



Addis Ababa Institute of Technology
School of Mechanical and Industrial Engineering
Graduate Program in Railway Engineering

**Analysis of Gear Train Structure of Electric Traction Train Using Finite
Element Method**

A thesis submitted to the School of Mechanical and Industrial Engineering, Addis Ababa
Institute of Technology in partial fulfillment of the Degree of Masters of Science in Mechanical
Engineering (Railway Engineering Stream)

By

Shuma Michael

Advisor

Dr. Ing Demiss Alemu

August 2014

Addis Ababa, Ethiopia



Addis Ababa Institute of Technology
School of Mechanical and Industrial Engineering
Graduate Program in Railway Engineering

**Analysis of Gear Train Structure of Electric Traction Train Using Finite
Element Method**

A thesis submitted to the School of Mechanical and Industrial Engineering, Addis Ababa Institute of Technology in partial fulfillment of the Degree of Masters of Science in Mechanical Engineering (Railway Engineering Stream)

By

Shuma Michael

Advisor

Dr.Ing Demiss Alemu

Co-Advisor

Tsegaye Feleke

September 2014

Addis Ababa, Ethiopia

Addis Ababa Institute of Technology

School of Mechanical and Industrial Engineering

**Analysis of Gear Train Structure of Electric Traction Train Using
Finite Element Method**

By

Shuma Michael

September 2014

Approved by Board of Examiners:

Dr.Birhanu Beshah

Head, Railway Centre

Signature

Date

Demiss Alemu (Dr-Ing)

(Advisor)

Signature

Date

Tsegaye Feleke

(Co-Advisor)

Signature

Date

Tollossa Deberie

Internal Examiner

Signature

Date

Fasil Gesessew

External Examiner

Signature

Date

DECLARATION

I hereby declare that the work which is being presented in this thesis entitled as “Analysis of Gear Train Structure of Electric Traction Train Using Finite Element Method” is original work of my own, had not been presented for a degree of in any other university, in any projects by any means, and all the resource materials used for this thesis had been accordingly acknowledged.

Shuma Michael

Date

This is to certify that the above declaration made by the candidate is correct to the best of my knowledge.

Dr.-Ing Demiss Alemu
(Advisor)

Date

Tsegaye Feleke
(Co-Advisor)

Date

ACKNOWLEDGEMENTS

I am indebted to my advisor Dr. Ing. Demiss Alemu and hereby would like to express my sincere appreciation and gratitude to him for his valuable guidance, help and encouragement during the process of this research work and in the preparation of this thesis. Without his guidance and support this work would have been impossible. I would also like to offer my sincere gratitude to Tsegaye Feleke for his valuable technical guidance and advice throughout the course of this research.

I am also grateful to the school and the staff of the department of Mechanical Engineering, AAit (Addis Ababa University Institute of Technology), who have provided the most cooperative assistance to my work.

I would also like to acknowledge the ERC (Ethiopian Railway Cooperation) for their provision of grateful assistance to all of my thesis work.

ABSTRACT

The gear trains are the critical components in a railway traction system as they transmit and withstand very high loads. Their primary function is to transmit power from one shaft to the other. In the railway traction system, there are several arrangements for the drives system. In this particular case the unit that takes power, from motor assembled to drives system through pinion on the shaft to the final spur or wheels by means of single stage or compound stage on the axle of the train, in such a way that power is transmitted. During the transmission of power from one shaft to another, for rail vehicles moving in high speed, torque is also transmitted. In this phenomenon, different kinds of stresses and deformations will occur on the structure of the gear train. FEM (Finite Element Method) is used to determine and analyze different kinds of stresses and deformations in the complex geometry. This thesis investigates the characteristics of gear trains under different kinds of loading, including bending and shear stresses, stresses due to vibration, contact stresses and deformations by FEM and proposes a better optimization for the gear units.

Two or three spur gear trains with and without idler gears are modeled in both solid works and ANSYS work bench for drives system to distinguish the behaviors of the stresses and vibration for the system, and optimization will be proposed for the design of the gears. Then the result obtained will be compared with the analytical method. Contact stresses are also analyzed and compared with the current method which is known as Hertzian contact stress; if both value agree very well, FEM is then the accurate method to analyze the effect of the contact stress for the gear units in the drives system. American Gear Manufactures Association (AGMA) is used for stress analysis but it is limited and many assumptions to be taken into account. Lewis equation is also used to determine the bending stress. After different kinds of analysis, the parameters which may affect the overall efficiency of the system will be optimized, by proper treatment and design of the gear structure for the drives system. These treatments will also include improving tribological property of the gears, profile modification of the gear tooth and so on. Therefore, FEM will predict the different kinds of stresses and deformations on the gear train structures of electric traction train .

CONTENTS

ABSTRACT.....	v
LIST OF TABLES.....	viii
LIST OF FIGURES	ix
CHAPTER 1	1
INTRODUCTION	1
1.1 Research Background	1
Fig1-1Drive Design [28].....	2
1.2 Statement of the Problem.....	3
1.3 Research Objective	4
1.4 Thesis Overview	5
1.5 Limitation of the Thesis	6
CHAPTER 2	7
LITERATURE REVIEW	7
2.1 Gear Bending Stress Analysis.....	7
2.2 Gear Train Contact Stress Analysis	10
2.3 Vibration Analysis of the Gear Train.....	11
2.4 Dynamic Effects of the Gear Train Analysis	13
CHAPTER 3	14
ANALYTICAL METHOD ANALYSIS OF THE GEAR TRAIN	14
3.1 Introduction.....	14
3.2 Standard Specifications for the Rail Vehicles.....	15
3.3 Operating Characteristics of Traction Motor for Trains	18
3.4 General Properties of Spur Gear	23
3.5 Gear Bending Stress Analysis Using AGMA and Lewis bending Equation	25
3.5.1 AGMA Stress Equation	25
3.5.2 The Lewis Bending Equation.....	28
3.6 Vibration Analysis of the Gear Train.....	29
3.6.1 Modal Analysis	30
3.6.2 Harmonic Response	34
3.7 Contact Stress Analysis.....	38
CHAPTER 4	40
FINITE ELEMENT METHOD PROCEDURE.....	40
4.1 Finite Element Analysis (FEA) for the Gear Units.....	40

4.2 Force Analysis of the Gears	41
4.3 Finite Element Analysis Results by ANSYS Work Bench	43
4.3.1 Static Analysis of Gear Units in ANSYS Work Bench	43
3.2 Static Analysis of the Gear Train without the Idler Gear in ANSYS	50
4.3.3 Fatigue Analysis of the Gear Train without Idler Gear.....	55
4.3.4 Modal and Harmonic Response Analysis of the Gear Train without the Idler Gear	57
4.3.5 Contact Stress Analysis of the Gear Train without Idler Gear in ANSYS	64
4.3.6 The Static Structural Analysis of the Gear trains with Idler gear	66
4.3.6 Fatigue Analysis for the Gear Trains with Idler Gear in ANSYS work bench.....	70
4.3.7 Harmonic Response and Modal Analysis for the Gear Train with Idler gear.....	70
4.6 Validation of FEM (Finite Element Method)	79
4.7 Results and Discussions.....	79
CHAPTER 5	81
OPTIMIZATION OF THE GEAR UNITS AND LIMITATION OF THE FINITE ELEMENT METHOD	81
5.1 Optimization of the Gear units.....	81
5.3 Limitation of Finite Element Method	84
CHAPTER 6	85
CONCLUSIONS AND RECOMMENDATIONS	85
6.1 Conclusions.....	85
REFERENCES	87
APPENDIX.....	90

LIST OF TABLES

Table 3-1 Speed and Axle load Range Value for Rail Vehicles in Ethiopia	15
Table 3-2 A list of AC Traction Motor for Trains	17
Table3-3 Rail Vehicles Speed and Power Range.....	18
Table 3-4 Power Rating of Some Motors	21
Table 3-5 Axle Specification [25]	21
Table3-6 General Properties of Gear train Parameters	22
Table 4-1 Specifications of gear units.....	40
Table 4-2 Material Properties and Design parameters of the test gears.....	43
Table 4-3 Input Parameters for gear train without Idler Gear.....	51
Table 4-4 Parameters for gear Train with Idler.....	66
Table 4-5 Some Useful ANSYS Work bench results	78
Table4-6 Comparison of Normal Stress for AGMA and FEM.....	79
Table5-1 Comparison of pinions profile.....	82

LIST OF FIGURES

Fig1-1 Drive Design [28]..... 2

Fig 1-2 Gear train [20] 3

Fig 3-1 Some parts of AC traction motor [21]..... 16

Fig 3-2 Frequent Operating Points of the Traction Motor for FHDS Electric Rail Vehicles 19

Fig 3-3 Operating Characteristics of Traction Motor for High Speed Train [24]..... 20

Fig 3-4 Transmitted power as a function of pinion speed [35]. 22

Fig3-5 Spur gear Nomenclature [3] 23

Fig3-6 Construction of involutes curve 24

Fig3-7 Base Circle Construction..... 24

Fig3-8 Lewis Cantilever [3]..... 27

Fig 3-9 Second Stage Gear Train..... 31

Fig 3-10 Variation of X and ϕ with frequency ratio $r=\omega/\omega_n$ [38]..... 36

Fig 3-11 The Hertz Contact [27]..... 38

Fig 4-1 Finite Element Procedure for obtaining Gear stresses and deformations [2] 41

Fig 4-2 (a) Geometry and (b) Boundary conditions for pinion 45

Fig 4-3 (c) Total deformation and (d) Equivalent stress for pinion 46

Fig 4-4 Normal and Maximum Principal stress 47

Fig 4-5 (a) Geometry and (b) Boundary conditions set up for Spur 48

Fig4-6 (c) Total deformation and (d) Equivalent stress for spur 49

Fig 4-7 Normal stress 50

Fig 4-8 (a) Geometry and (b) Mesh for the gear train..... 52

Fig 4-9 Boundary Conditions Set ups 53

Fig4-10 (a) Total deformation and (b) Equivalent von mises stress 55

Fig 4-11 Equivalent alternating stress..... 56

Fig4-12-factor of safety 56

Fig 4-13 Total deformation 1, 2 and 5 (modal)..... 59

Fig 4-14 Equivalent/Vonmises stress (Modal)..... 60

Fig 4-15 (a) Total deformation/harmonic excitation with varying phase angles 61

Fig 4-16 Equivalent stress/harmonic excitation with phase angle of 0° and 5° 62

Fig4-17 (a) and (b) Frequency response graph 63

Fig 4-18 Contact set up 64

Fig 4-19 Pressure due to contact..... 65

Fig 4-20 Frictional Stress due to Contact..... 65

Fig 4-21 (a) Geometry (b) Mesh for the gear train 67

Fig 4-22 Boundary condition set up..... 68

Fig 4-23 (a) Total deformation and (b) Equivalent stress 69

Fig 4-24 Fatigue/ Equivalent alternating stress/..... 70

Fig 4-25 Total deformation mode shape 1 and total deformation mode shape 2..... 72

Fig 4-26 Equivalent stress..... 73

Fig4-27 harmonic response total deformation 74

Fig 4-28 Harmonic response equivalent stress with phase angle of (a) 0 and (b) 5..... 75

Fig 4-29 Contact set up 76

Fig 4-30 (a) Pressure and (b) Frictional stress 77

Fig 5-1 (a) and (b) Pinion with more and less root fillet..... 83

LIST OF ABBREVIATIONS

3D	Three dimensional
AGMA	American Gear Manufacturers Association
ANSI	American National Standards Institute
ERC	Ethiopian Railway Corporation
EV	Electric Vehicle
FEA	Finite Element Analysis
FEM	Finite Element Method
FHDS	Federal Highway Driving Schedules
FUDS	Federal Urban Driving Schedules
HPSTC	Highest point of Single Tooth Contact
IMPSM	Interior Permanent Magnet Synchronous Motor
LRT	Light Rail Train
RPM	Revolution Per Minute
Kw	Kilowatt
X	Amplitude
φ	Phase angle

LIST OF NOTATIONS

K	Structural stiffness
u	Displacement vector
Φ	Pressure angle
σ_o	Maximum Hertz stress
r	Any radius to involute curve
r_b	Radius of base circle
P_d	Diametric pitch
Y	Lewis form factor
K_s	Size factor
$K_m (K_H)$	Load distribution factor
K_v	Dynamic factor
Y_j	Geometry factor
K_o	Overload Factor
K_θ	Temperature Factor
F_t	Transmitted Load
F_r	Radial Load
K_B	Rim thickness factor
m	Module
ν	Poisson ratio
ω	Natural frequency
n	Angular velocity(RPM)
N_1	Number of teeth for Pinion
N_2	Number of teeth for Idler
N_3	Number of teeth for Spur
H	Horse Power
b	Face width
E	Modulus of Elasticity
E_1	Modulus of Elasticity of body 1
E_2	Modulus of Elasticity of body 2
R_1	R_1 Radius of body 1
R_2	R_2 Radius of body 2

CHAPTER 1

INTRODUCTION

1.1 Research Background

Gearing is one of the most critical components in a mechanical power transmission like in a rail vehicle traction system. It is possible that gears will predominate as the most effective means of transmitting power in future rail vehicles traction system due to their high degree of reliability and compactness. Gear train is the unit which found in a rail vehicle drives system assembled between the wheels, axles and the bogie frames. The following fig1.1 shows a simple model of one of the drives design system with simple gear trains arrangements. There are several types of drives system in a rail vehicle. These are transverse drives, longitudinal drives, nose suspended drives and etc. In all cases, different types of gear units are used for transmitting the power, thus to reduce the speed of the motor. The motor assembled in the drives unit, as shown in the following fig1.2 with simple gear trains arrangements can have a rotational speed up to 6000RPM [20]. This maximum speed will be reduced by the gear train unit before it reaches to the wheels.

This power transmission unit must be reliable and perfectly designed to achieve the intended objective particularly, to carry the load and increase the torque. As we know, Ethiopia is building rail ways and launching several types of the ways for modern trains. All of these rail vehicle posses the gear trains for transmitting the torque and the power by proper gear ratio. There are several types of gears. These are spur gears, bevel gears, helical gears and worm gears. In this particular research: spur gears are considered. Spur gears have efficiency of about 96% up to 98% during the transmission of power. In addition, two or three gear units are considered for the analysis. The gear ratio depends on the speed of the rail vehicle.. Normally, locomotives with power rating of 100Kw to 1700Kw can have a gear ratio of 3 to 5 and the power range to be considered is also based on the speed of the rail vehicle and possibly varies from 125kw to 1700kw [31]. The speed of the rail vehicle for passengers' trains is from 120Km/hr to 180Km/hr, whereas for freight trains, it is from 80Km/hr to 120Km/hr. The assumed gear unit will give service for 1,500,000Km to 2,000,000Km without degradation of performance and the gears shall be highly treated. Most rail vehicles with the specified speed range have a pinion pitch diameter of 90 to 250mm and spur pitch

circle diameter of 300mm to 950mm for both passenger and freight rail vehicle [29]. The dimensions of the gears will be in the range between the specified dimension and different kind of structural analysis will be executed on them by FEM (Finite Element Method).

The increasing demand for quiet and smooth operation in transport industry is increasing. Therefore, the design of gears has been much improved by computer powers to reduce the problems caused by high loading, transmission error, noise and vibration. Using computer powers has also improved gear design.

Finite Element Method also called Finite Element Analysis is used to model, simulate and analyze the gear trains by using different kinds of commercial soft ware. This method is accurate and easy to analyze complex and difficult differential equations. We use this method because it provides us every solution we require for the gears. Moreover, it is an efficient technique. During the transmission of torque and power, the gear trains are under different kinds of stresses. These could be bending stresses, contact stresses, stresses due to vibration, shearing stresses and etc will occur. The combined action of these will affect the overall performance of the system. ANSYS work bench and Solid works are the method to be used for the analysis of the gear trains.

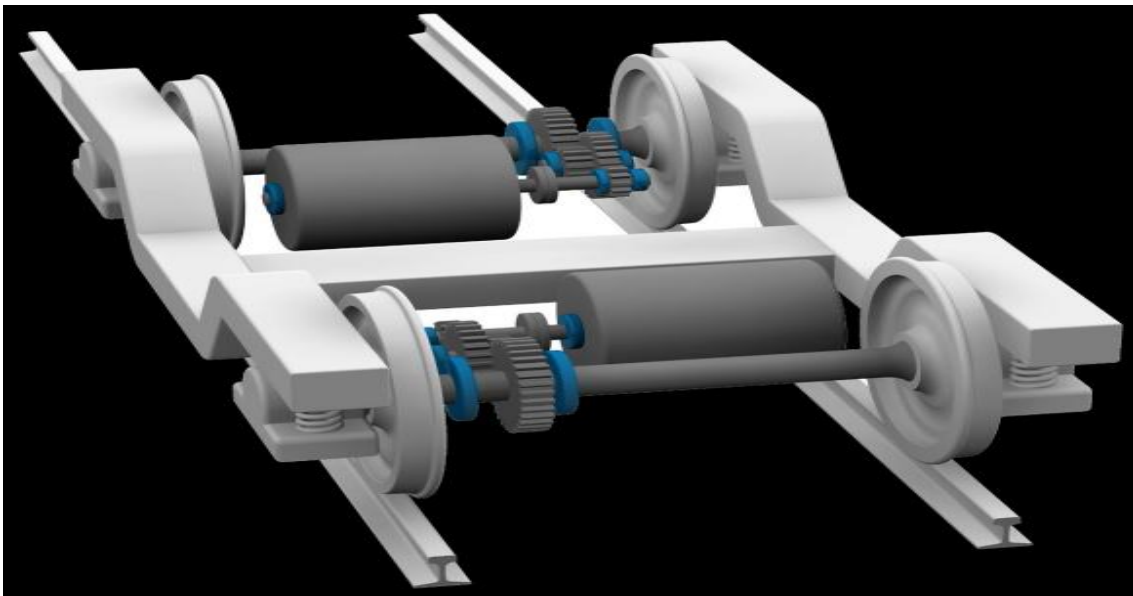


Fig1-1Drive Design [28]

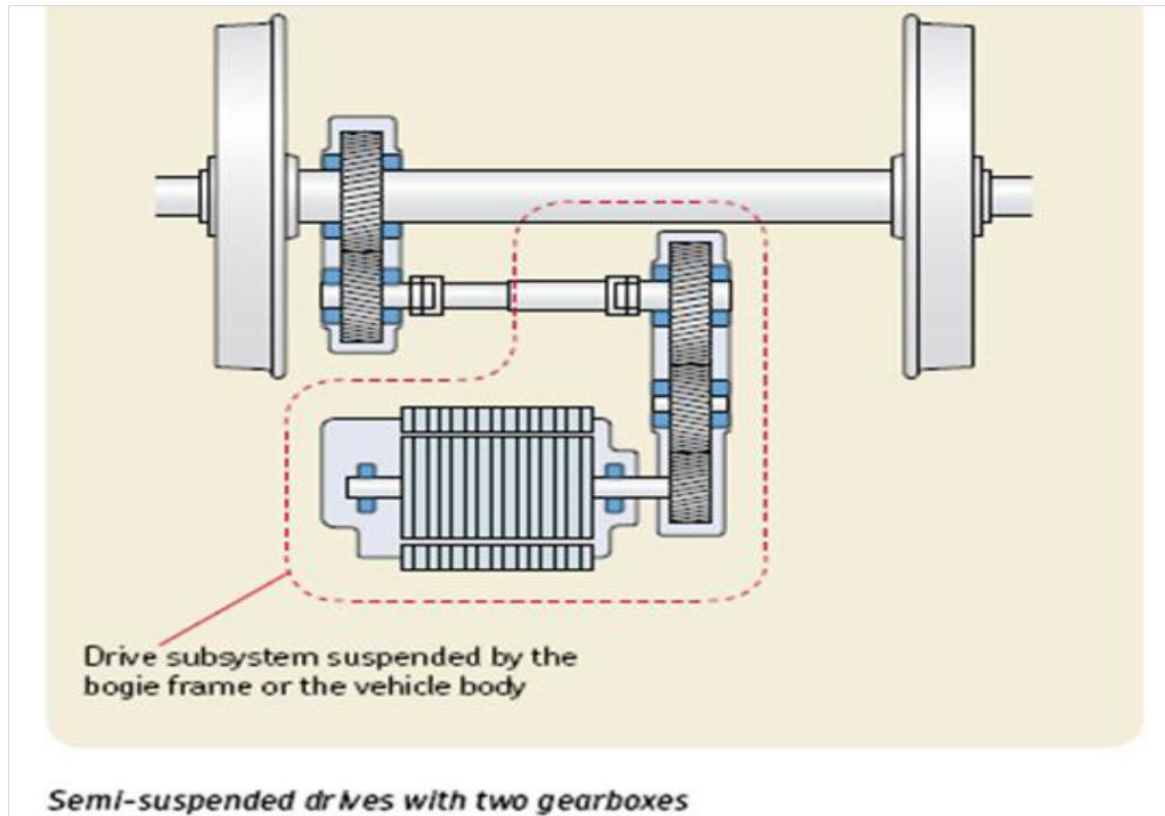


Fig 1-2 Gear train [20]

1.2 Statement of the Problem

Despite the fact that a number of investigation and research work have been done on gear train and gear design, there still requires a significant improvement in design for the gears. Gears are the most and efficient parts of transmitting power, ranging from a tiny machines to aero space, rail vehicles, ship building and etc. Therefore, their failures will result in catastrophic damage to the whole machines. The railway vehicles utilizes the gear train in its drives system of electric traction. Therefore, improvement in design, optimization by modeling, the 3D gear trains analysis is necessary by Finite Element Method, which uses commercial software such as ANSYS, Solid Works and etc to minimize the under design, over design and failure of the component.

1.3 Research Objective

In spite of the number of investigations devoted to gear research and analysis, there still remains to be developed, many numerical methods and analytical methods are used to investigate the characteristics of the gear units during transmission of power. These methods are investigating gear bending stresses, gear contact stresses, gear vibration, gear transmission error, gear shearing, deformation, torsion and etc. The objectives of this thesis are to use a Finite Element Method (FEM) approach to develop theoretical models of the behavior of spur gear trains in mesh, to help to predict the effect of gear stresses and transmission error, torsion and deformation on the performance and life of the specified gear on the whole drives system of the electric traction of the rail vehicle. Therefore, it is possible to predict the life of the unit and the whole effects by FEM. As a result better optimization will be proposed depending on the prediction of the Finite Element Method.

The main aims of this research as organized here are:-

- To collect the proper data about the specified rail vehicle, and then organize important data for its drives system, for modeling the gears separately and in mating.
- The dimensions of the gear units will be sorted out for modeling and asses its effect by FEM.
- To model the appropriate geometry of the rail vehicle traction gear trains by ANSYS, and then to predict the stresses and deformations.
- To model two or three gear train units and compare their different kinds of stresses and deformations results.
- Comparing the analytical methods of gear stresses calculations with that of the Finite Element Method.
- Depending on the result obtained by FEM, better optimization will be proposed or executed for the gear units and the whole drives system of the electric traction unit. This optimization or proposal even could be to treat materials, to propose a better design and so on.

Particularly the goals of the gear train modeling on the FEM soft ware are:

- Predicting different kinds of stresses on the gear units by FEM.
- To predict fatigue life of the assembled gear train on the rail vehicle and its efficiency for the total life cycle.

- To predict the different kinds of deformations on the gears by ANSYS.
- To perform vibration analysis of the gear trains.
- Proposing several optimization techniques for gear units depending on the results of FEM. Failure occurs frequently on the roots of the teeth, surfaces of the teeth. Failures can also occur on the surfaces of the gears and different kinds of surface treatment can be proposed. Lewis formula, American Gear Manufacturers Association (AGMA) and Hertzian contact stress will be utilized for comparing different kinds of the results obtained by FEM.

1.4 Thesis Overview

The thesis consists about six chapters which describes the analysis of the gear train of the drives system of electric traction of the rail vehicles. Chapter one presents the general introduction about the analysis of the gear train by Finite Element Method (FEM) and describing different kinds of results, and objectives to be achieved and the thesis lay out is explained. Chapter two covers a literature review which discusses the current approach used in analyzing and evaluating the gear train structure stresses, and the vibration behavior in various applications and their possible limitations. The background characteristics of the spur gears trains and the FEA model of the gear train developed by authors for evaluating the gear stresses, frequencies, mode shapes and the dynamic excitation characteristics are also explained. Chapter three is the analytical method analysis of the stress in the gearing system. In this chapter the main points of the American Gear Manufacturing Association (AGMA) equation, Lewis bending equation, vibration and contact stresses are explained. In this chapter data collection techniques are briefly outlined. Chapter four is the extensive analysis carried out by the Finite Element Method for the gear trains. The modeling steps are explained and the solution phases are executed. Different kinds of results are displayed. Two or three gear units are modeled for analysis. Extensive analysis including static, fatigue, vibration (harmonic and modal), and contact stresses are briefly outlined in this chapter. Moreover, mode shapes, total deformation and harmonic analysis at different types of frequency will be investigated. During the analysis, two types gear trains are used which possess the total transmission ratio of 7.47:1. In the first case three gears which possess the idler is considered and in the second case two gears are used. Chapter five is about the optimization of the gear units or trains for drives, based on different kinds of results and even

to propose gear modification and gear treatments, and limitation of the FEM. Chapter six is conclusion and recommendation are elaborated.

1.5 Limitation of the Thesis

The fact that the scarce material resources of the modern rail ways system in here, practical investigations of components in rail vehicles is at its infant stage. During the entire work of this research, gathering relevant data concerning rail vehicles drives system is so much exhaustive and it is all based on different kinds material resources such as web sites, books, journals and thesis and so on. Perhaps, the obtained data is doubtful whether it is currently used or not. For example, the type of gear used in this thesis is spur. Some articles mentioned that spur gears are also used in the rail vehicles drives system due to their superior efficiency in power transmission. In reality the spur gears are not preferable to the helical gear because they are nosier than helical. Therefore, the significant limitation of this thesis is that spur gears are used for the analysis.

CHAPTER 2

LITERATURE REVIEW

2.1 Gear Bending Stress Analysis

There has been a great deal of research on gear train analysis, and a large number of literatures on gear train modeling and structural analysis have been published. The gear stresses analysis, the transmission errors, and the prediction of gear dynamic loads, gear noise, and the optimal design for gear sets are always major concerns in gear design. Zeping Wei reviewed different kinds of literature and included the bending stress of the gear in mating [1]. Ooi Jong Boon has dealt extensively with gear train analysis and found out about gear bending stresses [2]. In the middle of the twentieth century most gear designs and structural analysis of the gear trains were based upon Lewis original bending equation [3]. Lewis based his analysis on a cantilever beam and assumed that failure will occur at the weakest point of this beam. However, failure due to flexural stresses on bodies with changing or asymmetrical cross-sections was proved inaccurate [5]. Their approach used photo elastic experiments to visualize the stress concentrations due to the fillets at the base of spur gears in meshing. By these visualization techniques, they were able to predict more accurately at what stress levels gears will fail due to high bending stresses. Much earlier work was done using photo elastic experiments to design spur gears based on the stresses observed at the most critical points. The use of photo elastic experiment is rare due to the high cost of the equipment and it requires experience and skills to determine the gear stresses. Although this method is useful in determining static stresses in spur gears, the photo elastic trend has become more popular toward its usage in gear dynamic analysis [6].

On the other hand, the bending stress for a standardized gear bending stress analysis can be estimated from numerous gear standards such as the AGMA standards and the ISO standards for gears [3]. The AGMA standards were established in 1982 and are still widely used in gear design and stress analysis today. The bending stress equations found in these standards are based on the Lewis's original equation with several gear factors for different kinds of gear in meshing [7]. The gear geometric factors found in the equations are critical in determining accurately the bending stresses for a wide variety of gears [8]. These geometric factors accounted for the changing shape of the gear tooth, the point where the load is applied, as well as the fillet radius at the tip and base of the tooth.

In general, the AGMA standard is only valid for standard gear design and stress analysis in which the gear must have 20° pressure angle and the gear tooth profile must be symmetrical [3]. Thus, these gear standards are not suitable for calculating the gear stresses for gear design with customized parameters. In this situation, when it comes to designing customized gear, gear designers need to approximate the strength and durability of their gears based on their own judgment and experience. This may lead to inaccurate evaluation of the gear strength.

The authors commented on the significant difference in results but concluded that it is in an acceptable range. In the AGMA analytical tooth model, the maximum load occurs at the highest point of single tooth contact (HPSTC). It was clearly shown the position of the load at HPSTC but the direction of the load is not defined. The direction of the load strongly depends on the surface contact between gears. However, the direction of the load applied to the tooth surface is uncertain and difficult to be determined. One of the solutions to determine the correct directional load is by using Finite Element Analysis (FEA) to model the gears and perform simulation on the contacting gears. Thus, the gear teeth bending stress should be evaluated with respect to varying angular position of the gear tooth.

David.V Hutton published his book about the Finite Element Method and expressed how the finite element works [4]. The finite element method (FEM), sometimes referred to as finite element analysis (FEA), is a computational technique used to obtain approximate solutions of boundary value problems in engineering. Simply stated, a boundary value problem is a mathematical problem in which one or more dependent variables must satisfy a differential equation everywhere within a known domain of independent variables and satisfy specific conditions on the boundary of the domain. Boundary value problems are also sometimes called field problems. The field is the domain of interest and most often represents a physical structure. The field variables are the dependent variables of interest, governed by the differential equation. The boundary conditions are the specified values of the field variables (or related variables such as derivatives) on the boundaries of the field. Depending on the type of physical problem being analyzed, the field variables may include physical displacement, temperature, heat flux, and fluid velocity to name only a few.

Although FEA has been around for over half a century, it was not until the increased in computing power that the real advantages of this method became apparent. In the area of gear research, the modeling of the gear tooth and simulation of the bending stress analysis using

FEM is in its infant stage. It was stated that FEA is one of the significant contributions to the understanding of gear stress behavior [9]. The modeling of spur gears and the analysis of the stress results using the FEA has led to many insights which may not have been immediately apparent. In the FEA modeling, the first challenge to overcome is to model the geometry and dimensions of the gear correctly. Once the geometry has been modeled, the type of elements and mesh to be applied is crucial. Areas where higher stresses and deformations occur needed to be meshed more densely so that the results were accurate.

The advantages of this method over experimental techniques are competitive, cost effectiveness and repeatable results. The accuracy of the FEA solution can be assessed by verifying that the FEA results correspond closely to experimental results. Hence, the validity of the FEA setup and technique applied can be confirmed. There are few authors who have validated their FEA models by comparing the gear stress results with the experimental results in their thesis work [1], [2].

The current trend of gear design and stress analysis is focused in designing different shaped gears to transmit higher loads without failure. The purpose of investigating the effect of the shaped gears is to precisely engineer these gears so that the maximum efficiency can be achieved and overdesign or under design of the gear can be avoided. In such a way that compact and reliable gears are assembled for rail vehicle drives system. By changing the shape of the gear tooth to an asymmetrical design, the authors have proven a decrease in both bending stress and contact pressure [10]. In the past, most 3D gear models developed was often a simplified model with many assumptions considered and some models are limited to analyzing the bending stress for a single involutes spur gear. When the gears are operating, the gear teeth are often meshed with one or more gear teeth depending on the gear contact ratio. The analysis of single gear tooth does not provide a full understanding of the actual gear meshing mechanism. Instead, a full gear bodies should be developed for a more comprehensive understanding of the gear stress analysis [11].

It was suggested that a finite element model of a single tooth was used to analyze the stress, deformation and fracture in gear teeth when subjected to dynamic loading and thus bending stress will occur [1].

2.2 Gear Train Contact Stress Analysis

In the gear train analysis, it was pointed out that the analysis of the contact pressure at the surfaces between the gears contacting teeth is also important so that one can understand how the wear resistance and the fatigue failure rate of the gear can be affected [2]. Heinrich Hertz was the first to analyze the contact stress of the gear tooth. His theory describes the contact pressure between two deformable cylinders [12]. The work of Hertz was expanded by Archard, who has proposed an experiment to investigate the contact pressure between two deformable bodies [13]. His work led to the evolution of many modern techniques and formulations that are present today for contact analysis. While the bending stress is dependent on the geometry and shape of the gear tooth, the contact pressure is mainly a function of the type of material in contact and the radius of curvature of the gear surface contact.

In recent years, there has been a number of proven research works done by authors; they investigate the contact pressure between two spur gears, helical gears and asymmetrical gearing. Moreover, they have pointed out that they applied the FEA approach in their research and the results are validated and well documented [14]. In solving contact problems using FEA, there is a vast amount of work done on the stiffness of spur gear teeth and the appropriate method used for developing a precise geometrical gear model using FEA. In the analysis of the gear contact stress done, they considered the effect of non-linearity at the beginning of contact between teeth, as well as the importance of the point of contact. The effect of the non-linearity in gear train contact stress was verified by the experimental test.

Although the contact pressure problem has been solved and the accuracy of the FEA solution was validated, the model used for the evaluation of contact stress is based on the contact surface between two cylindrical bodies that are identical. This model only provides the understanding of how contact stress is developed from the normal loading but it did not consider the friction effect due to the tangential load. One of the biggest challenges in analyzing the gear contact stress is to develop a gear tooth model that has smooth geometrical surface, precise alignment of the gear teeth to be in contact with another and solving contact problems. Properly aligned spur gears are designed to mesh with the pinion at a precise point. A slight change in the geometry or alignment of gear contact can cause a tremendous change in the contact stress thus on the whole drives system of the rail vehicles. Wei pointed out in his research that some scholars studied static contact stresses including the effect of friction between the mating gear teeth. Using the conventional finite element method, the element

stiffness matrices and the global stiffness matrix $[K]$ of the two gears in mesh were obtained. If the external forces at the various nodes are known, then the system of equations is written as:

$$[K]\{U\} = \{F\} \dots \dots \dots \text{Eq (2.1)}$$

Where

$$[K] = \begin{bmatrix} k_{11} & \dots & k_{1j} \\ \vdots & \ddots & \vdots \\ k_{i1} & \dots & k_{ij} \end{bmatrix} \text{ and}$$

$$\{U\} = \begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \\ \vdots \\ u_{i1} \end{bmatrix}$$

Where $\{U\}$ is the nodal displacement vector and $\{F\}$ is the nodal force vector. The system of equations is solved and $\{U\}$ is obtained. Then the stress can be calculated. Each gear is divided into a number of elements such that in the assumed region of contact, there is equal number of nodes on each gear. These contact nodes are all grouped together. The contact stresses between meshing gears was investigated by simulating them as two or more cylinders rolling over each other, by making the rotational speed of the gears equal to the corresponding cylinders [2]. In a way that all the results obtained from ANSYS are compared to that of the original Hertzian equation. Contact problems are of several types: point to point; point to surface and surface to surface. Generally, contact problems involve non linearity. FEM which uses ANSYS workbench estimates contact stresses.

2.3 Vibration Analysis of the Gear Train

Vibration is an imminent phenomenon in any machinery [3]. Vibration analysis of the gear train is important in ensuring the component is operating under an acceptable level of vibrations and noise level. In fast moving rail vehicles it is evident that torsional vibration is the case in gear train structure of the drives system of the electric traction train [32]. The vibration can be from unbalanced rotation of motor, misalignment, poor design and so on. It induces the resonance of the gear train excited by the dynamic input. Resonance can raise the

structural stress level to cause structural failure. There are quite a number of contributed research works by authors.

The study of the vibration is very few until recently. In the past few years, there are a few works devoted to study the vibrations by using analytical method and experimental test. It was presented a thorough summary of the gear dynamic models for gear train in the research work [2]. It was investigated that a single spur gear; pair of the rotor dynamics spurs geared system by experimental testing and was found that this particular system experienced severe coupled torsion, lateral, and axial vibrations. Other scholars pointed out and analyzed the vibratory excitation of gear systems of single stage and multi-stage theoretically. Lim and Singh presented study of the vibration analysis for complete gearboxes on three cases in which a single-stage rotor system with a rigid casing and flexible mounts, and a high-precision spur gear drive system with a flexible casing and rigid mounts were analyzed for noise analysis of the gear train experimentally [15].

The authors studied how the high excitation of vibration in the gears, shafts, bearings and the casing can cause high level of noise within the car or truck cab or in any kinds of moving machinery. In the application of FEA, in analyzing the gear train vibrations, the FEA program was also adopted by authors, to help simulate the stresses on a single stage, double stage, and gearboxes to investigate the gear failure and conduct parametric studies [16]. Draca investigated the model of double-stage helical gear reduction using finite element analysis [17]. The author analyzed modal analysis of the model using free and forced vibration response. The dimensional effects such as the shaft length, output shaft angle effect, and bearing stiffness effects were also considered in modal analysis and compared to the benchmark model.

A simple finite element model was developed to investigate the dynamic behavior of a spur gear rotor system. Even though there were many work contributed by authors in the research area of gear train modeling and simulation by using FEA, their investigations are limited to a specific gear train application where the operation behavior of the gear train can be very different. In the rail vehicle drives system of electric traction similar phenomenon will occur and one can investigate the vibration excited from motor unbalanced rotation. Two or three gear units are modeled for the drives system and its analysis will be carried out using FEM, particularly ANSYS and Solid Works. During this analysis, deformation due to vibration will be investigated and compared to the acceptable limit.

2.4 Dynamic Effects of the Gear Train Analysis

When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present [3]. One of the earliest efforts to account for an increase in the load due to velocity employed to a number of gears of the same size, material, and strength were investigated. Several of these gears were tested to destruction by meshing and loading them at zero velocity. The remaining gears were tested to destruction at various pitch-line velocities. For example, if a pair of gears failed at 2270N tangential loads at zero velocity and at 1135N at velocity V_1 , then a velocity factor, designated K_v , of 2 was specified for the gears at velocity V_1 . Then another, identical pair of gears running at a pitch-line velocity V_1 could be assumed to have a load equal to twice the tangential or transmitted load. The definition of dynamic factor K_v has been altered. AGMA standards ANSI/AGMA 2001-D04 and 2101-D04 contain this caution: Dynamic factor K_v has been redefined as the reciprocal of that used in previous AGMA standards. It is now greater than 1.0. In earlier AGMA standards, it was less than 1.0. Due to this effect stress and deformation will occur on the gear trains. It was determined that dynamic load factors for gears that were heavily loaded based on one and two degree of freedom models [2]. Using a torsion vibratory model, they considered the torsion stiffness of the shaft.

An interactive method was developed to calculate directly for variable gear mesh stiffness as a function of transmitted load, gear profile errors, gear tooth deflections and gear hub torsion deformation, and position of contacting profile points. These methods are applicable to both normal and high contact ratio gearing. Certain types of simulated sinusoidal profile errors and pitting can cause interruptions of the normal gear mesh stiffness function, and thus, increase the dynamic loads. In the research, the gear mesh stiffness is the key element in the analysis of gear train dynamics. The gear mesh stiffness and the contact ratio are affected by many factors such as the transmitted loads, profile modifications, gear tooth deflections, and the position of contacting points. Moreover, gear strength, gear wear, prediction of gear dynamic loads, and gear noise are always a major concern in gear design. However, one topic that has not been discussed is the effect of non-ideal loading conditions on the stress and vibration behavior of the gear. Non-ideal conditions are characterized by a change in the tolerances which define the acceptable assembly of a gear train. Some studies have been conducted on how the noise or vibration of spur gears is affected by manufacturing error. In addition it was pointed about in the gear design the maximum stress is to be considered [18], [19].

CHAPTER 3

ANALYTICAL METHOD ANALYSIS OF THE GEAR TRAIN

3.1 Introduction

The method to determine the gear stress analytically has been developed extensively by the American Gear Manufacturers Association (AGMA), Lewis bending equation, Hertz contact stress equation, some mathematical models such as modal analysis, harmonic excitation, eigen vectors and eigen value. It is necessary to carry out the analytical method calculation to compare the result with that of the finite element method to make sure of the accuracy of the results of FEM.

Designing highly loaded spur gears for power transmission systems that are both strong and quiet requires analysis methods that can easily be implemented and also provide information on contact and bending stresses, along with transmission errors. The AGMA method is limited to the tooth gear analysis and many assumptions need to be taken into account. The Lewis bending equation calculates only the bending stress in the single tooth and accounts the gear as a beam element. The Hertzian equation is used to determine contact stress in the mating gear teeth. It considers the gears as the two rollers or cylinders contact at a single point and neglect the effects of friction. In addition some mathematical models for harmonic and modal shape analysis for the gear trains are also conducted. Harmonic response involves the frequency response due to the impulsive force in the system, whereas the modal analysis involves the modal shapes of the system which involves the natural frequency of the system in different kinds of mode shapes as a free vibration. The prime source of vibration in gear trains is the transmission error. Transmission error is a term used to describe or is defined as the differences between the theoretical and actual positions between a pinion (driving gear) and a driven gear. It has been recognized as a main source for mesh frequency excited noise and vibration. With prior knowledge of the operating conditions of the gear set, it is possible to design the gears such that the vibration and noise is minimized. The illustrated methods are all based on some assumptions and all pointed toward in calculation of stresses and determine at what stress level gear will fail.

3.2 Standard Specifications for the Rail Vehicles

The gear train in the rail vehicle is located between the wheels, motor and bogie and suspended to the bogie frame. The purpose of the gear unit is to transmit power available to the wheels through proper gear ratio, so that required speed and torque are achieved. The power source is from motor assembled to it according to the speed and the capacity of the rail vehicles. Different countries manufacture motors of maximum rotational speed up to 10000RPM. The majority of them are applicable to the range between 3000RPM to 5000RPM. Here in Ethiopia the speed of the passenger rail vehicle to be introduced is 75Km/hr- 180Km/hr, whereas the freight rail vehicle is 60km/hr-120Km/hr. Table 3.1 shows the brief summary of the legacy of the new rail way in Ethiopia [34].

Table 3-1 Speed and Axle load Range Value for Rail Vehicles in Ethiopia

No	Specification	Range value for the specification
1	Passenger Car Speed	75km/hr-180Km/hr
2	Freight Car speed	60Km/hr-120Km/hr

In this particular thesis the specific passenger car speed is to be considered. Note that the specified speed is the output speed for the vehicles. These speeds are obtained through proper gear ratio available according to standard specifications. The motor and the gear units are incased in the gear box and suspended to the bogie frame. Drive systems are used to transmit the torque from the traction motor to the wheel set via a one, two or more shift gearbox.

Different countries and companies are manufacturing a large number of motors for rail way electric traction system .The motors have different kinds of speed and torque range as well as gear ratio depending on the capacity and speed of the passenger and freight rail vehicles. Most of the time, the pinion is assembled to the motor shaft with proper gear ratio. Moreover, drives system in a rail way have to be powerful, environmentally friendly, very reliable, cost-effective and have a low need for maintenance. In the drive systems there are gear units or gear train which is supposed to transmit the torque to the wheel sets. The requirement for such maintenance is much more stringent than those required of cars, for example, because the intervals between servicing need to be much longer. According to statics from different kinds of Railway Company, their major service is done after the locomotive has logged

between 1 million and 1.5 million km [30]. These locomotives operate, on average, more than 1000km a day. Fig 3.1 shows a photo of some parts AC traction motor for train.

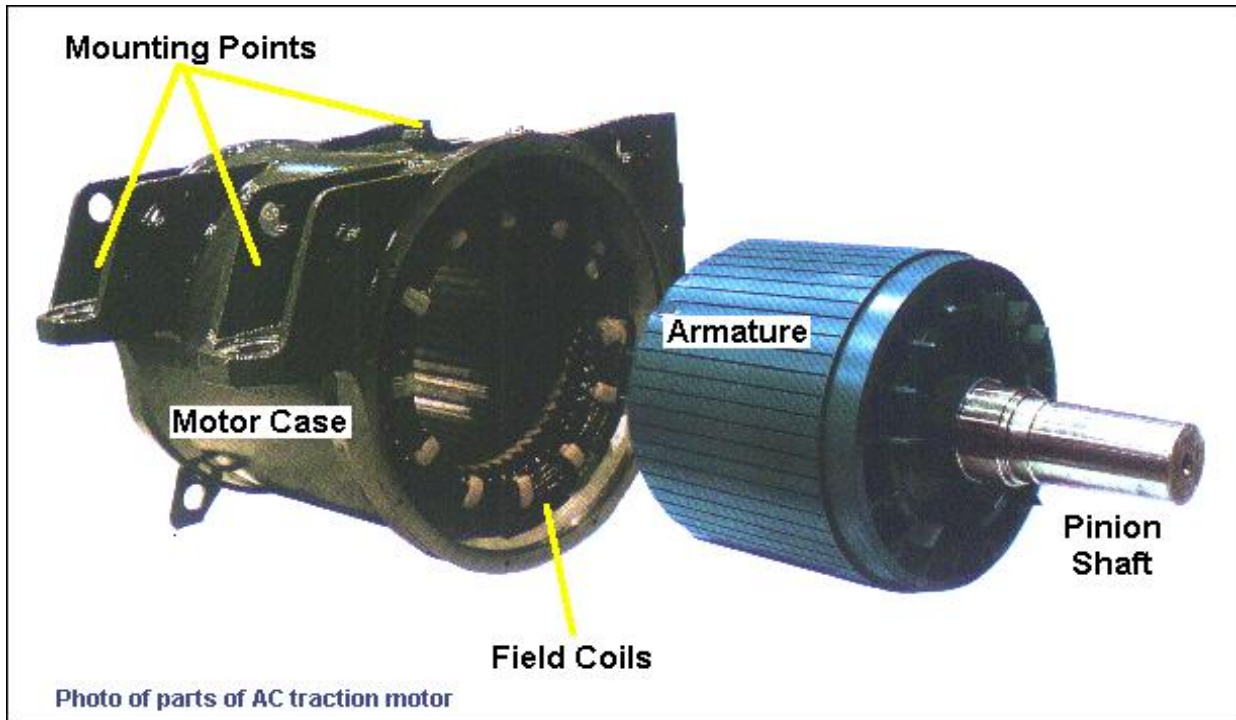


Fig 3-1 Some parts of AC traction motor [21]

In addition, the following table 3.2 could also be the basis for selecting the type of AC traction motor manufactured by different kinds of company at different times. The gear ratio is also provided along side with them. The table also provides a list of some specification of rail way traction motor provided by Siemens Company for different kinds of country to their rail ways organization [22].

Table 3-2 A list of AC Traction Motor for Trains [22]

Project	Country	Quantity	Motor Type	Gear ratio	Maximum Motor Speed (RPM)	Maximum Rail Vehicle Speed (Km/hr)	Working Since
GP-194/MRVC	India	900	1TB2022	5.71	3452	110	2007
AMTRAK F69	USA	10	1TB2626	3.59	3590	177	1988
MABI-RENFE	Spain	210	1TB2329	6.31	2950	90	1989-91
SD60-MAC	USA	2308	1TB2630	5.30	3210	113	Since1990
CP-SINTRA	Portugal	400	1TB2222	5.54	3970	120	1991-95
UTLIBERG-BAHN	Switzerland	36	1TB1921	7.40	4000	70	1991
SPOORNET	South Africa	40	1TB2820	6.31	3900	100	1993
SC80/90-MAC	USA	126	1TB2830	5.19	3435	128	1993
EMU500	Taiwan	692	1TB2021	4.36	3250	110	1994-97
SD70-MAC	USA	2580	1TB2630	5.30	3210	113	Since1994
NSB-DIB	Norway	90	1TB2325	4.94	3380	120	1995-99
BR152	Germany	680	1TB2822	6.30	4000	140	1996-2001
GTR6-MAC	India	361	1TB2622	5.30	3320	120	1997-99
DSB-LOK	Denmark	85	1TB2822	6.29	4000	140	1999-2000
GT46-PAC	India	42	1TB2622	5.30	3784	160	2000
FEVE-LOK	Spain	80	1TB2018	7.10	3300	80	2001-04

However, based on suspension arrangements and permissible speeds the manufactured motors are not providing with their full complete rotational speed. Most parts of the world are having motor speed less than the fabricated one. For example Indian Rail Way Company, GT46-PAC, Siemens provided a motor with a rotational speed of about 4500RPM but the motor is actually giving a service of about 3784RPM at the maximum speed of 160km/hr of

the rail vehicles. The following table 3.3 also provides the power rating of different kind of motor available and manufactured by rail way Company [23].

Table3-3 Rail Vehicles Speed and Power Range [23]

Project	Country	Rail vehicles speed	Power rating	Torque	Rotational speed(RPM)
TVG	France	570Km/hr	600Kw-900Kw	(continuous) 12,300Nm-	3435
TVG	France	130Km/hr	120Kw-200Kw	300Nm	3000
Vienna Metro type	Australia	160Km/hr	125Kw	350Nm	1230-3846
Low floor Tram ulf	Australia	120Km/hr	60Kw-80Kw	280Nm- 420Nm	4300
Tram Rome	Australia	140Km/hr	120Kw	300Nm	2285-4280
Tram City Runner Linz	Australia	130Km/hr	110Kw	300Nm	1680-5000
LRT Badner Bahn	Australia	140Km/hr	120Kw	350Nm	2369-5135
Tram Lodz refurbishment	Australia	130Km/hr	61Kw	420Nm	1634-4435
Motor Coach	Australia	140Km/hr	68Kw-80Kw	350Nm	1340-4520
Voith	Germany	140 Km/hr	130Kw	415Nm	3840
Regional rail car	China	80Km/hr	125Kw	433Nm	2750

In this thesis the last regional rail car is considered and at the present time operated in Ghana.

3.3 Operating Characteristics of Traction Motor for Trains

Traction motors are operating within the maximum torque-speed curve. Frequent operating points for electric vehicle on federal highway driving schedules (FHDS), electric vehicle on federal urban driving schedules (FUDES), high speed train(max. speed of 300 (km/h)) and low speed train(max. speed of 80 (km/h)) are shown in fig3. 2. As the traction motors run mostly in the high speed region for FHDS and high speed trains, it is clear that the performance in the high speed region should be mostly a concern. Fig3.3 also the frequent operating characteristics of the traction motor for the high speed train. Torque and speed ranges are also shown along side with them.

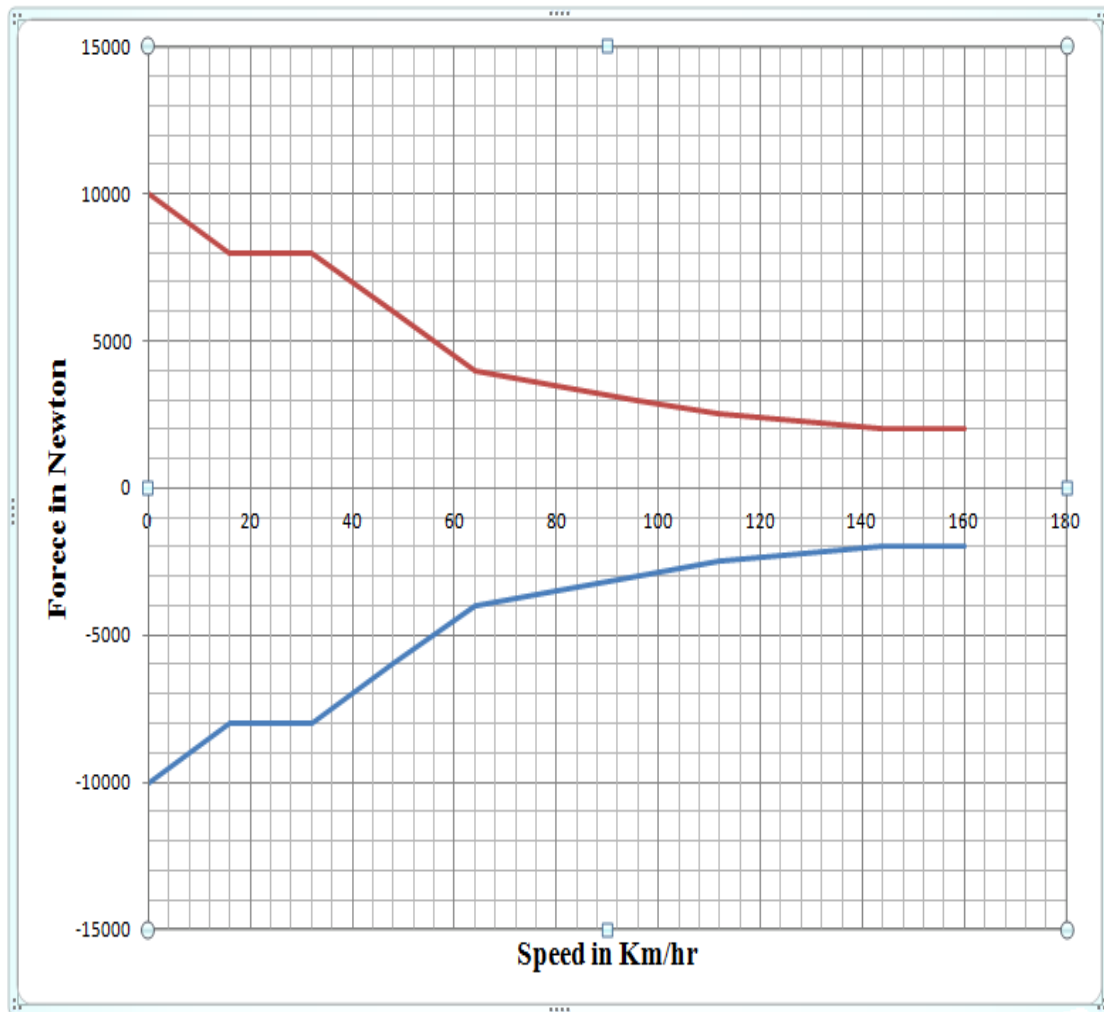


Fig 3-2 Frequent Operating Points of the Traction Motor for FHDS Electric Rail Vehicles

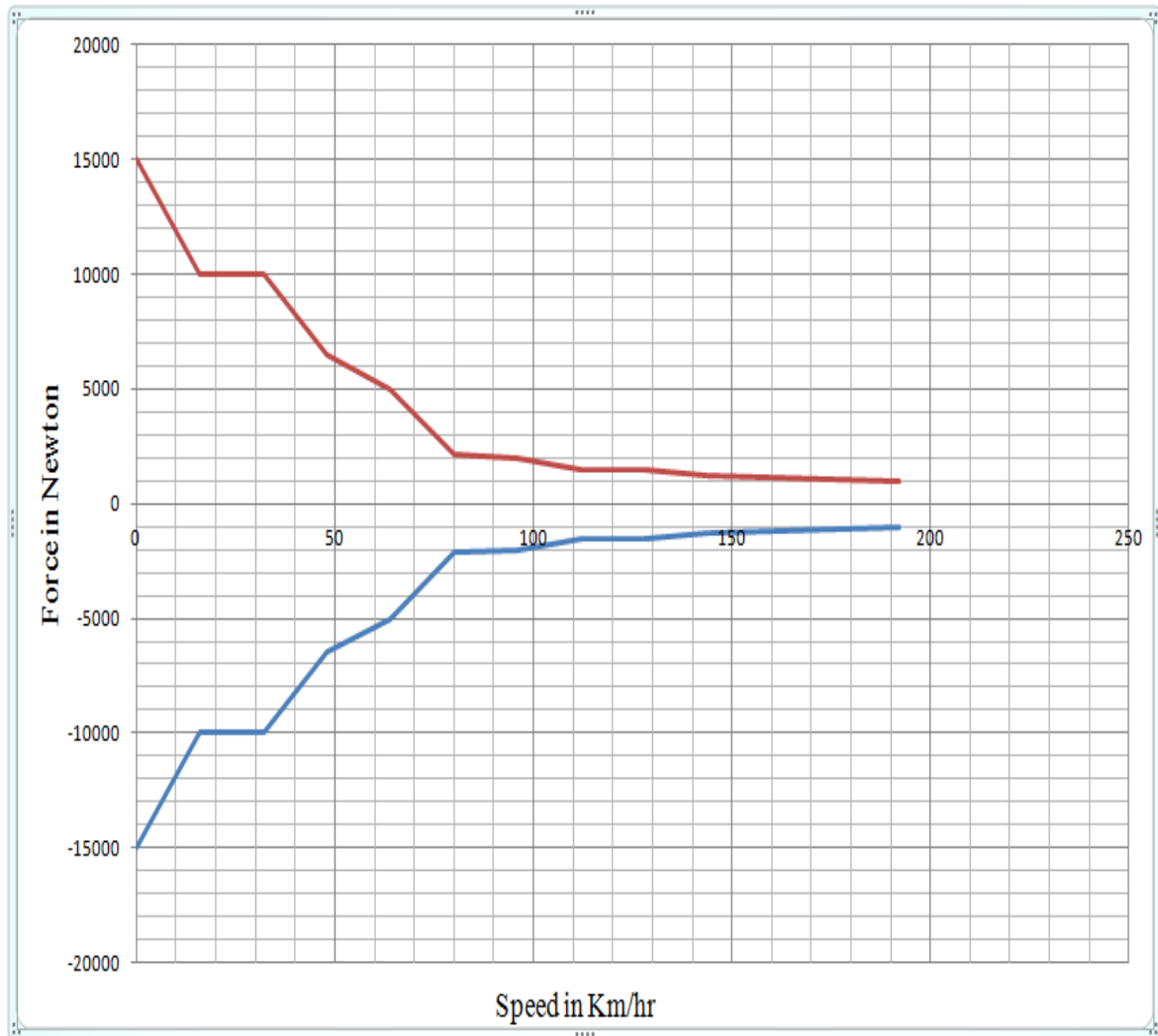


Fig 3-3 Operating Characteristics of Traction Motor for High Speed Train [24]

Generally, the rated power of electric motors for electric LRT is typically from 100 to 160 kW per axle and for rubber tire wheel trains from 70 to 110 kW per axle [31]. VEM is a company in Germany produces various types of traction motor with different kinds of power rating according to the following description shown in the table 3.4 [33]. In this thesis power rating of traction motor is 135Kw.

Table 3-4 Power Rating of Some Motors

Three-phase asynchronous drive motors (Monorails)	to 125Kw
Three-phase asynchronous drive motors (Trams)	to 130Kw
Three-phase asynchronous drive motors (Rapid transit railway)	to 250Kw
Three-phase asynchronous traction motors(Multiple units,electrical)	to 600Kw
Three-phase asynchronous traction motors (Railway locomotives)	to1,600kw

The gear ratio for the rail vehicle also depends on the stage of the gear units and power rating but different kinds of locomotives and cars use the gear ratios of 5:1, 4.5:1, 4:1, 8.775: 1, 5.023:1, 6.65, 4.091, 17:1, 84:17 3:1, 7.47 and so on depending on the torque requirements and related by the number of gear teeth calculated [33]. Here in Ethiopia the maximum speed is 120Km/hr for freight and 180Km/hr for passenger. Therefore, the gear ratio to be selected is 7.47:1.

Worldwide axle diameter for trains ranges from 120mm to 220mm for the bore diameter of the spur gear. Whereas for the pinion gear, the bore diameter ranges from 40mm to 175mm.Both pinion and spur gear possess the key way to prevent the relative movement. Proper hub is integrated to them too.

In addition, the following table 3.5 shows axle dimensions and its capacity ranges based on the speed of passenger rail cars and freight rail cars respectively.

Table 3-5 Axle Specification [25]

No	Size of journal (mm)	Capacity for axles for normal maximum Operating speed ranges (Kg)	
		Up to 136Km/hr	137Km/hr-160Km/hr
1	127x228.6	12,939	12,258
2	139.7x254	16,344	15,436
3	152.4x2794	20,430	19,295
4	165.1x304.8	24,516	23,154

In this thesis, axle diameter of 120mm is considered, whereas pinion shaft assembled to the motor varies from 40mm-60mm for the specified train. In addition the following data in the table are the overall parameters of the gear train for the specific electric traction train considered here in Ethiopia.

Table3-6 General Properties of Gear train Parameters

No	Component	Specification
1	Pinion1	0.2m
2	Pinion2	0.240m
2	Spur gear1	0.64m
3	Spur gear2	0.560m
4	Transmission ratio1	3.2:1
5	Transmission ratio2	2.33:1
6	Motor Speed	2750rev/min
7	Torque	433N-M
8	Tangential force on the gears	4.33KN
9	Radial force	1.580KN

Note that pinion1, spur gear1 and transmission ratio1 are in the case of gear train with idler gear, whereas pinion 2, spur gear 2 and transmission ratio 2 are in the case of gear train without idler gear. Note that number of teeth is determined in detail analysis. Module selection is based on the power transmitted and rotational speed of the motor. Once the speed of motor and power rating are known module can be selected from standard table [35]. The following graph shows how to select module for gears.

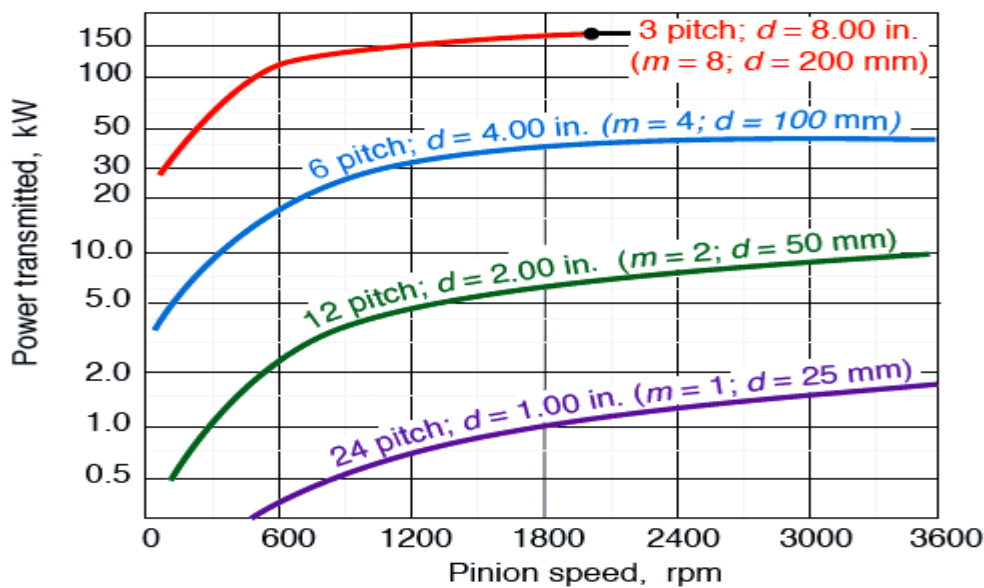


Fig 3-4 Transmitted power as a function of pinion speed [35].

3.4 General Properties of Spur Gear

The following figure shows nomenclature for spur gears:

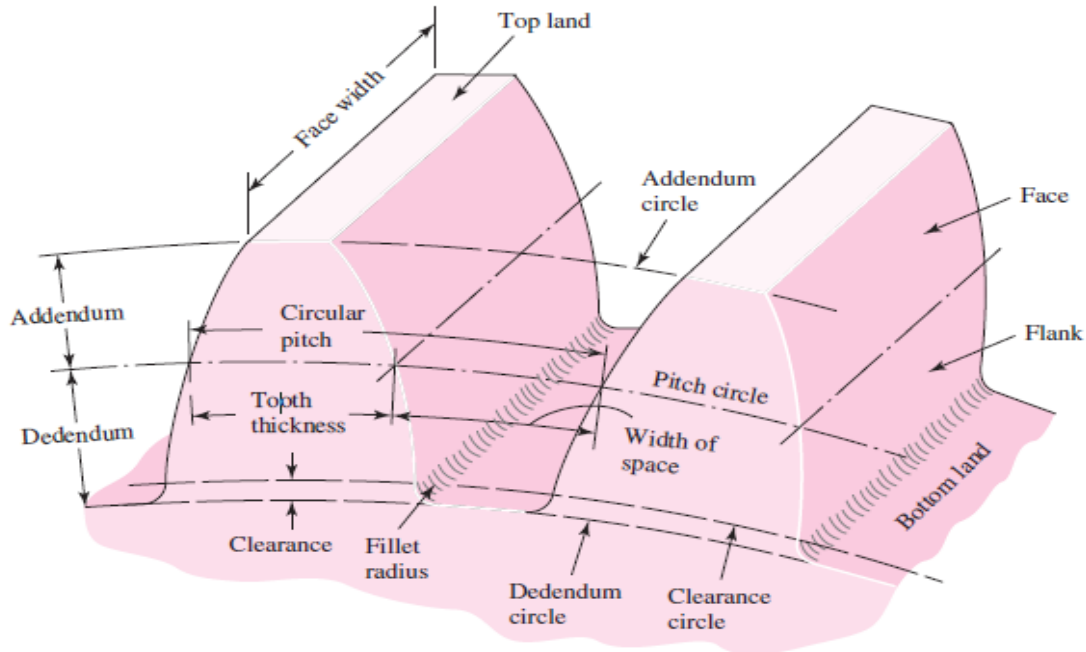


Fig3-5 Spur gear Nomenclature [3]

The expressions relating the nomenclature are as follows

$$P = \frac{N}{d},$$

$$m = \frac{d}{N},$$

$$P = p \times \frac{d}{N}$$

Where P =diametric pitch, teeth per mm, N =Number of teeth, d =pitch diameter in m =module in mm p =circular pitch. Using these relations, Pinion properties are

$$P = 20/200 = 0.1,$$

$$N = 200/8 = 25,$$

$p = 3.14 \times 200/25 = 25.12$ The gear ratio selected is 3.2:1 in the first case and by using the relationship of Pitch circle diameter and gear ratio:

$$d_{\text{spur}} = 3.2 \times d_{\text{pinion}} = 3.2 \times 200 = 640 \text{ mm}$$

Similarly for spur gears the number of teeth can be determined by using similar relationship above:

$$N = 640/8 = 80$$

$$p = 3.14 \times 640/80 = 25.12 \text{ mm.}$$

The following figure shows the properties of involutes curve which helps to construct the profile for the tooth for spur gears [3].

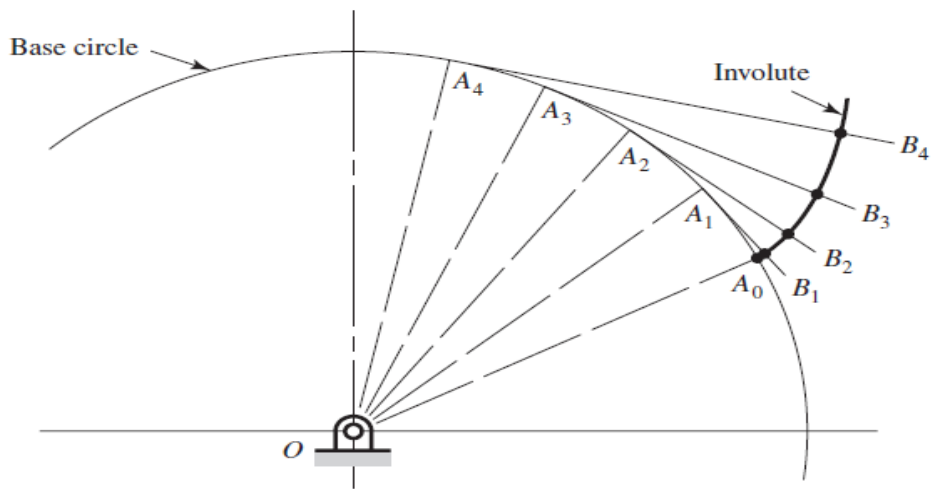


Fig3-6 Construction of involutes curve

The involutes curve helps us to construct a single tooth. The following figure shows a gear lay out and helps to construct the gearing system In addition base circle radius can be related to the pressure angle Φ and the pitch circle radius by:

$$r_b = r \cos \Phi \dots\dots\dots \text{Eq3.1}$$

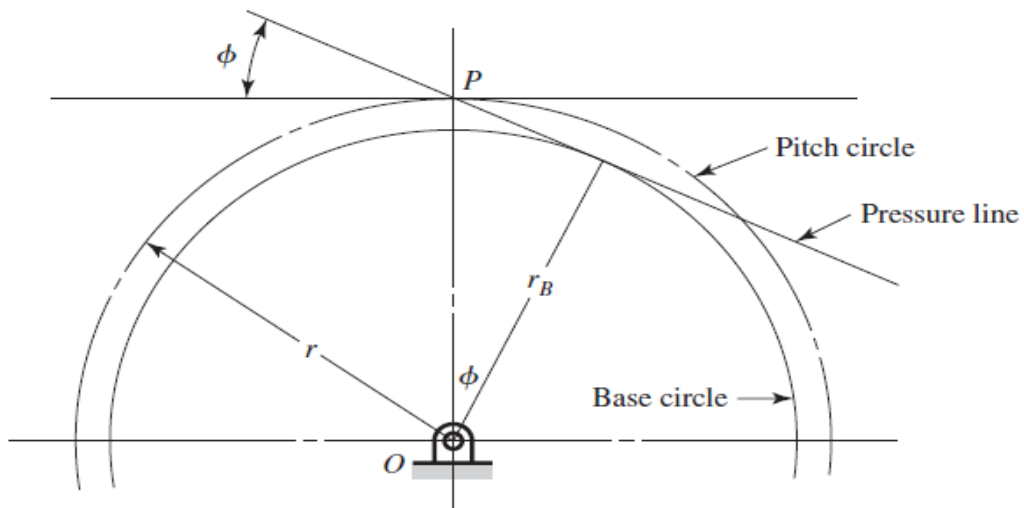


Fig3-7 Base Circle Construction

Depending on the above two figures, mating gears can be drawn by following step by step procedures as follows:

- Adjusting center distances of the mating teeth
- Outer diameters of both gears
- Base circle diameters of mating gears
- Pressure angle
- Face width
- Number of teeth for both gears
- Hub diameter and its length

Note that the hub length should not be less than

1.8×bore diameter of the given gear

$d_{hub}=1.8 \times 50=90\text{mm}$ $d_{hub}=\text{diameter of the hub of the gear.}$

$L_{hub}=1.25 \times d_{bore}=1.25 \times 50=62.5\text{mm}$

Note that all the mentioned values are minimum values.

3.5 Gear Bending Stress Analysis Using AGMA and Lewis bending Equation

3.5.1 AGMA Stress Equation

Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). In AGMA terminology, these are called stress numbers, as contrasted with actual applied stresses, and are designated by a lowercase letter *s* instead of the Greek lower case σ_b . The fundamental equations are:

$$\sigma_b = F_t \times K_o \times K_v \times K_s \times \left(\frac{1}{bm} \right) \times (K_H \times K_B) / Y_J \dots\dots \text{SI Units} \dots\dots \text{Eq3.2 [3]}$$

Where:-

F_t = the tangential transmitted load (N)

K_o =the overload factor

K_v =the dynamic factor

K_s =the size factor

P_d =the transverse diametral pitch

$F(b)$ =the face width

$K_m(K_H)$ =is the load distribution factor

K_B = is the rim thickness factor

$J(Y_J)$ =the geometric factor for bending strength

m =module in mm

σ_b =bending stress

Note that the factors are selected from the table available from Shigley Mechanical design based on the speed of the rotating gears and number of teeth required.

$K_v=1.25$ Shigley Mechanical design page 740

K_o =Overload factor for uniform power source, it is 1.25 .

$K_s=1.15$, Shigley mechanical design page 741

b =face width=0.100m,

Module (m) =0.008m,

$K_H=1 + C_{mc} (C_{pf} C_{pm} + C_{ma} C_e)=C_{mc}=1, C_{pf}=-.03, C_{ma}=0.247$

for commercially enclosed gear units,

$C_e=1, C_{pm}=0.8$ Shigley mechanical design page 742.

Upon substitution,

$$K_H=1+1 \times (-.3 \times .8 + .247 \times 1) = 1.01$$

$K_B=1.30$ from Shigley mechanical design hand page 746.

$Y_J=0.46$ for 25 toothed gear, Shigley Mechanical design page 736.

The necessary parameters can be calculated from the given data as follows:

Power (H) =135Kw,

$T=433N\cdot M$,

$P_d=0.2m$ for pinion

$N=25$, for pinion in the first case of the gear train.

Note that the rotational speed in all case is 2750RPM (288rad/sec).

Pressure angle(Φ) =20°

Torque= T acting on the pinion.

$$T = P_d/2 \times F_t = 0.2/2 \times F_t = F_t = 433 \times 10 = 4330N = \text{tangential force.}$$

To calculate the radial force or the normal force we use the relation

$$F_r = F_t \times \tan 20^\circ = 4330 \times \tan 20^\circ = 1576N.$$

The graphical depiction is shown below.

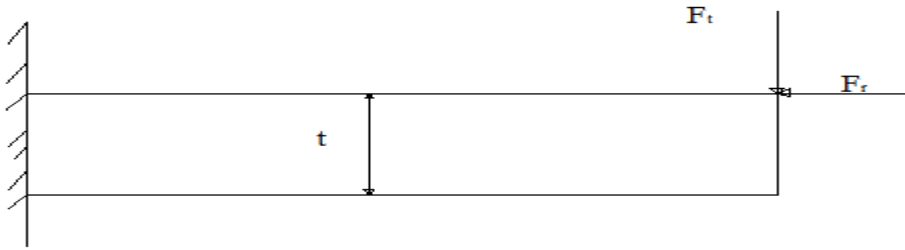


Fig3-8 Lewis Cantilever [3]

.Lewis considered the gear as a cantilever beam and both the tangential force and radial force are acting on the gear. In the first case of the gear train with the idler gear, the transmission ratio is 3.2:1.

Therefore, the speed of the pinion is reduced by half while increasing the torque available by double.

P_d of the spur gear is

$$P_d = 3.2 \times 0.2m = 0.64m = 640mm.$$

Number of teeth is

$$3.2 \times 25 = 80, \text{ where number of teeth for pinion is } 25.$$

The rest of the parameters are as calculated previously. In the second case of the gear train in the transmission system with the idler gear the minimum number of teeth for pinion is taken as 30 and its module is taken as $m=8mm$, therefore,

$$P_d \text{ (pitch circle diameter)} = 8 \times 30 = 240mm$$

The spur gear to mate in this case is obtained by using the relationship of the total gear ratio:

In the first case the gear ratio used is 3.2 and the total gear ratio selected is 7.47. By simple mathematical relationship: the gear ratio for the second gear train is

$$7.47/3.2 = 2.33$$

The number of teeth for the spur gear is:

$$2.33 \times 30 = 70$$

Note that power is constant throughout the transmission system. Therefore the module $m=8$. Pitch circle diameter of the final spur is

$$d_p = 8 \times 70 = 560mm \text{ where, } m = \text{module} = 8, \text{ and } N = 70 = \text{number of teeth.}$$

By using the AGMA (American Gear Manufacturing Association) for bending stress shown in the eq3.2 is helpful to calculate the bending stress or normal stress. In the first case of the gear train, the pinion the bending stress is as follows:

$$\sigma_b = F_t \times K_o \times K_v \times K_s \times 1/b \times m \times (K_H \times K_B) / Y_J = 4330 \times 1.25 \times 1.25 \times 1.15 \times (1/0.1 \times 0.008) \times (1.3 \times 1.01/0.46) = 27.16 \text{Mpa.}$$

In the second case of gear train , pinion with number of teeth of 30 and by using the AGMA bending equation:

$$K_v=1.25, \quad K_o=1.25, \quad K_s=1, \quad b=\text{face width}=0.100\text{m, module (m) }=0.008\text{m, } K_H= 1.01, \\ K_B=1.30 \quad Y_J=0.48 \text{ for 30 tooth gear.}$$

$$\sigma_b = F_t \times K_o \times K_v \times K_s \times 1/b \times m \times (K_H \times K_B) / Y_J = 4330 \times 1.25 \times 1.25 \times 1.15 \times (1/0.100 \times 0.008) \times (1.3 \times 1.01/0.48) = 26.88 \text{Mpa.}$$

In the case of gear train with idler gear, the idler gear with number of teeth of 40.

$$K_v=1.25, \quad K_o=1.25, \quad K_s=1.15, \quad K_H= 1.1, \quad K_B=1.30, \quad Y_J=0.49, \quad b=0.100\text{m, module (m) }=0.008\text{m}$$

The bending stress will be:

$$\sigma_b = F_t \times K_o \times K_v \times K_s \times (1/(b \times m)) \times (K_H \times K_B) / Y_J = 4330 \times 1.25 \times 1.25 \times 1.15 \times (1/0.100 \times 0.008) \times (1.3 \times 1.01/0.49) = 26.06 \text{Mpa.}$$

The spur gear bending stress the gear train with the idler gear. For Number of teeth=80.

$$K_v=0.8, \quad K_o=0.8, \quad K_s=1, \quad K_H=0.75, \quad K_B=0.8, \quad b=0.125\text{m } m=0.01, \quad Y_J=0.57$$

$$\sigma_b = F_t \times K_o \times K_v \times K_s \times 1/(b \times m) \times (K_H \times K_B) / Y_J = 4330 \times 0.8 \times 0.8 \times 1.15 \times 1/(0.100 \times 0.008) \times (0.8 \times 0.75)/(0.57) = 4.133 \text{Mpa.}$$

In the second case of the gear train the spur gear bending stress is for number of teeth=70.

$$\text{The only factor that is changed is } Y_J=0.57.$$

$$\sigma_b = F_t \times K_o \times K_v \times K_s \times 1/b \times m \times (K_H \times K_B) / Y_J = 4330 \times 0.8 \times 0.8 \times 1 \times 1/(0.125 \times 0.01) \times (0.8 \times 0.75)/(0.6) = 3.65 \text{Mpa}$$

3.5.2 The Lewis Bending Equation

The Lewis refers the loading on the gear as a cantilever beam. The equation for Lewis equation is given as: -

$$\sigma_b = F_t / (b \times P \times Y) \dots \dots \dots \text{Eq3.3 [3]}$$

Where F_t = tangential load,

$$F(b) = \text{Face width} = 0.100,$$

σ_b =bending stress. Y =Lewis factor taken from standard table of Shigley Mechanical design page 720. The pinion Lewis bending equation for the case of gear train with the idler gear (a-25-toothed gear), $Y=0.337$. By Substitution, the result of Lewis bending stress is as follows:

$$\sigma_b =F_t/ (b \times P \times Y) =4330/ (0.100 \times 0.0157 \times 0.337) =8.2\text{Mpa.}$$

For the second case of the gear train the pinion, Lewis bending equation for the case where (30-toothed gear), $Y=0.359$. Shigley Mechanical design page 720. By Substitution, the result of Lewis bending Stress is as follows:

$$\sigma_b =F_t/ (b \times P \times Y) =4330/ (0.100 \times 0.0157 \times 0.359) =7.68\text{Mpa}$$

The idler gear Lewis bending equation is :

$Y=0.390$ for 40-toothed gear from Shigley Mechanical design page 720.

$$\sigma_b =F_t/ (b \times P \times Y) =4330/(0.100 \times 0.0157 \times 0.390) =7.01\text{Mpa.}$$

The spur gear Lewis bending equation in the case of gear train without the idler gear.

For a-70-toothed gear, $Y=0.430$ by interpolation.

$$\sigma_b =F_t/ (b \times P \times Y) =4330/ (0.100 \times 0.0157 \times 0.430) =6.41\text{Mpa.}$$

The spur gear Lewis bending equation in the case of gear train with the idler gear is:

For 80-toothed gear, $Y=0.445$.

$$\sigma_b =F_t/ (F \times P \times Y) =4330/(0.100 \times 0.0157 \times 0.445) =6.13\text{Mpa.}$$

Both AGMA (American Gear Manufacturing Association) and the Lewis bending equation calculate the bending stress (Normal stress) on a single tooth once at a time. The other stresses are calculated through other equations. The Lewis bending equation neglects the effects for dynamic loading on the gear, the effects of stress concentration and the load is considered at the tip. Both the AGMA and Lewis bending equations neglect the effects of the radial force. Hence, they have limited the stress analysis of the gearing system. In similar manner the stress and deformation can be determined by using other analytical methods.

3. 6 Vibration Analysis of the Gear Train

All rotating machinery systems experience torsional oscillations to some degree during startup, shutdown, and continuous operation. Consequently, the torsional response characteristics of rotating component should be analyzed. In the gear train system, the axle and the shafts assembled to the drives system of the train rotates with high speed. Therefore, the system tends to oscillate from its equilibrium condition due to misalignments, poor damping condition and so on. The type of oscillation or vibrations result can be tilting,

twisting, out of plane misalignments, into plane misalignments and so on. In this section modal analysis and harmonic response analysis are to be considered. Severe torsional vibrations often occur with the only indication of a problem being gear noise or coupling wear. Excessive torsional vibrations can result in gear wear, gear tooth failures, key failures, shrink fit slippage, and broken axles in severe cases. The torsional vibration response of rotating machinery components is an important consideration in defining the operational reliability of a rotary equipment train. Accurate response prediction requires analysis techniques which consider all forcing functions in the system in addition to the mass-elastic properties of the shafts and gears. The severity of the torsional oscillations and stresses depends upon the relationship between the operating speed and excitation frequencies of unsteady torques and the torsional natural frequencies and mode shapes of the shaft system (critical speeds). The difference between these frequencies is referred to as the separation margin. The magnitude of the stress also depends upon the amplification factor on resonance (damping) and the stress concentration factors. In such a complex geometry of the gear train assembled to the motor and the axles experience such a high vibration [26]. Therefore, the FEM, which uses software and design modeler helps to determine the different kinds of deformation and mode shapes of the system. In this case only the frequency range, the fixed point and the support are provided from standard table.

3.6.1 Modal Analysis

Modal analysis is the study of the dynamic properties of structures under vibration excitation. If the system is poorly damped, the structures tend to vibrate or oscillate about its equilibrium position. For the gear train considered, the torsion effects of the axles and shafts are the main cause for the vibration. This torsional vibration is from unbalanced rotation, misalignments or so on. As a result of this kind of vibration the system particularly the gear train possesses different kinds of mode shapes as described above. In this modal analysis the gear trains vibrate freely due to natural structural frequency. The governing equation for free vibration with damping properties is given as follows:

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \dots\dots\dots \text{Eq3.4}$$

In this case, the gear train vibrates structurally without imposing any external force or energy upon it with sufficient damping properties for the gear box. The following figure shows the schematic drawing of the gear train in the second case possessing transmission ratio of 2.33:1

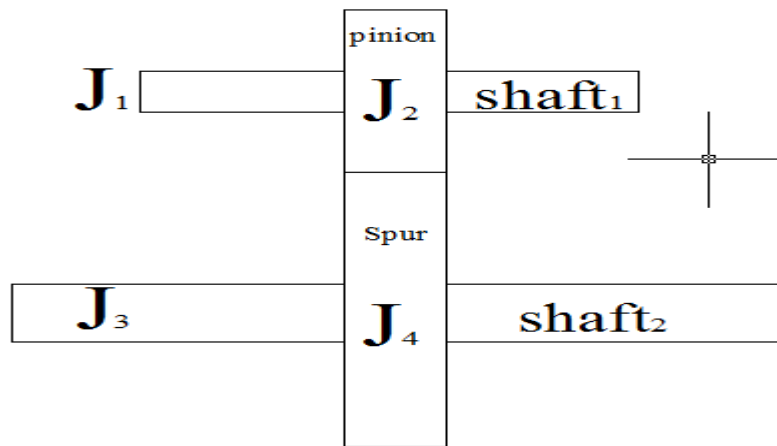


Fig 3-9 Second Stage Gear Train

J_1 = polar moment of inertia of the shaft assembled on the pinion, J_2 = polar moment of inertia of pinion, J_3 =polar moment of inertia of shaft assembled on the spur and J_4 =polar moment of inertia the spur. In addition polar moment of inertia and stiffness for the gear train in mate can be related by the following relationship [36]. Hence for the gear train the polar moment of inertia can be combined. Note that the speed ratio is in the second stage was 2.33:1. Therefore, in this case 1/2.33.

Polar moment of inertia can be calculated as follows: For pinion gear the known dimensions are bore diameter=0.06m, pitch circle diameter is 0.240m, material density=7850Kg/m³ and face width=0.1m. Therefore, polar moment of cylindrical body with inner hole is given as follows [37] :

$$J = \frac{\rho \times \pi \times b \times (D_o^4 - D_i^4)}{32} \dots \dots \dots \text{Eq3.5}$$

Where ρ = density of pinion material, b=face width of gears, D_o =pitch circle diameter of pinion and D_i = bore diameter of pinion. By using the relationship: the polar moment of inertia can be calculated as:

$$J_2 = \frac{7850 \times 3.14 \times 0.1 \times (0.24^4 - 0.06^4)}{32} = 0.254 \text{Kgm}^2$$

The parameters that are changed for the spur gears are as follows: D_i =0.120m and D_o =0.560m

$$J_4 = \frac{7850 \times 3.14 \times 0.1 \times (0.56^4 - 0.12^4)}{32} = 7.56 \text{Kgm}^2$$

Therefore, $J_G = 0.254 + 7.56 / (2.33)^2 = 1.6457 \text{Kgm}^2$. In addition the torsional stiffness of the shafts can be calculated as follows:

$$K_T = \frac{I_p \times G}{L} \dots\dots\dots \mathbf{Eq\ 3.6\ [36]}$$

Where K_T = torsional stiffness of the shafts, G =shear modulus of elasticity and L = length of the shafts, I_p =the area polar moment of inertia. D_o =outer diameter of shaft D_i = Inner diameter of shaft [36].

$$I_p = \pi \left(\frac{D_o^4 - D_i^4}{64} \right) \dots\dots\dots \mathbf{Eq\ 3.7}$$

Therefore, $I_p = \frac{\pi(0.05^4 - 0.01^4)}{64} = 3.06 \times 10^{-7}$

Shear modulus of elasticity and young modulus is related by:

$$G = \frac{E}{2(1+\nu)} \dots\dots\dots \mathbf{Eq3.8}$$

Where, ν = poisson ratio G =shear modulus E =Young modulus

$\nu = 0.3$, $E=2.1 \times 10^{11}$ and length of the shaft assembled to the pinion is 0.5m. Hence,

$$G = \frac{2.1 \times 10^{11}}{2(1+0.3)} = 8.1 \times 10^{10}$$

Therefore, K_T of the first shaft is calculated as

$$K_{T1} = \frac{3.06 \times 10^{-7} \times 8.1 \times 10^{10}}{0.5} = 49572 \text{N/m}$$

Again for the second shaft the torsional stiffness is calculated by similar procedures described above

$$K_{T2} = \frac{1.05 \times 10^{-5} \times 8.1 \times 10^{10}}{2} = 425250 \text{N/m}$$

Therefore, the total torsional stiffness is calculated as follows [37]:

$$K_T = \frac{K_{T1} K_{T2}}{n^2 K_{T1} + K_{T2}} \dots\dots\dots \mathbf{Eq3.9}$$

Where $n=D/d$ D =pitch circle diameter of spur gear and d = pitch circle diameter of pinion.

Hence $n=0.560/0.240=2.33$

$$K_T = \frac{49572 \times 425250}{2.33^2 \times 49572 + 425250} = 30359 \text{N/m}$$

By adding the polar moments of inertia for the two gears by using the relationship [36]:

$$J_T = J_2 + \frac{J_4}{\text{Speed ratio}^2} \dots\dots\dots \mathbf{Eq3.10}$$

Where J_T = total polar moment of inertia, J_2 = polar moment of inertia of pinion, J_4 = polar moment of inertia of spur. Therefore, by substituting the above results in the relationship:

$$J_T = 0.254 + \frac{7.56}{2.33^2} = 1.6547 \text{ Kg m}^2$$

Torsional stiffness of the gears can be calculated by similar procedure as for the shafts as follows: Where I_{pp} = polar moment of inertia of pinion I_{ps} = polar moment of area of spur.

$$I_{pp} = \pi \left(\frac{D_0^4 - D_i^4}{64} \right) = 3.14 \times \frac{0.24^4 - 0.060^4}{64} = 1.62 \times 10^{-4}$$

$$I_{ps} = 3.14 \times \frac{0.56^4 - 0.12^4}{64} = 4.815 \times 10^{-3}$$

Hence we can calculate the torsional stiffness of both gears by the following relationships as follows: Where K_{TP} = torsional stiffness for pinion and K_{TS} = torsional stiffness for spur gear.

$$K_{TP} = \frac{I_{pp} \times G}{L}$$

Where L = face width of gear G = shear modulus of elasticity. $G = 8.1 \times 10^{10}$, $I_{pp} = 1.62 \times 10^{-4}$

$$K_{TP} = \frac{1.62 \times 10^{-4} \times 8.1 \times 10^{10}}{0.1} = 131.2 \times 10^7$$

Similarly for spur gear

$$K_{TS} = \frac{4.815 \times 10^{-3} \times 8.1 \times 10^{10}}{0.1} = 390 \times 10^7$$

The total torsional stiffness of gears can be calculated as follows;

$$K_T = \frac{131.2 \times 10^7 \times 390 \times 10^7}{2.33^2 \times 131.2 \times 10^7 + 390 \times 10^7} = 46.44 \times 10^7 \text{ N/m}$$

To calculate the polar moment of inertia of shafts the following can be used for calculation:

$$J = \frac{\rho \times \pi \times b \times (D_0^4 - D_i^4)}{32} \quad \text{this is from equation 3.5}$$

For the first shaft: b = length of the first shaft = 0.5 meters

$$J_1 = \frac{7850 \times \pi \times 0.5 \times (0.05^4 - 0.01^4)}{32} = 2.4 \times 10^{-3} \text{ Kg m}^2$$

Similarly for the second shaft:

$$J_3 = \frac{7850 \times \pi \times 0.2 \times (0.12^4 - 0.025^4)}{32} = 0.032 \text{Kgm}^2$$

Therefore, the total polar moment of inertia of shafts can be calculated as:

$$J_T = J_1 + \frac{J_3}{\text{Speed ratio}^2} = 2.4 \times 10^{-3} + \frac{0.032}{2.33^2} = 6.234 \times 10^{-3} \text{Kgm}^2$$

Since both the gears and shafts are series the parameters are combined as follows

$$\frac{1}{K} = \frac{1}{K_{T1}} + \frac{1}{K_{T2}} \dots \dots \dots \text{Eq 3.11 [36]}$$

Where K_{T1} is the total sum of torsional stiffness for gears and K_{T2} is the torsional stiffness of the shafts. Hence: the governing total stiffness can be calculated as:

$$\frac{1}{K} = \frac{1}{46.44 \times 10^7} + \frac{1}{30359} = 3.23 \times 10^{-5} = 30357 \text{N/m}$$

The first natural frequency of the system can be calculated by using the following relationship [37]:

$$\omega_n = \frac{1}{2\pi} \sqrt{\frac{K(J_1+J_2)}{J_1J_2}} \text{ Hz} \dots \dots \dots \text{Eq3.12}$$

Where ω_n = the first natural frequency. Hence by substituting the above parameters:

$$\omega_n = \frac{1}{2\pi} \left(\sqrt{\frac{30357 \times (6.234 \times 10^{-3} + 1.6547)}{6.234 \times 10^{-3} \times 1.6547}} \right) = 352.05 \text{Hz}$$

The second natural frequency is given by:

$$\omega_n = \frac{1}{2\pi} \left(\sqrt{\frac{3 \times (30357) \times (6.234 \times 10^{-3} + 1.6547)}{6.234 \times 10^{-3} \times 1.6547}} \right) = 603.88 \text{Hz}$$

Note that the second natural frequency is given by:

$$\omega_n = \frac{1}{2\pi} \left(\sqrt{\frac{3k(J_1+J_2)}{J_1J_2}} \right) \dots \dots \dots \text{Eq 3.13}$$

The above analysis is said to be the modal characteristics under the natural structural vibration of the system without imposing any external energy.

3.6.2 Harmonic Response

Harmonic excitation refers to a sinusoidal external energy of a certain frequency applied to a system. The response of a system to harmonic excitation is a very important topic because it is encountered very commonly and also covers the concept of resonance. Resonance occurs when the external excitation has the same frequency as the natural frequency of the system. It

leads to large displacements and can cause a system to exceed its elastic range and fail structurally. Harmonic excitation is also commonly observed in systems that contain rotating masses such as gear train tires, engines, rotors, etc. This mode of vibration can be controlled by using proper damper. Particularly in this gear train structure the applied moment can be unbalanced external energy which might cause harmonic excitation and results in unbalanced rotation. When the motor rotates in unbalanced manner, the applied moment can also be unbalanced and results in harmonic response. Therefore, for damped forced harmonically excited system response equations are illustrated in the subsequent sections [38]:

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos\omega t \dots\dots\dots \mathbf{Eq3.14}$$

Where F_0 is initial applied tangential force of the gear train under consideration, ω is the frequency of the system and t is the response time. The particular solutions of the above equation are expected to be harmonic and arranged as follows:

$$x_p(t) = X\cos(\omega t - \varphi) \dots\dots\dots \mathbf{Eq 3.15}$$

Where X and φ are constants to be determined and denote the amplitude and phase angle of the response respectively. By substituting the Eq3.15 into Eq 3.14 we arrive at :

$$X[(k - m\omega^2) \cos(\omega t - \varphi) - c\omega \sin(\omega t - \varphi)] = F_0 \cos\omega t \dots\dots\dots \mathbf{Eq3.16}$$

Using the trigonometric relationship illustrated below

$$\cos(\omega t - \varphi) = \cos\omega t \cos\varphi + \sin\omega t \sin\varphi \dots\dots\dots \mathbf{Eq3.17}$$

$$\sin(\omega t - \varphi) = \sin\omega t \cos\varphi - \cos\omega t \sin\varphi \dots\dots\dots \mathbf{Eq3.18}$$

Using **Eq3.16** and equating the coefficients of $\cos\omega t$ and $\sin\omega t$ on both sides

$$X [(k - m\omega^2)\cos\varphi + c\omega \sin\varphi] = F_0 \dots\dots\dots \mathbf{Eq3.19}$$

$$X [(k - m\omega^2)\sin\varphi - c\omega \cos\varphi] = 0 \dots\dots\dots \mathbf{Eq3.20}$$

Solution of **Eq3.20** gives:

$$X = \frac{F_0}{[(K - m\omega^2)^2 + c^2\omega^2]^{1/2}} \dots\dots\dots \mathbf{Eq3.21}$$

$$\varphi = \tan^{-1}\left(\frac{c\omega}{k-m\omega^2}\right) \dots\dots\dots \text{Eq3.22}$$

Various relationships can be generated by using the above equations

$$\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{mk}} \dots\dots\dots \text{Eq3.23}$$

Where, ζ =constant damping ratio, c =damping constant c_c =critical damping ω_n =undamped natural frequency which is $\omega_n = \sqrt{\frac{k}{m}}$ by rearranging all the above equations the following relationship can be generated.

$$\varphi = \tan^{-1}\left\{\frac{2\zeta \times \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right\} \dots\dots\dots \text{Eq3.24}$$

For this particular gear train arrangements and using the defined parameters for the total mass of the system under consideration $m=256.63\text{Kg}$ from ANSYS work bench results. For further clarification it has been shown in the appendix. $\zeta = 0.6$ constant damping ratio assumption. It is also the most frequently used in many systems. ω_n be the first natural frequency of the system and calculated as above . For $\zeta=0.6$ the ratio for ω/ω_n can be estimated from the following graph as follows:

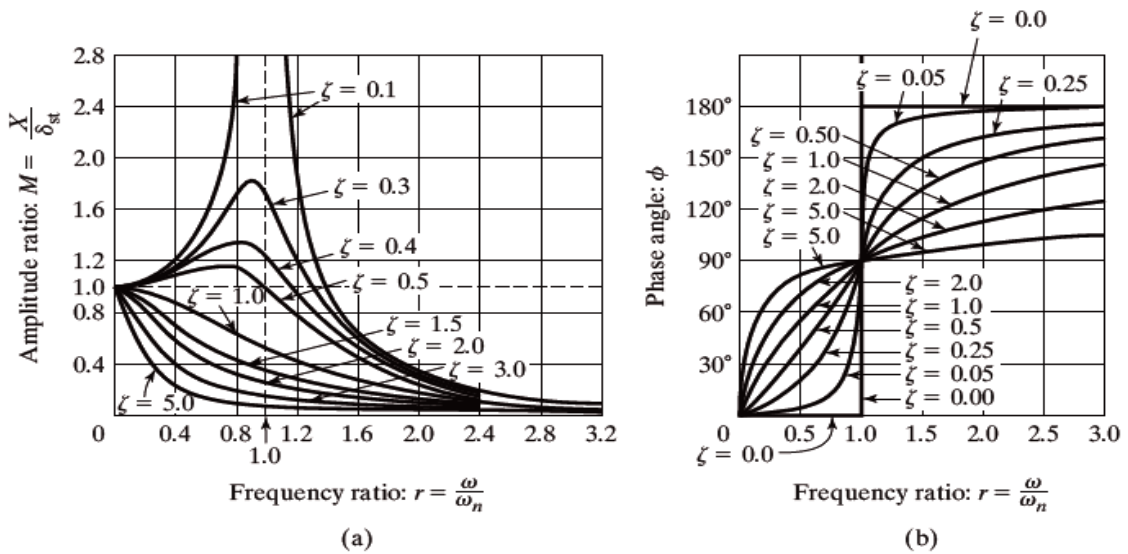


Fig 3-10 Variation of X and φ with frequency ratio $r=\omega/\omega_n$ [38].

Then from this graph for $\zeta=0.6$, ω/ω_n can be estimated as **0.45**. By substituting all the values in the equations, φ can be determined as follows:

$$\varphi = \tan^{-1}\left\{\frac{(2 \times 0.6 \times 0.45)}{1 - 0.45^2}\right\} = 0.617 = 34^\circ$$

is the phase angle response of the harmonic excitation. The phase angle states that the system response lags behind by sometime time later as soon as applied initial unbalanced harmonic force is applied. Hence the amplitude can be determined by using the following equation. For $F_0=4330\text{N}$, $K=30357\text{N/m}$ and ω can be determined as follows. Since the first natural frequency is already calculated in the previous section as $\omega_n=f_n=352.05\text{Hz}$. By using the frequency ratio **$r=0.45$** . The equation for frequency ratio is give by:

$$r = \frac{\omega}{\omega_n} \dots\dots\dots \text{Eq3.25}$$

Then $\omega = 0.45 \times 352.05 = 158.225\text{Hz}$ or 16.56rad/sec The amplitude of the system is

$$X = \frac{F_0}{[(K - m\omega^2)^2 + c^2\omega^2]^{1/2}} \quad \text{is from equation}$$

By using the relation stated in the following to obtain the value of c:

$$\frac{c}{m} = 2 \times \zeta \times \omega_n \dots\dots\dots \text{Eq3.26}$$

$m=256.63\text{Kg}$, $\zeta=0.6$, $\omega_n=16.56\text{rad/sec}$. Hence the value of c can be calculated as:

$$c = 2 \times 0.6 \times 16.56 \times 256.63 = 5099.75\text{Ns/m}$$

Therefore, the amplitude of the system can be determined:

$$X = \frac{4330}{[(30357 - 256.63 \times 16.56^2)^2 + 5099.75^2 \times 16.56^2]^{1/2}} = 0.04633\text{m}$$

Note that determining the amount of amplitude helps us to make decision on the vibration status of the system and then to adjust the damping components of the system. The phase angle and amplitude determination helps to adjust proper gear box for the drives system and it has a significant effect to control forced excitation response in the system. It helps to construct qualitative and quantitative physical interpretation. For example, the ratio of the amplitude of forced response to amplitude excitation has some physical phenomenon to make

decision on force developed in mass spring system together with damper. Moreover, it helps to design proper gearbox which can withstand severe vibration condition. Moreover maximizing the damping properties of the system keeps down the magnitude of amplitude

3.7 Contact Stress Analysis

The transfer of power between gears takes place at the contact between the acting teeth. The stresses at the contact point are computed by means of the theory of Hertz's. The theory provides mathematical expressions of stresses and deformations of curved bodies in contact. Fig below shows a model applied to the gear-two parallel cylinders in contact.

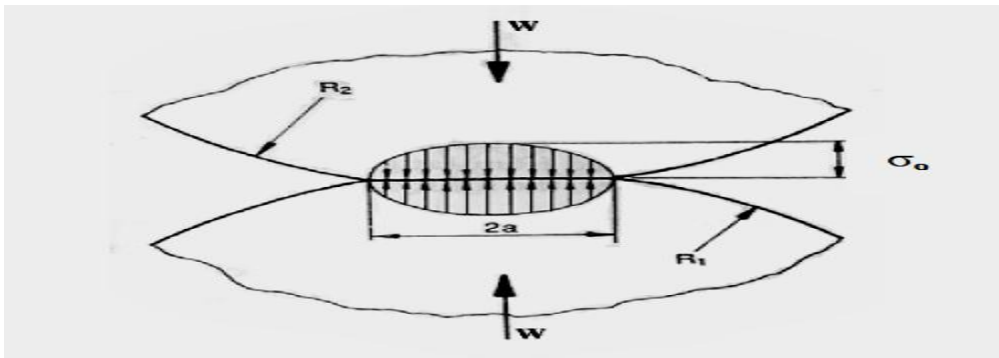


Fig 3-11 The Hertz Contact [27]

Hertz theory assumes an elliptic stress distribution, as seen in the fig above; the maximum stress is in the middle and equals:

- Where: -
- W=applied load
 - R₁ =radius of body 1
 - R₂= radius of body 2
 - a= major diameter of the elliptical shape

The equation for the contact stress is given as follows:

$$\sigma_o = \sqrt{W \left(\frac{1}{R_1} + \frac{1}{R_2} \right) / b \pi \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right]} \dots\dots\dots \text{Eq3.25 [27]}$$

Where W is the load, E₁ and E₂ are the modulus of elasticity of pinion and gear respectively, ν₁ and ν₂ are the poisson's ratios of pinion and gear respectively and b is the face width of pinion. The same equation can be applied for gears, assuming for R₁ and R₂ the respective radii of the involutes curve at the contact point. The contact stress calculation takes place for

the second case of the gear train. Hence to calculate the maximum contact stress by using the relation:

$$F_r = F_t \times \tan 20^\circ = 4330 \times \tan 20^\circ = 1576 \text{ N}$$

$W_t = 1576 \text{ N}$ = Normal force at the instant of contact.

$$E_1 = E_2 = 2.1 \times 10^{11}, \nu_1 = \nu_2 = 0.3. R_1 = 0.240 \text{ m}, R_2 = 0.560 \text{ m}, \text{Face width (b)} = F = 0.100 \text{ m}$$

σ_o = contact stress. By simple substitution in the equation,

$$\sigma_o = \sqrt{1576(4.17 + 1.79) / (0.100 \times 3.14) \times \left[\frac{0.91 \times 2}{2.1 \times 10^{11}} \right]} = 0.455 \text{ Mpa}$$

CHAPTER 4

FINITE ELEMENT METHOD PROCEDURE

4.1 Finite Element Analysis (FEA) for the Gear Units

Analysis by finite element method is carried out for the specified gear units each separately and in the gear trains with two or three gears are in mating for particular passenger rail vehicles which possess the following characteristics. The specifications are in the table 4.1 as shown below. Note that the following parameters are the selected and already designed structure for the particular rail vehicles to be analyzed by ANSYS soft ware. The geometry for the gears and gear box are modeled in both solid works and ANSYS work bench. In addition the rest of the analysis is extensively analyzed in ANSYS.

Table 4-1 Specifications of gear units

No	Parameters	Physical quantity
1	Pinion gear speed	2750rpm
2	Transmission ratio	3.2:1 in case of two gear mesh
3	Pitch circle diameter of pinion	200mm
4	Pitch circle diameter of spur	640mm
5	Pitch circle diameter of idler	320mm
5	Material for all types of gears	Structural steel
6	Number of teeth for pinion	25
7	Number of teeth for spur	80
8	Number of teeth for idler	40

The procedure for developing the model for the gear units are generally shown in the following fig 4.1. The steps show how to determine the stress and deformation on the modeled gear. In modeling of the gear unit, the gears required for the systems are modeled based on the design parameters. The material properties and gear parts assembly are also assigned in this stage. This is followed by the FEA pre-processing stage where the mesh element, the load, and constraint are set on the gear train model. In order to determine the stresses accurately, different mesh element settings and FEA solver are required in separate setup. In the FEA post-processing stage, the FEA solution for the gear unit model is completed. The maximum Vonmises stress is obtained from the contour plot of the gear train.

Thus, the bending and contact stress analysis can be performed by correlating the change in applied load and gear parameters.

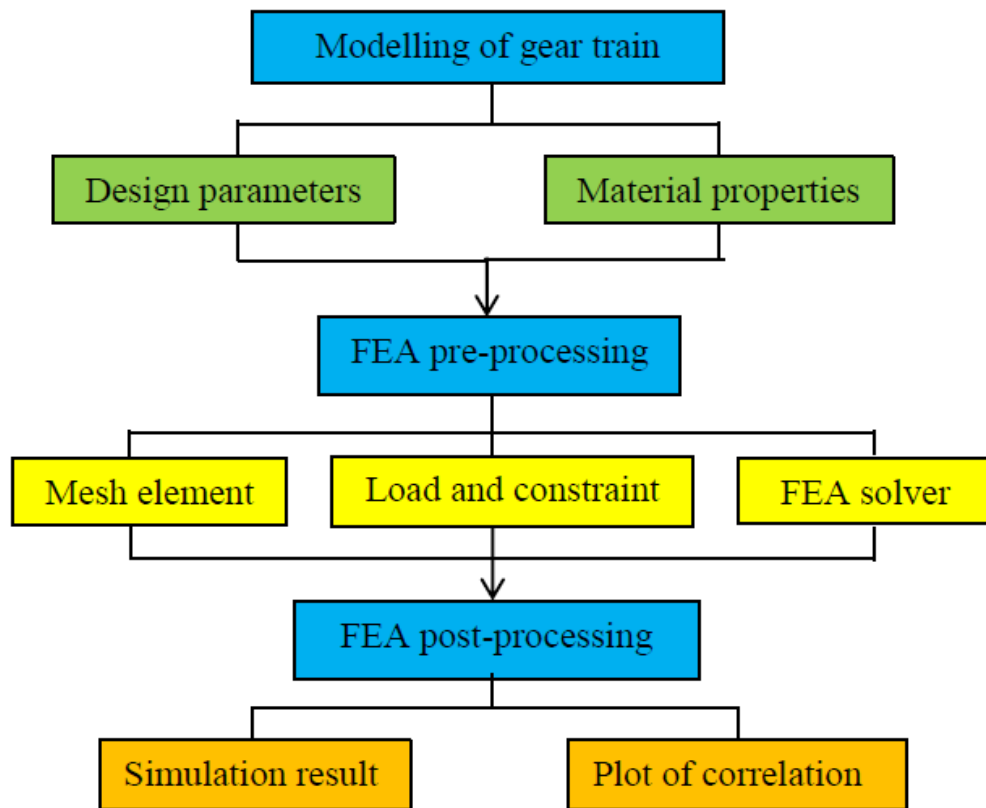


Fig 4-1 Finite Element Procedure for obtaining Gear stresses and deformations [2]

In the FEA, when each process is executed for the solution, the pre-processing for each step is selected according to the requirement of the solution phase. In order to determine the equivalent Vonmises stress and deformation, static structural is selected. In addition to analyze the vibration behavior of the structure, modal (ANSYS) or harmonic response analysis is selected based on the type of vibration. Moreover, contact stress and fatigue stress are found in static structural analysis system. Note that static structural is considered as a pre-stressed analysis for modal (ANSYS).

4.2 Force Analysis of the Gears

The type of gears considered here is the spur gear. This gears posses' efficiency of 96%-98%. The following are the typical force analysis of the rail vehicles. Transmitted load is calculated as

$$F_t = 2000T/d_p \dots\dots\dots\mathbf{Eq4.1}$$

Where T =Torque, d_p = pitch circle diameter of the gears. The following equation can also be alternative equation to evaluate the transmitted load:

$$F_t = 60000H/\pi n d_p \dots\dots\dots\mathbf{Eq4.2}$$

Where H=power (KW), d_p =pitch circle diameter in mm, and n=speed in revolution per minute (rev/min), F_t = transmitted load in kN and H = power (kW), Note that the equation is applicable for spur gears. In addition, from standard data the speed of the motor is taken as a maximum for $n=3000$ rpm and $p=200$ mm for pinion $H=130$ Kw. These data are for passenger rail vehicles possessing the maximum speed of 120Km/hr, here to operate in Ethiopia in the future. By substituting and rearranging the tangential force can be calculated as:

$$F_t = 60000 \times 125 / (3.14135 \times 2750 \times 200) = 4330\text{N} = 4.33\text{KN}$$

Therefore, the transmitted force is 4.33KN. Similarly by using the tangential force value, the radial force can be calculated. Hence:

$$F_r = F_t \tan \Phi \dots\dots\dots\mathbf{Eq4.3}$$

Where Φ = the pressure angle of the gear = 20° . Therefore, the radial force will be

$$F_r = 4330 \times \tan 20^\circ = 1576\text{N}$$

The moment reaction for the pinion is already selected from standard. Therefore, Moment (M)

$$M = 433\text{N-m}$$

Therefore, power (H) can be calculated as:

$$H = M \times \omega = 433 \times 288 = 125\text{kw}$$

Where M=moment and ω =rotational velocity in rad/sec. The gear ratio selected in the first stage is 3.2:1 for compound stage double reduction and in the second stage 2.33:1. Therefore, the total gear ratio is 7.47. Speed output for the spur gear is about 859rev/min because compound stage is to be used. Using the above formula and the following parameters can be input for the ANSYS.

$$F_t = 4330\text{N}$$

$$F_r = 1576\text{N}$$

$$\text{Moment} = M = 433\text{N-M}$$

$$\text{Speed of the pinion} = 2750\text{rev/min}$$

$$\text{Power} = 125\text{Kw}$$

Transmission ratio=3.2:1 in the first case:

Pinion diameter = 200mm

Spur gear diameter = 640mm

Pinion shaft diameter =50mm

Spur gear bore diameter =120mm in the without idler case

Number of teeth for pinion is 25

Number of teeth for spur is 80

Face width for gears =125mm will be shown by expression.

Module=8mm in all case.

The module is the capacity for gear units to transfer a given amount of power. For example to transfer 1.2Kw; at least 1mm of module is necessary depending on the speed of the system. The module selected in this case is based on the power rating and rotational velocity of motor. After the power rating and rotational velocity are known, the pinion dimension and module can be selected from table.

Table 4-2Material Properties and Design parameters of the test gears

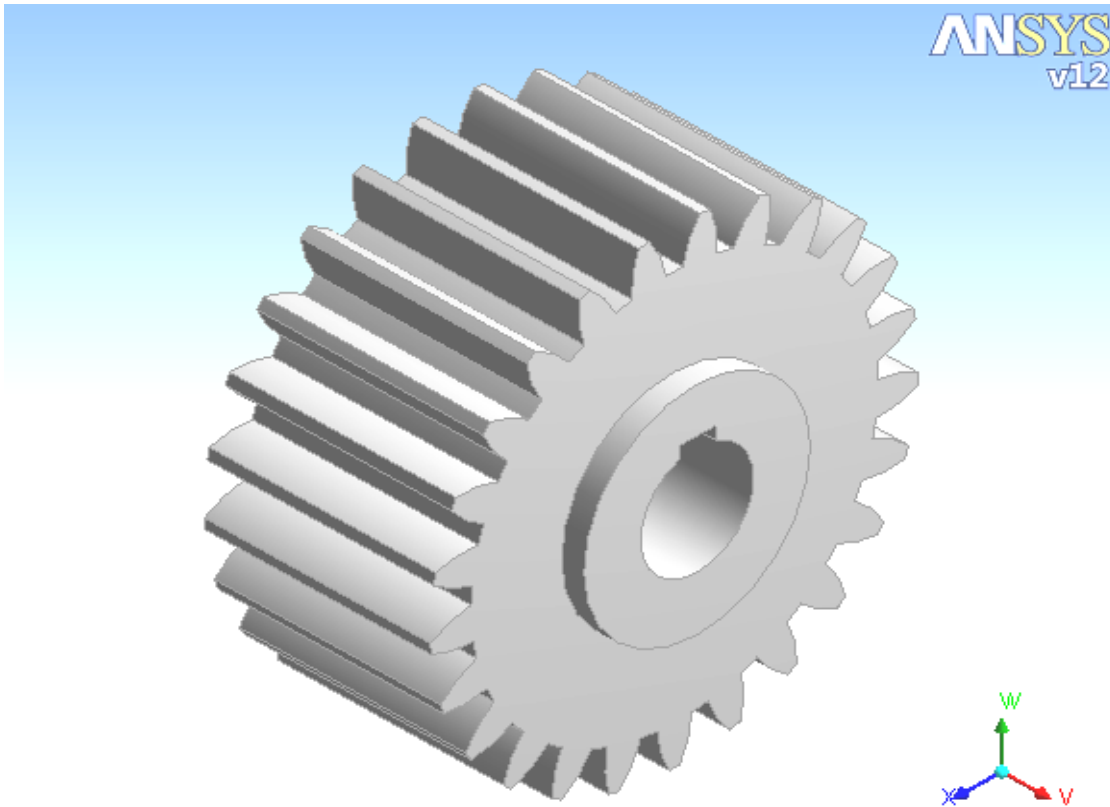
Parameters	Input gear	Output gear
Material	Structural steel	Structural steel
Modulus of elasticity	2E+11	2E+11
Yield strength	2.5E+08	2.5E+08
Ultimate strength	4.6E+08	4.6E+08
Poisson ratio	0.3	0.3
Number of teeth	20	40
Pitch circle diameter	200mm	640mm
Module	8	8
Pressure angle	20	20
Face width	100mm	100mm

4. 3 Finite Element Analysis Results by ANSYS Work Bench

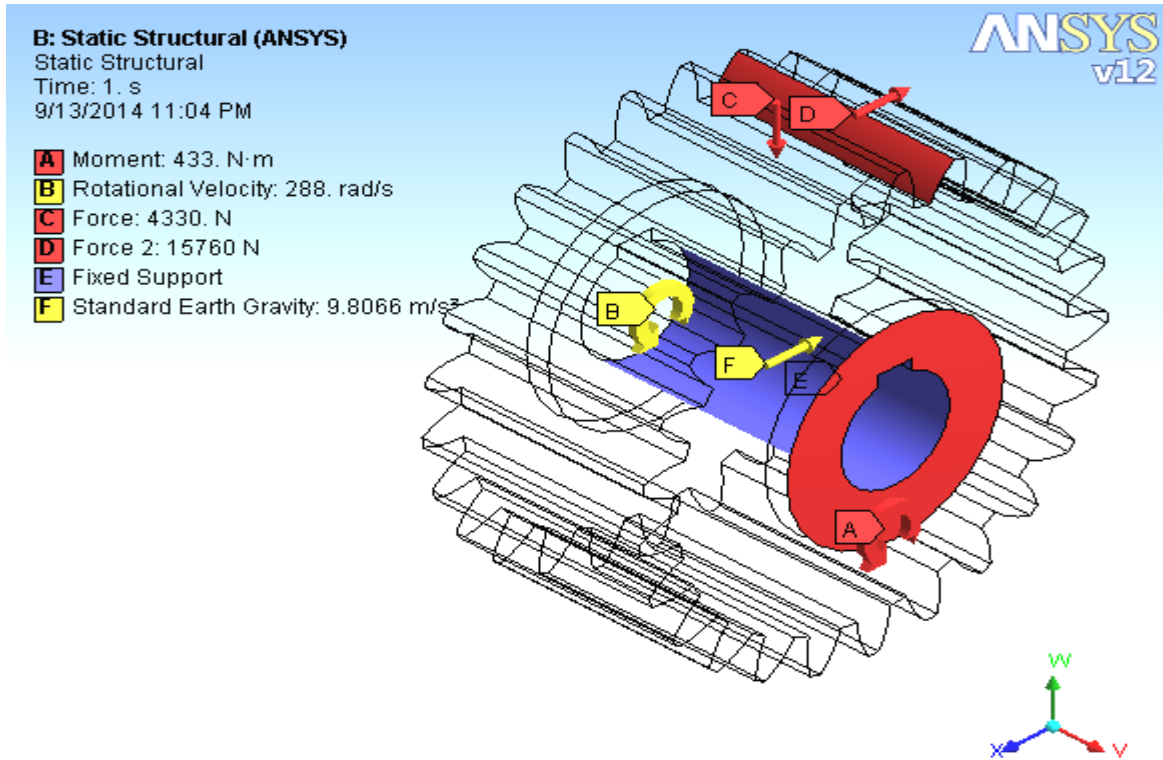
4.3.1 Static Analysis of Gear Units in ANSYS Work Bench

The gears are modeled in both Solid works and ANSYS work bench after the involutes curve of the gears are carefully calculated. The input and the output gears are well positioned and meshed accordingly. The boundary conditions are applied wherever necessary .The loads on

gear units are also applied both separately and in mating. After that solution phase is performed in ANSYS work bench. The following figures show the geometry and the boundary conditions set up procedures for a pinion respectively.



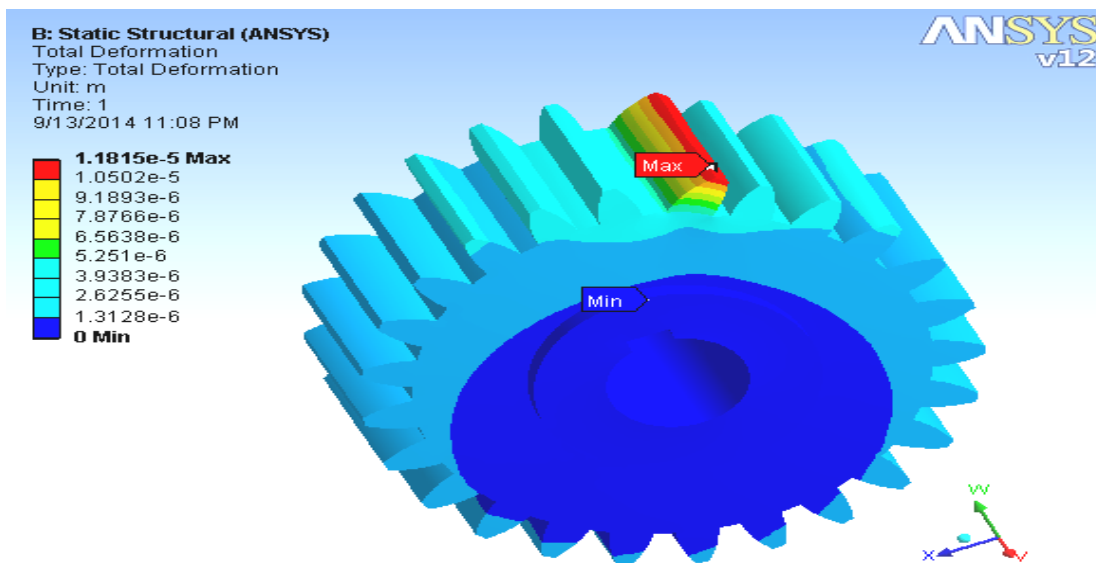
(a) Geometry



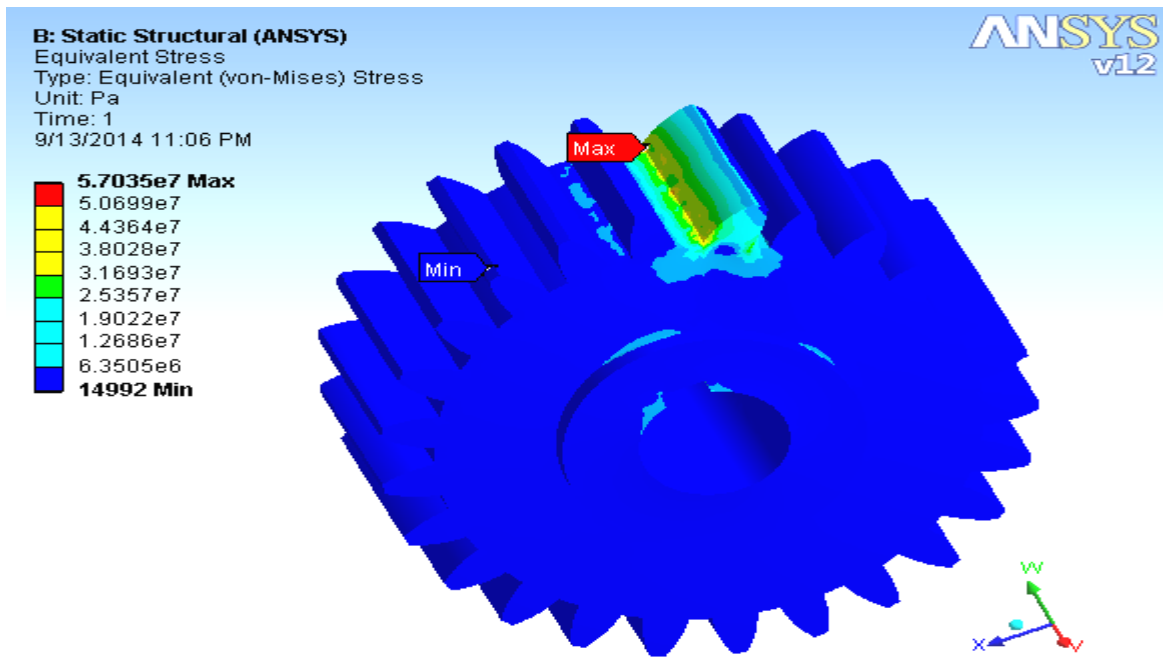
(b) Boundary Condition Set up

Fig 4-2 (a) Geometry and (b) Boundary conditions for pinion

As a result of the applied loads the unit is under stress, therefore ANSYS display different kinds of solution. The following two figures show the total deformation and equivalent von mises stress results respectively for the pinion.



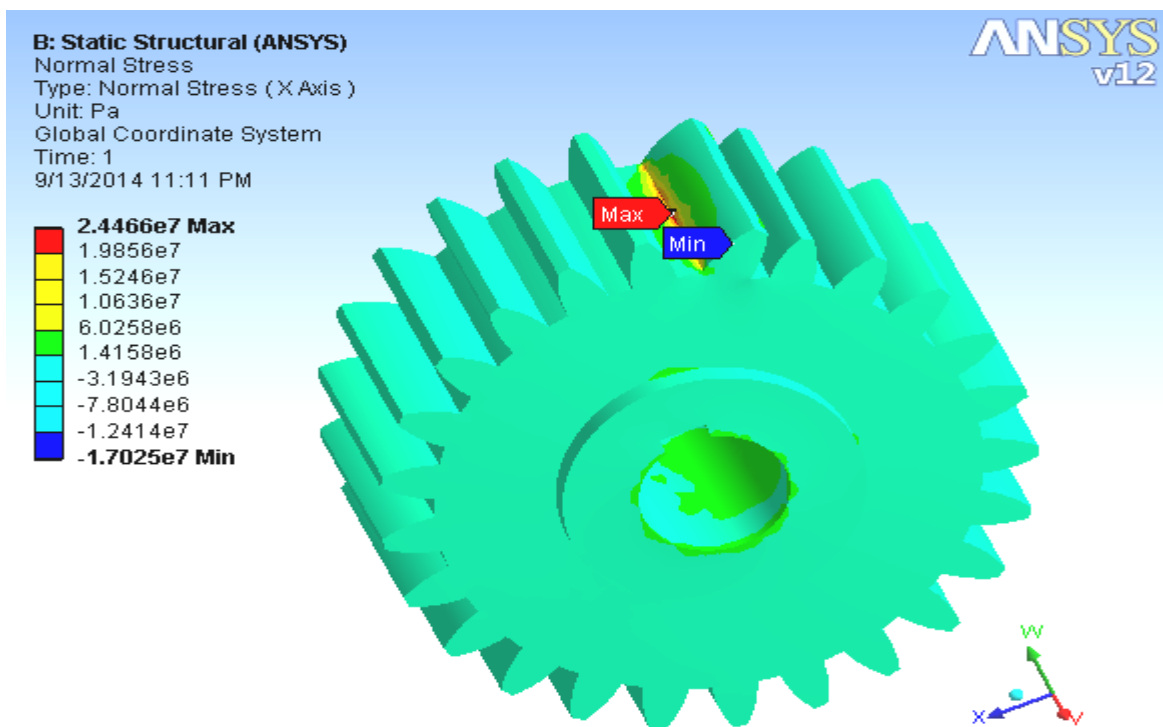
(c) Total deformation



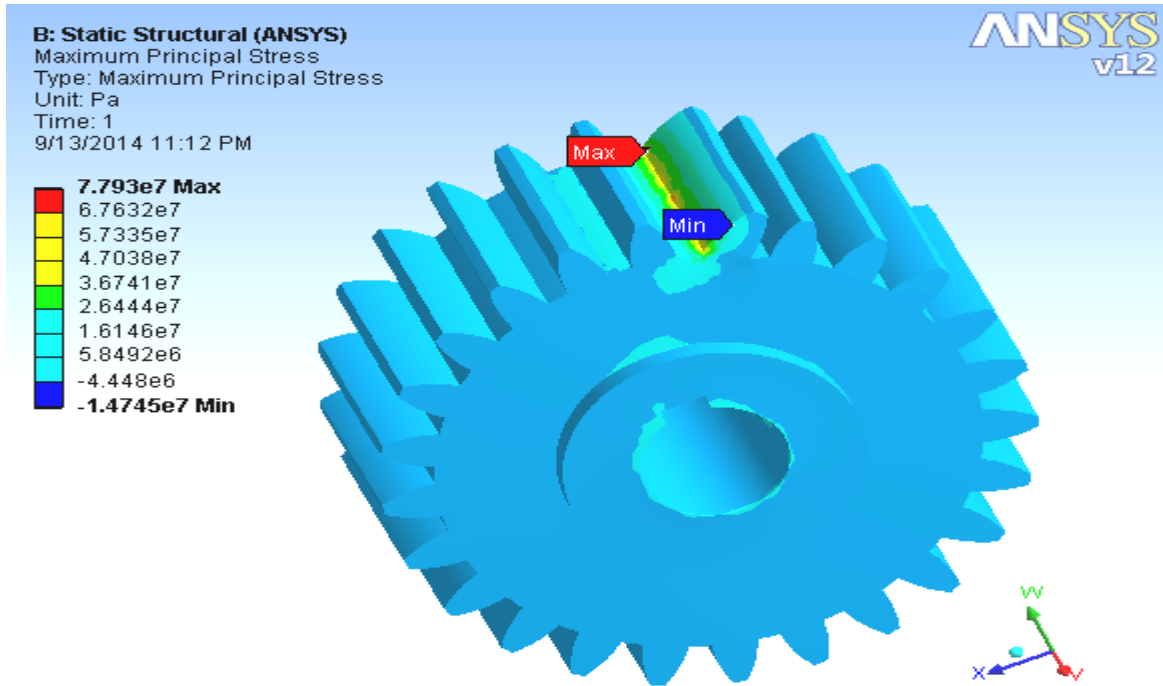
(d) Equivalent Stress

Fig 4-3 (c) Total deformation and (d) Equivalent stress for pinion

The results for normal stress and maximum principal stress are shown below.



(a) Normal Stress



(b)Maximum principal

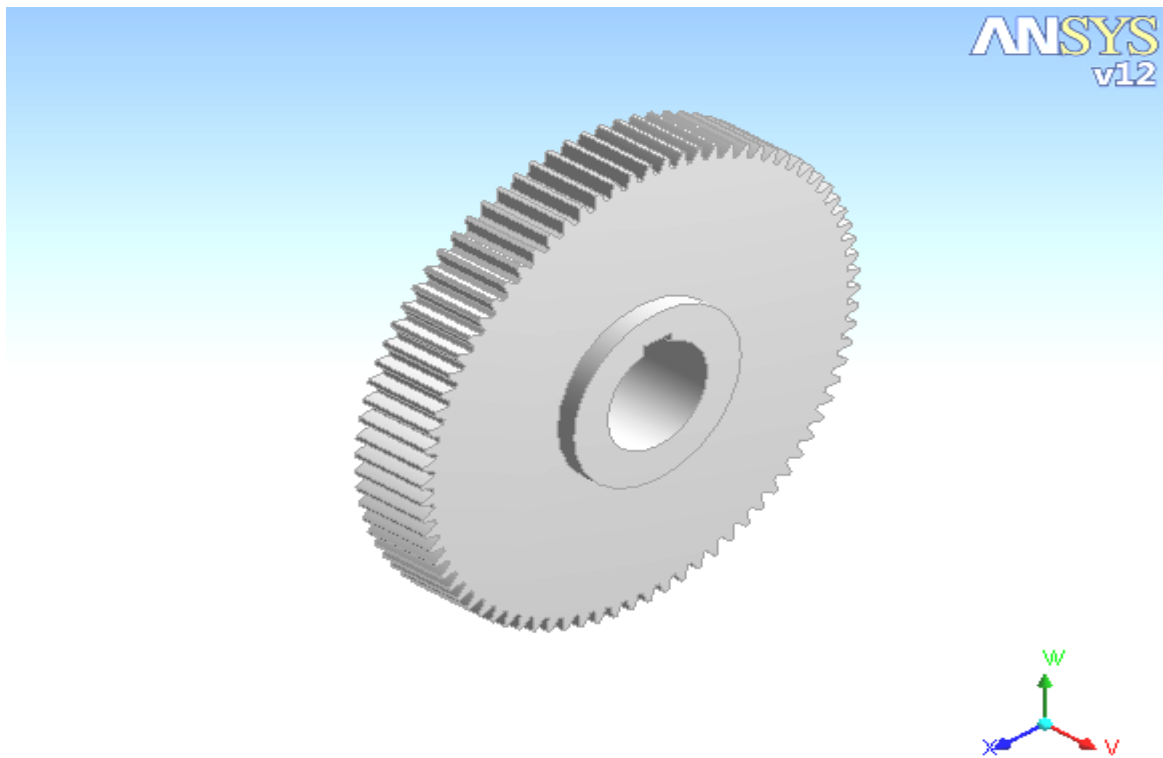
Fig 4-4 Normal and Maximum Principal Stress

The spur gear which is supposed to mate the pinion also modeled in the solid work and ANSYS work bench by the same procedures as for the pinion. The following figures show the geometry and the boundary condition set up for the spur gear. The dimension for the spur gear is modeled based on the gear ratio relationship between the pinion and the spur. In this case the gear ratio is in the second stage of reduction ratio is 2.33:1. Therefore, torque available at the final wheel is multiplied by 2.33. That is the following relationship shows the procedure of torque or moment multiplication at the spur.

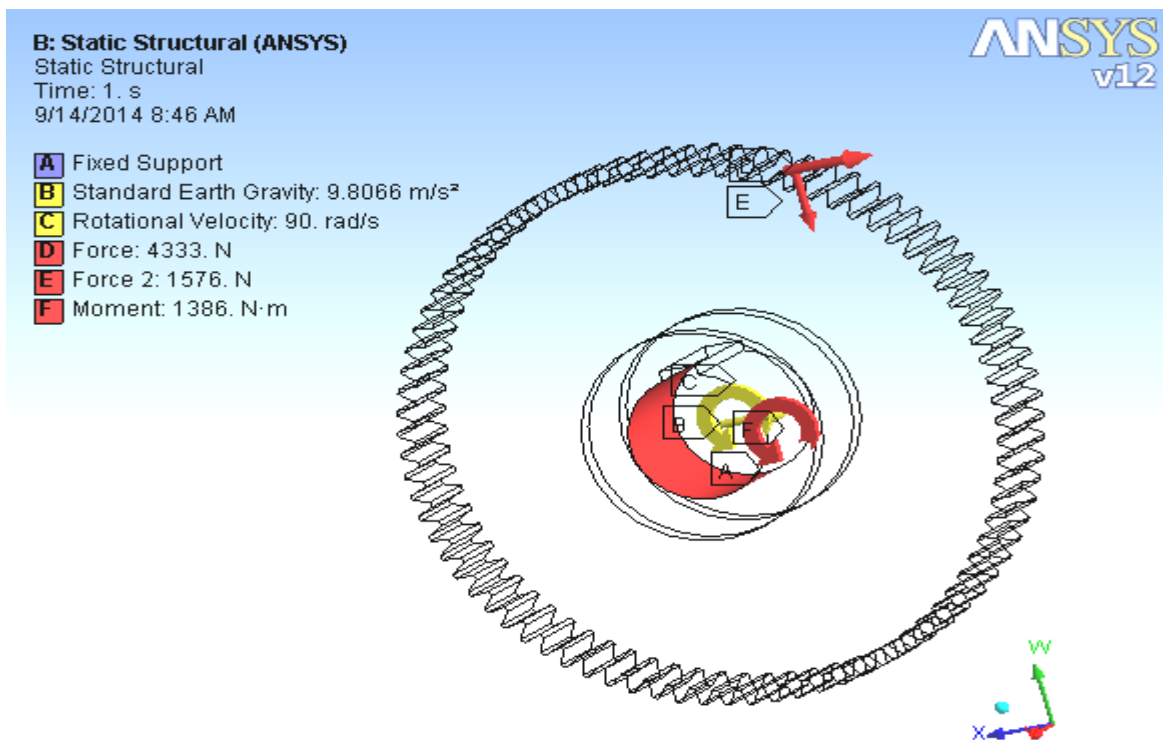
$M_o = \text{Initial moment taken from standard table} = 433\text{Nm}$, gear ratio in the first stage is 3.2

Torque at the spur is

$$3.2 \times 433 = 1386\text{Nm.}$$



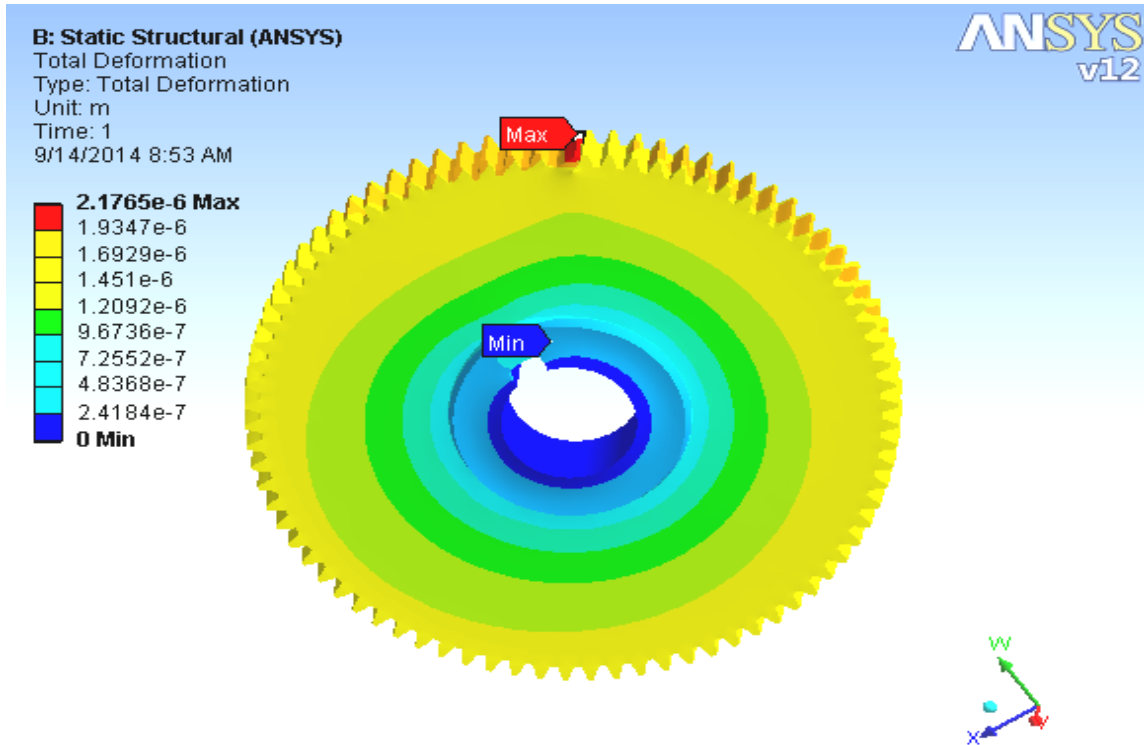
(a) Geometry for Spur



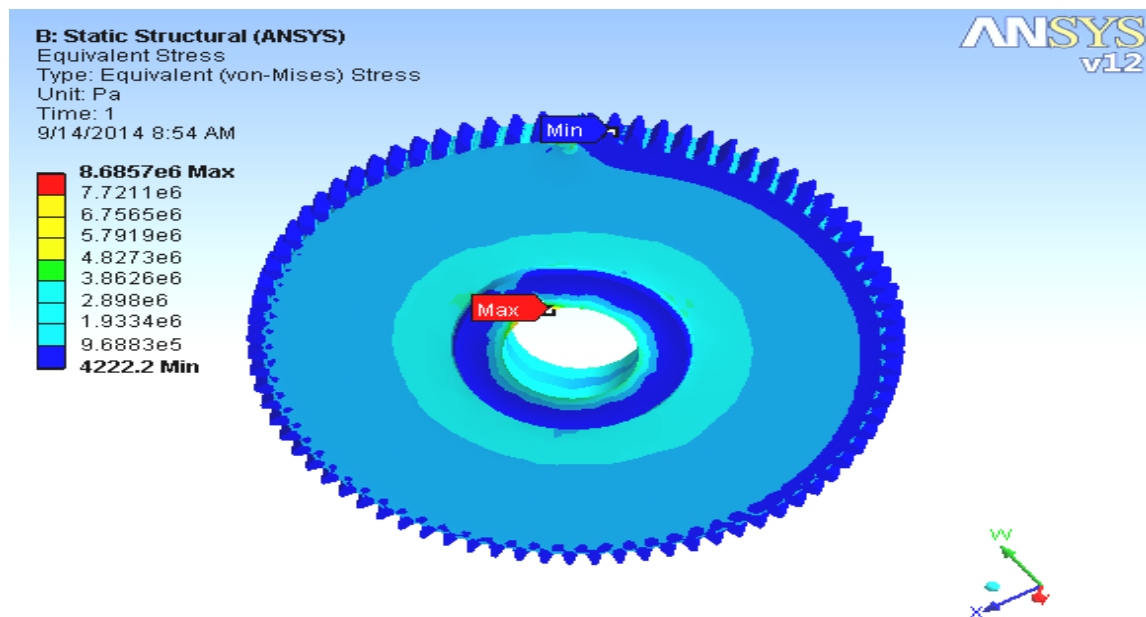
(b) Boundary Conditions for Spur

Fig 4-5 (a) Geometry and (b) Boundary conditions set up for Spur.

As a result of the applied loads the spur is also under different kinds of deformations and stresses. The following two figures show total deformation and equivalent von mises stress results respectively in the ANSYS work bench.



(c) Total deformation



(d) Equivalent stress

Fig4-6 (c) Total deformation and (d) Equivalent stress for spur

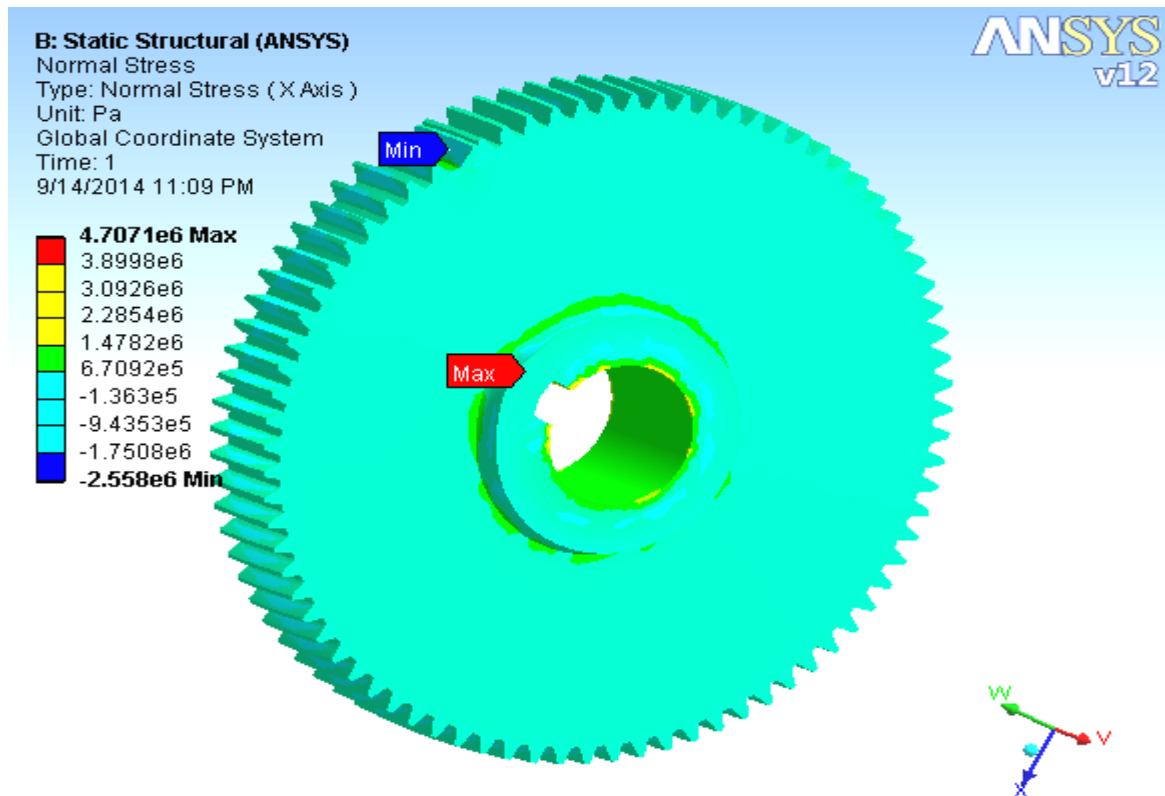


Fig 4-7 Normal stress

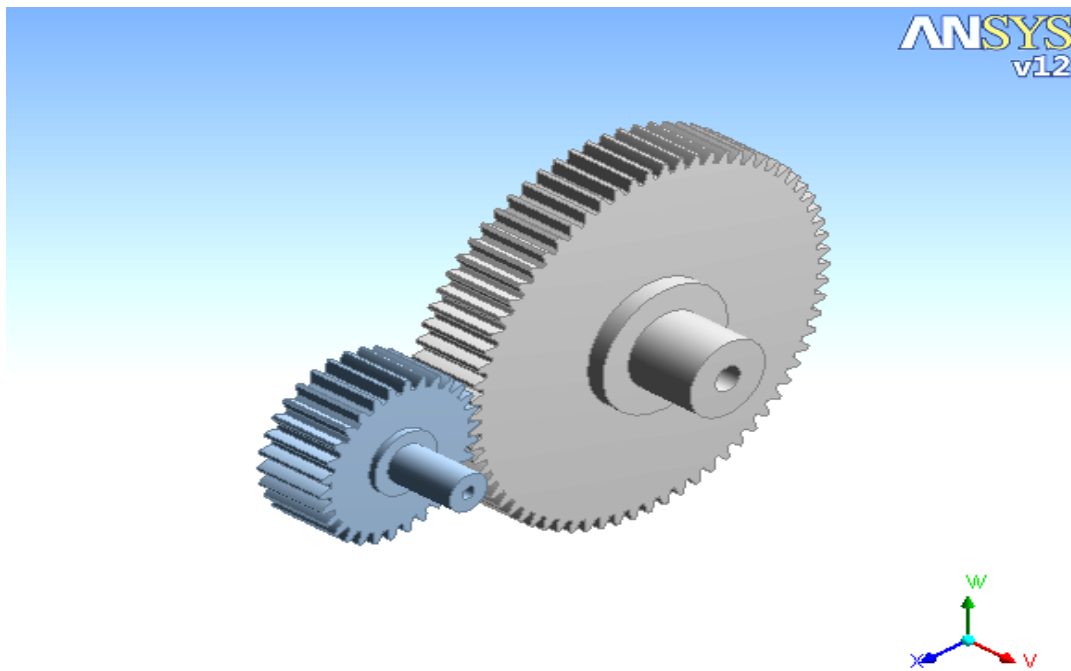
3.2 Static Analysis of the Gear Train without the Idler Gear in ANSYS

A gear train is formed by mounting gears on a frame so that the teeth of the gears engage. Gear teeth are designed to ensure the pitch circles of engaging gears roll on each other without slipping, providing a smooth transmission of rotation from one gear to the next so that they will transfer power efficiently. The next type of the gear train is the part of the drives system which is located nearer to the wheel. The following is the geometric modeling of the two gears in mesh which has a transmission ratio of 2.33:1. Number of teeth for the pinion is 30 and 70 for spur. Railway vehicle drives system uses such a type of transmission system with several types of arrangements. The following figure shows the geometry of the gear trains without the idler gear when the shafts are assembled together. ANSYS solver process executes the other solution in the subsequent section. The following table shows input parameters for the gear train without the idler gear.

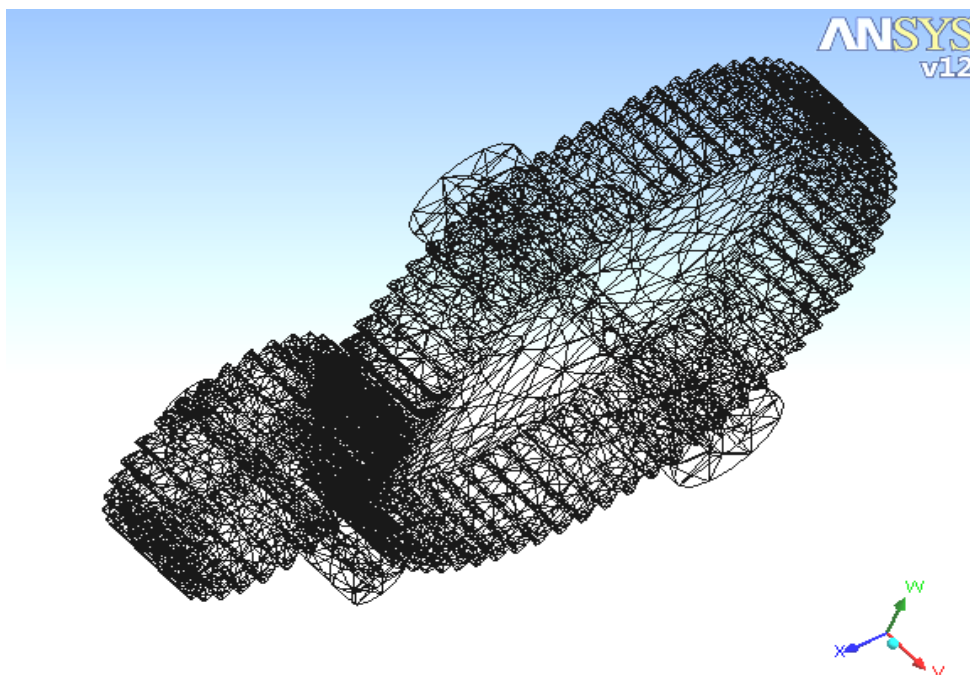
Table 4-3 Input Parameters for gear train without Idler Gear

No	Parts	Magnitude
1	Pinion pitch circle diameter	0.240m
2	Spur pitch circle diameter	0.560m
3	Number of teeth for pinion	30
4	Number of teeth for spur	70
5	Shaft diameter for pinion (hollow shaft is) assembled to the motor)	0.06m
6	Shaft diameter for spur (hollow shaft is used)	0.120m

The following figure illustrates the geometric model, the meshing procedure of the gear train without the idler in ANSYS workbench according the above parameters. After the geometry is well modeled the next step to take place is the mesh in ANSYS work bench. The default mesh controls that ANSYS program uses may produce a mesh that is adequate for the model that we are analyzing. Mesh controls allows us to establish such factors as the element shape, mid side node placement and element size to be used in meshing the solid model. This step is one of the most important of the entire analysis.



(a) Geometric model



(b) Mesh

Fig 4-8 (a) Geometry and (b) Mesh for the gear train

After this step all the necessary boundary conditions are well set up to execute the entire solution and analysis.

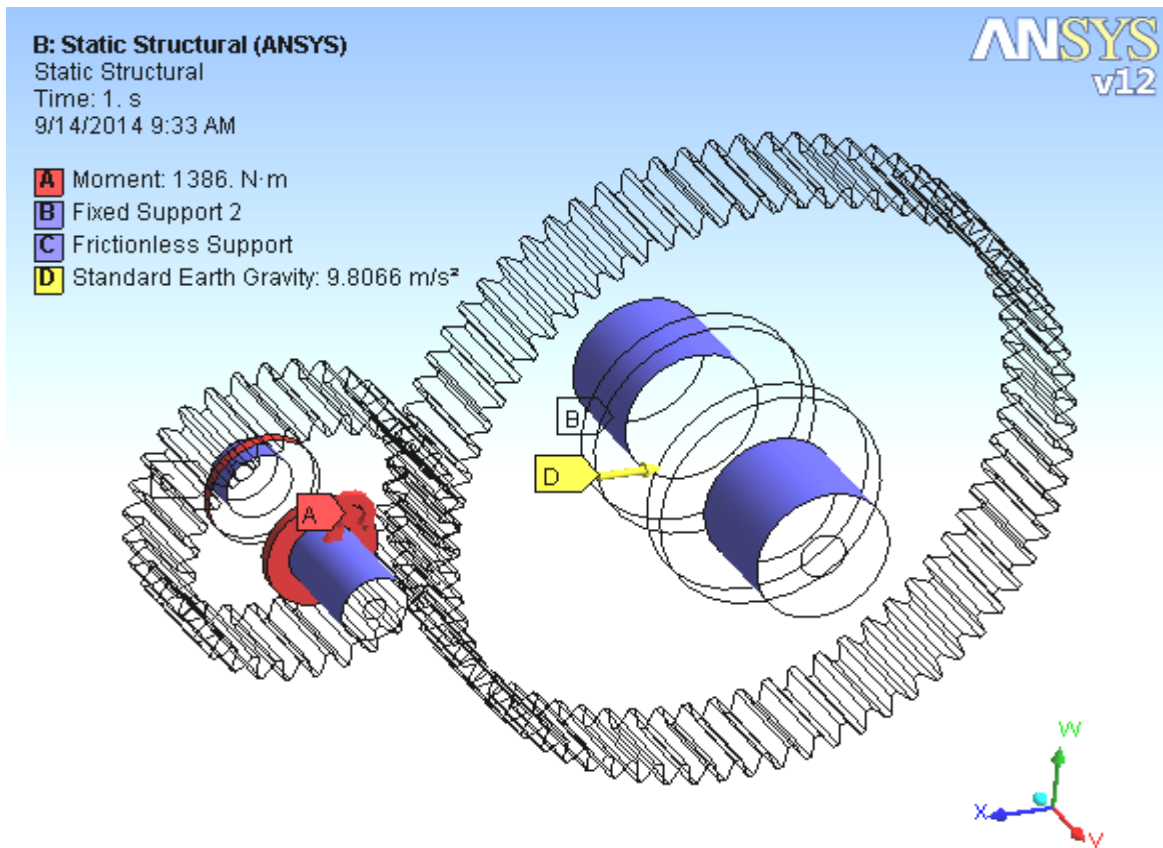
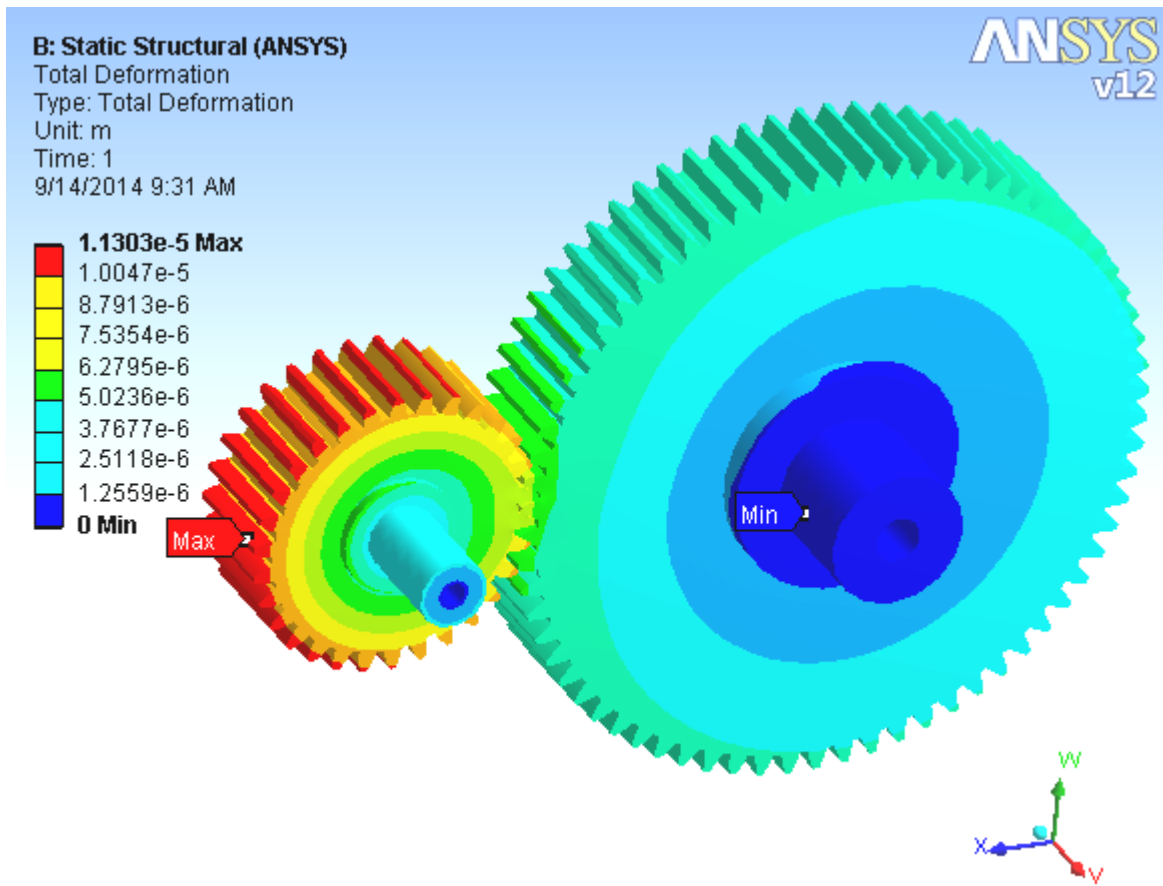
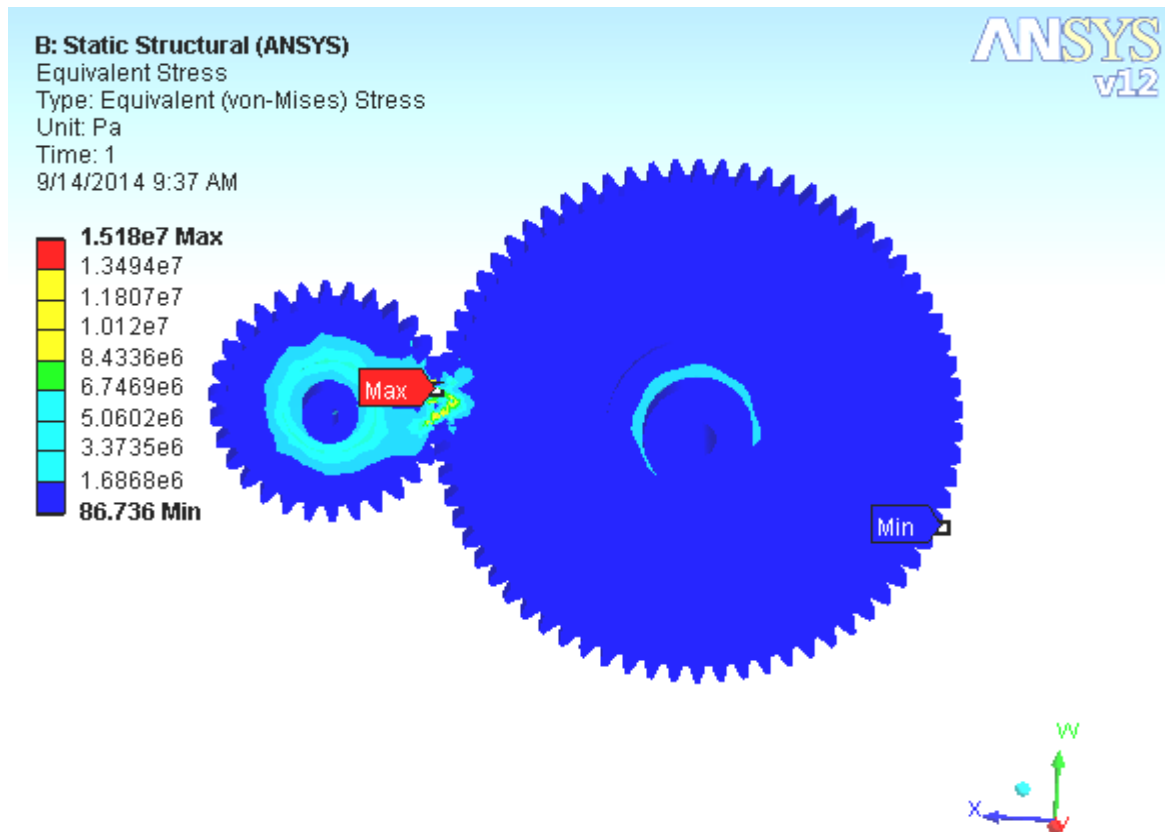


Fig 4-9 Boundary Conditions Set ups

Total deformation and equivalent/Von mises stress are shown as follows respectively in the solution phase of ANSYS work bench.



(a) Total deformation



(b) Equivalent/Vonmises stress

Fig4-10 (a) Total deformation and (b) Equivalent von mises stress

4.3.3 Fatigue Analysis of the Gear Train without Idler Gear

During the transmission phenomena, the gears are under cyclic and dynamic loading, hence they are under fatigue. Fatigue is the weakening of a material caused by repeatedly applied loads. It is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The nominal maximum stress values that cause such damage may be much less than the strength of the material typically quoted as the ultimate tensile stress limit, or the yield stress limit. FEM can predict the results of fatigue stress in the solution phase. The following are the results of the equivalent alternating stress and life that gears can have under cyclic loading respectively.

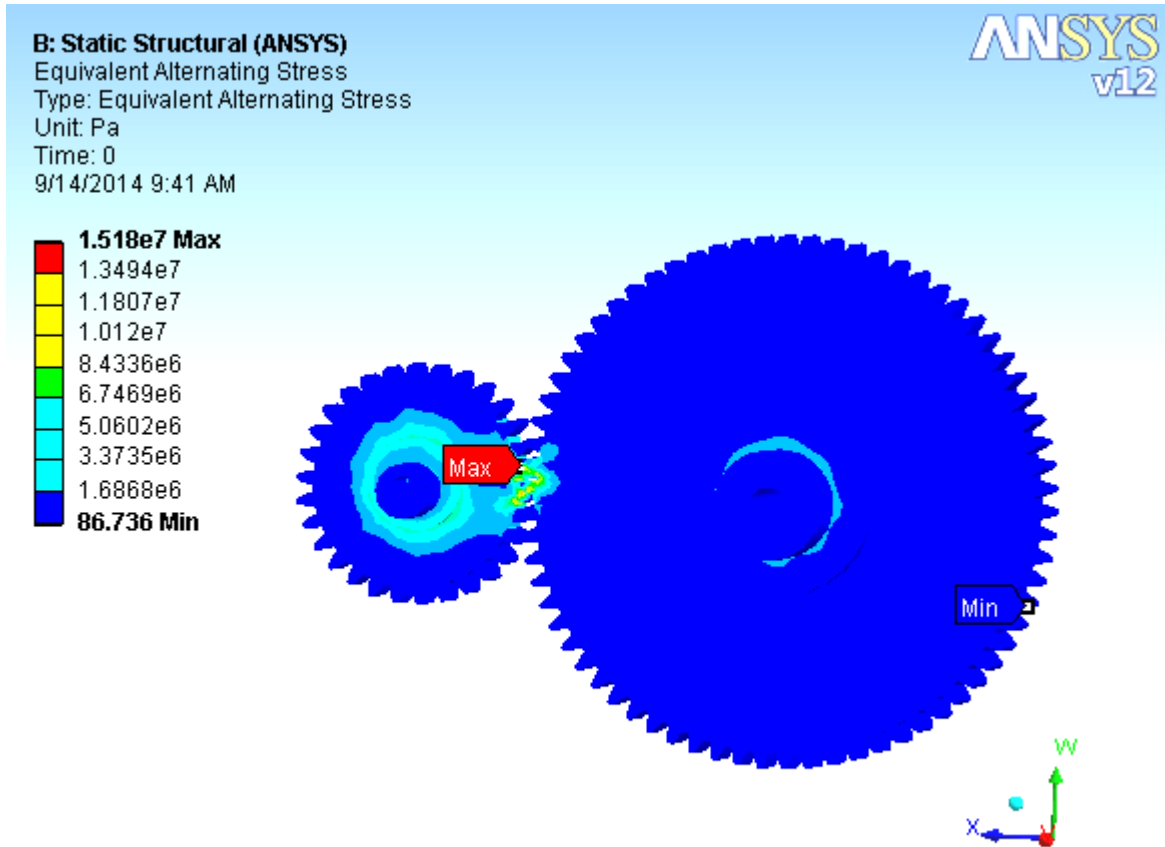


Fig 4-11 Equivalent alternating stress

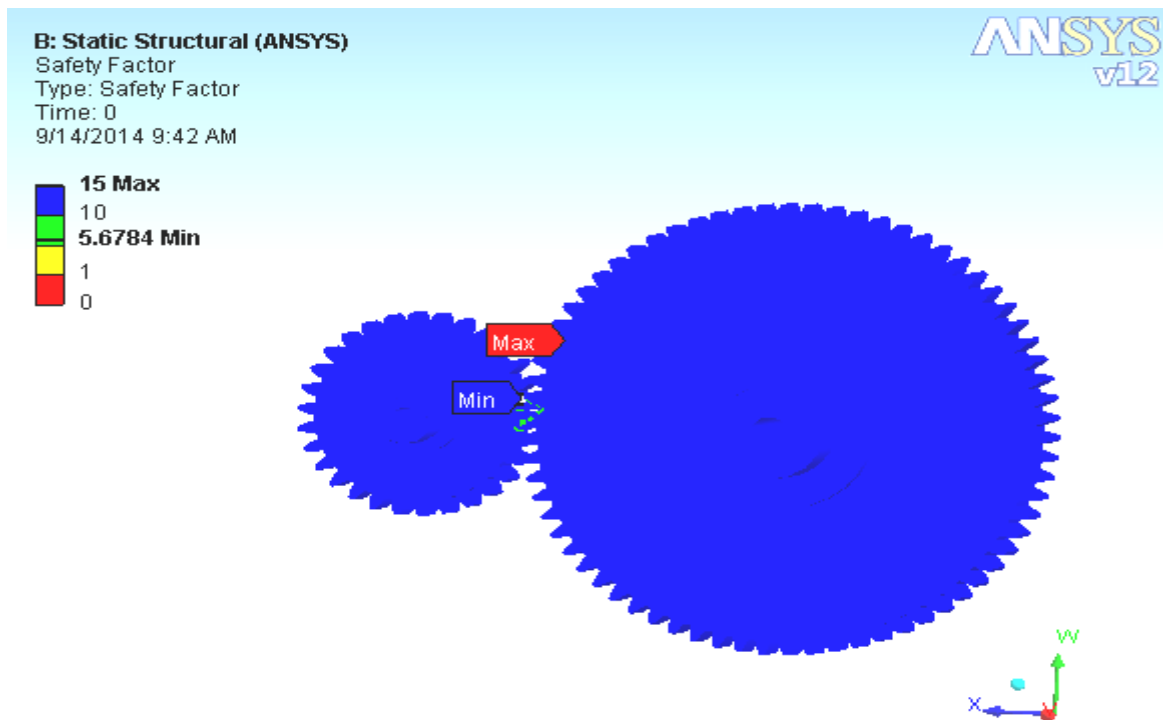
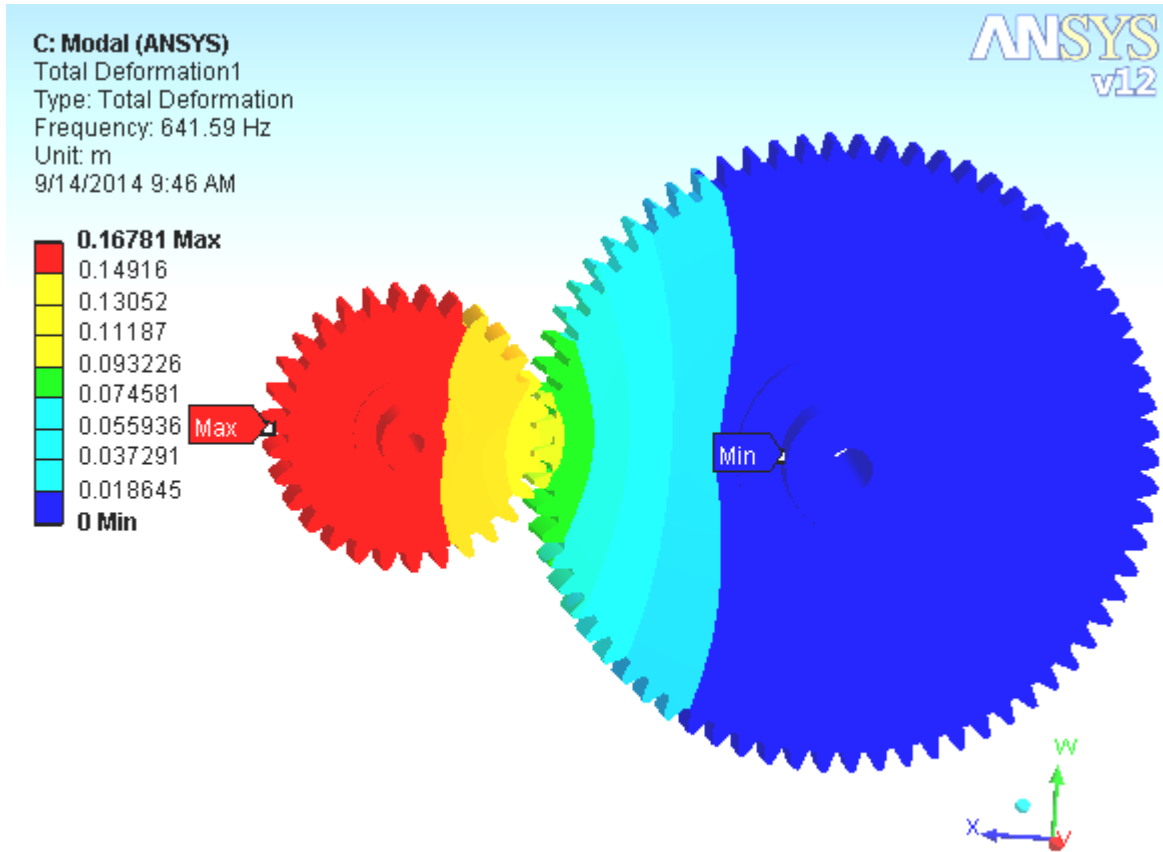


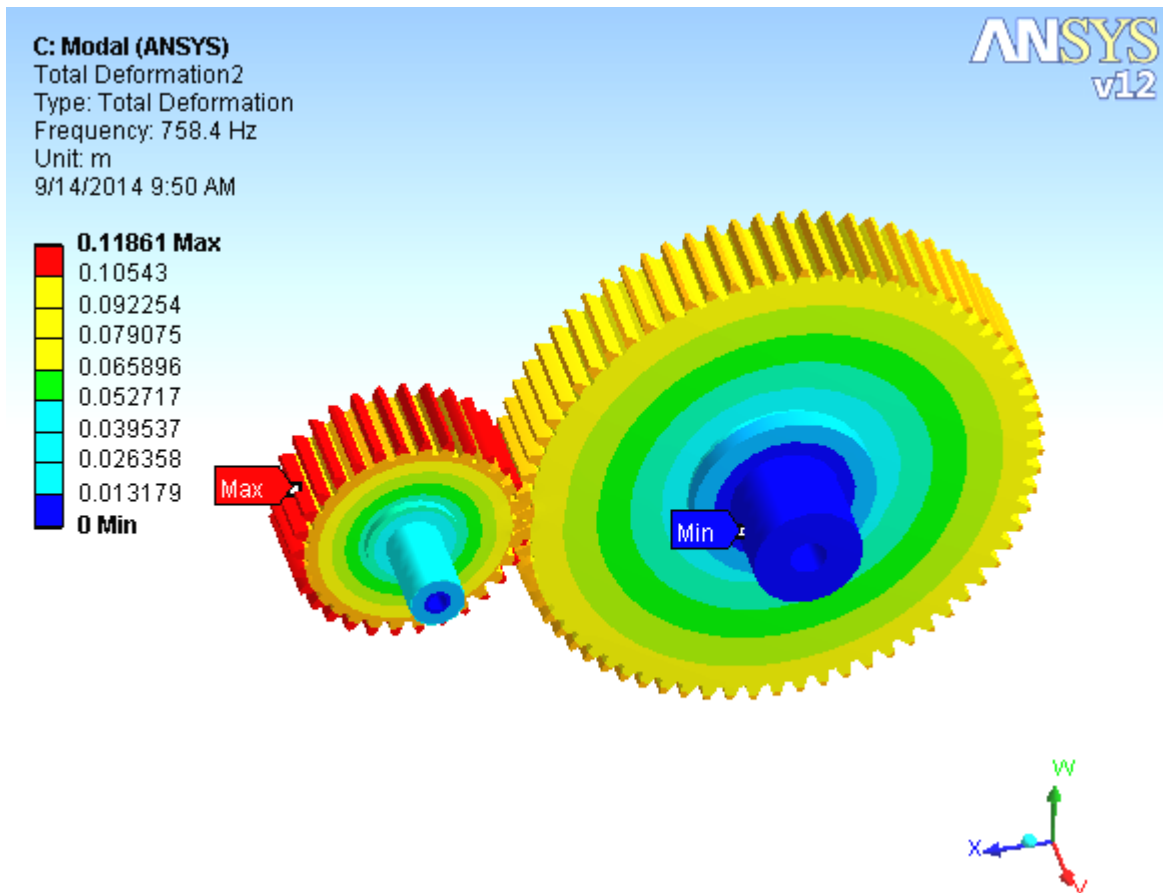
Fig4-12-factor of safety

4.3.4 Modal and Harmonic Response Analysis of the Gear Train without the Idler Gear

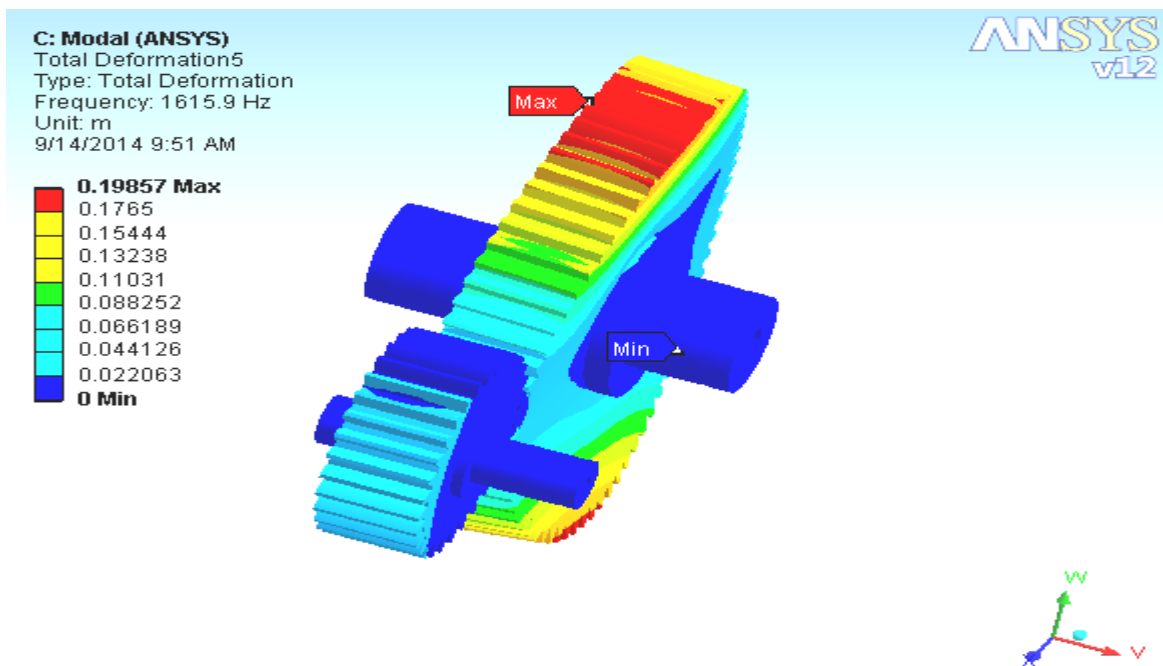
Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. Vibration can arise from track irregularities, misalignments, improper installation, unbalanced rotation and etc. Vibration analysis of the gear train is also important to ensure safe and quiet operation of the gear systems. The vibration behavior of the gear train models is closely dependent on the geometrical structure of the gear train, the source of load excitations and the constraints applied. In this thesis, gear train designs are modeled by using the Solid works and ANSYS software for vibration analysis. Moreover, it is helpful to ensure that the unit is in its operating frequency range. Therefore, modal and forced frequency responses are performed in the ANSYS modal and harmonic phase. In the modal phase, frequency range is never limited because the analysis will take place at any frequency range but in case of harmonic response, the frequency response is limited to its maximum as 10000Hz. Around the world for railway vehicle the excitation frequency is 20Hz-5000Hz [30]. In the harmonic response analysis the maximum frequency range is 10000Hz. The following ANSYS work bench result shows some results Modal (ANSYS). Total deformation 1, total deformation 2, total deformation 5 and equivalent stress are shown below.



(a) Total deformation 1 (modal)



(b) Total deformation 2 (modal)



Total deformation 5

Fig 4-13 Total deformation 1, 2 and 5 (modal)

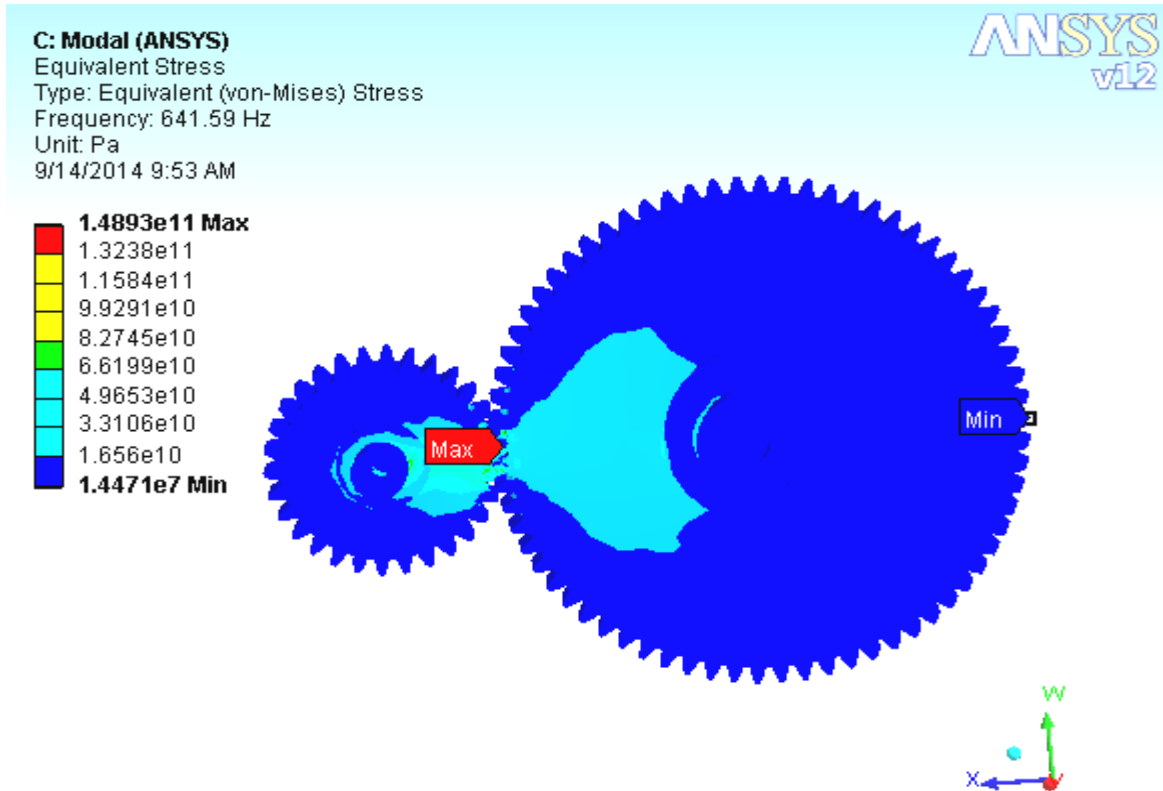
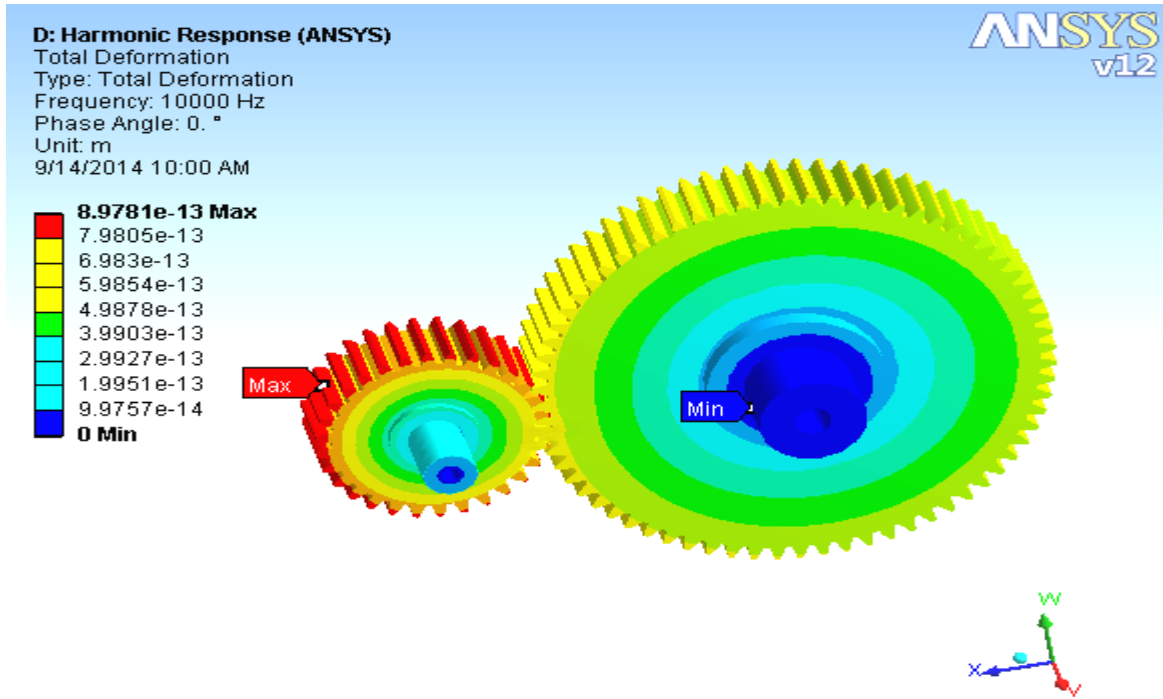
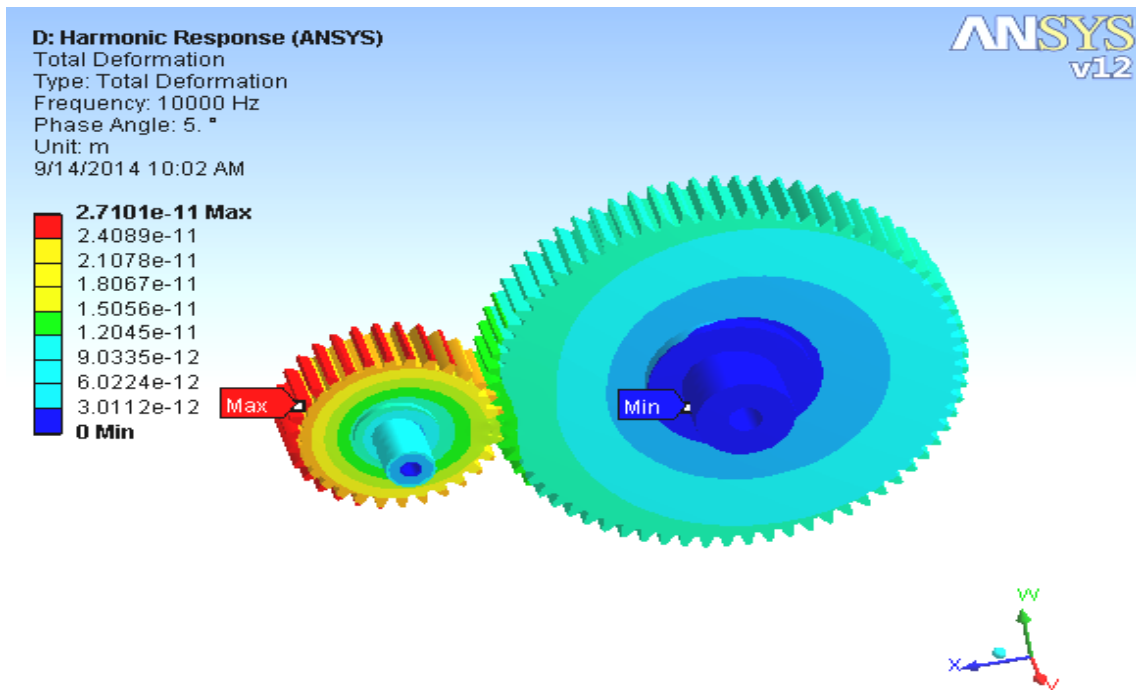


Fig 4-14 Equivalent/Vonmises stress (Modal)

The results of harmonic excitation response analysis with phase angle of 0 and 5 degrees for total deformation and equivalent stress are shown below. Note that the damping properties are all taken as 0.6. As the phase angle increases the total deformation and equivalent stresses increases. Whereas, as the damping ratios increases the deformations and the equivalent stresses decrease.

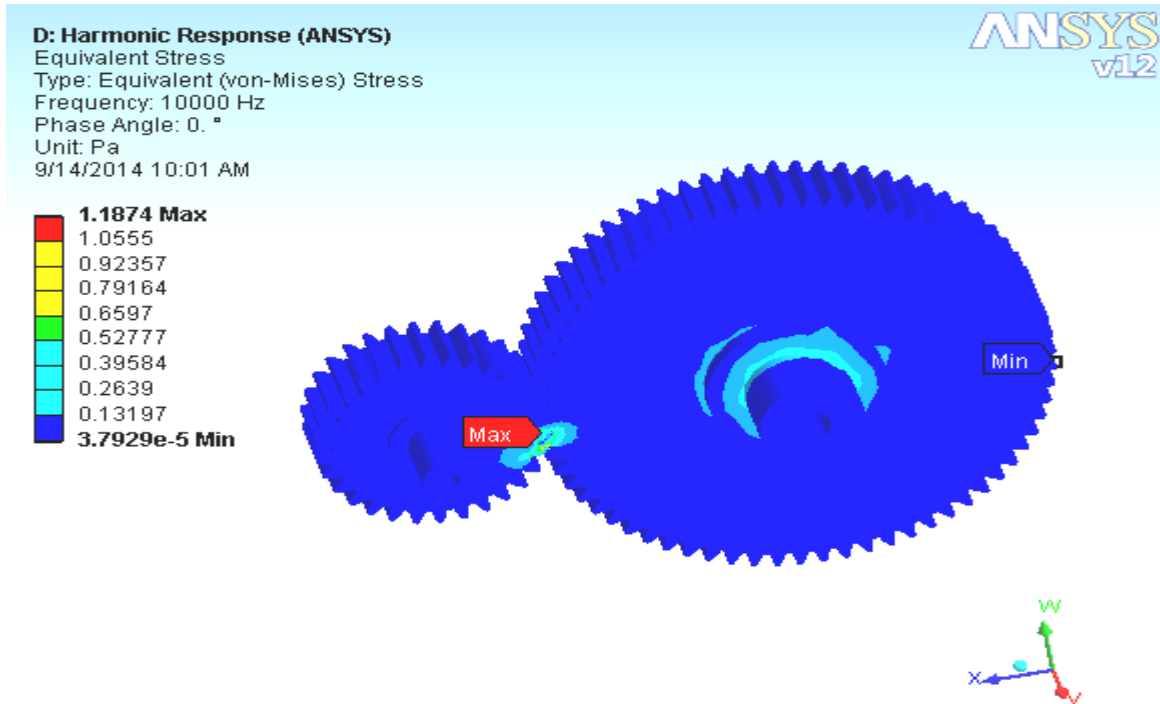


(a) Total deformation/harmonic excitation with phase angle of 0°

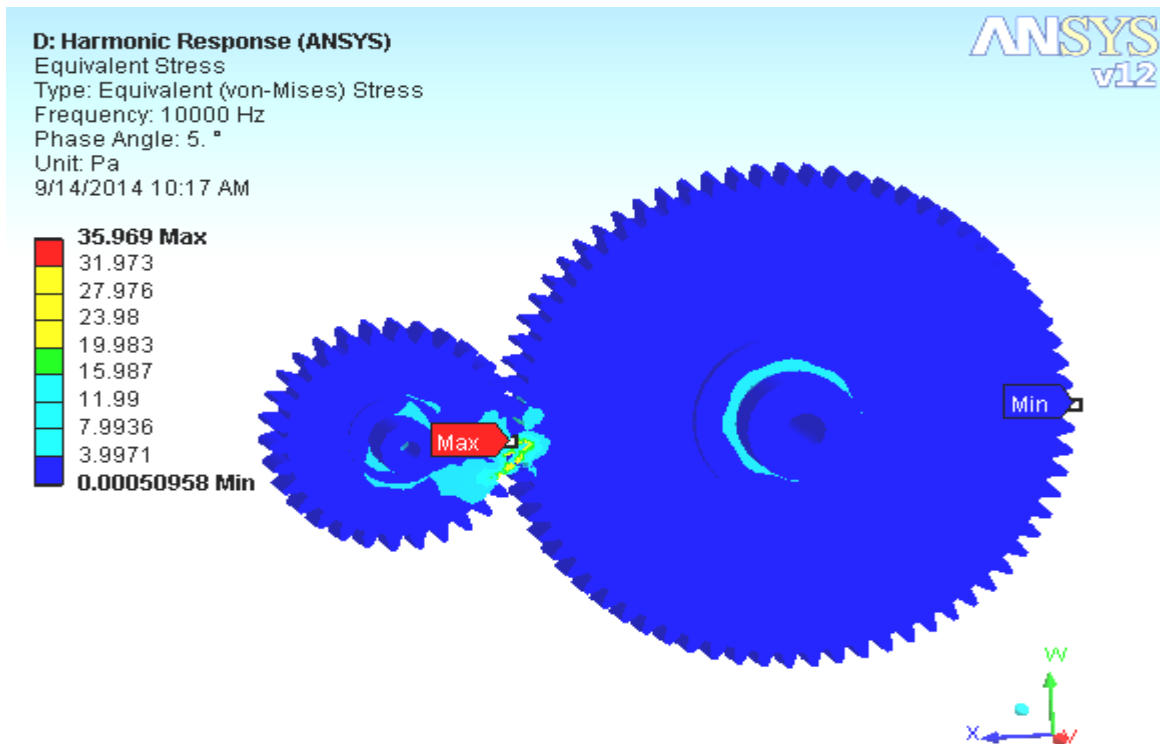


(b) Total deformation/harmonic excitation with phase angle 5°

Fig 4-15 (a)Total deformation/harmonic excitation with varying phase angles

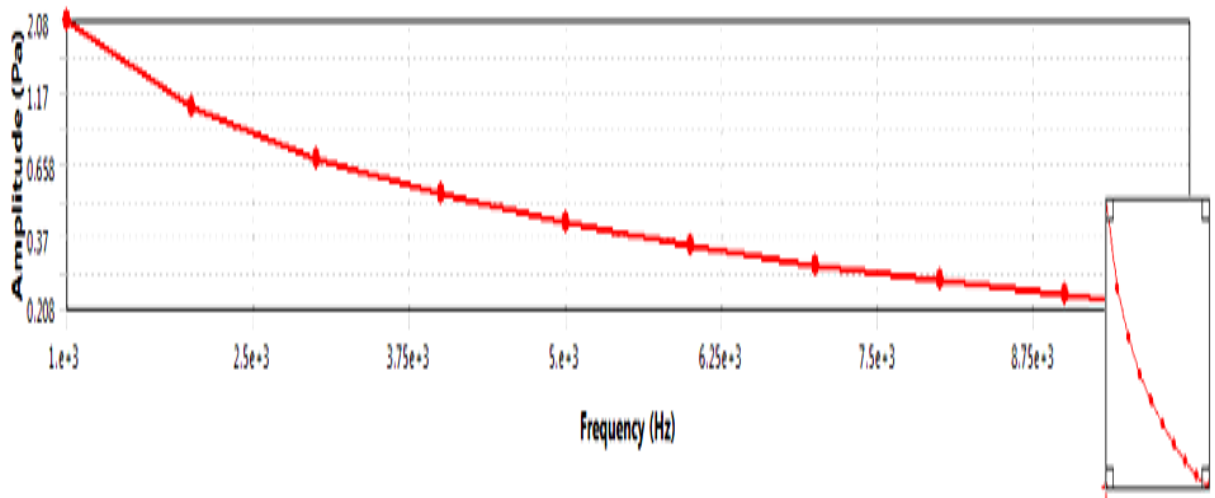


(a) Equivalent stress/harmonic excitation with phase angle 0°

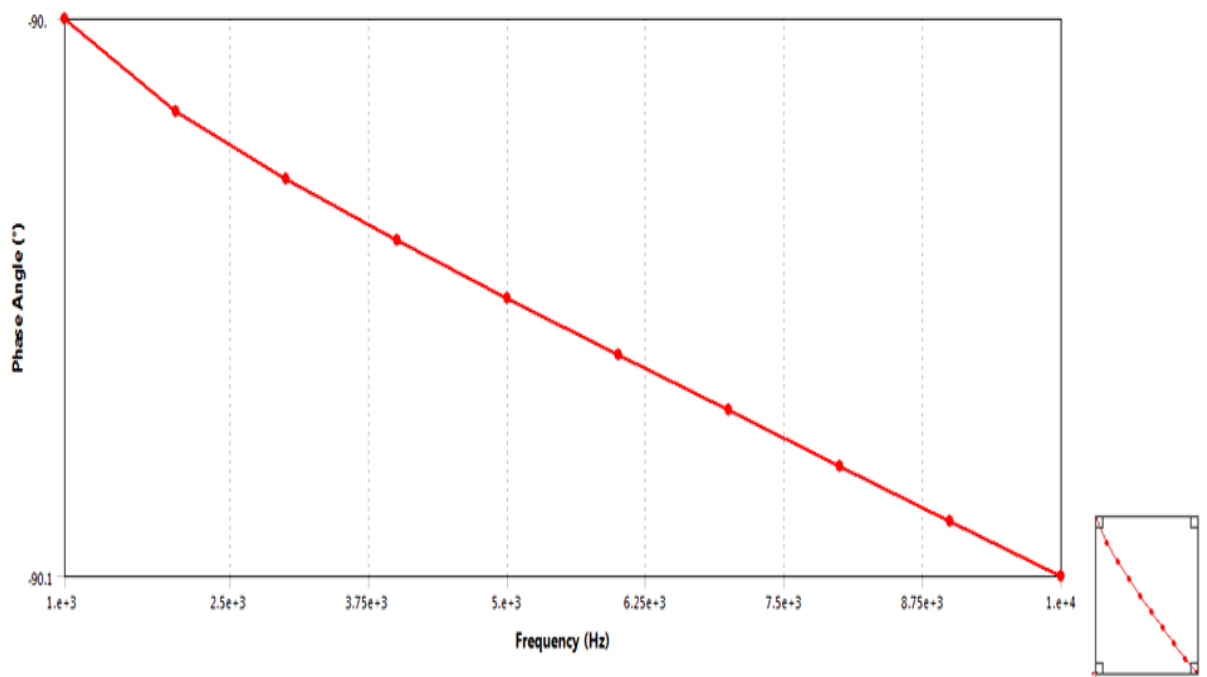


(b) Equivalent stress/harmonic excitation with phase angle of 5°

Fig 4-16 Equivalent stress/harmonic excitation with phase angle of 0° and 5°



(a) frequency response graph frequency vs amplitude



(b) frequency response: frequency vs phase angle

Fig4-17 (a) and (b) Frequency response graph

4.3.5 Contact Stress Analysis of the Gear Train without Idler Gear in ANSYS

Contact stresses are one of the factors which affect performance of gear transmission. The contact stresses induces surface contact fatigue. The surface contact fatigue leads to pitting on teeth's surface. The Hertzian contact stresses theory is used to determine the allowable contact stresses. ANSYS determines the contact stresses in the mating gear teeth as illustrated in the following figures. As usual all the steps executed in the other phases also taken place in the contact stress. The necessary boundary condition set ups are type of contact which is bonded, taking the friction between the mating gears as zero and the solution will be taken place as follows. The results are illustrated below

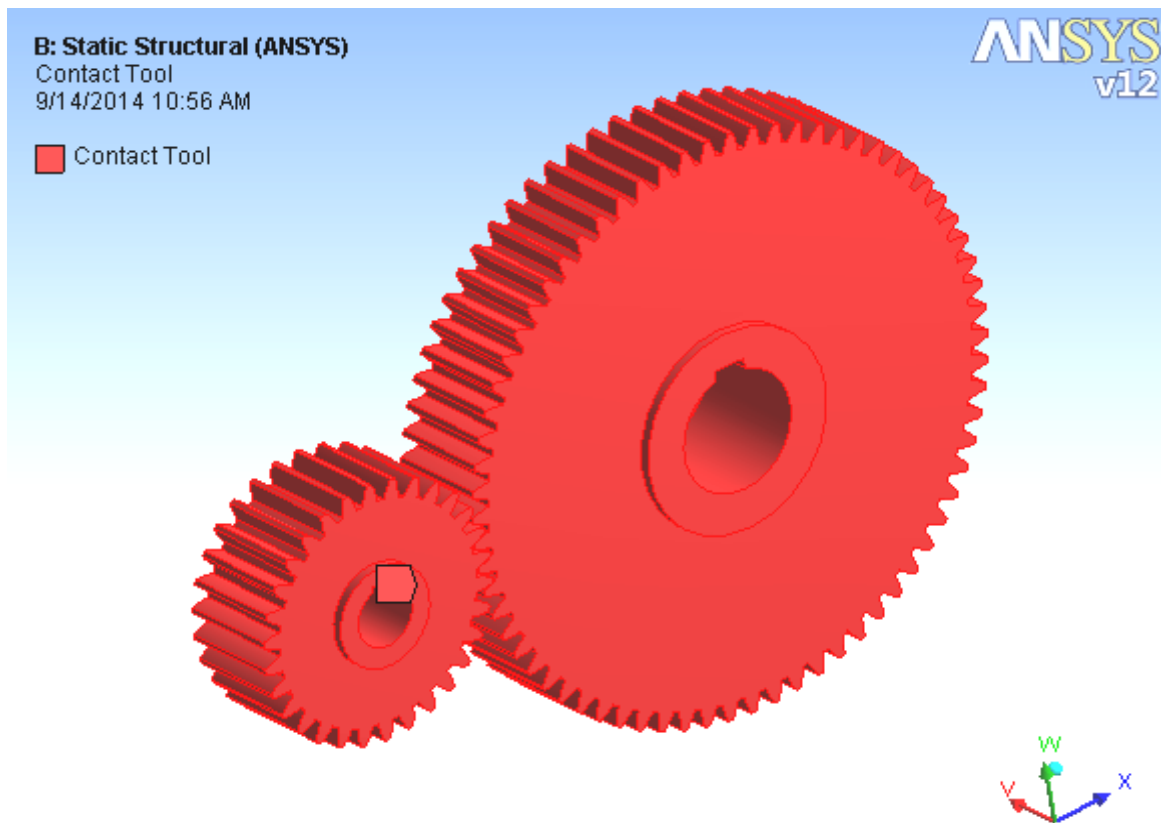


Fig 4-18 Contact set up

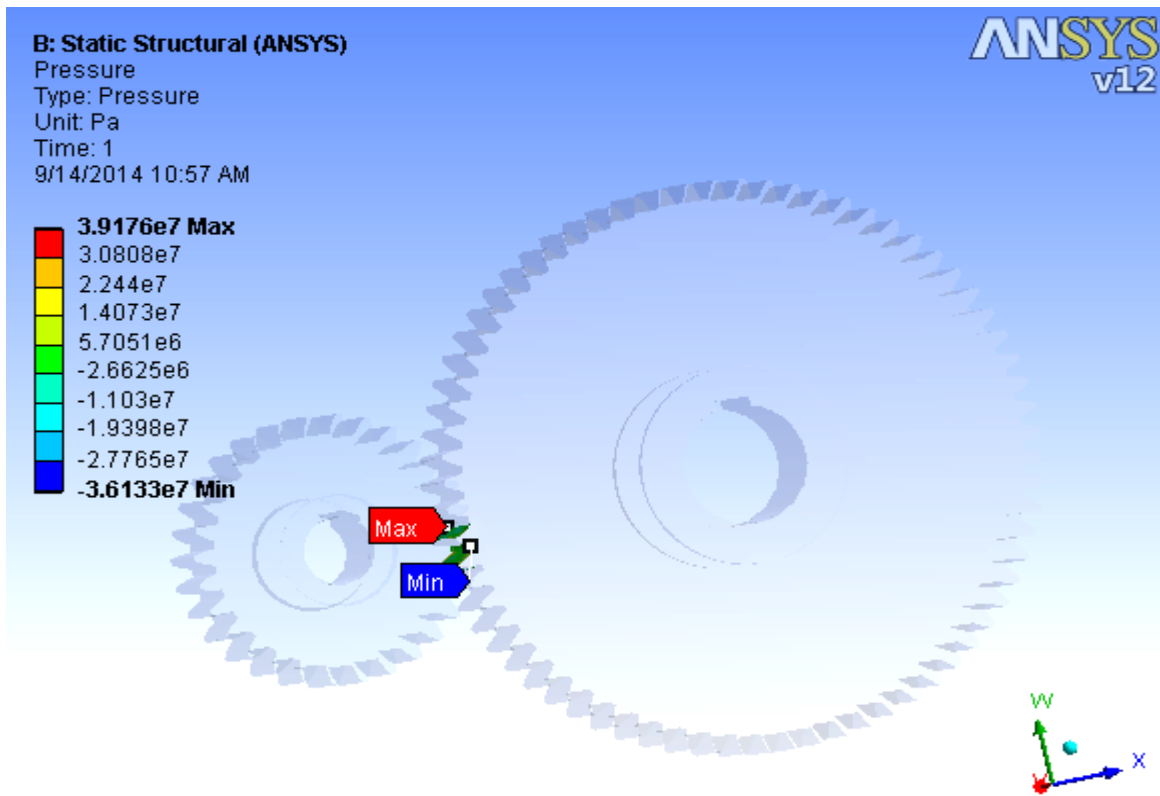


Fig 4-19 Pressure due to contact

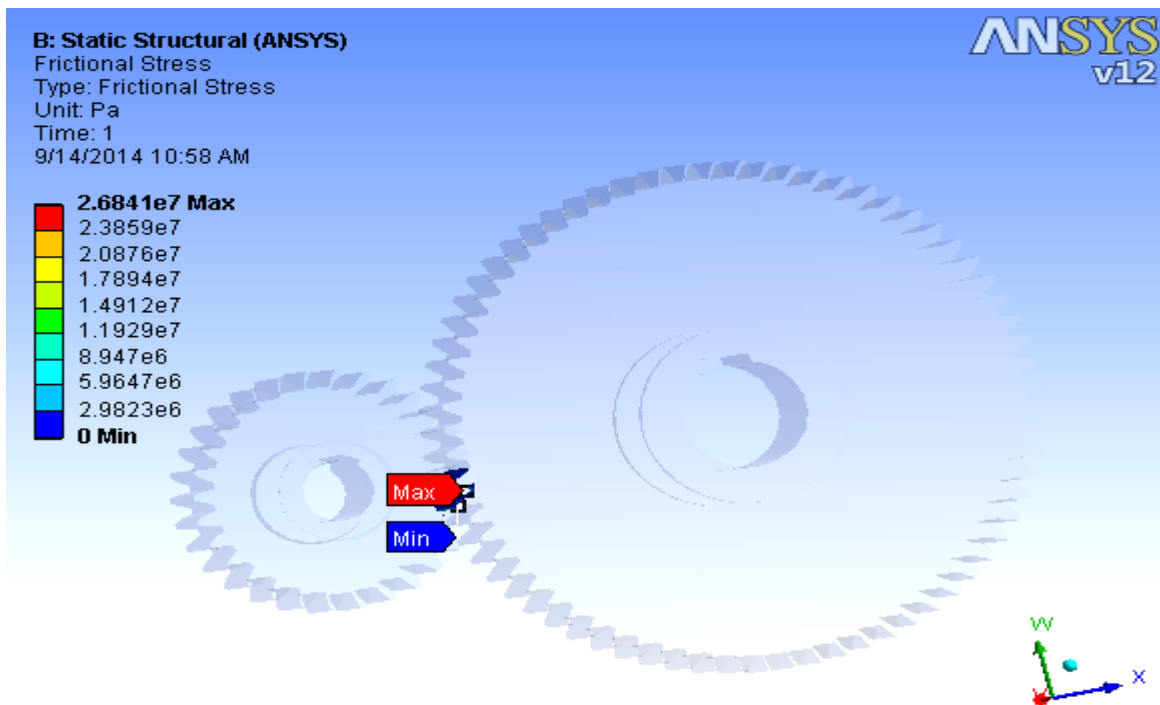


Fig 4-20 Frictional Stress due to Contact

4.3.6 The Static Structural Analysis of the Gear trains with Idler gear

The purpose of the idler gear in a gear train is to change the direction of rotation and to increase the load distributions areas. Different kinds of railway traction system use such a type gearing system in the transmission of power. The pinion is assembled to the motor, which is suspended to the bogie frame and one idler gear is used intermediately before the spur. The gear ratio used in this case is 3.2:1.

The following model shows the geometric model in solid works and ANSYS work bench. The parameters that are changed are illustrated in the following table.

Table 4-4 Parameters for gear Train with Idler

Parameters	Pinion	Idler gear	Spur gear
No of teeth	25	40	80
Face width	100mm	100mm	100mm
module	8 mm	8 mm	8 mm
Pitch circle diameter	200mm	320mm	640mm

The following relation is used to calculate the transmission ratio or the gear ratio

$$N_2/N_1 \times N_3/N_2 = (40/25) \times (80/40) = 3.2:1$$

The module and pitch circle diameter is related by

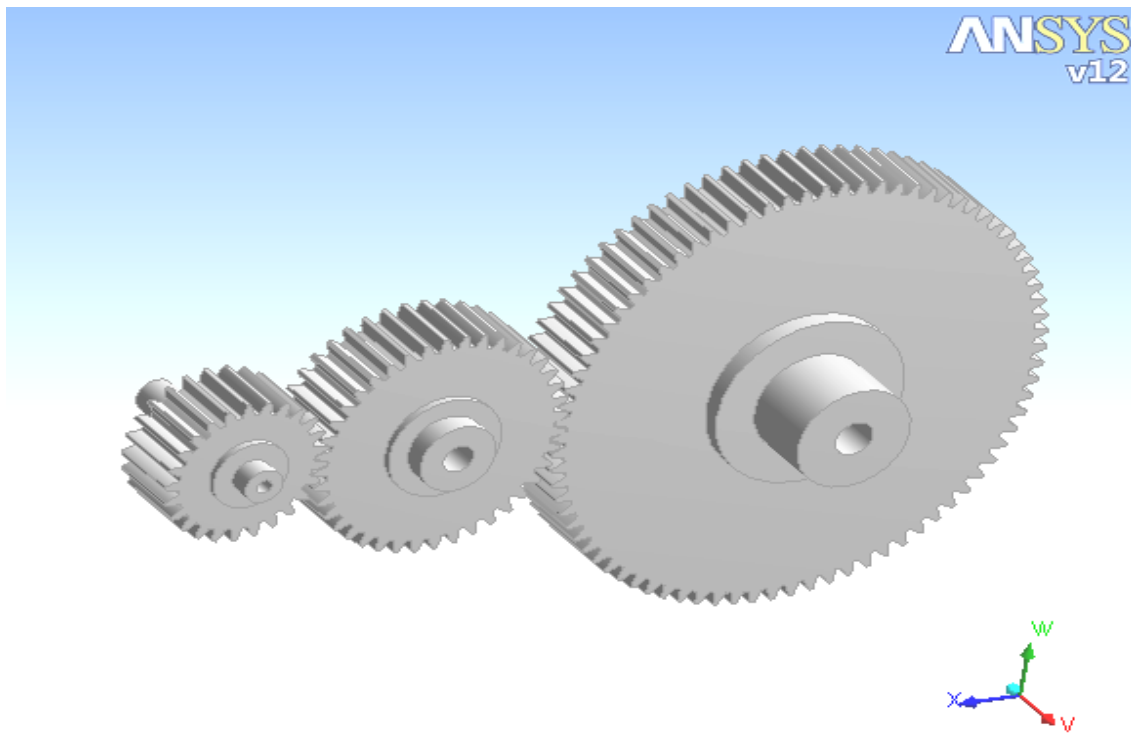
$$P_d = m \times N = 8 \times 25 = 200 \text{mm for pinion}$$

$$P_d = m \times N = 8 \times 40 = 320 \text{mm for idler gear}$$

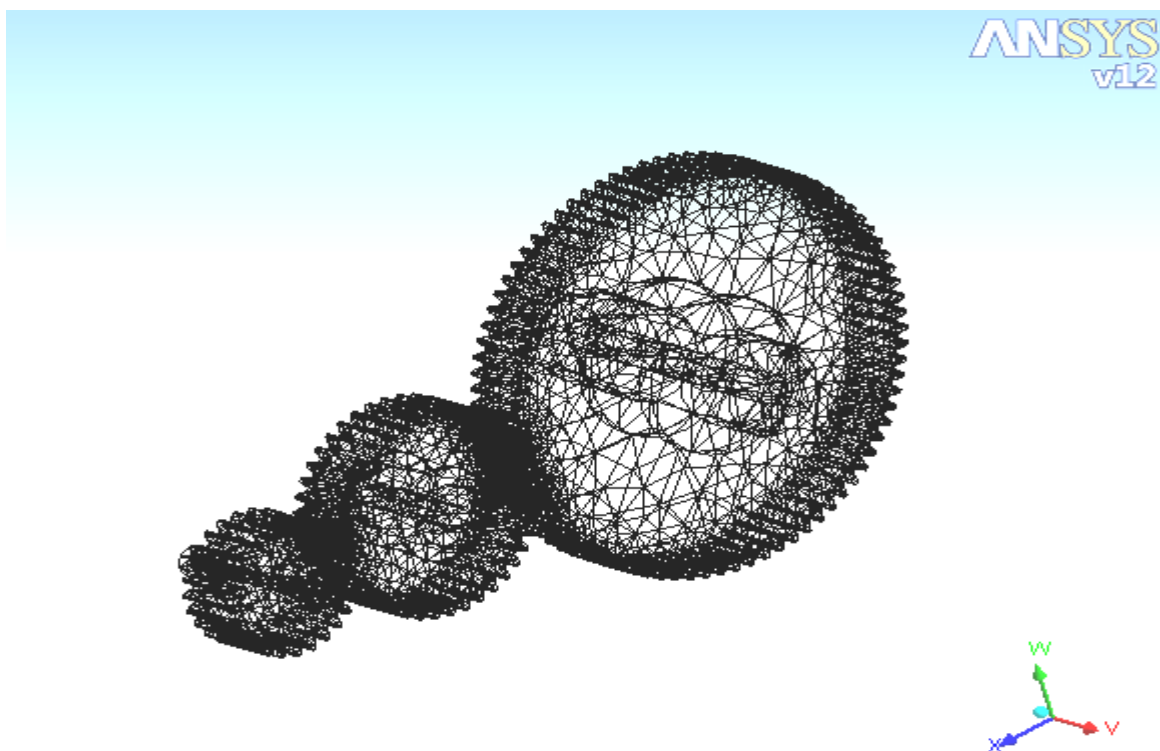
$$P_d = m \times N = 8 \times 80 = 640 \text{mm for spur gear}$$

Where m=module and N=number of teeth, P_d = Pitch circle diameter.

The steps for the analysis are briefly illustrated below. The following figures show the geometry, the meshing and the boundary condition set ups for the gear train in ANSYS work bench



(a) Geometric model of gear train



(b) Meshing

Fig 4-21 (a) Geometry (b) Mesh for the gear train

After the geometry is well modeled and meshed the next step is setting up of the boundary conditions.

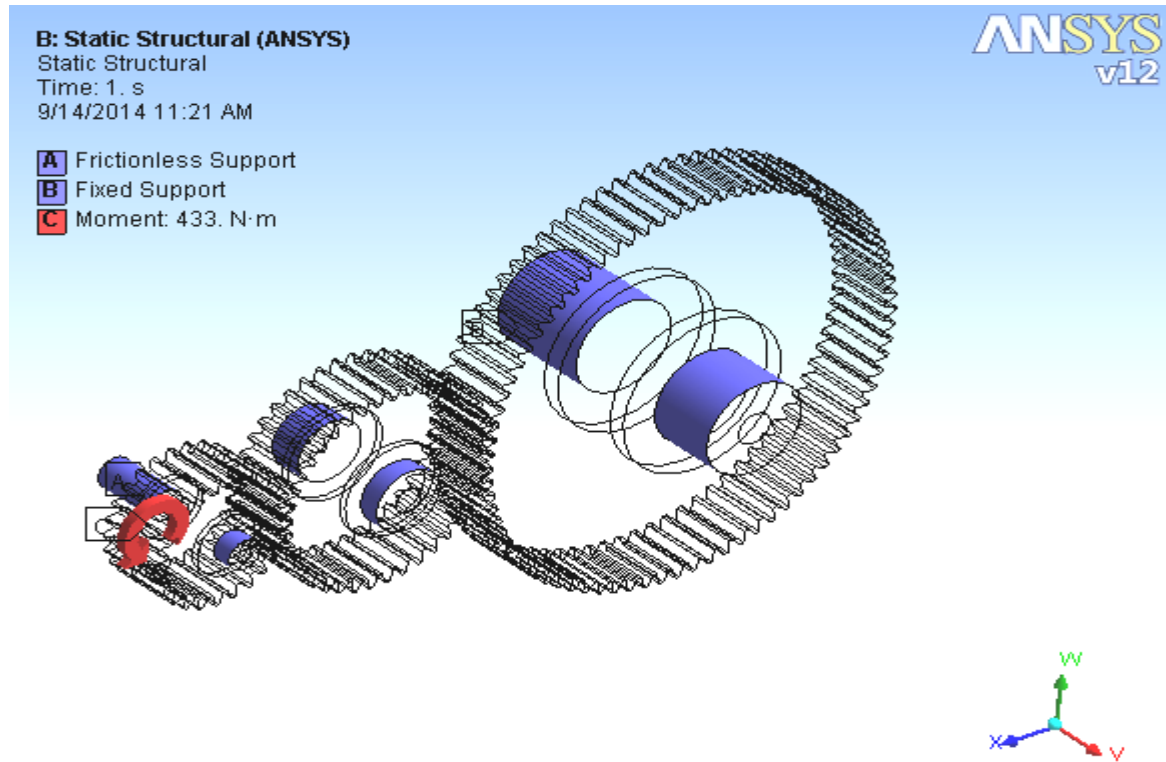
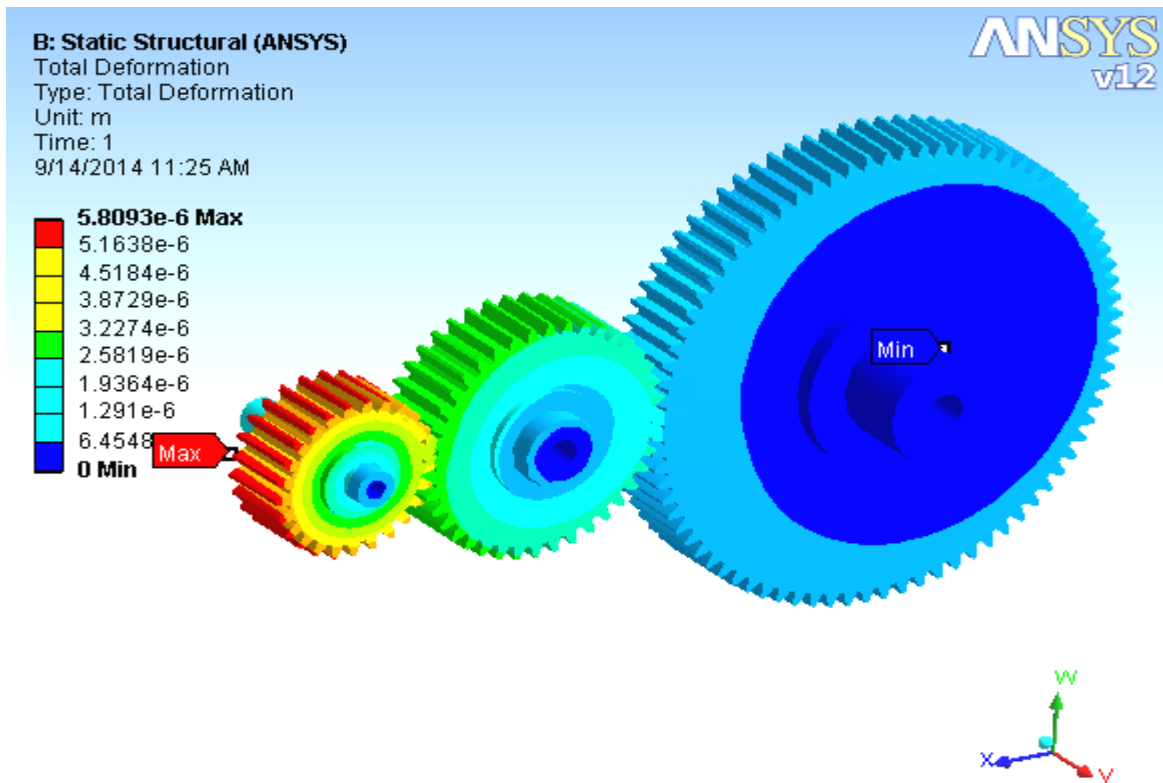
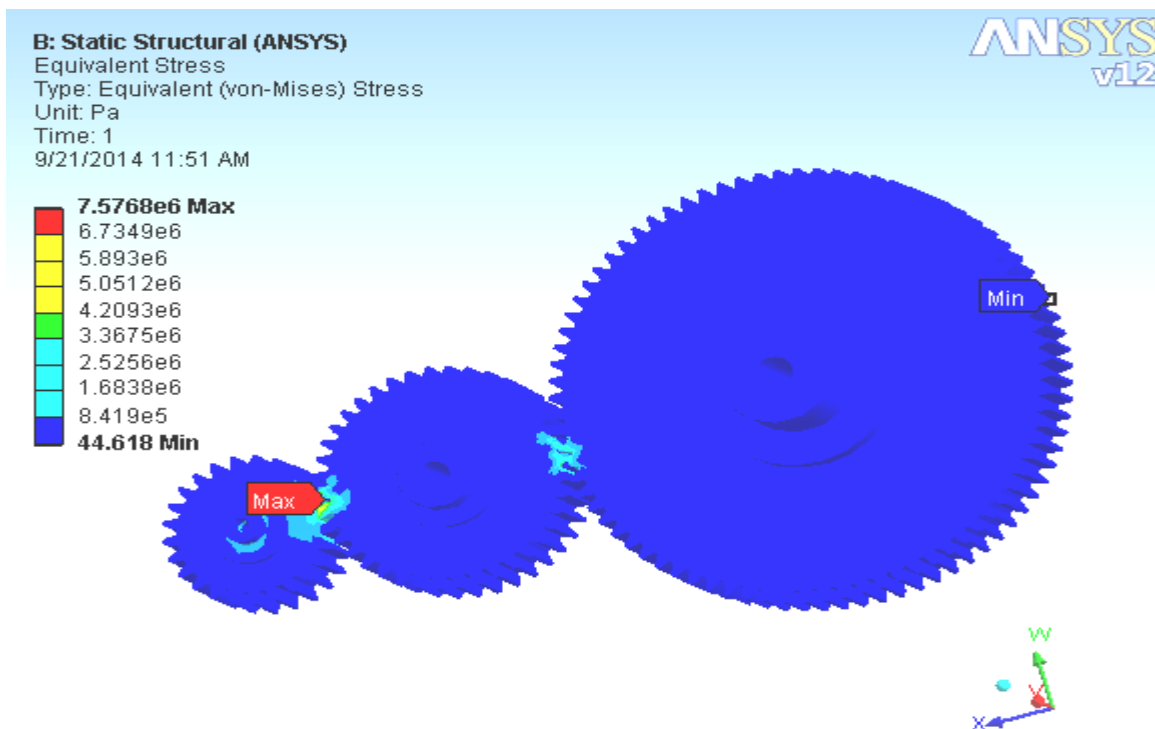


Fig 4-22 Boundary condition set up

The solution phases processes results for total deformation and equivalent stress are shown in the following figures for ANSYS work bench.



(a) Total deformation with idler gear



(b) Equivalent/Vonmises stress

Fig 4-23 (a) Total deformation and (b) Equivalent stress

4.3.6 Fatigue Analysis for the Gear Trains with Idler Gear in ANSYS workbench

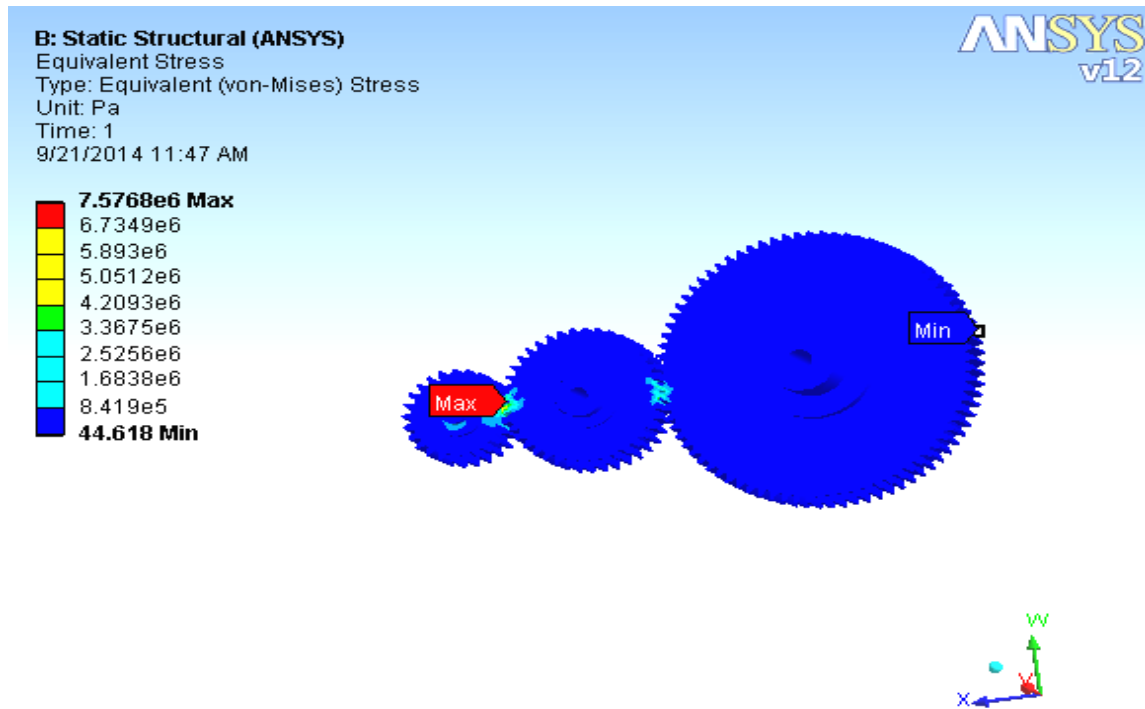
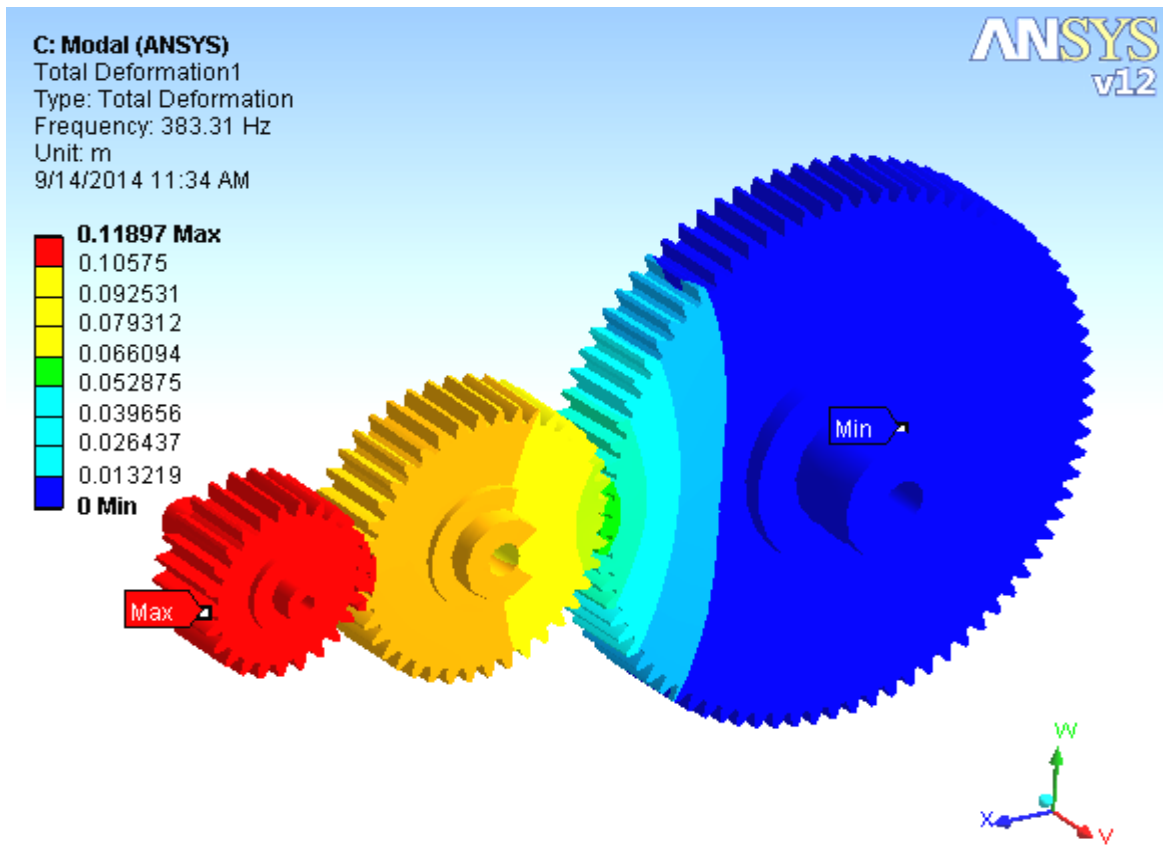


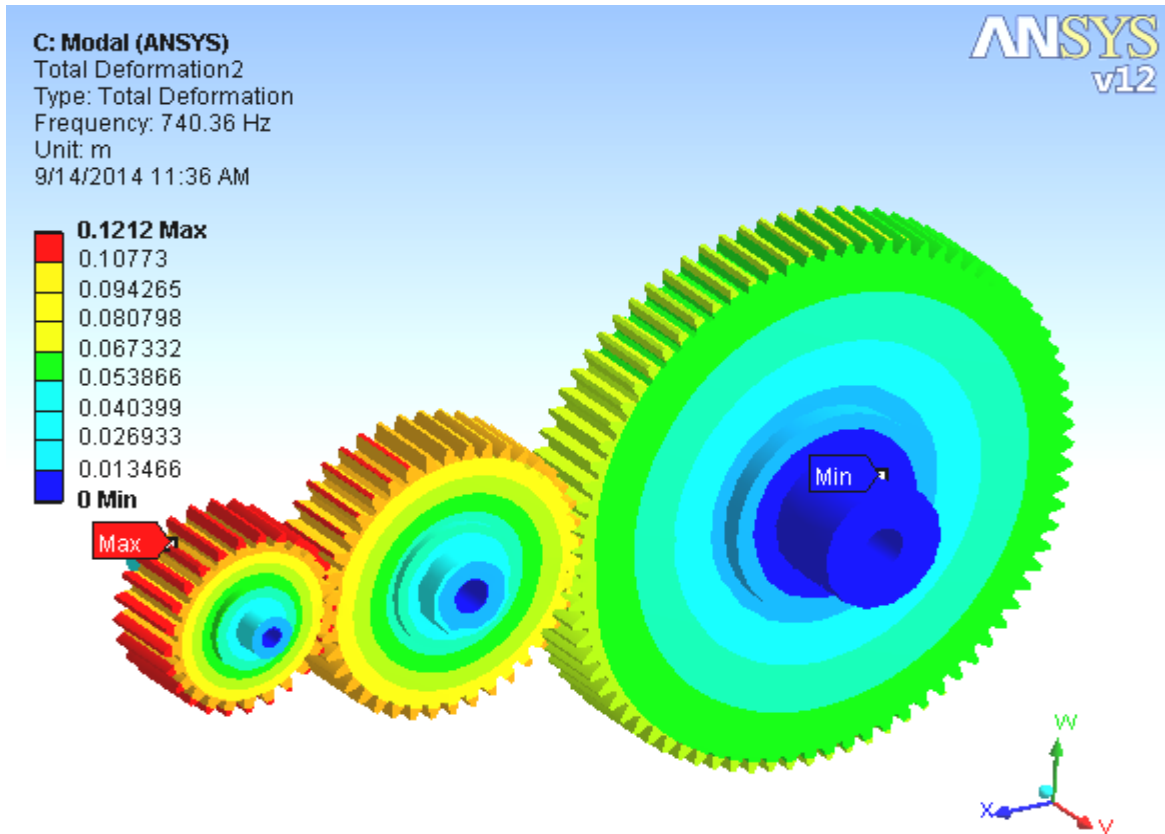
Fig 4-24 Fatigue/ Equivalent alternating stress/

4.3.7 Harmonic Response and Modal Analysis for the Gear Train with Idler gear

.The cause of vibration source is a sudden application of an impulsive force that lasts for a very short period of time or long period of time causing an impulsive excitation resulting in torsional vibration. The source of vibration can be from unbalanced rotation or variable load application. As a result of this kind of vibration, mechanical component will function abnormally. As illustrated above, stress and deformation results. The following solution phase of the ANSYS shows mode shapes and harmonic excitation. Total deformations mode shape and equivalent stress are shown below for modal analysis in ANSYS workbench. The total deformation is the summation of all the directional deformations mode shapes at the specified frequencies.



Total deformation mode shape 1



Total deformation mode shape 2

Fig 4-25 Total deformation mode shape 1 and total deformation mode shape 2

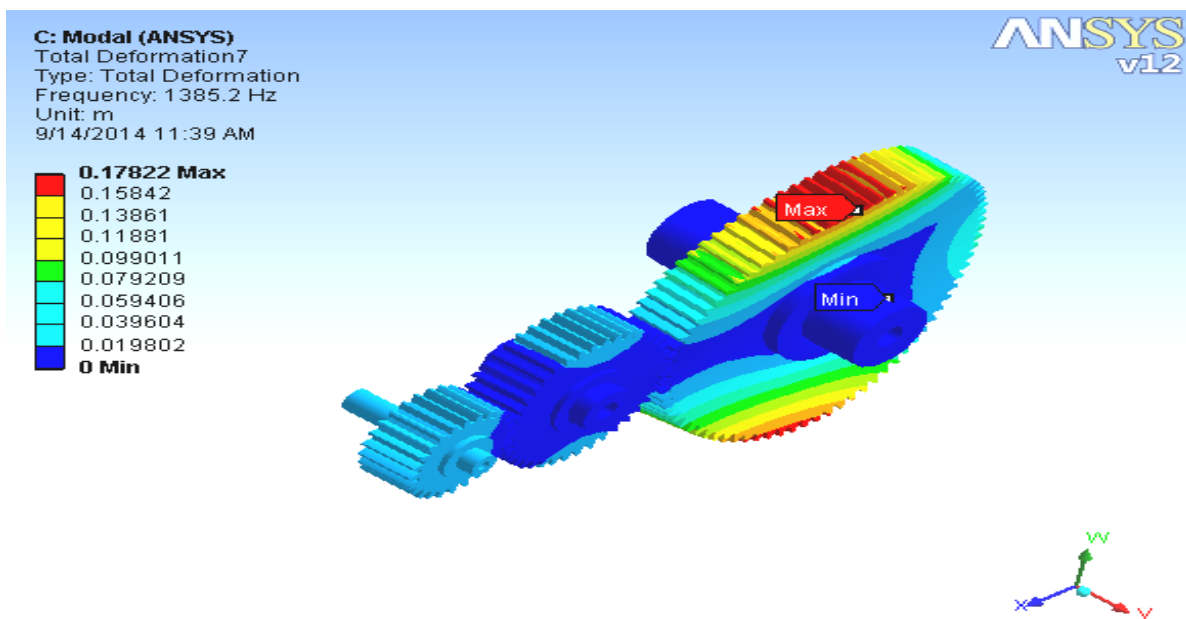


Fig 4-25 Total deformation 7

As a result of the total deformation mode shapes stress will also be induced in the system.

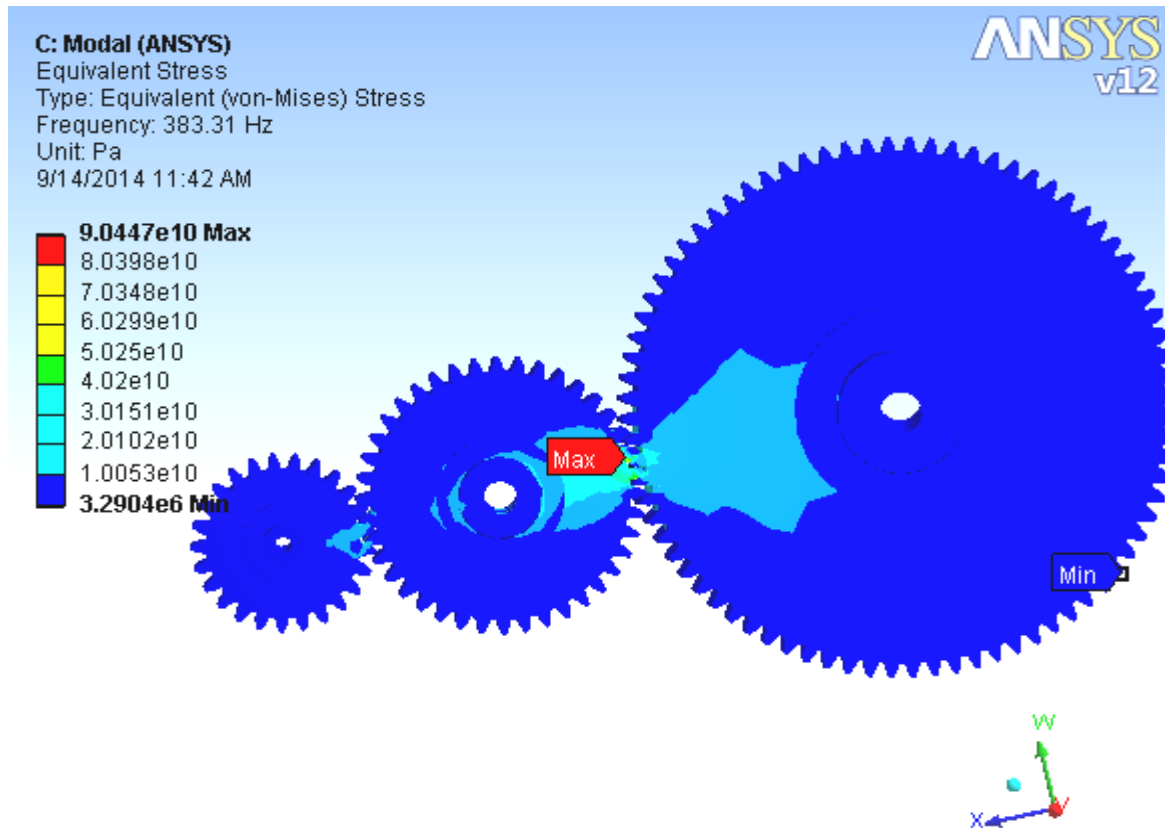


Fig 4-26 Equivalent stress

The maximum value occurred is much greater than the allowable material specification because the initial frequency range is so large that it can cause maximum stress. Moreover, the stress due to static condition is assumed as an initial condition or pre-stressed.

The next figure illustrates the equivalent Von-mises stress as a result of harmonic excitation in case the vibration will occur in the gear box at much higher frequencies and constant damping ratio of 0.6. Harmonic response occurs as sinusoidal; if the applied load varies as a sine or cosine function. Examples of harmonic motion include the vibration produced by an unbalanced rotating machine, the oscillations of a tall chimney due to vortex shedding in a steady wind, and the vertical motion of a train on sinusoidal track irregularities. Harmonic response total deformation and equivalent stress are shown below.

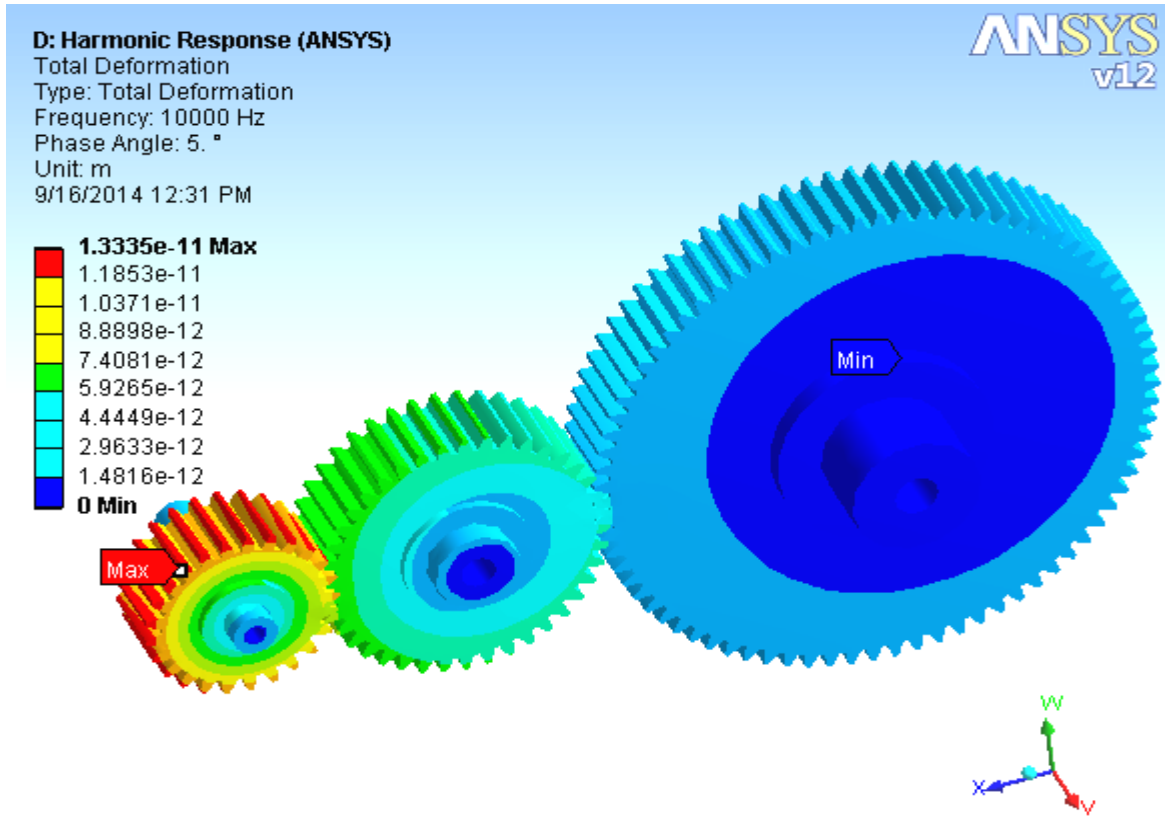
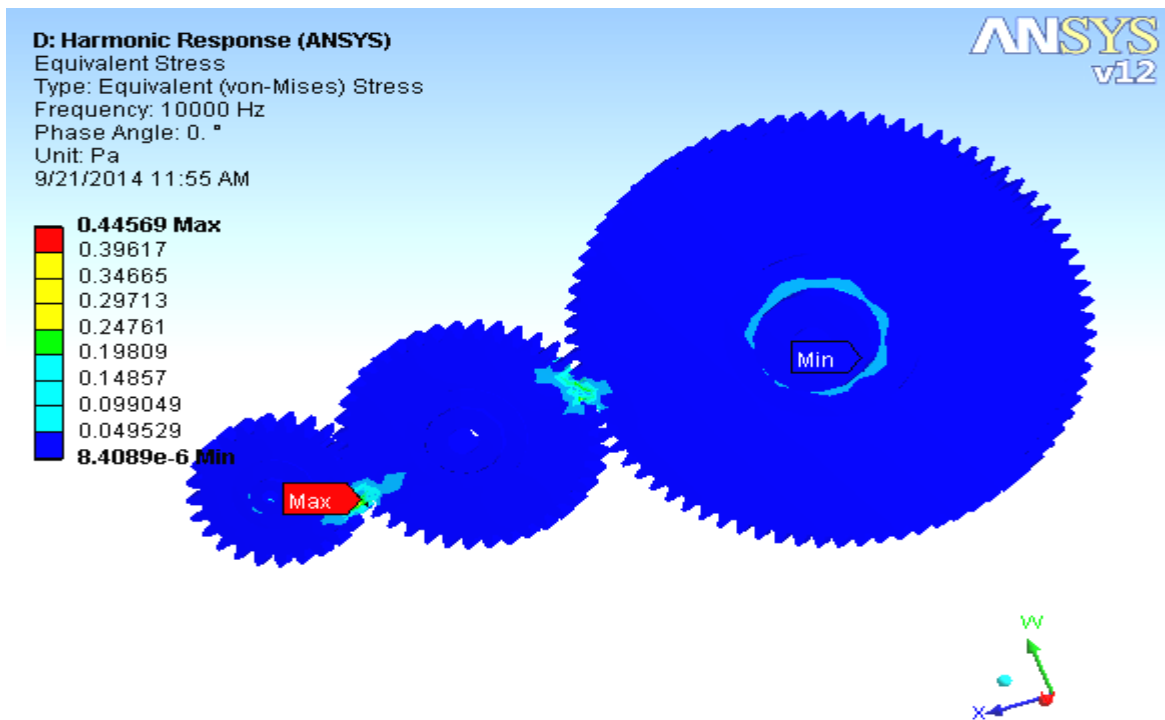
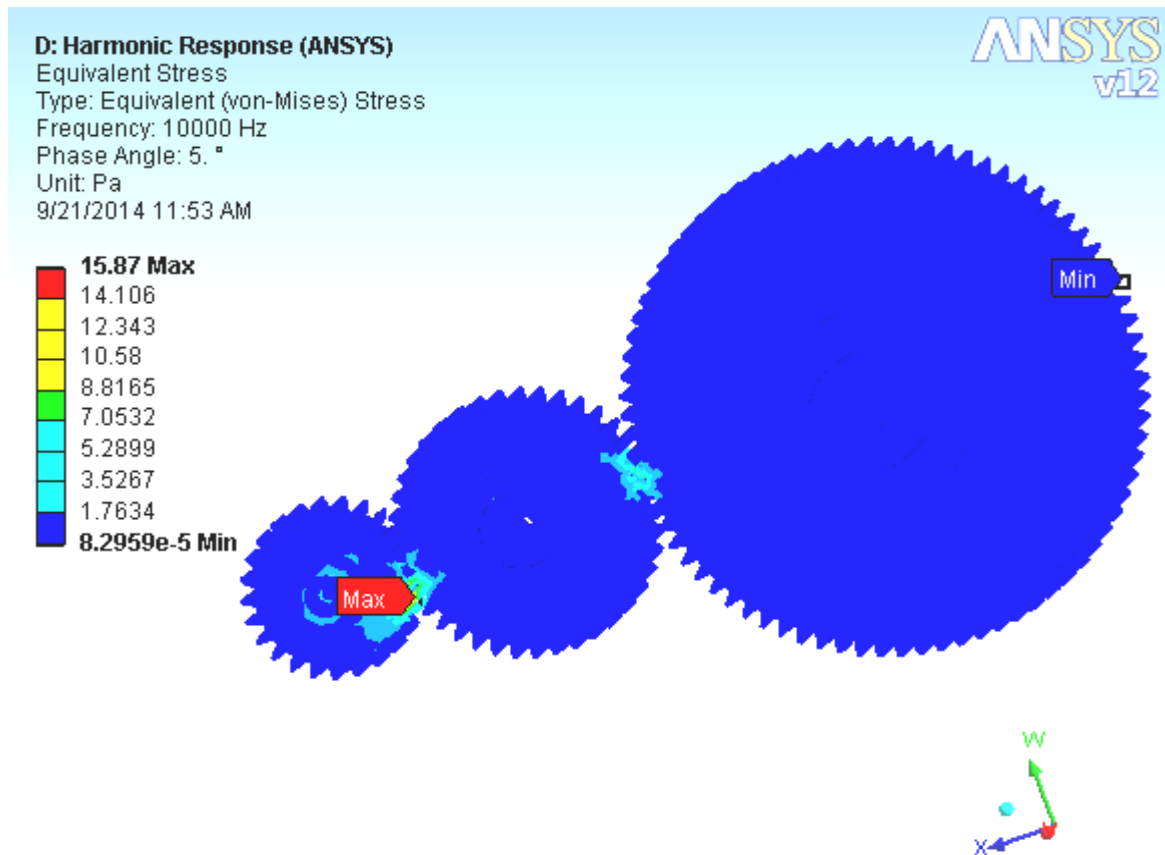


Fig4-27 harmonic response total deformation



Harmonic excitation equivalent stress with phase angle of 0.



(b) Harmonic response equivalent stress with phase angle of 5

Fig 4-28 Harmonic response equivalent stress with phase angle of (a) 0 and (b) 5

As compared to the modal analysis, harmonic excitation is small. However it requires quite careful vibration isolation. Note that in the analysis for harmonic excitation, the constant damping is taken as 0.6. In addition to that when damping constants have increased the result for total deformation will be much reduced. As a result vibration will also be reduced. As mentioned, as phase angle increases the deformation and equivalent stress increases.

4.4 Contact Stress Analysis of the Gear train with Idler Gear in ANSYS

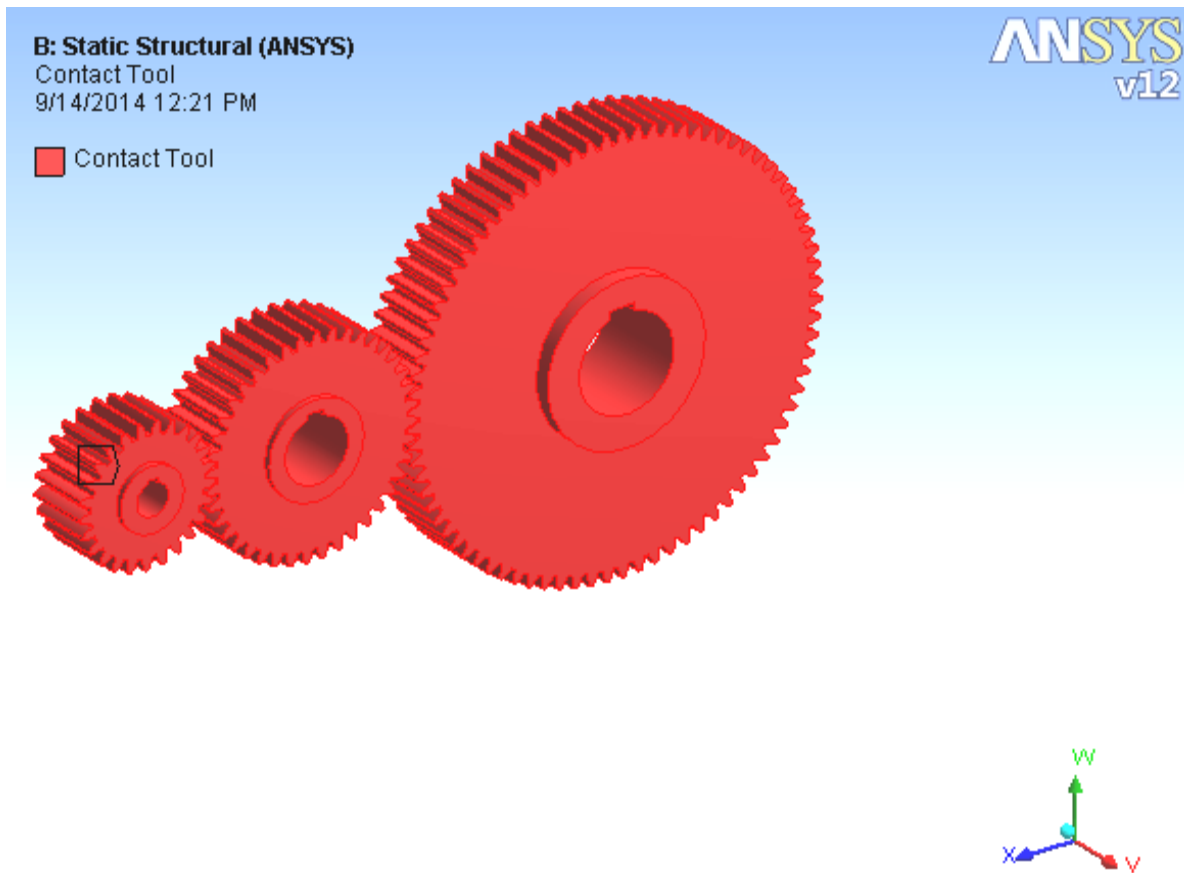
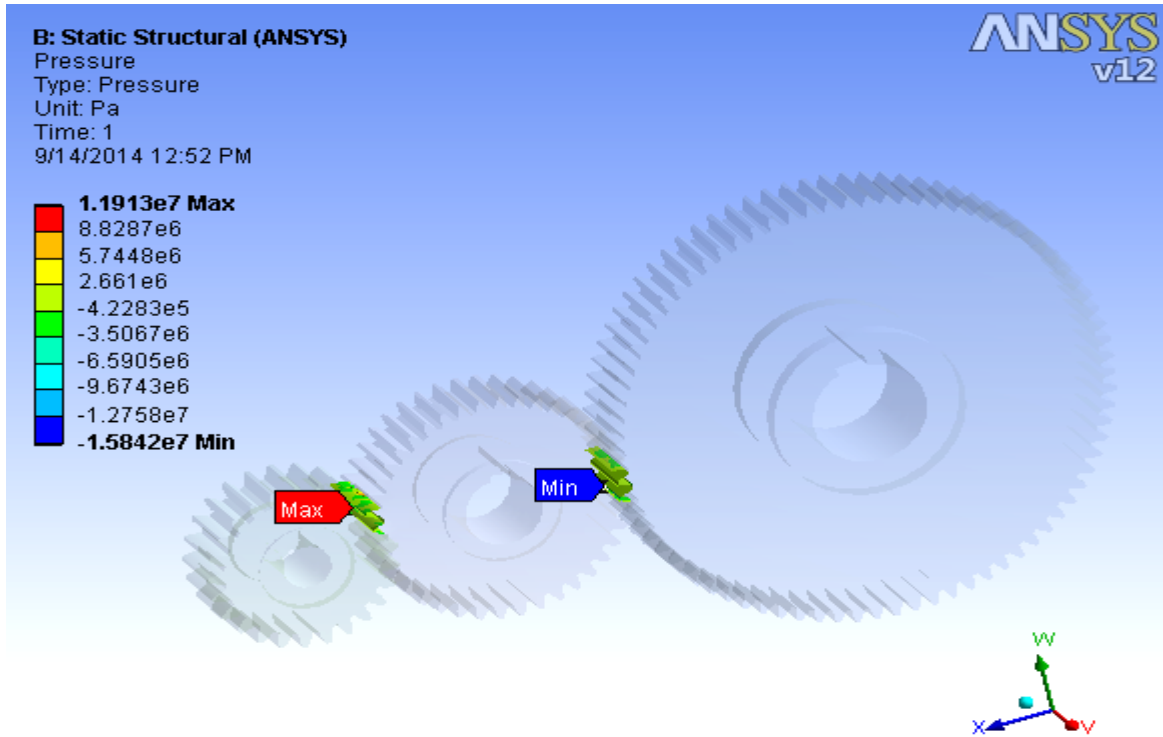
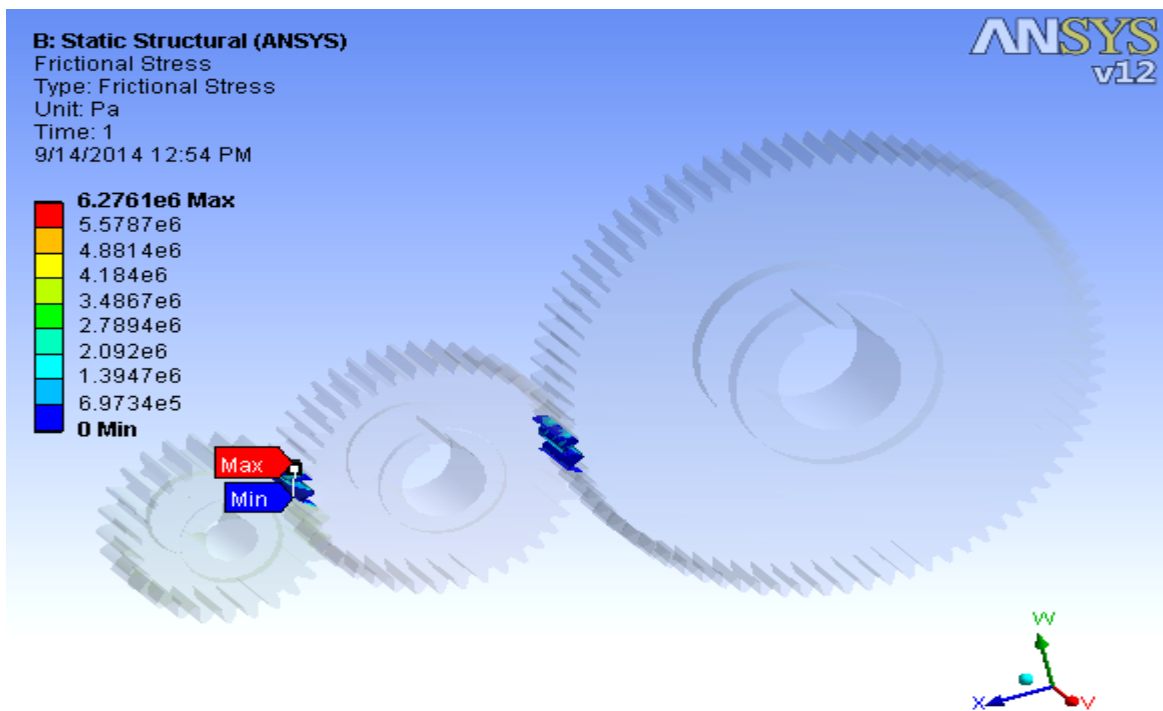


Fig 4-29 Contact set up

The following figures shows the results of pressure and frictional stress in the gears due to contact between the mating gears. Contact Stresses are severe at the contact surfaces.



(a) Pressure due to contact



(b) Frictional stress

Fig 4-30 (a) Pressure and (b) Frictional stress

The following table 4.5 briefly describes and compares some results of the ANSYS Work bench for both types of the gear trains.

Table 4-5 Some Useful ANSYS Work bench results

ANSYS Solution phase	Solution type (Maximum Result)	For the gear train without idler gear	For the gear train with idler gear
		Maximum result	Maximum result
ANSYS work bench static Analysis	Total deformation	0.0113mm	0.0058mm
	Equivalent stress/Vonmises stress	4.43Mpa	8.26Mpa
Modal(Ansys)	Total deformationI	210mm	130mm
	Equivalent Stress	19340 Mpa	6072Mpa
ANSYS harmonic response	Harmonic response total deformation	0.001mm	0.0002mm
	Harmonic response equivalent stress	2.54Mpa	0.76Mpa
Static analysis for contact surfaces	Frictional stress due to contact	4.25 Mpa	6.41Mpa
	Pressure due to contact	4.48Mpa	20Mpa
Static analysis for fatigue loading	Equivalent alternating stress due to fatigue loading	4.43Mpa	8.26Mpa

Note that type of material selected is structural steel having ultimate tensile strength 460 Mpa without any kind material treatment. If the structural steel is heat treated the material can have ultimate tensile strength of 2000Mpa. In addition, only modal equivalent stress at higher frequency is severe because static analysis result is considered as a pre stressed for modal ANSYS. Special treatment should be done for the material in order to resist the stress due to modal, fatigue and contact stress. For example, ANSI4340 normalized at 870°C alloy steel has ultimate tensile strength of 1279Mpa.

4.6 Validation of FEM (Finite Element Method)

The Finite Element Method result is validated for some results of the theoretical method executed for the pinion and the spur gear separately. The FEM is compared with that of the best known theoretical method of calculating gear root bending stress such as AGMA, Lewis bending equation and Hertz Contact stress. The following table illustrates some results of the Finite Element Method, particularly ANSYS work bench results in comparison with the theoretical AGMA methods for bending or normal stress for both pinion and spur gear when each unit is treated separately.

Table4-6 Comparison of Normal Stress for AGMA and FEM

	Pinion	Spur
	Normal Stress	
AGMA	14.8Mpa	2.3Mpa
FEM	15.5Mpa	3.2Mpa

The results are almost conformable and therefore, the FEM is validated through the theoretical method of AGMA. The difference is that AGMA neglects some effects on the gear tooth.

4.7 Results and Discussions

The 3D models of the gear train are conducted by FEM, which is used to predict and determine the different kinds of deformations and stresses in the gear train structure for the type of drives mentioned. Equivalent von mises stress, fatigue stress, alternating fatigue stress, contact stress and the different kinds of vibration such as Modal (Ansys) and harmonic excitation are determined in both gear trains without the idler gear and gear train with the idler gear. This helps to make an optimum design for traction of the rail vehicles in the design of gear trains. Photo elastic test has been used for determining the stress induced in the gears in order to design reliable gearing system. Photo elasticity is an experimental technique for stress and strain analysis that is particularly useful for members having complicated geometry, complicated loading conditions, or both. When analytical method which is strictly mathematical is cumbersome, photo elastic test is used for such complicated geometry like gear train and so on. Now FEA can be a replacement, reliable and accurate technique in order

to determine stress and strain in the complicated geometry. Moreover photo elastic test is an expensive technique. Even if the highest load sharing takes place when the idler gear is assembled in the gearing system some stresses exceeds that of the gear train without the idler gear this is because the pinion is smaller in size than that of the gear train without the idler gear. Vibration is also severe during the gear train without the ideal gear due to its lighter weight. To minimize the effect of vibration proper dampers at all directions should be assembled for the system. Experimental techniques to test the gear is so expensive according to different scholars, therefore FEM helps to predict the effects on the gear model. Analytical methods such as AGMA (American Gear Manufacturer association) are detailed and have so many limitations. Moreover, the current method which is used to determine contact stress is the Hertzian equation, which only considers contact without any friction but FEM which uses soft ware considers the effect friction too. That is why FEM which utilizes design modeler and solution phase is easy and accurate and conformable with the analytical method.

CHAPTER 5

OPTIMIZATION OF THE GEAR UNITS AND LIMITATION OF THE FINITE ELEMENT METHOD

The process and the method of optimization of gears and limitation of the Finite Element Method are here to be discussed in subsequent section.

5.1 Optimization of the Gear units

Gear optimization and treatments are well known to several gears manufacturing industry but the specific targets and parameters to be modified are not very much clear to them. The gears are power and motion carrier, their design and manufacturing require high precision. With the use of computer power, particularly FEM, which uses design modeler and solver process the under design and over design of gears can be avoided. For example, when ANSYS is used to determine the stress on gears it shows the particular part and target surface which is highly loaded. Hence, these specific parts require more modification than the rest of unaffected part. From engineering economic point of view, it is acceptable to modify the affected part.

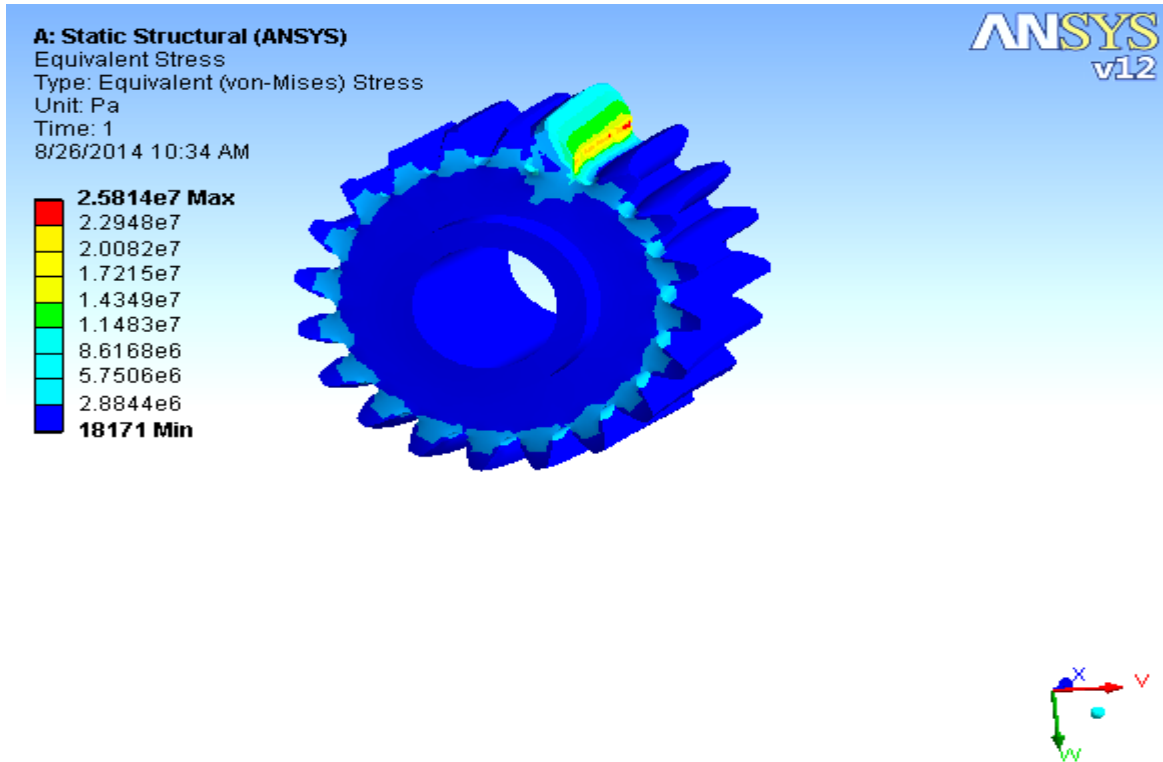
In the analysis which has been taken place in the FEM, the prediction states that the pinion is highly loaded and stressed than that of the spur gear; this means that the pinion requires optimization. Proposal can be made that increasing the size of the pinion can be a relief for induced stresses. The size of any gear can be increased by increasing its module; module increment involves addendum modification because this particularly optimizes the tooth part. As the module is increased the diameter of the gear is also increased too. To counter balance the effect, decreasing the number of teeth and increasing its module by some factor. The other parametric optimization is to make profile modification for root fillet to make relief for residual stress. This involves increasing the pressure angle while keeping the size of gears the same. This can be achieved during the design stage. The other optimization is different kinds of heat treatment, carburizing, nitriding, annealing, and selecting tribologically conformed material. For example structural steel before any execution of material treatment may fail during contact in heavy duty services in the gear train; because the material has ultimate strength of only 450Mpa. The same kind of material can have strength above 1000Mpa after proper heat treatments. To minimize the effect of vibration proper damper should be assembled for the unit. As the mass of a unit increases vibration will reduce. In the future, a

method has to be sought for polymer composite materials as the best gear material. Contact stress can be minimized by designing a suitable lubrication system for gear box. In the case of gear optimization as illustrated in the following figures which are shown by the results of FEM can be one kind of method to reduce the stress by increasing the root fillet. Both gears have the same size and material properties but they differ in their geometric profile around the root of the teeth. In the first figure the root fillet is 10mm whereas in the second case the root fillet is 5mm, therefore the maximum equivalent Vonmises stress in the first pinion is about 25.8Mpa and for the lesser root fillet it is 26.4Mpa. This implies that profile modification will help to reduce the stress in the gears. The following table shows that the engineering properties of both gears.

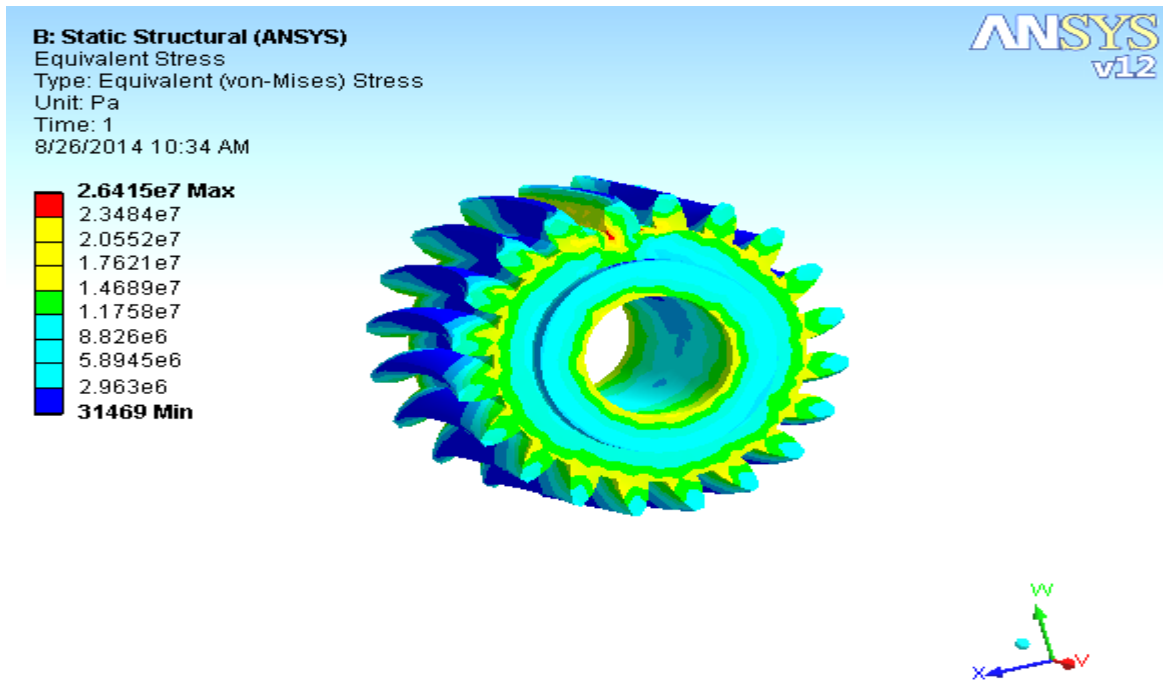
Table5-1 Comparison of pinions profile

Parameters	Pinion with more root fillet	Pinion with less root fillet
Pitch circle diameter	0.2m	0.2m
Face width	0.125m	0.125m
Root fillet	0.01m	0.005m
Material	Structural steel	Structural steel
module	0.01 m	0.01m
Bore diameter	0.05m	0.05m
Vonmises stress	25.8Mpa	26.4Mpa

Note that the rest of parameters are similar. Root fillet and Vonmises stress are different as shown.



(a) Pinion with more root fillet.



(b) Pinion with less root fillet.

Fig 5-1 (a) and (b) Pinion with more and less root fillet

5.3 Limitation of Finite Element Method

Due to the requirement of large computer memory and time, computer program based on FEM can be run only in high speed digital computers. For some problems, there may be considerable amount of input data. Errors may creep up in their preparation and the results thus obtained may also appear to be unacceptable which indicates deceptive state of affairs. Many problems lead to round off errors. Moreover, Finite element methods are extremely versatile and powerful and can enable designers to obtain information about the behavior of complicated structures with almost arbitrary loading. In spite of the significant advances that have been made in developing finite element packages, the results obtained must be carefully examined before they can be used. The most significant limitation of finite element methods is that the accuracy of the obtained solution is usually a function of the mesh resolution. Any regions of highly concentrated stress, such as around loading points and supports, must be carefully analyzed with the use of a sufficiently refined mesh. Special efforts must be made to analyze such problems. An additional concern for any user is that because current packages can solve so many sophisticated problems, there is a strong temptation to "solve" problems without doing the hard work of thinking through them and understanding the underlying mechanics and physical applications. Modern finite element packages are powerful tools that have become increasingly indispensable to mechanical design and analysis. However, they also make it easy for users to make big mistakes. Obtaining solutions with finite element methods often requires substantial amounts of computer and user time. Nevertheless, finite element packages have become increasingly indispensable to mechanical design and analysis.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The 3D models of the gear train in FEM, particularly ANSYS work bench are able to predict accurately and easily all the results of static analysis, mode shapes, contact stresses and harmonic response. It can also show the critical section or parts where failure will mostly occur. All the types of the gear trains are well analyzed in all types of analysis for the system. Therefore, complicated geometry can be analyzed for stresses and deformations by using FEM without the use of any experimental techniques. According to the results obtained in ANSYS work bench for both types of the gear trains, the pinion is highly loaded and stressed. Moreover, the contact surfaces specially the tooth part are severely stressed and the stresses are maximum at root fillet. Therefore special treatment and design are required for affected parts of the gears. The FEM also shows that highly stressed surface is the tooth.

The 3D models of the gear train with idler gear and without idler gear in ANSYS also predict the vibration condition of the unit for the case of harmonic analysis and mode shapes. In the modal analysis, bending and swaying are found to be more dominant mode shapes for the gear train without idler gear than the gear train with the idler gear; this is because additional weight increment will damp the system not to vibrate from its equilibrium point. In all cases of analysis system of the ANSYS work bench the stress results are not greater than the ultimate tensile strength of the material for gears. To sum up the failure condition of 3D models of the gear trains for the system can be predicted by finite element method. Hence the use of packages of FEM is indispensable in the area of mechanical analysis and design.

6.2 Recommendation

The increasing demand for efficient power transmission requires reliable and efficient gear design. Therefore, the use of the package of the FEM is helpful to analyze the loading and the failure condition of the power transmission, particularly the gear train design. The FEM is reliable, efficient and accurate technique to use and analyze the stresses and the deformations in the 3D models of the gear train. Several contributions have executed in this thesis for modeling the 3D gear train of the drives system of the rail vehicles. Therefore, FEM can be applied to the other types of the gears such as helical, bevel, and worm including suspension

arrangements and different kinds of drives system, such as transverse drives and longitudinal drives.

The use of ANSYS work bench is necessary to analyze the different kinds of analysis such as static analysis and vibration analysis. Therefore, the effects of parameters change in design can easily decided by designer.

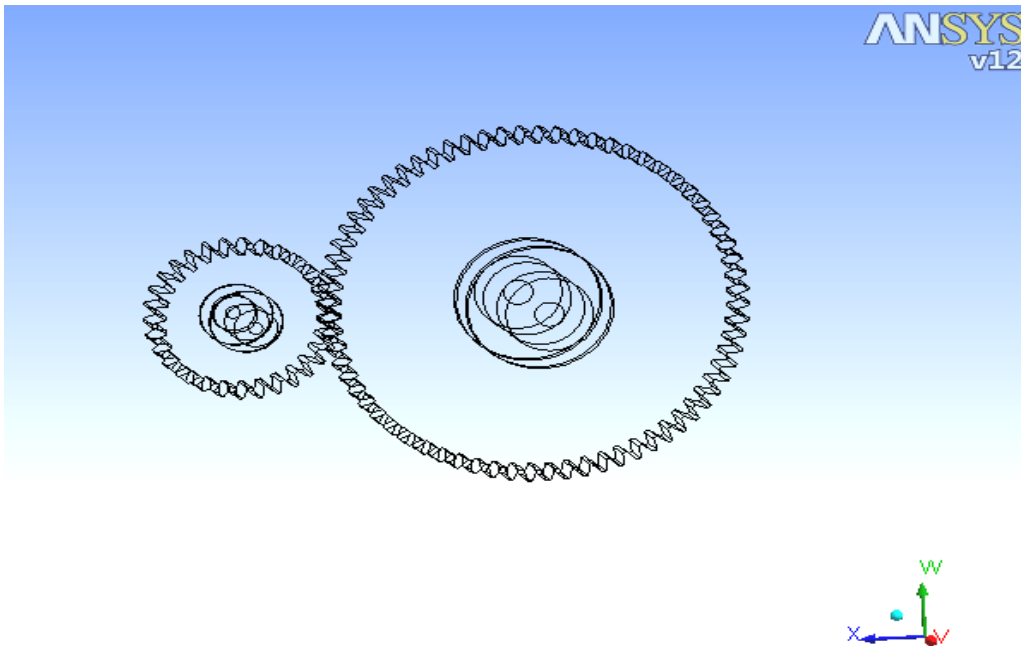
REFERENCES

- [1] Zeping Wei, October, 2004, “Stresses and Deformations in Involute Spur Gears By Finite Element Method”, M.Sc. Thesis, Department of Mechanical Engineering, University of Saskatchewan.
- [2] Ooi Jong Boon, January 2013, “Analysis and Optimization of Portal axle unit using Finite Element Modeling and Simulation”, Master of Engineering Science.Thesis, Faculty of Engineering Science, University Tunku Abdul Rahman.
- [3] Shigley, Copy Rights 2006, “Mechanical Engineering Design, Eighth Edition”, Budynas-Nisbet, Mc Budynas–Nisbett, McGraw Hill, PP 720-730
- [4] David V.Hutton, Copy Rights 2004, “Fundamentals of Finite Element Analysis”, McGraw Hill, pp 1-12
- [5] Lewis, W., 1893. “Investigation of the Strength of Gear Teeth”. Proceedings of Engineering Club, Philadelphia, pp16-23
- [6] Shimamura, S. and Noguchi, Y., 1965. “An Analytical Determination of Speed Factor in Gear Designs”,. Proceedings of the Second SESA International Congress on Experimental Mechanics, Washington, D.C.
- [7] Arikan, S., 2002, “Direct Calculation of AGMA Geometry Factor J by making use of Polynomial Equations” ,Journal of Mechanics Research Communications.
- [8] Chong, T.H., Bae, I. and Park, G., 2002, “A new and generalized methodology to design multi-stage gear drives by integrating the dimensional and the configuration design process”, Journal of Mechanism and Machine Theory, 37 (3), pp. 295-310.
- [9] Townsend, D.P. and Coy, J.J., 1985, “Gearing, NASA Reference Publication”, AVSCOM Technical Report 84-C-15.
- [10] Cavdar, K., Karpat, F. and Babalik, F., 2005, “Computer Aided Analysis of Bending Strength of Involute Spur Gears with Asymmetric Profile”, Journal of Mechanical Design, 127 (1), pp. 477-484.
- [11] Wang, J. and Howard, I., 2005. Finite Element Analysis of High Contact Ratio Spur Gears in Mesh. Journal of Tribology”, vol. 127 (3), pp. 469-483.
- [12] Sackfield, A. and Hills, D.A., 1983. “Some Useful Results in the Classical Hertz Contact Problem”, the Journal of Strain Analysis for Engineering Design, 18 (2), pp. 101-105.
- [13] Archard, J.F., 1953, “Contact and Rubbing of Flat Surfaces” *Journal of Applied Physics*, 24 (8), pp. 981-988.

- [14] Mao, K., 2007, "Gear tooth contact analysis and its application in the reduction of fatigue wear", *Journal of Wear*, 262 (11), pp. 1281–1288.
- [15] Lim, T.C. and Singh, R., 1991, "Vibration Transmission Through Rolling Element Bearings", Part III: Geared Rotor System Studies. *Journal of Sound and Vibration*, 151 (1), pp. 31–54.
- [16] Simon, V., 2000, "FEM stress analysis in hypoid gears", *Journal of Mechanism and Machine Theory*, 35 (9), pp. 1197-1220.
- [17] Draca, S., 2006, "Finite element model of a double-stage helical gear reduction", Master Thesis, University of Windsor, Canada.
- [18] Velez, P. and Maatar, M., 199, " A Mathematical Model for Analyzing the Influence of Shape Deviations and Mounting Errors on Gear Dynamic Behaviour", *Journal of Sound and Vibration*, 191 (5), pp. 629-660
- [19] Robert L. Norton, Copyright 2014, "Machine Design An Integrated Approach", fifth Edition, New Jersey: Prentice-Hall Inc.
- [20] The Rail way Technical hand book, 2012, "Drive Designs", Rail way Technical hand book, SKF group, pp25-75, Sweden
- [21] Railway Systems, technologies and operations across the world, 2014, "Electric Traction Drives", URL www.railway-technical.com/drives.shtml accessed on July20.
- [22] R.N.Lal, Senior Executive Director Standards, 2010, "Selection of Suspension Arrangement of Traction Motors", Ganesh, Director Standards, Electrical Directorate, Research Design and Standards Organization, Lucknow.
- [23] Mr.R.N.Lal, 2014, "Selection of Traction Arrangements of Traction Motors",Rail Electrica,India.
- [24] Chan-Bae Park, Byung-Song Lee and Hyung-Woo Lee, 2013, "Performance Comparison of the Railway Traction IPM motors between concentrated winding and distributed winding" , *J Electr Eng Technol* Vol. 8, Seoul,Korea.
- [25] APTA Commuter Rail Executive Committee, 1999, "Recommended Practice for Passenger Car Axle Design", The American Public Transportation Association 1666 K Street, N. W. Washington, DC, 20006, USA.
- [26] S.Graham Kelly, 2000, "Fundamentals of Mechanical Vibrations 2nd Edition",The University of Akron,McGraw-Hill Higher Education, USA pp 90-100 and pp250-297.

- [27] Dr. Ali Raad Hassan Assistant Professor, 2009, “Contact Stress Analysis of Spur Gear Teeth Pair”, International, University for Science and Technology, Basra, Iraq.
- [28] The Rail way Technical hand book, 2012, “Bogie Designs”, Rail way Technical hand book, SKF group, pp24-41, Sweden.
- [29] C.B. Chaudhar, Dr. R. N. Baxi,2012, “Experimental Investigation and Failure Analysis of Pinion Gear in WAG-9 Locomotive”, Research scholar, Department of Mechanical Engg.G.H. Rasoni of Engg. Nagpur University, India.
- [30] Stephen White, 2010, Overhaul Locomotive Traction Motor”,Rolling Stock Performance Standards,Rail Corpoaration.
- [31] Jacek F.Gieras,Nicola Bianchi,2004, “Electric Motor for Light Traction” ,United Technology Research Center,East Hart Ford,USA.
- [32] B. Soleimani, 2013, “Analytical Approach to Vibration Analysis of the Wheel-rail contact”, Department of Mechanical Engineering, Faculty of Engineering, Yazd University, Yazd, Iran
- [33] Toma Dordea, 2011, Direct-drive Induction Motor, For Railway Traction Applications”, Romanian Railway Company.
- [34] Ethiopian Railways Corporation, Operations and Services Division, Equipment Supply and Technical Services Department, Rolling Stocks Specifications.
- [35] Schmid,Hamrock and Jacobson, 2014, “Fundamentals of machine elements”, CRC Press
- [36] Malcom E Leader, P.E,Practical Implementation of of Torsional Analysis and Field Measurement, “Applied Machinery Dynamics Co. Dickinson,TX77539.
- [37] Ronald L. Eshleman, “Torsional vibration in Reciprocating and Rotating Machines”.
- [38] Raobv, “Harmonically Excited vibration”, Fluid Mechanics.ir.

APPENDIX



Contents

- [Units](#)
- [Model \(B4, C4, D4\)](#)
 - [Geometry](#)
 - [Parts](#)
 - [Coordinate Systems](#)
 - [Connections](#)
 - [Contact Region](#)
 - [Mesh](#)
 - [Refinement](#)
 - [Static Structural \(B5\)](#)
 - [Analysis Settings](#)
 - [Standard Earth Gravity](#)
 - [Loads](#)
 - [Solution \(B6\)](#)
 - [Solution Information](#)
 - [Results](#)
 - [Fatigue Tool](#)
 - [Results](#)
 - [Contact Tool](#)
 - [Results](#)
 - [Modal \(C5\)](#)
 - [Pre-Stress \(Static Structural\)](#)
 - [Analysis Settings](#)
 - [Solution \(C6\)](#)

- [Solution Information](#)
 - [Results](#)
 - [Harmonic Response \(D5\)](#)
 - [Analysis Settings](#)
 - [Loads](#)
 - [Solution \(D6\)](#)
 - [Solution Information](#)
 - [Results](#)
 - [Frequency Response](#)
- [Material Data](#)
 - [Structural Steel](#)

Units

TABLE 1

Unit System	Metric (m, kg, N, s, V, A) Degrees rad/s Celsius
Angle	Degrees
Rotational Velocity	rad/s
Temperature	Celsius

Model (B4, C4, D4)

Geometry

Model (B4, C4, D4)

Geometry

TABLE 2
Model (B4, C4, D4) > Geometry

Object Name	<i>Geometry</i>
State	Fully Defined
Definition	
Source	C:\Users\TOSHIBA\Documents\AWBex072\072_files\dp0\Geom\DM\Geom.agdb
Type	DesignModeler
Length Unit	Meters
Element Control	Program Controlled
Display Style	Part Color
Bounding Box	
Length X	0.35 m

Length Y	0.576 m
Length Z	0.81583 m
Properties	
Volume	3.2692e-002 m ³
Mass	256.63 kg
Scale Factor Value	1.
Statistics	
Bodies	2
Active Bodies	2
Nodes	47898
Elements	27286
Mesh Metric	None
Preferences	
Import Solid Bodies	Yes
Import Surface Bodies	Yes
Import Line Bodies	No
Parameter Processing	Yes
Personal Parameter Key	DS
CAD Attribute Transfer	No
Named Selection Processing	No
Material Properties Transfer	No
CAD Associativity	Yes
Import Coordinate Systems	No
Reader Save Part File	No
Import Using Instances	Yes
Do Smart Update	No

Attach File Via Temp File	Yes
Temporary Directory	C:\Users\TOSHIBA\AppData\Local\Temp
Analysis Type	3-D
Mixed Import Resolution	None
Enclosure and Symmetry Processing	Yes

TABLE 3
Model (B4, C4, D4) > Geometry > Parts

Object Name	<i>o7exsw2-1@spur gear_am-2</i>	<i>o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1</i>
State	Meshed	
Graphics Properties		
Visible	Yes	
Transparency	1	
Definition		
Suppressed	No	
Stiffness Behavior	Flexible	
Coordinate System	Default Coordinate System	
Reference Temperature	By Environment	
Material		
Assignment	Structural Steel	
Nonlinear Effects	Yes	
Thermal Strain Effects	Yes	
Bounding Box		
Length X	0.35 m	0.25 m
Length Y	0.576 m	0.25514 m
Length Z	0.57565 m	0.256 m
Properties		

Volume	2.7816e-002 m ³	4.8753e-003 m ³
Mass	218.36 kg	38.271 kg
Centroid X	-5.e-002 m	-4.5518e-002 m
Centroid Y	-6.1765e-007 m	-2.6143e-008 m
Centroid Z	-0.4 m	-2.5933e-009 m
Moment of Inertia Ip1	7.5185 kg·m ²	0.24941 kg·m ²
Moment of Inertia Ip2	4.2209 kg·m ²	0.1847 kg·m ²
Moment of Inertia Ip3	4.2209 kg·m ²	0.1847 kg·m ²
Statistics		
Nodes	29245	18653
Elements	16513	10773
Mesh Metric	None	

Coordinate Systems

TABLE 4
Model (B4, C4, D4) > Coordinate Systems > Coordinate System

Object Name	<i>Global Coordinate System</i>
State	Fully Defined
Definition	
Type	Cartesian
Ansys System Number	0.
Origin	
Origin X	0. m
Origin Y	0. m
Origin Z	0. m
Directional Vectors	
X Axis Data	[1. 0. 0.]
Y Axis Data	[0. 1. 0.]

Z Axis Data	[0. 0. 1.]
-------------	--------------

Connections

TABLE 5
Model (B4, C4, D4) > Connections

Object Name	<i>Connections</i>
State	Fully Defined
Auto Detection	
Generate Contact On Update	Yes
Tolerance Type	Slider
Tolerance Slider	0.
Tolerance Value	2.6456e-003 m
Face/Face	Yes
Face/Edge	No
Edge/Edge	No
Priority	Include All
Group By	Bodies
Search Across	Bodies
Revolute Joints	Yes
Fixed Joints	Yes
Transparency	
Enabled	Yes

TABLE 6
Model (B4, C4, D4) > Connections > Contact Regions

Object Name	<i>Contact Region</i>
State	Fully Defined
Scope	
Scoping Method	Geometry Selection
Contact	5 Faces
Target	5 Faces

Contact Bodies	o7exsw2-1@spur gear_am-2
Target Bodies	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW ---S30B100H120L60N-1
Definition	
Type	Bonded
Scope Mode	Automatic
Behavior	Symmetric
Suppressed	No
Advanced	
Formulation	Pure Penalty
Normal Stiffness	Program Controlled
Update Stiffness	Never
Pinball Region	Program Controlled

MeshTABLE 7

Model (B4, C4, D4) > Mesh

Object Name	<i>Mesh</i>
State	Solved
Defaults	
Physics Preference	Mechanical
Relevance	0
Sizing	
Use Advanced Size Function	Off
Relevance Center	Coarse
Element Size	Default
Initial Size Seed	Active Assembly
Smoothing	Medium
Transition	Fast
Span Angle Center	Coarse
Minimum Edge Length	4.8659e-003 m

Inflation	
Use Automatic Tet Inflation	None
Inflation Option	Smooth Transition
Transition Ratio	0.272
Maximum Layers	5
Growth Rate	1.2
Inflation Algorithm	Pre
View Advanced Options	No
Advanced	
Shape Checking	Standard Mechanical
Element Midside Nodes	Program Controlled
Straight Sided Elements	No
Number of Retries	Default (4)
Rigid Body Behavior	Dimensionally Reduced
Mesh Morphing	Disabled
Pinch	
Pinch Tolerance	Please Define
Generate on Refresh	No
Statistics	
Nodes	47898
Elements	27286
Mesh Metric	None

TABLE 8
Model (B4, C4, D4) > Mesh > Mesh Controls

Object Name	<i>Refinement</i>
State	Fully Defined
Scope	
Scoping Method	Geometry Selection

Geometry	17 Faces
Definition	
Suppressed	No
Refinement	1

Static Structural (B5)

TABLE 9
Model (B4, C4, D4) > Analysis

Object Name	<i>Static Structural (B5)</i>
State	Solved
Definition	
Physics Type	Structural
Analysis Type	Static Structural
Solver Target	ANSYS Mechanical
Options	
Environment Temperature	22. °C
Generate Input Only	No

TABLE 10
Model (B4, C4, D4) > Static Structural (B5) > Analysis Settings

Object Name	<i>Analysis Settings</i>
State	Fully Defined
Step Controls	
Number Of Steps	1.
Current Step Number	1.
Step End Time	1. s
Auto Time Stepping	Program Controlled
Solver Controls	
Solver Type	Program Controlled
Weak Springs	Program Controlled
Large Deflection	Off

Inertia Relief	Off
Nonlinear Controls	
Force Convergence	Program Controlled
Moment Convergence	Program Controlled
Displacement Convergence	Program Controlled
Rotation Convergence	Program Controlled
Line Search	Program Controlled
Output Controls	
Calculate Stress	Yes
Calculate Strain	Yes
Calculate Results At	All Time Points
Analysis Data Management	
Solver Files Directory	C:\Users\TOSHIBA\Documents\AWBex072\072_files\dp0\SYS\MECH\
Future Analysis	Prestressed analysis
Scratch Solver Files Directory	
Save ANSYS db	Yes
Delete Unneeded Files	Yes
Nonlinear Solution	No
Solver Units	Active System
Solver Unit System	mks

TABLE 11
Model (B4, C4, D4) > Static Structural (B5) > Accelerations

Object Name	<i>Standard Earth Gravity</i>
State	Fully Defined
Scope	
Geometry	All Bodies
Definition	
Coordinate System	Global Coordinate System

X Component	-0. m/s ² (ramped)
Y Component	-0. m/s ² (ramped)
Z Component	-9.8066 m/s ² (ramped)
Suppressed	No
Direction	-Z Direction

FIGURE 1
Model (B4, C4, D4) > Static Structural (B5) > Standard Earth Gravity

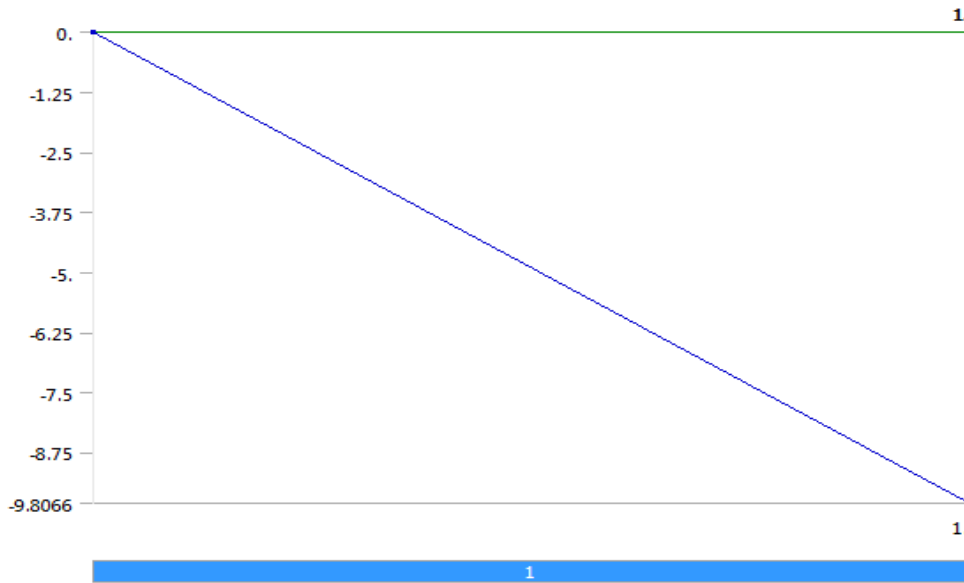
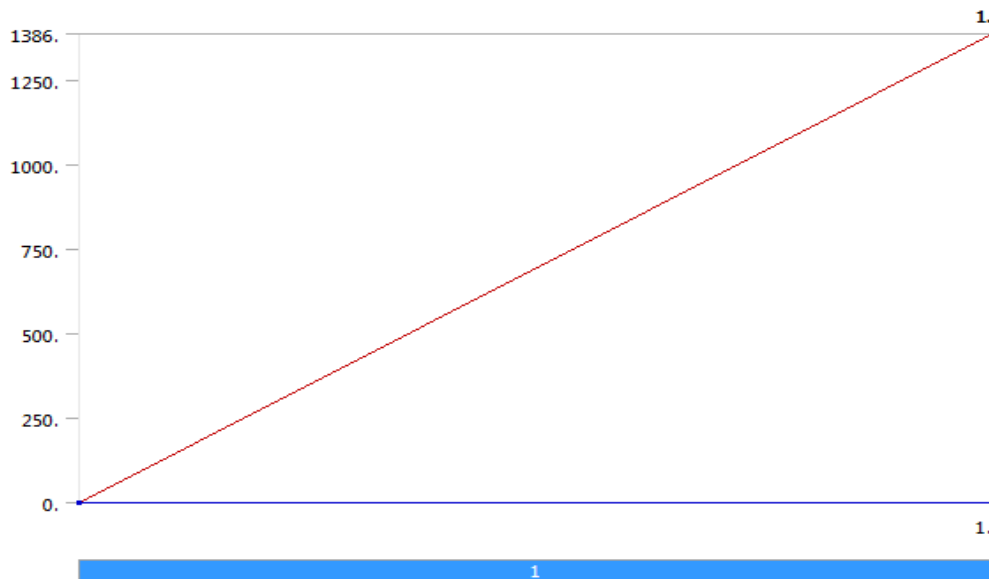


TABLE 12
Model (B4, C4, D4) > Static Structural (B5) > Loads

Object Name	<i>Moment</i>	<i>Fixed Support 2</i>	<i>Frictionless Support</i>
State	Fully Defined		
Scope			
Scoping Method	Geometry Selection		
Geometry	4 Faces	2 Faces	
Definition			
Type	Moment	Fixed Support	Frictionless Support
Define By	Components		
Coordinate System	Global Coordinate System		
X Component	1386. N·m (ramped)		

Y Component	0. N·m (ramped)	
Z Component	0. N·m (ramped)	
Suppressed	No	
Behavior	Deformable	
Advanced		
Pinball Region	All	

FIGURE 2
Model (B4, C4, D4) > Static Structural (B5) > Moment



Solution (B6)

TABLE 13
Model (B4, C4, D4) > Static Structural (B5) > Solution

Object Name	<i>Solution (B6)</i>
State	Solved
Adaptive Mesh Refinement	
Max Refinement Loops	1.
Refinement Depth	2.

TABLE 14
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Solution Information

Object Name	<i>Solution Information</i>
State	Solved

Solution Information	
Solution Output	Solver Output
Newton-Raphson Residuals	0
Update Interval	2.5 s
Display Points	All

TABLE 15
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Results

Object Name	<i>Total Deformation</i>	<i>Equivalent Stress</i>	<i>Maximum Shear Stress</i>	<i>Normal Stress</i>	<i>Maximum Principal Stress</i>
State	Solved				
Scope					
Scoping Method	Geometry Selection				
Geometry	All Bodies				
Definition					
Type	Total Deformation	Equivalent (von-Mises) Stress	Maximum Shear Stress	Normal Stress	Maximum Principal Stress
By	Time				
Display Time	Last				
Calculate Time History	Yes				
Identifier					
Use Average		Yes			
Orientation				X Axis	
Coordinate System				Global Coordinate System	
Results					
Minimum	0. m	86.736 Pa	50.066 Pa	-7.4846e+006 Pa	-6.9389e+006 Pa
Maximum	1.1303e-005 m	1.518e+007 Pa	8.3354e+006 Pa	8.8483e+006 Pa	2.2511e+007 Pa

Minimum Occurs On	o7exsw2-1@spur gear_am-2
Maximum Occurs On	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW ---S30B100H120L60N-1
Information	
Time	1. s
Load Step	1
Substep	1
Iteration Number	1

TABLE 16
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Fatigue Tools

Object Name	<i>Fatigue Tool</i>
State	Solved
Materials	
Fatigue Strength Factor (Kf)	1.
Loading	
Type	Fully Reversed
Scale Factor	1.
Definition	
Display Time	End Time
Options	
Analysis Type	Stress Life
Mean Stress Theory	None
Stress Component	Equivalent (Von Mises)
Life Units	
Units Name	cycles
1 cycle is equal to	1. cycles

FIGURE 3
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Fatigue Tool

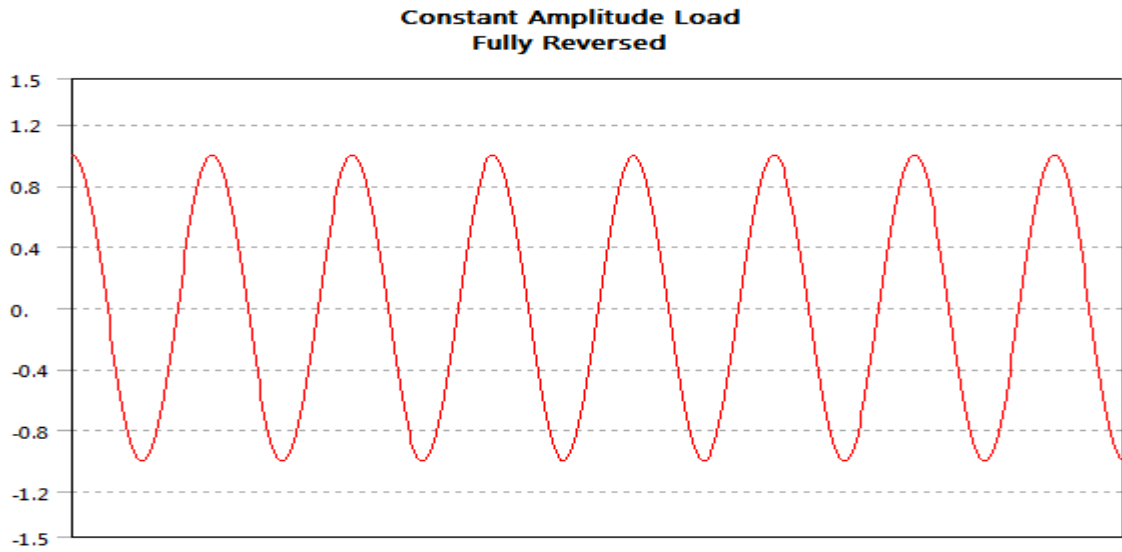


FIGURE 4
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Fatigue Tool
Mean Stress Correction Theory

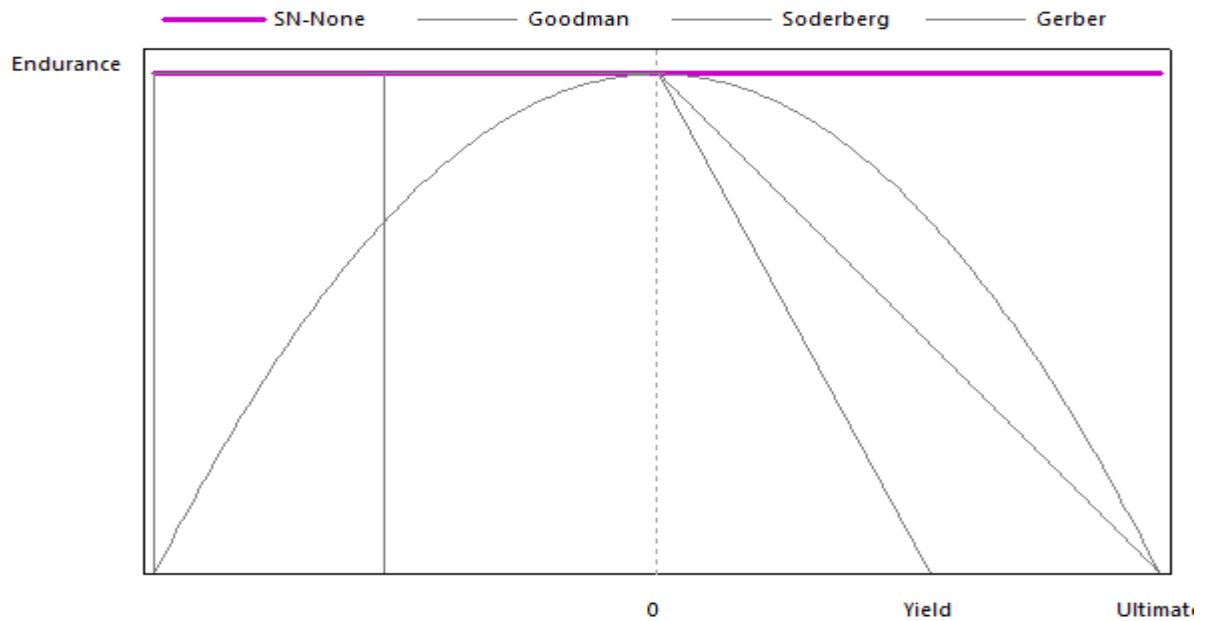


TABLE 17
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Fatigue Tool > Results

Object Name	Life	Safety Factor	Equivalent Alternating Stress
State	Solved		
Scope			
Scoping Method	Geometry Selection		

Geometry	All Bodies		
Definition			
Type	Life	Safety Factor	Equivalent Alternating Stress
Identifier			
Design Life		1.e+009 cycles	
Results			
Minimum	1.e+006 cycles	5.6784	86.736 Pa
Minimum Occurs On	o7exsw2-1@spur gear_am-2	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1	o7exsw2-1@spur gear_am-2
Maximum			1.518e+007 Pa
Maximum Occurs On			o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1

TABLE 18
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Contact Tools

Object Name	<i>Contact Tool</i>
State	Solved
Scope	
Scoping Method	Geometry Selection
Geometry	422 Faces

Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Contact Tool

Name	Contact Side
Contact Region	Both

TABLE 19
Model (B4, C4, D4) > Static Structural (B5) > Solution (B6) > Contact Tool > Results

Object Name	<i>Status</i>	<i>Frictional Stress</i>	<i>Pressure</i>
State	Solved		
Definition			
Type	Status	Frictional Stress	Pressure

By	Time	
Display Time	Last	
Calculate Time History	Yes	
Use Average	Yes	
Identifier		
Information		
Time	1. s	
Load Step	1	
Substep	1	
Iteration Number	1	
Results		
Minimum	0. Pa	-2.9146e+007 Pa
Maximum	2.7275e+007 Pa	4.0548e+007 Pa
Minimum Occurs On	o7exsw2-1@spur gear_am-2	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1
Maximum Occurs On	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1	o7exsw2-1@spur gear_am-2

Modal (C5)

TABLE 20
Model (B4, C4, D4) > Analysis

Object Name	Modal (C5)
State	Solved
Definition	
Physics Type	Structural
Analysis Type	Modal
Solver Target	ANSYS Mechanical
Options	

Generate Input Only	No
---------------------	----

TABLE 21
Model (B4, C4, D4) > Modal (C5) > Initial Condition

Object Name	<i>Pre-Stress (Static Structural)</i>
State	Fully Defined
Definition	
Pre-Stress Environment	Static Structural

TABLE 22
Model (B4, C4, D4) > Modal (C5) > Analysis Settings

Object Name	<i>Analysis Settings</i>
State	Fully Defined
Options	
Max Modes to Find	10
Limit Search to Range	No
Solver Controls	
Solver Type	Program Controlled
Output Controls	
Calculate Stress	Yes
Calculate Strain	No
Analysis Data Management	
Solver Files Directory	C:\Users\TOSHIBA\Documents\AWBex072\072_files\dp0\SYS-1\MECH\
Future Analysis	None
Scratch Solver Files Directory	
Save ANSYS db	No
Delete Unneeded Files	Yes
Solver Units	Active System
Solver Unit System	mks

Solution (C6)

TABLE 23
Model (B4, C4, D4) > Modal (C5) > Solution

Object Name	<i>Solution (C6)</i>
State	Solved
Adaptive Mesh Refinement	
Max Refinement Loops	1.
Refinement Depth	2.

The following bar chart indicates the frequency at each calculated mode.

FIGURE 5
Model (B4, C4, D4) > Modal (C5) > Solution (C6)

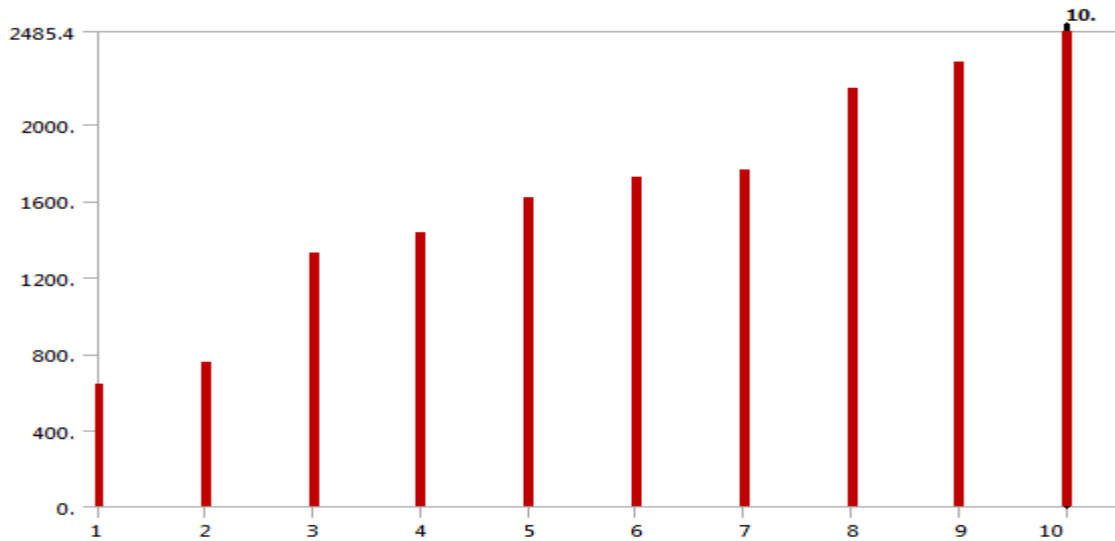


TABLE 24
Model (B4, C4, D4) > Modal (C5) > Solution (C6)

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3

9.	2325.8
10.	2485.4

TABLE 25
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Solution Information

Object Name	<i>Solution Information</i>
State	Solved
Solution Information	
Solution Output	Solver Output
Newton-Raphson Residuals	0
Update Interval	2.5 s
Display Points	All

TABLE 26
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Results

Object Name	<i>Total Deformation1</i>	<i>Total Deformation2</i>	<i>Total Deformation3</i>	<i>Total Deformation4</i>	<i>Total Deformation5</i>
State	Solved				
Scope					
Scoping Method	Geometry Selection				
Geometry	All Bodies				
Definition					
Type	Total Deformation				
Mode	1.	2.	3.	4.	5.
Identifier					
Results					
Minimum	0. m				
Maximum	0.16781 m	0.11861 m	0.17378 m	0.23851 m	0.19857 m
Minimum Occurs On	o7exsw2-1@spur gear_am-2				
Maximum	o7exsw2-1@Metric - Spur gear 8M 30T		o7exsw2-1@spur gear_am-2		

Occurs On	20PA 100FW ---S30B100H120L60N-1				
Information					
Reported Frequency	641.59 Hz	758.4 Hz	1330. Hz	1431.7 Hz	1615.9 Hz

TABLE 27
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation1

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8
10.	2485.4

TABLE 28
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation2

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8

10.	2485.4
-----	--------

TABLE 29
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation3

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8
10.	2485.4

TABLE 30
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation4

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8
10.	2485.4

TABLE 31
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation5

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8
10.	2485.4

TABLE 32
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Results

Object Name	<i>Equivalent Stress</i>	<i>Total Deformation9</i>	<i>Total Deformation10</i>
State	Solved		
Scope			
Scoping Method	Geometry Selection		
Geometry	All Bodies		
Definition			
Type	Equivalent (von-Mises) Stress	Total Deformation	
Mode	1.	9.	10.
Use Average	Yes		
Identifier			
Results			
Minimum	1.4471e+007 Pa	0. m	
Maximum	1.4893e+011 Pa	0.10838 m	0.15318 m
Minimum	o7exsw2-1@spur gear_am-2		

Occurs On			
Maximum Occurs On	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1	o7exsw2-1@spur gear_am-2	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW --- S30B100H120L60N-1
Information			
Reported Frequency	641.59 Hz	2325.8 Hz	2485.4 Hz

TABLE 33
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Equivalent Stress

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8
10.	2485.4

TABLE 34
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation9

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7

8.	2186.3
9.	2325.8
10.	2485.4

TABLE 35
Model (B4, C4, D4) > Modal (C5) > Solution (C6) > Total Deformation10

Mode	Frequency [Hz]
1.	641.59
2.	758.4
3.	1330.
4.	1431.7
5.	1615.9
6.	1723.8
7.	1761.7
8.	2186.3
9.	2325.8
10.	2485.4

Harmonic Response (D5)

TABLE 36
Model (B4, C4, D4) > Analysis

Object Name	<i>Harmonic Response (D5)</i>
State	Solved
Definition	
Physics Type	Structural
Analysis Type	Harmonic Response
Solver Target	ANSYS Mechanical
Options	
Environment Temperature	22. °C
Generate Input Only	No

TABLE 37
Model (B4, C4, D4) > Harmonic Response (D5) > Analysis Settings

Object Name	<i>Analysis Settings</i>
State	Fully Defined
Options	
Range Minimum	0. Hz
Range Maximum	10000 Hz
Solution Intervals	10
Solution Method	Mode Superposition
Cluster Results	No
Modal Frequency Range	Program Controlled
Store Results At All Frequencies	Yes
Output Controls	
Calculate Stress	Yes
Calculate Strain	Yes
Damping Controls	
Constant Damping Ratio	0.6
Beta Damping Define By	Direct Input
Beta Damping Value	0.6
Analysis Data Management	
Solver Files Directory	C:\Users\TOSHIBA\Documents\AWBex072\072_files\dp0\SYS-2\MECH\
Future Analysis	None
Scratch Solver Files Directory	
Save ANSYS db	No
Delete Unneeded Files	Yes
Solver Units	Active System
Solver Unit System	mks

TABLE 38
Model (B4, C4, D4) > Harmonic Response (D5) > Loads

Object Name	<i>Frictionless Support</i>	<i>Fixed Support</i>	<i>Moment</i>
State	Fully Defined		
Scope			
Scoping Method	Geometry Selection		
Geometry	2 Faces		4 Faces
Definition			
Type	Frictionless Support	Fixed Support	Moment
Suppressed	No		
Define By			Components
Coordinate System			Global Coordinate System
X Component			1386. N·m
Y Component			0. N·m
Z Component			0. N·m
Behavior			Deformable
Advanced			
Pinball Region			All

Solution (D6)

TABLE 39
Model (B4, C4, D4) > Harmonic Response (D5) > Solution

Object Name	<i>Solution (D6)</i>
State	Solved
Adaptive Mesh Refinement	
Max Refinement Loops	1.
Refinement Depth	2.

FIGURE 6
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6)

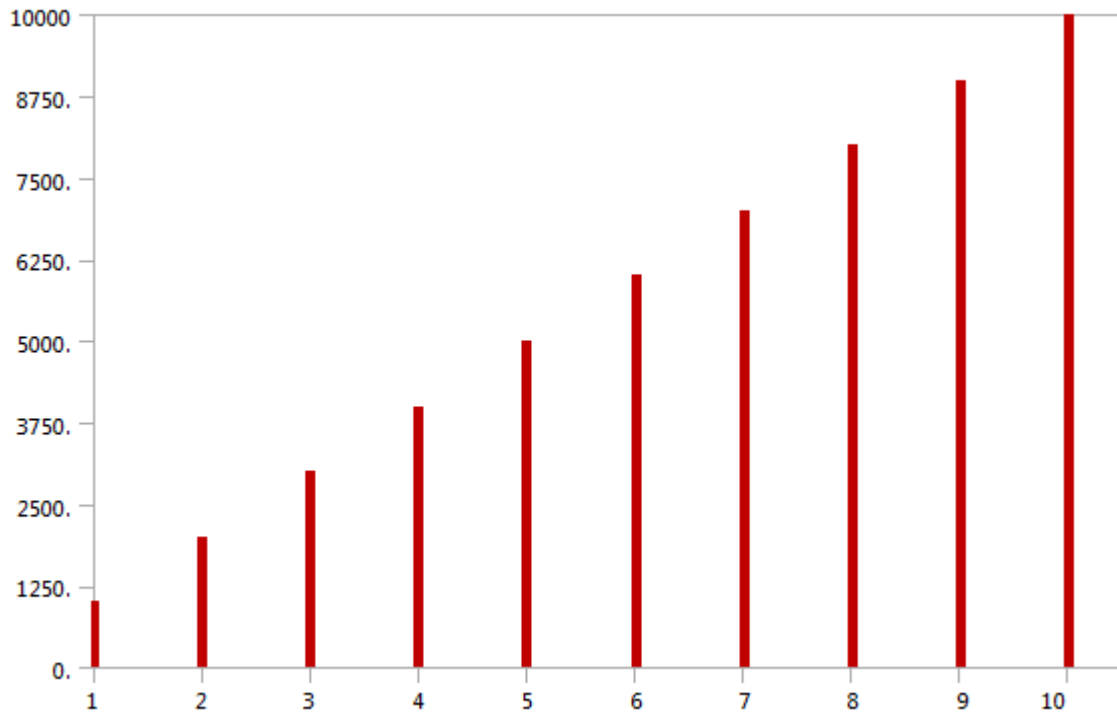


TABLE 40
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Solution Information

Object Name	<i>Solution Information</i>
State	Solved
Solution Information	
Solution Output	Solver Output
Newton-Raphson Residuals	0
Update Interval	2.5 s
Display Points	All

TABLE 41
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Results

Object Name	<i>Equivalent Stress</i>	<i>Total Deformation</i>
State	Solved	
Scope		
Scoping Method	Geometry Selection	
Geometry	All Bodies	

Definition		
Type	Equivalent (von-Mises) Stress	Total Deformation
By	Frequency	
Frequency	Last	
Phase Angle	5. °	
Use Average	Yes	
Results		
Minimum	5.0958e-004 Pa	0. m
Maximum	35.969 Pa	2.7101e-011 m
Minimum Occurs On	o7exsw2-1@spur gear_am-2	
Maximum Occurs On	o7exsw2-1@Metric - Spur gear 8M 30T 20PA 100FW ---S30B100H120L60N-1	
Information		
Reported Frequency	10000 Hz	

FIGURE 7
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Equivalent Stress

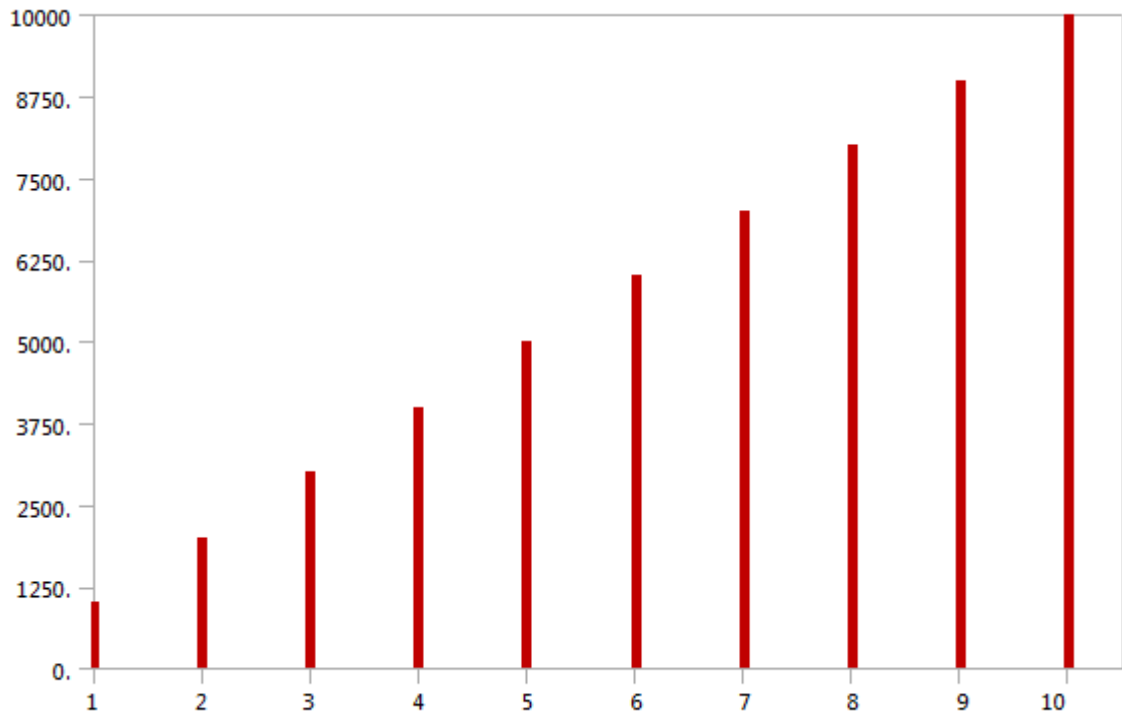


TABLE 42
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Equivalent Stress

Set	Frequency [Hz]
1.	1000.
2.	2000.
3.	3000.
4.	4000.
5.	5000.
6.	6000.
7.	7000.
8.	8000.
9.	9000.
10.	10000

FIGURE 8
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Total Deformation

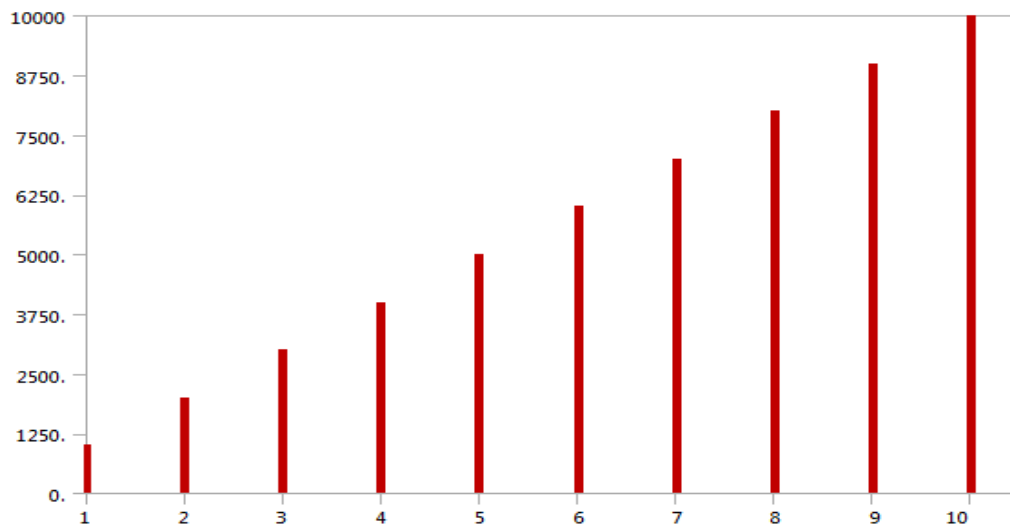


TABLE 43
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Total Deformation

Set	Frequency [Hz]
1.	1000.
2.	2000.
3.	3000.

4.	4000.
5.	5000.
6.	6000.
7.	7000.
8.	8000.
9.	9000.
10.	10000

TABLE 44
Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Result Charts

Object Name	<i>Frequency Response</i>
State	Solved
Scope	
Geometry	422 Faces
Spatial Resolution	Use Average
Definition	
Type	Normal Stress
Orientation	X Axis
Options	
Frequency Range	Use Parent
Minimum Frequency	0. Hz
Maximum Frequency	10000 Hz
Display	Bode
Results	
Maximum Amplitude	2.0796 Pa
Frequency	1000. Hz
Phase Angle	-89.995 °
Real	1.6933e-004 Pa
Imaginary	-2.0796 Pa

FIGURE 9
 Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Frequency Response

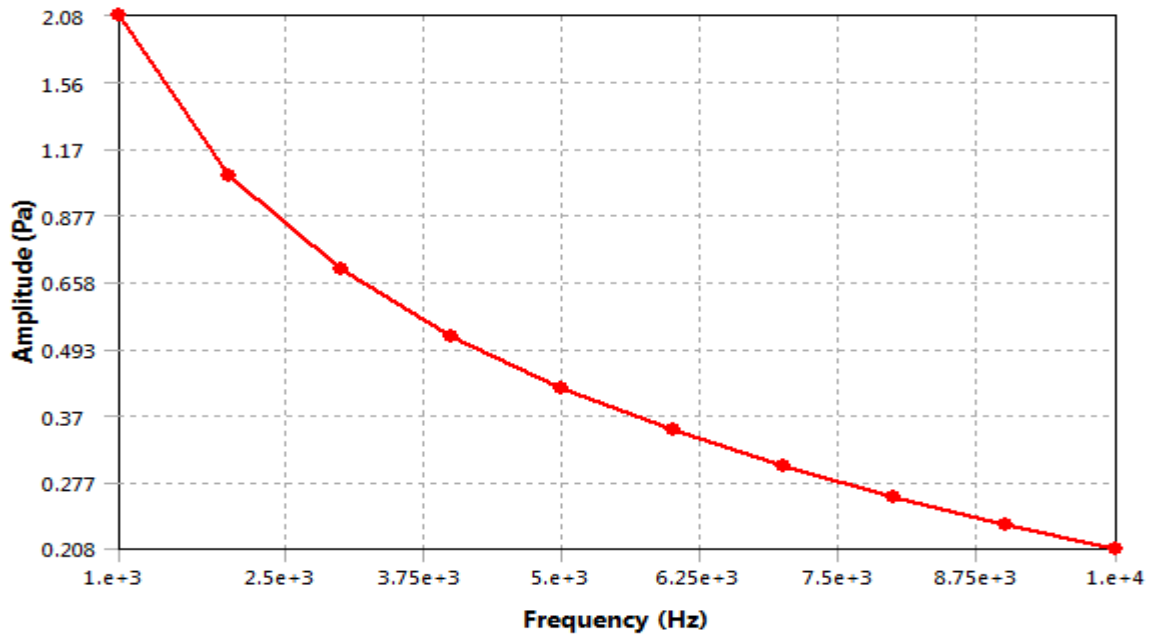
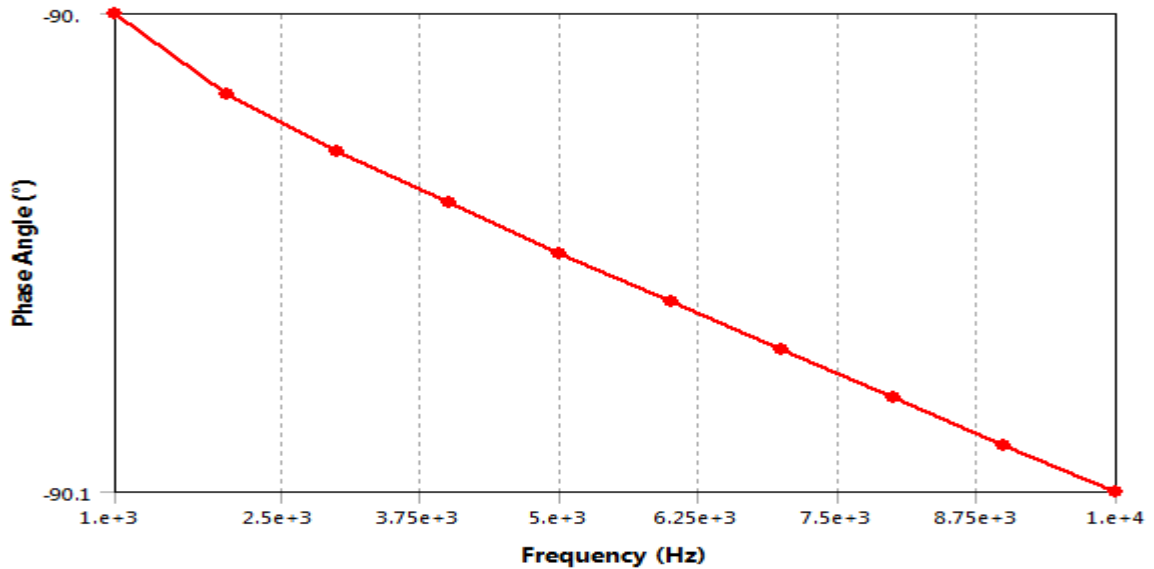


FIGURE 10
 Model (B4, C4, D4) > Harmonic Response (D5) > Solution (D6) > Frequency Response



Material Data

Structural Steel

TABLE 45
 Structural Steel > Constants

Density	7850 kg m ⁻³
Coefficient of Thermal Expansion	1.2e-005 C ⁻¹

Specific Heat	434 J kg ⁻¹ C ⁻¹
Thermal Conductivity	60.5 W m ⁻¹ C ⁻¹
Resistivity	1.7e-007 ohm m

TABLE 46
Structural Steel > Compressive Ultimate Strength

Compressive Ultimate Strength Pa
0

TABLE 47
Structural Steel > Compressive Yield Strength

Compressive Yield Strength Pa
2.5e+008

TABLE 48
Structural Steel > Tensile Yield Strength

Tensile Yield Strength Pa
2.5e+008

TABLE 49
Structural Steel > Tensile Ultimate Strength

Tensile Ultimate Strength Pa
4.6e+008

TABLE 50
Structural Steel > Alternating Stress

Alternating Stress Pa	Cycles	Mean Stress Pa
3.999e+009	10	0
2.827e+009	20	0
1.896e+009	50	0
1.413e+009	100	0
1.069e+009	200	0
4.41e+008	2000	0
2.62e+008	10000	0
2.14e+008	20000	0

1.38e+008	1.e+005	0
1.14e+008	2.e+005	0
8.62e+007	1.e+006	0

TABLE 51
Structural Steel > Strain-Life Parameters

Strength Coefficient Pa	Strength Exponent	Ductility Coefficient	Ductility Exponent	Cyclic Strength Coefficient Pa	Cyclic Strain Hardening Exponent
9.2e+008	-0.106	0.213	-0.47	1.e+009	0.2

TABLE 52
Structural Steel > Relative Permeability

Relative Permeability
10000

TABLE 53
Structural Steel > Isotropic Elasticity