ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

INSTITUTE OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF CIVIL AND ENVIROMENTAL ENGINEERING

Analysis Aid Preparation for Self Supporting Fixed Ended Helical Stairs

A thesis submitted to the school of Graduate Studies in Partial fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering (Structures)

By
Yonathan Gizaw

Advisor: **Dr. Bedilu Habte**

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Approved by Board of Examiners

Advisor	Signature	Date
Examiner	Signature	Date
Examiner	Signature	Date
Chairman	Signature	Date

Acknowledgment

My at most gratitude is to “THE ALMIGHTY” for his unquestionable supremacy and his help all along my journey. Secondly, I’m grateful to my advisor Dr. Bedilu Habte for his guidance, follow up and motivation. And thirdly, I won’t pass this chance with out sincerely thanking my father Gizaw for his endless support and understanding together with all of those who stood by my side to see through the completion of this research.

Abstract

As amazing its appearance is, spiral stair has complex geometric configuration. As a result of this complexity, its analysis is challenging. And our Building Code of Standard doesn't have analysis aid for these special structures, like the ones we have for slabs. Hence, the purpose of this study is to come up with a convenient and user friendly analysis tables and charts for the analysis of spiral stairs. This research follows analytical and finite element approach interdependently. The out come of the research is perceived by accompanying the comparative assessment of the two approaches.

The analysis of spiral stairs is done in different approaches. First detail formulation is derived for the stair using strain energy concept considering the stair as twisting girder and in plane curved beam. Secondly, modeling and analysis is carried out using finite element technique as 3D twisting beam and 3D twisting shell stair slab separately. Then, graphical and tabular comparison is done between all approaches. In addition, modification factors are formulated considering variation of internal actions along stair width. More over, valid width is demarcated to limit the acceptable width for the applicability of centerline model for stain energy solution of helical stairs.

Hence, the analysis tables and charts that are annexed in the write up together with the modification factors presented are very crucial tools for the analysis of these spectacular complex structures.

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List of Tables and Figures
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List of notations

EI: Flexural rigidity

GJ: Torsional rigidity

H: Horizontal shear force at mid-span (redundant)

M: Vertical moment at mid-span (redundant)

Mh: Lateral moment

Mv: Vertical moment

N: Axial force

R: Center line radius on horizontal projection

T: Torsion

U: Strain energy

Vh: Lateral shear force

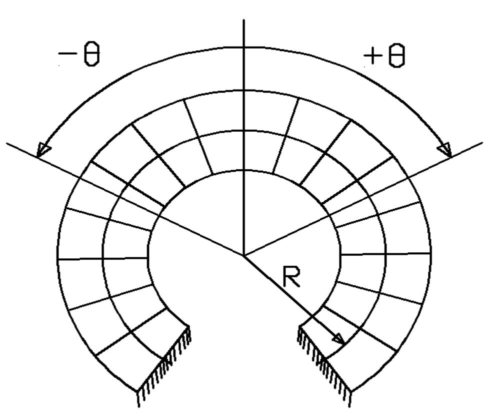
Vv: Vertical shear force

W: Design load

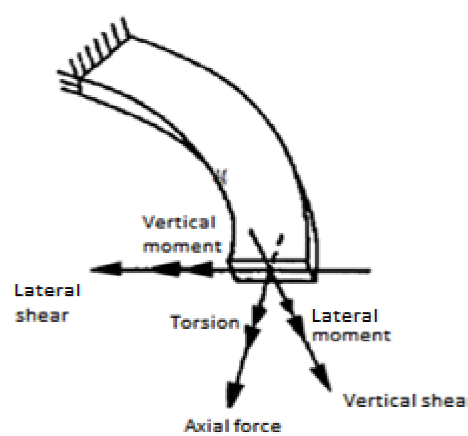
α : Flight slope

θ : Angular distance from mid-span on a horizontal plane

θ_{T}: Total angle of twist



Geometric Designation



Free Body Diagram

List of notations

EI: Flexural rigidity

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1. Introduction

1.1 Background

Ever since the start of wisdom, visual elegance has always been perused and sought by man kind. This urge to master the skill of possessing a better and appealing living environment also extends in the field of engineering, mainly in structural engineering. For years, engineers and architects have been trying to advance their achievements to level up with the most appealing and efficient structural components. Of these, one structural component that can be mentioned is helical stair. Spiral stair has an inherent fascinating appearance among different forms of stairs from architectural and functional point of view. For this reason, helical stairs are increasingly being used in many important buildings.

Currently, the construction sector in Ethiopia is dramatically enhancing. As the era of construction grows, the complexity of structure increases as well. And spiral stairs, as one of the most functional structural component, has complex structural configuration. This complexity is due to its geometry.

Geometrically, a helical surface is a three dimensional structure in space consisting of a warped surface which is generated by moving a straight line touching a helix so that the moving line is always perpendicular to the axis of the helix. In an oblique helix, the generating line always maintains a fixed angle with the helix. Since helix stair is a space member, all the six internal actions come in to play at any section, having varying directions and lines of actions. These internal actions are, Vertical moment (Mv), Lateral moment (Mh), Vertical shear (Vv), Lateral shear (Vh), Axial force (N) and Torsion (T), see figure 1.1.2.

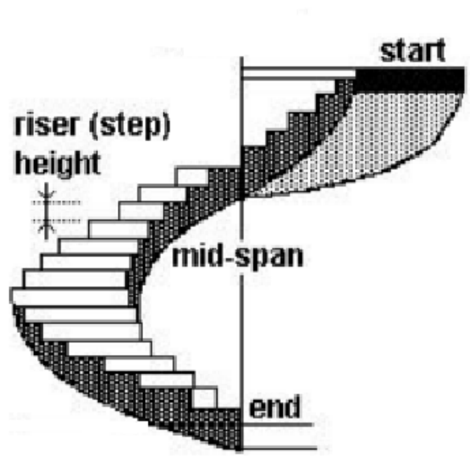

Figure1.1.1. 3D view of spiral stair

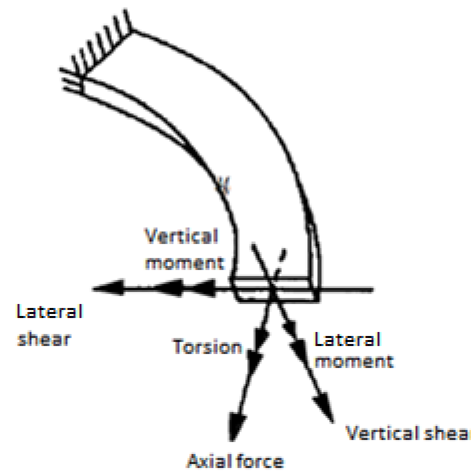

Figure 1.1.2. Sectioned helix with internal actions

But despite the existence of the six actions, helical stairs have been mainly designed only for vertical moment. In addition, due to their complexity, helical stairs have been designed by reducing the helix to its horizontal projection and resolving the problem into a fixed ended curved beam [8]. This approach is very conservative and approximate. It is not clear how the stair deals with the prevailing dangers of torsion and end moment, since the modeling doesn't represent the actual twisting nature of the member.

The aim of this study is therefore focused in the development of analytical approaches for spiral stairs using strain energy approach, studying the behavior of helical stair in finite element approach, carrying out comparative assessment between the different results and finally, preparation of analysis charts as a design aid for helical stair cases of various geometries.

1.2 Objective

The approaches for the analysis of helical stairs involve vigorous mathematical treatment and have created problem of solving simultaneous equations. This is very difficult and time taking for practical design. The main purpose of this research is, therefore, to find out a simpler approach to the analysis of fixed ended free standing helical staircase.

The manual calculation needed to analyze helix staircase is very complex, lengthy and tiresome. Therefore, preparation of analysis aid is believed to ease the difficulty to be faced. Analysis charts are very helpful as quick reference to analyze such complex systems. In addition to these, the comparison to be made between spiral girder solution and finite element solution leads to a good formulation of analysis aid. Hence this research makes it possible for designers to use an easy and user friendly analysis tables and charts for these complex structures.

1.2.1 Specific objectives

The specific objectives of this work include;

- Development of solution by strain energy approach.
- Observation of the behavior of helical stair by finite element approach.
- Comparative assessment between finite element and strain energy solutions.
- Carry out sensitivity analysis for the sensitivity of the internal actions to the different geometric parameters like radius, angle of rotation, flight slope.
- Preparation of analysis tables (charts) for helical stairs of various geometries.

1.3 Methodology and contents of research

In this research, analytical method is used applying different approaches. The in plane curved beam solution and spiral girder solution can be formulated in a similar formulation except that for in plane beam, flight slope is considered to be zero. Hence, first, the helix is modeled and analytically examined as in plane curved beam and spiral girder. For the spiral beam, the helix is modeled as twisting girder. This way of modeling gives a better representation of the system.

Because of symmetry in loading and geometry of a helical stair slab, the flight slope at the mid-span is zero and so is the horizontal deflection. And, according to the Castiglione's second theorem, the partial derivatives of the strain energy function with respect to the vertical moment (M) and lateral shear (H) at mid span is equal to zero. That is,

$$\frac{\partial U}{\partial M} = 0 \text{ and } \frac{\partial U}{\partial H} = 0 \qquad (1.3.1)$$

Solving these equations, M and H will be computed. Once these unknowns are determined, all actions will be computed at any section using equilibrium formulations.

And secondly, the helix can be dealt with by finite element approach. This approach is also a good representation of the spiral, since it depicts the actual 3D twist. For finite element approach, the modeling is done both as twisting beam and shell separately.

Finally, the results of the analytical methods are used for the preparation of analysis charts and tables. In addition, sensitivity analysis is also done to see the sensitivity of internal actions to geometric parameters. And for the analysis aid, the perception from all approaches is accounted, accompanied by formulation of modification factors.

Chapter 1 gives a brief introduction about the research and it incorporates background about research area and objective of research. Chapter 2 narrates on literature review including review of past works and concept of strain energy and its application for spiral stairs. In Chapter 3, detailed strain energy derivations are carried out for spiral stairs followed by analysis example solved by these derivations in chapter 4. Result comparison of different approaches is also included in chapter 4. And in chapter 5, sensitivity analysis is done to assess the responsiveness of the different internal actions to changes in geometric parameters. Next, in chapter 6, modification factors are formulated to take into account the variations observed. Finally chapter 7 gives the summarized conclusion and recommendation for future development.

2. Literature Review

Helical staircase is a three dimensional structure requiring calculation of moments, shears, axial and torsions in three dimensions. Therefore, deriving equations for the internal actions has always been a challenge.

2.1 Review of past works

The first helical stair was built in 1908 of reinforced concrete as an observation platform at the Franco British exhibition held at White City London. They used equations to relate load and stress of a helical spring made of rectangular cross sections, but the origin of the formula is untraceable and any material could not be found [2].

Bergman's method is an approximate analysis in which he reduces the problem to that of a horizontal bow girder, which fails to take into account the inherent structural strength of a helical beam. This method is called in plane beam solution [8].

The methods of Morgan and Scordelis are based on the analysis of the longitudinal elastic axis of the helical beam as a three dimensional structure indeterminate to the sixth degree. By selecting the redundant at mid span and using the principle of symmetry, all but two of the redundants become zero. The two redundant at mid span are the horizontal shear and the vertical moment [3 and 4].

A.R. Cusens and Supachai Trirojna have carried out a test on half scaled fixed ended helical staircase and confirmed the equilibrium equations derived by Morgan for helical stairs which are [4, 7],

- Vertical moment:

$$Mv = M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2(1-\cos\theta) \quad \text{(2.1.1)}$$

- Horizontal moment:

$$Mh = M\sin\theta\sin\alpha - HR\theta\tan\alpha\cos\theta\sin\alpha - HR\theta\sin\theta\cos\alpha + \left(WR^2\sin\theta - WR^2\theta\right)\sin\alpha$$ .... (2.1.2)

- Torsion:

$$T = \left(M\sin\theta - HR\theta\tan\alpha\cos\theta + WR^2\sin\theta - WR^2\theta\right)\cos\alpha + HR\sin\theta\sin\alpha$$ .... (2.1.3)

- Axial force:

$$N = -H\sin\theta\cos\alpha - WR\theta\sin\alpha$$ .... (2.1.4)

- Vertical Shear:

$$Vv = WH\theta\cos\alpha - H\sin\theta\sin\alpha$$ .... (2.1.5)

- Horizontal shear:

$$Vh = H\cos\theta$$ .... (2.1.6)

Where,

H: Horizontal shear force at mid-span (redundant)

M: Vertical moment at mid-span (redundant)

Mh: Lateral moment

Mv: Vertical moment

N: Axial force

R: Center line radius on horizontal projection

T: Torsion

Vh: Lateral shear force

Vv: Vertical shear force

W: Design load

α : Flight slope

Θ : Angular distance from mid-span on a horizontal plane

Arya and Prakash attempted to analyze the case of the helical stairs with intermediate landing. They used flexibility approach to analyze internal forces due to dead and live loads in fixed ended circular stairs having an intermediate landing. Like Scordeilis, they treated the structure as a linearly elastic member in space defined by its longitudinal centroidal axis. Influence lines were drawn at various cross sections for all the six stress resultants found at such sections for unit vertical load and unit moment about the axis of the structure [5].

Critical positions of loads were determined to obtain the maximum values of the internal forces. From this analysis they suggested some generalized behavior of helical stair slabs with intermediate landings.

Solanki analyzed the problem of intermediate landing using energy method to find the two unknown redundant at the mid-span section. Other redundant at mid-span have zero values because of symmetry of geometry and loads. He proposed two equations, the simultaneous solution of which gives the values of the redundant. Solankis findings were similar to those observed by Arya and Prakash. However, Arya and Prakash, as well as Solanki did not attempt to develop any simple design methods for helical stair slabs [6].

Among the few works done in this area, Reinhold's work is one of the best. Reynolds tried to put partial graphical solutions. As he stated in [1], strain energy approach was used even though the strain energy derivations are not explicitly provided. Besides, the provided solutions are only partial. And the solution he provides is only partial. There still remains a laborious work left to reach the final result. In addition to this, as it was stated in his book, the results are only satisfactory for preliminary analysis only since there exists a lot of interpolation the graphs he provided [1].

To summarize, most of the researches made are to create an approach for helical stair analysis. They were not intended to reduce the equations appropriate to each method which are somewhat lengthy. And also the strain energy applications were not explicitly detailed. Hence preparation of analysis charts for such structures developed by a detailed conceptual approach is found to be very essential.

2.2 The Strain Energy Method

Strain energy is an internal energy that occurs in response to an external work on a system and can be expressed as;

$$U = \frac{1}{2}\iiint_{v} \sigma\ \varepsilon\ dv \qquad \text{.. (2.2.1)}$$

The Castiglione's Second theorem states that the first partial derivative of the strain energy with respect to any particular force is equal to the displacement of the point of application of that force in the direction of its line of action. Mathematically,

$$\Rightarrow\ \frac{\partial U}{\partial P} = \delta \qquad \text{..(2.2.2)}$$

Where, U is the strain energy; P the force; and δ the deflection in the direction of force. The strain energy due to shear stress and axial force are neglected, because they are small as compared to strain energies due to moment and torsion. The strain energy stored by the bending moment is given by:

$$U = \int \frac{M^2}{2EI} dL \qquad \text{.. (2.2.3)}$$

And that by the twisting moment is:

$$U = \int \frac{T^2}{2GJ} dL \qquad \text{(2.2.4)}$$

2.3 The Strain Energy Method applied to the helical stair slabs

The following assumptions are made to facilitate the analysis procedure,

(i) Deformation due to shear and direct forces, being small in comparison to the deformations caused by twisting and bending moments, are neglected.

(ii) The cross section is symmetric about the two principal axes of the section.

(iii)The moment of inertia of the helical slab section with respect to a horizontal axis is negligible as compared to the moment of inertia with respect to the axis perpendicular to it.

(iv) System is considered to be elastic, homogeneous and isotropic.

Normally, for a space structure like twisting girder, there are 6 internal actions at any section which are Vertical moment(Mv), Lateral moment(Mh),Vertical shear(Vv), Lateral shear(Vh), Axial force(N) and Torsion (T). But at mid section, four of these six actions vanish due to symmetry and only Mv and Vh will be non zero. Making use of these, the unknown actions at any section can be expressed as function of Mv and Vh at mid span. The remaining challenge is to determine the two unknowns which are Mv and Vh. To do this, the concept of strain energy and second theorem of Castiglione will be used. Second theorem of Castiglione States that the partial derivative of strain energy to an acting load will give the deflection due to that load. Here the key is , at mid span, deflection and rotation is zero, so the partial derivative of strain energy to an acting load will be zero.

$$\frac{\partial U}{\partial H}=\int_{0}^{\theta} Mv\frac{\partial Mv}{\partial H}ds+\int_{0}^{\theta} Mh\frac{\partial Mh}{\partial H}ds+\frac{1}{2}\int_{0}^{\theta} T\frac{\partial T}{\partial H}ds=0..............case-1 \qquad (2.3.1)$$

$$\frac{\partial U}{\partial M}=\int_{0}^{\theta} Mv\frac{\partial Mv}{\partial M}ds+\int_{0}^{\theta} Mh\frac{\partial Mh}{\partial M}ds+\frac{1}{2}\int_{0}^{\theta} T\frac{\partial T}{\partial M}ds=0.............case-2 \qquad (2.3.2)$$

Where, U is strain energy, H & M are shear and moment at mid span, integration range is by angle of rotation. Solving these equations simultaneously, H and M will be computed. Once these unknowns are determined, all actions will be computed at any section.

3. Equation formulation and analysis approach

3.1 Introduction

The scope of this study is aimed to come up with a convenient approach for the analysis of fixed ended self-supporting helical stairs. As a result of complexity in geometry of helix, its analysis is difficult. The lack of analysis tables and charts in our design manual makes the process of hand calculation very long and tiresome. This chapter presents a series of explicit strain energy approach incorporating detailed equation formulation for helical stair.

3.2 Strain energy approach

As stated in the methodology section, strain energy principle is used in accordance with Castiglione's theorem. The derivation of strain energy solution follows the following steps.

- Formulation of strain energy equations for the internal actions.(using equations 1.3.1 , 2.2.1 , 2.2.3, 2.2.4)

- Application of Castiglione's second theorem.(using equations 2.3.1 and 2.3.2)

- Solution of the derived equations in (2) to compute the vertical moment and horizontal shear at mid span.

- Finally Formulation of equations for actions at any section in the system as a function of angle of rotation. (using 2.1.1 to 2.1.6)

3.2.1 Derivation using strain energy and Castiglione's theorem

In this part, the derivations as per the steps stated earlier are presented.

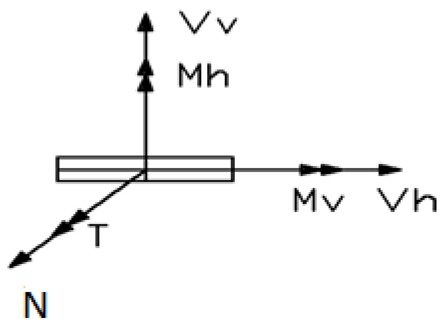

Figure 3.2.1.1 Internal action notation at a section

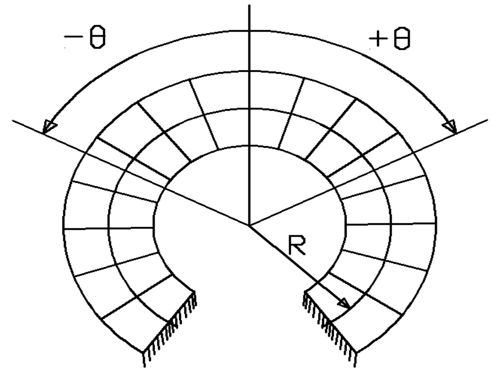

Figure 3.2.1.2 Geometric designations

Variable assignment

H: Horizontal shear force at mid-span (redundant)

M: Vertical moment at mid-span (redundant)

Mh: Lateral moment

Mv: Vertical moment

N: Axial force

R: Center line radius on horizontal projection

T: Torsion

U: Strain energy

Vh: Lateral shear force

Vv: Vertical shear force

W: Design load

α : Flight slope

Θ : Angular distance from mid-span on a horizontal plane

- **Formulation of strain energy for the internal actions**

The strain energy for moment and torsion are given in equations 2.2.3 and 2.2.4. And strain energy due shear and axial are neglected as explained in the assumptions made in section 2.3.

$$U = \int \frac{M^2}{2EI} dL \qquad\qquad U = \int \frac{T^2}{2GJ} dL$$

- **Application of Castiglione's theorem**

This theorem states that the partial derivative of strain energy with respect to an action gives the deflection caused by that action. To apply this concept for spiral system, let us consider two cases.

Case 1 : The partial derivative of strain energy with respect to the horizontal shear at mid span is equal to zero. That is;

$$\frac{\partial U}{\partial H} = 0 \qquad \text{......} \quad 3.2.1.1$$

$$\frac{\partial U}{\partial H} = \int_0^\theta Mv \frac{\partial Mv}{\partial H} ds + \tfrac{1}{2}\int_0^\theta T \frac{\partial T}{\partial H} ds = 0,$$
$$where, ds = R\sec\alpha d\theta$$

Here we can see that equation 3.2.1.1 can be segmented into two parts, which are the expression of moment and torsion. The expression involving moment is named Equation 3.2.1.1 A, and expression involving torsion is named equation 3.2.1.1 B. Thus, it can be stated as,

Equation 3.2.1.1 = Equation 3.2.1.1A + Equation 3.2.1.1B

Considering the first part in detail;

$$\int_0^\theta Mv \frac{\partial Mv}{\partial H} ds = \int_0^\theta Mv \frac{\partial Mv}{\partial H} R\sec\alpha d\theta$$

...... 3.2.1.1A

$$Mv = M\cos\theta + HR\,\theta\tan\alpha\sin\theta - WR^2(1-\cos\theta)$$ (From equation 2.1.1)

$$\frac{\partial Mv}{\partial H} = \frac{\partial}{\partial H}\left(M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2\left(1-\cos\theta\right)\right) = R\theta\tan\alpha\sin\theta$$

Now for simplicity labeling equation 3.2.1.1 A as just A, hence ,

$$A = \int_0^{\theta} Mv\frac{\partial Mv}{\partial H}R\sec a d\theta$$

$$= \int_0^{\theta}\left(M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2 + WR^2\cos\theta\right)\left(R\theta\tan\alpha\sin\theta\right)R\sec\alpha d\theta$$

$$= \int_0^{\theta}\left(\frac{\theta\sin 2\theta}{2}\right)MR^2\tan\alpha\sec\alpha d\theta + \int_0^{\theta}\left(\theta^2\sin^2\theta\right)HR^3\tan\alpha\sec\alpha d\theta -$$
$$\int_0^{\theta}\theta\sin\theta WR^4\tan\alpha\sec\alpha d\theta + \int_0^{\theta}\left(\frac{\theta\sin 2\theta}{2}\right)WR^4\tan\alpha\sec\alpha d\theta$$

$$= \left(\frac{-\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right)MR^2\tan\alpha\sec\alpha + \left(\frac{\theta^3}{6} - \frac{\theta^2\sin\theta}{4} - \frac{\theta\cos 2\theta}{4}\frac{\sin 2\theta}{8}\right)\left(HR^3\tan\alpha\sec\alpha\right) -$$
$$\left(-\theta\cos\theta + \sin\theta\right)WR^4\tan\alpha\sec\alpha + \left(-\theta\cos 2\theta + \sin 2\theta\right)WR^4\tan\alpha\sec\alpha$$

After mathematical manipulation,

$$\left\{\begin{array}{l} A = M\left(-\frac{\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right)R^2\tan\alpha\sec\alpha + HR^3\tan^2\alpha\sec\alpha\left(\frac{\theta^3}{6} - \frac{\theta^2\sin 2\theta}{4} - \frac{\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right) + \\ WR^4\tan\alpha\sec\alpha\left(\theta\cos\theta - \sin\theta - \frac{\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right) \end{array}\right\}$$

Similarly the second part reveals,

$$\frac{1}{2}\int_0^{\theta} T\frac{\partial T}{\partial H}ds = \frac{1}{2}\int_0^{\theta} T\frac{\partial T}{\partial H}R\sec\alpha d\theta \quad \text{..........} \quad 3.2.1.1\ B$$

$$T = \left(M\sin\theta - HR\theta\tan\alpha\cos\theta + WR^2\sin\theta - WR^2\theta\right)\cos\alpha + HR\sin\theta\sin\alpha$$

(From Equation 2.1.3)

$$\frac{\partial T}{\partial H} = -R\theta\tan\alpha\cos\theta\cos\alpha + R\sin\theta\cos\alpha\sin\alpha + R\sin\theta\sin\alpha$$

Now for simplicity, labeling equation 3.2.1.1 B as just B, hence

$$B = \tfrac{1}{2}\int_0^{\theta} T\frac{\partial T}{\partial H}R\sec\alpha\, d\theta$$

$$= \tfrac{1}{2}\int_0^{\theta}\left(M\sin\theta\cos\alpha - HR\sin\alpha\cos\theta + WR^2\sin\theta\cos\alpha - WR^2\theta\cos\alpha + HR\sin\theta\sin\alpha\right)$$
$$\left(-R\theta\sin\alpha\cos\theta + R\sin\theta\sin\alpha\right)R\sec\alpha d\theta$$

This equation is extremely long to be handled as a single equation, hence it is fragmented to five components to be treated separable then added to give the final equation. That is;

- B = B1 + B2 + B3 + B4 + B5

After these equations are mathematically manipulated the results will be,

$$\begin{pmatrix}
B1 = M\left\{\left(\cos\alpha R^2\sin\alpha\sec\alpha\right)\left(\frac{\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+\cos\alpha R^2\sin\alpha\sec\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)\right\} \\
B2 = H\left\{\left(R^3\sin^2\alpha\sec\alpha\right)\left(\frac{\theta^3}{12}+\frac{\theta^2\sin 2\theta}{8}+\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+\left(R^3\right)\sin^2\alpha\sec\alpha.\left(\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)\right\} \\
B3 = WR^4\left\{\sin\alpha.\left(\frac{\theta\cos 2\theta}{2}-\frac{\sin 2\theta}{16}\right)+\sin\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)\right\} \\
B4 = WR^4\left\{\sin\alpha.\left(\frac{\theta^2\sin\theta}{2}+\frac{2\theta\cos\theta}{2}-\frac{2\sin\theta}{2}\right)+\sin\alpha\left(\frac{\theta\cos\theta}{2}-\frac{\sin\theta}{2}\right)\right\} \\
B5 = H\left\{R^3\sin^2\alpha\sec\alpha\left(\frac{-\theta\cos 2\theta}{8}+\frac{\sin 2\theta}{16}\right)+R^3\sin^2\alpha\sec\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)\right\}
\end{pmatrix}$$

Final Computation Result of case 1

$$\frac{\partial U}{\partial H} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Case\ 1$$

$$\left(\begin{array}{l} M\left\{ \begin{array}{l} \left(\frac{-\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right)R^2\tan\alpha\sec\alpha + \left(\cos\alpha R^2\sin\alpha\sec\alpha\right)\left(\frac{\theta\cos 2\theta}{8} - \frac{\sin 2\theta}{16}\right) + \\ \cos\alpha R^2\sin\alpha\sec\alpha\left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right) \end{array} \right\} + \\ \\ H\left\{ \begin{array}{l} R^3\tan^2\alpha\sec\alpha\left(\frac{\theta^3}{6} - \frac{\theta^2\sin 2\theta}{4} - \frac{\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right) + \\ \left(R^3\sin^2\alpha\sec\alpha\right)\left(\frac{\theta^3}{12} + \frac{\theta^2\sin 2\theta}{8} + \frac{\theta\cos 2\theta}{8} - \frac{\sin 2\theta}{16}\right) + R^3\left(\sin^2\alpha\sec\alpha\right).\left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right) + \\ \left(R^3\sin^2\alpha\sec\alpha\right)\left(\frac{-\theta\cos 2\theta}{8} + \frac{\sin 2\theta}{16}\right) + R^3\sin^2\alpha\sec\alpha\left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right) \end{array} \right\} + \\ \\ W\left\{ \begin{array}{l} R^4\tan\alpha\sec\alpha\left(\theta\cos\theta - \sin\theta - \frac{\theta\cos 2\theta}{4} + \frac{\sin 2\theta}{8}\right) + R^4.\left(\sin\alpha\right)\left(\frac{\theta\cos 2\theta}{8} - \frac{\sin 2\theta}{16}\right) + \\ \cos\alpha\sin\alpha\left(\left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right)\right) + R^4\left(\sin\alpha\right)\left(\frac{\theta^2\sin\theta}{2} + \theta\cos\theta - \sin\theta\right) + \sin\alpha\left(\frac{\theta\cos\theta}{2} - \frac{\sin\theta}{2}\right) \end{array} \right\} = 0 \end{array} \right)$$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (*Result of equation 3.2.1.1A +3.2.1.1|B*)

Case 2 : The partial derivative of strain energy with respect to the vertical moment at mid span is equal to zero. That is;

$$\frac{\partial U}{\partial M}=0 \qquad \text{.........} \; 3.2.1.2$$

$$\frac{\partial U}{\partial M}=\int_0^{\theta} Mv\frac{\partial Mv}{\partial M}ds+\tfrac{1}{2}\int_0^{\theta} T\frac{\partial T}{\partial M}ds=0,$$
$$where, ds=R\sec\alpha d\theta$$

Here we can see that equation 3.2.1.2 can be segmented into two parts, which are the expression of moment and torsion. The expression involving moment is termed Equation 3.2.1.2 C, and expression involving torsion is termed equation 3.2.1.2 D. Thus, it can be stated as,

Equation 3.2.1.2 = Equation 3.2.1.2C + Equation 3.2.1.2D

$$\int_0^{\theta} Mv\,\frac{\partial Mv}{\partial M}R\sec\alpha d\theta \; \text{..........} \; 3.2.1.2\,C$$

$$Mv=M\cos\theta+HR\theta\tan\alpha\sin\theta-WR^2(1-\cos\theta)$$ (From equation 2.1.1)

$$\Rightarrow\frac{\partial Mv}{\partial M}=\cos\theta$$

Now for convenience labeling equation 3.2.1.2 C as just C, hence,

$$C=\int_0^{\theta}\left(M\cos\theta+HR\,\theta\tan\alpha\sin\theta-WR^2+WR^2\cos\theta\right)\cos\theta\,R\sec\alpha.d\theta\ .$$

$$=\int_0^{\theta}\left(M\cos^2\theta+HR\theta\tan\alpha\sin\theta\cos\theta-WR^2\cos\theta+WR^2\cos^2\theta\right)R\sec\alpha d\theta$$

$$=\int_0^{\theta}\cos^2\theta(MR\sec\alpha)d\theta+\int_0^{\theta}\left(\frac{\theta\sin 2\theta}{2}\right)HR^2\tan\alpha\sec\alpha d\theta-\int_0^{\theta}(\cos\theta)WR^3\sec\alpha d\theta+$$
$$\int_0^{\theta}\left(\frac{\cos 2\theta}{2}\right)WR^3\sec\alpha d\theta$$

$$\left\{\begin{array}{l} C = M\left\{R\sec\alpha\left(\frac{\theta}{2}+\frac{\sin\theta}{4}\right)\right\} + H\left\{R^2\tan\alpha\sec\alpha\left(\frac{-\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)\right\} + \\ WR^3\left\{-\sin\theta\sec\alpha + \sec\alpha\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)\right\} \end{array}\right\}$$

Similarly the next part gives,

$$\tfrac{1}{2}\int_0^\theta T\frac{\partial T}{\partial M}ds = \tfrac{1}{2}\int_0^\theta T\frac{\partial T}{\partial M}R\sec\alpha d\theta \quad \text{.................. 3.2.1.2 D}$$

$$\frac{\partial T}{\partial M} = \sin\theta\cos\alpha$$

Labeling equation 3.2.1.2 D as just D for convenience,

$$D = \tfrac{1}{2}\int_0^\theta T\frac{\partial T}{\partial M}R\sec\alpha d\theta$$

$$= \tfrac{1}{2}\int_0^\theta \left((M\sin\theta\cos\theta - HR\theta\tan\alpha\cos\theta\cos\alpha + WR^2\sin\theta\cos\alpha + R^2\theta\cos\alpha + HR\sin\theta\sin\alpha\right) \\ (\sin\theta\cos\alpha)R\sec\alpha\, d\theta$$

After mathematical manipulation, equation 3.2.1.2 D becomes,

$$\left\{\begin{array}{l} D = \tfrac{1}{2}\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)M\cos^2\alpha R\sec\alpha - \tfrac{1}{2}\left(\frac{-\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)HR\sin\alpha + \\ \quad \tfrac{1}{2}\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)WR^2\cos\alpha - \tfrac{1}{2}\left(-\theta\cos\theta+\sin\theta\right)WR^3\cos\alpha + \tfrac{1}{2}\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)HR^2\sin\alpha \end{array}\right\}$$

Final Computation Result of case 2

$$\frac{\partial U}{\partial M}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\textit{Case 2}$$

$$\left(\begin{array}{l} M.\left\{(R\sec\alpha)\left(\frac{\theta}{2}+\frac{\sin\theta}{4}\right)+(\cos\alpha\ R)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{4}\right)\right\}+ \\ H\left\{\left(R^2\tan\alpha\sec\alpha\right)\left(-\frac{\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)+\left(R^2\sin\alpha\right)\left(\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+R^2\sin\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)\right\}+ \\ W\left\{\begin{array}{l}\left(R^3\right)\left(-\sin\theta\sec\alpha+\sec\alpha\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)\right)+\left(R^3\right)(\cos\alpha)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)+\left(R^3\right)(\cos\alpha)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)+ \\ \left(R^3\right)(\cos\alpha)\left(\frac{\theta\cos\theta}{2}-\frac{\sin\theta}{2}\right)\end{array}\right\}=0 \end{array}\right)$$

……………………………………………………… (*Result of equation 3.2.1.2 C +3.2.1.2 D*)

- **Solving for mid span unknowns.**

Using final results of case 1 and case 2, the mid span unknowns (M and H) are computed by solving simultaneously.

Considering the final computation results of case 1; naming the expression multiplying M by C_1, the expression multiplying H by C_2,and lastly, Let the expression multiplying W by C_3, the final equation of case 1 can shortly be represented by,

$C_1M+C_2H+C_3W=0$……………………………………………………………………………………………………(*)

Considering the final computation results of case 2; naming the expression multiplying M by C_4, the expression multiplying H by C_5,and lastly, the expression multiplying W by C_6, the final equation of case 2 can shortly be represented by,

$C_4M+C_5H+C_6W=0$……………………………………………………………………………………………………(**)

Equations * and ** can be solved simultaneously to get the expressions for the mid span unknowns M and H. The expression after solving becomes;

$$\left.\begin{aligned} H &= \left(\frac{WC_1C_6 - WC_3C_4}{C_2C_4 - C_5C_1}\right) \\ M &= -\left(\frac{HC_5 + WC_6}{C_4}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (***)$$

- **Formulation of equations of internal actions for the system**

The values of M and H calculated in (***) will be inserted in the equilibrium equations. Then, internal actions at any section are determined as a function of Ɵ using (equations 2.1.1 to 2.1.6).In this research an internal actions are obtained in an interval of 5°. The set of tables and charts for spiral stairs of various geometries are provided as analysis aid in the Annex section. Using these derivations and formulations, a problem is solved in the next chapter; and, the results are presented in a tabular and graphical form.

4. Analysis example and result comparison

In this chapter, a spiral stair of flight slope 25° is considered for analysis to support a design load of 12.22kN/m2. The stair is to have a width of 1.0m. The radius to the center of stair is 1.5m and the angle turned through by the stair is 240°.

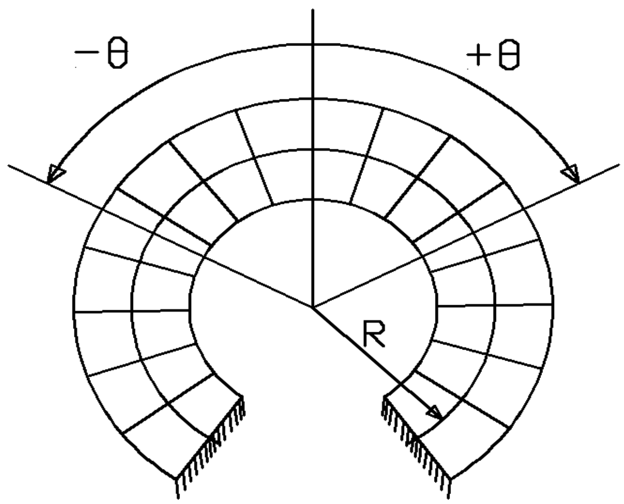

Figure 4: Plan view of Spiral stair

This problem is analytically solved using strain energy derivations made in this research following the steps in section 3.2 and final equations in section 3.2.1. The results are presented in a tabular and graphical form as follows in table 4.1 and figure 4.1. These results clearly show the pattern and values of each internal action along the span both qualitatively and quantitatively.

4.1 Strain energy solution

Table 4.1 : Strain energy solution for the example problem

Angle(°)	Mv(KNm)	Mh(KNm)	Vv(KN)	Vh(KN)	T(KNm)	N(KN)
-120	-2.13	41.09	-23.87	-14.93	-3.65	39.66
-115	-0.08	43.04	-21.91	-12.62	-3.74	40.07
-110	1.62	44.72	-20.04	-10.21	-3.67	40.30
-105	2.99	46.10	-18.26	-7.73	-3.49	40.33
-100	4.05	47.16	-16.57	-5.18	-3.21	40.17
-95	4.82	47.89	-14.98	-2.60	-2.86	39.80
-90	5.30	48.26	-13.48	0.00	-2.45	39.22
-85	5.55	48.28	-12.08	2.60	-2.02	38.45
-80	5.56	47.92	-10.77	5.18	-1.58	37.46
-75	5.38	47.19	-9.56	7.73	-1.15	36.27
-70	5.03	46.09	-8.44	10.21	-0.74	34.89
-65	4.55	44.61	-7.41	12.62	-0.36	33.31
-60	3.95	42.78	-6.47	14.93	-0.02	31.54
-55	3.27	40.60	-5.61	17.12	0.27	29.60
-50	2.54	38.08	-4.83	19.19	0.50	27.49
-45	1.79	35.25	-4.13	21.11	0.67	25.22
-40	1.04	32.12	-3.49	22.87	0.78	22.80
-35	0.31	28.73	-2.91	24.45	0.83	20.25
-30	-0.36	25.09	-2.39	25.85	0.83	17.58
-25	-0.97	21.24	-1.92	27.06	0.78	14.81
-20	-1.50	17.22	-1.48	28.05	0.68	11.96
-15	-1.92	13.04	-1.08	28.84	0.54	9.03
-10	-2.23	8.76	-0.71	29.40	0.38	6.05
-5	-2.42	4.40	-0.35	29.74	0.19	3.03
0	-2.48	0.00	0.00	29.85	0.00	0.00
5	-2.42	-4.40	0.35	29.74	-0.19	-3.03
10	-2.23	-8.76	0.71	29.40	-0.38	-6.05
15	-1.92	-13.04	1.08	28.84	-0.54	-9.03
20	-1.50	-17.22	1.48	28.05	-0.68	-11.96
25	-0.97	-21.24	1.92	27.06	-0.78	-14.81
30	-0.36	-25.09	2.39	25.85	-0.83	-17.58
35	0.31	-28.73	2.91	24.45	-0.83	-20.25
40	1.04	-32.12	3.49	22.87	-0.78	-22.80
45	1.79	-35.25	4.13	21.11	-0.67	-25.22
50	2.54	-38.08	4.83	19.19	-0.50	-27.49
55	3.27	-40.60	5.61	17.12	-0.27	-29.60
60	3.95	-42.78	6.47	14.93	0.02	-31.54
65	4.55	-44.61	7.41	12.62	0.36	-33.31
70	5.03	-46.09	8.44	10.21	0.74	-34.89
75	5.38	-47.19	9.56	7.73	1.15	-36.27
80	5.56	-47.92	10.77	5.18	1.58	-37.46
85	5.55	-48.28	12.08	2.60	2.02	-38.45
90	5.30	-48.26	13.48	0.00	2.45	-39.22
95	4.82	-47.89	14.98	-2.60	2.86	-39.80
100	4.05	-47.16	16.57	-5.18	3.21	-40.17
105	2.99	-46.10	18.26	-7.73	3.49	-40.33
110	1.62	-44.72	20.04	-10.21	3.67	-40.30
115	-0.08	-43.04	21.91	-12.62	3.74	-40.07
120	-2.13	-41.09	23.87	-14.93	3.65	-39.66

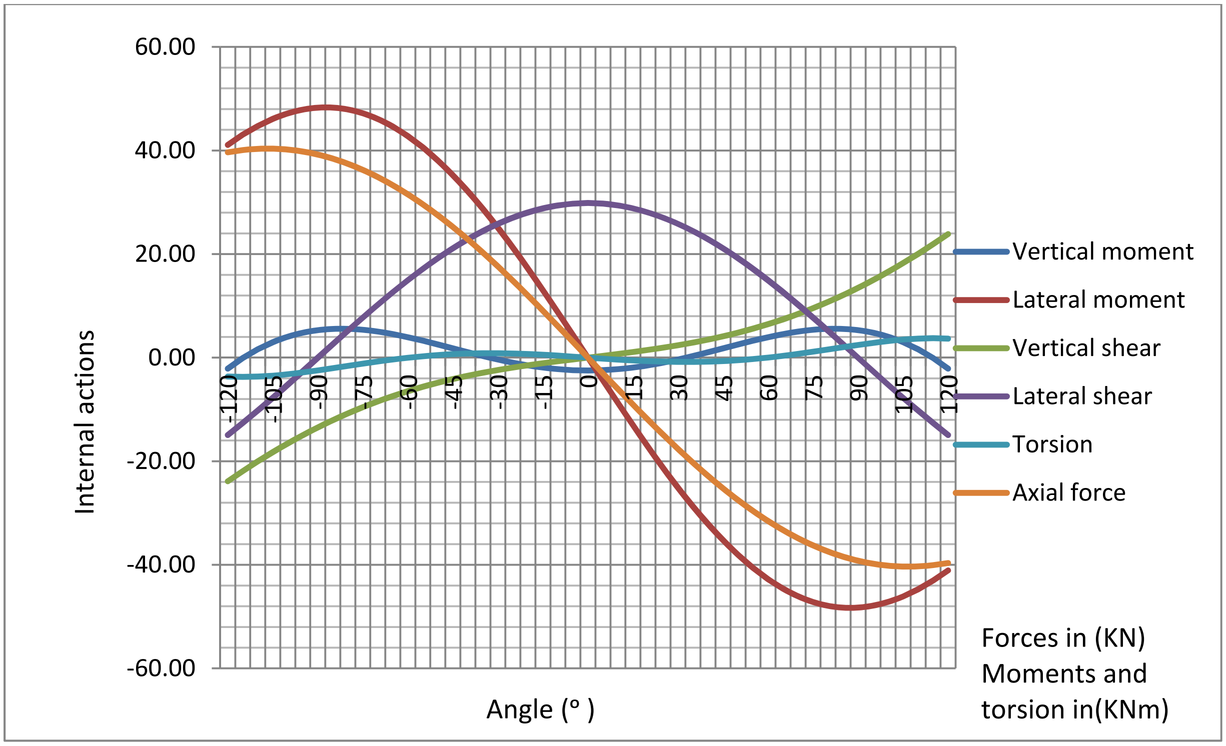

Figure 4.1 Graphical representation of strain energy solution

4.2 Finite element solution

Here the spiral stair is analyzed both as twisting frame simulating center line model and also as 3D shell structure. The shell element is a type of area object that is used to model shell behavior in three-dimensional structures. The modeling solution is done using SAP 2000 V.14.

4.2.1 Finite element solution as twisting frame

The same example that was solved previously will be treated in this section using finite element method by modeling it as twisting beam. The simulation follows the center line of the helical stair. The results obtained show the same pattern as the strain energy solution and are presented in a form of comparison in section 4.5.

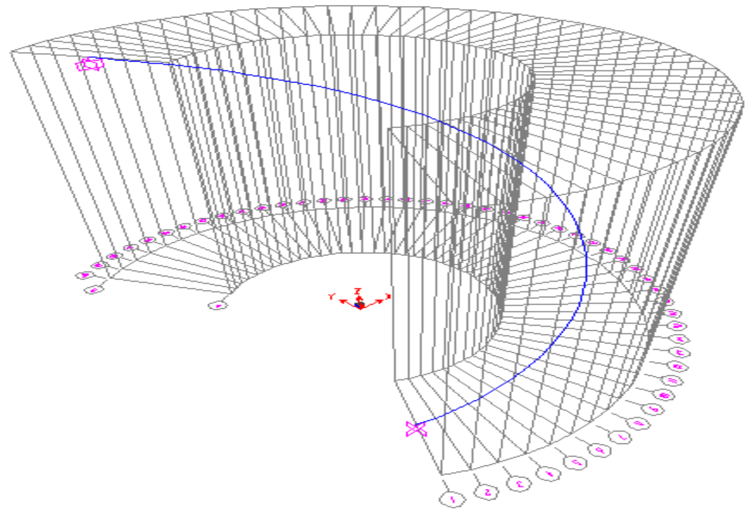

Figure 4.2.1 Finite element frame model

4.2.2 Finite element solution of helical shell slab

The helical stair is modeled in SAP as 3D surface structure, this time composed of shell elements. Corresponding values to the strain energy solutions are extracted from the analysis results and comparison is provided in section 4.5.

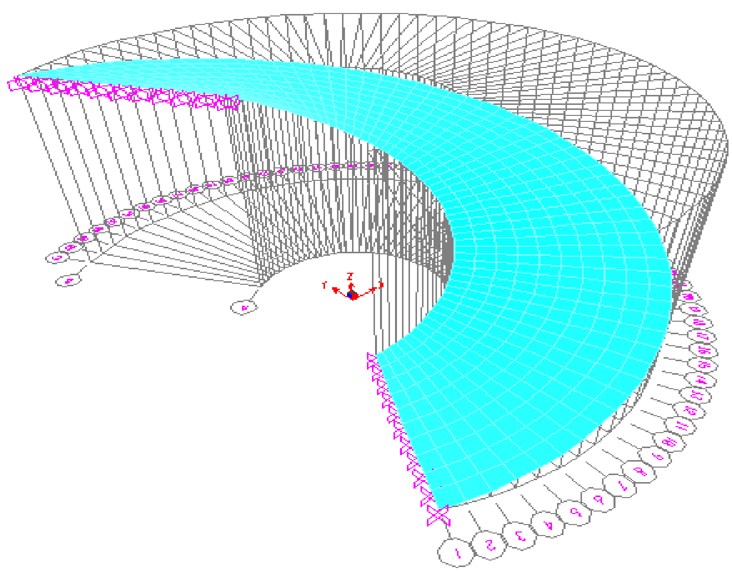

Figure 4.2.2 Finite element shell model

4.3 Reynolds solution

Reynolds work is based on strain energy concept. In his book [1], he presented equations and charts for spiral stairs. Even though his work is the best so far, the solution provided has limitations. The charts presented in the book give good information about internal actions but only qualitatively, and indicate set of constants to put in his equations then solved. And, as Reynolds himself stated in his book, his results are only satisfactory for preliminary analysis only as lots of interpolation is involved.

The previous example is solved by the equations provided by Reynolds then manipulated further to reach the final results. The results of Reynolds solution is very close to results of this research. Comparison is shown in section 4.5.

4.4 In plane beam solution

Due to the difficulty, helical stairs are analyzed as planar curved beam. This is made by reducing the system into its horizontal projection. Hence the modeling represents an in plane curved beam.

Hence, the strain energy derivations made in section 3.2.1 can be used for this technique by setting flight slope equal to zero, which give an in plane beam solution. In other words, when the flight slope in strain energy formulation is zero, it gives in plane beam solution. The results of in plane beam solutions highly differ from the results of the others. Results and comparison are presented in the next section 4.5.

4.5 Comparison of results

The example helical stair is analyzed using equations derived this research (Research results), as a framed structure using finite element (FE frame), as a 3D surface structure using finite element shell modeling (FE shell), using Reynolds equation (Reynolds) and in plane beam modeling (In plane). The results of these are put and compared in table and graph as follows.

Table 4.3.1Axial force comparison

	P (KN)				
Angle(°)	FE frame	FE shell	Reynolds	In plane	Research results
-120	37.53	47.59	38.07	0.00	39.66
-115	37.63	46.08	38.41	0.00	40.07
-110	37.58	46.34	38.58	0.00	40.30
-105	37.37	46.38	38.56	0.00	40.33
-100	36.99	46.19	38.36	0.00	40.17
-95	36.45	45.77	37.97	0.00	39.80
-90	35.74	45.11	37.39	0.00	39.22
-85	34.86	44.21	36.62	0.00	38.45
-80	33.82	43.08	35.66	0.00	37.46
-75	32.61	41.72	34.51	0.00	36.27
-70	31.24	40.12	33.17	0.00	34.89
-65	29.72	38.31	31.65	0.00	33.31
-60	28.04	36.28	29.96	0.00	31.54
-55	26.22	34.04	28.10	0.00	29.60
-50	24.27	31.61	26.08	0.00	27.49
-45	22.19	29.00	23.92	0.00	25.22
-40	19.99	26.22	21.62	0.00	22.80
-35	17.69	23.29	19.20	0.00	20.25
-30	15.29	20.22	16.67	0.00	17.58
-25	12.81	17.04	14.04	0.00	14.81
-20	10.26	13.75	11.33	0.00	11.96
-15	7.65	10.39	8.56	0.00	9.03
-10	5	6.96	5.73	0.00	6.05
-5	2.33	3.49	2.87	0.00	3.03
0	-0.37	0.00	0.00	0.00	0.00
5	-3.06	-3.49	-2.87	0.00	-3.03
10	-5.73	-6.96	-5.73	0.00	-6.05
15	-8.38	-10.39	-8.56	0.00	-9.03
20	-10.99	-13.75	-11.33	0.00	-11.96
25	-13.54	-17.04	-14.04	0.00	-14.81
30	-16.02	-20.22	-16.67	0.00	-17.58
35	-18.42	-23.29	-19.20	0.00	-20.25
40	-20.72	-26.22	-21.62	0.00	-22.80
45	-22.92	-29.00	-23.92	0.00	-25.22
50	-25	-31.61	-26.08	0.00	-27.49
55	-26.95	-34.04	-28.10	0.00	-29.60
60	-28.77	-36.28	-29.96	0.00	-31.54
65	-30.45	-38.31	-31.65	0.00	-33.31
70	-31.97	-40.12	-33.17	0.00	-34.89
75	-33.34	-41.72	-34.51	0.00	-36.27
80	-34.55	-43.08	-35.66	0.00	-37.46
85	-35.59	-44.21	-36.62	0.00	-38.45
90	-36.47	-45.11	-37.39	0.00	-39.22
95	-37.18	-45.77	-37.97	0.00	-39.80
100	-37.72	-46.19	-38.36	0.00	-40.17
105	-38.1	-46.38	-38.56	0.00	-40.33
110	-38.31	-46.34	-38.58	0.00	-40.30
115	-38.36	-46.08	-38.41	0.00	-40.07
120	-38.26	-47.59	-38.07	0.00	-39.66

Table 4.3.2 Vertical shear comparison

Vv (KN)					
Angle(°)	FE frame	FE shell	Reynolds	In plane	Research results
-120	-27.32	-23.19	-24.61	-38.39	-23.87
-115	-25.36	-16.63	-22.68	-36.79	-21.91
-110	-23.48	-10.54	-20.84	-35.19	-20.04
-105	-21.67	-5.45	-19.08	-33.59	-18.26
-100	-19.94	-4.27	-17.41	-31.99	-16.57
-95	-18.28	-1.57	-15.83	-30.39	-14.98
-90	-16.71	-0.78	-14.33	-28.79	-13.48
-85	-15.21	1.12	-12.93	-27.19	-12.08
-80	-13.79	1.78	-11.61	-25.59	-10.77
-75	-12.45	3.1	-10.38	-23.99	-9.56
-70	-11.18	3.56	-9.24	-22.39	-8.44
-65	-9.98	4.4	-8.19	-20.79	-7.41
-60	-8.86	4.62	-7.21	-19.20	-6.47
-55	-7.8	5.04	-6.31	-17.60	-5.61
-50	-6.8	5.01	-5.49	-16.00	-4.83
-45	-5.86	5.06	-4.73	-14.40	-4.13
-40	-4.98	4.79	-4.04	-12.80	-3.49
-35	-4.15	4.54	-3.40	-11.20	-2.91
-30	-3.36	4.05	-2.82	-9.60	-2.39
-25	-2.61	3.56	-2.28	-8.00	-1.92
-20	-1.89	2.9	-1.78	-6.40	-1.48
-15	-1.2	2.23	-1.30	-4.80	-1.08
-10	-0.53	1.47	-0.86	-3.20	-0.71
-5	0.13	0.69	-0.42	-1.60	-0.35
0	0.78	-0.11	0.00	0.00	0.00
5	1.44	-0.91	0.42	1.60	0.35
10	2.09	-1.68	0.86	3.20	0.71
15	2.77	-2.44	1.30	4.80	1.08
20	3.46	-3.11	1.78	6.40	1.48
25	4.18	-3.76	2.28	8.00	1.92
30	4.93	-4.24	2.82	9.60	2.39
35	5.72	-4.72	3.40	11.20	2.91
40	6.55	-4.96	4.04	12.80	3.49
45	7.43	-5.22	4.73	14.40	4.13
50	8.37	-5.15	5.49	16.00	4.83
55	9.36	-5.17	6.31	17.60	5.61
60	10.42	-4.73	7.21	19.20	6.47
65	11.55	-4.5	8.19	20.79	7.41
70	12.74	-3.64	9.24	22.39	8.44
75	14.01	-3.17	10.38	23.99	9.56
80	15.36	-1.84	11.61	25.59	10.77
85	16.78	-1.19	12.93	27.19	12.08
90	18.27	0.69	14.33	28.79	13.48
95	19.85	1.44	15.83	30.39	14.98
100	21.51	4.07	17.41	31.99	16.57
105	23.24	5.12	19.08	33.59	18.26
110	25.05	10.1	20.84	35.19	20.04
115	26.93	16.29	22.68	36.79	21.91
120	28.88	23.04	24.61	38.39	23.87

Table 4.3.3 Lateral shear comparison

Vh (KN)					
Angle(°)	FE frame	FE shell	Reynolds	In plane	Research results
-120	-11.72	-16.58	-13.92	0.00	-14.93
-115	-9.76	-12.79	-11.76	0.00	-12.62
-110	-7.73	-7.72	-9.52	0.00	-10.21
-105	-5.64	-5.83	-7.20	0.00	-7.73
-100	-3.51	-3.33	-4.83	0.00	-5.18
-95	-1.35	-0.73	-2.43	0.00	-2.60
-90	0.81	2.22	0.00	0.00	0.00
-85	2.97	5.08	2.43	0.00	2.60
-80	5.11	8	4.83	0.00	5.18
-75	7.21	10.82	7.20	0.00	7.73
-70	9.26	13.57	9.52	0.00	10.21
-65	11.24	16.2	11.76	0.00	12.62
-60	13.14	18.71	13.92	0.00	14.93
-55	14.94	21.06	15.96	0.00	17.12
-50	16.63	23.25	17.89	0.00	19.19
-45	18.2	25.24	19.68	0.00	21.11
-40	19.64	27.04	21.32	0.00	22.87
-35	20.94	28.62	22.80	0.00	24.45
-30	22.08	29.99	24.10	0.00	25.85
-25	23.06	31.12	25.23	0.00	27.06
-20	23.87	32.02	26.15	0.00	28.05
-15	24.51	32.65	26.88	0.00	28.84
-10	24.97	33.07	27.41	0.00	29.40
-5	25.25	33.21	27.73	0.00	29.74
0	25.34	33.13	27.83	0.00	29.85
5	25.25	32.76	27.73	0.00	29.74
10	24.97	32.21	27.41	0.00	29.40
15	24.51	31.35	26.88	0.00	28.84
20	23.87	30.35	26.15	0.00	28.05
25	23.06	29.02	25.23	0.00	27.06
30	22.08	27.62	24.10	0.00	25.85
35	20.94	25.85	22.80	0.00	24.45
40	19.64	24.1	21.32	0.00	22.87
45	18.2	21.92	19.68	0.00	21.11
50	16.63	19.91	17.89	0.00	19.19
55	14.94	17.36	15.96	0.00	17.12
60	13.14	15.18	13.92	0.00	14.93
65	11.24	12.27	11.76	0.00	12.62
70	9.26	10.03	9.52	0.00	10.21
75	7.21	6.77	7.20	0.00	7.73
80	5.11	4.61	4.83	0.00	5.18
85	2.97	1.01	2.43	0.00	2.60
90	0.81	-0.88	0.00	0.00	0.00
95	-1.35	-4.7	-2.43	0.00	-2.60
100	-3.51	-6	-4.83	0.00	-5.18
105	-5.64	-9.24	-7.20	0.00	-7.73
110	-7.73	-8.98	-9.52	0.00	-10.21
115	-9.76	-11.94	-11.76	0.00	-12.62
120	-11.72	-16.21	-13.92	0.00	-14.93

Table 4.3.4 Torsion comparison

T (KNm)					
Angle(°)	FE frame	FE shell	Reynolds	In plane	Research results
-120	-2.63	0.54	-0.56	-29.25	-3.65
-115	-1.78	0.40	-0.82	-25.53	-3.74
-110	-1.11	0.0	-0.93	-22.04	-3.67
-105	-0.58	-0.27	-0.93	-18.78	-3.49
-100	-0.17	-0.55	-0.83	-15.77	-3.21
-95	0.12	-0.73	-0.65	-12.99	-2.86
-90	0.32	-0.84	-0.43	-10.47	-2.45
-85	0.45	-0.88	-0.17	-8.20	-2.02
-80	0.51	-0.88	0.10	-6.17	-1.58
-75	0.53	-0.84	0.37	-4.39	-1.15
-70	0.51	-0.77	0.63	-2.85	-0.74
-65	0.46	-0.69	0.86	-1.54	-0.36
-60	0.4	-0.60	1.06	-0.46	-0.02
-55	0.33	-0.51	1.22	0.41	0.27
-50	0.25	-0.42	1.33	1.07	0.50
-45	0.18	-0.33	1.39	1.54	0.67
-40	0.12	-0.25	1.40	1.84	0.78
-35	0.07	-0.18	1.35	1.97	0.83
-30	0.03	-0.12	1.26	1.96	0.83
-25	0	-0.069	1.13	1.83	0.78
-20	-0.01	-0.027	0.95	1.59	0.68
-15	-0.02	0.007	0.74	1.27	0.54
-10	-0.02	0.033	0.51	0.88	0.38
-5	-0.01	0.054	0.26	0.45	0.19
0	0	0.071	0.00	0.00	0.00
5	0.01	0.086	-0.26	-0.45	-0.19
10	0.02	0.102	-0.51	-0.88	-0.38
15	0.02	0.12	-0.74	-1.27	-0.54
20	0.01	0.143	-0.95	-1.59	-0.68
25	0	0.173	-1.13	-1.83	-0.78
30	-0.03	0.21	-1.26	-1.96	-0.83
35	-0.07	0.26	-1.35	-1.97	-0.83
40	-0.12	0.32	-1.40	-1.84	-0.78
45	-0.18	0.39	-1.39	-1.54	-0.67
50	-0.25	0.47	-1.33	-1.07	-0.50
55	-0.33	0.56	-1.22	-0.41	-0.27
60	-0.4	0.66	-1.06	0.46	0.02
65	-0.46	0.77	-0.86	1.54	0.36
70	-0.51	0.87	-0.63	2.85	0.74
75	-0.53	0.97	-0.37	4.39	1.15
80	-0.51	1.06	-0.10	6.17	1.58
85	-0.45	1.10	0.17	8.20	2.02
90	-0.32	1.12	0.43	10.47	2.45
95	-0.12	1.06	0.65	12.99	2.86
100	0.17	0.95	0.83	15.77	3.21
105	0.58	0.71	0.93	18.78	3.49
110	1.11	0.48	0.93	22.04	3.67
115	1.78	0.46	0.82	25.53	3.74
120	2.63	0.69	0.56	29.25	3.65

Table 4.3.5 Vertical moment comparison

Mv (KNm)					
Angle	FE frame	FE shell	Reynolds	In plane	Research results
-120	-7.049	-3.9144	-3.08	-43.85	-2.13
-115	-5.565	-2.6118	-1.11	-41.32	-0.08
-110	-4.256	-1.7201	0.55	-38.69	1.62
-105	-3.115	-0.8395	1.90	-35.96	2.99
-100	-2.128	-0.4769	2.97	-33.18	4.05
-95	-1.302	-0.104	3.75	-30.35	4.82
-90	-0.616	-0.0319	4.29	-27.50	5.30
-85	-0.063	0.087	4.59	-24.64	5.55
-80	0.364	0.0376	4.68	-21.81	5.56
-75	0.679	0.0397	4.58	-19.03	5.38
-70	0.889	-0.0527	4.33	-16.30	5.03
-65	1.022	-0.0997	3.95	-13.67	4.55
-60	1.071	-0.1991	3.46	-11.14	3.95
-55	1.064	-0.2615	2.89	-8.73	3.27
-50	1.001	-0.3517	2.27	-6.46	2.54
-45	0.903	-0.4124	1.63	-4.36	1.79
-40	0.777	-0.4861	0.98	-2.43	1.04
-35	0.637	-0.5366	0.35	-0.69	0.31
-30	0.49	-0.5912	-0.24	0.84	-0.36
-25	0.35	-0.6276	-0.78	2.16	-0.97
-20	0.217	-0.6627	-1.23	3.25	-1.50
-15	0.098	-0.6838	-1.61	4.11	-1.92
-10	0	-0.7001	-1.88	4.73	-2.23
-5	-0.07	-0.7062	-2.04	5.10	-2.42
0	-0.119	-0.705	-2.10	5.22	-2.48
5	-0.119	-0.6974	-2.04	5.10	-2.42
10	0	-0.6803	-1.88	4.73	-2.23
15	0.098	-0.6607	-1.61	4.11	-1.92
20	0.217	-0.6293	-1.23	3.25	-1.50
25	0.35	-0.6	-0.78	2.16	-0.97
30	0.49	-0.5556	-0.24	0.84	-0.36
35	0.637	-0.5197	0.35	-0.69	0.31
40	0.777	-0.4637	0.98	-2.43	1.04
45	0.903	-0.4252	1.63	-4.36	1.79
50	1.001	-0.3595	2.27	-6.46	2.54
55	1.064	-0.3254	2.89	-8.73	3.27
60	1.071	-0.254	3.46	-11.14	3.95
65	1.022	-0.2371	3.95	-13.67	4.55
70	0.889	-0.1701	4.33	-16.30	5.03
75	0.679	-0.1962	4.58	-19.03	5.38
80	0.364	-0.1588	4.68	-21.81	5.56
85	-0.063	-0.2823	4.59	-24.64	5.55
90	-0.616	-0.3355	4.29	-27.50	5.30
95	-1.302	-0.6729	3.75	-30.35	4.82
100	-2.128	-0.9621	2.97	-33.18	4.05
105	-3.115	-1.7492	1.90	-35.96	2.99
110	-4.256	-2.5524	0.55	-38.69	1.62
115	-5.565	-3.7842	-1.11	-41.32	-0.08
120	-7.049	-4.7464	-3.08	-43.85	-2.13

Table 4.3.6 Lateral moment comparison

Mh (KN)					
Angle(°)	FE frame	FE shell	Reynolds	In plane	Research results
-120	39.2	29.04	39.63	0.00	41.09
-115	40.19	29.77	41.37	0.00	43.04
-110	40.97	30.35	42.85	0.00	44.72
-105	41.52	30.76	44.06	0.00	46.10
-100	41.83	30.99	44.98	0.00	47.16
-95	41.88	31.02	45.59	0.00	47.89
-90	41.67	30.87	45.87	0.00	48.26
-85	41.19	30.51	45.81	0.00	48.28
-80	40.44	29.96	45.41	0.00	47.92
-75	39.41	29.19	44.67	0.00	47.19
-70	38.11	28.23	43.58	0.00	46.09
-65	36.55	27.07	42.15	0.00	44.61
-60	34.72	25.72	40.39	0.00	42.78
-55	32.64	24.18	38.30	0.00	40.60
-50	30.32	22.46	35.91	0.00	38.08
-45	27.78	20.58	33.22	0.00	35.25
-40	25.03	18.54	30.26	0.00	32.12
-35	22.09	16.36	27.05	0.00	28.73
-30	18.99	14.07	23.62	0.00	25.09
-25	15.74	11.66	19.99	0.00	21.24
-20	12.37	9.16	16.20	0.00	17.22
-15	8.9	6.59	12.27	0.00	13.04
-10	5.37	3.98	8.24	0.00	8.76
-5	1.8	1.33	4.14	0.00	4.40
0	-1.79	-1.33	0.00	0.00	0.00
5	-5.37	-3.98	-4.14	0.00	-4.40
10	-8.9	-6.59	-8.24	0.00	-8.76
15	-12.37	-9.16	-12.27	0.00	-13.04
20	-15.75	-11.67	-16.20	0.00	-17.22
25	-19	-14.07	-19.99	0.00	-21.24
30	-22.11	-16.38	-23.62	0.00	-25.09
35	-25.05	-18.56	-27.05	0.00	-28.73
40	-27.81	-20.60	-30.26	0.00	-32.12
45	-30.35	-22.48	-33.22	0.00	-35.25
50	-32.67	-24.20	-35.91	0.00	-38.08
55	-34.75	-25.74	-38.30	0.00	-40.60
60	-36.58	-27.10	-40.39	0.00	-42.78
65	-38.14	-28.25	-42.15	0.00	-44.61
70	-39.42	-29.20	-43.58	0.00	-46.09
75	-40.43	-29.95	-44.67	0.00	-47.19
80	-41.16	-30.49	-45.41	0.00	-47.92
85	-41.61	-30.82	-45.81	0.00	-48.28
90	-41.79	-30.96	-45.87	0.00	-48.26
95	-41.69	-30.88	-45.59	0.00	-47.89
100	-41.33	-30.61	-44.98	0.00	-47.16
105	-40.72	-30.16	-44.06	0.00	-46.10
110	-39.88	-29.54	-42.85	0.00	-44.72
115	-38.81	-28.75	-41.37	0.00	-43.04
120	-37.55	-27.81	-39.63	0.00	-41.09

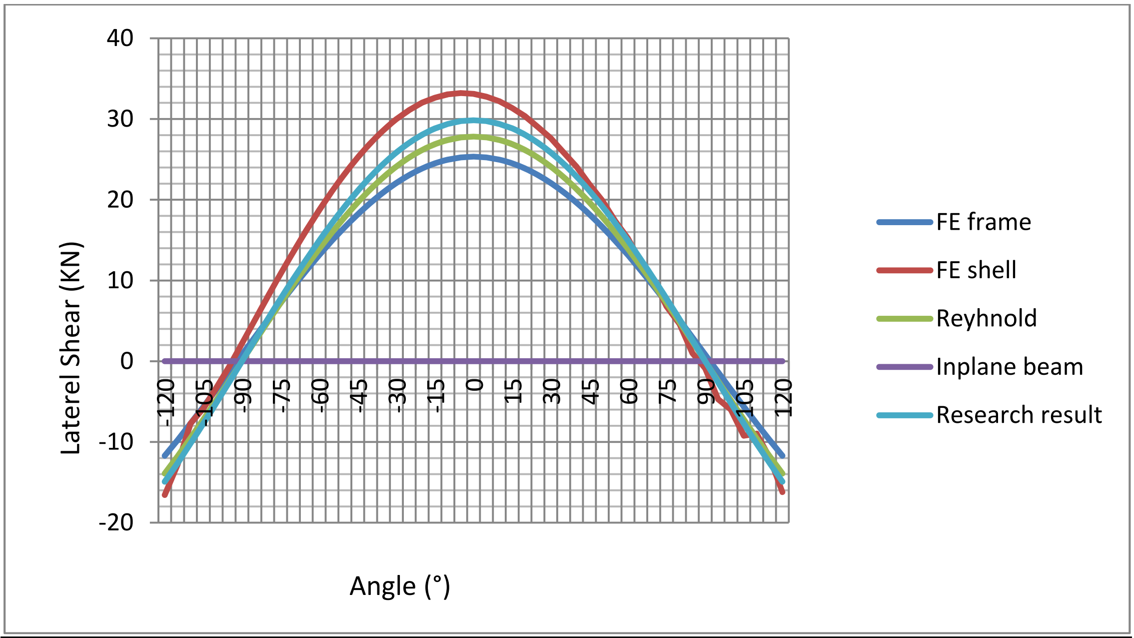

Figure 4.3.1 Graphical Comparison of lateral shear results

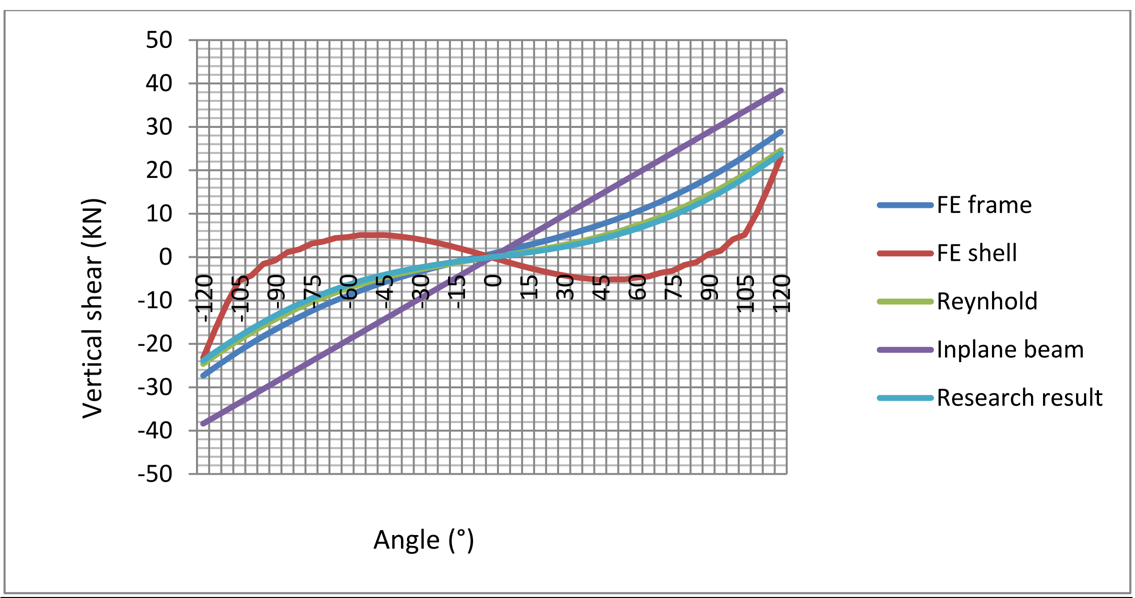

Figure 4.3.2 Graphical Comparison of vertical shear results

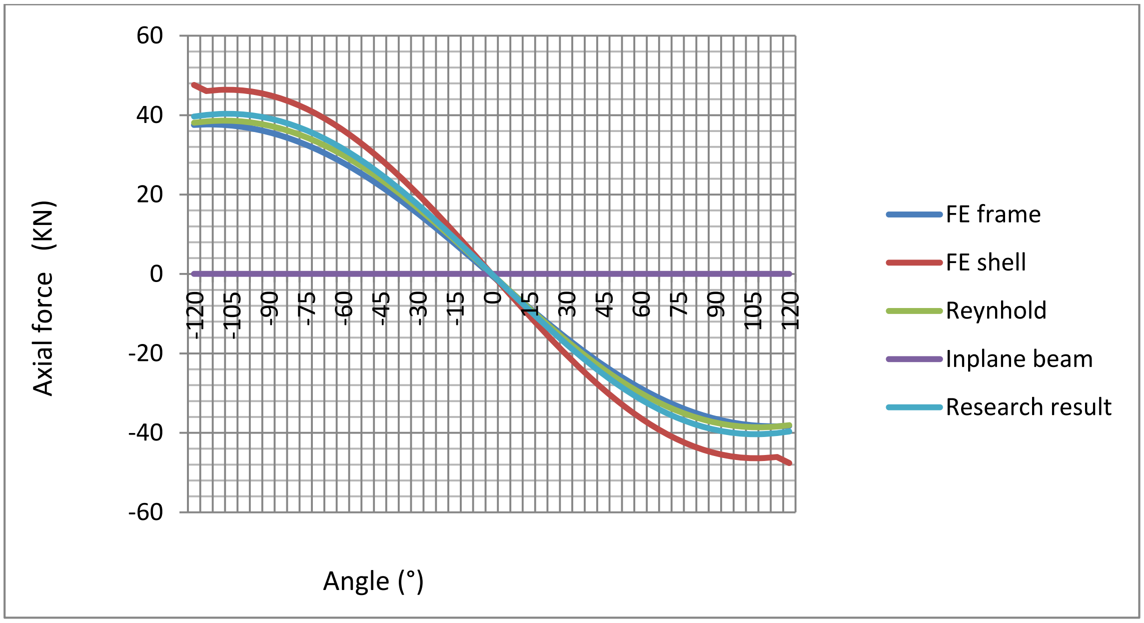

Figure 4.3.3 Graphical Comparison of axial force results

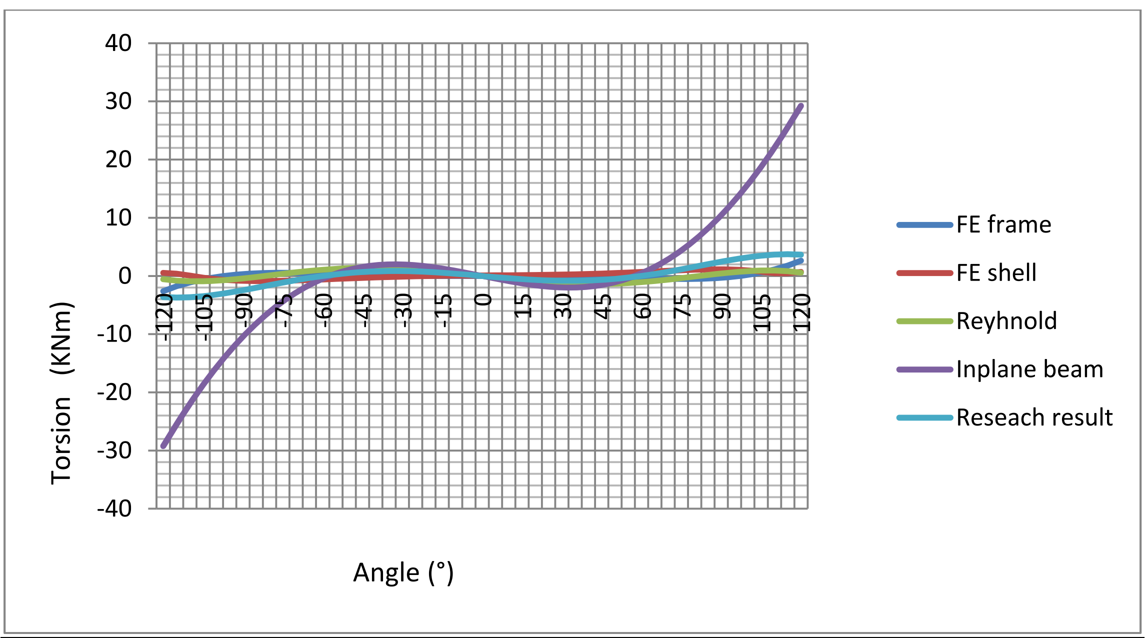

Figure 4.3.4 Graphical Comparison of Torsion results

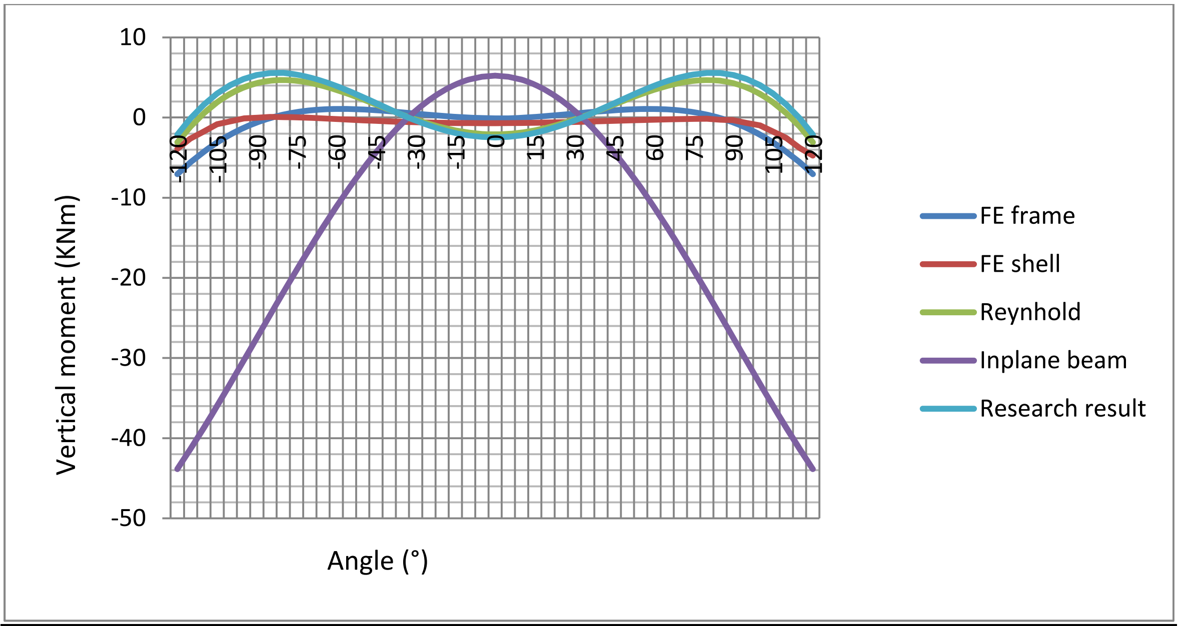

Figure 4.3.5 Graphical Comparison of vertical moment results

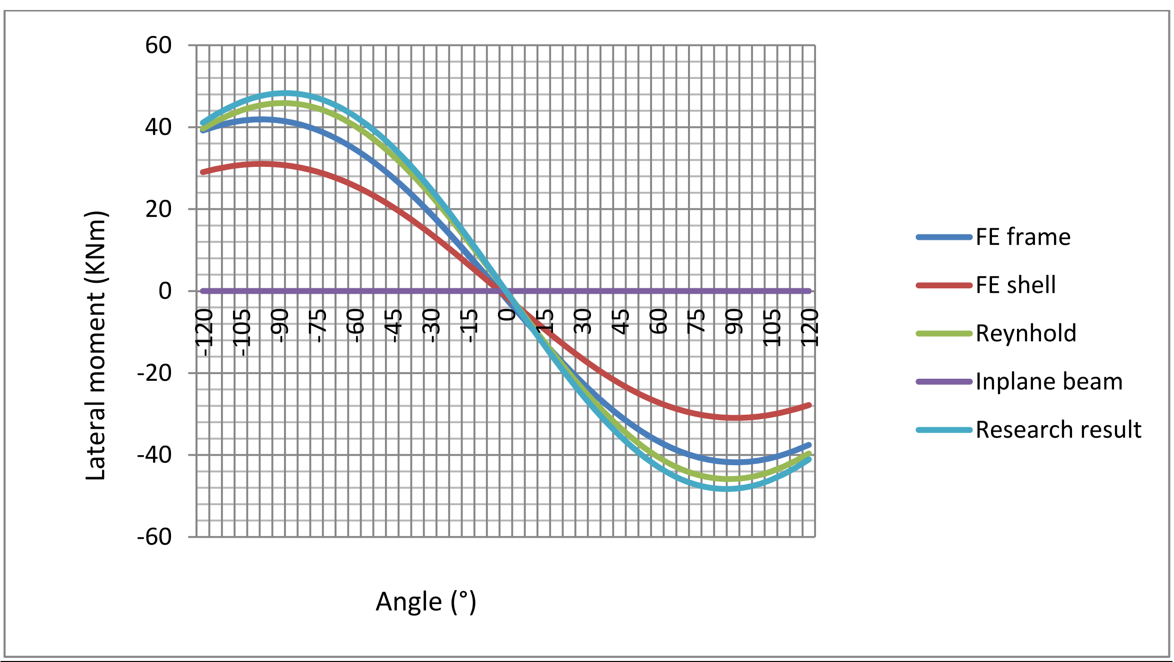

Figure 4.3.6 Graphical Comparison of lateral moment result

4.5.1 Observation on the comparison of results

From the graphical comparison, it is clear that there exist tremendous similarities as well as considerable differences between the results of the five approaches. In order to see through the variations, comparison is done in the following three categories.

A. Reynolds results Vs This research.
B. FE Frame Vs FE Shell Vs This research.
C. In plane Vs This research.

A. Reynolds results and This research.

The following observations are made between Reynolds and this research;

1. Lateral moment, Lateral shear, vertical shear and axial force results are very similar both in pattern and in values.
2. Vertical moment is also more or less similar, but around support Reynolds result is larger.
3. Torsion results of Reynolds are smaller as compared to the results of this research. The variation could have results because of the fact that lots of interpolations are involved in the chart provided by Reynolds.

B. FE frame, FE shell and This research

The following observations are made among FE frame, FE shell and This research;

1. Vh result is more or less the same with acceptable differences.
2. T and Mv values of the two results show difference even though the pattern is similar.
3. Vv and Mh values of FE frame and strain energy approach are similar but these values for FE shell are smaller.
4. Mh and Mv results of FE frame and strain energy are very close but the result of FE shell is smaller around support.

C. In plane beam solution Vs This research

The following observations are made between In plane solution and this research;

1. In plane beam result extremely exaggerates Mv and Mh especially around support.
2. The pattern of Vv of the two results is different. In case of in plane beam, it is straight line where as in the result of this research it is curve.
3. In plane beam solution completely ignores Vh, P and Mh.

In case of in plane beam solution, the 3D nature of the stair in approximated by a planar curve found by its in plane projection that is, Flight slope is considered to be zero. From this it can be concluded that In plane beam modeling of spiral stair is not recommended.

5. Sensitivity analysis

There are many engineering problems that are faced and these problems can be modeled mathematically, simulating the system at hand. The mathematical model may consist of different parameters which affect the out come. Hence, the process of examining the responsiveness of the systems out put to the variation of the various parameters is sensitivity analysis.

In the process of analysis, the study of sensitivity of internal actions to different geometric parameters is important. This is because geometric modifications are usually common by architects and when a geometric change occurs, the designer has to be aware of the sensitive internal actions due to those changes. In this section the sensitivity of the six internal actions of the spiral is studied against the geometric parameters of flight slope, radius and total angle of twist. The results are presented graphically. The sensitivity assessment is performed by varying one parameter and keeping two parameters constant, which will be done for all the three geometric parameters.

5.1 Varying Radius while keeping Alpha and Theta constant

Here by keeping flight slope alpha and angle of twist theta constant, the radius is varied from 1m to 2.5m with the interval of 0.5m, and the variations in the internal actions are observed.

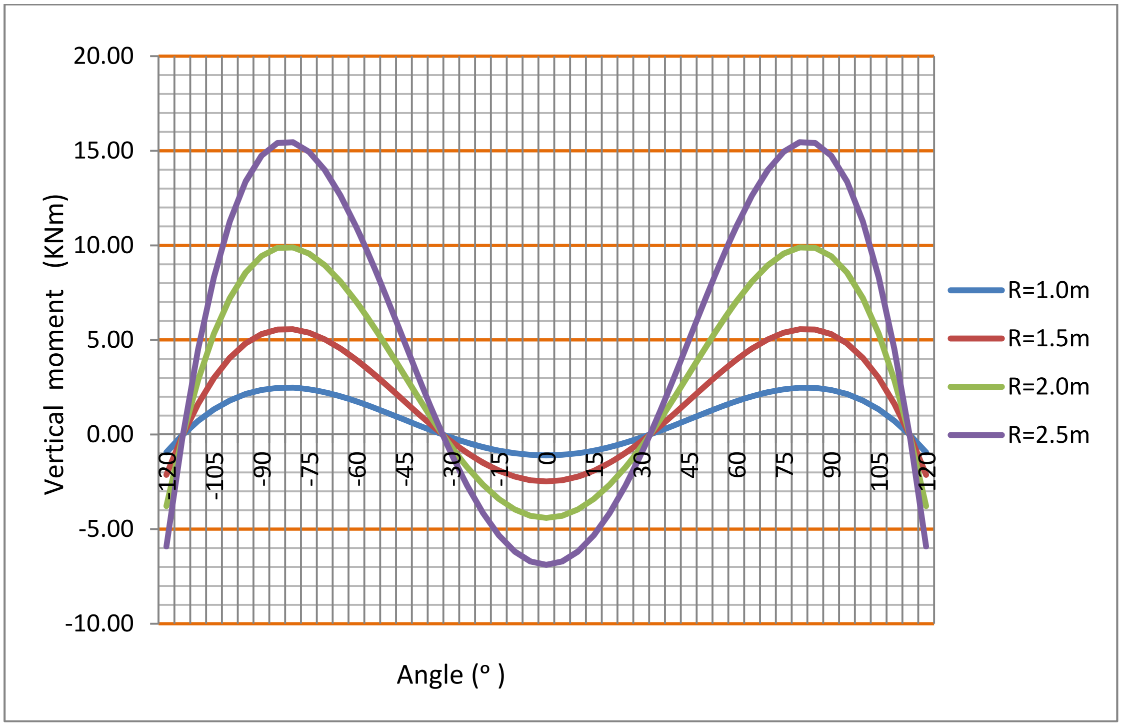

Figure 5.1.1 Sensitivity of vertical moment to radius

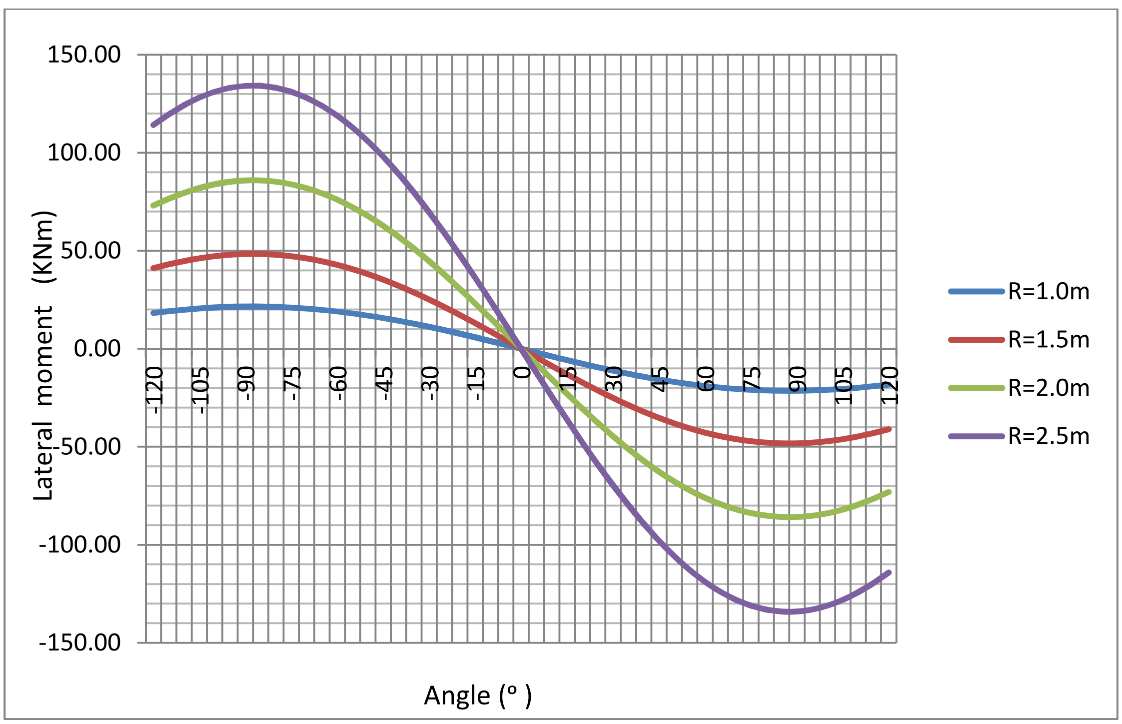

Figure 5.1.2 Sensitivity of lateral moment to radius

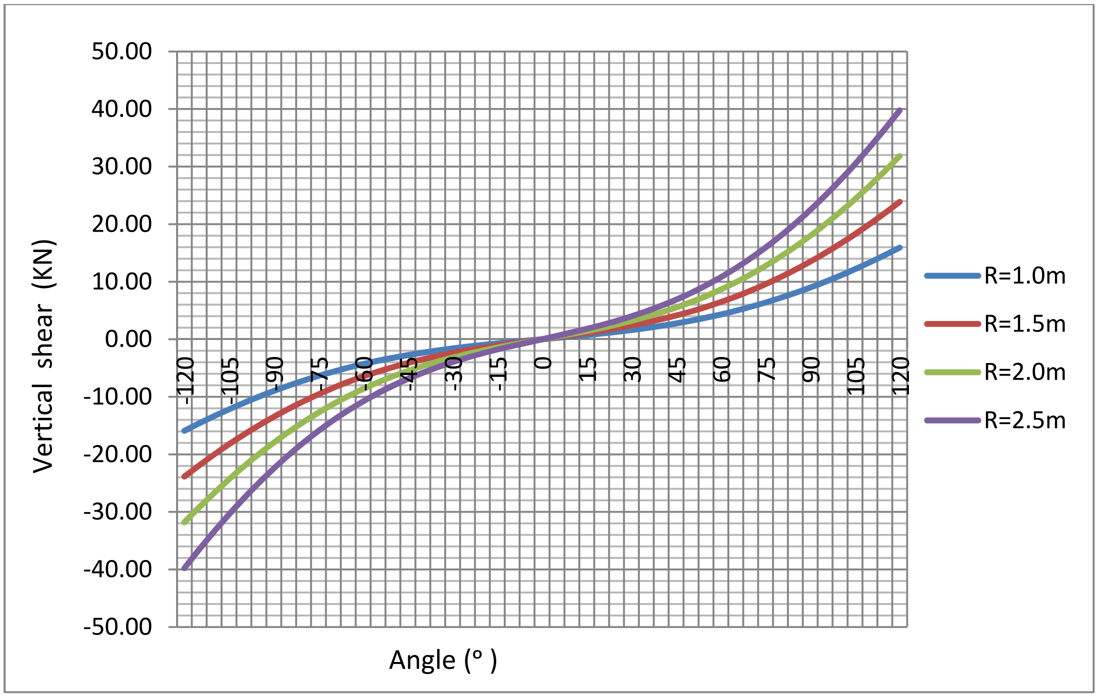

Figure 5.1.3 Sensitivity of vertical shear to radius

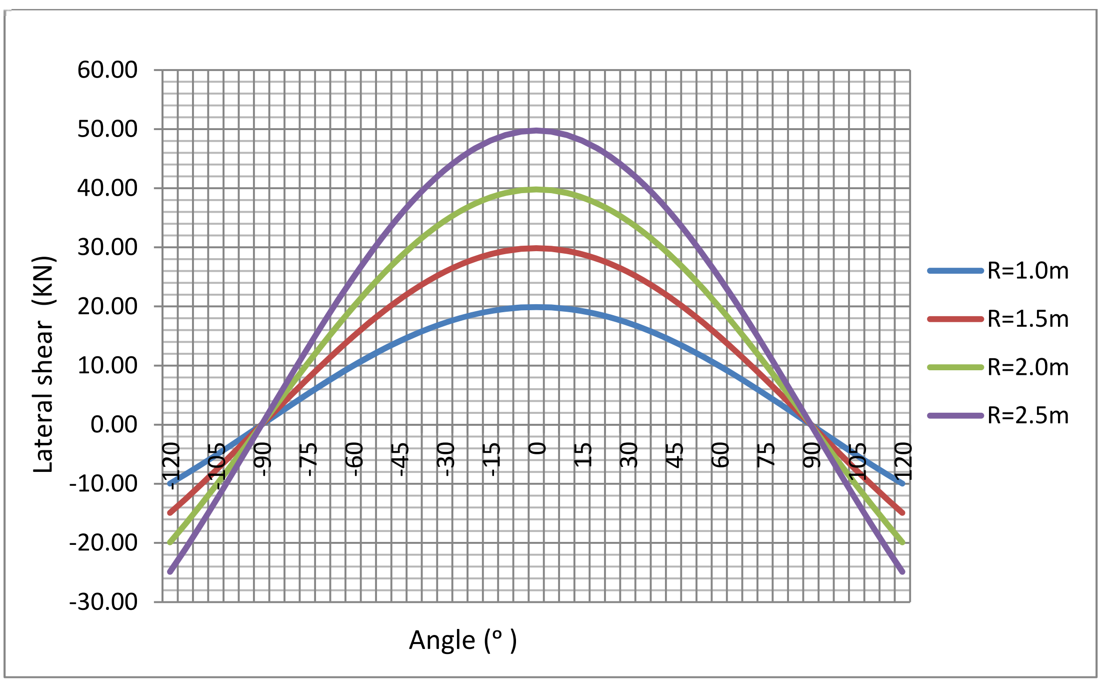

Figure 5.1.4 Sensitivity of lateral shear to radius

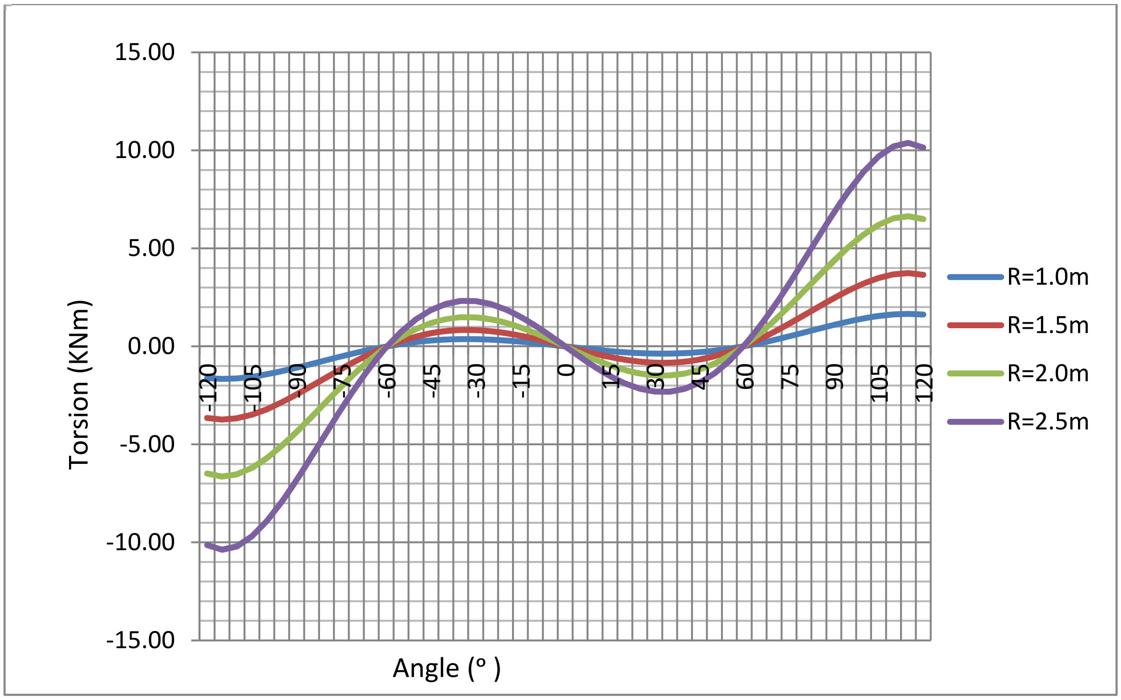

Figure 5.1.5 Sensitivity of twisting moment to radius

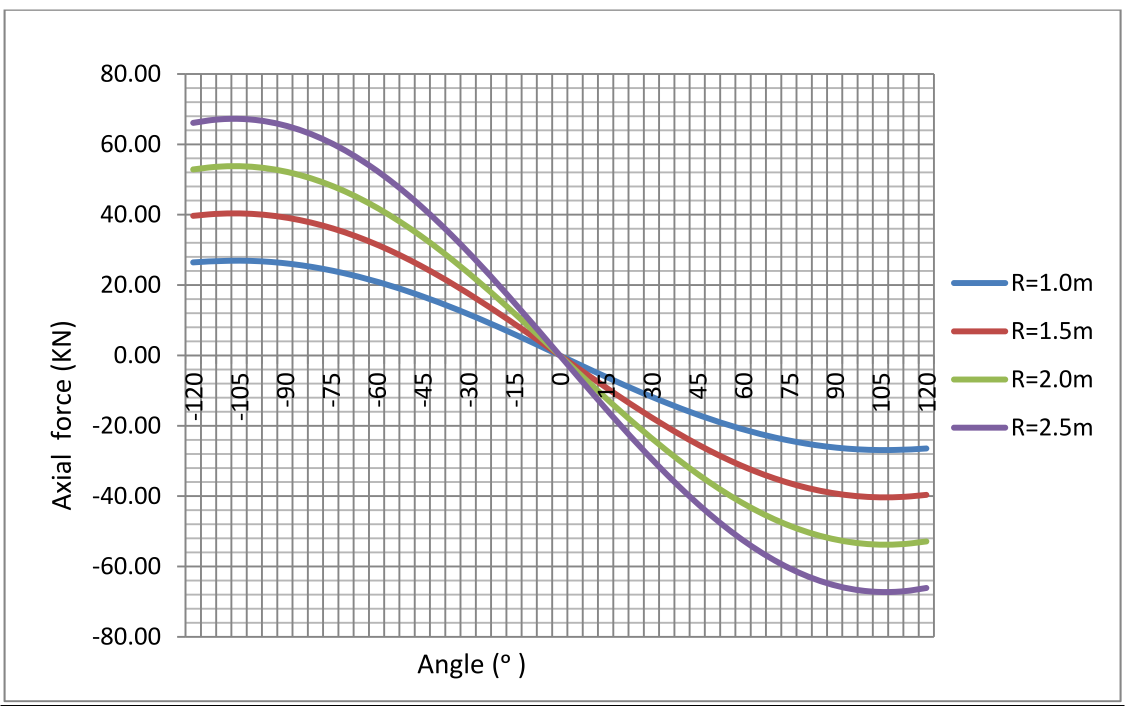

Figure 5.1.6 Sensitivity of axial force to radius

The results of this sensitivity analysis are summarized as follows:

- Increase in radius results in an increase in all of the internal actions but the rate of change is higher for lateral moment and vertical moment as can be seen in the graphs.
- As radius increases, significant increase in lateral shear at mid span where as for vertical shear, torsion and axial force, the case is true around support.
- Considerable increment is observed in vertical moment between mid span and support.

5.2 Varying Alpha while keeping Radius and Theta constant

Here by keeping radius and angle of twist constant, the flight slope alpha is varied from 20° to 35° with the interval of 5°, and the variations in the internal actions are observed.

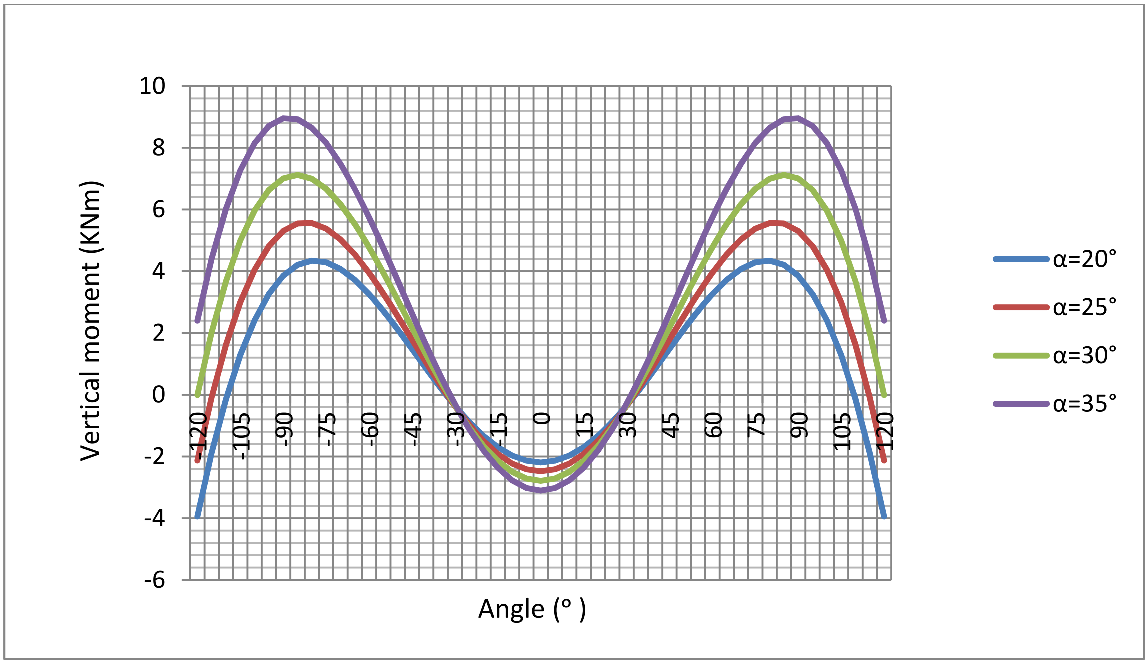

Figure 5.2.1 Sensitivity of vertical moment to flight slope

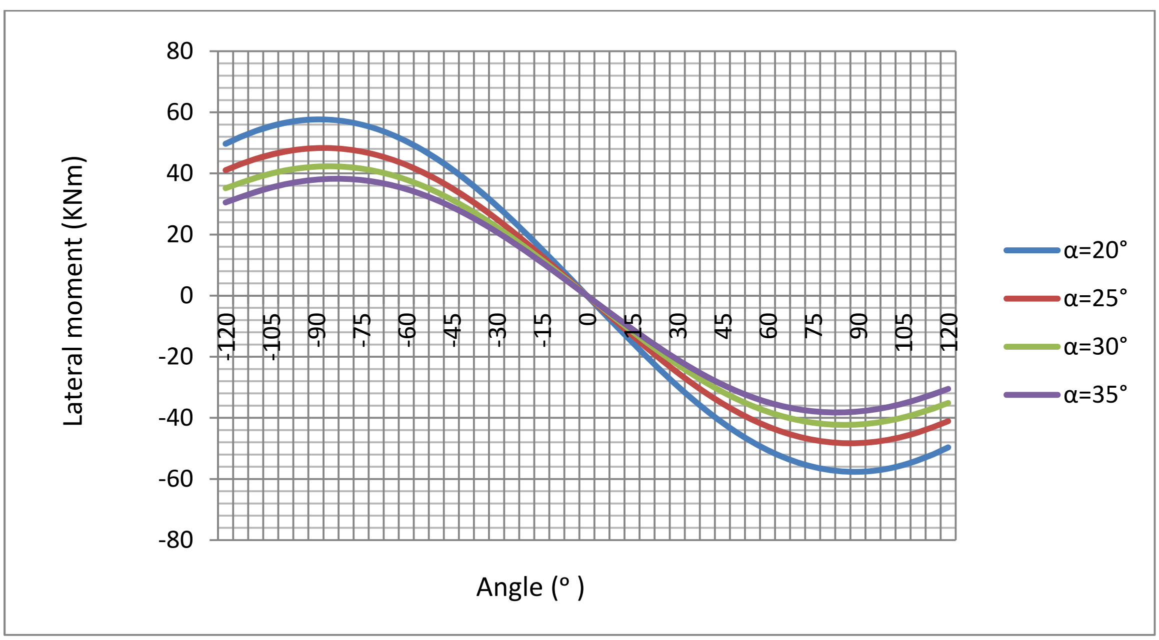

Figure 5.2.2 Sensitivity of lateral moment to flight slope

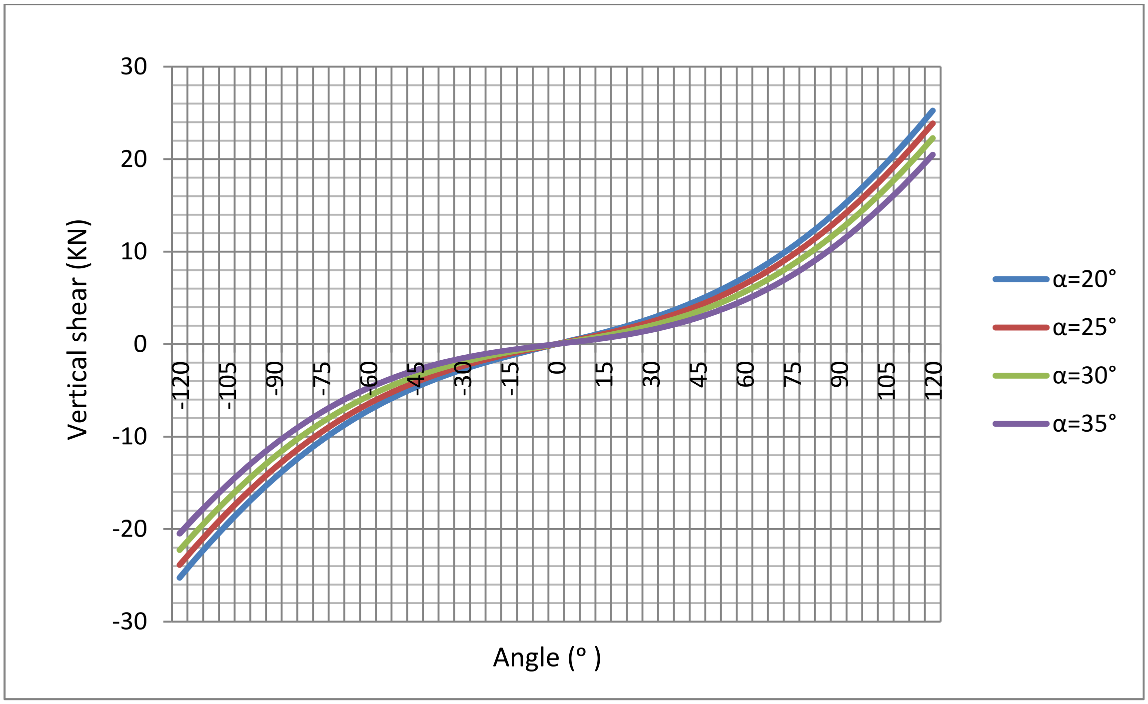

Figure 5.2.3 Sensitivity of vertical shear to flight slope

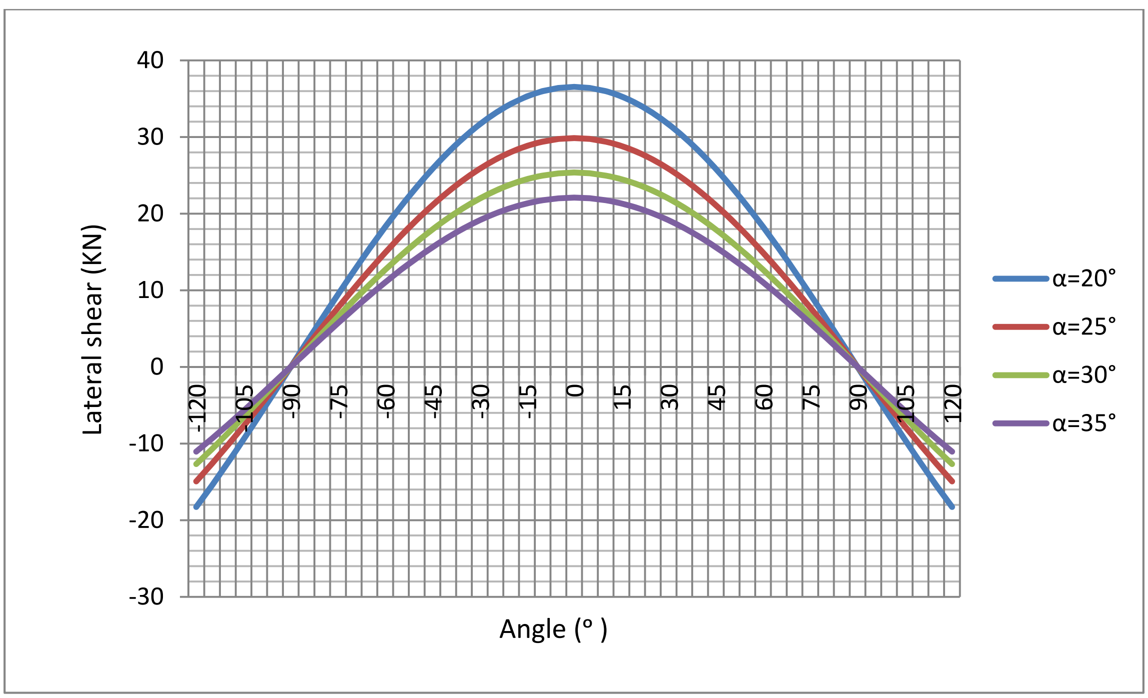

Figure 5.2.4 Sensitivity of lateral shear to flight slope

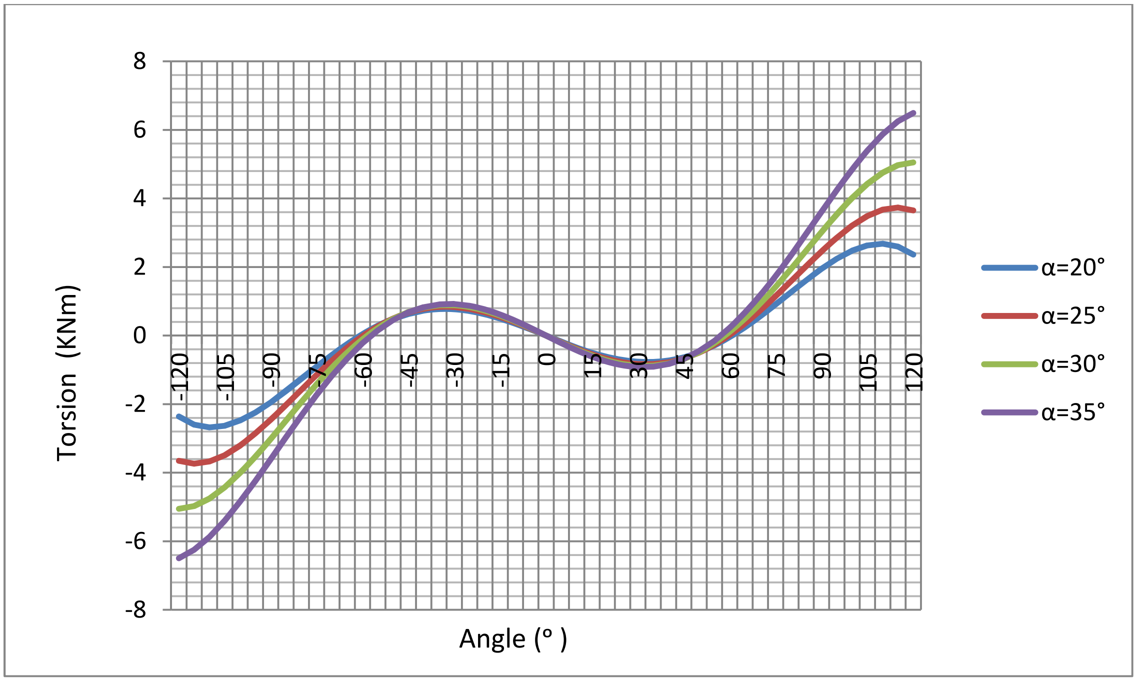

Figure 5.2.5 Sensitivity of twisting moment to flight slope

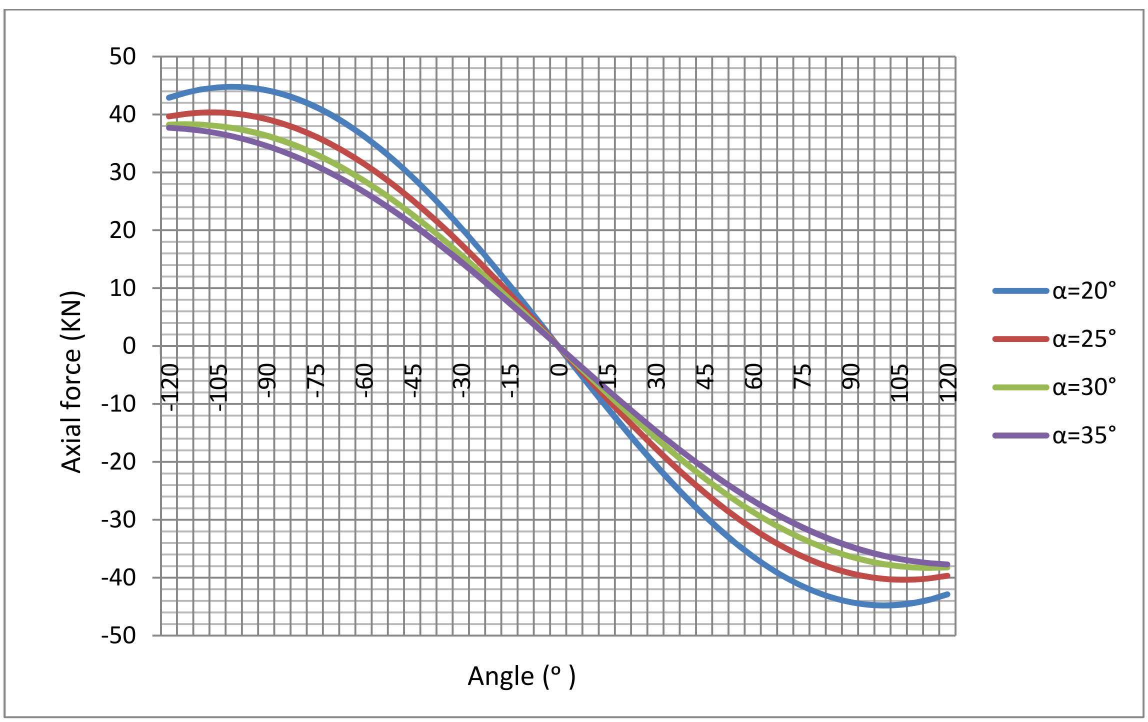

Figure 5.2.6 Sensitivity of axial force to flight slope

The results of this sensitivity analysis are summarized as follows:

- As flight slope increases, vertical moment and torsion increase whereas vertical shear, lateral moment, lateral shear and axial force decrease as it is well seen in the graphs.
- Change in torsion due to increment in flight slope around support is higher than around span.
- Change in vertical moment is high between support and span where as it is lower elsewhere.
- Due to an increment in flight slope, change in lateral shear is generally higher especially at mid span; where as change in vertical shear is very small.

5.3 Varying Theta while keeping Radius and flight slope constant

Here by keeping radius and flight slope alpha constant, the angle of twist is varied from 180° to 120° with the interval of 20°, and the variations in the internal actions are observed.

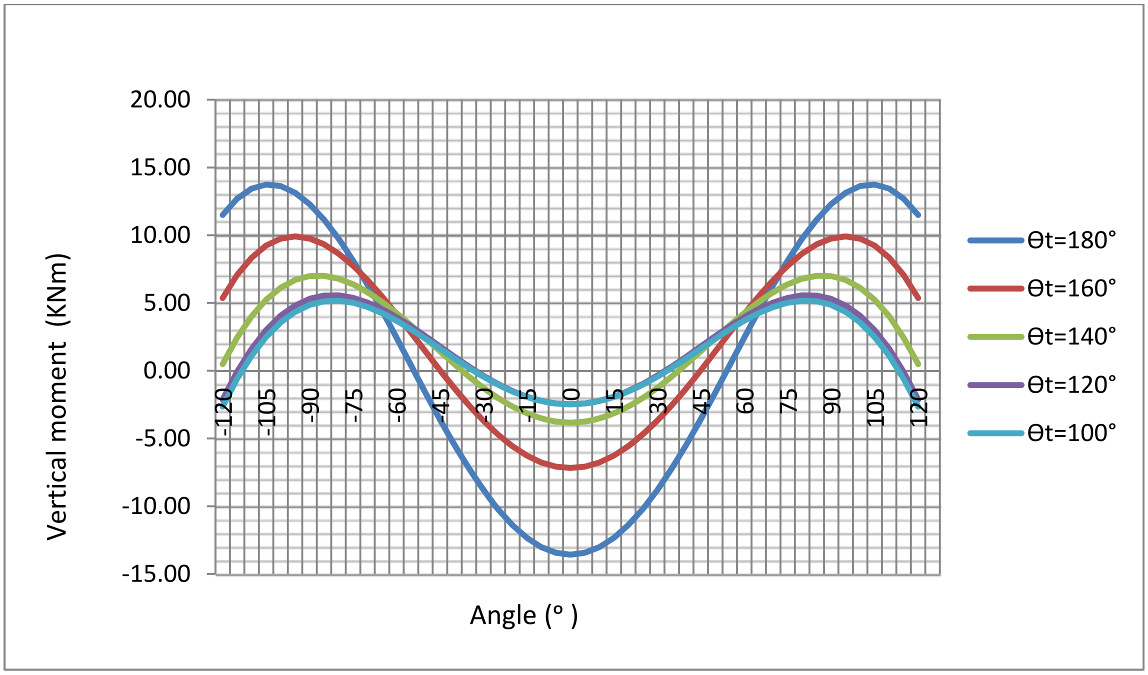

Figure 5.3.1 Sensitivity of vertical moment to twist angle

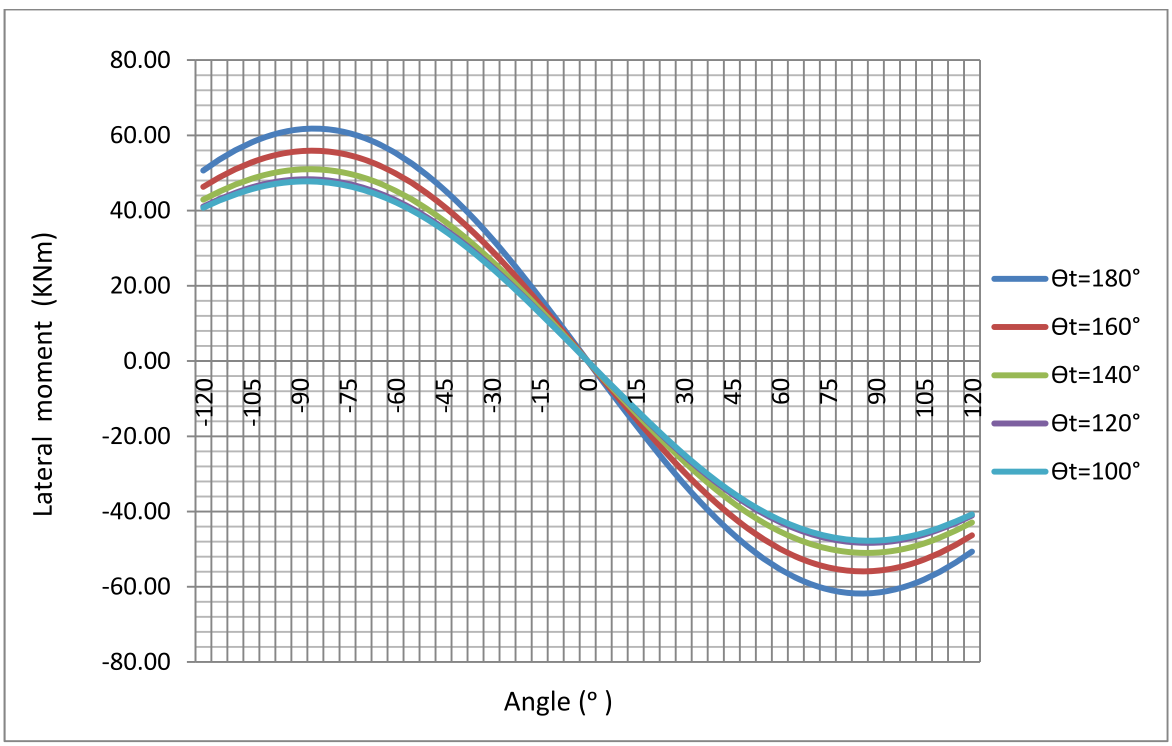

Figure 5.3.2 Sensitivity of lateral moment to twist angle

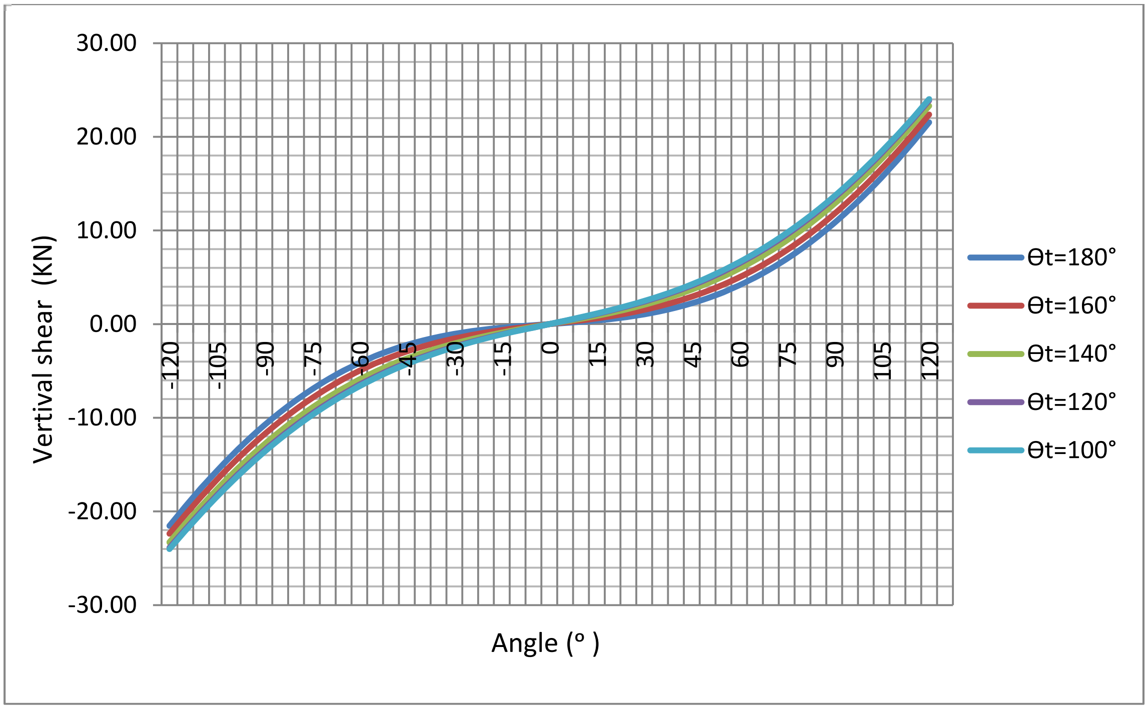

Figure 5.3.3 Sensitivity of vertical shear to twist angle

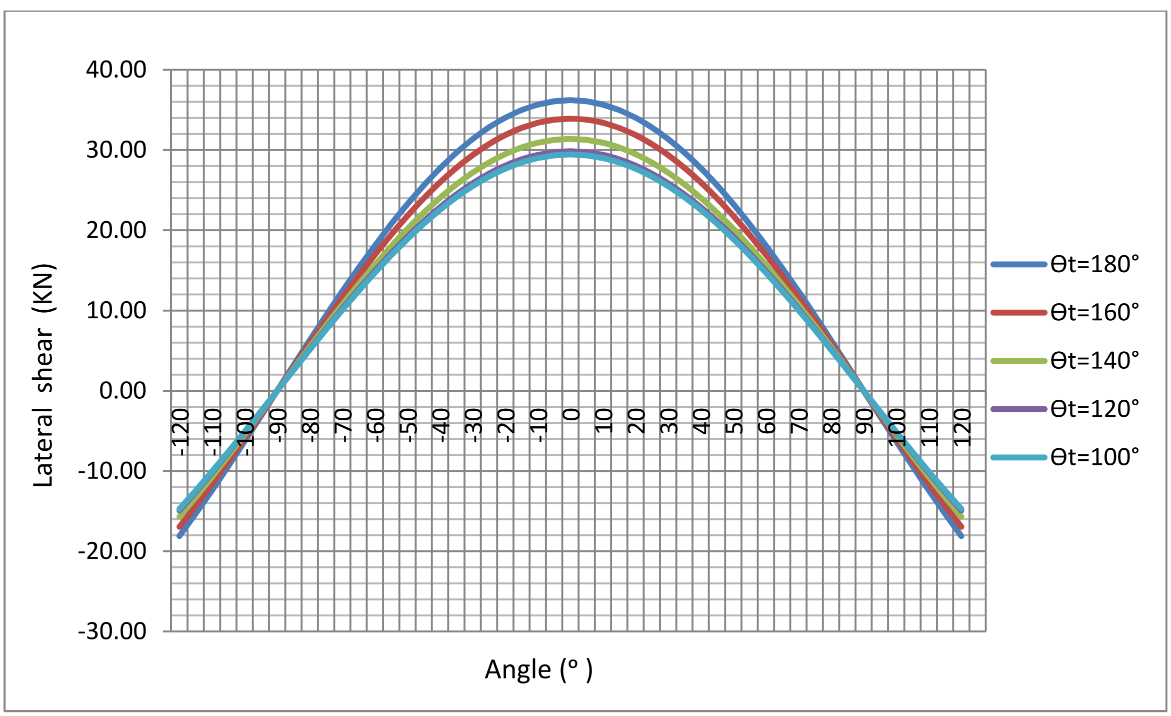

Figure 5.3.4 Sensitivity of lateral shear to twist angle

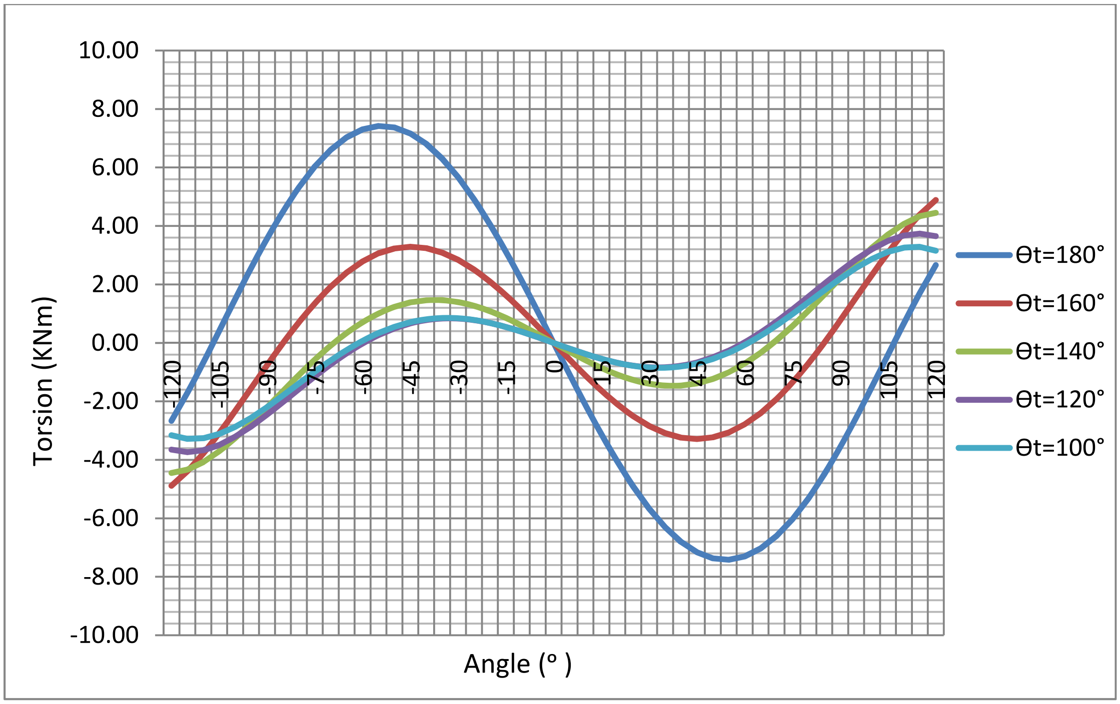

Figure 5.3.5 Sensitivity of torsion moment to twist angle

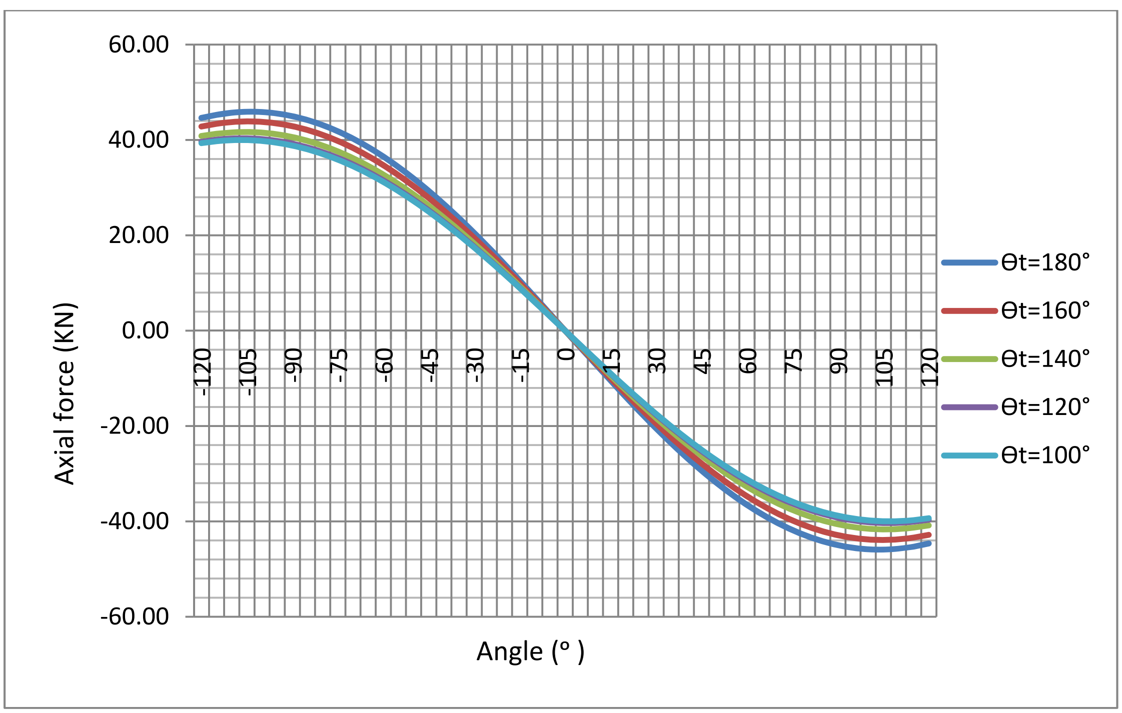

Figure 5.3.6 Sensitivity of axial force to twist angle

The results of this sensitivity analysis are summarized as follows:

- As total angle of twist decrease, only vertical shear increase, the rest of the five internal actions decrease.
- Decrement in torsion is much higher where as for vertical moment, lateral moment and lateral shear is small.
- Decrement in angle of twist has low effect on lateral moment and axial.

6. Modification factor formulation and valid width demarcation

6.1 Modification factor formulation for helical girder solution

The modification factors for helical girder solution are required mainly because the variation of actions along the stair width has to be taken into account. These factors will be applied for determination of the peak action along the span, for design purpose. Modification factors are computed by considering variation of action effects along stair width and the variation of results of strain energy solution with finite element results. The variation of action effects along stair width is best observed by shell finite element which simulates the closest to realistic model. Hence, the finite element technique for this purpose is done by modeling the stair as twisting shell.

The factors are formulated by comparing finite element shell results with the strain energy solutions. In order to compare the variations of action effects along the width, five lines of actions on shell finite element solution are considered. On each line of action, different points are taken to be compared with strain energy results. Hence many data points are analyzed to come up with the modification factors. Spiral stairs of various geometries are used to make the data representative enough. This is done for all 6 internal actions. Samples of these are presented in graphical form below for axial force and lateral shear.

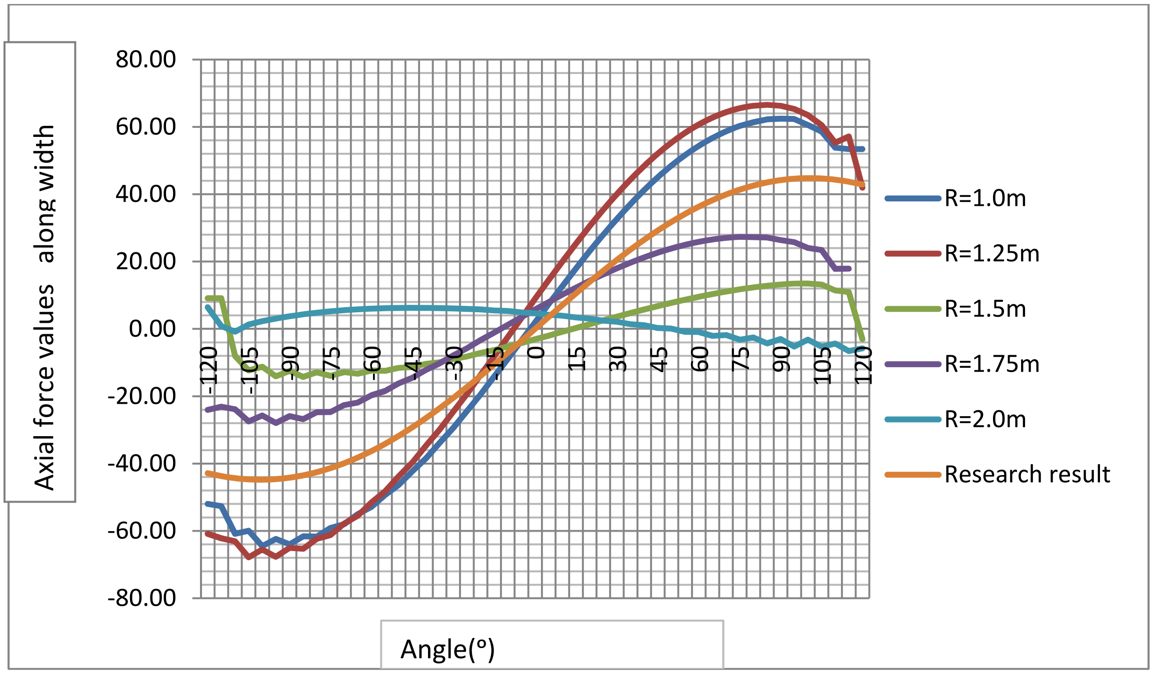

Figure 6.1.1: variation of axial force along width

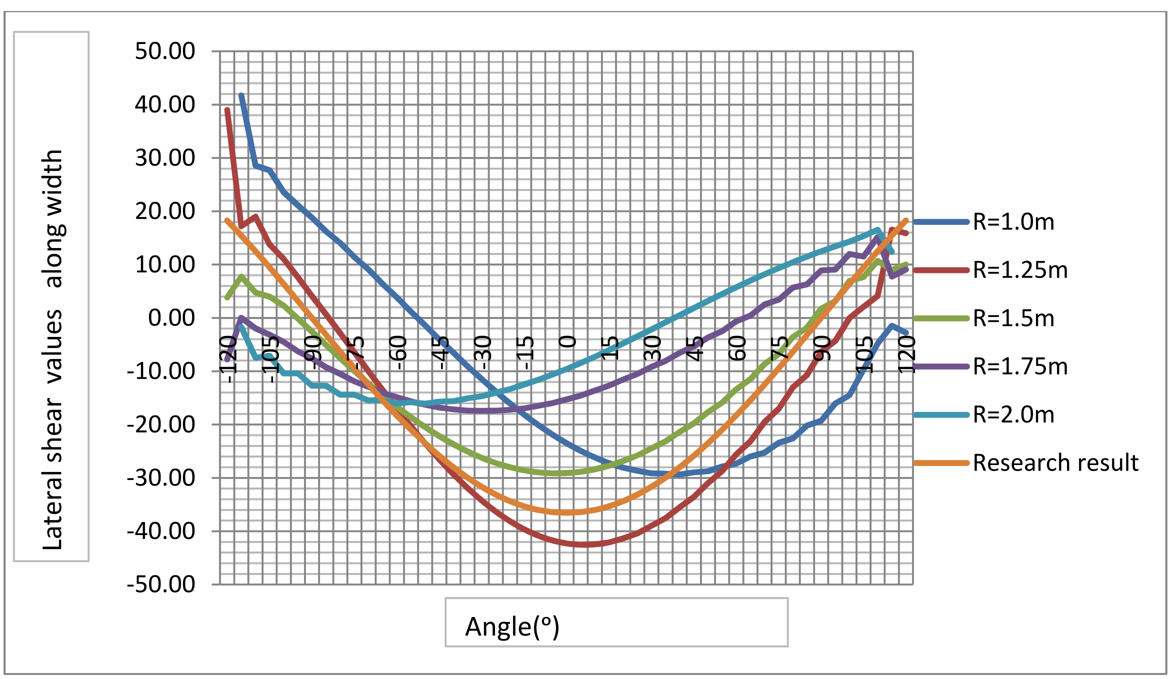

Figure 6.1.2: variation of lateral shear along width

Similarly, such tables will be prepared for vertical shear, vertical moment and torsion to compile problems with flight slope of 20°. By the same technique, tables will be prepared for flight slopes of 25°, 30° and 35°. Hence a total data of 24 groups is prepared for the six internal actions. Each internal action has many points to be compared with strain energy results to formulate the modification factors. This is done by taking the maximum factor of the fifty points in along a line then taking the maximum of the maximum from the 250 points in a group. Hence there will be four factor groups representing the 250 points. Finally by taking the weighted mean of the four values on behalf of the 1000 points, a representative modification factor is found. Using the same technique, modification factors are prepared for all six internal actions. The summary of the representative four groups of values are stated below.

Tables 6.1.1: Modification factors summaries

		Modification factors for Internal actions					
Flight slope(°)	Sign	P	Vh	Mv	T	Mh	Vv
20	for positive	1.39	1.14	1.00	1.02	1.00	1.29
	for Negative	1.39	1.14	1.25	1.00	1.00	1.29
25	for positive	1.40	1.23	1.00	1.00	1.14	1.36
	for Negative	1.40	1.23	1.32	1.00	1.31	1.36
30	for positive	1.26	1.27	1.00	1.00	1.59	1.33
	for Negative	1.26	1.27	1.26	1.00	1.32	1.33
35	for positive	1.13	1.29	1.00	1.00	1.48	1.31
	for Negative	1.13	1.29	1.02	1.00	1.52	1.31
Weighted mean	for positive	1.27	1.24	1.00	1.00	1.24	1.33
	for Negative	1.27	1.24	1.20	1.00	1.26	1.33

6.2 Stair width demarcation for center line model validity of helical stairs

The demarcation of width for center line model of helical stairs is important because for different width of stairs, the same center line model is used in these spiral girder solutions. Hence the valid stair width for the usage of center line model has to be clearly demarked. In other words, the stair width will be demarked for which strain energy solution will be sufficiently applicable. This demarcation is done by strain energy solutions in comparison with finite element analysis which considers problems of different stair width. This procedure is the same as the one used for modification factor except for varying the stair width from 1m to 2.5m with an interval of 0.5m. Hence the width demarcation is done by limiting the modification factors. The stair width having modification factor of more than 2 will be the demarcation width, as the variation along width is large. Sample graphs are given below.

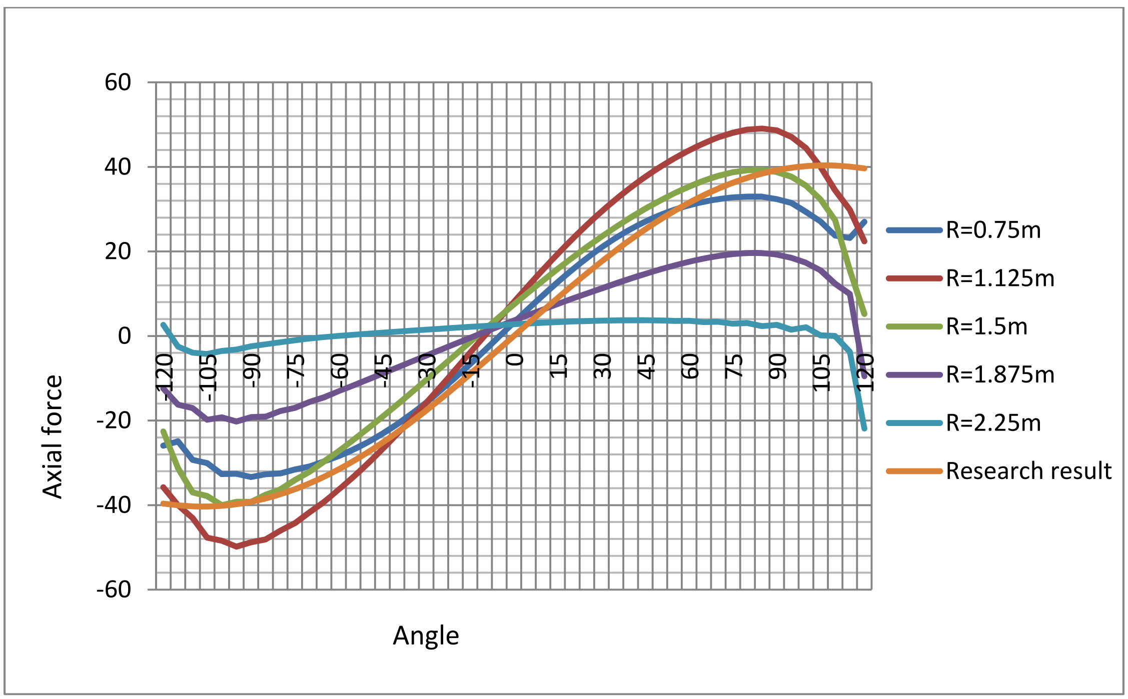

Figure 6.2.1: Variation of axial force along width

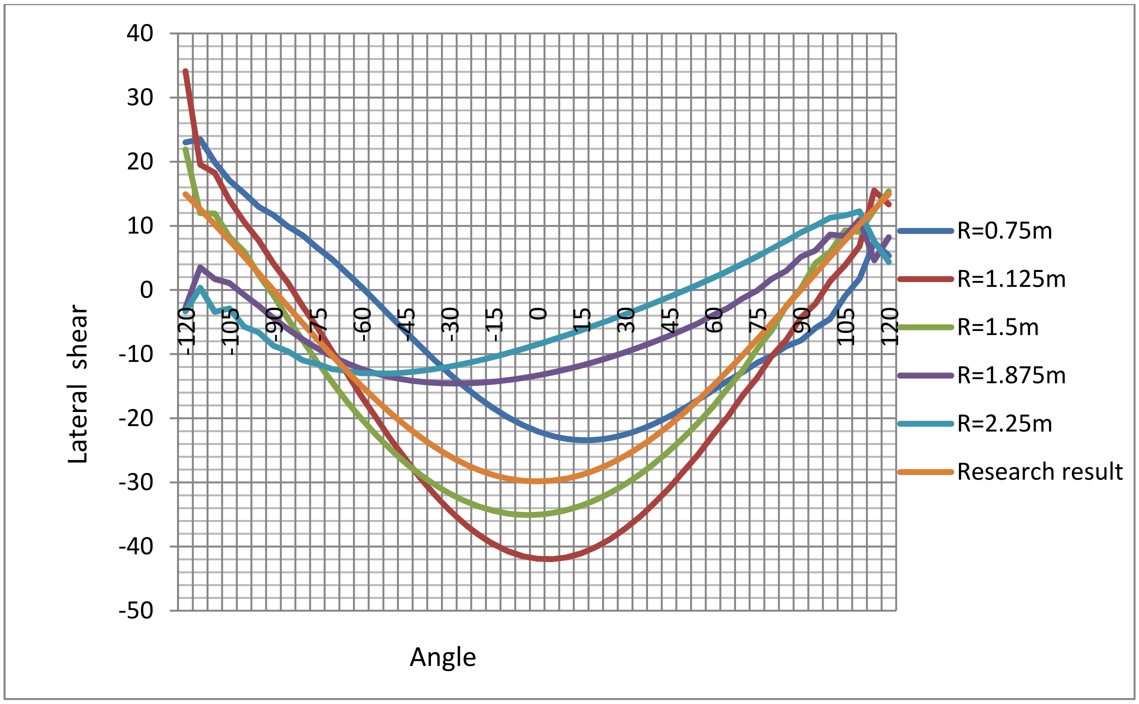

Figure 6.2.2: Variation of lateral shear along width

Tables 6.2.1: Summary of factors for width demarcation

		Modification factors for internal actions					
Stair width	Sign	P	Vh	Mv	T	Mh	Vv
W=1m	Positive	1.27	1.24	1.00	1.00	1.24	1.33
	Negative	1.27	1.24	1.20	1.00	1.26	1.33
W=1.5m	Positive	1.24	1.57	1.00	1.25	1.27	1.51
	Negative	1.24	1.57	1.09	1.25	1.21	1.51
W=2.0m	Positive	1.51	2.01	1.00	1.59	1.77	2.00
	Negative	1.51	2.01	2.10	1.55	1.53	2.00
W=2.5m	Positive	1.69	3.08	1.00	1.85	1.82	3.13
	Negative	1.69	3.08	3.03	1.83	1.82	3.13

From the summary of tables, it can be seen that for stair width more than 2m, the modification factors become very large like three times. Hence, the demarcation width of stair for a valid center line simulation is 2m. As a result, this system is valid for problems with not more than 2m stair width.

7. Conclusion and Recommendation

7.1 Conclusion

From the various investigations and numerous issues covered in this research, the following conclusions and recommendations are made.

- The analysis tables and charts presented in the annex section together with sensitivity analysis and modification factors formulated in this work are very helpful in analyzing these complex structural members.
- Strain energy technique is an applicable tool for Spiral stair analysis.
- Modeling of helical stair as in plane curved beam using their planar projection is inappropriate because it overestimates vertical moment, vertical shear and torsion where as it ignores lateral shear, lateral moment and axial force.
- Results of sensitivity analysis reveal that,
    - As radius increases, all internal actions increase especially vertical moment and torsion.
    - As flight slope increase, vertical moment and torsion increase where as vertical shear, lateral moment, lateral shear and axial force decrease
    - As the angle of twist decrease, only vertical shear increase, the rest of the five internal actions decrease.

- Helical girder solution is satisfactory only for stairs of width not more than 2m, due to the deviation of the off-center results from center line results.

7.2 Recommendation

- This research is focused on fixed ended circular spiral stairs subjected to uniformly distributed loads; hence for future study, it is recommended to develop such tools for elliptical spiral stair and non-uniform loading.
- From what was observed from sensitivity analysis and width demarcation , stairs with width of more than two meters are not advised to be analyzed by this technique ,hence for stairs with more than 2m width , specific study is recommended by observing the behavior.
- Vehicular loads are not considered as the research is focused on pedestrian helix, hence for the future it is advised to consider vehicular ramps with large radius.
- Displacement investigation is not included in this research; for the future, it is advised to further explore in this area.
- Experimental approach is also a great approach to make a research on this area even though it is expensive.

It is believed that the objectives of this research which are stated in section (1.2.1) are achieved and ultimately a helpful analysis charts and tables are produced. In addition, from the different points raised and covered in this research like sensitivity analysis, modification factor formulation and width demarcation, it is observed that spiral stair system is really complicated and an even further detail study involving experimental work is vital. The charts presented are useful as a quick reference to analyze spiral stairs. It should be noted that tremendous effort and time is spent to finalize this work. A message to who ever uses these tables and charts is that, it is of at most importance for the user to incorporate his/ her own engineering judgment in combination, instead of just picking up values from the charts**.**

8. Reference

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ANNEX A: Tabular and graphical analysis aid.

Table A 1: Flight slope =20° & Radius =1.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-18.43	0.95	-36.08	-31.13	2.62	13.13
-175	-15.39	3.34	-34.14	-31.02	1.24	15.32
-170	-12.50	5.79	-32.22	-30.66	0.09	17.48
-165	-9.77	8.28	-30.31	-30.07	-0.82	19.61
-160	-7.23	10.78	-28.42	-29.26	-1.51	21.68
-155	-4.89	13.27	-26.56	-28.22	-2.01	23.67
-150	-2.76	15.72	-24.74	-26.96	-2.32	25.57
-145	-0.86	18.11	-22.95	-25.50	-2.47	27.36
-140	0.82	20.40	-21.21	-23.85	-2.47	29.02
-135	2.27	22.58	-19.53	-22.02	-2.34	30.54
-130	3.48	24.62	-17.90	-20.01	-2.10	31.89
-125	4.47	26.49	-16.33	-17.86	-1.78	33.08
-120	5.23	28.19	-14.83	-15.57	-1.38	34.09
-115	5.77	29.69	-13.40	-13.16	-0.92	34.90
-110	6.10	30.98	-12.04	-10.65	-0.44	35.52
-105	6.23	32.03	-10.76	-8.06	0.07	35.92
-100	6.18	32.84	-9.55	-5.41	0.58	36.11
-95	5.96	33.40	-8.43	-2.71	1.08	36.08
-90	5.58	33.70	-7.39	0.00	1.55	35.82
-85	5.07	33.73	-6.43	2.71	1.99	35.35
-80	4.44	33.50	-5.55	5.41	2.38	34.65
-75	3.72	32.99	-4.75	8.06	2.72	33.73
-70	2.91	32.22	-4.02	10.65	2.99	32.60
-65	2.06	31.19	-3.38	13.16	3.19	31.26
-60	1.16	29.90	-2.80	15.57	3.33	29.71
-55	0.25	28.37	-2.30	17.86	3.38	27.98
-50	-0.65	26.61	-1.86	20.01	3.37	26.06
-45	-1.54	24.62	-1.49	22.02	3.28	23.97
-40	-2.38	22.43	-1.17	23.85	3.12	21.72
-35	-3.16	20.06	-0.91	25.50	2.89	19.33
-30	-3.88	17.51	-0.69	26.96	2.60	16.82
-25	-4.50	14.82	-0.51	28.22	2.25	14.19
-20	-5.03	12.01	-0.37	29.26	1.86	11.47
-15	-5.45	9.10	-0.25	30.07	1.43	8.67
-10	-5.76	6.11	-0.16	30.66	0.97	5.81
-5	-5.95	3.07	-0.07	31.02	0.49	2.91
0	-6.01	0.00	0.00	31.13	0.00	0.00
5	-5.95	-3.07	0.07	31.02	-0.49	-2.91
10	-5.76	-6.11	0.16	30.66	-0.97	-5.81
15	-5.45	-9.10	0.25	30.07	-1.43	-8.67
20	-5.03	-12.01	0.37	29.26	-1.86	-11.47
25	-4.50	-14.82	0.51	28.22	-2.25	-14.19
30	-3.88	-17.51	0.69	26.96	-2.60	-16.82
35	-3.16	-20.06	0.91	25.50	-2.89	-19.33
40	-2.38	-22.43	1.17	23.85	-3.12	-21.72
45	-1.54	-24.62	1.49	22.02	-3.28	-23.97
50	-0.65	-26.61	1.86	20.01	-3.37	-26.06
55	0.25	-28.37	2.30	17.86	-3.38	-27.98
60	1.16	-29.90	2.80	15.57	-3.33	-29.71
65	2.06	-31.19	3.38	13.16	-3.19	-31.26
70	2.91	-32.22	4.02	10.65	-2.99	-32.60
75	3.72	-32.99	4.75	8.06	-2.72	-33.73

80	4.44	-33.50	5.55	5.41	-2.38	-34.65
85	5.07	-33.73	6.43	2.71	-1.99	-35.35
90	5.58	-33.70	7.39	0.00	-1.55	-35.82
95	5.96	-33.40	8.43	-2.71	-1.08	-36.08
100	6.18	-32.84	9.55	-5.41	-0.58	-36.11
105	6.23	-32.03	10.76	-8.06	-0.07	-35.92
110	6.10	-30.98	12.04	-10.65	0.44	-35.52
115	5.77	-29.69	13.40	-13.16	0.92	-34.90
120	5.23	-28.19	14.83	-15.57	1.38	-34.09
125	4.47	-26.49	16.33	-17.86	1.78	-33.08
130	3.48	-24.62	17.90	-20.01	2.10	-31.89
135	2.27	-22.58	19.53	-22.02	2.34	-30.54
140	0.82	-20.40	21.21	-23.85	2.47	-29.02
145	-0.86	-18.11	22.95	-25.50	2.47	-27.36
150	-2.76	-15.72	24.74	-26.96	2.32	-25.57
155	-4.89	-13.27	26.56	-28.22	2.01	-23.67
160	-7.23	-10.78	28.42	-29.26	1.51	-21.68
165	-9.77	-8.28	30.31	-30.07	0.82	-19.61
170	-12.50	-5.79	32.22	-30.66	-0.09	-17.48
175	-15.39	-3.34	34.14	-31.02	-1.24	-15.32
180	-18.43	-0.95	36.08	-31.13	-2.62	-13.13

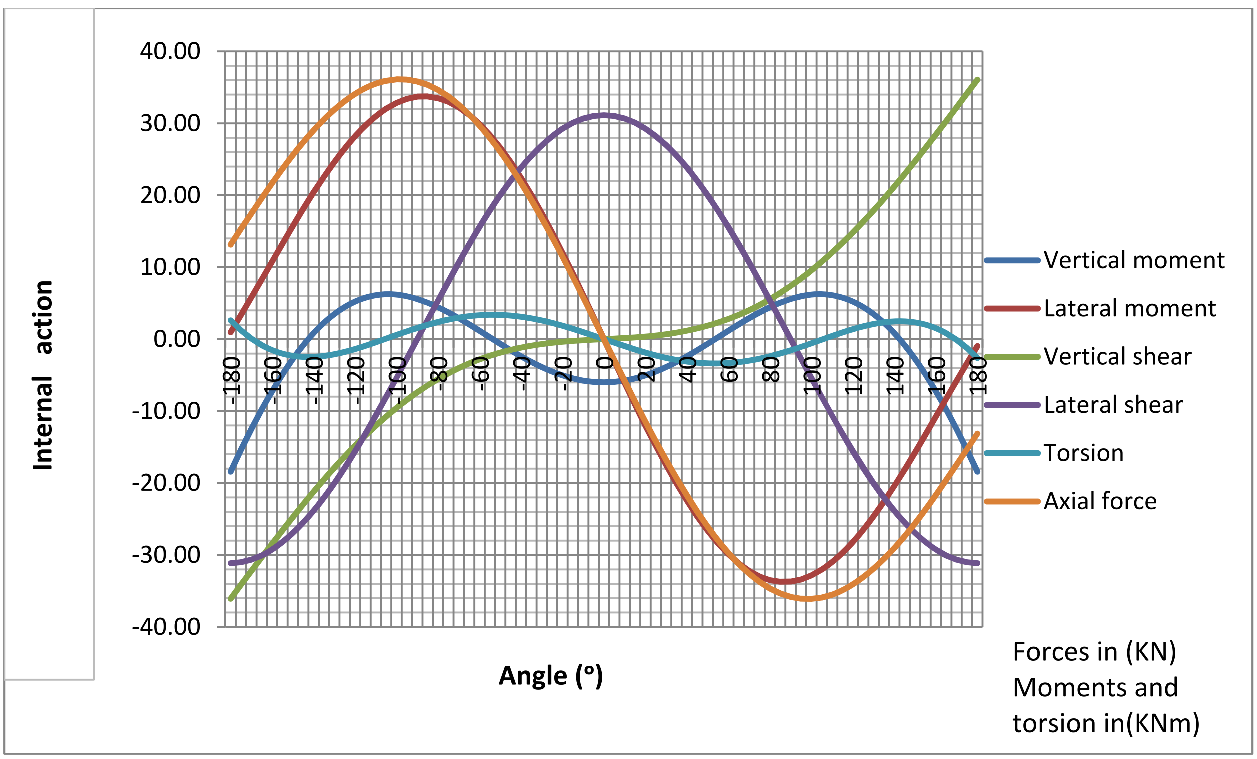

Figure A 1: Flight slope =20° & Radius =1.0m

Table A 2: Flight slope =20° & Radius =1.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-41.47	2.15	-54.11	-46.70	5.90	19.70
-175	-34.63	7.51	-51.22	-46.52	2.78	22.97
-170	-28.12	13.02	-48.33	-45.99	0.21	26.22
-165	-21.99	18.62	-45.47	-45.11	-1.84	29.41
-160	-16.27	24.26	-42.64	-43.88	-3.41	32.52
-155	-11.01	29.86	-39.85	-42.33	-4.52	35.51
-150	-6.22	35.37	-37.11	-40.44	-5.23	38.36
-145	-1.93	40.74	-34.43	-38.26	-5.56	41.04
-140	1.85	45.90	-31.82	-35.78	-5.56	43.53
-135	5.10	50.80	-29.29	-33.02	-5.27	45.80
-130	7.84	55.38	-26.85	-30.02	-4.73	47.84
-125	10.06	59.61	-24.49	-26.79	-4.00	49.63
-120	11.76	63.43	-22.24	-23.35	-3.10	51.14
-115	12.98	66.81	-20.10	-19.74	-2.08	52.36
-110	13.72	69.69	-18.06	-15.97	-0.98	53.27
-105	14.02	72.07	-16.14	-12.09	0.16	53.88
-100	13.90	73.89	-14.33	-8.11	1.31	54.16
-95	13.40	75.15	-12.65	-4.07	2.43	54.11
-90	12.56	75.82	-11.08	0.00	3.50	53.73
-85	11.40	75.89	-9.64	4.07	4.48	53.02
-80	9.99	75.37	-8.32	8.11	5.36	51.97
-75	8.36	74.23	-7.12	12.09	6.11	50.60
-70	6.56	72.50	-6.03	15.97	6.72	48.90
-65	4.63	70.18	-5.06	19.74	7.18	46.89
-60	2.61	67.28	-4.20	23.35	7.48	44.57
-55	0.57	63.84	-3.45	26.79	7.61	41.97
-50	-1.47	59.86	-2.80	30.02	7.57	39.09
-45	-3.46	55.40	-2.23	33.02	7.37	35.96
-40	-5.35	50.47	-1.76	35.78	7.01	32.59
-35	-7.12	45.12	-1.36	38.26	6.50	29.00
-30	-8.72	39.40	-1.03	40.44	5.85	25.22
-25	-10.13	33.35	-0.77	42.33	5.07	21.28
-20	-11.32	27.02	-0.55	43.88	4.19	17.20
-15	-12.27	20.47	-0.38	45.11	3.22	13.00
-10	-12.96	13.74	-0.23	45.99	2.19	8.71
-5	-13.38	6.90	-0.11	46.52	1.11	4.37
0	-13.52	0.00	0.00	46.70	0.00	0.00
5	-13.38	-6.90	0.11	46.52	-1.11	-4.37
10	-12.96	-13.74	0.23	45.99	-2.19	-8.71
15	-12.27	-20.47	0.38	45.11	-3.22	-13.00
20	-11.32	-27.02	0.55	43.88	-4.19	-17.20
25	-10.13	-33.35	0.77	42.33	-5.07	-21.28
30	-8.72	-39.40	1.03	40.44	-5.85	-25.22
35	-7.12	-45.12	1.36	38.26	-6.50	-29.00
40	-5.35	-50.47	1.76	35.78	-7.01	-32.59
45	-3.46	-55.40	2.23	33.02	-7.37	-35.96
50	-1.47	-59.86	2.80	30.02	-7.57	-39.09
55	0.57	-63.84	3.45	26.79	-7.61	-41.97
60	2.61	-67.28	4.20	23.35	-7.48	-44.57
65	4.63	-70.18	5.06	19.74	-7.18	-46.89
70	6.56	-72.50	6.03	15.97	-6.72	-48.90
75	8.36	-74.23	7.12	12.09	-6.11	-50.60
80	9.99	-75.37	8.32	8.11	-5.36	-51.97
85	11.40	-75.89	9.64	4.07	-4.48	-53.02
90	12.56	-75.82	11.08	0.00	-3.50	-53.73
95	13.40	-75.15	12.65	-4.07	-2.43	-54.11

100	13.90	-73.89	14.33	-8.11	-1.31	-54.16
105	14.02	-72.07	16.14	-12.09	-0.16	-53.88
110	13.72	-69.69	18.06	-15.97	0.98	-53.27
115	12.98	-66.81	20.10	-19.74	2.08	-52.36
120	11.76	-63.43	22.24	-23.35	3.10	-51.14
125	10.06	-59.61	24.49	-26.79	4.00	-49.63
130	7.84	-55.38	26.85	-30.02	4.73	-47.84
135	5.10	-50.80	29.29	-33.02	5.27	-45.80
140	1.85	-45.90	31.82	-35.78	5.56	-43.53
145	-1.93	-40.74	34.43	-38.26	5.56	-41.04
150	-6.22	-35.37	37.11	-40.44	5.23	-38.36
155	-11.01	-29.86	39.85	-42.33	4.52	-35.51
160	-16.27	-24.26	42.64	-43.88	3.41	-32.52
165	-21.99	-18.62	45.47	-45.11	1.84	-29.41
170	-28.12	-13.02	48.33	-45.99	-0.21	-26.22
175	-34.63	-7.51	51.22	-46.52	-2.78	-22.97
180	-41.47	-2.15	54.11	-46.70	-5.90	-19.70

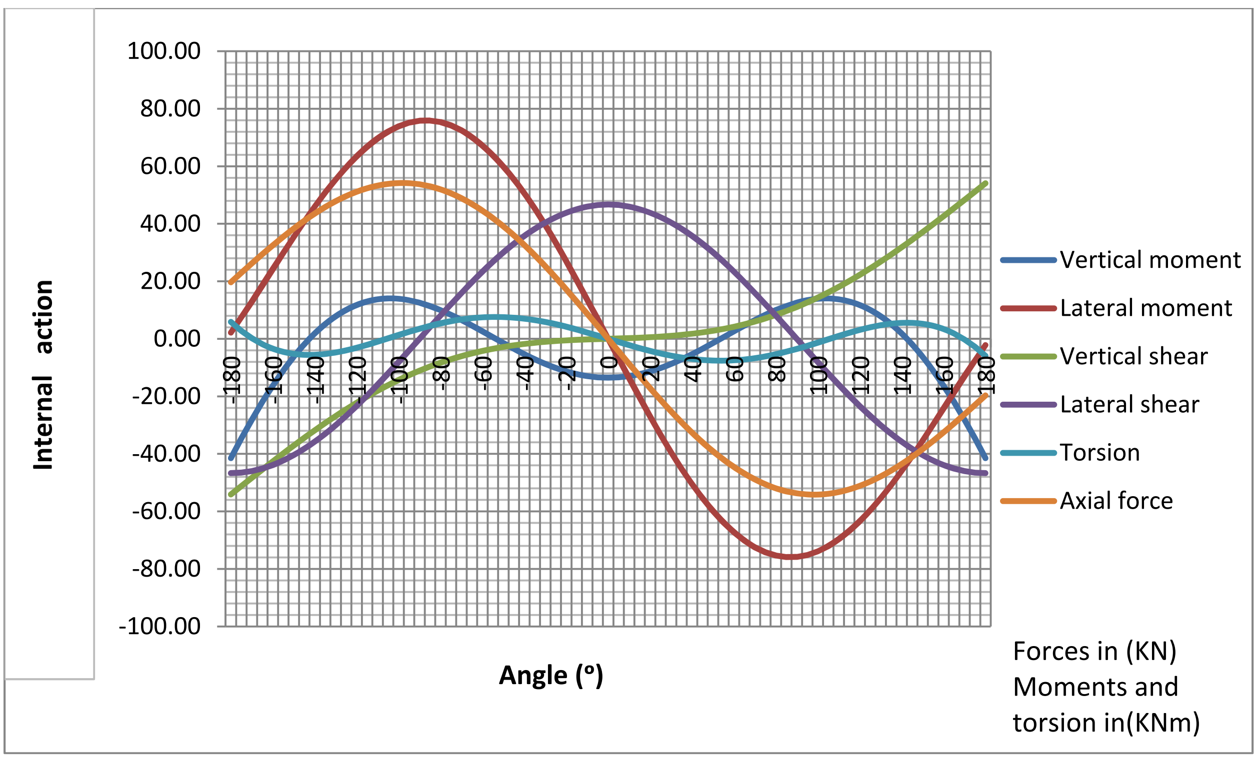

Figure A 2: Flight slope =20° & Radius =1.5m

Table A 3: Flight slope =20° & Radius =2.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-73.72	3.82	-72.15	-62.27	10.49	26.26
-175	-61.56	13.35	-68.29	-62.03	4.94	30.63
-170	-49.99	23.15	-64.44	-61.32	0.37	34.96
-165	-39.09	33.11	-60.63	-60.15	-3.27	39.22
-160	-28.93	43.12	-56.85	-58.51	-6.06	43.36
-155	-19.57	53.08	-53.13	-56.43	-8.04	47.34
-150	-11.06	62.88	-49.48	-53.93	-9.29	51.14
-145	-3.43	72.42	-45.91	-51.01	-9.88	54.72
-140	3.28	81.59	-42.43	-47.70	-9.88	58.04
-135	9.08	90.30	-39.05	-44.03	-9.36	61.07
-130	13.94	98.46	-35.79	-40.03	-8.41	63.79
-125	17.88	105.98	-32.66	-35.72	-7.10	66.17
-120	20.92	112.77	-29.66	-31.13	-5.51	68.18
-115	23.08	118.77	-26.79	-26.32	-3.70	69.81
-110	24.40	123.90	-24.08	-21.30	-1.74	71.03
-105	24.93	128.12	-21.52	-16.12	0.28	71.84
-100	24.72	131.36	-19.11	-10.81	2.32	72.21
-95	23.83	133.60	-16.86	-5.43	4.32	72.15
-90	22.32	134.79	-14.78	0.00	6.21	71.64
-85	20.27	134.92	-12.85	5.43	7.96	70.69
-80	17.76	133.98	-11.09	10.81	9.53	69.30
-75	14.86	131.97	-9.49	16.12	10.87	67.46
-70	11.65	128.89	-8.05	21.30	11.96	65.20
-65	8.22	124.76	-6.75	26.32	12.77	62.51
-60	4.65	119.61	-5.61	31.13	13.30	59.43
-55	1.01	113.49	-4.60	35.72	13.53	55.96
-50	-2.61	106.42	-3.73	40.03	13.47	52.12
-45	-6.14	98.48	-2.98	44.03	13.11	47.94
-40	-9.51	89.72	-2.34	47.70	12.46	43.45
-35	-12.65	80.22	-1.81	51.01	11.55	38.67
-30	-15.50	70.05	-1.38	53.93	10.40	33.63
-25	-18.01	59.29	-1.02	56.43	9.02	28.38
-20	-20.13	48.04	-0.73	58.51	7.45	22.93
-15	-21.82	36.39	-0.50	60.15	5.73	17.33
-10	-23.04	24.43	-0.31	61.32	3.89	11.62
-5	-23.79	12.27	-0.15	62.03	1.96	5.83
0	-24.04	0.00	0.00	62.27	0.00	0.00
5	-23.79	-12.27	0.15	62.03	-1.96	-5.83
10	-23.04	-24.43	0.31	61.32	-3.89	-11.62
15	-21.82	-36.39	0.50	60.15	-5.73	-17.33
20	-20.13	-48.04	0.73	58.51	-7.45	-22.93
25	-18.01	-59.29	1.02	56.43	-9.02	-28.38
30	-15.50	-70.05	1.38	53.93	-10.40	-33.63
35	-12.65	-80.22	1.81	51.01	-11.55	-38.67
40	-9.51	-89.72	2.34	47.70	-12.46	-43.45
45	-6.14	-98.48	2.98	44.03	-13.11	-47.94
50	-2.61	-106.42	3.73	40.03	-13.47	-52.12
55	1.01	-113.49	4.60	35.72	-13.53	-55.96
60	4.65	-119.61	5.61	31.13	-13.30	-59.43
65	8.22	-124.76	6.75	26.32	-12.77	-62.51
70	11.65	-128.89	8.05	21.30	-11.96	-65.20
75	14.86	-131.97	9.49	16.12	-10.87	-67.46
80	17.76	-133.98	11.09	10.81	-9.53	-69.30
85	20.27	-134.92	12.85	5.43	-7.96	-70.69
90	22.32	-134.79	14.78	0.00	-6.21	-71.64
95	23.83	-133.60	16.86	-5.43	-4.32	-72.15

100	24.72	-131.36	19.11	-10.81	-2.32	-72.21
105	24.93	-128.12	21.52	-16.12	-0.28	-71.84
110	24.40	-123.90	24.08	-21.30	1.74	-71.03
115	23.08	-118.77	26.79	-26.32	3.70	-69.81
120	20.92	-112.77	29.66	-31.13	5.51	-68.18
125	17.88	-105.98	32.66	-35.72	7.10	-66.17
130	13.94	-98.46	35.79	-40.03	8.41	-63.79
135	9.08	-90.30	39.05	-44.03	9.36	-61.07
140	3.28	-81.59	42.43	-47.70	9.88	-58.04
145	-3.43	-72.42	45.91	-51.01	9.88	-54.72
150	-11.06	-62.88	49.48	-53.93	9.29	-51.14
155	-19.57	-53.08	53.13	-56.43	8.04	-47.34
160	-28.93	-43.12	56.85	-58.51	6.06	-43.36
165	-39.09	-33.11	60.63	-60.15	3.27	-39.22
170	-49.99	-23.15	64.44	-61.32	-0.37	-34.96
175	-61.56	-13.35	68.29	-62.03	-4.94	-30.63
180	-73.72	-3.82	72.15	-62.27	-10.49	-26.26

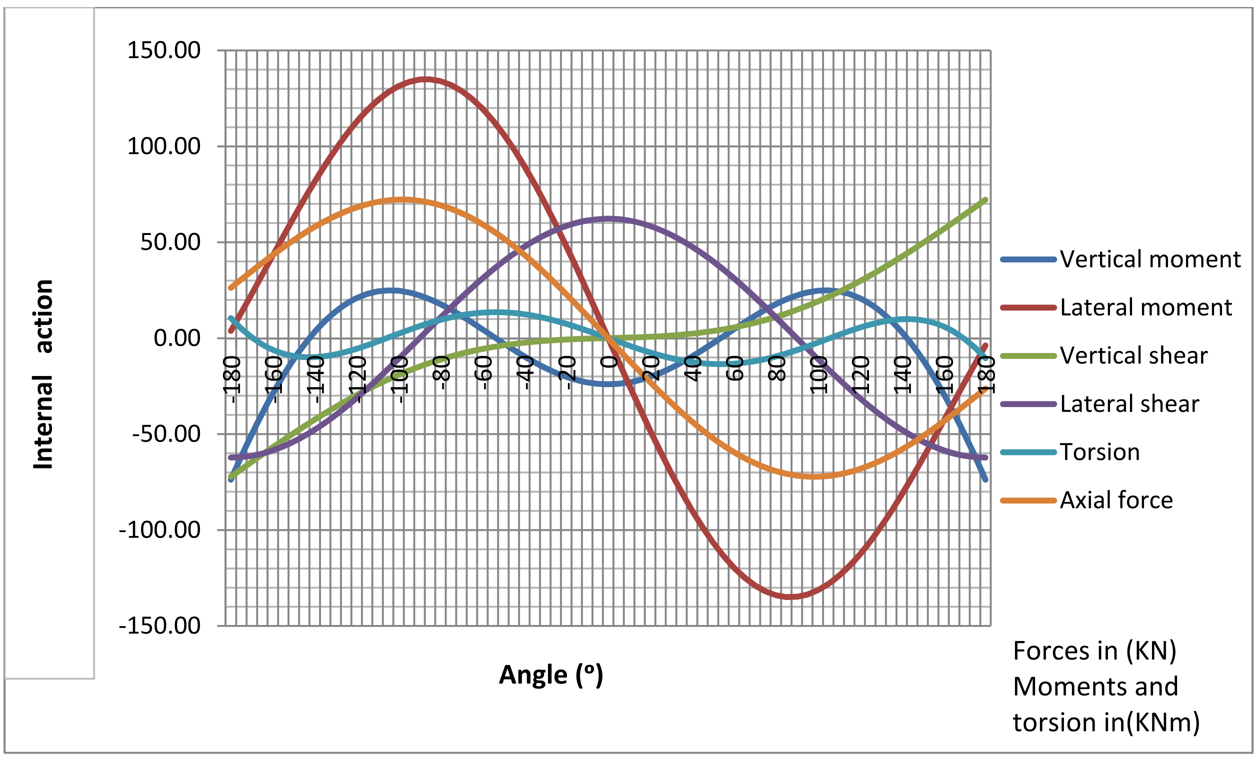

Figure A 3: Flight slope =20° & Radius =2.0m

Table A 4: Flight slope =20° & Radius =2.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-115.19	5.96	-90.19	-77.84	16.39	32.83
-175	-96.19	20.86	-85.36	-77.54	7.72	38.29
-170	-78.11	36.17	-80.55	-76.65	0.59	43.70
-165	-61.08	51.73	-75.78	-75.18	-5.11	49.02
-160	-45.20	67.38	-71.06	-73.14	-9.46	54.19
-155	-30.58	82.94	-66.41	-70.54	-12.56	59.18
-150	-17.28	98.26	-61.85	-67.41	-14.51	63.93
-145	-5.36	113.16	-57.38	-63.76	-15.43	68.40
-140	5.13	127.49	-53.03	-59.63	-15.43	72.55
-135	14.18	141.10	-48.82	-55.04	-14.63	76.34
-130	21.78	153.85	-44.74	-50.03	-13.15	79.74
-125	27.93	165.59	-40.82	-44.64	-11.10	82.71
-120	32.68	176.20	-37.07	-38.92	-8.60	85.23
-115	36.06	185.57	-33.49	-32.89	-5.78	87.26
-110	38.12	193.60	-30.10	-26.62	-2.73	88.79
-105	38.95	200.18	-26.90	-20.15	0.44	89.80
-100	38.62	205.25	-23.89	-13.52	3.63	90.27
-95	37.23	208.74	-21.08	-6.78	6.75	90.19
-90	34.88	210.61	-18.47	0.00	9.71	89.55
-85	31.68	210.82	-16.07	6.78	12.44	88.36
-80	27.75	209.35	-13.87	13.52	14.89	86.62
-75	23.22	206.20	-11.86	20.15	16.98	84.33
-70	18.21	201.39	-10.06	26.62	18.68	81.50
-65	12.85	194.94	-8.44	32.89	19.96	78.14
-60	7.26	186.90	-7.01	38.92	20.78	74.28
-55	1.58	177.32	-5.75	44.64	21.14	69.94
-50	-4.08	166.29	-4.66	50.03	21.04	65.15
-45	-9.60	153.88	-3.72	55.04	20.48	59.93
-40	-14.86	140.19	-2.93	59.63	19.47	54.31
-35	-19.77	125.34	-2.27	63.76	18.05	48.33
-30	-24.22	109.45	-1.72	67.41	16.24	42.04
-25	-28.14	92.64	-1.28	70.54	14.09	35.47
-20	-31.45	75.06	-0.92	73.14	11.65	28.66
-15	-34.09	56.85	-0.63	75.18	8.95	21.67
-10	-36.01	38.17	-0.39	76.65	6.08	14.52
-5	-37.17	19.17	-0.19	77.54	3.07	7.29
0	-37.56	0.00	0.00	77.84	0.00	0.00
5	-37.17	-19.17	0.19	77.54	-3.07	-7.29
10	-36.01	-38.17	0.39	76.65	-6.08	-14.52
15	-34.09	-56.85	0.63	75.18	-8.95	-21.67
20	-31.45	-75.06	0.92	73.14	-11.65	-28.66
25	-28.14	-92.64	1.28	70.54	-14.09	-35.47
30	-24.22	-109.45	1.72	67.41	-16.24	-42.04
35	-19.77	-125.34	2.27	63.76	-18.05	-48.33
40	-14.86	-140.19	2.93	59.63	-19.47	-54.31
45	-9.60	-153.88	3.72	55.04	-20.48	-59.93
50	-4.08	-166.29	4.66	50.03	-21.04	-65.15
55	1.58	-177.32	5.75	44.64	-21.14	-69.94
60	7.26	-186.90	7.01	38.92	-20.78	-74.28
65	12.85	-194.94	8.44	32.89	-19.96	-78.14
70	18.21	-201.39	10.06	26.62	-18.68	-81.50
75	23.22	-206.20	11.86	20.15	-16.98	-84.33
80	27.75	-209.35	13.87	13.52	-14.89	-86.62
85	31.68	-210.82	16.07	6.78	-12.44	-88.36
90	34.88	-210.61	18.47	0.00	-9.71	-89.55

95	37.23	-208.74	21.08	-6.78	-6.75	-90.19
100	38.62	-205.25	23.89	-13.52	-3.63	-90.27
105	38.95	-200.18	26.90	-20.15	-0.44	-89.80
110	38.12	-193.60	30.10	-26.62	2.73	-88.79
115	36.06	-185.57	33.49	-32.89	5.78	-87.26
120	32.68	-176.20	37.07	-38.92	8.60	-85.23
125	27.93	-165.59	40.82	-44.64	11.10	-82.71
130	21.78	-153.85	44.74	-50.03	13.15	-79.74
135	14.18	-141.10	48.82	-55.04	14.63	-76.34
140	5.13	-127.49	53.03	-59.63	15.43	-72.55
145	-5.36	-113.16	57.38	-63.76	15.43	-68.40
150	-17.28	-98.26	61.85	-67.41	14.51	-63.93
155	-30.58	-82.94	66.41	-70.54	12.56	-59.18
160	-45.20	-67.38	71.06	-73.14	9.46	-54.19
165	-61.08	-51.73	75.78	-75.18	5.11	-49.02
170	-78.11	-36.17	80.55	-76.65	-0.59	-43.70
175	-96.19	-20.86	85.36	-77.54	-7.72	-38.29
180	-115.19	-5.96	90.19	-77.84	-16.39	-32.83

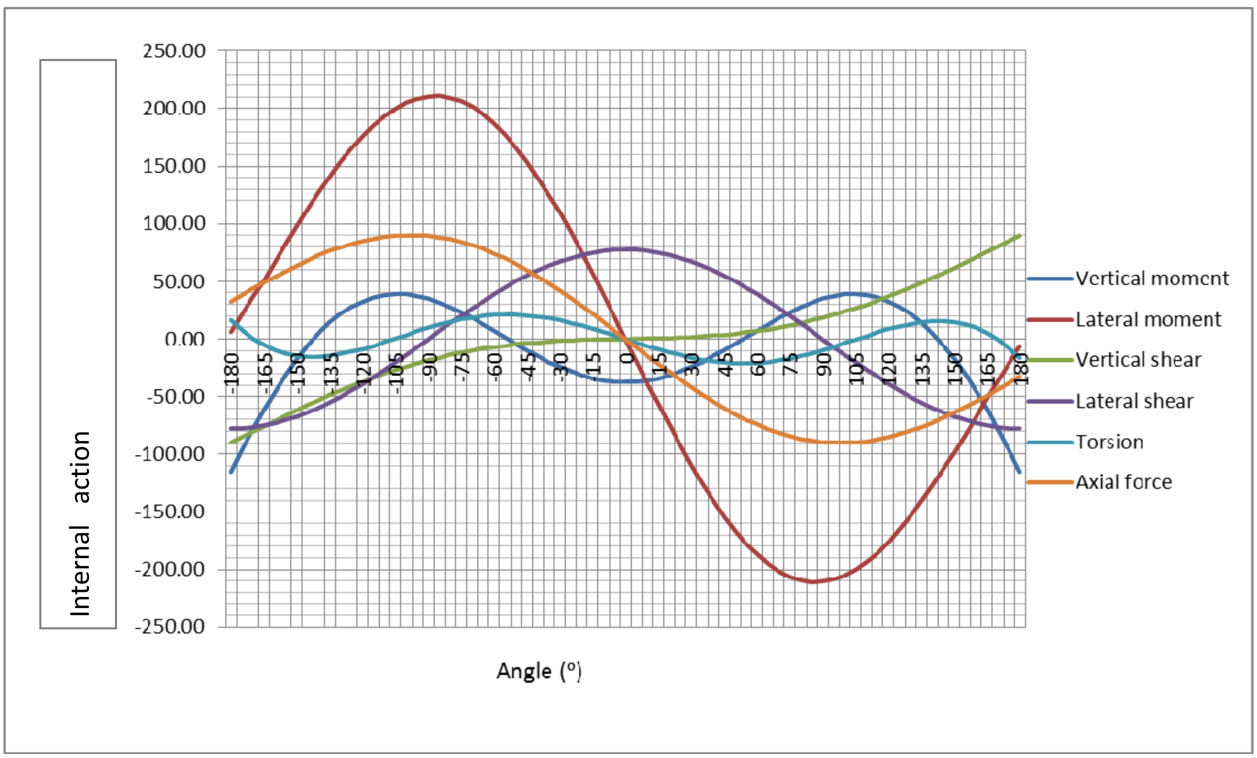

Figure A 4: Flight slope =20° & Radius =2.5m

Table A 5: Flight slope =25° & Radius =1.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-18.41	1.28	-34.79	-24.14	2.74	16.22
-175	-15.39	2.98	-32.94	-24.05	1.40	17.68
-170	-12.51	4.77	-31.09	-23.78	0.30	19.12
-165	-9.81	6.62	-29.25	-23.32	-0.58	20.54
-160	-7.28	8.53	-27.44	-22.69	-1.26	21.91
-155	-4.96	10.45	-25.65	-21.88	-1.74	23.22
-150	-2.84	12.37	-23.89	-20.91	-2.05	24.46
-145	-0.95	14.26	-22.18	-19.78	-2.19	25.62
-140	0.72	16.10	-20.50	-18.49	-2.20	26.68
-135	2.16	17.86	-18.88	-17.07	-2.09	27.64
-130	3.37	19.54	-17.31	-15.52	-1.87	28.48
-125	4.35	21.09	-15.80	-13.85	-1.56	29.19
-120	5.11	22.52	-14.36	-12.07	-1.18	29.77
-115	5.64	23.79	-12.98	-10.20	-0.76	30.20
-110	5.97	24.89	-11.67	-8.26	-0.30	30.48
-105	6.11	25.82	-10.44	-6.25	0.18	30.60
-100	6.06	26.54	-9.28	-4.19	0.66	30.56
-95	5.84	27.07	-8.20	-2.10	1.14	30.36
-90	5.46	27.38	-7.19	0.00	1.58	29.99
-85	4.96	27.47	-6.27	2.10	2.00	29.46
-80	4.34	27.34	-5.42	4.19	2.37	28.76
-75	3.62	26.98	-4.64	6.25	2.68	27.90
-70	2.82	26.40	-3.94	8.26	2.94	26.87
-65	1.97	25.60	-3.32	10.20	3.13	25.69
-60	1.08	24.58	-2.76	12.07	3.25	24.36
-55	0.18	23.36	-2.27	13.85	3.30	22.88
-50	-0.72	21.93	-1.85	15.52	3.28	21.27
-45	-1.59	20.32	-1.48	17.07	3.18	19.53
-40	-2.43	18.53	-1.17	18.49	3.02	17.67
-35	-3.21	16.59	-0.91	19.78	2.80	15.71
-30	-3.91	14.49	-0.70	20.91	2.52	13.64
-25	-4.53	12.28	-0.52	21.88	2.18	11.50
-20	-5.06	9.95	-0.38	22.69	1.80	9.29
-15	-5.48	7.54	-0.26	23.32	1.39	7.02
-10	-5.78	5.06	-0.16	23.78	0.94	4.70
-5	-5.97	2.54	-0.08	24.05	0.48	2.36
0	-6.03	0.00	0.00	24.14	0.00	0.00
5	-5.97	-2.54	0.08	24.05	-0.48	-2.36
10	-5.78	-5.06	0.16	23.78	-0.94	-4.70
15	-5.48	-7.54	0.26	23.32	-1.39	-7.02
20	-5.06	-9.95	0.38	22.69	-1.80	-9.29
25	-4.53	-12.28	0.52	21.88	-2.18	-11.50
30	-3.91	-14.49	0.70	20.91	-2.52	-13.64
35	-3.21	-16.59	0.91	19.78	-2.80	-15.71
40	-2.43	-18.53	1.17	18.49	-3.02	-17.67
45	-1.59	-20.32	1.48	17.07	-3.18	-19.53
50	-0.72	-21.93	1.85	15.52	-3.28	-21.27
55	0.18	-23.36	2.27	13.85	-3.30	-22.88
60	1.08	-24.58	2.76	12.07	-3.25	-24.36
65	1.97	-25.60	3.32	10.20	-3.13	-25.69
70	2.82	-26.40	3.94	8.26	-2.94	-26.87
75	3.62	-26.98	4.64	6.25	-2.68	-27.90
80	4.34	-27.34	5.42	4.19	-2.37	-28.76
85	4.96	-27.47	6.27	2.10	-2.00	-29.46
90	5.46	-27.38	7.19	0.00	-1.58	-29.99
95	5.84	-27.07	8.20	-2.10	-1.14	-30.36

100	6.06	-26.54	9.28	-4.19	-0.66	-30.56
105	6.11	-25.82	10.44	-6.25	-0.18	-30.60
110	5.97	-24.89	11.67	-8.26	0.30	-30.48
115	5.64	-23.79	12.98	-10.20	0.76	-30.20
120	5.11	-22.52	14.36	-12.07	1.18	-29.77
125	4.35	-21.09	15.80	-13.85	1.56	-29.19
130	3.37	-19.54	17.31	-15.52	1.87	-28.48
135	2.16	-17.86	18.88	-17.07	2.09	-27.64
140	0.72	-16.10	20.50	-18.49	2.20	-26.68
145	-0.95	-14.26	22.18	-19.78	2.19	-25.62
150	-2.84	-12.37	23.89	-20.91	2.05	-24.46
155	-4.96	-10.45	25.65	-21.88	1.74	-23.22
160	-7.28	-8.53	27.44	-22.69	1.26	-21.91
165	-9.81	-6.62	29.25	-23.32	0.58	-20.54
170	-12.51	-4.77	31.09	-23.78	-0.30	-19.12
175	-15.39	-2.98	32.94	-24.05	-1.40	-17.68
180	-18.41	-1.28	34.79	-24.14	-2.74	-16.22

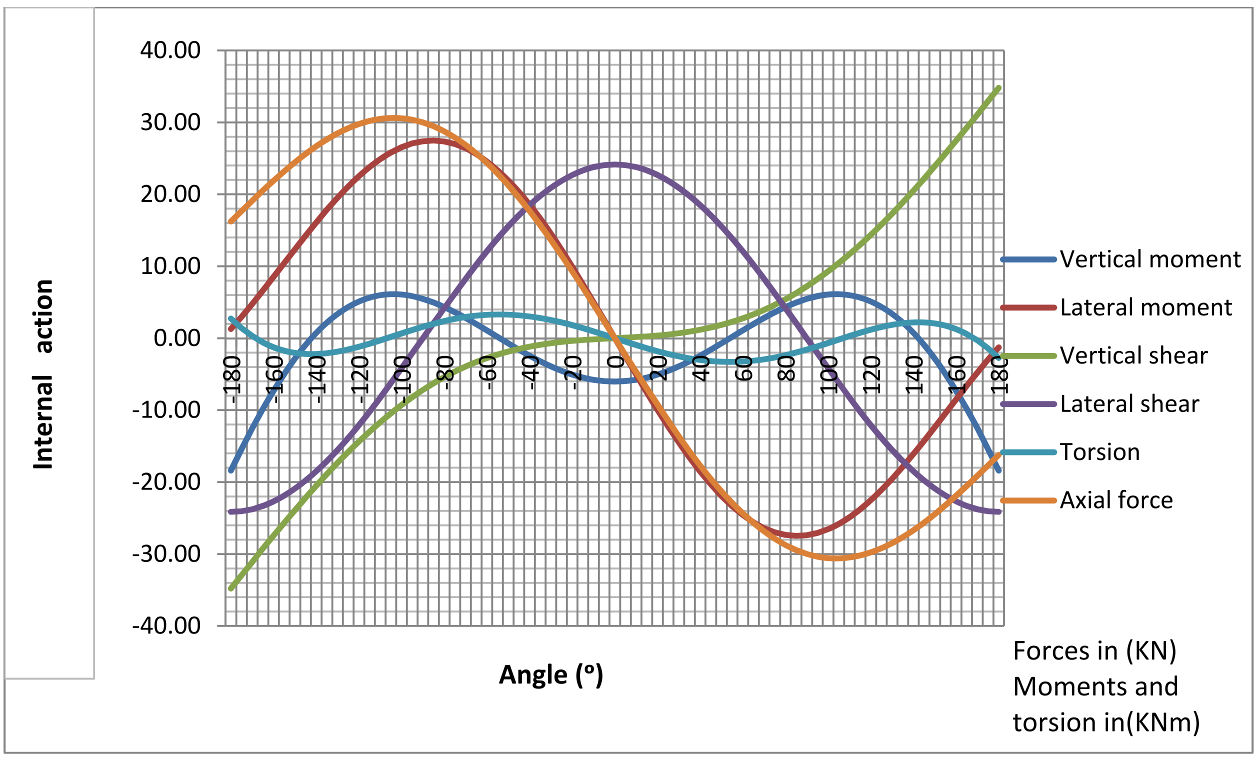

Figure A 5: Flight slope =25° & Radius =1.0m

Table A 6: Flight slope =25° & Radius =1.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-41.42	2.87	-52.19	-36.22	6.16	24.34
-175	-34.62	6.70	-49.41	-36.08	3.16	26.52
-170	-28.16	10.72	-46.63	-35.67	0.68	28.68
-165	-22.07	14.90	-43.88	-34.98	-1.31	30.80
-160	-16.39	19.18	-41.16	-34.03	-2.83	32.86
-155	-11.15	23.51	-38.47	-32.82	-3.91	34.83
-150	-6.40	27.82	-35.84	-31.36	-4.60	36.69
-145	-2.13	32.08	-33.26	-29.67	-4.94	38.43
-140	1.62	36.22	-30.75	-27.74	-4.95	40.03
-135	4.86	40.19	-28.32	-25.61	-4.69	41.46
-130	7.58	43.96	-25.97	-23.28	-4.20	42.72
-125	9.79	47.46	-23.71	-20.77	-3.51	43.79
-120	11.49	50.67	-21.54	-18.11	-2.66	44.65
-115	12.70	53.53	-19.47	-15.31	-1.70	45.30
-110	13.44	56.01	-17.51	-12.39	-0.67	45.72
-105	13.74	58.09	-15.66	-9.37	0.41	45.90
-100	13.63	59.73	-13.92	-6.29	1.50	45.84
-95	13.13	60.90	-12.30	-3.16	2.56	45.54
-90	12.30	61.60	-10.79	0.00	3.56	44.99
-85	11.16	61.81	-9.40	3.16	4.49	44.19
-80	9.75	61.51	-8.12	6.29	5.32	43.14
-75	8.14	60.71	-6.96	9.37	6.03	41.84
-70	6.35	59.40	-5.91	12.39	6.60	40.31
-65	4.44	57.60	-4.98	15.31	7.03	38.54
-60	2.44	55.31	-4.14	18.11	7.30	36.54
-55	0.41	52.56	-3.41	20.77	7.42	34.32
-50	-1.61	49.35	-2.77	23.28	7.37	31.90
-45	-3.58	45.72	-2.23	25.61	7.16	29.29
-40	-5.46	41.70	-1.76	27.74	6.81	26.51
-35	-7.21	37.32	-1.37	29.67	6.30	23.56
-30	-8.80	32.61	-1.05	31.36	5.67	20.47
-25	-10.20	27.62	-0.78	32.82	4.92	17.25
-20	-11.39	22.39	-0.56	34.03	4.06	13.93
-15	-12.33	16.97	-0.39	34.98	3.12	10.52
-10	-13.01	11.40	-0.24	35.67	2.12	7.05
-5	-13.43	5.72	-0.12	36.08	1.07	3.54
0	-13.57	0.00	0.00	36.22	0.00	0.00
5	-13.43	-5.72	0.12	36.08	-1.07	-3.54
10	-13.01	-11.40	0.24	35.67	-2.12	-7.05
15	-12.33	-16.97	0.39	34.98	-3.12	-10.52
20	-11.39	-22.39	0.56	34.03	-4.06	-13.93
25	-10.20	-27.62	0.78	32.82	-4.92	-17.25
30	-8.80	-32.61	1.05	31.36	-5.67	-20.47
35	-7.21	-37.32	1.37	29.67	-6.30	-23.56
40	-5.46	-41.70	1.76	27.74	-6.81	-26.51
45	-3.58	-45.72	2.23	25.61	-7.16	-29.29
50	-1.61	-49.35	2.77	23.28	-7.37	-31.90
55	0.41	-52.56	3.41	20.77	-7.42	-34.32
60	2.44	-55.31	4.14	18.11	-7.30	-36.54
65	4.44	-57.60	4.98	15.31	-7.03	-38.54
70	6.35	-59.40	5.91	12.39	-6.60	-40.31
75	8.14	-60.71	6.96	9.37	-6.03	-41.84
80	9.75	-61.51	8.12	6.29	-5.32	-43.14
85	11.16	-61.81	9.40	3.16	-4.49	-44.19

90	12.30	-61.60	10.79	0.00	-3.56	-44.99
95	13.13	-60.90	12.30	-3.16	-2.56	-45.54
100	13.63	-59.73	13.92	-6.29	-1.50	-45.84
105	13.74	-58.09	15.66	-9.37	-0.41	-45.90
110	13.44	-56.01	17.51	-12.39	0.67	-45.72
115	12.70	-53.53	19.47	-15.31	1.70	-45.30
120	11.49	-50.67	21.54	-18.11	2.66	-44.65
125	9.79	-47.46	23.71	-20.77	3.51	-43.79
130	7.58	-43.96	25.97	-23.28	4.20	-42.72
135	4.86	-40.19	28.32	-25.61	4.69	-41.46
140	1.62	-36.22	30.75	-27.74	4.95	-40.03
145	-2.13	-32.08	33.26	-29.67	4.94	-38.43
150	-6.40	-27.82	35.84	-31.36	4.60	-36.69
155	-11.15	-23.51	38.47	-32.82	3.91	-34.83
160	-16.39	-19.18	41.16	-34.03	2.83	-32.86
165	-22.07	-14.90	43.88	-34.98	1.31	-30.80
170	-28.16	-10.72	46.63	-35.67	-0.68	-28.68
175	-34.62	-6.70	49.41	-36.08	-3.16	-26.52
180	-41.42	-2.87	52.19	-36.22	-6.16	-24.34

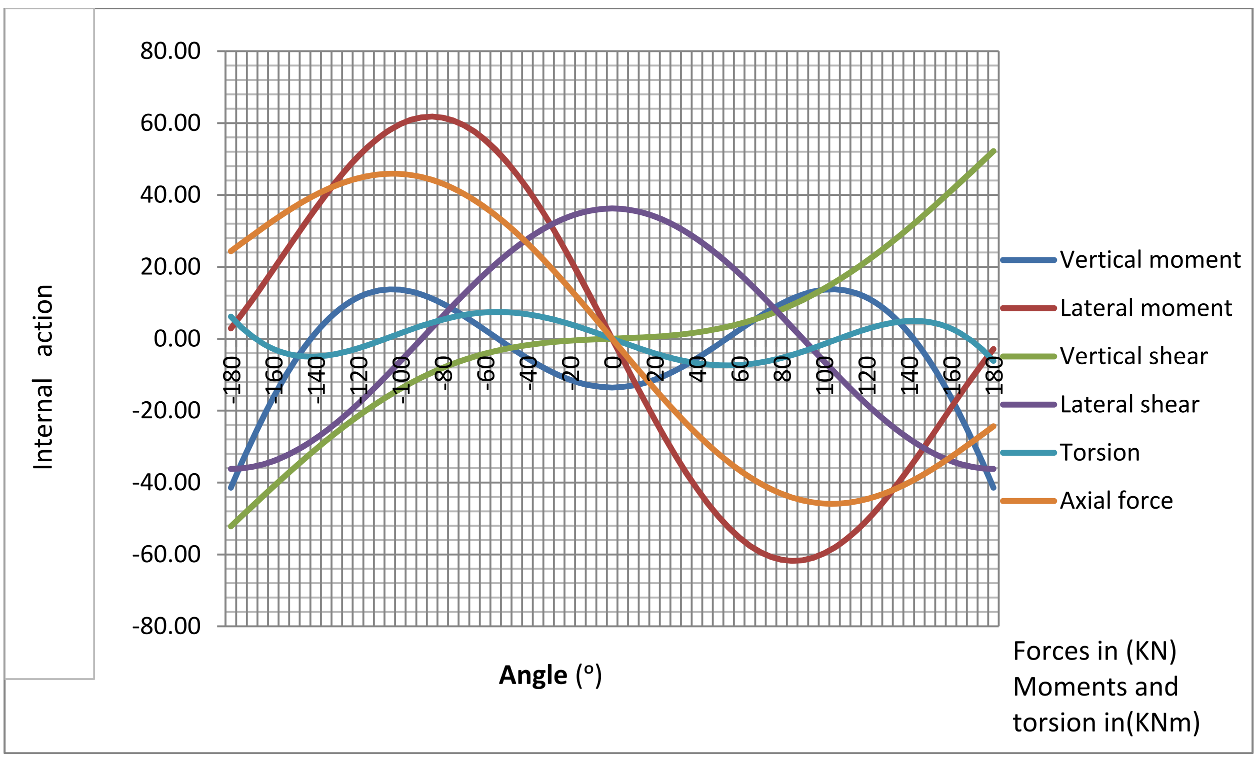

Figure A 6: Flight slope =25° & Radius =1.5m

Table A 7: Flight slope =25° & Radius =2.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-73.64	5.11	-69.59	-48.29	10.95	32.45
-175	-61.55	11.90	-65.88	-48.10	5.61	35.36
-170	-50.06	19.06	-62.18	-47.55	1.20	38.25
-165	-39.23	26.49	-58.51	-46.64	-2.33	41.07
-160	-29.13	34.10	-54.88	-45.37	-5.02	43.81
-155	-19.83	41.79	-51.30	-43.76	-6.95	46.44
-150	-11.37	49.46	-47.79	-41.82	-8.18	48.92
-145	-3.79	57.03	-44.35	-39.55	-8.78	51.24
-140	2.89	64.39	-41.01	-36.99	-8.81	53.37
-135	8.64	71.46	-37.76	-34.14	-8.34	55.28
-130	13.48	78.15	-34.62	-31.04	-7.46	56.96
-125	17.40	84.38	-31.61	-27.70	-6.24	58.38
-120	20.42	90.07	-28.72	-24.14	-4.73	59.53
-115	22.58	95.16	-25.96	-20.41	-3.03	60.39
-110	23.90	99.58	-23.35	-16.52	-1.19	60.95
-105	24.43	103.27	-20.88	-12.50	0.73	61.20
-100	24.22	106.18	-18.56	-8.38	2.66	61.13
-95	23.35	108.27	-16.40	-4.21	4.54	60.72
-90	21.86	109.51	-14.39	0.00	6.34	59.99
-85	19.83	109.88	-12.53	4.21	7.99	58.92
-80	17.34	109.35	-10.83	8.38	9.46	57.52
-75	14.47	107.93	-9.28	12.50	10.72	55.79
-70	11.29	105.61	-7.89	16.52	11.74	53.74
-65	7.88	102.40	-6.63	20.41	12.50	51.38
-60	4.34	98.34	-5.52	24.14	12.98	48.72
-55	0.73	93.44	-4.55	27.70	13.19	45.76
-50	-2.86	87.74	-3.70	31.04	13.10	42.54
-45	-6.36	81.29	-2.97	34.14	12.73	39.06
-40	-9.71	74.14	-2.35	36.99	12.10	35.34
-35	-12.82	66.34	-1.83	39.55	11.21	31.41
-30	-15.65	57.98	-1.39	41.82	10.08	27.29
-25	-18.14	49.11	-1.04	43.76	8.74	23.00
-20	-20.24	39.81	-0.75	45.37	7.22	18.57
-15	-21.92	30.17	-0.52	46.64	5.55	14.03
-10	-23.13	20.26	-0.32	47.55	3.76	9.40
-5	-23.88	10.17	-0.15	48.10	1.90	4.72
0	-24.12	0.00	0.00	48.29	0.00	0.00
5	-23.88	-10.17	0.15	48.10	-1.90	-4.72
10	-23.13	-20.26	0.32	47.55	-3.76	-9.40
15	-21.92	-30.17	0.52	46.64	-5.55	-14.03
20	-20.24	-39.81	0.75	45.37	-7.22	-18.57
25	-18.14	-49.11	1.04	43.76	-8.74	-23.00
30	-15.65	-57.98	1.39	41.82	-10.08	-27.29
35	-12.82	-66.34	1.83	39.55	-11.21	-31.41
40	-9.71	-74.14	2.35	36.99	-12.10	-35.34
45	-6.36	-81.29	2.97	34.14	-12.73	-39.06
50	-2.86	-87.74	3.70	31.04	-13.10	-42.54
55	0.73	-93.44	4.55	27.70	-13.19	-45.76
60	4.34	-98.34	5.52	24.14	-12.98	-48.72
65	7.88	-102.40	6.63	20.41	-12.50	-51.38
70	11.29	-105.61	7.89	16.52	-11.74	-53.74
75	14.47	-107.93	9.28	12.50	-10.72	-55.79
80	17.34	-109.35	10.83	8.38	-9.46	-57.52
85	19.83	-109.88	12.53	4.21	-7.99	-58.92
90	21.86	-109.51	14.39	0.00	-6.34	-59.99
95	23.35	-108.27	16.40	-4.21	-4.54	-60.72

100	24.22	-106.18	18.56	-8.38	-2.66	-61.13
105	24.43	-103.27	20.88	-12.50	-0.73	-61.20
110	23.90	-99.58	23.35	-16.52	1.19	-60.95
115	22.58	-95.16	25.96	-20.41	3.03	-60.39
120	20.42	-90.07	28.72	-24.14	4.73	-59.53
125	17.40	-84.38	31.61	-27.70	6.24	-58.38
130	13.48	-78.15	34.62	-31.04	7.46	-56.96
135	8.64	-71.46	37.76	-34.14	8.34	-55.28
140	2.89	-64.39	41.01	-36.99	8.81	-53.37
145	-3.79	-57.03	44.35	-39.55	8.78	-51.24
150	-11.37	-49.46	47.79	-41.82	8.18	-48.92
155	-19.83	-41.79	51.30	-43.76	6.95	-46.44
160	-29.13	-34.10	54.88	-45.37	5.02	-43.81
165	-39.23	-26.49	58.51	-46.64	2.33	-41.07
170	-50.06	-19.06	62.18	-47.55	-1.20	-38.25
175	-61.55	-11.90	65.88	-48.10	-5.61	-35.36
180	-73.64	-5.11	69.59	-48.29	-10.95	-32.45

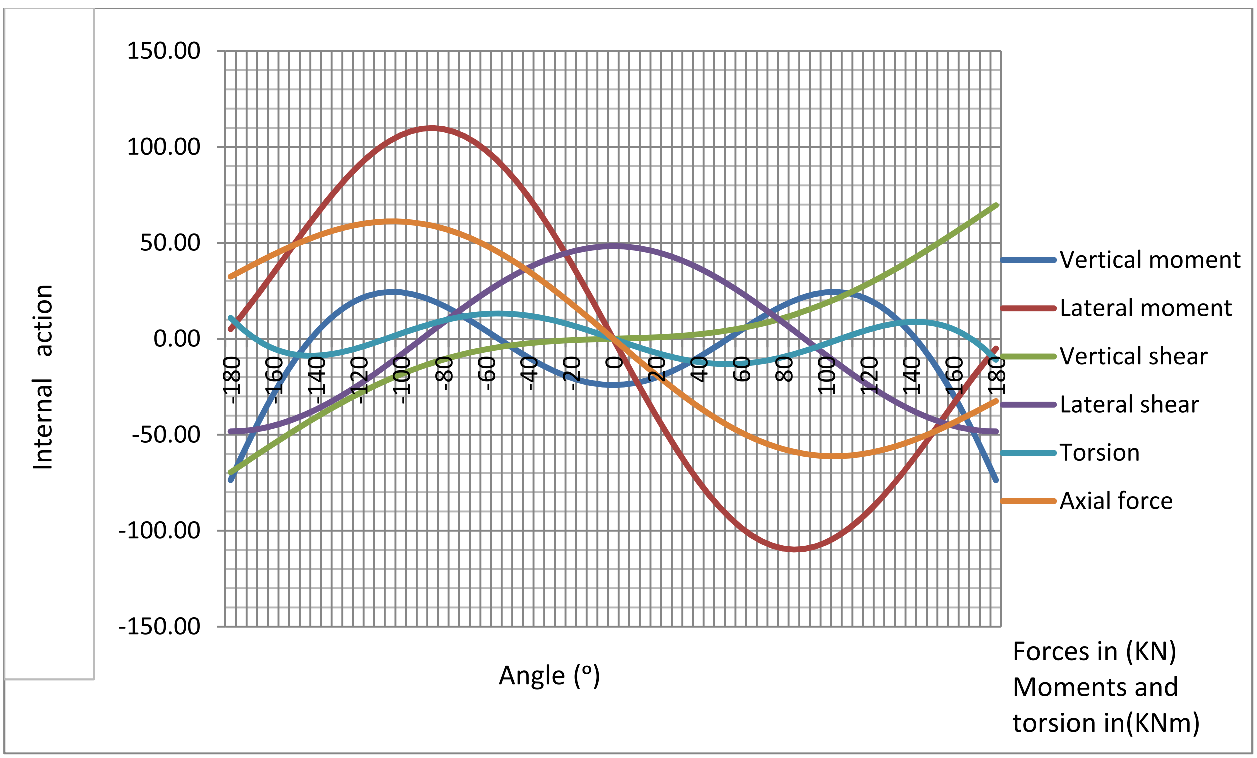

Figure A 7: Flight slope =25° & Radius =2.0m

Table A 8: Flight slope =25° & Radius =2.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-115.06	7.98	-86.98	-60.36	17.11	40.56
-175	-96.18	18.60	-82.34	-60.13	8.77	44.20
-170	-78.22	29.79	-77.72	-59.44	1.88	47.81
-165	-61.29	41.40	-73.13	-58.30	-3.63	51.34
-160	-45.52	53.28	-68.59	-56.72	-7.85	54.76
-155	-30.99	65.30	-64.12	-54.70	-10.87	58.05
-150	-17.77	77.29	-59.73	-52.27	-12.79	61.15
-145	-5.92	89.10	-55.44	-49.44	-13.71	64.05
-140	4.51	100.61	-51.26	-46.24	-13.76	66.71
-135	13.51	111.65	-47.20	-42.68	-13.04	69.10
-130	21.06	122.11	-43.28	-38.80	-11.66	71.20
-125	27.19	131.84	-39.51	-34.62	-9.74	72.98
-120	31.91	140.74	-35.90	-30.18	-7.40	74.42
-115	35.28	148.69	-32.45	-25.51	-4.73	75.49
-110	37.34	155.59	-29.19	-20.64	-1.85	76.19
-105	38.17	161.36	-26.10	-15.62	1.14	76.50
-100	37.85	165.90	-23.20	-10.48	4.15	76.41
-95	36.48	169.17	-20.50	-5.26	7.10	75.90
-90	34.15	171.11	-17.98	0.00	9.90	74.98
-85	30.99	171.68	-15.66	5.26	12.48	73.65
-80	27.10	170.86	-13.54	10.48	14.78	71.90
-75	22.60	168.63	-11.60	15.62	16.75	69.74
-70	17.64	165.01	-9.86	20.64	18.35	67.18
-65	12.32	160.00	-8.29	25.51	19.53	64.23
-60	6.78	153.65	-6.90	30.18	20.29	60.90
-55	1.14	145.99	-5.68	34.62	20.60	57.20
-50	-4.47	137.09	-4.62	38.80	20.47	53.17
-45	-9.95	127.01	-3.71	42.68	19.90	48.82
-40	-15.17	115.84	-2.93	46.24	18.90	44.18
-35	-20.03	103.66	-2.28	49.44	17.51	39.26
-30	-24.45	90.59	-1.74	52.27	15.75	34.11
-25	-28.34	76.73	-1.30	54.70	13.65	28.75
-20	-31.63	62.20	-0.94	56.72	11.28	23.22
-15	-34.24	47.13	-0.65	58.30	8.67	17.54
-10	-36.15	31.65	-0.40	59.44	5.88	11.75
-5	-37.31	15.90	-0.19	60.13	2.97	5.89
0	-37.69	0.00	0.00	60.36	0.00	0.00
5	-37.31	-15.90	0.19	60.13	-2.97	-5.89
10	-36.15	-31.65	0.40	59.44	-5.88	-11.75
15	-34.24	-47.13	0.65	58.30	-8.67	-17.54
20	-31.63	-62.20	0.94	56.72	-11.28	-23.22
25	-28.34	-76.73	1.30	54.70	-13.65	-28.75
30	-24.45	-90.59	1.74	52.27	-15.75	-34.11
35	-20.03	-103.66	2.28	49.44	-17.51	-39.26
40	-15.17	-115.84	2.93	46.24	-18.90	-44.18
45	-9.95	-127.01	3.71	42.68	-19.90	-48.82
50	-4.47	-137.09	4.62	38.80	-20.47	-53.17
55	1.14	-145.99	5.68	34.62	-20.60	-57.20
60	6.78	-153.65	6.90	30.18	-20.29	-60.90
65	12.32	-160.00	8.29	25.51	-19.53	-64.23
70	17.64	-165.01	9.86	20.64	-18.35	-67.18
75	22.60	-168.63	11.60	15.62	-16.75	-69.74
80	27.10	-170.86	13.54	10.48	-14.78	-71.90
85	30.99	-171.68	15.66	5.26	-12.48	-73.65
90	34.15	-171.11	17.98	0.00	-9.90	-74.98
95	36.48	-169.17	20.50	-5.26	-7.10	-75.90

100	37.85	-165.90	23.20	-10.48	-4.15	-76.41
105	38.17	-161.36	26.10	-15.62	-1.14	-76.50
110	37.34	-155.59	29.19	-20.64	1.85	-76.19
115	35.28	-148.69	32.45	-25.51	4.73	-75.49
120	31.91	-140.74	35.90	-30.18	7.40	-74.42
125	27.19	-131.84	39.51	-34.62	9.74	-72.98
130	21.06	-122.11	43.28	-38.80	11.66	-71.20
135	13.51	-111.65	47.20	-42.68	13.04	-69.10
140	4.51	-100.61	51.26	-46.24	13.76	-66.71
145	-5.92	-89.10	55.44	-49.44	13.71	-64.05
150	-17.77	-77.29	59.73	-52.27	12.79	-61.15
155	-30.99	-65.30	64.12	-54.70	10.87	-58.05
160	-45.52	-53.28	68.59	-56.72	7.85	-54.76
165	-61.29	-41.40	73.13	-58.30	3.63	-51.34
170	-78.22	-29.79	77.72	-59.44	-1.88	-47.81
175	-96.18	-18.60	82.34	-60.13	-8.77	-44.20
180	-115.06	-7.98	86.98	-60.36	-17.11	-40.56

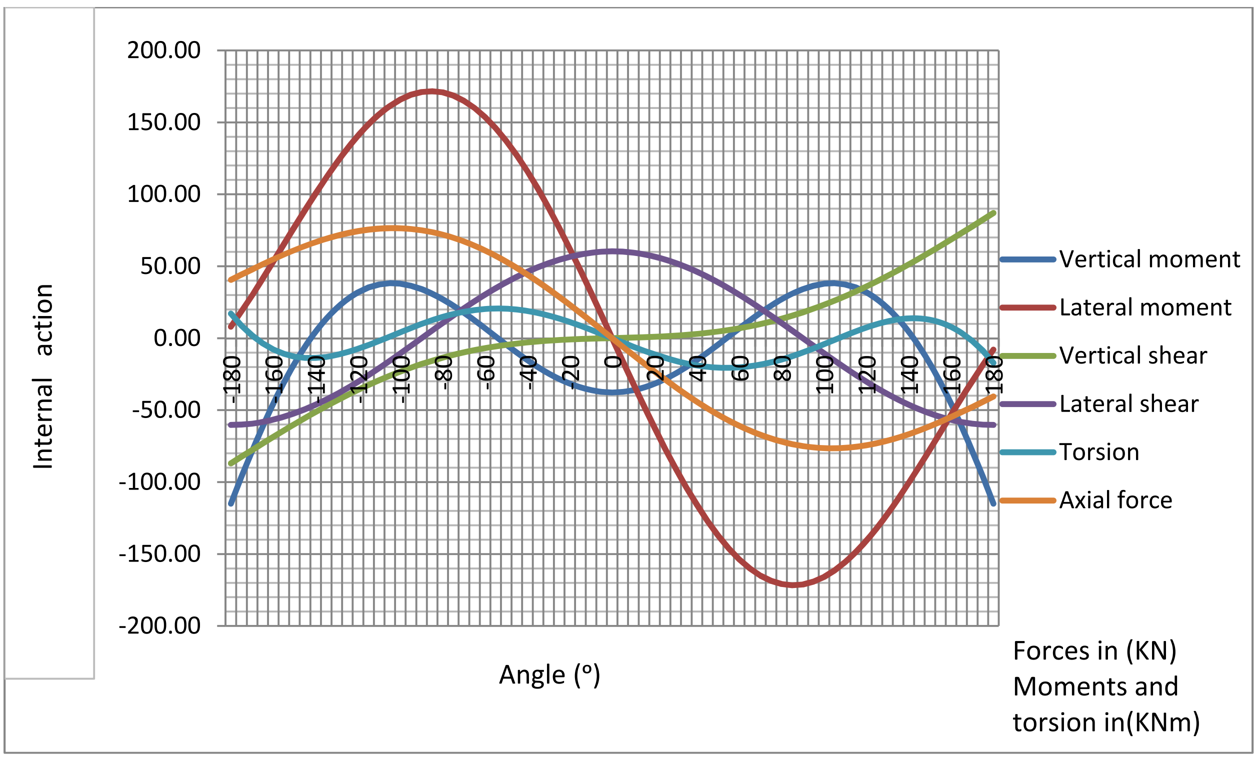

Figure A 8: Flight slope =25° & Radius =2.5m

Table A 9: Flight slope =30° & Radius =1.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-18.38	1.82	-33.25	-19.16	3.15	19.20
-175	-15.41	3.01	-31.49	-19.09	1.87	20.11
-170	-12.59	4.31	-29.74	-18.87	0.82	21.01
-165	-9.93	5.71	-28.00	-18.51	-0.03	21.89
-160	-7.45	7.17	-26.28	-18.00	-0.69	22.74
-155	-5.16	8.68	-24.58	-17.36	-1.16	23.54
-150	-3.08	10.21	-22.92	-16.59	-1.47	24.29
-145	-1.21	11.75	-21.29	-15.69	-1.63	24.98
-140	0.43	13.26	-19.70	-14.68	-1.66	25.60
-135	1.85	14.74	-18.16	-13.55	-1.57	26.13
-130	3.05	16.15	-16.67	-12.32	-1.39	26.57
-125	4.01	17.48	-15.24	-10.99	-1.12	26.92
-120	4.76	18.71	-13.87	-9.58	-0.79	27.17
-115	5.30	19.82	-12.56	-8.10	-0.40	27.30
-110	5.63	20.80	-11.32	-6.55	0.01	27.32
-105	5.77	21.62	-10.14	-4.96	0.44	27.22
-100	5.72	22.29	-9.04	-3.33	0.88	27.00
-95	5.51	22.79	-8.00	-1.67	1.30	26.66
-90	5.16	23.11	-7.04	0.00	1.71	26.19
-85	4.67	23.24	-6.16	1.67	2.08	25.59
-80	4.06	23.18	-5.34	3.33	2.41	24.87
-75	3.36	22.92	-4.60	4.96	2.69	24.03
-70	2.59	22.47	-3.93	6.55	2.92	23.06
-65	1.76	21.83	-3.32	8.10	3.08	21.97
-60	0.89	21.00	-2.79	9.58	3.18	20.77
-55	0.01	19.98	-2.31	10.99	3.21	19.46
-50	-0.86	18.79	-1.90	12.32	3.18	18.04
-45	-1.72	17.42	-1.54	13.55	3.08	16.53
-40	-2.54	15.91	-1.23	14.68	2.92	14.93
-35	-3.30	14.25	-0.97	15.69	2.70	13.25
-30	-3.99	12.46	-0.75	16.59	2.43	11.50
-25	-4.59	10.56	-0.57	17.36	2.10	9.68
-20	-5.11	8.57	-0.42	18.00	1.73	7.81
-15	-5.52	6.50	-0.29	18.51	1.33	5.89
-10	-5.82	4.36	-0.18	18.87	0.90	3.95
-5	-6.00	2.19	-0.09	19.09	0.46	1.98
0	-6.06	0.00	0.00	19.16	0.00	0.00
5	-6.00	-2.19	0.09	19.09	-0.46	-1.98
10	-5.82	-4.36	0.18	18.87	-0.90	-3.95
15	-5.52	-6.50	0.29	18.51	-1.33	-5.89
20	-5.11	-8.57	0.42	18.00	-1.73	-7.81
25	-4.59	-10.56	0.57	17.36	-2.10	-9.68
30	-3.99	-12.46	0.75	16.59	-2.43	-11.50
35	-3.30	-14.25	0.97	15.69	-2.70	-13.25
40	-2.54	-15.91	1.23	14.68	-2.92	-14.93
45	-1.72	-17.42	1.54	13.55	-3.08	-16.53
50	-0.86	-18.79	1.90	12.32	-3.18	-18.04
55	0.01	-19.98	2.31	10.99	-3.21	-19.46
60	0.89	-21.00	2.79	9.58	-3.18	-20.77
65	1.76	-21.83	3.32	8.10	-3.08	-21.97
70	2.59	-22.47	3.93	6.55	-2.92	-23.06
75	3.36	-22.92	4.60	4.96	-2.69	-24.03
80	4.06	-23.18	5.34	3.33	-2.41	-24.87
85	4.67	-23.24	6.16	1.67	-2.08	-25.59
90	5.16	-23.11	7.04	0.00	-1.71	-26.19
95	5.51	-22.79	8.00	-1.67	-1.30	-26.66

100	5.72	-22.29	9.04	-3.33	-0.88	-27.00
105	5.77	-21.62	10.14	-4.96	-0.44	-27.22
110	5.63	-20.80	11.32	-6.55	-0.01	-27.32
115	5.30	-19.82	12.56	-8.10	0.40	-27.30
120	4.76	-18.71	13.87	-9.58	0.79	-27.17
125	4.01	-17.48	15.24	-10.99	1.12	-26.92
130	3.05	-16.15	16.67	-12.32	1.39	-26.57
135	1.85	-14.74	18.16	-13.55	1.57	-26.13
140	0.43	-13.26	19.70	-14.68	1.66	-25.60
145	-1.21	-11.75	21.29	-15.69	1.63	-24.98
150	-3.08	-10.21	22.92	-16.59	1.47	-24.29
155	-5.16	-8.68	24.58	-17.36	1.16	-23.54
160	-7.45	-7.17	26.28	-18.00	0.69	-22.74
165	-9.93	-5.71	28.00	-18.51	0.03	-21.89
170	-12.59	-4.31	29.74	-18.87	-0.82	-21.01
175	-15.41	-3.01	31.49	-19.09	-1.87	-20.11
180	-18.38	-1.82	33.25	-19.16	-3.15	-19.20

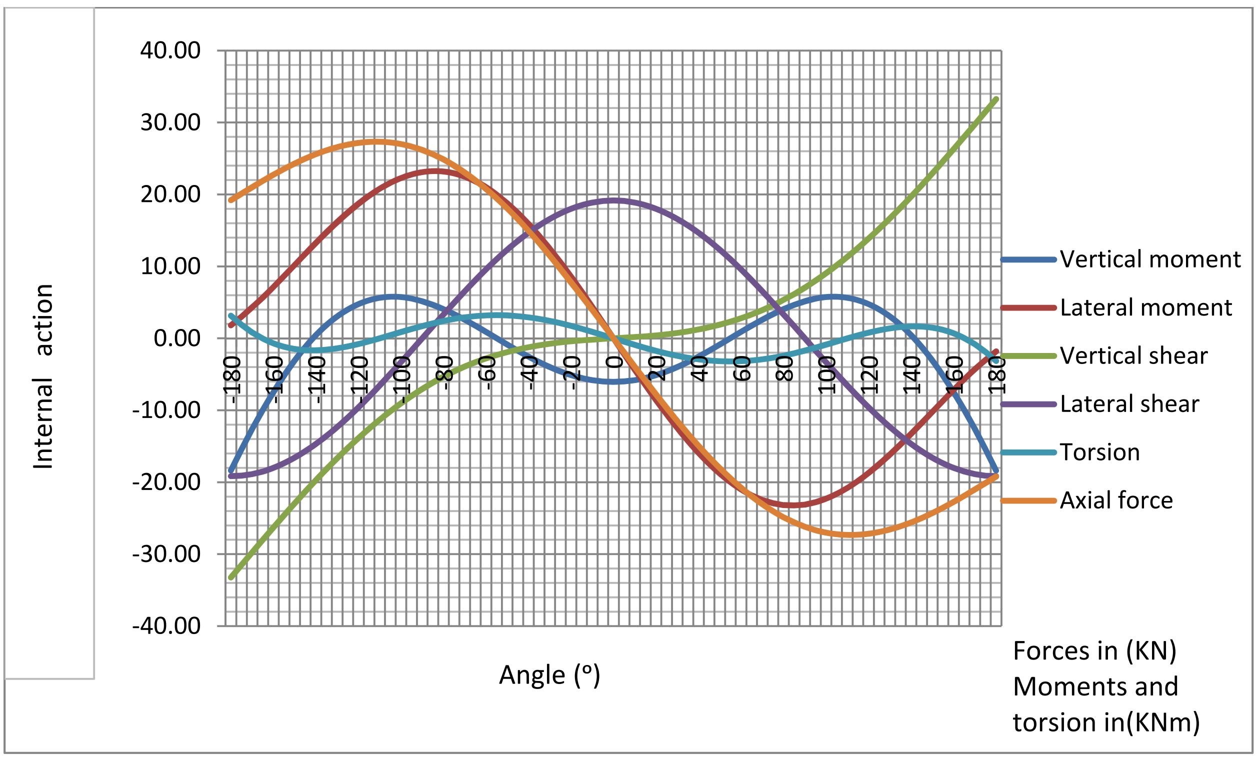

Figure A 9: Flight slope =30° & Radius =1.0m

Table A 10: Flight slope =30° & Radius =1.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-41.36	4.09	-49.87	-28.74	7.09	28.79
-175	-34.68	6.77	-47.23	-28.63	4.22	30.16
-170	-28.33	9.71	-44.60	-28.30	1.84	31.52
-165	-22.34	12.84	-42.00	-27.76	-0.07	32.84
-160	-16.75	16.13	-39.41	-27.01	-1.55	34.11
-155	-11.61	19.53	-36.87	-26.05	-2.62	35.31
-150	-6.92	22.98	-34.37	-24.89	-3.31	36.44
-145	-2.72	26.43	-31.93	-23.54	-3.68	37.47
-140	0.98	29.84	-29.55	-22.02	-3.74	38.39
-135	4.17	33.15	-27.24	-20.32	-3.54	39.19
-130	6.85	36.33	-25.01	-18.47	-3.12	39.86
-125	9.03	39.32	-22.86	-16.48	-2.52	40.38
-120	10.72	42.09	-20.80	-14.37	-1.77	40.75
-115	11.92	44.59	-18.84	-12.15	-0.91	40.95
-110	12.67	46.79	-16.97	-9.83	0.02	40.98
-105	12.97	48.66	-15.21	-7.44	0.99	40.84
-100	12.88	50.16	-13.55	-4.99	1.97	40.51
-95	12.41	51.28	-12.01	-2.50	2.93	39.99
-90	11.60	52.00	-10.57	0.00	3.84	39.29
-85	10.50	52.29	-9.23	2.50	4.68	38.39
-80	9.14	52.15	-8.01	4.99	5.42	37.31
-75	7.56	51.58	-6.90	7.44	6.05	36.04
-70	5.82	50.56	-5.89	9.83	6.56	34.59
-65	3.96	49.12	-4.99	12.15	6.93	32.95
-60	2.01	47.24	-4.18	14.37	7.15	31.15
-55	0.03	44.95	-3.47	16.48	7.23	29.19
-50	-1.94	42.27	-2.84	18.47	7.16	27.06
-45	-3.87	39.21	-2.31	20.32	6.94	24.80
-40	-5.70	35.79	-1.85	22.02	6.58	22.40
-35	-7.42	32.06	-1.45	23.54	6.08	19.87
-30	-8.97	28.04	-1.13	24.89	5.46	17.24
-25	-10.34	23.77	-0.85	26.05	4.73	14.52
-20	-11.49	19.28	-0.63	27.01	3.90	11.71
-15	-12.42	14.61	-0.44	27.76	3.00	8.84
-10	-13.09	9.82	-0.28	28.30	2.03	5.92
-5	-13.49	4.93	-0.13	28.63	1.03	2.97
0	-13.63	0.00	0.00	28.74	0.00	0.00
5	-13.49	-4.93	0.13	28.63	-1.03	-2.97
10	-13.09	-9.82	0.28	28.30	-2.03	-5.92
15	-12.42	-14.61	0.44	27.76	-3.00	-8.84
20	-11.49	-19.28	0.63	27.01	-3.90	-11.71
25	-10.34	-23.77	0.85	26.05	-4.73	-14.52
30	-8.97	-28.04	1.13	24.89	-5.46	-17.24
35	-7.42	-32.06	1.45	23.54	-6.08	-19.87
40	-5.70	-35.79	1.85	22.02	-6.58	-22.40
45	-3.87	-39.21	2.31	20.32	-6.94	-24.80
50	-1.94	-42.27	2.84	18.47	-7.16	-27.06
55	0.03	-44.95	3.47	16.48	-7.23	-29.19
60	2.01	-47.24	4.18	14.37	-7.15	-31.15
65	3.96	-49.12	4.99	12.15	-6.93	-32.95
70	5.82	-50.56	5.89	9.83	-6.56	-34.59
75	7.56	-51.58	6.90	7.44	-6.05	-36.04
80	9.14	-52.15	8.01	4.99	-5.42	-37.31
85	10.50	-52.29	9.23	2.50	-4.68	-38.39
90	11.60	-52.00	10.57	0.00	-3.84	-39.29
95	12.41	-51.28	12.01	-2.50	-2.93	-39.99

100	12.88	-50.16	13.55	-4.99	-1.97	-40.51
105	12.97	-48.66	15.21	-7.44	-0.99	-40.84
110	12.67	-46.79	16.97	-9.83	-0.02	-40.98
115	11.92	-44.59	18.84	-12.15	0.91	-40.95
120	10.72	-42.09	20.80	-14.37	1.77	-40.75
125	9.03	-39.32	22.86	-16.48	2.52	-40.38
130	6.85	-36.33	25.01	-18.47	3.12	-39.86
135	4.17	-33.15	27.24	-20.32	3.54	-39.19
140	0.98	-29.84	29.55	-22.02	3.74	-38.39
145	-2.72	-26.43	31.93	-23.54	3.68	-37.47
150	-6.92	-22.98	34.37	-24.89	3.31	-36.44
155	-11.61	-19.53	36.87	-26.05	2.62	-35.31
160	-16.75	-16.13	39.41	-27.01	1.55	-34.11
165	-22.34	-12.84	42.00	-27.76	0.07	-32.84
170	-28.33	-9.71	44.60	-28.30	-1.84	-31.52
175	-34.68	-6.77	47.23	-28.63	-4.22	-30.16
180	-41.36	-4.09	49.87	-28.74	-7.09	-28.79

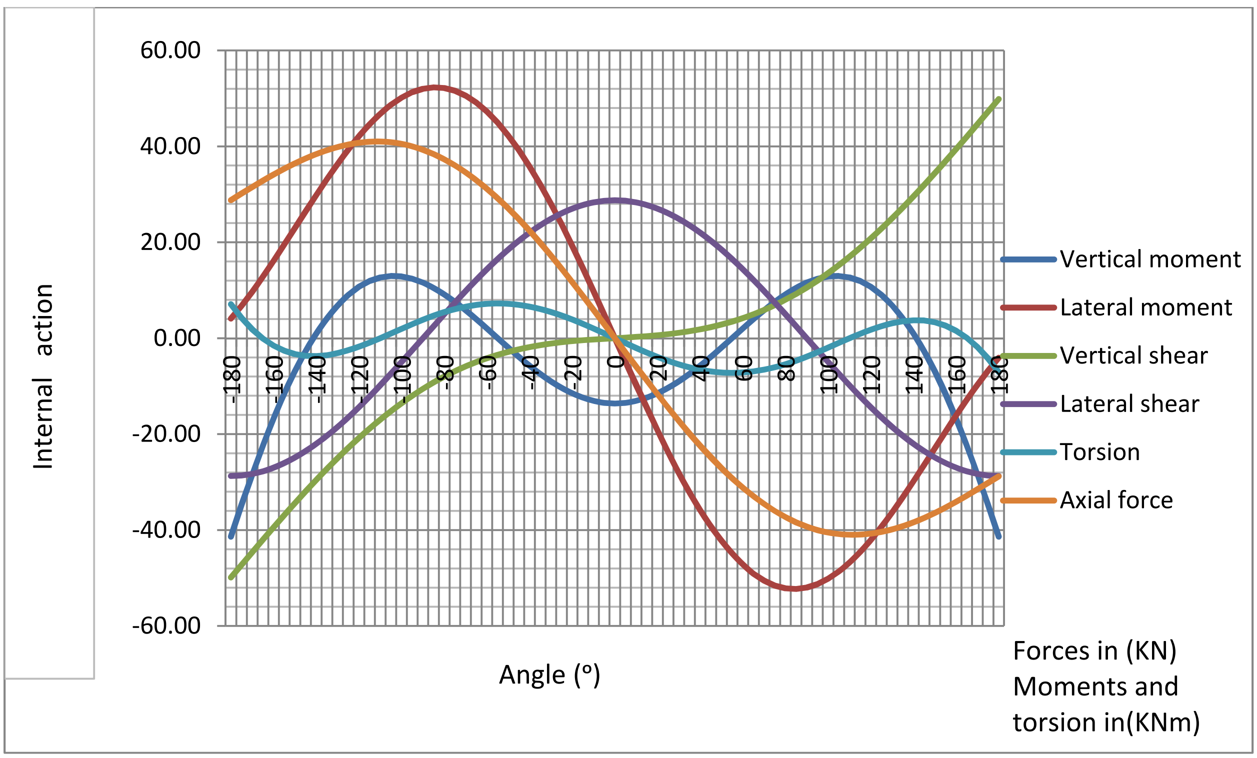

Figure A 10: Flight slope =30° & Radius =1.5m

Table A 11: Flight slope =30° & Radius =2.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-73.53	7.28	-66.49	-38.32	12.60	38.39
-175	-61.66	12.04	-62.98	-38.17	7.50	40.22
-170	-50.36	17.25	-59.47	-37.74	3.27	42.02
-165	-39.71	22.83	-55.99	-37.01	-0.13	43.78
-160	-29.78	28.68	-52.55	-36.01	-2.75	45.47
-155	-20.63	34.71	-49.16	-34.73	-4.65	47.08
-150	-12.31	40.85	-45.83	-33.19	-5.89	48.58
-145	-4.84	46.99	-42.57	-31.39	-6.53	49.96
-140	1.73	53.05	-39.40	-29.35	-6.65	51.19
-135	7.41	58.94	-36.32	-27.10	-6.29	52.26
-130	12.18	64.59	-33.35	-24.63	-5.55	53.15
-125	16.06	69.91	-30.48	-21.98	-4.48	53.84
-120	19.05	74.82	-27.74	-19.16	-3.14	54.33
-115	21.19	79.27	-25.12	-16.19	-1.62	54.60
-110	22.52	83.18	-22.63	-13.11	0.04	54.65
-105	23.07	86.50	-20.28	-9.92	1.77	54.45
-100	22.89	89.18	-18.07	-6.65	3.51	54.01
-95	22.06	91.17	-16.01	-3.34	5.21	53.32
-90	20.62	92.44	-14.09	0.00	6.83	52.38
-85	18.66	92.96	-12.31	3.34	8.31	51.19
-80	16.24	92.71	-10.68	6.65	9.64	49.74
-75	13.45	91.69	-9.20	9.92	10.76	48.05
-70	10.35	89.89	-7.85	13.11	11.66	46.11
-65	7.03	87.32	-6.65	16.19	12.32	43.94
-60	3.57	83.98	-5.57	19.16	12.72	41.54
-55	0.05	79.91	-4.62	21.98	12.86	38.91
-50	-3.46	75.14	-3.79	24.63	12.73	36.09
-45	-6.88	69.70	-3.08	27.10	12.34	33.06
-40	-10.14	63.63	-2.46	29.35	11.69	29.86
-35	-13.18	57.00	-1.94	31.39	10.81	26.50
-30	-15.95	49.85	-1.50	33.19	9.71	22.99
-25	-18.38	42.25	-1.14	34.73	8.41	19.36
-20	-20.43	34.27	-0.84	36.01	6.94	15.62
-15	-22.07	25.98	-0.58	37.01	5.33	11.79
-10	-23.26	17.45	-0.37	37.74	3.61	7.90
-5	-23.99	8.77	-0.18	38.17	1.83	3.96
0	-24.23	0.00	0.00	38.32	0.00	0.00
5	-23.99	-8.77	0.18	38.17	-1.83	-3.96
10	-23.26	-17.45	0.37	37.74	-3.61	-7.90
15	-22.07	-25.98	0.58	37.01	-5.33	-11.79
20	-20.43	-34.27	0.84	36.01	-6.94	-15.62
25	-18.38	-42.25	1.14	34.73	-8.41	-19.36
30	-15.95	-49.85	1.50	33.19	-9.71	-22.99
35	-13.18	-57.00	1.94	31.39	-10.81	-26.50
40	-10.14	-63.63	2.46	29.35	-11.69	-29.86
45	-6.88	-69.70	3.08	27.10	-12.34	-33.06
50	-3.46	-75.14	3.79	24.63	-12.73	-36.09
55	0.05	-79.91	4.62	21.98	-12.86	-38.91
60	3.57	-83.98	5.57	19.16	-12.72	-41.54
65	7.03	-87.32	6.65	16.19	-12.32	-43.94
70	10.35	-89.89	7.85	13.11	-11.66	-46.11
75	13.45	-91.69	9.20	9.92	-10.76	-48.05
80	16.24	-92.71	10.68	6.65	-9.64	-49.74
85	18.66	-92.96	12.31	3.34	-8.31	-51.19
90	20.62	-92.44	14.09	0.00	-6.83	-52.38
95	22.06	-91.17	16.01	-3.34	-5.21	-53.32

100	22.89	-89.18	18.07	-6.65	-3.51	-54.01
105	23.07	-86.50	20.28	-9.92	-1.77	-54.45
110	22.52	-83.18	22.63	-13.11	-0.04	-54.65
115	21.19	-79.27	25.12	-16.19	1.62	-54.60
120	19.05	-74.82	27.74	-19.16	3.14	-54.33
125	16.06	-69.91	30.48	-21.98	4.48	-53.84
130	12.18	-64.59	33.35	-24.63	5.55	-53.15
135	7.41	-58.94	36.32	-27.10	6.29	-52.26
140	1.73	-53.05	39.40	-29.35	6.65	-51.19
145	-4.84	-46.99	42.57	-31.39	6.53	-49.96
150	-12.31	-40.85	45.83	-33.19	5.89	-48.58
155	-20.63	-34.71	49.16	-34.73	4.65	-47.08
160	-29.78	-28.68	52.55	-36.01	2.75	-45.47
165	-39.71	-22.83	55.99	-37.01	0.13	-43.78
170	-50.36	-17.25	59.47	-37.74	-3.27	-42.02
175	-61.66	-12.04	62.98	-38.17	-7.50	-40.22
180	-73.53	-7.28	66.49	-38.32	-12.60	-38.39

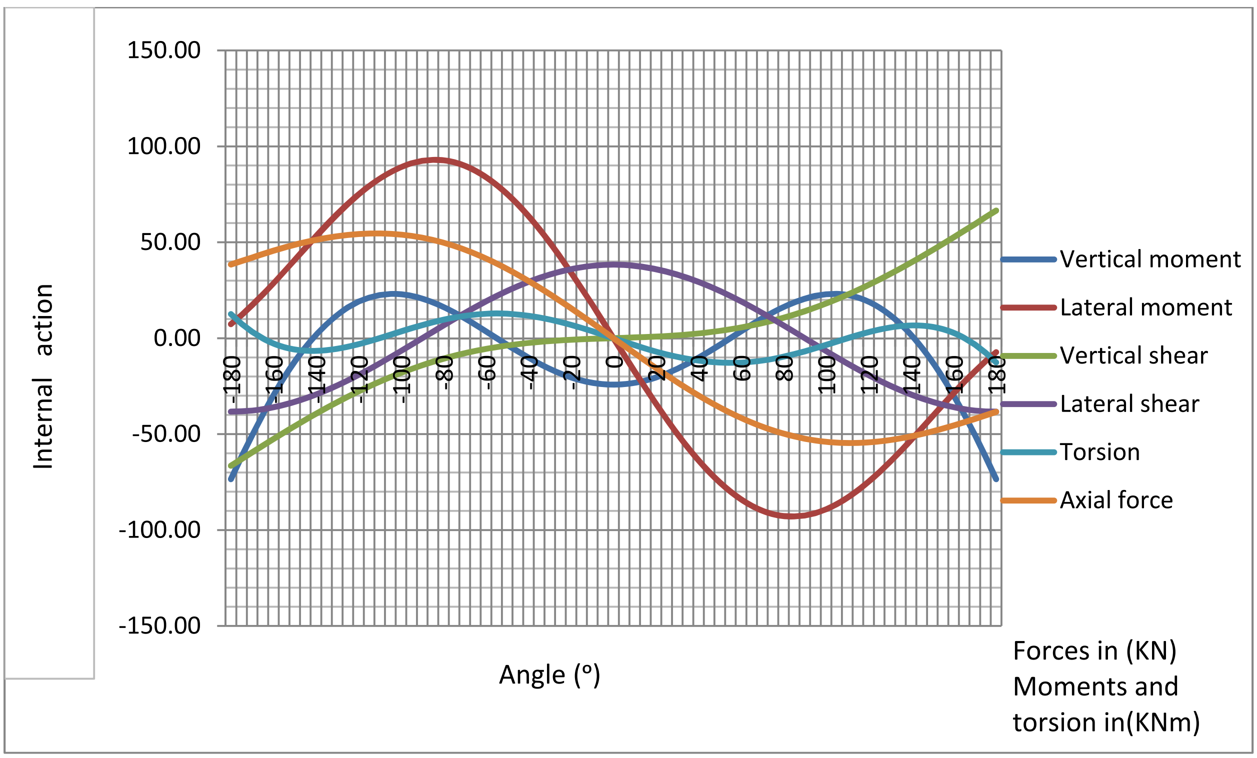

Figure A 11: Flight slope =30° & Radius =2.0m

Table A 12: Flight slope =30° & Radius =2.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-114.89	11.37	-83.12	-47.90	19.69	47.99
-175	-96.34	18.81	-78.72	-47.72	11.71	50.27
-170	-78.69	26.96	-74.34	-47.17	5.11	52.53
-165	-62.05	35.67	-69.99	-46.27	-0.20	54.73
-160	-46.54	44.81	-65.69	-45.01	-4.30	56.84
-155	-32.24	54.24	-61.45	-43.41	-7.27	58.85
-150	-19.23	63.82	-57.29	-41.48	-9.21	60.73
-145	-7.57	73.42	-53.22	-39.24	-10.21	62.45
-140	2.71	82.89	-49.25	-36.69	-10.38	63.99
-135	11.58	92.10	-45.40	-33.87	-9.84	65.32
-130	19.03	100.92	-41.68	-30.79	-8.67	66.44
-125	25.09	109.23	-38.10	-27.47	-6.99	67.31
-120	29.77	116.91	-34.67	-23.95	-4.91	67.92
-115	33.11	123.86	-31.40	-20.24	-2.53	68.25
-110	35.18	129.97	-28.29	-16.38	0.06	68.31
-105	36.04	135.16	-25.35	-12.40	2.76	68.06
-100	35.77	139.34	-22.59	-8.32	5.48	67.51
-95	34.47	142.45	-20.01	-4.17	8.14	66.65
-90	32.23	144.43	-17.61	0.00	10.67	65.48
-85	29.16	145.25	-15.39	4.17	12.99	63.99
-80	25.38	144.87	-13.36	8.32	15.06	62.18
-75	21.01	143.27	-11.50	12.40	16.81	60.06
-70	16.17	140.45	-9.82	16.38	18.22	57.64
-65	10.99	136.43	-8.31	20.24	19.25	54.92
-60	5.58	131.22	-6.96	23.95	19.87	51.92
-55	0.08	124.87	-5.78	27.47	20.09	48.64
-50	-5.40	117.41	-4.74	30.79	19.89	45.11
-45	-10.74	108.90	-3.84	33.87	19.28	41.33
-40	-15.84	99.43	-3.08	36.69	18.27	37.33
-35	-20.60	89.06	-2.42	39.24	16.89	33.12
-30	-24.92	77.89	-1.88	41.48	15.17	28.74
-25	-28.72	66.02	-1.42	43.41	13.14	24.20
-20	-31.93	53.55	-1.04	45.01	10.84	19.52
-15	-34.49	40.60	-0.73	46.27	8.33	14.74
-10	-36.35	27.27	-0.46	47.17	5.65	9.87
-5	-37.48	13.70	-0.22	47.72	2.85	4.95
0	-37.86	0.00	0.00	47.90	0.00	0.00
5	-37.48	-13.70	0.22	47.72	-2.85	-4.95
10	-36.35	-27.27	0.46	47.17	-5.65	-9.87
15	-34.49	-40.60	0.73	46.27	-8.33	-14.74
20	-31.93	-53.55	1.04	45.01	-10.84	-19.52
25	-28.72	-66.02	1.42	43.41	-13.14	-24.20
30	-24.92	-77.89	1.88	41.48	-15.17	-28.74
35	-20.60	-89.06	2.42	39.24	-16.89	-33.12
40	-15.84	-99.43	3.08	36.69	-18.27	-37.33
45	-10.74	-108.90	3.84	33.87	-19.28	-41.33
50	-5.40	-117.41	4.74	30.79	-19.89	-45.11
55	0.08	-124.87	5.78	27.47	-20.09	-48.64
60	5.58	-131.22	6.96	23.95	-19.87	-51.92
65	10.99	-136.43	8.31	20.24	-19.25	-54.92
70	16.17	-140.45	9.82	16.38	-18.22	-57.64
75	21.01	-143.27	11.50	12.40	-16.81	-60.06
80	25.38	-144.87	13.36	8.32	-15.06	-62.18
85	29.16	-145.25	15.39	4.17	-12.99	-63.99
90	32.23	-144.43	17.61	0.00	-10.67	-65.48

95	34.47	-142.45	20.01	-4.17	-8.14	-66.65
100	35.77	-139.34	22.59	-8.32	-5.48	-67.51
105	36.04	-135.16	25.35	-12.40	-2.76	-68.06
110	35.18	-129.97	28.29	-16.38	-0.06	-68.31
115	33.11	-123.86	31.40	-20.24	2.53	-68.25
120	29.77	-116.91	34.67	-23.95	4.91	-67.92
125	25.09	-109.23	38.10	-27.47	6.99	-67.31
130	19.03	-100.92	41.68	-30.79	8.67	-66.44
135	11.58	-92.10	45.40	-33.87	9.84	-65.32
140	2.71	-82.89	49.25	-36.69	10.38	-63.99
145	-7.57	-73.42	53.22	-39.24	10.21	-62.45
150	-19.23	-63.82	57.29	-41.48	9.21	-60.73
155	-32.24	-54.24	61.45	-43.41	7.27	-58.85
160	-46.54	-44.81	65.69	-45.01	4.30	-56.84
165	-62.05	-35.67	69.99	-46.27	0.20	-54.73
170	-78.69	-26.96	74.34	-47.17	-5.11	-52.53
175	-96.34	-18.81	78.72	-47.72	-11.71	-50.27
180	-114.89	-11.37	83.12	-47.90	-19.69	-47.99

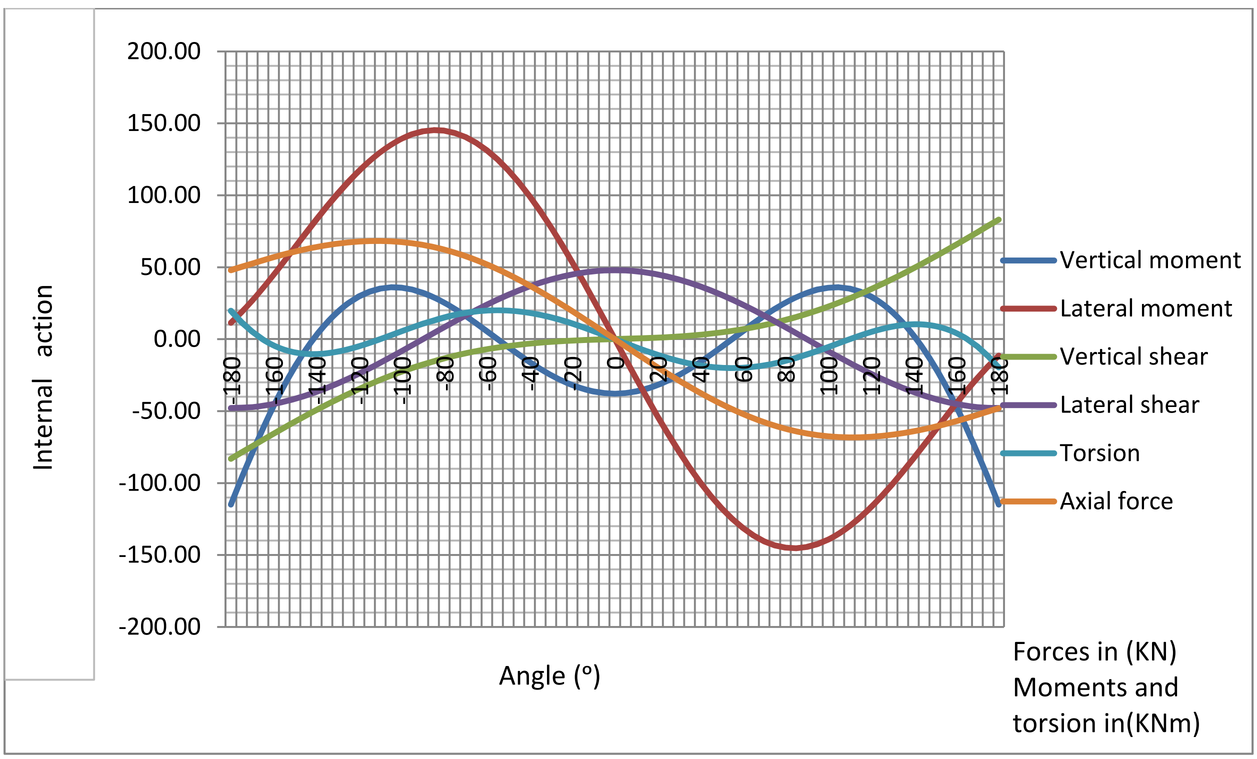

Figure A 12: Flight slope =30° & Radius =2.5m

Table A 13: Flight slope =35° & Radius =1.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-18.33	2.81	-31.45	-15.23	4.01	22.02
-175	-15.47	3.58	-29.81	-15.17	2.80	22.50
-170	-12.75	4.48	-28.18	-15.00	1.79	22.96
-165	-10.18	5.49	-26.57	-14.71	0.97	23.41
-160	-7.78	6.59	-24.97	-14.31	0.33	23.84
-155	-5.57	7.76	-23.39	-13.80	-0.14	24.23
-150	-3.56	8.97	-21.84	-13.19	-0.47	24.59
-145	-1.75	10.20	-20.32	-12.47	-0.66	24.89
-140	-0.16	11.44	-18.84	-11.67	-0.72	25.14
-135	1.22	12.67	-17.41	-10.77	-0.68	25.34
-130	2.38	13.85	-16.02	-9.79	-0.55	25.46
-125	3.33	14.98	-14.68	-8.73	-0.35	25.51
-120	4.06	16.04	-13.40	-7.61	-0.08	25.48
-115	4.59	17.01	-12.18	-6.44	0.23	25.37
-110	4.93	17.87	-11.01	-5.21	0.57	25.18
-105	5.07	18.61	-9.91	-3.94	0.93	24.89
-100	5.05	19.21	-8.87	-2.64	1.29	24.52
-95	4.86	19.67	-7.90	-1.33	1.64	24.05
-90	4.53	19.98	-6.99	0.00	1.98	23.48
-85	4.07	20.12	-6.15	1.33	2.29	22.82
-80	3.50	20.10	-5.37	2.64	2.56	22.07
-75	2.84	19.91	-4.67	3.94	2.79	21.22
-70	2.11	19.55	-4.02	5.21	2.97	20.29
-65	1.33	19.01	-3.44	6.44	3.09	19.26
-60	0.51	18.31	-2.92	7.61	3.15	18.14
-55	-0.33	17.44	-2.45	8.73	3.16	16.95
-50	-1.16	16.42	-2.04	9.79	3.11	15.67
-45	-1.98	15.24	-1.69	10.77	3.00	14.33
-40	-2.75	13.93	-1.37	11.67	2.83	12.91
-35	-3.48	12.49	-1.10	12.47	2.60	11.44
-30	-4.13	10.93	-0.87	13.19	2.33	9.91
-25	-4.71	9.27	-0.68	13.80	2.01	8.33
-20	-5.20	7.52	-0.51	14.31	1.66	6.71
-15	-5.59	5.70	-0.36	14.71	1.27	5.06
-10	-5.88	3.83	-0.23	15.00	0.86	3.39
-5	-6.05	1.92	-0.11	15.17	0.44	1.70
0	-6.11	0.00	0.00	15.23	0.00	0.00
5	-6.05	-1.92	0.11	15.17	-0.44	-1.70
10	-5.88	-3.83	0.23	15.00	-0.86	-3.39
15	-5.59	-5.70	0.36	14.71	-1.27	-5.06
20	-5.20	-7.52	0.51	14.31	-1.66	-6.71
25	-4.71	-9.27	0.68	13.80	-2.01	-8.33
30	-4.13	-10.93	0.87	13.19	-2.33	-9.91
35	-3.48	-12.49	1.10	12.47	-2.60	-11.44
40	-2.75	-13.93	1.37	11.67	-2.83	-12.91
45	-1.98	-15.24	1.69	10.77	-3.00	-14.33
50	-1.16	-16.42	2.04	9.79	-3.11	-15.67
55	-0.33	-17.44	2.45	8.73	-3.16	-16.95
60	0.51	-18.31	2.92	7.61	-3.15	-18.14
65	1.33	-19.01	3.44	6.44	-3.09	-19.26
70	2.11	-19.55	4.02	5.21	-2.97	-20.29
75	2.84	-19.91	4.67	3.94	-2.79	-21.22
80	3.50	-20.10	5.37	2.64	-2.56	-22.07
85	4.07	-20.12	6.15	1.33	-2.29	-22.82
90	4.53	-19.98	6.99	0.00	-1.98	-23.48

95	4.86	-19.67	7.90	-1.33	-1.64	-24.05
100	5.05	-19.21	8.87	-2.64	-1.29	-24.52
105	5.07	-18.61	9.91	-3.94	-0.93	-24.89
110	4.93	-17.87	11.01	-5.21	-0.57	-25.18
115	4.59	-17.01	12.18	-6.44	-0.23	-25.37
120	4.06	-16.04	13.40	-7.61	0.08	-25.48
125	3.33	-14.98	14.68	-8.73	0.35	-25.51
130	2.38	-13.85	16.02	-9.79	0.55	-25.46
135	1.22	-12.67	17.41	-10.77	0.68	-25.34
140	-0.16	-11.44	18.84	-11.67	0.72	-25.14
145	-1.75	-10.20	20.32	-12.47	0.66	-24.89
150	-3.56	-8.97	21.84	-13.19	0.47	-24.59
155	-5.57	-7.76	23.39	-13.80	0.14	-24.23
160	-7.78	-6.59	24.97	-14.31	-0.33	-23.84
165	-10.18	-5.49	26.57	-14.71	-0.97	-23.41
170	-12.75	-4.48	28.18	-15.00	-1.79	-22.96
175	-15.47	-3.58	29.81	-15.17	-2.80	-22.50
180	-18.33	-2.81	31.45	-15.23	-4.01	-22.02

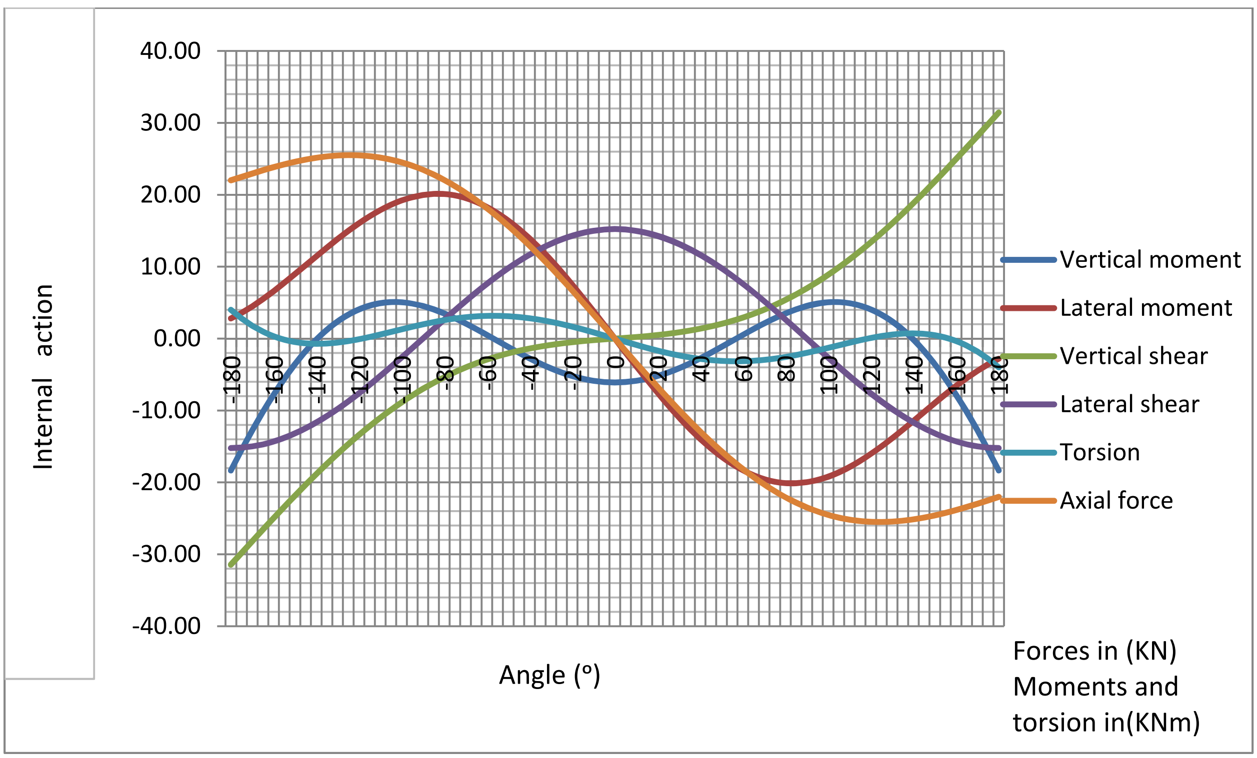

Figure A 13: Flight slope =35° & Radius =1.0m

Table A 14: Flight slope =35° & Radius =1.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-41.25	6.31	-47.17	-22.84	9.02	33.03
-175	-34.81	8.06	-44.72	-22.76	6.30	33.74
-170	-28.68	10.09	-42.28	-22.50	4.03	34.44
-165	-22.90	12.36	-39.85	-22.06	2.19	35.12
-160	-17.51	14.83	-37.45	-21.46	0.75	35.76
-155	-12.53	17.45	-35.08	-20.70	-0.32	36.35
-150	-8.00	20.18	-32.76	-19.78	-1.05	36.88
-145	-3.94	22.96	-30.48	-18.71	-1.48	37.34
-140	-0.35	25.75	-28.27	-17.50	-1.63	37.72
-135	2.75	28.50	-26.11	-16.15	-1.54	38.00
-130	5.36	31.17	-24.03	-14.68	-1.25	38.19
-125	7.49	33.71	-22.03	-13.10	-0.78	38.26
-120	9.14	36.09	-20.10	-11.42	-0.19	38.22
-115	10.33	38.27	-18.26	-9.65	0.51	38.06
-110	11.08	40.20	-16.52	-7.81	1.28	37.77
-105	11.41	41.86	-14.86	-5.91	2.09	37.34
-100	11.35	43.22	-13.30	-3.97	2.90	36.78
-95	10.93	44.26	-11.84	-1.99	3.70	36.07
-90	10.19	44.95	-10.48	0.00	4.46	35.23
-85	9.16	45.28	-9.22	1.99	5.15	34.24
-80	7.88	45.23	-8.06	3.97	5.76	33.11
-75	6.40	44.79	-7.00	5.91	6.27	31.84
-70	4.75	43.98	-6.03	7.81	6.67	30.43
-65	2.99	42.78	-5.16	9.65	6.95	28.89
-60	1.14	41.19	-4.38	11.42	7.10	27.21
-55	-0.74	39.24	-3.68	13.10	7.11	25.42
-50	-2.61	36.94	-3.07	14.68	6.99	23.51
-45	-4.44	34.30	-2.53	16.15	6.74	21.49
-40	-6.19	31.34	-2.06	17.50	6.36	19.37
-35	-7.82	28.09	-1.66	18.71	5.86	17.15
-30	-9.30	24.59	-1.31	19.78	5.24	14.86
-25	-10.60	20.85	-1.01	20.70	4.53	12.50
-20	-11.70	16.92	-0.76	21.46	3.73	10.07
-15	-12.58	12.83	-0.54	22.06	2.86	7.60
-10	-13.22	8.62	-0.35	22.50	1.94	5.08
-5	-13.61	4.33	-0.17	22.76	0.98	2.55
0	-13.74	0.00	0.00	22.84	0.00	0.00
5	-13.61	-4.33	0.17	22.76	-0.98	-2.55
10	-13.22	-8.62	0.35	22.50	-1.94	-5.08
15	-12.58	-12.83	0.54	22.06	-2.86	-7.60
20	-11.70	-16.92	0.76	21.46	-3.73	-10.07
25	-10.60	-20.85	1.01	20.70	-4.53	-12.50
30	-9.30	-24.59	1.31	19.78	-5.24	-14.86
35	-7.82	-28.09	1.66	18.71	-5.86	-17.15
40	-6.19	-31.34	2.06	17.50	-6.36	-19.37
45	-4.44	-34.30	2.53	16.15	-6.74	-21.49
50	-2.61	-36.94	3.07	14.68	-6.99	-23.51
55	-0.74	-39.24	3.68	13.10	-7.11	-25.42
60	1.14	-41.19	4.38	11.42	-7.10	-27.21
65	2.99	-42.78	5.16	9.65	-6.95	-28.89
70	4.75	-43.98	6.03	7.81	-6.67	-30.43
75	6.40	-44.79	7.00	5.91	-6.27	-31.84
80	7.88	-45.23	8.06	3.97	-5.76	-33.11
85	9.16	-45.28	9.22	1.99	-5.15	-34.24
90	10.19	-44.95	10.48	0.00	-4.46	-35.23

95	10.93	-44.26	11.84	-1.99	-3.70	-36.07
100	11.35	-43.22	13.30	-3.97	-2.90	-36.78
105	11.41	-41.86	14.86	-5.91	-2.09	-37.34
110	11.08	-40.20	16.52	-7.81	-1.28	-37.77
115	10.33	-38.27	18.26	-9.65	-0.51	-38.06
120	9.14	-36.09	20.10	-11.42	0.19	-38.22
125	7.49	-33.71	22.03	-13.10	0.78	-38.26
130	5.36	-31.17	24.03	-14.68	1.25	-38.19
135	2.75	-28.50	26.11	-16.15	1.54	-38.00
140	-0.35	-25.75	28.27	-17.50	1.63	-37.72
145	-3.94	-22.96	30.48	-18.71	1.48	-37.34
150	-8.00	-20.18	32.76	-19.78	1.05	-36.88
155	-12.53	-17.45	35.08	-20.70	0.32	-36.35
160	-17.51	-14.83	37.45	-21.46	-0.75	-35.76
165	-22.90	-12.36	39.85	-22.06	-2.19	-35.12
170	-28.68	-10.09	42.28	-22.50	-4.03	-34.44
175	-34.81	-8.06	44.72	-22.76	-6.30	-33.74
180	-41.25	-6.31	47.17	-22.84	-9.02	-33.03

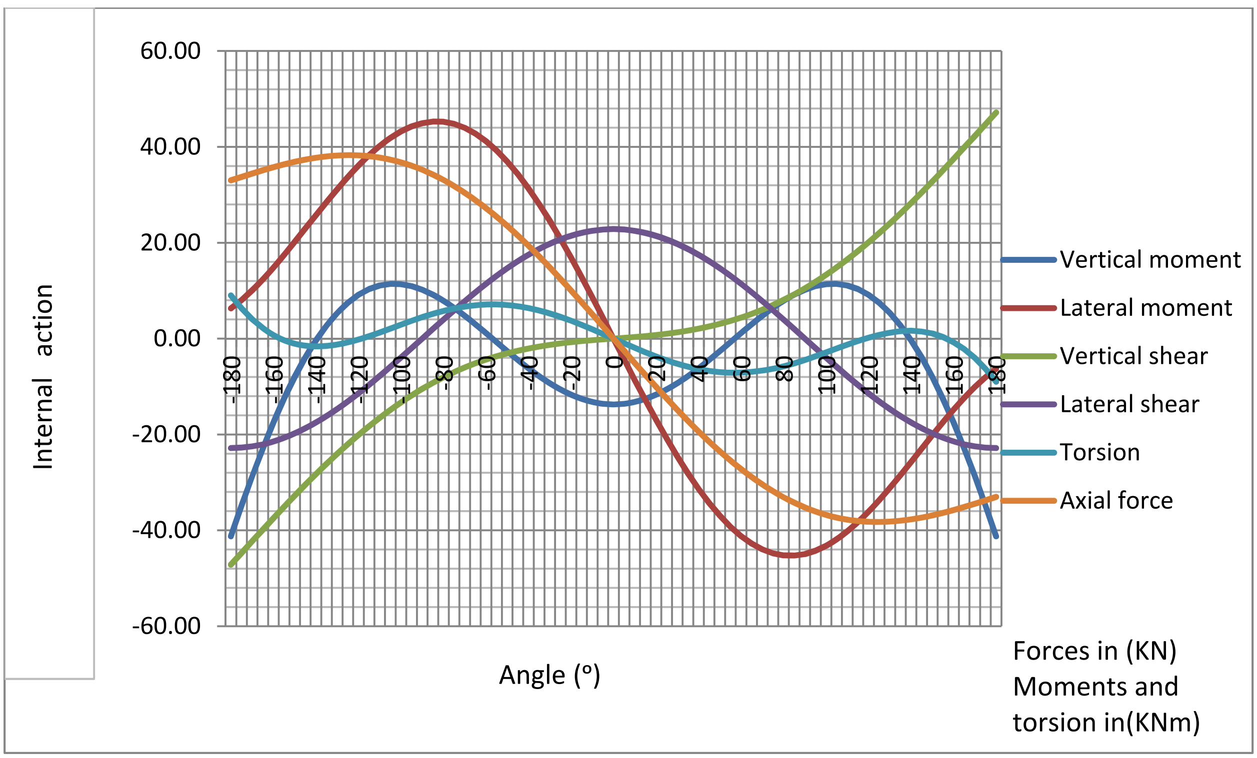

Figure A 14: Flight slope =35° & Radius =1.5m

Table A 15: Flight slope =35° & Radius =2.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-73.34	11.22	-62.89	-30.46	16.03	44.04
-175	-61.89	14.32	-59.63	-30.34	11.20	44.99
-170	-50.99	17.93	-56.37	-29.99	7.17	45.93
-165	-40.71	21.97	-53.13	-29.42	3.89	46.83
-160	-31.12	26.36	-49.93	-28.62	1.33	47.68
-155	-22.28	31.02	-46.78	-27.60	-0.57	48.47
-150	-14.23	35.87	-43.68	-26.38	-1.87	49.17
-145	-7.00	40.81	-40.65	-24.95	-2.63	49.79
-140	-0.62	45.77	-37.69	-23.33	-2.90	50.29
-135	4.89	50.66	-34.82	-21.54	-2.74	50.67
-130	9.53	55.41	-32.04	-19.58	-2.22	50.92
-125	13.32	59.94	-29.37	-17.47	-1.40	51.02
-120	16.25	64.17	-26.80	-15.23	-0.33	50.97
-115	18.37	68.03	-24.35	-12.87	0.91	50.75
-110	19.70	71.47	-22.02	-10.42	2.27	50.36
-105	20.29	74.42	-19.81	-7.88	3.71	49.79
-100	20.18	76.84	-17.74	-5.29	5.16	49.04
-95	19.44	78.68	-15.79	-2.65	6.58	48.10
-90	18.12	79.91	-13.98	0.00	7.92	46.97
-85	16.29	80.49	-12.30	2.65	9.16	45.65
-80	14.01	80.40	-10.75	5.29	10.24	44.14
-75	11.38	79.64	-9.33	7.88	11.15	42.45
-70	8.45	78.18	-8.04	10.42	11.86	40.57
-65	5.31	76.04	-6.88	12.87	12.35	38.51
-60	2.03	73.23	-5.84	15.23	12.62	36.29
-55	-1.31	69.77	-4.91	17.47	12.64	33.89
-50	-4.65	65.67	-4.09	19.58	12.43	31.34
-45	-7.90	60.97	-3.37	21.54	11.98	28.65
-40	-11.01	55.71	-2.75	23.33	11.30	25.82
-35	-13.90	49.94	-2.21	24.95	10.41	22.87
-30	-16.54	43.71	-1.75	26.38	9.32	19.81
-25	-18.85	37.07	-1.35	27.60	8.06	16.66
-20	-20.81	30.08	-1.01	28.62	6.64	13.43
-15	-22.37	22.81	-0.72	29.42	5.09	10.13
-10	-23.50	15.33	-0.46	29.99	3.45	6.78
-5	-24.19	7.70	-0.22	30.34	1.74	3.40
0	-24.42	0.00	0.00	30.46	0.00	0.00
5	-24.19	-7.70	0.22	30.34	-1.74	-3.40
10	-23.50	-15.33	0.46	29.99	-3.45	-6.78
15	-22.37	-22.81	0.72	29.42	-5.09	-10.13
20	-20.81	-30.08	1.01	28.62	-6.64	-13.43
25	-18.85	-37.07	1.35	27.60	-8.06	-16.66
30	-16.54	-43.71	1.75	26.38	-9.32	-19.81
35	-13.90	-49.94	2.21	24.95	-10.41	-22.87
40	-11.01	-55.71	2.75	23.33	-11.30	-25.82
45	-7.90	-60.97	3.37	21.54	-11.98	-28.65
50	-4.65	-65.67	4.09	19.58	-12.43	-31.34
55	-1.31	-69.77	4.91	17.47	-12.64	-33.89
60	2.03	-73.23	5.84	15.23	-12.62	-36.29
65	5.31	-76.04	6.88	12.87	-12.35	-38.51
70	8.45	-78.18	8.04	10.42	-11.86	-40.57
75	11.38	-79.64	9.33	7.88	-11.15	-42.45
80	14.01	-80.40	10.75	5.29	-10.24	-44.14
85	16.29	-80.49	12.30	2.65	-9.16	-45.65
90	18.12	-79.91	13.98	0.00	-7.92	-46.97

95	19.44	-78.68	15.79	-2.65	-6.58	-48.10
100	20.18	-76.84	17.74	-5.29	-5.16	-49.04
105	20.29	-74.42	19.81	-7.88	-3.71	-49.79
110	19.70	-71.47	22.02	-10.42	-2.27	-50.36
115	18.37	-68.03	24.35	-12.87	-0.91	-50.75
120	16.25	-64.17	26.80	-15.23	0.33	-50.97
125	13.32	-59.94	29.37	-17.47	1.40	-51.02
130	9.53	-55.41	32.04	-19.58	2.22	-50.92
135	4.89	-50.66	34.82	-21.54	2.74	-50.67
140	-0.62	-45.77	37.69	-23.33	2.90	-50.29
145	-7.00	-40.81	40.65	-24.95	2.63	-49.79
150	-14.23	-35.87	43.68	-26.38	1.87	-49.17
155	-22.28	-31.02	46.78	-27.60	0.57	-48.47
160	-31.12	-26.36	49.93	-28.62	-1.33	-47.68
165	-40.71	-21.97	53.13	-29.42	-3.89	-46.83
170	-50.99	-17.93	56.37	-29.99	-7.17	-45.93
175	-61.89	-14.32	59.63	-30.34	-11.20	-44.99
180	-73.34	-11.22	62.89	-30.46	-16.03	-44.04

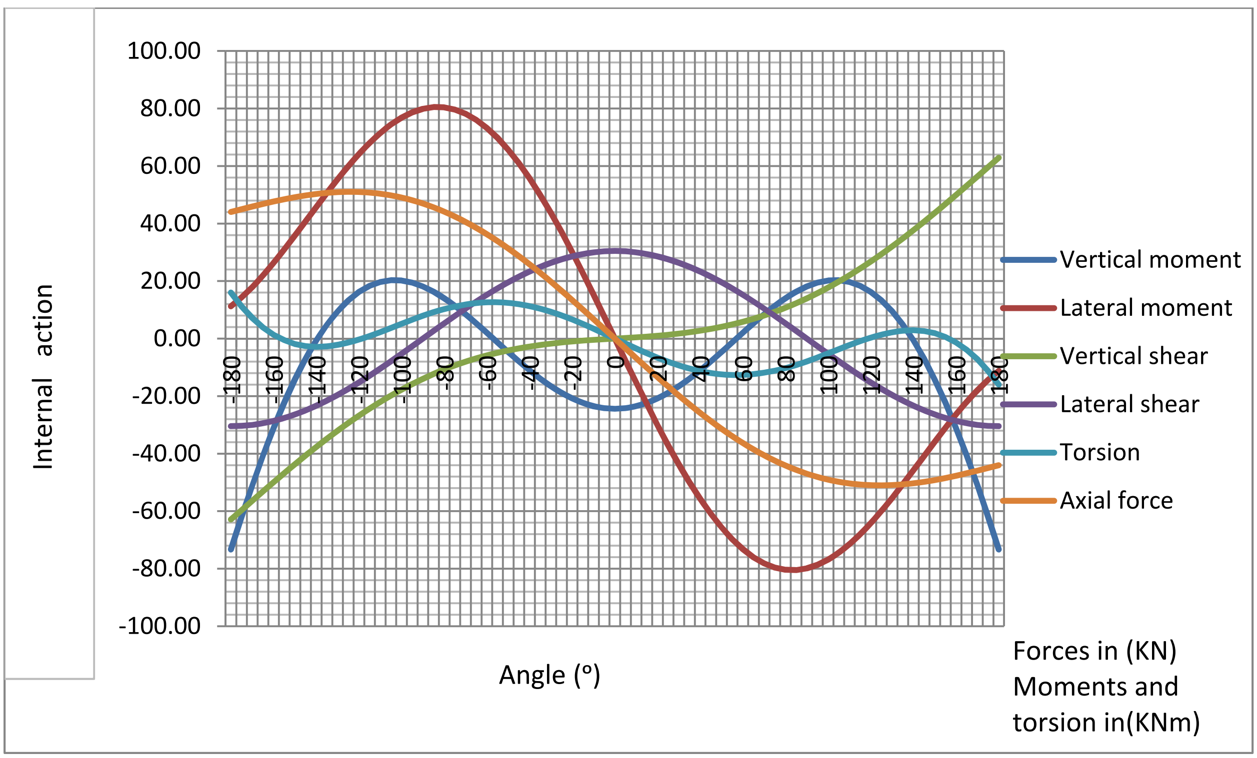

Figure A 15: Flight slope =35° & Radius =2.0m

Table A 16: Flight slope =35° & Radius =2.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-114.59	17.54	-78.62	-38.07	25.04	55.05
-175	-96.70	22.38	-74.53	-37.93	17.50	56.24
-170	-79.67	28.02	-70.46	-37.49	11.20	57.41
-165	-63.61	34.33	-66.42	-36.77	6.08	58.53
-160	-48.63	41.19	-62.41	-35.77	2.08	59.60
-155	-34.81	48.48	-58.47	-34.50	-0.90	60.58
-150	-22.23	56.04	-54.60	-32.97	-2.93	61.47
-145	-10.94	63.77	-50.81	-31.19	-4.11	62.23
-140	-0.98	71.52	-47.11	-29.16	-4.52	62.86
-135	7.64	79.16	-43.52	-26.92	-4.28	63.34
-130	14.90	86.58	-40.05	-24.47	-3.46	63.65
-125	20.81	93.65	-36.71	-21.84	-2.18	63.77
-120	25.40	100.26	-33.50	-19.04	-0.52	63.71
-115	28.71	106.30	-30.44	-16.09	1.42	63.43
-110	30.79	111.67	-27.53	-13.02	3.55	62.95
-105	31.70	116.29	-24.77	-9.85	5.79	62.24
-100	31.54	120.07	-22.17	-6.61	8.06	61.29
-95	30.37	122.94	-19.74	-3.32	10.28	60.12
-90	28.31	124.86	-17.47	0.00	12.38	58.71
-85	25.45	125.76	-15.37	3.32	14.31	57.06
-80	21.90	125.63	-13.44	6.61	16.00	55.18
-75	17.78	124.43	-11.67	9.85	17.42	53.06
-70	13.20	122.16	-10.05	13.02	18.53	50.71
-65	8.29	118.82	-8.60	16.09	19.30	48.14
-60	3.17	114.43	-7.30	19.04	19.71	45.36
-55	-2.05	109.01	-6.14	21.84	19.75	42.37
-50	-7.26	102.60	-5.11	24.47	19.42	39.18
-45	-12.34	95.27	-4.21	26.92	18.72	35.81
-40	-17.20	87.05	-3.43	29.16	17.66	32.28
-35	-21.72	78.03	-2.76	31.19	16.27	28.59
-30	-25.84	68.29	-2.18	32.97	14.57	24.77
-25	-29.45	57.92	-1.69	34.50	12.59	20.83
-20	-32.51	47.00	-1.27	35.77	10.37	16.78
-15	-34.95	35.64	-0.90	36.77	7.95	12.66
-10	-36.72	23.95	-0.58	37.49	5.39	8.47
-5	-37.80	12.03	-0.28	37.93	2.72	4.25
0	-38.16	0.00	0.00	38.07	0.00	0.00
5	-37.80	-12.03	0.28	37.93	-2.72	-4.25
10	-36.72	-23.95	0.58	37.49	-5.39	-8.47
15	-34.95	-35.64	0.90	36.77	-7.95	-12.66
20	-32.51	-47.00	1.27	35.77	-10.37	-16.78
25	-29.45	-57.92	1.69	34.50	-12.59	-20.83
30	-25.84	-68.29	2.18	32.97	-14.57	-24.77
35	-21.72	-78.03	2.76	31.19	-16.27	-28.59
40	-17.20	-87.05	3.43	29.16	-17.66	-32.28
45	-12.34	-95.27	4.21	26.92	-18.72	-35.81
50	-7.26	-102.60	5.11	24.47	-19.42	-39.18
55	-2.05	-109.01	6.14	21.84	-19.75	-42.37
60	3.17	-114.43	7.30	19.04	-19.71	-45.36
65	8.29	-118.82	8.60	16.09	-19.30	-48.14
70	13.20	-122.16	10.05	13.02	-18.53	-50.71
75	17.78	-124.43	11.67	9.85	-17.42	-53.06
80	21.90	-125.63	13.44	6.61	-16.00	-55.18
85	25.45	-125.76	15.37	3.32	-14.31	-57.06
90	28.31	-124.86	17.47	0.00	-12.38	-58.71
95	30.37	-122.94	19.74	-3.32	-10.28	-60.12

100	31.54	-120.07	22.17	-6.61	-8.06	-61.29
105	31.70	-116.29	24.77	-9.85	-5.79	-62.24
110	30.79	-111.67	27.53	-13.02	-3.55	-62.95
115	28.71	-106.30	30.44	-16.09	-1.42	-63.43
120	25.40	-100.26	33.50	-19.04	0.52	-63.71
125	20.81	-93.65	36.71	-21.84	2.18	-63.77
130	14.90	-86.58	40.05	-24.47	3.46	-63.65
135	7.64	-79.16	43.52	-26.92	4.28	-63.34
140	-0.98	-71.52	47.11	-29.16	4.52	-62.86
145	-10.94	-63.77	50.81	-31.19	4.11	-62.23
150	-22.23	-56.04	54.60	-32.97	2.93	-61.47
155	-34.81	-48.48	58.47	-34.50	0.90	-60.58
160	-48.63	-41.19	62.41	-35.77	-2.08	-59.60
165	-63.61	-34.33	66.42	-36.77	-6.08	-58.53
170	-79.67	-28.02	70.46	-37.49	-11.20	-57.41
175	-96.70	-22.38	74.53	-37.93	-17.50	-56.24
180	-114.59	-17.54	78.62	-38.07	-25.04	-55.05

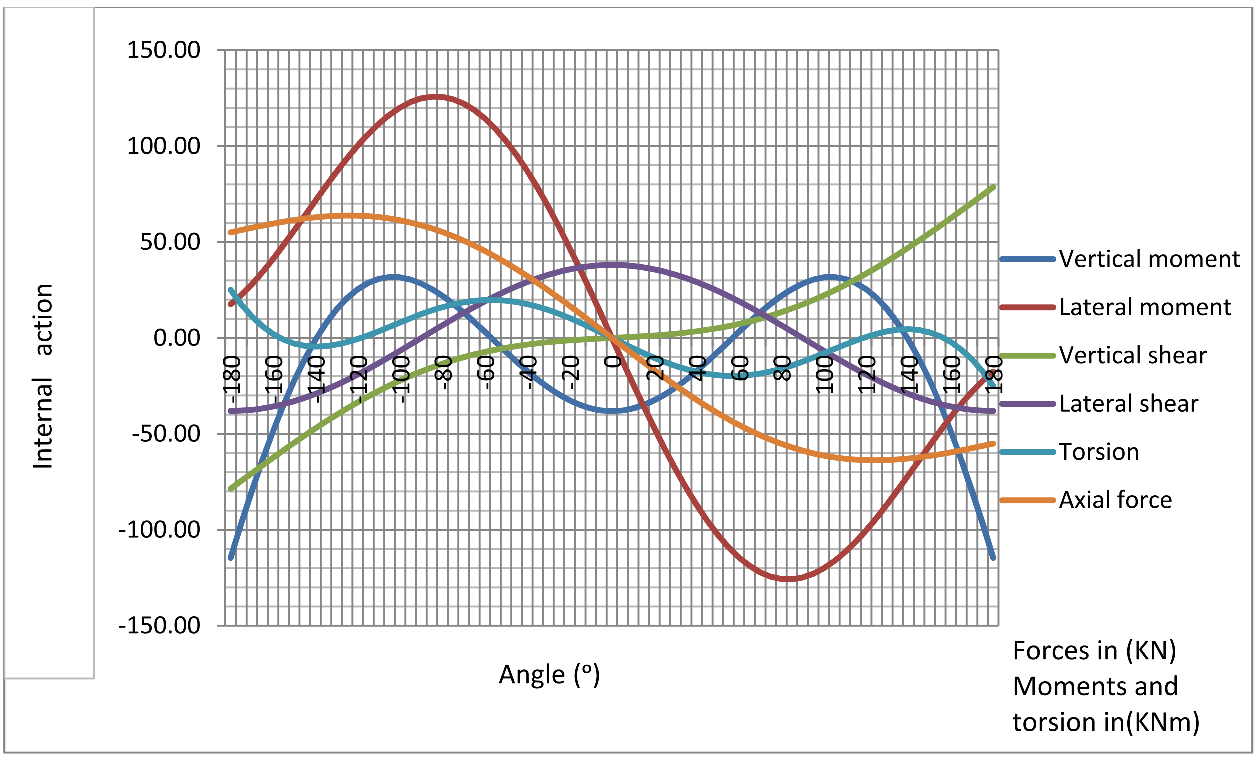

Figure A 16: Flight slope =35° & Radius =2.5m

Table A 17: Flight slope =40° & Radius =1.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-18.22	4.60	-29.41	-11.85	5.48	24.68
-175	-15.55	5.00	-27.93	-11.80	4.35	24.78
-170	-13.01	5.54	-26.45	-11.67	3.40	24.88
-165	-10.61	6.19	-24.99	-11.45	2.61	24.97
-160	-8.37	6.95	-23.54	-11.14	1.98	25.04
-155	-6.29	7.78	-22.11	-10.74	1.49	25.09
-150	-4.40	8.68	-20.70	-10.26	1.13	25.10
-145	-2.71	9.62	-19.32	-9.71	0.89	25.09
-140	-1.20	10.58	-17.98	-9.08	0.76	25.03
-135	0.10	11.55	-16.67	-8.38	0.73	24.93
-130	1.20	12.50	-15.40	-7.62	0.77	24.78
-125	2.11	13.41	-14.18	-6.80	0.88	24.57
-120	2.81	14.28	-13.01	-5.92	1.05	24.31
-115	3.33	15.07	-11.89	-5.01	1.26	23.99
-110	3.66	15.79	-10.81	-4.05	1.49	23.61
-105	3.83	16.40	-9.80	-3.07	1.74	23.16
-100	3.83	16.91	-8.84	-2.06	2.00	22.65
-95	3.68	17.30	-7.93	-1.03	2.25	22.07
-90	3.40	17.56	-7.09	0.00	2.49	21.42
-85	3.00	17.68	-6.30	1.03	2.70	20.70
-80	2.49	17.66	-5.57	2.06	2.89	19.91
-75	1.91	17.49	-4.90	3.07	3.03	19.05
-70	1.25	17.17	-4.28	4.05	3.14	18.13
-65	0.54	16.70	-3.72	5.01	3.20	17.14
-60	-0.20	16.09	-3.21	5.92	3.21	16.09
-55	-0.96	15.33	-2.75	6.80	3.17	14.98
-50	-1.71	14.44	-2.33	7.62	3.08	13.81
-45	-2.45	13.41	-1.97	8.38	2.94	12.59
-40	-3.16	12.26	-1.64	9.08	2.76	11.32
-35	-3.82	10.99	-1.35	9.71	2.52	10.00
-30	-4.42	9.62	-1.09	10.26	2.25	8.65
-25	-4.94	8.16	-0.87	10.74	1.93	7.26
-20	-5.39	6.62	-0.66	11.14	1.59	5.85
-15	-5.75	5.02	-0.48	11.45	1.21	4.41
-10	-6.01	3.38	-0.31	11.67	0.82	2.95
-5	-6.16	1.70	-0.15	11.80	0.41	1.48
0	-6.22	0.00	0.00	11.85	0.00	0.00
5	-6.16	-1.70	0.15	11.80	-0.41	-1.48
10	-6.01	-3.38	0.31	11.67	-0.82	-2.95
15	-5.75	-5.02	0.48	11.45	-1.21	-4.41
20	-5.39	-6.62	0.66	11.14	-1.59	-5.85
25	-4.94	-8.16	0.87	10.74	-1.93	-7.26
30	-4.42	-9.62	1.09	10.26	-2.25	-8.65
35	-3.82	-10.99	1.35	9.71	-2.52	-10.00
40	-3.16	-12.26	1.64	9.08	-2.76	-11.32
45	-2.45	-13.41	1.97	8.38	-2.94	-12.59
50	-1.71	-14.44	2.33	7.62	-3.08	-13.81
55	-0.96	-15.33	2.75	6.80	-3.17	-14.98
60	-0.20	-16.09	3.21	5.92	-3.21	-16.09
65	0.54	-16.70	3.72	5.01	-3.20	-17.14
70	1.25	-17.17	4.28	4.05	-3.14	-18.13
75	1.91	-17.49	4.90	3.07	-3.03	-19.05
80	2.49	-17.66	5.57	2.06	-2.89	-19.91
85	3.00	-17.68	6.30	1.03	-2.70	-20.70
90	3.40	-17.56	7.09	0.00	-2.49	-21.42

95	3.68	-17.30	7.93	-1.03	-2.25	-22.07
100	3.83	-16.91	8.84	-2.06	-2.00	-22.65
105	3.83	-16.40	9.80	-3.07	-1.74	-23.16
110	3.66	-15.79	10.81	-4.05	-1.49	-23.61
115	3.33	-15.07	11.89	-5.01	-1.26	-23.99
120	2.81	-14.28	13.01	-5.92	-1.05	-24.31
125	2.11	-13.41	14.18	-6.80	-0.88	-24.57
130	1.20	-12.50	15.40	-7.62	-0.77	-24.78
135	0.10	-11.55	16.67	-8.38	-0.73	-24.93
140	-1.20	-10.58	17.98	-9.08	-0.76	-25.03
145	-2.71	-9.62	19.32	-9.71	-0.89	-25.09
150	-4.40	-8.68	20.70	-10.26	-1.13	-25.10
155	-6.29	-7.78	22.11	-10.74	-1.49	-25.09
160	-8.37	-6.95	23.54	-11.14	-1.98	-25.04
165	-10.61	-6.19	24.99	-11.45	-2.61	-24.97
170	-13.01	-5.54	26.45	-11.67	-3.40	-24.88
175	-15.55	-5.00	27.93	-11.80	-4.35	-24.78
180	-18.22	-4.60	29.41	-11.85	-5.48	-24.68

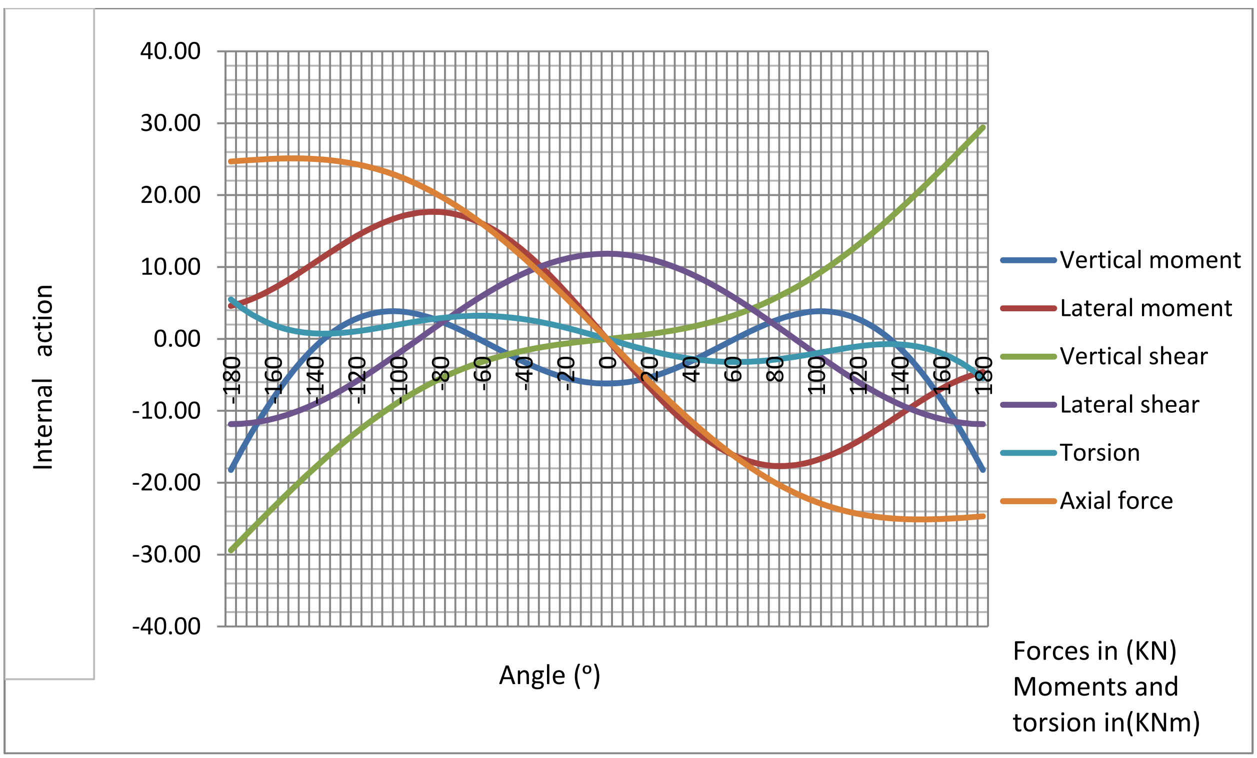

Figure A 17: Flight slope =40° & Radius =1.0m

Table A 18: Flight slope =40° & Radius =1.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-41.01	10.35	-44.11	-17.77	12.33	37.02
-175	-35.00	11.25	-41.89	-17.71	9.79	37.17
-170	-29.27	12.46	-39.68	-17.50	7.64	37.32
-165	-23.87	13.93	-37.48	-17.17	5.87	37.45
-160	-18.82	15.63	-35.30	-16.70	4.44	37.56
-155	-14.16	17.52	-33.16	-16.11	3.34	37.63
-150	-9.91	19.54	-31.05	-15.39	2.54	37.65
-145	-6.09	21.65	-28.98	-14.56	2.01	37.63
-140	-2.71	23.81	-26.97	-13.62	1.72	37.54
-135	0.22	25.98	-25.01	-12.57	1.64	37.39
-130	2.71	28.12	-23.11	-11.43	1.74	37.16
-125	4.74	30.18	-21.28	-10.20	1.99	36.86
-120	6.33	32.12	-19.51	-8.89	2.36	36.47
-115	7.49	33.91	-17.83	-7.51	2.83	35.99
-110	8.24	35.52	-16.22	-6.08	3.35	35.42
-105	8.61	36.91	-14.70	-4.60	3.92	34.74
-100	8.61	38.05	-13.26	-3.09	4.50	33.97
-95	8.28	38.92	-11.90	-1.55	5.06	33.10
-90	7.65	39.50	-10.63	0.00	5.60	32.12
-85	6.75	39.77	-9.45	1.55	6.08	31.04
-80	5.61	39.72	-8.35	3.09	6.49	29.86
-75	4.29	39.35	-7.34	4.60	6.83	28.58
-70	2.81	38.63	-6.42	6.08	7.06	27.19
-65	1.22	37.58	-5.57	7.51	7.20	25.71
-60	-0.45	36.20	-4.81	8.89	7.23	24.13
-55	-2.15	34.50	-4.12	10.20	7.14	22.46
-50	-3.86	32.48	-3.50	11.43	6.94	20.71
-45	-5.52	30.17	-2.95	12.57	6.62	18.88
-40	-7.11	27.58	-2.46	13.62	6.20	16.98
-35	-8.59	24.73	-2.02	14.56	5.68	15.01
-30	-9.94	21.64	-1.64	15.39	5.06	12.98
-25	-11.13	18.36	-1.30	16.11	4.35	10.90
-20	-12.13	14.90	-0.99	16.70	3.57	8.77
-15	-12.93	11.30	-0.72	17.17	2.73	6.61
-10	-13.51	7.59	-0.47	17.50	1.85	4.42
-5	-13.87	3.82	-0.23	17.71	0.93	2.21
0	-13.98	0.00	0.00	17.77	0.00	0.00
5	-13.87	-3.82	0.23	17.71	-0.93	-2.21
10	-13.51	-7.59	0.47	17.50	-1.85	-4.42
15	-12.93	-11.30	0.72	17.17	-2.73	-6.61
20	-12.13	-14.90	0.99	16.70	-3.57	-8.77
25	-11.13	-18.36	1.30	16.11	-4.35	-10.90
30	-9.94	-21.64	1.64	15.39	-5.06	-12.98
35	-8.59	-24.73	2.02	14.56	-5.68	-15.01
40	-7.11	-27.58	2.46	13.62	-6.20	-16.98
45	-5.52	-30.17	2.95	12.57	-6.62	-18.88
50	-3.86	-32.48	3.50	11.43	-6.94	-20.71
55	-2.15	-34.50	4.12	10.20	-7.14	-22.46
60	-0.45	-36.20	4.81	8.89	-7.23	-24.13
65	1.22	-37.58	5.57	7.51	-7.20	-25.71
70	2.81	-38.63	6.42	6.08	-7.06	-27.19
75	4.29	-39.35	7.34	4.60	-6.83	-28.58
80	5.61	-39.72	8.35	3.09	-6.49	-29.86
85	6.75	-39.77	9.45	1.55	-6.08	-31.04
90	7.65	-39.50	10.63	0.00	-5.60	-32.12

95	8.28	-38.92	11.90	-1.55	-5.06	-33.10
100	8.61	-38.05	13.26	-3.09	-4.50	-33.97
105	8.61	-36.91	14.70	-4.60	-3.92	-34.74
110	8.24	-35.52	16.22	-6.08	-3.35	-35.42
115	7.49	-33.91	17.83	-7.51	-2.83	-35.99
120	6.33	-32.12	19.51	-8.89	-2.36	-36.47
125	4.74	-30.18	21.28	-10.20	-1.99	-36.86
130	2.71	-28.12	23.11	-11.43	-1.74	-37.16
135	0.22	-25.98	25.01	-12.57	-1.64	-37.39
140	-2.71	-23.81	26.97	-13.62	-1.72	-37.54
145	-6.09	-21.65	28.98	-14.56	-2.01	-37.63
150	-9.91	-19.54	31.05	-15.39	-2.54	-37.65
155	-14.16	-17.52	33.16	-16.11	-3.34	-37.63
160	-18.82	-15.63	35.30	-16.70	-4.44	-37.56
165	-23.87	-13.93	37.48	-17.17	-5.87	-37.45
170	-29.27	-12.46	39.68	-17.50	-7.64	-37.32
175	-35.00	-11.25	41.89	-17.71	-9.79	-37.17
180	-41.01	-10.35	44.11	-17.77	-12.33	-37.02

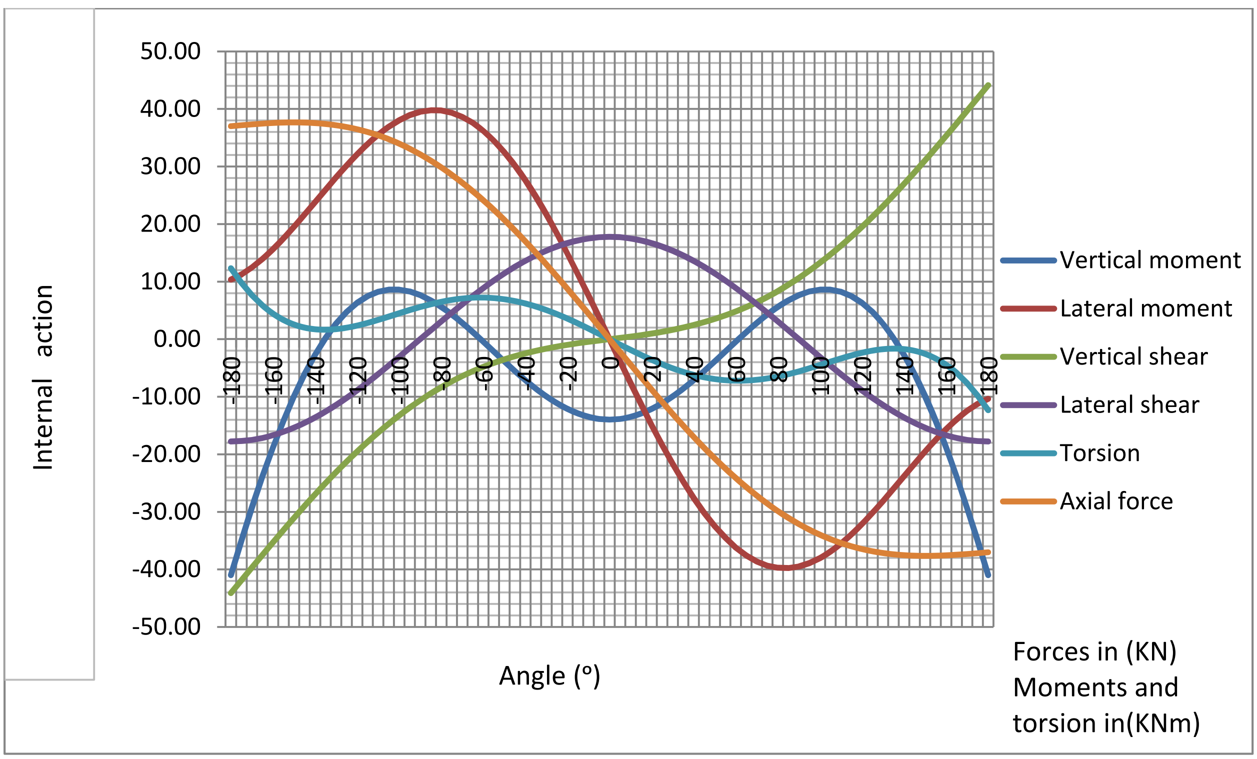

Figure A 18: Flight slope =40° & Radius =1.5m

Table A 19: Flight slope =40° & Radius =2.0m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-72.90	18.39	-58.82	-23.70	21.92	49.35
-175	-62.22	20.00	-55.86	-23.61	17.41	49.56
-170	-52.04	22.15	-52.90	-23.34	13.59	49.76
-165	-42.44	24.77	-49.97	-22.89	10.43	49.94
-160	-33.46	27.79	-47.07	-22.27	7.90	50.08
-155	-25.18	31.14	-44.21	-21.48	5.95	50.17
-150	-17.62	34.73	-41.40	-20.52	4.52	50.21
-145	-10.82	38.49	-38.64	-19.41	3.57	50.17
-140	-4.81	42.34	-35.95	-18.15	3.05	50.06
-135	0.40	46.19	-33.34	-16.76	2.91	49.85
-130	4.81	49.99	-30.81	-15.23	3.09	49.55
-125	8.42	53.65	-28.37	-13.59	3.54	49.14
-120	11.25	57.11	-26.02	-11.85	4.20	48.62
-115	13.32	60.29	-23.77	-10.02	5.02	47.99
-110	14.66	63.15	-21.63	-8.11	5.96	47.22
-105	15.31	65.61	-19.60	-6.13	6.97	46.33
-100	15.31	67.64	-17.67	-4.12	7.99	45.30
-95	14.72	69.19	-15.87	-2.07	9.00	44.13
-90	13.59	70.22	-14.18	0.00	9.95	42.83
-85	11.99	70.71	-12.60	2.07	10.81	41.39
-80	9.98	70.62	-11.14	4.12	11.55	39.81
-75	7.62	69.95	-9.79	6.13	12.14	38.10
-70	5.00	68.68	-8.56	8.11	12.56	36.25
-65	2.16	66.82	-7.43	10.02	12.80	34.28
-60	-0.80	64.36	-6.41	11.85	12.84	32.17
-55	-3.83	61.33	-5.49	13.59	12.69	29.95
-50	-6.85	57.75	-4.67	15.23	12.33	27.62
-45	-9.81	53.63	-3.93	16.76	11.78	25.18
-40	-12.63	49.02	-3.28	18.15	11.02	22.64
-35	-15.27	43.96	-2.70	19.41	10.09	20.01
-30	-17.67	38.48	-2.19	20.52	8.99	17.30
-25	-19.78	32.64	-1.73	21.48	7.73	14.53
-20	-21.56	26.49	-1.33	22.27	6.35	11.69
-15	-22.99	20.09	-0.96	22.89	4.86	8.81
-10	-24.02	13.50	-0.62	23.34	3.29	5.89
-5	-24.65	6.78	-0.31	23.61	1.66	2.95
0	-24.86	0.00	0.00	23.70	0.00	0.00
5	-24.65	-6.78	0.31	23.61	-1.66	-2.95
10	-24.02	-13.50	0.62	23.34	-3.29	-5.89
15	-22.99	-20.09	0.96	22.89	-4.86	-8.81
20	-21.56	-26.49	1.33	22.27	-6.35	-11.69
25	-19.78	-32.64	1.73	21.48	-7.73	-14.53
30	-17.67	-38.48	2.19	20.52	-8.99	-17.30
35	-15.27	-43.96	2.70	19.41	-10.09	-20.01
40	-12.63	-49.02	3.28	18.15	-11.02	-22.64
45	-9.81	-53.63	3.93	16.76	-11.78	-25.18
50	-6.85	-57.75	4.67	15.23	-12.33	-27.62
55	-3.83	-61.33	5.49	13.59	-12.69	-29.95
60	-0.80	-64.36	6.41	11.85	-12.84	-32.17
65	2.16	-66.82	7.43	10.02	-12.80	-34.28
70	5.00	-68.68	8.56	8.11	-12.56	-36.25

75	7.62	-69.95	9.79	6.13	-12.14	-38.10
80	9.98	-70.62	11.14	4.12	-11.55	-39.81
85	11.99	-70.71	12.60	2.07	-10.81	-41.39
90	13.59	-70.22	14.18	0.00	-9.95	-42.83
95	14.72	-69.19	15.87	-2.07	-9.00	-44.13
100	15.31	-67.64	17.67	-4.12	-7.99	-45.30
105	15.31	-65.61	19.60	-6.13	-6.97	-46.33
110	14.66	-63.15	21.63	-8.11	-5.96	-47.22
115	13.32	-60.29	23.77	-10.02	-5.02	-47.99
120	11.25	-57.11	26.02	-11.85	-4.20	-48.62
125	8.42	-53.65	28.37	-13.59	-3.54	-49.14
130	4.81	-49.99	30.81	-15.23	-3.09	-49.55
135	0.40	-46.19	33.34	-16.76	-2.91	-49.85
140	-4.81	-42.34	35.95	-18.15	-3.05	-50.06
145	-10.82	-38.49	38.64	-19.41	-3.57	-50.17
150	-17.62	-34.73	41.40	-20.52	-4.52	-50.21
155	-25.18	-31.14	44.21	-21.48	-5.95	-50.17
160	-33.46	-27.79	47.07	-22.27	-7.90	-50.08
165	-42.44	-24.77	49.97	-22.89	-10.43	-49.94
170	-52.04	-22.15	52.90	-23.34	-13.59	-49.76
175	-62.22	-20.00	55.86	-23.61	-17.41	-49.56
180	-72.90	-18.39	58.82	-23.70	-21.92	-49.35

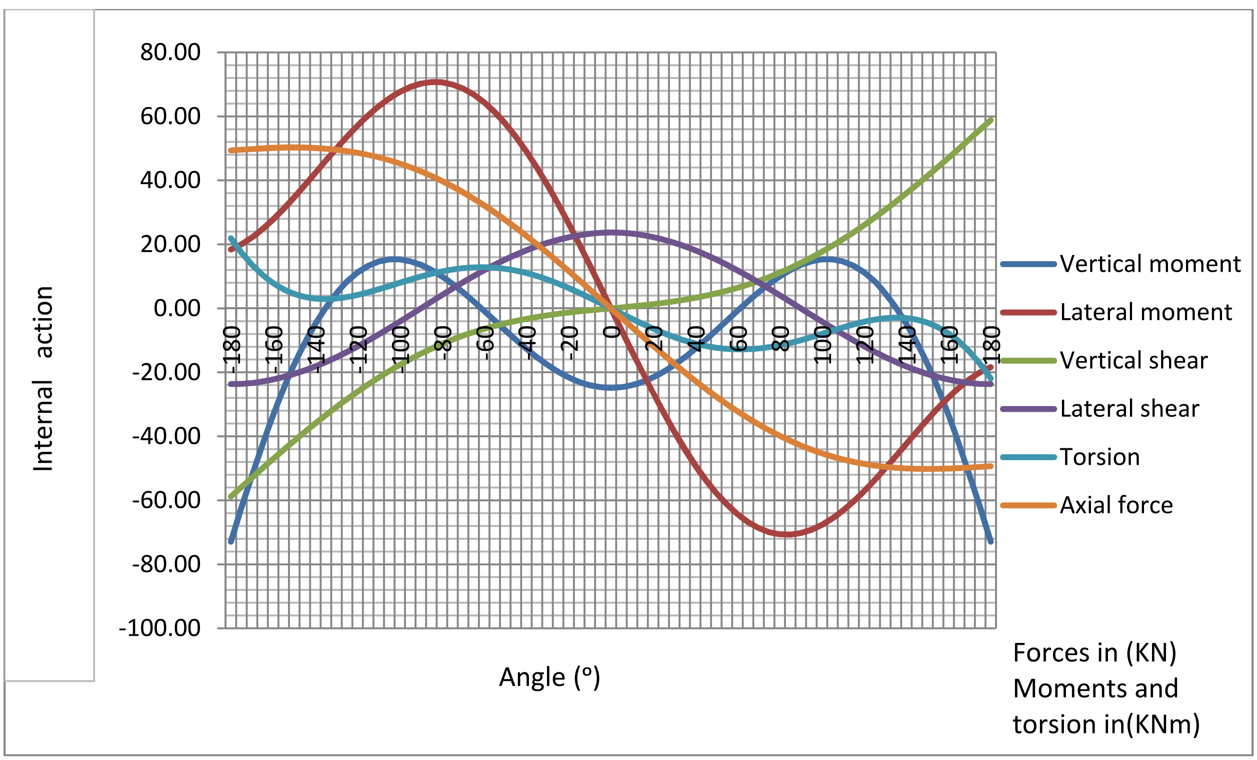

Figure A 19: Flight slope =40° & Radius =2.0m

Table A 20: Flight slope =40° & Radius =2.5m

Angle (°)	Mv (KNm)	Mh (KNm)	Vv (KN)	Vh (KN)	T (KNm)	N(KN)
-180	-113.90	28.74	-73.52	-29.62	34.25	61.69
-175	-97.22	31.25	-69.82	-29.51	27.20	61.96
-170	-81.32	34.60	-66.13	-29.17	21.23	62.21
-165	-66.31	38.70	-62.47	-28.61	16.30	62.42
-160	-52.29	43.43	-58.84	-27.84	12.35	62.60
-155	-39.34	48.65	-55.26	-26.85	9.29	62.71
-150	-27.53	54.26	-51.75	-25.66	7.06	62.76
-145	-16.91	60.14	-48.30	-24.27	5.58	62.71
-140	-7.52	66.15	-44.94	-22.69	4.77	62.57
-135	0.62	72.18	-41.68	-20.95	4.55	62.32
-130	7.51	78.11	-38.51	-19.04	4.83	61.94
-125	13.16	83.83	-35.46	-16.99	5.53	61.43
-120	17.58	89.23	-32.52	-14.81	6.56	60.78
-115	20.81	94.21	-29.71	-12.52	7.85	59.98
-110	22.90	98.67	-27.04	-10.13	9.32	59.03
-105	23.92	102.52	-24.49	-7.67	10.89	57.91
-100	23.92	105.69	-22.09	-5.14	12.49	56.62
-95	23.00	108.11	-19.83	-2.58	14.07	55.17
-90	21.24	109.73	-17.72	0.00	15.55	53.54
-85	18.74	110.48	-15.75	2.58	16.89	51.74
-80	15.59	110.35	-13.92	5.14	18.04	49.77
-75	11.91	109.29	-12.24	7.67	18.96	47.63
-70	7.80	107.31	-10.70	10.13	19.62	45.32
-65	3.38	104.40	-9.29	12.52	20.00	42.84
-60	-1.25	100.57	-8.02	14.81	20.07	40.22
-55	-5.98	95.83	-6.87	16.99	19.83	37.44
-50	-10.71	90.23	-5.84	19.04	19.27	34.52
-45	-15.33	83.80	-4.92	20.95	18.40	31.47
-40	-19.74	76.60	-4.10	22.69	17.23	28.30
-35	-23.86	68.68	-3.37	24.27	15.77	25.01
-30	-27.61	60.12	-2.73	25.66	14.04	21.63
-25	-30.90	51.00	-2.16	26.85	12.09	18.16
-20	-33.69	41.39	-1.66	27.84	9.92	14.62
-15	-35.91	31.39	-1.20	28.61	7.59	11.01
-10	-37.53	21.10	-0.78	29.17	5.14	7.37
-5	-38.52	10.60	-0.38	29.51	2.59	3.69
0	-38.85	0.00	0.00	29.62	0.00	0.00
5	-38.52	-10.60	0.38	29.51	-2.59	-3.69
10	-37.53	-21.10	0.78	29.17	-5.14	-7.37
15	-35.91	-31.39	1.20	28.61	-7.59	-11.01
20	-33.69	-41.39	1.66	27.84	-9.92	-14.62
25	-30.90	-51.00	2.16	26.85	-12.09	-18.16
30	-27.61	-60.12	2.73	25.66	-14.04	-21.63
35	-23.86	-68.68	3.37	24.27	-15.77	-25.01
40	-19.74	-76.60	4.10	22.69	-17.23	-28.30
45	-15.33	-83.80	4.92	20.95	-18.40	-31.47
50	-10.71	-90.23	5.84	19.04	-19.27	-34.52
55	-5.98	-95.83	6.87	16.99	-19.83	-37.44
60	-1.25	-100.57	8.02	14.81	-20.07	-40.22
65	3.38	-104.40	9.29	12.52	-20.00	-42.84
70	7.80	-107.31	10.70	10.13	-19.62	-45.32
75	11.91	-109.29	12.24	7.67	-18.96	-47.63
80	15.59	-110.35	13.92	5.14	-18.04	-49.77
85	18.74	-110.48	15.75	2.58	-16.89	-51.74
90	21.24	-109.73	17.72	0.00	-15.55	-53.54

95	23.00	-108.11	19.83	-2.58	-14.07	-55.17
100	23.92	-105.69	22.09	-5.14	-12.49	-56.62
105	23.92	-102.52	24.49	-7.67	-10.89	-57.91
110	22.90	-98.67	27.04	-10.13	-9.32	-59.03
115	20.81	-94.21	29.71	-12.52	-7.85	-59.98
120	17.58	-89.23	32.52	-14.81	-6.56	-60.78
125	13.16	-83.83	35.46	-16.99	-5.53	-61.43
130	7.51	-78.11	38.51	-19.04	-4.83	-61.94
135	0.62	-72.18	41.68	-20.95	-4.55	-62.32
140	-7.52	-66.15	44.94	-22.69	-4.77	-62.57
145	-16.91	-60.14	48.30	-24.27	-5.58	-62.71
150	-27.53	-54.26	51.75	-25.66	-7.06	-62.76
155	-39.34	-48.65	55.26	-26.85	-9.29	-62.71
160	-52.29	-43.43	58.84	-27.84	-12.35	-62.60
165	-66.31	-38.70	62.47	-28.61	-16.30	-62.42
170	-81.32	-34.60	66.13	-29.17	-21.23	-62.21
175	-97.22	-31.25	69.82	-29.51	-27.20	-61.96
180	-113.90	-28.74	73.52	-29.62	-34.25	-61.69

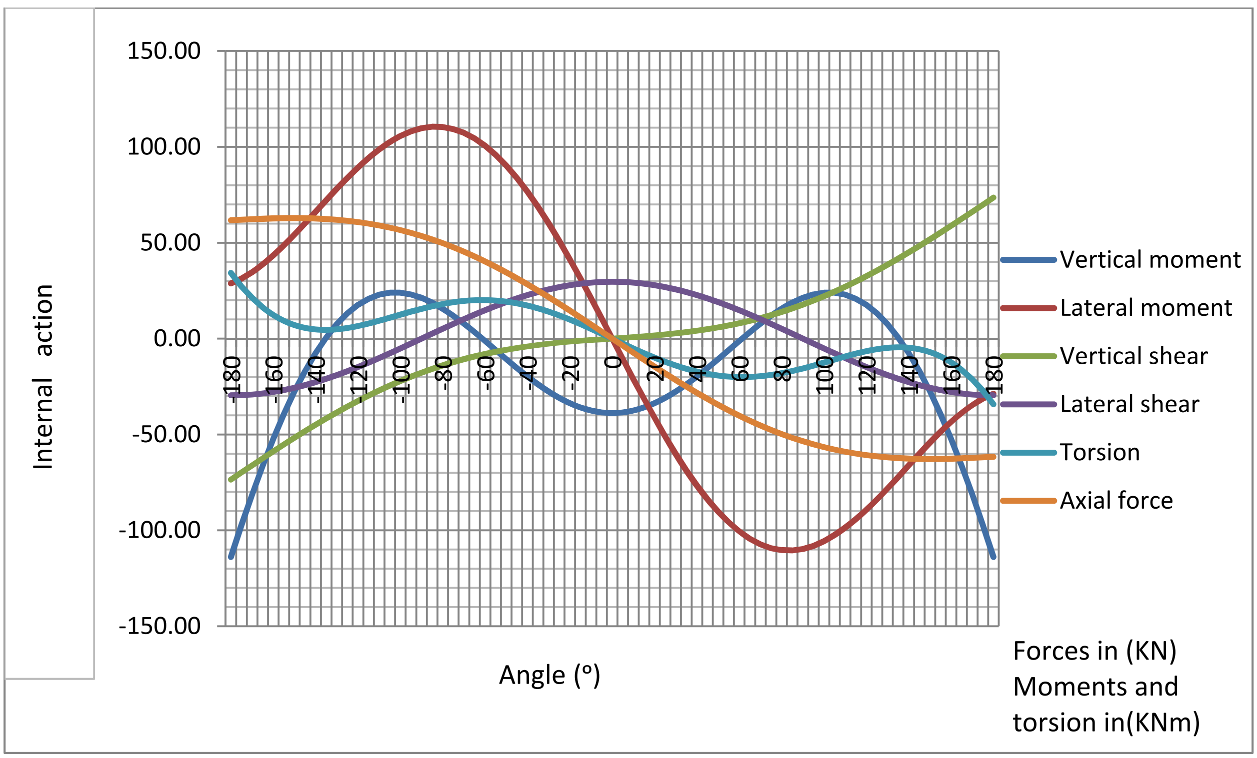

Figure A 20: Flight slope =40° & Radius =2.5m

ANNEX B:

Explicit mathematical derivation based on strain energy

The brief explanation of the steps and derivation concept is available in chapter 3.2.1. In this section, explicit mathematical formulation using strain energy and Castiglione's theorem is presented.

Case 1

$$\frac{\partial U}{\partial H} = 0$$ ……………………………………………………………………………………………….*Equation 3.2.1.1*

$$\frac{\partial U}{\partial H} = \int_0^{\theta} Mv\frac{\partial Mv}{\partial H}ds + \tfrac{1}{2}\int_0^{\theta} T\frac{\partial T}{\partial H}ds = 0,$$
$$where, ds = R\sec\alpha d\theta$$

Equation 3.2.1.1 = Equation 3.2.1.1A + Equation 3.2.1.1B

$$Mv = M\cos\theta + HR\ \theta\tan\alpha\sin\theta - WR^2(1-\cos\theta)$$

$$T = \left(M\sin\theta - HR\theta\tan\alpha\cos\theta + WR^2\sin\theta - WR^2\theta\right)\cos\alpha + HR\sin\theta\sin\alpha$$

$$\int_0^{\theta} Mv\frac{\partial Mv}{\partial H}ds = \int_0^{\theta} Mv\frac{\partial Mv}{\partial H}R\sec\alpha d\theta$$
……………………………………………………………………………………………………….*Equation 3.2.1.1A*

$$\frac{\partial Mv}{\partial H} = \frac{\partial}{\partial H}\left(M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2(1-\cos\theta)\right)$$

$$\frac{\partial Mv}{\partial H} = \frac{\partial}{\partial H}\left(M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2(1-\cos\theta\ )\right) = R\theta\tan\alpha\sin\theta$$

$$Mv = M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2 + WR^2\cos\theta$$

$$A = \int_0^{\theta} Mv\frac{\partial Mv}{\partial H}R\sec\alpha d\theta$$

$$= \int_0^{\theta}\left(M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2 + WR^2\cos\theta\right).\left(R\theta\tan\alpha\sin\theta\right).R\sec\alpha d\theta$$

$$= \int_0^{\theta}(M\cos\theta)(R\theta\tan\alpha\sec\theta)R\sec\alpha d\theta + \int_0^{\theta}(HR\theta\tan\alpha\sin\theta).(R\theta\tan\alpha\sin\theta)R\sec\alpha\ d\theta -$$
$$\int_0^{\theta} WR^2(R\theta\tan\alpha\sin\theta)R\sec\alpha d\theta + \int_0^{\theta} WR^2\cos\theta.(R\theta\tan\alpha\sin\theta).R\sec\alpha d\theta$$

$$=\int_0^\theta\left(\frac{\theta\sin 2\theta}{2}\right)MR^2\tan\alpha\sec\alpha d\theta+\int_0^\theta\left(\theta^2\sin^2\theta\right).HR^3\tan\alpha\sec\alpha d\theta-$$

$$\int_0^\theta\theta\sin\theta WR^4\tan\alpha\sec ad\theta+\int_0^\theta\left(\frac{\theta\sin 2\theta}{2}\right)WR^4\tan\alpha\sec ad\theta$$

$$=\left(\frac{-\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)MR^2\tan\alpha\sec\alpha+\left(\frac{\theta^3}{6}-\frac{\theta^2\sin\theta}{4}-\frac{\theta\cos 2\theta}{4}\frac{\sin 2\theta}{8}\right)\left(HR^3\tan\alpha\sec\alpha\right)-$$

$$\left(-\theta\cos\theta+\sin\theta\right)WR^4\tan\alpha\sec\alpha+\left(-\theta\cos 2\theta+\sin 2\theta\right)WR^4\tan\alpha\sec\alpha$$

$$=M\left(-\theta\cos 2\theta+\sin 2\theta\right)R^2\tan\alpha\sec\alpha+\left(\frac{\theta^3}{6}-\frac{\theta^2\sin 2\theta}{4}-\frac{\cos 2\theta}{4}\frac{\sin\theta}{8}\right)HR^3\tan^2\alpha\sec\alpha+$$

$$\left(\theta\cos\theta-\sin\theta-\theta\frac{\cos 2\theta}{4}-\frac{\sin 2\theta}{8}\right)WR^4\tan\alpha\sec\alpha$$

$$\left\{\begin{array}{l}A=M\left(-\theta\frac{\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)R^2\tan\alpha\sec\alpha+HR^3\tan^2\alpha\sec\alpha\left(\frac{\theta^3}{6}-\frac{\theta^2\sin 2\theta}{4}-\frac{\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)+\\ WR^4\tan\alpha\sec\alpha\left(\theta\cos\theta-\sin\theta-\frac{\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)\end{array}\right\}$$

....(3.2.1.1A)

$$\tfrac{1}{2}\int_0^\theta T\frac{\partial T}{\partial H}ds=\tfrac{1}{2}\int_0^\theta T\frac{\partial T}{\partial H}R\sec ad\theta$$

....................................*Equation 3.2.1.1 B*

$$T=\left(M\sin\theta-HR\theta\tan\alpha\cos\theta+WR^2\sin\theta-WR^2\theta\right)\cos\alpha+HR\sin\theta\sin\alpha$$

$$\frac{\partial T}{\partial H}=-R\theta\ \tan\alpha\cos\theta\cos\alpha+R\sin\theta\cos\alpha\sin\alpha+R\ \sin\theta\sin\alpha$$

$$B=\tfrac{1}{2}\int_0^\theta T\frac{\partial T}{\partial H}R\sec\alpha\, d\theta$$

$$=\tfrac{1}{2}\int_0^\theta\left(M\sin\theta\cos\alpha-HR\sin\alpha\cos\theta+WR^2\sin\theta\cos\alpha-WR^2\theta\cos\alpha+HR\sin\theta\sin\alpha\right).$$

$$\left(-R\theta\sin\alpha\cos\theta+R\sin\theta\sin\alpha\right).R\sec\alpha d\theta$$

- *Equation B=B1+B2+B3+B4+B5*

$$\begin{pmatrix} B1 = \frac{1}{2}\int_0^{\theta} -(M\sin\theta\cos\alpha)\, R\theta\sin\alpha\cos\theta . R\sec\alpha\; d\theta + \frac{1}{2}\int_0^{\theta}(M\sin\theta\cos\alpha)\, R\sin\theta\sin\alpha . R\sec\alpha\; d\theta \\ B2 = +\frac{1}{2}\int_0^{\theta} (H\; R\theta\sin\alpha\;\cos\theta) . R\theta\sin\alpha\cos\theta . R\sec\alpha d\theta - \\ \frac{1}{2}\int_0^{\theta}(HR\theta\sin\alpha\cos\theta)\, R\sin\theta\sin\alpha\cos\alpha\; R\sec\alpha d\theta \\ B3 = \frac{1}{2}\int_0^{\theta} -(WR^2\sin\theta\cos\alpha) R\theta\sin\alpha\cos\;\theta . R\sec\alpha d\theta + \int_0^{\theta}\frac{1}{2}(WR^2\sin\theta\cos\alpha) R\sin\theta\sin\alpha R\sec\alpha\; d\theta \\ B4 = +\frac{1}{2}\int_0^{\theta}(WR^2\theta\cos\alpha) R\theta\sin\alpha\cos\;\theta . R\sec\alpha d\theta - \frac{1}{2}\int_0^{\theta}(WR^2\theta\cos\alpha) R\sin\theta\sin\alpha R\sec\alpha d\theta \\ B5 = \frac{1}{2}\int_0^{\theta}(HR\sin\theta\sin\alpha) R\theta\sin\alpha\cos\theta . R\sec\alpha d\theta + \frac{1}{2}\int_0^{\theta}(HR\sin\theta\sin\alpha) . R\sin\theta\sin\alpha R\sec\alpha d\theta \end{pmatrix}$$

**Further mathematical modification on B1 to B5*

$$\begin{pmatrix} B1 = M\left\{(\cos\alpha R^2\sin\alpha\sec\alpha\;)\left(\frac{\cos 2\theta}{8} - \frac{\sin 2\theta}{16}\right) + \cos\alpha R^2\sin\alpha\sec\alpha\left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right)\;\right\} \\ B2 = H\left\{(R^3\sin^2\alpha\sec\alpha\;) . \left(\frac{\theta^3}{12} + \theta^2\frac{\sin 2\theta}{8} + \frac{\theta\cos 2\theta}{8} - \frac{\sin 2\theta}{16}\right) + (R^3) . \sin^2\alpha\sec\alpha . \left(\frac{\theta\cos 2\theta}{8} - \frac{\sin 2\theta}{16}\right)\;\right\} \\ B3 = WR^4\left\{\sin\alpha . \left(\frac{\theta\cos 2\theta}{2} - \frac{\sin 2\theta}{16}\right) + \sin\alpha . \left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right)\right\} \\ B4 = WR^4\left\{\sin\alpha . \left(\frac{\theta^2\sin\theta}{2} + \frac{2\theta\cos\theta}{2} - \frac{2\sin\theta}{2}\right) + \sin\alpha . \left(\frac{\theta\cos\theta}{2} - \frac{\sin\theta}{2}\right)\right\} \\ B5 = H . \left\{R^3\sin^2\alpha\sec\alpha\left(\frac{-\theta\cos 2\theta}{8} + \frac{\sin 2\theta}{16}\right) + R^3\sin^2\alpha\sec\alpha\left(\frac{\theta}{4} - \frac{\sin 2\theta}{8}\right)\right\} \end{pmatrix}$$

Case 2

$$\frac{\partial U}{\partial M} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\textit{Equation 3.2.1.2}$$

$$\frac{\partial U}{\partial M} = \int_0^{\theta} Mv\frac{\partial Mv}{\partial M}ds + \tfrac{1}{2}\int_0^{\theta} T\frac{\partial T}{\partial M}ds = 0, \ldots\ldots\ldots\ldots\ldots\ldots Equation - 3.2.1.2C$$

$$where, ds = R\sec a d\theta$$

Equation 3.2.1.2= Equation 3.2.1.2C+ Equation 3.2.1.2D

$$Mv = M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2(1-\cos\theta)$$

$$T = ((M\sin\theta - HR\theta\tan\alpha\cos\theta + WR^2\sin\theta - WR^2\theta\ )) + HR\sin\theta\sin\alpha$$

$$C = \int_0^{\theta} Mv\frac{\partial Mv}{\partial M}R\sec a d\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Equation\ 3.2.1.2\ C$$

$$\Rightarrow \frac{\partial Mv}{\partial M} = \cos\theta$$

$$C = \int_0^{\theta} (M\cos\theta + HR\theta\tan\alpha\sin\theta - WR^2 + WR^2\cos\theta).\cos\theta.R.\sec\alpha.d\theta\ .$$

$$= \int_0^{\theta} (\ M\cos^2\theta + HR\theta\tan\alpha\sin\theta\cos\theta - WR^2\cos\theta + WR^2\cos^2\theta\ ).R\sec\alpha d\theta$$

$$= \int_0^{\theta}\left(M\cos^2\theta R\sec\alpha d\theta\right) + \int_0^{\theta} HR\theta\tan\alpha\sin\theta\cos\alpha R\sec\alpha d\theta - \int_0^{\theta}\left(WR^2\cos\theta R\sec\alpha d\theta\right) +$$
$$\int_0^{\theta} WR^2\cos^2\theta R\sec\alpha d\theta$$

$$= \int_0^{\theta}\cos^2\theta(MR\sec\alpha)d\theta + \int_0^{\theta}\left(\frac{\theta\sin 2\theta}{2}\right)HR^2\tan\alpha\sec\alpha d\theta - \int_0^{\theta}(\cos\theta).WR^3\sec\alpha d\theta +$$
$$\left(\frac{\cos 2\theta}{2}\right)WR^3\sec\alpha d\theta$$

$$=\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)\left(MR\sec\alpha+\left(\left(\theta\frac{\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)HR^{2}\tan\alpha\sec\alpha-\sin\theta WR^{3}\sec\alpha+\right.\right.$$
$$\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)WR^{3}\sec\alpha$$

$$\begin{cases} C = M\left\{R\sec\alpha\left(\frac{\theta}{2}+\frac{\sin\theta}{4}\right)\right\}+H\left\{R^{2}\tan\alpha\sec\alpha\left(\frac{-\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)\right\} \\ WR^{3}\left\{-\sin\theta\sec\alpha+\sec\alpha\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)\right\} \end{cases}$$

$$D=\tfrac{1}{2}\int_{0}^{\theta}T\frac{\partial T}{\partial M}R\sec\alpha d\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Equation\ 3.2.1.2D$$

$$\frac{\partial T}{\partial M}=\sin\theta\cos\alpha$$

$$D=\tfrac{1}{2}\int_{0}^{\theta}T\frac{\partial T}{\partial M}R\sec\alpha d\theta=\tfrac{1}{2}\int_{0}^{\theta}\left(\left(M\sin\theta\cos\theta-HR\theta\tan\alpha\cos\theta\cos\alpha+WR^{2}\sin\theta\cos\alpha+\right.\right.$$
$$R^{2}\theta\cos\alpha+HR\sin\theta\sin\alpha\ \big)(\sin\theta\cos\alpha)R\sec\alpha d\theta$$

$$=\tfrac{1}{2}\int_{0}^{\theta}(M\sin\theta\cos\alpha)(\sin\theta\cos\alpha)R\sec\alpha d\theta-\tfrac{1}{2}\int_{0}^{\theta}(HR\theta\tan\alpha\cos\theta\cos\alpha)\sin\theta\cos\theta R\sec\alpha d\theta+$$
$$\tfrac{1}{2}\int_{0}^{\theta}(WR^{2}\sin\theta\cos\alpha)\sin\theta\cos\alpha R\sec\alpha d\theta-\tfrac{1}{2}\int_{0}^{\theta}(WR^{2}\theta\cos\alpha)(\sin\theta\cos\alpha)R\sec\alpha d\theta+$$
$$\tfrac{1}{2}\int_{0}^{\theta}(HR\sin\theta\sin\alpha)(\sin\theta\cos\alpha)R\sec\alpha d\theta$$

$$=\tfrac{1}{2}\int_{0}^{\theta}(\sin^{2}\theta)M\cos^{2}\alpha R\sec\alpha d\theta-\tfrac{1}{2}\int_{0}^{\theta}\left(\frac{\theta\sin 2\theta}{2}\right)HR\sin\alpha Rd\theta+\tfrac{1}{2}\int_{0}^{\theta}(\sin 2\theta)WR^{2}\cos\alpha Rd\theta-$$
$$\tfrac{1}{2}\int_{0}^{\theta}(\theta\sin\theta)WR^{2}\cos 2\sec\alpha d\theta-\tfrac{1}{2}\int_{0}^{\theta}(\sin 2\theta)HR^{2}\sin\alpha.\cos 2\alpha.\sec\alpha.d\theta$$

$$\begin{Bmatrix} D = \tfrac{1}{2}\left(\dfrac{\theta}{2} - \dfrac{\sin 2\theta}{4}\right) M \cos^2 \alpha R \sec \alpha - \tfrac{1}{2}\left(\dfrac{-\theta \cos 2\theta}{4} + \dfrac{\sin 2\theta}{8}\right) HR \sin \alpha + \\ \tfrac{1}{2}\left(\dfrac{\theta}{2} - \dfrac{\sin 2\theta}{4}\right) WR^2 \cos \alpha - \tfrac{1}{2}\left(-\theta \cos \theta + \sin \theta\right) WR^3 \cos \alpha + \tfrac{1}{2}\left(\dfrac{\theta}{2} - \dfrac{\sin 2\theta}{4}\right) HR^2 \sin \alpha \end{Bmatrix}$$

*Summary of expressions for A, B, C and D

$$\begin{Bmatrix} A = M\left(\dfrac{-\theta\cos 2\theta}{4} + \dfrac{\sin 2\theta}{8}\right) R^2 \tan\alpha \sec\alpha + H.R^3 \tan^2\alpha \sec\alpha\left(\dfrac{\theta^3}{6} - \dfrac{\theta^2 \sin 2\theta}{4} - \dfrac{-\theta\cos 2\theta}{4} + \dfrac{\sin 2\theta}{8}\right) + \\ WR^4 \tan\alpha \sec\alpha\left(\theta\cos\theta - \sin\theta - \dfrac{\theta\cos 2\theta}{4} + \dfrac{\sin 2\theta}{8}\right) \\ B1 = M\left(\cos\alpha R^2 \sin\alpha \sec\alpha\right)\left(\dfrac{\theta\cos 2\theta}{8} - \dfrac{\sin 2\theta}{16}\right) + \cos^2\alpha R^2 . \sin\alpha \sec\alpha\left(\dfrac{\theta}{4} - \dfrac{\sin 2\theta}{8}\right) \\ B2 = H\left(R^3 \sin^2\alpha \sec\alpha\right)\left(\dfrac{\theta^3}{12} + \dfrac{\theta^2 \sin 2\theta}{8} + \dfrac{\theta\cos 2\theta}{8} - \dfrac{\sin 2\theta}{16}\right) + R^2 . \left(\sin^2\alpha \sec\alpha\right)\left(\dfrac{\theta\cos 2\theta}{8} - \dfrac{\sin 2\theta}{16}\right) \\ B3 = WR^4 \sin\alpha\left(\dfrac{\theta\cos 2\theta}{8} - \dfrac{\sin 2\theta}{16}\right) + \sin\alpha\left(\dfrac{\theta}{4} - \dfrac{\sin 2\theta}{8}\right) \\ B4 = WR^3 \sin\alpha\left(\dfrac{\theta^2 \sin\theta}{2} + \dfrac{2\theta\cos\theta}{2} - \dfrac{2\sin\theta}{2}\right) + \sin\alpha\left(\dfrac{\theta\cos\theta}{2} - \dfrac{\sin\theta}{2}\right) \\ B5 = H.R^3 \sin^2\alpha \sec\alpha\left(\dfrac{-\theta\cos 2\theta}{8} + \dfrac{\sin 2\theta}{16}\right) + R^3 \sin^2\alpha \sec\alpha\left(\dfrac{\theta}{4} - \dfrac{\sin 2\theta}{8}\right) \\ C = M.R\sec\alpha\left(\dfrac{\theta}{2} + \dfrac{\sin\theta}{4}\right) + H.R^2 \tan\alpha \sec\alpha\left(\dfrac{-\theta\cos 2\theta}{4} + \dfrac{\sin 2\theta}{8}\right) + \\ WR^3 . \sin\theta \sec\alpha + \sec\alpha\left(\dfrac{\theta}{2} + \dfrac{\sin 2\theta}{4}\right) \\ D = M \cos\alpha R\left(\dfrac{\theta}{4} + \dfrac{\sin 2\theta}{8}\right) + H.R^2 \sin\alpha\left(\dfrac{\theta\cos 2\theta}{8} - \dfrac{\sin 2\theta}{16}\right) + R^2 . \sin\alpha\left(\dfrac{\theta}{4} - \dfrac{\sin 2\theta}{8}\right) + \\ WR^3 . \cos\alpha\left(\dfrac{\theta}{4} - \dfrac{\sin 2\theta}{8}\right) + WR^3 \cos\alpha\left(\dfrac{\theta\cos\theta}{2} - \dfrac{\sin\theta}{2}\right) \end{Bmatrix}$$

Final Computation Results of case 1

$$\frac{\partial U}{\partial H} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{Case 1}$$

A+B=0………………………………..Equation 3.2.1.1

$$\left(\begin{array}{l} M\left\{ \begin{array}{l} \left(\frac{-\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)R^2\tan\alpha\sec\alpha+\left(\cos\alpha R^2\sin\alpha\sec\alpha\right)\left(\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+ \\ \cos\alpha R^2\sin\alpha\sec\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right) \end{array} \right\} + \\ \\ H\left\{ \begin{array}{l} R^3\tan^2\alpha\sec\alpha\left(\frac{\theta^3}{6}-\frac{\theta^2\sin 2\theta}{4}-\frac{\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)+ \\ \left(R^3\sin^2\alpha\sec\alpha\right)\left(\frac{\theta^3}{12}+\frac{\theta^2\sin 2\theta}{8}+\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+R^3\left(\sin^2\alpha\sec\alpha\right)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)+ \\ \left(R^3\sin^2\alpha\sec\alpha\right)\left(\frac{-\theta\cos 2\theta}{8}+\frac{\sin 2\theta}{16}\right)+R^3\sin^2\alpha\sec\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right) \end{array} \right\} + \\ \\ W\left\{ \begin{array}{l} R^4\tan\alpha\sec\alpha\left(\theta\cos\theta-\sin\theta-\frac{\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)+R^4.\left(\sin\alpha\right)\left(\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+ \\ \cos\alpha\sin\alpha\left(\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)\right)+R^4\left(\sin\alpha\right)\left(\frac{\theta^2\sin\theta}{2}+\theta\cos\theta-\sin\theta\right)+\sin\alpha\left(\frac{\theta\cos\theta}{2}-\frac{\sin\theta}{2}\right) \end{array} \right\} = 0 \end{array} \right)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\textit{Result AB})$$

$$\frac{\partial U}{\partial M} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textit{Case 2}$$

C+D=0 ……………………..……..Equation 3.2.1.2

.

$$\left(\begin{array}{l} M.\left\{ \left(R\sec\alpha\right)\left(\frac{\theta}{2}+\frac{\sin\theta}{4}\right)+\left(\cos\alpha\ R\right)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{4}\right)\right\}+ \\ H\left\{ \left(R^2\ \tan\alpha\sec\alpha\right)\left(-\frac{\theta\cos 2\theta}{4}+\frac{\sin 2\theta}{8}\right)+\left(R^2\ \sin\alpha\right)\left(\frac{\theta\cos 2\theta}{8}-\frac{\sin 2\theta}{16}\right)+R^2\ \sin\alpha\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)\right\}+ \\ W\left\{ \begin{array}{l} \left(R^3\right)\left(-\sin\theta\sec\alpha+\sec\alpha\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)+\left(R^3\right)\left(\cos\alpha\right)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)+\left(R^3\right)\left(\cos\alpha\right)\left(\frac{\theta}{4}-\frac{\sin 2\theta}{8}\right)+ \right. \\ \left(R^3\right)\left(\cos\alpha\right)\left(\frac{\theta\cos\theta}{2}-\frac{\sin\theta}{2}\right) \end{array} \right\} = 0 \end{array} \right)$$

……………………………………………………………………………(*Result CD*)