2\pi h}{p_F}$. Using these relations (2.2.2) may be rewritten as:

$$\frac{2\pi h}{p_F} \ll \frac{cp_\perp}{eH}$$

or as:

$$\frac{2\pi h}{v_F} = \frac{p_\perp mc}{|e|H}$$

But since $\frac{|e|H}{mc} = \Omega$, it further reduces into:

$$h\Omega \ll \frac{p_\perp v_F}{2\pi}$$

or equivalently into

$$h\Omega \ll \epsilon_F$$

which proves the statement.

The requirement (2.2.2) means that the force set up by the field is small as compared with the interatomic or crystalline force; and it varies only slightly at distances of order <a> or is sufficiently smooth. The above statement can be shown to be correct as argued below. For an electron on the Fermi surface, $p_F \sim h/a$, and therefore, from the expression of de Broglie wavelength $\lambda_B = h/p_F$ follows: $\lambda_B \sim a$. Thus, (2.2.2) rewritten as

$$\lambda_B \ll \frac{cp_F}{|e|H}$$

takes on the form $a \ll \frac{c(h/a)}{|e|H}$ from which follows

$$H \ll \frac{ch}{|e|a^2} \qquad (2.2.3)$$

The quantity $\frac{ch}{|e|a^2} = H_a$ is the crystal field and is of the order of $10^8 - 10^9$ Oerested. Thus observation of GQO is possible for fields much less than this value which proves the statement. Notice that since magnetic field intensities really attainable in laboratory conditions are of the order 10^6 Oerested, (2.2.3) is delibrately satisfied. Moreover, temperature is seen to smear out oscillations as is evident in the discussion of sec.1.3. Thus, as a conclusion, one can state that the conditions for experimental observations of GQO in pT-2½ are the same conditions in (2.2.1) except that ε_F now has to be replaced by $|Z| = |\varepsilon_F - \varepsilon_k|$ which is the parameter of phase transition of order 2½ so that we get:

$$k_BT \ll h\Omega \ll |Z| \qquad (2.2.4)$$

Actually, for large values of $|Z|$ (2.2.4) reduces to (2.2.3). But on top of (2.2.4) we have also to consider

$$h\omega \ll \varepsilon_F \quad \text{and} \quad h\omega \ll h\Omega \qquad (2.2.5)$$

stated in sections 1.2 & 1.3 respectively (used actually to find the quasi - classical expression of the absorption coefficient) as additional conditions for observation of GQO.

2.3. Derivation of the formula for GQO in pT-2½ and arbitrary dispersion law

Eq. (1.3.8) was specifically derived under the assumption of the quadratic dispersion law as given by (1.3.1) or equivalently of (1.3.2). The direct application of (1.3.2) in the derivation of (1.3.8) is observed in the equations (1.3.3) and (1.3.4). But as the following steps demonstrate it can be shown that (1.3.8) also applies to an arbitrary dispersion law. As was mentioned in section (2.1), the Lifshitz - Onsager quantization rule can be readily applied to find quasiclassical energy levels of the conduction electron in the magnetic field. Assuming quadratic dispersion (not exactly as in 1.3.1), (2.1.10) before and after absorption of phonon will take on the form:

$$\left.\begin{aligned} S(\varepsilon_{n,\vec{p}_z},\vec{p}_z) &= \frac{2\pi eHh}{c}(n+\tfrac{1}{2}) \\ \text{and}\qquad S(\varepsilon'_{n,\vec{p}'_z},\vec{p}_z) &= \frac{2\pi eHh}{c}(n'+\tfrac{1}{2}) \end{aligned}\right\} \qquad (2.3.1)$$

respectively.

The conservation of energy and momentum principles (1.1.1) are now rewritten as:

$$\left.\begin{aligned} \vec{p}'_z &= \vec{p}_z + h\vec{k}_z \\ \varepsilon'_{n,\vec{p}'_z} &= \varepsilon_{n,\vec{p}_z} + h\omega \end{aligned}\right\} \qquad (2.3.2)$$

But, because

$$h\omega \ll |\varepsilon_{n,\vec{p}_z}| \text{ and } |h\vec{k}_z| \ll |\vec{p}_z|$$

the second part of (2.3.1) may be expanded as

$$S(\varepsilon'_{n,\vec{p}'_z},\vec{p}'_z) = S(\varepsilon_{n,\vec{p}_z},\vec{p}_z) + \frac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial\varepsilon_{n,\vec{p}_z}} h\omega + \frac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial p_z} hk_z$$

which since $\dfrac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial p_z} hk_z = \dfrac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial\varepsilon_{n,p_z}} \dfrac{\partial\varepsilon_{n,\vec{p}_z}}{\partial p_z} hk_z$

may be rewritten as

$$S(\varepsilon_{n',\vec{p}_z'},\vec{p}_z') = S(\varepsilon_{n,\vec{p}_z},\vec{p}_z) + \frac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial \varepsilon_{n,\vec{p}_z}} \left[h\omega + \frac{\partial \varepsilon(n,\vec{p}_z)}{\partial p_z} hk_z \right]$$

or as

$$S(\varepsilon_{n',\vec{p}_z'},\vec{p}_z') = S(\varepsilon_{n,\vec{p}_z},\vec{p}_z) + \frac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial \varepsilon_{n,\vec{p}_z}} (h\omega + V_z hk_z) \qquad (2.3.3)$$

Substituting (2.3.1) into (2.3.3) we finally get

$$\frac{2\pi eHh}{c}(n'-n) = \frac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial \varepsilon_{n,\vec{p}_z}} (h\omega + V_z hk_z)$$

which may be rewritten as

$$\frac{eHh}{\dfrac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial \varepsilon_{n,\vec{p}_z}} \left(\dfrac{c}{2\pi}\right)} (n'-n) = h\omega + V_z hk_z \qquad (2.3.4)$$

Letting $\frac{1}{2\pi}\left(\frac{\partial S(\varepsilon_{n,\vec{p}_z},\vec{p}_z)}{\partial \varepsilon_{n,\vec{p}_z}}\right) = m^*$ analogus to (2.1.14), Eq. (2.3.4) will reduce into:

$$\frac{eHh}{m^* c}(n'-n) = h\omega + V_z hk_z$$

or into

$$h\Omega^* \quad (n'-n) = h\omega + V_z hk_z \qquad (2.3.5)$$

where, $\Omega^* = \frac{eH}{m^* c}$ (Larmor Frequency)

For a magnetic field which is sufficiently large we have the conditions

$$h\omega \ll h\Omega$$

and $|V_z h k_z| \ll h\Omega$ satisfied; these conditions themselves being in accordance to condition (2.2.5). Consequently condition (2.3.5) can be satisfied only when $n' = n$ just as in section (1.3); which then makes it possible to rewrite (2.3.5) as:

$$V_z = -\frac{\omega}{k_z}$$

Using the relation $\omega = ks$ this may be reduced to

$$|V_z| \cos\theta = s \text{ (condition of absorption)}$$

which is exactly Eq. (1.3.6) for θ being the angle between the incident sound and the magnetic field directed along the z-axis. Now, after this, to arrive at the expression for the absorption coefficient all stepts after (1.3.6) may be repeated except of one minor change, namely, the substitution of $m_{\parallel}$ for m and hence redefine Ω as:

$$\Omega = \frac{eH}{m_{\parallel} c}$$

With this done, the absorption coefficient (1.3.3) reduces to exactly (1.3.8) which proves the stutement that (1.3.8):

$$\Gamma_H = \Gamma_0 \frac{h\Omega}{4k_B T} \sum_n \int dp_z \frac{\delta(p_z - \frac{m_{/\!/}\omega}{k_z})}{\cosh^2[(\varepsilon_{n,\vec{p}_z} - \mu)/2k_B T]}$$

is applicable to any arbitrary dispersion law.

The above expression may be integrated over p_z to give:

$$\Gamma_H = \Gamma_0 \frac{h\Omega}{4k_B T} \sum_n \cosh^{-2} [(\varepsilon_{n,p_{z0}} - \mu)/2k_B T] \qquad (2.3.6)$$

where,

$$p_{z0} = \frac{m_{/\!/}\omega}{k_z} = \frac{m_{/\!/}\omega}{k\cos\theta} = \frac{m_{/\!/}s}{\cos\theta}$$

Eq. (2.3.6) may be made to accomadate the case of phase transition of order 2½ by simply introducing the parameter of PT-2½:

$$Z = \mu - \varepsilon_c$$

In that case (2.3.6) transforms into:

$$\Gamma_H = \Gamma_0 \frac{h\Omega}{4k_B T} \sum_n \cosh^{-2} [(\varepsilon_{n,p_{z0}} - z)/2k_B T] \qquad (2.3.7)$$

Eq. (2.3.7) is the required formula for GQO in pT-2½ and arbitrary dispersion law.

CHAPTER III

THE ABSORPTION COEFFICIENT IN THE MAGNETIC FIELD IN THE APPEARANCE OF A SPHEROIDAL CAVITY

3.1. Validity of the expression for GQO in the case of pT-2½

It has already been stated in section 1.2 that near the point of critical energy where a spherical equal energy surface appears (disappears) the dispersion law is approximated by

$$\varepsilon_{(\vec{p})} = \varepsilon_k + \frac{p_x^2 + p_y^2 + p_z^2}{2m} .$$

Under the action of the magnetic field directed along the Z - axis it transforms into:

$$\varepsilon_{n,\vec{p}_z} = \varepsilon_k + h\Omega(n+\tfrac{1}{2}) + p_z^2/2m$$

where, $\Omega = \frac{eH}{mc}$ and $p_z = \frac{m\omega}{k_z}$

This when inserted into (2.3.6) delivers

$$\Gamma_H = \Gamma_0 \frac{h\Omega}{4k_BT} \sum_n \cosh^{-2} \left[\frac{h\Omega(n+\frac{1}{2})}{2k_BT} - \frac{Z-Z^*}{2k_BT}\right] \qquad (3.1.1)$$

where, $Z=\mu - \varepsilon_k$

and $Z^* = p_z^2/2m = \frac{ms^2}{2\cos^2\theta}$

In the limit as $H \to 0$, the summation in (3.1.1) may be replaced by integration so that

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \int_0^\infty \cosh^{-2}\left[\frac{h\Omega(x+\frac{1}{2})}{2k_BT} - \frac{Z-Z^*}{2k_BT}\right] dx \ .$$

Change of variable

$$y = \frac{h\Omega(x+\frac{1}{2}) - (Z-Z^*)}{2k_BT} \quad (dy = \frac{h\Omega}{2k_BT}\, dx)$$

simplifies the above expression into

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \int_{\frac{\frac{1}{2}h\Omega-(Z-Z^*)}{2k_BT}}^{\infty} \text{csh}^{-2} \left(\frac{2k_BT}{h}\right) dy$$

Thus as $H \to 0$ or equivalently $\Omega \to 0$ this may be replaced by

$$\Gamma_H = \Gamma_O/2 \int_{-\frac{Z-Z^*}{2k_BT}}^{\infty} \text{csh}^{-2} y \ dy$$

As the integral on the RHS is a table integral |15| (Dwight # 679.20) the above expression simplifies into

$$\Gamma_H = \Gamma_O/2 \ \tanh y \Big|_{-\frac{Z-Z^*}{2K_BT}}^{\infty}$$ which gives

$$\Gamma_H = \Gamma_O/_2 \ (1+\tanh \frac{Z-Z^*}{2k_BT}) \qquad (3.1.2)$$

But, under a further restriction of $T \cong 0$, Eq. (3.1.2) shows a jump of the ultrasonic absorption coefficient as given by

$$\Gamma = \begin{cases} \Gamma_O \text{ if } & Z>Z^* \\ 0 \text{ if } & 0<Z<Z^* \end{cases}$$

and it, of course, is in perfect agreement with (1.2.5) which was derived by V.N. Davydov and M.I. Kaganov. Thus this establishes the fact that (2.3.6) is undoubtdly valid as a formula of GQO under phase transition of order 2½.

Investigation of Γ_H near $Z \cong Z^*$ and discussion of results

Once the validity of (2.3.6) or equivalently (3.1.1) is established then it becomes reasonable to investigate the consequences of applying these formulas to the various regions of interest. Near the point $Z \cong Z^*$ (3.1.1) simplifies into:

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \sum_n \cosh^{-2} \frac{h\Omega(n+\frac{1}{2})}{2k_BT} \qquad (3.2.1)$$

The summation of (3.2.1) can be performed using the poisson summation formula [16] . For an arbitrary function f(x), where n<x<n+1, a complex fourier expansion can be made as:

$$f(x) = \sum_{k=-\infty}^{\infty} {}^{2\pi ikx} y_k$$

where the coefficients g_k are written as

$$g_k = \int_n^{n+1} f(\xi) \; e^{-2\pi i k \xi} \, d\xi \qquad (3.2.2)$$

For the case $x \to n+\frac{1}{2}$, (3.2.2) is replaced by

$$f(n+\tfrac{1}{2}) = \sum_{k=-\infty}^{\infty} e^{2\pi i k (n+\frac{1}{2})} g_k$$

Since $e^{2\pi i k n} = 1$ and $e^{\pi i k} = (-1)^k$, this further reduces into:

$$f(n+\tfrac{1}{2}) = \sum_{k=-\infty}^{\infty} (-1)^k g_k$$

which in using (3.2.2) takes on the form

$$f(n+\tfrac{1}{2}) = \sum_{k=-\infty}^{\infty} (-1)^k \left\{ \sum_{n=o}^{\infty} \int_n^{n+1} f(\xi) \; e^{-2\pi i k \xi} \, d\xi \right\}$$

Replacing the bracketed term by

$$\int_0^{\infty} f(\xi) \; e^{-2\pi i k \xi} \, d\xi$$

and summing over all possible values of ξ it results in

$$\sum_n f(n+\tfrac{1}{2}) = \sum_{k=-\infty}^{\infty} (-1)^k \int_0^{\infty} f(\xi) \; e^{-2\pi i k \xi} \, d\xi$$

which may be expanded as

$$\sum_n f(n+\tfrac{1}{2}) = \int_0^\infty f(\xi) \left| 1+\sum_{k=-\infty}^{-1} (-1)^k e^{-2\pi ik\xi} + \sum_{k=1}^{\infty} (-1)^k e^{-2\pi ik\xi} \right| d\xi$$

Noting that the cosine function is even and the sine function is odd, this further simplifies into:

$$\sum_n f(n+\tfrac{1}{2}) = \int_0^\infty f(x) + 2\sum_{k=1} (-1)^k \int_0^\infty f(x) \cos 2\pi kx \, dx \quad (3.2.3)$$

which is the Poisson summation formula.

In case of large values of the magnetic field so that $h\Omega \gg k_B T$ (3.2.3) may be used to rewrite (3.2.1) as

$$\Gamma_H = \Gamma_O \frac{h}{4k_B T} \left[\int_0^\infty \frac{dx}{\cosh^2 (\frac{h\Omega x}{2k_B T})} + 2 \sum_{k=1}^{\infty} (-1)^k \int_0^\infty \frac{\cos (2\pi kx)}{\cosh^2 (\frac{h\Omega x}{2k_B T})} dx \right] \quad (3.2.4)$$

Both integrals in (3.2.4) are table integrals and according to already mentioned formula in Dwight # 679.20 the first term on RHS gives $\Gamma_0/2$. Moreover according to Erdelyi (Tables of integral transforms) pp. 30#2, the cosine transform of sech^2 is given by

$$\int_0^\infty \mathrm{Sech}^2 (ay) \cos(xy) \, dy = \tfrac{1}{2}\pi a^{-2} x \, \mathrm{csch}(\tfrac{1}{2}\pi a^{-1} x) \quad (3.2.5)$$

Rewriting the integral of the second term as:

$$\int_0^\infty \frac{\cos(2\pi kx)}{\cosh^2(\frac{h\Omega x}{2k_BT})}\,dx = \frac{2k_BT}{h\Omega}\int_0^\infty \mathrm{Sech}^2 y\,\cos\left(\frac{4\pi kTk_B}{h\Omega}y\right)dy$$

where, $\frac{h\Omega x}{2k_BT} = y$, (3.2.5) may now be readily applied to it so that we get

$$\int_0^\infty \frac{\cos(2\pi kx)}{\cosh^2(\frac{h\Omega x}{2k_BT})}\,dx = \frac{k_BT}{h\Omega}\left(\frac{4\pi kTk_B}{h\Omega}\right)\mathrm{csch}\left(\frac{2\pi^2 kTk_B}{h\Omega}\right)$$

Consequently the second term on the RHS of (3.2.4) reduces into:

$$\Gamma_O \frac{h\Omega}{4k_BT}\left[2\sum_{k=1}^{\infty}(-1)^k\int_0^\infty \frac{\cos(2\pi kx)}{\cosh^2(\frac{h\Omega x}{2k_BT})}\,dx\right]$$

$$= 2\Gamma_O\,\pi^2\,(k_BT/h\Omega)\sum_{k=1}^{\infty}\frac{(-1)^k k}{\sinh\left(\frac{2\pi^2 kTk_B}{h\Omega}\right)}$$

Thus the whole of (3.2.4) may now be rewritten as

$$\Gamma_H = \Gamma_O/2 + 2\Gamma_O\,\pi^2\left(\frac{k_BT}{h\Omega}\right)\sum_{k=1}^{\infty}\frac{(-1)^k k}{\sinh\left(\frac{2\pi^2 kTk_B}{h\Omega}\right)} \qquad (3.2.6)$$

Letting, $2\pi^2\left(\frac{kTk_B}{h\Omega}\right) = x_k$ (3.2.6) may be rewritten as

$$\Gamma_H = \Gamma_O/2 + \Gamma_O \sum_{k=1}^{\infty} \frac{(-1)^k x_k}{\sinh x_k}$$

which since $x_k \ll 1$ (for large magnetic field) so that

$$\sinh x_k \doteq x_k + \frac{x_k^3}{6}$$

may be transformed into:

$$\Gamma_H = \Gamma_O/2 + \Gamma_O \sum_{k=1}^{\infty} \frac{(-1)^k 6}{6+x_k^2}$$

This on the other hand upon substitution of the value of x_k delivers:

$$\Gamma_H = \Gamma_O/2 + \Gamma_O \sum_{k=1}^{\infty} \frac{(-1)^k 6}{6+[2\pi^2 \frac{k_B T}{h\Omega})]^2 \; k^2}$$

which may be rewritten as

$$\Gamma_H = \Gamma_O/2 + \Gamma_O \sum_{k=1}^{\infty} \frac{(-1)^k 3/2\pi^4 \; (\frac{h\Omega}{k_B T})^2}{3/2\pi^4 \; (\frac{h\Omega}{k_B T})^2 + k^2}$$

When the substitution:

$$a^2 = 3/2\pi^4 \; (\frac{h\Omega}{k_B T})^2 \qquad (3.2.7)$$

is made the above expression simplifies into

$$\Gamma_H = \Gamma_O/2 + \Gamma_O\, a^2 \sum_{k=1}^{\infty} \frac{(-1)^k}{a^2+k^2} \qquad (3.2.8)$$

For the summation of the series in (3.2.8) we can employ the idea in Murray R. Spiegel (pp 190) [17]

$$\sum_{-\infty}^{\infty} (-1)^k f(k) = - \left\{ \begin{array}{l} \text{sum of the residues of } \pi \csc \pi Z\ f(z) \\ \text{at the poles of } f(z) \end{array} \right\}.$$

To make use of this statement, (3.2.8) has to be put in the form:

$$\Gamma_H = \Gamma_O/2 + \Gamma_O\, a^2/2 \left[\sum_{k=-\infty}^{\infty} \frac{(-1)^k}{a^2+k^2} - \frac{1}{a^2} \right]$$

$$\Gamma_H = \Gamma_O\, a^2/2 \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{a^2+k^2} \qquad (3.2.9)$$

If we now set $f(z) = \frac{1}{a^2+z^2}$, this function has simple poles at $Z = +ai$ and $Z = -ai$

Then the residue of $\frac{\pi \csc \pi z}{a^2+z^2}$ at, say, $Z = +ai$ is

$$\lim_{z \to ai} \frac{(z-ai)\,\pi \csc \pi z}{(z-ai)\ (z+ai)} = \frac{\pi \csc \pi (ai)}{2ai}$$

$$= \frac{-\pi i}{2ai\ \sin h \pi a}$$

$$= \frac{-\pi}{2a\ \sinh \pi a}.$$

In the same way it can be shown that the residue of $\frac{\pi \csc \pi z}{a^2+z^2}$ at $z = -ai$ is given also by $\frac{-\pi}{2a \sinh \pi a}$ so that $-\{$the sum of the residues$\}$ becomes $\pi/_{a \sinh \pi a}$. With this as the result of summation of the series, (3.2.9) takes now its final form as:

$$\Gamma_H = \Gamma_O \, \pi a/_{2 \sinh \pi a} \tag{3.2.10}$$

For large values of a (or large magnetic field) one can write

$\sinh \pi a \cong \frac{e^{\pi a}}{2}$ so that under such an assumption (3.2.10) may be rewritten as:

$$\Gamma_H = \Gamma_O \quad \pi a \; e^{-\pi a}$$

which after substitution of the value of <a> becomes:

$$\Gamma_H = \Gamma_o \sqrt{(3/_2)} \;\; (\tfrac{1}{\pi}) (\tfrac{h\Omega}{k_B T}) \;\; \exp \;\; -\sqrt{(3/2)} \; (\tfrac{1}{\pi}) (\tfrac{h\Omega}{k_B T}) \ll \Gamma_O. \tag{3.2.11}$$

Notice then that near the point $z \cong z^*$ and for the case of $h\Omega \gg k_B T$ the absorption coefficient is exponentially small.

If now we consider the opposite case of $k_B T \gg h\Omega$ or equivalently $a \ll 1$ (near the same point $z \cong z^*$) in which case $\sinh (\pi a) \cong \pi a$, (3.2.10) delivers:

$$\Gamma_H = \Gamma_O/_2 \tag{3.2.12}$$

and the absorption coefficient is seen to be a constant. Actually, the same result could have been obtained upon application of the Euler-Maclaurian formula:

$$\sum_n f(n+\tfrac{1}{2}) = \int_0^\infty f(x)\,dx - \frac{1}{24}\frac{df(x)}{dx}\Big|_{x=0}^{x=\infty} \quad \text{(Saalschitz 1893)}$$

to Eq. (3.2.1). The formula is applicable under the condition:

$$|f(n+\tfrac{1}{2}+1) - f(n+\tfrac{1}{2})| \ll f(n+\tfrac{1}{2})$$

which actually is equivalent to the condition

$$h\Omega \ll 2k_B T$$

Comparison of Eqtns (3.2.11) and (3.2.12) shows that, near $z \cong z^*$ in both cases of rise of the values of H or T from minimal values, the absorption coefficient given by (1.2.5) is smeared, the stronger effect being that of the magnetic field H. The actual smoothing of the jump Γ_0 at $z \cong z^*$ [i.e. $(\Delta = z - z^*) \ll k_B T$] by a small magnetic field ($h\Omega \ll k_B T$ can be calculated using the second order approximation of sinh a in (3.2.10) i.e.

$$\sinh \pi a \cong \pi a + \frac{(\pi a)^3}{3!} \quad (a \ll 1)$$

or $\operatorname{csch} \pi a \cong \dfrac{1}{\pi a} - \dfrac{\pi a}{6}$.

This when substituted into (3.2.10) delivers

$$r_H = r_0/2 - \frac{r_0(\pi a)^2}{12}$$

which upon replacement of the value of a^2 from (3.2.7) further reduces into

$$r_0/2\left[1 - \frac{1}{4\pi^2}\left(\frac{\hbar\Omega}{k_B T}\right)^2\right] \qquad (3.2.13)$$

Eq. (3.2.13) is the required equation which estimates the amount of smearing made by a small magnetic field.

As it is fully explained in section 1.3, the electrons that are rigorously involved in ultrasonic absorption are those whose velocities satisfy condition (1.3.6); and whenever a section of the Landau - level consisting of electrons with momentum p_{z0} satisfying (1.3.6) merges into the strip of width $k_B T$ (Figure 4) the absorption coefficient assumes a large value. As the magnetic field is uniformly increased this behaviour of absorption coefficient is periodically repeated which according to the language of section 1.3 is termed as Giant Quantum Oscillation. The region $Z \cong Z^*$ corresponds to electrons with Fermi velocity nearly along the magnetic field $\vec{H}$ (arbitrarily directed along the z- axis); and it consists not so appreciable number of electrons so that near this threshold of energy the coefficient of absorption is not expected to show pecurliar behaviours.

This is the whole content of informations delivered by Eqtns. (3.2.10 -13). Note that the absorption coefficient depends on the number of Landau levels below Z so that if the condition $h\Omega \gg k_BT$ is imposed on top of $Z \doteq Z^*$ (k_BT is the width of the strip) the absorption coefficient will be exponentially small. An increment of the magnetic field or even a shift of the parameter Z will aggravate the situation by taking away the effective section on a Landau level far out of the strip as can be easily observed from Eqtn. (3.2.11) (Figure 8).

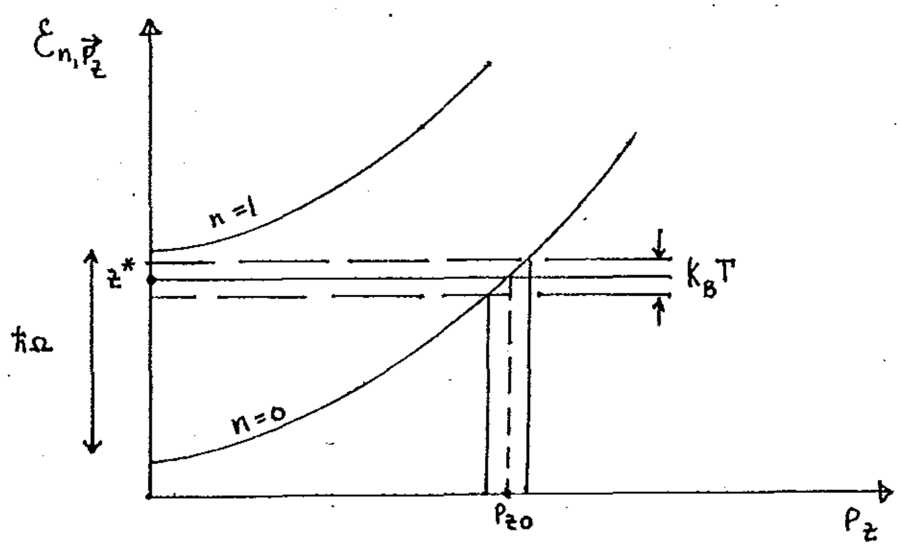


Fig. 8: Arrangement of Landau levels for $Z \doteq Z^*$ and the case $h\Omega \gg k_BT \leftrightarrow (\Gamma_{II} \propto \frac{h\Omega}{k_BT} \exp(\frac{-\alpha h\Omega}{k_BT})$

On the otherhand, if $h\Omega \ll k_BT$, the number of Landau levels below Z will be numerous so that the effective sections on the different Landau levels almost form a net with overlapping projections on the p_z - axis in such a way that the process of absorption is passed smoothly among levels for changes of $h\Omega$

($h\Omega \ll k_BT$) or Z which results in a uniform absorption coefficient just as Eq. (3.2.12) predicts (Figure 9).

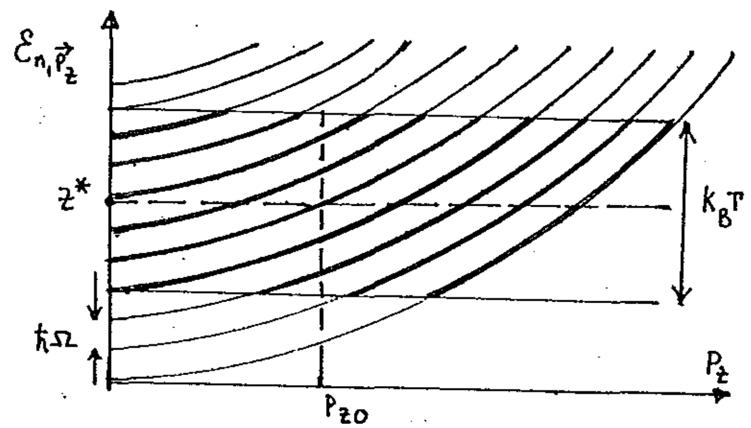


Fig. 9: Arrangement of Landau Levels for $Z = Z^*$ and $h\Omega \ll k_BT$ ($\Gamma_H = \Gamma_O/2$)

3.3 Investigation of Γ_H in the region $0<Z<Z^*$ and discussion of results:

In section (3.2), the behaviour of the absorption coefficient near $Z \cong Z^*$ was thoroughly discussed. In this section we pick another region, namely, the region $0<Z<Z^*$. For an arbitrary value of Z but still $k_BT \ll Z$, (3.2.3) may be imposed on (3.1.1) to produce:

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \int_0^\infty \cosh^{-2}\left(\frac{h\Omega x}{2k_BT} - \frac{Z-Z^*}{2k_BT}\right) dx +$$

$$+ 2(\Gamma_O \frac{h\Omega}{4k_BT} \sum_{k=1}^{\infty} (-1)^k \int_0^\infty \cosh^{-2}\left(\frac{h\Omega x}{2k_BT} - \frac{Z-Z^*}{2k_BT}\right) \cos(2\pi kx)\, dx$$

which may be rewritten as:

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \int_0^\infty \cosh^{-2} \left(\frac{h\Omega x}{2k_BT} - \frac{Z-Z^*}{2k_BT}\right) dx +$$

$$+ 2\left(\Gamma_O \frac{h\Omega}{4k_BT}\right) \text{Re} \sum_{k=1}^{\infty} (-1)^k \int_0^\infty \frac{e^{2\pi ikx}}{\cosh^2\left(\frac{h\Omega x}{2k_BT} - \frac{Z-Z^*}{2k_BT}\right)} dx$$

Using the usual change of variable

$$y = \frac{h\Omega x}{2k_BT} - \frac{Z-Z^*}{2k_BT}$$

and the techniques employed to find (3.1.2), Eq. (3.3.1) simplifies into:

$$\Gamma_H = \Gamma_O/2 \left[1+\tanh\left(\frac{Z-Z^*}{2k_BT}\right)\right] + \Gamma_O \text{ Re} \sum_{k=1}^{\infty} \int (-1)^k \int_{-\frac{Z-Z^*}{2k_BT}}^{\infty} \text{Sech}^2 y \; e^{\frac{4\pi kTk_B}{h\Omega}}$$

$$\times \left(y+ \frac{Z-Z^*}{2k_BT}\right) i dy \qquad (3.3.2)$$

Now since

$$\text{csh } y = \frac{e^y+e^{-y}}{2}$$

for $y>0$, we can write

$$e^y/_2 \leq \text{cshy} \leq e^y$$

or

$$2/e^{y} \geq (\cosh y)^{-1} \geq \frac{1}{e^{y}}$$

$$4/e^{2y} \geq (\cosh y)^{-2} \geq \frac{1}{e^{2y}}$$

Thus one can make the associated approximation

$$\frac{1}{\cosh^2 y} \cong \xi/e^{2y} \text{ , where } 1 \leq \xi \leq 4$$

$$\Gamma_E = \Gamma_0/2 \left| 1+\tanh \left(\frac{Z-Z^*}{2k_B T}\right) \right| + \Gamma_0 \; \xi \; \mathrm{Re} \sum_{k=1}^{\infty} (-1)^k \exp\left[\frac{4\pi k T k_B}{h\Omega}\left(\frac{Z-Z^*}{2k_B T}\right) i\right] \mathrm{X}$$

$$\mathrm{X} \int_{-\frac{Z-Z^*}{2k_B T}}^{\infty} \exp\left(-2+\frac{4\pi k T k_B i}{h\Omega}\right) y \; dy \qquad (3.3.3)$$

But,
$$\int_{-\frac{Z-Z^*}{2k_B T}}^{\infty} \exp\left(-2+\frac{4\pi k T k_B}{h\Omega} i\right) y \; dy = \left.\frac{\exp\left(-2+\frac{4\pi k T k_B}{h\Omega} i\right) y}{-2+\frac{4\pi k T k_B}{h\Omega} i}\right|_{-\frac{Z-Z^*}{2k_B T}}^{\infty}$$

Since $\exp(iy)$ is finite & $\exp(2y)$ is infinite for $y \to \infty$, this last expression finally gives

$$\frac{\exp\left(-2+\frac{4\pi k T k_B i}{h\Omega}\right)\left(\frac{Z^*-Z}{2k_B T}\right)}{2-\frac{4\pi k T k_B i}{h\Omega}}$$

so that (3.3.3) may now be put in the form:

$$\Gamma_H = \Gamma_0/2\ \left|1+\tanh\left(\frac{Z-Z^*}{2k_BT}\right)\right| + \Gamma_0\ \mathrm{Re}\sum_{k=1}^{\infty}(-1)^k\ \exp\ \frac{4\ kTk_B}{h\Omega}\ \left(\frac{Z-Z^*}{2k_BT}\right) i\ \mathrm{X}$$

$$\mathrm{X}\ \frac{\exp\ \left(-2+\frac{4\pi kTk_B}{h\Omega} i\right)\left(\frac{Z^*-Z}{2k_BT}\right)}{2-\frac{4\ kTk_B}{h\Omega}\ i}$$

This simplifies into:

$$\Gamma_H = \Gamma_0/2\ \left[1+\tanh\left(\frac{Z-Z^*}{2k_BT}\right)\right] + \Gamma_0\ \xi\ \mathrm{Re}\sum_{k=1}^{\infty}(-1)^k\ \frac{\exp\ \left(\frac{Z-Z^*}{k_BT}\right)}{2-\frac{4\pi kTk_B}{h\Omega} i} \tag{3.3.4}$$

Let us now consider the region $0<Z<Z^*$. Making the substitution $|Z-Z^*| = \Delta$, (3.3.4) may be rewritten as:

$$\Gamma_H = \Gamma_0/2\ \left[1-\tanh\ \frac{\Delta}{2k_BT}\right] + (\Gamma_0/2)\xi\ e^{-\Delta/k_BT}\ \mathrm{Re}\sum_{k=1}^{\infty}\frac{(-1)^k}{\left(1-\frac{2\pi kTk_B}{h\Omega}\ i\right)}$$

On multiplying and dividing the argument in the summation by $1+\frac{2\ kTk_B i}{h\Omega}$ this further reduces into:

$$\Gamma_H = \Gamma_0/2\ \left[1-\tanh\ \frac{\Delta}{2k_BT}\right] + (\Gamma_0/2)\xi\ e^{-\ /k_BT}\ \mathrm{Re}\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\frac{2\pi kTk_B i}{h\Omega}\right)}{\left(1+\frac{4\pi^2k^2T^2k_B^2}{h^2\ \Omega^2}\right)}$$

Taking only the real part we may from here write

$$\Gamma_H = \Gamma_O/_2\left[1-\tanh\frac{\Delta}{2k_BT}\right] + \Gamma_O/_2\,\xi\, e^{-\Delta/k_BT}\sum_{k=1}^{\infty}\frac{(-1)^k}{1+\dfrac{4\pi^2k^2T^2k_B^2}{h^2\Omega^2}}$$

or

$$\Gamma_H = \Gamma_O/2\,\left[1-\tanh\Delta/_{2k_BT}\right] + (\Gamma_O/2)\xi\, e^{-\Delta/k_BT}\sum_{k=1}^{\infty}\frac{(-1)^k h^2\Omega^2/4\pi^2T^2k_B^2}{(h^2\pi^2/4\pi^2T^2k_B^2)+k^2}$$

Letting
$$\frac{h^2\Omega^2}{4\pi^2T^2k_B^2} = a^2 \qquad (3.3.5)$$

this gives

$$\Gamma_H = \Gamma_O/2\,\left[1-\tanh\Delta/_{2k_BT}\right] + \Gamma_O/2\,\xi\, e^{-\Delta/k_BT}\, a^2\sum_{k=1}^{\infty}\frac{(-1)^k}{a^2+k^2} \qquad (3.3.6)$$

This is in exactly the same form as Eq. (3.2.8) and successive techniques used to derive (3.2.10) may be employed to derive a similar expression from (3.3.6), namely,

$$\Gamma_H = \Gamma_O/2\,\left[1-\tanh\frac{\Delta}{2k_BT}\right] + \Gamma_O/2\,\xi\, e^{-\Delta/k_BT}\left[-\tfrac{1}{2}+\frac{\pi a}{2\sinh\pi a}\right]$$

which upon substitution the value of <a> from (3.3.5) delivers:

$$\Gamma_H = \Gamma_O/2\,\left[1-\tanh\Delta/_{2k_BT}\right] + \Gamma_O/4\,\xi\, e^{-\Delta/k_BT}\left[-1+\frac{h\Omega}{2k_BT\sinh\frac{h\Omega}{2k_BT}}\right] \qquad (3.3.7)$$

Eq. (3.3.7) is actually the expression for the coefficient of ultrasonic absorption in the region $0<Z<Z^*$. A more exact formula may be obtained by considering (3.3.7) near $\Delta \cong 0$. At this point and moreover for the condition $h\Omega \gg k_B T$ so that

$$\sinh \frac{h\Omega}{2k_B T} = \frac{e^{\frac{h\Omega}{2k_B T}}}{2}$$

eq. (3.3.7) becomes:

$$\Gamma_H = \Gamma_O/2 + \Gamma_O/4 \; \xi \left[-1 + \frac{h\Omega}{k_B T} \; e^{-\frac{h\Omega}{2k_B T}}\right]$$

Comparison of this formula with that stated in (3.2.11) shows that $\xi = 2$. Thus (3.3.7) put in more exact form assumes:

$$\Gamma_H = \Gamma_O/2 \left[1 - \tanh \Delta/2k_B T\right] + \Gamma_O/2 \; e^{-\Delta/k_B T} \left[-1 + \frac{h\Omega}{2k_B T \sinh \frac{h\Omega}{2k_B T}}\right] \tag{3.3.8}$$

This being the general expression for the absorption coefficient in the region $0<Z<Z^*$ it can be shown to reduce to known expressions already derived. For instance for small magnetic fields ($h\Omega \ll k_B T$) in which case:

$$\sinh \frac{h\Omega}{2k_B T} \cong \frac{h\Omega}{2k_B T} \; ,$$

(3.3.8) dissolves into:

$$\Gamma_H = \Gamma_O/2 \left[1-\tanh\Delta/_{2k_BT}\right]$$

which near $\Delta \cong 0$ ($Z \cong Z^*$) itself dissolves into $\Gamma_H = \Gamma_O/2$ and this is in exact agrrement with (3.2.12). Moreover, for the case $\Delta \cong 0$ and $h\Omega \gg k_BT$, in which case

$$\sin \frac{h\Omega}{2k_BT} \cong \frac{e^{\frac{h\Omega}{2k_BT}}}{2}$$

eq. (3.3.8) takes on the form

$$\Gamma_H = \Gamma_O/2 \left(\frac{h\Omega}{k_BT}\right) e^{-\frac{h\Omega}{k_BT}} \ll \Gamma_O$$

which is in good agrrement with (3.2.11)

Finally for the condition $h\Omega \ll k_BT$ in which case

$$\sinh \frac{h\Omega}{2k_BT} \cong \frac{h\Omega}{2k_BT} + \frac{(h\Omega/_{2k_BT})^3}{6}$$

so that

$$\frac{h\Omega}{2k_BT\sinh(h\Omega/_{2k_BT})} \cong 1-\frac{1}{6}\left(\frac{h\Omega}{2k_BT}\right)^2$$

and a further condition of $\Delta \ll k_BT$ in which case

$$\tanh \frac{\Delta}{2k_BT} \to 0$$

(3.3.8) transforms into:

$$\Gamma_H = \Gamma_O/2 \left[1 - \frac{1}{6} \left(\frac{h\Omega}{2k_BT}\right)^2 \right]$$

But, this last expression is in a reasonable agreement with (3.2.13).

The region $0<Z<Z^*$ is in direct contradiction with condition (1.3.6). Thus in this region the absorption coefficient should remain of no particular property, low and monotonous which is, of course, rightly predicted by (3.3.8). Note that as Z is increased starting from zero towards Z^* the value of $\Delta = |Z-Z^*|$ decreases so that Γ_H shows considerable growth toward increasing value of Z. In other words, Γ_H monotonously increases from Γ_{min} at Z=0 to Γ_{max} at $Z \cong Z^*$ (Figure 10). The extremum values follows from (3.3.8) directly by substituting $\Delta = 0$ and Z=0 as a result of which we obtain

$$\Gamma_{max} = \Gamma_O/2 \frac{h\Omega}{2k_BT \sinh \frac{h\Omega}{2k_BT}}$$

and

$$\Gamma_{min} = \Gamma_O/2 \left(1-\tanh\frac{Z^*}{2k_BT}\right) + (\Gamma_{max}-\Gamma_O/2)\, e^{-Z^*/k_BT}$$

respectively.

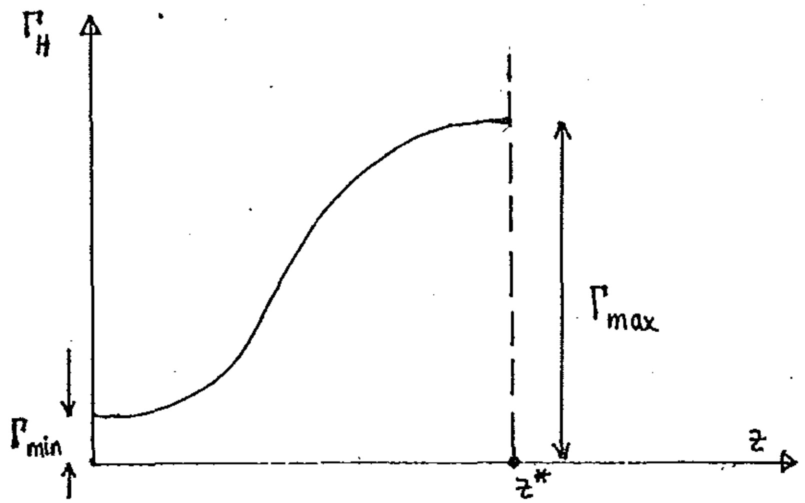


Fig. 10: Dependence of Γ_H on Z in the interval $0<Z<Z^*$.

3.4 Investigation of Γ_H in the region $Z \gg Z^*$ and discussion of results

Earlier, in section 3.3, Eq. (3.3.2) was derived as an expression of the absorption coefficient for an arbitrary value of Z; and it may be employed here without any reservation. First we try to evaluate the integral. Since $\Delta \gg k_B T$ the integral

$$\int_{-\Delta/2k_BT}^{\infty} \mathrm{Sech}^2 y \cos\left[\frac{4\pi kTk_B}{h\Omega}\left(y+\frac{Z-Z^*}{2k_BT}\right)\right] dy$$

can be rewritten as

$$\int_{-\infty}^{\infty} \mathrm{Sech}^2 y \cos\left[\frac{4\pi kTk_B}{h\Omega}\left(y+\frac{Z-Z^*}{2k_BT}\right)\right] dy$$

which after expansion gives

$$\int_{-\infty}^{\infty} \text{Sech}^2 y \cos \frac{4\pi kTk_B}{h\Omega} y \cos\left[\frac{4\pi kTk_B}{h\Omega} \left(\frac{Z-Z^*}{2k_BT}\right)\right] dy$$

$$-\int_{-\infty}^{\infty} \text{Sech}^2 y \sin \frac{4\pi kTk_B}{h\Omega} y \sin\left[\frac{4\pi kTk_B}{h\Omega} \left(\frac{Z-Z^*}{2k_BT}\right)\right] dy$$

But, the value of the second integral between symmetric limits is zero so that we are left only with

$$\int_{-\infty}^{\infty} \text{Sech}^2 y \cos \frac{4\pi kTk_B}{h\Omega} y \cos\left[\frac{4\pi kTk_B}{h\Omega} \left(\frac{Z-Z^*}{2k_BT}\right)\right] dy$$

$$= \cos \frac{2\pi k\Delta}{h\Omega} \int_{-\infty}^{\infty} \text{Sech}^2 y \cos \left[\left(\frac{4\pi kTk_B}{h\Omega}\right) y\right] dy$$

The integral is already evaluated in section 3.2 and using this result the above expression simplifies into:

$$2\left[\frac{\pi}{2} \cos \frac{2\pi k\Delta}{h\Omega} \left(\frac{4\pi kTk_B}{h\Omega}\right) \text{csch} \left(\frac{2\pi^2 kTk_B}{h\Omega}\right)\right]$$

$$=\left(\frac{4\pi^2 kTk_B}{h\Omega}\right) \cos \left(\frac{2\pi k\Delta}{h\Omega}\right) \text{csch} \left(\frac{2\pi^2 kTk_B}{h\Omega}\right)$$

Using this result Eq. (3.3.2) may be rewritten as

$$\Gamma_H = \Gamma_O/2 \left[1+\tanh\left(\frac{Z-Z^*}{2k_BT}\right)\right] +$$

$$+ \Gamma_O \left(\frac{4\pi^2 k_B T}{h\Omega}\right) \sum_{k=1}^{\infty} (-1)^k{}_k \cos\left(\frac{2\pi k\Delta}{h\Omega}\right) \operatorname{csch}\left(\frac{2\pi^2 kTk_B}{h\Omega}\right) \tag{3.4.1}$$

In case of when $\Delta \gg k_BT$ (or equivalently, since $Z \gg Z^*$, when $Z \gg k_BT$) under which $\tanh \frac{\Delta}{2k_BT} \cong 1$ (3.4.1) delivers

$$\Gamma_H = \Gamma_O + \Gamma_O \left(\frac{4\pi^2 k_B T}{h\Omega}\right) \sum_{k=1}^{\infty} (-1)^k k \cos\left(\frac{2\pi k\Delta}{h\Omega}\right) \operatorname{csch}\left(\frac{2\pi^2 kTk_B}{h\Omega}\right) \tag{3.4.2}$$

From this we discover that in the limit $\Delta \gg k_BT$ the presence of the magnetic field introduces the oscillatory term

$$\tilde{\Gamma} = \Gamma_O \left(\frac{4\pi^2 k_B T}{h\Omega}\right) \sum_{k=1}^{\infty} (-1)^k k \cos\left(\frac{2\pi k\Delta}{h\Omega}\right) \operatorname{csch}\left(\frac{2\pi^2 kTk_B}{h\Omega}\right)$$

on top of the threshold value of absorption coefficient of (1.2.5), Γ_O, so that the whole of (3.4.2) becomes oscillatory. The resulting oscillations are expected to be giant for the additional condition $h\Omega \gg k_BT$ as (2.2.4) predicts. The amplitude of oscillation can be estimated from an actual evaluation of (3.4.1) in the limits indicated.

Using the techniques employed in section (3.2) we can frist write

$$\frac{2\pi^2 kTk_B}{h\Omega} \ \text{csch} \ (\frac{2\pi^2 kTk_B}{h\Omega}) = \frac{a^2}{a^2+k^2}$$

where a^2 is given by (3.2.7).

Using this result (3.4.1) takes on the form

$$\Gamma_H = \Gamma_O/2 \ | \ 1+\tanh(\frac{Z-Z^*}{2k_BT}) \ | + 2\Gamma_O \sum_{k=1}^{\infty} (-1)^k \frac{a^2 \cos(\frac{2\pi k\Delta}{h\Omega})}{a^2+k^2}$$

under the condition $\tanh \frac{\Delta}{2k_BT} \simeq 1$ on effect this further reduces into

$$\Gamma_H = \Gamma_O + 2\Gamma_O \sum_{k=1}^{\infty} (-1)^k \frac{a^2 \cos(\frac{2\pi k\Delta}{h\Omega})}{a^2+k^2} \qquad (3.4.3)$$

The series in (3.4.3) rewritten in the form

$$\tfrac{1}{2}\left[\sum_{\infty}^{\infty}(-1)^k \ a^2 (\frac{\cos \frac{2\pi k\Delta}{h\Omega}}{a^2+k^2}) \ -1\right]$$

or as:

$$-\tfrac{1}{2} + a^2/2 \sum_{k=-\infty}^{\infty} (-1)^k \frac{\cos(\frac{2\pi k\Delta}{h\Omega})}{a^2+k^2}$$

may be summed using the formula in Murray R. Spiegel as indicated in section (3.2)

If we choose

$$f(Z) = \cos\left(\frac{2\pi Z\Delta}{h\Omega}\right)/a^2+Z^2$$

the function $\dfrac{\pi\csc\pi Z\cos\left(\frac{2\pi Z\Delta}{h\Omega}\right)}{a^2+Z^2}$ has simple poles at $Z = +ai$ and at $Z = -ai$.

The residue of the function at $Z = +ai$ is given by

$$\lim_{Z\to ai} \frac{(Z-ai)\pi\ \csc\pi Z\ \cos\left(\frac{2\pi Z\Delta}{h\Omega}\right)}{(Z-ai)(Z+ai)}$$

$$= \frac{\pi\csc\pi ai\ \cos\left(\frac{2\pi a\Delta i}{h\Omega}\right)}{2ai}$$

$$= \frac{\pi\cos\left(\frac{2\pi a\Delta i}{h\Omega}\right)}{2ai\ \sin\pi ai}$$

But, $\cos\left(\frac{2\pi a\Delta i}{h\Omega}\right) = \cosh\left(\frac{2\pi a\Delta}{h\Omega}\right)$

and $\sin(\pi ai) = i\ \sinh(\pi a)$

so that the residue at $Z = +ai$ finally becomes

$$\frac{\cosh\left(\frac{2\pi a\Delta}{h\Omega}\right)}{2ai\ (i\ \sinh\pi a)} = -\pi/_{2a}\left[\frac{\cosh\left(\frac{2\pi a\Delta}{h\Omega}\right)}{\sinh\pi a}\right]$$

In the same way the residue at $z = -ai$ can also be shown to be

$$-\pi/_{2a} \left[\frac{\cosh\left(\frac{2\pi a\Delta}{h\Omega}\right)}{\sinh \pi a} \right]$$

As the result (3.4.3) produces

$$\Gamma_H = \Gamma_O + 2\,\Gamma_O \left[-\tfrac{1}{2} + a^2/2\,(\pi/_a \times \frac{\cosh\left(\frac{2\pi a\Delta}{h\Omega}\right)}{\sinh \pi a}) \right]$$

$$\Gamma_H = \Gamma_O \;\; \pi a \left[\frac{\cosh\left(\frac{2\pi a\Delta}{h\Omega}\right)}{\sinh \pi a} \right]$$

Substituting the value of <a> from (3.2.7) this gives

$$\Gamma_H = \Gamma_O \frac{1}{\pi} (\sqrt{3/2}) \left(\frac{h\Omega}{k_B T}\right) \frac{\cosh\left(\frac{\sqrt{6}}{\pi} \frac{\Delta}{k_B T}\right)}{\sinh\left(\frac{1}{\pi} \sqrt{3/2}\, \frac{h\Omega}{k_B T}\right)}$$

which may be rewritten as

$$\Gamma_H = \Gamma_O \frac{1}{\pi} (\sqrt{3/2}) \left(\frac{h\Omega}{k_B T}\right) \frac{\exp(\sqrt{6}/\pi)\frac{\Delta}{k_B T} + \exp-(\sqrt{6}/\pi)\frac{\Delta}{k_B T}}{\exp\, 1/\pi\, \sqrt{(3/2)}\left(\frac{h\Omega}{k_B T}\right) - \exp -1/\pi\, \sqrt{(3/2)}\frac{h\Omega}{k_B T}} \qquad (3.4.4)$$

But it is already indicated that GQO is Observable under the conditions $k_B T \ll h\Omega \ll \Delta$.

From this follow the statements

$$e^{-\Delta/k_B T} \cong 0 \cong e^{-h\Omega/k_B T}$$

and $\Delta/k_BT \gg \frac{h\Omega}{k_BT}$

These statements imposed on (3.4.4) finally deliver

$$\Gamma_{H}\Big|_{k_BT \ll \Omega \ll \Delta} = \Gamma_0 \frac{1}{\pi} (\sqrt{3}/2) (\frac{h\Omega}{k_BT}) \exp (\frac{\sqrt{6}}{\pi}) \Delta/k_BT \qquad (3.4.5)$$

Eq. (3.4.5) is an estimate of the amplitude of oscillations under the conditions $k_BT \ll h\Omega \ll \Delta$; and we then conclude that under these conditions the absorption coefficient Γ_H shows Giant Quantum oscillations with an exponentially rising amplitude (in Z) which is in line with the ideas of Gurevich and other authors of GQO. The oscillations are periodic in inverse magnetic field (1/H) and in the parameter Z.

The periods of GQO of the coefficient of ultrasonic absorption in inverse magnetic field (1/H) or in the parameter (Z) are easily extracted from the period of cos $(\frac{2\pi k\Delta}{h\Omega})$. Note that for $Z \gg Z^*$ we have $\Delta = Z$; and since $\Omega = \frac{eH}{mc}$ we can write

$$\cos (\frac{2\pi k\Delta}{h\Omega}) = \cos \left[\frac{2\pi k(Z)}{h} \ (\frac{mc}{eH}) \right]$$

$$= \cos (\frac{2\pi kZmc}{eh}) \frac{1}{H}$$

But,

$$\cos (\frac{2\pi kZmc}{eh}) \frac{1}{H} = \cos \left[\frac{2\pi kZmc}{eh} \ (\frac{1}{H}) + \frac{2\pi kZmc}{eh} \Delta(\frac{1}{H}) \right]$$

where, $\Delta(1/H)$ is the period in inverse magnetic field. Thus we conclude that

$$\frac{2\pi kZmc}{eh}) \Delta(\frac{1}{H}) = 2\pi k$$

or $$\Delta\left(\frac{1}{H}\right) = \frac{eh}{mcZ} \tag{3.4.6}$$

where of course $Z = \epsilon_F - \epsilon_k$. But for $Z \gg 1$ in which case $Z \cong \epsilon_F$, the period can be rewritten as

$$\Delta\left(\frac{1}{H}\right) = \frac{eh}{mc\,\epsilon_F}$$

For the case $\Delta \cong Z$ we can also write

$$\cos\left(\frac{2\pi k\Delta}{h\Omega}\right) = \cos\left(\frac{2\pi k}{h\Omega} Z\right)$$

But,

$$\cos\left(\frac{2\pi k}{h\Omega}\Delta Z\right) = \cos\left(\frac{2\pi k}{h\Omega} Z + \frac{2\pi k}{h\Omega}\Delta Z\right)$$

where ΔZ is the period in the parameter Z. Thus we notice that

$$\frac{2\pi k}{h\Omega}\Delta Z = 2\pi k$$

or

$$\Delta Z = h\Omega \tag{3.4.7}$$

Under the conditions of GQO as given by (2.2.4) the "will be effective" sections are regularly placed with non-overlapping projections as shown in Figure (11).

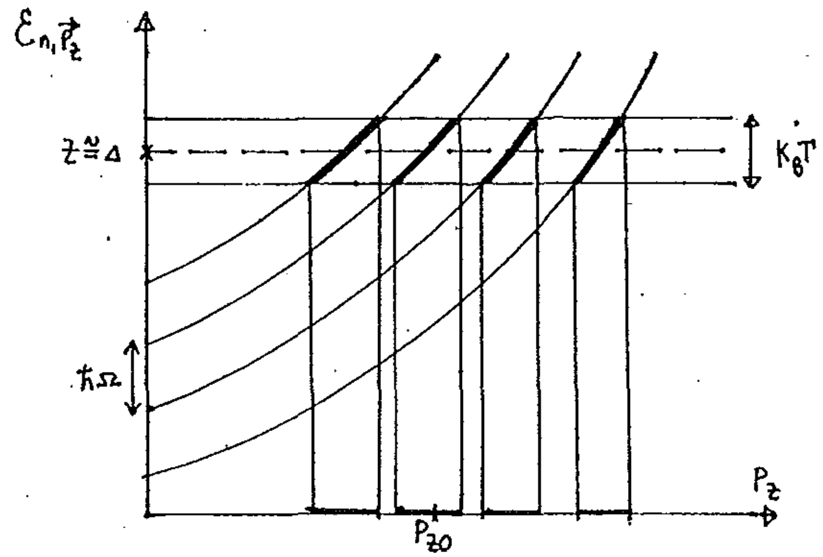


Fig. 11: Arrangement of Landau levels in GQO

So, as the magnetic field is increased which causes the upward movement of the levels and shift of the projections to the right, the absorption coefficient rises and falls between maximum and minimum values periodically together with periodic passage of the projections through p_{z0} as just (3.4.1) predicts (Figure 12).

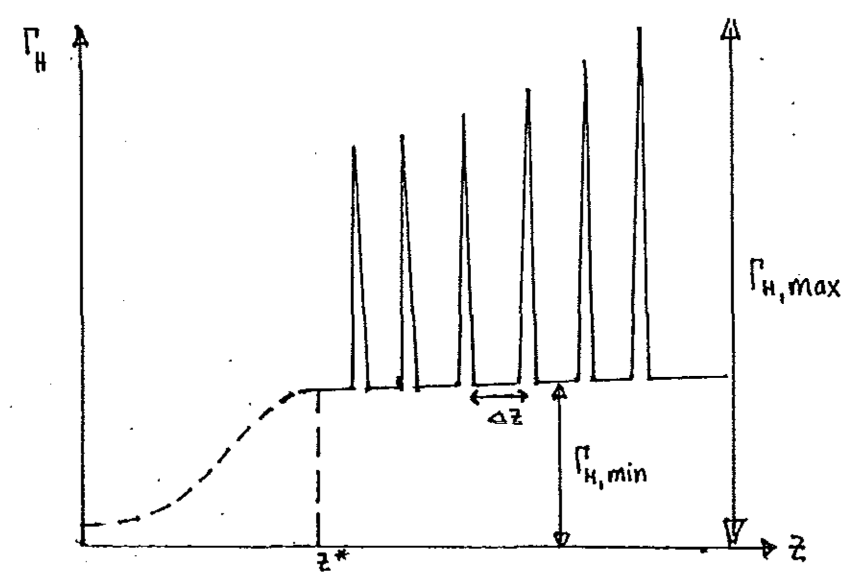


Fig. 12: Giant Quantum Oscillations in Z

$\Gamma_{H,max}$ is estimated by (3.4.5) and the estimation of $\Gamma_{H,min}$ can be made by referring to the work of T.V. Ivanova and M.I. kaganov [18]. In this work, it is shown that at $T = 0$, for the conditions,

$$Z \gg Z^* ,$$

$$Z^* \gg h\Omega$$

and $\omega\tau \gg 1$ (τ = Relaxation time of free electrons) all of them being in accordance with those required for GQO, the absorption coefficient Γ_H, which for low temperature is due to scattering from defects and impurities, is given by

$$\Gamma_H \cong \Gamma_0 \left(\frac{\Omega}{\omega}\right) \frac{1}{\tau}$$

where, $$\tau = \tau_0 \sqrt{\left(\frac{\varepsilon_0}{Z}\right)}$$

ε_0 = energy of order of the Fermi energy of the main band

τ_0 = Relaxation time of electrons from the main band

Hence, without loss of generality we can take up the above mentioned result for the estimation of $\Gamma_{H,min}$ i.e.

$$\Gamma_{H,min} = \Gamma_0 \left(\frac{\Omega}{\omega}\right) \frac{1}{\tau_0} \left(\frac{Z}{\varepsilon_0}\right)$$

It may be interesting in addition to know why the amplitude of oscillations increases with increasing magnetic field. At $H = 0$, the allowed states are distributed (in $\hat{p}$-space) uniformly inside the Fermi sphere and correspond to the elementary volumes $(2\pi h)^3$. For clarity, these volumes can be denoted by points spaced a distance $2\pi h$ from each other along the axes p_x, p_y and p_z. The points fill a circle of radius $\sqrt{(p_F^2 - p_z^2)}$ where $p_F = \sqrt{(2m\varepsilon_F)}$ in any section p_z = constant. A uniform filling of the Fermi sphere by the points depicting the allowed states corresponds to a quasi-contionous energy spectrum (Figure 13a).

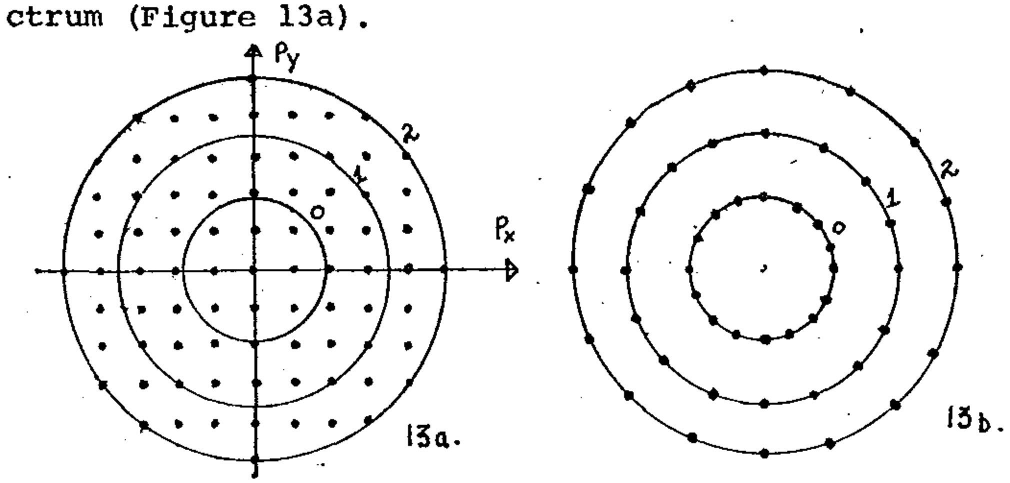


Fig. 13a: Section of the Fermi sphere by $p_z=0$

Fig. 13b: Allowed orbits

Under the action of a magnetic field along the Z-axis, the discrete levels given by $h\Omega(n+\frac{1}{2})$ become the allowed values of energy and these energy levels determine the discrete allowed orbits of electrons in the planes p_z = constant of $\hat{p}$ - space. (Figure 13b). Thus application of the magnetic field does not change the number of states in the plane but draws them onto the nearest orbits with radii given by

$$p_n = \sqrt{(2mh\Omega(n+\tfrac{1}{2}))} .$$

As the areas located between any two neighbouring orbits is the same (can be noticed from 4.1.1), the degree of degeneracy in each allowed orbit is $2\frac{2\pi h\Omega m}{(2\pi h)^2}$. The orbits considered are entirely identical to each other in all planes p_z = constant. This means that all allowed states in the Fermi surface in the magnetic field are condensed on the surface of coaxial cylinders parallel to the p_z - axis. The total number of states condensed onto the n-th cylinder will then be

$$N_n = \frac{8\pi m h\Omega \sqrt{2m\left[\varepsilon_F-(n+\frac{1}{2})h\Omega\right]}}{(2\pi h)^3}$$

when the magnetic field is increased to the extent that $\varepsilon_F = (n+\frac{1}{2})h\Omega$ the cylinder crosses over the Fermi surface in which case, as the above equation indicates, the cylinder gets depopulated from its electrons. These electrons are redistributed onto cylinders of a smaller radius which are located inside the Fermi sphere.

This process may also be observed in light of what happens to the Landau levels (parabolae). An energy state with the given value of p_z on a parabolae with the number n corresponds to all states on the discrete orbit with the same number which is located in the plane p_z = constant and hence has a multiplicity equal to $2\frac{2\pi h\Omega m}{(2\pi h)^2}$. The length of the parabolae between two symmetrical points corresponds to thelength of the n-th cylinder. Since the energy of an electron on the parabolae is only dependent on p_z, as in the model of a one dimensional gas the density of states on the parabolae can be written as

$$\nu_n(\varepsilon) = \frac{4\pi m h\Omega}{(2\pi h)^2} \frac{m^{\frac{1}{2}}}{\sqrt{2}\pi h} \left[\varepsilon - (n+\tfrac{1}{2})h\Omega \right]^{-\frac{1}{2}}$$

The total density of states of the the total electron system is then found by summing the above expression over all Landau levels. Notice that the density of states on the n-th parabola has an infinite singularity at $\varepsilon = (n+\frac{1}{2})h\Omega\,(n=0,1,2----)$. With an increase of the magnetic field the distance $h\Omega$ between the singularities grows and the infinite maxima on the curve of $\nu_n(\varepsilon)$ pass in succession through the Fermi level. Each time the bottom of a next Landau parabola coincides with the Fermi level, an infinite singularity of the density of states appears on the latter, which in turn causes a singularity of all thermodynamic and kinetic characterstics of the electron system, the absorption coefficient being one, which depend on the number of electrons on the Fermi level. The periodic repetition of these singularities at an increase of H is physically the cause of the oscillating effect of the magnetic field on these parameters. As the magnetic field is not expected to excite electrons outside the Fermi level, a Landau parabola as it clears off the Fermi level gets depopulated of its electrons which get redistributed among lower parabolae, which are still under. This eventually increases the number states in the effective section on the next parabola so that the absorption peak now appears with larger amplitude and this continues as the parabolae one by one skip over the Fermi level in the process of increasing the magnetic field.

One point that must not be forgotten is the fact that the Fermi level is itself a function of the magnetic field. It can be shown that the Fermi level increases with increasing magnetic field. This could have complicated the discussion of GQO had it not been for the condition of observing them, $h\Omega \ll Z \sim \varepsilon_F^o$ for large Z, where ε_F^o is the Fermi energy without magnetic field. For the condition stated above the relative variation of the Fermi energy in the magnetic field can be shown to be proportional to a small parameter $(\frac{h\Omega}{\varepsilon_F^o})$.

For that reason, in magnetic fields for which $h\Omega \ll \varepsilon_F^o$, the Fermi energy can be assumed practically constant and equal to ε_F^o.

CHAPTER IV

INFLUENLE OF RUPTURE OF THE CONNECTING NECK OF THE FERMI SURFACE ON GIANT WUANTUM OSCILLATIONS

4.1 Derivation of the formula of GQO in PT 2½

As already pointed out in section (1.2) the dispersion law near a conical point at which the Fermi - neck breaks is well approximated by (1.2.6). Below is made the description of the characterstics of the absorption coefficient Γ_H in relation to the above phenomenon. To this end, we start out with the assumption that the critical momentum p_k is at the conic point serving as origin of the momentum space in which the constant energy surface $\varepsilon_{(p)} = \varepsilon_k$ is drawn. This assumption actually simplifies the labor of mathematical manipulations. Let us also assume that the magnetic field is made to lie on the p_y-p_z plane making an angle θ with the p_z-axis so that , $\vec{H} = (o, H\sin\theta, H\cos\theta)$ (Figure 14).

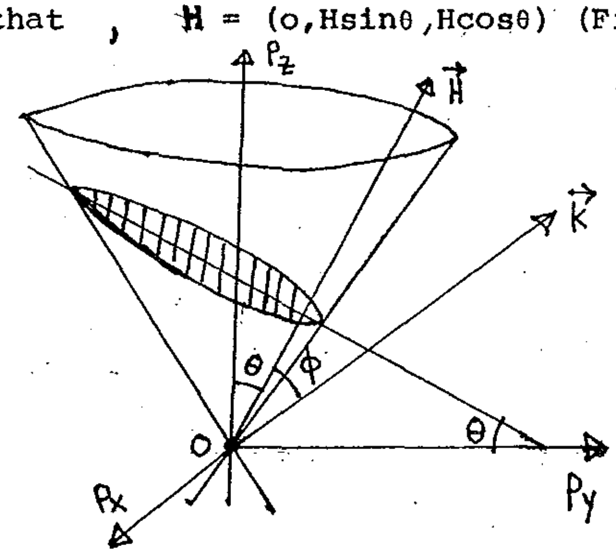


Fig.14: The constant energy surface just before rupture of the neck; and orientation of the magnetic field and incident sound.

ϕ is the angle the magnetic field $\vec{H}$ makes with the arbitrarily directed wave vector $\vec{K}$ of the incident sound.

The absorption coefficient can be fully determined from (2.3.7) provided the dispersion $\varepsilon_{n,p_{z0}}$ is known under the action of the applied magnetic field.

The dispersion law can be obtained from the Lifshitz - Onsager quantization rule (2.1.10) which in case of quardratic dispersion law under consideration may be rewritten as:

$$S(\varepsilon_{n,\vec{p}_z}) = \frac{2\pi|e|Hh}{c}(n+\tfrac{1}{2}) \qquad (4.1.1)$$

where, $S(\varepsilon_{n,p_z})$ is the area of the curve of orbit of the electron which is the intersection of

$$\varepsilon(\vec{p}) = \text{constant}$$

and

$$p_z = \text{cosntant}$$

Depending on the toplogy of the Fermi surface the orbit may be either closed or open; and in our case the orbit if closed will be elliptic for an arbitrary orientation of the magnetic field as the following geometrical consideration makes it clear. A geometric definition of the cones is usually given as follows: "Let F be a fixed point, called the focus, and 'd' a fixed line not through the point, called the directrix (Figure 15).

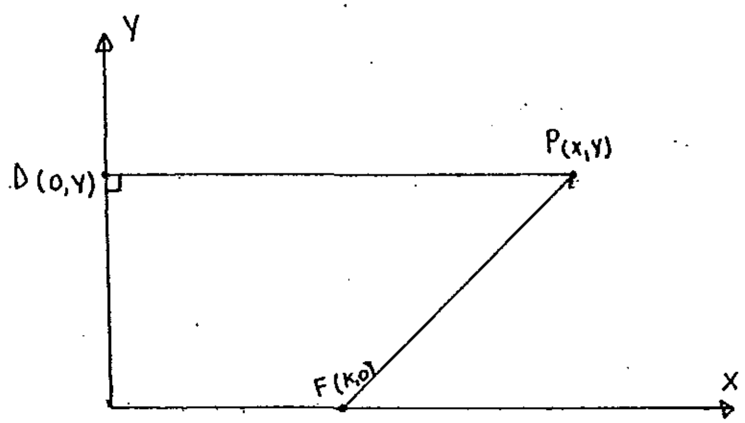


Fig. 15: The locus and directrix

"A conic is the locus of a point p such that the ratio of the distance FP to the distance of P from the line 'd' is a positive constant, e, called the eccentricity; it is a parabola when e=1, an ellipse when e<1 and a hyperbola when e>1".

Thus: $Fp/_{Dp}$ = constant, is the geometric definition of the conics.

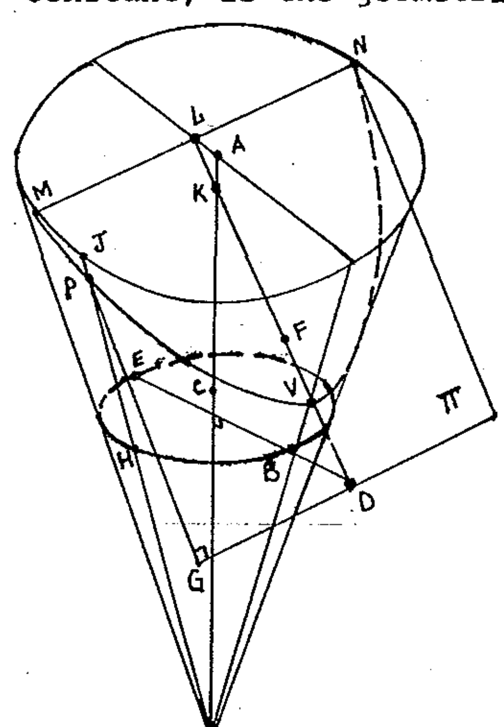


Fig. 16: The right circular cone and the cutting plane π.

Now, all conic sections can be shown to be plane sections of a right circular cone (to which our constant energy surface corresponds at thepoint of breaking of neck $E_F=E_k$) (Figure 16). Consider then this right circular cone with vertex O and a section MVN of the cone by a plane π; V is the point of the curve MVN on the intersection DVKL of π and the plane perpendicular to π through the axis OA; K is the intersection of π on OA, and C is the point in which the bisector of the angle KVO meets OA. The point C is at the same distance <r> from the element OV of the cone and from the line of symmetry KV of the curve MVN; and the perpendicular from C upon KV is normal to the plane π, being in a plane perpendicular to π . With C as center and <r> as radius describe a sphere; denote by F the point where it is tangent to the plane of the section and by B the point where it is tangent to the line OV. Since the cone is right circular, this sphere is tangent to each element of the cone, and all the points of tangency are on a circle BE. DG is the line of intersection of the plane of the circle and the plane π; this line is perpendicular to BD. Let P be any point of the curve in which the plane cuts the cone, and PHO the element of the cone through P, H being its point of tangency to the sphere. Since PF and PH are tangents to the sphere from an outside point O, they are equal, as shown in solid geometry. Since all elements of the cone make the same angle with the plane EHB, itself being a cross - section the line PH makes with this plane an angle equal to VBD. From P we draw PG, perpendicular to DG, which being parallel to the line KFD makes the same angle with the plane of the circle EHB as the latter line, that is the angle VDB. If we denote by Q (not shown in figure) the foot of the perpendicular from p on the plane of the circle, we have

$$PH = \frac{PQ}{\sin VBD}$$

$$PG = \frac{PQ}{\sin VDB}$$

Hence we get

$$\frac{PF}{PG} = \frac{PH}{PG} = \frac{\sin VDB}{\sin VBD} .$$

The angle VBD is the complement of the angle AOB, and the angle VDB is the angle which the plane of the section makes with the plane of the circle EHB, which is a plane normal to the axis of the cone. Since these angles do not depend in any way upon the position of the point p on the curve, it follows from the last equation that $\frac{pF}{pG}$ = constant, and thus the curve is a conic F being the focus and DG the directrix. For the curve to be an ellipse, the plane must intersect all the elements of the cone; that is the angle VDB must be less than the angle VBD, in which case the constant of the ratio is less than unity, as it should be.

When the angle VDB is equal to the angle VBD, that is, when the line VDL is parallel to the element OE, in which case the cutting plane is parallel to OE (Format of the cone) the above ratio equals +1, and the section by the plane is a parabola which corresponds to open orbit.

In addition, when the angle VDB is zero, the section is a circle (closed orbit) whereas when the angle VDB is greater than the angle VBD such that the ratio is greater than +1, the section is one part of a hyperbola (open orbit). The determination of the area of the closed orbit (ellipse) is better made in a reoriented set of axes so that the p_z' - axis is along H. The reorientation can, of course, be effected using the transformation matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

which transforms the primed axes into the unprimed ones. As a result of transformation we get:

$$p_x = p_x'$$

$$p_y = p_y' \cos\theta + p_z' \sin\theta$$

and

$$p_z = p_z' \cos\theta - p_y' \sin\theta \qquad (4.1.2)$$

Substitution of (4.1.2) into (1.2.6)

$$Z = \frac{p_x'^2}{2m_\perp} + \frac{(p_z' \sin\theta + p_y' \cos\theta)^2}{2m_\perp} - \frac{(p_z' \cos\theta - p_y' \sin\theta)^2}{2m_{/\!/}}$$

$$Z = \frac{p_x'^2}{2m_\perp} + p_y'^2 \left(\frac{\cos^2\theta}{2m_\perp} - \frac{\sin^2\theta}{2m_{/\!/}}\right) + p_z'^2 \left(\frac{\sin^2\theta}{2m_\perp} - \frac{\cos^2\theta}{2m_{/\!/}}\right) +$$

$$+ 2p_z' p_y' \sin\theta \cos\theta \left(\frac{1}{2m_\perp} + \frac{1}{2m_{/\!/}}\right) .$$

Factoring out $\frac{\cos^2\theta}{2m_{/\!/}}$ & $\frac{\cos^2\theta}{2m_\perp}$ from the second and third terms respectively, this further gives

$$Z = \frac{p_x'^2}{2m_\perp} + \frac{p_y'^2 \cos^2\theta}{2m_{/\!/}} \left(\frac{m_{/\!/}}{m_\perp} - \tan^2\theta\right) + \frac{p_z'^2 \cos^2\theta}{2m_\perp} \left(\tan^2\theta - \frac{m_\perp}{m_{/\!/}}\right) +$$

$$+ p_z' p_y' \sin\theta \cos\theta \left(\frac{m_\perp + m_{/\!/}}{m_\perp m_{/\!/}}\right) .$$

From here follows

$$Z + \frac{p_z'^2 \cos^2\theta}{2m_\perp} \left(\frac{m_\perp}{m_{/\!/}} - \tan^2\theta\right) = \frac{p_x'^2}{2m_\perp} + \frac{p_y'^2 \cos^2\theta}{2m_{/\!/}} \left(\frac{m_{/\!/}}{m_\perp} - \tan^2\theta\right) +$$

$$+ p_z' p_y' \sin\theta \cos\theta \left(\frac{m_\perp + m_{/\!/}}{m_\perp m_{/\!/}}\right) .$$

Factoring out $\frac{\cos^2\theta}{2m} \left(\frac{m_{/\!/}}{m_\perp} - \tan^2\theta\right)$ from the second and third terms, this further reduces into:

$$Z + \frac{p_z'^2 \cos^2\theta}{2m} \left(\frac{m_\perp}{m_{/\!/}} - \tan^2\theta\right) = \frac{p_x'^2}{2m_\perp} + \frac{\cos^2\theta}{2m_{/\!/}} \left(\frac{m_{/\!/}}{m_\perp} - \tan^2\theta\right) \Big[p_y'^2 +$$

$$+ \frac{2m_{\parallel}}{\cos^2\theta(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)} \times p'_z \, p'_y \sin\theta \cos\theta \left(\frac{m_{\perp}+m_{\parallel}}{m_{\perp}m_{\parallel}}\right) \Bigg]$$

Adding to both sides of the equation

$$p'^2_z \frac{\tan^2\theta\,(m_{\perp}+m_{\parallel})^2}{m_{\perp}^2(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)^2} \times \frac{\cos^2\theta}{2m_{\parallel}} \left(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta\right)$$

this may be changed into:

$$z + \frac{p'_z \cos^2\theta}{2m_{\perp}} \left(\frac{m_{\perp}}{m_{\parallel}} - \tan^2\theta\right) + p'^2_z \frac{\sin^2\theta\,(m_{\perp}+m_{\parallel})^2}{2m_{\parallel} m_{\perp}^2 (\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)}$$

$$= \frac{p'^2_x}{2m_{\perp}} + \frac{\cos^2\theta}{2m_{\parallel}} \left(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta\right) \mathfrak{p}'^2_y$$

where,

$$\mathfrak{p}'_y = p'_y + \frac{p'_z \tan\theta\,(m_{\perp}+m_{\parallel})}{m_{\perp}(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)}$$

This last equation may be rewritten as

$$z + z_{cr} = \frac{p'^2_x}{2m_{\perp}} + \frac{\cos^2\theta}{2m_{\parallel}} \left(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta\right) \mathfrak{p}'^2_y$$

where,

$$z_{cr} = \frac{p'^2_z \cos^2\theta}{2m_{\perp}} \left(\frac{m_{\perp}}{m_{\parallel}} - \tan^2\theta\right) + \frac{p'^2_z \sin\theta\,(m_{\perp}+m_{\parallel})^2}{2m_{\parallel} m_{\perp}^2 (\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)} \quad (4.1.3)$$

The area of the closed trajectory of the electron in the magnetic field is then from here given by πab where,

$$a = \left[2m_{\perp}(Z+Z_{cr})\right]^{\frac{1}{2}}$$

and

$$b = \left[\frac{2m\ (Z+Z_{cr})}{\cos^2\theta\,(\frac{m}{m} - \tan^2\theta)}\right]^{\frac{1}{2}}$$

thus:

$$S_{(Z,Z_{cr})} = \pi\left[2m_{\perp}(Z+Z_{cr})\right]^{\frac{1}{2}}\left[\frac{2m_{\parallel}(Z+Z_{cr})}{\cos^2\theta\,(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)}\right]^{1/2}$$

$$= 2\pi\ \frac{(m_{\perp}m_{\parallel})^{\frac{1}{2}}(Z+Z_{cr})}{\cos\theta\,(\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta)^{\frac{1}{2}}} \qquad (4.1.4)$$

The area of trajectory should necessarily be positive so that from (4.1.4) follow the conditions:

$$Z + Z_{cr} > 0$$

or

$$Z > - Z_{cr}$$

and

$$\frac{m_{\parallel}}{m_{\perp}} - \tan^2\theta > 0$$

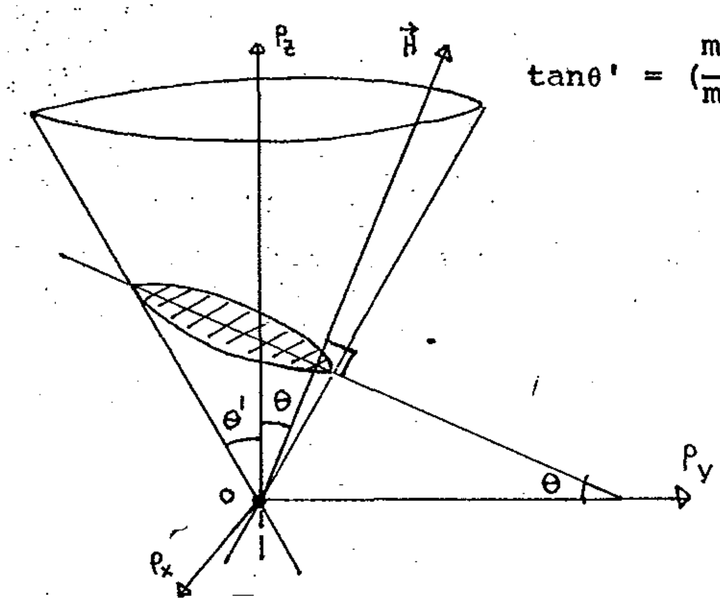
or

$$\tan^2\theta < \frac{m_{\parallel}}{m_{\perp}} \qquad (4.1.5\ a,b)$$

From (4.1.5 b) we conclude that there is a critical angle θ_{cr} above which θ cannot grow to so that the Bohr - Sommerfeld or equivalently the Lifshitz - Onsager quantization rule is applied; and this value of the critical angle is given by

$$\tan^2\theta_{cr} = \frac{m_{//}}{m_{\perp}} \quad \text{or} \quad \theta_{cr} = \tan^{-1}\left(\frac{m_{//}}{m_{\perp}}\right)^{\frac{1}{2}} \qquad (4.1.6)$$

Eq. (4.1.6) can even be obtained through pure geometrical argument. Figure 17 shows that since the plane of orbit is supposed to be perpendicular to the magnetic field $\vec{H}$, the orbit remains closed and elliptic for even past the angle of cone θ' which according to the formula in pp. 19 is given by



$$\tan\theta' = \left(\frac{m_{\perp}}{m_{//}}\right)^{\frac{1}{2}} .$$

Fig. 17: The angle of cone θ'

Notice that, as argued in pp 81-84, the orbit continues to be closed and elliptic until when it becomes parallel to the format of the cone. For such a case, the field $\vec{H}$ will stand perpendicular to the format of the cone. Then the angle θ under such a situation is the critical angle θ_{cr}. If the angle θ passes this limit the orbit becomes open. Hence we come to the conclusion that θ' and θ_{cr} are complementary angles to each other from which consequently follows (4.1.6).

Interms of (4.1.6) it is possible to rewrite (4.1.3) as

$$S_{(Z,Z_{cr})} = 2\pi \frac{(m_\perp m_{//})^{\frac{1}{2}}(Z+Z_{cr})}{\cos\theta\,(\tan^2\theta_{cr}-\tan^2\theta)^{\frac{1}{2}}}$$

and according to (4.1.1) this again delivers

$$2\pi \frac{(m_\perp m_{//})^{\frac{1}{2}}(Z+Z_{cr})}{\cos\theta\,(\tan^2\theta_{cr}-\tan^2\theta)^{\frac{1}{2}}} = \frac{2\pi|e|Hh}{c}(n+\tfrac{1}{2})$$

from which we get

$$Z = \frac{|e|Hh}{c}\,(n+\tfrac{1}{2})\,\cos\theta\,\frac{(\tan^2\theta_{cr}-\tan^2\theta)^{\frac{1}{2}}}{(m_\perp m_{//})^{\frac{1}{2}}} - Z_{cr}$$

Using now the fact

$$Z = \varepsilon_F - \varepsilon_c$$

with ε_c taken as origin, the above expression may be put in its final and required form:

$$\varepsilon_{n,p_{z0}} = \frac{|e|Hh}{c}(n+\tfrac{1}{2})\,\cos\theta\,\frac{(\tan^2\theta_{cr}-\tan^2\theta)^{\frac{1}{2}}}{(m_\perp m_{//})^{\frac{1}{2}}} - Z_{cr} \quad (4.1.7)$$

Eq. (4.1.6) is actually the dispersion of the electron under the action of the magnetic field; and it may be rewritten as

$$\varepsilon_{n,p_{zO}} = h\Omega_\theta \ (n+\tfrac{1}{2}) - Z_{cr} \qquad (4.1.8)$$

where,

$$\Omega_\theta = \frac{eH}{c} \cos\theta \ \frac{(\tan^2\theta_{cr} - \tan^2\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}} \qquad (4.1.9)$$

Finally substituting (4.1.8) into (2.3.7) we find the formula for the absorption coefficient Γ_H under pT-2½; namely,

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_B T} \sum_n \cosh^{-2}\left[\frac{h\Omega_\theta \ (n+\frac{1}{2}) - (Z+Z_{cr})}{2k_B T}\right] \qquad (4.1.10)$$

Eqtn (4.1.10) indicates the fact that Γ_H shows anisotropy in that it assumes different values for different orientations of the magnetic field in the interval

$$o < \theta \leq \theta_{cr} \ .$$

4.2 Investigation of the characterstics of Γ_H near the boundaries of the interval $o < \theta \leq \theta_{cr}$

Let us consider first the case that θ approaches zero i.e. the magnetic field is almost parallel to the axis of symmetry of the neck. In this case (4.1.3) may be rewritten as

$$Z_{cr} = \frac{p_z'}{2m_{/\!/}}$$

which upon using the condition of absorption $p_z' = m_{/\!/}s/\cos\phi$ may be transformed into

$$Z_{cr} = \frac{m_{/\!/}s^2}{2\cos^2\phi}$$

But, $\frac{m_{/\!/}s^2}{2\cos^2\phi} = Z^*$ as defined in pp 48. Thus in the limit indicated above $Z^* = Z_{cr}$

Moreover, in this limit, (4.1.9) reduces into

$$\Omega_\theta = \frac{eH}{c} \frac{\tan\theta_{cr}}{(m_\perp m_{/\!/})^{1/2}}$$

which further simplifies into

$$\Omega_\theta = \frac{eH}{m_\perp c} = \Omega \quad \text{if (4.1.6) is employed.}$$

These new values of Z_{cr} and Ω_θ may now be used to find the form of Γ_H from (4.1.10) in the limit as θ approaches zero, namely,

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \sum_n \cosh^{-2} \left[\frac{h\Omega(n+\frac{1}{2})}{2k_BT} - \frac{Z+Z^*}{2k_BT}\right] \qquad (4.2.1)$$

Eq. (4.2.1) is very similar to (3.1.1); and hence in this limit of θ approaching zero, all results of chapeter III may be totally transferred.

In the other limit of θ approaching θ_{cr} we start out the investigation of the characterstic of Γ_H by first finding the values of Z_{cr} and Ω_θ in this limit, Using $p_z' = m_{//}s/\cos\phi$ we can rewrite (4.1.3) as

$$Z_{cr}= \frac{m_{//}^2 s^2}{\cos^2\phi} \left(\frac{\cos^2\theta}{2m_\perp}\right) \left(\frac{m_\perp}{m_{//}} - \tan^2\theta\right) + \frac{m_{//}^2 s^2}{\cos^2\phi} \left(\frac{\sin^2\theta}{2m_{//}m_\perp^2}\right) \frac{(m_\perp + m_{//})^2}{(\tan^2\theta_{cr} - \tan^2\theta)}$$

$$\text{or} \quad = \frac{m_{//}^2 s^2}{\cos^2\phi} \left(\frac{\cos^2\theta}{2m_\perp}\right) \left(\frac{m_\perp}{m_{//}}\right) \left(1 - \frac{m_{//}}{m_\perp} \tan^2\theta\right) + \frac{m_{//}^2 s^2}{\cos^2\phi} \left(\frac{\sin^2\theta}{2m_{//}m_\perp^2}\right) \frac{m_\perp^2\left(1+\frac{m_{//}}{m_\perp}\right)^2}{(\tan^2\theta_{cr} - \tan^2\theta)}$$

After factoring out $\frac{m_{//}s^2}{2\cos^2\phi}$ this reduces into

$$Z_{cr} = \frac{m_{//}s^2}{2\cos^2\phi}\left[\cos^2\theta\left(1 - \frac{m_{//}}{m_\perp}\tan^2\theta\right) + \sin^2\theta \frac{\left(1+\frac{m_{//}}{m_\perp}\right)^2}{(\tan^2\theta_{cr} - \tan^2\theta)}\right]$$

Upon using (4.1.6) this finally delivers:

$$Z_{cr} = \frac{m_{//}s^2}{2\cos^2\phi}\left[\frac{\cos^2\theta\,(1-\tan^2\theta_{cr}\tan^2\theta)(\tan^2\theta_{cr} - \tan^2\theta) + \sin^2\theta\,(1+\tan^2\theta_{cr})^2}{(\tan^2\theta_{cr} - \tan^2\theta)}\right]$$

which in the above mentioned limit of $\theta \to \theta_{cr}$ simplifies into:

$$Z_{cr} = \frac{m_{//}s^2}{2\cos^2\phi} \left[\frac{\sin^2\theta_{cr}\,(1+\tan^2\theta_{cr})^2}{\theta_{cr} - \theta}\right]$$

But, from $\tan\theta_{cr} = \left(\frac{m_{//}}{m_\perp}\right)^{\frac{1}{2}}$ we get

$$\sin\theta_{cr} = \frac{m_{/\!/}^{\frac{1}{2}}}{(m_\perp + m_{/\!/})^{\frac{1}{2}}}$$

so that the above result may come out to be

$$z_{cr} = \frac{m_{/\!/} s^2}{2\cos^2\phi\,(\theta_{cr}-\theta)}\left[\frac{m_{/\!/}}{m_\perp + m_{/\!/}}(1+\frac{m_{/\!/}}{m_\perp})^2\right]$$

or

$$= \frac{s^2}{2\cos^2\phi\,(\theta_{cr}-\theta)}\;(\frac{m_{/\!/}}{m_\perp})^2\;(m_\perp + m_{/\!/}) \qquad (4.2.2)$$

Moreover, in this limit of θ approaching θ_{cr} (4.1.9) may be rewritten as

$$\Omega_\theta = \frac{eH}{c}\cos\theta_{cr}\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}$$

But from $\tan\theta_{cr} = (\frac{m_{/\!/}}{m_\perp})^{\frac{1}{2}}$ we get

$$\cos\theta_{cr} = \frac{m_\perp^{\frac{1}{2}}}{(m_\perp + m_{/\!/})^{\frac{1}{2}}}$$ which when applied to

the preceeding equation finally gives

$$\Omega_\theta = \frac{eH}{c}\;\frac{m_\perp^{\frac{1}{2}}}{(m_\perp + m_{/\!/})^{\frac{1}{2}}}\;\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_{/\!/}\cdot m_\perp)^{\frac{1}{2}}} \qquad (4.2.3)$$

Thus using (4.2.2) and (4.2.3) Eq. (4.1.8) takes on the form

$$\varepsilon_{n,p_{z0}} = \frac{eHh}{c}\;\frac{m_\perp^{\frac{1}{2}}}{(m_\perp + m_{/\!/})^{\frac{1}{2}}}\;\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}\;(n+\tfrac{1}{2}) - z_{cr} \qquad (4.2.4)$$

so that the absorption coefficient in this limit is given by

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \Sigma \cosh^{-2}\left[\frac{\frac{eHh}{c} \frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}} \frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp \cdot m_{/\!/})^{\frac{1}{2}}} (n+\frac{1}{2}) - (Z+Z_{cr})}{2k_BT} \right] \tag{4.2.5}$$

Notice that from (4.2.4) can be extracted the statement

$$\Delta\varepsilon = \varepsilon_{n+1} - \varepsilon_n = (\frac{eHh}{c}) \frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}} \frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}$$

from which, since, $\theta_{cr} = \theta$ follows the inequality

$$\Delta\varepsilon \ll \varepsilon_{n,p_{zO}}$$

the inequality invites us to use the Euler - Maclaurian formula to complete the summation in (4.2.5) regardless of the magnitude of the magnetic field. The Euler-Maclaurian formula is an approximate formula to the summation of the series of the type $f(\chi+\frac{1}{2})$. Consider an analytic and continous function $f(x)$ with continous derivatives. This function may be expanded over Δx as

$$f(x) = f(x_i^c) + \frac{1}{1} f'(x_i^c)\, \Delta x + \frac{1}{2} f''(x_i^c)(\Delta x)^2 + \text{------}$$

where, $\Delta x = x - x_i^c$

and $x_i^c = x_i + \frac{1}{2}$

If $f(x)$ is now integrated within one interval (x_i, x_i+1) it gives:

$$\int_{x_i}^{x_i+1} f(x)\,dx \doteq \int_{x_i}^{x_i+1} f(x_i^c)\,dx + \int_{x_i}^{x_i+1} f(x_i^c)\,\Delta x\,dx + \tfrac{1}{2}\int_{x_i}^{x_i+1} f''(x_i^c)(\Delta x)^2 dx$$

Performing the integration then delivers

$$\int_{x_i}^{x_i+1} f(x)\,dx = f(x_i^c) + f'(x_i^c)\frac{((\Delta x)^2}{2}\Big|_{x_i}^{x_i+1} + \tfrac{1}{2} f''(x_i^c)\frac{(\Delta x)^3}{3}\Big|_{x_i}^{x_i+1}$$

$$= f(x_i+\tfrac{1}{2}) + 0 + \tfrac{1}{2} f''(x_i^c)\ \tfrac{1}{3}[(\tfrac{1}{2})^3 - (-\tfrac{1}{2})^3]$$

$$= f(x_i+\tfrac{1}{2}) + 1/_{24}\ f''(x_i^c)$$

Performing the integration over all other intervals and taking a sum of these will eventually produce

$$\int_a^{x_1} + \int_{x_i}^{x_2} + \text{------} + \int_{x_i}^{x_i+1} f(x)\,dx + \text{------} + \int_{x_r}^{x_b} f(x)\,dx$$

$$= \sum_a^b f(x_i+\tfrac{1}{2}) + 1/_{24} \sum_a^b f''(x_i^c)$$

Thus we conclude from here

$$\int_a^b f(x)\,dx = \sum_a^b f(x+\tfrac{1}{2}) + \frac{1}{24}\sum_a^b f''(x_i)\ \Delta x_i \qquad (4.2.6)$$

where $\Delta x_i = 1$

and $f''(x_i^c) \cong f''(x_i)$ (since $\left| \frac{f''(x_i^c) - f''(x_i)}{f''(x_i)} \right| \ll 1$)

But, since $\Delta x_i \ll$ we can make the replacement

$$\sum_a^b f''(x_i)\Delta x_i = \int_a^b f''(x)\, dx = f'(x) \Big|_{x=a}^{x=b}$$

We then finally get the required formula from (4.2.6), namely, the Euler-Maclaurion formula:

$$\sum_a^b f(x+\tfrac{1}{2}) = \int_a^b f(x)dx - \frac{1}{24} f'(x) \Big|_{x=a}^{x=b}$$

When this formula is applied to (4.1.15) one gets

$$\Gamma_H = \Gamma_O \frac{h\Omega}{4k_BT} \left\{ \int_0^\infty \cosh^{-2} \frac{\frac{eHh}{c} \frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}} \frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}} x - z - z_{cr}}{2k_BT} \, dx \right.$$

$$\left. - \frac{1}{24} \frac{d}{dx} \left[\cosh^{-2} \frac{(\frac{eHh}{c} \frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}} \frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}} x - Z - Z_{cr})}{2k_BT} \right] \Bigg|_{x=0}^{x=\infty} \right\}$$

The integrated part may be expanded as:

$$- (\frac{1}{24})(-2) \cosh^{-3} \left(\frac{\frac{eHh}{c} \frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}} \frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}} x - Z - Z_{cr}}{2k_BT} \right) \times$$

$$\times\left[\sinh\left(\frac{\frac{eHh}{c}\frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}}\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}x-Z-Z_{cr}}{2k_BT}\right)\times\left(\frac{\frac{eHh}{c}\frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}}\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}}{2k_BT}\right.\right]_{x=0}^{x=\infty}$$

But, since $k_BT \ll Z$, this integrated part gives zero in both limits so that the preceeding equation after a change of variable

$$y = \frac{\frac{eHh}{c}\frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}}\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}x-Z-Z_{cr}}{2k_BT}$$

reduces into:

$$\Gamma_H = \Gamma_O\frac{h\Omega}{4k_BT}\left(\frac{2k_BT}{\frac{eHh}{c}\frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}}\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}}\right)\int_{-\frac{Z+Z_{cr}}{2k_BT}}^{\infty}\cosh^{-2}y\,dy$$

which upon integrating delivers

$$\Gamma_H = \Gamma_O\frac{h\Omega}{4k_BT}\left(\frac{2k_BT}{\frac{eHh}{c}\frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{/\!/})^{\frac{1}{2}}}\frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{/\!/})^{\frac{1}{2}}}}\right)\left[\tanh\frac{Z+Z_{cr}}{2k_BT}+1\right]$$

Using (4.2.6) and $\Omega = \frac{eH}{m_\perp c}$ this may be rewritten as:

$$\Gamma_H = \Gamma_O/2 \; \frac{\tan\theta_{cr}(1+\tan^2\theta_{cr})^{\frac{1}{2}}}{(\theta_{cr}-\theta)^{\frac{1}{2}}} \left[1+ \tanh \frac{Z+Z_{cr}}{2k_B T})\right] \qquad (4.2.7)$$

Notice that a formula similar to (4.2.7) was derived by V.N. Davydov and M.I. Kaganov for the ultrasonic absorption coefficient in the case of Zero magnetic field as expounded in section (1.2) pp (18-2..).

Since

$$Z+Z_{cr} = \frac{eHh}{c} \frac{m_\perp^{\frac{1}{2}}}{(m_\perp+m_{//})^{\frac{1}{2}}} \frac{(\theta_{cr}-\theta)^{\frac{1}{2}}}{(m_\perp m_{//})^{\frac{1}{2}}} (n+\tfrac{1}{2}) + \frac{s^2}{2\cos^2\phi\,(\theta_{cr}-\theta)^{\frac{1}{2}}} \left(\frac{m_{//}^2}{m_\perp}\right) (m_\perp+m_{//})$$

approaches ∞ in the combined limits

$$H \to o$$

and $\qquad \theta \to -\theta_{cr}$

(4.2.7) may be shown to reduce to the exact form of (1.2.11), namely, $\Gamma_H \propto \Gamma_O \, \Delta\theta^{-\frac{1}{2}}$ in this case.

Eq. (4.27) shows that with increasing of the angle θ that is in approaching the angle θ_{cr} the absorption coefficient reveals the same rise $\Gamma_H \propto 1/\sqrt{(\theta_{cr}-\theta)}$ as in the absence of the magnetic field. The latter means that there exists a band of values of θ where the effect of GQO vanishes. This is because in this band the growth of Γ_H of the type (4.2.7) dominates over the effects of GQO in the rupture of the connecting neck at pT-2½. It should also be noted that at angles very close to the critical value θ_{cr}, the obtained formula is invalid, and not because of failure of the classical quantization condition, but first of all, since the limit point to which the ellipse contracts as $Z \to -Z_{cr}$ is located very far from p_c where the dispersion law (1.2.6) is valid. Since, however, the value of Z_{cr} depends on $m_{//}s^2$ which is quite small when measured in energy scale, formula (4.2.7) is applicable right up to angles very close to θ_{cr}. Then one can state that in all cases when the Fermi surface has a narrow connecting neck (and not only in pT-2½) a relative sharp anisotropy of the ultrasonic absorption should be observable in the magnetic field.

SUMMARY AND CONCLUDING REMARKS

When ultrasonic wave is incident on a normal metal at low temperatures some of the wave energy gets attenuated. The process of absorption in this case is identified as some kind of electron phonon interaction; and in the limit of $k.\ell \gg 1$ (ultrasonic wave), where $<k>$ is the wave number of the incident sound and $<\ell>$ is the mean free path of the electrons, it is purely a quantum phenomenon. The conservation of energy and momentum principles together with the fact that theincident phonon energy is quite negligible as compared to the electron energy require that it is only a small belt of electrons on the Fermi surface which move in phase with the incident sound that are involved in sound attenuation. Since the process of absorption is mentioned to be quantum, the absorption coefficient of this pocket of electrons may be determined from time - dependent perturbation theory as (1.1.14).

The absorption coefficient, like all other kinetic and thermodynamic characterstics of the metal, shows an anomaly during a change in the topology of the Fermi surface which usually results in singularity of the density of states.

Anomalies associated with the topology of the Fermi surface are conventionally termed as electronic phase transitions or phase transitions of order 2½. Assuming quadratic dispersion, in both cases of appearance of a new spheroidal sheet and breaking of the Fermi neck, when there is no applied magnetic field, the absorption coefficient shows a jump of the type (1.2.5) and (1.2.11) respectively. An applied magnetic field quantizes the energy of the electron into levels known as Landau levels. In this case it is expected that Ω (Larmor frequency) is much greater than ω (frequency of incident phonon) so that phonon absorption does not change the level of conduction electrons. This fact together with the conservation principles still puts the same requirement, as in the case of no magnetic field, for the absorption of the incident sound.

In case when $h\Omega \ll k_BT$ and for quadratic dispersion law the absorption coefficient Γ_H shows a change of the type (1.2.5) whereas for $h\Omega \gg k_BT$ it oscillates with an amplitude proportional to the magnetic field.

The peaks are periodic in inverse magnetic field with the period as given by (1.3.11). The oscillations of Γ_H in applied magnetic field are termed as Giant Quantum Oscillations (GQO) and the condition of observation of GQO in experiment can be given as $k_BT \ll h\Omega \ll \mu$, where μ is the chemical potential.

In the present work GQO are studied in pT-2½. It is shown that observation of GQO in experiments in pT-2½ is possible only for $k_BT \ll h\Omega \ll |Z|$, where Z is the parameter of pT-2½ defined as $Z=\mu-\varepsilon_k$. The condition for ultrasonic absorption mentioned earlier is shown to persist even in this case with the absorption coefficient now taking on the form as in

(2 3.7) the validity of which is checked in that for an applied magnetic field directed along the Z-axis and quadratic dispersion law it reduces to already known formulae such as, (1.2.5) under the proper limit of $H \to 0$. As the absorption coefficient Γ_H is sensitive to the parameter Z, its characterstics are investigated in the various regions of Z. In pT-2½ associated with appearance of a new spheroidal covity, depending on whether $h\Omega \gg k_B T$ or $h\Omega \ll k_B T$ the absorption coefficient near $Z \simeq Z^*$, where Z^* is a threshold of energy as indicated in (3.1.1), is shown to be small exponentially and constant as given by (3.2.11) and (3.2.12) respectively the declination from the constant value being quite small and proportional to Ω^2 for small enough magnetic fields. In the region $0<Z<Z^*$ which is in direct contradiction to the condition of absorption, Γ_H is shown to remain low and monotonously rise towards increasing Z. But in the case of $Z \gg Z^*$, the absorption coefficient is found to show GQO with exponentially rising amplitude (3.4.5). The periods of the oscillations in inverse magnetic field and the parameter Z are as given in (3.4.6) and (3.4.7) respectively. The conditions for the observation of these oscillations are derived to be

$$k_B T \ll h\Omega \ll |Z|.$$

In pT-2½ associated with breaking of the Fermi neck and an arbitrarily oriented magnetic field (but with quadratic dispersion law still) Γ_H is found to show strict dependence on the angle of orientation (θ) of the field with the Z-axis, and it is shown that it vanishes beyond a critical value of the angle θ_{cr} (as given by (4.1.6)) where the orbit of the electron ceases to be closed. Near $\theta \simeq 0$, Γ_H is discovered to be in similar form with the one derived for the appearance of a new spheroidal cavity; whereas for the other case of $\theta \simeq \theta_{cr}$, Γ_H is shown to be inversely proportional to $(\theta_{cr} - \theta)^{\frac{1}{2}}$ indicating that in this particular limit there is a band of electrons on the Fermi surface which out match those responsible for GQO so that Γ_H shows the indicated sharp anisotropy.

Finally it may be commented that the knowledge of the period of GQO is quite important in that it leads (according to 2.1.10) to the knowledge of area of cross-section of the Fermi surface which on the otherhand gives a valuabe information about the metal which is under investigation.

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