



ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

ADDIS ABABA INSTITUTE OF TECHNOLOGY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

**Performance Analysis of Linear Precoding for Multiuser  
Multiple-Input and Multiple-Output Broadcast Channels**

By

Worku Tamene

Adviser

Dr. Murad Ridwan

A thesis Submitted to the School of Graduate Studies of Addis Ababa  
University in Partial Fulfilment of the Requirements for Degree of  
Masters of Science in Communication Engineering

September, 2023

Addis Ababa, Ethiopia

ADDIS ABABA UNIVERSITY  
SCHOOL OF GRADUATE STUDIES

**Performance Analysis of Linear Precoding for Multiuser  
Multiple-Input and Multiple-Output Broadcast Channels**

By

Worku Tamene

ADDIS ABABA INSTITUTE OF TECHNOLOGY

APPROVAL BY BOARD OF EXAMINER

_____	_____
Chairman, Dept. of Graduate Committee	Signature
Dr. Murad Ridwan	_____
Advisor	Signature
_____	_____
Internal Examiner	Signature
_____	_____
External Examiner	Signature

## **Dedication**

Dedicated to my wife Enguday Guadie and my son Dagmawi Worku.

## **Declaration**

I, the undersigned, declare that this thesis work is my original work, has not been presented for a degree in this or any other universities, and all source of materials used for the thesis work has been fully acknowledged.

Worku Tamene

Name

\_\_\_\_\_  
Signature

Place: Addis Ababa

\_\_\_\_\_  
Date of Submission

As the advisor to the university, I have given my consent for this thesis to be submitted for examination.

Dr. Murad Ridwan

Adviser

\_\_\_\_\_  
Signature

## **Abstract**

Multuser Multiple Input and Multiple Output is an antenna technology for wireless communication in which number of users or wireless terminals each with one or more number of antennas communicate with each other. Precoding in multuser MIMO systems is important to minimize or mitigate the multuser interference. As a consequence, the design of suitable precoding algorithms with a low computational complexity and a good overall performance is a challenging scenario when system dimensions are high.

A linear precoding technique such as regularized channel inversion (minimum mean square error), channel inversion (zero forcing), and Block diagonalization techniques for multi-user multiple input multiple-output broadcast channels are able to eliminate the multuser interference per antenna or sum power constraint.

After conducting this thesis an enhanced performance is measured from this thesis. In case, analysis of the MU-MIMO with fewer number of antennas may reduce the cost of antenna and some complexities in large antenna system.

Different researches are conducted in multuser MIMO with single antenna receivers and conducted mostly in Rayleigh channel conditions. Besides, the performance of multuser MIMO linear precoding under different channel conditions together with two or more antenna receivers have been investigated in this work.

In this research, the performance of linear precoding in multuser MIMO under Rayleigh, Rician and Deterministic channel conditions are illustrated in different performance metrics like data rate, channel capacity and spectral efficiencies.

The performance of linear precoding under multuser MIMO with two antenna users have a great performance due to the combined effect of the antennas. In addition, the Rician channels achieves minimum bit error rate than Rayleigh and deterministic channels. Furthermore, this study has advantage of detail comparative analysis when the users are more, and this analysis have a direct impact when the congested number of users are involved.

-Key words: MU-MIMO, broadcast channel, multuser Interference, linear precoding

## **Acknowledgments**

First and foremost, I would like to thank Almighty God who provide me patient, persistence and positive no matter what all the times, and supports me all the way to this phase of my life.

I want to express my appreciativeness to my Adviser Dr. Murad Ridwan for his positively support and excellent advice throughout my thesis work. I acknowledge that without his advice and critical support, this thesis would not have been completed at all.

I also like to thank my family who has always encourage me to do my best.

Finally, I would like to thank all Addis Ababa University AAiT Instructors who taught me during my graduate study.

# Table of Contents

<b>Dedication</b> .....	I
<b>Declaration</b> .....	II
<b>Abstract</b> .....	III
<b>Acknowledgments</b> .....	IV
<b>List of Tables</b> .....	VII
<b>List of Figures</b> .....	VIII
<b>List of Abbreviations</b> .....	IX
<b>Chapter 1</b> .....	1
<b>1. Introduction</b> .....	1
<b>1.1 Statement of the Problem</b> .....	5
<b>1.2 Literature Review and Related Works</b> .....	6
<b>1.3 Objectives</b> .....	8
<b>1.3.1 General Objectives</b> .....	8
<b>1.3.2 Specific Objectives</b> .....	8
<b>1.4 Contributions of the Research</b> .....	8
<b>1.5 Research Methodologies</b> .....	9
<b>1.6 Thesis Outlines</b> .....	9
<b>Chapter 2</b> .....	10
<b>2. MIMO Wireless Communication</b> .....	10
<b>2.1 Introduction</b> .....	10
<b>2.2 MIMO Transmission Scheme</b> .....	10
<b>2.2.1 Single-Input Single-Output (SISO)</b> .....	11
<b>2.2.2 Single Input Multiple Output (SIMO)</b> .....	11
<b>2.2.3 Multiple Input Single Output (MISO)</b> .....	12
<b>2.2.4 Multiple Input Multiple Output (MIMO)</b> .....	12
<b>2.3 MIMO Channel Capacity</b> .....	15
<b>2.3.1 Capacity of Deterministic MIMO Channel</b> .....	16
<b>2.3.2 Capacity of non-Deterministic or Random MIMO Channels</b> .....	16
<b>2.4 MIMO Channel Models</b> .....	17
<b>2.4.1 Fading and Channel State Information</b> .....	18
<b>2.5 Statistical Properties of the Channel Matrix</b> .....	21
<b>2.5.1 Degree of Freedom and Diversity</b> .....	21

2.5.2 Ricean Fading Channels.....	21
2.5.3 I.I. D Rayleigh Fading Channels.....	22
2.5.4 Spatial Correlation.....	23
2.5.4 Singular Values and Eigen Values.....	23
2.6 Analytical Models.....	24
2.6.1 Correlation-Based Analytical Models.....	24
Chapter 3 .....	28
3. Multiuser MIMO Systems.....	28
3.1 MU-MIMO vs SU-MIMO .....	28
3.2 Communication Schemes for MU-MIMO Systems .....	29
3.3 Multiuser MIMO System Model .....	30
3.3.1 Uplink Model (Multiple Access Channel).....	31
3.3.2 Downlink Model (Broadcast Channel).....	31
3.4 Transmission Method for Broadcast Channel .....	32
3.4.1 Single Antenna Receiver.....	32
3.4.2 Multiple Antenna Receiver .....	34
3.5 Multiuser MIMO Channel Decomposition.....	35
Chapter 4 .....	37
4. Result and Discussion .....	37
4.1 Introduction.....	37
4.2 Simulation Result and Discussion.....	38
Chapter 5 .....	51
5. Conclusion and Recommendation .....	51
5.1 Conclusion .....	51
5.2 Recommendation and Future Work .....	52
Bibliography.....	53

## List of Tables

Table 2. 1 Illustrates of various antennas types. ....	13
Table 4. 1 Simulation Parameters for CI, RCI and BD .....	37
Table 4. 2 Simulation parameters for BER performance for MU-MIMO under BD precoding .....	40
Table 4. 3 Simulation parameters for MIMO with different antennas .....	41
Table 4. 4 Simulation parameters for spectral efficiency of MIMO systems .....	43
Table 4. 5 Simulation parameters capacity of the different precoding transmissions.....	44
Table 4. 6 Simulation parameters for Capacity as a function of SNR with different receiver antenna configuration. ....	45
Table 4. 7 Simulation parameters channel capacity as a function of transmitter antenna .....	46
Table 4. 8 Simulation parameters for channel capacity as a function of channel correlation .....	47
Table 4. 9 Simulation parameters rate regions at different SNR .....	48
Table 4. 10 Simulation parameters rate regions at different SNR with attenuation .....	49

## List of Figures

Figure 1.1 Multiuser MIMO System Model (4) .....	2
Figure 1.2 Block diagram of multi-user MIMO downlink system [9].....	4
Figure 2.1 SISO [25] .....	11
Figure 2.2 SIMO [25].....	11
Figure 2.3 MISO [25].....	12
Figure 2.4 MIMO [25] .....	13
Figure 2.5 Decomposition of a MIMO channel with full CSI.....	24
Figure 2.6 Geometry of a Tx and an Rx linear antenna array. ....	25
Figure 3.1 (a) uplink and (b) downlink single cell Multiuser MIMO respectively. ....	29
Figure 3.2 Multiuser MIMO broadcast system with precoding .....	31
Figure 3.3 Channel Inversion .....	33
Figure 3.4 Multiuser MIMO decomposition.....	35
Figure 4.1 BER performance of channel inversion system .....	38
Figure 4.2 BER performance of regularized channel inversion method.....	39
Figure 4.3 BER comparison of RCI AND CI method .....	39
Figure 4.4 BER performance of block diagonalization method .....	40
Figure 4.5 BER performance of block diagonalization method b Rayleigh, Ricean and Deterministic Channels. .....	41
Figure 4.6 The CDF of MIMO channel capacity.....	42
Figure 4.7 Capacity of mimo channel for different antenna configuration.....	43
Figure 4.8 Complementary cumulative distribution functions of sum capacity for Gaussian channels for four transmitters.....	44
Figure 4.9 Capacity as a function of SNR with different receiver antenna configuration. ....	45
Figure 4.10 Capacity as a function of transmitter array size at a SNR of 20 dB. ....	46
Figure 4.11 Capacity as a function of channel correlation between Rx antennas at an SNR of 10 dB.....	47
Figure 4.12 Rate regions for a randomly generated H of dimension $\{2,2\} \times 4$ at various power constraints. ....	48
Figure 4.13 Rate regions for a “Near–Far” H of dimension $\{2,2\} \times 4$ with 10-dB .....	49

## **List of Abbreviations**

AP-Access Point

AWGN-Additive White Gaussian Noise

BS-Base Station

BER-Bit Error Rate

BT-Blind Transmitter

BD-Block Diagonalization

BC-Broadcast Channel

CDF-Cumulative Distribution Function

CCDF- Complementary cumulative distribution functions

CI-Channel Inversion

CSI-Channel State Information

DL-Downlink

DOA-Direction of Arrival

DOD-Direction of Departure

DL-MU-MIMO-Downlink Multiuser Multiple Input Multiple Output

DSL-Digital Subscriber Line

EMW-Electro Magnetic Wave

FDD-Frequency Division Duplexing

FX-Precoding Matrix at the Transmitter

GGP- Global Gradient Projection

GX-Combining at the Receiver

IEEE- International Electrical and Electronic Engineering

IID- Independent and Identically Distributed

IWF-Iterative Water Filling

LTE-Long Term Evolution

LTE-A-Long Term Evolution Advanced

MAC-Multiple Access Channel

MS-Mobile Station

MIMO- Multiple-Input Multiple-Output

MISO-Multiple Input Single Output

MU-MIMO-Multi-User Multiple-Input Multiple-Output

MIMO-MAC-Multiple Input Multiple Output Multiple Access Channel

MIMO-BC-Multiple Input Multiple Output Broadcast Channel

NT-Number of Transmit Antenna

NR-Number of Receive Antenna

OFDMA-Orthogonal Frequency Division Multiple Access

QPSK-Quadrature Phase Shift Keying

QRD-Quick Response Decomposition

RCI-Regularized Channel inversion

RSS-Covariance Matrix

SDMA- Space Division Multiple Access

SISO-Single Input Single Output

SIMO-Single Input Multiple Output

SNR-Signal to Noise Ratio

SVD-Singular Value Decomposition

SU-MIMO-Single User Multiple Input Multiple Output

S/I-Signal to Interference Ratio

TDD-Time Division Duplexing

3G-Third Generation

TDMA-Time Division Multiple Access

UL-uplink

UL-MU-MIMO-Uplink Multiuser Multiple Input Multiple Output

# Chapter 1

## 1. Introduction

Multiple-Input Multiple-Output (MIMO) systems have got a wide attention since Telatar [1] and Foschini et al. [2] verified the potential of high spectral efficiency achieved by multiple antenna systems. The MIMO capacity gain is achieved without cost of extra spectrum. Since its introduction, MIMO has penetrated commercial wireless markets and will likely become one of the pillars underlying transmission technologies for future wireless systems.

MIMO is the backbone of today's antenna technology, providing solutions for data rate, capacity, and the majority of wireless requirements [3]. To provide a variety of solutions, MIMO functions like as precoding, spatial multiplexing, and diversity coding are used. At the transmitter, precoding is employed to maximize gain through beamforming. Spatial multiplexing focuses on ways for increasing channel capacity. Signal diversity is improved through diversity coding. Single-user MIMO, multi-user MIMO, and Massive MIMO are examples of MIMO types.

MU-MIMO allows numerous users with access, similar to orthogonal frequency division multiple access (OFDMA). MU-MIMO was introduced in the year 2012 and the data rate is 100 Mbps. Because of the use of multi-user multiplexing, it delivers immediate benefit. MU-MIMO is less influenced by channel link failure. Advanced decoding and precoding techniques are required for enhanced MU-MIMO. Time division duplexing (TDD) and frequency division duplexing (FDD) are the two main applications of MU-MIMO.

Multi-user MIMO is a collection of multiple-input multiple-output wireless communication methods in which a group of users or wireless terminals, each with one or more antennas, connect with one another as shown in Figure 1.1

The base station (BS) sends various signals to MS1, MS2, and MS3 mobile users via the H1, H2, and H3 channels.

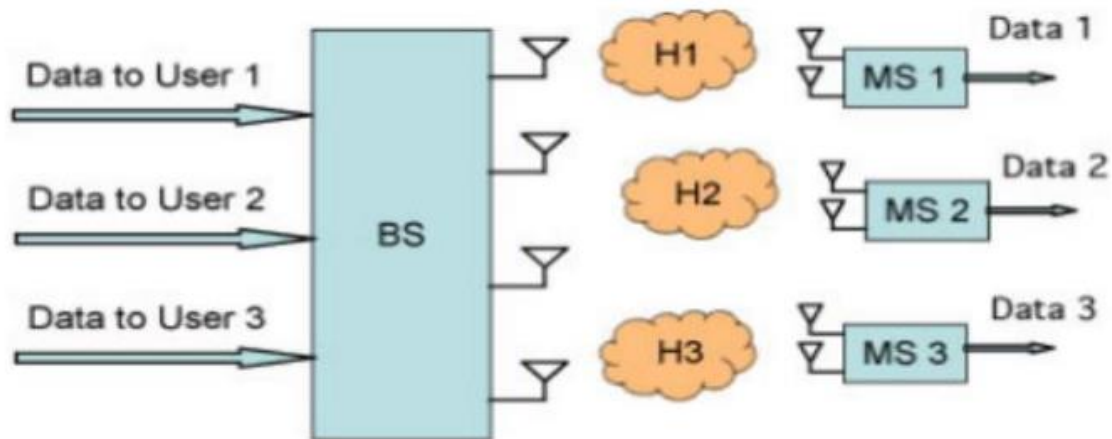


Figure 1.1 Multiuser MIMO System Model (4)

SU-MIMO in contrast, takes into account a single multi-antenna transmitter communicating with a single multi-antenna receiver. Multi-user (multiple access) capabilities are added to MIMO by MU-MIMO. Since the start of the investigation of multiple antenna communication, MU-MIMO has been studied.

The alarming rise in popularity of MIMO systems has resulted in an increase in data rate and the creation of several applications. Some technologies, such as IEEE 802.11, Third Generation (3G), and beyond network technologies rely on these systems.

The development of wireless communication technologies has aided in the creation of multiuser environment in MIMO systems. Such types of communication systems are known as MU-MIMOs. The development of new generations of wireless mobile radio system for future cellular radio networks mainly depends on these systems.

This introduction deals understanding the MIMO communication system under multiuser scenarios. Before proceeding to a more general concept, the features of MIMO communication system are highlighted. So, the MIMO system's channel model and MIMO communication are presented.

Following the MIMO system performances are highlighted to understand the concept in detail. Then, the concern is to give the reader a conceptual grasp of the multi-user MIMO technology's performance analysis. In order to accomplish this, it is crucial to introduce the framework of this systems and provide parts that illustrate the recent progress made in it.

Lastly, the multi-user MIMO under linear precoding is illustrated for nulling out the interference happened due to different users.

MU-MIMO systems can enhance capacity by merging MIMO processing with the advantages of space division multiple access (SDMA). In a MU-MIMO situation, an access point (AP) with multiple antennas communicates with a group of users at the same time. Each mobile user is also equipped with a number of antennas.

In a MU environment, capacity transforms into a  $K$ -dimensional space that fulfill the collection of all rate vectors  $(R_1, \dots, R_K)$  that can be achieved by all  $K$  users at the same time. Basically, there are two scenarios in MU-MIMO that are differentiated. The MU-MIMO uplink (UL) or broadcast channels (BC) which is multiple non-corporative terminals are transmitting to the single receiver. This scenarios in the information theory are named as MIMO multiple access channel (MAC). the second scenario, in which a single transmitter is transmitting to many non-corporative receivers is known as MU-MIMO downlink channel.

The downlink system for MU-MIMO system is shown in the figure 1.2 is also shown in [5] that the sum rate MIMO broadcast channel capacity equals the maximum sum rate dirty paper coding achievable region by demonstrating that the achievable rate meets the Sato upper bound [6].

The MU-MIMO information theory argues for users to share the channel spatially. Unlike time-division (TDMA) or code-division (CDMA) multiple access, such a multiple access protocol requires additional hardware (antennas and filters) but does not require bandwidth extension. The ensuing multiuser interference is managed by numerous antennas in spatial multiple access, which, in addition to providing per-link diversity, also provide the degrees of freedom required for spatial separation of the users.

Because of numerous important advantages over SU-MIMO communications, MU- MIMO techniques and performance have begun to be thoroughly researched and thanks to so-called multiuser multiplexing methods. MU-MIMO techniques allow for a direct increase in multiple access capacity (proportional to the number of base station (BS) antennas).

MU-MIMO looks to be more resistant to the majority of the propagation issues that plague single-user MIMO communications, such as channel rank loss and antenna correlation. Although greater correlation has an impact on per-user diversity that may not be significant if multiuser diversity [7] can be collected by the scheduler instead. Furthermore, in a multiuser setting, line of sight propagation which causes severe degradation in single user spatial multiplexing schemes is no longer an issue.

The "uplink" channel and the "downlink" channel are two different problems in MIMO communications in multi-user situations each with its own set of challenges [8].

When a group of users sharing the same channel (representing a unique time slot, frequency, or code sequence) transmits at the same time referred to as the uplink. This is known as the multiple access channel among information theorists. This scenario necessitates the multi-antenna base station properly separating all interfering signals which can be accomplished if the users transmit from various places and the receiver exploits the varying spatial properties of the corresponding channels.

### Downlink (Broadcast Channel, BC).

The MU-MIMO channel in downlink is a wireless system with a single transmitter is linked to multiple receiver system or terminals. This requires transmit handling as a precoding and space division multiple access user scheduling. The channel state information at transmitter is necessary to the transmitter or base station to empower enormous throughput enhancements over the traditional point to point MIMO frameworks. Specifically, when the antenna at the MS (receiver) surpassed by the quantity of transmit antenna.

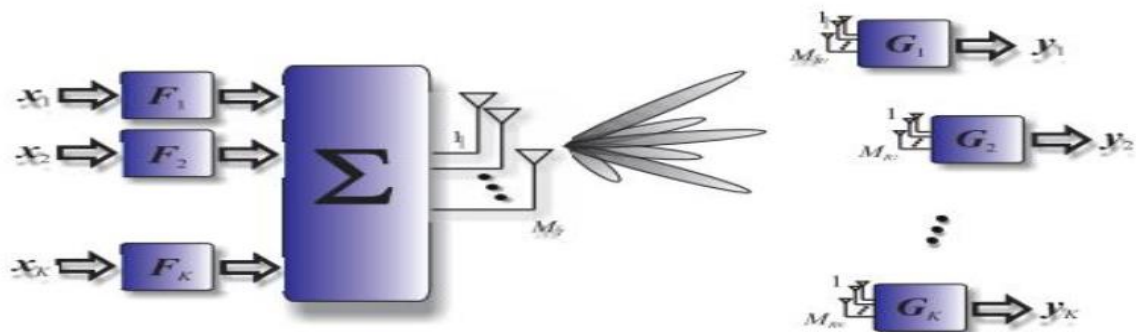


Figure 1.2 Block diagram of multi-user MIMO downlink system [9].

Linear MU-MIMO processing approaches are less computationally intensive than non-linear processing techniques, and they can precode or decode using either instantaneous channel knowledge or long-term channel data. In general, linear MU MIMO processing approaches cannot give the highest sum rate capacity, but in some instances, this is possible and the multiuser interferences can be kept to a minimum by using semi-orthogonal users for simultaneous transmission via SDMA [10].

To enhance signal power at the receiver, MIMO with many antennas transmits the same signal at the transmitter [3,11]. This method necessitates the use of appropriate gain and phase

parameters. This process is known as beamforming or single stream. Multiple streams are combined in precoding and known as multi-stream beamforming. To take use of precoding, both the transmitter and the receiver must be aware of the current state of the channel (CSI). By reducing the negative impacts of multipath propagation, different streams at the receiver add up constructively to improve receiver gain. Traditional beams are used in transmit beamforming which is unable to equalize the gain at the receiver. As a result, in modern wireless systems precoding is preferred.

The user's signal is multiplied with complex weights that adjust the magnitude and phase of the signal to and from each antenna in precoding or a multi-stream beamforming technique. The output from the array of antennas that forms a transmit/receive beam in the desired direction while minimizing the output in other directions is the result of precoding. The benefits of this are an increase in received signal gain and a reduction in multipath fading by making signals emitted from different antennas add up constructively. Precoding necessitates the transmitter's knowledge of channel state information (CSI).

## **1.1 Statement of the Problem**

The necessity of analysis of the linear precoder in terms of data rate and capacity in multiuser MIMO scenarios under different channel conditions needs attention because the received signals of different users in MU-MIMO systems not only suffer from the noise and the inter-antenna interference but are also affected by MUI.

Designing an appropriate precoding algorithm that exhibits excellent overall performance and minimal computational complexity is a formidable task in the area of MU-MIMO systems. In these scenarios, the users are spread out across different geographical locations. This creates a challenge in decoding the received symbols since joint decoding requires each user to have the data received from all the receive antennas of all the users. It is impractical to achieve this level of coordination between all users.

The need of analysis in the performance metrics like bit error rate and spectral efficiency in MU-MIMO scenarios. With the continued development of industry and business, the requirement for radio spectrum is increasingly strong, and thus the suitable radio spectrum is becoming scarcer and more expensive. Meanwhile, a complex propagation environment will always be encountered by wireless systems. The three influencing factors are noise, fading and interference.

The need to analysis linear precoders in case of MU-MIMO broadcast channels with the same transmit and receive antenna in different configurations.

## **1.2 Literature Review and Related Works**

The MU-MIMO is an advanced feature of a MIMO system that is used to increase spectrum efficiency and throughput. Because of several antennas are present at the base station or at the user terminal, it is predicted that a large number of users will use a single base station or numerous base stations with multiple antenna arrays for spatial multiplexing.

Some of the literature reviews showed some interesting technologies and interference mitigation techniques for the MU-MIMO broadcast system's transceiver side.

In [12] ZF precoding under uncorrelated Rayleigh channels in MU-MISO system with both perfect CSI and pilot-based CSI is examined. This paper shows that as SNR increases, the number of transmit antennas required for the ZF precoder to produce a certain percentage of the maximum possible sum rate decreases monotonically.

In [13] MU-MIMO systems assumed that ergodic sum spectral efficiency, tight closed-form approximation for the average SNR, and downlink ZF precoding are calculated along with correlated Rayleigh fading.

The possible sum rate is examined for two precoding strategies in [14] MU-MIMO systems with a finite number PSK input alphabet: ZF precoding and constructive interference (CI) precoding. This leads to the derivation of a new analytical equation for the average sum rate in the two scenarios, which is supported throughout by monte Carlo simulations.

In [15] Impact of Antenna spacing on Indoor MU-MIMO Channel Capacity by Feyissa Endebu under the guidance of Dr Murad Ridwan is done. This thesis discusses the antenna spacing effects on the channel capacity but due to small sizes of the wireless equipment's it recommends to use beamforming as a solution.

In [16] Investigating of SU-MIMO techniques for LTE Release 8 in highly scattering environment by Nebiyu Suleyman with the guidance of Dr. Murad Ridwan and the emphasis are to investigate SU-MIMO under release 8 and beyond release 8 the thesis recommends to use MU-MIMO. Design some pre-coding/beamforming mechanism which can be employed to translate a MISO or SIMO system into a mathematically equivalent SISO channel, Furthermore, it tries to translate a MIMO channel into a mathematically equivalent SIMO or MISO channel.

In multi-user MIMO systems with precoding, Kyung et al. investigated transmit antenna selection. Antenna selection techniques with optimum and decreased complexity sub-optimal complexity are presented. Antenna selection based on QR-decomposition (QRD) is explored, and the explanation for its sub-optimality is derived analytically. It introduces the traditional QRD-based algorithm and proposes a QRD-based transmit antenna system that is both efficient in terms of implementation and performance [17].

Jorswieck and Boche investigated the worst-case performance of a multiuser MIMO system with interference in their paper [18]. Demirkol and Ingram presented an iterative technique based on stream control in [19], [20]. This technique uses a trial-and-error method and only considers simple network topologies such as rectangular or hexagonal networks.

Chen and Gans investigated the network spectral efficiency of a MIMO ad hoc network with  $L$  simultaneous transmission pairs in their paper [21]. They showed that the network's asymptotic spectral efficiency is limited by  $n_r$  nats/s/Hz as  $L$  in the absence of channel state information (CSI) at the transmitters, and at least  $n_t + n_r + 2n_t n_r$  nats/s/Hz when CSI is available at the transmitters, where  $n_t$  and  $n_r$  are the numbers of transmitting and receiving antenna elements of each node respectively.

Unlike previous scaling law analyses, which reveal a tendency for extremely large networks, we are interested in constructing an algorithm that can compute exact maximum capacity for a given (limited size) network topology in this research. Blum demonstrated in [22] that the total of each node's mutual receiving antenna components respectively.

A multiuser MIMO system's information is neither a convex nor a concave function. A change in one user's covariance matrix will also result in a change in all users' mutual information. As a result, solving the problem analytically is extremely challenging and the majority of academics use iterative local optimization strategies to solve this problem.

Yu et al. suggested an iterative water-filling approach in [23]. (IWF). This method was first presented for DSL (digital subscriber line) networks. Because of its simplicity, proven convergence to global optimality and due to the convexity of MIMO-MAC channels, IWF was used to MIMO multiple access channel (MIMO-MAC) problems. Due to the lack of convexity, IWF has difficulty achieving convergence in MIMO ad hoc networks with mutually interfered links.

Jindal et al. presented several IWF variations for MIMO broadcast (MIMO-BC) channels [24]. They, too, cannot be applied directly to MIMO ad hoc networks. IWF is a non-cooperative game, and its convergence point is essentially a Nash equilibrium with no user can unilaterally raise their self-utility function by deviating from this stationary point.

Ye and Blum [25] proposed the global gradient projection (GGP) approach, which is a variation on the well-known steepest descent method with gradient projection. Unlike in IWF, where there is no cooperative game, all users work together to determine covariance matrices by computing gradients at each iteration. GGP is more sensitive to the choice of a starting point than IWF, and it is simpler to become stuck at a local optimum. Furthermore, as it approaches a local optimal solution GGP exhibits zigzagging [26]. GGP and IWF are both local optimization strategies that can identify a local optimal solution rapidly but cannot ensure global optimality for nonconvex optimization problems.

Precoding technique performance across various fading channel distributions, such as Rayleigh, Rician, and Nakagami fading channel distributions was evaluated in [27] in terms of capacity and BER with regard to SNR. Hence, the Rayleigh fading channel shows superior performance in MU-MIMO downlink than Rician and Nakagami channels.

## **1.3 Objectives**

### **1.3.1 General Objectives**

Performance analysis of different linear precoding for multiuser MIMO broadcast channels under the assumption of interference.

### **1.3.2 Specific Objectives**

- Using linear precoding to select superior sum rate performance under the assumption of interference.
- Finding the best bit error rate performance gains for the linear precoders in MU-MIMO broadcast channels.
- Analysis of average user spectral efficiency for MU-MIMO broadcast channels.

## **1.4 Contributions of the Research**

The differences in previous research work and the current work underlined in the following points.

- To provide insight to future researchers about linear precoding performance at different scenarios in MU-MIMO Broadcast Channels.
- To sum up the effect of interference by finding optimal linear precoder with different channel conditions.
- Making comparative performance when the receivers are using one or more antennas.
- To analyse spectrum efficiency variation due to transmitter preprocessing or precoder with different number of antennas.

## 1.5 Research Methodologies

The methodology used to do this thesis consists of different phases. The first step is survey and study different literature on MU-MIMO systems. the remaining parts shows such basic tasks are modelling, performance analysis and simulation are performed.

The methods used to achieve the desired objective of this thesis are as follows:

- 1) Literature reviews – focuses on gaining all the necessary background information required to understand the broader picture of MU-MIMO systems.
- 2) System modeling – involves how to characterize and model the different linear precoding scheme that overcome the multiuser interference appearing in a single cell multiuser MIMO.

Then, select appropriate algorithm to derive the performances analysis for linear precoding of MU-MIMO system.

- 3) Simulation – involves outline simulation workflows and programming the system in Matlab R2021a. The results will be explained and a conclusion will be made based on the obtained results. Finally, suggestions will be given to which linear precoding technique of MU-MIMO have a good performance.

## 1.6 Thesis Outlines

There are five chapters in this thesis work. These are, the theoretical overview, statement of the problem, literature review and related works, objectives, and research procedure in the introduction part. MIMO wireless communications are discussed in Chapter 2. Linear precoding in MU-MIMO broadcast channels is discussed in chapter 3. The result and discussion of the performance analysis for linear precoding for multiuser MIMO broadcast channels are discussed in Chapter 4. Conclusion and recommendation for future works is the final chapter.

## Chapter 2

### 2. MIMO Wireless Communication

#### 2.1 Introduction

The real performance of wireless communication systems is determined by the distance between the transmitter and receiver [28]. Both large and small scale fading are affected by signal propagation. In general, the signal follows multi-path propagation. Large separations between transmitters and receivers create large-scale fading, while lesser separations cause small-scale fading. Co-channel interference and neighboring channel effects increase the system's S/I and impair the MIMO antenna's actual performance metrics.

In wireless communication, several channel design and allocation procedures are utilized to cover each location with antenna installations. The system's capacity can be increased by controlling interference. The number of streams multiplied by the capacity of a single-input-single-output system determines the capacity of MIMO antennas.

Electromagnetic (EM) waves move through several pathways between transmitter and receiver, and are significantly hindered by high-rise buildings, hills, metal objects, and other obstacles. Because EM waves are random, the channel is unexpected, and analysis becomes difficult. In practice, two forms of fading are used: large scale and small scale fading. Reflection, diffraction, and scattering mechanisms are used to investigate large-scale fading.

The rapid variations in signal intensity over a short distance or time are referred to as small scale fading. This is produced by EM interference, which causes distinct signals to arrive to the receiver at different times. The resultant EM wave observes fluctuations in signal strength and angle due to multi-path propagation.

The signal strength (S) to interference (I) ratio determines the wireless system's capacity. By limiting the frequency reuse scheme, the S/I limit the coverage area. Finally, by increasing S the impacts of co-channel and adjacent channel interference are reduced.

Cell splitting, sectoring, repeaters, and microcells can all help to increase the capacity of wireless channels. MIMO is a multi-path propagation solution.

#### 2.2 MIMO Transmission Scheme

The different antenna system configuration for a wireless communication [29].

### 2.2.1 Single-Input Single-Output (SISO)

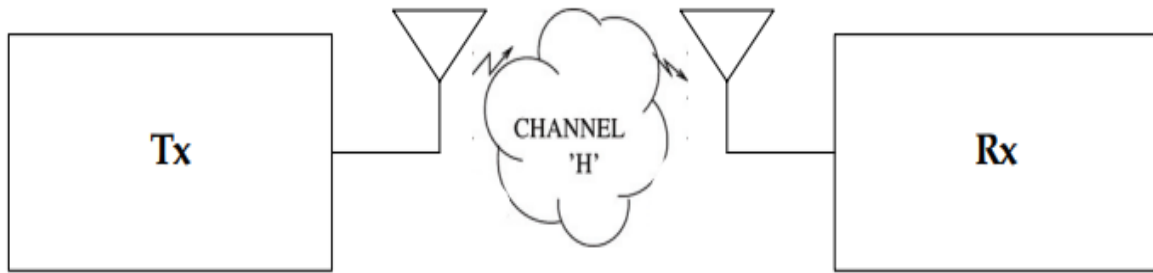


Figure 2.1 SISO [29]

The SISO wireless setup is well-known antenna configuration. SISO Systems are the simplest of the four types of communication systems in which a single transmitting antenna and a single receiving antenna are employed at the source and destination, respectively.

Figure 2.1 depicts a SISO wireless system. Because of SISO systems use fewer components and don't require any special coding techniques at the transmitter or receiver, they are less expensive to construct. In terms of usability, SISO is a good choice and is not necessary to process in terms of diversity schemes.

The system's throughput is determined by the channel bandwidth and signal to noise ratio. These systems are susceptible to concerns such as multipath effects in specific circumstances. When an electromagnetic wave collides with hills, buildings, and other barriers, the waveform scatters and travels down multiple paths to reach its goal. This results in issues such as fading, losses, and attenuation, as well as a decrease in data speed, packet loss, and errors.

### 2.2.2 Single Input Multiple Output (SIMO)

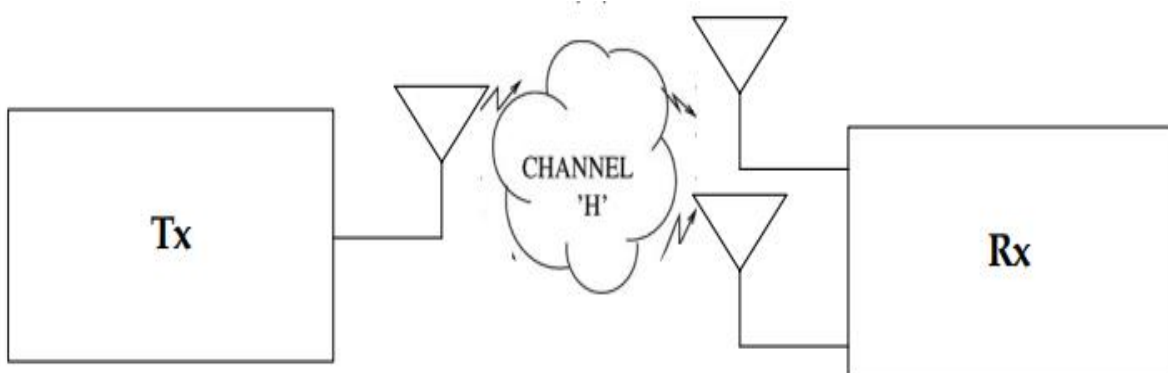


Figure 2.2 SIMO [29]

SIMO refers to a wireless communication method in which the receiver has several antennas and the source has a single transmitting antenna. Various receive diversity schemes, such as

selection diversity, maximum gain combining, and equal gain combining methods are used at the receiver to optimize the data scheme.

To combat the effects of ionosphere fading, SIMO systems were utilized for short wave listening and receiving stations. SIMO systems are suitable for a wide range of applications. However, where the receiving system is embedded in a mobile device such as a phone, performance may be constrained by size, cost, and battery life.

### 2.2.3 Multiple Input Single Output (MISO)

MISO antenna system has a feature of many transmitters antenna and a single receive antenna.

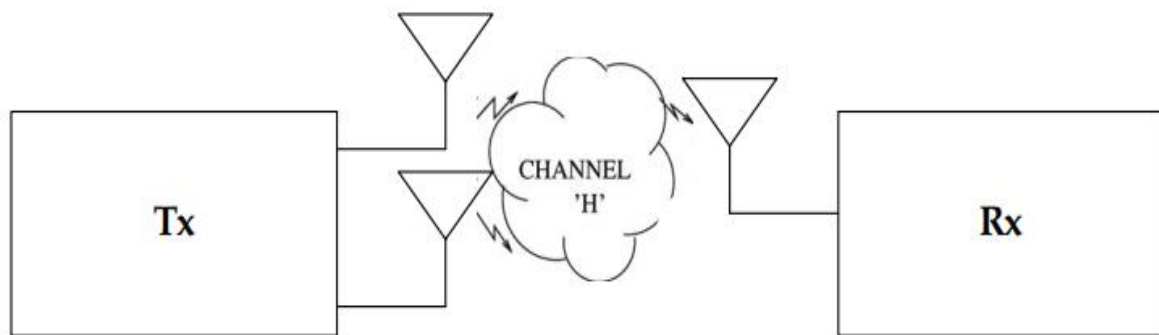


Figure 2.3 MISO [29]

MISO is an RF wireless communication system in which the source has several transmitting antennas and the system has a single receiving antenna, similar to SIMO, but the receiver has a single antenna at the destination. The consequences of multipath wave propagation, latency, packet loss, and other issues can be decreased when two or more antenna are used at the receiving end or at the destination. This system can be used in a different of methods.

MISO systems are advantageous because redundancy and coding have been relocated from the receiving end to the sending end which requires less power and processing at the user or receiver end as in the case of mobile phones.

### 2.2.4 Multiple Input Multiple Output (MIMO)

Multiple antennas are used on the sending end and multiple antennas are used on the receiving end in MIMO systems. Signals can travel several paths between a transmitter and a receiver in MIMO technology. We can use the different paths available to our advantage by shifting the antenna a short distance. MIMO can be used to take advantage of these additional

paths. They can be used to boost the link data capacity or improve the signal-to-noise ratio, making the radio link more robust.

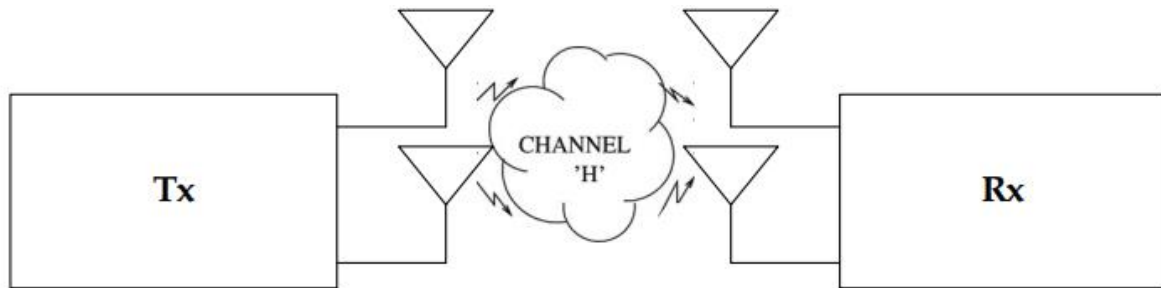


Figure 2.4 MIMO [29]

MIMO systems frequently use Spatial Multiplexing which allows signals to be sent across many spatial domains. A MIMO system can incorporate all of the benefits of SIMO and MISO systems by using a broadcast diversity strategy at the transmitter and a receive diversity scheme at the receiver at the same time.

Table 2. 1 Illustrates of various antennas types.

Multi-Antenna Types		
<b>SISO</b>	The radio system's transmitter and receiver have only one antenna, which is referred to as single-input-single-output.	
<b>SIMO</b>	The term "single-input-multiple-output" refers to the fact that the receiver has numerous antennas while the transmitter only has one.	
<b>MISO</b>	The term "multi-input-single-output" refers to the fact that the transmitter has numerous antennas while the receiver only has one.	
<b>MIMO</b>	The term "multi-input multiple-output" refers to the presence of numerous antennas on both the transmitter and receiver.	

## 2.2.4.1 The key benefits of this antenna configuration systems

### 1. Array gain

The term "array gain" refers to a signal's power gain produced when numerous antennas in the transceiver. An average increase in signal to noise ratio at the reception is the result of the transceiver's numerous antennas working together coherently to create a combining effect. If the sender is familiar with the media and has many antennas, it can apply a suitable weight to the transmission, which will cause coherent combining at the receiver.

To obtain this array gain, the state of the channels either in the sending, receiving, or transceivers side is essential in many antenna systems.

### 2. Diversity gain

For increasing the reliability of the message signals a diversity scheme is used for combining two or more communication channels with distinct characteristics. Diversity is essential for fading, co-channel interference, and preventing burst errors. The idea behind it is that there are different levels of disturbance and fades on different mediums.

The receiver has the ability to mix several copies of the same signal that are transceived. Alternatively, a redundant forward error correction code could be added, and various message portions could be sent in distinct wireless mediums. Multipath propagation can be exploited via diversity approaches, resulting in a diversity gain that is commonly quantified in dB.

Time diversity, frequency diversity, multiuser diversity, space diversity, transit diversity, and receive diversity are some of the several types of diversity schemes.

### 3. Multiplexing gain

In MIMO wireless communications, spatial multiplexing is a transmission technique that allows several data signals to be sent via any of the many antennas that transmit information. As a result, the space dimension is thus repeatedly used, or multiplied.

The maximal spatial multiplexing order is if the transmitter has  $N_T$  antennas and the receiver have  $N_R$  antennas.

$$N_s = \min(N_T, N_R) \quad (2.1)$$

If a linear receiver is employed,  $N_s$  streams can be sent in simultaneously and resulting an ideal  $N_s$  increase in spectral efficiency. Certain streams running simultaneously can have extremely low or negligible channel gains due to spatial correlation and the wireless mediums rank property, which might restrict the realistic multiplexing gain.

## 4. Interference reduction

The channel of the desired signal must be known for interference reduction. It may not be necessary to have a precise understanding of the interferer's channel. The purpose of interference reduction at the transmitter is to reduce the amount of interference energy supplied to cochannel individuals still transferring the information to the intended destinations.

Multicell capacity is increased as a result of aggressive reuse of frequencies produced possible by decreasing interferences. It is not possible to use all of the benefits of MIMO technology at the same time because of the competing needs on the spatial degrees of or number of antennas.

### 2.3 MIMO Channel Capacity

The highest transmission rate at which there is almost no possibility of error is known as the capacity. The MIMO channel's capacity is specified,

$$C = \max_{f(s)} I(s; y), \quad (2.2)$$

The mutual information between the transmit signal vector  $s$  and the receive signal vector  $y$  is denoted by  $I(s; y)$ , and  $f(s)$  is the probability distribution function (PDF) of the transmit signal vector  $s$ . The mutual of information is provided by

$$I(s; y) = H(y) - H(y|s), \quad (2.3)$$

where  $H(y)$  is the receive signal vector  $y$ 's entropy and  $H(y|s)$  is the receive signal vector  $y$ 's conditional entropy. Due to the independence of the transmit signal vector ( $s$ ) and noise vector ( $n$ ), the conditional entropy  $H(y|s)$  is equal to  $H(n)$ . Consequently, equation 2.3 above is expressed as,

$$I(s; y) = H(y) - H(n). \quad (2.4)$$

Maximizing  $H(y)$  is equivalent to enhancing the mutual information, or  $I(s; y)$ . Therefore, the information  $I(s; y)$  in equation 2.4 is provided by,

$$I(s; y) = \log_2 \det \left( I_{NR} + \frac{E_s}{N_T N_0} H R_{SS} H^H \right) \text{bps/Hz}, \quad (2.5)$$

where,  $I_{NR}$  is an  $NR \times NR$  identity matrix, The superscript  $H$  denotes conjugate transposition,  $R_{SS}$  is the covariance matrix for the transmit signal, and  $E_s$  is the power across the transmitter independent of the number of antennas ( $N_T$ ). The general capacity of the MIMO channel can be obtained from equation 2.5 as follows:

$$C = \max_{R_{SS}, \text{tr}(R_{SS})=N_T} \log_2 \det \left( I_{NR} + \frac{E_s}{N_T N_0} H R_{SS} H^H \right) \text{bps/Hz}. \quad (2.6)$$

### 2.3.1 Capacity of Deterministic MIMO Channel

At both the transmitter and receiver, the deterministic channel coefficient  $H$  is known. In practical MIMO systems, however, obtaining channel coefficient at the transmitter is quite challenging. If the MIMO system doesn't know the channel coefficient at the transmitter, it is a reasonable assumption that the sent signals from each transmit antenna have equal strength, which is also known as open loop system. As a result of this requirement, the covariance matrix,  $R_{SS} = I_{N_T}$ , is identical to the identity matrix. As a result of equation 2.6, equal powered mutual information can be calculated as follows:

$$I(s; y)_{eq} = \log_2 \det \left( I_{NR} + \frac{E_s}{N_T N_0} H H^H \right) \text{bps/Hz}, \quad (2.7)$$

the subscript "eq" indicates equal power Positive eigenvalues of the channel matrix  $H H^H$  can be used to determine mutual information in equation 2.7. When  $r$  represents the matrix  $H$ 's rank and  $\lambda_i$ ,  $i=1, 2, \dots, r$  is the matrix  $H H^H$ 's non-zero eigenvalues, then equation 2.7 for the mutual information becomes,

$$I(s; y)_{eq} = \sum_{i=1}^r \log_2 \left( 1 + \frac{E_s}{N_T N_0} \lambda_i \right) \text{bps/Hz}. \quad (2.8)$$

### 2.3.2 Capacity of non-Deterministic or Random MIMO Channels

The MIMO channels change randomly in general [30]. As a result,  $H$  is a random matrix, and its channel capacity varies randomly over time. In reality, the MIMO channel capacity can be defined by assuming that the random channel is an ergodic process.

$$C_{erg} = E \left[ \max_{R_{SS}} \log_2 \det \left\{ I_{NR} + \frac{SNR}{N_T} H R_{SS} H^H \right\} \text{bps/Hz}. \right] \quad (2.9)$$

### 2.3.2.1 Random MIMO Channel Capacity with CSI Unknown at the Transmitter

When CSI is unknown at the transmitter, The random MIMO channel's ergodic capacity can be obtained using,

$$C_{erg} = E \left[ \log_2 \det \left\{ I_{N_R} + \frac{SNR}{N_T} H R_{SS} H^H \right\} \right] \quad (2.10)$$

In terms of the positive eigenvalues,

$$C_{erg} = E \left[ \log_2 \det \left\{ I_{N_R} + \frac{SNR}{N_T} \lambda_i \right\} \right] \quad (2.11)$$

$C_{erg}$  is frequently known as an ergodic channel capacity.

## 2.4 MIMO Channel Models

The channel matrix  $H$  is a  $N_R \times N_T$  complex matrix with  $N_T$  transmit and  $N_R$  receive antennas for MIMO channels and represented as follows.

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,N_T} \\ h_{2,1} & h_{2,2} & \dots & h_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1} & h_{N_R,2} & \dots & h_{N_R,N_T} \end{bmatrix} \quad (2.12)$$

The coefficient of every channel from the  $j^{\text{th}}$  transmit antenna to the  $i^{\text{th}}$  receive antenna makes up the component of the matrix  $H$ ,  $h_{i,j}$ .

Assume that the total transmit power  $E_s$  equals the power of the received signal at each receive antenna. As a result, we obtain the channel matrix  $H$ 's normalization value for a deterministic channel condition as follows:

$$\sum_{j=1}^{N_T} |h_{i,j}|^2 = N_T, \quad i=1, 2, \dots, N_R. \quad (2.13)$$

The normalization value will be applied to the expected value in the event that the channel coefficients are random. At the  $i^{\text{th}}$  receive antenna, the received signal for time  $t$  is provided by,

$$y_i = \sum_{j=1}^{N_T} h_{i,j}(t) \cdot s_j(t) + n_i(t), \quad i=1, 2, \dots, N_R. \quad (2.14)$$

where  $n_i(t)$  is additive white Gaussian noise (AWGN) in the receiver with zero mean and  $\sigma^2$  variance, and  $s_j(t)$  is the transmit signal at the  $j^{th}$  transmit antenna. Each transmit antenna's signal,  $s_j(t)$ , is added to each receive antenna's signal in the equation above.

## 2.4.1 Fading and Channel State Information

Shannon was the first to derive capacity for time-invariant SISO channels in which the channel is constant in time and the transmitter is aware of its value [31]. The assumptions for wireless channels are different. First, the channel's schedule and frequency vary, Second, channel fluctuations imply that the transmitter and receiver have varying levels of channel knowledge. For example, if the channel varies rapidly, obtaining an accurate estimate of the channel is not always achievable. The capacity of a wireless channel varies depending on the type of the channel and channel knowledge.

- A wireless channel that varies in time is said to be fading, and depending on the duration of the codeword in relation to the channel variations, two separate situations can be distinguished: fast fading and slow fading channels.
- Channel State Information (CSI) refers to the information accessible about the channel and can take several forms, including channel coefficients, modulus, statistics, and noise variance at the receiver.

### 2.4.1.1 Fast and Slow Fading

#### Fast fading channel

Coding delay and coherence time are the two characteristics that define fast fading channels. Fast fading channels appear when the coherence time is substantially shorter than the coding delay. Capacity-achieving coding techniques are dispersed among a large number of fading states in general.

#### Slow fading channel

The coding delay is substantially smaller in a slow fading channel than it is in a fast-fading channel. Time for coherence. As a result, the channel is supposed to remain constant during the codeword. It's crucial to know the difference between a slow fading channel and a time invariant channel, for which capacity was first investigated. The channel is always constant in the latter scenario, and capacity is stated for this constant channel value.

### 2.4.1.2 Channel State Information

Channel State Information refers to the information about the channel that is available (CSI). The transmitter's and receiver's understanding of the channel determines performance. The acquisition of the channel is more difficult at the transmitter. It depends on the method used to divide communication from point A to point B and communication from point B to point A in the inverse way. In some circumstances, the channel information is updated on a regular basis at the transmitter, which is aware of the channel's current value.

In other circumstances, the transmitter is not updated frequently enough, but it has access to the channel's distribution.

#### CSI at the transmitter (CSIT)

In general, CSI at the transmitter is more difficult to achieve. Its acquisition is determined by the duplexing mode, or the way for separating transmission from point A to point B and in the opposite direction from B to A. Direct and inverse communications use distinct time slots in time division duplex (TDD), whereas they use different frequency bands in frequency division duplex (FDD). The processes for CSIT acquisition in wireless systems rely mostly on feedback or channel reciprocity.

The purpose is for A to calculate the length of the channel from point A to point B. To do this, one of the following two strategies could be used:

- Feedback: Pilot symbols encoded in the signal delivered from A to B are used to estimate the channel at B. The CSI is then passed back and forth between B and A.
- Channel Reciprocity: The channel reciprocity principle asserts that if a channel is measured at the same time and frequency from point A to point B, it is equal to the channel from point B to point A.

Both TDD and FDD can benefit from a feedback reporting method. Because the direct and inverse links do not use the same frequencies, only feedback and not channel reciprocity may be employed for FDD. TDD can make use of channel reciprocity.

Using pilot symbols included in the signal received from B, the channel from A to B can be calculated at A. This estimate is also an estimate for the channel from A to B, based on the reciprocity principle. In some systems, only B can estimate the channel from A to B.

The channel estimate from A to B is then fed back from B. For the following two scenarios, capacity is given.

1. The receiver is aware of the CSIR's current value. The transmitter is aware of the CSIT's current value.
2. The receiver is aware of the CSIR's current value. The transmitter is unaware of the CSIT's current value, but is aware of its distribution.

To achieve reliable communication, the coding scheme can be modified based on the available channel knowledge at the transmitter.

### CSI at the receiver (CSIR)

All of the channel coefficients of the links from the transmitter to the receiver are included in the CSI at the receiver. This information is essential in order to do the best ML decoding feasible. A simpler but still optimal receiver such as the minimum mean square error (MMSE) receiver can be used in some circumstances, but the noise variance must be specified. The receiver is expected to track the CSI accurately. In practical systems, pilot symbols embedded in the transmitter signal are widely employed to estimate channel and noise variation.

### Instantaneous and statistical channel state information

#### Single cell interference and cooperation

In the spatial domain, there are three main techniques to dealing with same-cell interference. First, the network can avoid the problem of interference by allocating time-frequency resources in an orthogonal manner, resulting in single-user MIMO communication within each cell [32]. This is the simplest and most popular method, although it is inefficient.

Interference cancellation in downlink is the third way stated above, in which streams are simply broadcast to each user and the receiver is left to decode them. This is unlikely to be a practical approach, especially in the downlink, because it necessitates each MS having on the order of  $nT$  adequately uncorrelated receive antennas and a willingness to accept significant processing complexity.

On the other hand, if the base station has at least  $KN$  antennas and each MS broadcasts  $N$  data streams, receiver multiuser interference cancellation for  $K$  mobile stations is theoretically possible in the uplink.

Precoding at the base station transmitter allows numerous customers to be served at the same time in most appealing and fascinating alternative.

## 2.5 Statistical Properties of the Channel Matrix

### 2.5.1 Degree of Freedom and Diversity

The spatial multiplexing capability of a MIMO channel can be quantified using a statistical model [33]. The rank of the random matrix  $H^a$  is given by  $\text{rank } H^a = \min \{N_r, N_c\}$ , where  $N_r$  is the number of non-zero rows and  $N_c$  is the number of non-zero columns with probability 1.

The number of non-zero rows and columns are determined by two factors:

- In a multipath environment, the amount of scattering and reflection. The number of non-zero elements in the random matrix  $H^a$  and the number of degrees of freedom increase as the number of scatterers and reflectors increases.
- The lengths of the broadcast and receive antenna arrays,  $L_t$  and  $L_r$ . Many different multipaths can be merged into a single resolvable path with short antenna array lengths. Increasing the array apertures allows more pathways to be resolved, resulting in more non-zero  $H^a$  entries and a higher number of degrees of freedom.

### 2.5.2 Ricean Fading Channels

In mobile cellular networks, there are instances when a strong coherent component is present. One or more specular contributions, for example, or a line-of-sight field are examples of this component that does not fade with time. In fixed wireless access scenarios, the bulk of reflected and diffracted contributions total coherently since the transmitter and receiver are fixed. All that produces non-coherent distributed contributions is the movement of a few obstacles (car, people, tree leaves, etc.).

In all of these cases, the received field amplitude has a Ricean distribution. The phase-shift-only entries in the matrix  $\bar{H}$  corresponding to the coherent component(s) are fixed.

$$\bar{H} = \begin{bmatrix} e^{j\alpha_{11}} & e^{\alpha_{12}} \\ e^{\alpha_{21}} & e^{\alpha_{22}} \end{bmatrix}. \quad (2.15)$$

The phase-shifts  $\alpha_{nm}$  are highly influenced by the array configuration and its orientation in relation to the dominating component's direction (s).  $\bar{H}$  is poorly conditioned when there is a single dominant component, such as line-of-sight, lowering the multiplexing gain for a given received SNR.

For broadside arrays, the Ricean matrix corresponding to a pure line-of-sight component is obtained by assuming a suitably large spacing between Tx and Rx.

$$\bar{H} = I_{n_r \times n_t} \quad 2.16$$

Combining  $\bar{H}$  with the non-coherent contribution  $\tilde{H}$  yields,

$$H = \sqrt{\frac{K}{1+K}} \bar{H} + \sqrt{\frac{1}{1+K}} \tilde{H} \quad (2.17)$$

Where the expected values of the channel matrix are given by,

$$E\{H\} = \sqrt{\frac{K}{1+K}} \bar{H} \text{ and } K \text{ is the Ricean } K\text{-factor.}$$

### 2.5.3 I.I. D Rayleigh Fading Channels

MIMO system designers frequently adopt the Rayleigh fading assumption, primarily because it is realistic in scatterer-rich situations [34]. It entails modeling narrowband transmission between a broadcast and receive antenna as the sum of a large number of contributions with random and statistically independent phases, departure, and arrival orientations.

The different channel correlations become very tiny and can be considered to be equal to zero when the antenna spacings and/or angular spreading of the energy on both sides of the link are large enough. The correlation matrix  $R$  is proportional to the identity matrix if all individual channels have the same average power (i.e., the antenna arrays should be balanced).

Actually,  $R = I_{n_t n_r}$ , with  $H_w$  denoting the channel matrix, which is a random fading matrix with unit variance and circularly symmetric complex Gaussian elements. When developing space-time codes, the so-called independent identically distributed (i.i.d.) Rayleigh assumption has been (and continues to be) widely utilized. However, we must emphasize that real-world channels occasionally diverge dramatically from this ideal channel for a variety of reasons:

The channels become coupled due to limited angular dispersion and/or smaller array sizes (channels are not independent anymore). The channel statistics may become Ricean as a result of a cohesive contribution (channels are not Rayleigh distributed anymore).

Using numerous polarizations causes gain imbalances in the channel matrix's various parts (channels are no longer identically distributed).

## 2.5.4 Spatial Correlation

MIMO techniques are distinct in that they take advantage of the spatial or double-directional nature of the channel. This architecture also has a significant impact on MIMO system performance. As a result, it's only natural to develop a way for describing the spatial characteristics of a multi-antenna channel, or, more precisely, the channel's space-only correlation.

The spatial correlation matrix for a MIMO channel with a limited number of antennas is defined as,

$$R = \varepsilon\{vec(H^H) vec(H)^H\}. \quad (2.18)$$

This matrix is a  $n_t n_t \times n_t n_t$  positive semi-definite Hermitian matrix, which describes the correlation between all pairs of transmit-receive channels.

## 2.5.4 Singular Values and Eigen Values

Using the singular value decomposition, a MIMO channel can be viewed as a collection of independent SISO channels (SVD) [35]. H is not usually full rank in the channel matrix. The singular value decomposition (SVD) of the  $n_r, n_t$  channel matrix reads as:

$$H = U_H \Sigma_H V_H^H \quad (2.19)$$

Where  $U_H$  and  $V_H$  are  $n_r \times r(H)$  and  $n_t \times r(H)$  unitary matrices ( $U^H U = I_{N_R}$  and  $V^H V = I_{N_T}$ ), and  $\Sigma_H = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_r(H)\}$  is the diagonal matrix containing the ordered singular value of H.

If H is a full-rank matrix, there are  $\min(N_R, N_T)$  of nonzero singular values and hence with the same number of independent channels.

If H is a full-rank matrix, there are  $\min(N_R, N_T)$  nonzero singular values, implying that there are the same number of independent channels.

The received signal  $\tilde{y}$  is calculated as follows:

$$\tilde{y} = U^H y$$

$$y = Hs + \eta$$

$$\tilde{y} = U^H (Hs + \eta), \text{ substitute equation of H in to } \tilde{y}.$$

$$\tilde{y} = U^H (U \Sigma V^H s + \eta) \quad (2.20)$$

Since  $s=V\tilde{s}$ ,  $\tilde{y}$  can be written as:

$$\begin{aligned}\tilde{y} &= U^H (U \Sigma V^H V \tilde{s} + \eta) \\ &= U^H U \Sigma V^H V \tilde{s} + U^H \eta \\ &= \Sigma \tilde{s} + \tilde{\eta}\end{aligned}$$

It can be observed that the output is the product of precoded input signal  $\tilde{s}$  and the singular value matrix  $\Sigma$ . The distribution of the noise does not change by multiplying the noise  $\eta$  by the unitary matrix  $U^H$ .

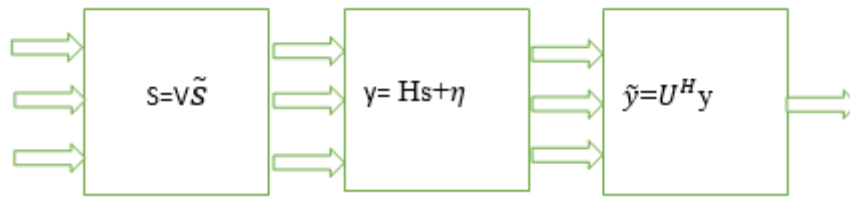


Figure 2.5 Decomposition of a MIMO channel with full CSI

## 2.6 Analytical Models

### 2.6.1 Correlation-Based Analytical Models

Several narrowband analytical models make use of a multivariate complex Gaussian distribution of MIMO channel coefficients (i.e., Rayleigh or Rician fading).

The channel matrix can be split into two parts: a zero-mean stochastic part  $H_s$  and a fully deterministic part  $H_d$ . Rician is the sum of a constant  $H_{LOS}$  and a scattering-induced variable Rayleigh component in the Rician MIMO channel [36,37].

$$H_{Ricean} = \sqrt{\frac{K_r}{1+K_r}} e^{j\phi_0} H_{LOS} + \sqrt{\frac{1}{1+K_r}} H_{Rayleigh} \quad (2.21)$$

The general configuration of a multiple transmitting and receiving antenna array is illustrated in figure below.

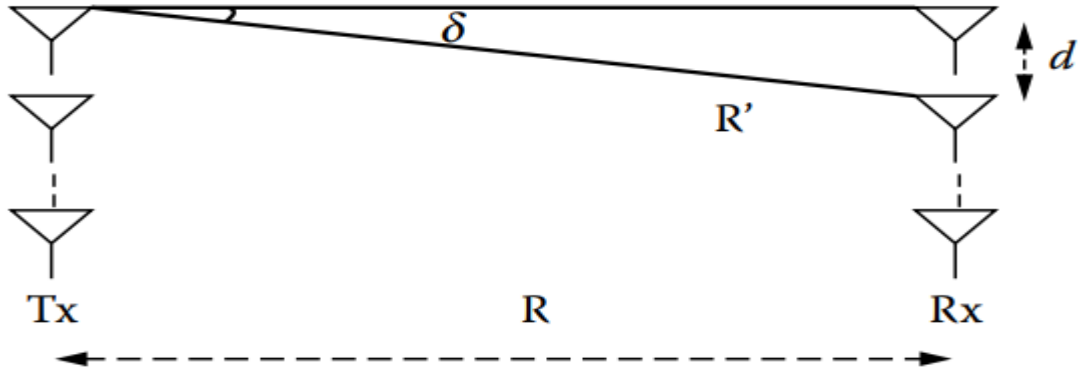


Figure 2.6 Geometry of a Tx and an Rx linear antenna array [36].

Where  $H_{\text{Ricean}}$  is the MIMO channel matrix,  $H_{\text{Rayleigh}}$  is the MIMO matrix for the variable component, HLOS is the MIMO matrix for the constant signal component, K is the Ricean–K factor, and  $\phi_0$  is the phase shift of the signal when propagating from a transmitting antenna element to the corresponding receiving antenna element.

The channel matrix for a given transmitter to receiver distance of R and the inter distance of d is given by:

$$H_{\text{LOS}} = \begin{bmatrix} 1 & e^{j\theta} & \dots & e^{j(n_t-1)\theta} \\ e^{-j\theta} & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & e^{j\theta} \\ e^{-j(n_r-1)\theta} & e^{-j(n_r-2)\theta} & \dots & 1 \end{bmatrix} \quad (2.22)$$

Where  $\theta$  is the angle corresponding to phase shift between the neighbour array elements.

$\phi_0$  affects the contribution of HLOS to  $H_{\text{Ricean}}$ . For simplicity,

$$\phi_0 = \pi/4, \text{ implies } e^{j\phi_0} = 1/\sqrt{2} + j1/\sqrt{2} \quad (2.23)$$

Consequently, after some manipulation the real and imaginary parts of  $H_{\text{Ricean}}$  is given as:

$$H_{\text{Ricean}} = \sqrt{\frac{K}{K+1}} \left( \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right) H(1) + \sqrt{\frac{1}{K+1}} H_w \quad (2.24)$$

### i.i.d Model

The degree of correlation between the individual nTnR channel gains that make up the MIMO channel is a complicated consequence of environmental scattering and transmitter and receiver antenna spacing.

With increasing antenna spacing, the decorrelation between the channel elements will rise. However, antenna spacing isn't enough to guarantee decorrelation.

Decorrelation of the MIMO channel elements is ensured by rich (i.e., omni-directional and isotropic) scattering in the environment combined with suitable antenna separation.

The typical antenna spacing required for decorrelation with rich scattering is roughly  $\lambda/2$ , where  $\lambda$  is the wavelength corresponding to the operating frequency.

## Kronecker Model

If we apply the Kronecker principle, the generation of correlated channel entries is largely simplified. Instead of dealing with a large matrix of dimension  $(N_{T_X} N_{R_X}) \times (N_{T_X} N_{R_X})$ , we can deal with a much smaller matrix of dimension  $(N_{T_X} \times N_{R_X})$ .

The Kronecker model simplifies the expression of the full correlation matrix by using a separability assumption as:

$$\mathbf{R} = \mathbf{R}_r \otimes \mathbf{R}_t, \quad (2.25)$$

where  $\mathbf{R}_t$  and  $\mathbf{R}_r$  are the transmit and receive correlation matrices, respectively.

The two conditions that jointly make the Kronecker model to be valid are,

- The transmit correlation coefficient are independent from the considered receive antenna.
- the cross-channel correlations must be equal to the product of corresponding transmit and receive correlations.

The Kronecker model is viable independently of antenna designs and intra-array spacings when the transmit and receive correlation coefficients are (in magnitude) independent of the considered receive and transmit antennas.

If all DoDs couple into all DoAs with the same power profile, and vice versa.

The channel matrix is expressed as,

$$\tilde{\mathbf{H}} = \mathbf{R}_r^{-1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \quad (2.26)$$

Where  $\mathbf{R}_t$  and  $\mathbf{R}_r$  are defined in the equation below.

$$\mathbf{R}_t = \varepsilon\{\mathbf{H}^H \mathbf{H}\} \quad (2.27)$$

$$\mathbf{R}_r = \varepsilon\{(\mathbf{H} \mathbf{H}^H)^T\} \quad (2.28)$$

## The Weichselberger Model

This model is the extended version of the Kronecker model without the assumption of separability.

The correlation matrices at the transmitter and receiver by eigenvalue decomposition is as follows:

$$R_{RX} = U_{RX} \Lambda_{RX} U_{RX}^H \quad (2.29)$$

$$R_{TX} = U_{TX} \Lambda_{TX} U_{TX}^H \quad (2.30)$$

Where  $\Lambda_{TX}$  and  $\Lambda_{RX}$  are eigenvalue matrices with corresponding orthonormal eigenvectors,  $U_{TX}$  and  $U_{RX}$ . The matrices  $R_{RX}^{1/2}$  and  $R_{TX}^{1/2}$  are as follows.

$$R_{RX}^{1/2} = U_{RX} \sqrt{\Lambda_{RX}} \quad (2.31)$$

$$R_{TX}^{1/2} = \sqrt{\Lambda_{TX}} U_{TX}^H \quad (2.32)$$

Where the magnitude of the eigenvalue matrices defined as,

$$\sqrt{\Lambda_{RX}} = \text{diag} (\lambda_{RX,1}, \lambda_{RX,2}, \dots, \lambda_{RX,\tilde{n}_x}), \quad (2.33)$$

$$\sqrt{\Lambda_{TX}} = \text{diag} (\lambda_{TX,1}, \lambda_{TX,2}, \dots, \lambda_{TX,\tilde{n}_x}), \quad (2.34)$$

$$H_{\text{Weich}} = U_{RX} \sqrt{\Lambda_{RX}} H_{\text{iid}} \sqrt{\Lambda_{TX}} U_{TX}^H \quad (2.35)$$

## Chapter 3

### 3. Multiuser MIMO Systems

As a result of the advancement of MIMO technology, new communication methods have emerged.

Multiuser MIMO is the name given to a wireless communication system in which a base station equipped with several transceiver antennas communicates with numerous users, each has one or more antennas.

The space-division multiple access (SDMA) system is usually seen as an extension of the MU-MIMO [38] technology (SDMA). With this method, multiple connections can be made over one conventional channel, and spatial signatures are used to distinguish between different users. To boost data speeds, SDMA uses spatial multiplexing. Various pathways could be used as separate data channels to achieve this. The reduction of interference from adjacent cells or users is another advantage of utilizing SDMA technology in cellular networks.

Conventional communication MIMO systems are commonly referred to as single-user MIMO systems or point-to-point MIMO systems. In MIMO systems, the base station establishes a connection with a single terminal. Multiple antennas are installed on both the base station and the end-user. The access point unlike the single-user can communicate with several mobile terminals. Multi-user communication systems can be configured in two ways: SU-MIMO and MU-MIMO.

#### 3.1 MU-MIMO vs SU-MIMO

The base station in SU-MIMO systems communicates only to one user, in contrast to MU-MIMO systems where a single base station can connect in numerous mobile terminals. Furthermore, numerous receivers are used in multi-user MIMO systems in order increase communication speed with maintaining the same level of guarantee. The total multiplexing gain, which is determined by subtracting the number of base station antennas from the number of antennas at every client, can be achieved by such systems. The system performs better because multiple users are able to communicate at the same time over the same frequency range. However, there is a significant amount of interference from neighboring channels in multiuser-MIMO systems, but SU-MIMO networks are not susceptible to such type of interference.

### 3.2 Communication Schemes for MU-MIMO Systems

For MU-MIMO systems, there are two communication methods: downlink multi-user MIMO and uplink multi-user MIMO. Users transfer information to the access point in the case of uplink communication. The access point, on the other hand delivers signals to users in downlink communication.

We will suppose that the access point has  $N$  antennas. In the case of downlink multiuser MIMO, the base station tries to send signals to  $K$  users  $U_1, \dots, U_K$ , which are having antennas with numbers  $M_1, \dots, M_K$ , respectively.

A notation denoting  $R_{xk}$  indicates whether antenna  $k$  is used as a receiving antenna. Alternatively, it is represented as  $T_{xk}$ .

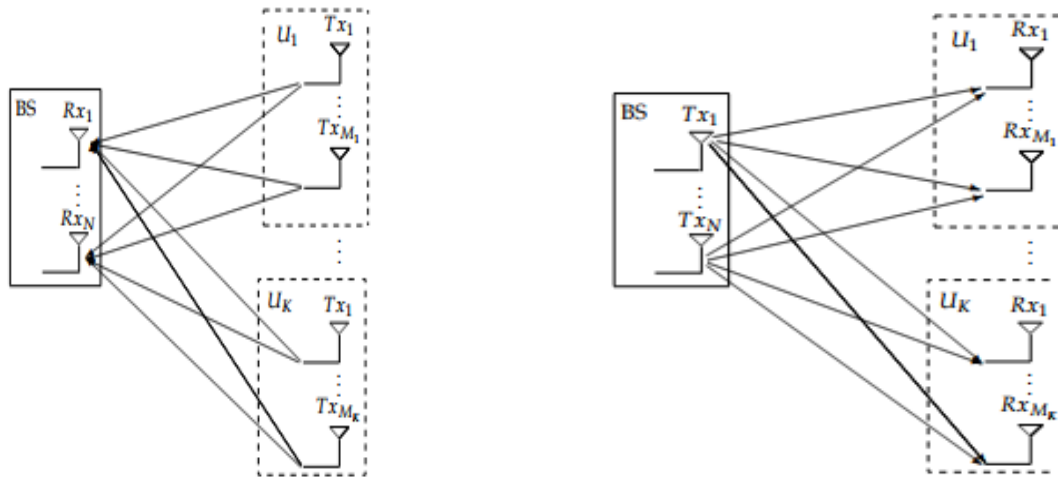


Figure 3.1 (a) uplink and (b) downlink single cell Multiuser MIMO respectively [38].

According to the downlink multiuser MIMO communication model,  $K$  users are expected to be receiving transmitting signals concurrently. The sum of signals meant for users  $U_1, \dots, U_K$  is the transmitted signal vector  $x$  ( $N \times 1$ ):

The downlink multiuser MIMO communication model implies that  $K$  users are receiving signals from the base station at the same time. The sum of signals intended for users  $U_1, \dots, U_K$  etc., is given as the transmitted signal vector  $x$  ( $N \times 1$ ).

$$x = \sum_{k=1}^K X_k \tag{3.1}$$

The channel matrix between user  $U_k$ ;  $k = 1, \dots, K$  and the base station is denoted by  $H_k(M_k \times N)$ . At each user, received signal vector of dimension ( $M_k \times 1$ );  $k = 1, \dots, K$  is given by:

$H_k(M_k \times N)$  represents the channel matrix between user  $U_k$ ;  $k = 1, \dots, K$  and the base station. The signal vector of dimension  $(M_k \times 1)$  at every user, where  $k = 1, \dots, K$  is determined as follows:

$$Y_k = H_k \cdot X + B_k; \quad (3.2)$$

The additive noise signal vector  $B_k$ ;  $k = 1, \dots, K$  has size  $(M_k \times 1)$ .

The above  $Y_k$  is written as:

$$\begin{aligned} Y_k &= H_k \cdot x + B_k; \quad (3.3) \\ &= H_k \cdot X_k + \sum_{j \neq k}^k (H_k \cdot X_j) + B_k; \quad \text{the index } k=1, \dots \end{aligned}$$

The interference signal from several users is represented by the summation term in eq (3.3).

### 3.3 Multiuser MIMO System Model

Consider a multiuser MIMO downlink system in which the base station transmits to  $K$  independent users at the same time, resulting in co-channel interference at all users, as depicted in Figure 1. For  $u=1, 2, \dots, U$ , the received signal is expressed as,

$$y_u = H_u^{DL} x + n_u \quad \text{for } u=1, 2, \dots, U. \quad (3.4)$$

In this system, user  $j$  has  $n_j$  having more than one receive antennas, and the access point has  $N_t$  transmit antennas, which are referred to as  $\{n_1, \dots, n_k\} \times N_t$  in the following.

Assume that a downlink MU-MIMO broadcast channel with  $U$  geographically sparse mobile stations communicates with a BS with  $M$  antennas. Where  $N_T = \sum_{u=1}^U N_u$  is the total number of users' antennas. With the independent channels of flat fading, the assumption is that  $N_T \leq M$ .

The system model for MU-MIMO downlink system in linear precoding with the transmitted data symbol vector  $S_s$ , the noise vector  $N_s$ , and the precoding matrix  $W_s$  for all users is shown in figure 3.2 as,

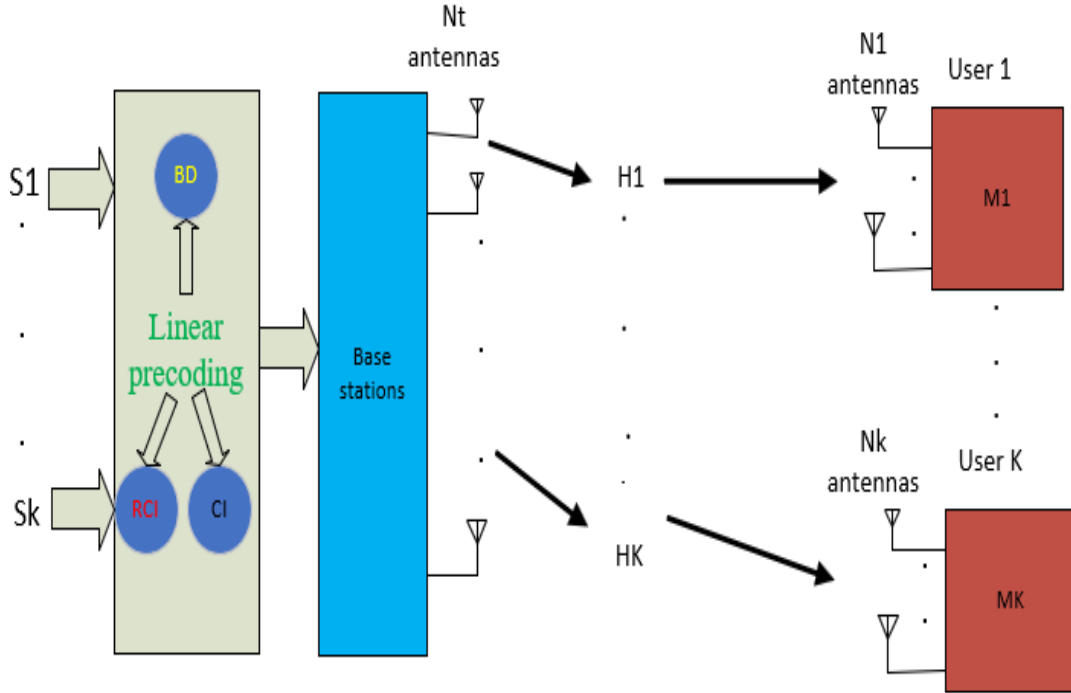


Figure 3.2 Multiuser MIMO broadcast system with precoding

### 3.3.1 Uplink Model (Multiple Access Channel)

The multiple access channel describes the data streams are transmitted towards from Mobile Stations to a Base Stations [39]. Let  $X_u \in \mathbb{C}^{N_M \times 1}$  be the transmitted signal from the  $U_{th}$ -user where  $u= 1, 2, \dots, k$ , and  $Y_{MAC} \in \mathbb{C}^{N_B \times 1}$  be the received signal from all the K users assuming  $N_B$  is the total number of base station antennas and  $N_M$  is each mobile station antennas, then the total received signal vector at the base station can be written as [35]:

$$Y_{MAC} = H_u X_U + Z \Rightarrow \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_K \end{bmatrix} = [H_1 \ H_2 \ \dots \ H_K] \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_K \end{bmatrix} + Z \quad (3.5)$$

Where  $H_u \in \mathbb{C}^{N_B \times N_M}$  is the channel matrix between the  $u_{th}$  MS and the BS and  $Z \in \mathbb{C}^{N_B \times 1}$  is a noise matrix.

### 3.3.2 Downlink Model (Broadcast Channel)

The Broadcast Channel (BC) [39] is the downlink model that explains the data streams that are routed from a BS to MSs. The received signal vector can be represented as [40]: using the same assumptions as the MAC channel.

$$Y_u = H_u X + Z \Rightarrow \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_K \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \cdot \\ \cdot \\ H_K \end{bmatrix} X + \begin{bmatrix} Z_1 \\ Z_2 \\ \cdot \\ \cdot \\ Z_K \end{bmatrix} \quad (3.6)$$

### 3.4 Transmission Method for Broadcast Channel

This work looks at methods for detecting data streams transmitted to the MS that is dependent on the number of antennas at the receiving unit. For reasons that will be discussed later, the detection methods utilized for single antenna receivers differ from those used for multiple antenna receivers. In all later treatments, the transmitter will always be a BS, while the receivers will always be MSs.

#### 3.4.1 Single Antenna Receiver

If the MS only has one receive antenna, it will be unable to reduce interference using receive diversity principles. As a result, prior to transmission, the transmitter must apply precoding techniques to decrease interference effects [40], which needs a thorough understanding of the channel state information. Channel inversion and regularized channel inversion are two proposed strategies for cancelling interference and noise effects in single antenna receivers.

Because of single-antenna receivers can't accomplish their own spatial interference reduction, they can only receive data over a single spatial channel.

The single-antenna techniques are a  $n_R \leq n_T$ , where  $n_R$  is the total number of receive antennas across all users. This basically treats each receive antenna as if it were a separate user, eliminating the requirement for collaborative processing between the antennas. However, because the problem is unduly constrained and the number of users is limited beyond what is required, performance is limited. In both channel inversion and regularized channel inversion, the number of users is limited to  $K \leq n_T$ .

##### 3.4.1.1 Channel Inversion

The linear processing-based multi-user transmission techniques are the first to be studied. The constraint that all interference terms be zero is a simple way to deal with inter-user interference.

By multiplying the data stream by the opposite channel response at the transmitter, this method negates the channel effect when the signal reaches the receiver [40]. As a result, before

the receiver can communicate this information back to the transmitter, the CSI must be entirely understood by the receiver. The figure below illustrates this strategy:

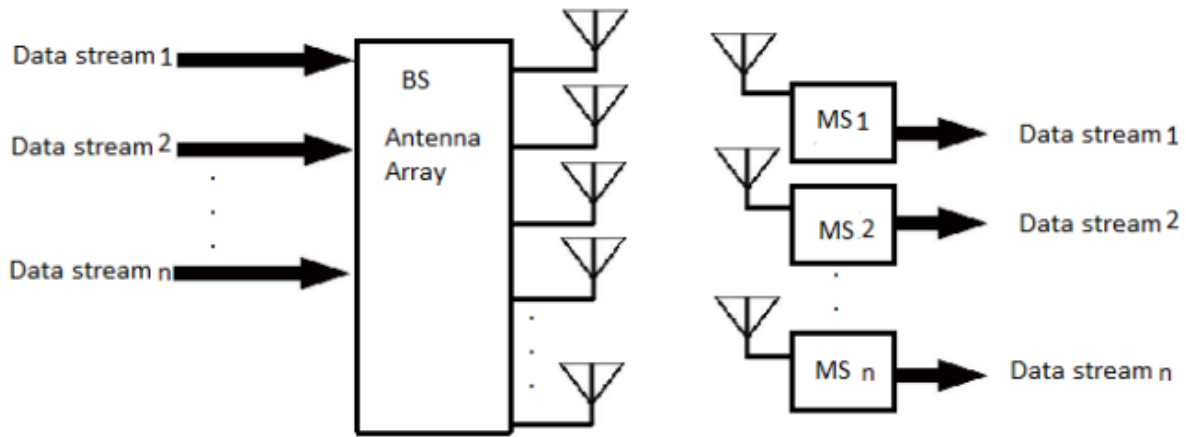


Figure 3.3 Channel Inversion [40]

It is a linear pre-coding technique that cancels out the effect of inter-user interference at the user level [41, 42]. Let  $W_k$  be the  $k$ th user's precoding vector, the received signal is given by,

$$y_u = H_u^{DL} \sum_{k=1}^K W_k \tilde{x}_k + n_u$$

$$y_u = H_u^{DL} W_u \tilde{x}_u + \sum_{k=1, k \neq u}^K H_u^{DL} W_k \tilde{x}_k + n_u \quad (3.7)$$

The precoder design forces zero interference is expressed as:

$$H_u W_k = 0 \text{ for } u \neq k \quad (3.8)$$

The precoding matrix is chosen as:

$$W_{ZF} = H^H (H H^H)^{-1} \quad (3.9)$$

Hence, the achievable sum rate and capacity respectively for this precoding technique is given as:

$$R_{CI} = \sum_{k=1}^K \log_2 \left( 1 + \frac{p}{K\sigma^2} |H_k W_k|^2 \right), \quad (3.10)$$

$$C_{CI} = \max_{H_k W_j = 0, j \neq k} \log_2 \left( 1 + \frac{p}{K\sigma^2} |H_k W_k|^2 \right) \quad (3.11)$$

### 3.4.1.2 Regularized Channel Inversion

The received signal is given by,

$$y_u = H_u^{DL} \sum_{k=1}^K W_k \tilde{x}_k + n_u = H_u^{DL} W_u \tilde{x}_u + \sum_{k=1, k \neq u}^K H_u^{DL} W_k \tilde{x}_k + n_u \quad (3.12)$$

$$W_{MMSE} = H^H \left( H H^H + \frac{\sigma_n^2}{\sigma_x^2} I \right)^{-1}, \quad (3.13)$$

where  $W_{MMSE}$  is the precoding matrix.

The achievable sum rate and the capacity is given respectively as:

$$R_{RCI} = \sum_{k=1}^K \log_2 \left( 1 + \frac{|H_k W_k|^2}{\sum_{j \neq k} |H_j W_j|^2 + K \sigma^2 / P} \right), \quad (3.14)$$

$$C_{RCI} = \max_{H_k W_j = 0, j \neq k} \log_2 \left( 1 + \frac{|H_k W_k|^2}{\sum_{j \neq k} |H_j W_j|^2 + K \sigma^2 / P} \right) \quad (3.15)$$

### 3.4.2 Multiple Antenna Receiver

If the receiver has many antennas, the multiuser MIMO channel is divided into separate single user MIMO channels, which can then be used to reduce noise and multipath fading using any of the space diversity techniques.

Multiple antennas overcome the constraints of a single antenna receiver, as long as the transmitter and receiver can coordinate their spatial processing and use the available spatial resources correctly.

#### 3.4.2.1 Block Diagonalization

The Multiuser MIMO system is decomposed into parallel single user MIMO systems using block diagonalization precoding, which lowers multiuser interference from the base station.

The aim is to design an optimal precoding vector  $W$  such that multi-user interference is zero. This is possible when  $W_j$  lies in the null space of  $H_k^*$ .

The received signal for  $u$  user is expressed as:

$$y_u = H_u^{DL} \sum_{k=1}^K W_k \tilde{x}_k + n_u = H_u^{DL} W_u \tilde{x}_u + \sum_{k=1, k \neq u}^K H_u^{DL} W_k \tilde{x}_k + n_u \quad (3.16)$$

The singular value decomposition of  $H_k^*$  is given by,

$$H_k^* = U_k^* S_k^* \left[ V_k^{*(1)} V_k^{*(0)} \right]^H \quad (3.17)$$

Any precoder  $W_k$  that is linear combination of columns of  $V_k^{*(0)}$  will satisfy the null constraint and will produce zero interference at other users.

$$H_k V_k^{*(0)} = U_k \begin{bmatrix} S_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_k^{(1)} & V_k^{(0)} \end{bmatrix}^* \quad (3.18)$$

The achievable sum rate and capacity of the BD is given respectively as:

$$R_{BD} = \sum_{k=1}^K \log_2 \left( I + \frac{P}{K} H_k W_k W_k^H H_k^H \right), \quad (3.19)$$

$$C_{BD} = \max_{H_k W_l = 0, l \neq k} \log_2 \left( I + \frac{P}{K} H_k W_k W_k^H H_k^H \right) \quad (3.20)$$

### 3.5 Multiuser MIMO Channel Decomposition

Choi and Murch [43] proposed an alternate strategy in 2004, dividing the multiuser channel into separate single user channels to avoid interference and then reducing noise effects using the channel inversion technique. This method is depicted in the diagram below.

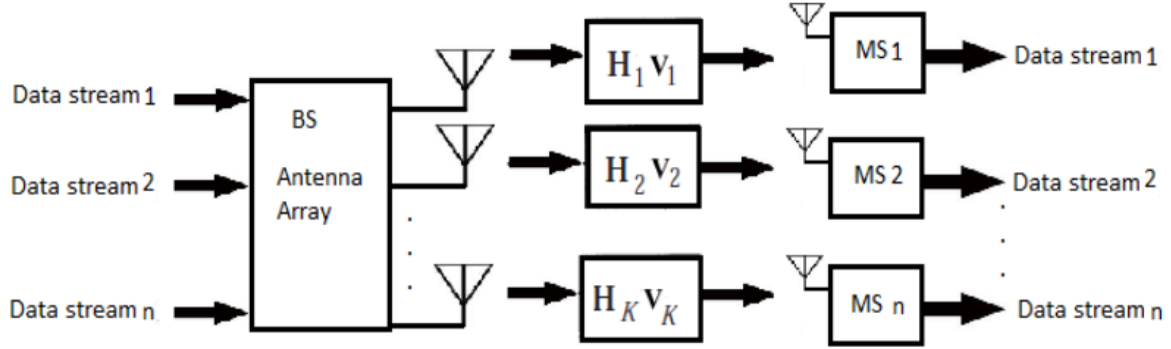


Figure 3.4 Multiuser MIMO decomposition [43]

$$y_u = H_u \sum_{k=1}^K w_k \tilde{x}_k + Z_u$$

$$y_u = \underbrace{H_u w_u \tilde{x}_u}_{\text{signal}} + \underbrace{\sum_{k=1, k \neq u}^K H_u w_k \tilde{x}_k}_{\text{interference}} + \underbrace{Z_u}_{\text{noise}} \quad (3.21)$$

Where:

$\tilde{x}_k$ : The data symbol vector of user K

$w_k$ : The precoding matrix

$H_u$ : The channel matrix

$Z_u$ : The noise matrix

The aim is to make the interference term equals to zero by selecting none zero precoding matrices. This condition can be expressed as [43]:

$$H_u w_k = 0, \forall u \neq k \quad (3.22)$$

For this purpose, a new channel matrix is constructed which contains the channel matrices of all users except the intended  $u_{th}$  user [43]:

$$H = [H_1^H \dots H_{u-1}^H H_{u+1}^H \dots H_K^H]^H \quad (3.23)$$

Based on the proposed method in [43], the singular value decomposition (SVD) on the matrix B is found [44]:

$$H=U[\Lambda 0][V_1 V_2]^H \quad (3.24)$$

Using matrix operations, if both sides are multiplied by w we get [44]:

$$HV_2=U[\Lambda 0] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^H \quad V_2=U\Lambda[V_1]^H V_2=U\Lambda \underline{0}=0 \quad (3.25)$$

Consequently, the best precoding matrix that cancels all the interference is  $w_u= V_2$  and this gives the following received signal vector assuming three users [39,44]:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1 & H_1 & H_1 \\ H_2 & H_2 & H_2 \\ H_3 & H_3 & H_3 \end{bmatrix} \begin{bmatrix} W_1 \tilde{X}_1 \\ W_2 \tilde{X}_2 \\ W_3 \tilde{X}_3 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1 W_1 & H_1 W_2 & H_1 W_3 \\ H_2 W_1 & H_2 W_2 & H_2 W_3 \\ H_3 W_1 & H_3 W_2 & H_3 W_3 \end{bmatrix} \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \quad (3.26)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1 W_1 & 0 & 0 \\ 0 & H_2 W_2 & 0 \\ 0 & 0 & H_3 W_3 \end{bmatrix} \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \quad (3.27)$$

This way the multiuser MIMO channel has been decomposed to single user MIMO channels [43].

### User selection/scheduling strategy

The system equation for the case of single-antenna UEs for the multi-user mode is,

$$y_1=h_1^+ p_1 x_1+h_1^+ p_2 x_2+n_1, \quad (3.28)$$

Where,  $p_1$  and  $p_2$  is the requested precoder.

The scheduling strategy is based on the principle of maximizing the desired signal strength while minimizing the interference strength. the eNodeB schedules two UEs on the same RBs which have requested opposite (orthogonal) precoders or out of phase 180°.

So, if UE-1 has requested  $\mathbf{p1} = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 \\ q \end{bmatrix}$ ,  $q \in \{\pm 1, \pm j\}$ , then eNodeB selects the second UE

which has requested  $\mathbf{p2} = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 \\ -q \end{bmatrix}$ .

## Chapter 4

### 4. Result and Discussion

#### 4.1 Introduction

In this chapter, illustration of simulation results on the basis of ideas discussed in previous chapter 3 using MATLAB 21.0 is going to be done.

In fact, a lot of programming languages are available today. Matlab is taken as a guarantee to the reliability and the algorithms are modified or changed. MATLAB has numerous built-in functions and is specially developed tool box for communication and signal processing which makes coding simple and straight forward. For this reason, MATLAB has been chosen as the underlying platform for all simulator and code development work.

In multi-user MIMO broadcast channel, the channel state information is assumed both known and unknown at the transmitter.

Table 4. 1 Simulation Parameters for CI, RCI and BD

Number of antennas at the transmitter (NT)	A Maximum of 8
Number of antennas at the receiver (NR)	A maximum of 8
Number of users in the multiuser case	A maximum of 4
Precoding type	Channel inversion, regularized channel inversion and block diagonalization.
Diversity	Spatial
Modulation type	QPSK
Number of frames/packets(N_frame)	20
Number of bits per QPSK symbol(b)	2
Number of packets	100
Number of bits in a packet(N_pbits)	$N\_frame * NT * b$
Number of total bits	Number of bits in a packet* Number of packets
Message bits	$randi(1, N\_pbits)$

## 4.2 Simulation Result and Discussion

- The simulation given in figure 4.1 considers four active users from the pool of 8 users where the base station and the mobile stations are equipped with  $N_B=4$  and  $N_M=1$  antennas.
- The Bit error rate performance of the channel inversion under a certain SNR is illustrated in the figure 4.1 is shows the error rate is decreases with increase of the SNR values.

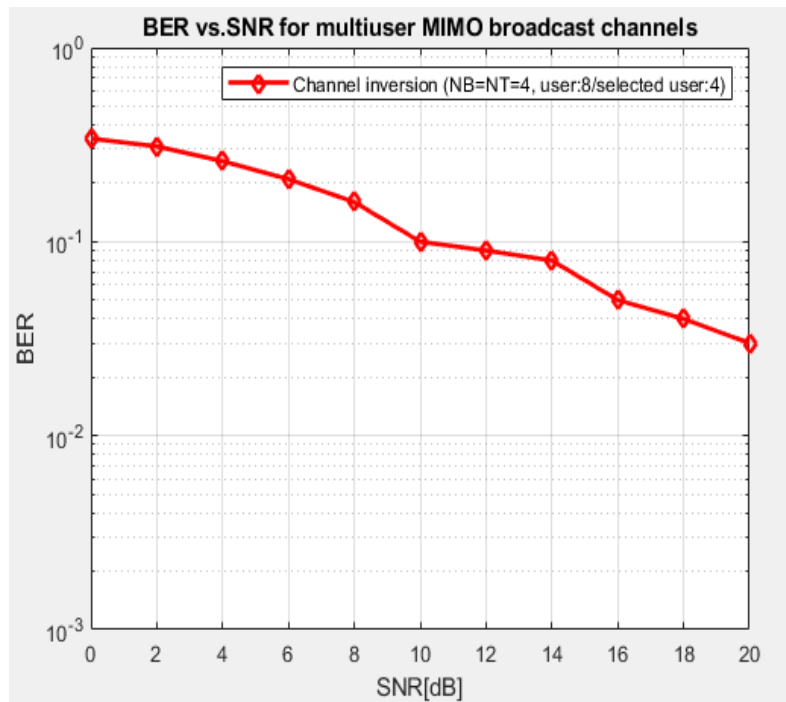


Figure 4.1 BER performance of channel inversion system

The channel inversion technique shown in figure 4.1 is solves the problem of interference due to different users used to transmit signals in same time frequency resources and have the capability of nulling out the interferences. The channel inversion techniques are assumed without the effect of noise enhancements.

The simulation given in figure 4.2 considers four active users from the pool of 8 users where the base station and the mobile stations are equipped with  $N_B=4$  and  $N_M=4$  antennas respectively.

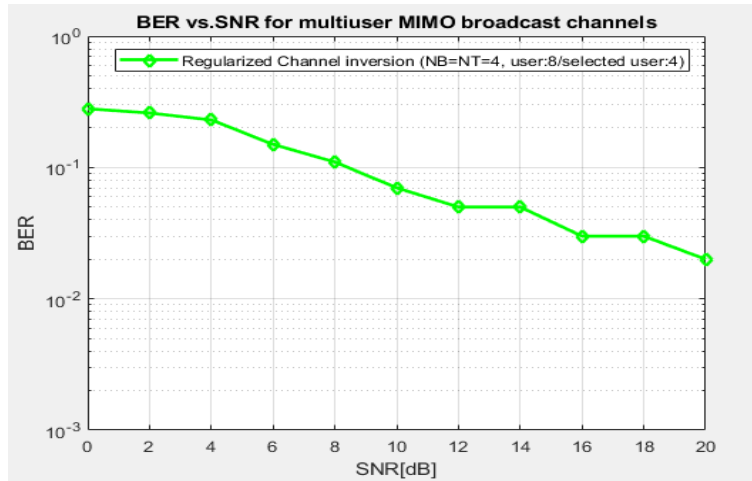


Figure 4.2 BER performance of regularized channel inversion method

The regularized channel inversion is a linear precoding with considering reduction of interference and considers the effect of noise appearing and have the capability of reducing this noise effects.

The channel inversion and regularized channel inversion is illustrated in figure below as:

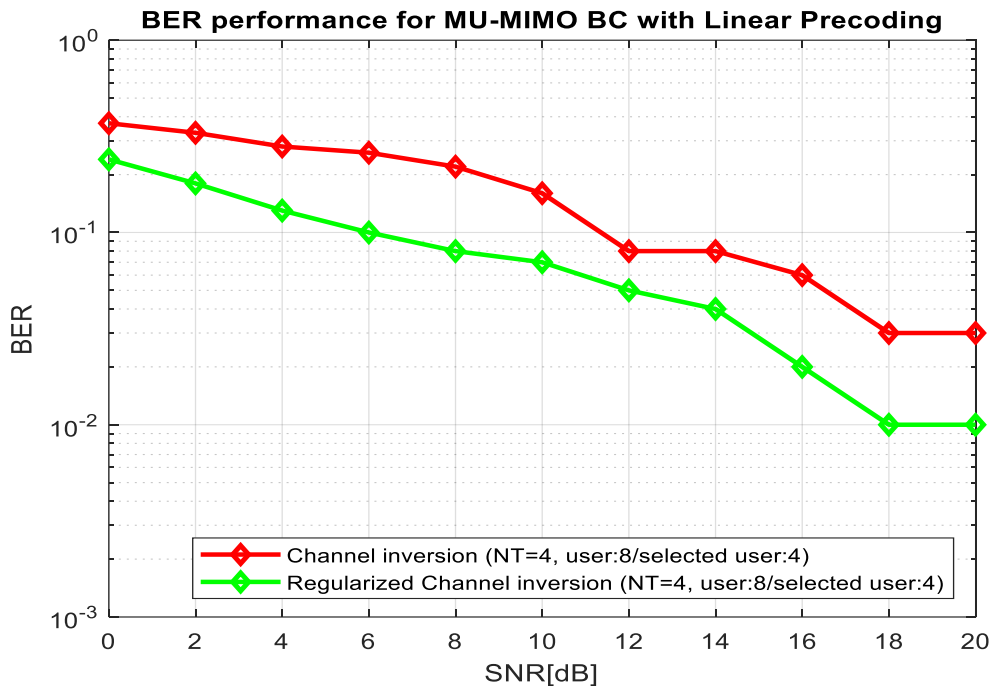


Figure 4.3 BER comparison of RCI AND CI method

Block diagonalization is for multiuser MIMO system are the best way of interference reduction for the special way since it considers receive antennas having more than a single antenna.i.e., $NM > 1$ .

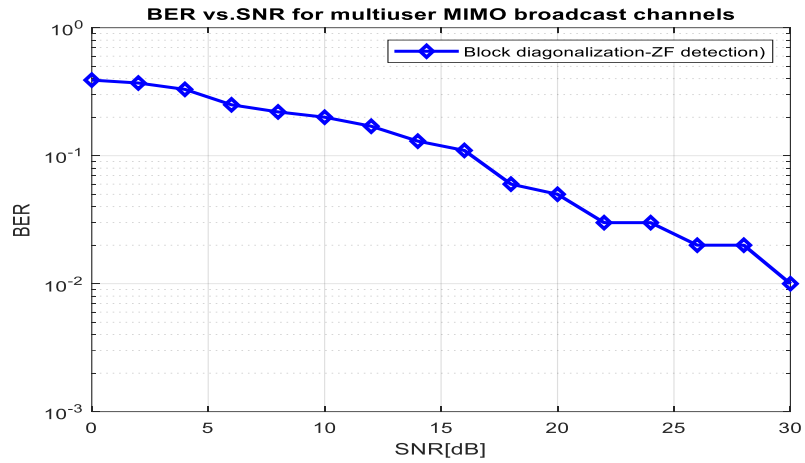


Figure 4.4 BER performance of block diagonalization method

System specifications for block diagonalization precoding under multiuser MIMO with number of antennas are greater than one per user is given by:

Table 4. 2 Simulation parameters for BER performance for MU-MIMO under BD precoding

Number of frames/packets	10
Number of packets	100
Number of bits per QPSK	2
Number of transmit antenna	4
Number of receive antenna	4
Number of users	2
Number of bits per packet	80
Number of total bits	8000
SNRdBs	[0:2:20] dB
Channels	Rician, Rayleigh and deterministic channels.

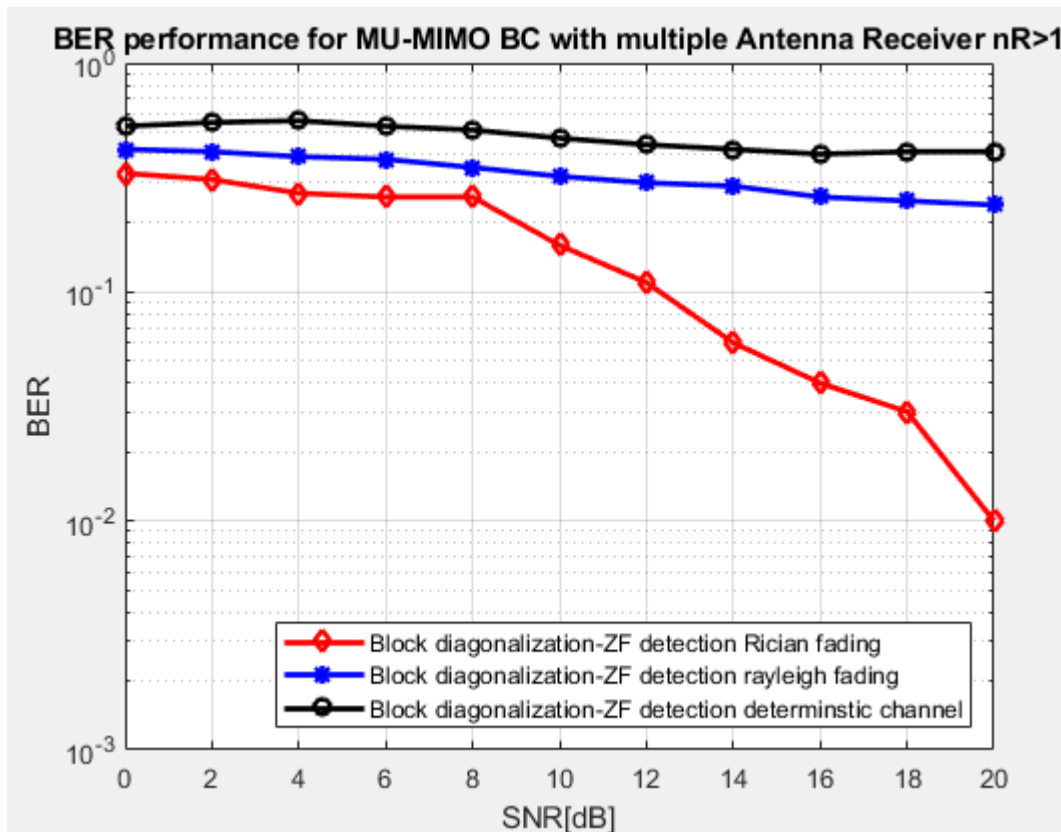
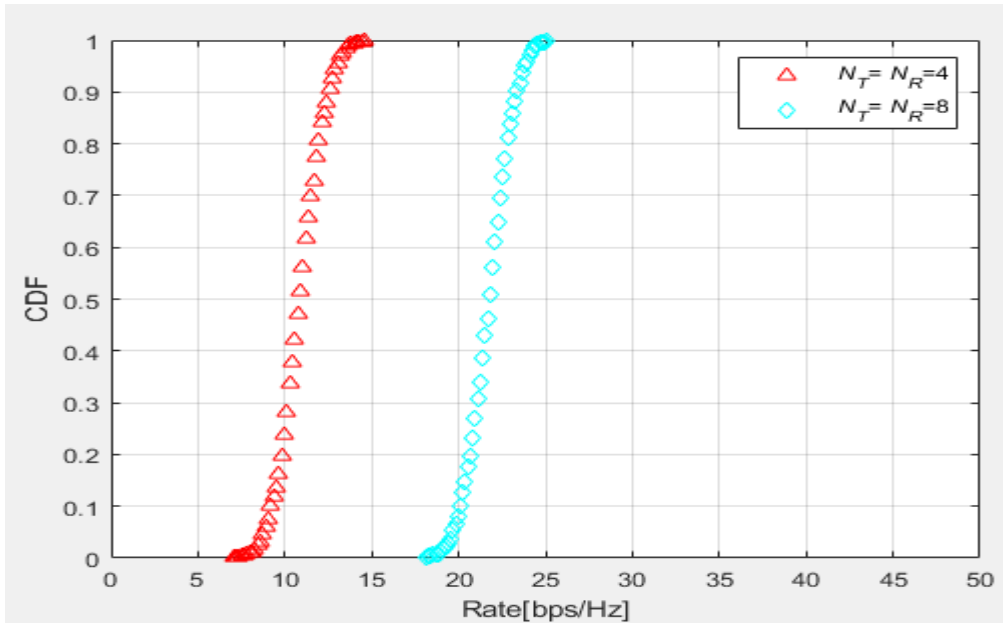


Figure 4. 5 BER performance of block diagonalization method b Rayleigh, Rician and Deterministic Channels.

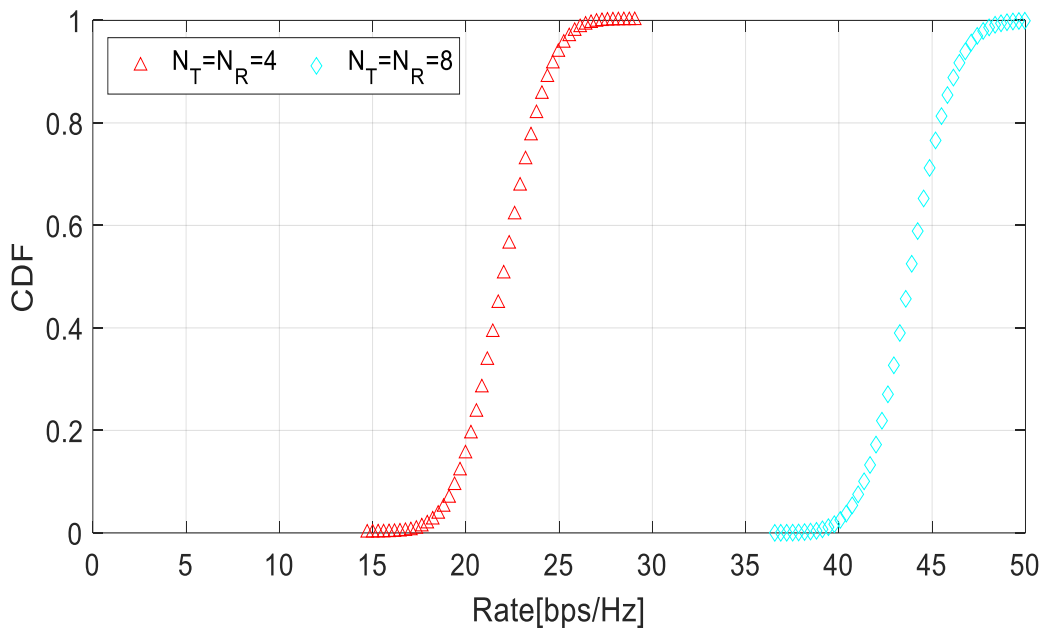
When the transmitter has no the states of the channel cumulative distribution function (CDF) of the capacity of the MIMO channel is produced as shown in figure 4.6.

Table 4. 3 Simulation parameters for MIMO with different antennas

Number of transmit antenna (NT)	4/8
Number of receive antenna (NR)	4/8
Function	Probability density function
SNR [dB]	10/20[dB]
CDF coefficients	0:0.1:1



a. SNR=10 dB



b. SNR=20 dB

Figure 4.6 The CDF of MIMO channel capacity

From the figure 4.6, it is clearly indicating the MIMO channel capacity improved with increasing the number of transmit and receive antenna and even influenced with the variation in the ratio of signal power to noise power. Capacity increases with increase in SNR values. The figure 4.7 shows spectral efficiency versus SNR at different transmitter and receiver antenna configuration.

Table 4. 4 Simulation parameters for spectral efficiency of MIMO systems

Number of transmit antenna	Maximum of 4
Number of receive antenna	Maximum of 4
SNR_dB	-10:5:20
Signal power(B)	1
Noise power(N0)	1e-4
Transmit power	$N_0 * SNR$
Spectral efficiency	pdf of elements in matrix $\lambda$ in SVD decomposition of matrix H.

### Spectral efficiency vs. SNR

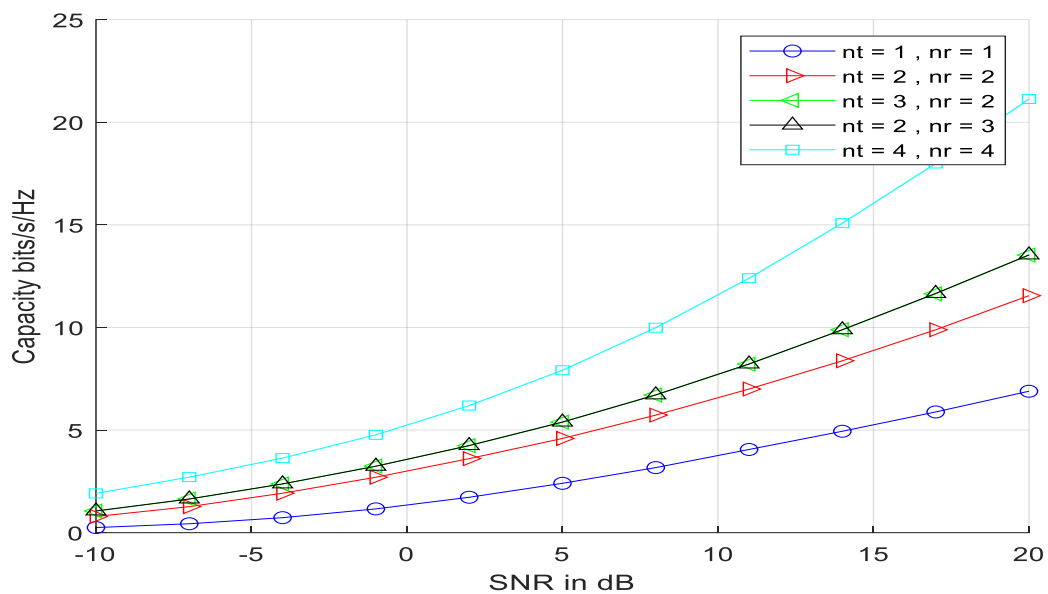


Figure 4.7 Capacity of MIMO channel for different antenna configuration

The spectral efficiency is the rate at a given transmission bandwidth. The spectral efficiency of the mimo channel is increased with in the increasement of the transmit and receive antenna and the SNR values too.

System specifications for capacity of the different linear precoding with different number of receive antenna configurations.

Table 4. 5 Simulation parameters capacity of the different precoding transmissions

Number of transmit antenna	4
Number of receive antenna	4(1,1,1,1) configurations
SNR	10dB
Outage probability	0.11 or 11%
Transmission system	Channel inversion, regularized channel inversion, block diagonalization, blind transmitter, single user.

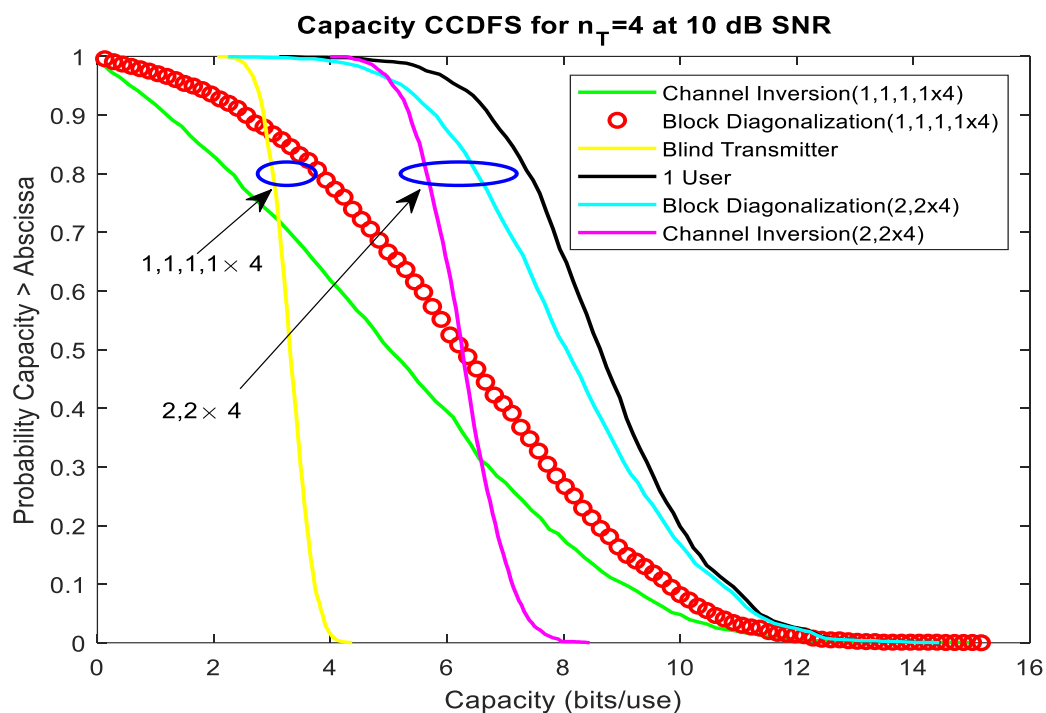


Figure 4.8 Complementary cumulative distribution functions of sum capacity for Gaussian channels for four transmitters.

Figure 4.8 shows the probability distribution function of the sum capacity of the three receiver antenna configurations with the at a given transmit antennas, namely the  $\{1,1,1,1\} \times 4$ ,  $\{2,2\} \times 4$ , and single-user  $4 \times 4$  channels.

The assumption of signal to noise ratio (SNR) is 10dB, and all the channels are assumed to be independent and identically distributed (IID) Gaussian.

It is interesting to note in Fig. 4.8 that at low outage probabilities, the case where each receiver has only one antenna produces better results when channel knowledge is not assumed and the users are simply time multiplexed. For the case of two antennas at each receiver, the

average capacity gain derived from exploiting channel knowledge using the BD algorithm is around 30%. Note that BD outperforms channel inversion at all outage probabilities.

The figure 4.9 shows the capacity vs. SNR at different transmitter shown in legend at outage( $R > C$ ) of 0.11(11%).

Table 4. 6 Simulation parameters for Capacity as a function of SNR with different receiver antenna configuration.

Number of transmit antenna	4
Number of receive antenna	4(1,1,1,1) or 4(2,2)
SNR	0:2:18[dB]
Transmission method	Channel inversion, regularized channel inversion, block diagonalization, blind transmitter, 1 user.

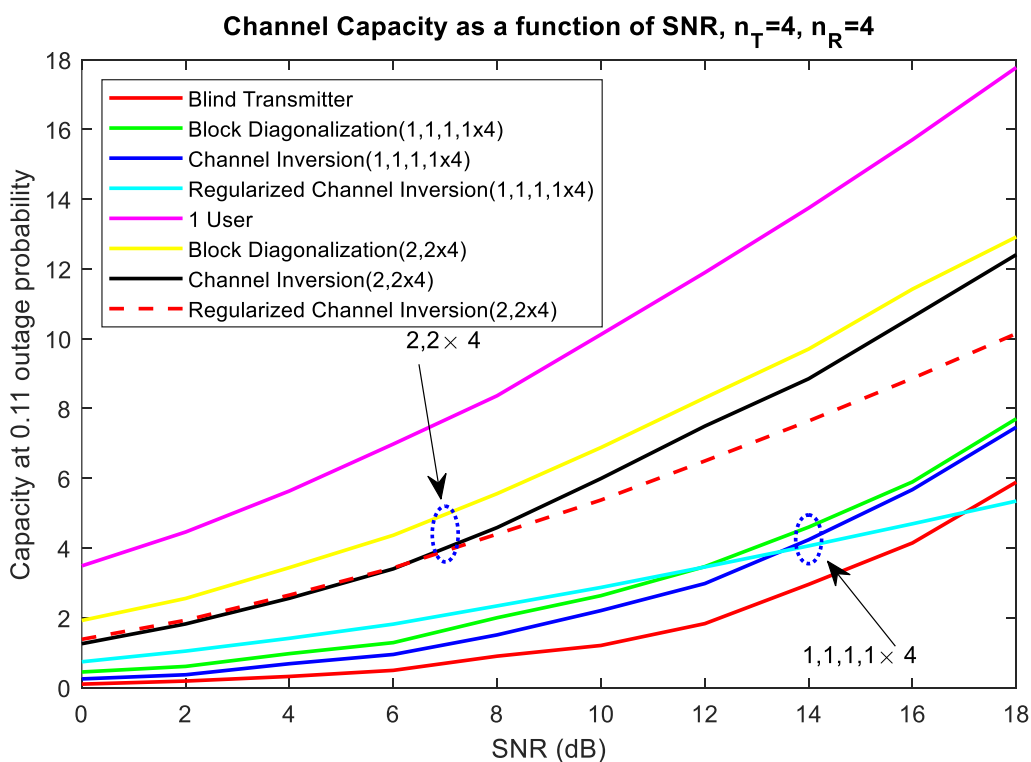


Figure 4.9 Capacity as a function of SNR with different receiver antenna configuration.

Figure 4.9 shows the different receiver antenna configuration of a different transmitter and with some outage probability. Thus, the configurations having two antenna per receiver shows some improvement in the capacity of the single antenna receivers.

From the figure 4.9 the capacity of a single user is better than the remaining configurations because of the receivers a single receiver is utilizing the shared resource from the given transmitter.

The figure 4.10 shows the capacity vs. the number of transmit antenna at different transmitter shown in legend.

Table 4. 7 Simulation parameters channel capacity as a function of transmitter antenna

Number of transmit antenna	4
Number of receive antenna	4[1,1,1,1] or 4 [2,2]
SNR	20 dB
Transmission method	Channel inversion, regularized channel inversion, block diagonalization, blind transmitter, 1 user.

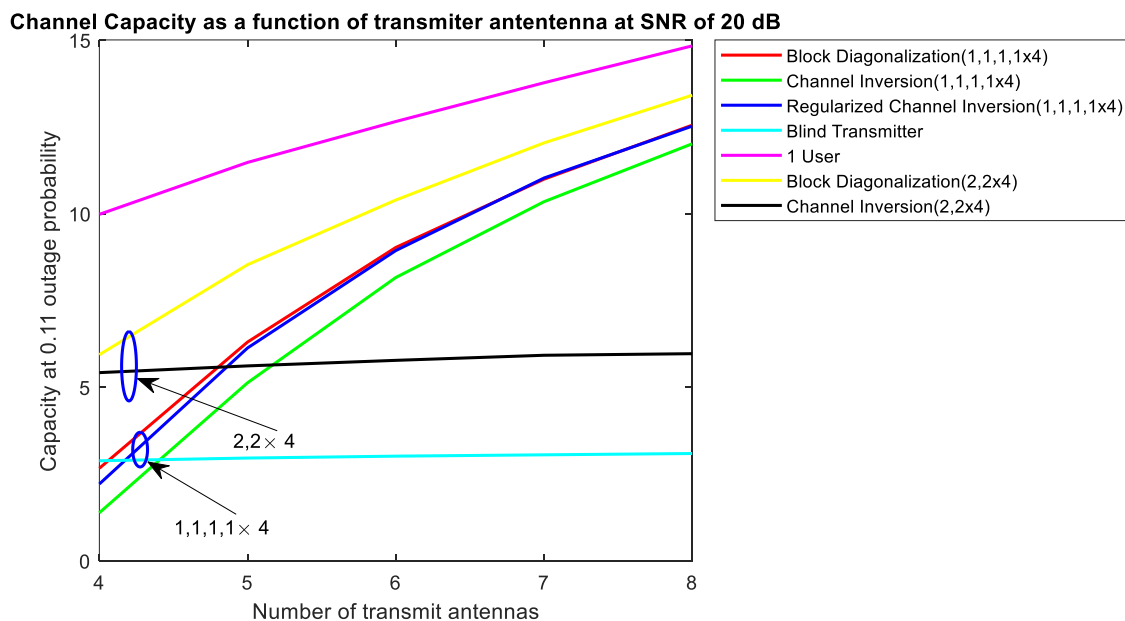


Figure 4.10 Capacity as a function of transmitter array size at a SNR of 20 dB.

Figure 4.10 shows the capacity as a function of the transmitter array size with the outage probability fixed at 0.11. The capacity gains of the BD algorithm are quite sizable here, up to a factor of 4 for the {1,1,1,1} channel, and a factor of 2 for the {2,2}x4 channel. This is due to the ability of the BD algorithms to optimally use the excess degrees of freedom available at the transmitter.

The result in figure 4.11 shows that capacity with different receiver configuration and with receiver antenna element correlation.

Table 4. 8 Simulation parameters for channel capacity as a function of channel correlation

Number of transmit antenna	4
Number of receive antenna	4[1,1,1,1] or 4[2,2]
SNR	10 dB
Receive antenna element correlation	0:0.1:1
Transmission type	Block-diagonalization, channel inversion, regularized channel inversion, blind transmitter, 1 user.

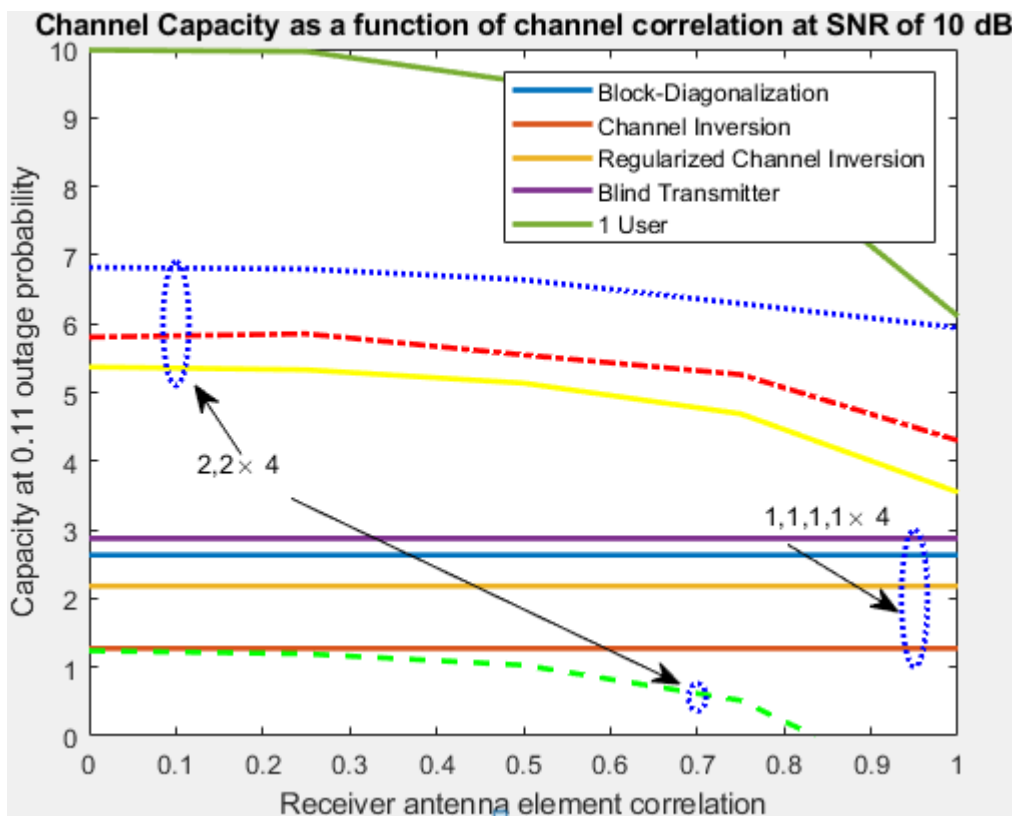


Figure 4.11 Capacity as a function of channel correlation between Rx antennas at an SNR of 10 dB.

From Figure 4.11 the performance of channel correlation with respect to the effect of correlated receive antenna but independently of the transmit antenna. The correlation of the channel arises because of the mobile stations having closely spaced antenna while the base station has significantly separated antennas.

The different users assumed to be uncorrelated and the channel inversion algorithm has two curves because the channel matrix for each user is independent, resulting in a completely independent channel  $H_j$  for  $\{1,1,1,1\} \times 4$  case and partially correlated  $H_s$  matrix for  $\{2,2\} \times 4$  case with the assumption of no correlation. As the channel becomes completely correlated, the capacity of the BD solution decreases slightly but less than the other algorithms.

This figure 4.12 shows rate of block diagonalization for 2 user case and their maximum sums achieved at given SNR and Blind Tx for comparison.

Table 4. 9 Simulation parameters rate regions at different SNR

Number of transmit antenna	4
Number of receive antenna	4(2,2)
Number of users	2
SNR	[5,10,15] dB
Transmission method	Block Diagonalization, Blind transmitters
Attenuation	No

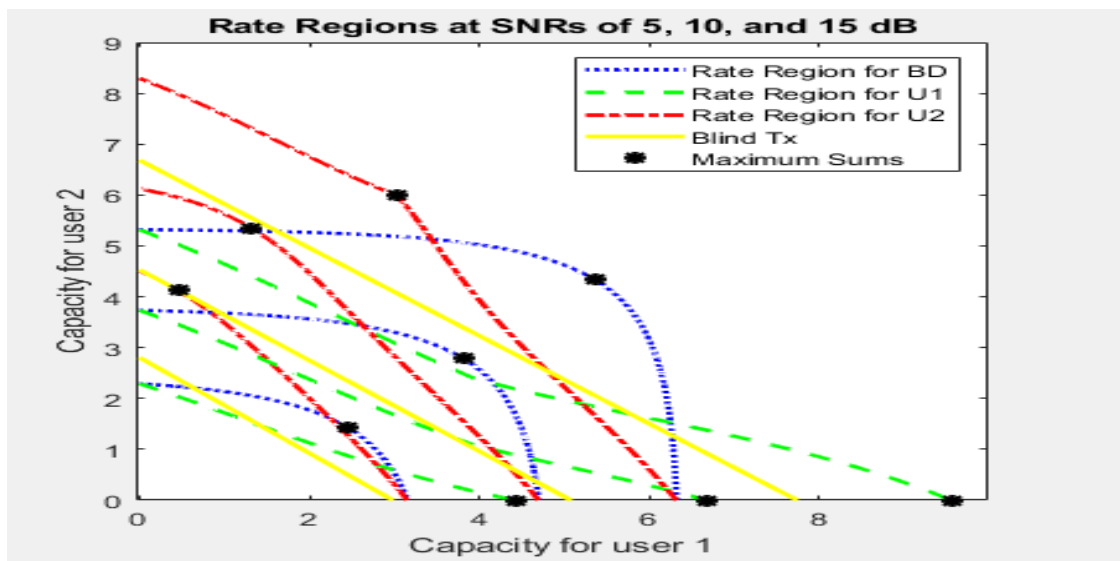


Figure 4.12 Rate regions for a randomly generated  $H$  of dimension  $\{2,2\} \times 4$  at various power constraints.

The figure 4.12 shows the two-dimensional rate region for a randomly chosen  $H$  matrix, and two users with two antennas of each.

The block diagonalization rate regions are derived by equally dividing the powers among the users and leading the power loading coefficients with ‘local’ water filling.

From the curves shown in the figure 4.12 an additional line is a blind transmitter means the channel is unknown at the transmitter which is set for comparison. The three curves are shown in figure 4.12, for SNR of 5dB, 10dB and 15 dB, respectively. The Asterix (\*) is used to indicate the maximum sum capacity on the point of each curve. The highest sum capacity in block diagonalization (BD) is in the outermost which is 15dB than the inner most 5dB.

In the figure 4.13, the assumption is for 2 user case with block diagonalization and their maximum sums achieved at given SNR with the presence of attenuation of 10 dB.

Table 4. 10 Simulation parameters rate regions at different SNR with attenuation

Number of transmit antenna	4
Number of receive antenna	4(2,2)
Number of users	2
Channel type	Rayleigh fading channel
SNR	[5,10,15] dB
Transmission method	Block Diagonalization, Blind transmitters
Attenuation	10dB

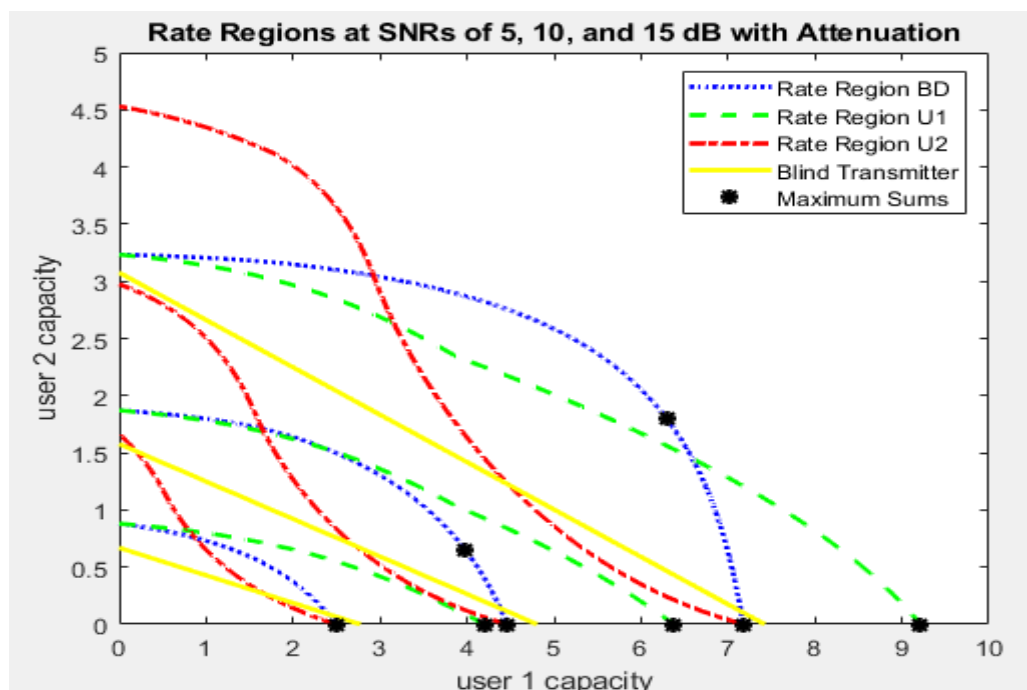


Figure 4.13 Rate regions for a “Near–Far” H of dimension  $\{2,2\} \times 4$  with 10 dB

The figure 4.13 shows the two-dimensional rate region for a randomly chosen  $H$  matrix, and two users with two antennas of each. In this figure the channel of user 2 is attenuated by 10 dB and thus creates the near far problem.

The block diagonalization rate regions are derived by equally dividing the powers among the users and leading the power loading coefficients with ‘local’ water filling.

The three curves are shown in figure 4.13, for SNR of 5dB, 10dB and 15 dB, respectively. The Asterix (\*) is used to indicate the maximum sum capacity on the point of each curve. The highest sum capacity in block diagonalization (BD) is in the outermost which is 15dB than the inner most 5dB.

## Chapter 5

### 5. Conclusion and Recommendation

#### 5.1 Conclusion

This thesis emphasizes the summary of the performance analysis of linear precoding for multiuser MIMO downlink systems. Depending on the given precoding strategies the performances of multiuser MIMO standards are compared in different metrics. However, Block Diagonalization precoding techniques are the best method of interference reduction in multiuser MIMO systems.

The use of linear precoding in Multiuser system are the best mechanism for interference reduction or nulling with the assumption of reduction of number of antennas with the higher generations. Furthermore, the use of precoding at the transmitter side makes the interference reduction easier because of the difficulties of interference nulling at the end devices or users.

Block diagonalization precoding have a better bit error rate and better channel capacity under Rician fading than Rayleigh fading and deterministic channels under the same number of total transmit and receive antennas.

Under the assumption of TX Antenna channel capacity is better when the receiver is having multiple antennas than the single antennas because of the combination of different antenna signals give a better performance than the single antenna reception.

## **5.2 Recommendation and Future Work**

This topic can be a hot research idea with the 5G and beyond networks because of a lot of users requires higher data rate and capacity. Besides, the capacity of the access point or base stations the interference are one issue. Another future research areas in this paper are investigation of interference mitigation at the end user side.

The results addressed in this thesis are based on the simulation results in which the number of antennas at the transmitting and receiving end are assumed to be equal with different configurations. The channel state information at both ends is assumed to be known and such kinds of assumptions which is stated above are a challenging task in real condition and interestingly solving such kind of problems need to be addressed.

All of the work in this paper is based on single cell multiuser phenomenon. As a result, another possible experiments like multicell multiuser MIMO systems taken as an account.

## Bibliography

- [1]. I. E. Telatar, Capacity of multi-antenna Gaussian channels, ‖ *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585–596, Nov. 1999.
- [2]. G. J. Foschini and M. J. Gans, —On limits of wireless communications in a fading environment when using multiple antennas, ‖ *Wireless Pers. Commun.*, vol. 6, pp. 311–355, Mar.1998.
- [3]. Tsoulos, G. (Ed.). (2018). *MIMO system technology for wireless communications*. CRC press.
- [4]. Professor R. W. Heath, *MIMO-communication*, <http://www.profheath.org/mimo-communication/multiple-user-mimo/>. [Online](November, 2014).
- [5]. G. Caire and S. Shamai, —On the achievable throughput of a multiantenna Gaussian broadcast channel, ‖ *IEEE Trans. on Inf. Theory.*, vol. 49, no. 7, pp. 1691–1706, July 2003.
- [6]. H. Sato, —An outer bound to the capacity region of broadcast channels, ‖ *IEEE Trans. on Inf. Theory*, vol. IT-24, pp. 374–377, May 1978.
- [7]. Knopp, R., & Humblet, P. A. (1995, June). Information capacity and power control in single-cell multiuser communications. In *Proceedings IEEE International Conference on Communications ICC'95* (Vol. 1, pp. 331-335). IEEE.
- [8]. Tsoulos, G. (Ed.). (2018). *MIMO system technology for wireless communications*. CRC press.
- [9]. V. Stankovic, Multi-user MIMO wireless communications, November, 2006.
- [10]. T. Yoo and A. Goldsmith, —On the optimality of multi-antenna broadcast scheduling using zero-forcing beamforming, ‖ *IEEE JSAC*, vol. 24, no. 3, pp. 528–541, March 2006.
- [11]. Malviya, L., Panigrahi, R. K., & Karthikeyan, M. V. (2020). *MIMO Antennas for Wireless Communication: Theory and Design*. CRC Press.
- [12]. X. Wu, D. Liu, and Z. Zheng, “Zero-forcing Precoding for the MU-MISO Downlink: How Many Antennas Do We Need?,” *IEEE Int. Symp. Pers. Indoor Mob. Radio Commun. PIMRC*, vol. 2018-September, pp. 1433–1439, 2018, doi:

- 10.1109/PIMRC.2018.8580814.
- [13]. H. Tataria et al., “Channel Correlation Diversity in MU-MIMO Systems - Analysis and Measurements,” *IEEE Int. Symp. Pers. Indoor Mob. Radio Commun. PIMRC*, vol. 2019-September, 2019, doi: 10.1109/PIMRC.2019.8904875.
- [14]. A. Salem and C. Masouros, “On the Finite Constellation Sum Rates for ZF and CI Precoding,” *IEEE Wirel. Commun. Netw. Conf. WCNC*, vol. 2019-April, pp. 1–6, 2019, doi: 10.1109/WCNC.2019.8885637.
- [15]. F. Endebu, “school of graduate studies Antenna Spacing Effects on Indoor MU-MIMO Channel Capacity Antenna Spacing Effects on Indoor MU-MIMO Channel Capacity,” 2016.
- [16]. N. Suleyman, “Investigating of SU-MIMO techniques for LTE- Release 8 in highly scattering environment Investigating of SU-MIMO techniques for LTE- Release 8 in highly scattering environment,” 2017.
- [17]. Mohaisen, M., & Chang, K. (2009, September). On transmit antenna selection for multiuser MIMO systems with dirty paper coding. In 2009 IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications (pp. 3074-3078). IEEE.
- [18]. E. A. Jorswieck and H. Boche, —Performance analysis of capacity of MIMO systems under multiuser interference based on worst-case noise behavior, || *EURASIP J. Wireless Commun. Networking*, vol. 2004, no. 2, pp. 273–285, Feb. 2004.
- [19]. M. F. Demirkol and M. A. Ingram, —Power-controlled capacity for interfering MIMO links, || in *Proc. IEEE Veh. Technol. Conf.*, Oct. 2001, pp. 187–191.
- [20]. ———, —Stream control in network with interfering MIMO links, || in *Proc. IEEE Wireless Commun. Networking Conf.*, Mar. 2003, pp. 343–348.
- [21]. B. Chen and M. J. Gans, —MIMO communications in ad hoc networks, || *IEEE Trans. Signal Processing*, vol. 54, no. 7, pp. 2773–2783, July 2006.
- [22]. R. S. Blum, —MIMO capacity with interference, || *IEEE J. Select. Areas Commun.*, vol. 21, no. 5, pp. 793–801, June 2003.
- [23]. W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, —Iterative water-filling for Gaussian vector multiple-access channels, || *IEEE Trans. Inform. Theory*, vol. 50, no. 1, pp. 145–152, Jan. 2004.
- [24]. N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, —Sum power iterative water-filling for multi-antenna Gaussian broadcast channels, || *IEEE Trans. Inform.*

Theory, vol. 51, no. 4, pp. 1570–1580, Apr. 2005.

[25]. S. Ye and R. S. Blum, optimized signalling for MIMO interference systems with feedback, || IEEE Trans. Signal Processing, vol. 51, no. 11, pp. 2839–2848, Nov. 2003.

[26]. M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms*, 3rd ed. New York, NY: John Wiley & Sons Inc., 2006.

[27]. A. Doneriya, M. Panchal, and J. Di. Lal, “Performance Analysis of Linear Precoding Techniques over the Fading Channel for MU-MIMO,” 2018 Int. Conf. Adv. Comput. Telecommun. ICACAT 2018, pp. 1–6, 2018, doi: 10.1109/ICACAT.2018.8933697.

[28]. Malviya, L., Panigrahi, R. K., & Kartikeyan, M. V. (2020). *MIMO Antennas for Wireless Communication: Theory and Design*. CRC Press.

[29]. Khatib, M. (Ed.). (2011). *Advanced Trends in Wireless Communications*. BoD–Books on Demand.

[30]. Rao, K. D. (2015). *Channel coding techniques for wireless communications* (pp. 1-20). Berlin, Germany: Springer India.

[31]. Brown, T., Kyritsi, P. and De Carvalho, E., 2012. *Practical guide to MIMO radio channel: With MATLAB examples*. John Wiley & Sons.

[32]. Honig, M.L., 2009. *Advances in multiuser detection* (Vol. 99). John Wiley & Sons.

[33]. *spatial multiplexing and channel modeling*

[34]. Oestges, C., & Clerckx, B. (2010). *MIMO wireless communications: from real-world propagation to space-time code design*. Academic Press.

[35]. Rao, K. D. (2015). *Channel coding techniques for wireless communications* (pp. 1-20). Berlin, Germany: Springer India.

[36]. A. Paulraj, R. Nabar, and D. Gore. *Introduction to Space-Time Wireless Communications*, Cambridge, U.K., Cambridge University Press.

[37]. 3GPP TR 25.876. 2005. Technical Specification Group Radio Access Network; Multiple-Input Multiple Output in UTRA, Release 7, Version 1.8.0, October 2005.

[38]. Zid, M.B. ed., 2013. *Recent trends in multi-user MIMO communications*. BoD–Books on Demand.

- [39]. Yong Soo Cho, Jaekwon Kim, Won Young Yang, Chung G. Kang (2010). *MIMO OFDM WIRELESS COMMUNICATIONS WITH MATLAB*. Singapore: John Wiley & Sons (Asia) Pte Ltd. Pages: 71-75, 278-279, 281-289, 294-297, 319-322, 327, 328, 373-382, 395-416
- [40]. A. B. Gershman, N. D. Sidiropoulos (2005). *Space-Time Processing for MIMO Communications*. England: John Wiley & Sons Ltd. Pages: 83, 84, 217-219.
- [41]. Sanguenetti, L., Bjornson, E., Debbah, M., & Moustakas, A. L. (2014). Optimal linear precoding in multiuser MIMO systems: A large system analysis. In Proceedings of IEEE Global communications conference (GLOBECOM) (pp. 3922–3927).
- [42]. Joham, M., Utschick, W., & Nossek, J. A. (2005). Linear transmit processing in MIMO communications systems. *IEEE Transactions on Signal Processing*, 53(8), 2700–2712.
- [43]. Lai-U Choi ; Murch, R.D. (2004). A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach. *IEEE Transactions on Wireless Communications*. Volume: 3 (Issue: 1), Pages 20 - 24.
- [44]. Yong Soo Cho, Jaekwon Kim, Won Young Yang, Chung G. Kang (2010). *MIMO OFDM WIRELESS COMMUNICATIONS WITH MATLAB*. Singapore: John Wiley & Sons (Asia) Pte Ltd. Pages: 71-75, 278-279, 281-289, 294-297, 319-322, 327, 328, 373-382, 395-416.