ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

COLLEGE OF EDUCATION AND BEHAVIORAL STUDIES

DEPARTMENT OF MATHEMATICS EDUCATION

AWI ZONE PREPARATORY SECONDARY SCHOOL STUDENTS' MISCONCEPTIONS ON THE CONCEPT OF LIMIT, CONTINUTY AND DIFFERENTIABLITY

By

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A Thesis

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Advisor: - Sileshi Yitebarek (PhD)

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Master of Education (MaEd) Thesis

This thesis has been evaluated and approved by the examining board consisted of:

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ACRONYMS AND ABBREVIATIONS

1) **MOE**: Minster of Education.
2) **ETP:** Education and Training Policy.
3) **NAEA**: National Agency for Examinations and Assessment.
4) **AAU**: Addis Ababa University.
5) **PSS:** preparatory secondary school.
6) **FNLA**: First National Learning Assessment.
7) **WPT**: Workout Problem Test.

ABSTRACT

The main purpose of this study was to investigate both the type and extent of Awi zone preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability. The research approach in the study was employed qualitative – quantitative approach which is mixed approach with descriptive survey design. Workout Problem Test (WPT) and structure interview were the data gathering instruments used for this study. Simple random sampling and purposive sampling techniques were used to select the data sources of the study for WPT and interviewing respectively. One-hundred eighty six (186) grade 12 students were involved in the study as a source of evidence and source of data. Accordingly, the result of the study indicates that students have misconceptions by considering existence of limit at a point as a sufficient condition for a function to be continuous at that point and there was a misunderstanding between limit, continuity, and differentiability due to the confusion of their relations, and confusion between limit and the function value. Furthermore, the findings shown that, students have misconception by perceiving a function has to be continuous and defined at a point to have a limit at that point, they thought that if the function f has a limit at a point, and then it must be continuous and defined at that point and they thought that a limit can be found by a method of substitution were the major types of misconceptions that students have formed . Majority of students in Awi zone have misunderstanding and confusions about the conditions of continuity (59%) and about differentiability (61%). A little less than half of the students (49%) confused on the concepts of the existence of limit of a function at a point and continuity of a function at that point.

CHAPTER ONE

1. INTRODUCTION

1.1. Background of the study

According to MOE (1994:1), education is defined as a process by which man transmits his experiences, new findings, and values accumulated over the years, in his struggle for survival and development, through generations. It enables individuals and society to make all-rounded participation in the development process by acquiring knowledge, ability, skills and attitudes.

Calculus which is a branch of mathematics that deals with change and rate of change is also a source of several difficulties to beginner students. Many researchers identified and indicated the sources of difficulties and students misconceptions in calculus. For example in his thesis Jordaan (2005) cited White & Mitchelmore as having concern about the growing number of students taking calculus while still having confusion about several concepts.

So far, an important role that calculus plays is acknowledged by most mathematics educators but it is a fact that the inherent nature of calculus is quite complex. Most students find it difficult to understand this idea. As the researcher read from different literature, for many years many researchers have been seeking for answers to why students have misconceptions and what they can do to help them develop mathematically correct conceptions but the amazing thing is that still there is students' misconceptions on the concept of mathematics particularly in calculus.

Different researchers (Tall, 1992; Jordaan, 2005; Meel, 1998 & Lauten et al., 1994) studied students' difficulties related in particular the concepts of limit, continuity and differentiability. These researchers well documented students' difficulties in learning these concepts. For example; Jordaan (2005:2) quoted from "**The History of Limits"** that:

> ". . . *Idea of "limit" is the most fundamental concept of calculus. Every major concept of calculus namely derivative, continuity, integral, convergence or divergence is defined in terms of limits. In fact, "limit" is what distinguishes at the*

most basic level what we call calculus, as the mathematics of change, variation, related rates and limits, from the other branches of mathematics, algebra, geometry and trigonometry".

Furthermore, Jordaan (2005:3) viewed that limit concept has long been considered fundamental to understanding of calculus and real analysis. He also mentioned that, many studies have confirmed that a complete understanding of the limit concept among students is comparatively rare. The areas of difficulties encountered by students in dealing with other concepts (continuity, differentiability) are related to their difficulties with limits. Jordaan generalized, as there is general agreement in the literature that students have trouble with the idea of limit, whether it is in the context of functions and continuity or of sequences and series (Davis & Vinner, 1986; Tall, 1992; Monaghan, 1991; Bezuidenhout, 2001; Williams, 1991).

1.2. Statement of the problem

Ethiopia has carried out three national learning assessments at the first and second cycles of primary education since 2000 (1993E.C). But, the First National Learning Assessment (FNLA) at secondary and the preparatory level (Grade 10 and 12, respectively) has just been conducted in 2010. The general findings by NAEA (2010:viii-ix) on overall achievement scores measured by the mean score of the five subjects namely English, Mathematics, Biology, Chemistry and Physics was found less than the 50% achievement level set by the Education and Training Policy (ETP). The national mean score was only 47.8 % for Grade 12.

In addition to this , the national mathematics achievement of grade 12 students at the end of the 2009/2010 academic year (2002 E.C) of Awi zone preparatory secondary schools namely , Anekesha, Dangela, and Injibara were also found less than 50% with compare to other subjects as the following .

Table 1: Grade 12 students' national achievement in four subjects at the three sample preparatory secondary schools in 2009/2010 academic year.

No	PSS	Subject	Average (%)
1	Ankesha	Mathematics	46.4
		Biology	65.5
		Physics	68.4
		chemistry	72.2
2	Dangela	Mathematics	36.5
		Biology	61.58
		Physics	43.59
		chemistry	53.56
3	Injibara	Mathematics	38.27
		Biology	64.74
		Physics	46.54
		chemistry	56.27

As observed from table 1, the average score (the average of what the students scored in mathematics subject was 38.27% for Injibara, 36. 5% for Dangela and 46.4% for Ankesha (Educational bureau of Awi zone). In general, from the preceding statistical data, one can understand, in these preparatory secondary schools of Awi zone, there is low national mathematics achievement with compare to other subjects. This indicates that students have a problem in mathematics subject.

From my little experience, many students have difficulties on the concept of limit, continuity and differentiability. Even when I taught at Enjibra preparatory secondary schools during summer time for some months, I observed students' difficulties on limit, continuity and differentiability. In addition to this, students are observed to be faced with lots of misconceptions on the concept of limit, continuity and differentiability when they are subjected to certain problems or exercises based on limit, continuity and differentiability concepts.

However, grade 12 mathematics curriculum is designed to enable all students to function successfully, to give students a basis to increase their mathematical knowledge and improve their quality of life, and to develop an understanding and appreciation of mathematics. But from the above information, the students did not achieve the expected objectives of ETP in these specific schools.

As discussed in the background of the study, there are also lots of misconceptions by students on the concept of limit, continuity and differentiability. So, the mathematics teachers of Awi zone preparatory secondary schools must give a great attention and full insights to these series problems. Really, I put myself under questions what types and extent of students' misconceptions do exist in these particular schools? And all the above discussions encourage me to undergo this study on one branches of mathematics so called calculus particularly on the concept of limit, continuity and differentiability in Awi Zone preparatory secondary schools.

I hope, for me it is an appropriate time to investigate students' misconceptions in Awi zone preparatory secondary schools on the concept of limit, continuity and differentiability which are really observed by students. So, by integrating the different research findings on students' misconceptions on the concept of limit ,continuity and differentiability with national achievement and Awi zone mathematics achievement level , the researcher encouraged to conduct this study.

1.2.1. Objectives of the study

The overall aim of this research is to investigate Awi zone preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability. To this end, the study has addressed the following specific objectives:

1) To identify the types of Awi zone preparatory secondary school students'

misconceptions on the concept of limit, continuity and differentiability.

2) To determine the extent of Awi zone preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability.

1.2.2. Research questions

In line with the above objectives of the study, the following basic research questions are formulated:

1) What types of misconceptions do have Awi zone preparatory secondary school students on the concept of limit, continuity and differentiability?
2) To what extent Awi zone preparatory secondary school students have misconceptions on the concept of limit, continuity and differentiability?

1.3. Significance of the Study

The identification of the Awi zone preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability will have the value in the following ways:

1) It can create worthwhile opportunities to enhance teaching - learning process on the concept of limit, continuity and differentiability at preparatory secondary school level.
2) It might lead to a better understanding of the students' thought process on the concept of limit, continuity and differentiability and the quality of learning that takes place.
3) It can be used for grade 12 mathematics teachers for planning more effective teaching strategies and methods to teach the concept of limit, continuity and differentiability.
4) It can be used as an initial point for further research to study the root causes and remedy of preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability.

1.4. Delimitation of the study

Due to economy and time constraints ,this study is delimited to grade 12 students only who had learned the concept of limit, continuity and differentiability in 2010/2011 (2003 E.C) academic year in Awi zone preparatory secondary schools and focus only on the concept of limit, continuity and differentiability.

1.5. Limitations of the Study

- Due to absence of sufficient and relevant materials related to the study in Ethiopian context, the researcher has been forced to rely mainly on foreign sources (Internet).

1.6. Definitions of important terms

The following terms were defined by the researcher in order to provide a clear and concise understanding as utilized in this study.

- **Concept**: an abstract or general idea inferred or derived from specific instances.
- **Conception**: the ability to form or understand mental concepts and abstractions.
- **Misconception**: is a mistaken idea or view resulting from a misunderstanding of concept of limit, continuity and differentiability.

1.7. Organization of the study

This study is divided into five chapters. In the first chapter ; the background of the study , the statement of the problem , objective of the study , research questions , the significance of the study , the delimitations and limitations of the study are clarified to enhance understanding of the readers of the study . The second chapter concentrates on reviewing related literature on what others study on the concepts of calculus. The third chapter (methodology & design) tries to describe in detail how the study was conducted. The fourth chapter (data analysis and interpretation) attempts to summarize and present the data collected from students. Finally, summary of important findings, conclusions and implications for further research are illustrated in chapter five.

CHAPTER TWO

2. REVIEW OF LITERATURE

Literature review search was done with the following core concepts of students' misconceptions on the concept of limit, continuity and differentiability which includes historical development of calculus, definitions of calculus, and students' difficulties in calculus, fundamental students' difficulties with limits and infinite processes, other difficulties in the calculus, basic concepts of calculus, definitions of misconceptions , limits of functions, continuity of function, misconceptions related to limits, continuity and limit of functions , continuity and differentiability and misconceptions and difficulties with the chain rule.

2.1. Historical development of calculus

Before going to see the concept of limit, continuity and differentiability, it is better to see some backgrounds of calculus that is who invented first calculus and when? As stated in Stewart (2008:102), historically, Isaac Newton was born on Christmas Day in1642, the year of Galileo's death. When he entered Cambridge University in 1661, Newton did not know much mathematics, but he learned quickly by reading Euclid and Descartes and by attending the lectures of Isaac Barrow. Cambridge was closed because of the plague in 1665 and 1666, and Newton returned home to reflect on what he had learned. Those two years were amazingly productive for at that time he made four of his major discoveries:

- His representation of functions as sums of infinite series, including the binomial theorem;
- His work on differential and integral calculus;
- His laws of motion and
- Law of universal gravitation; light and color.

Because of a fear of controversy and criticism, he was reluctant to publish his discoveries and it was not until 1687, at the urging of the astronomer Halley, that Newton published Principia Mathematica. In this work, the greatest scientific treatise ever written, Newton set forth his version of calculus and used it to investigate mechanics, fluid dynamics, and wave motion, and to explain the motion of planets and comets. Thus, the beginning of

calculus is founded with calculations of areas and volumes by ancient Greek scholars such as Eudoxus and Archimedes.Although aspects of the idea of a limit are implicit in their "method of exhaustion," Eudoxus and Archimedes never explicitly formulated the concept of a limit. Likewise, mathematicians such as Cavalieri, Fermat, and Barrow, the immediate precursors of Newton in the development of calculus, did not actually use limits. It was Isaac Newton, who was the first to talk explicitly about limits. He explained that the main idea behind limits is that quantities "approach nearer than by any given difference." Newton stated that the limit as the basic concept in calculus, but it was left to later mathematicians like Cauchy to clarify his ideas about limits (***ibid*).**

As cited in Wikipedia, the free encyclopedia, historically, there was much debate over whether it was Newton or Leibniz who first "invented" calculus. This argument, the Leibniz and Newton calculus controversy, involving Leibniz, who was German, and the English man Newton, led to a rift in the European mathematical community lasting over a century. Leibniz was the first to publish his investigations; however, it is well established that Newton had started his work several years prior to Leibniz and had already developed a theory of tangents by the time Leibniz became interested in the question. Much of the controversy centers on the question whether Leibniz had seen certain early manuscripts of Newton before publishing his own memoirs on the subject. Newton began his work on calculus no later than 1666, and Leibniz did not begin his work until 1673. Leibniz visited England in 1673 and again in 1676, and was shown some of Newton's unpublished writings. He also corresponded with several English scientists (as well as with Newton himself), and may have gained access to Newton's manuscripts through them. It is not known how much this may have influenced Leibniz. The initial accusations were made by students and supporters of the two great scientists at the turn of the century, but after 1711 both of them became personally involved, accusing each other of plagiarism.The priority dispute had an effect of separating English speaking mathematicians from those in the continental Europe for many years and, consequently, slowing down the development of mathematical analysis. Only in the 1820s, due to the efforts of the Analytical Society, Leibnizian analytical calculus became accepted in England. Today, the work of both Newton and Leibniz are given credit for independently developing the basics of calculus for us. It is Leibniz, however, who is credited with giving the new discipline the name it is known by today: "calculus" .Newton's name for it was "the science of fluent and fluxions". In addition to this, the work of both Newton and Leibniz is reflected in the notation used

today. For example, Newton introduced the notation $\dot{f}$ for the derivative of a function f; and Leibniz introduced the symbol $\int$ for the integral and wrote the derivative of a function y of the variable x as $\frac{dy}{dx}$ both of which are still in use.

In the same website, the author stated that before Newton and Leibniz, the word "calculus" was a general term used to refer to anybody of mathematics, but in the following years, "calculus" became a popular term for a field of mathematics based upon their insights (***History of calculus, Wikipedia, the free encyclopedia***[1]).Altogether, calculus was a great advancement which credit should be given to a number of mathematicians because without some of the other advancements made by Archimedes or other Greek mathematicians' calculus probably would not ever have been generated by the mind of Newton or Leibniz only.

From the above history, calculus is transformed from a very complicated process of different theorems to a form of calculus that we use often in our day today. Many Engineers use calculus to build numerous different things today and it is a basic for understanding of their job. Calculus is advanced so much that now we have numerous different classes just to teach the whole concept of calculus. Thus, we can conclude, without calculus the world would not be as it is today.

2.2. Definition of Calculus

According to Iverson (2002:6), calculus is based on the notion of studying any phenomenon (such as the position of a falling body) together with its rate of change, or velocity. This simple notion provides insight into a host of familiar things: the growth of trees or financial investments (whose rates of change are proportional to themselves); the vibration of a pendulum or piano string; the shape of the cables in a power line or suspension bridge; and the logarithmic scale used in music. In spite of the simplicity and ubiquity of its underlying notion, the calculus has long proven difficult to teach, largely because of the difficult notion of limits. We will defer this difficulty by first confining attention to the polynomials familiar from high-school algebra.

[1] **Available from:** http://en.wikipedia.org/wiki/History of calculus (1-19).

In an article, *the history of calculus,* the author states that*:*

> *"Calculus" as the study of change, in the same way that geometry is the study of shape and algebra is the study of operations and their application to solving equations. A course in calculus is a gateway to other, more advanced courses in mathematics devoted to the study of functions and limits, broadly called mathematical analysis.*

Thus, calculus is the study of how things change. It provides a framework for modeling systems in which there is change, and a way to deduce the predictions of such models[2]. Though some of them will eventually use calculus in their work in physics, chemistry, or economics, almost none of those people will ever need to prove anything about calculus. They are willing to trust the pure mathematicians whose job it is to certify the reliability of the theorems. Why, then, do we study epsilons and deltas, and all these other abstract concepts of proofs?[3]

The subject calculus, known historically as infinitesimal calculus, constitutes a major part of modern mathematics education. It has two major branches, differential calculus and integral calculus, which are related by the fundamental theorem of calculus**.**[4]

According to Tall et al. (2008:2), calculus is a rich subject with a varied cultural history. It serves not only as a basis of mathematical modeling and problem solving in applications, but also as a natural pinnacle of the beauty and power of mathematics for the vast majority of calculus students who take it as their final mathematics course. Calculus is only 350 years old. But in those three and one-half centuries it has been enriched by new perceptions and belief structures of successive enervations, starting with the original conceptions of Leibniz and Newton on infinitesimals and limits. Currently we have a range of modern formal methods, from intuitive dynamic approaches, through numerical, symbolic, and graphic approaches, culminating in theories ranging from formal epsilon-delta analysis, which banishes infinitesimals, to nonstandard analysis, which fully endorses them. The result is a wide range of viewpoints as to how the calculus should be conceived and taught. We therefore begin by considering some of these differing views to place our analysis in perspective.

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[2] **Available from:** http://www-math.mit.edu/indjk/calculus-beginners/ chapter 01/section OR:html
[3] **Available from:** http://www.math.vanerbilt.edu/ushectex/courses/why study.html
[4] **Available from:** History of calculus –Wikipedia, the free encyclopedia.

Tall (1992:1) emphasized that the calculus means a variety of different things in different countries in a spectrum form:

1. Informal calculus – informal ideas of rate of change and the rules of differentiation with integration as the inverse process, with calculating areas, volumes etc. as applications of integration,

2. Formal analysis – formal ideas of completeness, $\varepsilon - \delta$ definitions of limits, continuity, differentiation, Riemann integration, and formal deductions of theorems such as mean-value theorem, the fundamental theorem of calculus etc., with a variety of more recent approaches including,

3. Infinitesimal ideas based on non-standard analysis,

4. **Computer approaches** using one or more of the graphical, numerical, symbolic manipulation facilities with, or without, programming. In some countries the first of these is taught in secondary school and the second to mathematics majors in college. In others a subject somewhere along the spectrum between the two is taught as the first major college mathematics course. In a few countries (e.g. Greece), the formal ideas are taught from the beginning in secondary school. The details of these approaches, the level of rigor, the representations geometric, numeric, symbolic, using functions or independent and dependent variables), the individual topics covered, vary greatly from course to course.

So what is calculus already? Calculus is defined as follow[5];

- Calculus is an exploration of functions which are not constant.
- Calculus is dynamic; if it moves, calculus can describe it.
- Calculus is the study of infinitesimal quantities and instants.
- Calculus is the mathematics of change.
- Calculus is an integration of geometry and algebra.
- Calculus is algebra for continuous functions.
- Calculus is a form of capturing the changing world.
- Calculus is about rates of change and accumulation and how these two concepts are connected to one another.
- Calculus is not a subject because it is many subjects.

[5] **Available from**: *www.solonschools.org/accounts/.../8132009104312_Whatis**Calculus**.pdf*

Even if calculus is defined in many ways, in this study, calculus is defined as a branch of mathematics that treats the measurement of changing quantities, determining rates of change.

2.3. Application of calculus

Application of calculus in science, economics, and engineering and it is used to solve many problems for which algebra alone is insufficient. History of Calculus is part of the history of mathematics focused on limits, functions, derivatives, integrals, and infinite series. The concept of calculus is vastly important in all fields such as physics, engineering, chemistry, economics, and anything evolving change[6]. Overall, calculus is one of the most important fields in mathematics in understanding physical systems. Even if calculus is not difficult once students understand the central basics, many students have many difficulties and misconceptions in concept of calculus.

The first part, which is common to both natural science and social science streams students' is introduction to calculus where the basic concepts of calculus are introduced with intuitive definitions and examples followed by different exercises .

As the aim of preparatory program , mathematics syllabus in Ethiopia is to enable students to choose subjects or areas of training which prepare them adequately for higher education and the world of work , and introduction to calculus is a pre-requisite for most science fields of undergraduate program , it is crucial that this topic has to be taught for effective and efficient benefit of the goods of it ; for producing citizens who can involved in the production sector with advanced academic knowledge and vocational training (Ashebir, 2009:2-3).

Clearly, the most crucial courses in higher education are calculus driven. Introduction to calculus is an inseparable part of mathematics in preparatory secondary school. According to grade 12 syllabus, grade 12 students are usually expected to have a considerable knowledge and understanding of certain basic calculus concepts such as limit, continuity and differentiability. These basic concepts are frequently used in explanations and definitions of new ideas and concepts in calculus. For example, in the study of differentiability and continuity, the concepts of limit are the underlying concept. Without sufficient understanding and knowledge of these basic concepts, new concepts may be

[6]**Available from:** icsouza.wordpress.com/category/calculus.

falsely interpreted or difficult to accept by students. It is therefore important that students have the correct conception on the concepts which will act as tools or foundations in building new more complex concepts.

2.4. Students' Difficulties in Calculus

Calculus is a branch of mathematics that treats the measurement of changing quantities, determining rates of change (differential calculus) and quantities under changing conditions (integral calculus). The change (rate) is the derivative and determining volume is integration. Calculus is huge stepping stone in studying physics and chemistry. It is not difficult once we understand the central basics. The central themes in calculus are the derivative and integral.

Introduction to calculus has been introduced into the preparatory secondary school curriculum in Ethiopia. The secondary school calculus curriculum is what Tall (1992) refers to as "informal calculus", which involves "informal ideas of rate of change and the rules of differentiation with integration as the inverse process, with calculating areas, volumes etc. as application of integration."

Many students have many misconceptions about calculus concepts and its role. For example, as cited in Jordaan (2005:2-3), Cornu agrees that the mathematical concept of limit holds a central position which permeates the whole of mathematical analysis as a foundation of the theory of approximation, of continuity and of differential calculus and integral calculus. He is of the opinion that this mathematical concept is a particularly difficult idea, typical of the kind of thought required in advanced mathematics. He further mentions that one of the greatest difficulties in teaching and learning the idea of a limit does not only lie in its richness and complexities, but also in the extent to which the cognitive aspects cannot be generated purely from the mathematical definition.

Naturally, calculus is incorrectly viewed by many students as a collection of rigid rules and mysterious procedures that seem to be unrelated to each other and require total mastery with little or no understanding. Calculus is perceived by many students to be difficult and demanding and is considered to be a subject in which it is socially acceptable to do poorly. Mathematical thinking in calculus is regarded as essentially unimportant to people that do not actually "do" calculus in their employment capacity. The pervasive role of mathematics is underestimated in the world of everyday living. Not everybody is as excited as Issac

Newton was about calculus. For many students, calculus is a challenge and a gatekeeper. Successful completion of introduction to calculus at grade 12 level is considered not only a prerequisite for further study in mathematics and other subjects but also a door opening to many jobs and later opportunities.

The calculus represents the first time in which the student is confronted with the limit concept, involving calculations that are no longer performed by simple arithmetic and algebra, and infinite processes that can only be carried out by indirect arguments. Teachers often attempt to circumvent the problems by using an "informal" approach playing down the technicalities. However, whatever method is used, a general dissatisfaction with the calculus concept has emerged in various countries round the world in the last decade.

2.4.1. Fundamental students' difficulties with limits and infinite processes

Tall (1992:2) stated that whichever way the calculus is approached, there seem to be inherently difficult concepts which seem to cause problems no matter how they are taught. He concluded that limit concept creates a number of cognitive difficulties for students, including: difficulties embodied in the language; terms like "limit", "tends to", "approaches", "as small as we please" have powerful colloquial meanings that conflict with the formal concepts, the limit process is not be performed by simple arithmetic or algebra, infinite concepts arise and the whole thing becomes "surrounded in mystery", the process of " a variable getting arbitrarily small" is often interpreted as an "arbitrarily small variable quantity", implicitly suggesting infinitesimal concepts even when these are not explicitly taught, likewise, the idea of "N getting arbitrarily large", implicitly suggests conceptions of infinite numbers, students often have difficulties over whether the limit can actually be reached, There is confusion over the passage from finite to infinite, in understanding "what happens at infinity".Having considered the limit concept in some detail, he also listed some of the other difficulties students encounter in the calculus concept, each worthy of extended investigation, including:

- restricted mental images of functions,
- the Leibniz notation – a 'useful fiction' or a genuine meaning,
- difficulties in translating real-world problems into calculus formulation,
- difficulties in selecting and using appropriate representations,
- algebraic manipulation – or lack of it,
- difficulties in absorbing complex new ideas in a limited time,

- ❖ difficulties in handling quantifiers in multiply-quantified definitions,
- ❖ Consequent student preference for procedural methods rather than conceptual understanding.

He viewed and tried to explain the above difficulties as follow one by one:

Restricted mental images of functions: mental images of functions are not always seen as provoking a difficulty in elementary calculus particularly when the subject is seen as focusing on the differentiation and integration of standard functions given as formulae.

Difficulties in translating real-world problems into calculus formulation: difficulties in translating real-world problems into calculus formulation are part of the folk-lore of the subject (though there seems to be little cognitive research). Many examinations for calculus examinations focus on the symbolic manipulation rather than problem-solving.

The Leibniz notation:the Leibniz notation $\frac{dy}{dx}$ proves to be almost indispensable in the calculus. Yet it causes serious conceptual problems. Is it a fraction, or a single indivisible symbol? What is the relationship between the dx *in* $\frac{dy}{dx}$ and the dx *in* $\int f\,(x)dx$?

Can du be cancelled in the equation $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$?

Giving a modern meaning to these terms that allows a consistent meaningful interpretation for all contexts in the calculus is possible but not universally recognized. On the other hand, failing to give a satisfactory coherent meaning leads to cognitive conflict which is usually resolved by keeping the various meanings of the differential in separate compartments ($\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ in differentiation and dx means "with respect to x in integration). This can only exacerbate conceptualism between the notation and any possible coherent meaning. The very first definition of the derivative of Leibniz (1684) did not mention limits or infinitesimals. Instead, he began with the notion of tangent, and defined the derivative to be $\frac{dy}{dx}$ where dx and dy are the horizontal and vertical components of the tangent.

Difficulties in selecting and using appropriate representations: difficulties in selecting and using appropriate representations are known to be widespread. As cited in Tall (1992:5-6), Robert & Boschet (1984) reported that the students who were the most successful were invariably those who could flexibly use a variety of approaches: symbolic,

numeric, visual and others Dreyfus & Eisenberg (1986, 1991) report students' reluctance to visual concepts in calculus. They give examples where visual representations would solve certain problems almost trivially, yet students refrain from using them because the preference developed over the years is for a numerical, symbolic mode of approach. Yet research shows that visual images can provide vital insights. However, it may sometimes prove difficult for students to link the global gestalt to a sequential deductive form of thinking.

Algebraic manipulation: algebraic manipulation is the preferred mode of operation for many students. Wider accesses to calculus may have the effect of allowing students with less manipulative facility to take the calculus. In the United Kingdom, for instance, the latest school curricula require less algebraic manipulation. Students are beginning to take calculus with less facility in manipulating polynomials and with little knowledge of trigonometric formulae. As a result, routine manipulation in algebra can no longer be taken for granted.

Difficulties in absorbing complex new ideas in a limited time: difficulties in absorbing complex new ideas in a limited time occur throughout college mathematics. Yet the difficulties in calculus are often acute. The concepts change their nature as time passes. A limit might be initially an intuitive process of "getting close", then an "epsilon-delta" definition, but then after a few theorems have been proved, the definition is suppressed and the theorems are quoted instead. Thus students at different stages of coping with this transition are faced with different meanings for the concept. As cited in Tall (1992:5-6), Azcarate (1991) studied the changing concept of instantaneous speed, from a primitive ratio concept (distance or time), via an approximation idea, calculating the speed over an interval of decreasing size, to the notion of "instantaneous speed" represented as the gradient of the tangent to a time distance graph. Interviewing students before, during, and after the teaching showed students at various stages of development during the teaching.

2.5. Definitions of Misconceptions

As cited in Jordaan (2005:10-12) Olivier and Bezuidenhout agree that a teacher cannot transmit knowledge ready-made and intact to the students. They see errors and misconceptions as the natural result of people's efforts to construct their own knowledge.

These misconceptions are a mistaken idea or view resulting from a misunderstanding of

something. Misconceptions, therefore, cannot be avoided. They may however, hinder the development of more sophisticated understandings of fundamental mathematical concepts to a great extent.

Such a mistaken idea or view and misconceptions should not be treated as terrible things to be uprooted. This may confuse the learner and shake his or her confidence in his or her previous knowledge. Instead, making a mistake is best regarded as part of the process of learning.

This suggests that teachers become aware of their students' possible misconceptions. A classroom atmosphere that is tolerant of a mistakes and misconceptions ought to be created. Misconceptions should be taken into account when teaching strategies are developed. The identification of misconceptions can create worthwhile opportunities to enhance learning.

2.6. Basic concepts of calculus

2.6.1 .Limit of Sequences

In grade 12 students' text book, the concept of limit is introduced in many different mathematical contexts such as the limit of a sequence of numbers, and a limit of a function. Limit concept is also introduced in the notion of continuity of a function, discontinuity of a function, differentiability of a function and integration. In grade 12 mathematics syllabuses , under the topic of sequences ,students are expected to learn how to make predications, decisions and generalizations from given patterns. After learning the concepts of sequences of numbers , the students are expected to explain the concepts of sequences ,to compute any terms of sequences from given rule , to find out possible rules (formula) from a given terms , identify the types of sequences , draw graphs of finite sequences and apply the knowledge of sequences to solve practical and real life problems. Sequences are common examples in the study of limits. After completing the concept of limit of sequences of numbers, the students are highly expected to have a skill to find out limits of sequences of numbers. In particular, sequences that tends to a unique value when n increases indefinitely are important in the introductory part of limits of sequences of numbers. The terms of some of the sequences are tending to a unique real number L as n gets larger and larger.

Cornu in Jordaan (2005:16-17) has the opinion that the idea of a limit was introduced to resolve three types of difficulty:

- Geometric problems, for example, the calculations of area, 'exhaustion' and consideration of the nature of geometric lengths;
- The problem of the sum and rate of convergence of a series;
- The problems of differentiation that come from the relationship between two quantities that simultaneously tends to zero.

According to Tall (UD[7]:7) ,the idea of a sequence of points 'getting close' to a limit point or a sequence of graphs 'getting close' to a limiting graph both create the possibility of cognitive obstacles which cause deeply-held beliefs that are considered difficult to remediate.

In particular, the idea of a sequence of numbers tending to a limit often gives a view of a variable quantity that is 'arbitrarily small' so that the number 0.999… is conceived as 'just less than one' rather than precisely equal to one. The symbolic and visual aspects of convergence are here in conflict. While it is evident that the no term of this particular sequence of decimal approximations is ever equal to the limit, visually if one plots the points physically on a line, then they are soon indistinguishable from the limit. This can be seen more generally in a dynamic picture where a sequence of points a_1,a_2, a_3,… tend to a limit a. As they are added successively to a picture, the marked points eventually become indistinguishable from the limit a. When one focuses on the latter points, by successively removing $a_1, a_2, a_3, \ldots$ what is left reveals that after a certain stage all the later approximations are indistinguishable to our human eyes from the limit *"a"*. What is important here is to see the limit and then to see the later terms of the sequence becomes indistinguishable from this limit.

Numerically, beginning calculus students have been operating in a 'good enough' world of arithmetic. The sequence $3.1, 3.14, 3.141, 3.1415, \ldots$ tends to π in the sense that various approximations, such as $3.14, 3.1412, \frac{22}{7}$ are 'good enough' to be indistinguishable from π in a given practical context.

[7] Undated

2.6.2. Limits of functions

The limit concept is an important part of the foundations of mathematical analysis and not understanding it clearly might lead to problems when dealing with concepts such as convergence, continuity and derivatives. The limit concept is connected to many other concepts, example, continuity, discontinuity and differentiability. If an individual grasps the concept of limits, the connected concepts become easier to work with, but it is difficult for students to make sense of the concept. In fact, even many great mathematics researchers have found it hard to accurately handle limits through time.

Stewart, (2008:88) stated that limits of functions of the form $\lim_{x\to a} f(x) = L$ are also approached in manner, with an informal explanation, and then at a later stage a more formal definition may be given. Often the limit process is first introduced when differentiation is being discussed, in which case the concept of a limit may include a mental picture of a chord tending to a tangent. Other approaches to the derivative may affect the concept image via magnification factor or instantaneous velocity. As the limit of a sequence, the limit of a function is often considered as a dynamic process, where x approaches a, causing $f(x)$ to get close to L. Once again students may consider $f(x) \neq L$ as part of their concept image. On the other hand the formal concept definition may be given in an imprecise form. In the students' text, the example is discussed of $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$

Where $f(x) = \frac{x^2-1}{x-1}$ is not defined for $x = 1$.

The implication is that one does not consider the case $x = 1$.

In many calculus books, the authors defined limit of function in to two ways:

An informal definition of the limit is:

The intuitive definition of the limit of a function at point is clearly defined on the students text book on page 61, as follow :

Let $y = f(x)$ be a function defined on an interval surrounding $x_o \epsilon R$ (but f need not be defined at $x = x_o$). If f(x) gets closer and closer to a single real number L as x gets

closer and closer to (but not equal to) x_o, then we say that the limit of f(x) as x approaches x_o is L .

Symbolically, this is written as:

$$\lim_{x \to x_o} f(x) = L.$$

The formal epsilon-delta definition is as follows:

A function $f(x)$ is said to approach a limit 'L' as x approaches the value of 'a' if given any small positive 'a', it is possible to find a positive number δ such that

$|x - a| < \delta, \forall x$ Satisfying $0 < |f(x) - a| < \epsilon$.

2.6.3. Misconceptions related to limits

Tall's (1992) earliest research began with calculus and limits. This led him to the discovery of differences between mathematical theories and cognitive beliefs in many individuals. For example, the limit 'naught point nine recurring' has mathematical limit equal to one. Cognitively there is a tendency to view the concept as getting closer and closer to one, without actually reaching it **(Cited in limits, infinitesimals and Infinities)[8].**

He gives the following reason for this tendency. The primitive brain notices movement. Therefore the mental idea of a sequence of points tending to a limit is more likely to focus on the moving points than on the on fliclimit point. The idea of a limit is thus first conceived as a process and then as a concept. In the case of the limit, the process of tending to a limit is a potential process that may never reach its limit (it may not even have an explicit finite procedure to carry out the limit process). This gives rise to cognitive conflict in terms of cognitive images that act with the formal definition.

Tall & Vinner (1981:152) use the term concept image to describe the total cognitive structure (in an individual's mind) that is associated with a specific mathematical concept. This includes all the mental pictures and associated properties and processes. The concept image is built up over time through all kinds of experiences and changes as the individual meets new stimuli. Every individual has his own, unique concept image of a mathematical

[8] Available from**: http://www.warwick.ac.uk/staff/David.Tall/themes/limits-infinity.html**).

idea. The concept definition, on the other hand is quite a different matter. Regard the concept definition to be a form of words that specify that concept. Such a concept definition may be formal and given to the individual as part of a mathematical theory. It may be personal, invented by the individual to (partially) describe his concept image. A personal concept definition may be considered as part of the concept image.

2.6.4. The continuity of functions

Tall (UD[9]:4) stated that the perceptual notion of continuity is based on the idea of drawing a curve with a pencil in a stroke of the hand without taking the pencil off the paper.

According to Finney & Thomas cited in Jordaan (2005:27), continuity has become one of practical as well as theoretical importance. Continuous functions are the functions we normally use in the equations that describe numerical relations in the world around us. They are the functions we use to describe how a body moves through space or how the speed of a chemical reaction changes with time. It is important to know when continuity is called for what it entails and how to test for it .A function $y = f(x)$ that can be graphed throughout its domain with one continuous motion of the pen (that is without lifting the pen) is an example of a continuous function. A function is continuous if it is continuous at each point of its domain. The continuity test can be used to test for continuity at a point.

2.6.5. The continuity test at a point

Finney & Thomas cited in Jordaan (2005:27), stated that a function $y = f(x)$ is continuous at $x = c$ if and only if it meets all three of the following conditions:

1. $f(c)$ exists (is defined at c).

2. $\lim_{x \to c} f(x)$ $exists$ (f $has\ a\ limit\ as\ x \to c$).

3. $\lim_{x \to c} f(x) = f(c)$ (The limit equals the function value).

For any function $y = f(x)$ it is important to distinguish between continuity at $x = c$ and having a limit as $x \to c$. The limit is where the function is headed as $x \to c$.

Continuity is the property of arriving at the point where $f(x)$ has been heading when x

[9] Undated

actually gets to c. Tall &Katz (UD[10]: 7) the perceptual conception of continuity is now seen to be an embodied form of continuity. It relates to the dynamic movement of a pencil as one draws the graph to get a curve which is perceptually continuous without any gaps. Such an interpretation is often seen as an 'intuitive' form of continuity that lacks formal precision. However, such perceptual beginnings are an essential starting point for the later development of more sophisticated forms of mathematical thinking as it develops both in the individual and also in succeeding generations over the centuries.

2.6.6. Continuity and function values

There are many studies demonstrating the connections between students' conceptions of functions, and their conceptions of continuity. Some research studies have consistently shown students' difficulties in separating out the differences between considerations of continuity and function values. For instance, Bezuidenhout (2001) reported that students in his study did not consider the continuity as a function property rather they focused on the value of function at a given point. Vinner (1992), in his study, also came up with similar results. He conducted a study on college calculus students. In his study, students were given functions, which were represented either graphical or symbolic, and asked to determine whether given functions were continuous. Most of the students' responses demonstrated a common belief that "for a function to be continuous is the same as being defined and to be discontinuous is the same as being undefined at a certain point". Moreover, Lauten et al. (1994) also conducted a case study with a calculus student to understand the student's ideas of continuity and functions. Student's work on function construction tasks indicated that students had difficulties in combining ideas of continuity and functions. They reported that the student seemed to have difficulties in imagining a discontinuous function on a connected domain (Duru et al, 2010: 1520).

2.7. Students' Difficulties with the Derivative

Ferrini-Mundy and Graham cited in Hackworth (UD[11]:12-13)) studied connections students had made between the graphical and algebraic representations of a function and how these connections relate to their understanding of the limit, continuity, derivative and definite integral of a function. They stated that the students they interviewed had the most difficulty relating their graphical and formula-based understandings of a function to the

[10] undated
[11] Undated

derivative. However, their connections between procedural and conceptual knowledge of the derivative was weak since they had no idea how the derivative of a function related to the function itself or how the tangent line related to the derivative. Also, they could not relate the idea of differentiability to the idea of the smoothness of a graph.

Orton (1984) investigated elementary calculus students' basic algebra and algebraic manipulation, their understanding of rate of change and its relationship with differentiation, their use of limits, and their understanding of basic calculus' symbols using a wide range of questions which occur when calculus is being introduced. Students found eight questions difficult; four concerned with understanding integration as the limit of a sum and four concerned with understanding differentiation based on rate of change and limits. He noted that students' routine performance on differentiation items was adequate, but that little intuitive or conceptual understanding of the derivative is present. He argued that calculus should be introduced only after background work on limits and rate of change had been included within the curriculum over a period of years. Orton and Ferrini-Mundy had examined students' difficulties in understanding the derivative from a perspective that was highly symbolic. Another possibility was that students' difficulties with symbolic calculus was an expression of their inability to connect the symbolism with anything conceptual which would be naturally expressed by the notational methods they attempted to memorize. The present study investigates that possibility by examining students' conceptual understanding of rate of change before and after they study the derivative. If it turns out that their understanding of rate of change improves substantially, they probably connected their thinking, while studying the derivative, to intuitions they already had - thereby transforming both their conceptual understanding and its connections with the notational methods of calculus. If it turns out that their understanding of rate is poor both before and after studying the derivative, then it would be reasonable that they have difficulty using techniques of calculus skillfully and thoughtfully, for those techniques would have no basis in understanding (***ibid:13***).

2.8. Continuity and limit of functions

There are some research studies demonstrating students' tendency to equate the existence of a limit with continuity. One of these demonstrations appears in the study of Williams (1991). A questionnaire in this study was administered to calculus students. Then in-depth interviews were done with the participants in order to better understand the students' limit

conceptions. Results of this study revealed that students had difficulties in connecting the continuity and the existence of limits. For example, some students thought that limit was appropriate only for discontinuous functions. Thus, taking a limit of a continuous function was not possible. Consequently, for these students, continuity and existence of limits were inequitable rather than being related (Duru et al, 2010: 1521).

2.9. Continuity and differentiability

According to Tall and Vinner (1981), students had trouble distinguishing between the concepts of differentiability and continuity. For example, one student who claimed that a function was discontinuous since there was a sudden change in slope. Meel (1998) conducted a research with calculus students and the results of the study revealed that most of the students had beliefs that continuity was necessary and it is sufficient condition for deciding differentiability of a function. Tall and Vinner (1981) in their study obtained opposite results revealing that student used non-differentiability as a reason of discontinuity of a function. Both of the mentioned studies were agreed upon which students had trouble determining the relationship between these two ideas. Baker et al. (2000) conducted a study on college students, who had completed two semesters of calculus. The students were asked to sketch a graph of a continuous function satisfying several criteria related to the first and second derivatives on given intervals. It was observed that the students were unable to conceive what a function is, which was continuous but not differentiable, might look like. The authors then reported that the students were familiar with functions that were both continuous and differentiable. Although some students who were able to construct such a function, according to authors, they were not able to understand how removing the continuity might affect the graph of the function they have drawn. As a result, they did not fully understand how differentiability and continuity are related (***ibid: 1521***).

2.10. Misconceptions and Difficulties with the Chain Rule

Derivative is an inherently difficult concept for many students. Especially whenever the function considered is a composite function, the difficulties students often encounter increase and get worse.

As cited in Uygur and Özdaş (2005:209), Tall indicated "the Leibniz notation $\frac{dy}{dx}$ proves to be almost indispensable in the calculus. Yet it causes serious conceptual problems. Is it a

fraction, or a Single indivisible symbol?" According to Tall**,** one difficulty with the notion of the Chain Rule is the dilemma of whether the du can be cancelled in the equation $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$.As cited in Uygur and Özdaş (2005:209) Clark, Cordero, Cottrill, Czarnocha, Devries, John, Tolias, and Vidakovic studied understanding genetic decomposition of the Chain Rule and its applications. They came to the conclusion that the difficulties with the chain rule for a large number of students could be attributed to student difficulties in dealing with the composition and decomposition of functions. Cottrill (1999) studied also the correlation between a student's understandings of the composition of functions and understanding the chain rule. In his study, there was a small amount of evidence supporting the hypothesis which is states that the understanding of composition of functions is a key to understanding the chain rule.

The identified reasons for this are due to inappropriate and weak mental links between knowledge of limits and knowledge of other calculus concepts such as continuity and derivative. Well- constructed mental representations of the network of relationships among calculus concepts are essential for a thorough understanding of the conceptual understanding of the calculus. Another identified source of misconceptions in literature is the sheer complexity of the ideas. These ideas cannot appear instantaneously in complete and mature form, so that some parts of the idea will get adequate representations before other parts will. It would therefore be necessary for each student to gradually build mental representations of the limit, continuity and differentiability concepts (Jordaan; 2005:3).

CHAPTER THREE

3. RESEARCH METHODOLOGY AND DESIGN

The focus of this chapter is the research methodology and design which were used to conduct this study. It contains the research methodology, research design, data sources, study area, sample population and sampling technique, data collection instruments, strategies of data collection and methods of data analysis, validity and reliability of instruments.

3.1. Research methodology

This study was employed mixed research approach (i.e qualitative and quantitative) to answer the research questions asked in this study. According to Temechegn, (2008:137) such the structure is named as mixed model research design with which quantitative and qualitative approaches are mixed with in or across the stage of the research process. Since the study mainly concerns on preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability at Awi zone preparatory secondary schools , the data are to be analyzed qualitatively and quantitatively so that it is possible to answer my research questions.

3.2. Research Design

Descriptive survey design was used in this study as research design since this research study is an exploratory study aimed at investigating and describing the type and extent of preparatory secondary students' misconceptions on the concepts of limit, continuity and differentiability. Philliber et al. cited in Yin (2003:21) states that a research designs as a " blueprint" of research dealing with at least four problems :what questions' to study , what data are relevant , what data to collect , and how to analyze the results. In addition to this , Koul cited in Ashebir(2009:27) states that descriptive method helps to have general understanding of a problem by studying the current status , nature of prevailing conditions , practice and trends through relevant and precise information. The survey design is not concerned with characteristics of individuals as individuals; it is concerned with the generalized statistics that result when data are abstracted from a number of individual cases.

3.3. Data Sources

In this study, the main data sources are students from Ankesha, Danegela and Injibara preparatory secondary school.

3.4. Study area

Ankesha, Danegela and Injibara preparatory secondary schools were randomly selected as the study sites. These schools found in Awi zone .Agew Awi is one of 11 zones in the Amhara Region of Ethiopia which is located 452 km away from the capital city Addis Ababa.

3.5. Sample Population and Sampling Technique

The target population for this study was the whole Awi Zone Preparatory secondary school students .In Awi zone, there are six governmental preparatory secondary schools. From these preparatory secondary schools, the researcher was select randomly three preparatory secondary schools as accessible population to carry out the study because to address the whole preparatory secondary schools it needs high cost of money and time.

Thus, the accessible population for this study was three randomly selected Preparatory secondary school (Ankesha, Dangela & Injibara) students at the end of the 2010/2011 academic year (2003 E.C) in Awi zone. The total numbers of grade 12 students who are currently attending the preparatory program in those three selected preparatory secondary schools are 1807. There are 1164 male students and the remaining 642 are female students.

Nachmias and Nachmias cited in Solomon (2000) viewed as there are numerous suggestions about the necessary size of sample. One is that a sample size should be a regular proportion (often put at 5%) of the population. Another is that any increase in the sample size will increase the precision of the sample result.

Accordingly, the researcher had 186 (10%) students as the sample of the study, out of which 120 were male students and 66 were female students. The schools, streams and sex were used as strata so as to determine the sample that would be taken from each stratum.

Abebayehu et.al cited in Ashebir (2009) used formula to determine the sample size by strata from each stratum :

$n_k = \frac{N_k}{N} n$ Where;

n_k =The sample size for k^{th} strata

N_k =The population size of the strata

N =The total population size

n =The total sample size

Table 2: sample students taken from each school

Name of school	No of students					No of sample students				
	male		female		total	male		female		total
	natural	social	natural	social		natural	social	natural	social	
Ankesha PSS	210	54	76	64	405	21	6	8	7	42
Danegela PSS	422	132	146	157	857	43	14	15	16	88
Injibara PSS	252	94	103	96	545	26	10	10	10	56
Total	1164		642		1807	120		66		186

The individual respondent from each stratum was taken by simple random sampling technique. The researcher chooses, simple random sampling to reduce sampling error and bias.

3.6. Data collection instruments

Triangulation technique used to collect data which enhance the credibility of the results. Due to this reason, the researcher used concept based WPT on limit, continuity and differentiability. On the other hand, interview questions were also used to collect data which helps to cross check students' responses on limit, continuity and differentiability with concept WPT.

3.6.1. Workout problem Test

The purpose of the WPT was to identify the types of preparatory secondary school students' misconceptions on the concepts of limit, continuity and differentiability and to determine the extent of students' misconceptions on the concept of limit, continuity and differentiability. Although, WPT is difficult and time consuming for interpretation and analyzing the results, the researcher thought as WPT can be used deeply to identify the

types and to determine the extent of students' misconceptions on the concept of limit, continuity and differentiability and also it gives more time to think and write about their ideas.

The WPT (see appendix A) were developed by taking the students text book and teacher's guide of grade 12 mathematics as reference to investigate students' misconceptions on the concept of limit, continuity and differentiability The total number of WPT items were 9 (see appendix A).

3.6.2. Interview

The researcher used structured interview as a second major data collecting instrument. The researcher's rationale to use interview was that, it has particular strengths and it is also a useful tool to get data deeply. The aim of these content-based interviews was to find out more about each student's personal understanding of limit, continuity and differentiability in a way that the problem based test (questionnaire) did not necessarily reveal. In other words, to determine the nature of each student's concept image of a limit, continuity and differentiability. It is also used to assess the breadth and depth of his or her limit, continuity and differentiability understandings in general.

In face –to- face situation, the researcher can ask participants to repeat their answers, or may give interviewee more time to write down responses. Furthermore, this technique may offer the opportunity to ask follow - up questions based on participants' responses to previous questions. This allows deeper exploration of issues (Vanderstoep and Johnston, 2009:88).

3.6.3. Pilot testing of the instruments

Data collection instruments were piloted in March 20/2011 in Addiskidam preparatory secondary school which is found in Awi zone to check whether they can generate the expected information and to consider their internal consistency . The workout problem tests were distributed to 47 (male=30 & female=17) students of one section which was chosen by random sampling. Moreover, the analysis of the pilot test data was made to examine the reliability and the discrimination power of each item. Six students were chosen by their mathematics teacher based on their verbal abilities, sex based as well as their mathematics achievements (two high-achievers, two medium

achievers and two low-achievers), were individually interviewed by the researcher after the completion of the problem based test.

The researcher was also wanted to determine whether the directions and items are clear and whether the data was obtained from the test reflect the real understanding of the students. The respondents were asked to give comments on individual items as well as the test as a whole.

The samples of Pilot study's responses on the different items and the total WPT responses was read and was interpreted by the researcher and their mathematics teacher. In addition to this, the feedback of pilot study group was used to alter and improve the WPT questionnaire for the second phase development of the questionnaire to the actual study.

3.7. Strategies of data collection and methods of data analysis

3.7.1. Strategies of data collection

In relation to the administration of the WPT for the students , assistances are selected from teachers from each schools to cooperate and facilitate activities during the time of test administration . The main task of the assistances were to administer the test and control the students in the exam time . Brief orientation was given to the students by the researcher. The students were informed to answer all the questions in the test and they were told to ask any question which is not clear for them while doing the test. The test was collected while the students were complete it.

After the completion of the WPT and based on problem based test result, the predetermined structure interviews were conducted. The interviewees were asked one by one by the researcher.

3.7.2. Methods of data analysis

Depending on the nature of the research questions in this study and the nature of data collected, percentages were the main statistical tool for this study. Thus, on each item the students' responses was analyzed quantitatively as correct responses, incorrect responses and no responses in terms of percentages and possible types of misconceptions were categorized in different categories by identifying common students' misconceptions by looking steps when they work from their scripts for each item qualitatively. So these categories constitute types and extent of students' misconceptions that they had. The

researcher and three grade 12 mathematics teachers were read students responses in each WPT over and over again to get an overall picture to the type of responses that the students were given to examine the conceptions and misconceptions. The results obtained from WPT together with results from interview questions were used to answer my research questions. The informations gained through workout problem tests and those informations which are gained through interview questions by looking the students' solutions and steps were analyzed qualitatively. Therefore, both quantitative and qualitative aspects of data analysis were used in this study.

3.8. Validity and Reliability of instruments

After the items were prepared, to maintain the content validity of the WPT and interview questions, different bodies checked whether the items can measures what is intended to measure. According to this view, three post graduate mathematics education students in Addis Ababa university who taught grade 12 students for many years and two mathematics teachers who taught grade 12 students for about five years at Injibara preparatory secondary school were determine whether the items in the WPT were in the concept of limit, continuity and differentiability and appropriate for the purpose of obtaining information on grade 12 students understandings and misconceptions' on the concept of limit, continuity and differentiability.

Reliability of the instruments was established by administering the instruments in a pilot study that was held in one section with 47 (male =30 & female=17) students before the data collection. In this pilot study, items were tested in terms of consistency in students' responses given to each item and questions were also determined whether they were understandable or not. Using the result of the pilot test , the reliability of the WPT items was calculated using cronbach alpha using SPSS 15.0 window and the result was $\alpha = 0.78$. This result is reliable according to the standard test, cronbach alpha value greater than 0.5 is taken as adequate for research purpose and the average calculated discrimination index value was 0.5. Thus, items were reasonably good as discrimination index was more than 0.40. Each of the items functioned well.

CHAPTER FOUR

4. DATA ANALYSIS AND INTERPRETATION

4.1. Introduction

The purpose of this chapter is to present, analyze and interpret the data that were gathered through WPT and interview questions. The aim of the data collection was to identify the types of students' misconceptions on the concept of limit, continuity and differentiability and to determine the extent of students' misconceptions on the concepts of limit, continuity and differentiability. The results of the study are explained in two parts. First, the results of the workout problem tests and the summary of students' misconceptions on each item are presented in a table thereafter the discussion followed by showing students' evidence. In the second part, interview results are explained. The data are then analyzed and interpreted in order to reveal the type and extent of misconceptions that the students have.

4.2. Representation and analysis of the data from the workout problem test

4.2.1. Students' responses on the definition of limit, continuity and differentiability:

In grade 12 students' mathematics text book, one of the minimum learning competencies is expecting students to define the limit, continuity and differentiability of a function at a point. In line with this objective, the researcher prepared item number 1 to see how the students conceive the concept definitions of limit, continuity and differentiability of a function at a point.

Table 3: Summary of students' answers to items 1(a), 1(b), and 1(c) (see appendix A)

Item No.	Responses (N = 186)							
	Correct		Incorrect		No		Total	
	N	%	N	%	N	%	N	%
1(a)	72	39	108	58	6	3	186	100
1(b)	84	45	95	51	7	4		
1(c)	54	29	119	64	13	7		

Table 3 reveals, only 72 (39 %), 84 (45 %) and 54 (29%) of the students got the correct answer for items 1(a), 1(b) and 1(c) respectively. Majority of the students, 108 (58%), 95 (51 %) and 119 (64%) were tried to answer the items but did not get correct answer and some of them 6(3%), 7 (4 %) and 13 (7%) even never tried to find the answer for each items respectively.

Table 4: Summary of students' misconceptions on items 1(a), 1(b), and 1(c) (see appendix A)

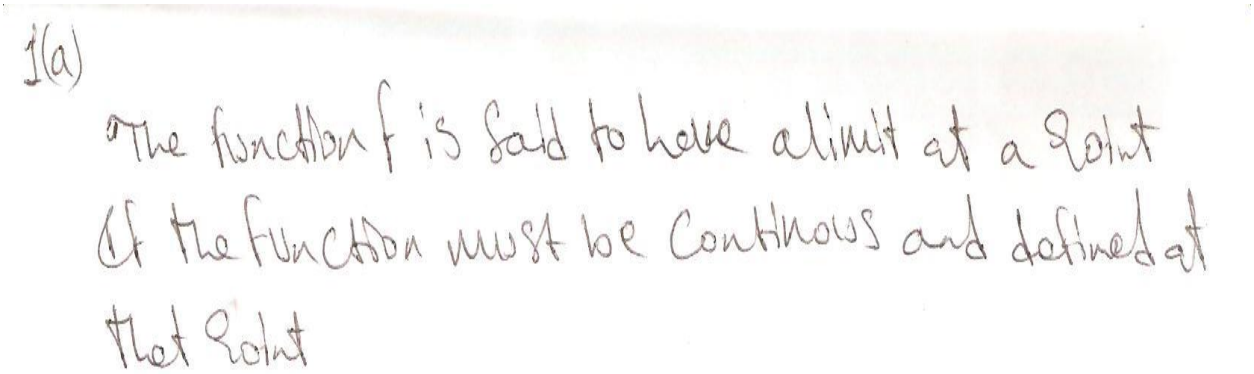
Item No	Incorrect responses (N=108)			
	categories	misconceptions	frequency	%
1(a)	1	Students misunderstood that a function f must be defined and continuous at a point to have a limit and they were under impression that discontinuous functions can not have a limit at the point of discontinuity.	47	44
	2	Students were associates a limit of a function at a point with the functional value.	38	35
	3	Students could not identify the difference between symbol and definition of limit of a function.	15	14
	4	Uncategorized	8	7
	total		108	100
1(b)	Categories	Incorrect responses (N=95)		
		misconceptions	frequency	%
	1	Students thought that being a function defined at a point is sufficient for a function to be continuous at that point.	56	59
	2	Students believed that if the left hand limit is equal to the right hand limit or if the limit of a function exists, then the function must be continuous.	34	36
	3	Uncategorized	5	5
	total		95	100
1(c)	Categories	Incorrect responses (N=119)	frequency	%
		misconceptions		
	1	Students thought that the continuity at a point is sufficient for a function to be differentiable at that point (students consistently thought that continuity and differentiability are equivalent).	41	35
	2	Students were not able to differentiate between symbol and definition of derivative of a function at a point (confusion of students to define the derivative a function in terms of symbols	60	50

		or formulas and Students were confused by quoting the quotient difference as a sum. (i.e. $f'(x) = \lim_{h\to 0} \frac{f(x+h)+f(x)}{h}$).		
	3	The students were not able identify between the symbol of derivative of a function and derivative of a function at point.	11	9
	4	uncategorized	7	6
total			119	100

Discussions:

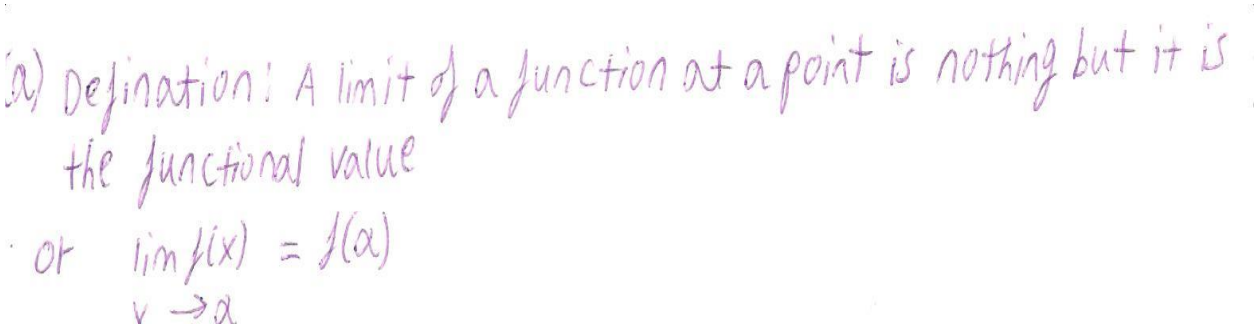
On the item number 1(a), students were asked to define the limit of a function at a point by their own words (see appendix A). Majority of students, 108 (58%) were experienced for different types of misconceptions (see table 4) to define this item. For instances, some incorrect definitions with certain misconceptions from students' scripts are the following:

Student 25 (male):



This student was under the impression that a function f must be defined and continuous at a point to have a limit and the student was under impression that discontinuous functions can not have a limit at the point of discontinuity. Clearly, $f(a)$ need not be defined in order for $\lim_{x\to a} f(x)$ to exist.

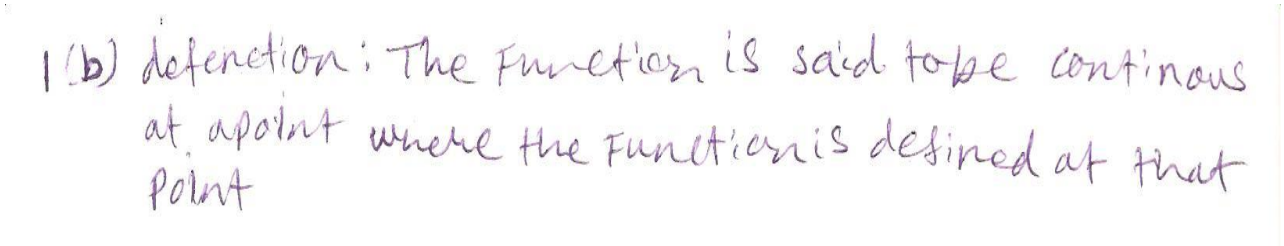
Student 32 (female):



She clearly associates a limit of a function at a point with the function value for a specific value of a and she has also great misunderstanding because she could not identify the difference between symbol and definition. A symbol " $\lim_{x \to a} f(x) = L$ " is nothing but used to abbreviate the concept "the limit of $f(x)$ as x approaches to a is L."

On item number 1(b) (see appendix A), many students 95(51%) faced to different types of misconceptions (see table 4) to define the continuity of a function at a point. For example, among those students, one student faced for the following misconception to define the continuity of a function at a point on student's script:

Student 68 (male):



This student thought that being defined at a point is sufficient for a function to be continuous at that point. This ideas taken from the student's definition indicated that the student has misconceptions in defining the correct definition of continuity of a function at a point. As mentioned in the literature review section 2.6.6 of this study, Vinner (1992) found that most of the students' responses demonstrated a common belief that "for a function to be continuous is the same as being defined and to be discontinuous is the same as being undefined at a certain point".

- Some other students, 34(36%) (See table 4) believed that whenever the left hand limit was equal to the right hand limit, the function was continuous at that point. That is $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$.Because of the lack of knowledge on continuity, it

was revealed that these students could not apply all of the conditions for a function to be continuous at a point:

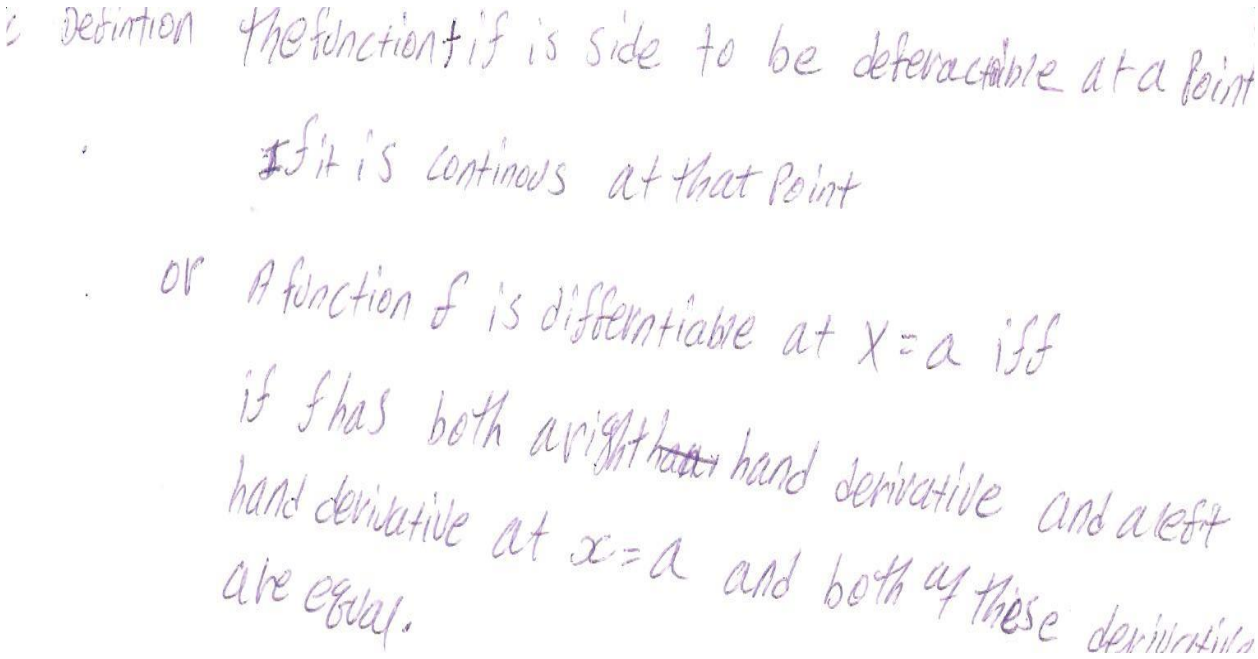
a) function f should be defined at the point;

b) $\lim_{x \to x_o} f(x)$ should be exist and;

c) $\lim_{x \to x_o} f(x) = f(x_o)$.

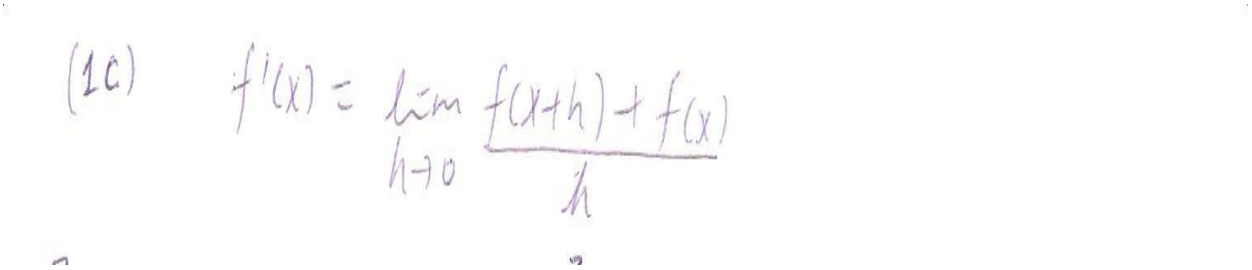
On item number 1(c) (see appendix A), as the students' result shown in Table 3, majority of the students , 119(64%) were tried to define the item but did not define correct definition and they admitted to different types of misconceptions (see table 4).Some of incorrect definition from students' script were put as follows:

Student 129 (female):



She thought that the **continuity** at a point is **sufficient** for a function to be **differentiable** at that point. She consistently thought that continuity and differentiability were equivalent. Sometimes a function can be continuous at a point and not be differentiable at that point. Continuity does not apply to differentiability. This result shows that she has weak mental representation on the relationship between continuity and differentiability. *Bezuidenhout cited in Duru et al (2010: 1521) also reported similar findings that students were unable to distinguish between continuity and differentiability*. Another student also answered as below:

Student 29 (male):



On this response, one can see that, this student has shown three misconceptions.

- The first type of student's misconception was that the student tried to define by using symbol, because the student was not able to differentiate between symbol and definition .

Unless students clearly conceive and understand the definition of derivative of a function at a point, knowing how to differentiate the given function at a given point by using symbol, formula or expression is all about playing with number.

Bezuidenhout cited in Jordaan (2005:25) argued that formula might make calculations easier, but did not promote understanding. He advises that the development of students' conceptual understanding should go hand in hand with the development of their manipulative skills.

The above student's misconception is series because if one student does not observe notation carefully, he or she will fail to appreciate the essence of mathematical argument. Derivative of a function at a point is not a symbol rather a name of quantity obtained from the expression $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ $or f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

Most grade 12 students may focus to pass in examination or to get a good mark and reward in mathematics subject by using expression or formula but this may not be good in understanding the concept definitions in mathematics . In this study , many students were tried to calculate and deal with item problems asked in questionnaire but they do not have a satisfactory understanding on the concept definitions.

As limit , continuity and derivative are a new concept to preparatory students , it is observed that students confuse the concept definitions and symbols and meaning of an expressions which in turn affect students understanding .

In general, the observed students' misconceptions under these items were, many students were misunderstood some very important concept definitions in calculus. Some of the misconceptions observed from the answer of students open ended test is generally confusion of concept definitions and their symbols, inability of using proper concept definitions.

*As mentioned in the review literature section, Tall & Vinner (198:152**)** stated that the concept definition as a form of words that specify that concept. Such a concept definition may be formal and given to the individual as part of a mathematical theory. It may be personal, invented by the individual to (partially) describe his concept image. A personal concept definition may be considered as part of the concept image.*

Therefore, the result obtained from item1(a, b & c) indicates that students' understanding the concept definitions on the concept of calculus is not satisfactory and not in line with the objectives stated in the mathematics syllabus of grade 12.

4.2.2. Students' responses on the concepts of limit

Table 5: Summary of students' answers to items 2 and 3(see Appendix A)

Item No.	Responses (N = 186)							
	Correct		Incorrect		No response		Total	
	N	%	N	%	N	%	N	%
2	59	31.7	126	67.8	1	0.5	186	100
3	69	37.1	113	60.8	4	2.1	186	100

As the data in Table 5 reveals that, only 59 (31.7 %) and 69 (37.1 %) of the students got the correct answer for items 2 and 3 respectively. Majority of the students 126 (67.8 %) and 113(60.8 %) were tried to answer the items but did not get correct answer and some of them 1(0.5%), 4(2.1 %) even never tried to find the answer for each items respectively. The difficulty encountered to do this item 2 for most of the students were unable to understand properly and misunderstanding of the definition of sequences of numbers and **item number 3** has also been answered by small number of students , but the majority of students were answered the incorrect answers and admitted to different types of

misconceptions (see table5) to this item. Common incorrect answers by students in item 3 were: 0, 1, and does not exist.

Table 6: Summary of students' misconceptions on items 2 and 3 (see appendix A)

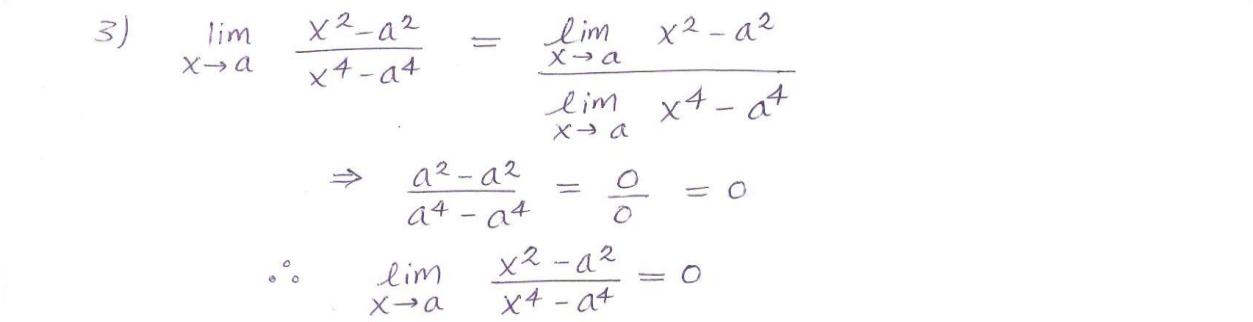
Item No.	Incorrect responses (N=126)		frequency	%
	Categories	misconceptions		
2	1	A limit of sequences can be found by a method of substitution.	82	65
	2	Confusion between the concept of limit of a sequences and limit of functions	36	29
	3	uncategorized	8	6
	total		126	100
3	Incorrect responses (N=113)			
	categories	misconceptions	frequency	%
	1	Confusion to understand the bounds where a quotient rule is applied to find the limit of a function and Confusion on the fraction $\frac{0}{0}$.	76	67
	2	students thought that limit of a function is obtained simply by substituting the value at which the limit of f is to be found into the expression and the students were considered the limit of a function is equal the functional value .	25	22
	3	Lack of knowledge of algebraic manipulation such as factorization and simplification problems.	9	8
	4	uncategorized	3	3
	total		113	100

Discussion:

On item 3, the majority of students, 113(60.8%) were answered the incorrect answer and admitted to different types of misconceptions (see table 6) to this item.

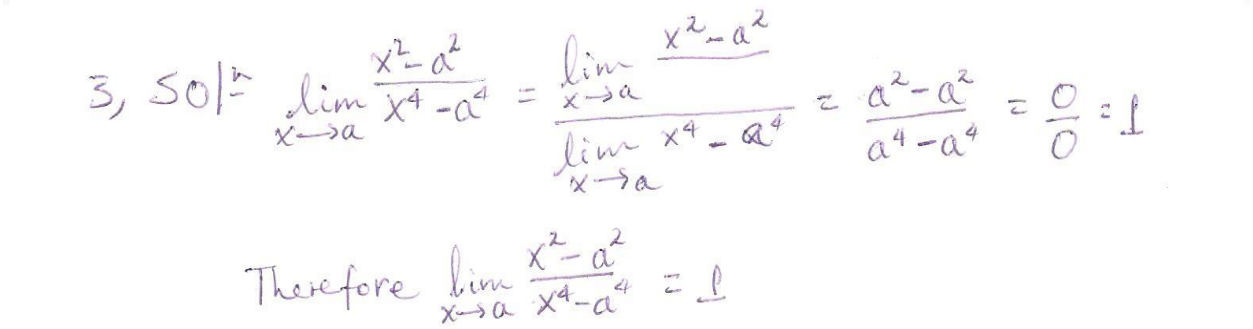
Some example of incorrect solutions from students' script on item number 3 was as the following:

Student 45 (male):



One can see from the above student's solution, the student was answered by using quotient rule and direct substitution method. One can see form the above solutions, this student fails to understand the bounds where a quotient rule is applied ($\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x)\neq 0$) or (the limit of a quotient is the quotient of the limits provided that the limit of the denominator is not 0). Although, in grade 12 mathematics text book , there are different techniques such as cancelling common factors in rational expressions' , like $\frac{(x-2)(x+5)}{(x-2)(x+1)}$, for $x\neq 2$, rationalization , like $\frac{(\sqrt{x}-1)}{x-1}\frac{(\sqrt{x}+1)}{\sqrt{x}+1}$, graphs , tables of values and other properties are used to find the limit of a function at a point , but the students missed this basic concepts and they also misunderstood where they use quotient rule .

Student 67(male):

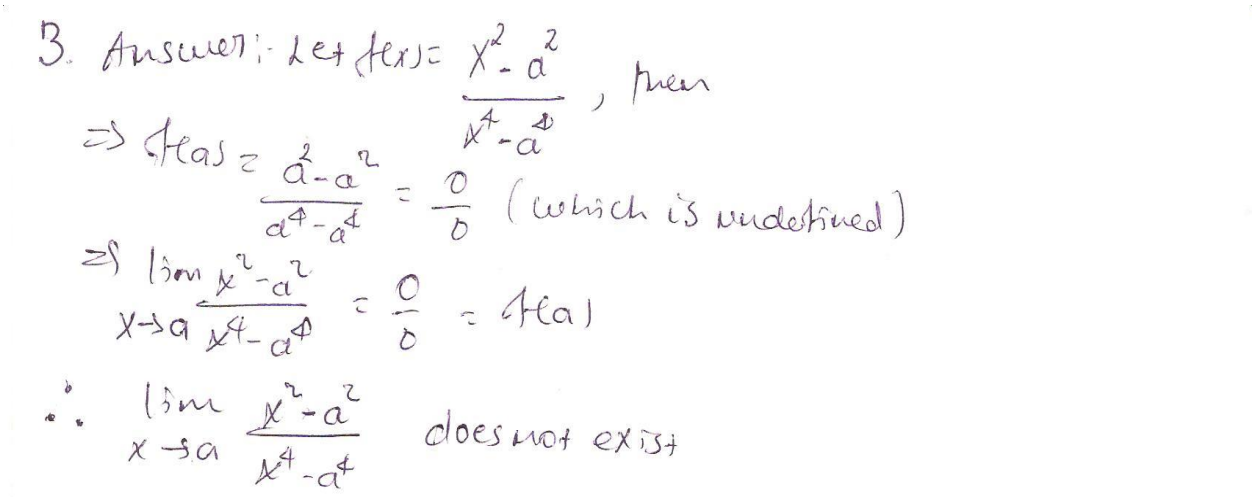


But this student, firstly misunderstood by trying to evaluate the limit of a given function by using the quotient rule (i.e $\frac{\lim_{x\to a} x^2-a^2}{\lim_{x\to a} x^4-a^4}$) . This student also fails to understand the bounds where a quotient rule is applied as student 45. And then the student was used direct substitution method and obtained the result $\frac{a^2-a^2}{a^4-a^4}$ and the student got $\frac{0}{0}$. In addition to this, the student was simplified $\frac{0}{0}$ to equal to1! (But cancellation only works

when the factor cancelled is nonzero.) It is not true that $\frac{0}{0}$ just give us 1! And don't give up other results. Instead, one must simplify the common factor of $\frac{x^2-a^2}{x^4-a^4}$.

Any time when the students get $\frac{0}{0}$, they should always try to simplify, hopping to get rid of one of the zeros. But many students didn't try like this.

Student 74(female):



The observed common misconception on the above solution by the student is ,she did not understand that the limit of a function is equal to the function value at a point that is a limit can be found by a method of substitution. This revealed that student views that a limit as a substitution process and she thought the limit value and function value are the same. But just substitution is always works for only a polynomial and continuous functions.

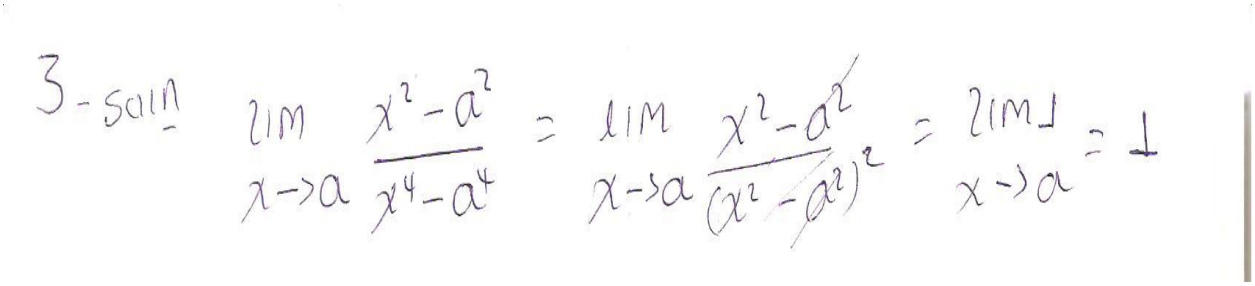
In item 3, the observed common students misconceptions were when the students were tried to substitute the value a in for x, many students , 76 (67%) got a fraction that looks like $\frac{0}{0}$. In this case the three common misconceptions were to say that the limit is zero, limit does not exist and an even worse misconception was to cancel the zeros and say that the limit equals 1. They do not realize the implications of the indeterminate forms.

It is clear that there is no problem with having a zero in the numerator. But the students were confused when there is zero in the denominator. In truth, the $\frac{0}{0}$ tells us that more work must be done to determine the limit, usually it needs some cancellation. Let us take

the simple example, like $\lim_{x\to 0}\frac{3x}{x}$. The answer is 3, and it is easy if we do some cancellation rather than direct substitution.

Generally for any real number c≠ 0, if we get something like $\frac{c}{0}$, we know the limit is infinite, if we get something like $\frac{0}{c}$ then the limit is 0, and if we get $\frac{0}{0}$ we know nothing , and more work is required . At any time when we get $\frac{0}{0}$ for a limit, it is a *BIG WARNING SIGN* that says YOU HAVE MORE WORK TO DO!

Student 11 (female):



From the solution of this student, one can see that she has misunderstood the concept of algebraic manipulation rather than the concept of limit of a function. Here in this case, the she was confused with the manipulation of the concepts $x^4 - a^4$ and $(x^2 - a^2)^2$ for any two real numbers x and$a$. It is clear that

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2) \neq (x^2 - a^2)^2$ except for a is zero or for both x and a are zero.

Hence as a result $x^4 - a^4$ is different from $(x^2 - a^2)^2$ except possibly when either a or both x and a are zero.

Similarly some students, 9(8%) results show that, students have common misconception due to lack of knowledge in preliminary algebra like factorization and simplification problems. Based on this result , one can conclude , in order to learn the concept of calculus , students must clearly understand the basic mathematical skills such as algebra and algebraic manipulation.

In addition to this ,the observed students' misconception on this item can also an implication to say success in calculus depends to a large extent on knowledge of the

mathematics that precedes calculus: algebra, analytic geometry , functions, and trigonometry.

In general , from the above result , one can see that students have no well constructed mental representations of algebra . This misunderstanding makes the comprehension of limit extremely difficult , particularly because it cannot be calculated directly using familiar methods of algebra and arithmetic.

4.2.3. Students' responses on the concept of continuity

Table 7: Summary of students' answers to item 4 and 5 (see appendix A)

Item No.	Responses (N = 186)							
	correct		incorrect		No		Total	
	N	%	N	%	N	%	N	%
4(A)	53	28.5	127	68.3	6	3.2	186	100
4(B)	65	35	111	60	10	5		
5	43	23	127	68	16	9		

As the data in table 7 reveals that, only 53(28.5 %), 65(35 %) and 43(23%) of the students got the correct answer for items 4(A), 4(B) and 5 respectively. Majority of the students, 127 (68.3 %), 111(60 %) and 127(68%) were tried to answer the items but did not get correct answer and some of them 6(3.2%) , 10(5 %) and 16 (9%) even never tried to find the answer for each items respectively .

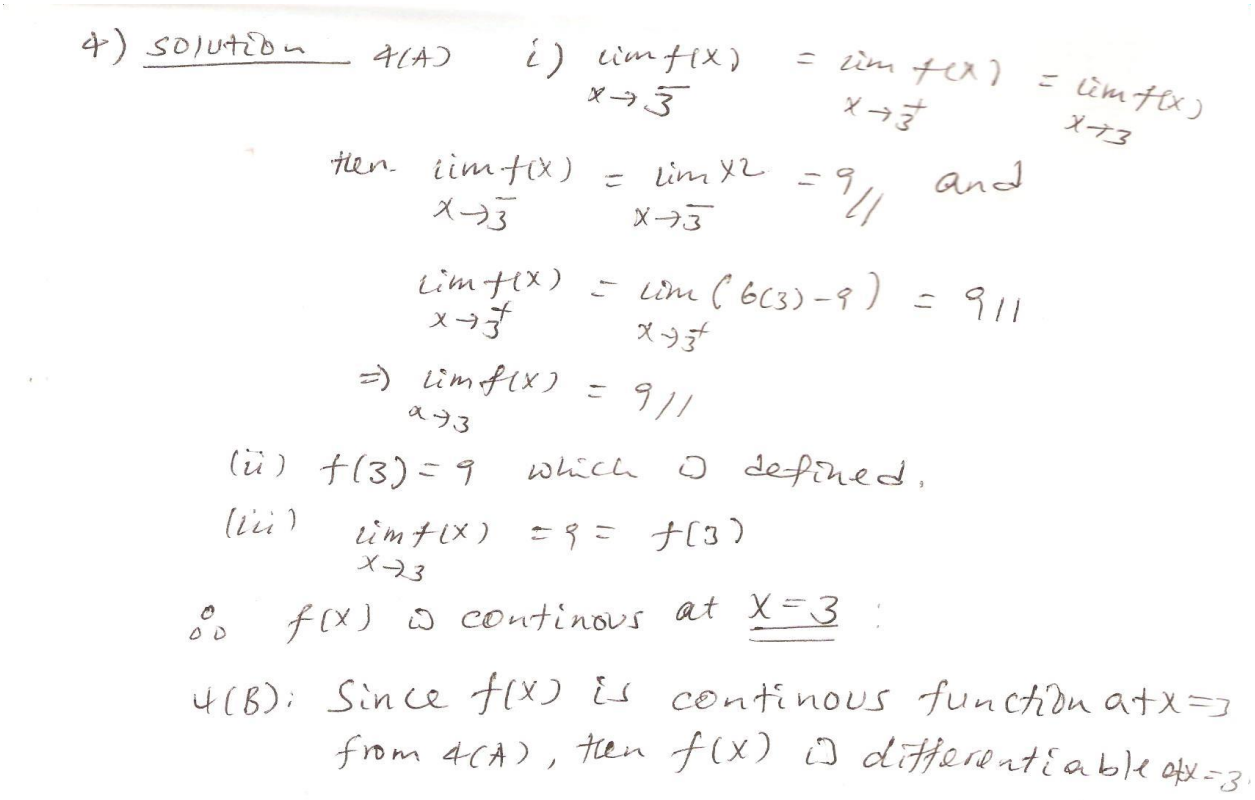
Table 8: Summary of students' misconceptions on item 4(A&B) and 5

Item No.	categories	Incorrect responses (N=127)		
		misconceptions	frequency	%
4(A)	1	Confuse the concepts of a function being defined, a function being continuous and a limit existing.	118	93
	2	uncategorized	9	7
	total		127	100
4(B)	categories	Incorrect responses (N=111)	frequency	%
		misconceptions		
	1	Confuse the concepts of continuity and differentiability of a function at a point (thought that being continuity of a function at a point is a sufficient condition for differentiability of a function at that point).	68	61
	2	Confuse the concepts of limit and differentiability of a function at a point.	33	30
	3	uncategorized	10	9
	total		111	100
5	categories	Incorrect responses (N=111)	frequency	%
		misconceptions		
	1	Confuse the concepts of the existence of limit of a function at a point and continuity of a function at that point.	54	49
	2	Students thought that being a function defined at a point is sufficient for a function to be continuous at that point.	40	36
	3	uncategorized	17	15
Total			111	100

Discussion:

Some of the students were put incomplete responses and incorrect reasons for items 4(A) and 4(B). Example, one student was put on his answer sheet as below:

Student 13(male):



Even if the student's answer for this item (4(B)) is correct, his reason is incorrect. This student thought that being continuity of a function at a point is a sufficient condition for differentiability of a function at that point. This student has misconceptions on the relationship between continuity and differentiability of a function at a point by thinking that the reverse of the theorem "**every differentiable function at a point is also continuous at that point**" is also true. In other words, this student in this study thought that if a function is continuous at a point then it is differentiable at that point.

> *This result is agreed with the result of Meel who conducted a research on calculus students and the results of the study revealed that most of the students had beliefs that continuity was necessary and it is sufficient condition for deciding differentiability of a function and Tall and Vinner (1981), were also identified students had trouble distinguishing between the concepts of differentiability and continuity (Duru et al ; 2010:1521).*

Indeed, there are many continuous functions at a point which are not differentiable functions at that point. For instance, the function $f(x) = |x|$ is continuous at $x = 0$ because $\lim_{x\to 0} f(x) = \lim_{x\to 0} |x| = 0 = f(0)$ but not differentiable at $x = 0$, because $f'(0) =$

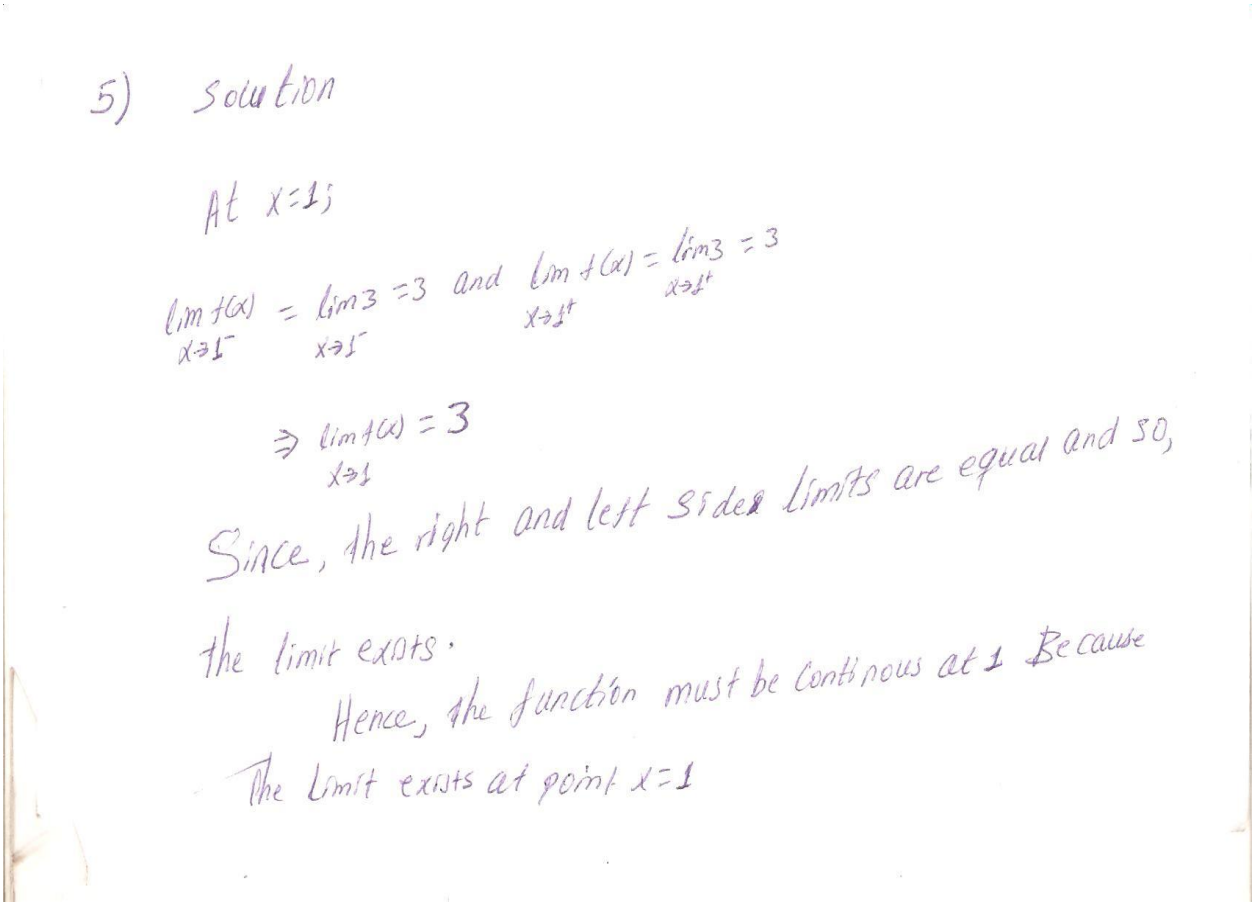
$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{|0+h|-|0|}{h}$ (if it exists).One can complete the left and right limits separately as the following.

$\lim_{h\to 0^+}\frac{|0+h|-|0|}{h}=\lim_{h\to 0^+}\frac{|h|}{h}=\lim_{h\to 0^+}\frac{h}{h}=\lim_{h\to 0^+}1=1$, (for h≥0) and

$\lim_{h\to 0^-}\frac{|0+h|-|0|}{h}=\lim_{h\to 0^-}\frac{|h|}{h}=\lim_{h\to 0^-}\frac{-h}{h}=\lim_{h\to 0^-}-1=-1$, (for h<0) .One can conclude from these results, the two side limits are different, thus $f'(0)$ does not exist. Therefore f is not differentiable at $x=0$. Indeed, if the function f is not continuous at a point, then it is not differentiable at that point. So at any discontinuity (for instance, a jump discontinuity) the function f fails to be differentiable.

Generally, on this study, many students were experienced for difficulties with the relationship between continuity and differentiability. Confusion between continuity and differentiability is an indicated as one of the main reasons for those students misconceptions. Some inappropriate conceptions of continuity and differentiability and of relationships among these concepts may be leaded to the deep – rooted misconceptions of the conceptual understanding of the calculus and may indeed interfere with future learning at university level.

From the incorrect solution of those students on item number 5, one can see that the students were confused on the criteria for continuity of a function. For example, if we observed the incorrect answers did by one student as below:



The identified student's misconceptions on this item were the phrases such as the "limit exists" and " the function is continuous at a point" were related in a wrong way to each other. This student was thought that the existence of limit of a function at a point is a sufficient conditions for continuity of a function at that point .This student's confusion might be with the idea that if f is continuous at a point , then the limit of a function at that point exists. Irrespective of the underlying cause of the misconception in this item , the misconception itself points to inappropriate mental representation of the relation between " the existence of the limit" and " continuity at that point". Erroneous principles that were formed in this way , is an indication of students' deficient conceptual understanding of the relationships between the concepts "limit" and " continuity".

4.2.4. Students' responses on the concept of differentiability

Table 9: Summary of students' answers to item number 6, 7, 8 & 9(see appendix A)

Item No.	Responses (N = 186)							
	Correct		Incorrect		No		Total	
	N	%	N	%	N	%	N	%
6	64	34	108	58	14	8	186	100
7	72	39	109	59	5	2		
8	66	35	111	60	9	5		
9	46	25	127	68	13	7		

As the data in Table 9 reveals, only 64(34 %), 72(39 %), 66 (35%) and 46(25) of the students got the correct answer for items 6, 7, 8 and 9 respectively. Majority of them,108 (58 %), 109(59 %) ,111(60%) and 127(68%) were tried to answer the items but did not get correct answer and some of them 14(8%) , 5(2 %) 9 (5%) and 13(7%) even never tried to find the answer for each items respectively

Table 10: Summary of students' misconceptions on item number 6, 7, 8 & 9(see appendix A)

Item No.	categories	Incorrect responses(N=108) misconceptions	frequency	%
6	1	Students were confused on the symbols of derivative of a function at a point and derivative of a function.	48	44
	2	Students were quoted the quotient difference as a sum. This misconception come due to lack of understanding the derivative as a gradient or a slope; a rise over a run; the vertical increase in f over the small x over the horizontal increase; h.	36	33
	3	Putting inappropriate formula ($f'(a)=\lim_{x\to a}\frac{f(a+h)-f(a)}{x-a}$) to determine $f'(a)$	18	17
	4	Equating the derivative of a function at appoint as the limit of a function ($f'(a) = \lim_{x\to a} f(x)$).	6	6
	total		108	100

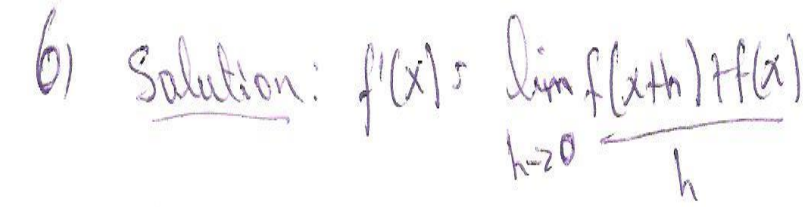
7	categories	Incorrect responses(N=109)	frequency	%
		misconceptions		
	1	Misunderstanding between exponential and power functions to use power rule. or confusion of students to identify the exponential function and power function	65	60
	2	There was confusion when students tried to use the rules for differentiation particularly chain rule for composition function.	39	36
	3	Uncategorized	5	4
	total		109	100
8	categories	Incorrect responses(N=111)	frequency	%
		misconceptions		
	1	Students missing some knowledge of property of logarithm that will allow them to easily get the derivative for a given function.	64	58
	2	Confusion on concept of derivative	45	40
	3	Uncategorized	2	2
	total		111	100
9	categories	Incorrect responses (N=127)	frequency	%
		misconceptions		
	1	Confusion on the concepts of existence of the limit at a point and the derivative of a function at that point.	68	54
	2	Students' have misconceptions in pre-calculus such as lack of knowledge in using the appropriate definition of absolute value functions and students' inability to restrict the domain.	52	41
	3	uncategorized	7	5
	total		127	100

Discussion:

The misconceptions encountered to do the item number 6, for most of the students were unable to understand the concept of derivative of a function at a point that is

misunderstanding of the definition of derivative of a function at a point. For example, the solution of one of the students who answered the item incorrectly is as follows:

Student 31 (male):



This type of misconception was also occur on item number 1(c) by other student (refer page 38)

- The first type of student's misconception on this item response is the student quoted the quotient difference as a sum. This result shows that this student's misconception might come due to lack of understanding the derivative as a gradient or a slope.
- The second type of student's misconception on this item shows that the student was not clearly identify derivative of a function at a point ($f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$) and derivative of a function ($f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$).

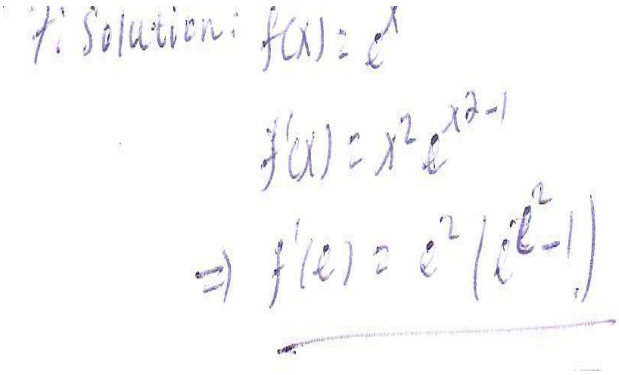
To answer item number 6, there are two alternative methods that students can choose the one that they are familiar. The first method is to use the symbol $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$. The second alternative method is to use the symbol $f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$.

The derivative of the function $y = f(x)$ at a point $x = a$, denoted by $f'(a)$.

The function f is said to be differentiable at a whenever the $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ exists. The $f'(a)$ represent the slope of the tangent line to the curve $y = f(x)$ at $x = a$. $f'(a)$ also represent's the instantaneous rate of change of $y = f(x)\ at\ x = a$. In particular, if $s = f(t)$ is the position function of an object, and then it represents the velocity of the object at time $t = a$.

On item 7, the students were asked to derivate the function e^{x^2} and also to evaluate first derivative at e. It was observed that more than half of the students109 (59%) failed to answer correctly and admitted for different misconceptions. One incorrect response of the student's was put directly from the student script as following.

Student 107(female):

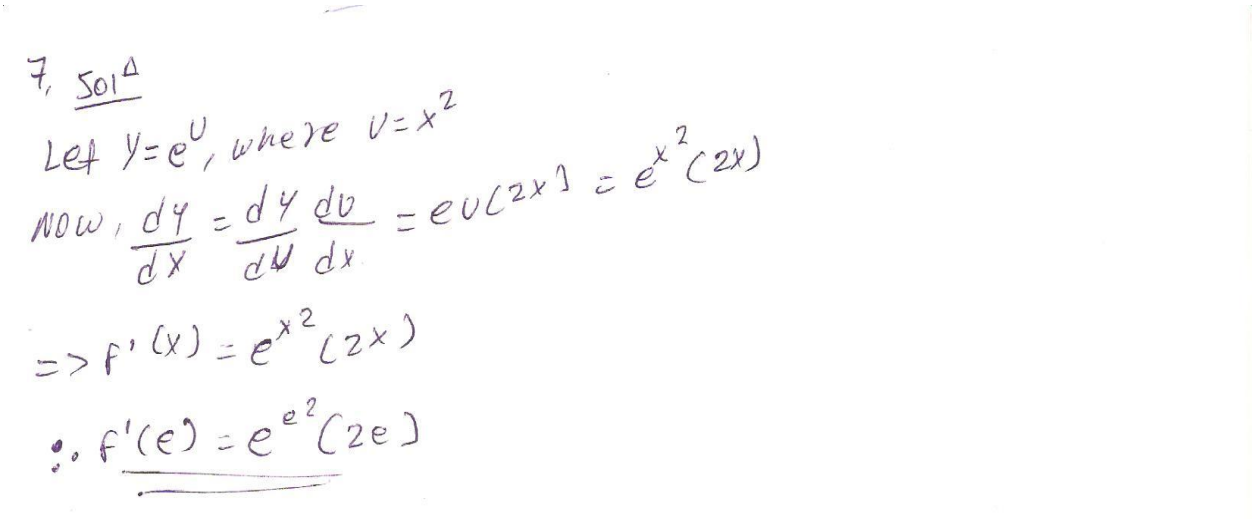


From this result, the observed student's misconception was that she was misunderstood to use the rules and the techniques of differentiation. Clearly, the given function f on this item was an exponential function. This student could not differentiate exponential functions and power function .An exponential function is not a power function. It is important to note that with the Power rule the exponent must be a constant and the base must be a variable while we need exactly the opposite for the derivative of an exponential function. For an exponential function the exponent must be a variable and the base must be a constant.

On this study, some students were confused on these two functions. Here, the above student was used the power rule. The power rule cannot be used to find the derivative of an exponential function. A power function has the independent variable (x in the context) in the base of the function as x^2. An exponential function has the independent variable in the exponent of the function, as in e^{x^2}.

To solve this item there might be different methods that students may choose the one that familiar to them . The first method is to apply derivative of composition function by using chain Rule and the other method is applying the definition of derivative of a function at a point (i.e. Limit of the difference quotient.)

Some students, 72 (39%) gave a correct answer to this item number 7 by using the concept of chain Rule. For example, one student did as follow.



This student has a clear concept image about derivative of a composition function.

Since f is composite function, first this student was used the chain rule. The student was first rewrite the function in variable form (which is more convenient when using the chain Rule), obtaining $y = e^{x^2}$ and the student was applied the chain Rule by introducing an intermediate variable , u as shown the above:

The item number 8 was needed some techniques and process of logarithmic differentiation. It was also needed the ability of differentiating algebraically since the item was a function for which the ordinary rules of differentiation do not apply directly but it needs some properties and methods for logarithm of base e , only the natural logarithm (base e, where $e \approx 2.718281828$), ln will be used to answer this item . In this item, the students were asked to differentiate a function f where $e^{f(x)} = 1 + x^2$.

As we can see from the table 8, majority of the students, 111 (60%) were done incorrect solution for item number 8. some students were put incorrect solution as the following.

Since $e^{f(x)} = 1 + x^2$ is equal to $f(x) = log_e 1 + x^2$

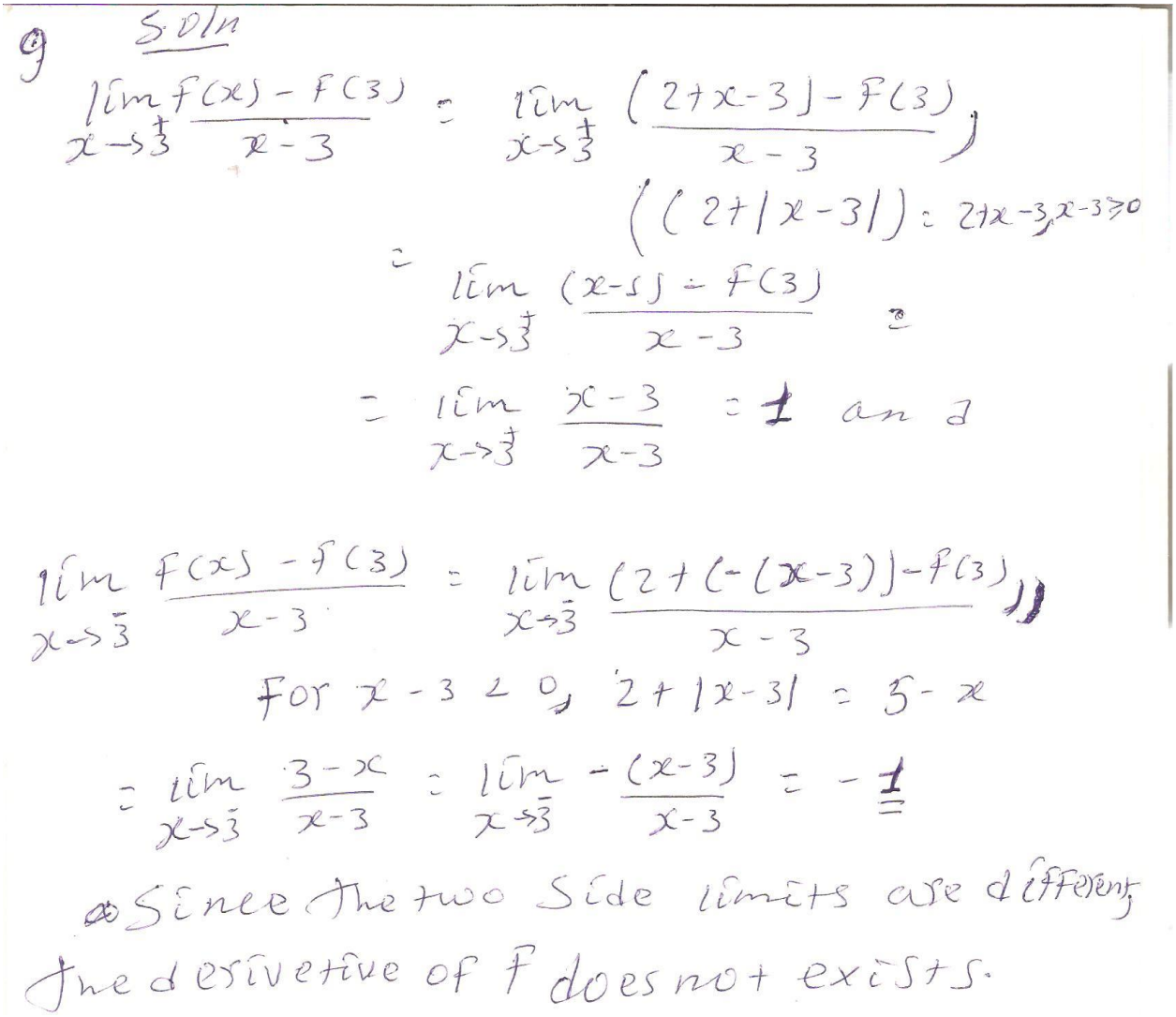
$\Rightarrow f(x) = 1 + x^2$

$\therefore f'(x) = 2x.$

This student has no full mental representation on the properties for logarithm functions. Particularly with the base e , only the natural logarithm (base e, where $e \approx 2.718281828$), ln will be used to answer this item. There is standard notation of logarithm if the base is e. That is $loge(1 + x^2) = ln(1 + x^2)$. At this point the student missing some knowledge of property of logarithm that will allow him to easily get the derivative for a given function.

On item 9 , the students were asked to determine whether f is differentiable or not differentiable at $x = 3$ .This item was included to measure the type of students' conceptions and misconceptions on the concept of derivative of a function at point and to see how the students can relate the concept of the existence of limit of a function at a point with the derivative of a function at that point .

As shown in table 10, some of students, 46(25%) were answered this item correctly. For example one student did by showing as below from his script:



Whereas majority of the students, 127 (68 %) were faced for great misconceptions in solving and writing the solution of this item.

Some of the incorrect solutions of the students' were put as follows directly from their script:

$f(x) = 2 + |x - 3|$

$\Rightarrow f(x) = 2 + |x - 3| = 2 + x - 3.$

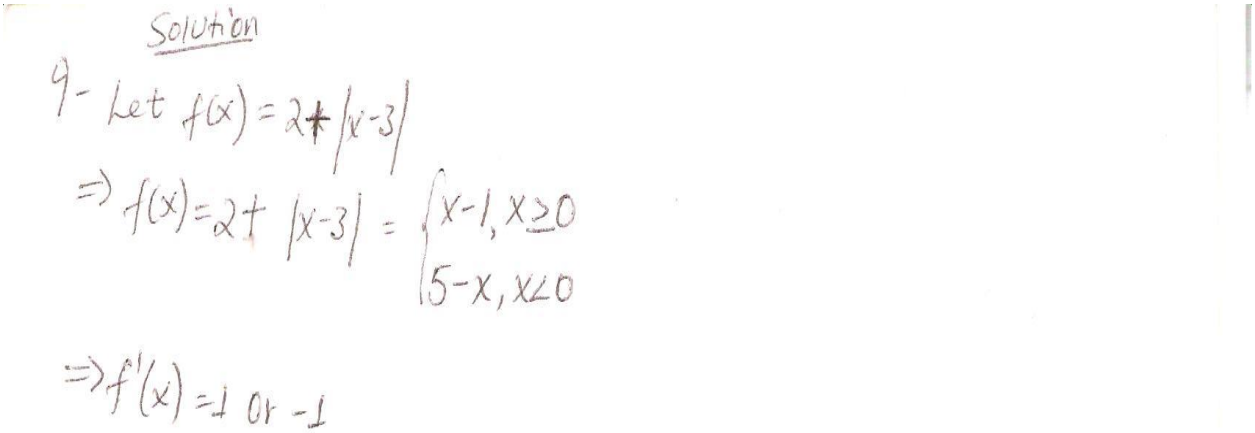
$\Rightarrow f'(x) = 0 + 1 - 0 = 1.$

$\therefore f'(3) = 1.$

From the above solutions , the students were truly misunderstood because the students have lack of the actual definition of the absolute value function . Here the

student may just thought it intuitively, saying to them self, well,$|x-3|$ is just means make $x-3$ positive. This misconception was leads them to trouble. Inability in use the definition $|x| = \begin{cases} x, if\ x \geq 0 \\ -x, if\ x < 0 \end{cases}$ might be just causes many students to pause to do this item. They could not perform well what they were required to do because they did not understand the idea of the absolute value functions. It was important for students to use the absolute definition to answer this item. For instance, $|3| = 3$ and $|-2| = 2$; no problem. But what the students do when $f(x) = 2 + |x-3|$? This was made the students to have misconception. Here the students could not break out this function in two piece wise function , because they misunderstood the definition of the absolute value function . So, they were not clearly understood and exactly what they were asked to do.

On the other hand, another incorrect answer by a student was done as the following:

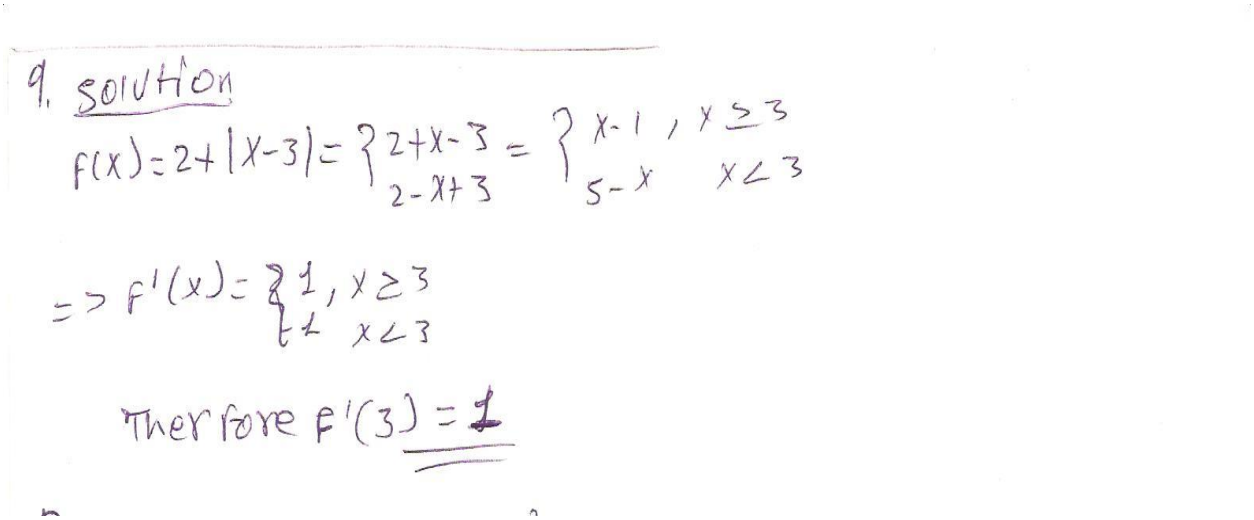


This student has no full misconception on the definition of absolute value of a function as the first student , but the first misunderstanding is the student's inability to restrict the domain and misunderstanding on the concept of derivative since the left side derivative and right side derivative are different, so, the derivative does not exist, but the student was misunderstood this concept . Especially, in restriction of the domain the students consider

$$2 + |x-3| = \begin{cases} x-1, x \geq 0 \\ 5-x, x < 0. \end{cases}$$

The wrong domain restriction may be the student related and simply recalled the definition of absolute value with $|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0. \end{cases}$

Some students start the solution correctly and end with incorrect result is due to problem of poor mental representation of the relationship between the limit concept and derivative .For example , the final misunderstanding by one of the student in this item was done as the following:



This student is under impression that the student cannot relate the existence of the limit at the point and the derivative of a function at that point. This student was experienced for difficulty with the relationship between the existence of the limit and differentiability of a function . Confusion between limit and differentiability is indicated as one of the main reasons for this student's misconceptions on this item. Since $2 + |x - 3|$ does not have a derivative at $x = 3$, it follows that the graph of $2 + |x - 3|$ does not have a tangent line at $(3, 2)$. Although $2 + |x - 3|$ is not a differentiable function at $x = 3$, it is differentiable at every number in $(-\infty, 3)$ and every number in $(3, \infty)$. if I is an open interval then we say that a function f is differentiable on I if f is differentiable at each point of I . As result, $2 + |x - 3|$ is differentiable on $(-\infty, 3)$ and on $(3, \infty)$. when I observed , the students could not determine whether f has a sharp corner and as it has no derivative at that corner .

4.3. Analysis of the data from the interviews

After the completion of the workout problem test, the predetermined structure interviews were conducted based on workout problem test (questionnaire) result that workout problem test did not necessarily reveal. Six students from each selected preparatory secondary schools (totally, 18) were selected to participate for the interview. They were interviewed

one by one. Each interview discussions were jotted down and lasted about 20-25 minutes with one student.

The interview questions were prepared based on students' misconceptions results on the workout problem test and aimed to triangulate with workout problem test result and further to find out how the students think about functional values, to examine how the students are able to relate continuity and differentiability of a function at a point with existence of limit of a function at a point as well as to determine how the students sketch the graphs of the functions which have a discontinuity point.

Each student's understanding was required and his or her understanding was revealed through interview questions. What the participant did as well as the process of doing the questions, the steps and focusing on how he or she was said and behaves during interview was jotted down by the researcher carefully.

Table 11: Students having been interviewed in actual study

Name of the school	Male	Female	Total
Anekesha PSS	3	3	6
Danegela PSS	3	3	6
Injibara PSS	3	3	6
Total	9	9	18

4.3.1. Excerpts from the interviews on item number 1

one can observe, obstacles to students' understanding on the concept of continuity in this study is their skill to sketch graph of functions. The aim of interview items was to examine whether students are able to sketch graph of a function which has a discontinuity point.

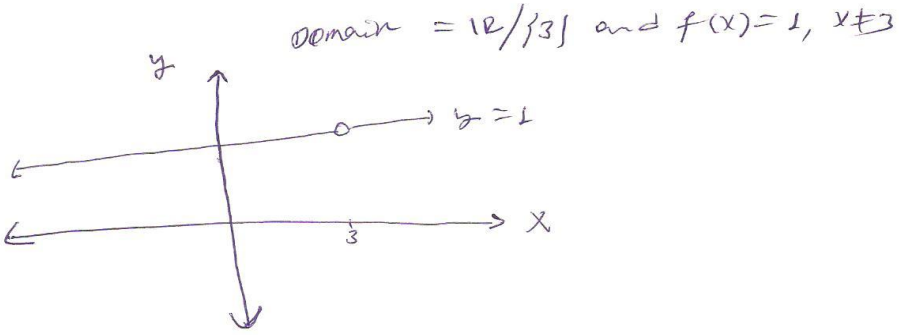
Under item number 1(A) (see appendix B), students were asked to sketch the graph of the function $f(x) = \frac{x-3}{x-3}$ which has a discontinuity at the point $x = 3$.

They were also asked what the value of $f(3)$ is (item number 1(B)) as well as whether the function has a limit at $x = 3$ (item number 1(C)).Some of the students, 6(33%) attempted

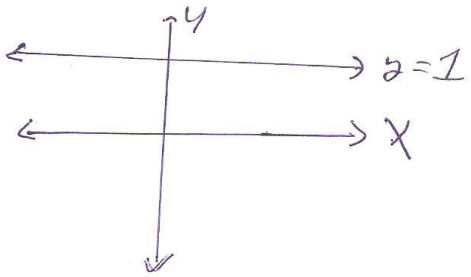
the graph by choosing values for x and calculating the corresponding $f(x)$ values. But at the point $x = 3$, they had some difficulty. Some students, 8 (44%) experienced difficulty in sketching the graph of $f(x)$.In order to try something, some of them made $y = 0$ which resulted in an x-intercept at $x = 3$.

Only 4(23%) of them could sketch the graph by restricting the domain of the function correctly.

For example, one student was sketch the graph as below:



The remaining students had the incorrect graph. For example, one student was sketch the graph as the following:



The reason which lead this student to sketch incorrect graph might be ignoring the restriction to the domain when joining points to construct the graph of a function .

On the question, what happens when $x = 3$, majority of the students answered that $f(x)$ becomes zero. At first, most students did not think of a discontinuity at the point $x = 3$ and they did their solution by showing as follow.

$f(x) = \frac{x-3}{x-3}$, then $f(3) = \frac{3-3}{3-3} = \frac{0}{0} = 0$.

Based on their solution, I was raised the new question, what $\frac{0}{0}$ is and 13 (72%) of them answered that it was zero. Only 5 (28%) students were correct in saying that it was undefined, although $f(3)$ remained equal to zero for them.

For example, one student, who is the top student in Ankesha preparatory secondary school, was said that the function was undefined at $x = 3$ (although he was one of those who said that $f(3) = 0$) and he elaborated on this by saying that the function is not continuous at the point $x = 3$. On the question does the limit could be exist, many students responses were no, because the function is undefined at $x = 3$. They were under the impression that there was no limit at the point because the function was undefined at the point. Accordingly, many students have tried to answer without sketching the graph and none of them could not use the graph to find the limit of the given function. These groups of students according to Stump in Ashebir (2009:58) views, seems missed the very good way to analyze limit of a function at a point . Accordingly, a very good way to analyze limit is to look at the graph of the function in a neighborhood of the limit point x_o.

After their conclusion, I was give for them a hint that the function could be discontinuous at $x = 3$, then the students realized that they had to make an open circle at $x = 3$ because the function was undefined at the point. 14(78 %) of the students could then say that the limit at $x = 3$ is equal to one, although they still believed that the value of $f(x)$ at $x = 3$ is equal to zero.

4.3.2. Excerpts from the interviews on item number 2

Under item number 2(see appendix B), on question 2(A) and 2(B) students were asked whether the function $f(x) = |x - 3|$ is continuous and differentiable at $x = 3$ respectively.They were also asked whether the limit of f exists at $= 3$. Finally the students were asked to sketch graph of a function.

The aim of interview item 2 was to triangulate the problem based test result (on students misconceptions on continuity and differentiability) and examine how the students are able to relate continuity and differentiability with existence of limit of a function at a point and further to determine how they could sketch the graph of the function. All of them could not sketch the graph of the function correctly and attempted the limit of a function.

Some students experienced great misconceptions with this question. For example if we look the two students' responses as indicated below:

***Researcher**: Is f continuous at $x = 3$?* ***Researcher**: Is f continuous at$x = 3$?*

***Student 1**: yes.* ***Student 2** : yes.*

***Researcher**: Why?* ***Researcher** : Why?*

***Student 1**: because f is continuous in its domain.* ***Student 2**: because f is satisfied conditions of continuity.*

***Researcher**: Is f differentiable at$x = 3$?* ***Researcher**: Is f differentiable at $x = 3$?*

***Student 1**: yes.* ***Student 2**: yes.*

***Researcher**: why?* ***Researcher**: why?*

***Student 1**: because f is continuous at$x = 3$.* ***Student 2**: because f is continuous at $x = 3$.*

These two students were misunderstood the relationship between continuity and differentiability of a function at a point by thinking that the reverse of the theorem "every differentiable function at a point is also continuous at that point" is also true. ***This result is similar to the result obtained from problem based test result by item 4.***

4.3.3. Students' explanations for interview item 3 and 4.

The item 3 (see appendix B) dealt with the idea of a limit in a different context namely that of $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. One of students' misconceptions on workout problem test was that they put the quotient difference as sum .To triangulate this problem, the students were asked what the meaning of the limit in this symbolic expression is as well as what this expression is usually used for. The students experienced great difficulty with this question as indicated by their responses:

S1*: $f'(h) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, and we use this formula to know the slope of a line that is*

tangent to the point $(h, f(h))$.

S2: It defines the limit of a function at h *approaches to* 0 *and this formula is used to calculate limit of any function.*

S3: It is difficult to explain this expression, but we use it to calculate the limit value for any function as h *approaches to 0.*

From these explanations and workout problem test results, it is clear that the students did not have a clear concept image of the idea of a limit in this context. They can calculate a derivative by using this formula if they are asked to do so, but they cannot describe the concept correctly in their own words.

CHAPTER FIVE

5. SUMMARY, CONCULUSIONS AND RECOMMENDATION

5.1. Summary

This study was intended to investigate the type and extent of students' misconceptions on the concept of limit, continuity and differentiability in Awi zone preparatory secondary schools. Accordingly, an attempt has been made to answer the two basic research questions asked in the study. A mixed research approach was the method of the study with descriptive survey design. In line with this, all grade 12 students in three randomly selected preparatory secondary schools of Awi zone were used as accessible population and data was collected through workout problem test and structured interview from sample students. A total of one hundred eighty six (186) students were participated as sample in the study. Based on the analysis made on the data gathered through the above instruments in the specified schools, the major findings of the study are summarized as follows (compare with chapter 4):

1) **Students' misconceptions regarding to concept definitions on the limit, continuity and differentiability:**
   - Confusion between the symbols and definitions of the concept of limit and differentiability at a point.
   - Confusion between the definitions of limit of sequences and limit of a function.
   - Confusion between the symbols of derivative of a function at a point and derivative of function.
2) **Students' misconceptions regarding to limit concept:**
   - Students thought that the limit value at a point is equal to the function value at that point (i.e.,$\lim_{x\to a} f(x) = f(a)$.) This is true for continuous functions but not for piecewise functions or discontinuous functions.
   - Students thought that a limit can be found by a method of substitution. These students view that finding limits and the substitution process as exactly the same thing.
   - Confusion to understand the bounds where a quotient rule is applied to find the limit of a function and confusion on the fraction $\frac{0}{0}$. They thought that dividing

zero by zero is zero; other says that dividing zero by zero is 1 and some of the students thought that dividing zero by zero is undefined.

3) Students' misconceptions regarding to limit and continuity:

- Students thought that existence of limit at a point is a sufficient for a function to be continuous at that point.
- Students thought that a function has to be continuous and defined at a point to have a limit at that point or a function that is undefined at a certain point does not have a limit or discontinuous functions can not have a limit at the point of discontinuity.

4) Students' misconceptions regarding to differentiability:

- Students' lack of knowledge in underlying calculus concepts, in which calculus techniques such as differentiation are instrumentally understood. There was confusion when students tried to use the rules for differentiation particularly chain rule for composition function and Confusion between exponential and power functions to use power rule

5) **Students' misconceptions regarding to continuity and differentiability**:

- Students have misconceptions on the relationship between continuity and differentiability of a function in a point by thinking that the reverse of the theorem "every differentiable function in a point is also continuous at that point" is also true. In other words, some of the students in this study thought that if a function is continuous at a point then it is differentiable at that point.

6) Other interesting findings :

- Knowledge gaps in algebra or weak pre-calculus skills such as factorization, cancellations, and simplifying.
- Lack of knowledge in using the appropriate definition of absolute value functions.

5.2. Conclusions

In line with summary of findings of this study**,** generally**,** many students had different types of misconceptions on the concept of calculus. Major type of students' misconception on the concept of limit, continuity and differentiability was that many students did not understand the relationship between these concepts. For example, out of 95 students who answered incorrect responses, 56(59%) students thought that being a function defined at a point is sufficient for a function to be continuous at that point (see table 4), out of 111 students who answered incorrect response ,68(61%) students thought that if a function is continuous at a point then it is differentiable at that point (see table 8) and out of 111 students who answered incorrect response ,54 (49%) of students confused on the concepts of the existence of limit of a function at a point and continuity of a function at that point(see table 8). They thought that existence of limit at a point is a sufficient for a function to be continuous at that point. This result revealed that, many students did not understand the relationships on the concepts of limit; continuity and differentiability. Out of 119 students who answered incorrect responses , 60 (50%) students used mathematical symbols or algebraic expressions to define the concepts of definitions of derivative of a function at a point and they also used inappropriate symbols to represent concept definition of derivative a function at a point, for example, they used $f'(x) = \lim_{h\to 0} \frac{f(x+h)+f(x)}{h}$ to represent the derivative of a function at a point (see table 4) and out of 113 students who answered incorrect responses ,76(67%) students confused to understand the bounds where a quotient rule is applied to find the limit of a function and confused on the fraction $\frac{0}{0}$ (see table6).

Finally, I concluded that some important findings in this study were in accordance with those found in different literature. For instance, the limit value is equal to the functional value at a point (Jordaan, 2005&Ashebir, 2009) and if a function is continuous at a point then it is differentiable at that point (Meel cited in Duru et al; 2010). In general, many students had different types of misconceptions on the concepts of limit, continuity and differentiability in this study.

5.3. Recommendation

Based on the major findings of the study, the following recommendations are proposed:

- Students should be equipped in algebra or pre-calculus skills before starting introduction to calculus at these particular schools.
- Grade 12 mathematics teachers of studied schools should aware in light of findings of this research and existing literature, it is clearly identified that students have different misconceptions in important concepts of calculus such as limit, continuity and differentiability and the relationship between these concepts.
- Further studies should be studied on the root causes of misconceptions or sources of misconceptions for those studied preparatory secondary school students on the concept of limit, continuity and differentiability.
- Further studies should be needed to possible remedy or possible solutions to minimize preparatory secondary school students' misconceptions on the concept of limit, continuity and differentiability at these particular schools in Awi zone.

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Appendices

Appendix A

Addis Ababa University

School of Graduate Studies

College of Education and Behavioral Studies

Department of Mathematics Education

Workout Problem Test

Dear student,

This test is designed to collect information for the study on students' misconceptions on the concept of limit, continuity and differentiability in Awi preparatory secondary schools.

Thus, your frankness and sincerity in the responses is highly appreciated for it will contribute to the accuracy of the study's achievement in the end.

Therefore, you are kindly requested to respond carefully to each of the items in the questionnaire. Please note that the information you provide will use only for research purpose and you are not required to write your name.

Thank you in advance for your cooperation!!

Background in formations:

Instruction I: The following items are related to your background information. Please put the mark "**X**" in space provided for the item, and write in words where necessary.

1. Name of your school_____
2. Sex ☐ male ☐ Female

Workout Problem questions:

Instruction II: Attempt all of the following questions as required in provided spaces by writing all the necessary steps (if any).

1) **Define** in your own words what is meant by:

a) The limit of a function at a point.

b) The continuity of a function at a point.

c) The derivative of a function at a point.

2) Determine $\lim_{n\to\infty} \frac{3n^3-5n}{n^3-2n^2+1}$?

3) If $x \neq a$, then what is $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$?

4) Let $f(x) = \begin{cases} x^2, x < 3 \\ 6x - 9, x \geq 3 \end{cases}$,then

(A) Determine whether $f(x)$ is **continuous** or **not continuous at** $\boldsymbol{x = 3}$?

(B) Determine whether $f(x)$ is **differentiable or not differentiable at** $\boldsymbol{x = 3}$?

5) Let $f(x) = \begin{cases} 3, for x \neq 1 \\ 0, for\ x = 1 \end{cases}$, then determine whether $f(x)$ is **continuous or not continuous at 1?**

6) If f is a **differentiable** function then determine $f'(a)$?

7) If $f(x) = e^{x^2}$, then what is $f'(e)$?

8) If $e^{f(x)} = 1 + x^2$, then find $f'(x)$?

9) If $f(x) = 2 + |x - 3|\ \forall x$, then find the value of $f'(x)$ at $x = 3$

Appendix B

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Interview Questions

1) Let $f(x) = \frac{x-3}{x-3}$

(A) Sketch the graph of the function f?

(B) What is the value of $f(3)$?

(C) Does $f(x)$ have a limit at $x = 3$?

(D) If yes, what is the limit?

2) Let $f(x) = |x - 3|$

(A) Is f continuous at $x = 3$?

(B) Why?

(C) Is f differentiable at $x = 3$?

(D) Why?

(E) Is the limit of f exists at $x = 3$?

(F) Sketch the graph of f.

3) What does the following expression mean?

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

4) For what purpose you use the above formula?

Appendix C

Table 12: Answer keys for workout problem test

Item No	Answer
1(a)	Let f be a function which is defined at all x near$a$. If, as x tends to a from both the left and the right, $f(x)$ tends to some number b, then we say that the limit of $f(x)$, as x tends to a, is b and write $\lim_{x\to a} f(x) = b$ (informal definition).
1(b)	A function f is said to be continuous at a point a when $a\ \varepsilon D$ (domain of $f(x)$) , $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} f(x) = f(a)$. Or a function f is said to be continuous at a point $x = a$ when the graph of f has no gap or hole, that is when f is continuous in its entire domain (initiative definition).
1(c)	The derivative of a function at a point $x = a$ is a slope of a function at a specific point.
2	$\lim_{n\to\infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ $\Rightarrow \lim_{n\to\infty} \frac{3 - \frac{5}{n^2}}{1 - \frac{2}{n} + \frac{1}{n^3}}$ $= 3$
3	$\lim_{x\to a} \frac{(x^2-a^2)}{(a^4-x^4)} = \lim_{x\to a} \frac{(x^2-a^2)}{(a^2-x^2)(a^2+x^2)}$ (by canceling common factors) we have the following $\lim_{x\to a} \frac{1}{x^2 + a^2} = \frac{1}{2a^2}$
4(A)	$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$ and $f(3) = 9$ $\Rightarrow 9 = 9$ $\Rightarrow \lim_{x\to 3} f(x) = f(3) = 9.$ $\Rightarrow f$ is continuous at $x = 3$.
4(B)	$\lim_{x\to 3^-} \frac{f(x) - f(3)}{x - 3} = \lim_{x\to 3^+} \frac{f(x) - f(3)}{x - 3}$

	$\Rightarrow \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \lim_{x\to 3^+} \frac{6x-18}{x-3}$ $\Rightarrow\ 6 = 6$.(left side derivative and right side derivative are equal) $\Rightarrow f$ is differentiable at $x = 3$..
5	f is not continuous at 1 because $\lim_{x\to 1} f(x) = \lim_{x\to 1^{\pm}} f(x)$ and $f(1) = 0\ \neq= 3 = \lim_{x\to 1} f(x)$
6	$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ or $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$
7	Let y= e^u, where $u = x^2$ $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = e^u(2x) = e^{x^2}(2x)$ $\Rightarrow f'(x) = e^{x^2}(2x). \Rightarrow f'(e) = e^{e^2}(2e)$
8	By multiplying both sides by natural logarithm 'ln', we have $\ln e^{f(x)} = \ln(1+x^2)$ $\Rightarrow\ f(x)\ln e = \ln(1+x^2)$ $\Rightarrow\ f(x) = \ln(1+x^2)$, since $\ln e = 1$. But $f'(x) = \frac{1}{1+x^2}(1+x^2)' \Rightarrow f'(x) = \frac{2x}{1+x^2}$
9	$f(x) = 2 + \lvert x-3\rvert = \begin{cases} x-1, x \geq 3 \\ 5-x, x < 3 \end{cases}$ then $f'(x) = \begin{cases} 1, x \geq 3 \\ -1, x < 3 \end{cases}$ Hence , f is not differentiable at $x = 3$ because it has a sharp corner at this point. Or the left hand derivative and right hand derivative are not equal.

Appendix D

Table 13: workout problem test items with corresponding objectives and research questions.

Concepts	Item No	Objectives of items	Research objectives	Research question s
Concept definitions of limit, continuity & differentiability of a function at a point.	1(a) 1(b) 1(c)	To identify the type and extent of students' misconceptions on the concept definition of limit, Continuity and derivative of a function at a point respectively	1&2	1&2
Limit of sequences &limit of a function	2&3	To identify the type and extent of students' misconceptions on the concept of limit.	1&2	1&2
Limit continuity & differentiability of a function at a point.	4(A),4(B)&5	To identify the type and to determine extent of students' misconceptions on how the students could relate the concept of limit, continuity & differentiability of a function at a point.	1&2	1&2
Limit& differentiability	6,7,8&9	To identify the type and extent of students' misconceptions on the concept of derivative at a	1&2	1&2

		point.		

Appendix E

Table 14: Interview questions with corresponding objectives and research questions

Concepts	Question No	objectives of items	research objectives	research question
sketching graphs	1(A),2(D)	To identify the type and to determine the extent of students' misconceptions on the sketching graphs.	1&2	1&2
limit	1(C),1(D),2(C)	To identify the type and extent of students' misconceptions on the concept of limit.	1&2	1&2
Continuity and functional value	1(B),2(A)	To identify the type and extent of students' misconceptions on the concept of continuity and functional value.	1&2	1&2
differentiability	2(B),3&4	To identify the type and to determine the extent of students' misconceptions on the concept of differentiability.	1&2	1&2

Appendix F

Pilot test result of grade 12 students of Addiskidam preparatory secondary school.

	1a	1b	1c	2	3	4A	4B	5	6	7	8	9	Total
1	1	1	1	1	1	1	1	1	1	1	1	1	12
2	1	1	1	1	1	1	0	1	1	1	1	1	11
3	1	1	1	1	1	1	1	1	1	1	1	1	12
4	0	1	1	1	1	1	0	1	1	1	1	1	10
5	1	1	0	0	0	1	1	0	0	1	0	0	5
6	0	0	0	1	1	0	1	1	0	1	0	0	5
7	1	1	0	1	1	1	1	1	1	1	0	1	10
8	1	0	0	1	1	1	1	1	1	1	0	0	8
9	0	0	0	1	0	0	0	0	1	0	0	0	2
10	0	0	0	1	1	1	1	0	0	1	0	1	6
11	0	0	0	1	1	1	1	1	0	1	0	0	6
12	1	1	0	0	0	0	0	0	0	1	0	0	3
13	0	1	0	1	1	0	0	1	0	1	1	1	7
14	0	0	0	1	0	1	1	1	1	1	0	1	7
15	0	0	1	1	1	1	1	1	1	1	1	1	10
16	1	1	0	0	0	1	1	1	0	0	0	0	5
17	0	0	0	1	1	1	1	1	1	1	0	0	7
18	0	1	0	1	0	1	0	0	1	1	0	0	5
19	1	1	0	1	0	1	1	1	0	1	0	1	5
20	0	0	0	0	0	1	1	1	0	1	0	1	5
21	1	1	1	1	1	1	0	1	1	1	0	1	10
22	0	1	1	1	1	1	1	1	0	1	0	0	8
23	0	0	0	1	1	0	0	1	1	1	0	0	5
24	0	1	1	1	1	1	1	1	0	1	1	1	10
25	1	1	1	1	0	1	0	0	0	1	0	0	6
26	1	1	1	1	1	1	1	1	1	1	1	1	12
27	0	0	0	0	0	0	0	1	0	1	0	0	2
28	1	0	0	1	0	0	0	0	0	1	0	0	3
29	0	0	0	0	0	0	0	0	0	1	0	0	1
30	0	0	0	1	1	0	1	1	0	1	1	1	7
31	1	1	1	1	1	1	1	1	1	1	1	1	12
32	1	1	1	1	0	0	1	0	1	0	1	1	8
33	1	1	1	0	0	1	1	0	0	1	1	1	8
34	1	1	1	0	0	1	1	1	1	1	1	1	10
35	1	1	1	0	1	1	1	1	1	1	1	1	11
36	1	1	1	1	1	0	0	1	1	1	1	1	10
37	0	0	1	1	1	0	0	1	1	1	1	1	8
38	0	0	0	1	0	1	0	1	1	0	1	0	5
39	1	1	0	1	1	1	1	1	1	1	1	1	11
40	0	1	0	1	1	1	1	0	1	1	1	1	9
41	0	0	0	1	1	0	0	0	1	1	1	1	6
42	0	0	0	0	0	0	0	1	1	1	0	0	3
43	1	1	1	1	1	0	1	0	1	1	1	1	10
44	0	0	0	0	1	0	0	0	1	0	0	1	3
45	0	0	0	0	0	0	0	1	0	1	0	0	2
46	0	1	0	1	1	1	1	1	1	1	1	0	9
47	0	0	0	0	0	0	1	1	1	0	1	0	4

Appendix G

Table 15: Discrimination indices for the workout problem test for pilot test items

item	Number of upper group students	Number of lower group students	Discrimination indices
1a	12	2	0.51
1b	14	1	0.55
1c	13	0	0.28
2	13	2	0.47
3	14	1	0.55
4A	13	0	0.55
4B	11	0	0.47
5	14	3	0.47
6	14	3	0.47
7	14	6	0.34
8	13	0	0.55
9	15	1	0.6
Average			0.5

$$D = \frac{U-L}{\frac{1}{2}T}$$

Where D= Index of discriminating power.

U= the number of students who get the item correct from upper group.

L= the number of students who get the item correct from lower group.

T= the total no of students in the analysis (the total sum of upper and lower group).

Declaration

I, the under signed, declare that this thesis is my original work and has not been presented for a degree in any other university, and that all sources of material used for the thesis have been duly acknowledged.

_____ _____

Name of author Signature

_____ _____

Name of Advisor Signature