TIME SERIES ANALYSIS ON THE OUTGOING INTERNATIONAL TELEPHONE CALLS TO SAUDI ARABIA (1989 G.C. - 1996 G.C.)

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TABLE OF CONTENTS

CHAPTER 1
INTRODUCTION

1.1. Objective Of The Research Project

According to a magazine* Ethiopia was communicating with the outside world by telephone and telegraph lines through Addis Ababa, Asmara and Kasala. From 1933 to 1979 the country's links with foreign countries were limited by High Frequency (HF) radio communication services. To bring about a radical improvement in the quality and efficiency of international services facility the HF radio communication facility was replaced by a "Standard A" satellite earth station at Sululta with a 300 channel Semi-automatic telephone exchange. When the station was set up it had a 36- channel international telephone communication facilities. Out of the many outgoing international telephone communications the second largest in number of calls is the outgoing international telephone calls to Saudi Arabia.

The principal objective of this project is to analyse the performance of this service on the outgoing international telephone calls to Saudi Arabia. Accordingly, an attempt shall be made to:

a. Identify the model which represent the series
b. Study the relation of the outgoing telephone calls to Saudi Arabia with time
c. Make forecasts based on the tested model
d. Check whether the series is increasing or decreasing.

1.2. Source And Nature of Data

This study is based on the secondary data of the outgoing international telephone calls to Saudi Arabia from 1989 to 1996. The source of the data is the Ethiopian Telecommunication Corporation (ETC). The data was obtained from a document where the monthly data had been recorded.

* Ethiopian telecommunication corporation magazine issued to commemorate the centenary of introduction of telecommunication to Ethiopia,

1.3. The Time Plot

In any time series analysis the first step is to plot the observations against time. The plots show up important features of the series such as trend seasonality outliners and soon.

The time plot of the telephone calls data is given in fig. 1. The figure shows some interesting features. First, we may note that there is an upward trend from January 1989 up-to December 1992. Secondly, the plot shows a marked decreasing trend starting from December 1992 up-to the last month of the last year, December 1996. From the plot the lowest observation occurs on February 1990 and the highest is on February 1993.

Also observing the maximum and minimum values for each year gives us information whether or not to assume marked seasonal variation.

Year	1989	1990	1991	1992	1993	1994	1995	1996
Maximum Value & Corresponding Month	9636 August	18331 May	22026 Jan.	36908 Sept.	39119 Feb.	35729 Jan.	27407 Jan.	26588 Feb.
Minimum Value & Corresponding Month	5303 Oct.	5179 Feb.	10335 June	17711 Jan.	31465 June	25462 Aug.	24097 Nov.	20690 Nov.

From the above table more yearly maximum values occurred at the beginning of each year, on Januarys and Februarys, 3 and 2 times respectively. And more yearly minimum values occurred on Junes and Novembers 2 times on each. We note also the largest of all observation occurred on February. Since there is no regularity in the patterns they suggest the absence of a significant seasonal pattern. Moreover, the graph of the autocorrelation coefficients (figure 4) ascertain that the series has little or no seasonal variation (see section 4.1).

1.4. Test Of Randomness

Application of time series analysis needs that the series be non random i.e. the time order of the observations are dependent. The objectives of checking randomness are:

- to know whether the actual series is random or not
- to know whether the residual series (a series whose systematic components are removed) are random for applying further modeling.

A series is random if the observations are independent of time, could have occurred in any order, and further no technique of time series analysis can be applied. Hypothetically, the observations in a random series are independent of each other. The four widely used tests of randomness (see kendal, 1976) are:

1. Turning point test
2. Phase length test
3. Rank test
4. Sign test

In this study the rank test is used to test for randomness.

Rank Test

According to kendall (1976) the rank test is a powerful test because it involves and takes into consideration every observation by comparing not only neighbouring valves but also by taking each value in the series and compare it with all other values.

For a series Y_t for $t = 1,2,\ldots, n$ count the number of cases where $Y_j > Y_i$ for $j > i = 1,2,\ldots,n$ and let this number be P. The expected number of P in a random series is $\frac{n(n-1)}{4}$

If $P > E(P)$ then a rising trend is suspected in the series and

if $P < E(P)$ then a falling trend is suspected.

Therefore, rank test is more appropriate test where a trend movement is suspected. The number P is related to kendall's rank correlation coefficient given by:

$$\tau = \frac{4P}{n(n-1)} - 1, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [1]$$

where $-1 < \tau < 1$ ...

In a random series, $E(\tau) = 0$ and $Var(\tau) = \frac{2(2n+5)}{9n(n-1)}$ .... [2]

For large n, $\tau \overset{app}{\sim} N\left[o, \frac{2(2n+5)}{9n(n-1)} \right]$

The decision rule is to accept Ho if $\tau = o$ or reject Ho if

$/\tau/$ is not in the interval $\left[o, \frac{2(2n+5)}{9n(n-1)} \right]$

For the outgoing international telephone calls to Saudi Arabia data the number of observation = 96, and P = 3131, calculated using Table 2 which is given in appendix Á. By substituting the Values for n and P in equations **[1]** and **[2]** we get $\tau = 0.37325$ and $Var(\tau) = 0.0048$. Therefore, reject the null hypothesis, since $\tau = 0.373$ is not in the interval (-0.0048, 0.0048). This shows that the data on the monthly outgoing international telephone calls to Saudi Arabia are not random and hence the analysis of time series is justifiable.

CHAPTER 2
CLASSICAL METHOD

Time series analysis is concerned with changes in a variable over a period of time which are due to the resultant force of four main components, namely trend, seasonal, cyclical and irregular (random) fluctuations. Croxton and Cowden (1965) have suggested that one or two of these may overshadow the others in some series. In the classical method of decomposing a series into its components.

2.1. Model Assumptions

There are two common models:

a. Additive Model

$$Y_t = M_t + S_t + C_t + U_t \ , \ t = 1,2,\ldots, n$$

Where	Y_t	-	the effect at time t
	M_t	-	trend effect at time t
	S_t	-	seasonal effect at time t
	C_t	-	cyclical effect at time t
	U_t	-	Irregular effect at time t

An additive relationship is said to exist between the components of the series if the seasonal variation stays roughly the same size regardless of the trend throughout the series. This model implies the various components operate with equal absolute effect and independent of the general level of the series.

b. Multiplication Model

$$Y_t = M_t \ . \ S_t \ . \ C_t \ . \ U_t$$

Where the terms are as defined above.

If the amplitude of seasonal effect is directly proportional to the mean level, then it is said to be multiplication. This model implies that various components operate proportional to the general level of the series.

2.2. Estimation Of Trend

Trend component is one of the most important element of time series. It is the tendency of the series to increase or decrease over a long period of time. The reason for analysing it could be:

i. To measure it inorder to describe variability due to it, to compare it with other time period and to forecast its future value.

ii. To remove it inorder to study the remaining component free of the trend effect.

There are a number of methods discussed for estimation of trend in different time series literature's (see for example Montgomery and Johnson 1976).

The commonest simple moving average, double moving average, simple exponential smoothing and double exponential smoothing can be applied only for a constant series and for a series which has increasing or decreasing linear trend. However, the time plot (Fig 1) of our observed data suggests a higher degree polynomial model which can be handled by the least squares method.

2.2.1. Least Squares Method

The method of least squares may be used to estimate a linear trend or higher order polynomials. Its mathematical way with the appropriate model of the data can be expressed as first, partially differentiating the sum of squares error with respect to each parameter, then estimating the parameters from the normal equations obtained by solving simultaneously.

Suppose that a time series data is generated by a K^{th} degree polynomial represented by:

$$Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \ldots + \beta_k t^k + U_t$$

Where Bi , i = 0,1,..., k are unknown constant parameters

U_t - is a random error with mean zero and variance σu^2.

Y_t - is the observed value at time t

t - is the independent time point , t = 1,2,...,n

The least squares method is chosen so as to minimize the sum of square errors (SSE)

$$SSE = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2$$

Where $Y_t = \beta_0 + \beta_1 t + \ldots + \beta_k t^k$

and $\hat{Y}_t$ - the estimate of the corresponding observed data, t = 1,...,n

βi - the estimate of the unknown constant parameters,

i = 0,1,...,k

Then, partially differentiating SSE with respect to each parameters, β_i, i,...k and equating the result to zero gives K normal equations,

i.e. $\frac{\partial SSE}{\partial \beta i} = 0$ for i = 0,1,...,k

Then, solving these equations for β_i , i = 1,...k simultaneously gives the required solution.

The hypothesis testing to test the significance of the fitted model parameters is:

Ho : $\beta_i = 0$, i = 0,1,...,k

against H_1 : $\beta_i \neq 0$

The test statistic is:

$$Z = \frac{\beta_i}{\sqrt{Var(\beta_i)}}$$

and the decision rule is to reject Ho if $(Z) > Z$ at $\propto$ level of significance. computer print out also gives the appropriate variables in the equation.

2.2.1.1. Coefficient Of Determination

The quantity:

$$R^2 = 1 - \frac{SSE}{Syy} = \frac{Syy - SSE}{Syy}$$

$o \leq R^2 \leq 1$ is called coefficient of determination.

$$\text{Since } Syy = \sqrt{\frac{\Sigma (Y_t - \bar{Y})^2}{n - 1}}$$

is a measure of the variability in Y_t without considering the effect of the regressor variable and SSE is a measure of the variability in Y_t . R^2 is often called the proportion of variation explained by the regressor. It should be used with caution, since it is always possible to make R^2 larger by adding terms to the model. To correct for this defect we adjust R^2 by taking into account the degrees of freedom, which clearly decrease as new regressors are introduced in the function.

Adjusted coefficient of determination is given by:

$$R^2 = 1 - (1 - R^2) \left(\frac{n - 1}{n - k}\right)$$

Where n = nº of sample observations

k = nº of parameters estimated

It tells us the fraction of total variability in Y_t explained by the relationship between Y and t. The more it approaches to 1 the better it becomes.

CHAPTER 3
BOX - JENKIN'S METHOD

There are different methods that are developed for forecasting purpose which are based on the assumption that random error terms $\{U_t\}$ and observation $\{Y_t\}$ are independent. But in many time series successive observations are highly dependent. Hence forecasting methods based on such models may be inappropriate because they don't take advantage of the dependence in the observations in the most effective way. The models used to take advantage of the dependence between errors are collectively known as Box-Jenkin's model. The detailed theory on this models may be found in Box and Jenkin's (1976). The Box-Jenkin's forecasting models are often reffered to as Auto- Regressive Integrated Moving Average (ARIMA) models. They are specially suited to short term forecasting.

A time series is said to be stationary if there is no systematic change in mean (no trend), if there is no systematic change in variance, and if strictly periodic variation have been removed. In contrast, if the series fluctuates without maintaining a constant mean level or variance and there are more than one mean for the different time intervals of the series and infinite variance then the series is said non stationary.

Nonstationarity can be detected by analysing the auto correlation function of the series from a certain pattern. The efficient methods suggested for stationarizing a nonstationary series is to use differencing until it become stationary. The important tools to study certain properties and functions such as stationarity and nonstationarity of a series are Autocorrelation and partial auto correlation.

3.1. Autocorrelation And Partial Autocorrelation

A set of autocorrelation coefficients graph is used to indicate whether the series being modeled is stationary or not and to identify which type of ARIMA model gives the best representation of an observed time series. The autocorrelation at lag K measures the correlation between two observations that are K lags apart, and is defined as:

$$\rho_k = \frac{E\left((Y_t - E(Y_t))\left(Y_{t+k} - E(Y_{t+k})\right)\right)}{\sqrt{E\left((Y_t - E(Y_t))^2\left(Y_{t+k} - E(Y_{t+k})^2\right)\right)}}$$

The correlation matrix associated with a stationary process for observations $\{Y_i ' S\}$ made at n successive times is:

$$\rho_n = \begin{bmatrix} 1 & \rho_1, & \dots & \rho_{n-1} \\ \rho_1 & \rho_2, & \dots & \rho_{n-2} \\ \rho_2 & \rho_3, & \dots & \rho_{n-3} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \dots & 1 \end{bmatrix}$$

The sample estimate Γ_k of ρ_k is given by: $\Gamma_k = \frac{C_k}{C_o}$

Where $C_k = \frac{1}{N} \sum_{t=1}^{N-k} (Y_t - \bar{Y}) (Y_{t+k} - Y)$, $K = 0,1,2,\dots,$ k is the autocorrelation coefficient at lag K that measures the covariance between two values Y_t and Y_{t+k} separated by K intervals of time. The standard error of the K^{th} sample autocorrelation coefficient is given as:

$$S(\Gamma_k) = \frac{1}{\sqrt{n}} \left(1 - 2 \sum_{j=1}^{k-1} \Gamma_j^2\right)^{½} \quad \text{Where } n = \text{number of observation}$$

The partial autocorrelation at lag k is the correlation between Y_t and Y_{t+k} with the effects of the intervening observations $\{ Y_{t+1}, Y_{t+2}, \dots, Y_{t+k-1}\}$ removed. The K^{th} partial autocorrelation coefficient, ϕ_{kk}, can be estimated by placing ρ_j by its estimate Γ_j, obtaining:

$$\Gamma_j = \phi_{k-1} \Gamma_{j-1} + \phi_{k-2} \Gamma_{j-2} + \dots + \phi_{kk}, \Gamma_{j-k}, \; j = 1,2, \dots, k.$$

The standard error of the estimated partial autocorrelation ϕ_{kk}, is $S(\phi_{kk}) = \frac{1}{\sqrt{n}}$

3.2. Box-Jenkin's Models For Non-Seasonal Processes

Auto Regressive Process (AR)

The process Y_t is said to be an Auto Regressive of order P (AR (p)) for purely random process U_t with mean zero and variance σu^2 if :

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p}$$

Where $\phi_1, \phi_2, \ldots, \phi_p$ are constants

The two very important cases of AR (P) are the first order auto regressive, AR(1) and the second order auto regressive, AR(2).

Moving Average Process (MA)

The process Y_t is said to be moving average of order of q (MA (q)) for purely random process U_t with mean zero and variance σu^2 if :

$$Y_t = \theta_1 U_{t-1} + \theta_2 U_{t-2} + \ldots + \theta_q U_{t-q} + U_t$$

Where θ_i's are constants

Since there are only a finite number of non-zero weights in the MA (q) process, any MA (q) process will be stationary regardless of the values chosen for the weights. The variance of the MA (q) process is:

$$\text{Var}(Y_t) = \sigma u^2 \sum_{i=o}^{q} \theta_i^2$$

The widely applied moving average models for modeling of time series are MA (1) and MA (2) .

Mixed Auto regressive - Moving Average Process (ARMA)

The mixed auto regressive moving average model of order (p,q) are a powerful class of stationary time series models of the form:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \theta_1 U_{t-1} + \theta_2 U_{t-2} + \ldots + \theta_q U_{t-q} + U_t$$

Auto Regressive Integrated Moving Average Process (ARIMA)

The ARIMA process can be extended to the analysis of a wide variety of non stationarity time series by a simple operation called differencing. It is a mechanism by which a non-stationary series can be reduced to a stationary series by taking the difference of successive observations.

3.3. Stages For Model Fitting

The Box-Jenkin's methodology of model fitting involves four stages: model identification, estimation, diagnostic checking and forecasting.

Model Identification

Tentative model identification of an ARIMA time series needs a minimum sample size of once the primary tools, autocorrelation and partial autocorrelation functions, have been computed they may be exhibited on a graph and a tentative model is identified by comparing the observed patterns with the theoretical autocorrelation patterns. These theoretical patterns are as shown in the following table.

Table *

Model	Auto Correlation Function	Partial Auto correlation Function
AR (p)	Tails off	Cuts of after lag p
MA (q)	Cuts off after lag q	Tails off
ARIMA (p,q)	Tails off	Tails off

Tails off: means the function decays in an exponential, sinusoidal or geometric fashion, approximately, with a relatively large number of zero values;

Cuts off: means the function truncates abruptly with only a very few non-zero values.

The standard errors of the sample auto correlation and partial auto correlation functions are useful in identifying non-zero values. As a general rule, we would assume an auto correlation or partial auto correlation coefficient to be zero if the absolute valves of its estimate is less than twice its standard error. So it is useful to plot the limits $\pm$ 2S (ρk) and $\pm$ 2S (ϕ_{kk}).

If the time series is not stationary, the sample auto correlation function will die down extremely slowly, If these functions behave according to the theoretical patterns in Table * (on page 12), we try successive order differencing until stationary behavior is achieved.

Model Estimation

After a tentative time series model has been identified, the least squares estimates of the model parameters are obtained. The usual approach is to apply an iterative search procedure directly to the residual sum of squares function. This can be easily done using a suitable computer program. To test the significance of the parameters approximate t-test can be employed.

Diagnostic Checking

After estimation of model parameters we must examine its adequacy, and if necessary, suggest potential improvements. The two methods suggested by Box and Jenkins (1990) to examine model adequacy are auto correlation check and the portmanteau lack fit test.

a. Autocorrelation Check

This method involves computation of the residuals, say $U_t = Y_t - \hat{Y}_t$, and then estimate and examine their autocorrelation function { $\rho_k (U_t)$ }

If the model is appropriate, then the residual sample autocorrelation function {$\hat{\rho}_k (U_t)$ } should have no structure to identify; that is, the auto correlation should not differ significantly from zero for all lags greater than one. If the form of the model were correct and if we know the true parameter values, then the standard error of the residual autocorrelation's would be $N^{-½}$. However at low lags the standard error of $\sigma_k (U_t)$ may be substantially less than $N^{-½}$, plotting the limits $\pm$ 2 S {$\hat{\rho}_k (U_t)$' directly on the graphs is useful to see which of the $\hat{\rho}_k (U_t)$ 'S are equal to zero.

b. Portmanteau Lack of Fit Test

This method involves obtaining an indication of whether the first K residual auto correlation considered together indicate adequacy of the model rather than consider the $\hat{\rho}_k(U_t)$ individually. The hypothesis to be tested in this case is:

Ho	:	The residuals are random
against H_1	:	The residuals are not random

The test statistic is $Q = (N - d) \sum_{k=1}^{k} \hat{\rho}_k^2 (U_t)$ which is approximately distributed as chi-square with k-p-q degrees of freedom

Where N	=	length of time series
ρ	=	AR order
q	=	MA order
d	=	number of differencing

Thus using the above method we reject Ho if Q exceeds an approximately small upper tail point of the chi-square distribution with k-ρ-q degrees of freedom. However, it was suggested that the test based on Q seems to have rather poor power (see chatfield, 1980).

CHAPTER 4
FORECASTING AND ITS ACCURACY

4.1. Forecasting

Once a time series model has been fit to an observed series, the next step may be to use the model to forecast future valves of the series. Forecasting has a great purpose of reducing risk in decision making process.

Forecasting Using a polynomial Regression Model

Assume that the fitted time series model on the observed time series X_1 , ... , X_n has the form:

$$\hat{X}_t = \hat{\beta}_o + \hat{\beta}_1 t + \ldots + \hat{\beta}_k t^k + \hat{U}_t$$

Where the β_i ' S are constants and U_t is the random component having mean zero and variance σ^2. After the coefficients are estimated from the observed series, we obtained a forecasting equation of the form

$$\hat{X}_{n+\tau} = \hat{\beta}_o + \hat{\beta}_1 (n + \tau) + \ldots + \hat{\beta}_k (n + \tau)^k + \hat{U}_{n+\tau}$$

Which provides a point estimate for period $n + \tau$ where n is the forecasting period and τ is the lead time.

Forecasting Using Box-Jenkin's Method

As discussed in a simplified way by Montgomery and Johnson (1976) once an appropriate model has been fit, it may be used to generate forecasts of future observations that are optimal in a minimum mean square error sense. Let the current period be denoted by n and suppose we wish to forecast the series in period $n+\tau$.

Let $\hat{X}_{n+\tau}(n)$ represent the forecast for period n+τ made at origin n. The forecast is generated by taking expectation at origin n of the model written at time n+τ. Generally, the forecasts for period T+ τ are built successively from the forecasts for period T+ τ are built successively from the forecasts for period. T+1, T+2, ... , T+ τ - 1. In this procedure, the $X_{T+\tau}$ that: have not occurred are replaced by the zero, and $\hat{U}_{\tau+j}$ that have occurred are replaced by the single period forecast error $U_{T-J} = X_{T-j} - \hat{X}_{T-j}$ (T-j-1). In starting the forecasting process it will be necessary to assure that:

$$U_{T-\tau} = o \quad \text{for} \quad T-\tau \leq o$$

4.2. Forecasting Accuracy

As discussed in Makridakis, Wheel wright and McGee (1983), there are various methods for measuring the adequacy of a particular forecasting method for a given data. Out of them the Theil's U-statistic considers the disproportionate cost of large errors and provides a relative basis for comparison with naive methods.

Theil's U-statistic is given by:

$$U = \sqrt{\frac{\sum_{i=1}^{n-1} \left(\frac{F_{i+1} - X_{i+1}}{X_i}\right)^2}{\sum_{i=1}^{n-1} \left(\frac{X_{i+1} - X_i}{X_i}\right)^2}} \qquad (1)$$

Where F denotes forecaster values

X " observed "

n " number of observations

The Theil's u-statistic allows a relative comparison of formal forecasting methods with naive approaches (a method which uses the X_{i-1} observation as a forecast value of X_i) and squares the errors involved so that large errors are given much more weight than small errors.

The ranges of the u-statistic can be summarized as:

u = 1 - the naive method is as good as the forecasting technique being evaluated

u < 1 - the forecasting technique being used is better than the naive method. The smaller the u-statistic, the better the forecasting technique is relative to the naive method

u > 1 - there is no point in using a formal forecasting, since using a naive method will produce better results.

CHAPTER 5
RESULTS

5.1. Estimate Of The Polynomial Regression Model

In the attempt of an appropriate model using SPSS the computer gave the following adjusted coefficients of determination (adjusted R square) from regression analys is of our observed data.

Equations	Adjusted R Square
Linear	0.44079
Quadratic	0.79774
Cubic	0.82239
Quadratic	0.81387
Fifth degree polynomial	0.87642
Sixth degree polynomial	0.87754
Seventh degree polynomial	0.8700

It can be observed from this table that the fitted model with the lightest adjusted R square is the sixth degree polynomial. The following test statistics values were obtained for this from a computer printout. Tests of significance in the β_i declared that only β_4 and β_5 are zero with $p > 0.05$.

Parameter		T Value
β_0	-	5.177
β_1	-	- 2.604
β_2	-	6.571
β_3	-	- 7.538
β_4	-	- 0.899
β_5	-	- 0.505
β_6	-	2.8457

Accordingly the parameter estimates are shown in the following computer print out were obtained for the reduced model.

... Variables in the equation...

Variable	B	SEB	T
Time	- 512.693211	196.870618	-2.604
$(Time)^2$	43.334591	6.594793	6.571
$(Time)^3$	- 0.498379	0.066116	-7.538
$(Time)^6$	1.35654E -07	2.0857E-08	6.504
Constant	8398.576248	1622.244497	5.177

... Variables in the equation...

Variable	Beta in partial mine Toiler		T
$(Time)^4$	-10.291052	5.777E-06	- 0.899
$(Time)^2$	-4.711305	1.494E-05	- 0.505

Therefore, the fitted model becomes:

$$\mathbf{Y} = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_6 t^6$$

$$= 8398.576 + (-512.693)t + (43.33459)t^2 + (-0.498379)t^3 + 1.35654E - 07t^6$$

5.2. Estimated Box - Jenkin's Model

Following the steps discussed in section 3.3 for selecting a tentative model for the series on the outgoing international telephone calls to Saudi Arabia, the following results were obtained.

Model Identification

The main concern of the identification process is selection of a tentative model out of the possible models which best represent the series. Fig. 2 of appendix B shows the graph of the auto correlation with two standard error limits for the original data. From this we see that the auto correlation exhibit a very slowly declining scheme. this indicates that the original series is not stationary, therefore, it needs differencing. the auto correlation and partial auto correlation functions for the non seasonal component of the first differenced series is given in figures 4 and 5 of appendix B. From Fig. 4 we can observe that the function decays with a relatively large number of zero values after lag 1 and from fig 5 we note that the function truncates abruptly with only a very few non-zero values after lag 1. Therefore the tentative model seems to be ARIMA (1,1,0) and it is of the form:

$$\hat{Y}_t = (1 + \hat{\phi}_1) \hat{Y}_{t-1} - \hat{\phi}_1 \hat{Y}_{t-2} + \hat{U}_t \quad \cdots\cdots (3)$$

Estimation

Using SPSS the following estimate of the parameter and related statistics were obtained after 13 iterations.

Parameter	Lag	Estimate	Std-error	t - Value
AR (ϕ_1)	1	- 0.53046	0.0874	- 6.0688

Since the + - ratio for the parameter is large enough we reject the null hypothesis of zero for value for ϕ_1 - Therefore, the fitted model after substituting the calculated parameters in equation (3) becomes:

$$\hat{Y}_t = 0.46954 \hat{Y}_{t-1} + 0.53046 \hat{Y}_{t-2} + \hat{U}_t$$

Diagnostic Checking

The auto correlation of the residual series after fitting the above model is given in Fig. 6 of appendix B. The auto correlation function values that lie inside the two standard error for all lags, imply that they can be assume to be zero. Therefore, we conclude that the fitted model is adequate.

5.3. Forecasting And Accuracy

Fore Casts Using the Polynomial Regression

Based on the estimated equation (equation 2)

$$\hat{Y}_t = 8398.576 - 512.693211\, t + 43.33459\, t^2 - 0.49837\, t^3 + 1.35654E\text{-}07\ t^6$$

The forecasts using the above equation are displayed in table 3.

Fore Casts Using The ARIMA (1,1,0) Model

The computer print out gave forecasts with origin at lag 96 (Dec. 1996) and with 95% confidence limit and standard error. Table 4 shows the forecasts with origin at Dec. 1996 and 95% confidence limit of ARIMA (1,1,0). These forecaster valves show the outgoing international telephone calls to Saudi Arabia will decrease even for the coming year.

Forecasting Accuracy

When we turn to measure the accuracy of each forecasting method i.e. the polynomial regression and the ARIMA (1,1,0) model we get the U-statistic values 0.7492 and 0.7202 respectively using equation 1.

Since the U valve of the ARIMA (1,1,0) model is less than that of the polynomial regression model it seems that the ARIMA (1,1,0) forecasts are better than the regression forecasts for this series.

CHAPTER 6
SUMMARY AND CONCLUSION

The classical and Box-Jenkin's methodologies were employed on the data of the outgoing international telephone calls to Saudi Arabia from 1989 up to 1996. In this section the results are summarized. In chapter 1 introduction of the research project and its objectives were presented. The historical development of international communication through the ETC with the outside world was introduced and source and collection of data were also discussed.

The nature of the data were examined through its time plot and test of randomness using the rank test. The time plot shows an upward trend from January 1989 up to December 1992 and then a marked decreasing trend thereafter. On the other hand the rank test shows that the series is non random.

In chapter 2 based on the evidence obtained from the time plot the least square method was discussed as a means to choose an appropriate equation for the commonest simple moving average, double moving average, simple exponential and double exponential methods cannot be applied because our series is non-linear. The least square method suggested fitting a sixth degree polynomial.

In chapter 3 the main steps to be taken in analysing a time series using Box-Jenkin's method are discussed. These are identifying an appropriate ARIMA model by making use of the auto correlation and partial auto correlation functions. Moreover the method for estimating parameter for a chosen model and applying a diagnostic check so as to know the adequacy of the model chosen were discussed. The stationarity condition of the series was checked by looking at the observed pattern of the auto correlation for the actual series. A slowly decaying scheme of the auto correlation (Fig. 2) showed that the series is non-stationary and needs differencing to achieve stationarity and there by identify an appropriate model. Accordingly a tentative model ARIMA (1,1,0) was identified after the first degree non-seasonal differencing had been applied.

Chapter 4 focused on forecasting of future values based on the polynomial regression model and the ARIMA (1,1,0) model. A discussion was also made on how the accuracy of a particular forecasting method can be measured using the theil's u-statistic.

Then the best forecasting method was identified. It appears that the ARIMA (1,1,0) model is better than the polynomial regression. Model in generating forecasts for the series although their U-statistic values show no marked difference (the U-statistic values are 0.74 and 0.72 for the polynomial regression and the ARIMA (1,1,0) models respectively.)

As the graph of the autocorrelation and the time plot of the observed data suggest the absence of a significant seasonality pattern no attempt has been made to estimate the seasonality components. No attempt has been made also to estimate the cyclical component as it is impossible to pick out useful information on cycles without data from several cycles.

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APPENDIX

APPENDIX A

List Of Tables

APPENDIX B

List of Figures

Table 1. The Original Data Of The Outgoing International Telephone Calls To Saudi Arabia

YEAR	1989	1990	1991	1992	1993	1994	1995	1996
MONTH								
JAN	5834	12385	22026	17711	35647	35729	27407	25332
FEB	5627	5179	14726	25743	39119	30971	25220	23289
MAR	6159	10570	17618	20848	34208	31485	27362	26588
APR	6731	8416	17876	22732	32731	28991	25839	24588
MAY	6400	18331	16294	25246	34166	29991	24342	23889
JUN	6227	5293	10335	23432	31465	26765	24373	23836
JUL	9106	11784	1228	27248	34608	30457	26136	22779
AUG	9636	15391	15446	30147	31577	25462	24843	22439
SEP	6845	11051	15521	36908	33309	27728	24465	20782
OCT	5303	14891	18880	30647	37944	27169	25874	23351
NOV	8624	16589	20366	31292	36705	27164	24097	20690
DEC	7044	16606	20825	32862	33849	27437	24772	21357

TABLE 2. VALUES FOUND FOR RANK TEST

MONTH / YEAR	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
1998	--	0	2	3	3	3	6	7	6	0	8	8
1990	12	0	13	10	16	1	16	18	16	19	21	22
1991	24	19	24	25	22	15	0	24	25	32	33	34
1992	30	37	36	38	39	39	42	43	44	44	45	46
1993	47	49	47	46	48	46	51	47	50	56	55	51
1994	56	45	48	43	44	42	47	41	45	44	44	47
1995	47	40	48	44	40	41	47	42	42	49	40	44
1996	48	39	55	45	41	41	39	38	35	43	35	39

Table 3. Forecasted Values Using Least Square Method

YEAR 1997

MONTH	FORECASTED VALUE
JANUARY	8398.576
FEBRUARY	8398.576
MARCH	8398.576
APRIL	8398.576
MAY	8398.576
JUNE	8398.576
JULLY	8398.576
AUGUST	8398.576
SEPTEMBER	8398.576
OCTOBER	8398.576
NOVEMBER	8398.576
DECEMBER	8398.576

YEAR 1998

MONTH	FORECASTED VALUE
JANUARY	8398.576
FEBRUARY	8398.576
MARCH	8398.576
APRIL	8398.576
MAY	8398.576
JUNE	8398.576
JULLY	8398.576
AUGUST	8398.576
SEPTEMBER	8398.576
OCTOBER	8398.576
NOVEMBER	8398.576
DECEMBER	8398.576

Table 4. Forecasted Values using Box-Jenkins method

YEAR 1997

MONTH	FORECASTED VALUE
JANUARY	21252.51
FEBRUARY	21557.27
MARCH	21644.93
APRIL	21847.76
MAY	21989.5
JUNE	22163.64
JULLY	22320.59
AUGUST	22486.66
SEPTEMBER	22647.9
OCTOBER	22811.7
NOVEMBER	22974.13
DECEMBER	23137.29

YEAR 1998

MONTH	Forecasted Value
JANUARY	23300.07
FEBRUARY	23463.05
MARCH	23625.93
APRIL	23788.86
MAY	23951.76
JUNE	24114.67
JULLY	24277.58
AUGUST	24440.49
SEPTEMBER	24603.4
OCTOBER	24766.31
NOVEMBER	24929.22
DECEMBER	25092.13

Table 5. Estimated and Residual Values Using Least Square Method

YEAR	TIMI	MONTH	CALLS	ESTIMATE(Y)	RESIDUAL
1989	1	JANUARY	5834	7542.534987	-1708.535
	2	FEBRUARY	5627	7237.045786	-1610.0458
	3	MARCH	6159	7009.255472	-850.25547
	4	APRIL	6731	6856.174565	-125.17456
	5	MAY	6400	6774.814383	-374.81438
	6	JUNE	6227	6762.187483	-535.18748
	7	JULY	9106	6815.308193	2290.69181
	8	AUGUST	9636	6931.193253	2704.80675
	9	SEPTEMBER	6845	7106.862544	-261.86254
	10	OCTOBER	5303	7339.339918	-2036.3399
	11	NOVEMBER	8624	7625.654129	998.345871
	12	DECEMBER	7044	7962.839853	-918.83985
1990	13	JANUARY	12385	8347.938818	4037.06118
	14	FEBRUARY	5179	8778.001019	-3599.001
	15	MARCH	10570	9250.08604	1319.91396
	16	APRIL	8416	9761.264471	-1345.2645
	17	MAY	18331	10308.61942	8022.38058
	18	JUNE	5293	10889.24811	-5596.2481
	19	JULY	11784	11500.26364	283.736364
	20	AUGUST	15391	12138.79671	3252.20329
	21	SEPTEMBER	11051	12801.9976	-1750.9976
	22	OCTOBER	14891	13487.03813	1403.96187
	23	NOVEMBER	16589	14191.11378	2397.88622
	24	DECEMBER	16606	14911.44588	1694.55412
1991	25	JANUARY	22026	15645.28389	6380.71611
	26	FEBRUARY	14726	16389.90784	-1663.9078
	27	MARCH	17618	17142.63076	475.369241
	28	APRIL	17876	17900.8013	-24.801304
	29	MAY	16294	18661.80644	-2367.8064
	30	JUNE	10335	19423.0742	-9088.0742

Table 5. continued ...

	31	JULY	1228	20182.07662	-18954.077
	32	AUGUST	15446	20936.33265	-5490.3327
	33	SEPTEMBER	15521	21683.41128	-6162.4113
	34	OCTOBER	18880	22420.9347	-3540.9347
	35	NOVEMBER	20366	23146.58156	-2780.5816
	36	DECEMBER	20825	23858.09034	-3033.0903
1992	37	JANUARY	17711	24553.26287	-6842.2629
	38	FEBRUARY	25743	25229.9678	513.032203
	39	MARCH	20848	25886.14434	-5038.1443
	40	APRIL	22732	26519.80601	-3787.806
	41	MAY	25246	27129.04445	-1883.0445
	42	JUNE	23432	27712.03343	-4280.0334
	43	JULY	27248	28267.03288	-1019.0329
	44	AUGUST	30147	28792.39303	1354.60697
	45	SEPTEMBER	36908	29286.55869	7621.44131
	46	OCTOBER	30647	29748.07356	898.926441
	47	NOVEMBER	31292	30175.58471	1116.41529
	48	DECEMBER	32862	30567.84709	2294.15291
1993	49	JANUARY	35647	30923.7282	4723.2718
	50	FEBRUARY	39119	31242.21278	7876.78722
	51	MARCH	34208	31522.40771	2685.59229
	52	APRIL	32731	31763.54688	967.453118
	53	MAY	34166	31964.99626	2201.00374
	54	JUNE	31465	32126.25902	-661.25902
	55	JULY	34608	32246.98073	2361.01927
	56	AUGUST	31577	32326.95473	-749.95473
	57	SEPTEMBER	33309	32366.1275	942.8725
	58	OCTOBER	37944	32364.60422	5579.39578
	59	NOVEMBER	36705	32322.65436	4382.34564
	60	DECEMBER	33849	32240.71741	1608.28259
1994	61	JANUARY	35729	32119.40868	3609.59132

Table 5. continued ...

	62	FEBRUARY	30971	31959.5252	-988.5252
	63	MARCH	31485	31762.05175	-277.05175
	64	APRIL	28991	31528.16697	-2537.167
	65	MAY	29991	31259.24953	-1268.2495
	66	JUNE	26765	30956.88444	-4191.8844
	67	JULY	30457	30622.86946	-165.86946
	68	AUGUST	25462	30259.2216	-4797.2216
	69	SEPTEMBER	27728	29868.18368	-2140.1837
	70	OCTOBER	27169	29452.23105	-2283.2311
	71	NOVEMBER	27164	29014.07839	-1850.0784
	72	DECEMBER	27437	28556.68655	-1119.6866
1995	73	JANUARY	27407	28083.26956	-676.26956
	74	FEBRUARY	25220	27597.30173	-2377.3017
	75	MARCH	27362	27102.5248	259.475205
	76	APRIL	25839	26602.9552	-763.9552
	77	MAY	24342	26102.8915	-1760.8915
	78	JUNE	24373	25606.92178	-1233.9218
	79	JULY	26136	25119.9313	1016.0687
	80	AUGUST	24843	24647.11007	195.889929
	81	SEPTEMBER	24465	24193.9607	271.039297
	82	OCTOBER	25874	23766.30622	2107.69378
	83	NOVEMBER	24097	23370.29802	726.701983
	84	DECEMBER	24772	23012.42396	1759.57604
1996	85	JANUARY	25332	22699.51649	2632.48351
	86	FEBRUARY	23289	22438.76091	850.239085
	87	MARCH	26588	22237.70374	4350.29626
	88	APRIL	24588	22104.26113	2483.73887
	89	MAY	23889	22046.72742	1842.27258
	90	JUNE	23836	22073.78384	1762.21616
	91	JULY	22779	22194.50714	584.49286
	92	AUGUST	22439	22418.37854	20.6214574
	93	SEPTEMBER	20782	22755.29261	-1973.2926
	94	OCTOBER	23351	23215.56629	135.433714
	95	NOVEMBER	20690	23809.94807	-3119.9481
	96	DECEMBER	21357	8398.57	12958.43

Table 6 Estimated And Residual Value Using Box- Jenkin's Method

YEAR	TIME	MONTH	CALLS	ESTIMATE	RESIDUAL
1989	1	JANUARY	5834	.	.
	2	FEBRUARY	5627	5996.91046	-369.91046
	3	MARCH	6159	5986.13216	172.86784
	4	APRIL	6731	6126.12422	604.87578
	5	MAY	6400	6676.90592	-276.90592
	6	JUNE	6227	6824.90886	-597.90886
	7	JULLY	9106	6568.09661	2537.90339
	8	AUGUST	9636	7828.14094	1807.85906
	9	SEPTEMBER	6845	9604.18513	-2759.1851
	10	OCTOBER	5303	8574.83382	-3271.8338
	11	NOVEMBER	8624	6370.29265	2253.70735
	12	DECEMBER	7044	7111.67881	-67.67881
1990	13	JANUARY	12385	8131.45003	4253.54997
	14	FEBRUARY	5179	9801.15507	-4622.1551
	15	MARCH	10570	9250.80279	1319.19721
	16	APRIL	8416	7959.63221	456.36779
	17	MAY	18331	9807.93252	8523.06748
	18	JUNE	5293	13320.8434	-8027.8434
	19	JULLY	11784	12458.4298	-674.42975
	20	AUGUST	15391	8590.12918	6800.87082
	21	SEPTEMBER	11051	13726.968	-2675.968
	22	OCTOBER	14891	13602.5122	1288.48783
	23	NOVEMBER	16589	13103.3715	3485.62853
	24	DECEMBER	16606	15937.611	668.38899
1991	25	JANUARY	22026	16846.3097	5179.69028
	26	FEBRUARY	14726	19400.2489	-4674.2489
	27	MARCH	17618	18847.6658	-1229.6658
	28	APRIL	17876	16333.245	1542.75501
	29	MAY	16294	17988.4695	-1694.4695
	30	JUNE	10335	17382.5109	-7047.5109

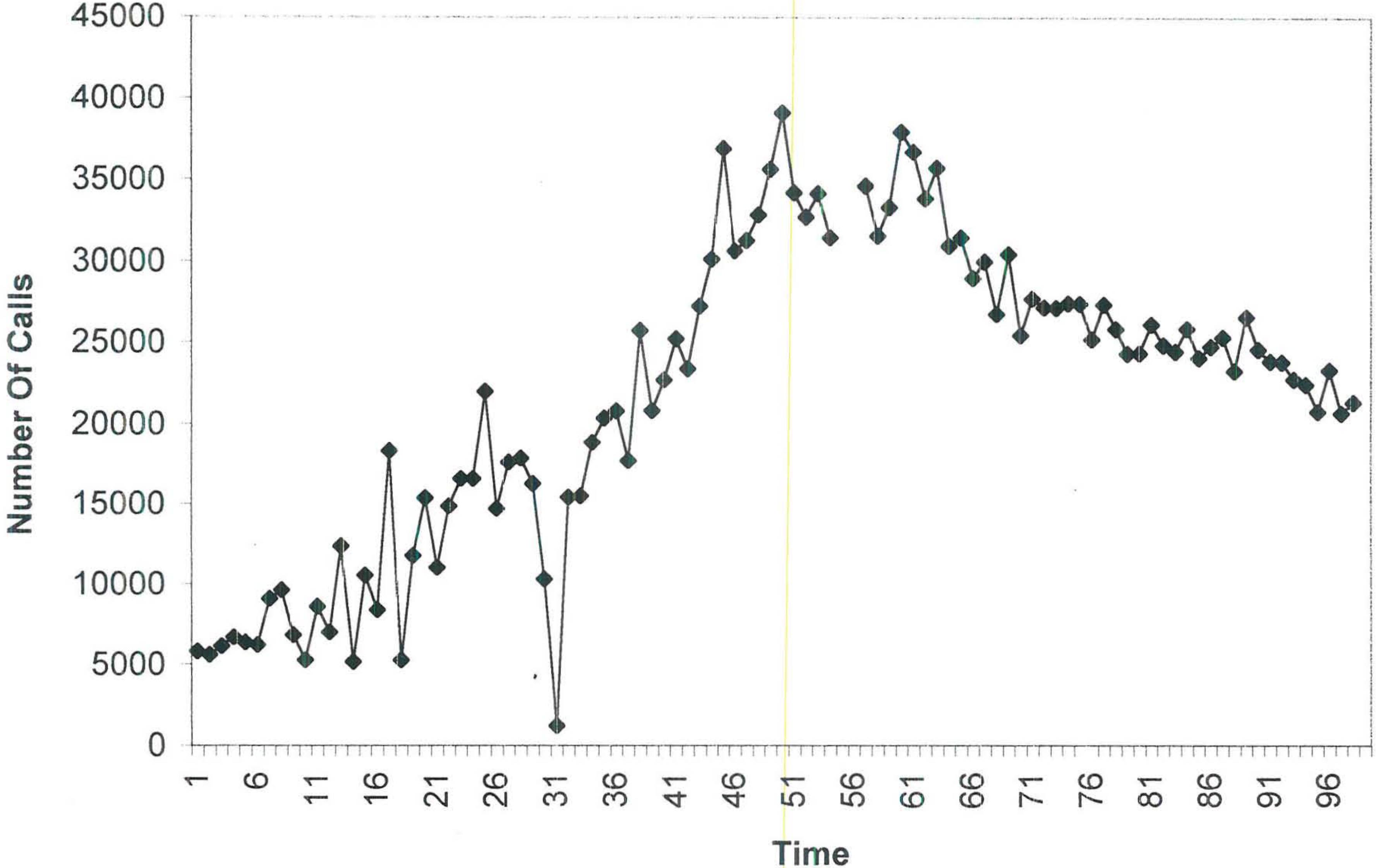
Table 6 Continued ...

	31	JULLY	12228	13745.3225	-1517.3225
	32	AUGUST	15446	11473.1718	3972.82817
	33	SEPTEMBER	15521	13988.3159	1532.68409
	34	OCTOBER	18880	15730.5432	3149.4568
	35	NOVEMBER	20366	17347.5214	3018.47857
	36	DECEMBER	20825	19827.068	997.93205
1992	37	JANUARY	17711	20830.8476	-3119.8476
	38	FEBRUARY	25743	19612.1715	6130.82848
	39	MARCH	20848	21731.6945	-883.69448
	40	APRIL	22732	23693.916	-961.91597
	41	MAY	25246	21981.946	3264.05405
	42	JUNE	23432	24161.7579	-729.75785
	43	JULLY	27248	24643.577	2604.42296
	44	AUGUST	30147	25473.1025	4673.89755
	45	SEPTEMBER	36908	28858.5318	8049.46821
	46	OCTOBER	30647	33570.9057	-2923.9057
	47	NOVEMBER	31292	34217.5206	-2925.5206
	48	DECEMBER	32862	31199.1825	1662.81746
1993	49	JANUARY	35647	32278.5095	3368.49046
	50	FEBRUARY	39119	34419.0039	4699.99608
	51	MARCH	34208	37526.5798	-3318.5798
	52	APRIL	32731	37062.4033	-4331.4033
	53	MAY	34166	33763.8129	402.18707
	54	JUNE	31465	33654.1213	-2189.1213
	55	JULLY	34608	33147.0927	1460.90734
	56	AUGUST	31577	33190.1002	-1613.1002
	57	SEPTEMBER	33309	33434.1436	-125.14357
	58	OCTOBER	37944	32639.5755	5304.42454
	59	NOVEMBER	36705	35734.6579	970.34208
	60	DECEMBER	33849	37611.5641	-3762.5641

Table 6 Continued ...

1994	61	JANUARY	35729	35613.3135	115.68646
	62	FEBRUARY	30971	34981.0678	-4010.0678
	63	MARCH	31485	33744.2433	-2259.2433
	64	APRIL	28991	31461.6725	-2470.6725
	65	MAY	29991	30563.288	-572.288
	66	JUNE	26765	29709.8702	-2944.8702
	67	JULLY	30457	28725.5827	1731.41726
	68	AUGUST	25462	28747.8792	-3285.8792
	69	SEPTEMBER	27728	28360.9617	-632.9617
	70	OCTOBER	27169	26775.3113	393.68874
	71	NOVEMBER	27164	27714.8531	-550.85313
	72	DECEMBER	27437	27415.9798	21.02021
1995	73	JANUARY	27407	27541.5127	-134.51266
	74	FEBRUARY	25220	27672.2412	-2452.2412
	75	MARCH	27362	26629.4376	732.56239
	76	APRIL	25839	26475.088	-636.08797
	77	MAY	24342	26896.214	-2554.214
	78	JUNE	24373	25385.4221	-1012.4221
	79	JULLY	26136	24605.8833	1530.11668
	80	AUGUST	24843	25450.1313	-607.13128
	81	SEPTEMBER	24465	25778.2088	-1313.2088
	82	OCTOBER	25874	24914.8404	959.15964
	83	NOVEMBER	24097	25375.9132	-1278.9132
	84	DECEMBER	24772	25288.9501	-516.95012
1996	85	JANUARY	25332	24663.2688	668.73118
	86	FEBRUARY	23289	25284.2714	-1995.2714
	87	MARCH	26588	24622.0518	1965.94824
	88	APRIL	24588	25087.3489	-499.34887
	89	MAY	23889	25898.2421	-2009.2421
	90	JUNE	23836	24509.1172	-673.11715
	91	JULLY	22779	24113.4417	-1334.4417
	92	AUGUST	22439	23589.0209	-1150.0209
	93	SEPTEMBER	20782	22868.683	-2086.683
	94	OCTOBER	23351	21910.2952	1440.70476
	95	NOVEMBER	20690	22237.5827	-1547.5827
	96	DECEMBER	21357	22350.8744	-993.87437

Time Plot Of Original Data



```
        Auto-    Stand.
Lag     Corr.    Err. -1   -.75   -.5 -.25   0    .25   .5    .75   1       Box-Ljung   Prob.
                     š"""""."""""."""""."""""."""""."""""."""""."""""›
  1     .913    .100                     .    "***.**************            82.524     .000
  2     .901    .100                     .    "***.**************           163.758     .000
  3     .865    .099                     .    "***.*************            239.446     .000
  4     .839    .099                     .    "***.*************            311.444     .000
  5     .800    .098                     .    "***.************             377.539     .000
  6     .764    .098                     .    "***.***********              438.618     .000
  7     .728    .097                     .    "***.***********              494.612     .000
  8     .706    .097                     .    "***.**********               547.856     .000
  9     .670    .096                     .    "***.*********                596.398     .000
 10     .626    .096                     .    "***.*********                639.306     .000
 11     .580    .095                     .    "***.********                 676.521     .000
 12     .550    .094                     .    "***.*******                  710.420     .000
 13     .500    .094                     .    "***.******                   738.718     .000
 14     .450    .093                     .    "***.*****                    761.954     .000
 15     .410    .093                     .    "***.****                     781.448     .000
 16     .369    .092                     .    "***.***                      797.431     .000
 17     .339    .092                     .    "***.***                      811.087     .000
 18     .286    .091                     .    "***.**                       820.944     .000
 19     .241    .090                     .    "***.*                        828.050     .000
 20     .216    .090                     .    "****                         833.815     .000
 21     .187    .089                     .    "****                         838.185     .000
 22     .145    .089                     .    "***.                         840.849     .000
 23     .116    .088                     .    "** .                         842.593     .000
 24     .082    .087                      .   "**.                          843.481     .000
 25     .069    .087                      .   "* .                          844.120     .000
 26     .027    .086                      .   "* .                          844.218     .000
 27    -.007    .086                      .   *  .                          844.225     .000
 28    -.034    .085                      .  *"  .                          844.387     .000
 29    -.073    .084                      .  *"  .                          845.138     .000
 30    -.113    .084                       .**"  .                          846.949     .000
 31    -.160    .083                       ***"  .                          850.637     .000
 32    -.190    .082                      *.**"  .                          855.930     .000
 33    -.219    .082                      *.**"  .                          863.065     .000
 34    -.250    .081                     **.**"  .                          872.561     .000
 35    -.278    .081                    ***.**"  .                          884.521     .000
 36    -.299    .080                    ***.**"  .                          898.552     .000
 37    -.330    .079                   ****.**"  .                          915.917     .000
 38    -.340    .079                   ****.**"  .                          934.706     .000
 39    -.376    .078                  *****.**"  .                          958.053     .000
 40    -.397    .077                  *****.**"  .                          984.595     .000

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  96     Computable first lags:  95
```

Fig.2 Autocorrelations of original data

```
Pr-Aut-  Stand.
Lag  Corr.   Err. -1   -.75   -.5 -.25   0   .25  .5   .75   1
                  š"""""."""""."""""."""""."""""."""""."""""."""""›
  1   .913   .102                   .    "***.*************
  2   .406   .102                   .    "***.****
  3   .042   .102                   .    "*   .
  4   .009   .102                   .     *   .
  5  -.080   .102                   .  **"    .
  6  -.061   .102                   .   *"    .
  7  -.032   .102                   .   *"    .
  8   .078   .102                   .    "**  .
  9  -.017   .102                   .     *   .
 10  -.129   .102                   .***"     .
 11  -.127   .102                   .***"     .
 12   .036   .102                   .    "*   .
 13  -.073   .102                   .   *"    .
 14  -.104   .102                   .  **"    .
 15   .017   .102                   .     *   .
 16   .000   .102                   .     *   .
 17   .055   .102                   .    "*   .
 18  -.108   .102                   .  **"    .
 19  -.082   .102                   .  **"    .
 20   .100   .102                   .    "**  .
 21   .068   .102                   .    "*   .
 22  -.064   .102                   .   *"    .
 23   .014   .102                   .     *   .
 24  -.041   .102                   .   *"    .
 25   .072   .102                   .    "*   .
 26  -.100   .102                   .  **"    .
 27  -.091   .102                   .  **"    .
 28   .020   .102                   .     *   .
 29  -.151   .102                   .***"     .
 30  -.111   .102                   .  **"    .
 31  -.084   .102                   .  **"    .
 32   .003   .102                   .     *   .
 33   .008   .102                   .     *   .
 34  -.026   .102                   .   *"    .
 35   .023   .102                   .     *   .
 36   .072   .102                   .    "*   .
 37  -.138   .102                   .***"     .
 38   .073   .102                   .    "*   .
 39   .009   .102                   .     *   .
 40  -.064   .102                   .   *"    .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  96     Computable first lags:  95
```

Fig. 3 Partial Autocorrelations of original data

Transformations: difference (1)

Lag	Auto-Corr.	Stand. Err.	-1 -.75 -.5 -.25 0 .25 .5 .75 1	Box-Ljung	Prob.
1	-.536	.101	*******.***" .	28.144	.000
2	.176	.100	. "****	31.199	.000
3	-.066	.100	. *" .	31.638	.000
4	.090	.099	. "** .	32.457	.000
5	-.035	.099	. *" .	32.586	.000
6	.049	.098	. "* .	32.833	.000
7	-.097	.098	. **" .	33.817	.000
8	.054	.097	. "* .	34.131	.000
9	.032	.097	. "* .	34.244	.000
10	.062	.096	. "* .	34.658	.000
11	-.142	.095	.***" .	36.868	.000
12	.219	.095	. "****	42.216	.000
13	-.111	.094	. **" .	43.601	.000
14	.009	.094	. * .	43.610	.000
15	-.029	.093	. *" .	43.706	.000
16	.061	.093	. "* .	44.140	.000
17	-.026	.092	. *" .	44.220	.000
18	.031	.091	. "* .	44.338	.001
19	-.087	.091	. **" .	45.261	.001
20	-.036	.090	. *" .	45.421	.001
21	.142	.090	. "***.	47.936	.001
22	-.074	.089	. *" .	48.625	.001
23	.038	.088	. "* .	48.810	.001
24	-.073	.088	. *" .	49.501	.002
25	.103	.087	. "**.	50.893	.002
26	-.022	.087	. * .	50.958	.002
27	-.043	.086	. *" .	51.207	.003
28	.055	.085	. "* .	51.621	.004
29	-.080	.085	.**" .	52.512	.005
30	.074	.084	. "* .	53.283	.006
31	-.074	.083	. *" .	54.074	.006
32	-.011	.083	. * .	54.092	.009
33	.065	.082	. "* .	54.726	.010
34	-.005	.081	. * .	54.729	.014
35	-.048	.081	. *" .	55.083	.017
36	.024	.080	. * .	55.173	.021
37	-.033	.079	. *" .	55.342	.027
38	.108	.079	. "**.	57.213	.023
39	-.080	.078	.**" .	58.256	.024
40	-.036	.077	. *" .	58.473	.030

Plot Symbols: Autocorrelations * Two Standard Error Limits .

Total cases: 96 Computable first lags after differencing: 94

Fig. Autocorrelation of non-seasonal Differenced Series (d = 1)

```
Transformations:  difference (1)

   Pr-Aut- Stand.
Lag Corr.    Err. -1   -.75   -.5 -.25   0   .25  .5   .75   1
                  š""""."""".""""."""".""""."""".""""."""".""""›
  1  -.536   .103          *******.***"   .
  2  -.156   .103                 .***"   .
  3  -.059   .103                 .  *"   .
  4   .073   .103                 .   "*  .
  5   .073   .103                 .   "*  .
  6   .089   .103                 .   "** .
  7  -.057   .103                 .  *"   .
  8  -.061   .103                 .  *"   .
  9   .044   .103                 .   "*  .
 10   .164   .103                 .   "***.
 11  -.031   .103                 .  *"   .
 12   .160   .103                 .   "***.
 13   .107   .103                 .   "** .
 14  -.048   .103                 .  *"   .
 15  -.099   .103                 . **"   .
 16  -.004   .103                 .   *   .
 17   .038   .103                 .   "*  .
 18   .039   .103                 .   "*  .
 19  -.055   .103                 .  *"   .
 20  -.195   .103                 ****"   .
 21  -.003   .103                 .   *   .
 22   .007   .103                 .   *   .
 23   .112   .103                 .   "** .
 24  -.015   .103                 .   *   .
 25   .040   .103                 .   "*  .
 26   .058   .103                 .   "*  .
 27  -.056   .103                 .  *"   .
 28   .034   .103                 .   "*  .
 29  -.021   .103                 .   *   .
 30   .021   .103                 .   *   .
 31  -.043   .103                 .  *"   .
 32  -.064   .103                 .  *"   .
 33  -.027   .103                 .  *"   .
 34   .032   .103                 .   "*  .
 35  -.036   .103                 .  *"   .
 36   .027   .103                 .   "*  .
 37  -.064   .103                 .  *"   .
 38   .048   .103                 .   "*  .
 39   .046   .103                 .   "*  .
 40  -.069   .103                 .  *"   .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  96     Computable first lags after differencing:  94
```

Fig. **Partial Autocorrelation of non-seasonal Differenced Series (d =1)**

Lag	Auto-Corr.	Stand. Err.	-1 -.75 -.5 -.25 0 .25 .5 .75 1	Box-Ljung	Prob.
1	-.088	.101	. **" .	.760	.383
2	-.132	.100	.***" .	2.482	.289
3	.078	.100	. "** .	3.093	.378
4	.087	.099	. "** .	3.853	.426
5	.040	.099	. "* .	4.017	.547
6	-.010	.098	. * .	4.028	.673
7	-.097	.098	. **" .	5.017	.658
8	.050	.097	. "* .	5.287	.727
9	.145	.097	. "***.	7.538	.581
10	.029	.096	. "* .	7.630	.665
11	-.045	.095	. *" .	7.853	.726
12	.206	.095	. "****	12.570	.401
13	-.029	.094	. *" .	12.667	.474
14	-.088	.094	. **" .	13.541	.484
15	.000	.093	. * .	13.541	.561
16	.069	.093	. "* .	14.098	.591
17	.022	.092	. * .	14.156	.656
18	-.028	.091	. *" .	14.252	.713
19	-.160	.091	.***" .	17.354	.566
20	-.023	.090	. * .	17.420	.626
21	.174	.090	. "***.	21.186	.448
22	.001	.089	. * .	21.186	.509
23	-.040	.088	. *" .	21.390	.557
24	-.026	.088	. *" .	21.480	.610
25	.114	.087	. "**.	23.194	.566
26	.005	.087	. * .	23.198	.622
27	-.053	.086	. *" .	23.575	.654
28	.007	.085	. * .	23.583	.703
29	-.047	.085	. *" .	23.889	.734
30	.017	.084	. * .	23.932	.775
31	-.086	.083	.**" .	24.991	.768
32	-.027	.083	. *" .	25.095	.802
33	.106	.082	. "**.	26.777	.769
34	.005	.081	. * .	26.781	.806
35	-.071	.081	. *" .	27.545	.811
36	-.017	.080	. * .	27.589	.842
37	.041	.079	. "* .	27.851	.862
38	.110	.079	. "**.	29.794	.827
39	-.091	.078	.**" .	31.172	.810
40	-.146	.077	***" .	34.740	.706

Plot Symbols: Autocorrelations * Two Standard Error Limits .

Total cases: 96 Computable first lags: 94

Fig.6 Autocorrelations of Error for calls from ARIMA(1,1,0)