**ADDIS ABABA UNIVERSITY**

**SCHOOL OF GRADUATE STUDIES**

**INSTITUTE OF TECHNOLOGY**

**DEPARTMENT OF CIVIL ENGINEERING**

# An assessment of Pre-stressing in post-tensioned Concrete Box Girder Railway Elevated Structures: a case study on Comparative Analysis of Continuous Vs Simply Supported Elevated Structure of Addis Ababa's Light Rail Transit

**A Thesis Submitted to the Graduate School of Addis Ababa University in Partial Fulfillment of the Requirements for the Degree of Masters of Science In Civil Engineering**


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**April, 2015**

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## Abstract

Elevated structures are the key element in metropolitan city railways like Addis Ababa's Light railway Transit. This is because it is much more expensive, controls the system's capacity and so many more. This Paper which mainly focuses on the comparative analysis of the values of pre-stressing is done to predict percentages of the relative prestressing values if the simply supported segments need to be continuous in the future.

The objective of this research is to estimate the quantitative percentage values of pre-stress for continuously supported elevated structures in a reference to the simply supported ones and the resulting span increments, all are being straight line girders. Consideration of the varying cross-sectional area along the span makes the study more realistic and the results dependable.

In this study the main governing manual is AREMA 2010 - Volume II, manual for Railway Engineering. Again, almost all input values are exhaustively taken from Addis Ababa Light Rail Transit Construction Project Design Drawing (AA-LRT's CDD, 2013). The four basic loadings considered are Dead load, Secondary dead load, Live load and pre-stress load. Others loadings like wind load, earth quake load, etc could be considered. Yet, they do not have major influence on the relative percentage values of pre-stressing. The modeling and analysis are done for a total of four structures. On the one hand 20m (one-span simply supported) & 23.002m (two-span continuous) single cell each. And on the other hand 30m (one-span simply supported) & 97.758m (three-span continuous) double cell each. But, full pre-stress design is done for the double cell ones. The reason is that the single cell one's areal cross-section geometry does not meet the requirements of AREMA 2010's specification independently.

The maximum span increment is calculated by making the 20m simply supported segment two span continuous and the 30m simply supported segment three span continuous. The parameter used to determine span increment is deflection, caused by the structure weight, prior to the application of pre-stress and is calculated to be 4.7mm for the single cells and 15.6mm for the double cells. The deflection caused by Live load after the application of pre-stress also meets the requirement of not greater than l/640 of their all respective span lengths. The relative percentage value of pre-stressing for each continuous structure is compared with the respective simply supported segment. Single cells are stressed from both ends only. But, the double cells are stressed using two prestressing schemes: tensioning from the start end and tensioning from both ends. From SAP 2000 modeling results the 20m spans are increased to 23.002m. Again, by keeping both end span 30m, the middle span of three span continuous box girders is increased to 37.758m.

The Prestress analysis involves inputting values and comparing the results and reiterating the process until convergence.

Studying the comparative values of pre-stressing, making the simply supported one's continuous, help us to know and figure out the relative advantages of it. For the same size of cross-section, the span in a continuous ones made increased than in a simply supported one. Generally, increasing the span has a key role in a significant reduction of the number of piers that are likely to be a key source of obstruction to the traffic flow.

**Key words:** simply supported elevated structures, continuously supported elevated structures, pre-stressing, span increment, deflection and the relative percentage values of prestressing.

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## Nomenclatures

$A_s$ = area of non-prestressed tension reinforcements ($mm^2$)
$A^*_s$ = area of prestressing steel ($mm^2$)
$A_{ps}$ = area of pre-stressing steel ($m^2$)
$A_g$ = gross area of section ($m^2$)
b =width of flange of flanged member or width of rectangular member (mm)
$b'$ = width of a web of a flanged member (mm)
d = distance from extreme compression fiber to centroid of the prestressing force (mm)
$d_b$ = nominal diameter of prestressing strand (mm)
$d_p$ = distance from extreme compression fiber to centroid of prestressing steel (mm)
e = base of Naperian logarithms
$e_m$ = average prestressing steel eccentricity at critical sections (m)
$E_c$ = modulus of elasticity of concrete (MPa)
$E_{ci}$ = modulus of elasticity of concrete at transfer (MPa)
$E_p$ = modulus of elasticity for pre-stressing tendons (MPa)
$E_s$ = modulus of elasticity of steel reinforcement (MPa)
$f_b$ = concrete stress at the bottom fiber of the girder beam ($KN/m^2$)
$f_{cds}$ = stress in concrete at centroid of prestressing reinforcement, due to all dead load not included in calculation of $f_{cir}$ (MPa)
$f_{cir}$ = stress in concrete at the centroid of pre-stressing reinforcement immediately after transfer due to total pre-stress force and dead load acting at transfer(MPa)
$f_{cit}$ = extreme fiber stress in tension of concrete immediately after prestress transfer (MPa)
$f_{ct}$ = extreme fiber stress in tension of concrete at service loads (MPa)
$f_{pbt}$ = stress in prestressing steel immediately prior to transfer (MPa)
$f_{pu}$ = ultimate tensile strength of post-tensioned tendon (MPa)
$f_{pj}$ = jacking stress, (MPa)
$f_{su}$ = average stress in prestressing steel at ultimate load (MPa)
$f_y$ = yield stress of steel (MPa)
$f'_s$ = tensile strength of the pre-stressing steel (1860 MPa)
$f'_c$ = compressive strength of concrete at 28 days (MPa)
$f'_{ci}$ = compressive strength of concrete at time of initial prestress (MPa)
$f'_{cir}$ = average concrete stress at the c.g. of the prestressing steel at time of release (MPa)
$FC_{ES}$ = Coefficient of loss due to elastic shortening (instantaneous loss)
$FC_{TL}$ = Coefficient of loss due to time dependant loss
h = overall depth of member in (mm)
I = moment of inertia about the centroid of the cross section in ($m^4$)
$I_g$ = moment of inertia of the gross concrete section ($m^4$)

K = friction wobble coefficient meter of prestressing steel (/m)

l = length of prestressing steel element from jack end to point x in (mm)

$M_{cr}$ = moment causing flexural cracking at section due to externally applied loads (KN-m)

$M_{LL+I}$ = Unfactored BM due to live load plus impact (KN-m)

$M_g$ = Unfactored bending moment at critical sections due to self weight (KN-m)

$M_{SDL}$ = Unfactored BM due to Superimposed dead load

$M_{max}$ = maximum factored moment at section due to externally applied loads in (KN-m)

$M_n$ = nominal moment strength of a section, (KN-m)

N= number of tendons

p = $As/bd_t$, ratio of non-prestressed tension reinforcements

$p^*$= $A^*s/bd$, ratio of prestressing steel

$P_{se}$ = effective prestress force after allowing for all losses (KN)

$S_b$ = bottom fiber section modulus ($m^3$)

t = compression flange thickness (m)

$w_c$ = weight of concrete ($Kg/m^3$)

x = distance from stressing location (m)

μ = friction curvature coefficient (unit less)

α= total angular change of prestressing steel profile in radians from jacking end to point x (radian)

$\beta_1$ = factor for concrete strength

γ = factor for type of prestressing steel

φ= Strength Reduction Factor

$\Delta f_{PF}$ = friction loss, ($KN/m^2$)

$\Delta f_s$=total prestress loss, excluding friction

$\Delta f_{se}$ = effective stress prestress after losses

$f'_y$ = yield strength of non-prestressed conventional reinforcement in compression (MPa)

## Abbreviations

AA-LRT CDD – Addis Ababa Light Rail Transit Construction Project Design Drawing

AREMA – American Railway Engineering and Maintenance-of-way Association

AASHTO - American Association of State Highway and Transportation Officials

ADL = Additional or secondary Dead Load

CGC = Center of gravity of the concrete section

CGS = Center of gravity of the stressing steel

$CR_c$ = loss of prestress due to creep of concrete (MPa)

$CR_s$ = loss of prestress due to relaxation of prestressing steel (MPa)

DL = Dead Load

ES = loss of prestress due to elastic shortening (MPa)

FR = friction loss below $0.70f'_s$ at the point being considered.

GPa = Giga Pascal ($10^9$ N/m$^2$)

KPa – Killo Pascal ($10^3$ N/m$^2$)

LL or ML = Moving or Live Load

LL+IM = Live load including impact force

MPa = Mega Pascal ($10^6$ N/m$^2$)

SLS = strength limit state

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# CHAPTER 1

## 1. Introduction

## 1.1 Background of the research

### 1.1.1 History of rail transport of the world in brief

According to the History of Rail Transport [23], Railway transport history dates back to as early as Greek times, nearly 500 years, including systems with man or horse power, and tracks or guides made of stone or wood. The first true railway has been a funicular railway made at Broseley, England around 1605. And, the modern rail transport systems first appeared again in England in the 1820s. These systems which were mechanized and made use of the steam locomotive were critical to the Industrial revolution and to the development of export economies across the world. They have remained the primary form of land transport ever since for most of the world. The history of rail transport gives credit for prominent personalities like William James (1771 - 1837), the Father of Railway. He was inspired by the development of the steam locomotive to suggest a national network of railways and proposed long term development of railway. Electric railways revolutionized urban transport and in 1881, the world's first electric tram line opened in Lichterfelde, Germany. Accordingly, the trains began to be powered by electricity instead of steam. Rail transport had originally been conceived to carry industrial goods but the value of carrying railway passengers overtook goods during 1842 through 1850 for the first time.

### 1.1.2 History of rail transport of Ethiopia in brief

According to the Rail Transport in Ethiopia [24], the construction of railway lines in the country first began in October 1897 from Djibouti during the reign of Emperor Menelek II. The first commercial service began in July 1901, from Djibouti to Dire Dawa. It is, then, 18 years later that the line reached Akaki in 1915, only 23 kilometers from the capital. And, after two years was that it reached Addis Ababa itself. However AAU Civil Engineering Department, TCDC and others [1] states that the move in modern railway construction sparked after the national railway development launched by Council of Ministers, Regulation No. 141/2007 which established the public enterprise called Ethiopian Railways Corporation in 2007. Following this, the government of the Federal Democratic Republic of Ethiopia (FDRE) has launched a 5 year Growth and Transformation Plan (GTP) in September, 2010. The policy statement aimed the construction of the National Railway Networks of Ethiopia which extends to about 5000Km and 34 km Light Rail Transit (LRT) system for the capital city, Addis Ababa to be constructed in two phases.

### 1.1.3 Addis Ababa Light Rail Transit, AA-LRT

According to Addis Ababa Light Rail [26], AA-LRT is a light rail system which is currently under construction in Addis Ababa, Ethiopia. The project was commenced in December, 2011 and to be completed in three years time. It will be a standard gauge of 34.25 Km long double track electrified for the whole route. It extends to two rail lines which render a transportation service to 60,000 people per hour in all four directions. The north to south line i.e. 16.9 Km long, from Menilik Square to Kaliti, will pass through Merkato, Lideta, Legehar, Meskel Square, Gotera. The east to west line i.e. 17.35 Km long, from Ayat Square to Tor Hailoch, will pass through Megenagna, Legehar, Mexico Square. Both lines which share a common track of about 2.8Km have a total of 41 stations each with a capacity to carry 286 passengers.

With the increment in the number of populations and expansion of urbanization, it is obvious that the transportation needs increases too. Now a day's railway transport has become dependable mode of transportation in addressing such mass transport need than any other modes of transportation. Railway transport is the most efficient mechanized land transport because it has a high level of passenger or cargo utilization over a limited space. It is also very cheapest, safest and has some other advantages over other transportation modes. Accordingly, these days there is high demand of railway transportation in the world including in our country Ethiopia for the transportation of both passengers and goods. Such demand needs to be backed with advanced design and analysis of functional components which have a direct or an indirect impact on the users demand. Among those many complex analyses, this paper analyzes pre-stressing (essentially post-tensioning) and compares the relative values of it on simply supported segments Vs continuous ones. Again, it deals with increment of the spans. Span increment means the maximum possible distance that the simply supported spans could be increased when made continuous i.e. by keeping cross-sectional dimensions the same for both cases. This research mainly focuses on investigating in which type of structure that post-tensioning gets more advantageous. This is done by conducting analysis and using different parameters to compare.

First of all span increment, maximizing the spans, is dealt prior to post-tensioning. It is done considering only deflection due to the structure weight. Then, immediately after the concrete casted attained its initial compressive strength, initial tension is exercised. When the concrete strength and its elastic modulus reach the design values final tension applied. The post-tensioning force is applied to resist all the loading cases. Because of different structural behavior the post-tensioning properties like pre-stress losses, jacking forces, number of tendons and some other factors get differ in each type of the structures. To put it in a nut shell, in order to know in which type of the structure does post-tensioning takes more advantage it is better to analyze and understand post-tensioning relatively following spanincrement.

## 1.2 Railway Transport Advantages and Disadvantages

### 1.2.1 Advantages

- It is the most efficient transportation mode that has a high level of passenger and goods utilization over a relatively small space
- Most efficient energy consumer
- environmentally friendly (electric powered mostly and less emission of $co_2$)
- Less use of land for transportation
- Safest transport mode with lowest accident rate
- Cheapest transportation mode
- It offers comfortable ride options such as meal, sleeping and entertainment

### 1.2.2 Disadvantages

- High initial investment and operation costs
- Difficult to function in a severe alignments like steep slopes and sharp curves
- High noise emission
- Low flexibility (very rigid regarding network operations)
- Accident is sever once it happens
- Security threat: susceptible to terror attacks

## 1.3 Statement of the Problem

Post-tensioning is required to increase the load carrying capacity of prestressed box-girder than ordinary reinforced box girder. During the comparison of simply supported Vs continuous box-girder (with the same cross-section) the main problem is identifying the relative values of Post-tensioning and related issues. Because the very structural frame work differs, it is obvious that the Post-tensioning values would not be similar. And both structures meant to bear the same design loadings being different structurally. Accordingly, there are two main questions that need to be answered, like:

I. In which type of elevated structure does post-tensioning takes more advantage in:
   - Having lower tensioning force
   - Having lower number of tendon or pre-stressing strands
   - Provision of relevant end anchorages

II. To which maximum distance can the span be increased in continuous ones keeping the same cross-sectional dimensions with that of the simply supported ones?

Such and other kinds of questions divert the interest of the study to focus on an assessment of comparative nature of post-tensioning in different structures, specifically in simpler ones: continuous Vs simply supported.

### 1.4 Significance of the Research

This study has a great impact on future analysis and application of post-tensioning in general and in the Ethiopian context of Light Rail Transit construction projects. Specifically, it directs the analysis and estimation of the relative percentage values of post-tensioning like Tensioning force, pre-stress losses, Area of pre-stressing steel and span increment. As simple spans are made continuous post-tensioning values decrease quantitatively. The reason is that for the same cross section there is increased structural integrity of the spans in continuous ones making the need of pre-stressing significantly lower.

This paper provides optimization of the prevailing simple spans of AA-LRT's elevated structures of both single cell and double cells. Post-tensioning analysis has been done for those span increments in a relative to their respective simple spans.

### 1.5 Scope and Objective of the Study

#### 1.5.1 Scope of the study

The scope of this study includes the outputs of pre-stress design. The emphasis is on both simply supported and continuous box-girder elevated superstructures of AA-LRT utilizing bonded tendons but the basic principles are also applicable to external tendons. The scope of this study does not include such items as anchorage zone design and couplers. In addition, conceptual design and overall structural design is not addressed.

#### 1.5.2 General Objectives

The general objective is that

- ✓ To investigate the comparative nature of post-tensioning in simply supported Vs continuous box-girder elevated structures and identifying the one through which it's advantageous
- ✓ Identifying the maximum possible distance that the span can be increased in continuous one keeping the same cross-sectional dimension with the simply supported one

#### 1.5.3 Specific Objectives

This is done by comparing the following values:

- ✓ The relative tensioning force (the initial pre-stressing force) required to be applied
- ✓ The relative maximum increment of the span when simply supported ones are made continuous keeping both cross-sectional dimensions the same
- ✓ The relative area of tendons, $A_p$ & the corresponding maximum eccentricity, $e_{max}$
- ✓ The relative prevailing deflection at various stages of loadings
- ✓ The relative provisions of end anchorages

### 1.6 Design Theory

The post-tensioned concrete box girders shall be designed as follows:

- ✓ The prestressing force shall be determined by Allowable Stress Design using elastic theory for loads at the service level considering the AA-LRT CDD Live load.
- ✓ The ultimate moment capacity shall be checked by Load Factor Design using ultimate strength theory (Strength Limit State) for loads at the factored level.
- ✓ Shear design shall be based on Strength Limit State (Load Factor Design) using ultimate strength theory with factored loads.

### 1.7 General Conditions

During the formulations of this paper the general conditions considered are basic theory of the main distinguishing characteristics of prestressed concrete as compared to ordinary reinforced concrete. These are that

- ✓ The stresses for concrete and prestressing steel at each stages of prestressing is investigated on the basis of elastic theory
- ✓ The prestressing force is determined by concrete stress limits under service load by meeting stress limits in the service limit states, and then checked in the strength limit states for ultimate capacity
- ✓ Flexure and shear capacities are determined based on the ultimate strength theory (Load Factor Design)
- ✓ To design the pre-stressed members all stress limits should be satisfied: Temporary Stress before loss, Final Stress after loss at service load

# Chapter 2

## 2. Literature Reviews

### 2.1 Introduction

This research paper is one part of previously related works which are guide for the start of the current work. Some of them have a direct relation with this work whereas the others have indirect relations. However, the main principles, methodologies and approaches they precede will be selected generally and applied for the formulation of specific model and analysis.

Generally, there are many journals, manuals, design works, conference papers, proceedings, and books related to simply supported box-girders, continuously supported box-girders and particularly post-tensioned concrete box-girders etc. Yet, there are limited resources regarding railway elevated structures which provides necessary grade separation at roads and have nearly the same nature with those bridges. To save time and to manage the paper work the review of literatures mainly focuses better related works. This intensifies the deep analysis of the previously related woks and selection of appropriate ways for the successful accomplishment of the paper.

The analysis of relative post-tensioning requires detail analysis of increased span of concrete box-girder elevated structure and post-tensioning properties like pre-stress losses, Jacking force and the resulting number of tendons and other factors affecting all these parameters. Therefore, to understand the general functions and properties of these parameters it is preferable to see each sub components separately as follows.

#### 2.1.1 Pre-stressed Concrete Box Girder Beam

Pre-stressed concrete box girder beam is a hollow rectangular section and may be placed against one another, similar to pre-cast planks, or may be spaced apart. A reinforced concrete deck slab is cast on top. Close contact eliminates or minimizes the need for deck slab formwork. Span ranges are similar to those of standard I-girders but, in comparison to I-girders, the section itself utilizes more material and requires more complex forms. Cast-in-place closed box girder sections usually have multiple webs. The torsion rigidity of closed box sections makes this type of construction well suited to sharply curved viaducts and interchange ramp structures [4]. Piers used to elevate this structure.

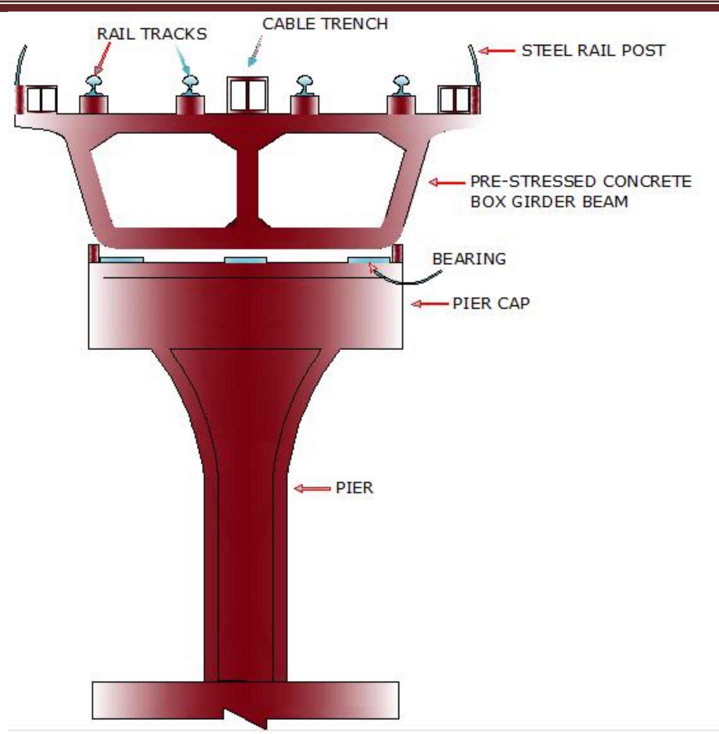



Figure 1.1: Typical cross-section of Double Cell, AA-LRT's Elevated structure

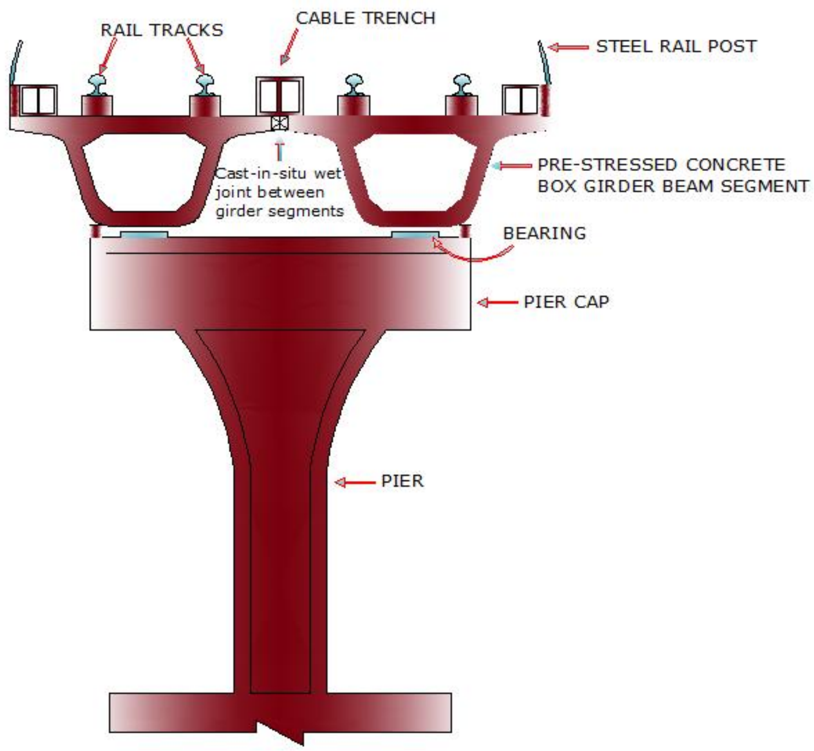



Figure 1.2: Typical cross-section of Two Single Cell Connected with Epoxy Joint (cast-in-situ wet joint), AA-LRT's Elevated structure

There are substantial numbers of elevated crossing structures in the AA-LRT project. These structures isolate the light rail transit (LRT) from other conflicting transportation modes prevailing in the city. It also provides AA-LRT's way upper than the rest transportation systems so that decreasing interference in traffic operation. Post-tensioned Concrete Box Girder Railway Elevated Crossings being the preferred type in the project, they provide a solid deck for track laying. Pre-cast, Post-tensioned segmental concrete construction is the fast and efficient method for structure assembly these days, elevated crossing structures essentially. This method allows the construction of very long spans. In AA-LRT's project post-tensioned concrete segments with a span length of 20m, 25m, 30m, etc are used for straight station girders.

### 2.1.2 The stressing Tendon

The stressing tendon is one of the basic elements of a post-tensioning system. A tendon consist one or more strands, constrained at both ends by anchorage and encapsulated throughout within a duct. In the Figure 1.3 a general scheme is shown of a tendon consisting of two part tendons joined by a coupler.

All tendons can either be pre-assembled or pulled into the duct or the strands pushed individually into the duct with the aid of a strand pusher, before or after concreting to suit the construction sequence. All tendons are stressed with the aid of hydraulic jacks.

The strands used for post tensioning tendons are comprised of 7-wires low relaxation steel. The most common diameters are 15.2/15.7 mm and 12.7/12.9 mm corresponding to tensile strengths of 1770/1860 N/mm$^2$ and 1860 N/mm$^2$ respectively [16].

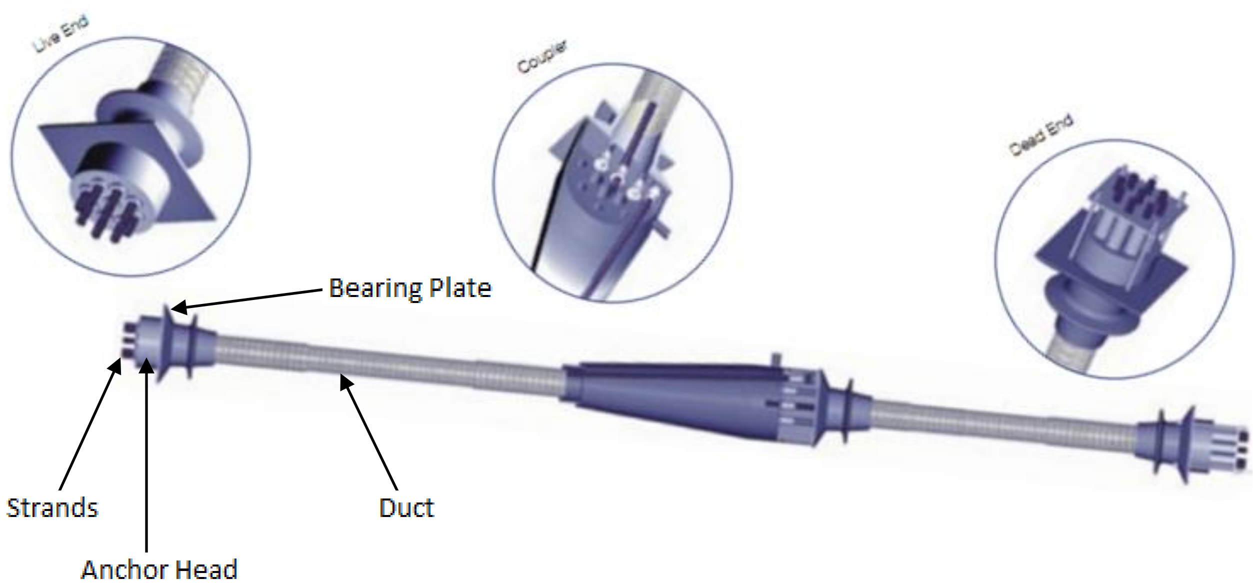



Figure 1.3: The stressing tendon

### 2.1.3 Pre-stress Losses

As per NYSDOT Bridge Design Manual [17] Loss of prestress is the difference between the initial tensile stress in prestressing tendons at the time the strands were seated in their anchorages, and the effective prestress at a particular time at the considered location.

Losses that apply to both pre-tensioned and post-tensioned elements are Concrete Shrinkage, Elastic Shortening, Concrete Creep, and Steel Relaxation. Losses that apply only to post-tensioned elements are Anchorage Set and Friction (for drape and wobble). Computation of the losses shall be as per the applicable provisions of the design specifications. Concrete Shrinkage, Shrinkage after hardening of concrete, is the decrease with time of concrete volume. The decrease is due to changes in the moisture content of the concrete and physical-chemical changes, which occur without stresses attributable to actions external to the concrete. Shrinkage is conveniently expressed as a dimensionless strain under steady conditions of relative humidity and temperature.

**Elastic Shortening** - The concrete beam shortens at transfer when the prestressed strands are released and the force in them is transferred to the concrete. This elastic shortening is immediate and results in a reduction in the strain of the prestressing steel and therefore a prestress loss. The loss from elastic shortening should be included in both initial and total loss computations.

**Concrete Creep** - The time dependent increase of strain in hardened concrete subjected to sustained stress is defined as concrete creep.

**Steel Relaxation** - Steel relaxation is very similar to concrete creep. With steel relaxation the length of the strand is held constant under stress and there is a time dependent loss in stress.

**Anchorage Set** - Some loss of prestress occurs to post-tensioned tendons as the anchorage hardware deforms and sets at the transfer of tension. The amount of set is a function of the type of anchorage system used. The amount of prestress loss is a function of this anchorage set and the length between anchorages. Power seating of the chucks tends to reduce this loss.

**Friction** - Tendons also lose some prestress due to friction inside the ducts during stressing operations.

**Total Losses** - Some of the losses mentioned above are interdependent. Shrinkage and concrete creep reduce the strain in the prestressing steel, which reduces the force in the prestressing steel. The reduction in force in the prestressing steel affects elastic shortening, future concrete creep and steel relaxation.

According to [6] some of the imposed strain on the strand is lost when the wedge seats in the plate.

Friction and anchorage losses are as depicted in the figures below. The stress diagrams of all the rest losses are directly proportional or gets down in a linear manner with the friction and anchorage losses.

Figure 1.4: Jacking from one end:

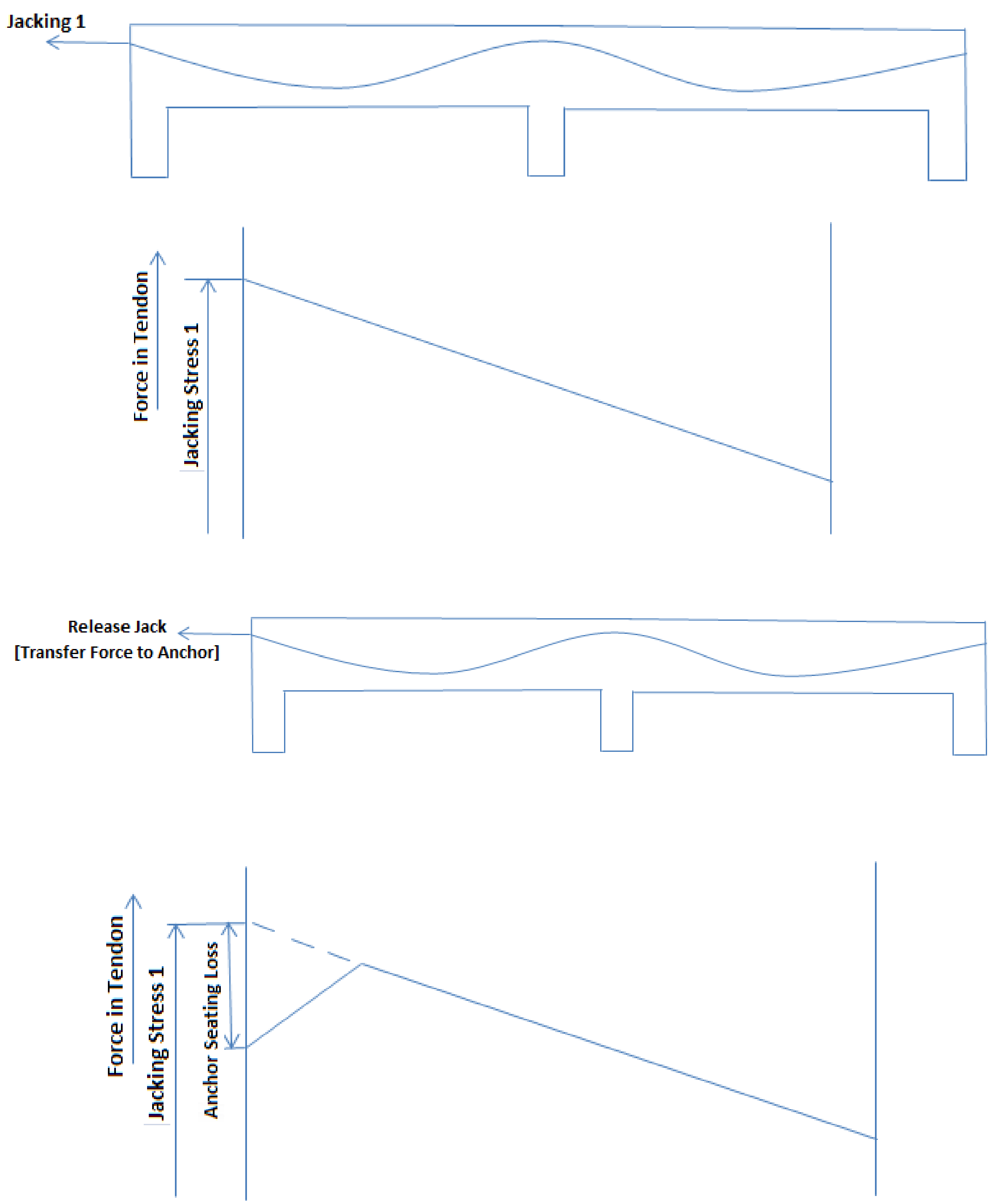
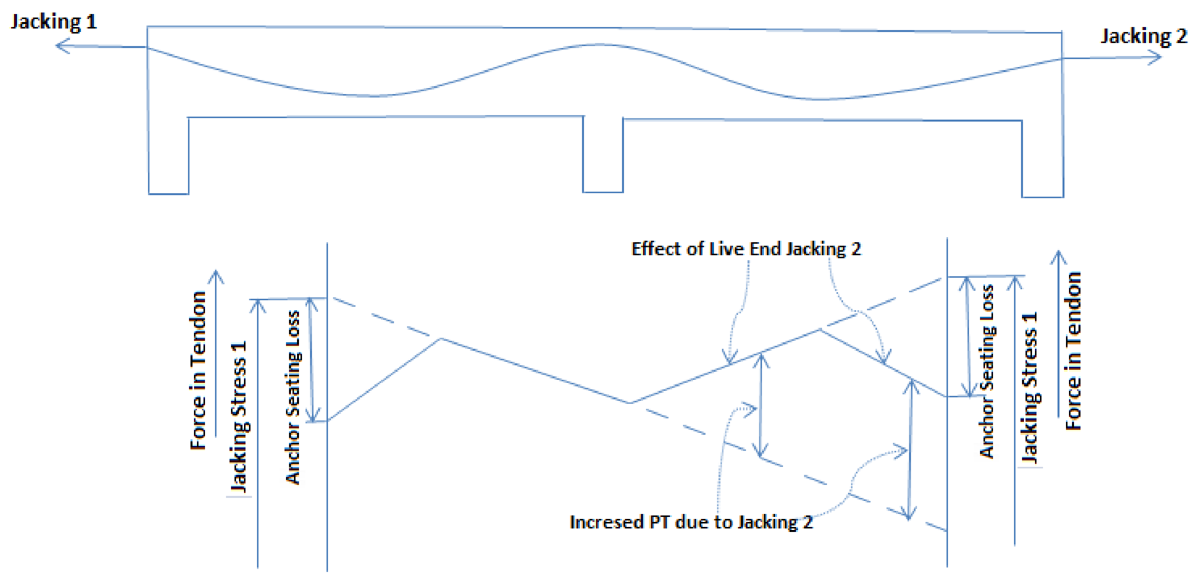


Figure 1.5: Jacking from both ends consecutively




### 2.1.4 The prestressing force

The pre-stressing force is the force that needed to be applied to tension the pre-stressing steels based on the minimum allowable tension requirement. Bridge Design Practice [4] states that there are two methods of applying prestressing force. Pre-tensioning indicates that tensioning of the steel is done before the concrete is cast in the forms. Post-tensioning means that the steel is tensioned after the concrete has been cast and attained the required strength. In the former, the force is transmitted by bond between the steel and concrete. The initial prestress is immediately reduced due to the deformation and shrinkage of the concrete. Gradually, these losses are increased by further shrinkage and creep of the concrete. In post-tensioning there are no immediate losses but there is a gradual loss due the shrinkage and creep of the concrete and the relaxation of the steel.

So, post-tensioning could generally be defined as an application of prestressing force by which tendons stressed in the post-tensioning ducts or tubes of cast-in-pace concrete at the time when the concrete attain its initial strength. In a post-tensioning operation the tendons are stressed using two types of tensioning schemes: either from one end or from both ends.

### 2.1.5 Advantages of prestressed concrete over ordinary reinforced concrete

There are significant advantages of prestressed concrete over ordinary reinforced concrete. Some of them best described in a Bridge Design Practice [4] as follows

- Reduction of concrete and steel quantities
- Considerable reduction in depth of section, not only relative to reinforced concrete, but also relative to structural steel

- Crack less concrete within a known range of load. This results in greater durability under severe conditions of exposure
- Possesses maximum rigidity under working loads and maximum flexibility under excessive overloads
- Provides capacity to support a load in excess of the design load in which cracks appear but disappear completely on removal of the excess load
- Provides resistance to repeating and alternating loads even when exceeding the design load
- Produce definite reduction in diagonal tension which leads to fewer stirrups needed for shear
- Makes it possible to control and/or reduce long term deflections
- Added flexibility for construction of continuous multi-span frame

### 2.1.6 Disadvantages of prestressed concrete over ordinary reinforced concrete

The Bridge Design Practice [4] also states the disadvantages of prestressed concrete over ordinary reinforced concrete:

- The prestressed structure is more sensitive to quality of workmanship and materials
- Creep and shrinkage of the concrete and relaxation of the prestressing steel are important considerations which need to be considered by the designer

### 2.1.7 Comparison of continuous Vs simply supported Post-tensioned Concrete Box-Girder

The main difference that exists among the design of continuous versus simply supported precast members is that secondary moments are induced in the continuous ones due to prestressing. Precast members are generally designed to be simply supported for dead load, and continuous for additional dead load and live load, whereas the cast-in-place box girder is designed as continuous for all loadings. Also, the cable path is usually much longer, and the effects of friction loss due to the angle change play a role in the design process. Certain design criteria and specifications also differ between the two types of design [4].

As per Dr. Amlan K Sengupta and Prof. Devdas Menon [9] Continuous beams are made to increase the structural integrity so as to have more redundancy in the structure. In case of a failure at a particular section, there will be an alternative load path for the load to go down to support of the structure. In addition, there are advantages and disadvantages of continuous beam over simply supported beams.

#### 2.1.7.1 The advantages of a continuous beam as compared to a simply supported beam are as follows:

- For the same span and section; vertical load capacity is more

- The mid span deflection is less for continuous beam as compared to a simply supported beam. Since there is a strain at the two ends, the deflection at the middle is reduced in the continuous beam
- The depth at a section can be less than a simply supported beam for the same span
- A continuous beam requires less number of anchorages of tendons.

#### 2.1.7.2 The disadvantages in continuous beams as compared to a simply supported beam are

- Continuous beam is difficult to analyze and design.
- Increased frictional loss due to changes of curvature in the cable profile. If you have a cable running through the spans and the supports, there will be more changes in curvature and this will have increased frictional loss throughout the length of the beam. That is why at times the tendons are interrupted so as to reduce the frictional loss.
- Increased shortening of beam leading to lateral force on the supporting columns. As the beam shortens due to the effect of creep and shrinkage, this will induce some forces in the support and these forces need to be considered during the design of the support of the beams.
- Secondary stresses develop due to time dependent effects like creep and shrinkage, settlement of support and variation of temperature. The continuous beam is a statically indeterminate structure and hence it is subjected to secondary stresses which come due to the moment, creep and shrinkage

## 2.2 Prestressing

Prestressing is an operation which is sub-divided in to two as stated in Michael,P.,&Denis,M [14]: Pre-Tensioning & Post-Tensioning. In Pre-Tensioning the tendon is tensioned prior to casting the concrete, while in Post-Tensioning the tendon is tensioned after the concrete has been cast.

Prestressed Concrete as defined in WisDOT Bridge manual [22] is a concrete in which there has been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loadings are counteracted to a desired degree. In reinforced concrete members the prestress is commonly introduced by tensioning the steel reinforcement.
This internal stress is induced into the member by either of the following prestressing methods: Pre-Tensioning & Post-Tensioning.

## 2.3 Post-Tensioning

Post-tensioning is a method of prestressing in which tendons are tensioned after concrete has hardened or reached a required strength.

Post-tensioning has got its own procedures as stated in WisDOT Bridge manual [22]. The concrete member is first cast with one or more post-tensioning ducts or tubes for future insertion of tendons. Once the concrete is sufficiently strong, the tendons are stressed by jacking against the concrete. When the desired prestress level is reached, the tendons are locked under stress by means of end anchorages or clamps. Subsequently, the duct is filled with grout to protect the steel from corrosion and give the added safeguard of bond. In contrast to pre-tensioning, this is usually incorporated in pre-casting (casting away from final position), post-tensioning lends itself to cast-in-place construction.

### 2.4 Calculating the prestress force

There are different ways of calculating the prestress force for the two basic types of structures: statically determinate structure and continuous structure

#### 2.4.1 Calculating the prestress force for statically determinate structure

Calculating the prestress force for statically determinate structure is very simple. Robert B. [18] studied as to how to calculate the prestressing force with the statically determinate beam, it was possible to calculate the prestress force directly at any location along the beam; the only data required are the total bending moment, the eccentricity of the prestress centroid and the geometric properties of the cross section.

So that the tensioning force found by dividing the total bending moment due to prestress by the vertical eccentricity of tendons provided like this: $P_j = \frac{M_{total}}{e}$.

#### 2.4.2 Calculating the prestress force for continuous structure

The prestress force computation for continuous beams cannot be done directly for the reason that the total bending moment includes the secondary or parasitic moment. Robert B. [18] states that this parasitic moment depends on the prestress force and on the locus of the prestress centroid along the beam. Consequently, one has to proceed by trial and error. At each design section there are three unknowns, the prestress force P, the eccentricity e and the parasitic moment. The following techniques allow the designer to control the trial and error process.
In a continuous beam, there are several options for the arrangement of the prestress.
Then it considers two methods of pre-stressing. These are Prestress Scheme 1 which assumes that the prestress will be constant over the full length of the continuous beam and Prestress Scheme 2 in which the friction losses are dealt to make the option economical.

Unlike the simply supported ones, during the design of continuous post-tensioned concrete box girders secondary moments are introduced due to prestressing.
The Bridge Design Practice [4] states that precast members are generally designed to be simply supported for dead load, and continuous for additional dead load and live load, whereas the cast-in-place box girder is designed as continuous for all loadings. Also, the cable path is usually much longer, and the effects of friction loss due to the angle change play a role in the design process. Certain design criteria and specifications also differ between the two types of design.
Accordig to Dr. Amlan K Sengupta and Prof. Devdas Menon[10] secondary moments in the continuous beams are reactions at the supports generated by pre-stressing. The reactions at the

intermediate supports cause moment at a section of the continuous beam and the resultant moment ($M_2$) at a location due to the prestressing force can be written:

$$M_2 = M_1 + M'_1$$

Where: $M_1$ (the primary moment) = moment due to the eccentricity of the prestressing force neglecting the intermediate supports

= $P_e e$

$P_e$ = effective prestress

e = eccentricity of CGS with respect to CGC

$M'_1$(the Secondary moment) = moment due to the reactions at intermediate supports

The moment due to the external loads ($M_0$) that is obtained from the envelop moment diagrams is added to $M_2$ to get the total resultant moment ($M_3$) at a location.

$$M_3 = M_2 + M_0$$

$$M_3 = M_1 + M'_1 + M_0$$

# Chapter 3

## 3. Analysis Overview

This chapter describes the analysis model, material properties, applied loads, strength and serviceability limit states design requirements. The analysis is performed using the program SAP2000 and excel spreadsheet calculations.

The purpose of the analysis is to determine the maximum distance that the spans in a simply supported structure be increased and also to determine the amount of prestressing (essentially post-tensioning) values for each structures. These values are compared among each type of structures of both single cells and double cells. By examining the optimization and relative percentage values, the study determines in which structure is that all these parameters get advantageous quantitatively.

The general step by step procedures followed in the SAP 2000 bridge wizard for detail analysis to investigate the values of post-tensioning in all four structures are

- Initially assumed pre-stressing force and its area of pre-stressing steel values are entered based on the AA-LRT CDD leaving all friction and other losses equal to zero and run the analysis
- From the results of the analysis the bending moment diagrams and prestress force requirement for each section along the discretized span lengths found. For simple spans the prestress moment is the result of $P_j$ and e only. Accordingly, to get the prestress force the prestress moment is divided by e. Unlike the simply supported box girders, in the continuous ones there is secondary moment. So, in order to find the prestress force for the continuous box-girders the secondary moment has to be deducted from the total prestress moment before it's divided by e. Then, the resulting values of prestress force and losses would be re-entered from the sections where maximum prestress force occurs.
- On the next subsequent analyses, the elastic shortening losses, time dependant losses and the pre-stressing force with its prestressing steel area would be made an input until all the values converge

### 3.1 Analysis Models

The analysis models are straight station girders which could be classified as single cell and double cells. The single cell: 20m-span simply supported & 46.004m two-span continuous and the double cells: 30m-span simply supported & 97.758m three-span continuous. Typical plans and elevations are shown in the figures below.

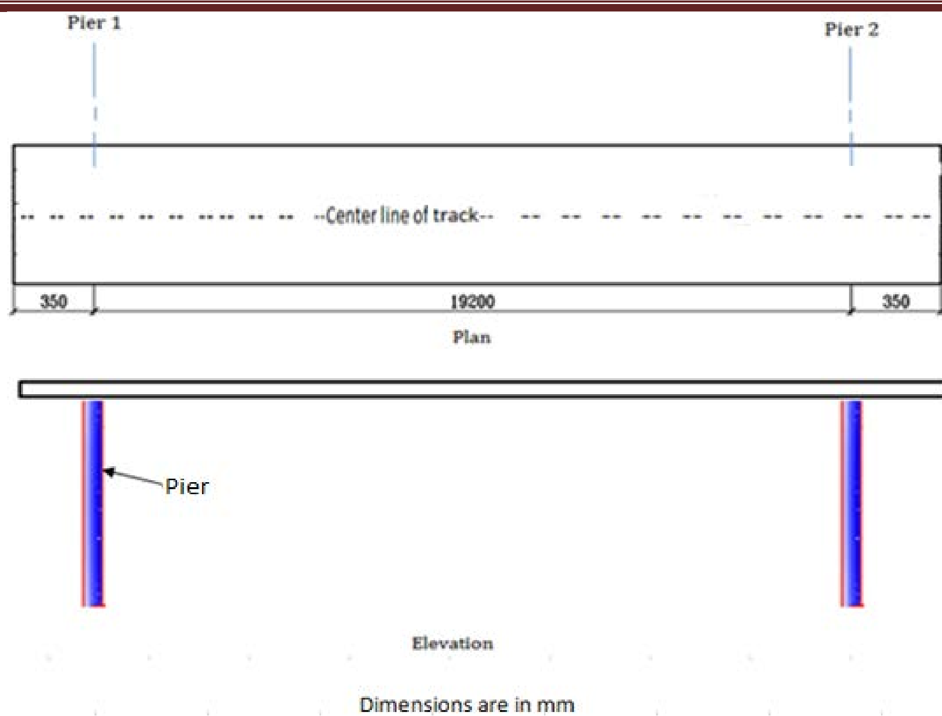



Figure 3.1: Typical Plan and Elevation, 20m simply supported span

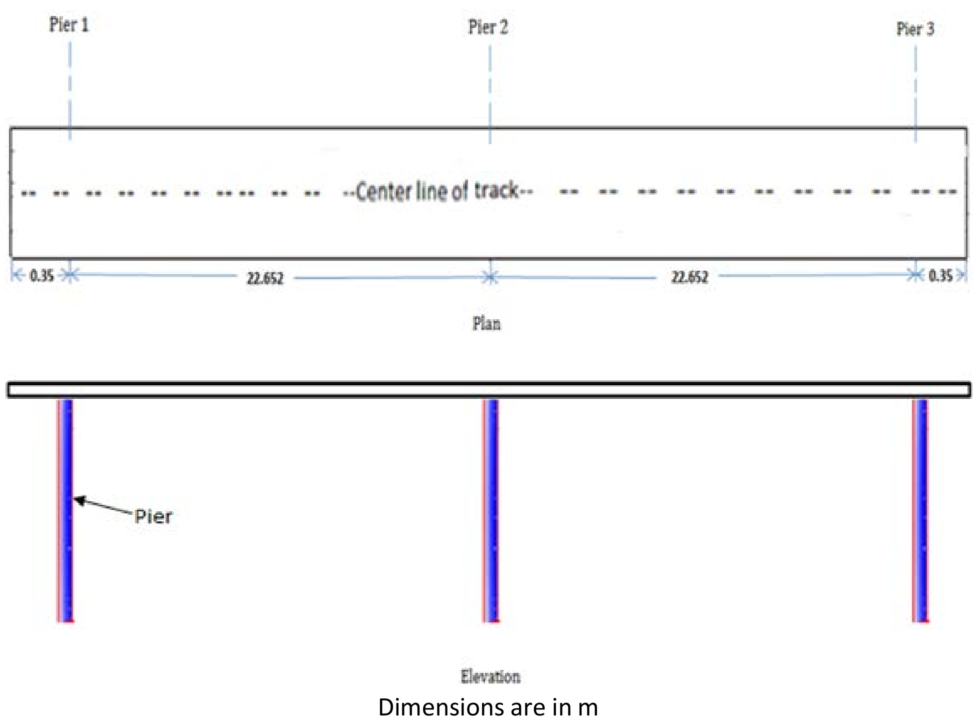
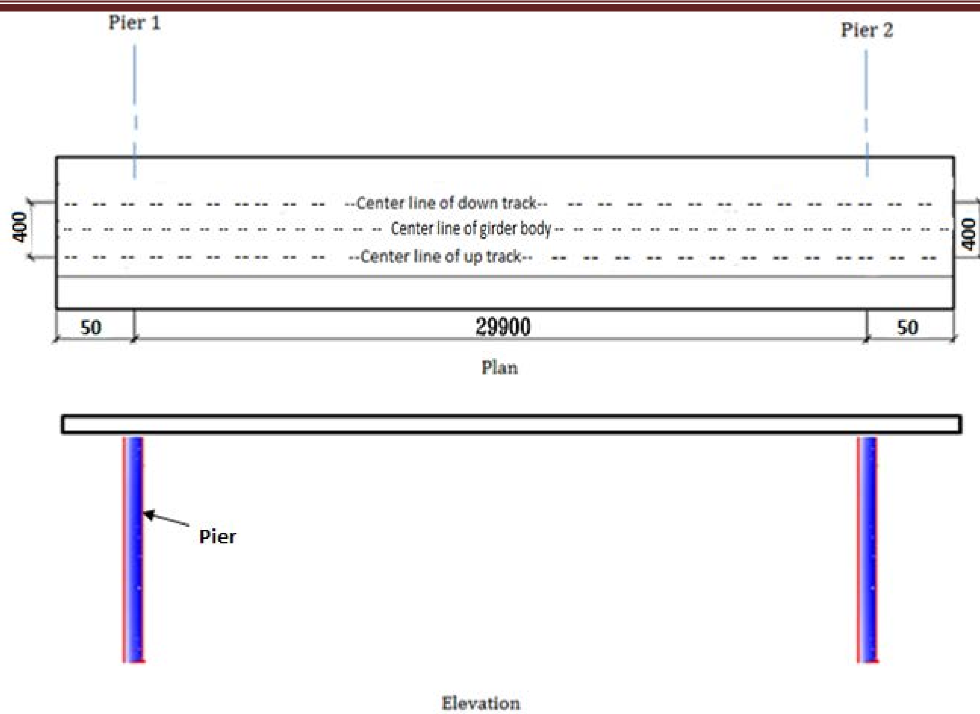
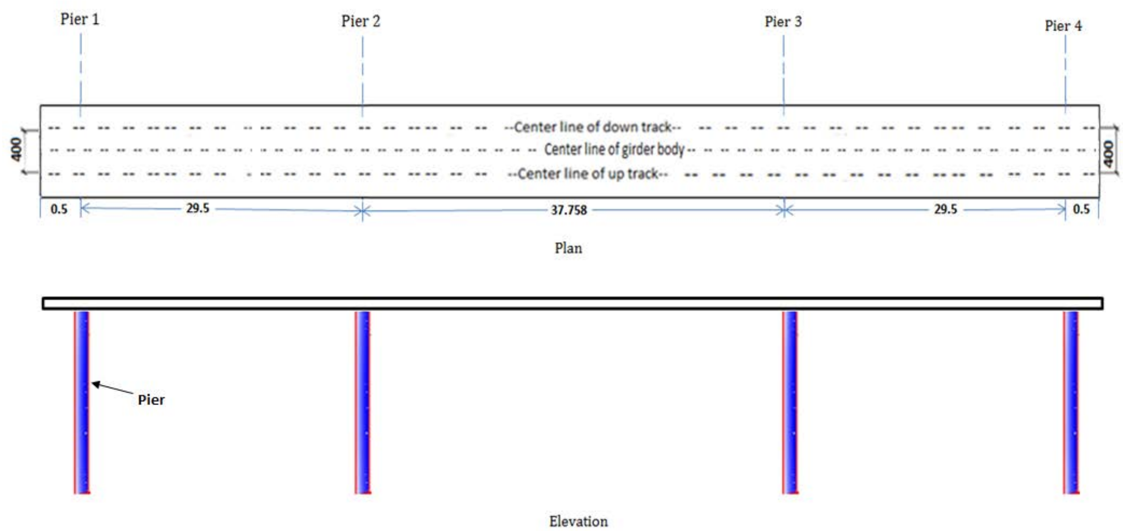



Figure 3.2: Typical Plan and Elevation, 46.004m two-span continuous




Dimensions are in mm

Figure 3.3: Typical Span and Elevation, 30m simple span




Dimensions are in m

Figure 3.4: Typical Span and Elevation, 97.758m three-span continuous

### 3.2 Materials

The following table summarizes the material properties all are taken from the AA-LRT CDD and verified as per AREMA 2010 to use in the analysis. A concrete compressive strength of 50MPa is used.

Table 3.1: Material Properties

| Material Property | | Values |
|---|---|---|
| Concrete | compressive strength | $f'_c$ = 50 MPa, [1]$f'_{ci}$ = 0.6x50 = 30MPa (for the 30m simple span) |
| | Tensile strength | $f_{ct}$ = 0MPa, $f_{cit} = 0.25\sqrt{f_{ci}}$ = 1.369MPa |
| | Elastic modulus | $E_c=0.043w_c^{1.5}\sqrt{f_c}$=42,689,387.17MPa,<br>$E_{ci}=0.043w_c^{1.5}\sqrt{f_{ci}}$=33,067,057.12MPa |
| Pre-stressing tendons | Ultimate strength | $f_{pu}$ = 1860MPa |
| | Yield stress | $f_{py} = 0.9f_{pu}$ = 1674MPa |
| | Effective prestress after all losses | $f_{pj} = 0.7f_{pu}$ = 1,302MPa |
| | Elastic modulus | $E_p$ = 195,000MPa |
| Reinforcing steel bars | Yield stress | $f_y$ = 235MPa & $f_y$ = 335MPa |
| | Elastic modulus | $E_s$ = 210 GPa |

[1]Verified in detail and found to be conformal with the AREMA 2010's requirement

### 3.2.1 Prestressing Tendons

In the beginning of analysis, it's assuming values of friction loss coefficients directly from AA-LRT's CDD [8]. Accordingly the same pre-stressing tendon properties are applied here too. All analysis cases utilize size 15.24mm seven-wire low-relaxation steel as prestressing tendons.

Table 3.2: Pre-stressing tendon properties for the single cell box girders, strand Ø 15.2mm (AA-LRT CDD, 2013)

| Property | Tensioning from both ends | | | |
|---|---|---|---|---|
| | 20m simply supported | | 46.004m two span continuous | |
| | Tendon type 1 | Tendon type 2 | Tendon type 1 | *Tendon type 3 |
| Number of strands | 6 | 8 | 6 | 13 |
| Number of bundle | 6x1 | 2x2 | 3x1 | 2x3 |
| Nominal Cross-section | 840mm$^2$ | 1,120mm$^2$ | 840mm$^2$ | 1,820mm$^2$ |
| Mass | 6.606Kg/m | 8.808Kg/m | 6.606Kg/m | 6.606Kg/m |

*Freyssinet, 2010

Table 3.3: Pre-stressing tendon properties for the 30m simple span, strand Ø 15.2mm (AA-LRT CDD, 2013)

| Property | Tensioning from the start end | | Tensioning from both ends | |
|---|---|---|---|---|
| | Tendon type 4 | Tendon type 5 | Tendon type 4 | Tendon type 5 |
| Number of strands | 9 | 15 | 9 | 15 |
| Number of bundle | 17x1 | 3x3 | 18x1 | 3x3 |
| Nominal Cross-section | 1,260mm2 | 2,100mm2 | 1,260mm2 | 2,100mm2 |
| Mass | 9.909Kg/m | 16.515Kg/m | 9.909Kg/m | 16.515Kg/m |

Table 3.4: Pre-stressing tendon properties for the 97.758m three-span continuous, strand Ø 15.2mm (freyssinet, 2010)

| Property | Tensioning from the start end | | Tensioning from both ends | |
|---|---|---|---|---|
| | Tendon type 3* | Tendon type 6* | Tendon type 3* | Tendon type 6* |
| Number of strands | 13 | 22 | 13 | 22 |
| Number of bundle | 22x1 | 3x4 | 17x1 | 3x4 |
| Nominal Cross-section | 2,850mm2 | 3,750mm2 | 2,850mm2 | 3,750mm2 |
| Mass | 22.42Kg/m | 29.5Kg/m | 22.42Kg/m | 29.5Kg/m |

*Freyssinet, 2010

The ducts for multiple wire, strand, or bar tendons shall have an inside cross sectional area not less than two times the net area of tendons. And the minimum clear distance between prestressing tendons at each end of a member shall not be less than 1-1/3rd times the maximum size of the coarse aggregate. The minimum spacing center-to-center of tendon size of 15.24mm shall be 50mm. Again, the minimum clear distance between adjacent post-tensioning ducts is 40mm. The horizontal spacing among the duct diameter is used to provide sufficient space for concrete placement and vibration AREMA 2010 (CL.8.17.5.5).

### 3.2.2 Concrete Covers

For the Cast-in-Place Concrete the following minimum concrete covers provided for prestressing tendons and non-prestressed reinforcements.

Table 3.5: Concrete cover requirements, AREMA 2010 [CL.8.17.5.2]

| Reinforcement or steel ducts | Concrete covers (mm) | |
|---|---|---|
| | Cast-in-place | Pre-cast |
| Post-tensioning ducts | 75 mm | 40 mm |
| Non-prestressed reinforcement | 50mm | 40 mm |
| Stirrups, ties and spirals | 50mm | 25mm |

## 3.3 Loads

The basic loadings that used in the analysis are Dead Load, Additional dead load, Live load and prestress load. Others loadings like wind load, earth quake load, etc could be considered. Yet, they do not have major influence on the relative percentage values of pre-stressing.

### 3.3.1 Dead Load (DL):

The dead load carried by the box girder consists of its own self weight which is based on the volume weight of concrete, 26.5kN/m3.

### 3.3.2 Secondary Dead Load (SDL):

They are loads of auxiliary facilities such as the track side equipment, cable ducts, railings, pipelines and the supporting devices, cast-in-place wet joints of bridge deck boards, cross slabs, waterproof layers, and paving layers of bridge decks, etc. It is taken to be 25KN/m (for the single cell) & 77KN/m (for the double cells).

### 3.3.3 Live Load

The Live load schemes are taken from AA-LRT's CDD [8] and verified as per AREMA 2010 [2] Live load requirement and found to be conformal. The loads of tram cars are shown in the figure below:

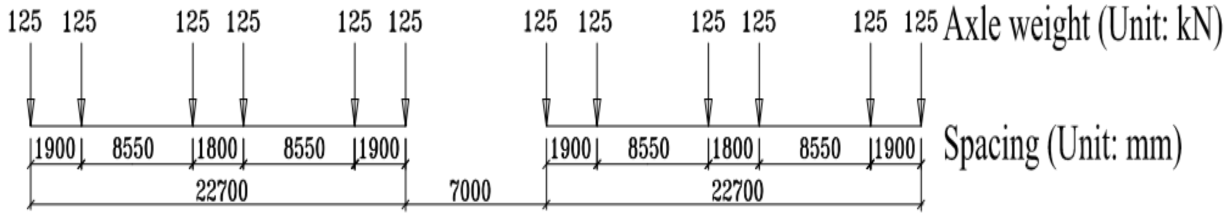



Figure 3.5: Schematic diagram of Live Load (AA-LRT CDD, 2013)

- **Coefficient of dynamic force**: 1+u=1+0.8x0.6xa/(30+L), where a is taken as 2 and L is the calculated span of Girder: 1+u = 1.20
- **The Braking force or tractive force**: calculated based on 15% of vertical static live load.
- **Lateral Swaying force**: calculated based on 15% of the total weight of 4 axles of adjacent two cars. It acts on the rail top in the form of concentrated force.

AREMA [CL.8.2.2.c] given freedom to use other types of loading stating the recommended live load for each track of main line structure is Cooper E 80 (EM 360) loading. On branch lines and in other locations where the loading is limited to the use of light equipments (just like AA-LRT's cars), or cars only, the live load may be reduced, as directed by the engineer.

For LL cooper E-80 loading the section should be capable enough so that the deflection has to be limited to 1/1800 span which has a relatively nearer limit in the AA-LRT's construction design: 1/2000 which makes the limit much smaller. Accordingly, it is safer as per AREMA's over all section limits.

## 3.4 Load Combinations and Load Factors

There are several limit states that need to be considered in the design of the box-girders. The following groups from AREMA [CL.8.2.2.4] represent various combinations of loads and forces to which a structure may be subjected. Each component of the structure shall be proportioned for the group of loads that produce the most critical design condition.

**Strength limit states:** a basic load combination by which flexural, shear strength are investigated.

**Service limit state:** a Load combination relating to normal operational use of the bridge. Concrete stresses, deflection and camber are investigated at service limit states.

**Service I:** Crack control and limiting compression in prestressed concrete

**Service II:** Crack control and tension in prestressed concrete

The load combinations and load factors used in the analysis under strength and service limit states are summarized in the table below. Extreme event limit state is not considered in the analysis for the reason that earthquake and appropriate collision forces are out of the scope of this study. Again, fatigue limit state is not considered because fatigue of the reinforcement need not be checked for fully pre-stressed concrete members satisfying the requirements of service limit state.

Table 3.6: Load Combinations for the simple spans

| Loads | Permanent Loads | | Transitory Load |
|---|---|---|---|
| | DL | ADL | LL |
| Strength limit state (Strength I) | 1.40 | 1.40 | 2.33 |
| Service limit state (Service I) | 1.00 | 1.00 | 1.00 |
| Service limit state (Service II) | 1.25 | 0 | 0 |

The secondary moments caused by the pre-stressing in continuous structures must be considered in the Strength Limit States. These moments are permanent loads and have a load factor equal to 1.00.

Table 3.7: Load Combinations for the continuously supported spans

| Loads | Permanent Loads | | | Transitory Load |
|---|---|---|---|---|
| | DL | ADL | P | LL |
| Strength limit state (Strength I) | 1.40 | 1.40 | 1.00 | 2.33 |
| Service limit state (Service I) | 1.00 | 1.00 | 0 | 1.00 |
| Service limit state (Service II) | 1.25 | 0 | 0 | 0 |

**Legend:**

DL = dead load

ADL = Additional Dead Load or secondary dead load

LL = live load

P = secondary pre-stress effects

As shown in the table above, the load combinations Service II only consider permanent loads. Without live loads, the tendons push the bridge upward to make camber, longitudinally, while there might not be enough dead load to weigh the bridge down. Therefore, these load combinations must be checked whether they fulfill Strength limit state and Service limit state requirements. This, specifically, is critical for slender bridges with large prestressing forces and small dead loads. Considering only live load can be used for superstructure vibration check.

**Strength Capacity Reduction Factors**

The following strength capacity reduction factors used according to AREMA [CL.8.17.15.1]

For flexure: $\varphi = 0.95$

For shear: $\varphi = 0.90$

### 3.5 Design Requirements

The analyses are done to satisfy AREMA 2010's Load factor (Strength limit state) and Service load design requirements. The strength limit state deals with flexure and shear strength requirements where as the Service limit state deals with stress, vibration and deflection limitations. The design check considers only longitudinal behavior.

### 3.5.1 Strength limit state

#### 3.5.1.1 Flexural strength

According to AREMA [CL.8.17.18] prestressed concrete members may be assumed to act as uncracked members subjected to combined axial and bending stresses within specified service loads. In calculations of section properties, the transformed area of bonded reinforcement may be included in pre-tensioned members and in post-tensioned members after grouting; prior to bonding of tendons, areas of the open ducts shall be deducted.

Rectangular Sections

For rectangular or flanged sections having prestressing steel only, in which the depth of the equivalent rectangular stress block, defined as $(A_s\ f_{su})/(0.85f'_{cb})$, is not greater than the compression flange thickness t, and which satisfy the reinforcement index for prestressing steel only, the design flexural strength shall be assumed as:

$$\varphi M_n = \varphi[A_s\ f_{su}\ d\ \{1\text{-}0.6(p\ f_{su}/\ f'_c)\}]$$

Ductility Limits

Maximum Prestressing Steel

Prestressed concrete members shall be designed so that the steel is yielding as ultimate capacity is approached. In general, the reinforcement index for prestressing steel only ($f_{su}$ is for bonded Members with prestressing only) shall be such that:

$\varphi M_n \geq 1.2M_{cr}$

For rectangular sections: $pf_{su}/f'_c$ &

For flanged sections: $A_{sr}f_{su}/b'df'_c$, does not exceed $0.36\beta_1$

For members with reinforcement indices greater than $0.36\ \beta_1$, the design flexural strength shall be assumed not greater than:

For rectangular sections: $\varphi M_n = \varphi[0.36\ \beta_1 - 0.08\ \beta_1^2)f'_c bd^2]$

For flanged sections: $\varphi M_n = \varphi[0.36\ \beta_1 - 0.08\ \beta_1^2)f'_c bd^2 + 0.85f'_c(b-b')t(d-0.5t)]$

Minimum Prestressing Steel

The total amount of prestressed and non-prestressed reinforcement shall be adequate to develop an ultimate moment at the critical section at least 1.2 times the cracking moment $M_{cr}$.

$$\varphi M_n \geq 1.2M_{cr}$$

$M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

Appropriate values for $M_d/_{nc}$ and $S_b$ shall be used for any intermediate composite sections. Where beams are designed to be non composite, substitute $S_b$ for $S_c$ in the above equation for the calculation of $M_{cr}$

So that the ubove equation becomes $M_{cr} = (f_r + f_{pe})S_c$

Where $f_r = 0.623\sqrt{f_{ci}}$

$S_c = 2.471m^3$ (from SAP2000 output)

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}]$

Steel Stress

As an alternative to a more accurate determination of $f_{su}$ based on strain compatibility, the following approximate values of $f_{su}$ shall be permitted to be used:

For bonded Members with prestressing only: $f_{su} = f'_s [1-(\gamma/\beta_1)( pf'_s/ f'_c)]$

For bonded Members with non-prestressed tension reinforcement included:

$$f_{su} = f'_s \{1-(\gamma/\beta_1)[( pf'_s/f'_c)+d_t/d(pf_{sy}/f'_c)]\}$$

Where:

γ = factor for type of prestressing steel and it is 0.28 for low-relaxation steel here

$f'_s$ = ultimate strength of prestressing steel (MPa)

$\beta_1$ = factor for concrete strength

For rectangular or flanged sections with non-prestressed tension reinforcement included, in which the depth of the equivalent rectangular stress block, defined as $(A_s f_{su} + A_s f_{sy})/(0.85f'_c b)$, is not greater than the compression flange thickness "t" and which satisfy the reinforcement index with non-prestressed tension reinforcement included, the design flexural strength shall be assumed as:

$$\varphi M_n = \varphi\{A_{ps} f_{su}\, d[1-0.6((p f_{su}/ f_c)+(d_t/d)( p f_{sy}/ f_c))]+A_s f_{sy}\, d_t[1-0.6((d/d_t)(pf_{su}/f_c)+(pf_{sy}/f_c))]\}$$

The reinforcement indices of sections with non-prestressed reinforcement tension reinforcement included ($f_{su}$ is for bonded Members with non-prestressed tension reinforcement included):

Non-Prestressed Reinforcement

Non-prestressed reinforcement may be considered as contributing to the tensile strength of the beam at design flexural strength in an amount equal to its area times yield strength, provided that:

For rectangular sections: $(pf_{sy}/f'_c)d_t/d+(pf_{su}/f'_c)-(pf'_y/f'_c) \leq 0.36\beta_1$

For flanged sections: $(A_s f_{sy})/(b'df'_c) + (A_{sr} f_{su})/(b'df'_c) - (A'_s f'_y)/(b'df'_c) \leq 0.36\beta_1$

For members with reinforcement indices greater than 0.36ß1, the design flexural strength shall be assumed not greater than:

For rectangular sections: $\varphi M_n = \varphi[0.36\beta_1 - 0.08\beta_1^2)f'_c bd^2]$

For flanged sections: $\varphi M_n = \varphi[0.36\beta_1 - 0.08\beta_1^2)f'_c bd^2 + 0.85f'_c(b-b')t(d-0.5t)]$

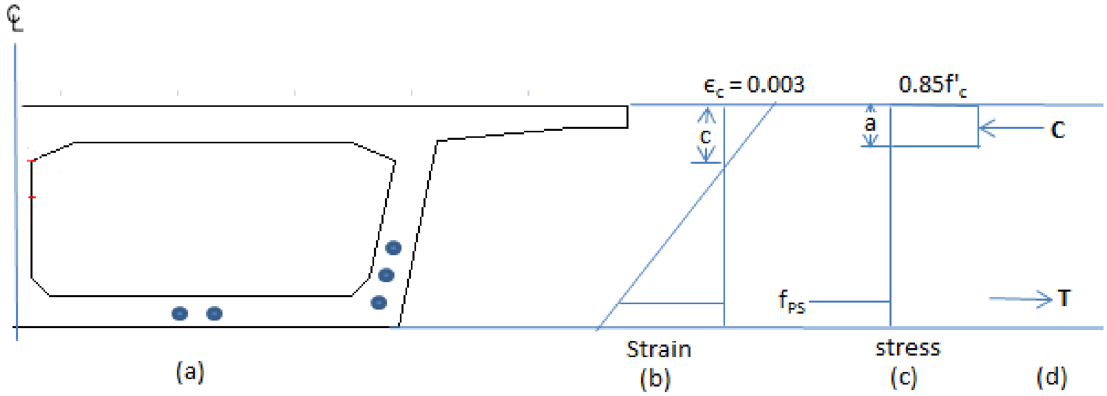



Figure 3.6: Flexural resistance: a) cross-section, b) concrete stains, c) equivalent concrete stresses, d) concrete forces

### 3.5.1.2 Shear Strength

Pre-stressed concrete flexural members shall be reinforced for shear and diagonal tension stresses as per AREMA [CL.8.17.21]. Members subject to shear shall be designed so that

$V_n = \varphi(V_c+V_s)$.

Shear strength provided by Concrete

For members with effective prestress force not less than 40 percent of the total tensile strength of flexural reinforcement:

$$V_c = (\frac{\sqrt{f_c'}}{20}+5\frac{V_u d_p}{M_u})b_w d$$

But, $V_c$ need not be taken less than $\frac{1}{6}\sqrt{f_c' b_w}$d and nor shall $V_c$ be taken greater than $0.4\sqrt{f_c' b_w}$d

Shear strength provided by web reinforcement

Shear reinforcement shall consist of stirrups perpendicular to axis of member

The shear strength provided by web reinforcement shall be taken as $V_s = (A_v f_{sy} d)/s$ where Av is the area of web reinforcement with in a distance s. $V_s$ shall not be taken greater than $0.66\sqrt{f_c'}$b'd and d need not be taken less than 0.8h.

The spacing of web reinforcing shall not exceed 0.75h or 600mm when $V_s$ exceeds $0.332\sqrt{f_c'}$b'd

### 3.5.1.3 Stress in Concrete

The basic equation for stress in concrete: $f = P_{eff}(\frac{1}{A}\pm\frac{e_m y}{I})+\sum\frac{\gamma My}{I}$ -------------------------------eq (3.5.1)

From eq (3.5.1):

For bottom fiber: $f_b = P_j FC_{ES,mid}(1/A+e_m\, y/I)-\sum\gamma My/I$

For top fiber: $f_t = P_j FC_{ES,mid}(1/A+e_m\, y/I)-\sum\gamma My/I$

Initial Concrete Strength

Service I limit state is used to determine the concrete compressive stress. According to AREMA [CL.8.17.16.2] the concrete stresses in concrete immediately after pre-stress transfer (before time-dependent prestress losses-Creep and Shrinkage) is limited to 0.55 $f'_{ci}$.

For extreme fiber stress in tension again not to exceed $0.25\sqrt{f'_{ci}}$

Final Concrete Strength

Service I and Service II limit state are used to determine the concrete compressive stress. Live load only also used to check the vibration.

Stresses in the concrete at service loads (after allowance for all prestress losses) shall not to exceed $0.4f'_{ci}$ in Compression and 0 in tension.

Cracking Stress

Modulus of rupture for concrete $f_r = 0.623\sqrt{f'_{ci}}$

## 3.6 Other analysis assumptions

- Tendon eccentricity in duct is neglected. The centre of gravity of tendons is assumed to coincide with the centroid of the duct
- The increment in concrete to accommodate intermediate tendon anchors and deviators is assumed to be negligible
- Every span is assumed to have the same number of prestressing tendons
- Substructure design is not considered in these analyses

# Chapter 4

## 4. Post-tensioning analysis of single cells and double cells concrete box-girders of AA-LRT elevated super structures

### 4.1 Introduction

First of all, what has been done is checking the AA-LRT's elevated structures cross-sections whether they meet the geometry requirements of AREMA 2010 or not. Single cells do not meet the requirement. However, the double cells meet the entire requirement so that flexure and shear strength check has been done for them.

The design of post-tensioned concrete elevated structures which has been done for the critical locations along the span involves taking input values from AA-LRT CDD [8]. After that calculating results, comparing the results to the values initially taken and reiterating the process until convergence. The step by step procedures followed for the post-tensioning analysis of single cells and double cells concrete box-girders of AA-LRT elevated super structures is by the determination of the following parameters:

- ✓ Optimization
- ✓ Cross sectional geometry
- ✓ Cable path
- ✓ Pre-stress losses
- ✓ Post-tensioning force, $P_j$ and area of pre-stressing steel, $A_{ps}$
- ✓ Concrete strength using Service Limit State
- ✓ Flexural strength using Strength Limit State
- ✓ Shear strength using Strength Limit State

### 4.2 Span increment

Increasing the spans is done by widening of the span in the simply supported box-girders to its respective maximum possible span in the continuously supported box-girders. In order to optimize, the sole parameter used is deflection. First the deflection is determined from the simple spans and the same deflection is used to determine the maximum possible increment of the respective span in a continuous ones.

#### 4.2.1 Span increment of the 20m simple span to 46.004m two-span continuous

Their relative increase in the length of each section is proportional. i.e. for example the deflection that used to optimize 20m simple span to two span continuous (each being 23.002m span) was 4.7mm. For the reason that the cross-section vary across the simple span, its respective optimized span also varies proportionally with it. Each section across the length of 20m simple span is optimized to 23.002m in a two-span continuous one by multiplying it with a factor of 23.002/20 = 1.1501. The detail optimization is shown in the table below:

Table: 4.1 Length of each varying cross-section along the length of the spans in the single-cells

| Length of each varying cross-section along the simply supported span, 20m | Length of each varying cross-section along each optimized spans of the 46.004m continuous box-girder |
|---|---|
| 0 | 0 |
| 1.45 | 1.668 |
| 4 | 4.600 |
| 16 | 18.402 |
| 18.55 | 21.334 |
| 20 | 23.002 |

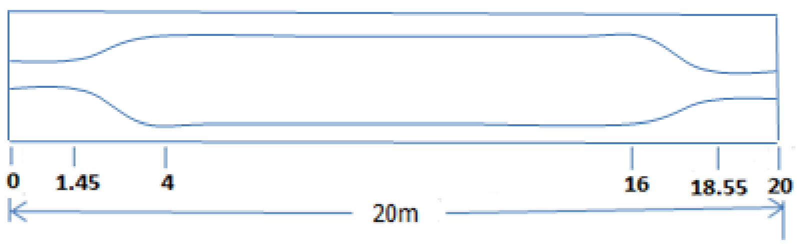



Figure 4.1: Points for the length of varying cross-section along the 20m simply supported post-tensioned box-girder

### 4.2.2 Span increment of the 30m simple span to 97.758m three-span continuous

The relative increase in the length of each section in the continuous box-girder is proportional with that of its simply supported counterpart i.e. for example the deflection that used to optimize 30m simple span to three span continuous (keeping the end spans 30m and optimizing the middle one to 37.758m) was 15.6mm. So that because the Crossection vary across the simple span, it's respective optimized span also varies proportionally with it. Each section across the length of 30m simple span is optimized to 37.758m span in a three-span continuous one multiplying it by a factor of 37.758/30 = 1.2586. The detail optimization is shown in the table below:

Table: 4.2 Length of each varying cross-section along the length of the spans in the double-cells

| Length of each varying cross-section along the simply supported span, 30m | Length of each varying cross-section along each increased spans of the 37.758m middle-span in a three-span continuous box-girder |
|---|---|
| 0 | 0 |
| 1.85 | 2.328 |
| 4.05 | 5.097 |
| 25.95 | 32.661 |
| 28.15 | 35.43 |
| 30 | 37.758 |

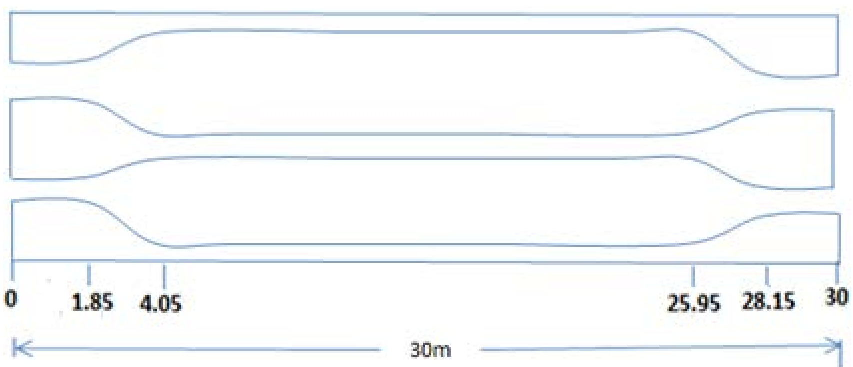



Figure 4.2: Points for the length of varying cross-section along the 30m simply supported post-tensioned box-girder

### 4.2.3 Deflection at various stages of loadings and check as per AREMA

Flexural members of bridge structures shall be designed to have adequate stiffness to limit deflections or any deformations that may adversely affect strength and serviceability of the structure at service load. Members having simple or continuous spans shall be designed so that the deflection due to service live load plus impact does not exceed l/640 of the span, AREMA 2010 [CL.8.17.13].

Deflections at different stages of loadings i.e. deflections that occur immediately on set of application of prestressing (right after instantaneous loss) and additional long-time deflection to be computed considering stresses in concrete and steel under sustained load and including effects of creep and shrinkage of concrete and relaxation of prestressing steel (time dependant losses).

The vertical deflection caused by the live load is shown in the table below for each stage of loadings and Tensioning schemes. Every deflection meets the requirement not to exceed l/640 of the span. It also conforms to the requirement in the AA-LRT CDD [8] i.e. l/2000 of span which is more safe in relative to AREMA 2010.

Table: 4.3: Prevailing deflection due to Live load of all box-girders at various stages of loadings

| Post-tensioned box-girders | Deflection due to Live load at different stages of loadings | | | | | | |
|---|---|---|---|---|---|---|---|
| | Immediately after post-tensioning | | | | Long-time deflection | | |
| | Jacking conditions | Deflection (mm) | Meets AREMA's requirement? | Meets AA-LRT's CDD requirement? | Deflection (mm) | Meets AREMA's requirement? | Meets AA-LRT's CDD requirement? |
| 20m simply supported | $P_j$, from both ends | 3.83 | yes | yes | 3.9 | yes | yes |
| 46.004m two span continuous | $P_j$, from both ends | 4.44 | yes | yes | 4.77 | yes | yes |
| 30m simply supported | $P_j$, from start end | 7.16 | yes | yes | 7.18 | yes | yes |
| | $P_j$, from both ends | 7.16 | yes | yes | 7.18 | yes | yes |
| 97.758m three-span continuous | $P_j$, from start end | 10.71 | yes | yes | 10.55 | yes | yes |
| | $P_j$, from both ends | 10.66 | yes | yes | 10.55 | yes | yes |

NB: Def for 97.758m three-span continuous are for the middle span at which the maximum deflection occur.

## 4.3 Cross-sectional geometry

### 4.3.1 Cross-sectional geometry determination for the single cells

The post-tensioning analysis has been done for both types of single cell box girders namely 20m simply supported and 46.004 two span continuous. The analysis is done by keeping their cross-sectional dimensions the same. Because they don't fulfill AREMA [CL.8.26.17] geometry requirement independently, flexure and shear strength checks cannot be done. It is only done for the double cell box-girders only. The following cross-sectional geometry determination check is based on AREMA 2010 and it works for both types of box girders.

#### 4.3.1.1 Minimum flange thickness

Top and bottom flange thickness shall not be less than any of the following:

- 1/30 the clear span between webs or haunches, a lesser dimension will require transverse ribs at a spacing equal to the clear span between webs or haunches.

Table: 4.4: cross-sectional geometry check for the single cells, minimum flange thickness

| The three cross-sections | | | |
|---|---|---|---|
| | **Support** | **Near support (varying)** | **Middle** |
| Top flange | 0.35> (1.2865/30=0.043) ok! | 0.35> (1.2865/30=0.043) ok! | 0.22> (1.6868/30=0.056) ok! |
| Bottom flange | 0.4> (1.0369/30=0.035) ok! | 0.4> (1.0369/30=0.035) ok! | 0.22> (1.3079/30=0.044) ok! |

#### 4.3.1.2 Minimum web thickness

- Webs with only longitudinal (or vertical) post-tensioning tendons - 12 inches=0.305m, hence the 3$^{rd}$ cross-section, at the mid span, do not meets this requirement.

#### 4.3.1.3 Length of top flange cantilever

The cantilever length of the top flange measured from the centerline of web should, preferably, not exceed 0.45 the interior span of the top flange measured between the centerline of the webs.

Table: 4.5: cross-sectional geometry check for the single cells, length of top flange cantilever

| The three cross-sections | | | |
|---|---|---|---|
| | **Support** | **Near support (varying)** | **Middle** |
| Right flange | 1.008 > (0.45x1.794=0.807) ok! | 0.941 > (0.45x1.934=0.870) ok! | 0.894 > (0.45x2.007=0.903) ok! |
| Left flange | 0.678 < (0.45x1.794=0.807) NOT ok! | 0.589 < (0.45x1.934=0.870) NOT ok! | 0.559 < (0.45x2.007=0.903) NOT ok! |

As such the dimension in the whole length of top flange of single cell box-girders do not meet the requirement.

#### 4.3.1.4 Overall cross section dimensions

Overall dimensions of the box girder cross section should preferably not be less than required to limit live load plus impact deflection calculated using the gross section moment of inertia and the secant modulus of elasticity to 1/1800 of the span.

To lower the center of gravity of the superstructure at the face of the pier cap in the Cast in Place post-tensioned box girder, the thickness of soffit is flared as shown in the figures below.

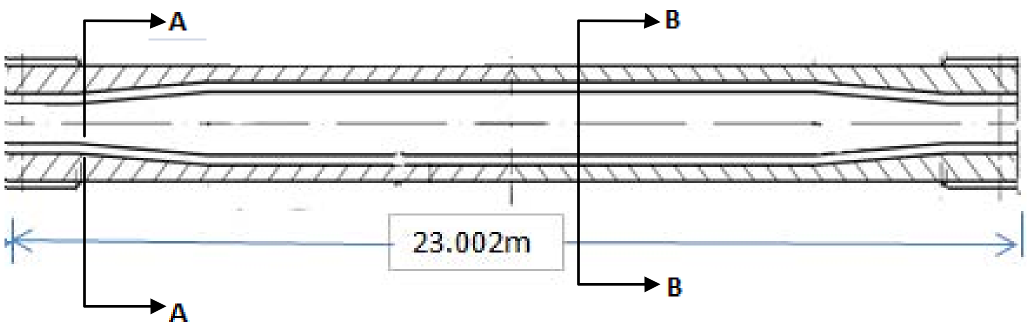



Figure 4.3: the longitudinal cross-section of the 20m simply supported box-girder

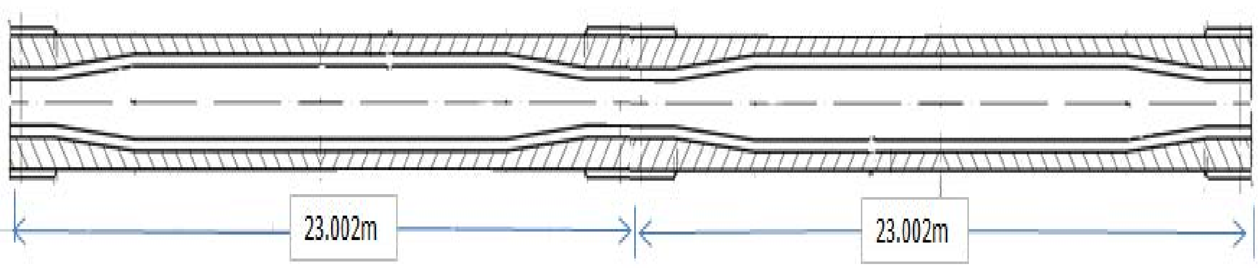



Figure 4.4: the longitudinal cross-section of the 46.004m two-span continuous box-girder

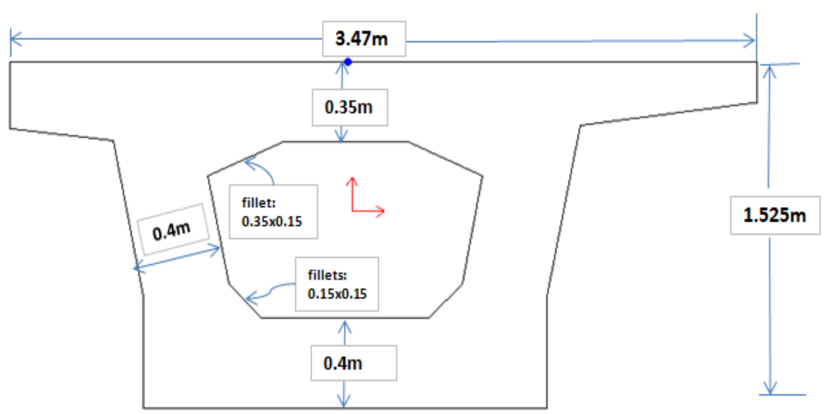



Figure 4.5: Section A-A of figure 4.3

Section Properties:

- Gross area of the section = 2.5598m$^2$
- Moment of inertia about horizontal axis = 0.6667m$^4$
- Section modulus (negative face) = 0.7685m$^3$
- Section modulus (positive face) = 1.0114m$^3$
- Center of gravity of the section from base point = 0.8675m

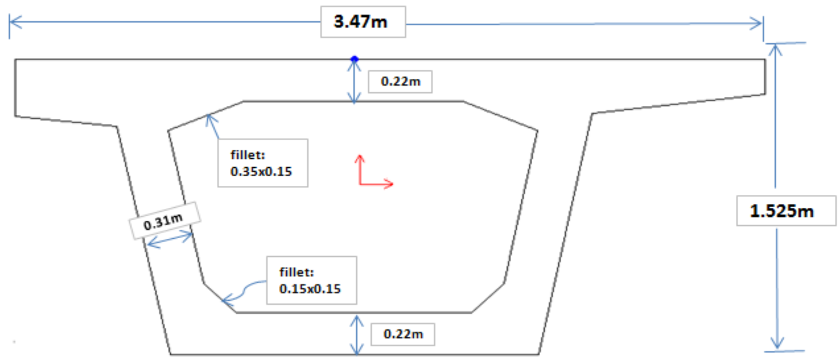



Figure 4.6: Section B-B of figure 4.3

Section Properties:

- Gross area of the section = 1.7838$m^2$
- Moment of inertia about horizontal axis = 0.5254$m^4$
- Section modulus (negative face) = 0.5667$m^3$
- Section modulus (positive face) = 0.8788$m^3$
- Center of gravity of the section from base point = 0.9272m

### 4.3.2 Cross-sectional geometry determination for the double cells

The post-tensioning analysis has been done for both types of double cell box girders namely 30m simply supported and 97.758 three span continuous. The analysis is done by keeping their cross-sectional dimensions the same. Because they both fulfill AREMA [CL.8.26.17] geometry requirement independently, flexure and shear design has been done. The following cross-sectional geometry determination works for both types of double cell box girders.

#### 4.3.2.1 Minimum flange thickness

Top and bottom flange thickness shall not be less than any of the following:

- 1/30 the clear span between webs or haunches, a lesser dimension will require transverse ribs at a spacing equal to the clear span between webs or haunches.

Table: 4.6: cross-sectional geometry check for the double cells, minimum flange thickness

| **The three cross-sections** | | | |
|---|---|---|---|
| | **Support** | **Near support (varying)** | **Middle** |
| Top flange | 0.55> (1.3632/30=0.0454) ok! | 0.425> (2.6114/30=0.087) ok! | 0.3> (2.9352/30=0.0978) ok! |
| Bottom flange | 0.55> (1.0369/30=0.0454) ok! | 0.4> (2.4681/30=0.0823) ok! | 0.25> (2.7408/30=0.0914) ok! |

#### 4.3.2.2 Minimum web thickness

Webs with only longitudinal (or vertical) post-tensioning tendons – 12 inches=0.305m, hence the web of 3$^{rd}$ cross-section at the mid span is 0.3m nearly equal.

#### 4.3.2.3 Length of top flange cantilever

The cantilever length of the top flange measured from the centerline of web should preferably not exceed 0.45 the interior span of the top flange measured between the centerline of the webs.

Table: 4.7: cross-sectional geometry check for the double cells, length of top flange cantilever

| The three cross-sections | | | |
|---|---|---|---|
| | **Support** | **Near support (varying)** | **Middle** |
| Right flange | 2.194> (0.45x2.781=1.251) ok! | 1.809> (0.45x3.18=1.431) ok! | 1.677> (0.45x3.312=1.490) ok! |
| Left flange | 2.194> (0.45x2.781=1.251) ok! | 1.809> (0.45x3.18=1.431) ok! | 0.22> (0.45x2.781=1.251) ok! |

### 4.3.2.4 Overall cross section dimensions

Overall dimensions of the box girder cross section should preferably not be less than required to limit live load plus impact deflection calculated using the gross section moment of inertia and the secant modulus of elasticity to 1/1,800 of the span.

To lower the center of gravity of the superstructure at the face of the pier cap in the cast in place post-tensioned box girder, the thickness of soffit is flared as shown in the figures below.

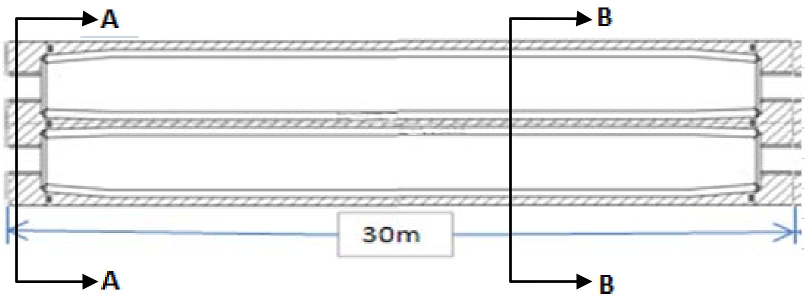
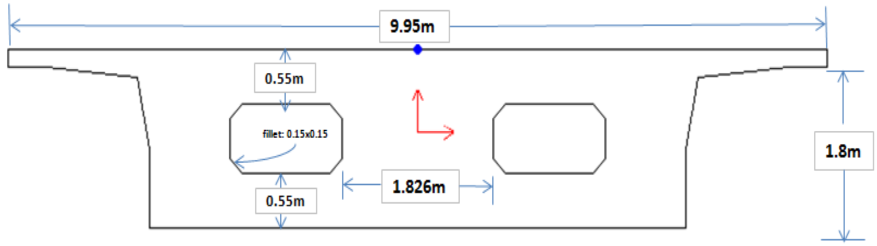



Figure 4.7: the longitudinal cross-section of the 30m simply supported box-girder

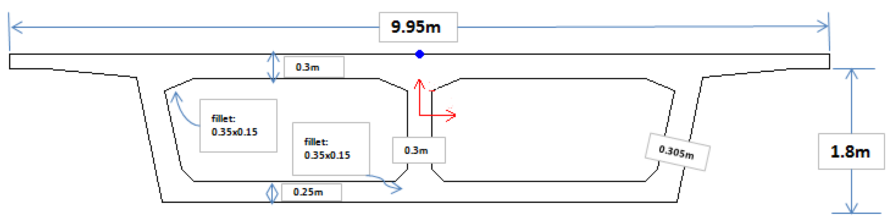

Figure 4.8: the longitudinal cross-section of the 97.758m three-span continuous box-girder




Section Properties:

- Gross area of the section = 10.7681$m^2$
- Moment of inertia about horizontal axis = 3.5484$m^4$
- Section modulus (negative face) = 3.6991$m^3$
- Section modulus (positive face) = 4.2206$m^4$
- Center of gravity of the section from base point = 0.9593m

Figure 4.9: Section A-A of figure 4.7




Section Properties:

- Gross area of the section = 5.5802$m^2$
- Moment of inertia about horizontal axis = 2.6051$m^4$
- Section modulus (negative face) = 2.4713$m^3$
- Section modulus (positive face) = 3.4929$m^4$
- Center of gravity of the section from base point = 1.0542m

Figure 4.10: Section B-B of figure 4.7

## 4.4 The Cable Path

A cable path is generally controlled by the maximum dead-load moment and the position of the jack at the end section. Duan, L. [10] states that maximum eccentricities should occur at points of maximum dead load moments and almost no eccentricity should be present at the jacked end section. Tensioning schemes undertaken after the cable paths are set.

The location of the center of gravity of the cable at the ends is very important for the anchor zone design. Placing the center of gravity at the neutral axis results in a uniform stress distribution at the ends but the top tendons will probably be too high to have sufficient top edge clearance. Placing the cable path near the geometric center of the section is usually a good compromise. At the mid-span the cable path should be as low as possible. However, care must be taken to ensure that the cable path can be physically located where assumed. A check on the center of gravity at the ends and at the mid-span is required once the area of prestressing steel is determined.

### 4.4.1 The cable path for the single cells, 20m simply supported box-girder

The maximum dead-load moments occur at the mid span. In order for the analysis to be comparable the same cable profile with the AA-LRT CDD has been followed.

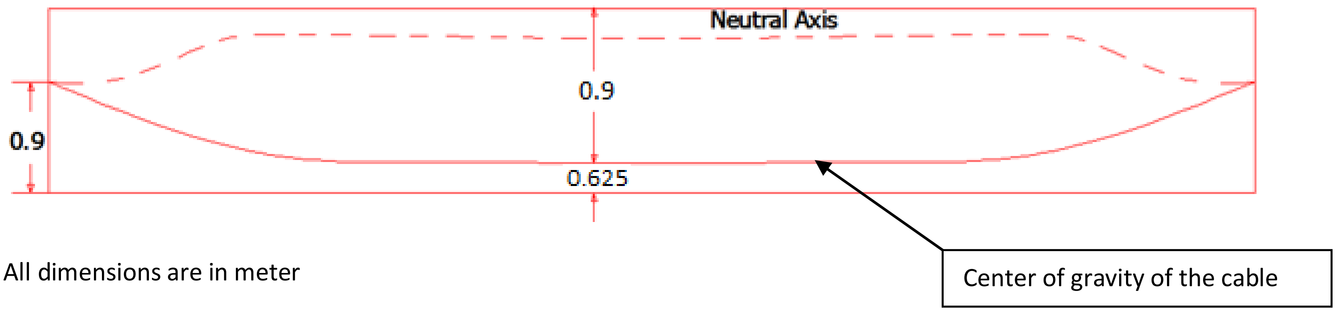



Figure 4.11: Vertical profile of the cable path for 20m simply supported box-girder

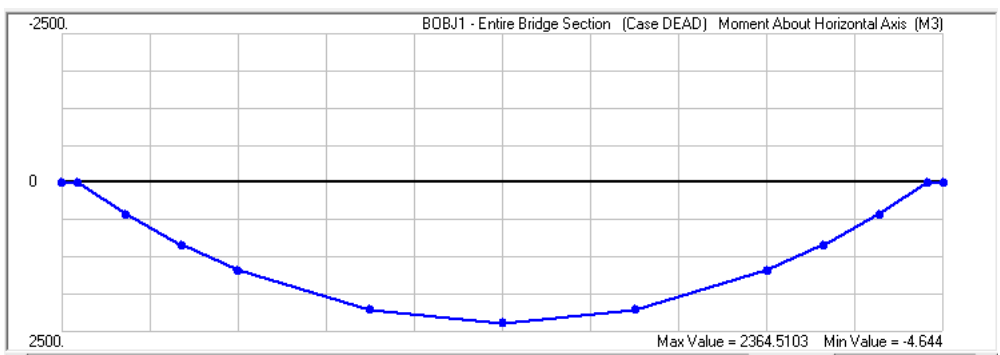



Figure 4.12: the bending moment diagram for the dead load of 20m simple span box-girder, prior to application of pre-stressing force

### 4.4.2 The cable path for the single cells, 46.004m two-span continuous box-girder

For the case of two spans continuous, the maximum dead-load moments occur at three locations: at the pier cap, at the locations close to 0.4L for Span 1 and 0.6L for Span 2. A parabolic cable path is chosen as shown in Figure below.

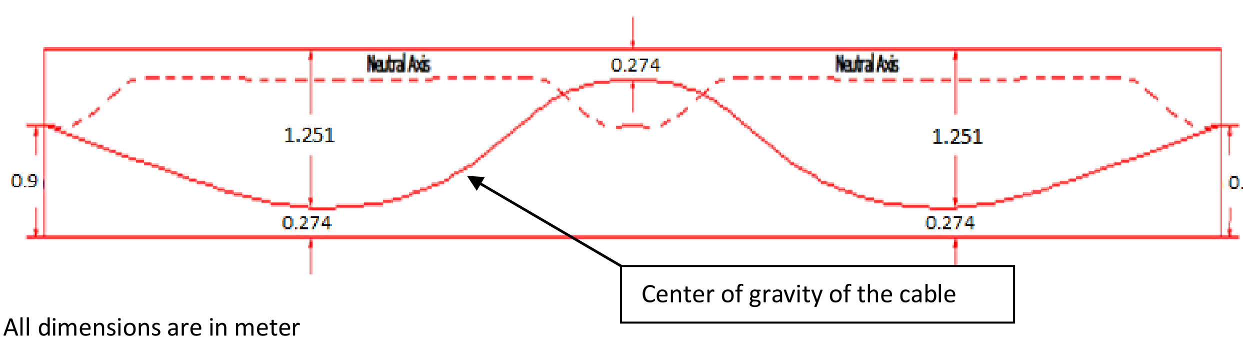



Figure 4.13: Vertical profile of the cable path for 46.004m two-span continuous box-girder

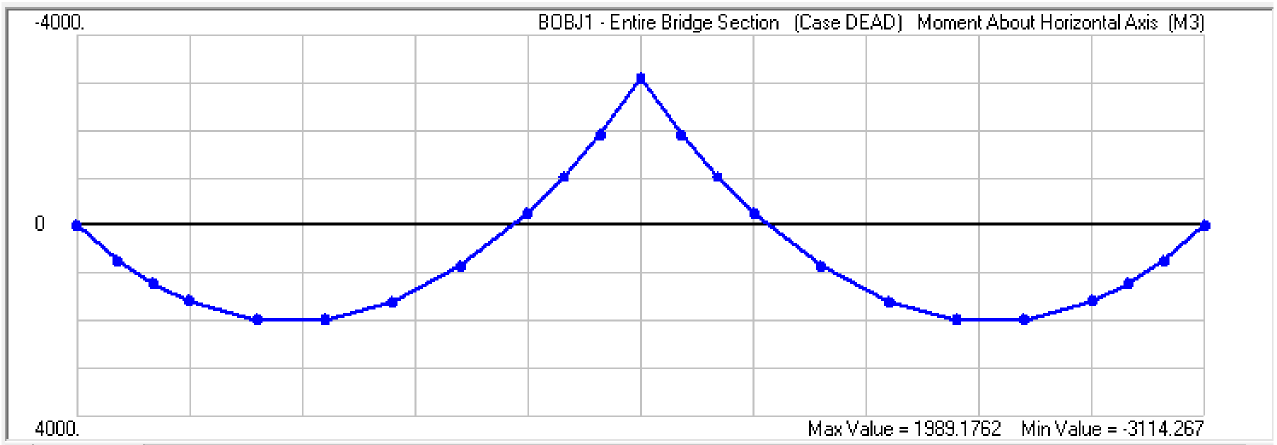



Figure 4.14: the bending moment diagram for the dead load of 46.004m two-span continuous box-girder prior to application of pre-stressing force

### 4.4.3 The cable path for the double cells, 30m simply supported box-girder

The maximum dead-load moments occur at the mid span. In order for the analysis to be comparable the same cable profile with the AA-LRT CDD has been followed.

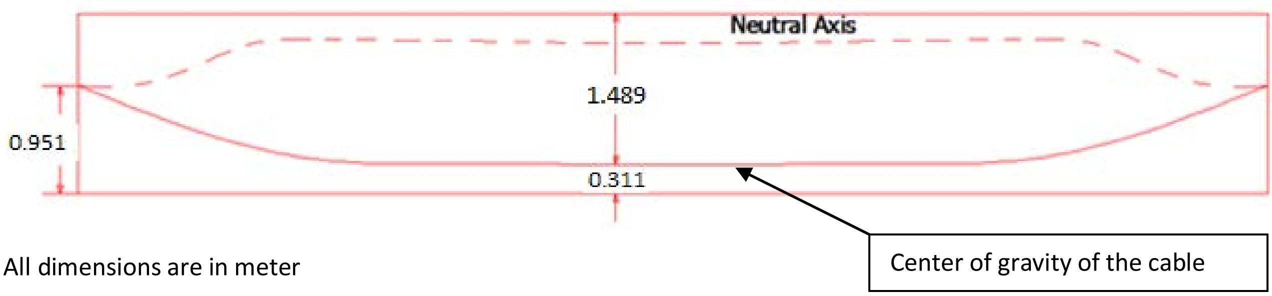



Figure 4.15: Vertical profile of the cable path for 30m simply supported box-girder

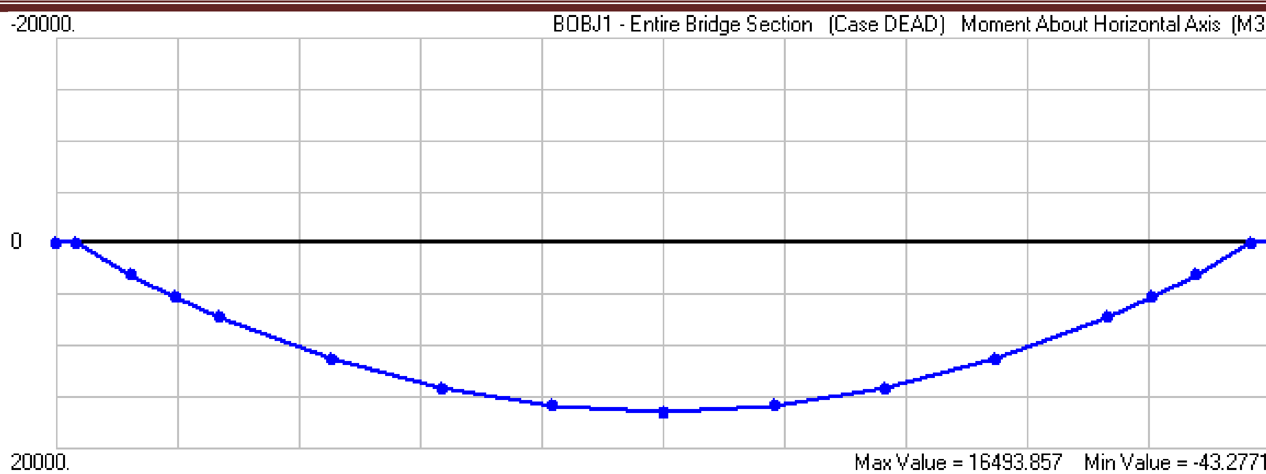



Figure 4.16: the bending moment diagram for the dead load of 30m simple span box-girder, prior to application of pre-stressing force

### 4.4.4 The cable path for the double cells, 97.758m three-span continuous box-girder

For the case of three spans continuous, the maximum dead-load moments occur at five locations: at the pier caps, at the locations close to 0.41L for Span 1, 0.5L for Span 2 and 0.59L for span 3. A parabolic cable path is chosen as shown in Figure below.

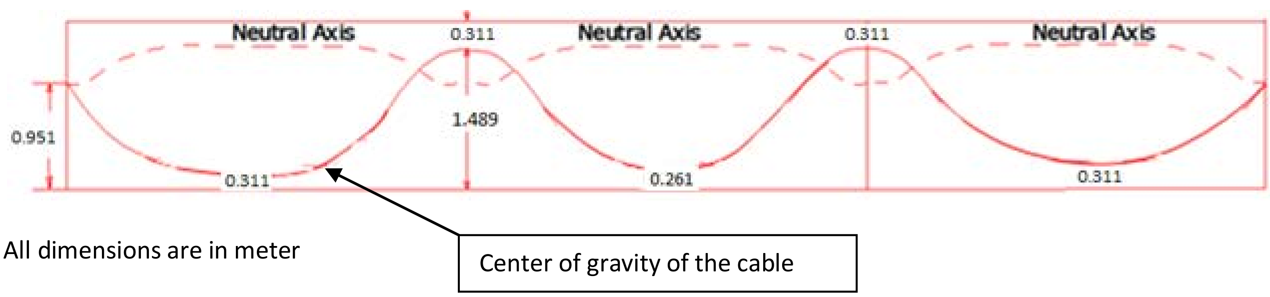



Figure 4.17: Vertical profile of the cable path for 97.758m three-span continuous box-girder

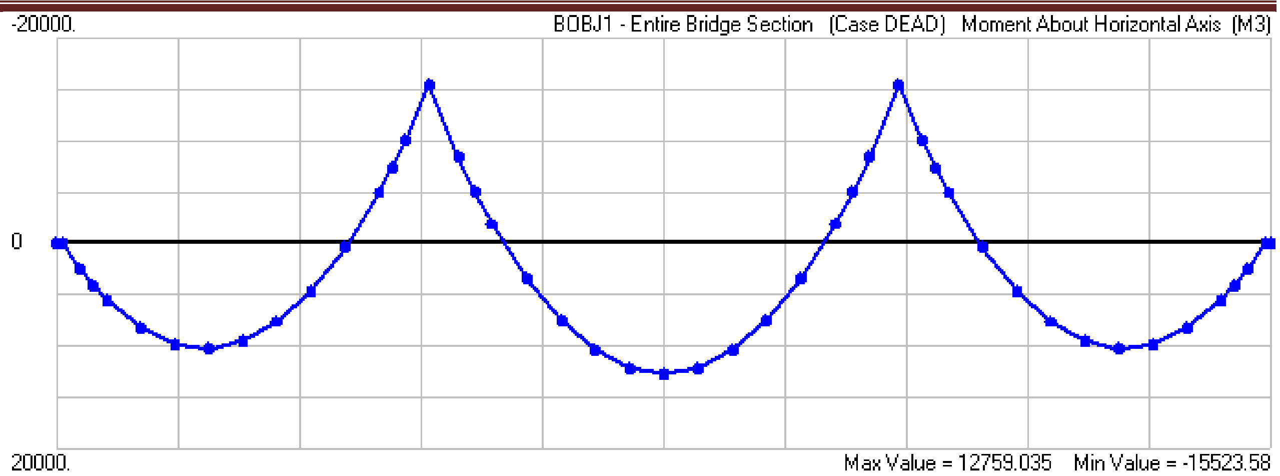



Figure 4.18: the bending moment diagram for the dead load of 97.758m three-span continuous box-girder prior to application of pre-stressing force

## 4.5 Pre-stress Losses

### 4.5.1 Allowable Stress in Strands

In the AA-LRT CDD normal design indexes, control stress of steel strand under pre-stress after all losses = $0.75f_{pk}$ but the AREMA 2010 requirement differs and it is $0.7f'_s$. So that the allowable stress in strands to conduct the analysis is used as per AREMA's requirement. AREMA 2010 [CL.8.17.16] states the limits for stress in prestressing strands. Tensile stress in prestressing tendons shall not exceed the limits specified. Accordingly, the tensile stresses in the strand used in this study are checked against these limits:

- Due to tendon jacking force or Prior to seating: $0.7f'_s$; $0.7\ f_{pk} \leq 0.7f'_s$, hence OK!
- At anchorages, immediately after tendon anchorage or anchor set: $0.7f'_s$; $0.7\ f_{pk} \leq 0.7f'_s$, hence OK!
- The stress at the end of the seating loss zone must not exceed 0.7f's immediately after seating: $0.7f'_s$; $0.7\ f_{pk} \leq 0.7f'_s$, hence OK!

Since the criteria for stress in the strand are met, the jacking coefficient of 0.7 is satisfactory!

### 4.5.2 Friction and Anchor Set Losses

Total losses in prestress are due to instantaneous and time-dependent losses. The instantaneous losses include friction loss, anchor set loss and elastic shortening loss. Time-dependent losses include shrinkage, creep and relaxation. When the strands are pulled through the ducts, losses occur. Some loss is due to a friction along the length of the path and some is due to angle changes in the cable path.

Friction loss equation: $\Delta f_{pF} = f_{pj}\,(1-e^{Kx+\mu\alpha})$

Anchor set loss equation: $L_{PA} = \sqrt{\frac{E(\Delta L)L_{PF}}{\Delta f_{pF}}}$

Standard friction coefficients for post-tensioning tendons simple span and continuous are taken from AA-LRT CDD and are as follows:

Table 4.8: Standard friction coefficients for post-tensioning tendons that are used in the analysis

<table>
<tr><th colspan="2" rowspan="2">Type of elevated structure</th><th colspan="3">Standard friction coefficients</th></tr>
<tr><th>μ (unit less)</th><th>K (/m)</th><th>(ΔL) anchorage set slip (m)</th></tr>
<tr><td>20m simply supported box-girder</td><td>Tensioned from both ends</td><td>0.25</td><td>0.0015</td><td>0.006</td></tr>
<tr><td>46.004m two-span continuous box-girder</td><td>Tensioned from both ends</td><td>0.25</td><td>0.0015</td><td>0.01</td></tr>
<tr><td rowspan="2">30m simply supported box-girder</td><td>Tensioned from one end</td><td rowspan="2">0.23</td><td rowspan="2">0.0025</td><td rowspan="2">0.006</td></tr>
<tr><td>Tensioned from both ends</td></tr>
<tr><td rowspan="2">97.758m three-span continuous box-girder</td><td>Tensioned from one end</td><td rowspan="2">0.25</td><td rowspan="2">0.0025</td><td rowspan="2">0.01</td></tr>
<tr><td>Tensioned from both ends</td></tr>
</table>

### 4.5.3 The equation used to apply curvature and wobble coefficients in SAP 2000

Frictional losses due to curvature and wobble effects are calculated in terms of the tendon force $P_x$ at a distance x from the jacking end, expressed as follows: $P_x = P_o.e^{-(\mu\alpha+Kx)}$ [27].

Where:

- $P_o$ = prestressing force at the jacking end
- $P_x$ = effective prestressing force, at location x, after frictional loss
- $\mu$ = curvature friction coefficient
- $\alpha$ = cumulative angle (in radians) of change in the tendon-profile tangent between x and the jacking end
- $K$ = wobble friction coefficient
- x = position along tendon from jacking end

### 4.5.4 Elastic Shortening, Creep, Shrinkage and Relaxation

Final losses from elastic shortening, shrinkage, creep and relaxation loses are calculated using the formulas from AREMA [CL.8.17.17] of which the elastic shortening loss occurs immediately. Final answerers of the iterations have been used in determining final stress. These losses are added to the friction and anchor set losses.

To determine effective pre-stress $f_{se}$, allowance for the following sources of pre-stress and their considered: $\Delta f_s = ES + CR_s + SH + CR_s$. In addition to these anchorage seating and friction due to intended and unintended curvature in the tendons shall be considered. AREMA also details each loss stress as follows:

#### 4.5.4.1 Elastic shortening of concrete, ES

For Post-tensioned members:

$$ES = 0.5(\frac{E_s}{E_{ci}})f_{cir} \text{ --------------------------------------------------------------- eq (4.5.4.1)}$$

Where $E_{ci}=0.043w_c^{1.5}\sqrt{f_c}$ in MPa

However, $f_{cir}$ from eq (4.5.4.1) in AASHTO LRFD [CL.5.9.5.2.3] is detailed as follows:

$$\Delta f_{pES} = \frac{N-1}{2N}\frac{E_P}{E_{ci}}f_{cgp} \text{ ---------------------------------------------------------- eq (4.5.4.2)}$$

Where $f_{cgp}$= Concrete stress at pre-stressing centroid ($KN/m^2$)

And also it re-determines the loss due to elastic shortening in post-tensioned members, other than slab systems by the following equation:

$$\Delta f_{pES} = \frac{N-1}{2N}\frac{A_{ps}(FC_i)f_{pbt}(I_g+e_m^2A_g) - e_mM_gA_g}{A_{ps}(I_g+e_m^2A_g)+\frac{A_gI_gE_{ci}}{E_p}} \text{ ------------------------ eq (4.5.4.3)}$$

#### 4.5.4.2 Creep of Concrete

For pre-tensioned and post-tensioned members: $CR_c = 12f_{cir} – 7f_{cds}$ ------------------------eq (4.5.4.4)

#### 4.5.4.3 Shrinkage of Concrete

For post-tensioned members:

SH = 0.8(117-1.03R)

Where: R = annual average ambient relative humidity in percent.

The annual average ambient relative humidity for Addis Ababa city in percent is stated to be R = 60.4%. [25]

#### 4.5.4.4 Relaxation of tendon stress

For post-tensioning tendons: 1860MPa low relaxation strand anchored at $0.75f'_s$

$CR_s$ = 25% of $CR_s$ = 20 - 0.3FR - 0.4ES - 0.2(SH + $CR_c$) ---------------------------------------------eq (4.5.4.6)

The resulting prestress losses of the analysis at controlling critical locations are shown in the tables below.

Table 4.9: Instantaneous and time dependent losses for the 20m simple span box-girder for each cable

| | Tensioning from both end |
|---|---|
| Elastic shortening ($KN/m^2$) | 3,649.56 |
| Creep ($KN/m^2$) | 12,951.46 |
| Shrinkage ($KN/m^2$) | 21,791.60 |
| Relaxation ($KN/m^2$) | 15,147.89 |

Table 4.10: Instantaneous and time dependent losses for the 46.004m two-span box-girder each cable

| | Tensioning from both end |
|---|---|
| Elastic shortening ($KN/m^2$) | 8,717.74 |
| Creep ($KN/m^2$) | 56,618.97 |
| Shrinkage ($KN/m^2$) | 21,915.20 |
| Relaxation ($KN/m^2$) | 12,451.52 |

Table 4.11: Instantaneous and time dependent losses for the 30m simple span box-girder each cable

| | Tensioning from the starting end | Tensioning from both end |
|---|---|---|
| Elastic shortening ($KN/m^2$) | 5,858.91 | 6,207.87 |
| Creep ($KN/m^2$) | 17,188.54 | 18,789.04 |
| Shrinkage ($KN/m^2$) | 14,527.73 | 14,527.73 |
| Relaxation ($KN/m^2$) | 9,328.29 | 9,213.37 |

The figures 4.23 up to 4.26 the loss stress diagrams are clearly depicted to visualize the comparisons for each tensioning schemes. The friction loss coefficient that occurred at the location where maximum demand moment occurs is 0.829 for the tensioning from one end and 0.825 for the tensioning from both ends. Accordingly, though the elastic shortening value is the same for both tensioning schemes the resulting loss coefficient will be higher for the tensioning from one end. This is because the same value is deducted from the different friction loss values of the tensioning schemes.

Table 4.12: Instantaneous and time dependent losses for the 97.758m three-span box-girder each cable

| | Tensioning from the starting end | Tensioning from both end |
|---|---|---|
| Elastic shortening ($KN/m^2$) | 2,700.42 | 8,549.47 |
| Creep ($KN/m^2$) | 8,148.75 | 31,477.84 |
| Shrinkage ($KN/m^2$) | 14,527.73 | 14,527.73 |
| Relaxation ($KN/m^2$) | 628.57 | 3,265.86 |

The following figures show instantaneous loss stress diagram (friction losses, anchor set losses, elastic shortening losses) and time dependent losses (creep, shrinkage and relaxation losses) for all the single cells and double cells.

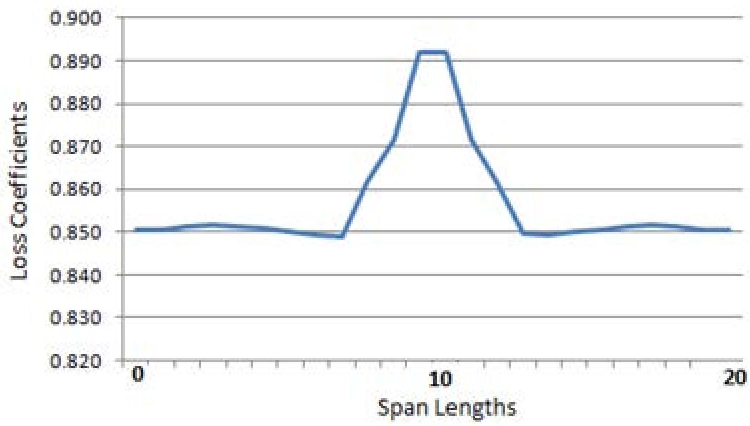



Figure 4.19: Instantaneous loss stress diagram, 20m simply supported box-girder tensioning from both end

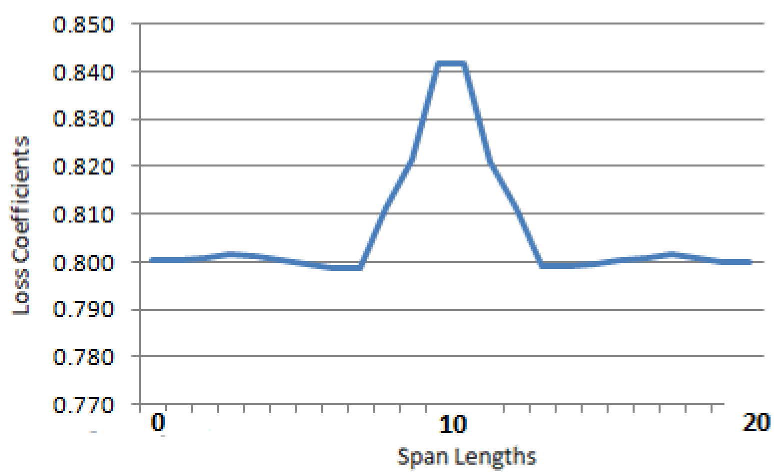



Figure 4.20: Total Loss stress diagram, 20m simply supported box-girder tensioning from both end

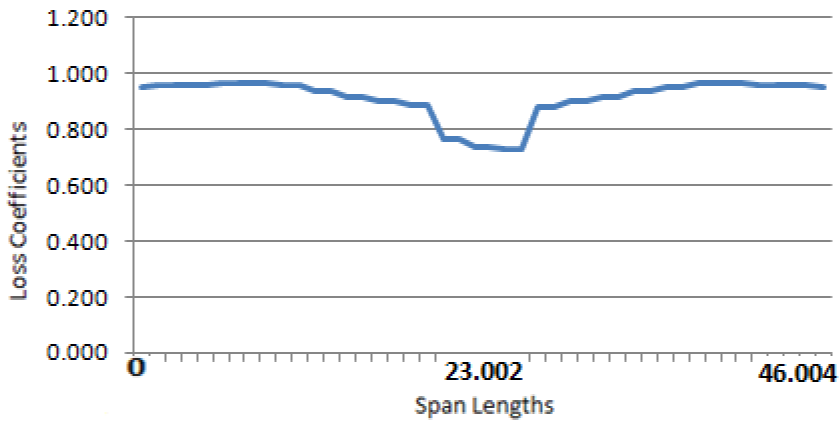



Figure 4.21: Instantaneous loss stress diagram, 46.004m two-span continuous box-girder tensioning from both end

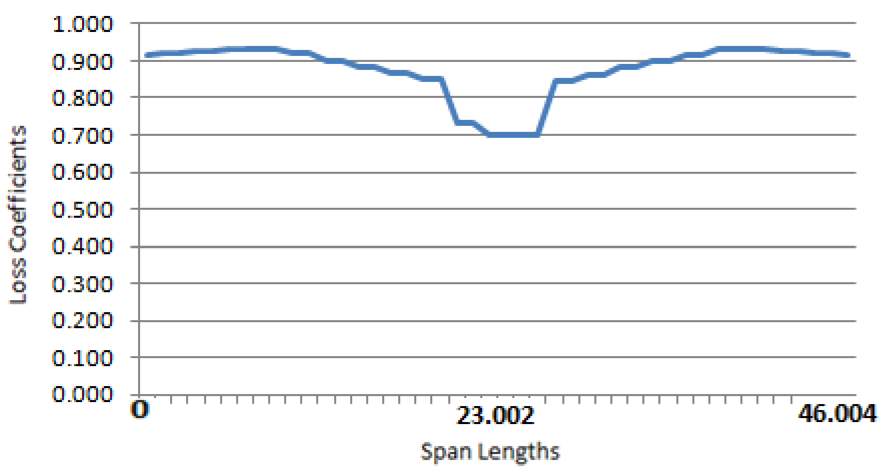



Figure 4.22: Total loss stress diagram, 46.004m two-span continuous box-girder tensioning from both end

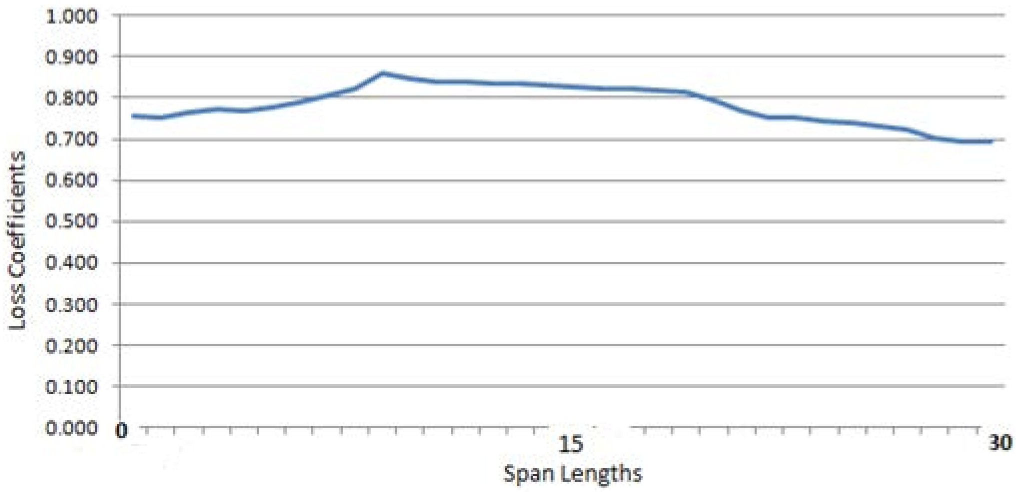



Figure 4.23: Instantaneous loss stress diagram, 30m simply supported box-girder tensioning from start end

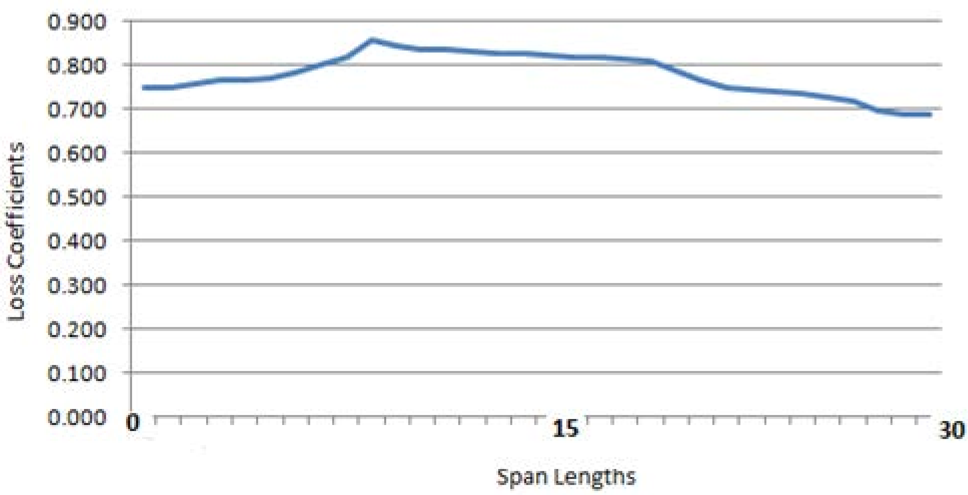



Figure 4.24: Total loss stress diagram, 30m simply supported box-girder tensioning from the start end

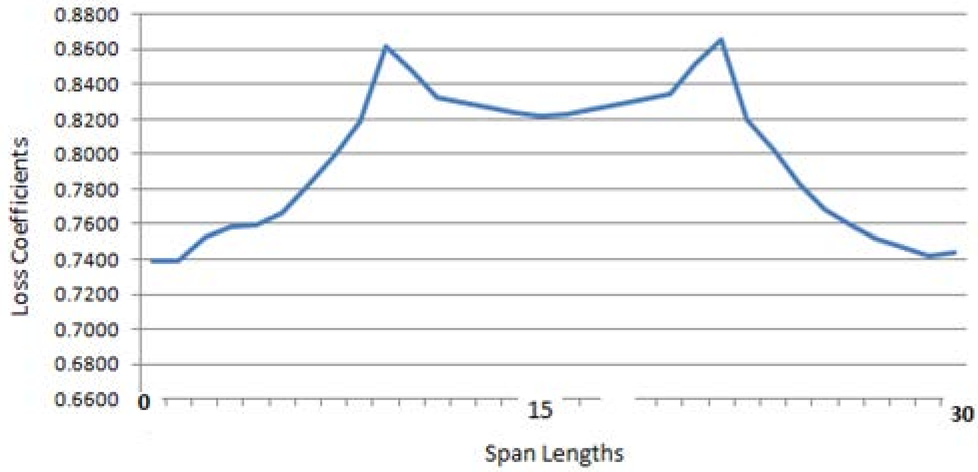



Figure 4.25: Instantaneous loss stress diagram, 30m simply supported box-girder tensioning from both end

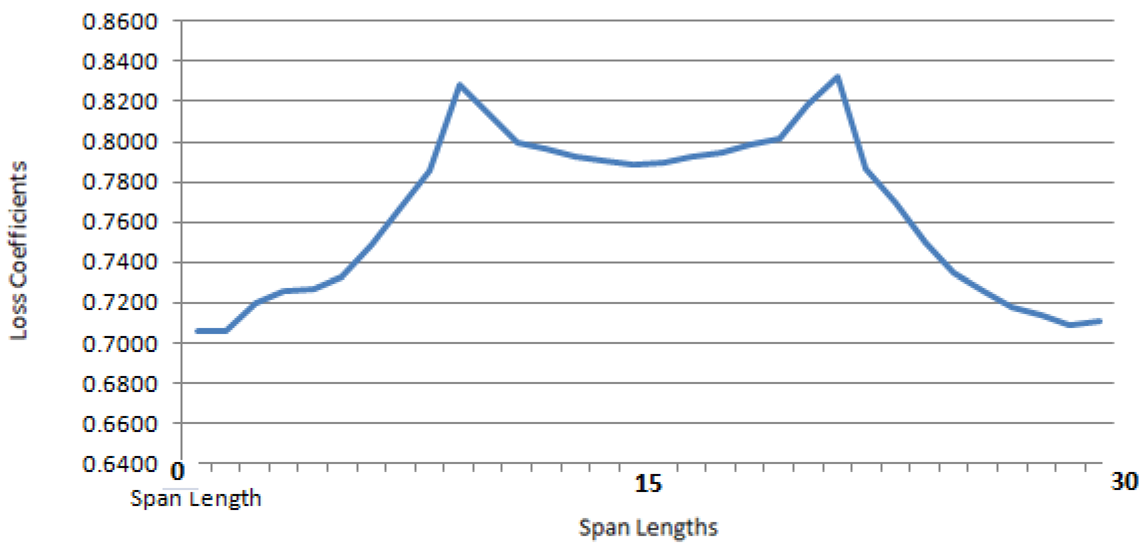



Figure 4.26: Total Loss stress diagram, 30m simply supported box-girder tensioning from both end

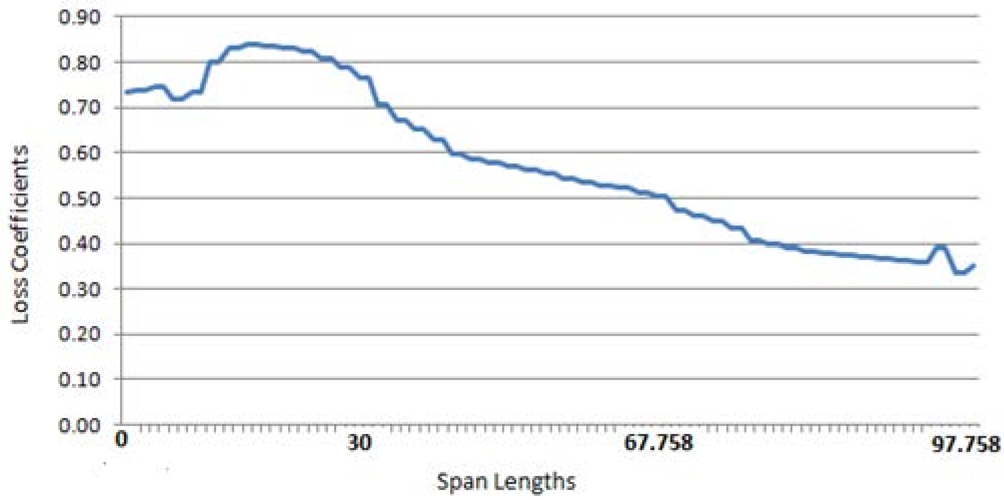



Figure 4.27: Instantaneous loss stress diagram, 97.758m three-span continuous box-girder tensioning from the start end

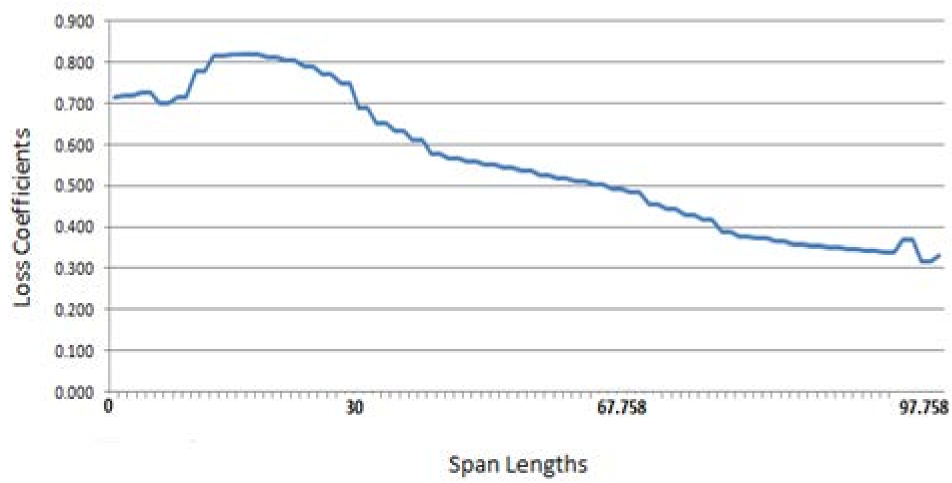



Figure 4.28: Total loss stress diagram, 97.758m three-span continuous box-girder tensioning from the start end

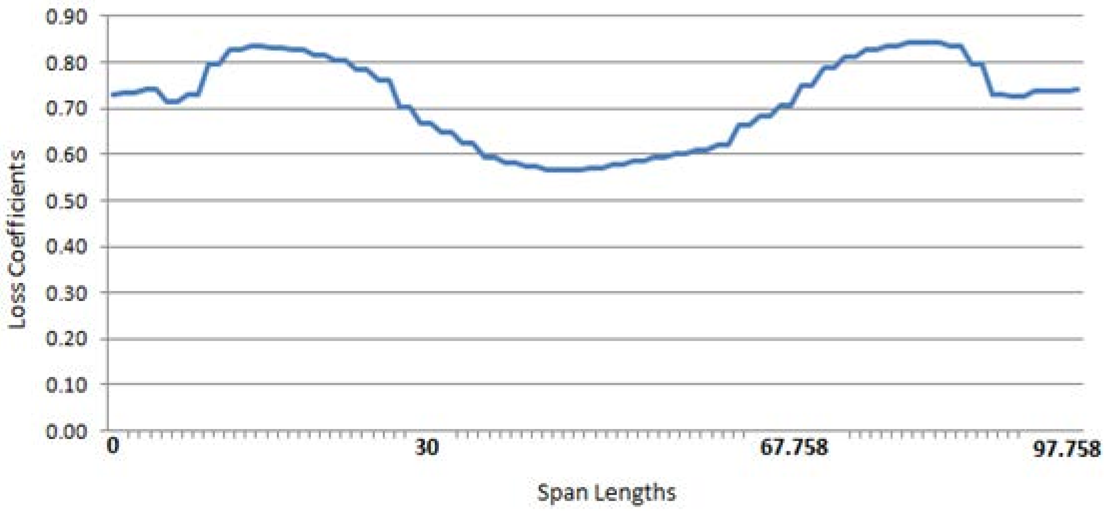



Figure 4.29: Instantaneous loss stress diagram, 97.758m three-span continuous box-girder tensioning from both ends

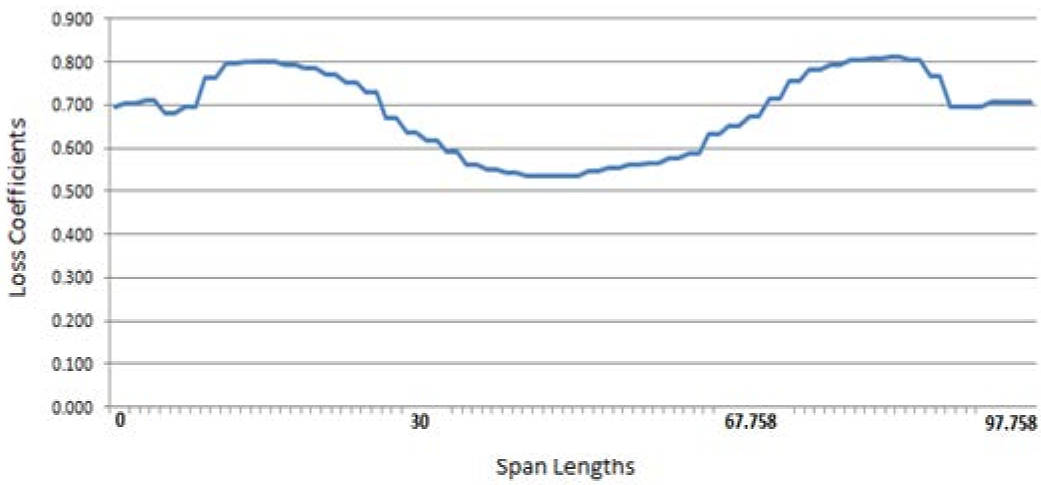



Figure 4.30: Total loss stress diagram, 97.758m three-span continuous box-girder tensioning from both ends

## 4.6 Jacking force & Area of Pre-stressing Steel

The amount of prestressing steel required is controlled by the tension in the bottom fiber at the critical locations. For simple spans PCI Bridge Manual [CL.17.7.7] states the way how to estimate the required prestressing force. The prestressing force is found by equating two equations: Bottom tensile stress due to applied loads at service load,

$$f_b = \frac{M_g + M_{SDL} + M_{LL+I}}{S_b} \quad \text{------------------------------------------------} \quad 4.6.1$$

And Bottom fiber stress due to prestress after all losses,

$$f_b = \frac{P_{se}}{A} + \frac{P_{se} e_c}{S_b} \quad \text{------------------------------------------------} \quad 4.6.2$$

For continuously supported post-tensioned structures the way how pre-stressing force determined is somehow different than the simple spans. The reason lies on the continuity of the spans which gives rise to secondary moment.

According to Duan, L., Chen [10] (CL.10.10.4), for the prestressed concrete member section, the stress at various load stages can be expressed by: f = $\frac{P_j}{A} \pm \frac{P_j ey}{I} \mp \frac{M_y}{I}$ --------------- 4.6.3

This is the basic stress equation for prestressed concrete members. The basic stress equation later modified to $f_{psF} = \frac{F_{pCF} P_j}{A} + \frac{M_{psCF} P_j C}{I_x}$ ----------------------------------------------------- 4.6.4

Where: $F_{pCF}$ final pre-stress force coefficient.

Finally, the prestressing force derived from eq (4.6.4) as

$$P_j = \frac{-f_{DL} - f_{LL+I} - Allowable\ tension}{\frac{F_{pCF}}{A} + \frac{M_{psCF} C}{I_x}}$$ ------------------------------------------------------- 4.6.5

Where C is $y_b$ or $y_t$ and $f_{DL}$ & $f_{LL+IM}$ are stress due to dead load and live load including impact factor respectively. The allowable tension in AREMA (CL.8.17.19) is $0.25\sqrt{f_c'}$.

The area of steel is calculated by assuming a final loss. The tension reinforcing is also controlled by the requirement of zero tension from the effective prestress and dead loads. The results of the prestressing forces analyzed using the ubove equations are shown in the tables below.

$A_{ps}$ is found by dividing $P_j$ by $0.7xf_{pu}$

Table 4.13: demand of tensioning force and area of pre-stressing steel per each cable, 20m simple-span box-girder

| | | Jacking from both end |
|---|---|---|
| $P_j$ (KN) | Initial | 5,955.44 |
| | Final | 6,221.79 |
| $A_{ps,}$ ($m^2$) | | 0.00478 |

Table 4.14: demand of tensioning force and area of pre-stressing steel per each cable, 46.004m two-span continuous box-girder

| | | Jacking from both end |
|---|---|---|
| $P_j$ (KN) | Initial | 7,926.21 |
| | Final | 8,736.24 |
| $A_{ps,}$ ($m^2$) | | 0.00671 |

Table 4.15: demand of tensioning force and area of pre-stressing steel per each cable, 30m simple-span box-girder

| | | Jacking from the starting end | Jacking from both end |
|---|---|---|---|
| $P_j$ (KN) | Initial | 15,981.05 | 15,999.55 |
| | Final | 17,053.68 | 17,884.3 |
| $A_{ps,}$ ($m^2$) | | 0.01309 | 0.01374 |

Table 4.16: demand of tensioning force and area of pre-stressing steel per each cable, 97.758m three-span continuous box-girder

| | | Jacking from the starting end | Jacking from both end |
|---|---|---|---|
| $P_j$ (KN) | Initial | 27,231.19 | 26,274.61 |
| | Final | 35,067.8 | 29,441.83 |
| $A_{ps,}$ ($m^2$) | | 0.02539 | 0.02261 |

Pre-stressing force and concrete strength are determined by meeting stress limits in the service limit states, and then checked in the strength limit states for ultimate capacity.

### 4.7 Flexural strength, Shear strength and Concrete strength

Because all the elevated structures are symmetrical representative critical locations are taken as shown in the tables below.

Table 4.17: Summary of structural response for the 30m simple span box girder

| | | Strength Limit State | | | | | Serviceability Limit States | |
|---|---|---|---|---|---|---|---|---|
| | | Flexural Strength | | | | Shear Strength | Concrete Stresses | |
| | | | | | | | Initial | Final |
| Tensioning Scheme | Critical locations | $M_{SLS}$(KNm) | $M_r$(KNm) | $\frac{M_{SLS}}{M_r}$ | $f'_c$ (KN/$m^2$) | $V_n$ (kN) | $f_{SLS}$ (KN/$m^2$) | $f_{SLS}$ (KN/$m^2$) |
| Tensioning from start end | mid span (0.5L) | 50,180.03 | 97,886.69 | 51.26% | 50,000 | 8,888.25 | $f_b$=12,463.26<br>$f_t$ =3,304.07 | [1]$f_{t1}$=3,213.76<br>[2]$f_{t2}$ =3,849.2<br>[3]$f_{t3}$ =7,354.41 |
| Tensioning from both end | mid span (0.5L) | 50,180.03 | 101,436.26 | 49.47% | 50,000 | 9,393.97 | $f_b$=12,393.34<br>$f_t$ =3,303.00 | [1]$f_{t1}$ =3,207.40<br>[2]$f_{t2}$ =3,845.42<br>[3]$f_{t3}$ =7,352.13 |

**Note**: [1]compressive strength of concrete at service I

[2]compressive strength of concrete at service II

[3]compressive strength of concrete considering Live load only, vibration check

Allowable compression Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses-Creep and Shrinkage) is 0.55 $f'_{ci}$ = 0.55x30 = 16.5MPa ($f'_{ci}$ = 0.6x50MPa = 30MPa). As such all the initial concrete stress in the table 4.17 are less than 16.5MPa. Therefore, the initial assumption $f'_{ci}$ = 30MPa is acceptable.

Maximum allowable stresses in concrete at service loads (after allowance for all prestress losses) is 0.4 $f'_c$ = 0.4x50 = 20 MPa. As such all the final concrete stress in the table 4.17 are less than 20 MPa. So that the final stresses are below the maximum allowable value.

Table 4.18: Summary of structural response for the 97.758m three span continuous box girder

| Strength Limit State | | | | |
|---|---|---|---|---|
| | | Flexural Strength | | |
| Tensioning Scheme | Critical locations | $M_{SLS}$(KNm) | $M_r$(KNm) | $\frac{M_{SLS}}{M_r}$ (%tage) |
| Tensioning from the start end | Span 1 (0.41L) | 41,957.11 | 166,490.48 | 25.20 |
| | Pier II | 32,486.86 | 126,105.75 | 25.76 |
| | Span 2 (0.5L) | 56,232.44 | 173,492.20 | 32.41 |
| | Pier III | 37,867.2 | 126,105.75 | 30.03 |
| | Span 3 (0.6L) | 39,846.74 | 166,490.48 | 23.93 |
| Tensioning from both ends | Span 1 (0.41L) | 41,489.52 | 150,273.36 | 27.61 |
| | Pier II | 33,710.56 | 123,761.17 | 27.24 |
| | Span 2 (0.5L) | 57,654.95 | 156,771.67 | 36.78 |

| Serviceability Limit States (Concrete Stresses) | | | | |
|---|---|---|---|---|
| Tensioning Scheme | Critical locations | $f'_c$ (KN/m$^2$) | Initial | Final |
| | | | $f_{SLS}$ (KN/m$^2$) | $f_{SLS}$ (KN/m$^2$) |
| Tensioning from start end | Span 1 (0.41L) | 50,000 | $f_b$ =26,094.31<br>$f_t$ =3,281.19 | $^1f_{t1}$ =2,872.18<br>$^2f_{t2}$ =3,119.55<br>$^3f_{t3}$ =4,555.64 |
| | Pier II | 50,000 | $f_b$ =749.21<br>$f_t$ =10,583.13 | $^1f_{t1}$ =1,245.84<br>$^2f_{t2}$ =1,274.22<br>$^3f_{t3}$ =651.30 |
| | Span 2 (0.5L) | 50,000 | $f_b$ =11,659.24<br>$f_t$ =6,128.69 | $^1f_{t1}$ =5,867.09<br>$^2f_{t2}$ =5,195.04<br>$^3f_{t3}$ =8,128.58 |
| Tensioning from both ends | Span 1 (0.41L) | 50,000 | $f_b$ =24,333.18<br>$f_t$ =3,291.17 | $^1f_{t1}$ =3,121.09<br>$^2f_{t2}$ =3,244.01<br>$^3f_{t3}$ =4,804.55 |
| | Pier II | 50,000 | $f_b$ =291.63<br>$f_t$ =9,616.61 | $^1f_{t1}$ =476.79<br>$^2f_{t2}$ =1,658.74<br>$^3f_{t3}$ =1,420.35 |
| | Span 2 (0.5L) | 50,000 | $f_b$ =10,374.67<br>$f_t$ =6,615.51 | $^1f_{t1}$ =6,525.65<br>$^2f_{t2}$ =5,524.32<br>$^3f_{t3}$ =8,787.14 |

**Note**: [1]compressive strength of concrete at service I
[2]compressive strength of concrete at service II
[3]compressive strength of concrete considering Live load only, vibration check

Allowable compression Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses-Creep and Shrinkage) is 0.55 $f'_{ci}$ = 0.55x30 = 16.5MPa ($f'_{ci}$ = 0.6x50MPa = 30MPa). But, all the initial concrete stresses in the table 4.18 are not less than 16.5MPa. Therefore, the initial assumption $f'_{ci}$ = 30MPa is not correct. So that $f'_{ci}$ is found by dividing the maximum value, 26,094.31 KN/m$^2$ by 0.55 which is equal to 47,444.19 KN/m$^2$

Stresses in concrete at service loads (after allowance for all prestress losses) is 0.4 $f'_c$ = 0.4x50 = 20 MPa. As such all the final concrete stress in the table 4.18 are less than 20 MPa. So that the final stresses are within acceptable range.

# Chapter 5

## 5. Comparisons of results and discussion

### 5.1 The relative maximum increment of spans when simply supported box girders made continuous

Result comparisons and discussions are expressed in terms of percentage difference of each prestressing parameter. The optimization has been done keeping all the cross-sectional dimensions the same. In order to optimize the sole parameter used is deflection. First the deflection is determined from the simple spans and the same deflection is used to determine the maximum possible increment of the respective span in a continuous ones.

Regarding 20m simply supported span and 46.004m two span continuous box girders, both superstructures analysis has been done in consideration of only one post-tensioning scheme: stressing from both ends only. The reason is that the same scheme only has been applied in AA-LRT CDD.

The 20m simple span has been made increased to 23.002m two spans continuous box girder. So that the percentage increment is $\frac{3.002\text{m}}{20m}$ = 0.1501 ≈ 15.01%.

Again the 30m simple span has been made increased to 37.758m span which is in the middle of three span continuous box girder keeping the two end spans 30m long. So that the percentage increment is $\frac{7.758\text{m}}{30m}$ = 0.2586 ≈ 25.86%.

### 5.2 The relative Prestressing force required to be applied, the corresponding area of prestressing steel, $A_p$ and maximum eccentricity, $e_{max}$

The area of steel is calculated by assuming a final loss. The results of the prestressing forces analyzed are

Table 5.1: percentage difference of prestressing values for single cells

| | | 20m simply supported | 46.004m two span continuous | Percentage increment | *percentage left over |
|---|---|---|---|---|---|
| Jacking force, Pj from both ends (kN) | Initial | 4,956.41 | 7,449.74 | 50.31 | 49.69 |
| | Final | 5,007.54 | 8,316.69 | 66.08 | 33.92 |
| Area of pre-stressing steel ($m^2$) | | 0.00385 | 0.00639 | 65.97 | 34.03 |
| maximum eccentricity, $e_{max}$ (m) | | 0.6372 | 0.867 | 36.06 | 63.94 |

***100 – percentage increment**

By increasing the eccentricity, $e_{max}$ the prestressing force, $P_j$ can be decreased significantly. From the table it can be paraphrased that by optimizing the 20m simple span to two 23,002 spans continuous the prestress values increased to the fourth column percentage increment. It's inevitable for increased prestressing values due to the span increment. For example: $P_j$ =5,007.54kN would have been more than 2x5,007.54kN = 10,015.08kN. But, the resulting value is 8,316.69kN which is even lesser than 10,015.08kN. The reasons behind this emanates from the way how prestressing force is calculated. The prestressing force determination is dependent on the profile of tendons i.e. the nature of parabolas differs greatly in each girders. $\Delta\alpha$ is 0 in the critical location of simply supported one and $\neq 0$ in the case of the continuous one. This favors the decrement of $P_j$ by making the friction relatively lesser in the continuous one as compared to the simply supported one. Again the eccentricity is higher in the continuous one than the simply supported one. This significantly decrease the amount of $P_j$ needed. The maximum $P_j$ occurred right in the middle support of the continuous one. The secondary moment due to continuity which is linear between the supports increase moment in the mid span and decrease moment in the middle. As a result this is also one good reason for the decrement of $P_j$ in the continuous one.

Table 5.2: percentage difference of prestressing values for double cells, tensioning from the start end

| | | 30m simply supported | 97.758m three span continuous | Percentage increment | *percentage left over |
|---|---|---|---|---|---|
| Jacking force, $P_j$ from the start end (kN) | Initial | 15,981.05 | 27,231.19 | 41.31 | 58.69 |
| | Final | 17,053.68 | 33,067.8 | 48.43 | 51.57 |
| Area of pre-stressing steel ($m^2$) | | 0.01309 | 0.02539 | 48.44 | 51.56 |
| maximum eccentricity, emax (m) | | 0.743 | 0.793 | 6.3 | |

***100 - percentage increment**

Table 5.3: percentage difference of prestressing values for double cells, tensioning from both ends

| | | 30m simply supported | 97.758m three span continuous | Percentage increment | *percentage left over |
|---|---|---|---|---|---|
| Jacking force, Pj from both ends (kN) | Initial | 15,999.55 | 26,274.61 | 39.11 | 60.89 |
| | Final | 17,884.3 | 29,441.83 | 39.26 | 60.74 |
| Area of pre-stressing steel ($m^2$) | | 0.01374 | 0.0226 | 39.26 | 60.74 |
| maximum eccentricity, emax (m) | | 0.743 | 0.793 | 6.3 | |

***100 - percentage increment**

The same concept is true here with the double cells. For the reason that increased middle span as compared to the single one it's inevitable for increased percentage value. But due to the continuity of the structure, for the three-span continuous, it gets lesser than expected. Another important point here is that even if the eccentricity increased in the middle span, the prestressing force and the corresponding area of steel keep on decreasing. The main reason still lies on the nature of the continuity of 97.758m three span continuous structure. The secondary moment is high enough to swallow the effect of eccentricity.

## 5.3 The relative prevailing deflections at various stages of loadings and relative provisions of end anchorages

Table 5.4: percentage difference of prevailing deflections at various stages of loadings

| Single cell | | | | |
|---|---|---|---|---|
| Post-tensioned box-girders | Tensioning Schemes | Immediately after post-tensioning | Long-time deflection | Percentage increment |
| | | Deflection (mm) | Deflection (mm) | |
| 20m simply supported | $P_j$, from both end | 3.83 | 3.9 | 1.79 |
| 46.004m two span continuous | $P_j$, from both end | 4.74 | 4.8 | 1.25 |
| Double cell | | | | |
| 30m simply supported | $P_j$, from start end | 7.16 | 7.18 | 0.28 |
| | $P_j$, from both end | 7.16 | 7.18 | 0.28 |
| 97.758m three-span continuous | $P_j$, from start end | 10.55 | 10.71 | 1.49 |
| | $P_j$, from both end | 10.55 | 10.66 | 1.032 |

If a box girder is made of simple spans, then anchorage is needed at each end of each span. However, the number of bearings and end anchorages could be decreased by making the simple spans continuous. The provision of end anchorages are as follows relatively:

For the simple spans: at the two endings

For two-span continuous: at the two endings (provision of two end anchorages has been saved in a relative to the two respective simple spans)

For three-span continuous: at the two endings (provision of four end anchorages has been saved in a relative to the three respective simple spans)

## 5.4 Material cost comparisons

This material cost investigation addresses the relative values of material cost and compares the material costs per deck area for the analysis cases of the double cells. The cost study also demonstrates their relative cost benefits.

#### 5.4.1 Material Unit Prices

The unit price for concrete and prestressing tendons are listed in the Table 5.5 below in accordance with Sandy Shuk-Yan Poon [19]. All unit prices include installation costs (i.e. concrete placement and vibration, grouting of tendons).

Table 5.5: Material unit prices

| Material | Unit | Unit price |
|---|---|---|
| Concrete ($f'_c$=50MPa) Material only | Per $m^3$ | $ 250 |
| Longitudinal prestressing tendons | Per Kg | $ 8.5 |

##### 5.4.1.1 Concrete Cost Comparison

The volume of concrete that is needed to build up a 30m simply supported double cell box girder tensioned from the start end is equal to 124.83$m^3$ - 1.21$m^3$ = 123.62$m^3$. It's found by deducting the volume of pre-stressing steel from the total volume of concrete. As such the cost is calculated by multiplying the resulting volume by its unit price: 123.62 x 250 = $ 30,905.

The above calculation has been done for a single simple span. Yet, the same computation is followed for the cost comparison. Accordingly, the volume of concrete that is needed to build up three 30m simply supported girders:

- For tensioning from the start end: 3 x 124.83 – 3 x 1.21 = 370.85$m^3$ and the cost is 370.85 x 250 = $ 92,712.5
- For tensioning from both ends: 3 x 124.83 – 3 x 1.25 = 370.73$m^3$ and the cost is 370.73 x 250 = $ 92,682.5

However, the volume of concrete that is needed to build up 97.758m three spans continuously supported girder:

- For tensioning from the start end: 406.75 – 7.53 = 399.22$m^3$ and the cost is 399.22 x 250 = $ 99,805.58
- For tensioning from both ends: 406.75 – 6.64 = 400.11$m^3$ and the cost is 400.11 x 250 = $ 100,027.98

Therefore, the concrete cost increases directly with its volume. But, this is not true for prestressing cost as it dependant on the supporting conditions of the superstructures.

##### 5.4.1.2 Pre-stressing cost comparison

Unlike concrete, the cost of pre-stressing tendons varies as simply supported girders appear to be continuous. As such the total number of prestressing steels and it's cost is lower for the 97.758m three spans continuously supported box girder as shown below:

- For tensioning from the start end: A total of 9,512.64 Kg prestressing steel is needed for each three simply supported and the cost is 9,512.64 x 8.5 = $ 80,857.44

- For tensioning from both ends: A total of 9,809.91 Kg prestressing steel is needed for each three simply supported and the cost is 9,809.91 x 8.5 = $ 83,384.24

Whereas the prestressing cost for the 97.758m three spans continuously supported box girder:

- For tensioning from the start end: A total of 9,512.64 Kg prestressing steel is needed for each three simply supported and the cost is 9,512.64 x 8.5 = $ 80,857.44
- For tensioning from both ends: A total of 9,809.91 Kg prestressing steel is needed for each three simply supported and the cost is 9,809.91 x 8.5 = $ 83,384.24

### 5.5 Summary

An assessment of post-tensioning conducted for a total of four elevated structures, two single cells and two are double cells. The modeling and analysis is done for a total of four structures of both types: 20m (one-span simply supported) & 23.002m (two-span continuous) single cell each and 30m span (one-span simply supported) & 97.757m (three-span continuous) double cell each (optimizing only the middle one can highly be applied in areas where there is a need to interchange heavy traffic flow) like the junction right infront of Estifanos church. The double cells analyzed in consideration of two types of prestressing schemes: tensioning from the start end and both ends whereas the single cells analyzed using only one prestressing scheme: tensioning from both ends.

The objective of this research is to estimate the quantity of percentage values of pre-stress for continuous ones in a relative to the simply supported ones and the resulting span increments.

In this study only three basic loadings are considered: Dead load, Secondary dead load and Live load. Full pre-stress design is done for the double cell ones as the single cells do not meet the geometry requirements of AREMA's specification independently.

The maximum span increment is calculated by making the 20m simply supported segment two span continuous and the 30m one three span continuous i.e. by keeping two ends 30m and allowing the middle span to maximum optimization possible. The parameter used to determine span increment is deflection prior to application of pre-stress and is calculated to be 5mm for 20m and 15.6mm for the 30m.

The relative percentage values of pre-stressing for each continuous structure are compared with the respective single segments. Cross-sections and pre-stress loss parameters are basically taken from construction drawing of Addis Ababa E-W & N-S (Phase I) Light Rail Transit Project. From SAP2000 modeling results the 20m spans are increased to 23.002m. Again, by keeping both end span 30m, the middle span of three span continuous box girders is increased to 37.758m.

In order to investigate the relative prestressing values the analysis involved inputting values and comparing the results and reiterating the process until convergence.

# Chapter 6

## 6. Conclusion and future works

### 6.1 Conclusion

A continuous box girder is a statically indeterminate structure whose continuity supports a great deal in the reduction of prestressing values despite additional construction costs in a relative to the simply supported ones. In addition to the optimization, there is significant reduction of jacking force and steel area when two simply supported spans are made continuous.

The horizontal and vertical profiles of the cable or tendons affect all the prestress loss values and parameters. The same profiles in AA-LRT CDD, 2013 have been used for this study for the simple box girders. Again the vertical cable profiles for the continuous box girders also adjusted to have the same eccentricity at the critical points to make a room for the provision of non prestressed reinforcements. The horizontal profiles are adjusted to keep the cable on the center of the webs for all the box girders.

There are two types of tensioning schemes that has been used to analyze all box girders namely: tensioning from one end and tensioning from both ends. The main difference between these tensioning schemes lies on the resulting loss which affects the prestressing force. Because all spans of the box girders under this study are symmetrical the friction loss that occurs when all box girders tensioned from both ends are symmetrical and higher as compared to the tensioning from the start end for the simply supported box girders. The opposite happened for the continuous box girders. The friction loss that occurs when all the box girders tensioned from the start end is not symmetrical. It is from these friction loss patterns that the instantaneous and time dependant loss are deducted to result the loss coefficients which later used in the computation of the prestressing force and the concrete stresses for each structures.

The three basic loadings that have been used in this study are Dead load, Additional dead load and Live load of train. The Prestressing force that is required if the 20m simply supported segment made two span, each 23.002m long, continuous box girder is 17,472.473 kN which is only 40.414% increment relatively. In other words, it means that 59.586% of prestressing force would have been saved per each segment. Again, the Prestressing force that is required if the 30m cast in place simply supported ones made three span, keeping the end span 30m long and the middle one 37.758m, continuous box girder is 99,203.4 kN (for tensioning from the start end) and is 88,325.49 kN (for tensioning from both ends) which means only 48.43% and 39.26% increment respectively. In other words, the resulting post-tensioning forces for the three span continuous do not reach even two folds of the post-tensioning forces of the respective simply supported box girders. So that 54.72% (for tensioning from one end) 82.23% (for tensioning from both ends) of prestressing force would have been saved per each cast in place simply supported box girder. The area of prestressing steels and the number of tendons are directly depend on the amount of prestressing force for each box girders. The numbers of strands in the post-tensioning tendons has been done in a reference from the post-tensioning system of AA-LRT CDD, 2013 for the simply supported box-girders and freyssinet prestressing, 2010. The provision of end anchorages for the two-span continuous is in such a way that provision of two end anchorages has been saved in a relative to the two respective simple spans. And again provision of four end anchorages has been saved in a relative to the three respective simple spans for the case of three-span continuous box girder.

## 6.2 Future works

During the analysis of post-tensioning of concrete box girder railway elevated structures, there are a lot of problems to be solved. However, due to the broad applications of post-tensioning of Concrete Box girder railway elevated Structures, in this paper the analysis is limited to the relative percentages of post-tensioning values of AA-LRT just to determine the relative or comparable values of post tensioning values if the simply supported ones are to be made continuous in the future.

During the work of this paper and review of literatures there are a lot of related problems that are to be addressed and solved parallel or to start from the finding of this paper. However, expecting that, investigation of relative percentages of post-tensioning values if simply supported box girders to be made continuous has significant role in the future works of optimization of prestressed concrete box girders railway elevated structures, this paper gives priority for the relative percentages of post-tensioning values.

Depending on the current and future railway demands and the upcoming shortage of resources in the metropolitan cities like Addis Ababa, the broad study of post-tensioning invites the future works of researchers, specifically on the relative percentages of post-tensioning values. Though there are so many problems the following points needs to be addressed very importantly.

- ✓ The relationship between pre-stressing and optimization of the spans i.e. optimization using prestressing values as a parameter with in a limit described in the AREMA 2010.
- ✓ The Impact of couplers which are used to give continuity to the tendons due to their length on further decreasing of post-tensioning losses and further improvement of post-tensioning values
- ✓ The complex relationship between cable profile, pre-stressing force and pre-stress losses
- ✓ The way how relative prestressing done in cases of absence of enough jacking equipments i.e. when one jacking equipment used to tension tendons in both ends of box girder consecutively?
- ✓ Comparison of the way how post-tensioning addressed in a bridge design manuals, AREMA and Chinese code for design of railway line, especially for Light rail Transits elevated structures.

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# APPENDIX

# Appendices

Appendix A: Supporting Calculations

A.1 Flexural Strength for bonded tendons at strength limit state (SLS)

A.2 Shear Strength at SLS

Appendix B: Summary of Analytical Results

20m simple span single cell simply supported box-girder elevated structure

Unfactored Moment envelope for the entire bridge section (tensioning from both ends)

46.004 two-span continuous single cell box-girder elevated structure

Unfactored Moment envelope for the entire bridge section (tensioning from both ends)

B.3. 30m simple span double cell simply supported box-girder elevated structure

Unfactored Moment envelope for the entire bridge section (tensioning from the start end)

Unfactored Shear envelope for the entire bridge section (tensioning from the start end)

Unfactored Moment envelope for the entire bridge section (tensioning from both ends)

Unfactored Shear envelope for the entire bridge section (tensioning from both ends)

B.4. 97.758m three-span continuous double cell box-girder elevated structure

Unfactored Moment envelope for the entire bridge section (tensioning from the start end)

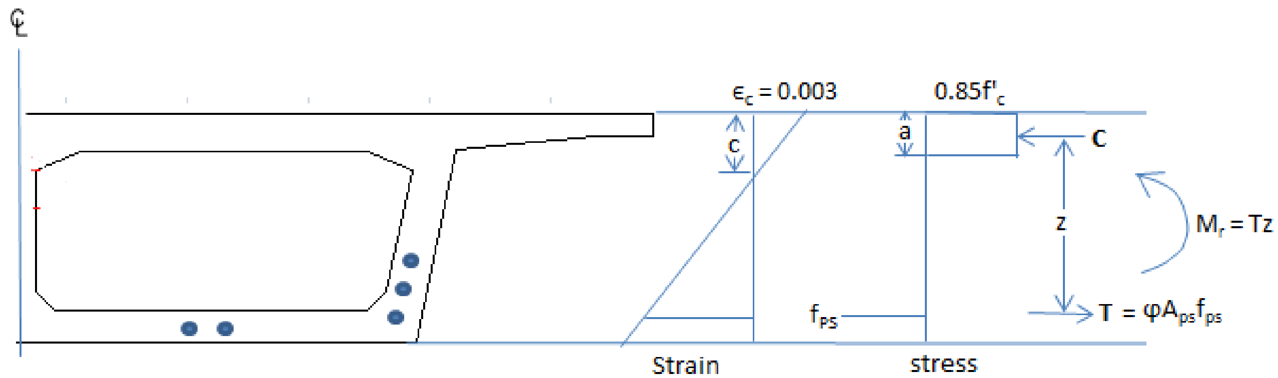
Unfactored Shear envelope for the entire bridge section (tensioning from the start end)

Unfactored Moment envelope for the entire bridge section (tensioning from both ends)

Unfactored Shear envelope for the entire bridge section (tensioning from both ends)

## Appendix A: Supporting Calculations

### 7. A.1 Flexural Strength for bonded tendons at strength limit state (SLS)




This calculation is performed for cast-in-place concrete box girders of 30m simple span and 97.758 three span continuous box girders

Concrete properties

$f'_c$ = 50 MPa

Cross-sectional properties

D = Total depth = 1.8m

b = width of compressive component = 9.95m

$y_b$ = distance from base to centroid of the section = 1.054m

$I_g$ = moment of inertia about horizontal axis at the section = 2.605m$^4$

**30m simple span box girder**

Flexural strength design done for the two types of tensioning schemes namely: tensioning from the start end and tensioning from both ends.

$M_{SLS}$ = Moment demand = 50,180.026 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel

= 1.51m (for tensioning from the start end)

= 1.52m (for tensioning from both ends)

Prestressing tendon properties

$f_{py}$ = 1860 MPa

Number of strands = 288 (for tensioning from the start end)

= 297 (for tensioning from both ends)

$A_{ps}$ = area of prestressing tendons

= (288strands) x (140 $mm^2$) = 0.0403$m^2$ (for tensioning from the start end)

= (297strands) x (140 $mm^2$) = 0.0416$m^2$ (for tensioning from both end)

Flexural strength requirements

For $(A_{ps} f_{su})/(0.85 f'_c b) < t$, $\varphi M_n = \varphi[A_{ps} f_{su} d \{1-0.6(p f_{su}/ f'_c)\}] = \varphi[A_{ps} f_{su} (d – a/2)]$

(AREMA, CL.8.17.18.2)

$c = (A_{ps} f_{su})/(0.85 f'_c b)$

For tensioning from the start end = (0.0403x0.966x1,860,000)/(0.85x50,000x9.95)

= 0.17m < 0.3m

Rectangular section!

→ $f_{su} = f'_s [1-(\gamma/\beta_1)( pf'_s/ f'_c)]$

= 1,860,000[1-(0.28/0.825)x(0.0403/(9.95x1.51))x(1,860,000/50,000)] = 0.966x1,860,000

→ a = c$\beta_1$= 0.17x0.825 =0.14

→ $T = \varphi A_{ps} f_{su}$ = 0.95x0.0403x0.966x1,860,000 = 68,788.96 kN

→ $z = d - \frac{a}{2} = 1.51 – 0.6 \frac{A_{PS}}{b}\frac{f_{su}}{f'_c} = 1.52 – 0.6x\frac{0.0403}{9.95}x\frac{0.966x1860,000}{50,000} = 1.423m$

→ $\varphi M_n = M_r = T \cdot z$ = 68,788.96x1.423 = 97,886.69 kNm ≥ $M_{SLS}$ = 50,180.026 kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$p f_{su}/ f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

For tensioning from the start end:

(0.0403/(9.95x1.51))x0.966x1860,000/50,000 = 0.096 ≤ 0.36x0.825 = 0.297

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2 M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{30,000}$ = 107.91 kNm (AREMA, CL.8.17.162.3)

$S_c = 3.1762m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}] = 51,161.042x0.827x[\frac{1}{5.58} + \frac{0.743x1.054}{2.61}]$ = 20,301.87 kN/$m^2$

$M_{cr} = (f_r + f_{pe})S_c$
= (107.91+ 20,301.87) x 3.1762
= 64,825.54 kNm

1.2 $M_{cr}$ = 1.2x64,825.54 = 77,790.65 kNm < 97,886.69 kNm

∴ Requirement is satisfied!

For tensioning from both ends:

For tensioning from the start end = (0.0416x0.965x1,860,000)/(0.85x50,000x9.95)
= 0.18m < 0.3m
Rectangular section!

→ $f_{su} = f'_s\,[1-(\gamma/\beta_1)(\,pf'_s/f'_c)]$
= 1,860,000[1-(0.28/0.825)x(0.0416/(9.95x1.52))x(1,860,000/50,000)] = 0.965x1,860,000

→ a = $c\beta_1$= 0.18x0.825 =0.149

→ T = $\varphi A_{ps} f_{su}$ = 0.95x0.0416x0.965x1,860,000 = 70,934.45 kN

→ $z = d - \frac{a}{2} = 1.52 - 0.6\,\frac{A_{PS}}{b}\frac{f_{su}}{f'_c} = 1.52 - 0.6\text{x}\frac{0.0416}{9.95}\text{x}\frac{0.965x1860{,}000}{50{,}000}$ = 1.43m

→ $\varphi M_n = M_r = T\cdot z$ = 70,934.45x1.43 = 101,436.26 kNm ≥ $M_{SLS}$ = 50,180.026 kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$p\,f_{su}/f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

For tensioning from the start end:

(0.0416/(9.95x1.52))x0.965x1860,000/50,000 = 0.099 ≤ 0.36x0.825 = 0.297

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623\text{x}\sqrt{30{,}000}$ = 107.91 kNm (AREMA, CL.8.17.162.3)

$S_c$ = 3.1762 $m^3$

For tensioning from the start end:

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}]$ = 53,652.886x0.788x$[\frac{1}{5.58} + \frac{0.743x1.054}{2.61}]$ = 20,286.65 kN/$m^2$

$M_{cr} = (f_r + f_{pe})S_c$
= (107.91+ 25,713.59) x 3.1762

= 64,777.22 kNm

1.2 $M_{cr}$ = 1.2x64,777.22 = 77,732.66 kNm $\leq$ 101,436.26 kNm

∴ Requirement is satisfied!

**97.758m three span continuous box girder**

Flexural strength design done for the two types of tensioning schemes namely: tensioning from the start end and tensioning from both ends.

Prestressing tendon properties

$f_{py}$ = 1860 MPa

Number of strands = 563 (for tensioning from the start end)

= 498 (for tensioning from both ends)

$A_{ps}$ = area of prestressing tendons

= (550 strands) x (140 $mm^2$) = 0.077$m^2$ (for tensioning from the start end)

= (485 strands) x (140 $mm^2$) = 0.0679$m^2$ (for tensioning from both ends)

Flexural strength requirements

For ($A_{ps}$ $f_{su}$)/(0.85 $f'_c$b) < t, $\varphi M_n = \varphi[A_{ps} f_{su}$ d {1-0.6(p $f_{su}$/ $f'_c$)}] = $\varphi[A_{ps} f_{su}$ (d – a/2)]

(AREMA, CL.8.17.18.2)

For tensioning from the start end

Span 1 (0.41L)

$M_{SLS}$ = Moment demand = 41,957.11 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.483m

→ $f_{su} = f'_s$ [1-(γ/$\beta_1$)( p$f'_s$/ $f'_c$)]

= 1,860,000[1-(0.28/0.723)x(0.077/(9.95x1.483))x(1,860,000/50,000)] = 0.925x1,860,000

c = ($A_{ps}$ $f_{su}$)/(0.85 $f'_c$b)

= (0.077x0.925x1,860,000)/(0.85x50,000x9.95) = 0.313m > 0.3m

Flanged section!

$\varphi M_n = \varphi\{A_{sr} f_{su}$ d [1-0.6($A_{sr}$ $f_{su}$/b′d$f'_c$)] + 0.85$f'_c$ (b-b′)(t)(d-0.5t)}

where $A_{sr}$ = $A_{ps}$ – 0.85$f_c'$(b-b′)t/$f_{su}$

= 0.077 - 0.85x50,000x(9.95 - 0.91)x0.3/(0.925x1,860,000)

= $0.01m^2$

→ $\varphi M_n$ = 0.95x(0.01x0.925x1,860,000x1.483[1-0.6x((0.01x0.925x1,860,000)/ (0.91x1.483x50,000)]+0.85x50,000x(9.95-0.91)x0.3x(1.483-0.5x0.3)

= 166,490.48 kNm > $M_{SLS}$ = 41,957.25 kNm

∴ the section is adequate for flexural strength!

<u>Maximum prestressing steel</u>

$A_{sr}f_{su}/b^{'}d\ f^{'}_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

= (0.01x0.925x1860,000)/(0.91x1.483x50,000)

= 0.18 < 0.36x0.723 = 0.26

∴ requirement is satisfied!

<u>Minimum reinforcement</u>

$\varphi M_n \geq 1.2M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{44{,}864.18}$ = 131.96 kN/m$^2$ (AREMA, CL.8.17.162.3)

$S_c = 3.1762m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}]$ = 99,203.4x0.821x$[\frac{1}{5.58} + \frac{0.738x1.054}{2.605}]$ = 38,915.80 kN/m$^2$

$M_{cr} = (f_r + f_{pe})S_c$

= (131.96 + 38,915.80) x 3.1762

= 124,023.49 kNm

1.2 $M_{cr}$ = 1.2x124,023.49 = 148,828.19 kNm < 166,490.48 kNm

∴ Requirement satisfied!

<u>Pier II</u>

$M_{SLS}$ = Moment demand = -32,486.86 kNm

<u>Cross-sectional properties</u>

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.489m

→ $f_{su} = f^{'}_s\ [1-(\gamma/\beta_1)(\ pf^{'}_s/\ f^{'}_c)]$

= 1,860,000[1-(0.28/0.825)x(0.077/(6.228x1.489))x(1,860,000/50,000)] = 0.88x1,860,000

$c = (A_{ps} f_{su})/(0.85 f'_c b)$

= (0.077x0.88x1,860,000)/(0.85x50,000x6.228) = 0.476m > 0.3m

Flanged section!

$\varphi M_n = \varphi\{A_{sr} f_{su} d [1-0.6(A_{sr} f_{su}/b' d f'_c)] + 0.85 f'_c (b-b')(t)(d-0.5t)\}$

where $A_{sr} = A_{ps} - 0.85 f_c'(b-b')t/f_{su}$

= 0.077 - 0.85x50,000x(6.228- 0.91)x0.3/(0.88x1,860,000)

= 0.036m²

→ $\varphi M_n$ = 0.95x(0.036x0.88x1,860,000x1.489x[1-0.6x((0.036x0.88x1,860,000)/(0.91x1.489x50,000)]+0.85x50,000x(6.228-0.91)x0.3x(1.489-0.5x0.3)

= 126,105.75 kNm > $M_{SLS}$ = 32,486.86 kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$A_{sr} f_{su}/b' d f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

= (0.036x0.88x1860,000)/(0.91x1.489x50,000)

= 0.27 < 0.36x0.825 = 0.297

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2 M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623 \times \sqrt{44{,}864.18}$ = 131.96 kN/m² (AREMA, CL.8.17.162.3)

$S_c$ = 3.6991m³

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}] = 99{,}203.4 \times 0.655 \times [\frac{1}{10.768} + \frac{0.53 x 0.841}{3.548}]$ = 11,639.78 kN/m²

$M_{cr} = (f_r + f_{pe})S_c$

= (131.96 + 11,639.78) x 3.6991

= 43,544.86 kNm

1.2 $M_{cr}$ = 1.2x43,544.86 = 52,253.83 kNm < 126,105.75 kNm

∴ Requirement satisfied!

Span 2 (0.5L)

$M_{SLS}$ = Moment demand = 56,232.44 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.539m

$\rightarrow f_{su} = f'_s\,[1-(\gamma/\beta_1)(\,pf'_s/\,f'_c)]$

$= 1,860,000[1-(0.28/0.723)x(0.077/(9.95x1.539))x(1,860,000/50,000)] = 0.927x1,860,000$

$c = (A_{ps}\,f_{su})/(0.85\,f'_c b)$

$= (0.077x0.927x1,860,000)/(0.85x50,000x9.95) = 0.314m > 0.3m$

Flanged section!

$\varphi M_n = \varphi\{A_{sr}\,f_{su}\,d\,[1-0.6(A_{sr}\,f_{su}/b'df'_c)] + 0.85f'_c\,(b-b')(t)(d-0.5t)\}$

where $A_{sr} = A_{ps} - 0.85f_c'(b-b')t/f_{su}$

$= 0.077 - 0.85x50,000x(9.95 - 0.91)x0.3/(0.927x1,860,000)$

$= 0.01m^2$

$\rightarrow \varphi M_n = 0.95x(0.01x0.927x1,860,000x1.489[1-0.6x((0.01x0.927x1,860,000)/$
$(0.91x1.539x50,000)]+0.85x50,000x(9.95-0.91)x0.3x(1.539-0.5x0.3)$

$= 173,492.20$ kNm $> M_{SLS} = 56,232.44$ kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$A_{sr}f_{su}/b'd\,f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

= (0.01x0.927x1860,000)/(0.91x1.539x50,000)

= 0.173 < 0.36x0.825 = 0.297

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{44,864.18} = 131.96$ kN/m$^2$ (AREMA, CL.8.17.162.3)

$S_c = 3.1762m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}]$ = 99,203.4x0.537x$[\frac{1}{5.58} + \frac{0.738x1.054}{2.605}]$ = 25,454.06 kN/m$^2$

$M_{cr} = (f_r + f_{pe})S_c$
= (131.96 + 25,454.06) x 3.1762
= 81,266.32 kNm

1.2 $M_{cr}$ = 1.2x81,266.32 = 97,519.58 kNm < 173,492.20 kNm

∴ Requirement satisfied!

Pier III

$M_{SLS}$ = Moment demand = -37,867.2 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.489m

→ $f_{su} = f'_s\ [1-(\gamma/\beta_1)(\ pf'_s/ f'_c)]$
= 1,860,000[1-(0.28/0.723)x(0.077/(6.228x1.489))x(1,860,000/50,000)] = 0.88x1,860,000

c = $(A_{ps}\ f_{su})/(0.85\ f'_c b)$
= (0.077x0.88x1,860,000)/(0.85x50,000x6.228) = 0.476m > 0.3m
Flanged section!

$\varphi M_n = \varphi\{A_{sr}\ f_{su}\ d\ [1-0.6(A_{sr}\ f_{su}/b'df'_c)] + 0.85f'_c\ (b-b')(t)(d-0.5t)\}$
where $A_{sr} = A_{ps} - 0.85f_c'(b-b')t/f_{su}$
= 0.077 - 0.85x50,000x(6.228- 0.91)x0.3/(0.88x1,860,000)
= 0.036m$^2$

→ $\varphi M_n$ = 0.95x(0.036x0.88x1,860,000x1.489x[1-0.6x((0.036x0.88x1,860,000)/ (0.91x1.489x50,000)]+0.85x50,000x(6.228-0.91)x0.3x(1.489-0.5x0.3)
= 126,105.75 kNm > $M_{SLS}$ = 37,867.2 kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$A_{sr}f_{su}/b'd\ f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)
= (0.036x0.88x1860,000)/(0.91x1.489x50,000)
= 0.27 < 0.36x0.825 = 0.297
∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2 M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623\text{x}\sqrt{44{,}864.18} = 131.96$ kN/m$^2$ (AREMA, CL.8.17.162.3)

$S_c = 3.6991\text{m}^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}] = 99{,}203.4\text{x}0.445\text{x}[\frac{1}{10.768} + \frac{0.53x0.841}{3.548}] = 9{,}645.63$ kN/m$^2$

$M_{cr} = (f_r + f_{pe})S_c$

$= (131.96 + 9{,}645.63) \text{ x } 3.6991$

$= 36{,}168.28$ kNm

$1.2\ M_{cr} = 1.2\text{x}36{,}168.28 = 43{,}401.94 \text{ kNm} < 126{,}105.75$ kNm

∴ Requirement satisfied!

Span 3 (0.59L)

$M_{SLS}$ = Moment demand = 39,846.74 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.483m

→ $f_{su} = f'_s\ [1-(\gamma/\beta_1)(\ pf'_s/ f'_c)]$

= 1,860,000[1-(0.28/0.723)x(0.077/(9.95x1.483))x(1,860,000/50,000)] = 0.925x1,860,000

$c = (A_{ps}\ f_{su})/(0.85\ f'_c b)$

= (0.077x0.925x1,860,000)/(0.85x50,000x9.95) = 0.313m > 0.3m

Flanged section!

$\varphi M_n = \varphi\{A_{sr}\ f_{su}\ d\ [1-0.6(A_{sr}\ f_{su}/b'df'_c)] + 0.85f'_c\ (b-b')(t)(d-0.5t)\}$

where $A_{sr} = A_{ps} - 0.85 f_c'(b-b')t/f_{su}$

= 0.077 - 0.85x50,000x(9.95 - 0.91)x0.3/(0.925x1,860,000)

= 0.01m$^2$

→ $\varphi M_n$ = 0.95x(0.01x0.925x1,860,000x1.483[1-0.6x((0.01x0.925x1,860,000)/(0.91x1.483x50,000)]+0.85x50,000x(9.95-0.91)x0.3x(1.483-0.5x0.3)

= 166,490.48 kNm > $M_{SLS}$ = 39,846.74 kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$A_{sr}f_{su}/b^{'}d\ f^{'}_{c} < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

= (0.01x0.925x1860,000)/(0.91x1.483x50,000)

= 0.18 < 0.36x0.723 = 0.26

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{44{,}864.18} = 131.96$ kN/m$^2$ (AREMA, CL.8.17.162.3)

$S_c = 3.1762m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}]$ = 99,203.4x0.357x$[\frac{1}{5.58} + \frac{0.738x1.054}{2.605}]$ = 16,921.97 kN/m$^2$

$M_{cr} = (f_r + f_{pe})S_c$

= (131.96 + 16,921.97) x 3.1762

= 54,166.70 kNm

1.2 $M_{cr}$ = 1.2x54,166.70 = 65,000.04 kNm < 166,490.48 kNm

∴ Requirement satisfied!

For tensioning from both ends

Span 1 (0.41L)

$M_{SLS}$ = Moment demand = 41,489.52 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.483m

→ $f_{su} = f^{'}_{s}\ [1-(\gamma/\beta_1)(\ pf^{'}_{s}/\ f^{'}_{c})]$

= 1,860,000[1-(0.28/0.723)x(0.0697/(9.95x1.483))x(1,860,000/50,000)] = 0.934x1,860,000

$c = (A_{ps}\ f_{su})/(0.85\ f^{'}_{c}b)$

= (0.0679x0.934x1,860,000)/(0.85x50,000x9.95) = 0.279m < 0.3m

Rectangular section!

→ $a = c\beta_1 = 0.279 x 0.723 = 0.2$

→ $T = \varphi A_{ps} f_{su} = 0.95x0.0679x0.934x1,860,000 = 112,060.67$ kN

→ $z = 1.483x(1 - 0.6\frac{A_{PS}}{bd}\frac{f_{su}}{f'_c}) = 1.483x(1 - 0.6x\frac{0.0697}{9.95x1.483}x\frac{0.934x1860,000}{50,000}) = 1.341m$

→ $\varphi M_n = M_r = T \cdot z = 112,060.67x1.341 = 150,273.36$ kNm $\geq M_{SLS} = 41,489.52$ kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$p\, f_{su}/ f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

For tensioning from the start end:

(0.0679/(9.95x1.483))x0.934x1860,000/50,000 = 0.16 $\leq$ 0.36x0.723 = 0.26

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{42,805.96} = 128.89$ kNm (AREMA, CL.8.17.162.3)

$S_c = 3.1762m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}] = 88,325.49x0.798x[\frac{1}{5.58} + \frac{0.738x1.054}{2.605}] = 33,677.91$ kN/m$^2$

$M_{cr} = (f_r + f_{pe})S_c$

= (128.89 + 33,677.91) x 3.1762

= 107,377.17 kNm

1.2 $M_{cr}$ = 1.2x107,377.17 = 128,852.60 kNm $\leq$ 150,273.36 kNm

∴ Requirement is satisfied!

Pier II

$M_{SLS}$ = Moment demand = -33,710.56 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.489m

→ $f_{su} = f'_s\, [1-(\gamma/\beta_1)(\, p f'_s/ f'_c)]$

= 1,860,000[1-(0.28/0.723)x(0.0679/(6.228x1.489))x(1,860,000/50,000)] = 0.895x1,860,000

$c = (A_{ps}\, f_{su})/(0.85\, f'_c b)$

$= (0.0679x0.88x1{,}860{,}000)/(0.85x50{,}000x6.228) = 0.42m > 0.3m$

Flanged section!

$\varphi M_n = \varphi\{A_{sr}\, f_{su}\, d\, [1\text{-}0.6(A_{sr}\, f_{su}/b' d f'_c)] + 0.85 f'_c\, (b\text{-}b')(t)(d\text{-}0.5t)\}$

where $A_{sr} = A_{ps} - 0.85 f_c' (b\text{-}b')t/f_{su}$

$= 0.0679 - 0.85x50{,}000x(6.228- 0.91)x0.3/(0.88x1{,}860{,}000)$

$= 0.026m^2$

$\rightarrow \varphi M_n = 0.95x(0.026x0.88x1{,}860{,}000x1.489x[1\text{-}0.6x((0.026x0.88x1{,}860{,}000)/$
$(0.91x1.489x50{,}000)]+0.85x50{,}000x(6.228\text{-}0.91)x0.3x(1.489\text{-}0.5x0.3)$

$= 123{,}761.17\ kNm > M_{SLS} = 33{,}710.56\ kNm$

∴ the section is adequate for flexural strength!

<u>Maximum prestressing steel</u>

$A_{sr}f_{su}/b' d\, f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

$= (0.026x0.88x1860{,}000)/(0.91x1.489x50{,}000)$

$= 0.24 < 0.36x0.825 = 0.297$

∴ requirement is satisfied!

<u>Minimum reinforcement</u>

$\varphi M_n \geq 1.2 M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{42{,}805.96} = 128.89\ kN/m^2$ (AREMA, CL.8.17.162.3)

$S_c = 3.6991m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}] = 88{,}325.49x0.632x[\frac{1}{10.768} + \frac{0.53x0.841}{3.548}] = 12{,}196.84\ kN/m^2$

$M_{cr} = (f_r + f_{pe})S_c$

$= (128.89 + 9{,}645.63) \times 3.6991$

$= 45{,}594.09\ kNm$

$1.2\ M_{cr} = 1.2x45{,}594.09 = 54{,}712.91\ kNm < 123{,}761.17\ kNm$

∴ Requirement satisfied!

Span 2 (0.5L)

$M_{SLS}$ = Moment demand = 57,654.95 kNm

Cross-sectional properties

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.539m

→ $f_{su} = f'_s\ [1-(\gamma/\beta_1)(\ pf'_s/ f'_c)]$

= 1,860,000[1-(0.28/0.723)x(0.0679/(9.95x1.539))x(1,860,000/50,000)] = 0.936x1,860,000

$c = (A_{ps}\ f_{su})/(0.85\ f'_c b)$

= (0.0679x0.936x1,860,000)/(0.85x50,000x9.95) = 0.28m > 0.3m

Rectangular section!

→ a = cß₁= 0.28x0.723 =0.2

→ $T = \varphi A_{ps} f_{su}$ = 0.95x0.0679x0.936x1,860,000 = 112,300.62 kN

→ $z = 1.539x(1 - 0.6\,\frac{A_{PS}}{bd}\frac{f_{su}}{f'_c}) = 1.539x(1 - 0.6x\frac{0.0679}{9.95x1.539}x\frac{0.936x1860{,}000}{50{,}000}) = 1.396m$

→ $\varphi M_n = M_r = T\cdot z$ = 112,300.62x1.396 = 156,771.67 kNm ≥ $M_{SLS}$ = 57,654.95 kNm

∴ the section is adequate for flexural strength!

Maximum prestressing steel

$p\ f_{su}/ f'_c < 0.36\beta_1$ (AREMA, CL.8.17.19.1)

For tensioning from the start end:

(0.0679/(9.95x1.539))x0.936x1860,000/50,000 = 0.15 ≤ 0.36x0.723 = 0.26

∴ requirement is satisfied!

Minimum reinforcement

$\varphi M_n \geq 1.2M_{cr}$ (AREMA, CL.8.17.19.2)

Where $M_{cr} = (f_r + f_{pe})S_c - M_{d/nc}(S_c/S_b - 1)$

The structure is designed for the monolithic section to resist all loads and $S_c = S_b$ resulting in the second term equal to zero.

$f_r = 0.623\sqrt{f_{ci}} = 0.623x\sqrt{42{,}805.96}$ = 128.89 kNm (AREMA, CL.8.17.162.3)

$S_c = 3.1762m^3$

$f_{pe} = P_f[\frac{1}{A} + \frac{e_m y_b}{I}] = 88{,}325.49x0.531x[\frac{1}{5.58} + \frac{0.738x1.054}{2.605}] = 22{,}409.74\ kN/m^2$

$M_{cr} = (f_r + f_{pe})S_c$

= (128.89 + 22,409.74) x 3.1762

= 71,587.19 kNm

1.2 $M_{cr}$ = 1.2x107,377.17 = 85,904.63 kNm $\leq$ 156,771.67 kNm

∴ Requirement is satisfied!

**8. A.2 Shear Strength at SLS**

This calculation is performed for the cast-in-place concrete box-girders of double cell types. The following calculation aims to determine the minimum stirrup spacing‘s’ needed to meet shear requirements.

**30m simple span box girder**

The critical shears

$V_{SLS}$ = shear demand = 6,774.08 kN (for tensioning from the start end)

= 6,756.54 kN (for tensioning from both ends)

$M_{SLS}$ = moment demand = 10,083.824 kN (for tensioning from the start end)

= 9,452.671 kN (for tensioning from both ends)

Concrete properties

$f'_c$ = 50 MPa

Prestressing tendon properties

$f_{py}$ = 1860 MPa

Number of strands = 288 (for tensioning from the start end)

= 297 (for tensioning from both ends)

$A_{ps}$ = area of prestressing tendons

= (288strands) x (140 mm$^2$) = 0.0403m$^2$ (for tensioning from the start end)

= (297strands) x (140 mm$^2$) = 0.0416m$^2$ (for tensioning from both end)

$E_p$ = 195,000MPa

Non-prestressed reinforcement properties

$f_y$ = 335 MPa

$E_s$ = 210,000 MPa

$A_v$ = area of stirrup = 2 x 3 x ($\frac{\pi x 0.016^2}{4}$) = 0.0012 m$^2$ (using two leg 16mm stirrups)

Cross-section properties

b = width of compressive component = 9.95 m

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.489m

h = height of the section = 1.8 m

$d_v$ = 0.5+0.8h = 1.94m

$b_v$ = Available web width = 0.84m

For tensioning from the start end:

$V_c = (\frac{\sqrt{f_c'}}{20}+5\frac{V_u d_p}{M_u})b_w d$ = 9,453.59 kN; yet the maximum value i.e. $V_c = 0.4\sqrt{f_c'}b_w d$ = 4,993.31 kN is used.

$V_s = V_u/\Phi - V_c$ = 2,533.45 kN

$s = (A_v f_y d_v)/(V_u/\Phi - V_c)$ = 309.31mm ≈ 310mm $\leq$ maximum spacing of 600mm

∴ s = 310mm is used

For tensioning from both ends:

$V_c = (\frac{\sqrt{f_c'}}{20}+5\frac{V_u d_p}{M_u})b_w d$ = 10,018.75 kN; yet the maximum value i.e. $V_c = 0.4\sqrt{f_c'}b_w d$ = 4,993.31 kN is used.

$V_s = V_u/\Phi - V_c$ = 2,513.96 kN

$s = (A_v f_y d_v)/(V_u/\Phi - V_c)$ = 311.71mm ≈ 312mm $\leq$ maximum spacing of 600mm

∴ s = 312mm is used

**97.758m three span continuous box girder**

The critical shears

$V_{SLS}$ = shear demand = 7,623.34 kN (for tensioning from the start end)

= 6,927.12 kN (for tensioning from both ends)

$M_{SLS}$ = moment demand = 22,515.824 kN (for tensioning from the start end)

= 18,588.86 kN (for tensioning from both ends)

Concrete properties

$f'_c$ = 50 MPa

Prestressing tendon properties

$f_{py}$ = 1860 MPa

Number of strands = 563 (for tensioning from the start end)

= 498 (for tensioning from both ends)

$A_{ps}$ = area of prestressing tendons

= (550 strands) x (140 $mm^2$) = 0.077$m^2$ (for tensioning from the start end)

= (485 strands) x (140 $mm^2$) = 0.0679$m^2$ (for tensioning from both ends)

$E_p$ = 195,000MPa

Non-prestressed reinforcement properties

$f_y$ = 335 MPa

$E_s$ = 210,000 MPa

$A_v$ = area of stirrup = 2 x 3 x ($\frac{\pi \text{x} 0.016^2}{4}$) = 0.0012 $m^2$ (using two leg 16mm stirrups)

Cross-section properties

b = width of compressive component = 9.95 m

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel = 1.489m

h = height of the section = 1.8 m

$d_v$ = 0.5+0.8h = 1.94m

$b_v$ = Available web width = 0.84m

For tensioning from the start end:

$V_c$ = ($\frac{\sqrt{f_c'}}{20}$+5$\frac{V_u d_p}{M_u}$)$b_w d$ = 9,453.59 kN; yet the maximum value i.e. $V_c$ = 0.4$\sqrt{f_c'}b_w d$ = 4,993.31 kN is used.

$V_s = V_u/\Phi - V_c$ = 2,533.45 kN

s = $(A_v f_y d_v)/(V_u/\Phi - V_c)$ = 309.31mm ≈ 310mm ≤ maximum spacing of 600mm

∴ s = 310mm is used

For tensioning from both ends:

$V_c$ = ($\frac{\sqrt{f_c'}}{20}$+5$\frac{V_u d_p}{M_u}$)$b_w d$ = 5,522.04 kN; yet the maximum value i.e. $V_c$ = 0.4$\sqrt{f_c'}b_w d$ = 4,993.31 kN is used.

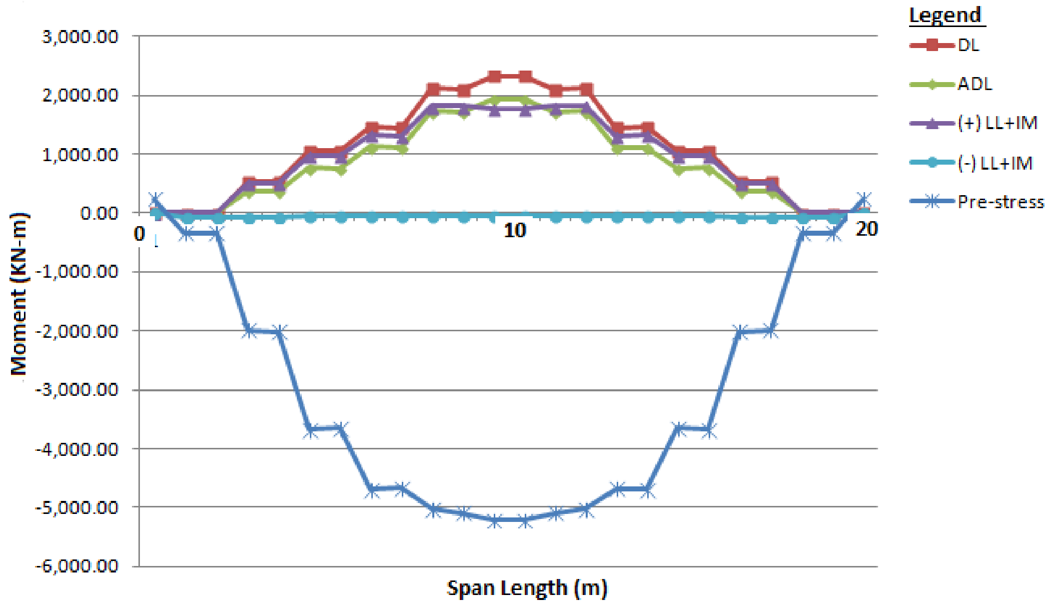
$V_s = V_u/\Phi - V_c = 450.58$ kN

$s = (A_v f_y d_v)/(V_u/\Phi - V_c) = 289.86$mm $\approx$ 290mm $\leq$ maximum spacing of 600mm
$\therefore$ s = 290mm is used

# Appendix B: Summary of Analytical Results

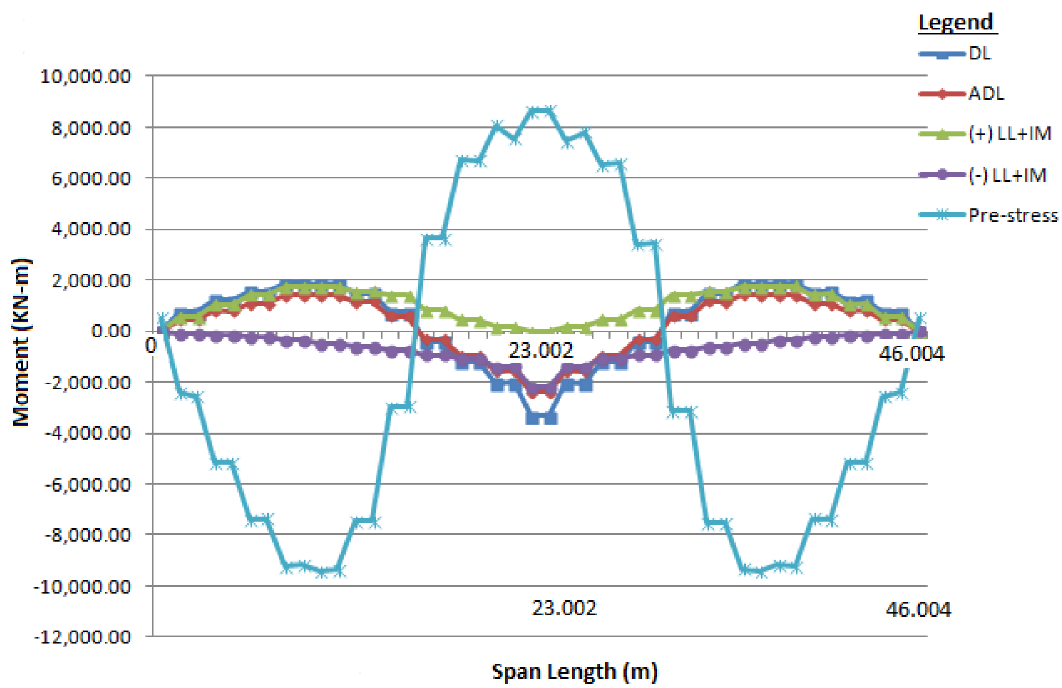
## B.1. 20m simple span single cell simply supported box-girder elevated structure

### Unfactored Moment envelope for the entire bridge section (tensioning from both ends)



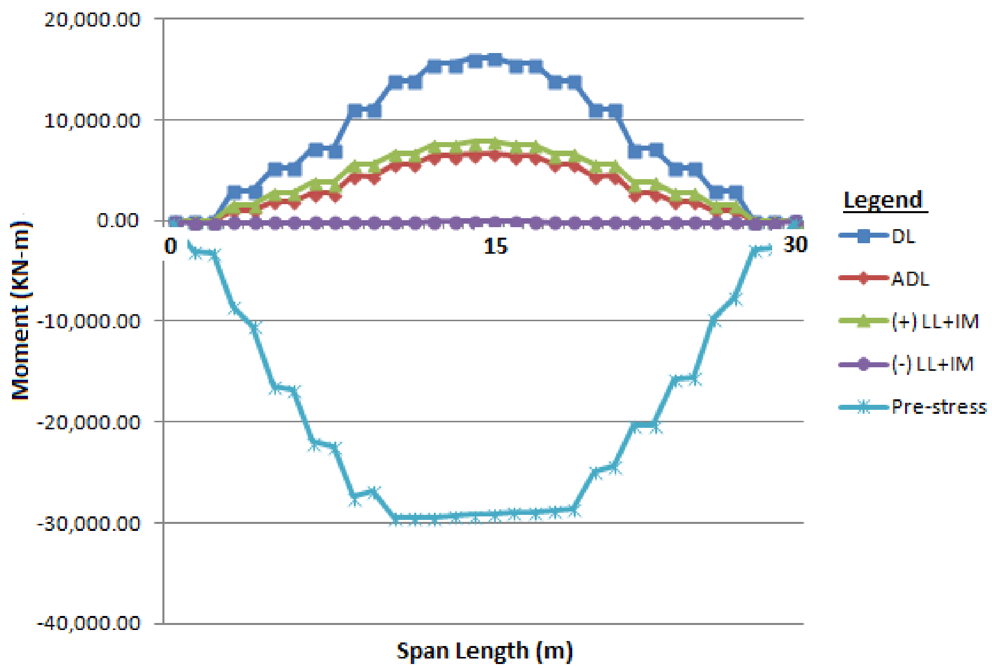

## B.2. 46.004 two-span continuous single cell box-girder elevated structure

### Unfactored Moment envelope for the entire bridge section (tensioning from both ends)

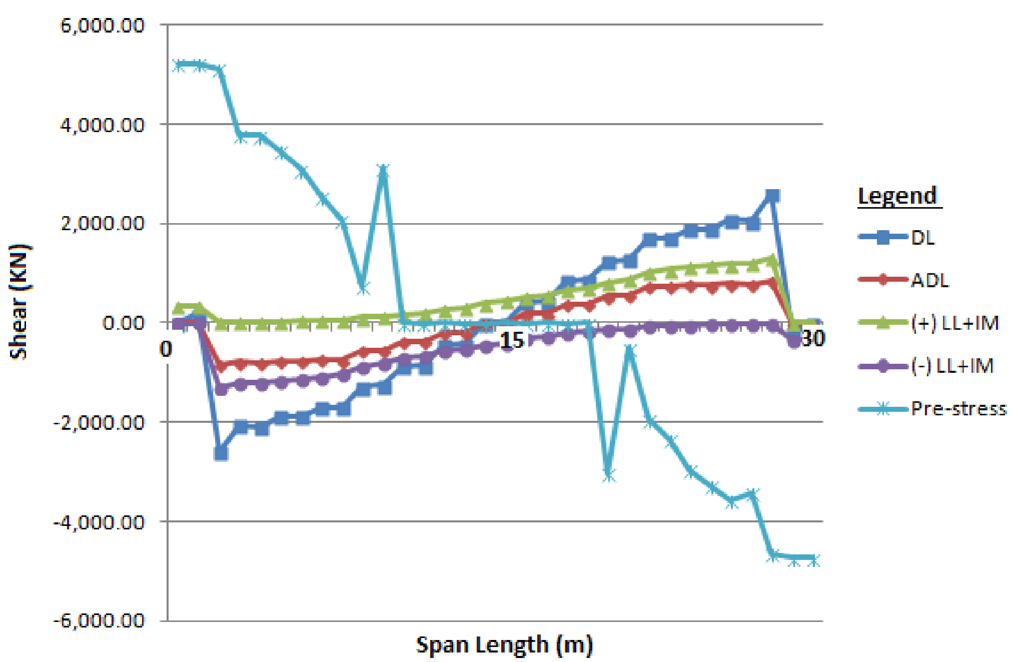


### B.3. 30m simple span double cell simply supported box-girder elevated structure

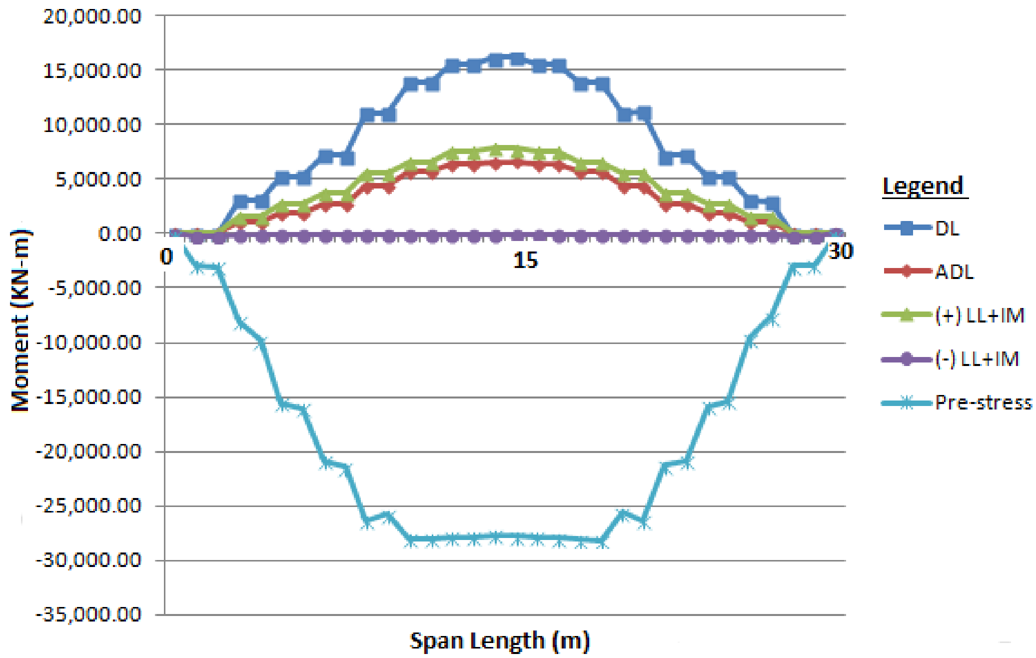
**Unfactored Moment envelope for the entire bridge section (tensioning from the start end)**



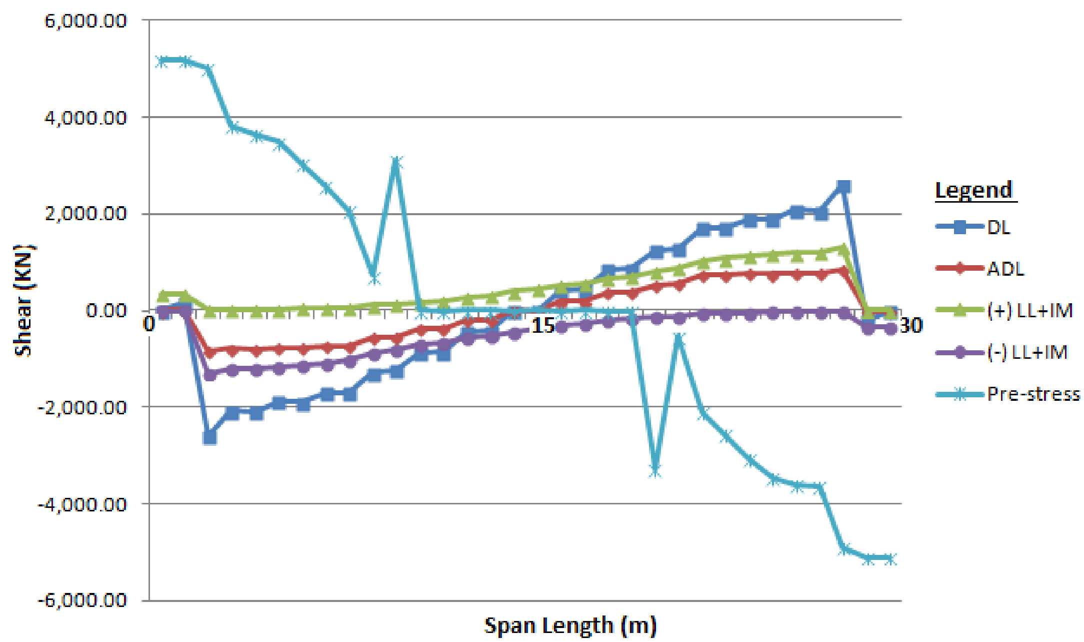

**Unfactored Shear envelope for the entire bridge section (tensioning from the start end)**



**Unfactored Moment envelope for the entire bridge section (tensioning from both ends)**

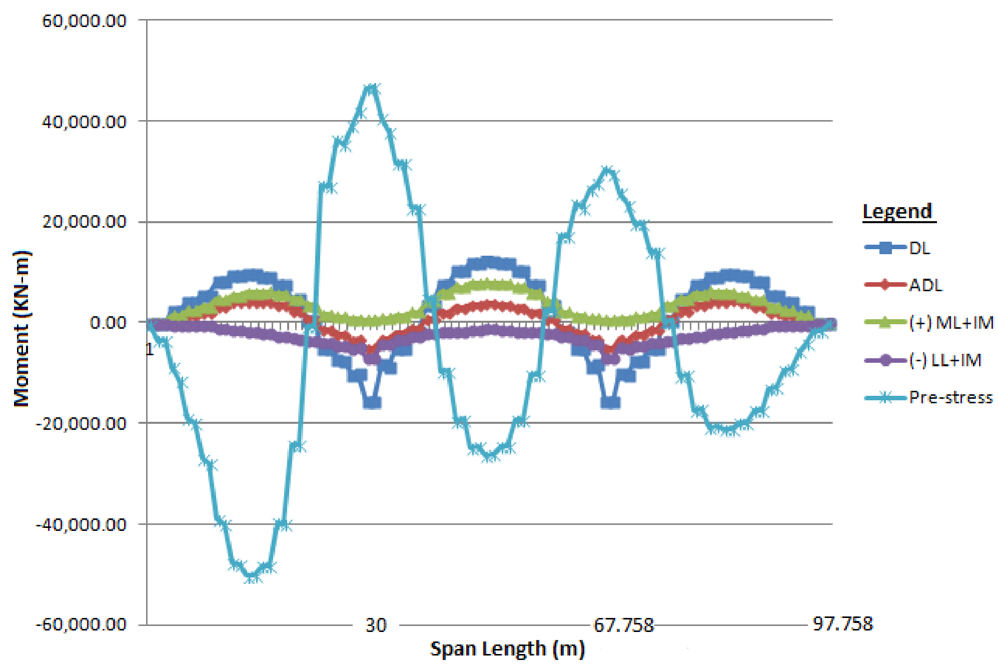



**Unfactored Shear envelope for the entire bridge section (tensioning from both ends)**

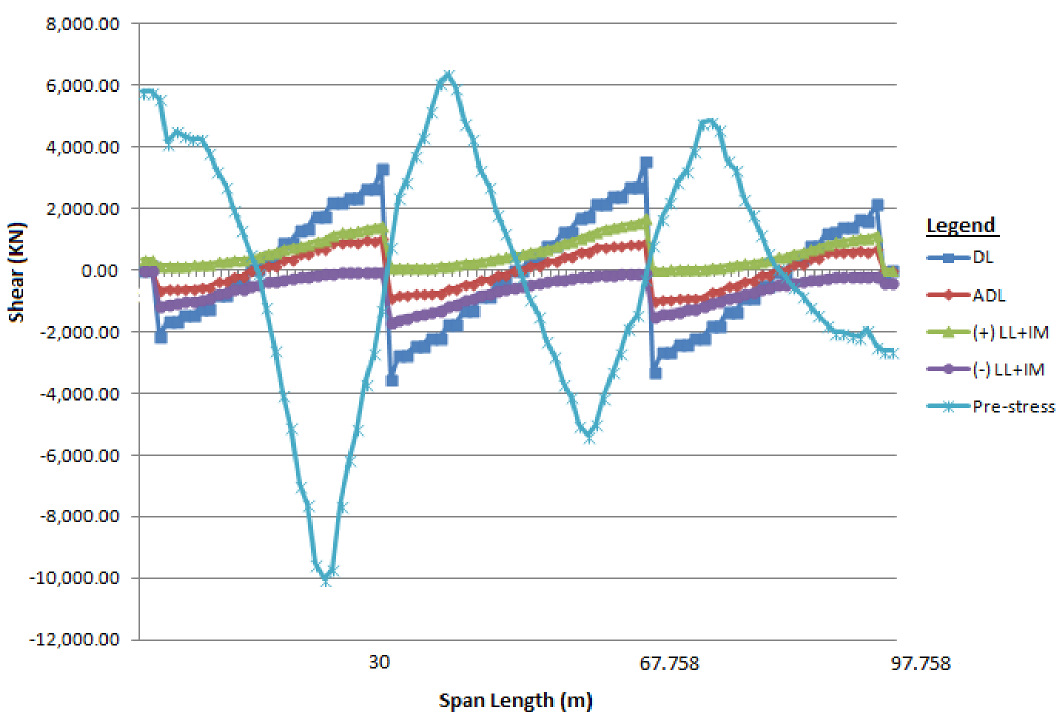


### B.4. 97.758m three-span continuous double cell box-girder elevated structure

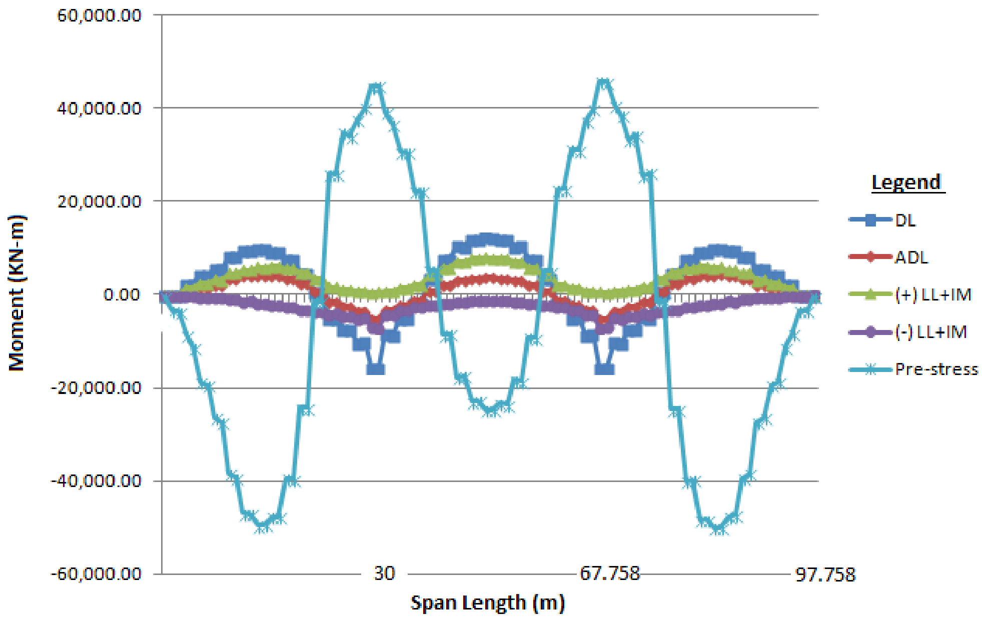
**Unfactored Moment envelope for the entire bridge section (tensioning from the start end)**



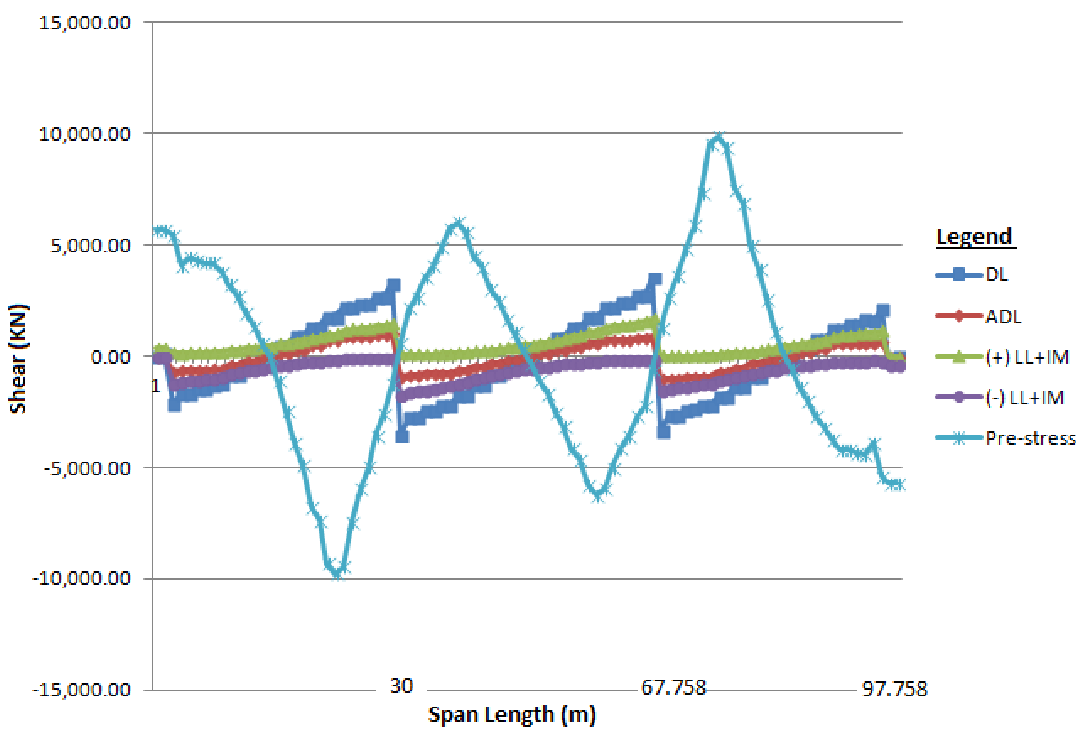

**Unfactored Shear envelope for the entire bridge section (tensioning from the start end)**



**Unfactored Moment envelope for the entire bridge section (tensioning from both ends)**




**Unfactored Shear envelope for the entire bridge section (tensioning from both ends)**



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**INSTITUTE OF TECHNOLOGY**

**DEPARTMENT OF CIVIL ENGINEERING**

## DECLARATION

I, the undersigned, declare that this thesis is the original work of mine and has not been presented for any degree in any university and all the sources of materials used for the thesis have been duly acknowledged.

| | | |
|---|---|---|
| Dagim Tadesse | __________________ | __________________ |
| Name | Signature | Date |

April, 2015