



ANALYSIS OF ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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Approval

This thesis has been examined and approved as meeting the requirements for the partial fulfillment of Master of Science in Mathematics.

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Declaration

I, **TEREFE WODAJENEH KIFLE** with student ID **GSK/6060/05**, hereby declare that this thesis entitled “**ANALYSIS OF ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS**” has been compiled and organized by myself under the supervision of **Dr. Mengistu Goa** and that no part of the thesis has formed the basis for the award for any degree, associateship, fellowship or any other similar title for me. All relevant sources of materials have been duly acknowledged.

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Notation

- $\mathbb{R}^n$ n-dimensional Euclidian space
- $\mathbb{R}^{n \times n}$ nxn symmetric matrix
- Ω an open subset of R^n
- ∇u $(\partial_1 u, \dots, \partial_n u)$ = gradient of u.
- $D^\alpha u = v$ v is the α^{th} weak partial Derivatives of u
- $C^k(\Omega)$ function continuously k-differentiable in Ω .
- $C(\Omega)$ the space set of all function that are continuous everywhere in Ω
- $L^2(\Omega)$ the space of all squares integrable function on Ω .
- PDE partial differential equation .
- A^T Transpose of matrix A.
- α multi-index (ad-tuple): $\alpha=(\alpha_1, \alpha_2, \dots, \alpha_d)$.
- $|\alpha|$ order or length of the multi-index .
- $D^\alpha w$ weak derivative notation .
- $\mathbb{N}_0$ The set of non-negative integers.