ADDIS ABABA UNIVERSITY

School of Graduate Studies

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A STUDY OF UNDRAINED BEHAVIOR OF SATURATED SAND AND THE CONCEPT OF STEADY STATE

A Thesis submitted to School of Graduate Studies of Addis Ababa University in Partial Fulfillment of Masters of Science in Civil Engineering

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Next, I wish to thank my sponsor, Jimma University, and my country as well for giving me the opportunity to learn by covering all my expenses and build up my capacity for my better contribution towards the development of the home land country.

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ABSTRACT

It has been very common to see collapse of structures, slopes, and foundations due to flow of ***foundation soils*** without observing any structural failure during earthquake. This can also occur frequently and cause severe damages in this country following the occurrence of earthquake, particularly in the ***Rift Valley*** region where there is a considerable deposit of saturated sand and silty sand. This problem has been studied in various countries and attributed to ***Liquefaction Phenomenon***. The phenomenon occurs in saturated sand, which either completely loses its strength and flow as a liquid (Flow Liquefaction, **F.L**) or shows a progressive softening (Cyclic Mobility, **C.M.**) under cyclic loading. Since liquefaction of sand is entirely related to its undrained behavior, the undrained behavior of saturated sand under cyclic or equivalent monotonic loading has been studied in the laboratory by different researchers, simulating field conditions properly for the better understanding of liquefaction phenomenon. The undrained behavior of saturated sand is useful to predict the occurrence of the probable type of liquefaction (**C.M**. or **F.L**) and to provide suitable solutions for liquefaction problem.

The objective of this research is therefore to study the undrained behavior of saturated sand in relation to liquefaction phenomenon based on intensive literature survey. This is best explained by identifying factors affecting the behavior and studying their effects observing the resulting ***stress-strain*** and ***p-q*** diagrams. The concept of ***Liquefaction*** and ***steady state*** are then discussed based on the undrained behavior of sand. Finally, the application of ***Steady State Line*** (**SSL**) on the assessment of liquefaction potential and in the identification of ***partial liquefaction, limited liquefaction, and complete liquefaction*** is discussed.

NOTATIONS

Greek letters	Descriptions
γ	Shear strain
$\sigma_1^{'}$	Effective major principal stress
$\sigma_3^{'}$	Effective minor principal stress
ε_a	Axial strain
τ	Shear stress

Latin letters	Descriptions
C_u	Uniformity coefficient
D_{50}	Grain size of sand sample corresponding to 50% cumulative percentage finer.
D_r	Relative density
e	Void ratio
e_c	Void ratio after consolidation
e_{max}	The maximum void ratio
e_{min}	The minimum void ratio
G_s	Specific gravity of sand
S_{su}	Residual shear strength
q	$= \dfrac{(\sigma_1^{'} - \sigma_3^{'})}{2}$
p’	$= \dfrac{(\sigma_1^{'} + \sigma_3^{'})}{2}$

Chapter One
Introduction, Scope and Organization of the Thesis

1.1 INTRODUCTION

Many failures of earth structures, slopes, and foundations on saturated sands have been attributed to ***Liquefaction*** of sand. Classical examples of liquefactions are the flow slides that have occurred in the province of Zealand in Holland and in the point bar deposits along the Mississippi River, the failures of Fort Peck Dam in Montana in 1938, the Calaveras Dam in California in 1920, and the Lower Lan Norman Dam during the 1971. San Fernando earthquake in California provides typical examples of liquefaction failures of ***hydraulic fill Dams***. The best-known cases of ***foundation*** failures due to liquefaction are those that occurred during the 1964 earthquake in Niigata, Japan.

Many researchers have studied the phenomenon of liquefaction. Among those, A. Casagrande and Gonzalo Castro have contributed a lot in the area of the study. They have found that liquefaction occurs in saturated cohesionless soil, particularly in saturated sands, when subjected to ground vibrations like earthquake or blasting. If loose saturated sand is subjected to ground vibrations, it tends to compact and decrease in volume; if drainage is unable to occur, the tendency to decrease in volume results in an increase in pore water pressure, and if the pore water pressure builds to the point at which it is nearly equal to the overburden pressure, the effective stress rapidly decreases and approaches a minimum undrained strength; where the sand loses its strength, and flows like a fluid. At this state, the soil undergoes a steady flow deformation and causes catastrophic failure of foundations. The other important finding of the researchers is that the phenomenon of liquefaction is entirely related to the undrained behavior of sand. For example, the saturated sand must be loose enough to be compacted and decrease in volume when subjected to ground vibrations. Otherwise, it tends to dilate and lead the pore pressure to decrease instead of increasing. This then improves the shear strength of sand and it can hardly to flow. Therefore, the undrained behavior of saturated sand has been studied under monotonic and cyclic loading for the past four decades for the better understanding of liquefaction phenomenon.

The undrained behavior of sand is best studied by testing sand in triaxial /simple shear equipment under different conditions that can affect the behavior and observing the resulting responses. It has been found out that the ***initial confining pressure*** and the ***in situ void ratio*** of the sand are the main factors affecting the undrained behavior of the saturated sand under monotonic and cyclic loading. In addition to the initial conditions, the type of sand, particularly the grain size distribution, is also found to affect the undrained behavior of sand [15, 16].

Accordingly, three basic types of responses of sand are observed when tested under different conditions, Fig. 1.1, and have been used for the discussion of the phenomenon of liquefaction and the concept of steady state.

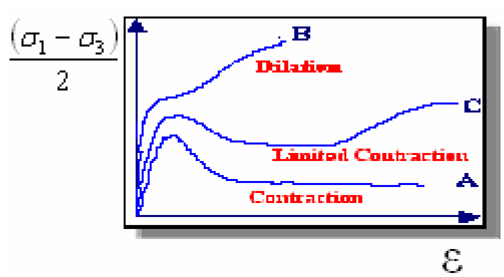


Fig.1.1, Basic types of responses of saturated sand under undrained condition [5]

The first type of response, type A, is observed in saturated loose sands which continuously contract during shear test and let the pore pressure to increase. This type of sand is, therefore, termed as contractive sand. The continuous increase of pore pressure reduces the effective shear strength of sand to a minimum constant value as shown in the Fig. 1.1. At this state, the sand loses its strength and flows steadily like a fluid. This state is therefore termed as a ***steady state*** of sand and the over all phenomena is referred to ***flow liquefaction*** Casagrande [2].

The second type of response, type B, is observed in dense sands. Dense sands, when subjected to exciting load, start to dilate after a small amount of contraction. Therefore, this type of sand is referred to as dilative sand. The corresponding pore pressure decreases even to negative values and hence increases the effective shear strength of sand. As a result, this type of sand is not susceptible to flow type of liquefaction; however, it can develop moderate deformation progressively when subjected to vibratory loading and is referred to ***cyclic mobility*** by Casagrande [2].

The third and final type of response, type C, is observed in medium dense sand which contract initially and then start to dilate later when it is sheared. Correspondingly, the pore pressure increases during the contraction of sand and then starts to decrease during the dilation of sand. Therefore, it is the limited type of liquefaction that can occur in this type of sand.

1.2 SCOPE OF THE STUDY

The thesis contains detail discussions on undrained behavior of saturated sand and the concept of steady state using intensive literature survey. The phenomenon of liquefaction is also best explained based on the undrained behavior of sand. Finally, the construction and application of steady state line for the assessment of liquefaction potential is thoroughly discussed.

Here, the specific objectives of the thesis are summarized as follows:-

1. Study of undrained behavior of saturated sand conducting intensive literature survey.
2. Discussion of phenomenon of liquefaction based on undrained behavior of saturated sand.
3. Discussion of concept of steady state in reference to undrained behavior of sand.
4. The construction and application of steady state line for the assessment of liquefaction potential of sand and its types.

The out put of the study could be used

I. for the better understanding of liquefaction phenomenon and to create awareness for the problem.
II. as a reference for further detail studies of undrained behavior of local sands to determine the extent of liquefaction problem in our country and to device for the possible solutions.
III. to introduce the concept of steady state so that it could be used in modeling of soil for designing purpose.

1.3 ORGANIZATION OF THE THESIS

Chapter two discusses undrained behavior of saturated sand in well-organized form. The factors affecting the undrained behavior are stated. Stress-strain and p-q diagrams of sand samples which are reported by different researchers are collected and the observed responses are discussed accordingly.

Chapter three describes the phenomenon of liquefaction in reference to the undrained behavior of sand. Liquefaction types are discussed, and the differences and similarities are outlined in detail. The phenomenon of liquefaction is further demonstrated using movie of laboratory model and pictures taken at actual sites.

The concept of steady state of sand is discussed in chapter four based on the undrained behavior of sand. The construction and application of steady state line for the assessment of liquefaction potential and its types is also discussed in this chapter.

Finally, conclusions and recommendations are drawn in chapter five.

Chapter Two

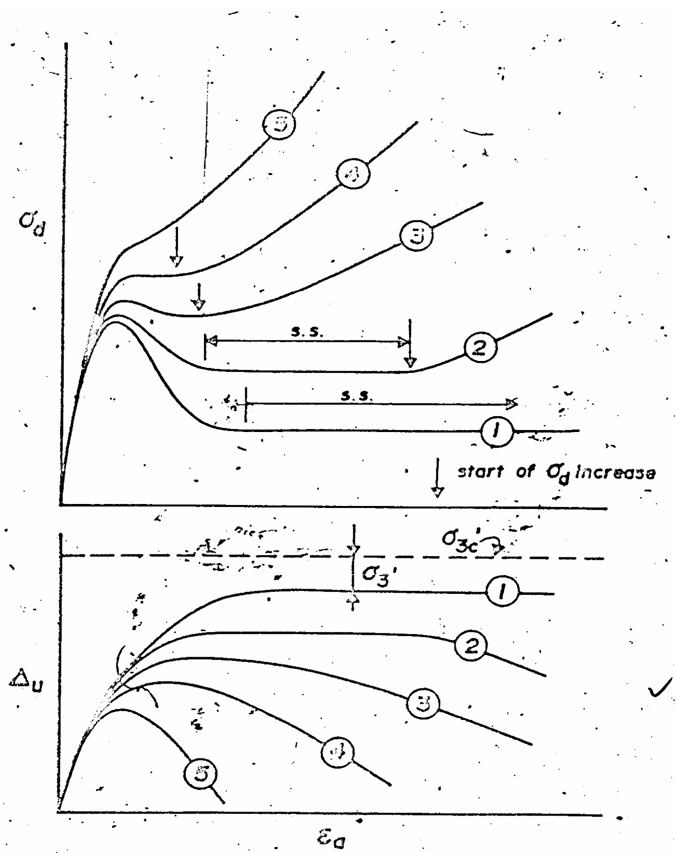
Undrained Behavior of Saturated Sand

2.1 GENERAL

Several studies have been conducted to provide a better understanding of the undrained behavior of saturated sand under different types of loading. Most of the pioneering works have been performed by Casagrande, Seed and Lee, Castro, Ishihara, and Poulos. In addition, several recent researchers like Mohammed and Dobry, Guzman, and Valid have made significant contributions in the area of the study.

Undrained response of saturated sand is traditionally considered separately under monotonic and cyclic loading conditions. Monotonic undrained triaxial tests were introduced by the Corps of Engineers at the late 1950's for the measuring of pore pressure and for liquefaction characterization studies of very loose soils. Later Seed and Lee first introduced constant volume cyclic triaxial tests into the laboratory in the mid 1960's, from which they evaluated pore pressure and effective stress relationships in different density sands during dynamic loading [19]. Interest in monotonic loading has generally been related to undrained failure associated with flow slides. The characteristics feature of such behavior is extremely large deformation under very small shear resistance. Conditions which could bring about such a response from soil could be rapid increase in stresses due to earthquake loading, shock loading or even static loading. Interest in cyclic undrained loading behavior has been related to the susceptibility of sand to accumulate undesirable deformation during earthquake shaking.

Under monotonic loading, isotropically consolidated saturated sand would show range of typical undrained triaxial compression behaviors as shown in Fig. 2.1. The variations in stress-strain curves from type 1 to type 5 are associated with increasing relative density. The same types of characteristic behaviors are also obtained if the sand is initially anisotropically consolidated.



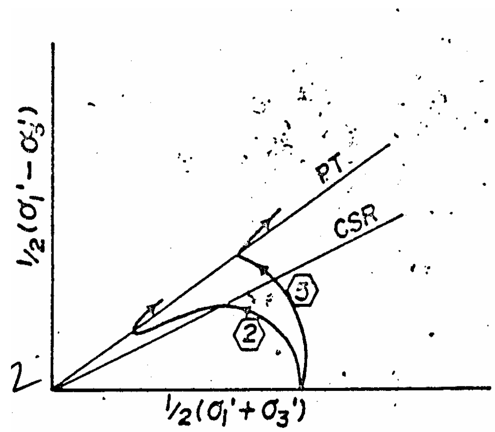


Fig. 2.1, Characterstic behavior of saturated sand under undrained monotonic loading

Fig. 2.2, Effective stresspaths of contract ive and dilative response

Types 1, 2 and 3 are ***strain softening*** responses – a behavior associated with loss of shear resistance after the occurrence of peak value. Sand showing such behavior is called ***contractive.*** The characteristic feature of type 1 response is finally a continuous deformation with constant void ratio, confining stress and shear resistance, which has been called ***steady***

state deformation or ***flow deformation***, since it resembles flow of fluid [7]. However, the shear resistance during such deformation is of a frictional nature, instead of zero, as would be the case for a fluid, and is called ***residual strength***. On the other hand, type 2 and 3 responses are strain softening with limited unidirectional strain. Instead of deforming continuously at reduced constant shear resistance, the shear resistance of sand increases with further deformation after attaining a minimum, and simultaneously the pore pressure decreases after attaining its maximum value. However, over some finite range of strain prior to the commencement of increase in shear resistance, the sand deforms at essentially constant void ratio, effective confining stress and shear resistance, which would be considered as the steady state condition of the case type 1. The difference in response represented by type 2 and type 3 is a lesser degree of strain softening and associated smaller strain until the start of increase in shear resistance or decrease in pore pressure in type 3 compared to that in type 2.

The arrows in Fig. 2.1 indicate the arrest of strain softening response, i.e., the start of increase in shear resistance and decrease in pore pressure with further straining. On effective stress path, this condition is reflected by a sharp turnaround of the effective stress path. Such a condition has been called ***phase transformation*** **(PT)** state by Ishihara [19].

Type 4 response is associated with a terminal case of strain softening response in which the degree of strain softening can be considered as zero. Such a behavior is represented by a flat plateau in stress-strain curve over a certain strain range before the shear resistance starts to increase and pore pressure starts to decrease with further straining.

Type 5 response represents the strain hardening behavior with no loss of shear resistance. Sand showing such behavior is called ***dilative***. For such a response, a sharp turnaround in the effective stress path is not well defined, Fig. 2.2. However, the condition of start of decrease in pore pressure after its maximum value is well defined. Such a condition has been called ***characteristic threshold*** **(CT)** by Luong, which is the same as the PT state described before [19].

Triaxial tests on several sands with rounded to angular particles under a wide range of confining pressure and initial void ratio have been conducted by the different researchers to study the undrained behavior of sand and to support the above concepts. This chapter also discusses the undrained behavior of sand presenting some representative triaxial test results of different researchers together with their concrete discussions and conclusions.

2.2 FACTORS AFFECTING THE UNDRAINED BEHAVIOR OF SATURATED SAND

The undrained behavior of saturated sand is best explained by observing test results conducted by varying several factors, which affect the response. The initial conditions of a sample,

1. ***initial confining pressure,*** and
2. ***in situ void ratio***,

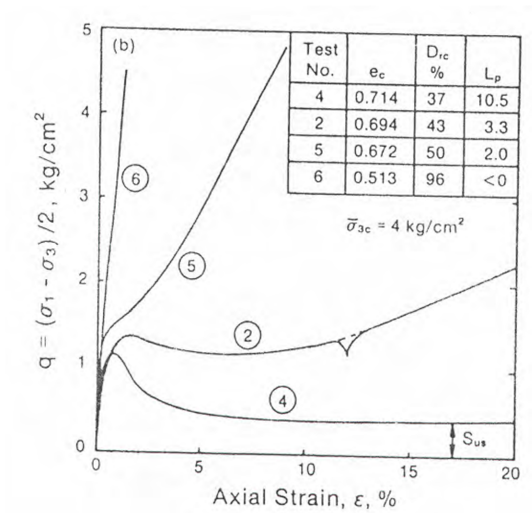
have been observed as the major factors affecting the stress-strain curves. By consolidating different types of saturated sand under different values of confining pressure and void ratio, it is possible to assess the ***variation of peak undrained strength***, the ***residual stress*** of the saturated sand, and the associated ***levels of strain***.

In addition to the initial states, the following factors have been also observed affecting the undrained behavior of saturated sand.

3. Type of sand
   - Grain size distribution
   - Shape of the particles
   - Compressibility characters of particles
4. Type of test (strain-controlled or load-controlled tests)

2.2.1 Effect of Initial Void Ratio on the Undrained Behavior of Saturated Sand

Researchers have conducted varies consolidated undrained tests on saturated sands by varying the initial void ratio from loose to dense, while keeping other factors constant, and have studied the effect of initial void ratio on the undrained behavior of saturated sand. Some of the representative test results are shown in Fig. 2.3-2.6



Fig, 2.3(a), Stress-Strain Response of Banding Sand in CIU Tests, (D_{10} = 0.097mm; D_{50} = 0.16mm; and C_u = 1.8) [16]

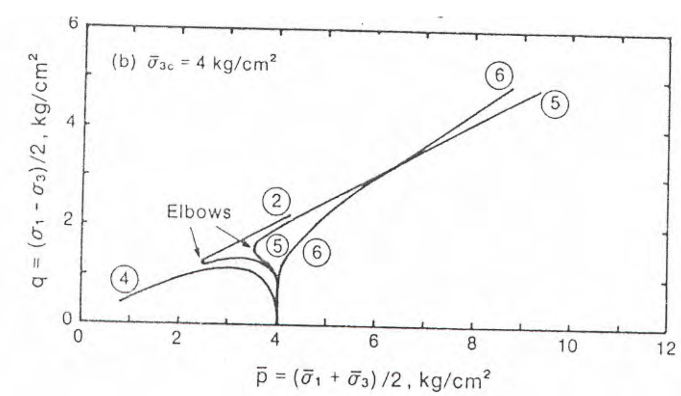


Fig. 2.3(b), Effective Stress Paths for Banding Sand in CIU Tests, (D_{10} = 0.097mm; D_{50} = 0.16mm; and C_u = 1.8) [16]

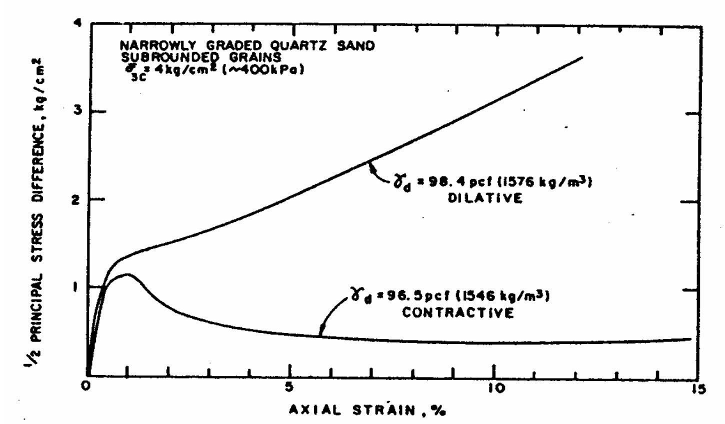


Fig. 2.4, Effect of Minor Density change on Stress-Strain Curves for Consolidated Undrained Triaxial Tests on Clean Sand [17]

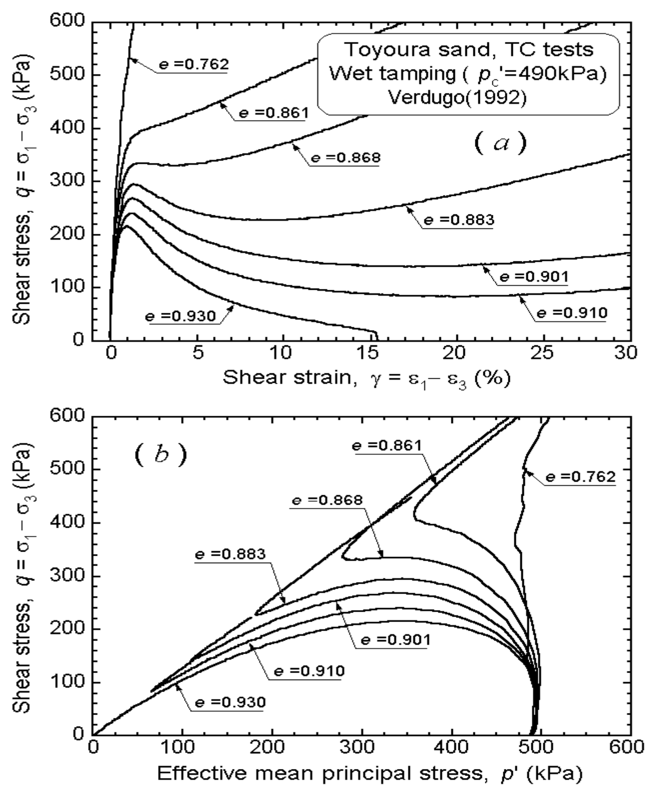


Fig.2.5, Toyoura sand, Triaxial Compression tests, Wet tamping (Pc'=490kPa); [15]

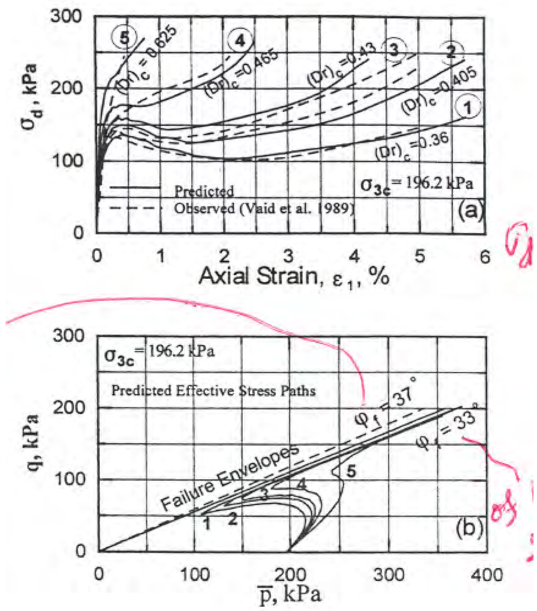


Fig.2.6, Undrained Response of Ottawa Sand under Monotonic Loading [14]

In test 4 of Fig. 2.3(a), the applied shear stress, $q = ½(\sigma_1-\sigma_3)$, first increased and reached to its ultimate value within small amount of axial deformation. It is then dropped rapidly and reached to a minimum constant value, with the corresponding rapid increment of axial deformation. The test result of the contractive sand in Fig. 2.4, and the test results reported in Fig. 2.5 for void ratios of 0.93, 0.91, and 0.901 have also showed the same phenomena. The corresponding effective stress path for test 4 of Fig. 2.3(a) is shown in Fig. 2.3(b), and for Verdugo's result in Fig. 2.5. It is observed from these figures that the applied shear stress, q, first increased and then rapidly decreased with the corresponding decrement of mean effective normal stress, p'[16].

This phenomenon is attributed to the contractive behavior of sand specimens. When the deviator stress is applied, because the sand was loose and confined, it deformed continuously with out dilating so that no excess pore water can dissipate, and this resulted in the corresponding increment of pore pressure. In the mean time, the shear resistance of the sand mobilized and led to the increment of the applied shear stress, q, towards its peak value. But as the pore pressure reached its peak value, it reduced the effective pressure substantially, which in turn decreased the shear resistance of loose sand. Therefore the applied shear stress dropped rapidly and reached to a minimum constant value. At this stage, the loose sand flowed steadily and reached to a state, known as a ***steady state*** **(Critical Steady State)** and it is said to be ***liquefied*** [14]**.** The behavior of loose sand to reach at a peak stress in small axial deformation, and then reduce in strength until the stress stabilizes at a residual strength reaching to steady state is termed as ***Strain-Softening behavior*** [1]. The steadily flowing of sand grains at steady state is termed as ***Flow deformation*** and the corresponding residual strength of sand at this stage is known as ***Steady State Strength*** [1]. Since loose sands can contract continuously, this behavior is typically observed in such type of sands, and they are referred to as ***contractive*** sands [6].

Test 2 of Fig. 2.3(a), test results for void ratios of 0.883 and 0.868 in Fig 2.5, and tests 1, 2 and 3 of Fig.2.6, were conducted on medium dense sands and exhibited similar behavior in their stress-strain and effective stress path diagrams. In the stress-strain diagram, the applied shear stress, q, first increased and reached to its ultimate value within small amount of axial deformation. It was then dropped slowly and reached to a constant value (started to liquefy) for a while. But then it started increasing with the corresponding increment of axial deformation. The corresponding effective stress paths of the tests shown in Fig. 2.3(b), Fig 2.5, and Fig. 2.6 also explained similar behavior. The applied deviator stress, q, was increased at the beginning and then started to decrease and showed that the sand was already liquefied. But then it turned to the right with the corresponding increment of both the applied deviator stress and the mean effective normal stress, explained that the shear resistance of sand started to regained ***(strain-hardening behavior)*** [1].

This phenomenon is attributed to the ***dilatency*** behavior of the sand specimen. When the deviator stress was applied, the medium dense sand initially started to contract and made the pore pressure to increase. Correspondingly, the applied shear stress, q, increased and reached to its maximum value to which the shear resistance of the sand was fully mobilized and resisted. At this stage, the amount of pore pressure became large enough to decrease the effective stress and consequently to drop the shear resistance of the sand. Therefore the applied shear stress decreased from its ultimate value and reached to steady state as the shear resistance of the sand decreased substantially. At this level, since the sample was medium dense, it exhausted contraction and started to dilate. The pore pressure, therefore, began to decrease from its maximum value. This contributed to the increment of effective stress and as a result the shear resistance was regained back. The point where the dilatancy behavior of the sand changes from contractive to dilative was named as ***Phase Transformation*** by **Ishihara** and the corresponding transient steady-state at the phase transformation has been called as the ***quasi steady-state***, Fig. 2.7, [15]. Medium dense sands initially contract for sometimes and then start to dilate because they reach to the critical density before liquefying, and this behavior is typically observed in such type of sands which can partly ***contract*** and partly ***dilate*** [6].

Test 5 and 6 of Fig. 2.3(a), the test result of dilative sand specimen shown in Fig 2.4, the test result for void ratio of 0.861 and 0.762 given in Fig 2.5, and tests 4 and 5 of Fig. 2.6 were conducted on dense sands and obeyed similar behavior which is different from the type of behaviors observed previously. The stress-strain diagrams showed that the applied shear stress continuously increased with the corresponding increment of axial strain. The corresponding effective stress paths also showed that the applied shear stress, q, and the mean effective normal stress, p, increased continuously.

This phenomenon was again attributed to the ***dilatency*** behavior of dense sand. The dense sand specimen when sheared, it started to dilate immediately after the deviator stress was applied, and caused reduction of the corresponding pore pressure. This contributed to the improvement of the effective shear resistance of the sand ***(strain-hardening behavior)***. Therefore, the undrained shear resistance of sand increased continuously with the corresponding increment of axial deformation and reached to a steady state known as ***Ultimate Steady State***, Fig. 2.7, [15]. Since dense sands dilate continuously till fail, this behavior is typically observed in such type of sands, and they are referred to as ***dilative*** sands [6].

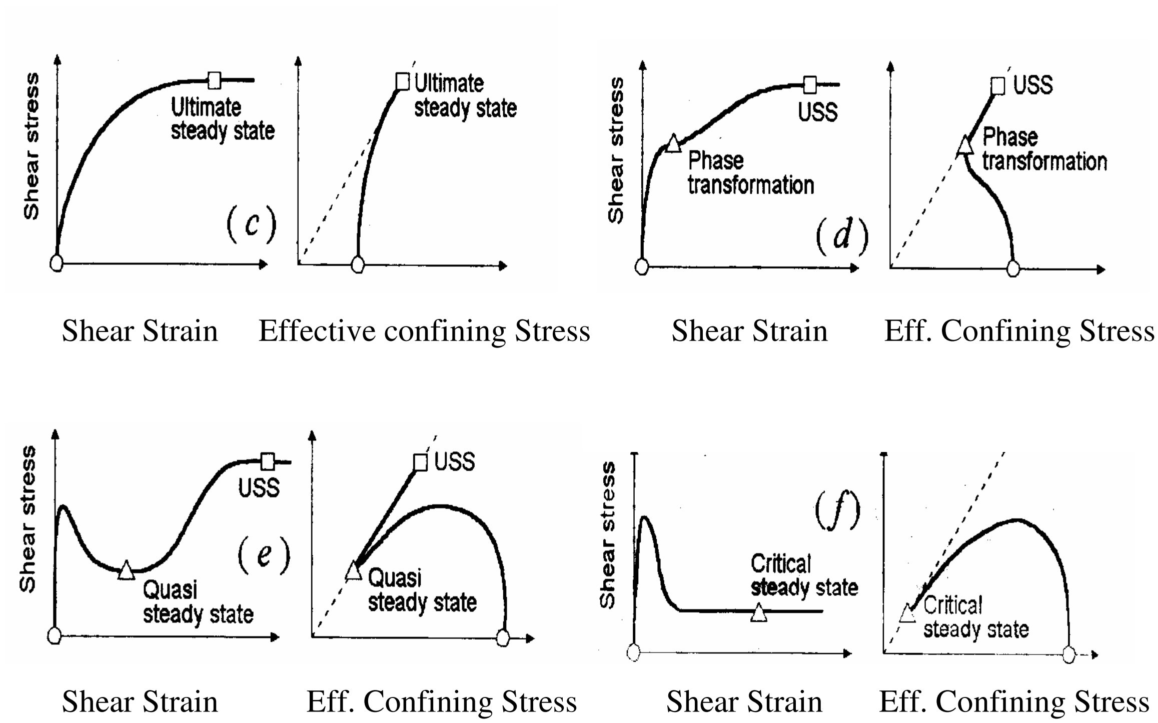


Fig. 2.7, Ultimate steady state, Quasi steady state, and Critical steady state on Stress-Strain and Stress Path Curves [15].

2.2.2 Effect of Confining Pressure on the Undrained Behavior of Saturated Sand

Many tests on undrained saturated sands have been conducted to explain the effect of confining pressure on the undrained behavior of sand. Some representative test results are presented in Fig. 2.8-Fig.2.9 for further discussion.

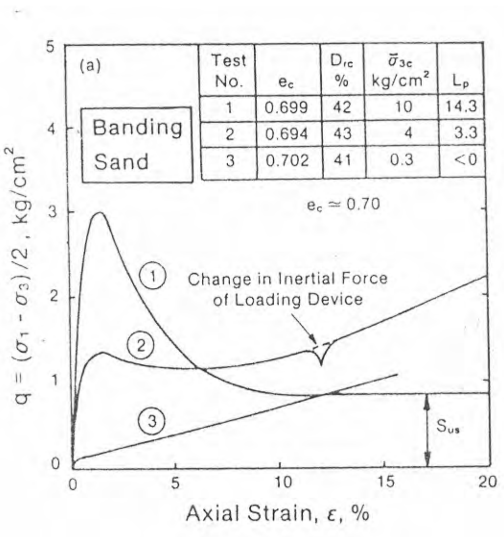


Fig. 2.8(a), Stress-Strain Response of Banding Sand in CIU Tests, ($D_{10} = 0.097$mm; $D_{50} = 0.16$mm; and $C_u = 1.8$) [15].

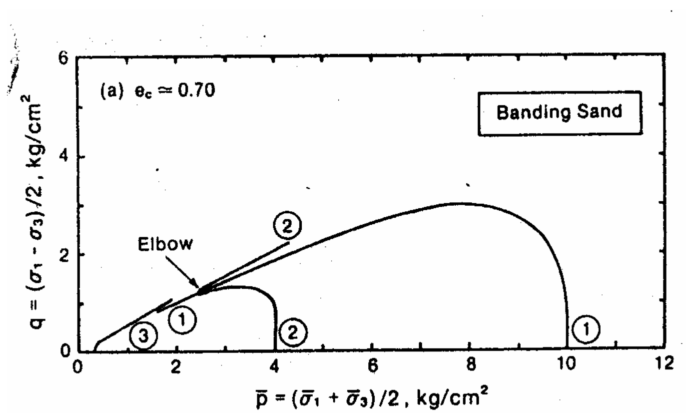


Fig. 2.8(b), Effective Stress Paths for Banding Sand in CIU Tests, ($D_{10} = 0.097$mm; $D_{50} = 0.16$mm; and $C_u = 1.8$) [15].

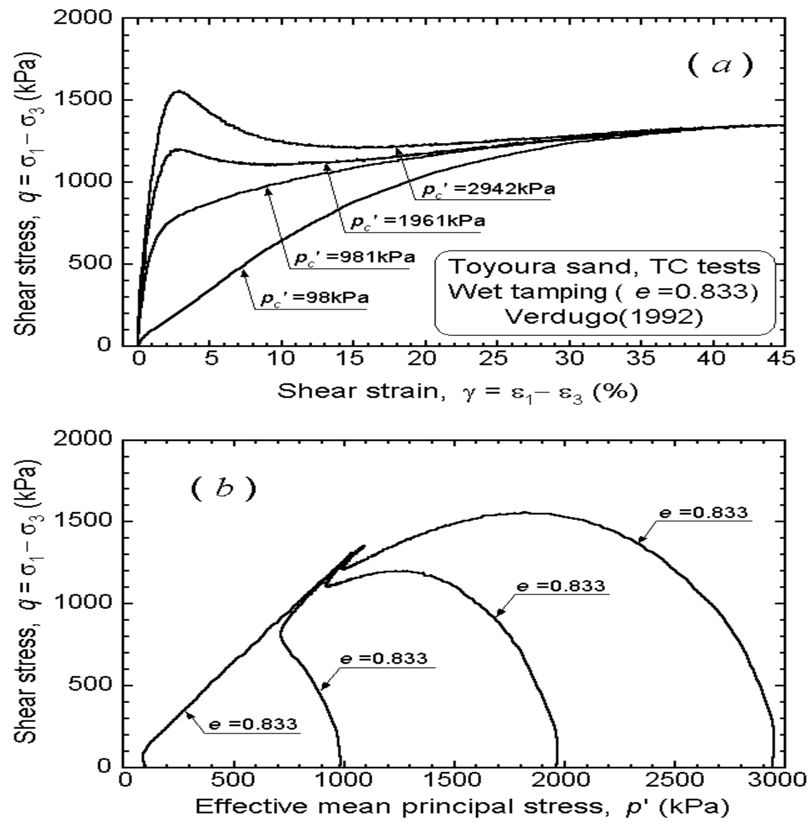


Fig. 2.9, Toyoura sand, Triaxial Compression tests, Wet tamping (e=0.833); [15]

From the test results it is easily observed that a strain softening behavior is exhibited for larger confining pressures. This has been explained that since dilatation tendencies are smaller at high confining pressures, highly confined sands are enabled to deform continuously with the corresponding continuous increment of pore pressure. As a result the effective pressure is decreased and the corresponding shear resistance of sand is lowered [9].

It is also observed that all the specimens of Fig 2.9 have reached to the same steady state strength irrespective of the confining pressures to which they are consolidated. In the test result of Fig 2.8, a minor difference in the steady state strengths is observed due to a minor difference in their void ratio. The initial void ratios of Tests 1 and 3 are so close that the corresponding steady state strengths are also so closed to each other. These enabled many researchers to conclude that the steady state position is solely a function of initial void ratio.

On the study of undrained behavior of saturated sand under monotonic loading, it has been concluded that saturated sand at any given density is potentially less stable under high confining pressure and can undergo flow deformation if subjected to exciting load, observing the occurrence of strain softening behavior for high confining pressure. But, in the study of the behavior of sand under cycling loading, it is observed that even though samples under the same initial void ratio have ultimately reached to the same steady state when consolidated in different confining pressures, samples confined under high confining pressures needed large number of cyclic loading (large amount of deviator stress in the case of monotonic loading) to reach to the steady state while samples consolidated under low confining pressures are reached to the steady state with the application of small number of cyclic loading [9, 10].

This result enabled Bolton Seed, Kenneth L. Lee, Gonzalo Castro, and other researchers to conclude that saturated sand at any given density is potentially highly stable under high confining pressure and has more resistant against flow deformation [9, 10].

2.2.3 The Effect of Type of Sand on the Undrained Behavior of saturated sand

In order to study the influence of material characteristics upon the undrained behavior of sand, a variety of materials were tested by different researchers. The tests were intended to see the effect of ***grain size distribution***, ***particle shape***, and ***compressibility characteristics of particles*** on the undrained behavior of sand. Some representatives of many test results are presented here for discussion.

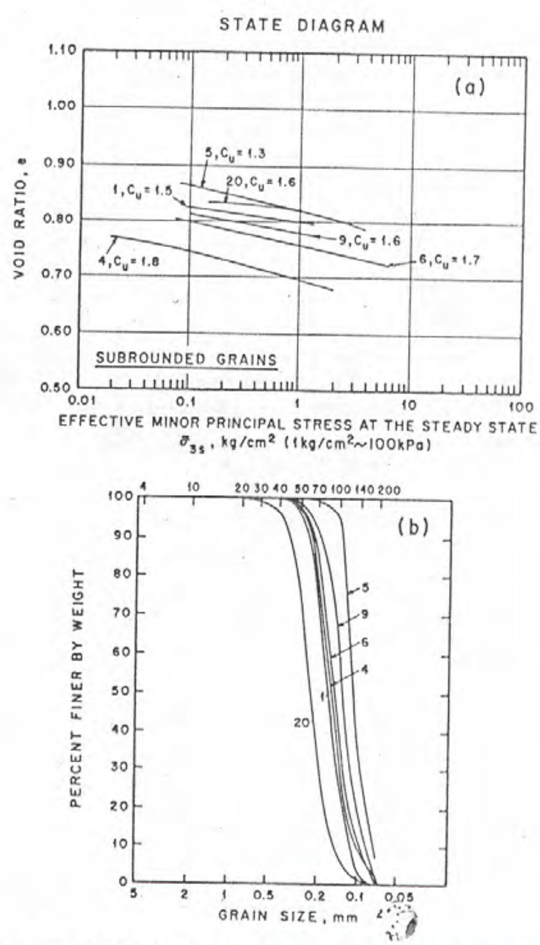


FIG. 3.—Steady-State Lines for Sands—Subrounded Grains (Castro, et al., 1982)

Fig. 2.10, Steady-State Lines for Sands-Subrounded Grains [17].

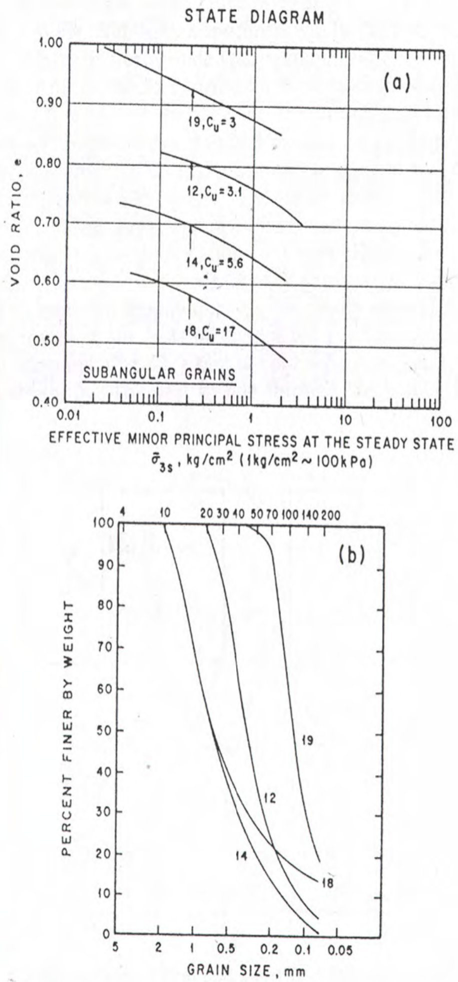


Fig.2.11. Steady-State Lines for Sands-Subangular Grains [17].

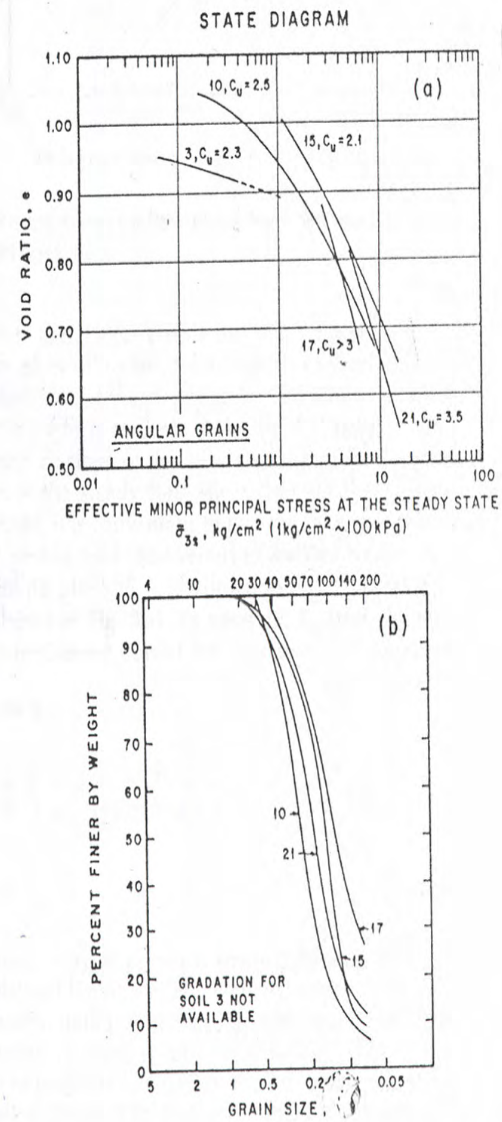


Fig. 2.12. Steady-State Lines for Sands--Angular Grains [17].

The test results in the Fig. 2.10, Fig. 2.11, and Fig. 2.12 are reported by Castro to show the effect of ***grain size distribution*** on the undrained behavior of sands by drawing the corresponding steady state diagrams for each type of grain size distribution [17]. As it is observed from the results, the grain size distribution has affected the steady state lines* and hence the undrained behavior of sand samples. If the grain size distribution had not affected the behavior of sand specimens, the respective steady state lines would have been coincide.

To understand the qualitative effect of grain size distribution, recently, Kokusho performed a series of stress-controlled undrained tests on clean sands and gravels with different particle gradations and different relative densities. Despite large differences in gradation, for a given relative density, only small differences between materials were observed in the cyclic stress required to initiate liquefaction which implies that sands under the same void ratio are equally liquefiable irrespective of particle gradations. This was also observed by Dongdong and during the study of failure of Merriespruit gold tailings dam in 1994 in South Africa [5]. However, they also found the undrained monotonic shear strength at much larger strains to be affected by gradation. This effect of gradation was also understood and studied by Kuerbis who identified apparent effects of C_U and D_{50} on the undrained behavior of sands subjected to monotonic and cyclic triaxial testing. He reported his findings that soil was more contractive as uniformity increases and grain size decreases. It is also reported in ASCE that a series of undrained tests were performed on granular soils consisting of sand and gravel with different particle gradations and different relative densities reconstituted in laboratory, and undrained monotonic shear strength defined at larger strains was observed at least eight times larger for well-graded soils than poorly graded sand despite the same relative density [13]. This is then analyzed as devastating failures with large post-liquefaction soil strain are less likely to develop in well-graded granular soils compared to poorly graded sands with the same relative density, although they are almost equally liquefiable. Finally, Alarcon-Guzman and Leonards reported that the smoother, more rounded, finer the particles, and the more uniform the gradation, the higher the collapse potential and the higher the susceptibility of soil for flow liquefaction [1].

* The concept of steady state line is discussed well in chapter four..

Tests to investigate the influence of ***particle shape*** were conducted by C.C. HIRD and F.A.K Hassona on the Leighton Buzzard sand, glass beads and carborundum, the sphericities of which are included in Table 2.1. Leighton Buzzard sand was uniform medium quartz with subrounded to subangular grains, while glass bead particles were well-rounded and carborundum particles were highly angular. Their grain size distributions were also similar as observed in Fig. 2.13.

In Fig.2.14, the SSL's established on the basis of these tests are plotted in both e-log p' space and Dr-log p' space where Dr is the relative density. It is seen from this figure that the SSL of angular particles is plotted above the SSL of rounded particles. This implies that at a given effective confining pressure the void ratio required for sand to be plotted above its SSL increases (and the relative density decreases) with increasing angularity. Therefore, angular particles should be loose enough to undergo steady state flow (liquefaction) than rounded particles**. This is because at steady state flow angular particles are more likely to interlock than rounded ones, and more space is required for the particles to flow past one another and develop a flow structure. Thus the results of Fig. 2.14 suggest that, other things being equal, rounded sands are more susceptible to liquefaction than angular ones [3].

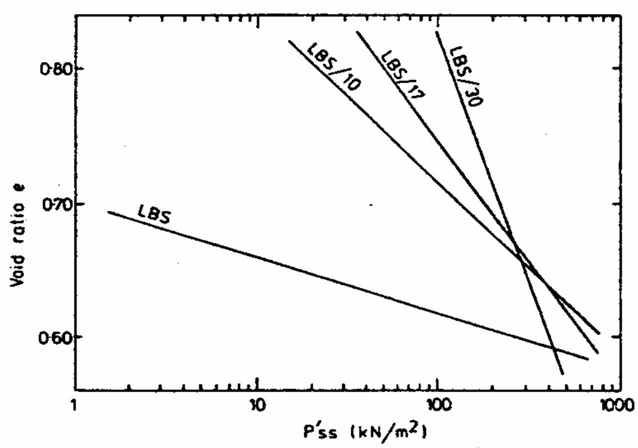


Fig. 2.15, SSL's for materials tested by C.C. HIRD and F.A.K Hassona [3].

** It is clearly discussed in chapter three that sand with initial void ratio and confining pressure plotted above its SSL would develop flow liquefaction while sand plotted below its SSL would not develop liquefaction though can develop large deformation progressively.

Table 2.1, Index properties of materials tested by C.C. HIRD and F.A.K Hassona [3].

Index properties of materials

Material	Leighton Buzzard sand	Carborundum granules	Glass beads	Sand–mica mixtures		
Abbreviation	LBS	CBM	GLB	LBS/10	LBS/17	LBS/30
Mica content (%)	—	—	—	10	17	30
Sphericity	0.83	0.65	1.00	—	—	—
Median grain size, D_{50} (mm)	0.50	0.48	0.47	0.50*1	0.47*1	0.45*1
Uniformity coefficient, D_{60}/D_{10}	1.41	1.70	1.72	1.53*1	1.67*1	[illegible]*1
Maximum void ratio, e_{max}*2	0.790	0.741	0.772	1.070	1.320	1.789
Minimum void ratio, e_{min}*3	0.515	0.561	0.534	0.591	0.615	0.823

*1Denotes correction of erroneous value quoted by Hird and Hassona (1986).
*2Test method as in ASTM D2049.
*3Non-standard test methods. Materials compacted in dry state in cylindrical mould (105 mm dia × 115 mm high). LBS compacted by vibrating hammer (900 W, 60 Hz), 3 layers, 1 min/layer. CBM and GLB compacted by vibrating table (50 Hz, amplitude 0.1–0.2 mm) 3 layers, 3 min/layer, no surcharge.
For further details see Hassona (1986).

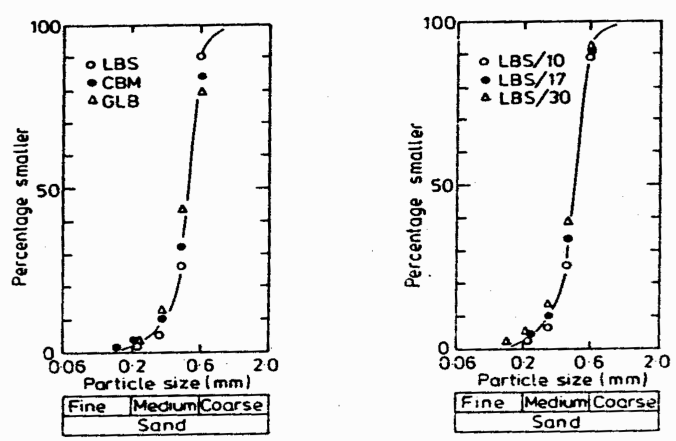


Fig.5. Particle-size distributions of materials tested.

Fig.2.13, Particle-size distributions of materials tested by C.C. HIRD and F.A.K Hassona [3].

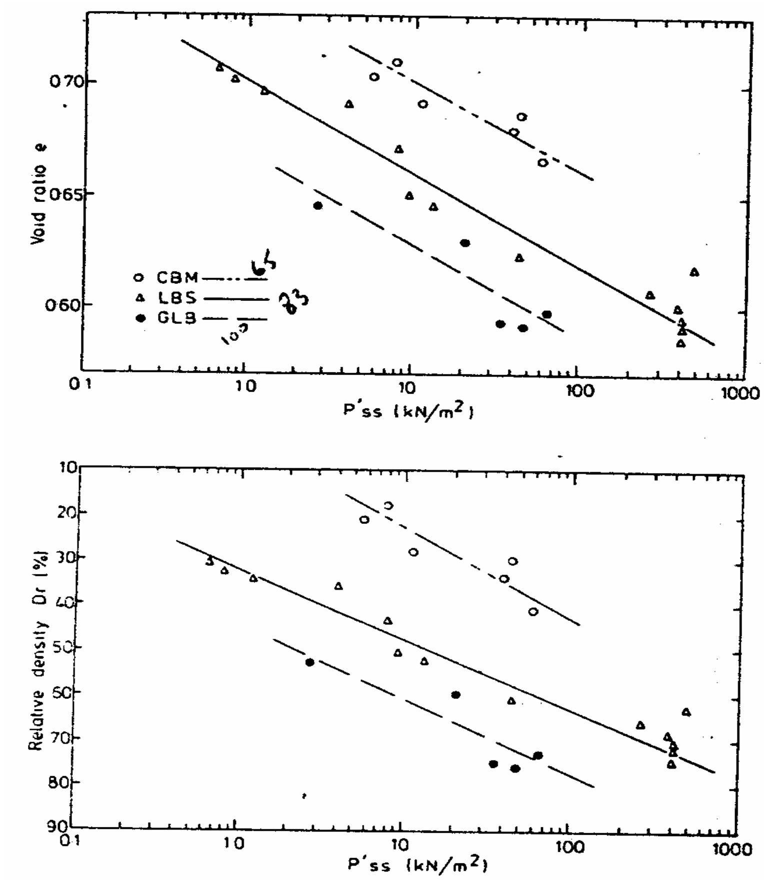


Fig. 2.14, SSL's for materials tested by C.C. HIRD and F.A.K Hassona [3].

To study the influence of ***compressibility characteristics of particles*** upon the undrained behavior of sand, the compressibility of Leighton Buzzard sand was enhanced by adding 10%, 17%, and 30% of mica and producing three derivatives of LBS namely, LBS/10, LBS/17, and LBS/30; see Table 2.1. The addition of mica do not affected the gradation curves of the derivatives; see Fig. 2.13. The SSL's obtained for these materials are shown in Fig.2.15. From this figure, it is observed that SSLs are plotted above and their slopes increase with the corresponding increment of compressibility (mica content). For example, the SSL of LBS/30 is plotted above the SSL of LBS/17 which is again plotted above the SSL of LBS/10. Of course, this fact is changed for higher values of confining pressures and lower values of void ratios; however, it is true for practical values and hence can be used for the intended study.

At the same initial confining pressure, say 100kPa, the void ratio of LBS/17 must be at least 0.75 to be plotted above its SSL while a value of 0.725 or greater is sufficient for LBS/10. LBS/30 also requires a minimum void ratio of 0.85 again to be plotted above its SSL. Therefore, at the same initial confining pressure compressible particles require high void ratio to be plotted above their SSL and hence to be liquefied. It can be concluded from this result that liquefaction is much less severe in the micaceous specimen and the severity of liquefaction was found to decrease progressively with increasing mica content [3].

The above results may be understood by thinking about particle movements during consolidation, prior to undrained loading. By increasing the compressibility of the material the mica promotes a greater degree of particle movement and re-orientation during undrained loading and causes consolidation instead of particles flowing one on the other surface and form a flowing structure at steady state. Therefore, stable plastic deformation occurs as the material is sheared and obeys a clay behavior [3].

2.2.4 Effect of Strain Rate on the Undrained Behavior of Saturated Sand

Although the undrained behavior of sand is normally investigated using load-controlled tests, similar behavior patterns may be observed in strain-controlled tests. Typical results for Leighton Buzzard sand are compared in Fig. 2.16. The corresponding steady state lines for Leighton Buzzard sand and Banding sand are also shown in Fig. 2.17. The figures have shown that the SSL's obtained from load- and strain-controlled tests, though parallel in e-log p' space, don't coincide. Instead, the critical void ratios are higher under strain-control condition than under load-control condition.

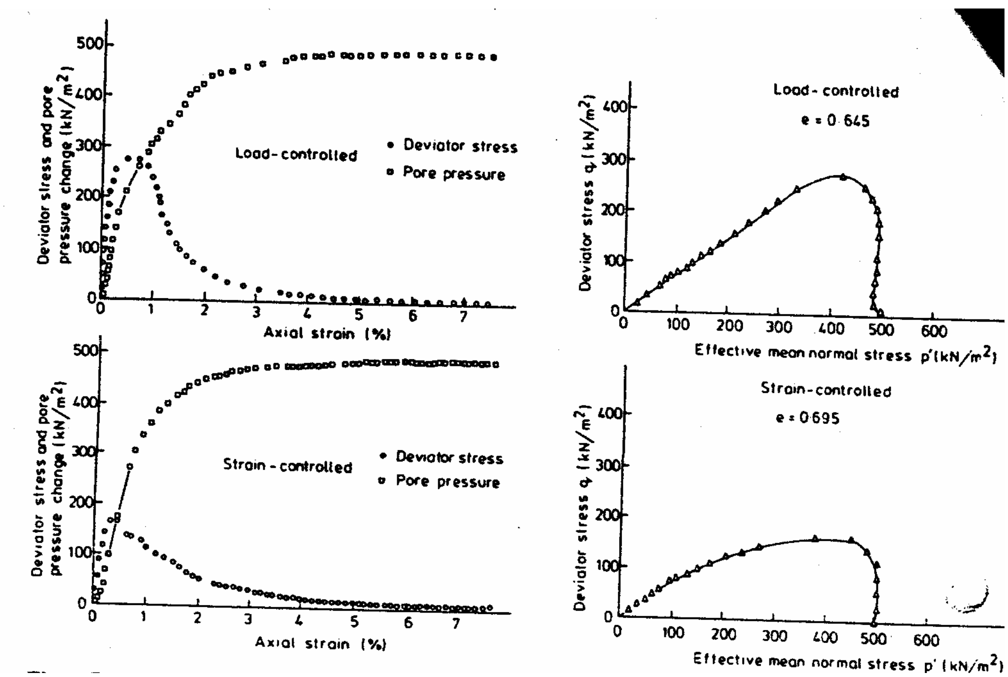


Fig. 2.16, Results of load- and strain-controlled tests on Leighton Buzzard sand [3].

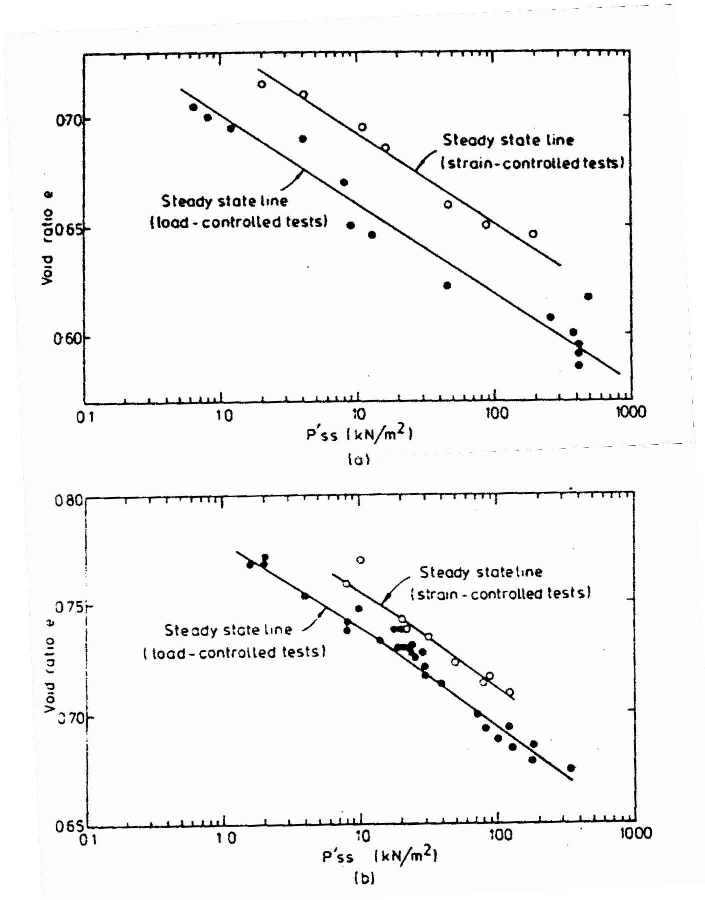


Fig. 2.17, Comparison of SSL's from load- and strain-controlled tests on: (a) Leighton Buzzard sand, and (b) Banding sand [3].

The difference between the SSL's under load- and strain- controlled conditions was attributed by Casagrande to the large difference in the rate of deformation [2]. At steady state, in a load-controlled test the rate of deformation has developed about 10000 times faster than when conventional strain-controlled test is used. Hence the occurrence of large amount of rate of deformation in the load-controlled test has favored towards the development of flow deformation and hence liquefaction of sand even at lower critical void ratio. Therefore, at a given void ratio and confining pressure, sands tested under load-controlled conditions have showed instability when compared with sands tested under strain-controlled conditions. Furthermore, it is postulated that the development of a flow structure in sand is observed at high strain rates. In this respect it was argued that load-controlled testing permitted a better representation of field conditions [3].

Chapter Three
Liquefaction

3.1 GENERAL

Predominant interest in undrained behavior of saturated sand has been with the occurrence of the phenomenon of liquefaction. Castro and Casagrande studied such behavior in relation to the problem of flow slide. Several recent researchers like Verdugo, Dongdong, and others have also studied the undrained behavior of saturated sand under monotonic and cyclic loading and explained the phenomenon of liquefaction and its types using the observed results.

The basic types of undrained behavior of saturated sand are best explained in the previous chapter. Accordingly, the types are sketched in Fig. 3.1 for the convenience of the forgoing discussion.

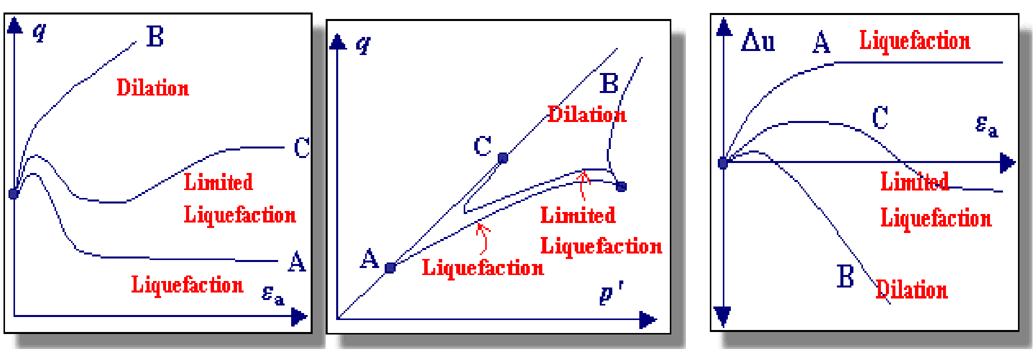


Fig.3.1, Sketch for static triaxial test stress paths for three specimens of different densities (very loose, medium, and dense) [5].

In the strain softening response, type A, the effective shear strength, $q/2$, and pore pressure, Δu, are observed first to increase continuously towards their peak values. The corresponding strain that develop during this phase is very small usually in the order of 1% or less. The pore pressure then reaches to its maximum value and becomes nearly equals to the initial confining pressure. Hence, the effective shear strength drops rapidly to the residual (minimum) value. Here, the sand starts to undergo flow deformation and thus the state corresponding to the point where the pore pressure equals the confining pressure for the first time is termed as ***"initial liquefaction"*** [7].

After the initiation of liquefaction, the deformation of sand accelerates very rapidly with further application of deviator stress and the sand flows like a fluid. Finally, it collapses and causes failure of foundations where the sand is said to be completely liquefied and the corresponding state is referred to as ***"complete liquefaction"***. When sand flows and develops a strain value of about 20%, it is said to be completely liquefied; if the developed deformation is within this value, i.e. less than 20%, it is said only partially liquefied and is termed as ***"partial liquefaction"*** of sand [7, 10].

The point of initial liquefaction is also observed in the response type C. In this type of response, the pore pressure at the point of initial liquefaction reaches to its maximum value and would attain this value constant for further application of deviator stress. However, this can no longer continue up to a strain level of 20% or more. Instead, the pore pressure starts to decrease some where before complete liquefaction as the sand begins to dilate after a certain amount of contraction. As a result, the effective shear strength begins to revive and increase to the ultimate value. Thereafter, strain hardening is taken place and further loading can be sustained by the sand. Correspondingly, ***flow deformation*** of sand is exhibited while the pore pressure attains its maximum value constant and this flow of sand is arrested during the reduction of pore pressure. As a result, it can only a limited type of liquefaction exists in this type of response and Castro termed this phenomenon of liquefaction as ***"limited liquefaction"*** [7].

In the strain hardening response, type B, since the sand starts to dilate immediately after the application of deviator stress, the pore water pressure decreases continuously and results in the continuous increment of effective shear strength. As a result, the flow deformation of sand is unlikely to occur and flow type of liquefaction which discussed above can not exist. However, the investigation of sand behavior under cyclic loading have shown that sands which exhibit a strain hardening behavior can develop a moderate deformation progressively when subjected to cyclic type of loading. This is observed by Castro and Casagrande, and termed as ***cyclic mobility*** [8].

3.2 INTERNAL CAUSE OF LIQUEFACTION

Liquefaction occurs when the structure of ***loose*** saturated sand breaks down due to some rapidly applied loading. As the structure breaks down, the loosely packed individual soil particles attempt to move into a denser configuration. In an earthquake, however, there is not enough time for the water in the pores of the soil to be squeezed out. Instead, the water is "trapped" and prevents the soil particles from moving closer together. This is accompanied by an increase in water pressure, which reduces the contact forces between the individual soil particles, thereby softening and weakening the soil deposit [5].

To understand liquefaction, it is important to recognize the conditions that exist in a soil deposit before an earthquake. A soil deposit consists of an assemblage of individual soil particles. If we look closely at these particles, we can see that each particle is in contact with a number of neighboring particles. The weight of the overlying soil particles produce contact forces between the particles - these forces hold individual particles in place and give the soil its strength [5].

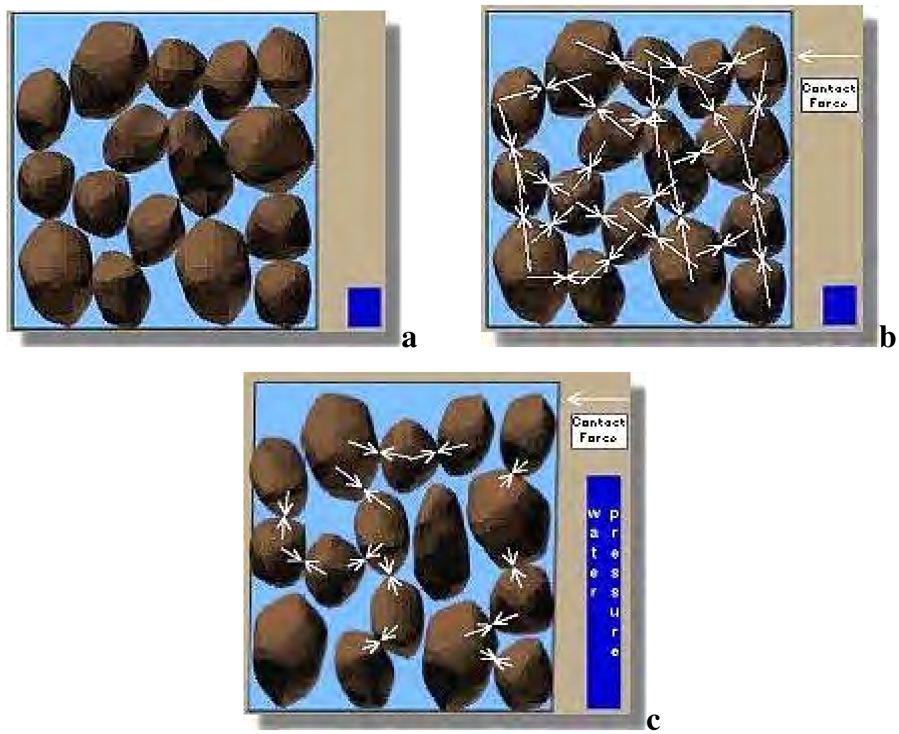


Fig.3.2, Particle structure and arrangement in a soil deposit [5].

Fig. 3.2 a shows soil grains in a natural soil deposit. The height of the black column to the right represents the level of pore water pressure in the soil. The length of the arrows in Fig. 3.2 b represents the size of the contact forces between individual soil grains before an earthquake. The contact forces are large since the pore water pressure is low. After the occurrence of dynamic load, we can see in Fig. 3.2 c how small the contact forces are because of the development of high water pressure in the soil deposit. In an extreme case, the pore water pressure may become so high that many of the soil particles loose contacts with each other. In such cases, the soil will have very little strength, and will behave more like a liquid than a solid - hence, it is said to be liquefied and the overall phenomenon is named as ***"flow liquefaction"*** [5].

Various models are prepared in the laboratory by different researchers for further understanding of the occurrence of liquefaction phenomenon. One of the best models which developed in the University of Washington is attached in this document as ***movie 1*** and it demonstrates well the increment of pore pressure and the loss of contacts between particles during an earthquake and the occurrence of liquefaction finally.

In the case of ***dense*** sand however, this type of liquefaction phenomenon hardly occurs. Instead, the continuous application of monotonic loading makes dense sands to dilate and decrease the pore pressure. As a result, the effective shear strength increases and the flow deformation of sand would be inhibited. Nevertheless, after the investigation of undrained behavior of dense sand under cyclic loading, it is observed that large deformation can be developed ***progressively*** under the application of cyclic stress and it is much smaller than those developed by flow liquefaction, but sufficiently large to cause damage.

Castro noted this phenomenon as, dense sands exhibited positive pore pressure generation at small strains when loaded cyclically [19]. The accumulation of these small increments of positive pore pressure reaches the confining pressure and eventually brought the soil to momentary points of zero effective stress when the cyclic stress passes the value of zero, and it drops substantially and increases the effective stress when either the axial extension or axial compression load is applied. It was suggested by Castro that these momentary lapses of high pore pressures were attributed to the redistribution of the void ratio and water contents in the sample. This principle of void ratio redistribution was further addressed and confirmed by Casagrande through a series of laboratory gyratory/ cyclic simple shear tests and stated that void ratio redistribution and soil softening allow for conditions where cyclic pore pressure is equal to the effective confining pressure [19]. Therefore, this type of liquefaction phenomenon is principally due to the redistribution of voids under cyclic loading, and it is referred to as ***"cyclic mobility"*** by Casagrande and Castro [19]**.**

However, the behavior of the soil after this point, i.e. after the development of momentary high pore pressure, depends on the magnitude and duration of the induced cyclic load. If the cyclic loading conditions are substantial enough to reverse the shear stress direction and possibly bring the soil to zero effective stress, the soil will undergo large deformations. Once the cyclic loading stops, however, the soil exhibits a strain hardening behavior and deformation cease. If the cyclic loading conditions are not substantial enough to reverse the shear stress direction, the shear stress do not pass through value of zero and momentary zero effective stress do not occur; therefore, the soil exhibits a strain hardening behavior after the limiting strain condition is reached the resulting deformations are generally small [19].

3.3 TYPES OF LIQUEFACTION

As per the discussions in sec. 3.1 and 3.2; excitation of dynamic loads like earthquake can cause softening and weakening of ***loose*** saturated sand stratum which is accompanied by a loss of shear strength that may lead to large shear deformation or even flow failure. This phenomenon has been observed in situ on many occasions and it has been reproduced in the laboratory. Hazen first used the term ***"liquefies"*** in 1920 to express this phenomenon after observing it in the failure of the Calaveras Dam in California [8, 19], while Terzaghi was the first to coin the term ***liquefaction*** at 1925, where he stated:

"Liquefaction can occur if saturated soil collapses, resulting in a transfer of the collapsed weight of the solid particles to the surrounding water. As a consequence, the hydrostatic water pressure at any depth increases, which is then close to the submerged unit weight of the soil." [19].

Terzaghi originally introduced the term ***liquefaction*** into the engineering community in the classical book ***Erdbaumechanik***, and in 1936 Casagrande used the term to explain the massive soil failures at the Fort Peck Dam. The concept of liquefaction gathered worldwide attention in the 1960's, when in 1964 large magnitude of earthquakes located near Anchorage, Alaska and Niigata, Japan caused massive structural through soil failure [6, 19].

It is also observed that in ***moderately dense to dense*** materials, liquefaction leads to transient softening and increased cyclic shear strains, but a tendency to dilate during shear inhibits major strength loss and large ground deformations, [13]. This type of liquefaction phenomenon is observed by Casagrande and Castro in 1960's, after their investigation of undrained behavior of saturated sand under cyclic loading and they referred it as ***"cyclic mobility"*** [19].

Up to the investigation of cyclic mobility, it was believed that there existed only one type of liquefaction. For this reason, Casagrande and Castro have classified the type of liquefactions as ***flow liquefaction*** and ***cyclic mobility*** and defined them as follows.

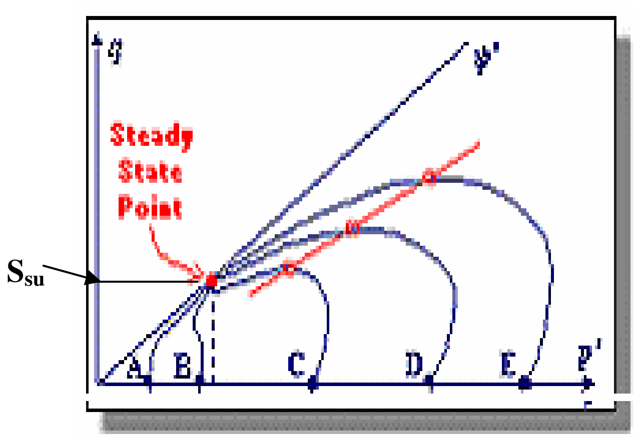
Flow liquefaction is a phenomena wherein a saturated sand loses a large percentage of its shear resistance (due to monotonic or to cyclic loadings) and flows in a manner resembling a liquid until the shear stresses acting on the mass are as low as its reduced shear resistance [8].

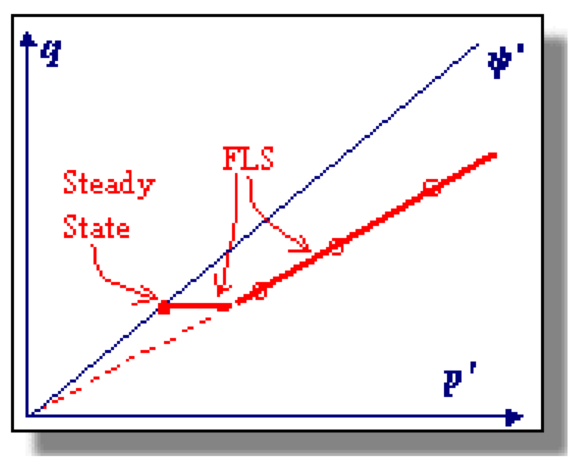
Cyclic mobility is the progressive softening of a saturated sand specimen when subjected to cyclic loading at constant water content [8].

Flow liquefaction is a phenomenon in which the static equilibrium is destroyed by static or dynamic loads in a soil deposit with low residual strength. For example, new buildings on a slope can exert static loads on the soil beneath the foundations while earthquakes, blasting, and pile driving can be examples of dynamic loads that could trigger flow liquefaction. It can occur when the shear stress required for static equilibrium of a soil mass (the static shear stress) is greater than the shear strength of the soil in its liquefied state [20]. Once triggered, the strength of a soil susceptible to flow liquefaction is no longer sufficient to withstand the static stresses that were acting on the soil before the disturbance. Failures caused by flow liquefaction are often characterized by large and rapid movements which can produce disastrous effects [19, 20]. Various models are prepared in the laboratory by different researchers for further understanding of flow liquefaction and one of the best models which developed in the University of Washington is attached in this document as ***movie 2*** [12]**.**

Cyclic mobility is a liquefaction phenomenon, triggered by cyclic loading, occurring in soil deposits even when the static shear stress is less than the shear strength of liquefied soil. Deformations due to cyclic mobility develop ***incrementally*** because of static and dynamic stresses that exist during an earthquake [5, 20]. Castro et al. defined cyclic mobility as the softening of saturated sand when subjected to cyclic loading at constant void ratio. The softening is accompanied with high pore pressures, increasing cyclic deformation, and in some cases, permanent deformations, but it does not lead to loss in shear strength nor to continuous deformation, both of which essential aspects of liquefaction [5].

The types of liquefaction can be further explained using the stress path curves of materials. In Fig.3.3 (a), plots of stress paths for five undrained shear tests are shown. Three test specimens (C, D, and E) were subjected to loads greater than their residual strengths, and experienced flow liquefaction. The ***initial liquefaction*** points of test specimens are located in the respective stress paths and they are observed to lie on a straight line which passes through the origin when projects back, Fig.3.3 (b). This line is called the ***Flow Liquefaction Surface (FLS)*** because it joins the points where flow liquefaction was initiated. Since flow liquefaction cannot take place if the static shear stress is lower than the steady state strength, the flow liquefaction surface is truncated by a horizontal line through the steady state point, Fig.3.3 (b). Flow liquefaction will be initiated if the stress path crosses the flow liquefaction surface during undrained shear regardless of whether the loading is cyclic or monotonic loading [5].





(a) (b)

Fig.3.3, Graphical explanation of flow liquefaction surface [5].

The flow liquefaction process under both monotonic and cyclic loading can be described in two stages. First, the excess pore pressure that develops at low strains moves the effective stress path to the flow liquefaction surface, at which point the soil becomes unstable, Fig.3.4. When the soil reaches this point of instability under undrained condition, its shear strength drops to the residual strength. As a result the static shear stresses drive the large strains that develop as the soil "collapses". A great amount of strain-softening takes place when the stress path moves toward the steady state point, Fig.3.4 [5].

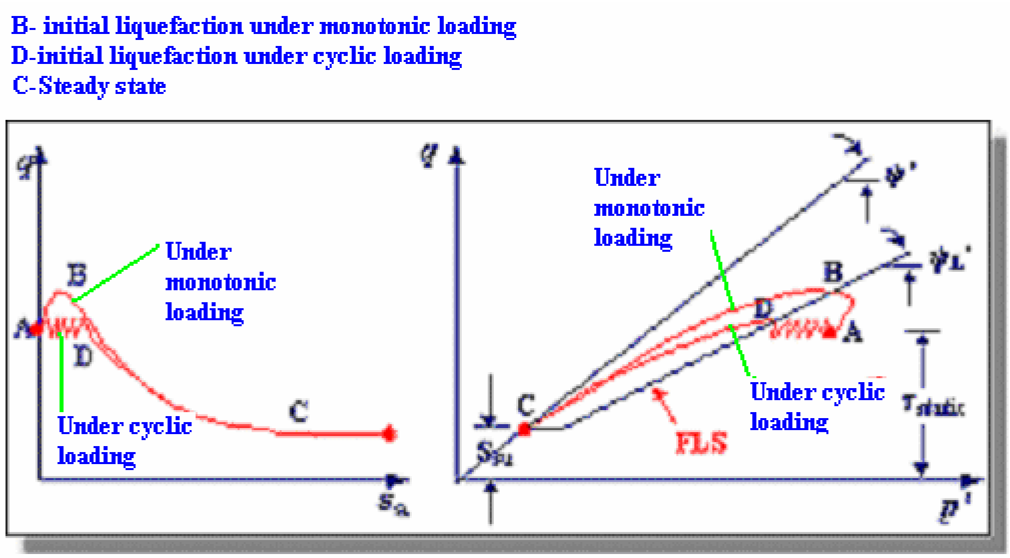


Fig.3.4, A schematic diagram of stress-strain and p-q curves of loose sand under monotonic and cyclic loading [5]

A key to understanding cyclic mobility is the phase transformation line. Medium dense to dense sands which have a potential to develop cyclic mobility when subjected to monotonic loading will initially exhibit contractive behavior, but then exhibit dilative behavior as they strain toward the steady state. A plot of the stress path points at which the transformation from contractive to dilative behavior takes place reveals a ***phase transformation line (PTL)*** that appears to project back through the origin, Fig.3.5.

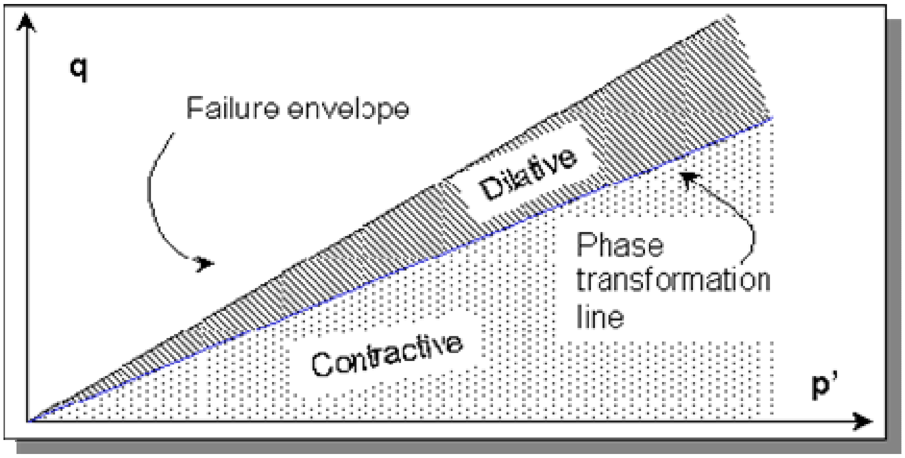


Fig.3.5, A p'-q plot of the phase transformation line [5]

The stress paths of these sands, in the contractive region, will tend to move to the left as the tendency for contraction causes pore pressure to increase and p' to decrease, Fig.3.6. As the stress path approaches the phase transformation line, the tendency for contraction reduces and the stress path becomes more vertical. When the stress path reaches the phase transformation line, there is no tendency for contraction or dilation, hence p' is constant and the stress path is vertical. After the stress path crosses the phase transformation line, the tendency for dilation causes the pore pressure to decrease and p' to increase, and the stress path moves to the right, Fig.3.6.

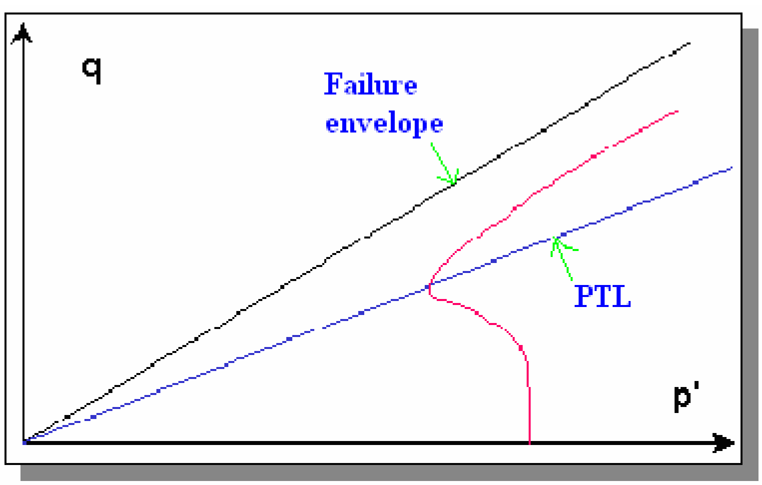


Fig.3.6, A stress path example [5]

SUMMARY

In general, the differences between flow liquefaction and cyclic mobility according to Castro, Poulos, and Verdugo are summarized and given below [8, 18].

Flow Liquefaction	Cyclic Mobility
It involves a loss in shear strength	It doesn't entail a loss in shear strength
Only such states above the SSL in the e-p' plane are susceptible to flow liquefaction	Loose as well as dense sands under any pressure can develop cyclic Mobility
It may be triggered only when driving forces are greater than the undrained steady state strength	Deriving forces smaller than steady state strength may cause cyclic mobility
Static load can cause liquefaction. Cyclic loads causing shear stresses larger than the steady-state strength also can cause liquefaction.	Any soil in any state can develop cyclic mobility in the laboratory if the cyclic stresses are large enough.
During flow liquefaction, the effective stresses drop to constant values which are equal to solely in the case of extremely loose sandy soil.	For reversed amplitude of cyclic stress, cyclic mobility is associated with momentarily zero effective stresses.
During flow liquefaction, the soil mass deforms continuously under its residual shear strength.	During cyclic mobility, the soil mass undergoes cyclic deformation without mobilize necessarily its ultimate shear resistance.
Flow liquefaction involves remarkably large deformation of several kilo meters, mainly depending on the difference (shear stress-steady state strength) and the geometry of the problem itself.	Cyclic mobility usually compromise a moderate level of deformation, but sufficiently large to cause damage

The higher the effective overburden pressure, the larger the deformation is but need high intensity triggering load to induce flow liquefaction.	The higher the effective overburden pressure, the more difficult to build up pore pressure, and therefore to develop cyclic mobility.
The residual strength is known if the void ratio is known	Only the deformation that takes place in a sample are known

3.4 OCCURRENCE AND CONSEQUENCE OF LIQUEFACTION

Liquefaction usually occurs to saturated sand during excitation of dynamic loads like earth quake so that it commonly happens to the areas near water body like lakes, rivers, oceans and bays. It also occurs in sloping areas during construction of infrastructures which induce large static loads or during blasting and/ or occurrence of earthquake.

The liquefaction phenomenon by itself may not be particularly damaging or hazardous. Only when, it is accompanied by some form of ground displacement or ground failure that it becomes destructive to all types of engineering structures which are founded or buried in saturated sands. Adverse effects of liquefaction can take many forms. These include lateral spreads, ground oscillation, loss of bearing capacity, settlement, and increased lateral pressure on retaining structures.

Failures caused by flow liquefaction are often characterized by large and rapid movements which can produce the type of disastrous effects experienced by, like the Kawagishi-cho apartment buildings, which suffered a remarkable bearing capacity failure during the Niigata Earthquake 1964 [5]. See Fig. 3.7

Cracked highway, Alaska, USA
1964

Retaining wall damage and
lateral spreading, Kobe 1995

Sand Boils, Loma Prieta, USA, 1989

Kawagishi-cho apartments sank in and tipped,
Niigata, Japan, 1964

Lateral displacement of quay wall,
Kobe, Japan, 1995

Collapsed bridge, Kobe, Japan, 1995

Lateral spreading caused 1.2-2 meter drop of paved surface and local flooding, Kobe 1995

Collapsed bridge, Niigata, Japan, 1964

Sanfernando dam failure [12]

Fig. 3.7, Damages caused by liquefaction during some earthquakes [12].

Lateral spreading is a common result of cyclic mobility, which can occur on gently sloping and on flat ground close to rivers and lakes, Fig. 3.8 [5].

Fig. 3.8, 1976 Guatemala earthquake caused lateral spreading [5].

Chapter Four

Concept of Critical Void Ratio and Steady State of Soil

4.1 GENERAL

The critical void ratio and the steady state of soil are investigated during the study of drained and undrained behavior of sands. The critical void ratio is first introduced by Casagrande in 1936 using drained direct shear tests while Castro advanced the concept of critical void ratio to the steady state concept using series of undrained shear tests [5, 19].

The critical state refers to the failure state of a material at a constant void ratio when sheared in drained condition and the constant void ratio is termed as ***critical void ratio*** by Casagrande. It is the void ratio beyond which loose sand can't contract more and dense sand can't dilate more; however, both types of the specific sand develop large strain at critical void ratio till point of failure.

The steady state refers to the state of steadily flow of loose material after the development of flow structure and the reduction of effective shear strength to the residual (minimum) value. It can be located in the stress-strain curve of loose sand that is obtained from undrained test, Fig. 4.1.

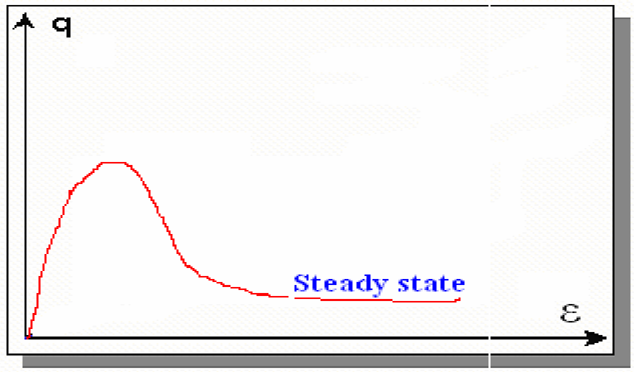


Fig. 4.1, A schematic diagram of stress-strain curve for loose sand

4.2 THE CONCEPT OF CRITICAL VOID RATIO

The concept of critical void ratio was first postulated by A. Casagrande in the 1930's [2]. By means of stress-strain curves obtained from drained direct shear tests, Fig. 4.2, he observed that during shearing, dense sands dilated gradually with the corresponding increment of shear strength towards to the peak value. The peak shear strength of sand was then reduced substantially and reached to a constant value while the void ratio of sand increased abruptly and approached a constant value of void ratio. On the other hand, loose sands were observed to contract continuously until a constant void ratio reached with the corresponding continuous increment of shear strength towards to the ultimate value. He also observed that both loose and dense specimens that were consolidated under the same initial confining pressure, reached finally to the same constant void ratio and shear strength values at large deformations, Fig. 4.2 [2, 18].

Based on these observations Casagrande concluded that; "when dense and loose sands are sheared in a drained condition, they change their void ratios until a common constant value is eventually reached. This ultimate common void ratio was termed by him as the ***critical void ratio***". In this state, i.e. in the critical state, the soil continues to deform under constant strength and constant volume change, and hence the soil behaves as a frictional fluid [2, 18].

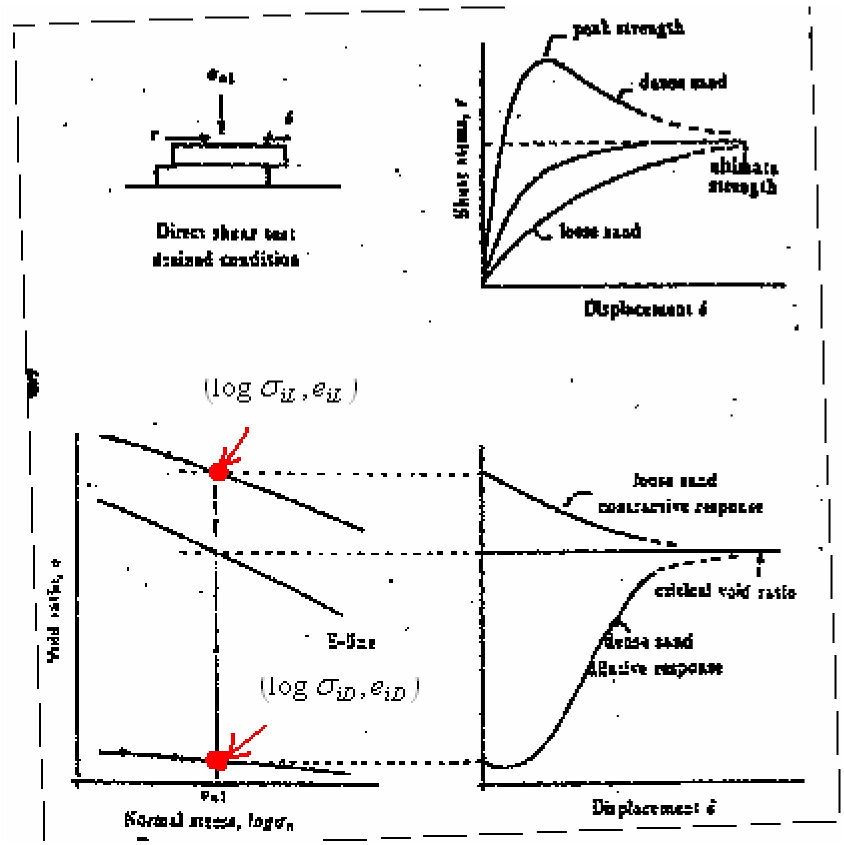


Fig. 4.2, Hypothesis of critical void ratio explained by means of direct shear tests [18].

By means of series of stress-strain curves derived from direct shear tests, he showed that shear strength and void ratio of sand at constant deformation (critical state) are dependent only on effective normal stress, where as the shape of the stress-strain curves is dependent on the initial void ratio. Casagrande then plotted the ***critical void ratios*** with the corresponding ***effective normal stresses*** of different sand specimen as seen in Fig. 4.2, and observed that the critical void ratio of sand decreases with increasing effective normal stress, with an approximately straight-line relationship between critical void ratio and the logarithm of normal effective stress. This line is then termed as ***critical void ratio line (E line)*** by him [2].

It is easily seen from Fig. 4.2 that a critical void ratio (CVR) line in e-logp' space constitute the boundary between dilative and contractive sands in drained triaxial compression tests. This can be easily demonstrated using Fig. 4.2 as follows.

The initial state of the dense (dilative) sand, i.e. the initial void ratio, e_{iD} and the initial normal stress, σ_{iD} are plotted below the E-line while the initial void ratio, e_{iL} and the initial normal stress, σ_{iL} of loose (contractive) sand are plotted above this line. It is also seen in the Fig. 4.2 that the initial normal stresses at which both sand specimen consolidated were the same. During shear, the initial state of loose sand moved vertically down ward to the critical void ratio line while the initial state of dense sand moved vertically upward to the same line. The respective stress-strain curves of the sand specimen also moved in the same manner to the critical state, but not right vertically. Finally the states of the specimen are observed to reach at a common point in the E-line and the stress-strain curve. Therefore, the critical void ratio and the effective normal stress at constant deformation (critical state) of the specimen which were consolidated at the same initial normal stress are the same and independent of the initial void ratios at which they were consolidated. Further more, it is also observed that the shape of stress-strain curves of the specimen varied with the initial void ratios and this supports the observation done by Casagrande; "the shape of the stress-strain curves is dependent on the initial void ratios".

As a result, Professor Casagrande's original hypothesis was that sands, which plot above this critical void ratio line in their in situ state, are loose and are susceptible to liquefaction under undrained conditions, while those plotted below are dense and are safe against this type of failure. Investigations of the Fort Peck Dam slide, however, proved that sands, which according to such test results should be safe against liquefaction, had actually liquefied with a very large loss in strength. Therefore, he concluded that either his method for determining the critical void ratio or his entire concept of liquefaction of sands was faulty [2].

It was not, until the apparatus capable of inducing liquefaction in the laboratory was developed by Castro at Harvard, explained the reason why Casagrande's approach failed after the phenomenon observed in the Fort Peck Dam slide [2]. But Castro performed numerous load-controlled CU and CD triaxial tests on different sands at various void ratios and consolidation stresses and demonstrated the existence of two critical void ratio lines, namely ***F line*** and ***S line***, from CU and CD test results respectively. See Fig. 4.3. It is easily observed from this figure that at the same void ratio, the undrained steady-state strength (F line) may be considerably smaller than the strength that would have been obtained from constant volume drained tests (S line). The reason for this is that the pore-water pressure response of sand specimens in undrained shear does not depend only on the potential volume changes, as the case of drained shear, but also on the tendency to collapse, i.e., to a sudden change in particle arrangement. The pore pressure increment resulting from such a collapse determines the difference between the S and F lines. If the sand structure is not inherently brittle, no collapse will take place; thus, the pore pressure response will be due solely to sand compressibility, and the S and F lines will tend to merge. On the other hand, the smoother, more rounded, the finer the particles, and the more uniform the gradation, the higher the collapse potential and the farther apart the F and S lines tend to be [1]. In general, the difference between these lines was attributed to the existence of what Casagrande called a "flow structure", in which the grains orient themselves and flow like a fluid so small amount of steady state strength will be mobilized to resist least amount of the applied stress and it is due to the frictional resistance during flow [5, 18, 19]. This all was the reason for the failure of sand in the Fort Peck Dam slide, and Casagrande and Castro have also given similar reason [2]. Therefore, Castro and Casagrande proposed the F line, not the S line, as the line to predict the resistance of soil against liquefaction because it provides the minimum strength that a soil could develop during flow liquefaction, and the sand in the Fort Peck Dam would show flow liquefaction had it been assessed using the F line [18].

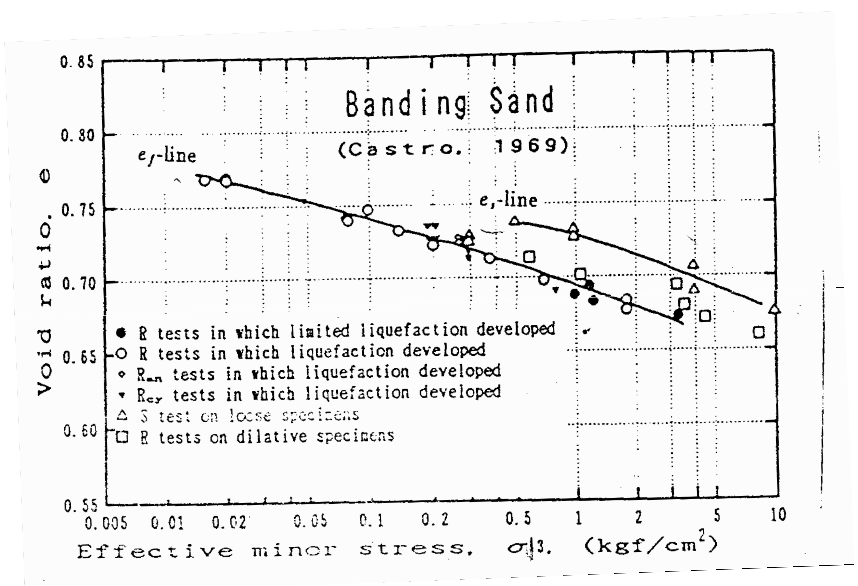


Fig. 4.3, Summary of triaxial tests on Banding Sand [18].

Castro also defined the upper critical void ratio line by the S line, which is parallel to the F line. The S line is essentially the same as the E line, the critical void ratio line, obtained by A. Casagrande [18].

4.3 THE CONCEPT OF STEADY STATE

The concept of steady state was introduced by Castro to explain large strain soil behavior during drained and undrained shear. The concept was based on the earlier works of Casagrande, who showed that all soils tested at the same confining pressure in drained triaxial compression approached the same density when sheared to large strains [19]. The void ratio at this density was termed the ***critical void ratio*** (as discussed in the previous section).

Castro advanced the critical void ratio concept to the concept of steady state through the use of stress-controlled undrained triaxial compression tests to study soil behavior during undrained shear. His stress-controlled tests allowed for the identification of the large post-peak strength loss in loose soils and the potential for pore pressure accumulation during cyclic loading in dense soils. Loose soils, after the loss of the large peak shear strength, undergo large deformation with constant void ratio and constant residual strength, and develop *flow structure*. Dense sands, in the presence of accumulated pore pressure, can also develop large strain after redistribution of void ratio. Castro suggested the soil in the *flow structure* was in its ***steady-state***, which he defined as a "state of deformation of a soil mass where the deformation is occurring at a constant volume, pore pressure, normal effective stress, and shear stress" [7]. Later the concept of steady state was again studied by Poulos and he redefined it as "a state in which the soil is continuously deforming at constant volume, pore pressure, normal effective stress, shear stress and constant velocity" and introduced that the soil in the steady state also deforms at constant velocity [19]. They noted that the steady-state deformation is only achieved after all particle orientation has reached a steady-state 'flow' type of condition and after all particle breakage is complete. A soil can only exist in this type of condition if the velocity and shear stress needed for continuous deformation are constant. If any variation from these 'constant' conditions is encountered, the soil will develop shear strength and the large shear induced deformations will cease [19].

As discussed in section 4.2, Castro observed that the locus of the steady-state strength in a void-ratio logarithm of effective minor stress diagram defines a unique line that is referred to as the F line. It relates the ***void ratio at the steady state*** with the ***effective confining pressure at the same state***. The letter F stands for flow, since at failure, a flow structure develops in these undrained tests. The F line is introduced as the ***steady-state line*** by Castro and Poulos into the engineering community through a landmark paper presented at the ASCE Annual Convention in Philadelphia, PA [8, 19]. They identified the steady-state line (SSL) as the 3-D graphical representation of the locus of states in which a soil will flow at the constant shear stress, volume, effective stress, and velocity conditions mentioned above. They noted that the axes of the plot would include shear stress (τ), void ratio (e), and minor effective principle stress (σ_3) in 3-D space, but could also be plotted in a pair of 2-D plots with one common axes. Examples of these relation ships are presented in Fig. 4.4. [5].

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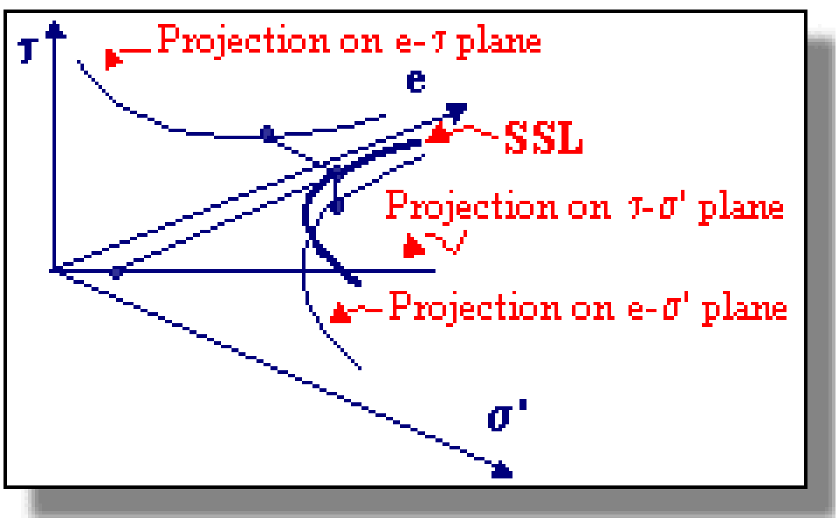

Fig. 4.4 (a), 3-D representation of steady state line [5]

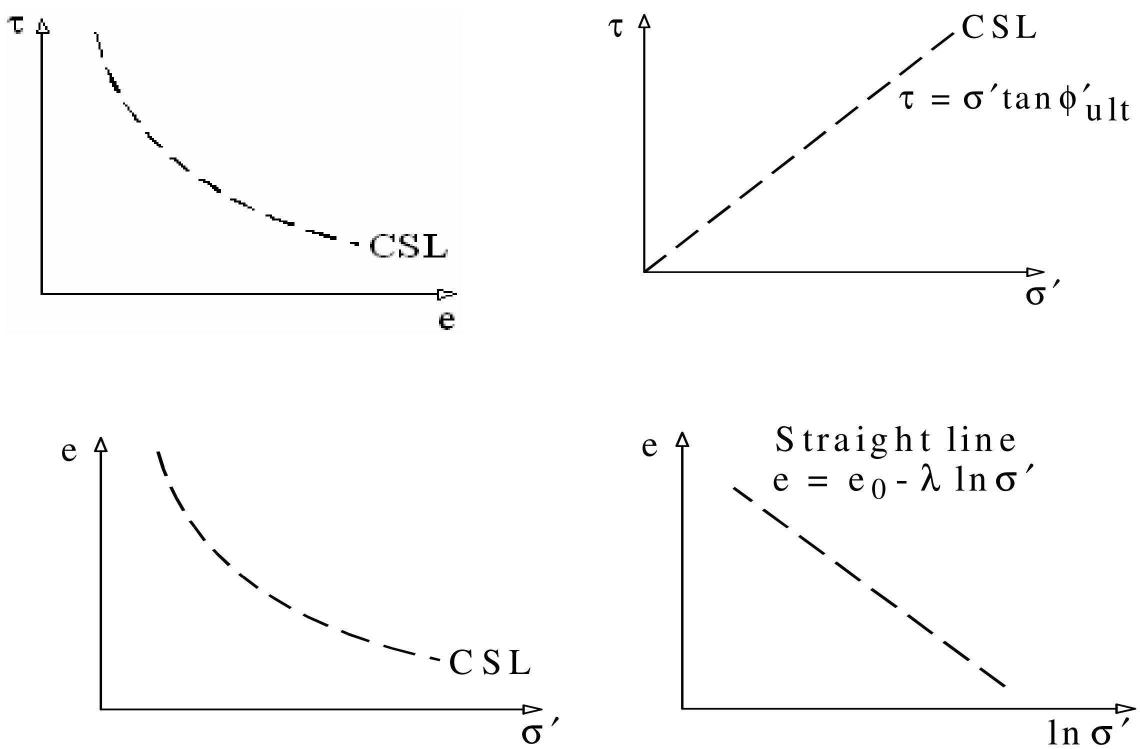

Fig. 4.4 (b), 2-D representation of steady state line [5]

They suggested that the steady-state line concept can also be used to evaluate the soil response during undrained monotonic and cyclic shear by considering the position of the in-situ soil relative to the SSL; and the 2-D plot of steady state line in e-logp space, Fig. 4.5, can be used for this purpose, discussed in sec. 4.4.

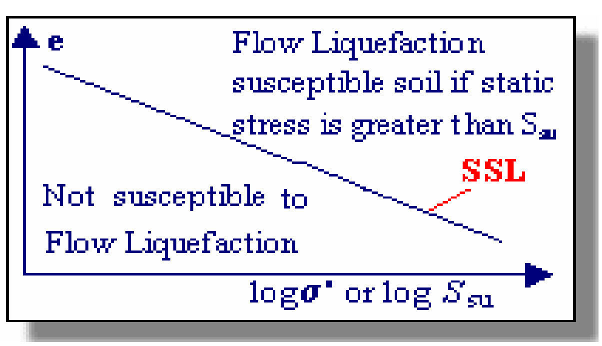

Fig. 4.5, A schematic diagram of steady state line in using for assessment of liquefaction susceptibility of soil deposit [5].

Castro, Verdugo, Poulos, and others found that steady-state strength and as well the steady state were solely a function of ***initial void ratio*** of sand [17], though others like J. M. Konard and Jain Chu proposed that the steady state strength of given sand at a given void ratio was not unique, but that it was dependent upon the effective confining stress, and therefore they divided the F line as the upper F line (UF) and the lower F line (LF) [11].

DIVERGENT OPINION

Lots of works have been done to relate the critical void ratio line (S line) with the steady state line (F line) which are obtained by Castro. Accordingly, most have agreed in the uniqueness of these lines. For example, T. Samuel, in his PhD thesis work, has conducted series of undrained and drained tests and concluded the following; "at large strain level, the same steady state line is obtained from monotonic undrained and drained compression tests. This indicates that the large strain level needed to develop the steady state is able to transform the initial soil structure to a new soil structure which is the same for all samples tested under these two conditions of loading. The result also confirms that the steady state is an ultimate condition absolutely common to both undrained and drained loading conditions", Fig. 4.6 [18].

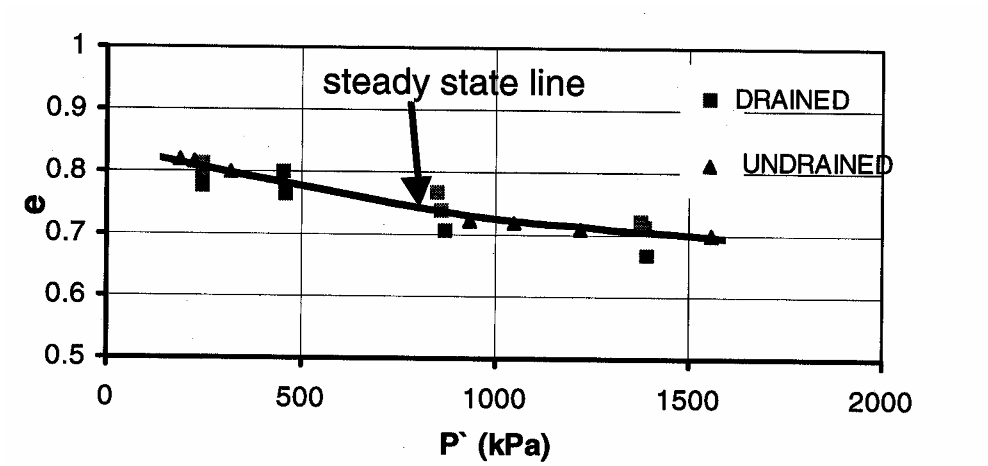

Fig. 4.6, Steady State data from drained and undrained triaxial compression tests and steady state line [18].

Dongdong Chang also stated that the location of the steady state line has been suggested to be influenced by the following factors [5]:

1. Strain rate
2. Sample preparation procedures
3. Stress path (extension, plain stress, or compression)
4. Consolidation stress prior to shear

Numerous tests done by Castro, Sladen, and others at different rates have in fact shown that the location of the steady state line is independent of strain rate [5].

Seed, Been, and Poulos have studied the effect of sample preparation procedures and have shown that although behaviors before the steady state can be influenced by sample preparation procedures, true steady state conditions are not [5].

Vaid and his co-wokers have shown that, for given sand, behaviors prior to the steady state can be influenced by stress path, especially extension versus compression. They also suggested that quasi-steady state conditions in extension are different from those in compression. However, the stress path doesn't influence the true steady state conditions [5].

Konrad has shown that the quasi-steady state is influenced by consolidation stress prior to shear. Consolidation stress prior to shear, however, does not influence true steady state as shown by Mcroberts [5].

Mentioning the above facts, Dongdong concluded that true steady state or critical state is unique state for a specific type of sand [5].

Sladen, and others have also concluded the fact that the critical state and the steady state are essentially the same [1, 5]. Gonzalo Castro and Steve J. Poulos also stated that the void ratio at the steady state is the same as the critical void ratio as defined by Casagrande [8].

However, some do not agree in this statement and believe in the existence of two steady state lines of which the S line is similar to the critical void ratio line obtained in drained test. Among them, A. Alarcon-Guzman and G. A. Leonards have said that "steady-state" (line F from undrained tests) and "critical void ratio" (line S from drained tests) are not necessarily one and the same line [1].

4.4 APPLICATION OF SSL ON THE ASSESSMENT OF LIQUEFACTION POTENTIAL AND IDENTIFICATION OF TYPES OF LIQUEFACTIONS

Different approaches are devised for the assessment of liquefaction susceptibility of soil deposit. One of the methods is using the steady state line as a boundary for contractive sands versus dilative sands.

Gonzalo Castro and Steve J. Poulos showed the application of steady state line on the assessment of liquefaction potential, Fig. 4.7 stating the uniqueness of steady state and critical state [8].

When fully saturated highly contractive (loose) sand is sheared in undrained condition, it develops a strain-softening behavior with the characteristic of large deformation at failure and exhibits the type of ***flow liquefaction***. If the initial states of sand, i.e. the initial void ratio and initial confining pressure of sand before it is sheared, are plotted in e-logp space, they lie above the steady state line of the respective sand, for example at point C in Fig. 4.7. If the locus of the states of the sand, i.e. the void ratio and effective confining pressure, during shear are plotted in this plane, it travels from the initial state, i.e. point C, horizontally to the left (as shown in Fig. 4.7) and ends after it arrives to the respective SSL of sand at point A where the sand flows at steady-state with constant volume and constant confining pressure during undrained flow.

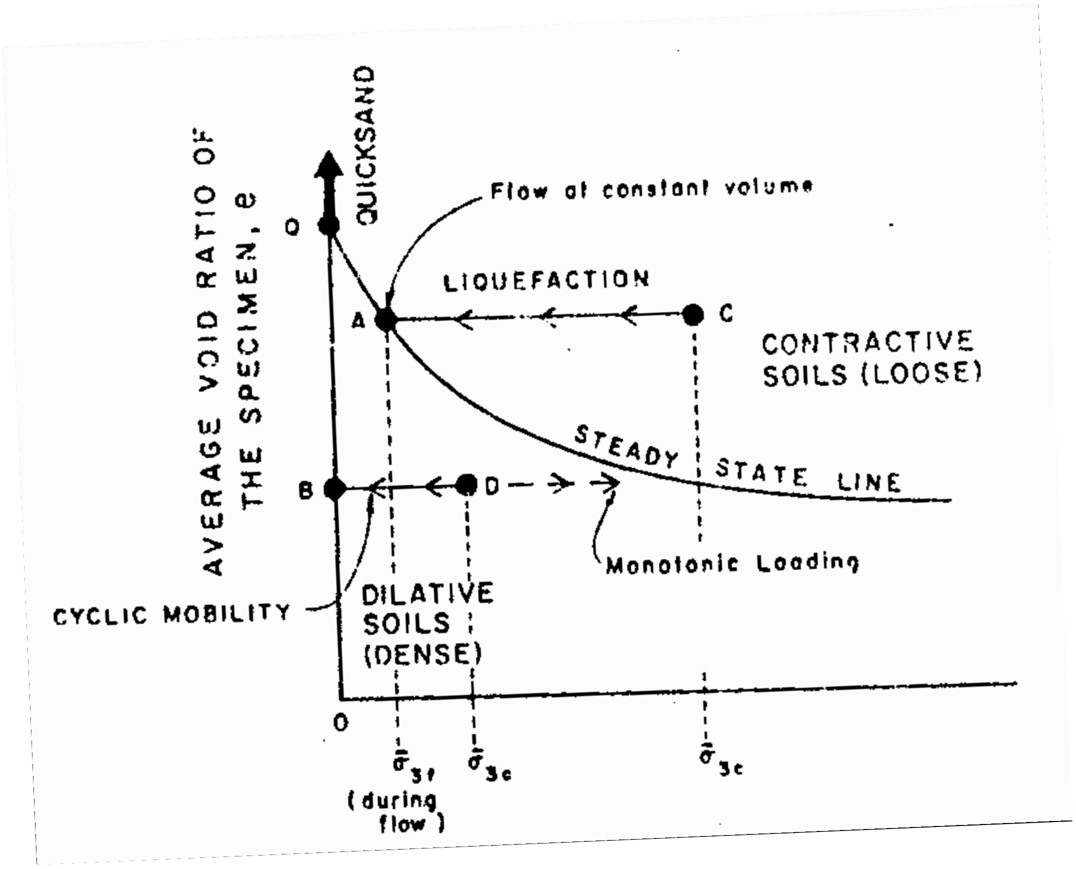

Fig. 4.7, Undrained tests of fully saturated sands depicted on state diagram [8].

This is because, the void ratio of sand would never change from the initial value since the sand is sheared in undrained condition while the effective confining pressure decreases continuously as the pore pressure during undrained shear of contractive sand increases and attains its ultimate value constant. Therefore, sands with initial states plotted above the respective SSLs' are contractive sands which are susceptible for flow liquefaction [8].

If the effective confining pressure, $\sigma_3^{'}$, during flow liquefaction of sand is zero instead of minimum residual value, the phenomenon called quick sand condition is taken place. Sands exhibiting such phenomenon can be identified by comparing the initial void ratio of sand with the void ratio corresponding to point Q in the respective state diagram, Fig. 4.7. At void ratios above Q the sand grains are not in close contact at all times. In this state, sand has zero strength and is also neither dilative nor contractive [8].

The mechanics of ***cyclic mobility*** may also be explained with the aid of this figure. Consider first the behavior on Fig. 4.7 when a fully saturated dilative sand starting, for example, at point D is loaded monotonically ("statically") in the undrained condition. In that case the point on the state diagram may move slightly to the left of point D but then it will move horizontally toward the steady-state line and intersect it as the monotonic load is applied. However, if cyclic loading is applied instead of monotonic, one can follow the behavior by plotting the average void ratio and the effective stress each time the applied cyclic load passes through value of zero. In this case the state point moves horizontally to the left, because the average void ratio is held constant and the pore pressure rises due to cyclic loading. In particular, it has been observed in the laboratory that in triaxial tests for which the applied cyclic load passes through value of zero, and if a large enough number of cycles of sufficient size are applied, the state point for the average conditions in the specimen eventually reaches zero effective stress at point B each time the cyclic load passes through value of zero where the sand is said to develop ***cyclic mobility***. Subsequent application of undrained monotonic loading after this point, however, can move the state point to the right toward the steady-state line, and the resistance of the specimen increases [8].

In summary then, specimens that lie above the steady-state line on Fig. 4.7 can liquefy, show ***complete liquefaction***, if the load applied is large enough. Such liquefaction can be triggered by monotonic or cyclic undrained loading. The further to the right of the steady-state line that the starting point is, the greater will be the deformations associated with the liquefaction. If the initial point is above Q, the strength after liquefaction will be zero. If the starting point is below Q, the strength after liquefaction will be small but finite. Saturated sands starting at points on or below the steady-state line will be ***dilative*** during undrained monotonic loading in the triaxial cell and the state point will move to the right. If cyclically loaded the state points will shifts to the left as strains occur and the specimen softens. If enough cycles are applied, if they are large enough, and if the applied cyclic loading passes through a value of zero during each cycle, then the zero effective stress condition, ***cyclic mobility***, can ultimately be reached in the laboratory [8].

PROBLEMS OF THIS APPROACH

The problem with the steady state approach for the assessment of liquefaction potential is mainly the difficulty in determining the in situ void ratio. Any small inaccuracy of the parameter would cause largely different result, so it's very important to get accurate in situ void ratio, while it is difficult to obtain in practical cases [5].

SUGGESTIONS

It is widely discussed about the application of SSL for the study of liquefaction analysis. However, it has a limitation in determining the in situ void ratio and can arrive sometimes with wrong conclusions. Therefore, other methods like field tests, i.e. SPT, should also be employed additionally to cross check and verify the results obtained using SSL.

Chapter Five
Conclusions and Recommendations

5.1 CONCLUSIONS

The undrained behavior of saturated sand is studied in detail in relation to liquefaction phenomenon. The different factors that can affect the undrained behavior of sand are investigated and their influences are studied to the better understanding of the response of sand under undrained condition. Furthermore, the concept of steady state is discussed observing the stress-strain curves of sands that are obtained from the shear test results. The relation ship of the steady state and the corresponding shear strength is also established as a steady state line and its application on assessing the liquefaction potential is demonstrated. The liquefaction phenomenon is elaborated based on the undrained behavior of sand and its types are defined and discussed well. On the overview of these, some important conclusions are drawn from the whole discussion.

1. The convenient way of studying undrained behavior of saturated sand, as followed by different researchers, is by testing sands under different conditions that can affect the responses and then observing the results obtained. Accordingly, the initial conditions of sand, i.e. ***initial void ratio*** and ***initial confining pressure*** are determined to be the main factors affecting the response of saturated sand under undrained condition. In addition to the initial conditions, the undrained behavior of sand also varies according to the type of sand, particularly the ***grain size distribution, particle shape***, and ***compressibility characteristics of particles.***

2. Three basic types of undrained behavior of saturated sand are found out after series of laboratory studies by different researchers. The first type is a ***strain-softening*** behavior where the sand losses its shear strength substantially to a residual value after the occurrence of peak value. The characteristic of such type of response at failure is a steadily flow of particles with the development of large deformation. The second type is a ***strain-hardening*** behavior where the shear strength of sand continuously increases to the ultimate value. The ultimate value of shear strength is then attained constant at failure and the sand deforms continuously. The final type of response is a ***limited strain- softening*** behavior followed by a ***strain- hardening*** behavior. In this type of response the shear strength of sand decreases to a residual value after the occurrence of peak and attain this value (residual value) constant for a while, i.e. ***strain-softening*** behavior. But then, the shear strength increases back to the ultimate value and attains it constant during failure where the sand is deforming continuously though the amount of deformation is small, i.e. ***strain-hardening*** behavior.

3. Loose sands are able to contract continuously when subjected to axial load in undrained condition. This results in the increment of the corresponding pore pressure which causes a substantial decrease of the shear strength of sand to residual value. Therefore, loose sands exhibit strain-softening behavior during undrained shear and it is more emphasized as the sand is very loose. On the other hand, dense sands are able to dilate after a small amount of contraction when subjected to axial load in undrained condition. As a result, the corresponding pore pressure decreases and improves the shear strength of sand. Therefore, dense sands exhibit strain-hardening behavior during undrained shear and it is more emphasized as the sand is very dense.

4. Highly confined sands are able to contract continuously when subjected to axial load in undrained condition. This results in the increment of the corresponding pore pressure which causes a substantial decrease of the shear strength of sand to residual value. Therefore, highly confined sands exhibit strain-softening behavior during undrained shear and it is more emphasized as the amount of confining pressure is very high. On the other hand, sands confined under low overburden pressure can dilate after a small

amount of contraction when subjected to axial load in undrained condition. As a result, the corresponding pore pressure decreases and improves the shear strength of sand. Therefore, these sands exhibit strain-hardening behavior during undrained shear and it is more emphasized as the overburden pressure is very small. However, for the same inititial void ratio, both highly and loosely confined sands arrive ultimately at the same steady state. Though, loosely confined sands show strain-hardening behavior under monotonic loading and seem to have a better resistant for flow liquefaction, investigation under cyclic loading has revealed that highly confined sands required large number of cycles to develop flow liquefaction when they are compared with loosely confined sands and hence they are more resistant to flow liquefaction.

5. The phenomenon of liquefaction is best explained based on the undrained behavior of saturated sand. Accordingly, loose (contractive) sands which exhibit strain-softening behavior can undergo steady flow at failure with the development of large deformation. This is attributed to the loss of contacts due to the decrement of effective overburden pressure as the pore pressure increases and results in the reduction of shear strength. In such cases, the sand flows continuously and behaves more like a liquid – hence, it is said to be liquefied and the over all phenomenon is named as ***"flow liquefaction"*** by Casagrande and Castro. On the other hand, the investigation of undrained behavior of sand under cyclic loading has revealed that positive pore pressure can be accumulated progressively when dense (dilative) sand is subjected to cyclic loading and results in the reduction of shear strength and development of large deformation which can cause considerable failure of soil deposit. The development of pore pressure is attributed to the redistribution of void ratio and water content during cyclic loading and hence this phenomenon is referred to as ***"cyclic mobility"*** by Casagrande and Castro.

6. The steadily flowing of sand is observed by different researchers when loose (contractive) sand which exhibits a strain-softening behavior undergo shear failure and the corresponding state is defined as ***steady state*** by Castro and Poulos. At this state the sand is flowing at constant pore pressure, constant effective shear stress, constant effective normal stress, constant volume and constant velocity developing a ***flow***

structure. Castro has shown that this state of sand, the failure state of sand obtained from the undrained test, plotted below the corresponding state of sand obtained from drained test. However, many have supported with the test results that the ***critical state*** obtained from the drained shear test is identical to the ***steady state*** obtained from undrained shear test. But then, some have not yet agreed on the uniqueness of these states.

7. Totally, as a static approach, the steady state concept is useful in understanding the basic mechanics of true liquefaction, but it's still difficult to apply in practice for the assessment of liquefaction potential because of its difficulties and limitations such as highly sensitive to its parameters for sands and extremely difficult to measure required parameter with sufficient accuracy.

5.2 RECOMMENDATIONS

An intensive literature review is conducted to the better understanding of the undrained behavior of saturated sand. The phenomenon of liquefaction and the concept of steady state are well discussed based on the undrained behavior of sand. Divergent opinions of different researchers on the related concepts are also presented here and discussed well. As per the objective of the thesis, this document presents the basic theoretical background of the undrained behavior of saturated sand, the phenomenon of liquefaction and the concept of steady state. However, series of laboratory and field tests must be conducted for detail study of the undrained behavior of local sands, for the assessment of the extent of liquefaction problem in the country, and to device for the possible solutions for the problem of liquefaction. Accordingly, the following recommendations are given for the completeness of the work.

1. A proper type of **triaxial** and/or **direct shear** equipment which can produce monotonic and cyclic load should be available in the country and detail laboratory study must be performed to establish the undrained behavior of local sands in the areas like rift valley region where liquefaction problem is likely to be prone.

2. Future studies using field tests can be attempted on the assessment of liquefaction potential of sand deposits in susceptible areas of the country for liquefaction; studies shall also be extended to correlate the results obtained from laboratory study with the field test results.

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