ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES
Department of Civil Engineering

SOFTWARE DEVELOPMENT FOR CONSOLIDATION ANALYSIS UNDER VARIOUS LOADING CONDITIONS USING FINITE DIFFERENCE METHOD

By

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B.Sc. in Civil Engineering
Addis Ababa University, 2004

A Thesis Submitted to School of Graduate Studies in
Partial Fulfillment of the Requirement for Degree of
Master of Science
in
Geotechniques

Advisor
Dr. Hadush Seged

May, 2010
Addis Ababa, Ethiopia

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DECLARATION

I, the undersigned, declare that this thesis is my original work performed under the supervision of my research advisor Dr. Hadush Seged and has not been presented as a thesis for a degree in any other university. All sources of materials used for this thesis have also been duly acknowledged.

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Date May, 2010

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ABSTRACT

Consolidation analysis of soils can be undertaken using empirical and numerical methods. The problems related to empirical analysis of consolidation are the involvement of the determination for time factor using empirical equation obtained from curve fitting method. The empirical correlation of the time factor and the load application do not represent the actual situation where soil properties are assumed to be spatially and temporally invariant and thus, average property values are used. In reality, however, these soil parameters usually vary from point to point (heterogeneous) and these parameters may vary in time while a consolidation process is occurring due to an external influence such as surface pressure. Moreover, it is difficult to model and make consolidation analysis for variable loading conditions.

In order to solve this problem, numerical analysis using Finite Difference Method is proposed. The finite difference method uses the original governing consolidation equation and provides the solution of a continuous problem by representing it in terms of a discrete set of elements such that there is a number of points in depth and time at which one calculates the field variables, in this case the excess pore water pressure, consolidation settlement and average degree of consolidation.

Thus, this thesis involves development of user friendly application software for consolidation analysis using finite difference method. In the hope of achieving this goal, a computer program named ***ConsolAnalysis*** has been developed. The analysis capabilities of the software comprises of consolidation analysis for homogeneous and stratified soils subjected to constant loading, variable loading, and abrupt change of loading with constant and variable coefficient of consolidation.

LIST OF SYMBOLS

A_n	-	constant coefficient (where n is an integer)
a_v	-	Coefficient of compressibility
C_v	-	Coefficient of consolidation
dx, dy, dz	-	elemental soil mass dimensions
e	-	Void ratio
F	-	Function of
G	-	Function of
H	-	Thickness of the soil layer
H_d	-	Thickness of drainage path
i	-	Hydraulic gradient
k	-	Coefficient of permeability
log	-	logarithm
m	-	Any number
M	-	Any number
m_v	-	Compressibility of soil on the rate of volume change
N	-	Any number
n	-	Any number
q	-	Intensity of uniform distributed load
q_x, q_y	-	Rate of flow or rate of discharge (discharge per unit time)
S	-	Settlement
sin	-	Sine (Trigonometric function)
cos	-	Cosine (Trigonometric function)
t	-	Time
T_v	-	Time factor
T	-	Total Time
U	-	Degree of consolidation in percentage
u	-	Pore pressure, hydrostatic pressure
U_{av}	-	Average degree of consolidation in percentage
u_i	-	Initial excess hydrostatic (pore) pressure
v	-	Velocity of flow or discharge velocity

V_s	-	Volume of solids in the soil element
V_v	-	Volume of voids in the soil element
z	-	Thickness; depth
β (beta)	-	Constant
γ_w	-	Unit weight of water
Δ	-	Change or increment (or decrement) used
π (pi)	-	3.1416
σ	-	Consolidation pressure
σ'	-	Effective stress
∞	-	Infinity

1 INTRODUCTION

1.1 Background

For some time it has been realized that the analytical approach to solution of many engineering problems limits considerably in scope and in some cases prejudices the practical significance of the conclusion drawn from idealized analytical solution. One of the earliest issues in soil mechanics to be examined mathematically was that of consolidation analysis of soils under load, and since the solution to a problem of this nature was obtained, further practical advance has been made.

When a loading is applied on a saturated soil stratum, the incompressible pore water initially supports this loading and no volume change takes place. As time passes by, water from locations with higher heads flows towards locations with lower heads, and the excess pore water pressure dissipates. In this process, loading that was originally supported by the water is transmitted to the soil grains. This results in the increase of the effective stress of soils. As no holes are to develop in the ground, the volume of the pore water that flows away equals to the volume reduction of soils, and the ground settles. As the permeability of a cohesive soil is rather small, the process of excess pore water pressure dissipation and the ground settlement takes time to develop. This time-dependent process is called consolidation.

The average degree of consolidation can be predicted by using empirical equations where the coefficient of rate of consolidation is assumed as a constant value. However, it is known empirically and from laboratory experiments that the coefficient of consolidation changes with time in the process of consolidation. Thus, when the coefficient of consolidation is assumed as a constant value, the limitations are that the following conditions should be satisfied:

1. The coefficient of permeability remains constant during consolidation;
2. The relationship between void ratio or vertical strain and effective vertical stress is linear,
3. The relationship between void ratio and effective vertical stress is independent of time.

Normally, empirical expressions which relate time factor, T_v to consolidation ratio, *U* are used to calculate the settlement at time *t*. Problems arise when the coefficient of consolidation is, in reality, not constant. In addition to this, consolidation theory assumed that the load is applied at once on the soil. However, in reality the variation in the application of the load includes stage loading, load increasing by a certain rate, and abrupt change of load such as the case of surcharge preloading.

1.2 Problem Statement

There are three problems related to conventional method of consolidation analysis i.e.,

1) The analytical solution involves the determination of time factor T_v using empirical equation obtained from curve fitting method which obviously is inexact and tedious
2) It is difficult to model variation of loading as the consolidation process progresses.
3) It is also difficult to obtain a solution for stratified soil profile conditions.

Therefore, the objective of this study is to overcome these problems by making use of numerical solution based on finite difference method for consolidation analysis. The method does not require the empirical correlation for T_v because it can be done by using the governing equation and load application and other variations can be modeled almost correctly.

1.3 Aim and Objectives

This study is aimed at the development of application software for numerical analysis for the theory of consolidation based on finite difference method.

The following objectives are set forth in order to reach the aim of the study:

1. To understand the process involved in consolidation analysis.
2. To model the process using finite difference method and to apply the model into a computer application software.
3. To build the application of the software for different loading cases, variation of coefficient of consolidation and soil profile stratification.
4. To analyze and compare the results of the software output with the conventional method for the analysis of consolidation.

1.4 Scope of Study

This study is limited to the application of finite difference method to solve the governing theory of consolidation. Application software is developed based on finite difference method to model consolidation process with various conditions. The software can be used for providing a solution to consolidation analysis for

i. various types of loading,
ii. constant and variable coefficient of consolidation with time as the consolidation process proceeds and
iii. Homogeneous and stratified soil profiles.

It is inevitable to mention that the software developed will also provide accurate finite difference solution to any combination of the above three cases for various drainage conditions.

1.5 Thesis Outline

This thesis is divided into five chapters. The first chapter gives the basic introduction to the research background, outlines the scope, objectives of the research and structure of this thesis.

The basic principles of analysis of consolidation are given in the second chapter. Chapter two also presents the detail review of previous researches done to determine consolidation parameters.

Methodology of the research is presented in chapter three, where detailed discussion are made regarding theoretical background, software models and methods that are adapted in development of the software.

Chapter four analyzes the output of the software for various loading and coefficient of consolidation cases. Besides, detailed comparison and discussion will also be made between the results obtained from the software and that of empirical approach.

Chapter five closes the whole theme of the thesis by making conclusions and recommendations. In appendices, finite difference solution or the software output for various initial pore water pressure variation and stratification are presented.

2 LITERATURE REVIEW

2.1 General

When a soil layer is subjected to a compressive stress, such as during construction of a structure, it will exhibit a certain amount of compression. This compression is achieved through a number of ways, including rearrangement of the soil solids or extrusion of the pore air and/or water. According to Terzaghi (1943), "a decrease of water content of a saturated soil without replacement of water by air is called process of consolidation". [Das, B. M, 1987]

Consolidation is a gradual process involving drainage, compression, and stress transfer. It refers to the adjustment of soil to an applied loading. It may require a long time for a soil formation to come to equilibrium under load. During this time, it is said in engineering that the soil is consolidating under a given load. When the equilibrium is reached, it is taken that the soil is fully consolidated. [Leikun, M. and Teffera, A., 1999]

Analysis of consolidation involves determination of

- Pore water pressure,
- Effective stress,
- Average degree of consolidation and
- Consolidation settlement at any time during the process of consolidation.

Consolidation is a time dependent process that depends on:

- Magnitude of consolidating pressure
- Hydraulic conductivity of the soil
- Soil type, thickness and stratification
- Drainage condition

The rate of consolidation is governed by the speed of pore water dissipation and this is related to the permeability of the soil. The coefficient of permeability of soil is dependent on several factors such as fluid viscosity, pore-size distribution, grain-size distribution, void ratio, roughness of the mineral particles and degree of soil saturation.

In clayey soils, structure plays an important role in the coefficient of permeability. Other major factors that affect the permeability of clays are the ionic concentration and the thickness of the layers of water held to the clay particles. The higher the permeability the higher the rate of consolidation due to the facts that pore water can easily escape. For coarse-grained soil, consolidation completes as rapidly as loads are applied on it owing to the structure of coarse-grained soils that is relatively pervious. On the other hand, for fine-grained soils, consolidation process requires longer time as a result of low permeability.

Regarding the analysis of consolidation, numerical methods such as finite difference, finite element and boundary element provide appropriate solutions in which analytical solutions are not possible [Budhu, M., 2009].

This study will attempt to develop finite difference method based software for consolidation analysis because of the fact that finite difference method involves only expansion of differential equations and can easily be adopted for development of application software.

2.2 Theory of consolidation

The theory of one dimensional consolidation was first proposed by Terzaghi (1925). The theory, which is expressed in mathematical form, becomes the basis for all the studies of consolidation as well as other subjects related to the deformation of the flow of fluids through the porous media.

The theory states that the second derivation of pore water pressure to depth is proportional to the first derivation of pore water pressure to time and the governing equation for one-dimensional consolidation is [Das, B. M, 1987]:

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = C_V\frac{\partial^2 u}{\partial z^2} \qquad Eq. (2.1)$$

This equation describes the variation of excess pore water pressure with time (t) and depth (z) and

$$C_V = coefficient\ of\ consolidation = \frac{k}{\gamma_w m_v} \qquad Eq. (2.2)$$

The coefficient C_V has unit of length2/time, for example, m2/year and k is the coefficient of permeability and m_v is coefficient of volume compressibility.

Terzaghi suggested the following empirical equations for U_{av} to approximate the values obtained from Eq. (2.1):

For U_{av} = 0 – 53%

$$T_v = \frac{\pi}{4}\left(\frac{U_{av}(\%)}{100}\right)^2 \qquad Eq. (2.3)$$

For U_{av} = 53 – 100%

$$T_v = 1.781 - 0.933[\log(100 - U_{av}(\%))] \qquad Eq. (2.4)$$

Sivaram and Swamee (1977) gave the solution for U_{av} varying from 0 to 100%:

$$\frac{U_{av}(\%)}{100} = \frac{(4T_v/\pi)^{0.5}}{[1 + (4T_v/\pi)^{2.8}]^{0.179}} \qquad Eq. (2.5)$$

$$T_v = \frac{(\pi/4)(U_{av}(\%)/100)^2}{[1 - (U_{av}(\%)/100)^{5.6}]^{0.375}} \qquad Eq. (2.6)$$

Eq. (2.5) and (2.6) give an error in T_v of less than 1% for $0 < U_{av} < 90\%$ and less than 3% for $90\% < U_{av} < 100\%$. It must be pointed out that, Eq. (2.1) is still valid for a situation of one way drainage. Note, however, that the length of drainage path is equal to the total thickness of the soil layer.

2.3 Finite Difference Solution of the Governing Consolidation Equation

Finite difference solution of the governing consolidation equation is obtained by expansion of the differential consolidation equation using Taylor's theorem [Budhu, M., 2007]. The governing finite difference consolidation equation valid for nodes that are not boundary nodes is (derivation of the finite difference equation is presented in the Appendix):

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (2.7)$$

For an impermeable boundary, no flow across it can occur and hence

$$\frac{\partial u}{\partial z} = 0 \qquad Eq.(2.8)$$

The finite difference equation will then be:

$$\frac{\partial u}{\partial z} = 0 = \frac{1}{2\Delta z}\left(u_{i-1,j} - u_{i+1,j}\right) = 0 \qquad Eq.(2.9)$$

Hence, by applying $\boldsymbol{u_{i-1,j}} = \boldsymbol{u_{i+1,j}}$ from Eq. (2.9) in Eq. (2.7), the simplified governing consolidation equation which are valid for nodes at the impermeable boundary becomes:

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(2.10)$$

2.4 Finite difference solution for Consolidation in a layered Soil

According Das, B. M, (1987), it is not always possible to develop a closed form solution for consolidation in layered soils. This is due to the fact that several variables are involved in the analysis of consolidation for a layered soil, such as different coefficient of permeability, different thickness of layers, and different values of coefficient of consolidation.

In view of the above, numerical solutions provide a better approach for the analysis of consolidation. Thus, for the calculation of excess pore water pressure at the interface of a layered soil profile with different types of soils (i.e., different values of C_v), the governing consolidation equation in Eq. (2.1) will have to be modified to some extent and the finite difference equation for calculating pore water pressure in a layered soil will be as shown in Eq. (2.11) below.

$$u_{0,t+\Delta t} = \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{1,t} + \frac{2k_2}{k_1+k_2}u_{3,t} - 2u_{0,t}\right) + u_{0.t} \quad Eq.(2.11)$$

3 METHODOLOGY

3.1 General

This study involved development of finite difference method based application software for consolidation analysis for the following cases:

- Homogeneous Soil Profile
    - Constant loading and constant coefficient of consolidation case,
    - Constant loading and variable coefficient of consolidation (variability of C_V according to Ronald, F.S. (1953)) case,
    - Constant loading and constant rate variation of coefficient of consolidation case,
    - Constant rate variability in loading and constant coefficient of consolidation case,
    - Abrupt change in load application and constant coefficient of consolidation case,
    - Constant rate variability in loading and constant rate variation of coefficient of consolidation case,
    - Abrupt change in loading and constant rate variation of coefficient of consolidation case,
- Two Layered Soil profile case
    - Constant loading and constant coefficient of consolidation case,
    - Constant loading and constant rate variation of coefficient of consolidation case,
    - Constant rate variability in loading and constant coefficient of consolidation case,
    - Abrupt change in load application and constant coefficient of consolidation case,
    - Constant rate variability in loading and constant rate variation of coefficient of consolidation case,
    - Abrupt change in loading and constant rate variation of coefficient of consolidation case,
- Multi-Layered Stratified Soil case
    - Constant loading and constant coefficient of consolidation case,
    - Constant loading and constant rate variation of coefficient of consolidation case,
    - Constant rate variability in loading and constant coefficient of consolidation case,
    - Abrupt change in load application and constant coefficient of consolidation case,

- Constant rate variability in loading and constant rate variation of coefficient of consolidation case,
- Abrupt change in loading and constant rate variation of coefficient of consolidation case,

This research was implemented in five stages. The first stage is the literature review of related field and the study on using the programming software (Visual Basic.Net). Literature review is important for insight into techniques, which have been employed in previous study on consolidation. In order to carry out this task, searching into journal, proceedings, technical notes, and textbooks is needed. Literature review will focus on gathering knowledge of consolidation, the governing equation of the consolidation theory and detail study of finite difference method of numerical analysis. In addition, knowledge concerning the usage of Visual Basic.Net is studied to help in the development of the application software later on in the process.

The second stage is to identify and understand the problems and the early development of the software for constant loading, constant coefficient of consolidation and homogeneous soil profile condition. In this stage, the problems associated with the limitation of the original solution are studied. The purpose of this study is to obtain clear objectives on what has to be done to further improve the application of Terzaghi's one-dimensional consolidation theory. At this stage, the understanding of the finite difference method will help in solving some of the limitation posed by the original solution.

Once the software is developed for the simple case in the second stage, modifications will be made for different load case scenario and variation in C_V in homogeneous as well as stratification soil profile conditions in the third stage of implementation. Here, the different load cases which occur in reality are identified. In addition, the relationships between the coefficients of consolidation, C_V, with consolidation at different time interval are studied. At this stage, the behavior of the consolidation of soil due to different loading application, variable C_V and stratification are identified and incorporated into the software.

The fourth stage will be dealt with the validation of the software by comparing the result of the analysis from the software with empirical solution for a hypothetical problem. This part is the core of the whole thesis where the result of the computed settlement and average degree of consolidation are compared with the result from the empirical solution. Extensive discussion is made at this stage to reach the conclusion.

Finally, conclusion and thesis finalization work are done based on the result of the analysis. The conclusion is based on the outcome from the analysis and computation stage. The relevant recommendations will also be proposed to improve this research in the coming future. Generally, the whole process of the thesis implementation is given in flowchart as shown in Figure **3.1**.

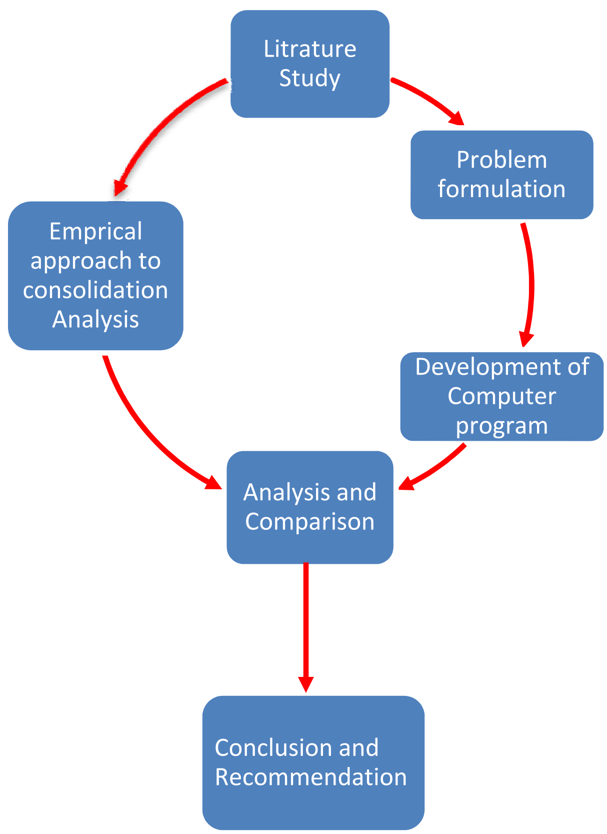

Figure 3.1: Chart for processes involved in undertaking the research.

3.2 Development of the Computer Application Software

Computers are utilized in solving a wide range of problems, which arise in such diverse fields as science, engineering, business, management and administration. A programmer's task is solving problems by the computer which involves several steps such as problem definition, problem analysis, formulation of a method of solution, flowchart preparation, program coding, debugging, and running as can be observed from Figure 3.2.

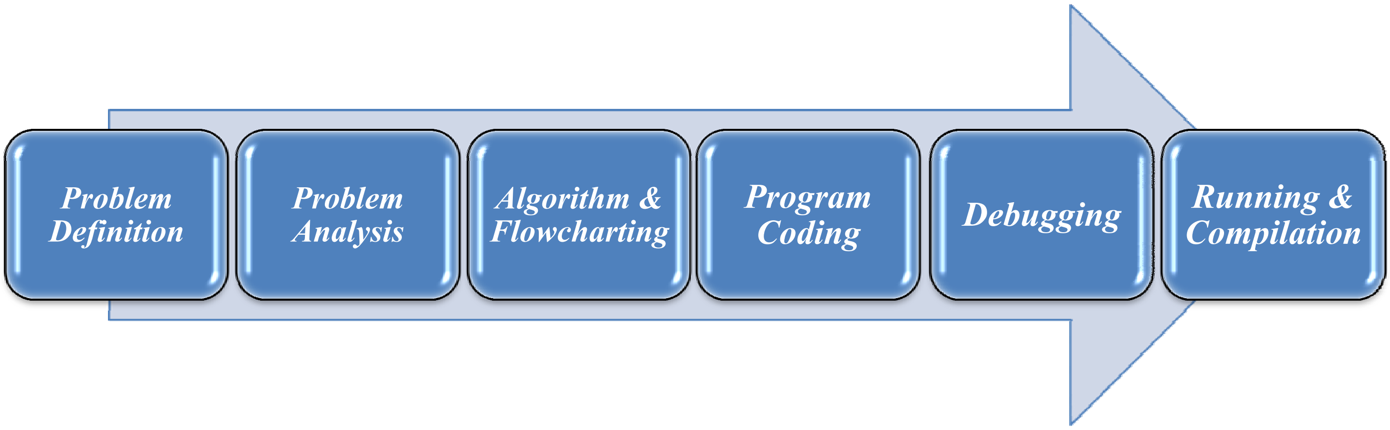

Figure 3.2: Flow chart for software development.

The first step in the process of problem solving by a computer is the problem definition. Problem definition is precise and well-defined statement of the problem in terms, which can be understood clearly by people involved in the problem area. Problem analysis and formulation of a method of solution is the most difficult and crucial step. To obtain a correct and efficient solution to a problem, existing mathematical and other pertinent techniques and tools relevant to the problem must be used and whenever necessary new techniques must be developed. At this step, the type and organization of the data to be handled, the mathematical procedures are determined. The method of solution is expressed as a sequence of precisely described logical steps. Such a description of method of solution is called an algorithm. (T. Cay a, F. Iscan., 2006)

Once a method of solution is formulated, the logical steps and the sequence of operations to be performed at each step are next represented in a pictorial form, which is called a flowchart. A flowchart is just a visual aid, but a computer cannot understand it. The method of solution must be expressed in a computer language to be understood by it. Many languages have been developed in order to communicate with the computers. Expressing the method of solution in terms of a computer language is called programming. (T. Cay a, F. Iscan., 2006)

In this study visual basic 2008 programming language has been used and the codes for finite difference equations for consolidation analysis are prepared for all the cases discussed in section 3.4 following the coding requirement of visual basic 2008.

Model of how the application software will look like is designed at program coding stage where the facility for the user to enter input data and how the results will be depicted is designed. Then, the derived finite difference equation is coded into the visual basic module to calculate the pore water pressure, average degree of consolidation and settlement at any time, t. Besides that, custom dialog box also help users to input the necessary data in an orderly sequence for the computation of consolidation analysis.

The last step is to run the program on a computer to get the desired answers, to correct the mistakes made in formulating the method of solution and/or programming until valid results are obtained and to document all the work done so that the same program can be used over and over again by different people to solve the same problem with different data values.

3.3 Numerical Solution Using Finite Difference Method

Finite difference approximation is an efficient way to solve consolidation problems. The governing equation in consolidation has one additional independent variable i.e. time. If the solution of all the $u(z,t)$ up to time t has been obtained, then a scheme for finding the solution for time , $t = t + \Delta t$ can be developed. The finite difference method consist of substituting finite ranges for the differential governing consolidation equation, factoring the resulting equation into a form suitable for analysis, ensuring that the method of solution is stable and achieve a solution. The solution has two steps:

1. The excess pore water pressure is calculated at a series of points (nodes), which are distributed vertically throughout the deposit, at a series of times.
2. The effective stresses, average degree of consolidation, and settlement are calculated from the known total stresses and pore-water pressure.

3.3.1 Excess Pore water pressure calculation

In this method the solution is evaluated at a number of points at different times as indicated on the **Figure 3.3**.

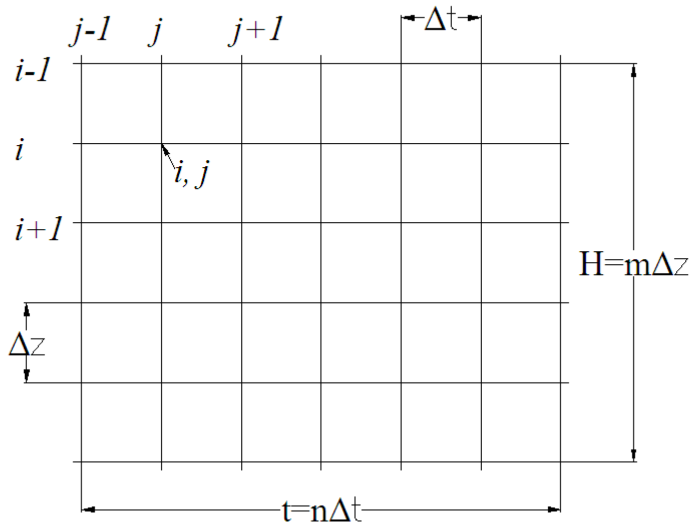

Figure 3.3: Nodes showing points at which solution are calculated.

The governing one dimensional consolidation equation as discussed above in literature review section is:

$$\frac{\partial u}{\partial t} = C_V \frac{\partial^2 u}{\partial z^2} \qquad Eq.(3.1)$$

Using Taylor's theorem,

$$\frac{\partial u}{\partial t} = \frac{1}{\Delta t}\left(u_{i,j+1} - u_{i,j}\right) \qquad Eq.(3.2)$$

And

$$\frac{\partial^2 u}{\partial t} = \frac{1}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.3)$$

Where *(i, j)* denotes a nodal position at the intersection of row *i* and column *j*. Columns represent time division and rows represent soil depth divisions. The assumption implied in Eq. (3.2) is that the excess pore water pressure between two adjacent nodes changes linearly with time. This assumption is reasonable if the distance between the two nodes is small. Substituting Eq. (3.2) and Eq. (3.3) in the governing consolidation equation, Eq. (3.1) and rearranging gives

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.4)$$

Eq. (3.4) is valid for nodes that are not boundary nodes. For example, at an impermeable boundary, no flow across it can occur and subsequently,

$$\frac{\partial u}{\partial z} = 0 \qquad Eq. (3.5)$$

For which the finite difference equation is:

$$\frac{\partial u}{\partial z} = 0 = \frac{1}{2\Delta z}\left(u_{i-1,j} - u_{i+1,j}\right) = 0 \qquad Eq. (3.6)$$

And the governing consolidation equation at an impermeable boundary becomes

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq. (3.7)$$

To determine how the pore water pressure is distributed within a soil at a given time, the initial excess pore water pressure at the boundaries should be established. Once this is done, the variation of the initial excess pore water pressure within the soil can be estimated. One may, for example, assume a linear distribution of initial excess pore water pressure with depth if the soil layer is thin or a triangular distribution for a thick soil layer. If one cannot estimate the initial excess pore-water pressure, then one can assume reasonable values for the interior of the soil or use linear interpolation. Then, successively applying Eq. (3.4) to each interior nodal point and replace the old value by the newly calculated value until the old value and the new value differ by a small tolerance. At impermeable boundaries, one should apply Eq. (3.7).

The procedure to apply the finite difference form of the governing consolidation equation to determine the variation of excess pore-water pressure with time and depth is as follows [Budhu. M., 2007]:

1. Divide the soil layer into a depth-time grid (Figure 3.3). Rows represent subdivisions of the depth, columns represent subdivisions of time. Let us say we divide the depth into *m* rows and the time into *n* columns, then $\Delta z = H/m$ and $\Delta t = t/n$, where *H* is the thickness of the soil layer and *t* is the total time. A nodal point represents the i^{th} depth position and the j^{th} elapsed time.

   To ensure that the finite-difference solution is an acceptable approximation to the “true result”, the differential equation must be mathematically stable. The interpretation concerning this is to mean that round off or other error introduced at some point in time in the solution must not increase in magnitude as t increases. The approximate solution must converge to exact solution as $\Delta z \rightarrow 0$ and as $\Delta t \rightarrow 0$**.** To avoid convergence problems, it has been found out that

$$\beta = \frac{C_v \Delta t}{\Delta z^2} \leq \frac{1}{2} \qquad Eq. (3.8)$$

This places a limit on the number of subdivisions in the grid. Often, the depth is subdivided arbitrarily and the time step Δt is selected so that $\beta < 1/2$. In many practical situations, $\beta = 0.25$ usually ensures convergence. If β is greater than 0.5, the scheme become unconditionally stable thus the numerical solution will oscillate.

2. Identify the boundary conditions. For example, if the top boundary is a drainage boundary then the excess pore-water pressure there is zero. If, however, the top boundary is an impermeable boundary, then no flow can occur across it and Eq. (3.7) applies.

   At a free draining boundary (top drainage boundary in Figure **3.4**) there is no barrier to the flow and so the pore pressure remains constant, thus the excess pore water pressure is zero. This happen when it is at the soil surface or the adjacent layer is very permeable.

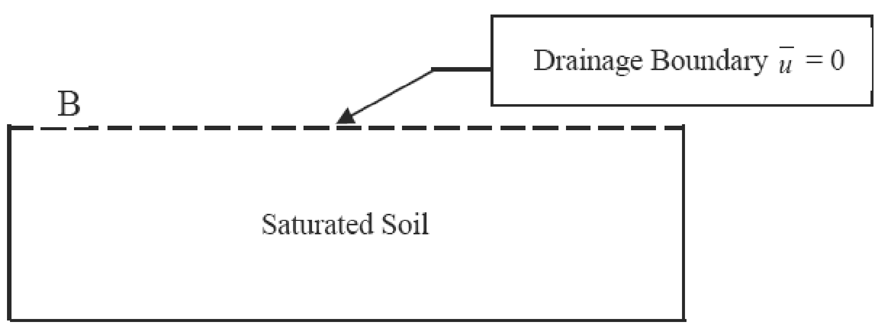

Figure 3.4: Drainage boundaries for finite difference approximation.

When the adjoining stratum at one boundary is impervious (Figure 3.5) there is no flow across the boundary. Hence, Eq. (3.7) applies.

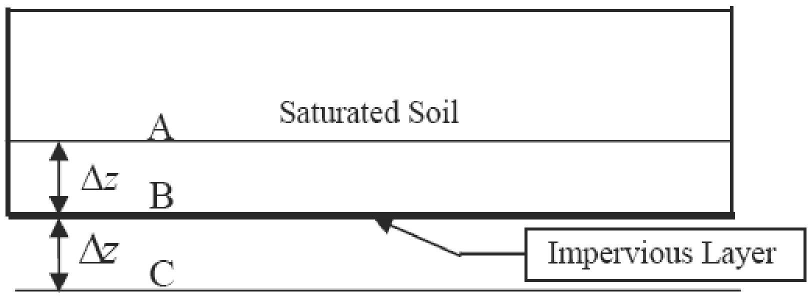

Figure 3.5: Impermeable boundary for finite difference approximation

3. Estimate the distribution of initial excess pore-water pressure and determine the nodal initial excess pore-water pressures.
4. Calculate the excess pore-water pressure at interior nodes using Eq. (3.4) and at impermeable boundary nodes using Eq. (3.7). If the boundary is permeable, then the excess pore-water pressure is zero at all nodes on this boundary.

Eq. (3.4) and (3.7) allow the value of the excess pore pressure at time $t+\Delta t$ to be calculated from those at time t. This means, that all the quantities on the right hand side of Eq. (3.4) are known and thus that u at time $t+\Delta t$ can be calculated.

Thus the value of u at time t is required in order to calculate u at time $t+\Delta t$. The solution is advanced without the necessity of equation solving. One starts from $t = 0$ for at that time $u\ (z, 0)$ at all z are given as the initial conditions. Knowing the boundary conditions, u (z, $t+\Delta t$) are then determined. By repeating this procedure, the solution over time is found. These equations (Eq. (3.4) and (3.7)) can be used for calculating u at each point (depth) in the solution grid as there is no variable that is governed by the depth value. Now the initial distribution of u can always be determined and thus the solution can be found by "marching" forward in time.

3.3.2 Settlement and Average Degree of consolidation calculation

The downward movement of the surface of a consolidating layer due to a decrease in its voids is called the consolidation settlement. Considering the soil element of thickness Δz, the change in thickness, ΔS of the soil element at any time $\boldsymbol{t}$ is given by Eq. (3.10).

$$m_v = -\frac{\Delta H}{H}\frac{1}{\Delta\sigma'} \qquad Eq. (3.9)$$

$$\Delta S = m_v \sigma \Delta z \qquad Eq.(3.10)$$

Where: Δσ' = Δσ – u = effective stress increment at time *t*

Integrating between the limits 0 to *H* the settlement *S* of the full thickness of the soil layer at time *t* is given by:

$$S = \int_0^H m_v(\Delta\sigma - u)dz \qquad Eq.(3.11)$$

$$S = m_v \sigma H - m_v \int_0^H u dz \qquad Eq.(3.12)$$

In Eq. (3.12), the integral of the excess pore pressure cannot be evaluated exactly because the excess pore pressure is only at the grid points. However, the integral can be evaluated approximately using numerical techniques (Figure **3.6**). The simplest approach is to use the Trapezoidal Method (Singiresu 1979):

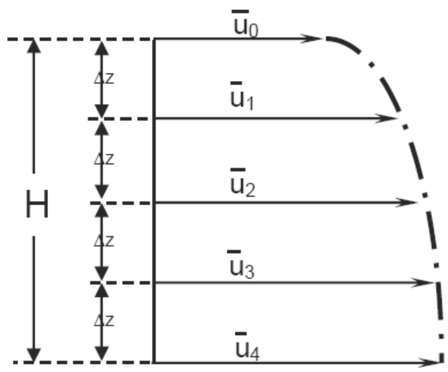

Figure 3.6: Excess pore water at grids points

$$\int_0^H u dz \approx \left[\left(\frac{u_o + u_n}{2}\right) + \sum_{i=1}^{n-1} u_i\right]\Delta z \qquad Eq.(3.13)$$

Where:

$$u_i = u_i(\Delta z, t)$$

Therefore the "Average Degree of Consolidation", U_{av}, which refers to the percentage of the total excess pore water pressure that has been dissipated within a whole deposit up to time *t*, can be calculated as:

$$U_{av} = 1 - \left(\frac{\int_0^H u_{t=ti} dz}{\int_0^H u_{t=0} dz} \right) \qquad Eq.(3.14)$$

3.4 Description of different Models

In reality, a loading is never placed instantaneously. For some problems, the approximation that assumes loading to be instantaneous may be acceptable. For others, it may be not. In this latter situation, the problems can be easily handled through finite difference formulation. The following sub sections presents the details of calculation methods and equations for pore water pressure, consolidation settlement and average degree of consolidation for various conditions.

3.4.1 Calculation of pore water pressure

3.4.1.1 Homogeneous Soil profiles

In a homogeneous soil profile case, it is assumed that the soil under investigation has the same property throughout its depth. Based on this assumption, the finite difference equation has been coded into the Visual basic 2008 programming language for various cases as described herein under,

i. Constant Load and Constant C_V

Constant loading and constant C_V case is the simplest case where there is no variation in loading, coefficient of consolidation and soil profiles. The calculation of pore water pressure is undertaken using the following equations:

For interior nodes

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2} \left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j} \right) \qquad Eq.(3.16)$$

For impermeable nodes

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2} \left(2u_{i-1,j} - 2u_{i,j} \right) \qquad Eq.(3.17)$$

ii. Constant Load and Variable C_V according to Ronald, F.S. (1953, Massachusetts Institute of Technology)

The coefficient of consolidation is usually considered a constant in conventional consolidation analysis although it is a variable quantity. It is known that the coefficient of consolidation changes during consolidation as the void ratio of the soil changes causing a decrease in permeability and compressibility of the soil. The coefficient of consolidation decreases with the increase in effective stress and it varies in some fashion with inter-granular pressure (or the hydrostatic excess pore-water pressure). It has been observed that the hydrostatic pore-water pressure at a point after a small time increment bears a relation to the pressure at that point and the two points surrounding it, at the pervious time [Ronald, F.S., 1953]. The relationship depends on the value chosen for β $(=C_v\Delta t/(\Delta z^2)$ which contains the coefficient of consolidation. Therefore, by changing the value of coefficient of consolidation in β, one can calculate the pore-pressure affected by the varying coefficient of consolidation as:
For interior nodes

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.18)$$

For impermeable nodes

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq. (3.19)$$

Where according to Ronald, F.S. (1953, Massachusetts Institute of Technology), it has been explained that when the results of a laboratory consolidation tests are shown, a plot of coefficient of consolidation versus inter-granular pressure is as shown in Figure **3**.**7**.

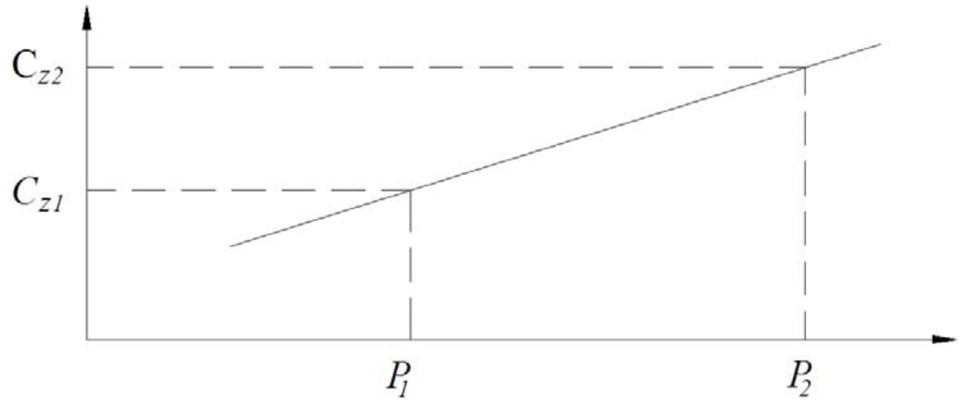

Figure 3.7: Variation of coefficient of consolidation with inter-granular pressure

Consider the soil is subjected to an initial inter-granular pressure of P_1, and then when a load giving a pressure of P_2 is applied at the surface, the soil begins to consolidate. Hence it can be observed that the degree of consolidation may be considered to be 0% at points where an inter-granular pressure of P_1, exists and 100% where a pressure of P_2 exists. Thus the revised diagram, Figure **3.8**, can be drawn to represent the case being considered.

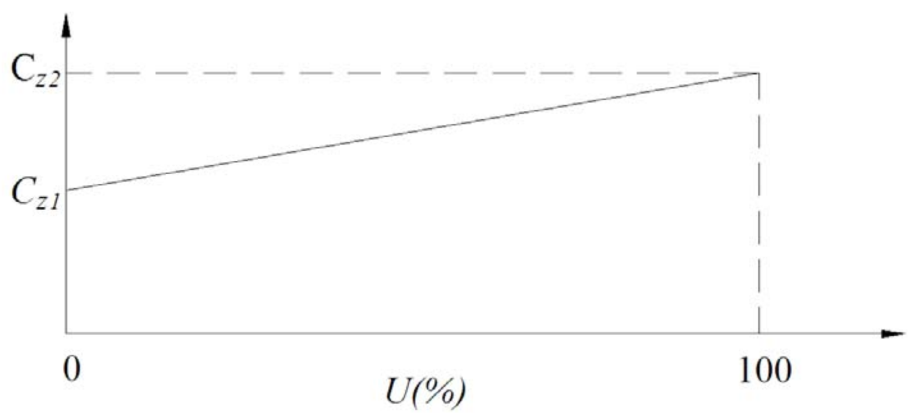

Figure 3.8: Variation of coefficient of consolidation with degree of consolidation

The initial conditions are

$U = 0 \; and \; C_z = C_{z1}$

And finally

$U = 100 \; and \; C_z = C_{z2}$

The equation giving the coefficient of consolidation C_z, at any degree of consolidation, U, is [Ronald, F.S., 1953]

$$C_z = C_{z1} + (C_{z2} - C_{z1})\frac{U}{100} \qquad Eq.(3.20)$$

In which, this equation can be expressed in finite difference equation form as;

$$C(i,j) = C_V + \left(\frac{C_V \times u_{(Imax,j)}}{u_o} - C_V\right) \times \frac{U_{(i,j)}}{100} \qquad Eq.(3.21)$$

Where: *C(i, j)* = the coefficient of consolidation at any interior node

C_V = initial coefficient of consolidation

$u_{(imax,\, j)}$ = the final node pore water pressure in the calculation

u_o = the initial pore water pressure

$U_{(i,\, j)}$ = the average degree of consolidation at the node where the coefficient of consolidation is to be calculated.

iii. Constant Load and Variable C$_V$ with constant rate

In this case, the calculation was modeled in such a manner that the rate for variation of coefficient of consolidation is to be specified by the user and the pore water pressure calculation will be accomplished Eq. (3.22) and Eq. (3.23) for interior and boundary nodes respectively.

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.22)$$

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.23)$$

Where *C (i, j)* [the coefficient of consolidation at any interior node] = C_V - rate of variability of C_V which is to be specified by the user

iv. Variable Loading and Constant C$_V$

Here, there is a constant increment of loading with constant coefficient of consolidation and Eq. (3.24) and Eq. (3.25) will be applicable for interior and impermeable nodes respectively.

For interior nodes

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.24)$$

For impermeable nodes

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq. (3.25)$$

Where: Δq = constant rate change in load to be specified by the user.

v. <u>Abrupt change in application of loading and Constant C_V</u>

When the existing soil layers are too weak, it has to be loaded in stages. Loading in stages will increase the shear strength and bearing capacity of the soil as it consolidates. When the additional surcharges, Δq is loaded onto the pervious fill, the instantaneous application of the load will cause the excess pore water pressure to be equal to Δq. In this case, the calculation of pore water pressure can be accomplished using Eq. (3.26) and Eq. (3.26) for interior and boundary nodes respectively.

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.26)$$

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq. (3.27)$$

Where: Δq = the abrupt change in load to be specified by the user.

vi. <u>Variable Loading and Variable C_V with constant rate</u>

For the case of constant rate change of loading and variable C_V with constant rate case, Eq. (3.28) and Eq. (3.29) will be used for the calculation of pore water pressure.

For interior nodes

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.28)$$

For impermeable nodes

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.29)$$

Where: Δq = the change in load which will be specified by the user.

C(i, j) = the coefficient of consolidation at any intermediate node where variability in C_V is to be specified by the user.

vii. Abrupt change of loading and Variable C_V with constant rate case

In this case, the calculation is done as follows:
For interior nodes

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.30)$$

For impermeable nodes

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j) \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.31)$$

Where: Δq = the abrupt change of loading which will be specified by the user

C(i, j) = the coefficient of consolidation at any interior node where variability in $\boldsymbol{C_V}$ is to be specified by the user.

3.4.1.2 Two Layered Soil profiles

i. Constant Load and Constant C_V Case

Here, a two layered soil profile with constant loading and constant C_V for each layer is considered. The calculation of pore water pressure requires the determination of pore pressure for the two layers using their respective soil parameters and determination of pore water pressure at the interface between the two layers. The whole consolidation analysis calculation is carried out as follows:

For first soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C_{V1}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.32)$$

For interior nodes of the second soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.33)$$

At the interface between the two soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{(i-1,j)} + \frac{2k_2}{k_1+k_2}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.34)$$

For impermeable nodes of the second soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.35)$$

Where: C_{V1}, C_{V2} = the coefficient of consolidation of the first and second soil layer respectively

K_1, K_2 = the coefficient of permeability of the first and second soil layer respectively

ii. <u>Constant Load and Variable C_V with constant rate in variability</u>

Here, the consolidation analysis with constant load and variable C_V with constant rate in variability is carried out as follows:

For first soil layer,

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j)_1 \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.36)$$

For interior nodes of the second soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j)_2 \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.37)$$

At the interface between the two soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_1}u_{(i-1,j)} + \frac{2k_2}{k_1+k_1}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.38)$$

For impermeable nodes of the second soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j)_2 \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.39)$$

Where: C_{V1}, C_{V2} = the coefficient of consolidation of the first and second soil layer respectively

K_1, K_2 = the coefficient of permeability of the first and the second soil layer respectively

$C(i,j)_1$, $C(i,j)_2$ = the coefficient of consolidation at the first and second soil layer where variability in C_v is to be entered by the user.

iii. Constant Rate Change of Loading and Constant C_V Case

Here, the consolidation analysis is carried out as follows:

For first soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V1}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.40)$$

For interior nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.41)$$

At the interface between the two soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{(i-1,j)} + \frac{2k_2}{k_1+k_2}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.42)$$

For impermeable nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.43)$$

Where: Δq = constant rate change in load to be specified by the user.

C_{V1}, C_{V2} = the coefficient of consolidation of the first and second soil layer respectively

K_1, K_2 = the coefficient of permeability of the first and the second soil layer respectively

iv. Abrupt change of loading and Constant C_V

Here, the consolidation analysis is carried out as follows:

For first soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V1}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.44)$$

For interior nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.45)$$

At the interface between the two soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{(i-1,j)} + \frac{2k_2}{k_1+k_2}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.46)$$

For impermeable nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.47)$$

Where: Δq = the abrupt change in load to be specified by the user.

C_{V1}, C_{V2} = the coefficient of consolidation of the first and second soil layer respectively

K_1, K_2 = the coefficient of permeability of the first and the second soil layer respectively

v. Constant Rate Change of Loading and Variable C_V with constant rate

Here, the consolidation analysis is carried out as follows:

For first soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_1 \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.48)$$

For intermediate nodes of the second soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j)_2 \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.49)$$

At the interface between the two soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{(i-1,j)} + \frac{2k_2}{k_1+k_2}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq. (3.50)$$

For impermeable nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_2 \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq. (3.51)$$

Where: Δq = constant rate change in load to be specified by the user.

C_{V1}, C_{V2} = the coefficient of consolidation of the first and second soil layer respectively

K_1, K_2 = the coefficient of permeability of the first and the second soil layer respectively

$C(i,j)_1$, $C(i,j)_2$ = the coefficient of consolidation at the first and second soil layer where variability in C_V is to be specified by the user.

vi. <u>Abrupt change of loading and Variable C_V with constant rate</u>

Here, the consolidation analysis is carried out as follows:

For first soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_1 \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.52)$$

For interior nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_2 \times \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.53)$$

At the interface between the two soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{(i-1,j)} + \frac{2k_2}{k_1+k_2}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.54)$$

For impermeable nodes of the second soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_2 \times \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.55)$$

Where: Δq = abrupt change in load to be specified by the user.

C_{V1}, C_{V2} = the coefficient of consolidation of the first and second soil layer respectively

K_1, K_2 = the coefficient of permeability of the first and the second soil layer respectively

$C(i,j)_1$, $C(i,j)_2$ = the coefficient of consolidation at the first and second soil layer where variability in C_V is to be specified by the user.

3.4.1.3 Multi-Layered Stratified Soil profiles

i. <u>Constant Load and Constant C_V Case</u>

Here, we will consider a multi-layered soil profile with constant loading and constant C_V case. The calculation of pore water pressure requires the determination of pore pressure for the multi-layers using their respective soil parameters and determination of pore water pressure at the interface between the layers. For this case, the consolidation analysis is carried out as follows:

For interior nodes of different soil layers

$$u_{i,j+1} = u_{i,j} + \frac{C_{Vn}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.56)$$

At the interface between the different soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \frac{\Delta t C_{vn}}{(\Delta z_n)^2}\left(\frac{1 + k_{n+1}/k_n}{1 + (k_{n+1}/k_n)(C_{vn}/C_{vn+1})}\right)\left(\frac{2k_n}{k_n+k_{n+1}}u_{(i-1,j)} + \frac{2k_{n+1}}{k_n+k_{n+1}}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.57)$$

For impermeable nodes of the bottom soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.58)$$

Where: C_{Vn+1}, C_{Vn} = the coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer respectively

K_{n+1}, K_n = the coefficient of permeability of the n+1th soil layer and the preceding soil layer respectively

ii. ***Constant Load and Variable C_V with constant rate***

Here, the consolidation analysis calculation is carried out as follows:

For interior nodes of different soil layers

$$u_{i,j+1} = u_{i,j} + \frac{C_{Vn}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.59)$$

At the interface between the different soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \frac{\Delta t C(i,j)_{vn}}{(\Delta z_n)^2}\left(\frac{1 + k_{n+1}/k_n}{1 + (k_{n+1}/k_n)(C(i,j)_{vn}/C(i,j)_{vn+1})}\right)\left(\frac{2k_n}{k_n+k_{n+1}}u_{(i-1,j)} + \frac{2k_{n+1}}{k_n+k_{n+1}}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.60)$$

For impermeable nodes of the bottom soil layer

$$u_{i,j+1} = u_{i,j} + \frac{C(i,j)_{Vn+2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.61)$$

Where: C_{Vn+1}, C_{Vn} = the coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer respectively

$C(i,j)_{Vn+1}$, $C(i,j)_{Vn}$ = the variable coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer in which the rate of decrement of C_V are to be specified by the user.

K_{n+1}, K_n = the coefficient of permeability of the n+1th soil layer and the preceding soil layer respectively

iii. Constant Rate Change of Loading and Constant C_V

Here, the whole consolidation analysis is carried out as follows:

For interior nodes of different soil layers

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{Vn}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq. (3.62)$$

At the interface between the different soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C_{vn}}{(\Delta z_n)^2}\left(\frac{1 + k_{n+1}/k_n}{1 + (k_{n+1}/k_n)(C_{vn}/C_{vn+1})}\right)\left(\frac{2k_n}{k_n + k_{n+1}} u_{(i-1,j)} + \frac{2k_{n+1}}{k_n + k_{n+1}} u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq. (3.63)$$

For impermeable nodes of the bottom soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq. (3.64)$$

Where: Δq = constant rate change in load to be specified by the user.

C_{Vn+1}, C_{Vn} = the coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer respectively

K_{n+1}, K_n = the coefficient of permeability of the n+1th soil layer and the preceding soil layer respectively

iv. Abrupt change of loading and Constant C_V Case

Here, the whole consolidation analysis is carried out as follows:

For intermediate nodes of different soil layers

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{Vn}\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.65)$$

At the interface between the different soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C_{vn}}{(\Delta z_n)^2}\left(\frac{1 + k_{n+1}/k_n}{1 + (k_{n+1}/k_n)(C_{vn}/C_{vn+1})}\right)\left(\frac{2k_n}{k_n + k_{n+1}}u_{(i-1,j)} + \frac{2k_{n+1}}{k_n + k_{n+1}}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.66)$$

For impermeable nodes of the bottom soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_{V2}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.67)$$

Where: Δq = abrupt change in load to be specified by the user.

C_{Vn+1}, C_{Vn} = the coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer respectively

K_{n+1}, K_n = the coefficient of permeability of the n+1th soil layer and the preceding soil layer respectively

v. Constant Rate Change of Loading and Variable C_V with constant rate case

Here, the whole consolidation analysis is carried out as follows:

For interior nodes of different soil layers

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_n\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.68)$$

At the interface between the different soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C(i,j)_{vn}}{(\Delta z_n)^2}\left(\frac{1 + k_{n+1}/k_n}{1 + (k_{n+1}/k_n)(C(i,j)_{vn}/C(i,j)_{vn+1})}\right)\left(\frac{2k_n}{k_n + k_{n+1}}u_{(i-1,j)} + \frac{2k_{n+1}}{k_n + k_{n+1}}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.69)$$

For impermeable nodes of the bottom soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_{Vn+1}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.70)$$

Where: Δq = constant rate change in load to be specified by the user.

$C(i,j)_{Vn+1}$, $C(i,j)_{Vn}$ = the variable coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer in which the rate of decrement of C_V are to be specified by the user.

K_{n+1}, K_n = the coefficient of permeability of the n+1th soil layer and the preceding soil layer respectively

vi. Abrupt change of loading and Variable C_V with constant rate

For the case of abrupt change of loading and variable C_V with constant rate, the pore water pressure calculation will be undertaken as follows:

For interior nodes of different soil layers

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_n\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(3.71)$$

At the interface between the different soil layers, pore pressure is calculated as:

$$u_{(i,j+1)} = \Delta q + \frac{\Delta t C(i,j)_{vn}}{(\Delta z_n)^2}\left(\frac{1 + k_{n+1}/k_n}{1 + (k_{n+1}/k_n)(C(i,j)_{vn}/C(i,j)_{vn+1})}\right)\left(\frac{2k_n}{k_n + k_{n+1}}u_{(i-1,j)} + \frac{2k_{n+1}}{k_n + k_{n+1}}u_{(i+1,j)} - 2u_{(i,j)}\right) + u_{(i,j)} \qquad Eq.(3.72)$$

For impermeable nodes of the bottom soil layer

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C(i,j)_{Vn+1}\Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(3.73)$$

Where: Δq = abrupt change in load to be specified by the user.

$C(i,j)_{Vn+1}$, $C(i,j)_{Vn}$ = the variable coefficient of consolidation of the n+1th soil layer and the preceding soil layer to the n+1th soil layer in which the rate of decrement of C_V are to be specified by the user.

K_{n+1}, K_n = the coefficient of permeability of the n+1th soil layer and the preceding soil layer respectively

3.4.2 Calculation of Consolidation Settlement

The calculation of consolidation settlement is accomplished using the following formula:

$$S_t = m_v \sigma H - m_v \int_0^H u dz \qquad Eq. (3.74)$$

Where $$\int_0^H u dz \approx \left[\left(\frac{u_o + u_n}{2}\right) + \sum_{i=1}^{n-1} u_i\right] \Delta z \qquad Eq. (3.75)$$

Hence, $$S_t = m_v \sigma H - m_v \left[\left(\frac{u_o + u_n}{2}\right) + \sum_{i=1}^{n-1} u_i\right] \Delta z \qquad Eq. (3.76)$$

Where: - S_t = the settlement at time t

m_v = coefficient of volume compressibility

u_o = pore water pressure at a time t = 0.

u_n , u_i = pore water pressure at the nth and i^{th} time respectively

Δz = finite difference depth grid

σ = applied stress

H = height of the soil layer under consideration

3.4.3 Calculation of Average degree of consolidation

The calculation of average degree of consolidation is accomplished using the following formula:

$$U_{ave} = 1 - \left(\frac{\int_0^H u_{t=ti} dz}{\int_0^H u_{t=0} dz}\right) \qquad Eq. (3.77)$$

$$OR \qquad U_{ave} = \frac{S_t}{S_T} \times 100(\%) \qquad Eq. (3.78)$$

Where S_t is the consolidation settlement at time where average degree of consolidation is to be calculated and S_T is the total consolidation settlement. The total consolidation settlement is given by:

$$S_T = m_v \sigma H \qquad Eq. (3.79)$$

4 ANALYSIS AND DISCUSSION

4.1 General

This study concentrates on the application of finite difference method based numerical methods for the solution of Terzaghi's consolidation theory. The solution proposed using the finite difference method is then written into programming language using Visual Basic.Net 2008 and application software, ***ConsolAnalysis***, is developed for the analysis of consolidation.

The solution for different load case scenarios, variable coefficient of consolidation and stratified soil conditions are designed as separate window forms to increase the user friendly nature of the software.

In this chapter, the solutions obtained from the numerical method and/or the output of the software will be compared with the calculations made using the empirical method. Only the results of the settlement and average degree of consolidation will be used as a medium for comparison.

4.2 Finite Difference Solution (Output of the Software)

The settlement and average degree of consolidation of the hypothetical problem due to different load case scenarios are discussed in this section. The outline for presentation of the software output is as summarized in the Table 4.1.

Table 4.1: Summary of different scenarios to be analyzed using Finite Difference method

Section 4.2.1.1	Calculation for constant load and constant C_V.
Section 4.2.1.2	Calculation for constant loading and variable C_V with inter-granular pressure.
Section 4.2.1.3	Calculation for constant loading and constant rate change of C_V.
Section 4.2.1.4	Calculation for constant rate change of loading and constant C_V.
Section 4.2.1.5	Calculation for abrupt change of change of loading and constant C_V.
Section 4.2.1.6	Calculation for constant rate change of loading and constant rate change of C_V.
Section 4.2.1.7	Calculation for abrupt change of loading and constant rate change of C_V.
Section 4.2.1.8	Double drainage condition.

4.2.1 Calculation for Constant Load and Constant C_V Case

A hypothetical problem (Figure **4.1**) will be considered in order to study the consolidation analysis using both the numerical method and the empirical method. For the case of constant loading and constant coefficient of consolidation, a 4m layer of clay, which is free to drain at its upper boundary, and rest on an impermeable base is considered. The soil properties are represented by the value of coefficient of volume compressibility, m_v and the coefficient of consolidation C_V as given in the Figure **4.1**.

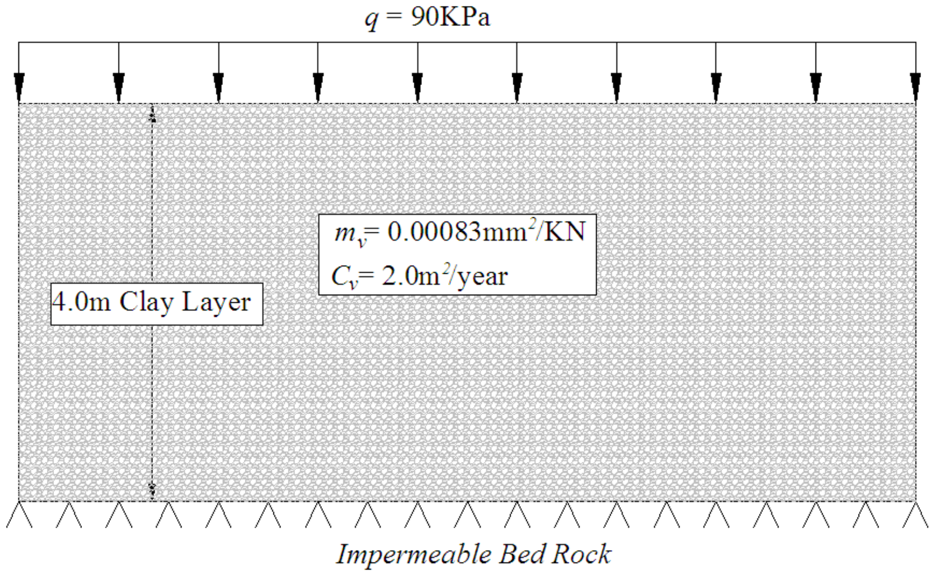

Figure 4.1: Clay soil subjected to constant loading.

In this section, the calculation is presented for two different conditions, i.e. for β $(=C_v\Delta t/(\Delta z^2)$ equal to 0.5 and for β less than 0.5.

Case (a): β = 0.5

As discussed in Chapter 3, if the value of β is set to 0.5, the finite difference equation takes the following simple form:

$$u_{i,j+1} = \Delta q + u_{i,j} + \frac{C_V\Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(4.1)$$

$$u_{(i,j+1)} = \Delta q + u_{(i,j)} + 0.5\left[u_{(i+1,j)} - 2u_{(i,j)} + u_{(i-1,j)}\right] \qquad Eq.(4.2)$$

$$u_{(i,j+1)} = \Delta q + \frac{1}{2}\left[u_{(i+1,j)} + u_{(i-1,j)}\right] \qquad Eq.(4.3)$$

Initially the pore-water pressure is subjected to a hydrostatic excess pressure of 90kPa everywhere in the clay layer. The pore water pressure at the drainage surface becomes zero instantaneously. In the case under consideration the surcharge is applied at $t = 0$ and remains constant thereafter so that $\Delta q = 0$. The solution then proceeds as follow:

Step 1: Divide the deposit into sub layers.

In this case the deposit is divided into 4 sub-layers of the same thickness and thus $\Delta z = 1$m.

Step 2: Select $\Delta t = 0.25$ to fix β ($C_v\Delta t/(\Delta z^2)$) = 0.5

For the case under consideration:

$$\beta = \frac{C_v \Delta t}{\Delta z^2} = \frac{2 \times 0.25}{1^2} = 0.5$$

Step 3: Calculate the initial pore pressure

Because there cannot be an instantaneous volume change it follows that

$$u(t = 0) = q(t = 0) = 90kPa \quad \text{Except at the top drainage surface}$$

Step 4: March the solution forward using the finite difference equation and introducing the boundary conditions.

Sample calculation after one year and at a depth of 2m is as follows:

Governing consolidation equation valid for nodes that are not boundary nodes

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right)$$

Hence at t=1year and z=2m

$$u_{2m,1yr} = u_{2m,0.75yr} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{1m,0.75yr} - 2u_{2m,0.75yr} + u_{3m,0.75yr}\right)$$

$$u_{2m,1yr} = 67.5 + \frac{2 \times 0.25}{(1)^2}(33.75 - 2 \times 67.5 + 78.75) = 56.25$$

Similarly for nodes at the boundary, the pore water pressure after 2years and at the depth of 4m becomes:

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right)$$

$$u_{4m,2yr} = u_{4m,1.75yr} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{3m,1.75yr} - 2u_{4m,1.75yr}\right)$$

$$u_{4m,2yr} = 67.5 + \frac{2 \times 0.25}{(1)^2}(2 \times 57.66 - 2 \times 67.5) = 57.66$$

Step 6: Calculate settlement

The consolidation settlement at the end of two years is calculated as:

$$S_t = m_v \sigma H - m_v \left[\left(\frac{u_o + u_n}{2} \right) + \sum_{i=1}^{n-1} u_i \right] \Delta z$$

$$S = 0.00083 \times 90 \times 4 - 0.00083 \times \left[\left(\frac{0 + 57.66}{2} \right) + 23.9 + 40.78 + 57.66 \right] \times 1$$

$$S = 0.17333\text{m or } 173.33\text{mm}$$

Step 6: Calculate Average degree of Consolidation

The average degree of consolidation after 2 years then becomes,

$$U_{ave} = \frac{S_i}{S_{tot}} \times 100(\%)$$

When S_i is the settlement at 2years = 0.17333m

S_{tot} is the total Settlement

$$S_{tot} = m_v q H = 0.00083 \times 90 \times 4 = 0.2988m$$

$$U_{ave} = \frac{0.17333m}{0.2988m} \times 100 = 58.01\%$$

The settlement after two years is 173.33mm and the average degree of consolidation is 58.01%. The complete result of consolidation analysis for the present case is tabulated in **Table 4.2**.

Table 4.2: Solution for constant load and constant C_v applied to clay layer with β= 0.5

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	45.00	45.00	33.75	33.75	28.12	28.12	23.90	23.90
	2.00		90.00	90.00	67.50	67.50	56.25	56.25	47.81	47.81	40.78
	3.00		90.00	90.00	90.00	78.75	78.75	67.50	67.50	57.66	57.66
	4.00		90.00	90.00	90.00	90.00	78.75	78.75	67.50	67.50	57.66
Settlement(mm)			0.00	74.70	93.38	112.05	126.06	140.07	151.74	163.41	173.33
Average Degree of Consolidation (%)			0.00	25.00	31.25	37.50	42.19	46.88	50.78	54.69	58.01

Case (b) β < 0.5

Here, the time step is changed to 0.20 year which gives β $(C_v\Delta t/(\Delta z^2)$ equal to 0.4. Table **4.3** shows the results of consolidation for this case.

Table 4.3: Numerical solution for constant load and C_V applied to clay with β ≠ ½

Time in years			0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	54.00	46.80	39.60	35.86	32.57	30.26	28.12	26.33	24.65	23.13
	2.00		90.00	90.00	75.60	69.84	63.50	59.36	55.18	51.77	48.46	45.50	42.69
	3.00		90.00	90.00	90.00	84.24	80.78	75.71	71.57	67.15	63.19	59.32	55.74
	4.00		90.00	90.00	90.00	90.00	85.39	81.70	76.91	72.64	68.25	64.20	60.30
Settlement(mm)			0.00	67.23	85.16	100.70	113.85	125.75	136.56	146.61	155.95	164.70	172.88
Average Degree of Consolidation (%)			0.00	22.50	28.50	33.70	38.10	42.09	45.70	49.07	52.19	55.12	57.86

For this case the settlement after two years (Table **4.3**) is 172.88mm with the average degree of consolidation equal to 57.86%. The results for both the analyses are quite close even though the β value used is different. After 2 years the settlement predicted from using β equal to 0.5 is 173.33 mm, which compares well with the settlement calculated using β equal to 0.4 which is 172.88mm.

Case (c) β = 0.5 and double drainage case

This case is similar to the hypothetical problem in Section 4.2.1 case (a) except that both top and bottom boundaries are pervious. The finite difference solution of this case is as tabulated in Table **4.4**.

Table 4.4: Numerical solution for constant loading and C_V case with double drainage

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	45.00	45.00	22.50	22.50	11.25	11.25	5.62	5.62
	2.00		90.00	90.00	45.00	45.00	22.50	22.50	11.25	11.25	5.62
	3.00		90.00	45.00	45.00	22.50	22.50	11.25	11.25	5.62	5.62
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	149.40	186.75	224.10	242.78	261.45	270.79	280.13	284.81
Average Degree of Consolidation (%)			0.00	50.00	62.50	75.00	81.25	87.50	90.63	93.75	95.32

4.2.2 Constant Load and Variable C_V Case based on Ronald Fraser Scott thesis Equation

Table **4.5** shows the results of the software output for hypothetical problem of Figure **4.1** where the variability of C_V is as per equation described under section 3.4.1.1(ii).

Table 4.5: Numerical solution for constant load and variable C_V according to Ronald, F.S.(1953)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
C_V (m^2/yr)			2.00	1.78	1.59	1.52	1.45	1.40	1.37	1.32	1.26
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	52.20	46.48	39.14	35.82	32.59	30.45	28.35	26.65
	2.00		90.00	90.00	73.65	69.11	62.40	58.78	54.43	51.29	47.97
	3.00		90.00	90.00	90.00	82.72	79.93	73.88	70.19	65.43	61.76
	4.00		90.00	90.00	90.00	90.00	83.48	80.34	74.69	70.77	66.18
Settlement(mm)			0.00	68.72	87.04	102.94	116.29	128.30	139.10	149.02	158.14
Average Degree of Consolidation (%)			0.00	23.00	29.13	34.45	38.92	42.94	46.55	49.87	52.93

After 2 years the coefficient of consolidation decreases from initial value of 2.0m^2/year to a value of 1.3m^2/year. As a result of this, the settlement and average degree of consolidation becomes 158.14mm and 52.93% respectively.

4.2.3 Constant Load and Variable C_v with constant rate Case

In this analysis, taking the hypothetical problem of Figure **4.1**, the coefficient of consolidation is assumed to decrease by 0.4m^2/yr per year linearly with time. Choosing 4 layers with a time step of 0.25 years gives β = 0.5 as before. The results are summarized in Table **4.6**. The decreased coefficient of consolidation will only decrease the β value, thus still providing a more accurate β value for the analysis of the consolidation at later time.

Table 4.6: Numerical solution for constant load with variable C_V decreasing at 0.4m^2/year/year

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
C_V (m^2/yr)			2.00	1.90	1.80	1.70	1.60	1.50	1.40	1.30	1.20
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	47.25	45.22	36.86	34.60	31.56	29.80	28.11	26.76
	2.00		90.00	90.00	70.76	68.08	61.09	58.10	54.41	51.73	49.27
	3.00		90.00	90.00	90.00	81.82	79.60	74.11	70.76	67.16	64.17
	4.00		90.00	90.00	90.00	90.00	83.46	80.56	76.04	72.61	69.34
Settlement(mm)			0.00	72.83	90.49	106.44	118.67	129.44	138.62	146.66	153.66
Average Degree of Consolidation (%)			0.00	24.37	30.28	35.62	39.72	43.32	46.39	49.08	51.43

The results in Table **4.6** show that a settlement of 153.66mm and average degree of consolidation of 51.43% after 2 years.

Suppose that the layer of clay is free to drain at its upper boundary and rests on a permeable base which allows it to drain freely. Then the pore water pressure at the upper and lower boundary is equal to zero. The finite difference solution for this case is as summarized in Table **4.7**.

Table 4.7: Numerical solution for constant load and constant C_v with double drainage

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	47.25	45.22	28.68	24.20	18.12	14.61	11.79	9.72
	2.00		90.00	90.00	51.52	46.16	32.18	26.20	20.54	16.69	13.75
	3.00		90.00	47.25	45.22	28.68	24.20	18.12	14.61	11.79	9.72
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	145.67	180.97	212.88	231.92	246.97	257.50	265.38	271.25
Average Degree of Consolidation (%)			0.00	48.75	60.57	71.24	77.62	82.65	86.18	88.82	90.78

The results show that a settlement of 271.25 mm and average degree of consolidation of 90.78% after 2 years.

Table 4.8: Numerical solution for constant load with double drainage for duration of 6 months with smaller time step

Time in years			0.00	0.10	0.20	0.30	0.40	0.50
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	72.36	61.85	54.24	48.11	42.94
	2.00		90.00	90.00	83.23	75.19	67.48	60.51
	3.00		90.00	72.36	61.85	54.24	48.11	42.94
	4.00		0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	103.98	127.05	146.35	162.93	177.30
Average Degree of Consolidation (%)			0.00	34.80	42.52	48.98	54.53	59.34

From Table **4.8**, we can see that the settlement predicted after half a year is 177.30 mm and average degree of consolidation of 59.34%. Comparing these results with the results in Table **4.7** which predicted consolidation settlement of 180.97 mm and average degree of consolidation of

60.57% after half a year, it can be concluded that the uses of finer time step is vital in obtaining a more accurate results of consolidation analysis.

4.2.4 Constant Rate Change of Loading and Constant C_V Case

Constant rate variability in loading with constant coefficient of consolidation throughout the consolidation process is studied by assuming that the fill having a unit weight of 15kN/m^3 is overlaid at a rate of 0.5 m/month for one year. The final load would be 90kPa and application of this load starts after a year and will remain constant thereafter.

Case (a) β = 0.5

Choosing four depth grid at z = 1m with a time step of 0.25 years as before gives β = 0.5. The analysis only differs from that of constant loading and constant C_V case in that the value of Δq needs to be included in the finite difference equation. In this case, the usefulness of the superposition is demonstrated, in a way that only the values obtained from the simple instantaneous loading case Δq are utilized, and the value at each point are merely added to the preceding value. The result of the finite difference analysis is shown in Table **4.9**.

Table 4.9: Numerical solution for variable load applied to clay with β = ½

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			0.00	22.50	45.00	67.50	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		0.00	22.50	33.75	45.00	53.44	39.38	35.16	30.94	28.48
	2.00		0.00	22.50	45.00	61.88	78.75	70.32	61.88	56.96	52.04
	3.00		0.00	22.50	45.00	67.50	87.19	84.38	78.76	73.13	67.86
	4.00		0.00	22.50	45.00	67.50	90.00	87.19	84.38	78.76	73.13
Settlement(mm)			0.00	9.34	28.01	51.35	79.36	101.53	117.87	132.46	145.30
Average Degree of Consolidation (%)			0.00	12.50	18.75	22.91	26.56	33.98	39.45	44.33	48.63

It can be seen from Table **4.9** that settlement after two years is 145.3 mm.

Case (b) β ≠ 0.5

If the calculation is repeated for the case in which there are 5 sub-layers, and a time step of 0.1 years is adopted, giving β = 0.3125 the result will be as shown in Table **4.10**. Here it is shown that the β value can be changed by changing the value for time increment or the thickness of the sub-layer. Nevertheless, the β value must always comply with the condition in which it must be in the range of more than 0 to equal or less than 0.5.

Table 4.10: Numerical solution for constant rate variability in load and constant C_V (β = 0.3125)

Time in years			0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Load(KN/m2)			0.00	9.00	18.00	27.00	36.00	45.00	54.00	63.00	72.00	81.00	90.00
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.80		0.00	9.00	15.19	20.32	24.78	28.79	32.45	35.85	39.03	42.03	44.88
	1.60		0.00	9.00	18.00	26.12	33.58	40.50	46.98	53.08	58.87	64.39	69.67
	2.40		0.00	9.00	18.00	27.00	35.72	44.14	52.24	60.05	67.58	74.85	81.88
	3.20		0.00	9.00	18.00	27.00	36.00	44.91	53.70	62.32	70.76	79.00	87.03
	4.00		0.00	9.00	18.00	27.00	36.00	45.00	53.94	62.79	71.50	80.04	88.39
Settlement(mm)			0.00	2.99	7.84	13.98	21.19	29.32	38.29	48.01	58.44	69.53	81.24
U_{av} (%)			0.00	10.01	13.12	15.60	17.73	19.63	21.36	22.95	24.45	25.86	27.19

Constant rate variability in load and constant C_V table continued

Time in years			1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.80		38.60	35.02	32.39	30.33	28.64	27.22	25.98	24.89	23.91	23.01
	1.60		65.74	61.61	58.19	55.26	52.72	50.47	48.46	46.63	44.94	43.37
	2.40		79.67	77.25	74.61	72.05	69.60	67.28	65.07	62.97	60.96	59.03
	3.20		85.85	84.45	82.83	81.00	79.05	77.02	74.96	72.88	70.81	68.77
	4.00		87.54	86.48	85.21	83.72	82.02	80.16	78.20	76.18	74.12	72.05
Settlement(mm)			90.55	98.56	105.82	112.55	118.84	124.79	130.43	135.81	140.98	145.94
U_{av} (%)			30.30	32.99	35.41	37.67	39.77	41.76	43.65	45.45	47.18	48.84

From Table **4.10**, the settlement at 2 years is 145.94 mm. Again the settlement at 2 years is quite similar with the case made for β = 0.5 which predicted settlement of 145.3 mm and the difference in the results of consolidation settlement is 0.44%.

4.2.5 Abrupt change in application of loading and Constant C_V Case

Suppose the surface loading for the hypothetical problem is filled with surcharge of 30kPa after one year to the existing fill of 60kPa. Table **4.11** shows the result of finite difference analysis for this case.

Table 4.11: Numerical solution for abrupt change of loading and constant C_V

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		60.00	30.00	30.00	22.50	52.50	33.75	33.75	27.19	27.19
	2.00		60.00	60.00	45.00	45.00	67.50	67.50	54.38	54.38	45.94
	3.00		60.00	60.00	60.00	52.50	82.50	75.00	75.00	64.69	64.69
	4.00		60.00	60.00	60.00	60.00	82.50	82.50	75.00	75.00	64.69
Settlement(mm)			0.00	49.80	62.25	74.70	96.49	118.28	132.28	146.28	157.56
Average Degree of Consolidation (%)			0.00	25.00	31.25	37.50	32.29	39.59	44.27	48.96	52.73

Since the addition of the desired load to the hydrostatic excess pore water pressure was done after some time, the consolidation due to the load increment was proceeded independently of the consolidation existing previously. This is clearly shown at time 1.0 year when all the hydrostatic pore water pressure along the nodes was added with Δq= 30 *kPa*. The results show that a settlement of 157.56mm and average degree of consolidation of 52.73% after 2 years.

4.2.6 Constant Rate Change of Loading and Variable C_V with constant rate

The case for constant rate variability in loading is studied by assuming that the fill having a unit weight of 15kN/m^3 is overlaid at a rate of 0.5 m/month for one year. The final load would be 90kPa and application of this constant load starts after a year. The coefficient of consolidation is assumed to decrease by 0.4m^2/yr/yr linearly with time. Choosing 4 sub layers with a time step of 0.25 years gives β = 0.5. The results for this case are shown as Table **4.12**.

Table 4.12: Numerical solution for constant rate change of load and constant rate decrease in C_v

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			0.00	22.50	45.00	67.50	90.00	90.00	90.00	90.00	90.00
C_V (m^2/yr)			2.00	1.90	1.80	1.70	1.60	1.50	1.40	1.30	1.20
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		0.00	22.50	34.88	46.86	57.15	44.62	39.55	36.01	33.52
	2.00		0.00	22.50	45.00	63.20	80.88	74.76	68.20	63.72	60.14
	3.00		0.00	22.50	45.00	67.50	88.28	86.15	83.06	79.48	76.23
	4.00		0.00	22.50	45.00	67.50	90.00	88.71	86.92	84.41	81.45
Settlement(mm)			0.00	9.34	27.07	48.71	73.61	91.40	104.36	115.03	123.99
Average Degree of Consolidation (%)			0.00	12.50	18.12	21.74	24.64	30.59	34.93	38.50	41.50

4.2.7 Abrupt change of loading and Variable C_V with constant rate case

Suppose the surface loading for the hypothetical problem in Figure **4.1** is filled with surcharge of 30kPa after one year to the existing fill of 60kPa and subjected to a constant decrease in coefficient of 0.4m^2/yr per year linearly with time. Table **4.13** shows the result of finite difference analysis for this case.

Table 4.13: Numerical solution for abrupt change of loading and constant rate decrement of C_v

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
C_V (m^2/yr)			2.00	1.90	1.80	1.70	1.60	1.50	1.40	1.30	1.20
Depth(m)	**0.00**	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		60.00	31.50	30.15	24.57	53.07	39.79	36.00	32.86	30.72
	2.00		60.00	60.00	47.18	45.39	70.73	68.74	62.34	58.60	55.35
	3.00		60.00	60.00	60.00	54.55	83.07	79.41	77.18	73.50	70.50
	4.00		60.00	60.00	60.00	60.00	85.64	83.71	80.70	78.41	75.46
Settlement(mm)			0.00	48.55	60.32	70.96	91.56	108.07	119.63	129.34	137.53
Average Degree of Consolidation (%)			0.00	24.37	30.28	35.62	30.64	36.17	40.04	43.29	46.03

4.3 Empirical Solution

Based on the hypothetical problem of Figure **4.1**, the settlement and average degree of consolidation for various load case scenario and coefficient of consolidation cases will be determined using empirical methods. In determination of consolidation settlement and average degree of consolidation, the following table of degree of consolidation versus time factor was established using Eq. (2.41) and (2.42) which will be used for empirical analysis of consolidation problems in the following sections.

Table 4.14: Variation of Time factor (T_v) with average degree of consolidation

U_{av} (%)	T_v
0	0.000
10	0.008
20	0.031
30	0.071
40	0.126
50	0.196
60	0.286
70	0.403
80	0.567
90	0.848
99	1.781

For the purpose of comparison with the numerical solutions discussed above, the following cases, as presented in Table **4.15**, will be analyzed using empirical method of consolidation analysis.

Table 4.15: Summary of different scenarios to be analyzed using Empirical method

Section 4.3.1	Calculation for constant load and constant C_V.
Section 4.3.2	Calculation for constant loading and constant rate change of C_V.
Section 4.3.3	Calculation for constant rate change of loading and constant C_V.
Section 4.3.4	Calculation for abrupt change of change of loading and constant C_V.
Section 4.3.5	Calculation for constant rate change of loading and constant rate change of C_V.
Section 4.3.6	Calculation for abrupt change of loading and constant rate change of C_V.
Section 4.3.7	Double drainage condition.

4.3.1 Calculation for Constant Load and Constant C_V Case

The empirical calculation will be done for the hypothetical problem in Figure **4.1** with their respective loading and coefficient of consolidation variation cases which were discussed for finite difference calculation (Section 4.3).

For the present case of constant loading and constant coefficient of consolidation the empirical calculation will be undertaken as follows:

Step 1: Calculate the total settlement

$S_c = m_v \Delta\sigma H$

$S_c = 0.00083 \times 90 \times 4 = 0.2988m$

Step 2: Select t = 2.0years and calculate the Time Factor at that time.

$$T_v = \frac{C_v t}{(H_d)^2}$$

$$T_v = \frac{2 \times 2.00}{(4)^2} = 0.25$$

Step 3: Using the time factor obtained in step 2, find the value for degree of consolidation from Table **4.14**.

$U_{ave} = 50\%$ $\quad\quad$ $T_v = 0.196$

$U_{ave} = 60\%$ $\quad\quad$ $T_v = 0.287$

By linear interpolation, U_{av} corresponding to $T_v = 0.25$ is

$$U_{av} = 50 + \left(\frac{10}{(0.287 - 0.196)} \times (0.25 - 0.196)\right) = 55.93\%$$

Thus,

$U_{av} = 55.93\%$

Step 4: Calculate settlement by multiplying the average degree of consolidation obtained in step 3 and the total settlement obtained in step 1.

Thus, settlement after 2 years:

$$Settlement = U_{ave} \times total\ Settlemet$$

$$Settlement = 0.56 \times 298.8 = 167.33mm$$

Table 4.16 shows the calculation of the settlement and average degree of consolidation with time using the empirical methods.

Table 4.16: Empirical solution for constant load and constant C_V

t(years)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	90	90	90	90	90	90	90	90	90
T_v	0.000	0.031	0.063	0.094	0.125	0.156	0.188	0.219	0.250
U_{av} ***(%)***	0	20.00	28.00	34.18	39.82	44.29	48.86	52.56	56.00
Settlement(mm)	0.00	59.76	83.66	102.13	118.98	132.34	145.99	157.05	167.33

The settlement after 2 year is 167.33mm and the average degree of consolidation is 56.0%.

4.3.2 Constant Load and Variable C_v with constant rate Case

The variable value of C_v will be taken into consideration in the time factor, T_v, calculations. After obtaining the time factor value, the average degree of consolidation and consolidation settlement is calculated. For this case, the empirical solution is summarized in Table **4.17**.

Table 4.17: Empirical solution for constant loading and variable C_V

C_v	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2
t(years)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	90	90	90	90	90	90	90	90	90
T_v	0.00	0.030	0.056	0.080	0.100	0.117	0.131	0.142	0.150
U_{av} ***(%)***	0	19.57	26.25	31.64	35.27	38.36	40.71	42.29	43.43
Settlement(mm)	0.00	58.48	78.44	94.54	105.39	114.62	121.64	126.36	129.77

The settlement after 2 year is 129.77mm and the average degree of consolidation is 43.43 %.

4.3.3 Constant Rate Change of Loading and Constant C_v Case

For the case of constant increment in loading with time, the integral of the loading increment cannot be evaluated exactly. Hence, to calculate the settlement, the integral can be evaluated approximately by a series of steps-load increment. In order to calculate the settlement due to step load increment, it is mandatory to find the settlement induced by the previous load and at the same time find the settlement induced by the additional load. Moreover, the average degree of consolidation will start from zero whenever there is an increment of load applied onto the soil layer. Table **4.18** shows the calculation of the ultimate consolidation settlement for each load increment. The result of empirical analysis for this case is as tabulated in Table **4.19**.

Table 4.18: Calculation of ultimate settlement against load

Load(*Kpa*)	Settlement (mm)
0	0.0
22.5	74.7
45	149.4
67.5	224.1
90	298.8

Table 4.19: Empirical solution for variable load at constant rate and constant C_V

t(years)	0.00	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	0.00	22.5	45.00	67.50	90.00	90.00	90.00	90.00	90.00
Calculation of settlement and U_{av} assuming that 22.5KPa load is applied from start up to 0.5 year									
T_v	0.00	0.031	0.063	0.063	0.063	0.063	0.063	0.063	0.063
$U_{av}(\%)$	0.00	20.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00
Settlement(mm)	0.00	14.94	20.92	20.92	20.92	20.92	20.92	20.92	20.92
Calculation of settlement and U_{av} assuming that 45KPa load is applied from 0.5 to 0.75 year									
T_v	0.00	0.00	0.00	0.031	0.063	0.063	0.063	0.063	0.063
$U_{av}(\%)$	0.00	0.00	0.00	20.00	28.00	28.00	28.00	28.00	28.00
Settlement(mm)	0.00	0.00	0.00	29.88	41.83	41.83	41.83	41.83	41.83
Calculation of settlement and U_{av} assuming that 67.5KPa load is applied from 0.75 to 1 year									
T_v	0.00	0.00	0.00	0.000	0.031	0.063	0.063	0.063	0.063
$U_{av}(\%)$	0.00	0.00	0.00	0.00	20.00	28.00	28.00	28.00	28.00
Settlement(mm)	0.00	0.00	0.00	0.00	44.82	62.75	62.75	62.75	62.75
Calculation of settlement and U_{av} assuming that 90KPa load is applied from 1 to 2 year									
T_v	0.00	0.00	0.00	0.00	0.00	0.031	0.063	0.094	0.125
$U_{av}(\%)$	0.00	0.00	0.00	0.00	0.00	20.00	28.00	34.18	39.82
Settlement(mm)	0.00	0.00	0.00	0.00	0.00	59.76	83.66	102.13	118.98
Cumulative settlement and U_{av} using super position of the above four cases									
Settlement(mm)	0.00	14.94	20.92	50.80	107.57	185.26	209.16	227.63	244.48
$U_{av}(\%)$	0.00	20.00	28.01	34.00	48.00	62.00	70.00	76.18	81.82

The settlement after 2 year is 244.48mm and the average degree of consolidation is 81.82%.

4.3.4 *Abrupt change in application of loading and Constant C_v Case*

In the case of stage loading, the calculation of settlement after the application of the additional surcharge is calculated in the same manner as that of section 4.3.3. When the first loading, 60KPa load, is applied onto the soil layer, the settlement at any time interval was computed in the same fashion as the calculation for constant load case. At t=1year when the additional surcharge is applied, it will induce a new settlement curve due to the 90KPa load. At the same time, the settlement due to the 60KPa load is S_1. Thus, for the total settlement at this time is equal to S_1 plus the settlement due to the 90KPa load, which is S_2. The ultimate settlement due to the load applied is shown below:

$$S_1 = m_v \times \Delta q \times H$$

$$S_1 = 0.00083 \times 60 \times 4 = 199.2mm$$

$$S_2 = m_v \times \Delta q \times H$$

$$S_2 = 0.00083 \times (90 + 30) \times 4 = 298.8mm$$

Table 4.20: Empirical solution for abrupt change of load and constant C_v

t(years)	0.00	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	60	60	60	60	90	90	90	90	90
Calculation of settlement and U_{av} assuming that 60KPa load is applied from start up to 1.0 year									
T_v	0.00	0.031	0.063	0.094	0.125	0.125	0.125	0.125	0.125
U_{av}(%)	0.00	20.00	28.00	34.18	39.82	39.82	39.82	39.82	39.82
Settlement(mm)	0.00	39.84	55.78	68.09	79.32	79.32	79.32	79.32	79.32
Calculation of settlement and U_{av} assuming that 90KPa load is applied from 1.0 to 2.0 year									
T_v	0.00	0.00	0.00	0.000	0.00	0.031	0.063	0.094	0.125
U_{av}(%)	0.00	0.00	0.00	0.00	0.00	20.00	28.00	34.18	39.82
Settlement(mm)	0.00	0.00	0.00	0.00	0.00	59.76	83.66	102.13	118.98
Cumulative settlement and U_{av} using super position of the above two cases									
Settlement(mm)	0.00	39.84	55.78	68.09	79.32	139.08	162.98	181.45	198.30
U_{av}(%)	0.00	20.00	28.00	34.18	39.82	46.55	54.54	60.73	66.37

The settlement after 2 year is 198.30 mm and the average degree of consolidation is 66.37%.

4.3.5 *Constant Rate Change of Loading and Variable C_v with constant rate case*

Using the same hypothetical problem and methodology as section 4.3.3 except the coefficient of consolidation decreases at rate of 0.4m^2/yr/yr, the result of empirical analysis is as shown in Table **4.21**.

Table 4.21: Empirical solution for constant rate change of load and constant rate change in C_v

t(years)	0.00	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	0.00	22.5	45.00	67.50	90.00	90.00	90.00	90.00	90.00
C_v	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2
Calculation of settlement and U_{av} assuming that 22.5KPa load is applied from start upto 0.5 year									
T_v	0.00	0.030	0.056	0.056	0.056	0.056	0.056	0.056	0.056
U_{av}(%)	0.00	19.57	26.25	26.25	26.25	26.25	26.25	26.25	26.25
Settlement(mm)	0.00	14.62	19.61	19.61	19.61	19.61	19.61	19.61	19.61
Calculation of settlement and U_{av} assuming that 45KPa load is applied from 0.5 to 0.75 year									
T_v	0.00	0.00	0.0000	0.0266	0.0500	0.0500	0.0500	0.0500	0.0500
U_{av}(%)	0.00	0.00	0.00	18.09	24.75	24.75	24.75	24.75	24.75
Settlement(mm)	0.00	0.00	0.00	27.03	36.98	36.98	36.98	36.98	36.98
Calculation of settlement and U_{av} assuming that 67.5KPa load is applied from 0.75 to 1 year									
T_v	0.00	0.00	0.00	0.000	0.025	0.047	0.047	0.047	0.047
U_{av}(%)	0.00	0.00	0.00	0.00	17.39	24.00	24.00	24.00	24.00
Settlement(mm)	0.00	0.00	0.00	0.00	38.97	53.78	53.78	53.78	53.78
Calculation of settlement and U_{av} assuming that 90KPa load is applied from 1 to 2 year									
T_v	0.00	0.00	0.00	0.00	0.00	0.023	0.044	0.061	0.075
U_{av}(%)	0.00	0.00	0.00	0.00	0.00	16.52	23.25	27.50	30.73
Settlement(mm)	0.00	0.00	0.00	0.00	0.00	49.36	69.47	82.17	91.82
Cumulative settlement and U_{av} using super position of the above four cases									
Settlement(mm)	0.00	14.62	19.61	46.64	95.56	159.73	179.84	192.54	202.19
U_{av}(%)	0.00	19.57	26.25	31.22	42.64	53.46	60.19	64.44	67.67

The settlement after 2 year is 202.19 mm and the average degree of consolidation is 67.67%.

4.3.6 Abrupt change of loading and Variable C_V with constant rate case

The case for Abrupt change of loading and Variable C_v with constant decrement rate was analyzed Empirically using the same hypothetical problem and method of calculation as done for the above five cases and the results of Empirical solution is as presented in Table **4.22**.

Table 4.22: Empirical solution for abrupt change of load and constant rate change in C_v

t(years)	0.00	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	60	60	60	60	90.00	90.00	90.00	90.00	90.00
C_v	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2
Calculation of settlement and U_{av} assuming that 60KPa load is applied from start up to 1 year									
T_v	0.00	0.030	0.056	0.080	0.100	0.100	0.100	0.100	0.100
U_{av}*(%)*	0.00	19.57	26.25	31.64	35.27	35.27	35.27	35.27	35.27
Settlement(mm)	0.00	38.98	52.29	63.03	70.26	70.26	70.26	70.26	70.26
Calculation of settlement and U_{av} assuming that 90KPa load is applied from 1 to 2 year									
T_v	0.00	0.00	0.00	0.000	0.000	0.023	0.044	0.061	0.075
U_{av}*(%)*	0.00	0.00	0.00	0.00	0.00	16.52	23.25	27.50	30.73
Settlement(mm)	0.00	0.00	0.00	0.00	0.00	49.36	69.47	82.17	91.82
Cumulative settlement and U_{av} using super position of the above two cases									
Settlement(mm)	0.00	38.98	52.29	63.03	70.26	119.62	139.73	152.43	162.08
U_{av}*(%)*	0.00	19.57	26.25	31.64	35.27	40.03	46.76	51.01	54.24

The settlement after 2 year is 162.08 mm and the average degree of consolidation is 54.24%.

4.3.7 Calculation for Constant Load and constant C_v with Double Drainage

When the soil layer is able to drain freely through top and bottom boundary surfaces, the height of the drainage path is half of the total thickness. Table **4.23** shows the settlement and average degree of consolidation for duration of 2 years whereas Table **4.24** depicts the consolidation process for half a year.

Table 4.23: Empirical solution for constant load and C_v with double drainage for 2 years

t(years)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
q(Kpa)	90	90	90	90	90	90	90	90	90
T_v	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
U_{av}***(%)***	0	39.82	56.00	67.61	75.91	82.06	86.51	90.48	92.70
Settlement(mm)	0.00	118.98	167.33	202.02	226.82	245.20	258.49	270.35	276.99

The settlement after 2 year is 276.99 mm and the average degree of consolidation is 92.70%.

Table 4.24: Empirical solution for constant load and C_v with double drainage for half a year

t(years)	0	0.1	0.2	0.3	0.4	0.5
q(Kpa)	90	90	90	90	90	90
T_v	0.000	0.050	0.100	0.150	0.200	0.250
U_{av}(%)	0	24.75	35.27	43.43	50.44	56.00
Settlement(mm)	0.00	73.95	105.39	129.77	150.71	167.33

The settlement after half a year is 167.33 mm and the average degree of consolidation is 56.0%.

4.4 Comparison and discussion

Table **4.25** and Table **4.26** summarize the consolidation settlement and the average degree of consolidation for the hypothetical problem as predicted from finite difference solution using the application software developed for consolidation analysis and empirical solution for different cases respectively.

Table 4.25: Calculation of settlement after 2 years with different methods

S. No	*Description of different scenario*	*Settlement (mm)*		
		Numerical Solution	*Empirical Solution*	*Difference (%)*
1	Constant loading and constant C_v	173.33	167.33	3.46
2	Constant loading and constant rate change of C_v	153.66	129.77	15.55
3	Constant rate change of loading and constant C_v	145.30	244.48	68.26
4	Abrupt change of loading and constant C_v	157.56	198.30	25.86
5	Constant rate change of load and constant rate change of C_v	123.99	202.19	63.07
6	Abrupt change of loading and constant rate change of C_v	137.53	162.08	17.85
7	Double drainage with constant loading and constant C_v	284.81	276.99	2.75

Table 4.26: Calculation of average degree of consolidation after 2 years with different methods

S. No.	Description of different scenario	Average Degree of Consolidation		
		Numerical Solution	Empirical Solution	Difference (%)
1	Constant loading and constant C_v	58.01	56.00	3.46
2	Constant loading and constant rate change of C_v	51.43	43.43	15.56
3	Constant rate change of loading and constant C_v	48.63	81.82	68.25
4	Abrupt change of loading and constant C_v	52.73	66.37	25.87
5	Constant rate change of load and constant rate change of C_v	41.50	67.67	63.06
6	Abrupt change of load and constant rate change of C_v	46.03	54.24	17.84
7	Double drainage with constant loading and constant C_v	95.32	92.70	2.75

The results from the analysis using both methods are relatively close for the constant load and constant coefficient of consolidation with all boundary drainage condition. After two years, the consolidation settlement predicted using finite different method for constant loading and constant coefficient of consolidation case is 173.33 mm, which compares well with the consolidation settlement calculated using the empirical method, 167.33 mm. Whereas in the double drainage condition of the same case, the consolidation settlement predicted is 284.81 mm and 276.99 mm using finite difference and empirical methods respectively.

As can be shown in the Table **4.25** and Table **4.26**, the difference between the results obtained from both methods is also comparable for the case of constant loading and constant coefficient of consolidation with single and double drainage conditions in which the difference is limited to an average of 3%.

In the case of constant loading and variable coefficient of consolidation, the difference was considerable. The finite difference solution predicted settlement of 153.66 mm whereas the empirical solution shows a settlement of 129.77 mm after 2 year. This gives a difference of 15.56%.

However, highly significant variation from the results of numerical solutions and that of the empirical solution was observed for variable loading and variable coefficient of consolidation cases which is in excess of 60%.

The variation in results especially for variable loading and variable coefficient of consolidation cases are due to the method of calculation used by finite difference method and that of empirical methods. In empirical methods of consolidation analysis, the calculation of consolidation settlement and average degree of consolidation involves interpolation from table of degree of consolidation versus time factor or curve fitting in which it is only an approximate solution and the load application especially for variable loading and variability in coefficient of consolidation cannot be modeled accurately as the empirical solution of consolidation process involves the assumption of instantaneous loading and constant coefficient of consolidation in its derivation.

While in the case of finite difference method of consolidation analysis, the above drawbacks can be enhanced due to the fact that finite difference method of consolidation analysis makes use of the original consolidation equation using time and depth grid to acquire a solution. Hence variability in load application and coefficient of consolidation can be modeled accurately without involvement of empirical and/or approximate methods.

Moreover, when the coefficient of a finite difference equation is evaluated at new time step, an iteration scheme has been employed as a part of the solution. This is because the software uses the solution at the new time step to evaluate the coefficients at each iteration level. The solution for the pore water pressure at this iteration level is obtained and it is used the coefficient of succeeding calculation.

Besides this, the way the software (finite difference method) calculates the excess pore water pressure in the case of load scenarios for variable loading is in such a manner that any instantaneous loadings applied during the duration of consolidation are merely added to the preceding value in each node at the time of exertion. Thus, the excess pore water pressure at the time of application is initial load exerted deduct with excess pore water pressure that has dissipated plus the instantaneous load. This means that the changes in vertical stress, $\Delta\sigma$ that influences the ultimate settlement at the time of addition of instantaneous load is not the total of existing load plus the instantaneous load, but lesser.

This was not the case considered in the empirical analysis in which the application of subsequent load during consolidation is added to the changes in the vertical stress without considering that the excess pore water pressure at the time of exertion of load is not the same as the initial excess pore water pressure due to the fact that part of the consolidation pressure due to the initial load has transferred to the soil skeleton.

5 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

Based on the study of consolidation analysis using the finite different method, the conclusions that can be drawn from this study are as follows:

1. Calculation using the numerical solution (Finite Difference Method) gives more accurate results than the solution done by the empirical method.
2. Finite difference method has a simpler form of equation which makes it easier to be written into module in the application software.
3. The accuracy of the finite difference solution is very sensitive to the value chosen for β $(C_v\Delta t/(\Delta z^2))$. The value of β should be between zero and 0.5 to maintain the stability of the computation and the use of smaller mesh spacing will provide more accurate results.
4. In order to apply the finite difference solution, the boundary drainage conditions and the initial distribution of excess pore water pressures must be clearly identified. This is to render the results obtained from numerical solution are more accurate and practical.
5. In a nut shell, with the careful study in the literature review and coding into visual basic programming language, the application software, ***ConsolAnalysis***, is developed for consolidation analysis of soils so that it is possible to undertake consolidation analysis for homogeneous and stratified soil profile under various loading and variability in coefficient of consolidation scenarios, with different boundary drainage condition.

5.2 Recommendation

The recommendation suggested here basically focus on the upgrading of the software to extend its application for the employment to wider problems. In this regard, incorporation of two and three dimensional consolidation analysis into the software to make it possible for calculation of other loading option like circular loads with the ability to calculate soil stress under different surface loading from Boussinesq Equation. Besides, integrating the effect of sand drain on the settlement of the layer will enhance the software.

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Appendix

1 Appendix 1 - Derivation of Consolidation Equation

The theory of one dimensional consolidation was first proposed by Terzaghi (1925). The derivation of the governing consolidation equation is presented in this section which is taken from Das, B. M, (1987). The underlying assumptions in the derivation of mathematical equations are as follows:

1. The soil layer is homogeneous.
2. The soil layer is fully saturated.
3. The compression of the soil layer is due to change in volume only, which in turn, is due to the squeezing out of water from the void spaces.
4. Darcy's law is valid.

$$v = \frac{q}{A} = ki$$

Where: v = velocity of flow or discharge velocity.
q = rate of flow or rate of discharge (discharge per unit time)
A = total cross section area of soil mass perpendicular to the direction of flow
i = hydraulic gradient
k = Darcy's coefficient of permeability

5. Deformation of the soil occurs only in the direction of the load application.

With the above assumption, let us consider a clay layer of thickness H_t as shown in Figure 1.1A. This clay layer is located between two highly permeable sand layers. When the soil is subjected to an increase in vertical pressure, $\Delta\sigma$, the pore water pressure at any point inside the clay layer will increase by u.

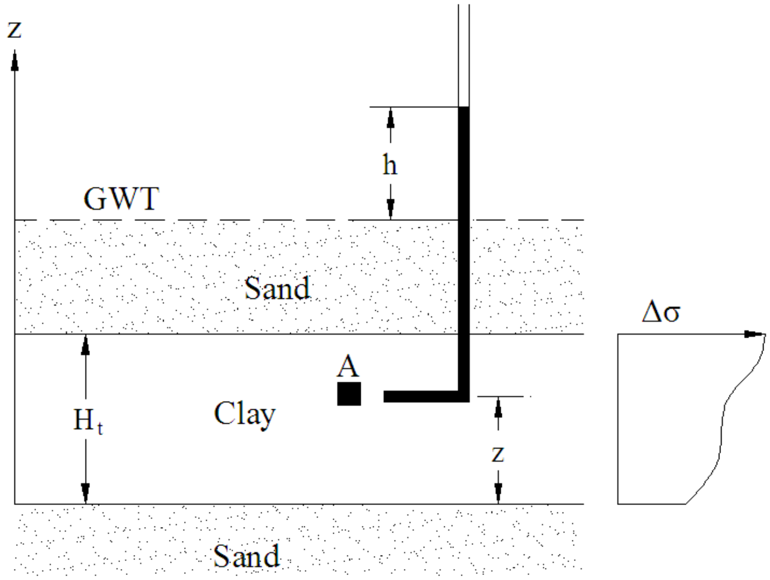

Figure 1.1A: Clay layer undergoing consolidation

Consider an elemental soil mass with a volume of *dx.dy.dz* at A. In the case of one dimensional consolidation, the flow of water into and out of the soil element is in one direction only, i.e., in the *z* direction. This means that q_x, q_y, dq_x, and dq_y in Figure 1.**2**A are equal to zero, and the rate of flow into and out of the soil element can be given by Eq. (1A.1) and Eq. (1A.2), respectively.

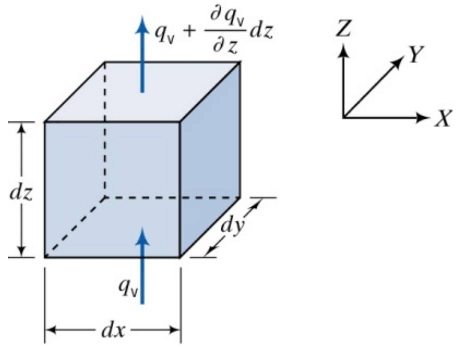

Figure 1.2A: One dimensional flow through a two dimensional soil element

$$q_z = k_z i_z A_z = k_z \frac{\partial h}{\partial z} dxdy \qquad (inflow) \qquad Eq.(1A.1)$$

$$q_z + dq_z = k_z \left(\frac{\partial h}{\partial z} + \frac{\partial^2 h}{\partial z^2} dz \right) dxdy \qquad (outflow) \qquad Eq.(1A.2)$$

Where q_z = flow entering in z direction

k_z = coefficient of permeability in the z direction (k)

h = hydraulic head

Hence, the rate of volume change of the soil element is given by:

$$\frac{\partial V}{\partial t} = (q_z + dq_z) - q_z \qquad Eq.(1A.3)$$

Where $$V = dxdydz \qquad Eq.(1A.4)$$

Substituting the right-hand sides of Eq. (1A.1) and (1A.2) into right-hand side of Eq. (1A.3), one obtains,

$$k\frac{\partial^2 h}{\partial z^2}dxdydz = \frac{\partial V}{\partial t} \qquad Eq.(1A.5)$$

Where k is the coefficient of permeability [k_z in Eq. (1A.1) and (1A.2)].

However,

$$h = \frac{u}{\gamma_w} \qquad Eq.(1A.6)$$

Where γ_w is the unit weight of water.

Substitution of Eq. (21A.6) into Eq. (1A.5) and rearranging gives

$$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{\partial V}{\partial t} \qquad Eq.(1A.7)$$

During consolidation, the rate of change of volume is equal to the rate of change of the void volume. So,

$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} \qquad Eq.(1A.8)$$

Where V_v is the volume of voids in the soil element

However,

$$V_v = eV_s \qquad Eq. (1A.9)$$

Where V_S is the volume of soil solids in the element, which is constant, and e is the void ratio. So,

$$\frac{\partial V}{\partial t} = V_s \frac{\partial e}{\partial t} = \frac{V}{1+e}\frac{\partial e}{\partial t} = \frac{dxdydz}{1+e}\frac{\partial e}{\partial t} \qquad Eq. (1A.10)$$

Substituting the result of Eq. (1A.10) into Eq. (1A.7) and rearranging gives:

$$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e}\frac{\partial e}{\partial t} \qquad Eq. (1A.11)$$

The change in void ratio, ∂e, is due to the increase in effective stress, assuming that these are linearly related, then

$$\partial e = -a_v \partial(\Delta\sigma') \qquad Eq. (1A.12)$$

Where a_v is the coefficient of compressibility

Again, the increase of effective stress is due to the decrease of excess pore water pressure, ∂e. Hence

$$\partial e = a_v \partial u \qquad Eq. (1A.13)$$

Combining Eq. (2.11) and (2.13) gives

$$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{a_v}{1+e}\frac{\partial e}{\partial t} = m_v\frac{\partial u}{\partial t} \qquad Eq. (1A.14)$$

Where

$$m_v = coefficient\ of\ volume\ compressibility = \frac{a_v}{1+e} \qquad Eq. (1A.15)$$

Rewriting and substitution of Eq. (1A.15) and Eq. (1A.17) into Eq. (1A.14), provides the governing one dimensional consolidation equation as shown in Eq. (1A.16) below.

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = C_V\frac{\partial^2 u}{\partial z^2} \qquad Eq. (1A.16)$$

Where

$$C_V = coefficient\ of\ consolidation = \frac{k}{\gamma_w m_v} \qquad Eq. (1A.17)$$

The coefficient C_V is called the coefficient of consolidation and its unit is length^2/time, for example, m^2/year.

1.1 Analytical (Fourier series) Solution of Consolidation Equation

Eq. (1A.16) is the basic differential equation of Terzaghi's consolidation theory and can be solved with proper boundary condition. To solve the equation, we assume u to be a product of two functions, i.e., the product of a function of z and a function of t, or

$$u = F(z)G(t) \qquad Eq. (1A.18)$$

So,

$$\frac{\partial u}{\partial t} = F(z)\frac{\partial}{\partial t}G(t) = F(z)G'(t) \qquad Eq. (1A.19)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2}{\partial z^2}F(z)G(t) = F''(z)G(t) \qquad Eq. (1A.20)$$

From Eq. (1A.16), (2.19), and (1A.20),

$$F(z)G'(t) = C_V F''(z)G(t) \qquad Eq. (1A.21)$$

Or

$$\frac{F''(z)}{F(z)} = \frac{G'(t)}{C_V G(t)} \qquad Eq. (1A.22)$$

The right-hand of Eq. (1A.22) is a function of z only and is independent of t; the left-hand of the equation is a function of t only and independent of z. Therefore they must be equal to a constant, say, $-B^2$. So,

$$F''(z) = -B^2 F(z) \qquad Eq. (1A.23)$$

A solution to Eq. (1A.23) can be given by

$$F(z) = A_1 \cos(Bz) + A_2 \sin(Bz) \qquad Eq. (1A.24)$$

Where A_1 and A_2 are constants

Again, the right hand side of Eq. (1A.23) may be written as

$$G'(t) = -B^2 C_V G(t) \qquad Eq. (1A.25)$$

The solution of Eq. (1A.25) is given by

$$G(t) = A_3 \exp\,(-B^2 C_V t) \qquad Eq. (1A.26)$$

Where A_3 is a constant.

Combining Eq. (1A.16), (1A.24), and (1A.26),

$$u = (A_1 \cos(Bz) + A_2 \sin(Bz)) A_3 \exp\,(-B^2 C_V t)$$

$$u = (A_4 \cos(Bz) + A_5 \sin(Bz)) \exp\,(-B^2 C_V t) \qquad Eq. (1A.27)$$

Where $A_4 = A_1 A_3$ and $A_5 = A_2 A_3$.

The constants in Eq. (1A.27) can be evaluated from the boundary conditions, which are as follows:

1. At time $t = 0$, $u = u_i$ (initial excess pore water pressure at any depth).
2. $u = 0$ at $z = 0$.
3. $u = 0$ at $z = H_t = 2H$

Note that H is the length of the longest drainage path. In this case, which is a two way drainage conditions (top and bottom of the clay layer), H is equal to half the total thickness of the clay layer.

The second boundary condition dictates that $A_4 = 0$, and from the third boundary condition one obtains:

$$A_5 \sin(2BH) = 0 \; or \; 2BH = n\pi$$

Where *n* is an integer.

Considering the above equation, a general solution of Eq. (1A.27) can be given in the form of

$$u = \sum_{n=0}^{n=\infty} A_n \sin\frac{n\pi z}{2H} \exp\left(\frac{-n^2\pi^2 T_V}{4}\right) \qquad Eq. (1A.28)$$

Where T_v is the non-dimensional time factor and is equal to $C_v t/H^2$.

To satisfy the first boundary condition, one must have the coefficient A_n such that

$$u_i = \sum_{n=0}^{n=\infty} A_n \sin\frac{n\pi z}{2H} \qquad Eq. (1A.29)$$

Eq. (1A.29) is a Fourier sine series, and A_n can be given by

$$A_n = \left[\frac{1}{H}\int_0^{2H} u_i \sin\frac{n\pi z}{2H} dz\right] \qquad Eq. (1A.30)$$

Combining Eq. (2.28) and (2.30),

$$u_i = \sum_{n=0}^{n=\infty} \left[\frac{1}{H}\int_0^{2H} u_i \sin\frac{n\pi z}{2H} dz\right] \sin\frac{n\pi z}{2H} \exp\left(\frac{-n^2\pi^2 T_V}{4}\right) \qquad Eq. (1A.31)$$

So far, no assumptions have been made regarding the variation of the initial distribution of pore water pressure u_i with depth of the soil layer. Some possible types of variation for u_i are discussed below.

Constant u_i with Depth

If u_i is constant with depth-i.e., if $u_i = u_o$ then, referring to Eq. (1A.31),

$$\left[\frac{1}{H}\int_0^{2H} u_i \sin\frac{n\pi z}{2H} dz\right] = \frac{2u_o}{n\pi}(1 - \cos n\pi) \qquad Eq. (1A.32)$$

$$u = \sum_{n=0}^{n=\infty} \frac{2u_o}{n\pi}(1-\cos n\pi)\sin\frac{n\pi z}{2H}\exp\left(\frac{-n^2\pi^2T_V}{4}\right) \qquad Eq.(1A.33)$$

Note that the term *1-cosnπ* in the above equation is zero for a case when *n* is even; therefore *u* is also zero. For the nonzero terms it is convenient to substitute *n=2m+1*, where *m* is an integer. So Eq. (1A.33) will now read as:

$$u = \sum_{m=0}^{m=\infty} \frac{2u_o}{(2m+1)\pi}(1-\cos(2m+1)\pi)\sin\frac{(2m+1)\pi z}{2H}\exp\left(\frac{-(2m+1)^2\pi^2T_V}{4}\right) \quad Eq(1A.34)$$

Or

$$u = \sum_{m=0}^{m=\infty} \frac{2u_o}{M}\sin\frac{Mz}{H}\exp\left(-M^2T_V\right) \qquad Eq.(1A.35)$$

Where $M = (2m+1)\,\pi/2$.

At a given time the degree of consolidation at any depth *z* is defined as

$$U_z = \frac{excess\ pore\ water\ pressure\ dissipated}{initial\ excess\ pore\ water\ pressure} \qquad Eq.(1A.36)$$

$$U_z = \frac{u_i - u}{u_i} = 1 - \frac{u}{u_i} = \frac{\Delta\sigma'}{u_o} \qquad Eq.(1A.37)$$

Where $\Delta\sigma'$ is the increase of effective stress at a depth *z* due to consolidation.

From Eq. (1A.35) and (1A.37),

$$U_z = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M}\sin\frac{Mz}{H}\exp\left(-M^2T_V\right) \qquad Eq.(1A.38)$$

The variation of U_z with depth can be plotted for various values of the non-dimensional time factor T_v; these curves are called isochrones.

In most cases, however, one needs to obtain the average degree of consolidation for the entire layer. This is given by

$$U_{av} = \frac{(1/H_t)\int_0^{H_t} u_i dz - (1/H_t)\int_0^{H_t} u dz}{(1/H_t)\int_0^{H_t} u_i dz} \qquad Eq.(1A.39)$$

The average degree of consolidation is also the ratio of consolidation settlement at any time to maximum consolidation settlement. Note, in this case, that $H_t = 2H$ and $u_i = u_o$.

Combining Eq. (1A.35) and (1A.39)

$$U_{av} = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2 T_v) \qquad Eq.(1A.40)$$

Terzaghi suggested the following equations for U_{av} to approximate the values obtained from Eq. (1A.40):

For U_{av} = 0 – 53%

$$T_v = \frac{\pi}{4}\left(\frac{U_{av}(\%)}{100}\right)^2 \qquad Eq.(1A.41)$$

For U_{av} = 53 – 100%

$$T_v = 1.781 - 0.933[\log(100 - U_{av}(\%))] \qquad Eq.(1A.42)$$

Sivaram and Swamee (1977) gave the solution for U_{av} varying from 0 to 100%:

$$\frac{U_{av}(\%)}{100} = \frac{(4T_v/\pi)^{0.5}}{[1 + (4T_v/\pi)^{2.8}]^{0.179}} \qquad Eq.(1A.43)$$

$$T_v = \frac{(\pi/4)(U_{av}(\%)/100)^2}{[1 - (U_{av}(\%)/100)^{5.6}]^{0.375}} \qquad Eq.(1A.44)$$

Eq. (1A.43) and (1A.44) give an error in T_v of less than 1% for $0 < U_{av} < 90\%$ and less than 3% for $90\% < U_{av} < 100\%$. It must be pointed out that, Eq. (1A.40) is still valid for a situation of one way drainage. Note, however, that the length of drainage path is equal to the total thickness of the soil layer.

Linear Variation of u_i

The linear variation of initial pore water pressure may be written as

$$u_i = u_o - u_1 \frac{H - z}{H} \qquad Eq.(1A.45)$$

Substitution of the above relation for u_i into Eq. (1A.31) yields

$$u = \sum_{n=1}^{n=\infty} \left[\frac{1}{H} \int_0^{2H} \left(u_o - u_1 \frac{H-z}{H} \right) \sin \frac{n\pi z}{2H} dz \right] \sin \frac{n\pi z}{2H} \times exp\left(\frac{-n^2 \pi^2 T_v}{4} \right) \qquad Eq.(1A.46)$$

The average degree of consolidation can be obtained by solving Eq. (21A.46) and (1A.39)

$$U_{av} = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2 T_v)$$

This is identical to Eq. (1A.40), which was the case where the excess pore water pressure is constant with depth.

Sinusoidal Variation of u_i

Sinusoidal variation can be represented by the equation

$$u_i = u_o \sin \frac{\pi z}{2H} \qquad Eq.(1A.46)$$

The solution for the average degree of consolidation for this type of excess pore water pressure distribution is of the form

$$U_{av} = 1 - exp\left(\frac{-\pi^2 T_v}{4} \right) \qquad Eq.(1A.47)$$

1.2 Finite Difference Solution of the Governing Consolidation Equation

Finite difference solution of the governing consolidation equation is obtained by expansion of the differential consolidation equation using Taylor's theorem [Budhu, M., 2007]. Consider a function *u(x)* and its derivative for the governing consolidation equation of Eq. (1A.16).

$$\frac{\partial u(x)}{\partial x} = \lim_{x \to \infty} \frac{u(x + \Delta x) - u(x)}{\Delta x} \qquad Eq.(1A.48)$$

Using Taylor's series expansion for $u(x+\Delta x)$ gives

$$u(x+\Delta x) = u(x) + \Delta x \frac{\partial u(x)}{\partial x} + \frac{(\Delta x)^2}{2}\frac{\partial^2 u(x)}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 u(x)}{\partial x^3} + \cdots \qquad Eq. (1A.49)$$

Using Eq. (1A.49), neglecting small errors and rearranging provides the solution of Eq. (1A.50) as:

$$\frac{u(x+\Delta x) - u(x)}{\Delta x} = \frac{\partial u(x)}{\partial x} \qquad Eq. (1A.50)$$

Now using Taylor's theorem and taking three nodal points at *i-1*, *i* and i+1 which are separated by distance Δx, one can calculate u_{i+1} and u_{i-1} as

$$u_{i+1} = u_i + \Delta x \left(\frac{\partial u}{\partial x}\right)_i + \frac{(\Delta x)^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_i + \frac{(\Delta x)^3}{3!}\left(\frac{\partial^3 u}{\partial x^3}\right)_i + \cdots \qquad Eq. (1A.51)$$

$$u_{i-1} = u_i - \Delta x \left(\frac{\partial u}{\partial x}\right)_i + \frac{(\Delta x)^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_i + \frac{(\Delta x)^3}{3!}\left(\frac{\partial^3 u}{\partial x^3}\right)_i + \cdots \qquad Eq. (1A.52)$$

Rearranging Eq. (1A.51) and applying forward difference from node *i+1* to node *i* gives the following equation

$$\left(\frac{\partial u}{\partial x}\right)_i = \frac{u_{i+1} - u_i}{\Delta x} \qquad Eq. (1A.53)$$

Similarly rearranging Eq. (1A.52) and applying backward difference from node *i* to node *i-1* gives the following equation

$$\left(\frac{\partial u}{\partial x}\right)_i = \frac{u_i - u_{i-1}}{\Delta x} \qquad Eq. (1A.54)$$

Subtracting Eq. (1A.52) from Eq. (1A.51) gives Eq. (1A.55).

$$\left(\frac{\partial u}{\partial x}\right)_i = \frac{u_{i+1} - u_i}{\Delta x} \qquad Eq. (1A.55)$$

Finally adding Eq. (1A.51) and Eq. (1A.52) gives

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x} \qquad Eq. (1A.56)$$

Hence, from Eq. (1A.53) to Eq. (1A.56), the finite difference solution of the first derivative of pore water pressure with respect to time and second derivative of pore water pressure with respect to depth of the governing consolidation equation in Eq. (1A.16) becomes as shown in Eq. (1A.57) and Eq. (1A.58) respectively:

$$\frac{\partial u}{\partial t} = \frac{1}{\Delta t}\left(u_{i,j+1} - u_{i,j}\right) \qquad Eq.(1A.57)$$

And

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(1A.58)$$

Therefore, governing finite difference consolidation equation valid for nodes that are not boundary nodes is:

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad Eq.(1A.59)$$

For an impermeable boundary, no flow across it can occur and hence

$$\frac{\partial u}{\partial z} = 0 \qquad Eq.(1A.60)$$

The finite difference equation will then be:

$$\frac{\partial u}{\partial z} = 0 = \frac{1}{2\Delta z}\left(u_{i-1,j} - u_{i+1,j}\right) = 0 \qquad Eq.(1A.61)$$

Hence, by applying $\boldsymbol{u_{i-1,j} = u_{i+1,j}}$ from Eq. (1A.61) in Eq. (1A.59), the simplified governing consolidation equation which are valid for nodes at the impermeable boundary becomes:

$$u_{i,j+1} = u_{i,j} + \frac{C_V \Delta t}{(\Delta z)^2}\left(2u_{i-1,j} - 2u_{i,j}\right) \qquad Eq.(1A.62)$$

1.3 Finite difference solution for Consolidation in a layered Soil

For the calculation of excess pore water pressure at the interface of a layered soil profile with different types of soils (i.e., different values of C_v), the governing consolidation equation in Eq. (1A.16) will have to be modified to some extent.

$$\frac{k}{C_v}\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial z^2} \qquad Eq.(1A.63)$$

In Eq. (1A.63), the left side indicates that the change in volume while the right side indicates the difference between the rate of flow.

Based on the derivation of layered soil equation

$$k\frac{\partial^2 u}{\partial z^2} = \frac{1}{2}\left[\frac{k_1}{(\Delta z)^2} + \frac{k_2}{(\Delta z)^2}\right]\left(\frac{2k_1}{k_1+k_2}u_{1,t} + \frac{2k_2}{k_1+k_2}u_{3,t} - 2u_{0,t}\right) \qquad Eq.(1A.64)$$

Where k_1 and k_2 are the coefficient of permeability in layers 1 and 2, respectively, and $u_{0,1}$, $u_{1,t}$ and $u_{3,t}$ are the excess pore water pressures at time t for points at top of the first layer, interface, and bottom of the second layer, respectively. Also, the average volume change for the element at the boundary is

$$\frac{k}{C_v}\frac{\partial u}{\partial t} = \frac{1}{2}\left(\frac{k_1}{C_{v1}} + \frac{k_2}{C_{v2}}\right)\frac{1}{\Delta t}\left(u_{0,t+\Delta t} - u_{0.t}\right) \qquad Eq. (1A.65)$$

Where $u_{(0,t)}$, and $u_{(0,t+\Delta t)}$ are the excess pore water pressures at a point at the top of the first layer at times t and $t+\Delta t$, respectively.

Equating the right hand side of Eq. (1A.64) and (1A.65) gives Eq. (1A.66) as follows:

$$\left(\frac{k_1}{C_{v1}} + \frac{k_2}{C_{v2}}\right)\frac{1}{\Delta t}\left(u_{0,t+\Delta t} - u_{0.t}\right) = \frac{1}{(\Delta z)^2}[k_1 + k_2]\left(\frac{2k_1}{k_1+k_2}u_{1,t} + \frac{2k_2}{k_1+k_2}u_{3,t} - 2u_{0,t}\right) \qquad Eq. (1A.66)$$

Hence, after rearranging Eq. (1A.66), the finite difference equation for calculating pore water pressure in a layered soil will be as shown in Eq. (1A.67) below.

$$u_{0,t+\Delta t} = \frac{\Delta t C_{v1}}{(\Delta z)^2}\left(\frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})}\right)\left(\frac{2k_1}{k_1+k_2}u_{1,t} + \frac{2k_2}{k_1+k_2}u_{3,t} - 2u_{0,t}\right) + u_{0.t} \qquad Eq. (1A.67)$$

2 APPENDIX 2 - Finite difference solution (software output) for homogeneous soil profile cases

1.1 Constant Loading and Constant C_v case

The software output or finite difference method solution of constant load and constant coefficient of consolidation (C_v) for the homogeneous soil profile cases will be presented using the hypothetical problem Table **2.1**A which is the same as the problem in section 4.1.

Table 2.1A: Input data for homogeneous soil under constant loading

Input Data	
Depth of Soil layer (m)	3.0
time (year)	3.2
dt (time grid) in m	0.4
dz (depth grid) in m	1.00
C_v (coefficient of consolidation)	1.95
u_i (Initial pore water pressure)	110.00
M_v (coefficient of compressibility)	0.0009

The solutions that are appended below from Table **2.2**A to Table **2.9**A which are carried out for various initial pore water pressure distributions. The plots of pore water pressure versus depth, consolidation settlement versus time and average degree of consolidation versus time are also presented in the respective headings.

i. Uniform initial pore water pressure distribution (u_i is uniform with depth)

Table 2.2A: Constant load and C_v under triangular initial pore water pressure (single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load (KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		67.50	22.50	22.50	11.25	14.06	8.44	11.25	7.04	9.49
	2.00		45.00	45.00	22.50	28.12	16.88	22.50	14.07	18.98	11.96
	3.00		22.50	22.50	33.75	22.50	30.94	19.69	26.72	16.88	22.85
	4.00		0.00	22.50	22.50	33.75	22.50	30.94	19.69	26.72	16.88
Settlement (mm)			0.00	214.76	224.10	233.44	238.10	243.94	247.44	252.10	255.03
U_{avg} (%)			0.00	71.87	75.00	78.13	79.69	81.64	82.81	84.37	85.35

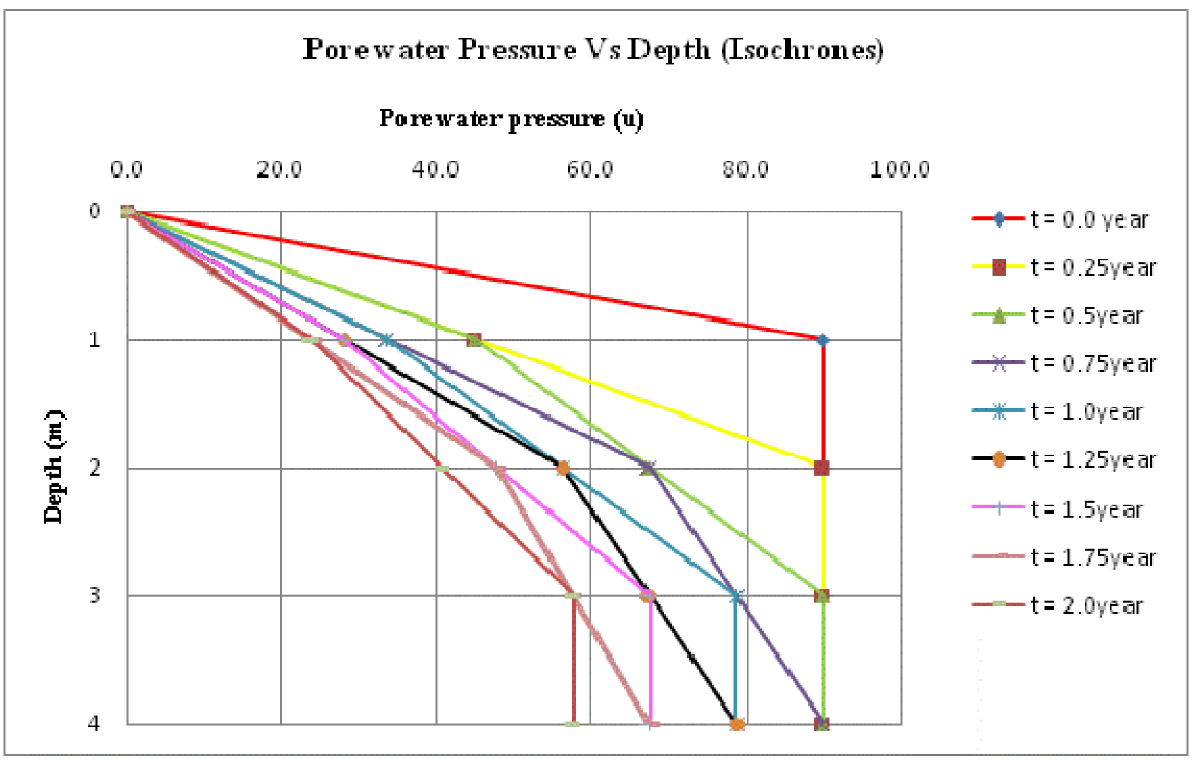

Figure 2.1A: Isochrones for Constant load and C_v (Uniform u_i and single drainage)

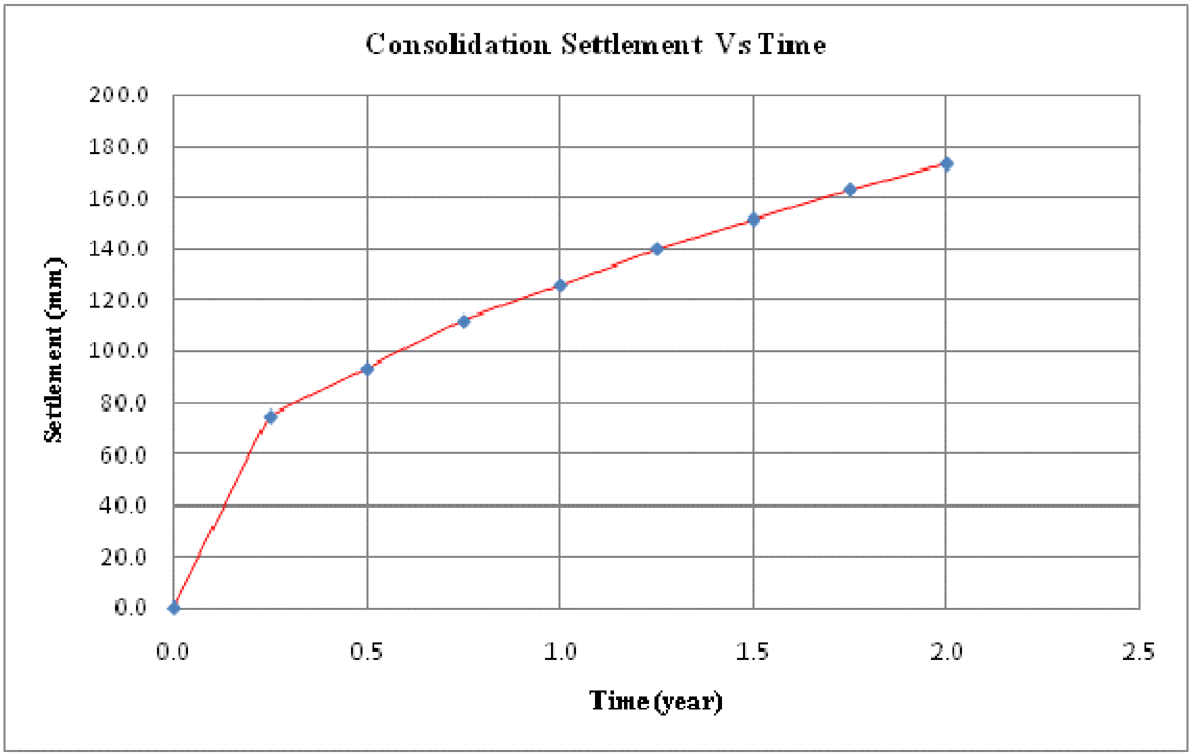

Figure 2.2A: Settlement Vs time for Constant load and C_v (Uniform u_i and single drainage)

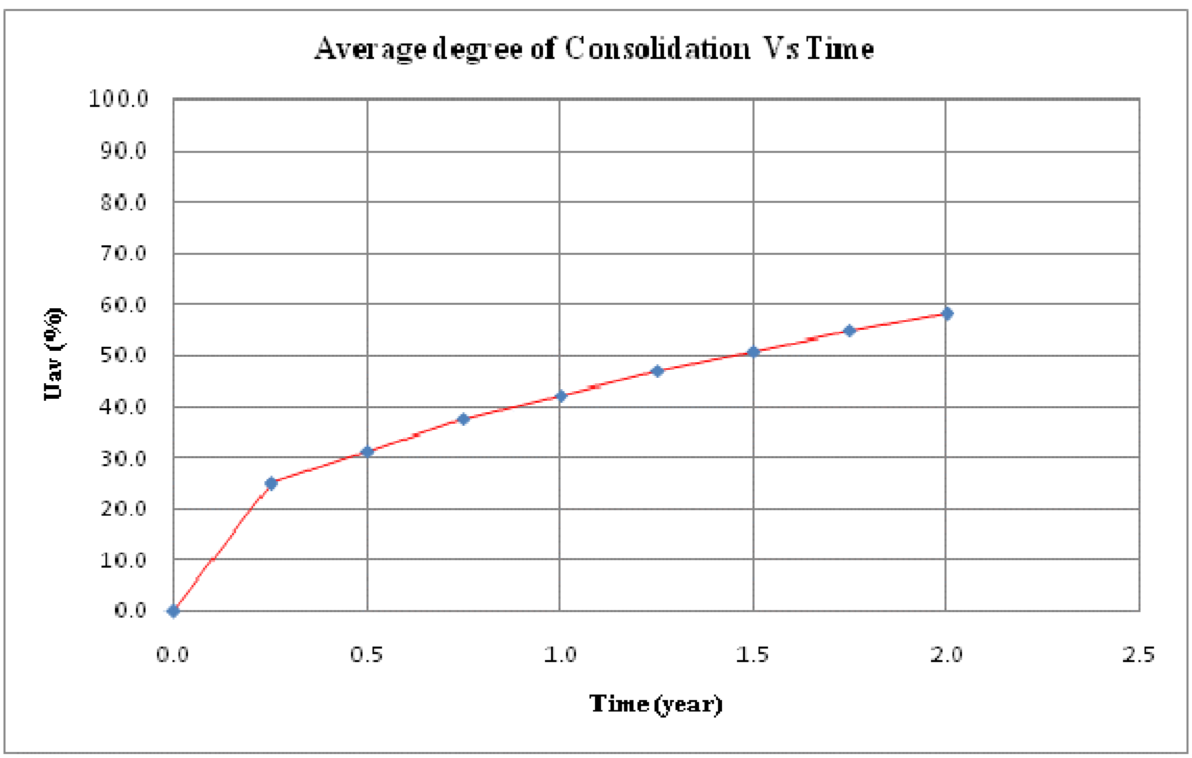

Figure 2.3A: U_{av} Vs time for Constant load and C_v (Uniform u_i and single drainage)

Table 2.3A: - Constant load and C_v under triangular initial pore water pressure (double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Applied Load(KN/m2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		90.00	45.00	45.00	22.50	22.50	11.25	11.25	5.62	5.62
	2.00		90.00	90.00	45.00	45.00	22.50	22.50	11.25	11.25	5.62
	3.00		90.00	45.00	45.00	22.50	22.50	11.25	11.25	5.62	5.62
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	149.40	186.75	224.10	242.78	261.45	270.79	280.13	284.81
Average Degree of Consolidation (%)			0.00	50.00	62.50	75.00	81.25	87.50	90.63	93.75	95.32

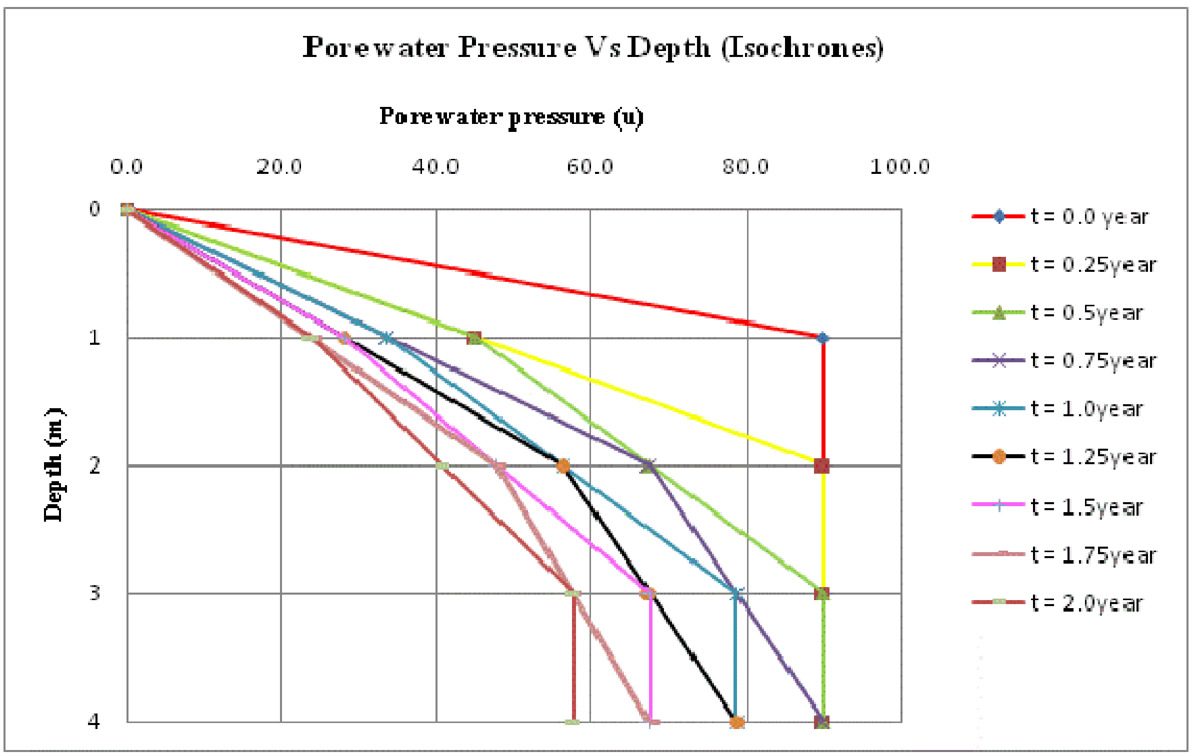

Figure 2.4A: Isochrones for Constant load and C_v (Uniform u_i and double drainage)

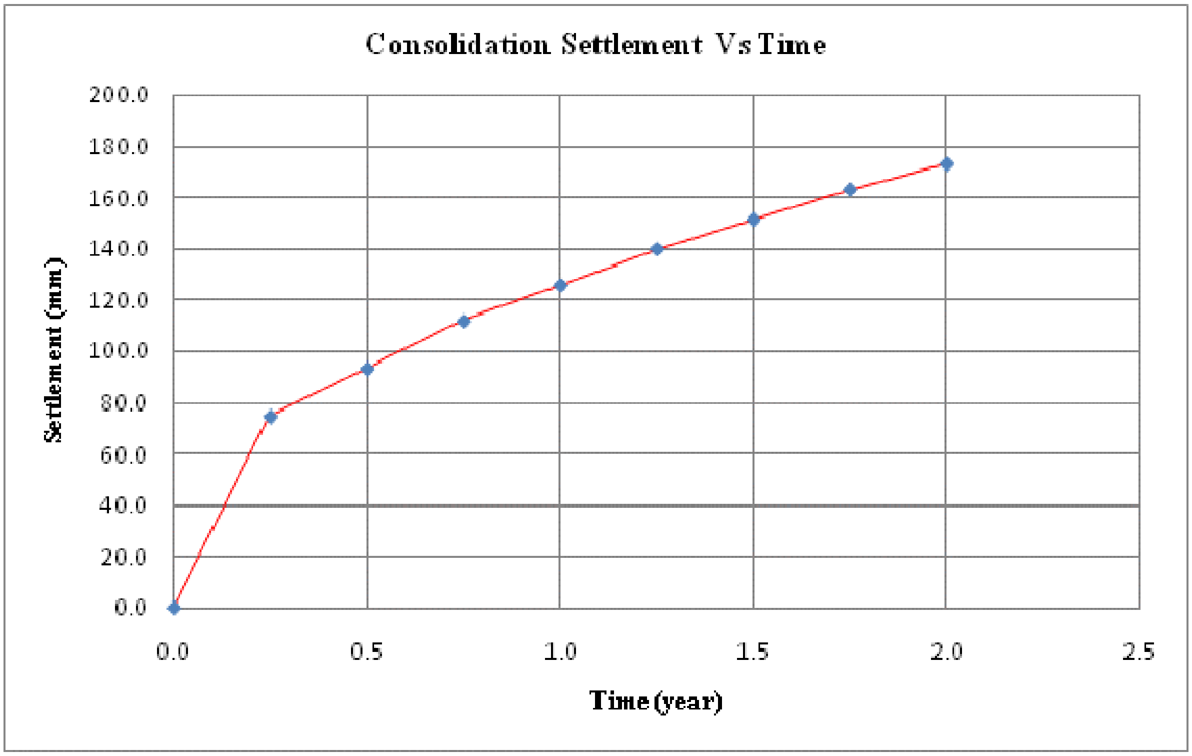

Figure 2.5A: Settlement Vs time for Constant load and C_v (Uniform u_i and double drainage)

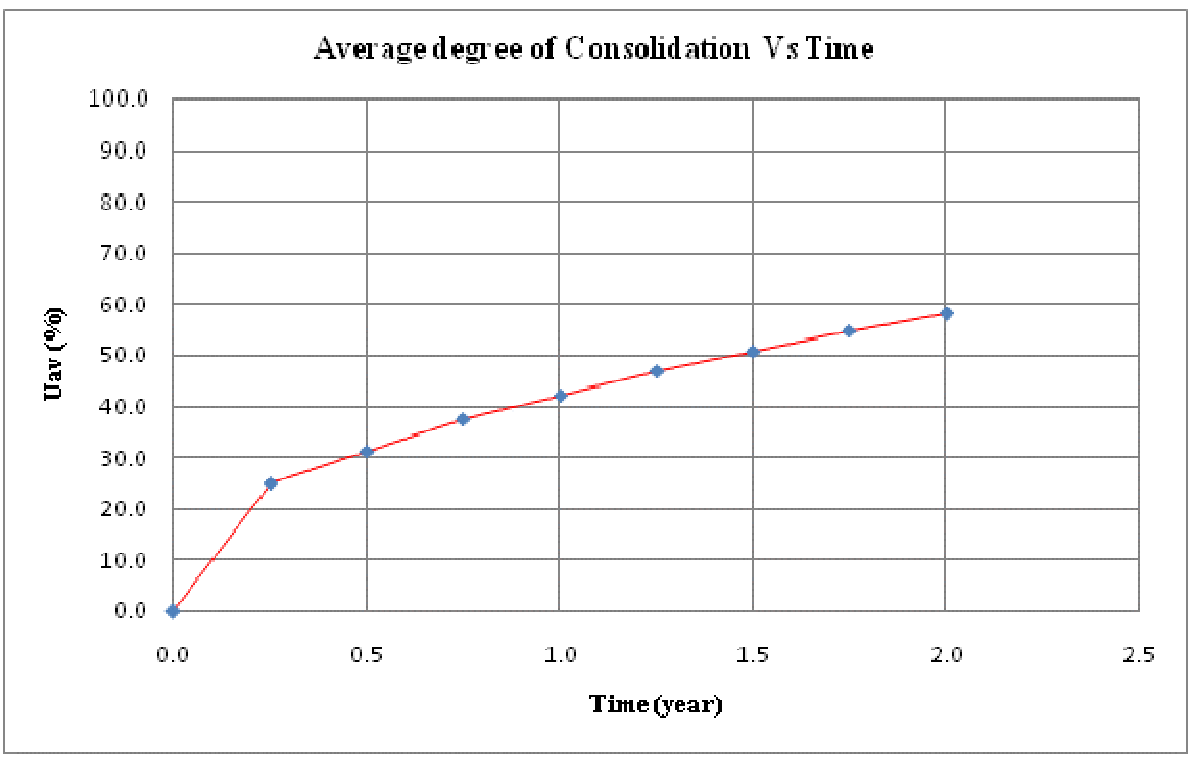

Figure 2.6A: U_{av} Vs time for Constant load and C_v (Uniform u_i and double drainage)

ii. Triangular initial pore water pressure distribution (u_i decreasing with depth)

Table 2.4A: - Constant load and C_v under triangular initial pore water pressure (single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load (KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		67.50	22.50	22.50	11.25	14.06	8.44	11.25	7.04	9.49
	2.00		45.00	45.00	22.50	28.12	16.88	22.50	14.07	18.98	11.96
	3.00		22.50	22.50	33.75	22.50	30.94	19.69	26.72	16.88	22.85
	4.00		0.00	22.50	22.50	33.75	22.50	30.94	19.69	26.72	16.88
Settlement(mm)			0.00	214.76	224.10	233.44	238.10	243.94	247.44	252.10	255.03
U_{avg} (%)			0.00	71.87	75.00	78.13	79.69	81.64	82.81	84.37	85.35

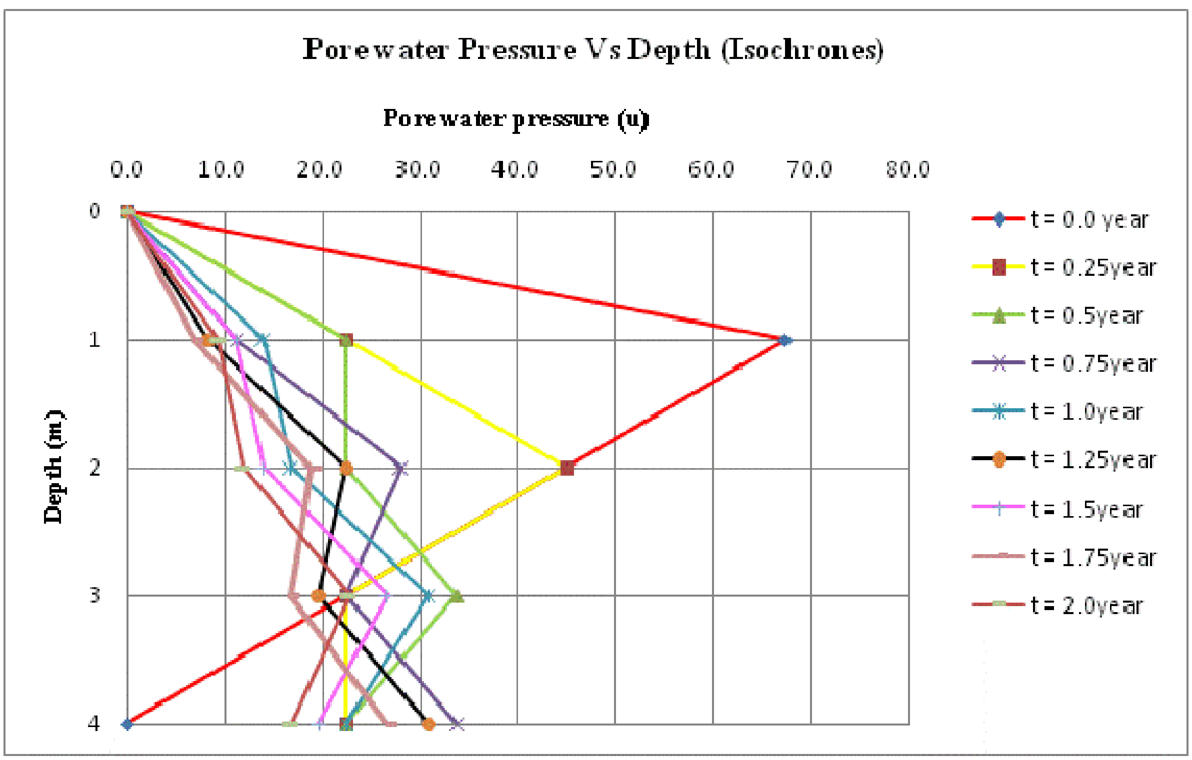

Figure 2.7A: Isochrones for Constant load and C_v (Triangular u_i and single drainage)

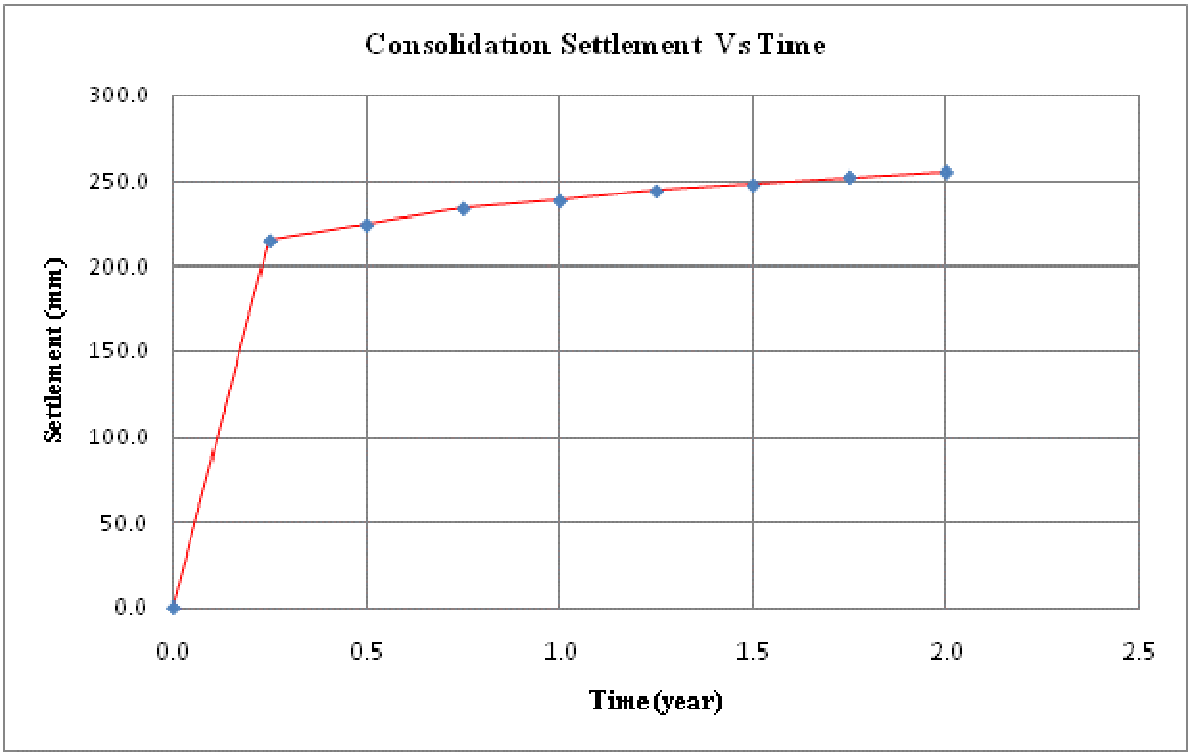

Figure 2.8A: Settlement Vs time for Constant load and C_v (Triangular u_i and single drainage)

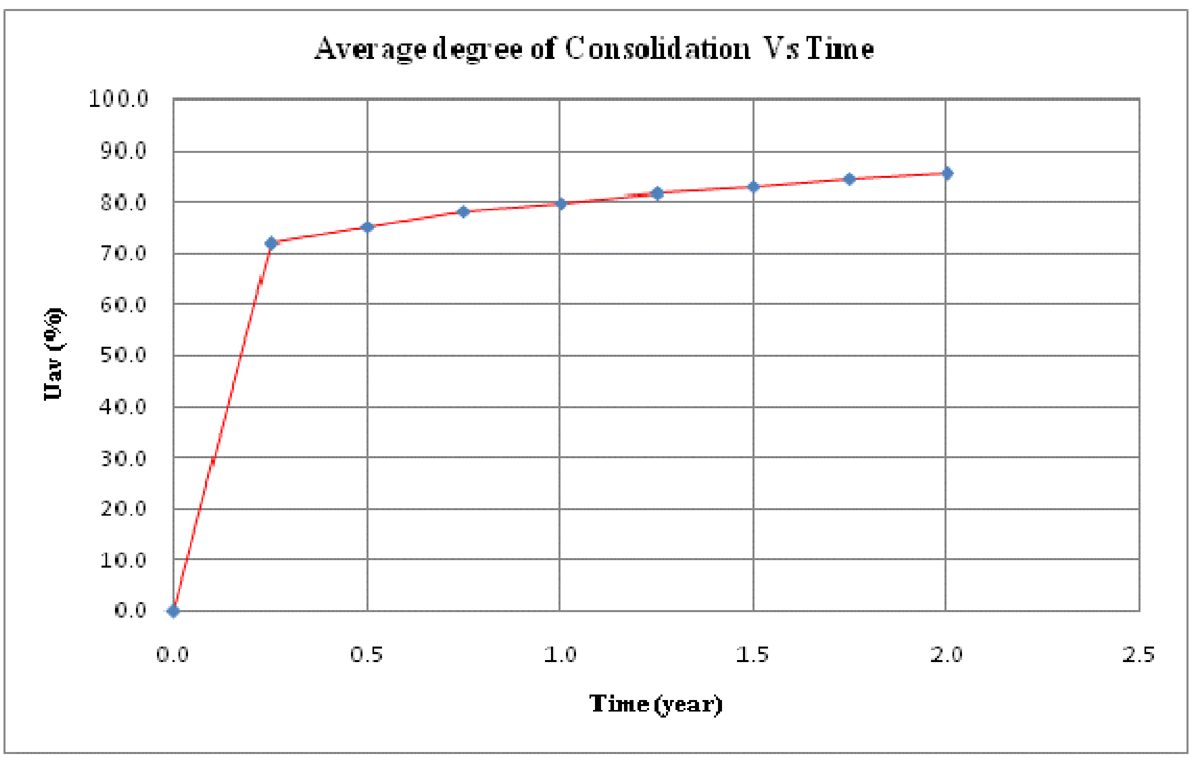

Figure 2.9A: U_{av} Vs time for Constant load and C_v (Triangular u_i and single drainage)

Table 2.5A: Constant load and C_v under triangular initial pore water pressure (double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	0.00 22.50 45.00 22.50 0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	67.50		22.50	11.25	11.25	5.62	5.62	2.81	2.81	7.04	9.49
	45.00		22.50	22.50	11.25	11.25	5.62	5.62	2.81	18.98	11.96
	22.50		22.50	11.25	11.25	5.62	5.62	2.81	2.81	16.88	22.85
	0.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	26.72	16.88
Settlement(mm)			0.00	224.10	242.78	261.45	270.79	280.13	284.81	289.47	291.80
U_{avg} (%)			0.00	75.00	81.25	87.50	90.63	93.75	95.32	96.88	97.66

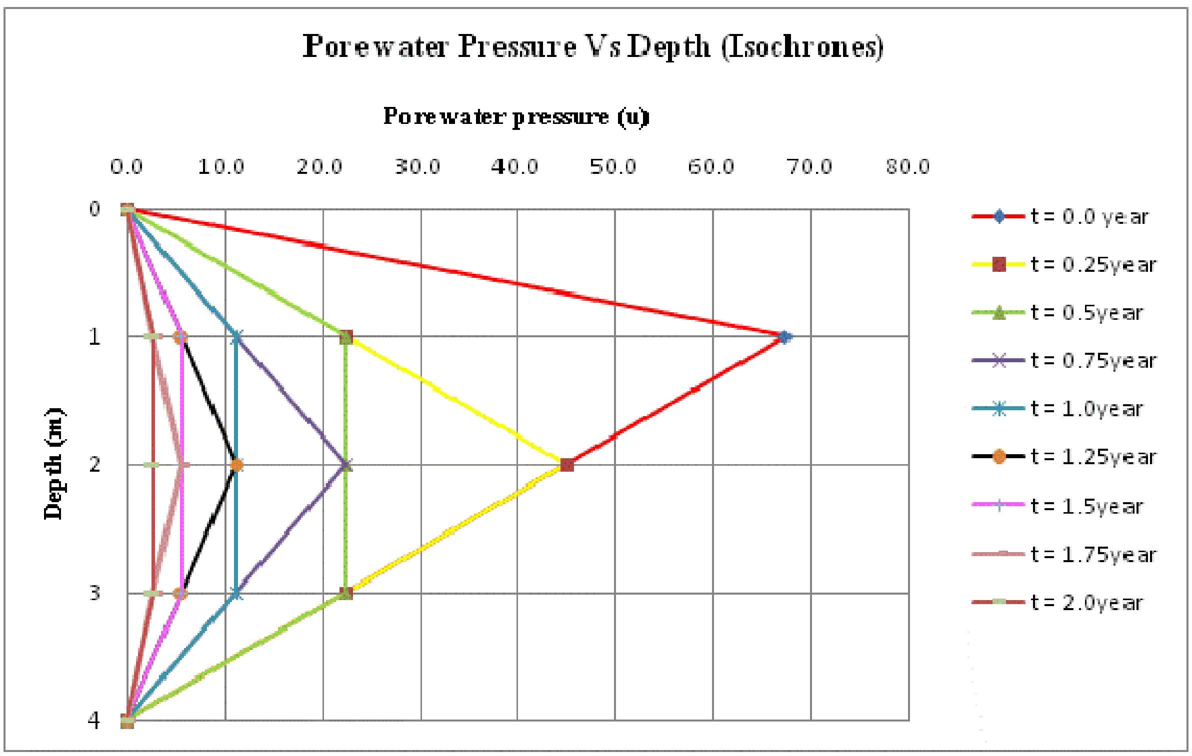

Figure 2.10A: Isochrones for Constant load and C_v (Triangular u_i and double drainage)

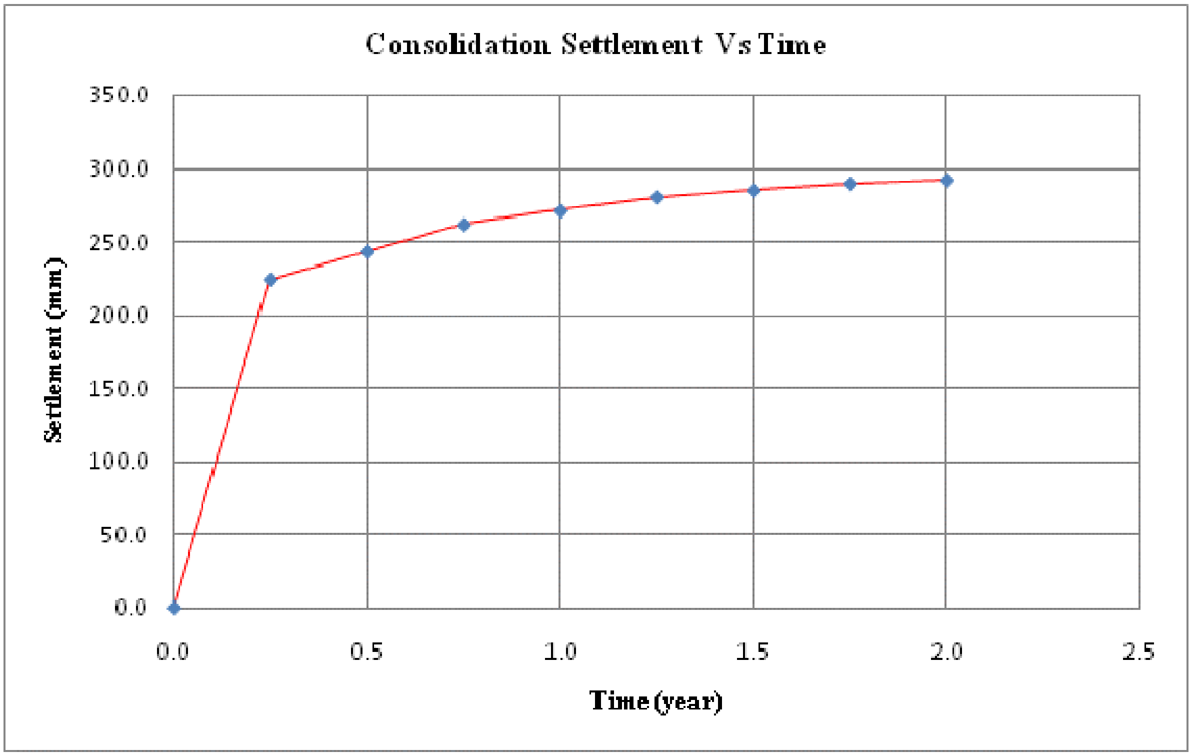

Figure 2.11A: Settlement Vs time for Constant load and C_v (Triangular u_i and double drainage)

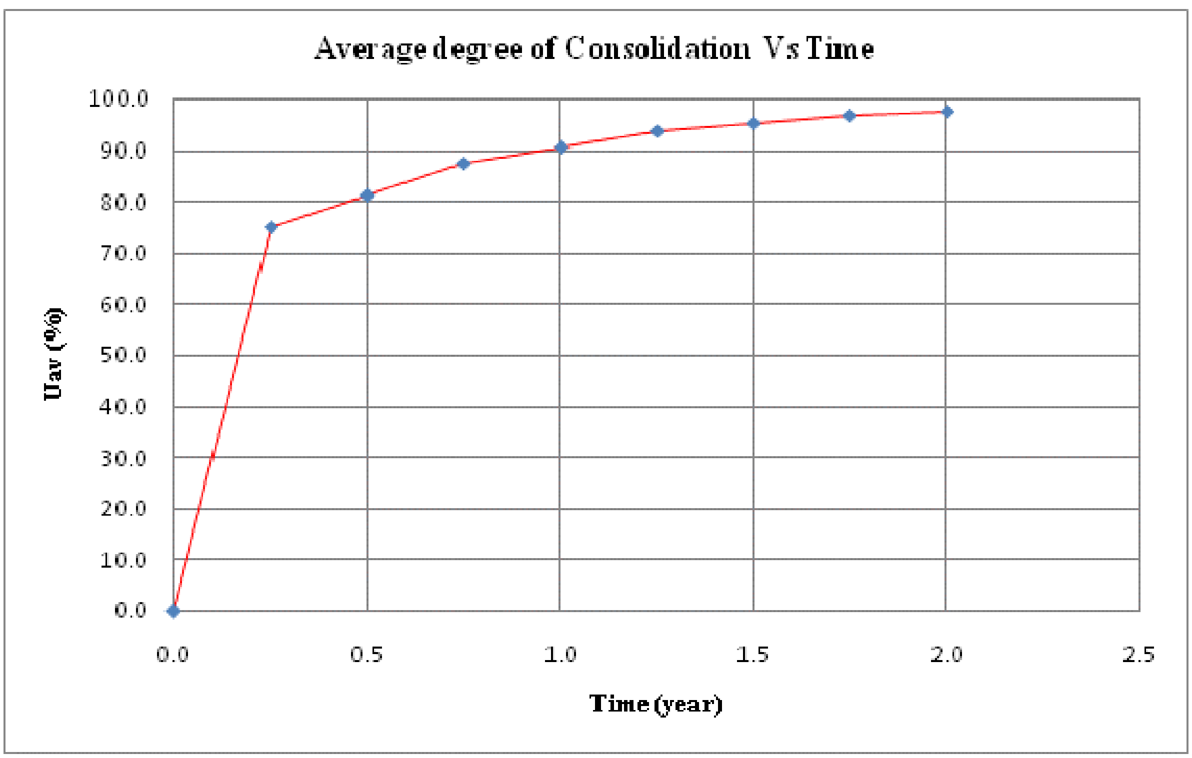

Figure 2.12A: U_{av} Vs time for Constant load and C_v (Triangular u_i and double drainage)

iii. Trapezoidal initial pore water pressure distribution (u_i decreasing with depth)

Table 2.6A: Constant load and C_v under trapezoidal initial pore water pressure (single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		76.00	31.00	31.25	19.88	21.75	16.00	17.84	13.52	15.12
	2.00		62.00	62.50	39.75	43.50	32.00	35.68	27.03	30.24	23.03
	3.00		49.00	48.50	55.75	44.12	49.62	38.06	42.65	32.54	36.44
	4.00		35.00	49.00	48.50	55.75	44.12	49.62	38.06	42.65	32.54
Settlement(mm)			0.00	160.60	173.47	186.44	194.69	203.72	210.36	217.77	223.39
U_{avg} (%)			0.00	53.75	58.06	62.40	65.16	68.18	70.40	72.88	74.76

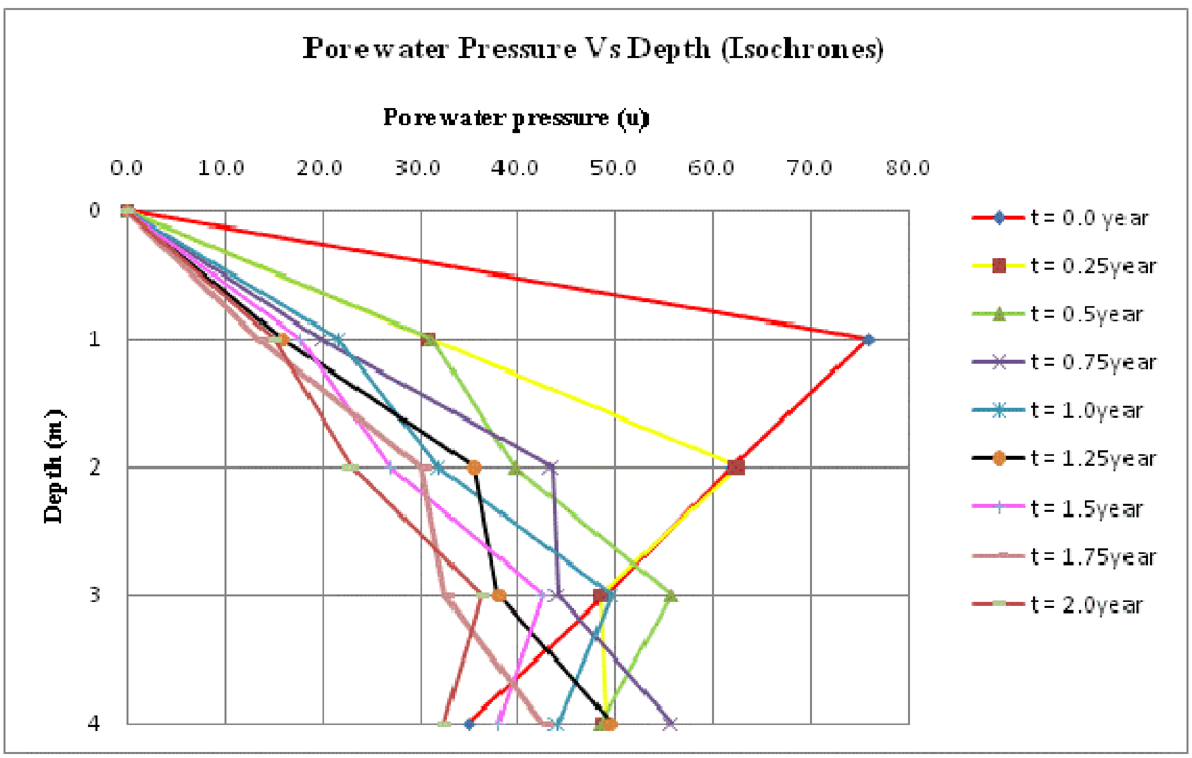

Figure 2.13A: Isochrones for Constant load and C_v (Trapezoidal u_i and single drainage)

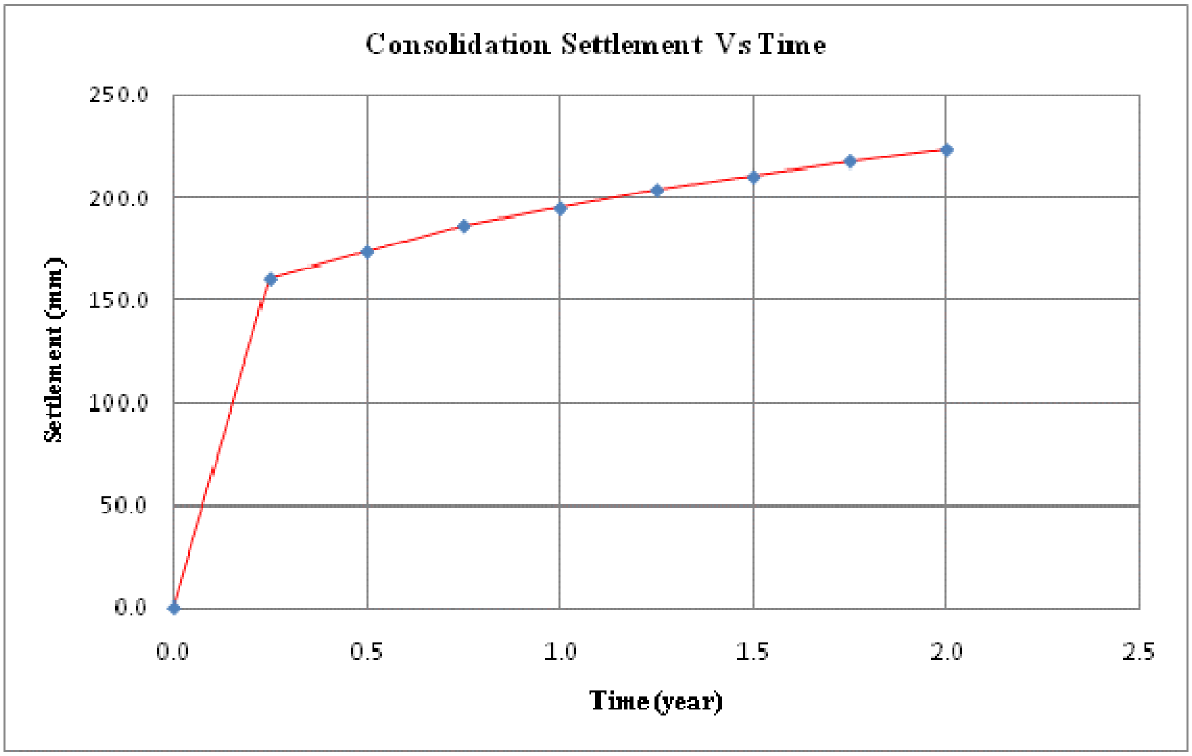

Figure 2.14A: Settlement Vs time for Constant load and C_v (Trapezoidal u_i and single drainage)

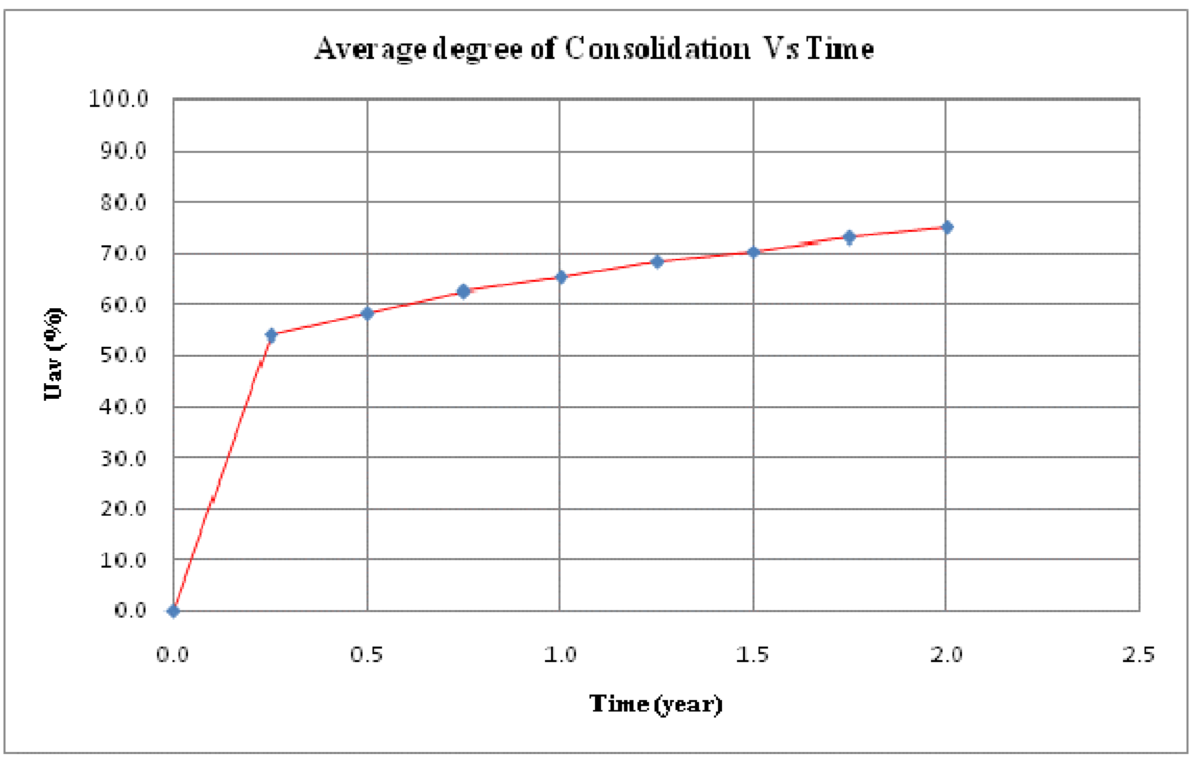

Figure 2.15A: U_{av} Vs time for Constant load and C_v (Trapezoidal u_i and single drainage)

Table 2.7A: Constant load and C_v under trapezoidal initial pore water pressure (double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		76.00	31.00	31.25	15.50	15.62	7.75	7.81	3.88	3.90
	2.00		62.00	62.50	31.00	31.25	15.50	15.62	7.75	7.81	3.88
	3.00		49.00	31.00	31.25	15.50	15.62	7.75	7.81	3.88	3.90
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	195.46	221.20	247.13	260.01	272.97	279.40	285.88	289.11
U_{avg} (%)			0.00	65.41	74.03	82.71	87.02	91.36	93.51	95.68	96.76

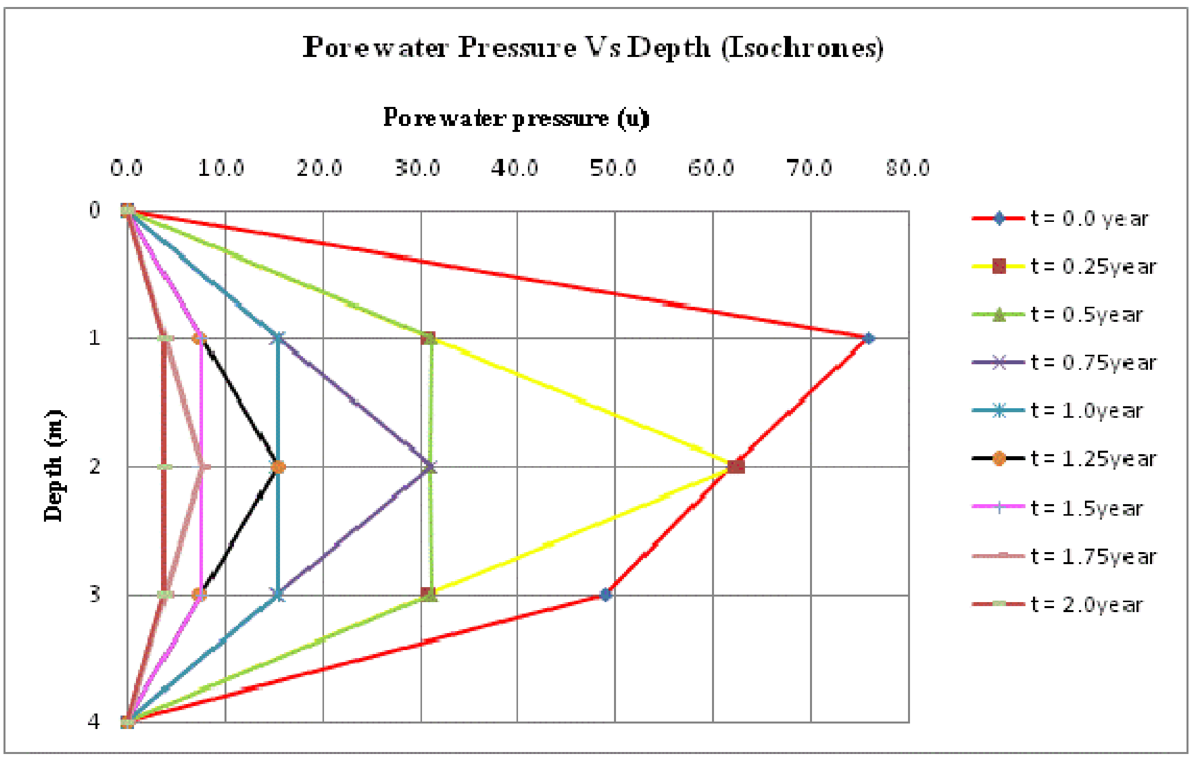

Figure 2.16A: Isochrones for Constant load and C_v (Trapezoidal u_i and double drainage)

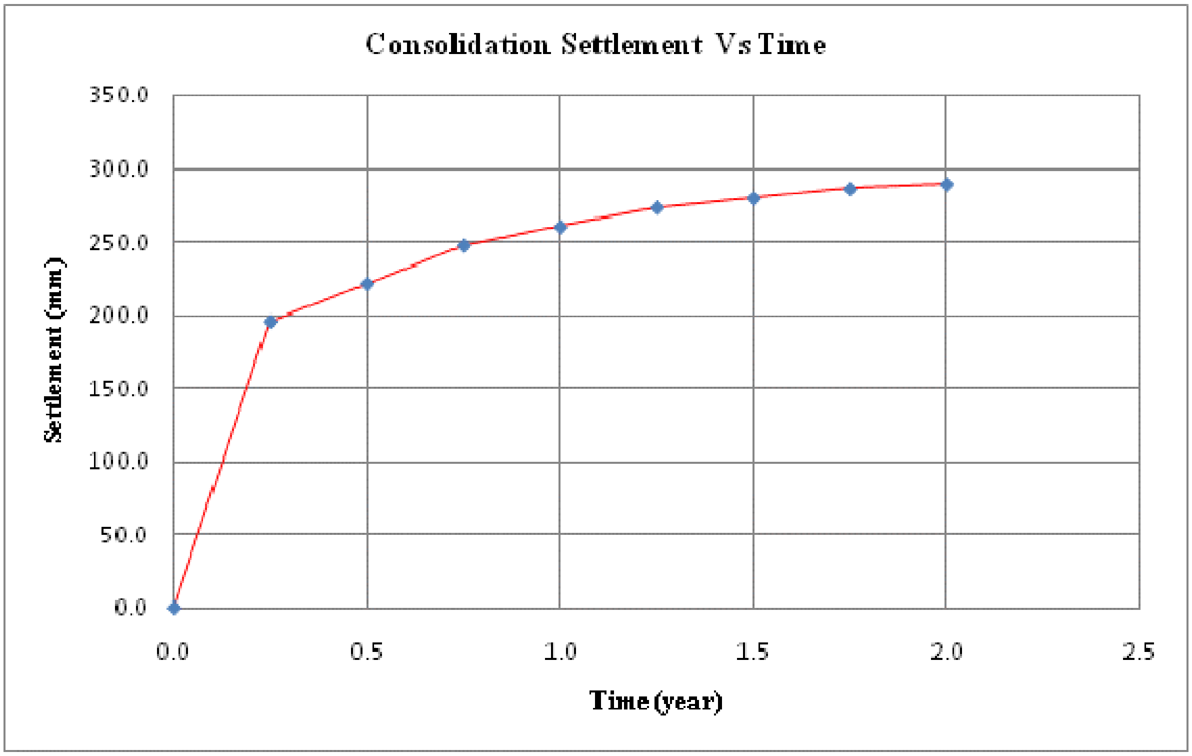

Figure 2.17A: Settlement Vs time for Constant load and C_v (Trapezoidal u_i and double drainage)

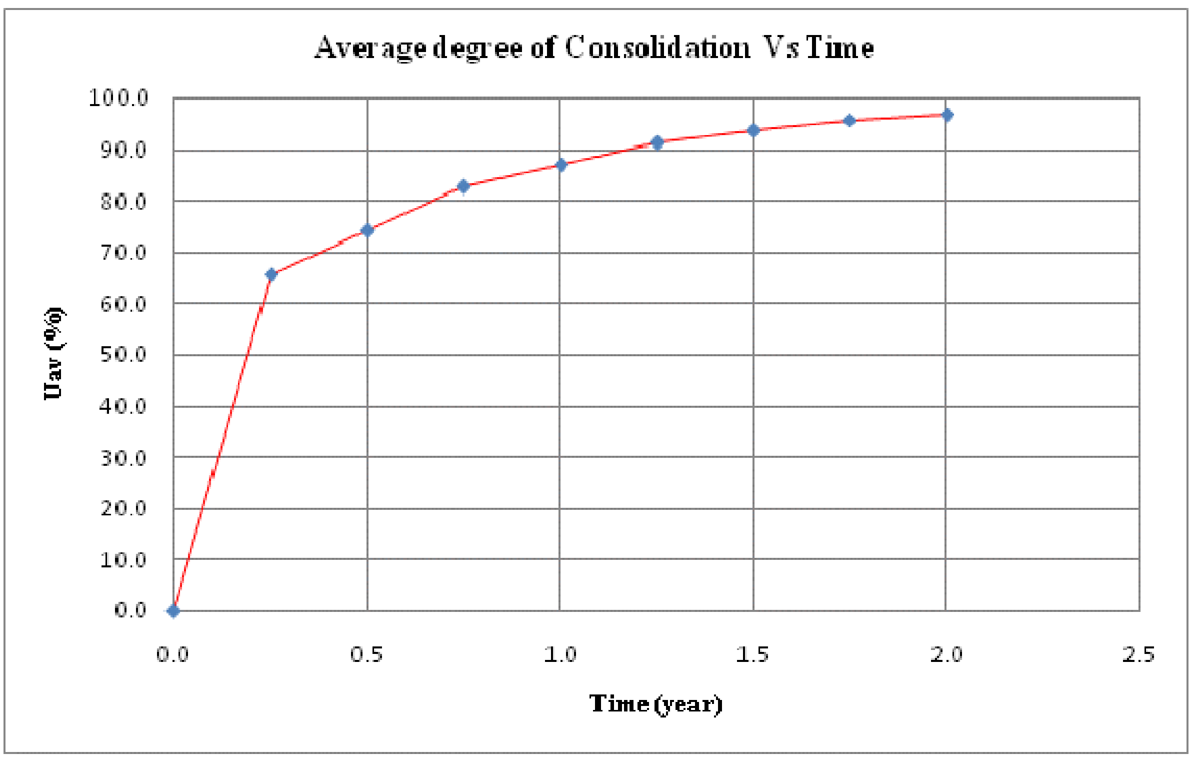

Figure 2.18A: U_{av} Vs time for Constant load and C_v (Trapezoidal u_i and double drainage)

iv. Sinusoidal initial pore water pressure distribution

Table 2.8A: Constant load and C_v under Sinusoidal initial pore water pressure (single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m²)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		63.64	45.00	31.82	22.50	23.86	16.88	19.89	14.06	16.90
	2.00		90.00	63.64	45.00	47.73	33.75	39.77	28.13	33.80	23.91
	3.00		63.64	45.00	63.64	45.00	55.68	39.38	47.72	33.76	40.76
	4.00		0.00	63.64	45.00	63.64	45.00	55.68	39.38	47.72	33.76
Settlement(mm)			0.00	144.87	163.54	176.75	186.09	195.99	202.99	211.25	217.09
U_{avg} (%)			0.00	48.48	54.73	59.15	62.28	65.59	67.94	70.70	72.65

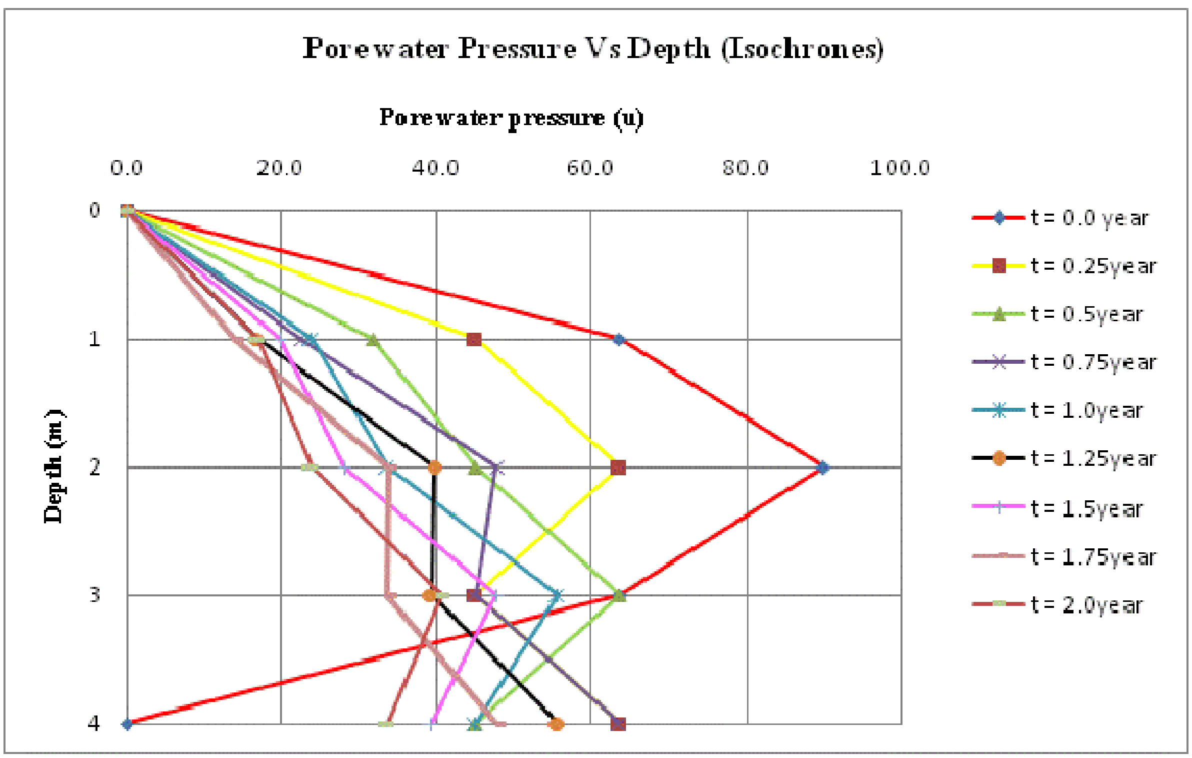

Figure 2.19A: Isochrones for Constant load and C_v (Sinusoidal u_i and single drainage)

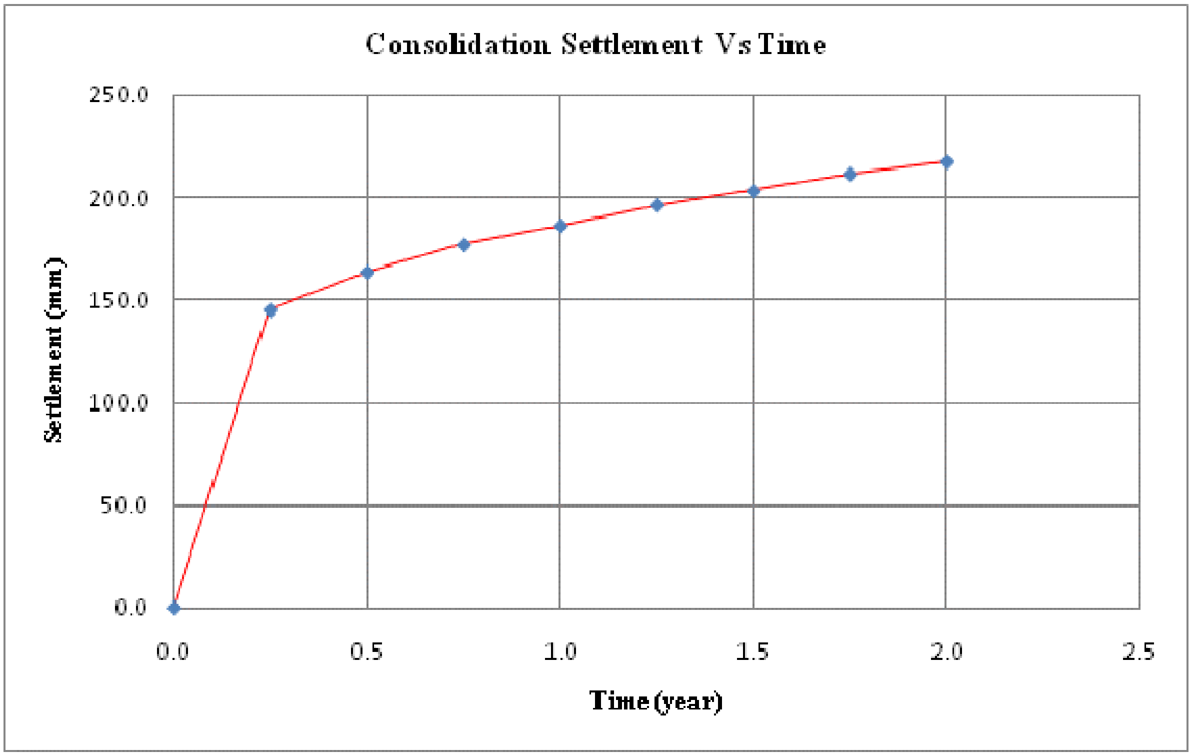

Figure 2.20A: Settlement Vs time for Constant load and C_v (Sinusoidal u_i and single drainage)

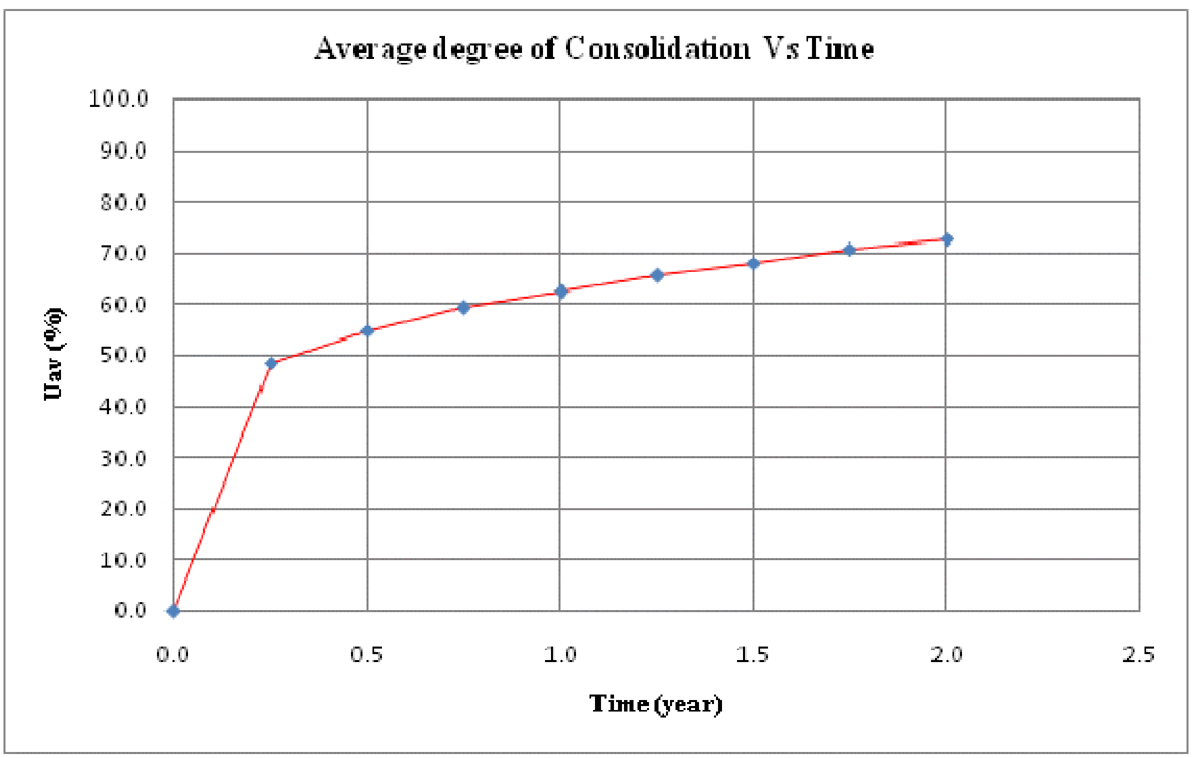

Figure 2.21A: U_{av} Vs time for Constant load and C_v (Sinusoidal u_i and single drainage)

Table 2.9A: Constant load and C_v under Sinusoidal initial pore water pressure (double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		63.64	45.00	31.82	22.50	15.91	11.25	7.96	5.62	3.98
	2.00		90.00	63.64	45.00	31.82	22.50	15.91	11.25	7.96	5.62
	3.00		63.64	45.00	31.82	22.50	15.91	11.25	7.96	5.62	3.98
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	171.28	208.63	235.04	253.71	266.92	276.25	282.86	287.53
U_{avg} (%)			0.00	57.32	69.82	78.66	84.91	89.33	92.45	94.67	96.23

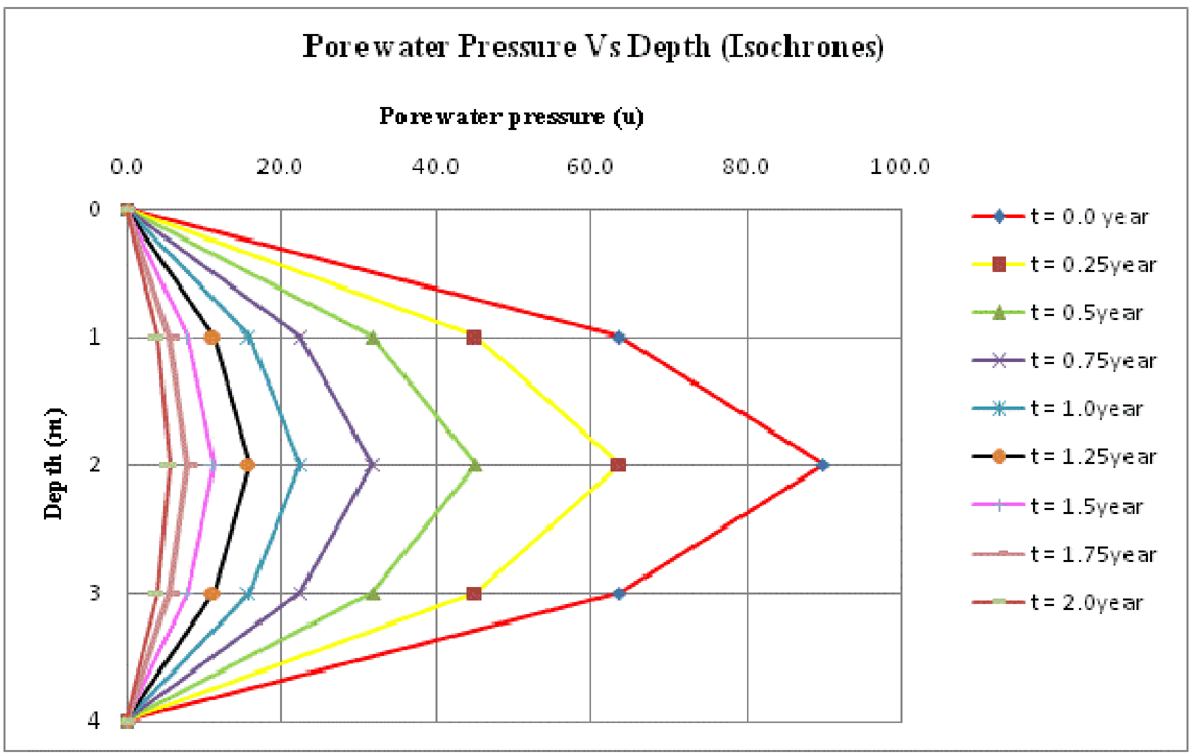

Figure 2.22A: Isochrones for Constant load and C_v (Sinusoidal u_i and double drainage)

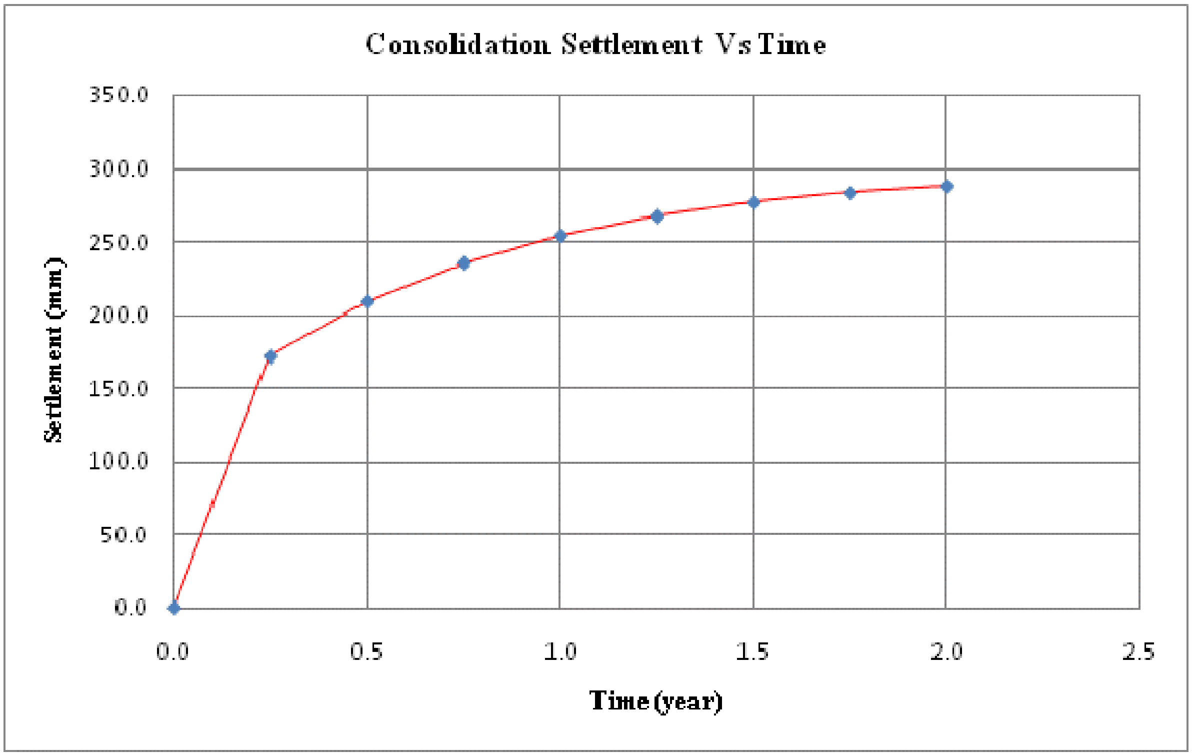

Figure 2.23A: Settlement Vs time for Constant load and C_v (Sinusoidal u_i and double drainage)

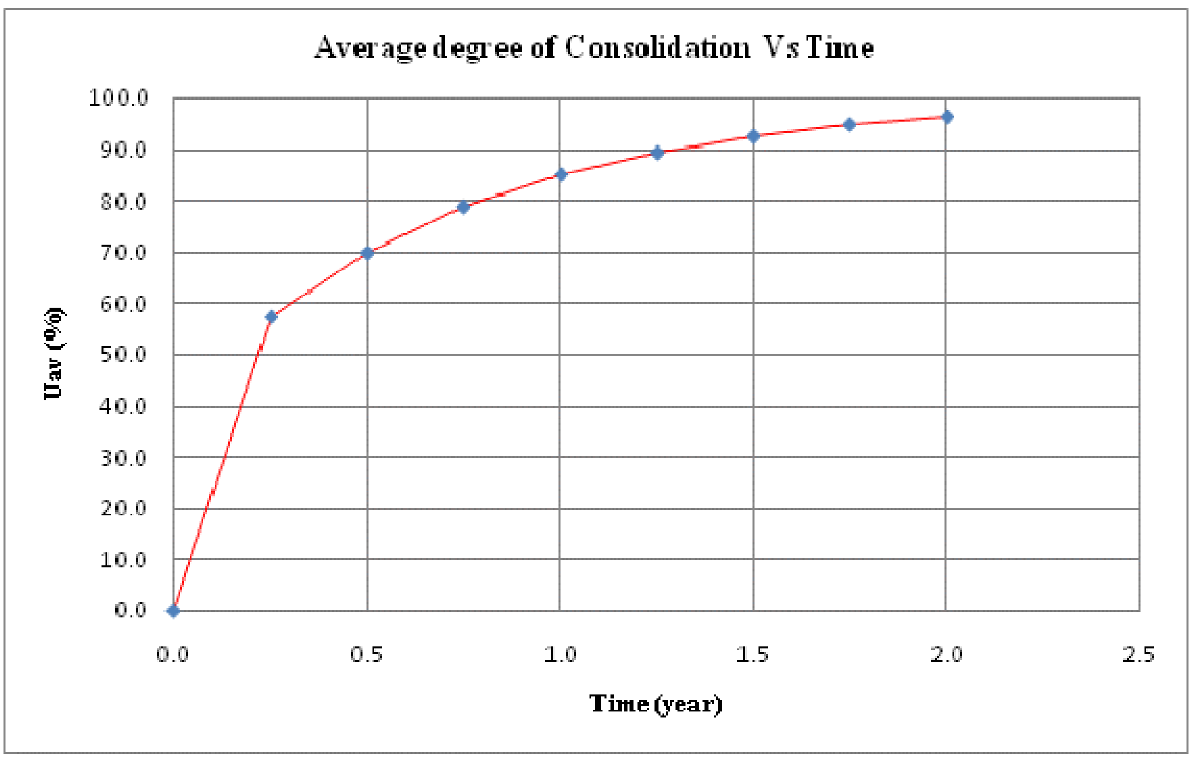

Figure 2.24A: U_{av} Vs time for Constant load and C_v (Sinusoidal u_i and double drainage)

1.2 Abrupt Change of Loading and Constant Rate change of C_v case

For this case also the hypothetical problem in section 4.1 is used. The abrupt change in loading and the variation of C_v with time is also taken similar to section 4.2 where 60Kpa load is applied for in the first year and then the load becomes 90Kpa from the start of the first year up to the end of the given time period. The coefficient of consolidation was assumed to decrease at a rate of 0.4m^2/yr/yr. The solutions of the same for various initial pore water distributions are tabulated as shown from Table **2.10**A to Table **2.15**A below:

i. *Triangular initial pore water pressure distribution (u_i decreasing with depth)*

Table 2.10A: Abrupt load and variable C_v (Sinusoidal initial pore water pressure & single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		45.00	16.50	15.15	9.57	39.16	26.36	23.32	20.76	19.17
	2.00		30.00	30.00	17.17	18.11	44.19	44.03	38.75	36.21	33.96
	3.00		15.00	15.00	21.41	16.85	48.79	46.61	46.36	44.15	42.52
	4.00		0.00	14.25	14.92	20.44	47.57	48.48	47.17	46.64	45.15
Settlement(mm)			0.00	142.24	148.41	153.76	169.38	181.57	189.23	195.51	200.67
U_{avg} (%)			0.00	71.41	74.50	77.19	56.69	60.77	63.33	65.43	67.16

Table 2.11A: Abrupt load and variable C_v (Sinusoidal initial pore water pressure & double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	**Pore pressure**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		45.00	16.50	15.15	9.57	38.07	24.79	21.00	16.77	13.84
	2.00		30.00	30.00	17.17	15.39	40.73	38.74	28.98	23.79	19.58
	3.00		15.00	15.00	15.00	9.55	38.07	24.79	21.00	16.77	13.84
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	148.15	159.92	170.56	201.80	225.49	239.89	251.22	259.57
U_{avg} (%)			0.00	74.37	80.28	85.62	67.54	75.47	80.28	84.08	86.87

ii. Trapezoidal initial pore water pressure distribution (u_i decreasing with depth)

Table 2.12A: Abrupt load and variable C_v (trapezoidal initial pore water pressure & single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		52.00	23.98	22.65	17.07	46.15	33.10	29.69	26.84	24.97
	2.00		45.00	45.00	32.17	31.84	57.50	56.45	50.60	47.46	44.71
	3.00		38.00	37.52	40.92	35.76	66.05	63.08	61.86	58.90	56.58
	4.00		30.00	37.60	37.53	40.41	66.69	66.21	64.02	62.62	60.39
Settlement(mm)			0.00	95.20	104.16	112.15	130.27	144.64	154.25	162.26	168.94
U_{avg} (%)			0.00	47.79	52.29	56.30	43.60	48.41	51.62	54.30	56.54

Table 2.13A: Abrupt load and variable C_v (trapezoidal initial pore water pressure & double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		52.00	23.98	22.65	14.35	42.11	27.81	23.43	18.73	15.46
	2.00		45.00	45.00	25.77	23.09	46.09	43.10	32.40	26.57	21.87
	3.00		38.00	23.28	22.58	14.34	42.10	27.81	23.43	18.73	15.46
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	122.62	140.27	156.22	190.65	216.86	233.01	245.66	254.98
U_{avg} (%)			0.00	61.56	70.42	78.42	63.81	72.58	77.98	82.22	85.33

iii. Sinusoidal initial pore water pressure distribution

Table 2.14A: Abrupt load and variable C_v (Sinusoidal initial pore water pressure & single drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		42.43	30.62	22.55	16.94	46.05	32.64	29.30	26.40	24.55
	2.00		60.00	43.31	31.89	31.66	56.34	55.73	49.67	46.62	43.87
	3.00		42.43	30.62	40.69	33.08	65.01	61.50	60.65	57.68	55.45
	4.00		0.00	40.31	31.59	39.32	64.33	64.84	62.50	61.30	59.13
Settlement(mm)			0.00	95.69	107.13	115.09	133.16	147.50	156.98	164.88	171.45
U_{avg} (%)			0.00	48.04	53.78	57.78	44.56	49.36	52.54	55.18	57.38

Table 2.15A: Abrupt load and variable C_v (Sinusoidal initial pore water pressure & double drainage)

Time in years			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Load(KN/m^2)			60.00	60.00	60.00	60.00	90.00	90.00	90.00	90.00	90.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		42.43	30.62	22.55	16.94	42.97	28.87	24.17	19.35	15.97
	2.00		60.00	43.31	31.89	23.95	48.34	44.31	33.50	27.44	22.59
	3.00		42.43	30.62	22.55	16.94	42.97	28.87	24.17	19.35	15.97
	4.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	112.42	135.30	151.20	187.35	214.10	230.87	243.90	253.54
U_{avg} (%)			0.00	56.44	67.92	75.90	62.70	71.65	77.27	81.63	84.85

3 APPENDIX 3 - Finite difference solution (Software Output) for a multi layered soil profile case

2.1.Three Layered soil profile with Constant Loading and Constant C_V case

Calculation for Constant Load and Constant Coefficient of Consolidation (C_v) for the case of two layered stratified soil profile will be presented using a hypothetical problem with the data as shown in Table **3.1A**.

Table 3.1A: Soil and loading data for three layered soil profile

Input Data	
Depth Soil layer 1	3.0
Depth Soil layer 2	4.5
Depth Soil layer 3	2.5
time in yr	3.2
dt (time grid) in m	0.4
dz1 (depth grid for soil layer 1) in m	1.00
dz2 (depth grid for soil layer 2) in m	1.50
dz3 (depth grid for soil layer 3) in m	1.25
K1 (coefficient of permeability for soil layer 1)	2.80
K2 (coefficient of permeability for soil layer 2)	2
K3 (coefficient of permeability for soil layer 3)	1.8
C_v1 (coefficient of consolidation for soil layer 1)	1.95
C_v2 (coefficient of consolidation for soil layer 2)	1.77
C_v2 (coefficient of consolidation for soil layer 3)	1.56
u_i (Initial pore water pressure)	110.00
M_v1 (coefficient of compressibility for soil layer 1)	0.0009
M_v2 (coefficient of compressibility for soil layer 2)	0.00056
M_v3 (coefficient of compressibility for soil layer 3)	0.00048

And the results of finite difference solution are as tabulated below for various initial pore water pressure distributions from **Table 3.2A** to **Table 3.9A**.

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i. Uniform initial pore water pressure distribution (u_i decreasing with depth)

Table 3.2A: Constant load and C_v (uniform initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		110.00	59.84	55.43	44.61	41.82	36.70	34.53	31.72	30.07
	2.00		110.00	110.00	87.13	83.10	72.42	68.64	62.90	59.82	56.30
	3.00		110.00	110.00	110.00	98.16	94.74	87.99	84.52	80.19	77.21
	4.50		110.00	110.00	110.00	110.00	107.75	105.70	103.07	100.65	98.16
	6.00		110.00	110.00	110.00	110.00	110.00	109.57	108.92	108.00	106.93
	7.50		110.00	110.00	110.00	110.00	110.00	110.00	109.92	109.74	109.45
	8.75		110.00	110.00	110.00	110.00	110.00	110.00	110.00	109.98	109.93
	10.00		110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	109.99
Settlement(mm)			0.00	84.39	106.28	127.70	143.06	157.61	169.44	180.48	190.16
U_{avg} (%)			0.00	11.95	15.05	18.08	20.26	22.32	23.99	25.56	26.93

Table 3.3A; Constant load and C_v (uniform initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		110.00	59.84	55.43	44.61	41.82	36.70	34.53	31.71	30.02
	2.00		110.00	110.00	87.13	83.10	72.42	68.64	62.87	59.72	56.06
	3.00		110.00	110.00	110.00	98.16	94.74	87.93	84.31	79.71	76.36
	4.50		110.00	110.00	110.00	110.00	107.58	105.14	101.88	98.65	95.18
	6.00		110.00	110.00	110.00	109.12	107.57	105.21	102.45	99.37	96.17
	7.50		110.00	110.00	105.38	100.08	95.13	90.75	86.82	83.25	79.95
	8.75		110.00	82.97	69.22	61.10	55.66	51.68	48.58	46.04	43.87
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	150.22	186.85	219.75	244.83	267.99	287.74	306.19	322.92
U_{avg} (%)			0.00	21.27	26.46	31.12	34.67	37.95	40.74	43.36	45.73

ii. Triangular initial pore water pressure distribution (u_i decreasing with depth)

Table 3.4A: Constant load and C_v (triangular initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		96.25	46.09	41.68	31.28	28.62	23.74	21.69	19.04	17.50
	2.00		82.50	82.50	60.55	56.72	46.53	42.99	37.56	34.70	31.40
	3.00		68.75	70.78	71.01	59.81	56.68	50.34	47.15	43.13	40.42
	4.50		55.00	55.00	55.39	55.68	53.76	52.01	49.70	47.61	45.46
	6.00		41.25	41.25	41.30	41.44	41.66	41.54	41.27	40.79	40.21
	7.50		27.50	27.76	27.93	28.34	28.93	29.63	30.31	30.94	31.47
	8.75		13.75	13.75	15.48	17.24	18.88	20.39	21.80	23.11	24.31
	10.00		0.00	6.76	10.20	12.80	14.98	16.90	18.62	20.18	21.62
Settlement(mm)			0.00	432.94	450.65	467.93	479.45	490.26	498.52	506.05	512.33
U_{avg} (%)			0.00	61.31	63.81	66.26	67.89	69.42	70.59	71.66	72.55

Table 3.5A: Constant load and C_v (triangular initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		96.25	46.09	41.68	31.28	28.62	23.74	21.69	19.04	17.50
	2.00		82.50	82.50	60.55	56.72	46.53	42.99	37.56	34.70	31.39
	3.00		68.75	70.78	71.01	59.81	56.68	50.34	47.15	43.11	40.39
	4.50		55.00	55.00	55.39	55.68	53.76	52.00	49.66	47.52	45.28
	6.00		41.25	41.25	41.30	41.44	41.60	41.36	40.90	40.18	39.31
	7.50		27.50	27.76	27.93	28.06	28.18	28.30	28.34	28.29	28.13
	8.75		13.75	13.75	13.81	13.89	13.96	14.02	14.08	14.12	14.13
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	435.65	456.08	475.98	490.06	503.37	514.10	524.06	532.76
U_{avg} (%)			0.00	61.69	64.58	67.40	69.39	71.28	72.80	74.21	75.44

iii. Trapezoidal initial pore water pressure distribution (u_i decreasing with depth)

Table 3.6A: Constant load and C_v (trapezoidal initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		101.00	50.38	46.14	35.46	32.82	27.81	25.75	23.03	21.47
	2.00		91.00	91.46	68.85	65.12	54.65	51.10	45.53	42.63	39.25
	3.00		82.00	82.96	83.38	71.83	68.70	62.18	58.94	54.81	52.03
	4.50		72.00	72.19	72.49	72.76	70.75	68.94	66.54	64.36	62.10
	6.00		63.00	63.00	63.04	63.13	63.28	63.06	62.67	62.05	61.32
	7.50		54.00	54.00	54.04	54.28	54.66	55.12	55.55	55.91	56.18
	8.75		44.00	44.25	45.46	46.67	47.78	48.81	49.77	50.65	51.45
	10.00		35.00	39.42	41.79	43.59	45.10	46.42	47.59	48.66	49.64
Settlement(mm)			0.00	322.65	341.63	360.29	373.01	385.02	394.39	403.04	410.38
U_{avg} (%)			0.00	45.69	48.38	51.02	52.82	54.52	55.85	57.07	58.11

Table 3.7A: Constant load and C_v (trapezoidal initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		101.00	50.38	46.14	35.46	32.82	27.81	25.75	23.02	21.45
	2.00		91.00	91.46	68.85	65.12	54.65	51.10	45.52	42.59	39.16
	3.00		82.00	82.96	83.38	71.83	68.70	62.16	58.87	54.64	51.73
	4.50		72.00	72.19	72.49	72.76	70.70	68.75	66.13	63.65	61.03
	6.00		63.00	63.00	63.04	62.85	62.47	61.56	60.37	58.90	57.29
	7.50		54.00	54.00	52.57	50.94	49.43	48.11	46.88	45.71	44.55
	8.75		44.00	35.64	31.39	28.88	27.20	25.98	25.03	24.25	23.56
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	345.37	370.87	394.96	412.51	428.96	442.54	455.21	466.44
U_{avg} (%)			0.00	48.91	52.52	55.93	58.41	60.74	62.66	64.46	66.05

iv. Sinusoidal initial pore water pressure distribution

Table 3.8A: Constant load and C_v (Sinusoidal initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		42.10	39.17	36.46	33.46	31.36	29.29	27.63	26.06	24.71
	2.00		77.78	72.39	66.35	62.31	58.17	54.93	51.82	49.16	46.67
	3.00		101.63	92.37	87.37	82.09	77.88	73.74	70.17	66.79	63.77
	4.50		110.00	106.82	102.52	98.39	94.28	90.41	86.73	83.32	80.15
	6.00		101.63	98.69	95.96	93.10	90.35	87.76	85.37	83.16	81.12
	7.50		77.78	76.21	74.38	73.36	72.83	72.56	72.40	72.28	72.15
	8.75		42.10	40.52	44.42	48.35	51.78	54.73	57.26	59.42	61.25
	10.00		0.00	20.69	30.44	37.31	42.74	47.18	50.89	54.02	56.67
Settlement(mm)			0.00	275.65	286.75	297.15	306.54	315.37	323.55	331.27	338.53
U_{avg} (%)			0.00	39.03	40.60	42.08	43.41	44.66	45.82	46.91	47.94

Table 3.9A: Constant load and C_v (Sinusoidal initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		42.10	39.17	36.46	33.46	31.36	29.29	27.63	26.06	24.71
	2.00		77.78	72.39	66.35	62.31	58.17	54.93	51.82	49.15	46.65
	3.00		101.63	92.37	87.37	82.09	77.88	73.74	70.15	66.75	63.66
	4.50		110.00	106.82	102.52	98.39	94.28	90.38	86.62	83.04	79.62
	6.00		101.63	98.69	95.96	93.10	90.19	87.23	84.29	81.37	78.50
	7.50		77.78	76.21	74.38	72.49	70.56	68.60	66.62	64.63	62.63
	8.75		42.10	40.52	39.33	38.28	37.28	36.30	35.32	34.33	33.34
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	283.95	303.05	320.90	337.27	352.72	367.18	380.92	393.94
U_{avg} (%)			0.00	40.21	42.91	45.44	47.76	49.95	51.99	53.94	55.78

2.2. Three Layered soil profile with Abrupt Change in Loading and Variable C_V case

A hypothetical problem of Table **3.10**A is used for the calculation of the present section.

Table 3.10A: Soil and loading data for three layered soil profile

Input Data	
Depth Soil layer 1	3.0
Depth Soil layer 2	4.5
Depth Soil layer 3	2.5
time in yr	3.2
dt (time grid) in m	0.4
dz1 (depth grid for soil layer 1) in m	1.00
dz2 (depth grid for soil layer 2) in m	1.50
dz3 (depth grid for soil layer 3) in m	1.25
K1 (coefficient of permeability for soil layer 1)	2.80
K2 (coefficient of permeability for soil layer 2)	2
K3 (coefficient of permeability for soil layer 3)	1.8
C_v1 (coefficient of consolidation for soil layer 1)	1.95
C_v2 (coefficient of consolidation for soil layer 2)	1.77
C_v2 (coefficient of consolidation for soil layer 3)	1.56
M_v1 (coefficient of compressibility for soil layer 1)	0.0009
M_v2 (coefficient of compressibility for soil layer 2)	0.00056
M_v3 (coefficient of compressibility for soil layer 3)	0.00048

Besides the data in Table **3.10**A, the following data are used:-

- Application of loading changed from constant load of 110KPa to an abrupt change in application of loading such that
    - A constant load of 75Kpa is applied throughout the first year only (0 up to 1year)
    - In the remaining time, a constant load of 110Kpa is applied (1 up to 2year)
- The coefficient of consolidation is variable and decreases at a rate of 0.4m2/year per year.

Using the above soil and loading data, the calculation of pore water, consolidation settlement and average degree of consolidation was calculated for different initial pore water pressure distribution and are tabulated from Table **3.11**A to Table **3.18**A.

i. Uniform initial pore water pressure distribution (u_i decreasing with depth)

Table 3.11A: Abrupt load and variable C_v (uniform initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		25.00	14.40	47.91	33.52	54.40	44.06	64.44	55.68	50.96
	2.00		25.00	25.00	55.84	54.48	73.86	70.61	91.11	87.74	84.02
	3.00		25.00	25.00	60.00	58.31	82.34	80.11	103.24	101.21	99.43
	4.50		25.00	25.00	60.00	60.00	84.77	84.51	109.10	108.64	108.16
	6.00		25.00	25.00	60.00	60.00	85.00	84.97	109.92	109.85	109.77
	7.50		25.00	25.00	60.00	60.00	85.00	85.00	110.00	109.99	109.98
	8.75		25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
	10.00		25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Settlement(mm)			0.00	18.54	37.12	51.11	69.92	82.85	102.07	113.87	122.53
U_{avg} (%)			0.00	11.55	9.64	13.27	12.81	15.18	14.45	16.12	17.35

Table 3.12A: Abrupt load and variable C_v (uniform initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		25.00	14.40	47.91	33.52	54.40	44.06	64.44	55.68	50.95
	2.00		25.00	25.00	55.84	54.48	73.86	70.61	91.11	87.73	84.01
	3.00		25.00	25.00	60.00	58.31	82.34	80.11	103.23	101.18	99.38
	4.50		25.00	25.00	60.00	60.00	84.76	84.46	108.99	108.46	107.90
	6.00		25.00	25.00	60.00	59.88	84.69	84.35	108.93	108.45	107.97
	7.50		25.00	25.00	59.19	58.31	81.78	80.25	103.59	102.10	100.70
	8.75		25.00	19.37	51.56	43.46	63.77	57.21	78.01	72.64	69.10
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	33.09	68.61	89.91	123.91	143.61	177.91	195.62	208.64
U_{avg} (%)			0.00	20.62	17.81	23.34	22.71	26.32	25.19	27.70	29.54

ii. Triangular initial pore water pressure distribution (u_i decreasing with depth)

Table 3.13A: Abrupt load and variable C_v (triangular initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		21.88	10.28	44.50	28.48	50.20	38.72	59.39	49.66	44.89
	2.00		18.75	18.75	48.94	48.04	66.23	63.21	83.00	79.46	75.14
	3.00		15.62	16.08	51.13	49.10	73.27	70.55	93.59	91.26	89.27
	4.50		12.50	12.50	47.57	47.63	72.40	72.15	96.70	96.21	95.69
	6.00		9.38	9.38	44.39	44.41	69.44	69.44	94.42	94.37	94.31
	7.50		6.25	6.30	41.33	41.39	66.47	66.55	91.63	91.69	91.74
	8.75		3.12	3.12	38.42	38.71	63.97	64.18	89.36	89.50	89.61
	10.00		0.00	1.41	37.11	37.59	62.96	63.25	88.48	88.66	88.80
Settlement(mm)			0.00	98.62	116.49	131.16	149.70	163.37	182.71	195.42	204.60
U_{avg} (%)			0.00	61.45	30.24	34.05	27.43	29.94	25.87	27.67	28.97

Table 3.14A: Abrupt load and variable C_v (triangular initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		21.88	10.28	44.50	28.48	50.20	38.72	59.39	49.66	44.89
	2.00		18.75	18.75	48.94	48.04	66.23	63.21	83.00	79.46	75.14
	3.00		15.62	16.08	51.13	49.10	73.27	70.55	93.59	91.26	89.27
	4.50		12.50	12.50	47.57	47.63	72.40	72.15	96.69	96.18	95.63
	6.00		9.38	9.38	44.39	44.41	69.43	69.33	94.15	93.88	93.58
	7.50		6.25	6.30	41.33	41.36	65.63	64.77	88.66	87.60	86.54
	8.75		3.12	3.12	38.13	31.69	53.08	47.27	68.61	63.64	60.39
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	99.18	131.62	151.91	184.38	203.83	237.47	255.45	268.48
U_{avg} (%)			0.00	61.79	34.17	39.44	33.79	37.35	33.63	36.17	38.02

iii. Trapezoidal initial pore water pressure distribution (u_i decreasing with depth)

Table 3.15A: Abrupt load and variable C_v (trapezoidal initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		24.00	11.94	46.33	30.21	51.95	40.45	61.12	51.38	46.61
	2.00		22.00	22.46	52.35	51.55	69.68	66.66	86.43	82.88	78.55
	3.00		21.00	21.14	56.38	54.23	78.41	75.67	98.69	96.34	94.34
	4.50		20.00	20.00	55.02	55.07	79.81	79.54	104.07	103.56	103.03
	6.00		19.00	19.00	53.98	53.96	78.95	78.92	103.87	103.79	103.70
	7.50		18.00	17.86	52.81	52.80	77.80	77.81	102.83	102.84	102.85
	8.75		16.00	16.23	51.40	51.54	76.66	76.75	101.83	101.89	101.94
	10.00		15.00	15.45	50.77	51.00	76.18	76.32	101.43	101.51	101.57
Settlement(mm)			0.00	51.08	69.29	84.37	103.24	117.20	136.78	149.72	159.05
U_{avg} (%)			0.00	31.83	17.99	21.90	18.92	21.48	19.37	21.20	22.52

Table 3.16A: Abrupt load and variable C_v (trapezoidal initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		24.00	11.94	46.33	30.21	51.95	40.45	61.12	51.38	46.61
	2.00		22.00	22.46	52.35	51.55	69.68	66.66	86.43	82.88	78.55
	3.00		21.00	21.14	56.38	54.23	78.41	75.67	98.69	96.33	94.32
	4.50		20.00	20.00	55.02	55.07	79.80	79.51	104.00	103.44	102.84
	6.00		19.00	19.00	53.98	53.89	78.76	78.50	103.16	102.75	102.32
	7.50		18.00	17.86	52.32	51.77	75.53	74.25	97.80	96.47	95.20
	8.75		16.00	12.85	46.24	38.84	59.59	53.33	74.35	69.14	65.71
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	59.99	94.20	115.91	149.49	169.83	204.20	222.78	236.30
U_{avg} (%)			0.00	37.38	24.45	30.09	27.39	31.12	28.92	31.55	33.46

iv. Sinusoidal initial pore water pressure distribution

Table 3.17A: Abrupt load and variable C_v (Sinusoidal initial pore water pressure & single drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		9.57	8.89	43.31	28.73	50.75	39.65	60.55	50.97	46.31
	2.00		17.68	16.43	50.17	49.35	68.47	65.80	85.87	82.52	78.34
	3.00		23.10	21.05	54.99	54.00	78.23	75.74	98.89	96.64	94.70
	4.50		25.00	24.33	58.50	57.75	82.08	81.49	105.79	105.12	104.46
	6.00		23.10	22.48	56.95	56.45	81.00	80.61	105.28	104.99	104.73
	7.50		17.68	17.35	52.00	51.79	76.66	76.58	101.53	101.49	101.46
	8.75		9.57	9.24	44.89	45.54	71.09	71.53	96.88	97.15	97.35
	10.00		0.00	4.31	41.33	42.64	68.59	69.31	94.86	95.27	95.58
Settlement(mm)			0.00	62.66	79.07	92.34	110.28	123.54	142.67	155.27	164.40
U_{avg} (%)			0.00	39.04	20.53	23.97	20.21	22.64	20.20	21.99	23.28

Table 3.18A: Abrupt load and variable C_v (Sinusoidal initial pore water pressure & double drainage)

Time in years			0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20
Load(KN/m^2)			25.00	25.00	60.00	60.00	85.00	85.00	110.00	110.00	110.00
Depth(m)	0.00	Pore pressure	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00		9.57	8.89	43.31	28.73	50.75	39.65	60.55	50.97	46.31
	2.00		17.68	16.43	50.17	49.35	68.47	65.80	85.87	82.52	78.33
	3.00		23.10	21.05	54.99	54.00	78.23	75.74	98.89	96.63	94.69
	4.50		25.00	24.33	58.50	57.75	82.08	81.49	105.78	105.08	104.38
	6.00		23.10	22.48	56.95	56.45	80.99	80.48	104.96	104.42	103.89
	7.50		17.68	17.35	52.00	51.68	75.63	74.49	98.13	96.86	95.63
	8.75		9.57	9.24	44.01	37.37	58.59	52.63	73.85	68.77	65.44
	10.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Settlement(mm)			0.00	64.38	96.36	116.10	148.67	168.30	202.21	220.49	233.79
U_{avg} (%)			0.00	40.11	25.02	30.14	27.24	30.84	28.63	31.22	33.11

4 *APPENDIX 4 - Sample codes that are used for development of the software*

In this part of the appendix, sample visual basic codes that are used for the development of the application software named, *ConsolAnalysis*, will be presented. It is inevitable to mention that the program codes that are presented here in under are very small portion of the total programming codes (approximately 1/20th of the total codes written for the complete program) and only for the main parts of the simple case of constant loading and coefficient of consolidation case.

N.B. The sentences that start with double astrix (**) are the brief description of the codes and they are not the part of the program codes.

***** *The following are codes used to import or call Visual basic libraries and other tools***

```
Imports System.Windows.Forms.Panel
Imports System.Drawing
Imports System.Drawing.Drawing2D
Imports System.Drawing.Imaging
Imports System.Drawing.Text
Imports System
Imports System.IO
Imports System.Windows.Forms.TextBox
Imports System.Windows.Forms.TextFormatFlags
Imports System.Drawing.Printing
```

***** *The following code starts the main body of the program for constant loading, constant C_V and homogeneous soil profile case.***

```
Public Class Homogeneous_soil_Constant_Loading_and_Constant_Cv
```

***** *Declaration of variables (Variable names are arbitrarily assigned in such a manner that they are short and can easily be understood and/or identified)***

```
Dim TextBx As New TextBox
Dim ObjTbx(0 To 100, 0 To 100) As Object
Dim PObjTbx(0 To 100, 0 To 100) As Object
Dim ALObjTbx(0 To 100, 0 To 100) As Object
Dim CVObjTbx(0 To 100, 0 To 100) As Object
Dim LD(0 To 100, 0 To 100) As Double
Dim UU(0 To 100, 0 To 100) As Double
Dim C(0 To 100, 0 To 100) As Object
Dim ObjTbxEnd(0 To 100) As Object
Dim ZObjTbxEnd(0 To 100) As Object
Dim PObjTbxEnd(0 To 100) As Object
Dim TextBoxCreated As Boolean
Dim PrvCol, PrvRow As Integer
Dim TT As Double
Dim ZZ As Double
Dim CV As Double
Dim CVV As Double
Dim dt As Double
Dim dz As Double
Dim Uo As Double
Dim Mv As Double
Dim x As Double
Dim y As Double
Dim T As Double
Dim Z As Double
```

** ***<u>Following is the initial code for assigning task when button, in this case Create Grid Button, is clicked.</u>***

```
Private Sub CreatGridBtn_Click(ByVal sender As System.Object, ByVal e As
System.EventArgs) Handles CreatGridBtn.Click

Try

ZZ = TBZZ3.Text

TT = TBTT3.Text

dt = TBdt3.Text

dz = TBdz3.Text

CV = TBCV3.Text

Uo = TBUo3.Text

Mv = TBMv3.Text
```

***** Codes that are used to check the stability of the given calculation.***

```
beta = (CV * dt / (dz * dz))

 If beta > 0.5 Then

 MessageBox.Show("Beta =(CV * dt / (dz * dz)) should be less than 0.5 ")

 MessageBox.Show("Beta =(CV * dt / (dz * dz))shall be less than 0.5 for best result",
"Notification Box", MessageBoxButtons.OKCancel, MessageBoxIcon.Warning)

 End If
```

***** Codes for Calculation table development***

```
            Imax = ((TT / dt) + 1)

            Jmax = (((ZZ) / dz) + 3)

           Dim Left As Integer = 0

            Dim Top As Integer = 40

  If TextBoxCreated Then

                For i As Integer = 0 To PrvCol - 1
```

```
            For j As Integer = 0 To 0
                Panel2.Controls.Remove(PObjTbx(i, j))
            Next
        Next
        For j As Integer = 0 To PrvRow - 1
            Panel2.Controls.Remove(ZObjTbxEnd(j))
        Next
        For j As Integer = 0 To 0
            Panel2.Controls.Remove(PObjTbxEnd(j))
        Next
        For i As Integer = 0 To PrvCol - 1
            Panel2.Controls.Remove(ObjTbxEnd(i))
        Next
        For i As Integer = 0 To PrvCol - 1
            For j As Integer = 0 To PrvRow - 1
                Panel2.Controls.Remove(ObjTbx(i, j))
            Next
        Next
    End If

    For j As Integer = 0 To 0
        Dim TextBx As New TextBox
        PObjTbxEnd(j) = TextBx
        Panel2.Controls.Add(PObjTbxEnd(j))
        PObjTbxEnd(j).Left = Left
```

```
        PObjTbxEnd(j).Top = Top
        PObjTbxEnd(j).Width = 60
        PObjTbxEnd(j).Height = 25
        PObjTbxEnd(j).Visible = True
        PObjTbxEnd(j).enabled = False
        PObjTbxEnd(j).Text = "dP(KN/m2)= ".ToString
        'Top += 20
    Next
    Top = 60
    For j As Integer = 0 To Jmax - 3
        Dim TextBx As New TextBox
        ZObjTbxEnd(j) = TextBx
        Panel2.Controls.Add(ZObjTbxEnd(j))
        ZObjTbxEnd(j).Left = Left
        ZObjTbxEnd(j).Top = Top
        ZObjTbxEnd(j).Width = 60
        ZObjTbxEnd(j).Height = 25
        ZObjTbxEnd(j).Visible = True
        ZObjTbxEnd(j).enabled = False
        ZObjTbxEnd(j).Text = "Z(m) = " + ((dz * j)).ToString
        Top += 20
    Next
    For j As Integer = Jmax - 2 To Jmax - 2
        'Top = (Jmax - 1) * 20
        Dim TextBx As New TextBox
```

```
    ZObjTbxEnd(j) = TextBx
    Panel2.Controls.Add(ZObjTbxEnd(j))
    ZObjTbxEnd(j).Left = Left
    ZObjTbxEnd(j).Top = Top
    ZObjTbxEnd(j).Width = 60
    ZObjTbxEnd(j).Height = 25
    ZObjTbxEnd(j).Visible = True
    ZObjTbxEnd(j).enabled = False
    ZObjTbxEnd(j).Text = "Sc(mm) = ".ToString
    Top += 20
Next
For j As Integer = Jmax - 1 To Jmax - 1
    Dim TextBx As New TextBox
    ZObjTbxEnd(j) = TextBx
    Panel2.Controls.Add(ZObjTbxEnd(j))
    ZObjTbxEnd(j).Left = Left
    ZObjTbxEnd(j).Top = Top
    ZObjTbxEnd(j).Width = 60
    ZObjTbxEnd(j).Height = 25
    ZObjTbxEnd(j).Visible = True
    ZObjTbxEnd(j).enabled = False
    ZObjTbxEnd(j).Text = "Uav(%)=".ToString
    Top += 20
Next
Left = 60
```

```
Top = 20
For i As Integer = 0 To Imax - 1
    Dim TextBx As New TextBox
    ObjTbxEnd(i) = TextBx
    Panel2.Controls.Add(ObjTbxEnd(i))
    ObjTbxEnd(i).Left = Left
    ObjTbxEnd(i).Top = Top
    ObjTbxEnd(i).Width = 60
    ObjTbxEnd(i).Height = 25
    ObjTbxEnd(i).Visible = True
    ObjTbxEnd(i).enabled = False
    ObjTbxEnd(i).Text = "t(y) = " + (dt * i).ToString
    Left += 60
Next
Left = 60
Top = 20
For i As Integer = 0 To Imax - 1
    Top = 20
    For j As Integer = 0 To 0
        Dim TextBx As New TextBox
        PObjTbx(i, j) = TextBx
        Panel2.Controls.Add(PObjTbx(i, j))
        Top += 20
        PObjTbx(i, j).Left = Left
        PObjTbx(i, j).Top = Top
```

```
        PObjTbx(i, j).Width = 60
        PObjTbx(i, j).Height = 25
        PObjTbx(i, j).Visible = True
        PObjTbx(i, j).Text = "0"
    Next
    Left += 60
Next
Left = 60
Top = 40
For i As Integer = 0 To Imax - 1
    Top = 40
    For j As Integer = 0 To Jmax - 1
        Dim TextBx As New TextBox
        ObjTbx(i, j) = TextBx
        Panel2.Controls.Add(ObjTbx(i, j))
        Top += 20
        ObjTbx(i, j).Left = Left
        ObjTbx(i, j).Top = Top
        ObjTbx(i, j).Width = 60
        ObjTbx(i, j).Height = 25
        ObjTbx(i, j).Visible = True
        ObjTbx(i, j).Text = "0"
    Next
    Left += 60
Next
```

```
TextBoxCreated = True
```

***** Codes that are used for Catching or removing program ERRORS***

```
Catch formatException As FormatException
    MessageBox.Show(formatException.Message, _
    "Invalid Number Format", MessageBoxButtons.OK, _
    MessageBoxIcon.Error)
Catch invalidcastexception As InvalidCastException
    MessageBox.Show(invalidcastexception.Message, _
    "Invalid Number Format", MessageBoxButtons.OK, _
    MessageBoxIcon.Error)
End Try
PrvCol = Imax : PrvRow = Jmax
Catch invalidcastexception As InvalidCastException
    MessageBox.Show(invalidcastexception.Message, _
    "Invalid Cast Exception", MessageBoxButtons.OK, _
    MessageBoxIcon.Error)

Catch NullReferenceException As NullReferenceException
    MessageBox.Show(NullReferenceException.Message, _
    "Invalid Reference", MessageBoxButtons.OK, _
    MessageBoxIcon.Error)
Catch OverflowException As OverflowException
MessageBox.Show(OverflowException.Message, "Invalid Overflow Exception"_
, MessageBoxButtons.OK, MessageBoxIcon.Error)
```

```
End Try

End Sub
```

***** Codes that assign a task for calculation of pore water pressure, consolidation settlement and average degree of consolidation***

```
Private Sub CalcMainBtn_Click(ByVal sender As System.Object, ByVal e As System.EventArgs) Handles CalcMainBtn.Click

    Dim Sum1 As Double = 0

    Dim Sum2 As Double = 0

    Dim Sum3 As Double = 0

    Dim Sum4 As Double = 0

    Dim Sum5 As Double = 0

    Dim Sum6 As Double = 0

    Dim Sum7 As Double = 0

    Dim Sum8 As Double = 0

    Dim Uoi As Double = 0

    Dim Count As Integer = 0

    Dim Avg As Double = 0
```

***** Various Initial pore water pressure modeling using "If" clause and "For….Next loop", the initial pore water pressure distribution considered includes uniform, triangular, trapezoidal and sinusoidal type of initial pore water distribution***

```
    Try

      If RecUi3.Checked Then

        For i As Integer = 0 To 0

          For j As Integer = 0 To Jmax - 3

            PObjTbx(0, 0).Text = Math.Round(CDbl(Uo), 2).ToString
```

```
                If i = 0 And j = 0 Then
                    If ImperviousTop.Checked Then
                        Uoi = Math.Round(CDbl(PObjTbx(0, 0).Text))
                    ElseIf PerviousTop.Checked Then
                        Uoi = 0
                    End If
                ElseIf i = 0 And j = Jmax - 3 Then

                    If ImperviousBottom.Checked Then
                        Uoi = Math.Round(CDbl(PObjTbx(0, 0).Text))
                    ElseIf PerviousBottom.Checked Then
                        Uoi = 0
                    End If
                Else
                    Uoi = Math.Round(CDbl(PObjTbx(0, 0).Text))
                End If
                ObjTbx(i, j).Text = Math.Round(Uoi, 2).ToString
            Next
        Next

    ElseIf TriangIN.Checked Then
        For i As Integer = 0 To 0
            For j As Integer = 0 To Jmax - 3
                PObjTbx(0, 0).Text = Math.Round(CDbl(Uo), 2).ToString
                If i = 0 And j = 0 Then
```

```
            Uoi = 0
          ElseIf i = 0 And j = Jmax - 3 Then
            If ImperviousBottom.Checked Then
              Uoi = Math.Round(CDbl(PObjTbx(0, 0).Text))
            ElseIf PerviousBottom.Checked Then
              Uoi = 0
            End If
          Else
            Uoi = Math.Round((CDbl(PObjTbx(0, 0).Text) * ((j * dz) / (ZZ))), 2)
          End If
          ObjTbx(i, j).Text = Math.Round(Uoi, 2).ToString
        Next
      Next

ElseIf TrapIN.Checked Then
Dim Uii As Double
Uii = InputBox("Enter a number for Initial pore pressure @ top")
For i As Integer = 0 To 0
For j As Integer = 0 To Jmax - 3
PObjTbx(0, 0).Text = Math.Round(CDbl(Uo), 2).ToString
If i = 0 And j = 0 Then
If ImperviousTop.Checked Then
Uoi = Math.Round(CDbl(PObjTbx(0, 0).Text))
ElseIf PerviousTop.Checked Then
Uoi = 0
```

```
End If
ElseIf  i = 0 And j = Jmax - 3 Then
If ImperviousBottom.Checked Then
Uoi = Math.Round(Uii)
ElseIf PerviousBottom.Checked Then
Uoi = 0
End If
Else
Uoi = Math.Round((((Uii - CDbl(PObjTbx(0, 0).Text)) / ((ZZ))) * (j * dz)) + CDbl(PObjTbx(0, 0).Text))
End If
ObjTbx(i, j).Text = Math.Round(Uoi, 2).ToString
Next
Next

ElseIf Sino.Checked Then
For i As Integer = 0 To 0
For j As Integer = 0 To Jmax - 3
PObjTbx(0, 0).Text = Math.Round(CDbl(Uo), 2).ToString
If i = 0 And j = 0 Then
Uoi = 0
Else
Uoi = Math.Round((CDbl(PObjTbx(0, 0).Text) * (Math.Sin(CDbl((3.141592 * j * dz) / ((ZZ)))))), 2)
End If
ObjTbx(i, j).Text = Math.Round(Uoi, 2).ToString
```

```
Next

Next

Else

MessageBox.Show("Please choose initial pore water distribution", "Notification Box", _

MessageBoxButtons.OKCancel, MessageBoxIcon.Warning)

For i As Integer = 0 To Imax - 1

For j As Integer = 0 To Jmax - 1

PObjTbx(i, j).Text = "0"

ObjTbx(i, j).Text = "0"

Next

Next

End If
```

***** ***<u>Following are codes which are used for modeling of various types of drainage condition</u>***

```
If ImperviousBottom.Checked And PerviousBottom.Checked Then

MessageBox.Show("Please choose only one top and one bottom drainage conditions",
"Notification Box", MessageBoxButtons.OKCancel, MessageBoxIcon.Warning)

For i As Integer = 0 To Imax - 1

For j As Integer = 0 To Jmax - 1

PObjTbx(i, j).Text = "0"

ObjTbx(i, j).Text = "0"

Next

Next

ElseIf ImperviousTop.Checked And PerviousTop.Checked Then

MessageBox.Show("Please choose only one top and one bottom drainage conditions",
"Notification Box", MessageBoxButtons.OKCancel, MessageBoxIcon.Warning)
```

```
For i As Integer = 0 To Imax - 1

For j As Integer = 0 To Jmax - 1

PObjTbx(i, j).Text = "0"

ObjTbx(i, j).Text = "0"

Next

Next

ElseIf ImperviousBottom.Checked = False And PerviousBottom.Checked = False And
ImperviousTop.Checked = False And PerviousTop.Checked = False Then

MessageBox.Show("Please choose top and bottom drainage conditions", "Notification Box", _

MessageBoxButtons.OKCancel, MessageBoxIcon.Warning)

For i As Integer = 0 To Imax - 1

For j As Integer = 0 To Jmax - 1

PObjTbx(i, j).Text = "0"

ObjTbx(i, j).Text = "0"

Next

Next

End If
```

***** Calculation of pore water pressures, average degree of consolidation and consolidation settlement sample codes for homogeneous soil profile under constant loading and constant coefficient of consolidation***

```
For k As Integer = 1 To 100

        For i As Integer = 1 To Imax - 1

          For j As Integer = 0 To 0

            PObjTbx(i, j).Text = Math.Round(Uo, 2)

          Next
```

```
        Next
        For i As Integer = 0 To Imax - 1
          For j As Integer = 0 To Jmax - 3
            UU(i, j) = Math.Round((CDbl(PObjTbx(i, 0).Text)), 2)
          Next
        Next
        For i As Integer = 1 To Imax - 1
          For j As Integer = 0 To 0
            If i <> 0 Then
              Sum8 = Math.Round((CDbl(ObjTbx(i, 0).Text)), 2)
              Sum1 = Math.Round((CDbl(ObjTbx(i - 1, j).Text)), 2) : Count += 1
              Sum2 = Math.Round((CDbl(ObjTbx(i - 1, j + 1).Text)), 2) : Count += 1
            End If
            If ImperviousTop.Checked Then
Avg = (UU(i, j) - UU(i - 1, j) + ((Sum1 + (2 * Sum2 - 2 * Sum1) * (CV * dt / (dz *dz)))))
ObjTbx(i, j).Text = Math.Round(Avg, 2).ToString
ElseIf PerviousTop.Checked Then
Avg = 0
ObjTbx(i, j).Text = Math.Round(Avg, 2).ToString
End If
Next
Next
For i As Integer = 1 To Imax - 1
          For j As Integer = 1 To Jmax - 4
            If i <> 0 Then
```

```
                Sum8 = Math.Round((CDbl(ObjTbx(i, 0).Text)), 2)
                Sum1 = Math.Round((CDbl(ObjTbx(i - 1, j).Text)), 2) : Count += 1
                Sum2 = Math.Round((CDbl(ObjTbx(i - 1, j + 1).Text)), 2) : Count += 1
                Sum3 = Math.Round((CDbl(ObjTbx(i - 1, j - 1).Text)), 2) : Count += 1
            End If
            If i = 1 Then
                Avg = (((Sum1 + (Sum2 - 2 * Sum1 + Sum3) * (CV * dt / (dz * dz)))))
                ObjTbx(i, j).Text = Math.Round(Avg, 2).ToString
            Else
Avg = (UU(i, j) - UU(i - 1, j) + ((Sum1 + (Sum2 - 2 * Sum1 + Sum3) * (CV * dt / (dz * dz)))))
ObjTbx(i, j).Text = Math.Round(Avg, 2).ToString
            End If
        Next
    Next

    For i As Integer = 1 To Imax - 1
        For j As Integer = Jmax - 3 To Jmax - 3
            If i <> 0 Then
                Sum8 = Math.Round((CDbl(ObjTbx(i, 0).Text)), 2)
                Sum1 = Math.Round((CDbl(ObjTbx(i - 1, j).Text)), 2) : Count += 1
                Sum3 = Math.Round((CDbl(ObjTbx(i - 1, j - 1).Text)), 2) : Count += 1
            End If

If ImperviousBottom.Checked Then
Avg = (UU(i, j) - UU(i - 1, j) + (Sum1 + (2 * Sum3 - 2 * Sum1) * (CV * dt / (dz * dz))))
```

```
ObjTbx(i, j).Text = Math.Round(Avg, 2).ToString

ElseIf PerviousBottom.Checked Then

Avg = 0

ObjTbx(i, j).Text = Math.Round(Avg, 2).ToString

End If

Next

Next

For i As Integer = 1 To Imax - 1

For j As Integer = 1 To Jmax - 1

ObjTbx(i, Jmax - 1).Text = Math.Round(CDbl((ObjTbx(i, Jmax - 2).Text) / (10 * Mv * UU(i, 0)
* (ZZ))), 2).ToString

Next j

Next

Next

    Catch formatException As FormatException

       MessageBox.Show(formatException.Message, _

       "Invalid Number Format", MessageBoxButtons.OK, _

       MessageBoxIcon.Error)

    Catch invalidcastexception As InvalidCastException

       MessageBox.Show(invalidcastexception.Message, _

       "Invalid Cast Exception", MessageBoxButtons.OK, _

       MessageBoxIcon.Error)

    Catch NullReferenceException As NullReferenceException

       MessageBox.Show(NullReferenceException.Message, _

       "Invalid Reference", MessageBoxButtons.OK, _
```

```
        MessageBoxIcon.Error)
    Catch OverflowException As OverflowException
        MessageBox.Show(OverflowException.Message, _
        "Invalid Overflow Exception", MessageBoxButtons.OK, _
        MessageBoxIcon.Error)
End Try
End Sub
```

***** Sample codes used to display specific results at any time and depth for pore water pressure, consolidation settlement and average degree of consolidation***

```
Private Sub DesplaySepcificResultsBtn_Click(ByVal sender As System.Object, ByVal e As System.EventArgs) Handles DesplaySepcificResultsBtn.Click
    Try
        Dim sp As Double
        Dim ss As Double
        Dim sua As Double
        Dim scv As Double
        Dim depth As Double
        Dim time As Double
        'Dim tes As Double
        depth = InputBox("Enter a number for depth")
        time = InputBox("Enter a number for time")
        'tes = InputBox("Enter a numbertes")
        If Not IsNumeric(depth And time) Then
            Err.Raise(1)
        End If
```

```
        For i As Integer = 0 To ((time / dt))
            For j As Integer = 0 To ((depth / dz))
                sp = CDbl(ObjTbx(i, j).Text)
            Next
        Next
        For i As Integer = 0 To (time / dt)
            For j As Integer = (((ZZ) / dz) + 1) To (((ZZ) / dz) + 1)
                ss = CDbl(ObjTbx(i, j).Text)
            Next
        Next
        For i As Integer = 0 To (time / dt)
            For j As Integer = (((ZZ) / dz) + 2) To (((ZZ) / dz) + 2)
                sua = CDbl(ObjTbx(i, j).Text)
            Next
        Next
For i As Integer = 1 To (time / dt)
            For j As Integer = 1 To (depth / dz)
                scv = Math.Round(CV, 3)
            Next
        Next
MessageBox.Show("Pore water pressue at" & depth & "m = " & sp & "KN/m2" & _
vbCrLf & "Settlement at time" & time & "year = " & ss & "mm" & _
vbCrLf & " Average Degree of consolidation" & time & "year = " & sua & "%" & _
vbCrLf & "Coefficient of consolidation" & time & "year = " & scv & "m2/year", "Specific
Consolidation Analysis Results")
```

```
    Catch formatException As FormatException
      MessageBox.Show(formatException.Message, _
      "Invalid Number Format", MessageBoxButtons.OK, _
      MessageBoxIcon.Error)
    Catch invalidcastexception As InvalidCastException
      MessageBox.Show(invalidcastexception.Message, _
      "Invalid Number Format", MessageBoxButtons.OK, _
      MessageBoxIcon.Error)
End Try
End Sub
```

***** ***Sample codes for displaying drawings in the software windows***

```
Private Sub DrawSoilProfileBtn_Click(ByVal sender As System.Object, ByVal e As
System.EventArgs) Handles DrawSoilProfileBtn.Click
    Try
      For i As Integer = 1 To Imax - 1
        For j As Integer = 1 To Jmax - 1
          ' Get the Graphics object corresponding to the form's surface.
          Dim gr As Graphics = GroupBox1.CreateGraphics
          Dim str As Graphics = GroupBox1.CreateGraphics
          ' Clear the background with azure color.
          gr.Clear(Color.Azure)
          gr.ResetTransform()
          gr.TranslateTransform(120, 40)
          ' Create multiple shapes that vary in angle and scale.
```

```
        str.TranslateTransform(20, 40)
        str.ScaleTransform((1), (1))
        Dim gvs As Double
        Dim ghs As Double
        ghs = (500 / 500)
        gvs = (300 * 0.7 / CInt(((ZZ) * 20) * 1.25))
        gr.ScaleTransform((ghs), (gvs))
        gr.ScaleTransform(1, 1)

Dim br1 As New HatchBrush(HatchStyle.LightVertical, Color.White, Color.Black)
Dim br2 As New HatchBrush(HatchStyle.BackwardDiagonal, Color.White, Color.Yellow)
Dim br3 As New HatchBrush(HatchStyle.BackwardDiagonal, Color.White, Color.Cyan)

Dim grid As New Pen(Color.Blue, 1)
Dim points1() As Point = {New Point(dt * 100, 20), New Point((dt * 100), ZZ * 20 + 20)}
gr.DrawLines(grid, points1)
grid.Dispose()
gr.FillRectangle(br1, New Rectangle(0, 0, 400, 20))
gr.FillRectangle(br2, New Rectangle(0, 20, 400, CInt((ZZ * 20) + 20)))
br1.Dispose()
br2.Dispose()
        ' Create a font from a family and a size in points.
        Dim font1 As New Font("Arial", 9)
        ' Create a font from a family, a size, and a style.
        Dim font2 As New Font("Arial", 14, FontStyle.Bold)
```

```
            ' Create a font from a family, a size, and two combined styles.
            Dim font3 As New Font("Arial", 16, FontStyle.Italic Or FontStyle.Underline)
            ' Create a font from a family and a size in another unit.
            Dim font4 As New Font("Arial", 10, FontStyle.Regular, GraphicsUnit.Millimeter)
            ' Create a font family and then a font of that family.
            Dim fontFam As New FontFamily("Courier New")
            Dim font5 As New Font(fontFam, 18, FontStyle.Italic)
str.DrawString("Applied Load = " & Uo & "KN/m2", font1, Brushes.Black, 120, -15)
            str.DrawString("soil layer 1", font1, Brushes.Black, 5, 20 * gvs)
            str.DrawString("Cv1 = " & CV & "m2/yr", font1, Brushes.Black, 5, 20 * gvs + 15)
            str.DrawString("Depth = " & ZZ & "m", font1, Brushes.Black, 10, 20 * gvs + 30)
          Next
        Next
      Catch invalidcastexception As InvalidCastException
        MessageBox.Show(invalidcastexception.Message, _
        "Invalid Cast Exception", MessageBoxButtons.OK, _
        MessageBoxIcon.Error)
      Catch NullReferenceException As NullReferenceException
        MessageBox.Show(NullReferenceException.Message, _
        "Invalid Reference", MessageBoxButtons.OK, _
        MessageBoxIcon.Error)
      Catch OverflowException As OverflowException
        MessageBox.Show(OverflowException.Message, _
        "Invalid Overflow Exception", MessageBoxButtons.OK, _
        MessageBoxIcon.Error)
```

```
    End Try
  End Sub
```

***** **_Sample codes for importing input data from other application software's_**

```
Private Sub ImportOutput_Click(ByVal sender As System.Object, ByVal e As
System.EventArgs) Handles ImportOutput.Click
    ' create dialog box enabling user to save file
    Dim fileChooser As OpenFileDialog = New OpenFileDialog()
    Dim result As DialogResult = fileChooser.ShowDialog()
    Dim fileName As String ' name of file to save data
    ' allow user to create file
    fileChooser.CheckFileExists = False
    ' exit event handler if user clicked "Cancel"
    If result = DialogResult.Cancel Then
      Return
    End If
    fileName = fileChooser.FileName ' get specified file name
    ' show error if user specified invalid file
    If (fileName = "" OrElse fileName = Nothing) Then
      MessageBox.Show("Invalid File Name", "Error", _
      MessageBoxButtons.OK, MessageBoxIcon.Error)
    Else
      ' save file via FileStream if user specified valid file
      Try
        Dim r As New StreamReader(fileName)
```

```
    Dim FirstLine As String, _
    thirdLine As String, _
    fourthLine As String, _
    fivththLine As String, _
    ninethLine As String, _
    tenthLine As String, _
    eleventhLine As String
    FirstLine = r.ReadLine()
    thirdLine = r.ReadLine()
    fourthLine = r.ReadLine()
    fivththLine = r.ReadLine()
    ninethLine = r.ReadLine()
    tenthLine = r.ReadLine()
    eleventhLine = r.ReadLine()
    r.Close()
    TBZZ3.Text = FirstLine
    TBTT3.Text = thirdLine
    TBdz3.Text = fourthLine
    TBdt3.Text = fivththLine
    TBCV3.Text = ninethLine
    TBUo3.Text = tenthLine
    TBMv3.Text = eleventhLine
  Catch fileException As FileNotFoundException
    MessageBox.Show("File Does Not Exits", "Error", _
    MessageBoxButtons.OK, MessageBoxIcon.Error)
```

```
        End Try
      End If
    End Sub
```

Sample codes for exporting computation results of the software to Microsoft Excel

```
Private Sub ExportOutputtoExcel_Click(ByVal sender As System.Object, ByVal e As System.EventArgs) Handles ExportOutputtoExcel.Click
      Dim XLApp As Object
      Dim wb As Object
      XLApp = CreateObject("Excel.Application")
      XLApp.Visible = True
      wb = XLApp.Workbooks.Add
      For i As Integer = 3 To 3
        For j As Integer = 1 To 1
XLApp.Cells(j, i).Value = "Consolidation analysis Software Report"
        Next
      Next
      For i As Integer = 3 To 3
        For j As Integer = 2 To 2
XLApp.Cells(j, i).Value = "Pore water pressure, settlement, Average Degree of Consolidation Result"
        Next
      Next
      For i As Integer = 3 To 3
        For j As Integer = 3 To 3
```

```
XLApp.Cells(j, i).Value = "Analysis Output for Homogeneous Soil Constant Loading and
Constant Cv Case"
            Next
        Next
        For i As Integer = 3 To 4
            For j As Integer = 5 To 12
                XLApp.Cells(5, 3).Value = "Input Data"
                XLApp.Cells(6, 3).Value = "Depth"
                XLApp.Cells(6, 4).Value = ZZ
                XLApp.Cells(7, 3).Value = "time in yr"
                XLApp.Cells(7, 4).Value = TT
                XLApp.Cells(8, 3).Value = "dt"
                XLApp.Cells(8, 4).Value = dt
                XLApp.Cells(9, 3).Value = "dZ"
                XLApp.Cells(9, 4).Value = dz
                XLApp.Cells(10, 3).Value = "Cv"
                XLApp.Cells(10, 4).Value = CV
                XLApp.Cells(11, 3).Value = "Initial U"
                XLApp.Cells(11, 4).Value = Uo
                XLApp.Cells(12, 3).Value = "Mv"
                XLApp.Cells(12, 4).Value = Mv
            Next
        Next
        For i As Integer = 1 To 1
            For j As Integer = 14 To 14
```

```
        XLApp.Cells(j, i).Value = "time in years"
    Next
Next
For i As Integer = 3 To Imax + 2
    For j As Integer = 16 To Jmax + 15
        XLApp.Cells(j, i).Value = ObjTbx(i - 3, j - 16).Text()
    Next
Next
For i As Integer = 3 To Imax + 2
    For j As Integer = 15 To 15
        XLApp.Cells(j, i).Value = PObjTbx(i - 3, j - 15).Text()
    Next
Next
For i As Integer = 3 To Imax + 2
    For j As Integer = 14 To 14
        XLApp.Cells(j, i).Value = (dt * (i - 3))
    Next
Next
For i As Integer = 1 To 1
    For j As Integer = 15 To 15
        XLApp.Cells(j, i).Value = PObjTbxEnd(j - 15).Text()
    Next
Next
For i As Integer = 1 To 1
    For j As Integer = Jmax + 14 To Jmax + 14
```

```
                XLApp.Cells(j, i).Value = "Settlement(mm)"
            Next
        Next
        For i As Integer = 1 To 1
            For j As Integer = Jmax + 15 To Jmax + 15
                XLApp.Cells(j, i).Value = "Deg.of Conso.(%)"
            Next
        Next
        For i As Integer = 2 To 2
            For j As Integer = 16 To Jmax + 13
                XLApp.Cells(j, i).Value = (dz * (j - 16))
            Next
        Next
        For i As Integer = 1 To 1
            For j As Integer = 16 To Jmax + 13
                XLApp.Cells(j, i).Value = "Depth(m)"
            Next
        Next
    End Sub
End Class
```