ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

CIVIL ENGINEERING DEPARTMENT

A STUDY ON THE DESIGN AND ADVANTAGE OF CONICAL TYPE SHELL FOUNDATION USING ANALYTICAL AND FEM

By

Endalkachew Taye Chekol

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A thesis submitted to the school of graduate studies, Addis Ababa University, in partial fulfillment of the requirements for the Degree of Masters of Science in Civil Engineering

By

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Advisor

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Notations

P_s : Design load

N_s : Meriodonal compression

$N\theta$: Hoop tension

V : Vertical load

H : Horizontal load

M : Moment

D : Diameter of footing

γ : Unit weight

p_{uv} : ultimate strength due to vertical soil pressure.

p_{un} : ultimate strength due to normal soil pressure.

s : Slope of cone

f : Cone rise

r_2 : Radius of footing

r : Radius of column

f_d : Depth of foundation

A_s : Total area of reinforcement

a_s : Area of single bar

S : Spacing between bars

ρ : Reinforcement ratio

V_{Rd} : Shear resistance of concrete

f_{yk} : Characteristics strength of steel

f_{ck} : Characteristics strength of concrete

EW_h : external work done due to horizontal soil pressure

EW_v : external work done due to vertical soil pressure

IW_{n_θ} : Internal work done

σ_{st} : permissible stress in tension in the steel

σ_{sy} : yield strength.

N : ultimate capacity of the shell section per unit width

M : moment capacity of the plastic hinge, per unit width

q_a : Allowable bearing pressure

q_{ul} : Ultimate bearing pressure

P_u :Ultimate strength

S.F : Factor of safety

d : Effective depth

D : Total depth

P_n : Normal soil pressure

P_v : Vertical soil pressure

c : Cohesion

ϕ : Angle of internal friction

C_u : the design compression force

A_c : area of concrete (alone

ABSTRACT

This thesis introduces shell foundations in general and conical type shell foundation in particular as an alternative to the conventional plain foundation. The different types of shells that may be employed in foundations are introduced with their geometry and applications under different situations. Shell foundations can be used as combined or isolated footing. The simplest form of shell appropriate for isolated footing is conical shells. Conical shells are employed as an alternative to plain circular footings.

The design of conical shell foundation is based on membrane theory in order to determine membrane stresses, and ultimate strength theory to obtain the ultimate load which enables to compute the load factors involved in the design. A design example is provided and comparison is made with plain circular footing. The result clearly indicates that conical shells save more material than do so plain circular footings.

A finite element analysis is carried out using the finite element software PLAXIS for different soil properties and footing size. The analysis was performed for conical shells and plain circular footing in order to compare the results. The finite element analysis results prove that conical shells have high load carrying capacity than circular footing under same soil properties.

Shell foundations need smaller quantities of material than the conventional plain foundations. In a country like Ethiopia where material to labor cost ratio is high, shell foundations may be used as an economic alternative.

1.0 Introduction

1.1 General

The purpuse of providing foundations is to transmit the load of structures safely and economically by serving as a media between the structure and the sub-soil. Different types of foundations developed and came in to practice after a great deal of scientific research and innovations are employed to achieve this goal. Foundations are generally classified as shallow and deep foundation. In the developed parts of the world alternative foundations came in to practice in addition to conventional type of foundations. In this case the use of shells as a structural foundation could be mentioned.

The concept of shell as foundation is not new . Shells in modern foundation engineering however are relatively new .Moreover; shells are essentially thin structures, structurally more efficient than flat structures. This is an advantage in situation involving heavy super structural loads to be transmitted to weaker soils.

Shell footings require significantly smaller quantities of concrete and reinforcing steel than do common isolated footings of equivalent load carrying capacities. Because they are favored in areas having high material-to-labor cost ratios, shell footings have gained popularity more rapidly in Mexico and several European countries than in the United States Shell footing is limited to a few geometries, such as conical, pyramidal; hypar and spherical footings. The conical shell footing is the simplest form of shell, which can be employed in foundation engineering due its singly curved surface. Due to its circular plan, the use of conical shell footing is restricted to an isolated footing only.

The design of shell foundation is based on the membrane, and ultimate strength theory. The membrane theory enables us to determine the membrane stresses as a function of the soil reaction and the geometry of the shell foundation. The ultimate strength theory provides the maximum load that a foundation sustains under a given set of loading conditions. The geotechnical design of shell foundation in the current practice is the same as the design of conventional plain foundations, using the available bearing capacity equations (Kurian , 2006).

1.2 Objective of the thesis

Tha main objective of this study is to introduce shell foundations in general and conical shell in particular as alternative to the conventional plain foundation.The main purpuse is to study the design and advantage of conical shell foundation,and to came up with basic conclusions and recommendations.

1.3 Organization of the thesis

The thesis has been organized in the following chapters. Chapter one ,deals with the introduction of shells as foundation.

Chapter two, is the literature review which deals about the use of shells as a structural foundation, ,and the different type of shells that can be employed on foundations.

Chapter three, deals with the current practice on the geotechnical design of shell foundation, and soil structure interaction under shell foundation.

Chapter four, deals with the membrane analysis of conical shell foundation. This chapter illustrates how the membrane stresses can be obtained in terms of the contact pressure distribution and the geometry of the footing.

Chapter five, deals with the ultimate strength analysis of conical shell foundation ,this chapter provides the principles behind the computation of the ultimate loads that the foundation could sustain .

Chapter six, deals with the Finite Element analysis of Conical shell foundation using the software PLAXIS. In this chapter the load displacement curves are presented in order to examine the load carrying capacity of conical shell footing and circular footings for comparison.

Chapter seven, deals with the design of shell foundations. This chapter illustrates some important points on the design of conical shell foundation, and a design example is also presented.Finally, chapter eight deals with conclusions and recommendations as out-put of this particular study.

2.0 Lterature review

2.1 Shells in structural foundations

Even though shells have been enjoying wide and varied use in roofs they are new comers to the family of structural foundations and it is only five decade since Candela 1963) poured his first shell footing on the Mexican soil. However, what may appear strange is the fact that the concept of shells itself is not new in foundations, if only one would consider the old inverted arch foundation as belonging to this group. It may be noted that use of brick arches in foundation has been in practice for a long time in India, and many buildings with such foundations are in existence in many parts of India.

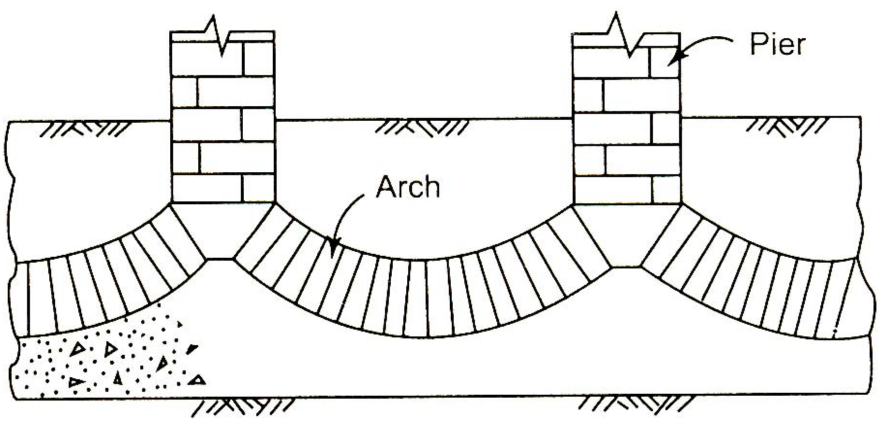


Fig.2.1 Inverted Arch Foundation

The twin attributes of a shell used in roofs are economy and aesthetics. Since the latter aspects is of no concern in the case of a buried structure such as the foundations, it is the aspect of economy which holds the key to the acceptance and use of shells in foundations. In other words, shells hold prospect for adoptions in foundations only if they can project themselves as economic alternatives to the conventional plain foundations. It has been found in respect of foundations that in situations involving heavy column loads to be transmitted to weaker soils, large areas of foundations become necessary and a situation analogous to large spans in roofs develops. The scope of economy with shells becomes immediately apparent under these circumstances. Shells are potential economic alternatives to plain shallow foundation in such situation. Where actual estimates have been made, builders have come out with impressive statistics in favor of shells, which have been reported as exceeding even 50% in certain cases. It should be noted that shells in foundations call for no formwork, except possibly along the sides, as the main body of the foundation shell can be cast directly on the soil, profiled to the required shape of the soil. It is apparent that the latter will not be a difficult operation, particularly in the case of axisymmetrical shell and shells with the straight-line property.

2.2 Shells in roofs vs. foundations

Even though shells lend themselves for use in roofs and foundations, foundation shells differ from roof shells in the following important respects

1. Shells used in foundation are invariable characterized by smaller size, but greater thickness and depth compared to those used in roofs. This fact assumes considerable significance when it comes to the analysis of shells.
2. Since foundation shells bear directly on soil at their bottom and carry backfill on top, besides being deep and thick, the problem of buckling is of lesser concern in foundations shells when compared to roof shells.
3. Shell foundations can be cast in situ on earth cut to the required profile, no form work is necessary except perhaps at the sides. This fact has an important bearing when considering the relative cost of shells in roofs vs, foundations.
4. Since the self weight of the foundation shell is directly transmitted to the soil, no significant stresses are induced in the shell on account of it, (this is true also of plain foundations)unlike in roofs, where it may constitute the major part of the loading.
5. Since the loading on the foundation shell is essentially the soil reaction, it is indeterminate to the extent that its actual distribution is the result of the complex shell-soil interaction, unlike in the case of the roof shell where the loading is determinate in as much as its magnitude and distribution are known prior to analysis.

2. 3 Different types of shells used in foundations

Even though a variety of shells lend themselves for adoption in roof, those that can be adopted for use in foundations are far too few. The following is an count of some of the more common types of shells used in foundations, and their geometric characteristics which enable them to perform their assigned functions efficiently and effectively in foundations under different circumstances.

2.3.1 Cone

Few shells can match the cone in the simplicity of its shape. Reinforced concrete, rotationally symmetric truncated conical footings of the type show in (Fig 2.2a) is probably the simplest form in which a shell can be put to use in foundations. The provision of radial and circumferential reinforcement is as simple as for a circular flat footing; while the construction is probably only a little more difficult. The shell may be of uniform thickness, or the same can be made to vary along the slope . However, on account of its circular plan the use of the conical shell is limited to individual footings, unlike hyperbolic paraboloid which could be used for combined footings or rafts as well .

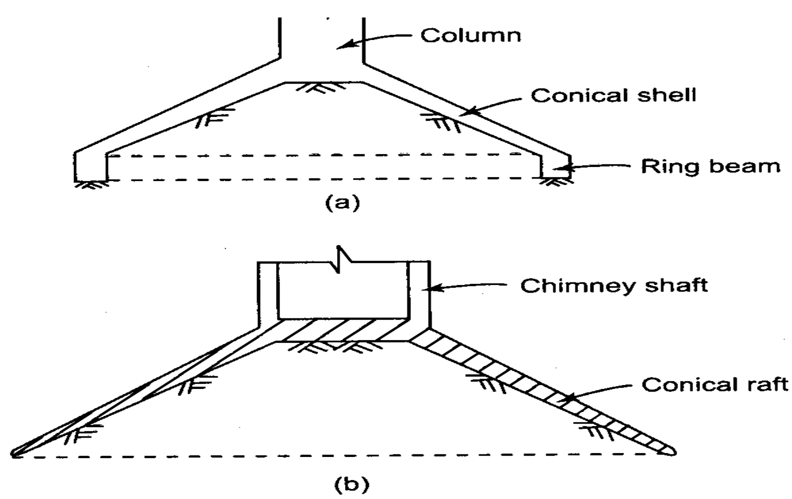


Fig.2.2 Conical Shell foundations.(a) column footing,(b) chimney raft

Cones of substantially larger dimensions can also serve as foundations for tall structures like chimney shafts, in place of the conventional circular or octagonal rafts, (Fig. 2.2b). The cone should be in perfect contact with soil throughout its bottom surface, besides the surcharge that comes on top of it.

The shell of (Fig .2.2a) in the inverted position is suitable as footings for structures such as guyed masts. A large inverted conical shell;either in full or truncated form can also serve as rafts for cylindrical structures, such as ground storage tanks or overhead structures, such as water tanks, supported on a circular row of column, depending up on the contact area requirement from the soil side.

While the cone is a suitable type of foundation for the uses mentioned above, the majority of instance in which conical shells have been adopted are for tall telecommunication towers (television, radio, telephone, etc) where they serve not as regular foundations of the type described above, but as substructures linking the tower shaft to the annular raft (or ring) at the bottom (Fig 2.3), which is the actual foundation bearing on soil and transmitting loads to it . Such a transition from the shaft above to the ring below has been necessitated on account of the fact that the dimensions of the ring foundation, as demanded by the soil conditions, are substantially bigger than that of the shaft. The space within the flanged conical substructure is free and is often utilized for services.

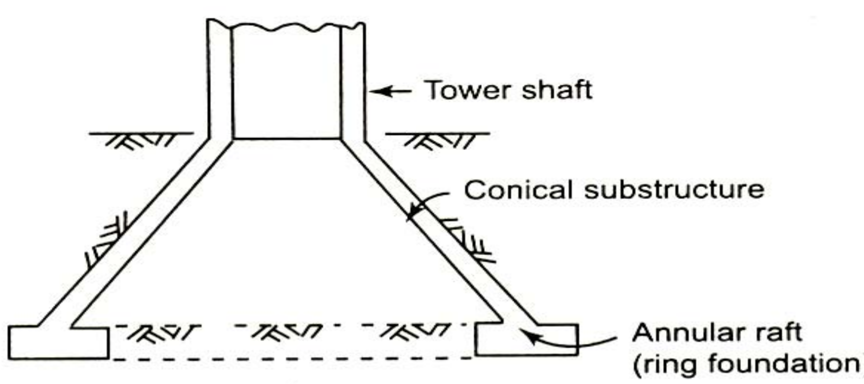


Fig. 2.3 Conical substructure for tower

2.3.2 Inverted dome

For circular structures, or overhead structures like water tanks supported on a circular row of columns, thin inverted domes can serve as economic alternatives to thick circular or annular raft foundations (Fig 2.4). The transfer of column load to the inverted dome can be effected through a ring beam at top as shown in the figure.

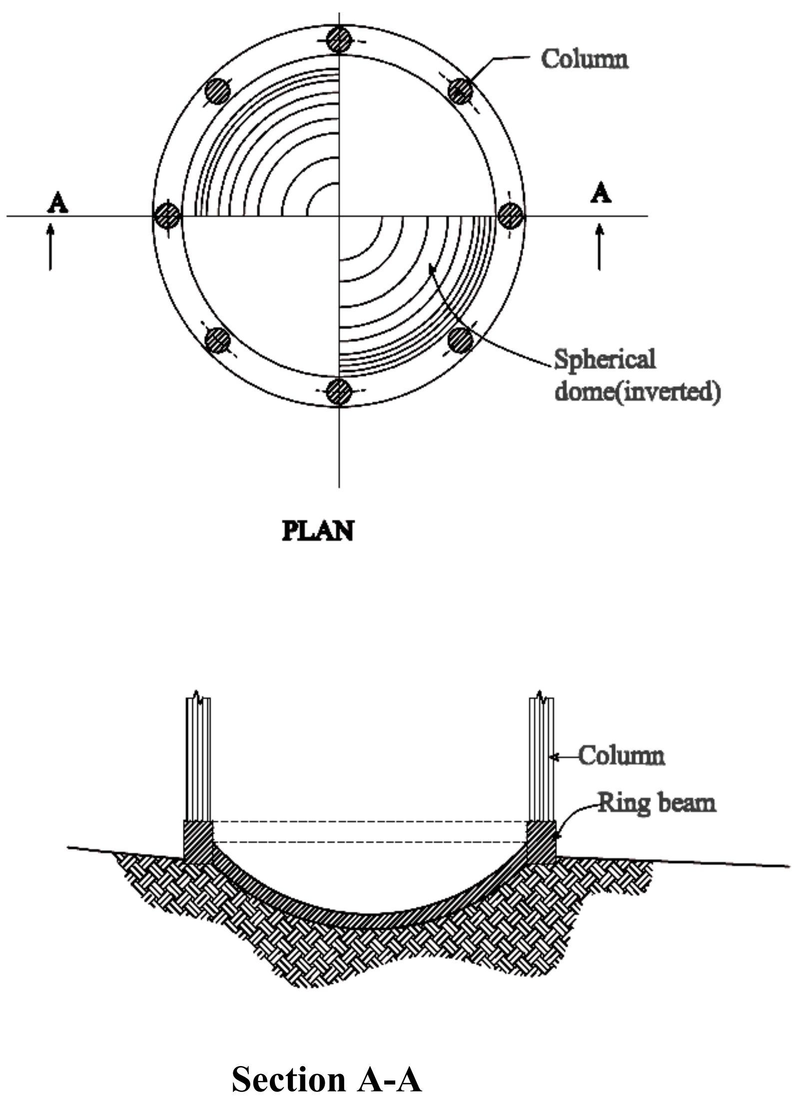


Fig. 2.4 Inverted spherical Dome Raft

2.3.3 Hyperbolic paraboloid

The Hyperbolic shell is a doubly curved, non developable anticlastic shell of translation with ruled surfaces, having straight line property(Fig. 2.5). Such shells are often used for roofs on account of their elegance and versatility. They were introduced in foundations by Candela (1963), in order to transmit highly concentrated loads to weaker soils. Yperbolic paraboloid shells are suited for supporting single-column loads, because of their single point of discontinuity (Paliwal D.N,Sinha S.N,and Ahmad A.,1992). Their use results in saving concrete. The cost of construction is also less, since they can be generated with straight lines.

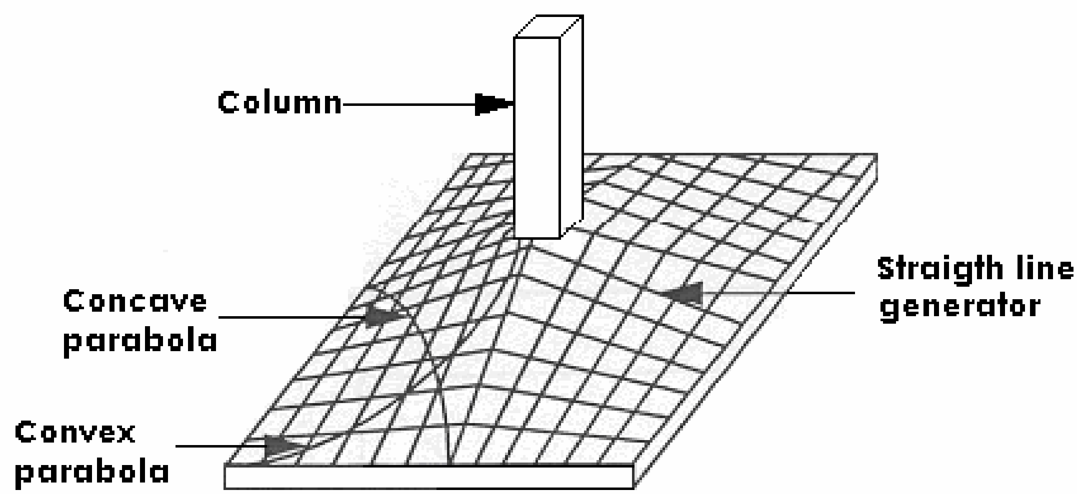


Fig. 2.5 Hyperbolic paraboloid Shell foundation

From the engineering point of view, the most versatile aspect of the geometry of the shell is its straight-line property, which gives it all the advantages of a shell and at the same time that of a plain surface. This property is effectively exploited in making the formwork, laying the reinforcement, casting and finishing the shell in the case of roof, and profiling the soil,laying the reinforcement, casting concrete and finishing the shell in the case of the foundation.

Hyperbolic paraboloidal shell elements either in the form bounded by parabola or straight lines, besides giving scope for use as single shells, lend themselves to be combined in an amazing number of ways. This results in surfaces of the most outstanding configurations, to suit the widely varying architectural and structural requirements that may be demanded in the case of roofs. Among the combinations of hyperbolic paraboloidal shell used in roofs, one of the early favorites has been the inverted umbrella roof resting on central columns. It is the success with this form that has given the clue for trying this combination in foundation, where in an upright position they can serve as foundations for columns foot.

The hyperbolic paraboloidal shell, whether in roof or foundation, owes much of its present-day popularity to the pioneering efforts of the famous Mexican engineer-architect, Felix Candela. He is also the father of the concept of modern shell foundations, and demonstrated it for the first time by constructing hypar footings of the above type for the Mexico City Customs House in 1953. Since then he has poured a large number of such footings in Mexico and elsewhere in Latin America all of which are reported to have performed exceedingly well (Abdel-Rahman, M.M, and A.M.Hanna ,1990) . It may be interesting to note that many of his columns supporting inverted umbrella roofs are in turn supported on umbrella footings as shown in (Fig. 2.6 a,b,and c).

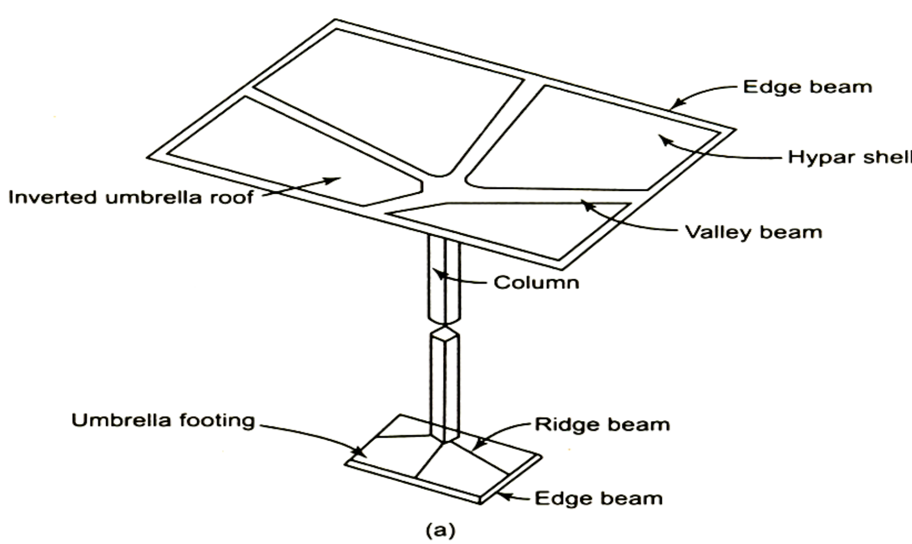
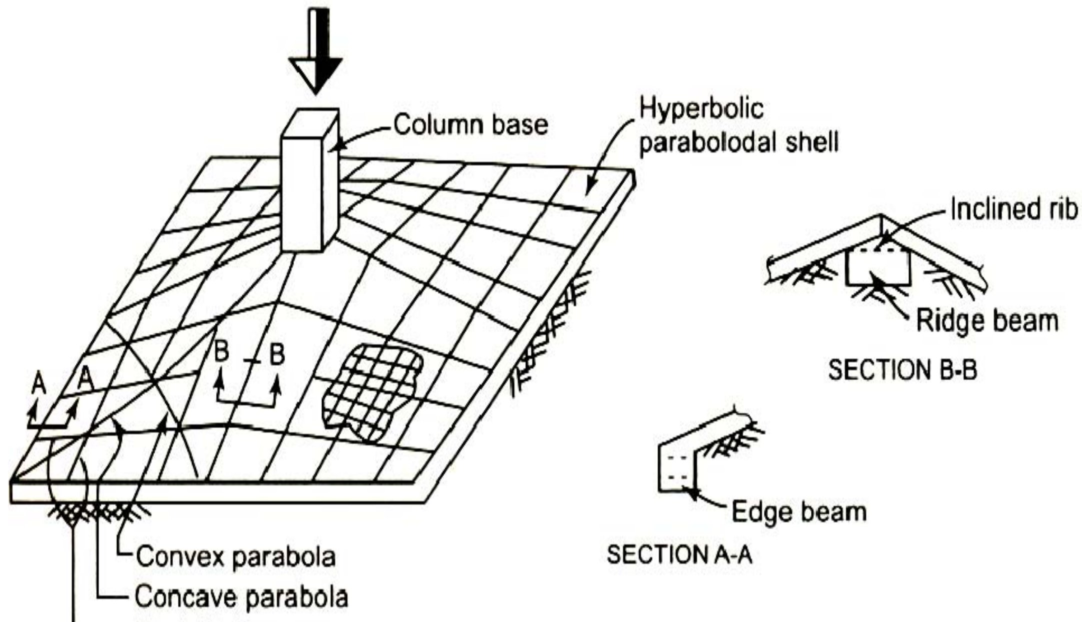
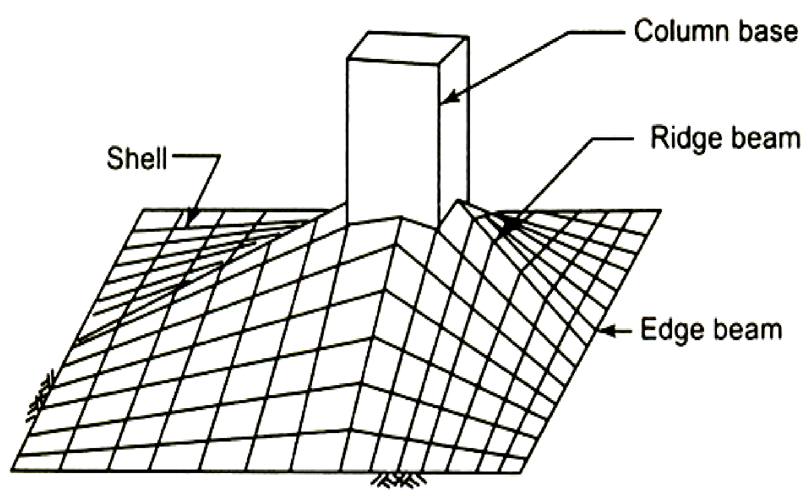


Fig. 2.6(a) Inverted umbrella roofs supported on umbrella footings



(b)



(c)

Fig.2.6(b,c) Details of hypar footing supporting umbrella roof

The use of hyperbolic paraboloidal shell, in the umbrella type of combination described above, is by no means limited to individual footings of the types described above. They can be extended as combined footings for two or more columns in one row, or as rafts for columns in several rows.

2.3.4 Folded plate foundations

A typical folded plate that can be considered for use as foundation is the pyramidal combination of four inclined trapezoidal plate elements, that can support a column at its centre (Fig. 2.7).Note that the term pyramidal footing is frequently used for the solid pyramid used as footing. When this is made hollow one gets the folded plate type of footing described above. Since these pyramidal folded plates can be rendered square or rectangular in plan, they can be combined to form multiple units to serve as combined footings or rafts, as could be done with hyperbolic paraboloid. Another possibility is as illustrated in (Fig. 2.8) and (Fig. 2.9.),in which 2D folded plate are shown serving as a continuous (strip) footing for a continuous load-bearing wall.

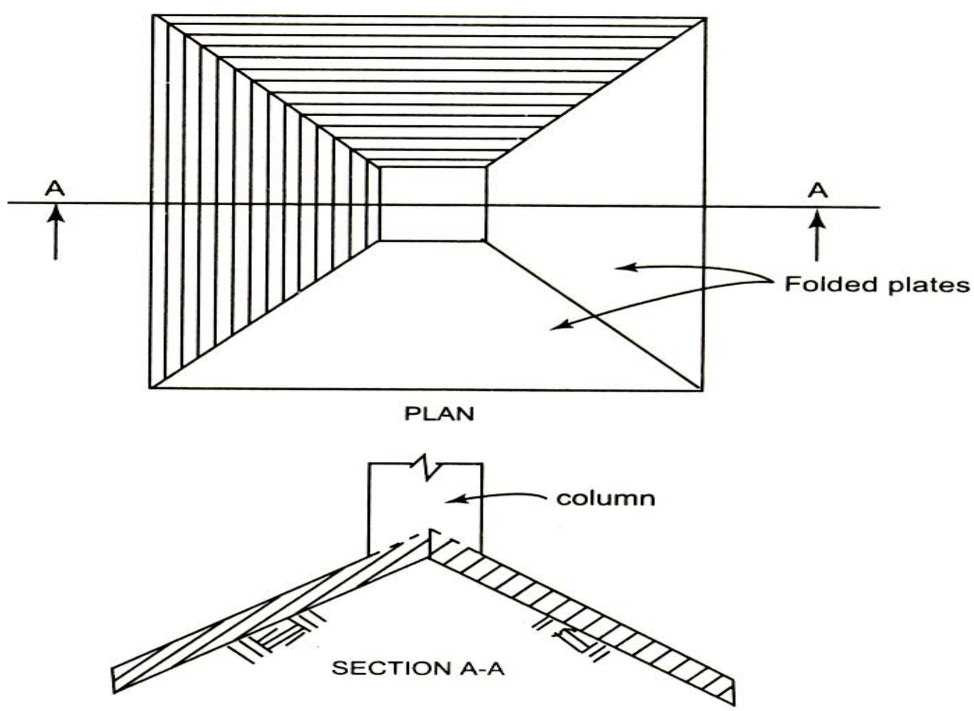


Fig. 2.7 Folded plate Footing

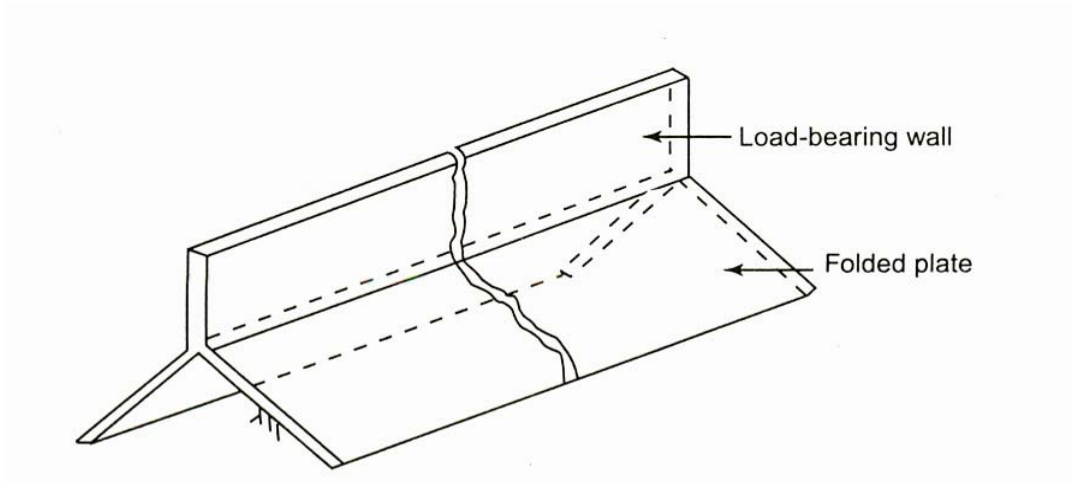


Fig. 2.8 Folded Plate Strip footing

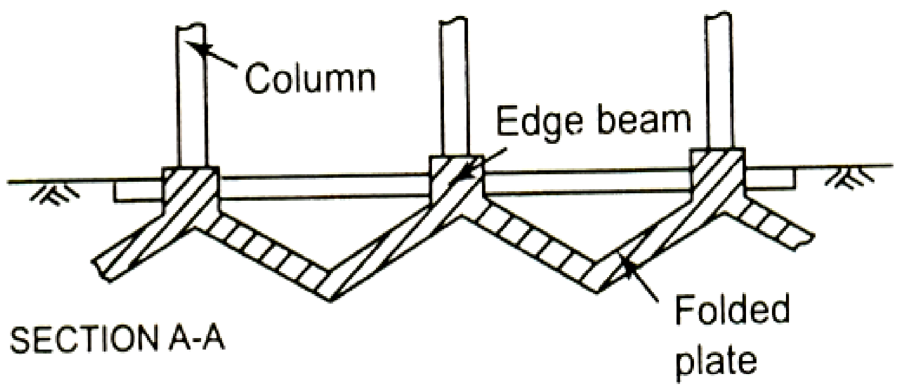


Fig. 2.9 Folded Plate Strip Raft

Shell foundations are potentially economic under conditions of heavy loads to be transmitted to weaker soils. Since their economy is mainly the result of the savings in materials they offer, it is obvious that overall economy with them should be more pronounced in countries where material costs are high compared to labor costs. This is a situation typically prevalent in many of the developing countries.

Shell foundation enjoy the widest use in countries of Latin America, in particular Mexico, East European countries, the former Soviet Union, some countries of Africa, and

India. On the research front, however, Russian and Indian engineers in particular have carried out detailed investigations on shell foundations of various types. A number of these foundations have already been constructed in India, and is perhaps the only country having a code of practice on the subject.

2.3.5 Combined shell foundations

The requirement with regard to the area of contact between the shell and the soil is a matter of geotechnical design and is based on bearing capacity and settlement considerations. However, if the area of support needed from the soil is larger than the plan dimensions of the structure, an ingenious solution lies in combining the inverted dome with the frustum of an upright cone (Fig. 2.10) in such a manner that the latter will make up for the remaining part of the foundation area required. Depending upon the situation, however, the inner shell need not be full, but can be partial .Combinations of this type are called combined shell foundations

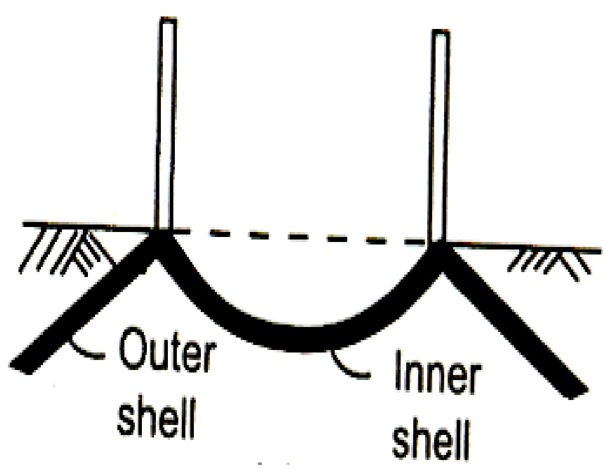


Fig. 2.10 Combined Shell Foundation with inverted spherical sector and Upright Frustrum of Cone

3. Geotechnical design of conical shell foundation and soil-structure interaction

The previous chapter introduces the different types of shell foundation, their advantage and application under various situations. This chapter will address some basic points on the geotechnical design ,and soil-structure interaction of conical shell foundation; but it would be appropriate to start with the general term foundation, which according to definition, is part of a structure whose function is to receive the load from the superstructure and transmit it to the soil or rock below . Foundation actually consists of two components, ie., the structural foundation, and the supporting soil below. While the former is the structural component of the foundation, such as a footing or a pile, the latter indicates the supporting soil below, which in the majority of cases ultimately receives and bears the load.

3.1 The two phases of foundation design

The foundation being a two-component system, the complete design of a structural foundation consists of two phases,ie., the geotechnical and the structural. Between them, the aim of the geotechnical phase of design, or geotechnical design, is to arrive at the plan dimensions of the foundation satisfying the two requirements from the soil side, bearing capacity and settlement, independently. The latter two are therefore, called geotechnical design parameters. It is obvious that when two requirements are to be satisfied independently, one of them will be just satisfied, and the other, over satisfied, in a general case. That is to say, if bearing capacity is just satisfied, settlement will be over-satisfied and vice versa Kurian (2004).

The aim of structural design is to cater to the structural action induced in the foundation-which is shear and flexure.The adequacy of a footing in the process of load transfer from the foundation to the soil will be ensured in the geotechnical phase of design. It is evident from the above that geotechnical design must necessarily precede structural design.

Geotechnical design is carried out for net-load which is the gross load transmitted at the base of the foundation in excess of the load to which it was originally subjected to. In other words ,the net-load is the gross load transmitted at the base of the foundation which includes the column load and the self weight of the footing and that of the backfill, subtracted for the weight of the soil removed from the trench. In calculations, we more often use the net loading intensity (nli),which is the unit value of the net load, that is the net load divided by the plan area of the foundation.. The basic reason for the consideration of net load is the fact that actual settlement is net settlement, which is produced by net load transmitted to the soil through the foundation.

3.2 Geotechnical design

The geotechnical design of a foundation is organized by comparing the property of the foundation with the property of the soil. The property of the foundation so chosen is the net loading intensity (nli) and the property of the soil to which it is compared is the allowable soil pressure (asp).Geotechnical design is organized by satisfying the following requirement, the net loading intensity (nli) is less than or equal to the allowable soil pressure (asp).

In the above allowable soil pressure is defined as the smaller of (1) safe bearing pressure which is related to bearing capacity and (2) soil pressure for a given permissible settlement which is related to settlement. By taking asp as the smaller of the two , we are satisfying the requirement on bearing capacity and settlement independently.

Geotechnical design is not a direct exercise, but interactive in most instances. This is because the terms net loading intensity , safe bearing pressure and permissible settlement are variously functions of the width of the foundation (B) which necessitates the assumption of a trial value of the plan dimensions to start with and converge to the correct design value by several cycle of iteration. In other words, the quantities using which we want to determine the plan dimensions are themselves dependent on the latter. In the place of sps it is more convenient to go by settlement (S) in a general approach to geotechnical design applying to c, ϕ, and c-ϕ soils. In a manual approach to design nli is first satisfied with sbp and S is checked on the design so arrived at. The design is modified, if necessary, till settlement is satisfied.

3.3 Geotechnical design of shell foundations

If we assumes that the plane passing through the base of the shell footing corresponds to the same plane under a flat footing there is little difference between the geotechnical design of shell and plain footings except that the core soil underneath the shell footing now enters into the calculation for net loading intesity (nli) and allowable soil pressure (asp). It should be noted that this approach is sufficiently sound so long as the core soil can be assumed to act integrally with the footing, but because of the friction available between the core soil and the foundation soil in the case of shell footing than between concrete and the foundation soil in the case of plain footings, to resist horizontal loads this approach has its own side-effect.

The question in respect of shell foundations and their plain counterparts should be formulated as what is the influence of contact shape (in the vertical direction) on the bearing capacity and settlement of foundations. The earliest attempt by Szechy,C.(1965) to address this question appears that a surface curved downwards (concave) aids a wider spread of load, resulting in a decrease in stresses and a consequent reduction in settlement as shown in (Fig.3.1).However, further studies conducted by Szechy,C.(1965), revealed that this tendency holds good at small foundation depths only, and the facilitation of lateral displacement of soil by the concave contact surface adversely influences bearing capacity..

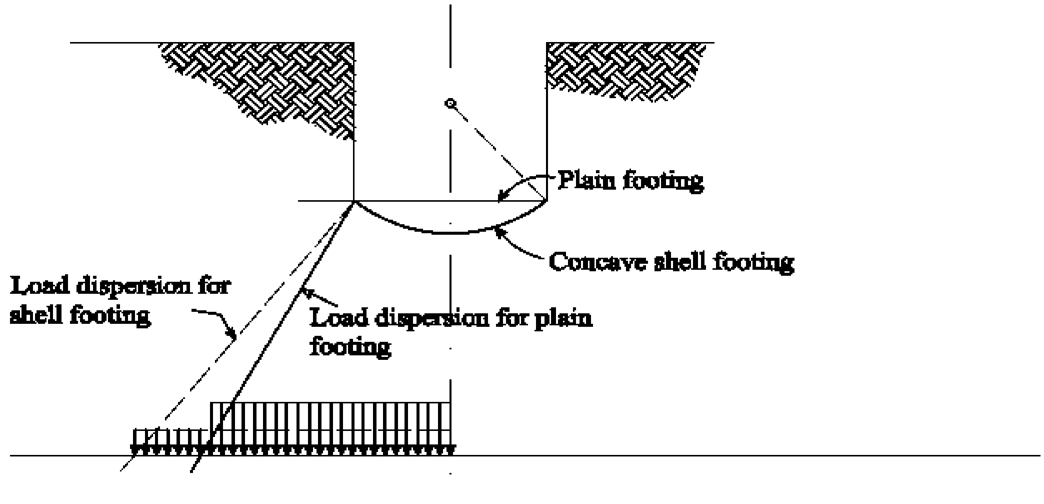


Fig. 3.1 Load dispersion under plain and shell strip footings

More recently, Hanna and Abdel-Rahma(1990) conducted experimental and theoretical investigations on triangular strip shell footings, on sand, varying the peak angle of the footings (θ) from 60^0 to 180^0 (flat footing). Fig 3.2 shows the theoretical model used by the authors. The tests were conducted under both surface and embedded condition.

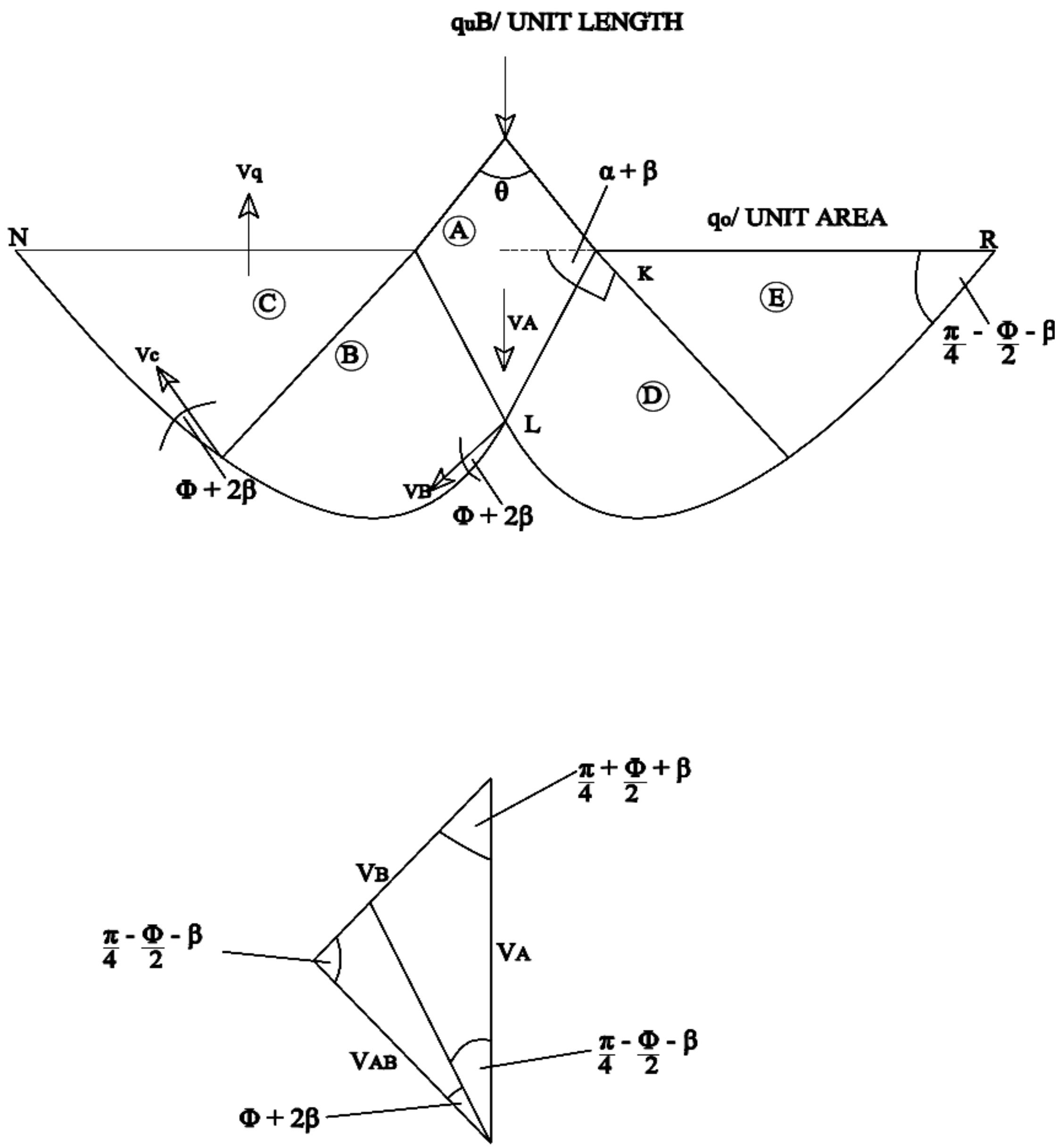


Fig. 3.2Theoretical model for bearing capacity (Hana,and Abdel-Rahama ,1990)

From the velocity diagram shown in Fig. 3.2, the following expressions can be obtained as functions of *VA,* assuming that zone A moves down with velocity *VA.*

Rate of external work = BV_A $\{q_u - q_o \exp[\pi \tan(\phi + 2\beta)] \tan^2(\frac{\pi}{4} + \frac{\phi}{2} + \beta)\}$ (3.1)

And,

Total dissipation rate = $c\ BV_A\ cot(\phi + 2\beta)\ \{\exp[\pi \tan(Ø + 2\beta)] \tan^2(\frac{\pi}{4} + \frac{\phi}{2} + \beta) - 1\}$

(3.2)

BY equating Eqs. 3.1 and 3. 2 we can get

$$q_u = c\ N_{ct} + q_o\ N_{qt} \quad (3.3)$$

Where,

$$N_{qt} = \exp[\pi \tan(\phi + 2\beta)] \tan^2(\frac{\pi}{4} + \frac{\phi}{2} + \beta) \quad (3.4)$$

$$\mathrm{Nct} = \cot(\phi + 2\beta)\ (Nq\text{-}1) \quad (3.5)$$

The factors Nq and Nc for the case of flat foundations (Prandtl 1920; Reissner 1924) are:

$$Nq = \exp(\pi \tan \phi)\ \tan^2(\frac{\pi}{4} + \frac{\phi}{2}) \quad (3.6)$$

$$Nc = (Nq\text{-}1)\ cot\ \phi \quad (3.7)$$

Definitions for new bearing capcity coefficients of triangular strip shell footing can be found by comparing Eqs. 3.4 and 3.5 with 3.6 and 3.7 as follows:

$$Nqt = NqFq \quad (3.8)$$

$$Nct = cot\ \phi\ (Nqt - 1)\ Fc \quad (3.9)$$

Where Fc and Fq are factors that depend on the foundation peak angle θ,the angle of shearing resistance of the soil ϕ,and the angle β..It should be noted that for the case of flat foundation (θ = o) ,Fc = Fq = 1,and the deduced bearing capacity coefficients *Nc* and *Nq* are the same as those from classical theory of bearing capacity.

By applying the principle of superposition for triangular shell strip footing on soil having weight and by using the bearing capacity coefficient $N\gamma$ for the case of a flat footing, the following general equation for the ultimate bearing capacity of triangular shell strip foundation can be obtained:

$$qu = cNct + q_0 Nqt + -\frac{1}{2}\gamma B N\gamma t \qquad (3.10)$$

Where,

$$Nct = \{FqNq - 1) \cot \phi \, \mathrm{Fc}$$

$$Nqt = Nq\ Fq$$

$$N\gamma t = 1.80\ (FqNq - 1) \tan\phi$$

Hanna and Abdel-Rahma (1990) defined modified bearing capacity factors as functions of ϕ and θ,and concluded that a footing of the above type gives higher bearing capacity and better settlement characteristics in comparison to its flat counterpart.

The same authors conducted tests on conical shell footings at the working load stage under both surface and embedded conditions, and supplemented the same by linear finite element analysis. They found a reduction of vertical displacements in favour of the shell, under both the surface and embedded conditions. They further took up a detailed testing (Fig. 3.3) of triangular (*2D*),pyramidal (*3D*) and conical (axisymmetrical) footings and their plain counterparts, on dry sand at three densities, viz., loose, medium, dense, and both at the surface and at a depth of 0.75 times the width of the footing extending ,the tests, up to failure of the soil.

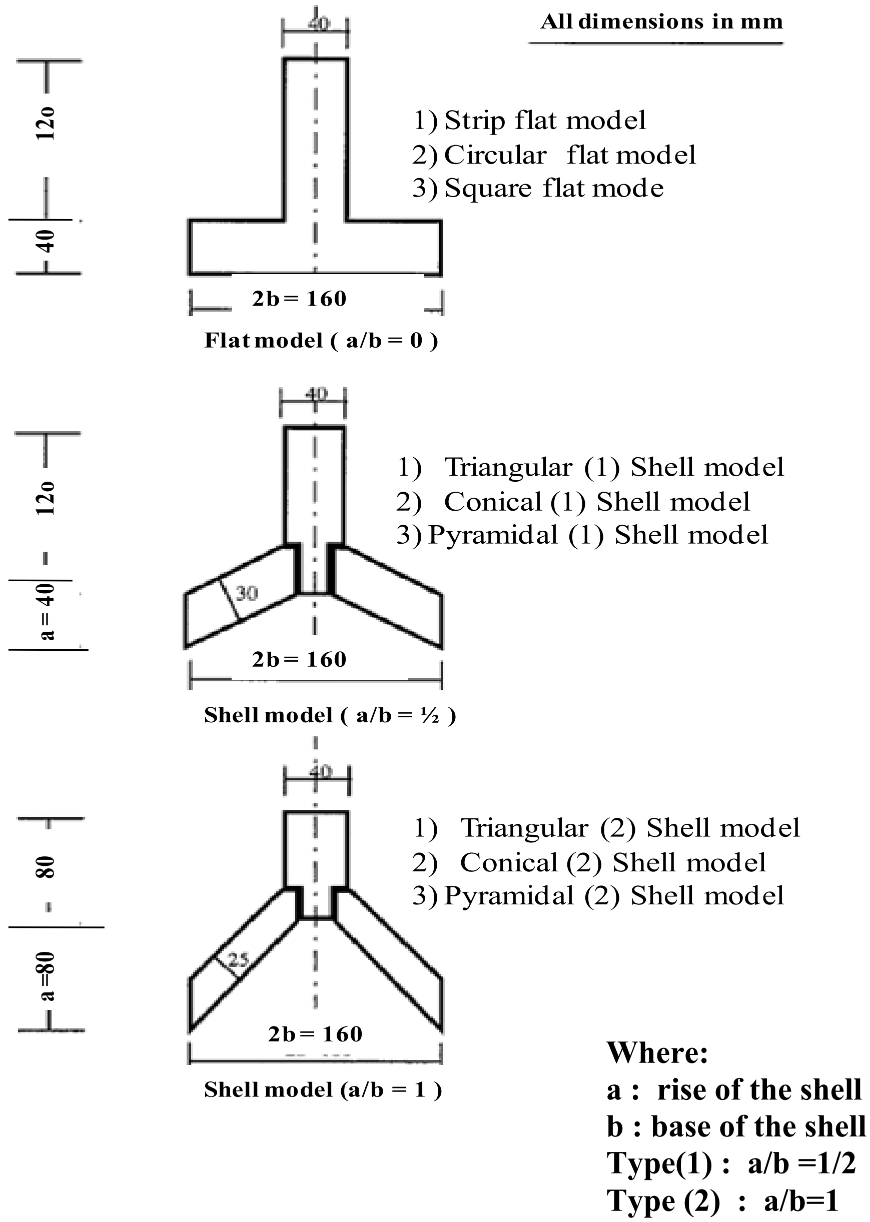


Fig.3.3 Overall view of the foundation models

In order to express the difference in bearing capacity and settlement between the shell and plain cases, a shell gain factor and settlement factor were defined.

$$\eta = \frac{Qus - Quf}{Quf}$$

Table 3.1. Shell gain factor, conical and pyramidal footings.

	Angle of shearing resistance (ϕ °)					
	Surface footing (D/B =0)			Surface footing (D/B =0.75)		
	34	38	41	34	38	41
Conical (1)	20.0	15.0	14.0	11.6	9.3	7.6
Conical (2)	37.0	29.2	24.1	20.2	17.2	14.9
Pyramidal (1)	20.4	14.7	12.7	11.3	9.2	7.2
Pyramidal (2)	38.1	28.1	23.0	20.0	16.9	14.1

The shell footings consistently gave higher bearing capacities, but the gain was found to decrease with increasing ϕ of the soil, implying that the effect of shell configuration diminishes when the soil becomes denser. The increase in bearing capacity was attributed to rupture surfaces which were found to go deeper in the shell case. The same decrease was also observed with the depth of embedment of the footing. On the settlement side, settlements were lesser for the shells: however, the same was higher for embedded footing.

3.4 Analytical Studies on the geotechnical performance of shell foundations

Rigorous analytical solutions in closed form on the geotechnical performance of shell foundations in terms of bearing capacity and settlement are extremely complex to develop, due to the non-planar interface between the shell and the soil. This problem, however, can be overcome by the powerful numerical technique called the finite element method (FEM).

An extensive study was undertaken recently on the geotechnical performance of shell of diverse types (cone,Hypar,and Spherical) by the finite element method, using the NISA (Numerically Integrated Elements for Systems Analysis) package. NISA has been developed and maintained by the Engineering Mechanics Research Corporation of Troy, Michigan by Nainan P.Kurian,and V.M Jayakrishna (2005).

Nonlinear results were obtained invoking the Mohr-coulomb model for the soil, and using interface elements characterized by the coefficient of interface friction. The study being on the response of the soil under the shell to load, it was necessary to use perfectly rigid models (which was also simulated in the analysis) so that they would undergo no structural deflection till the soil fails in bearing. In the analysis the load was applied in equal increments with a maximum of 200 iterations with in each increment. The incremental-iterative technique ensured high degree of accuracy of the results.

.

For all shell configurations, the bearing capacity is found to increase with the rise of the shell in the upright position, showing a corresponding decrease in the inverted position when compared to the plain counterparts. The above analysis was extended in the form of a parametric study, but covering only the conical, spherical and hypar shells, and the parameters chosen are interface roughness ,type of soil c, ϕ and c - ϕ and type of loading (Vertical load, horizontal load and moment). As would be expected, bearing capacity increases with interface roughness and the cohesion (c) and angle of internal friction (ϕ) of the soil. As regards ultimate load, the upright shell (cone and hypar) are found to be far better over their plain counterparts under the same loading combinations, unlike the inverted spherical shell which suffer a decline in its load carrying capacity.

Kurian and Jeyachandran,(2005) undertook an extensive experimental investigation on four types of shell foundation, ie., V-shaped folded plate, cylindrical, conical and hypar, using cast iron models which would not deflect structurally, thereby retraining the shape of contact at tall stages of loading, besides ensuring the failure of the soil before the footing fails. The first two shapes, even though *2D*, were tested as independent footings *(3D)* by giving a length equal to the width of the footing. For each shell, models were cast with rise to half-span-rations of ½ and 1 and each model was tested in the upright and invented position, on dense sand, at the surface. Plain square and circular models were also tested for comparison of the results.

Even though bearing capacity was found to be higher for the upright shells, a marked tendency for reduction was noticed on the negative side of rise. The latter was attributed to the punching effect of the footing facilitated by the shape.

Kurian and Jeyachandran (2005) concluded that, in view of the fact that the differences are not very substantial in favor of shells, and in the absence of conclusive evidence to the contrary, it was considered prudent to ignore the difference so that the bearing capacity and settlement under plain and shell footings are treated as identical, under identical field conditions, for the purpose of geotechnical design.

3.5 Soil structure interaction

It has been stated in the previous section that foundation is a two-component system consisting of the structural foundation and the natural foundation (soil) on which the former is supported. By soil structure interaction is meant the mechanical (i.e in terms of mechanics) interaction between the foundation (structure) and the soil, which determines the response of the soil to the load that is transmitted to it through the foundation. By response is meant the distribution of contact pressure (soil reaction) on the foundation

The resultants of the applied vertical load and soil reactions must be equal and collinear, to satisfy the two condition of equilibrium ,i.e., $\sum V=0$ and $\sum M=0$, respectively. However, consistent with the above, the contact pressure distribution can take diverse forms and the actual distribution is important in determining the moments and shears for the more realistic structural design of the foundation. The same can be determined only by an interaction analysis which is highly involved, being statically indeterminate.

Instead of such a rigorous approach, foundations are normally designed for linear soil reaction distributions (uniform or rectangular under a concentric load, and trapezoidal under an eccentric load). It is obvious that whatever the actual distribution, the uniform value, for example, under a central vertical load is the average of the actual non-uniform distribution resulting from soil-structure interaction as shown in (Fig 3.4).

Conventional design of foundation based on the uniform average value is called rigid design against which, design based on the actual interactive contact pressure distribution is called flexible design.

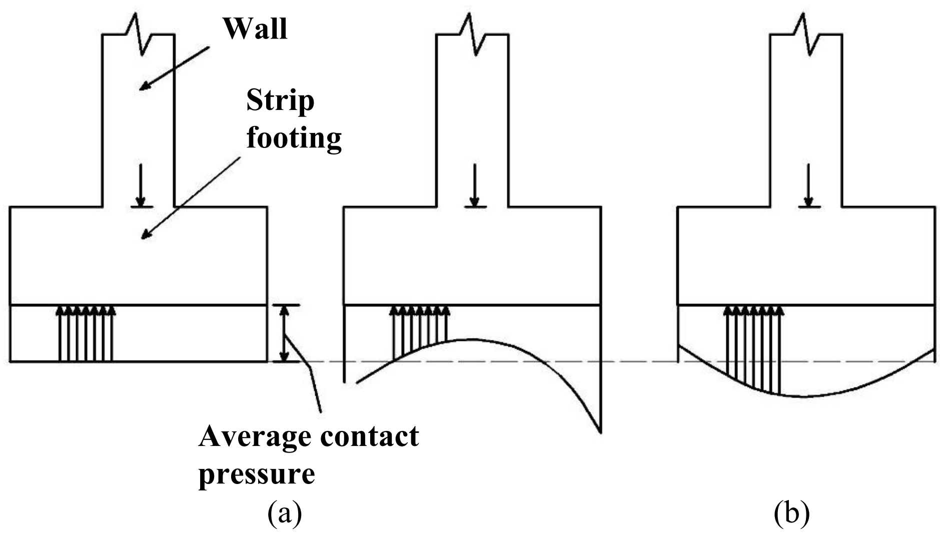


Fig.3.4 Contact pressure Distribution:(a) Rigid,(b) Flexible

3.6. Contact pressures under shell foundations

The previous discussion on contact pressures and soil-structure interaction was so far confined to plain foundations, i.e., foundations having planar soil foundation interfaces. The problem in relation to shell foundations, is more complex on account of the non-planar nature of the soil foundation interface.

If we examine the shell problem at any point of contact between the shell and the soil the contact pressure (i.e., the soil reaction) will in general have a normal component and a tangential component., the tangential component, being likely along the direction of the lateral deformation of the shell at the point. The maximum value of the tangential component is obviously limited to the coefficient of wall friction multiplied by the normal component, both being acting on the same elemental area.At any point on the surface (of contact) of a shell of any regular geometric shape, we have a normal to the shell surface, and a tangential plane which is tangential to the shell surface .

Through any point on the shell two curves having the maximum curvature and the minimum curvature are lying on the shell surface. The planes carrying these curves also carry the shell normal, and are mutually perpendicular. If we draw tangents to the two curves at the point, the tangents will both lie on the tangential plane, and will be perpendicular, and together with the normal constitute an orthogonal system consisting of three mutually perpendicular straight lines.The contact pressure at any point in general has there components, one along the normal and two along the directions of the principal tangents. We shall call these components N , T_1 and T_2 as shown in (Fig 3.5a).

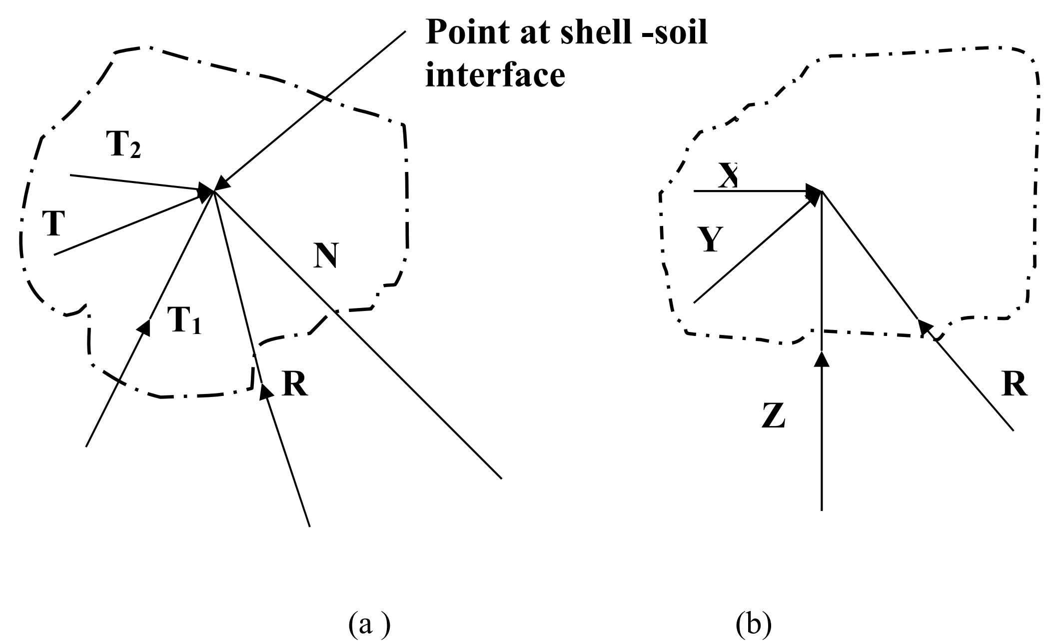


Fig.3.5 Contact Pressure Components

The frame consisting of the normal and the two tangents, though orthogonal in itself, will change direction as we pass from point to point on the shell surface. Therefore it shall be convenient to resolve the resultant contact pressure (resultant of N ,T_1 and T_2) along the coordinate directions x ,y and z which remain unchanged for all points on the shell surface. We shall call these coordinate components, X,Y and Z (Fig 3.5b). But one thing that is certain is that, whether we consider N, T_1, T_2, or X, Y,Z, the actual magnitudes of these contact pressure components ,and the distributions over the shell surface are highly indeterminate because of the highly complex interaction that takes place between the shell and the soil, under the load.

If the soil beneath the shell is a soft clay ($\phi \approx 0$) it is obvious that the tangential components (T1, T2) will not develop, as they can not be sustained and we have a nearly hydrostatic case in which the contact pressures are purely normal to the shell surface at every point. On the other hand, if the soil is highly frictional, like sand, the induction of these tangential components may give a shift in the direction of the resultant contact pressure form normal to vertical or nearly vertical. And if the resultant contact pressure becomes ideally vertical at all point, it means X=Y=O, and R= Z at all points. On top of this, if we now make the assumption that R is uniformly distributed it means the normal assumption made in the conventional structural design of shell foundations in frictional soils

$$R = Z = \frac{P}{A_p} = p_v \tag{3.11}$$

.

If we revert to the assumption that the soil reaction is purely normal (i.e. unaccompanied by any tangential component) and uniform as sown in (Fig 3.6) , its value is obtained as:

$$P_n = p_v = \frac{P}{A_p} \tag{3.12}$$

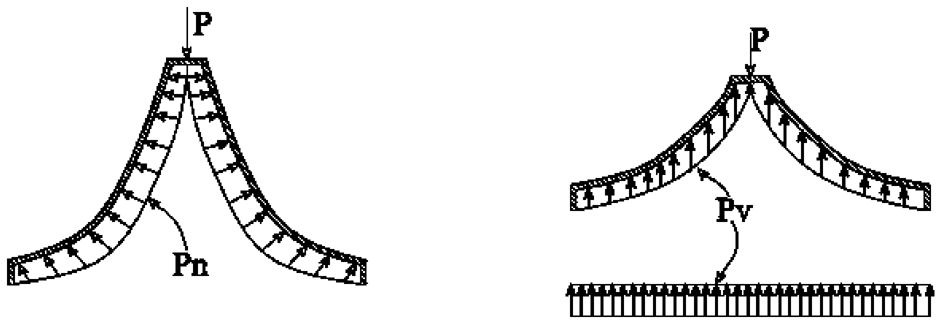


Fig.3.6 Normal and Vertical Contact Pressure Distributions

This follows from the simple fact that a hydrostatic pressure situation has the same intensity of pressure in all directions.

3.7 Parametric studies

3.7.1Core subsidence

Kurian(2004) analyzed the influence of sub grade modulus on conical shell foundation. The two parameters selected from the soil side for the investigation are : 1) the quality of the soil (represented by its sub grade modulus k), and 2) partial contact underneath the footing (representing core subsidence).

It is well established that a shell is not an efficient structural system under concentration of loads. The possible subsidence of the core soil at the centre (Fig. 3.9) if it is shrinkage type can lead to loss of support at the centre and consequent concentration of soil reaction at the edges, a matter over which serious concern has been expressed by designers. This prompted a study in which the cone, the double cone and the hypar were subjected to partial soil contact to examine the extent to which their safety and stability are affected .

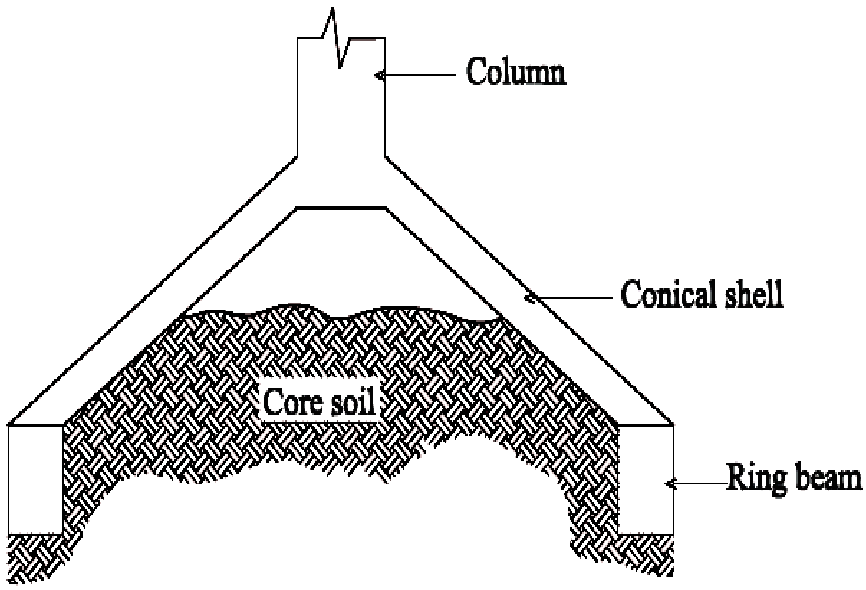


Fig. 3.7 Core Subsidence under a Shell Footing

In the finite element analysis by Kurian (2004) the soil under the centrally loaded shell foundation has been modeled by Winkler springs, and the system analyzed for various degrees of contact between the shell and the soil. The contact parameter chosen was, the ratio of the height over which the soil is present, to the full rise of the shell as shown in (Fig. 3.8) The cases analyzed consist of 100% , 75% and 25% soil contact, in the which 100% corresponds to the full contact. In order to model partial contacts the springs have been progressively removed, which is achieved by assigning them negligible stiffness.

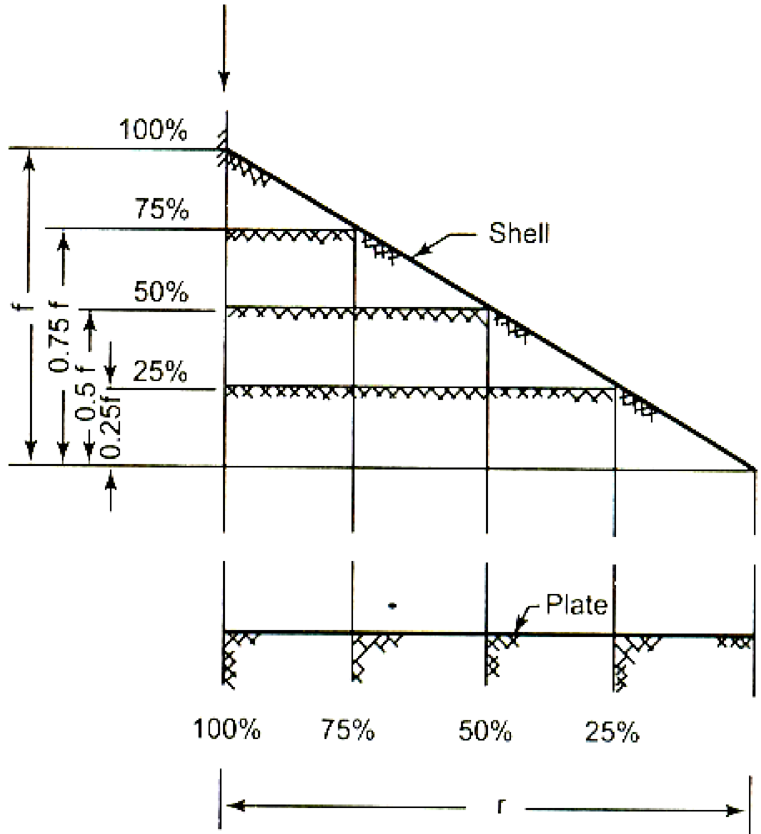


Fig. 3.8 Partial Soil Contacts considered in the Analysis

The behavior of the shells under partial soil contacts has been studied in terms of the variations in the vertical deformations (w) and the membrane stress (N) and bending moment (M) resultants in shell and the edge beams. The size of the shell models and other relevant data used for the analysis are given together with the results. Among the shell cases investigated, the results pertaining to the conical shell are presented in (Fig. 3.9) for various soil contacts.

This is followed by the results of a flat circular plate of the same thickness analyzed under identical conditions. It is seen that the stress resultants and displacements increase in the shell with decreasing soil contact. The meridonal compression is hardly influenced by the degree of contact unlike the hoop tension .As regards to moments ,the same in both directions get redistributed progressively from the region of no contact to the region where the contact is intact ,without affecting ,the absolute maximum values.

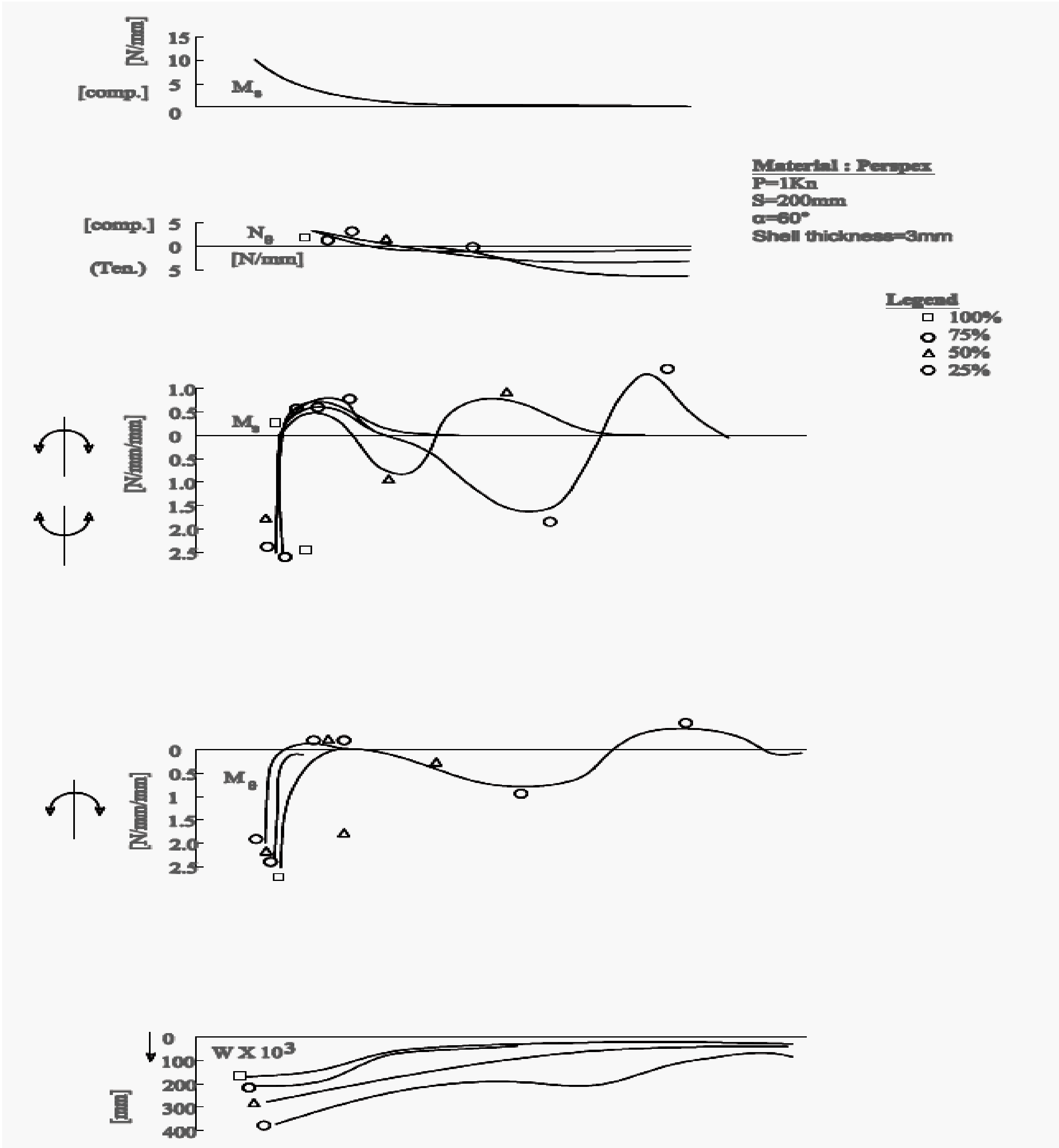


Fig. 3.9 Conical Shell Foundation – System Response vs. Degree of Soil Contact (Kurian,1994)

In the case of flat plate, even at 100% contact, the edges lose contact with the soil and lift up which required iterative analysis to arrive at a no- tension state between the shell and the soil. In sharp contrast to the performance of the shell under partial contact, the deformations and moments in the circular plate shoot up phenomenally with decrease in contact.

The fact that the shell system is far more stable than its plain counterpart is indeed a great redeeming feature in respect of the behavior of the shell. This is contrary to the fear expressed by designers in this regard, and as such these results should go a along way in popularizing the use of shell foundations even under possible minor core separation.

3.7. 2 Additional considerations

Elastic instability (buckling) is a consideration that assumes significance in the context of thin shells predominantly subject to membrane compression. However, buckling is not a matter of serious concern in shell foundation owing to its attributes of depth and thickness, besides being restrained by soil, both at top and bottom.

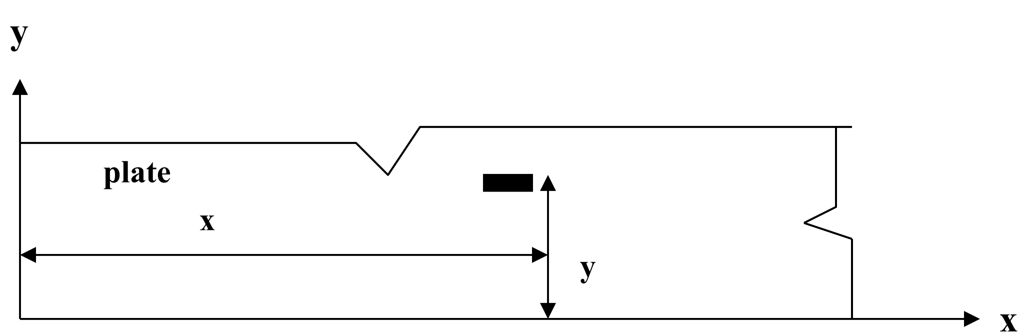
4.0 Membrane analysis of foundation shells

4.1 Introduction

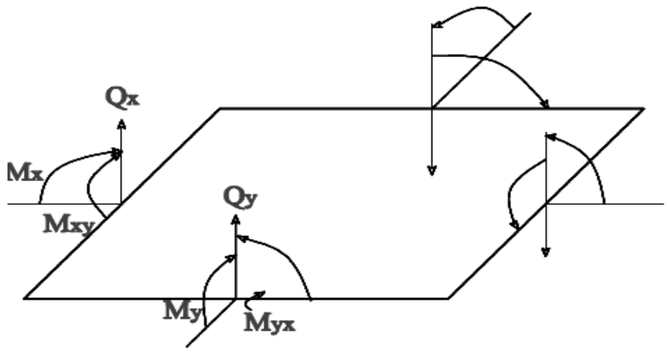
Shells, whether for roof or foundation, like all other structural elements, should be designed for the state of stress induced in them by the system of applied loads and reaction, against their geometry and boundary condition. In this and the following two chapters, our concern shall be the analysis of shells by the membrane theory, and ultimate load theory, respectively. While the first one is concerned with the working stress design of these foundation shells, the ultimate strength theory gives us the maximum loads these shells can sustain under the given set of conditions. Since several standard works are available for ready reference on the theory of shells, instead of repeating those theoretical derivations here, we shall rather confine ourselves to results using an intuitive approach suited to the subject of foundations.

It is known that, while a plain structure like a slab resists applied loads in flexure, which it can, on account of its high flexural rigidity contributed by thickness, shell by virtue of their geometry and small flexural rigidity, tend to carry applied loads primarily by direct or in-plane stresses accompanied by little or no bending.

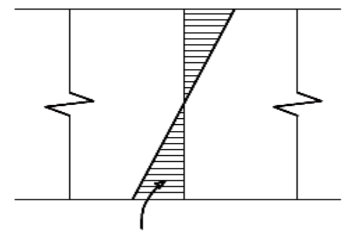
The flexural mode of resisting applied loads in the case of plates, can be fully described in terms of two transverse shear forces, two transverse bending moments and one twisting moment.



(a) Plan



(b) Bending stress resultants at point (x,y)



(c) Bending stress due to M_x

Fig. 4.1 System of loads in the case of plates

Referring to Fig 4.1 at any point (x ,y) in the plate, we have the shear force Q_x and bending moment M_x in the x-direction, Q_y and M_y the shear force and bending moment, respectively, in the y-direction, and the twisting moment M_{xy} ($=-M_{yx}$), constituting the effects which fully describe the state of flexural stresses in sections perpendicular to the co-ordinate directions. While the transverse moments M_x ,and M_y produce flexural stresses linearly varying intensities across the thickness of the section (Fig 4.1) ,the twisting moment Mxy tends to warp the section, and induces shear stresses at the section as do the transverse shear forces Qx and Qy.

If we now consider a point (x,y) on the middle surface of a shell as shown in (Fig 4.2) and assuming that the shell resists the applied loads purely in terms of the in-plane stresses, the stress picture at the point can be fully described by three membrane stress resultants, which are the two normal stress resultants N_x and N_y and the membrane shear stress resultants produce normal and shear stresses (horizontal) of uniform intensity across the shell thickness.

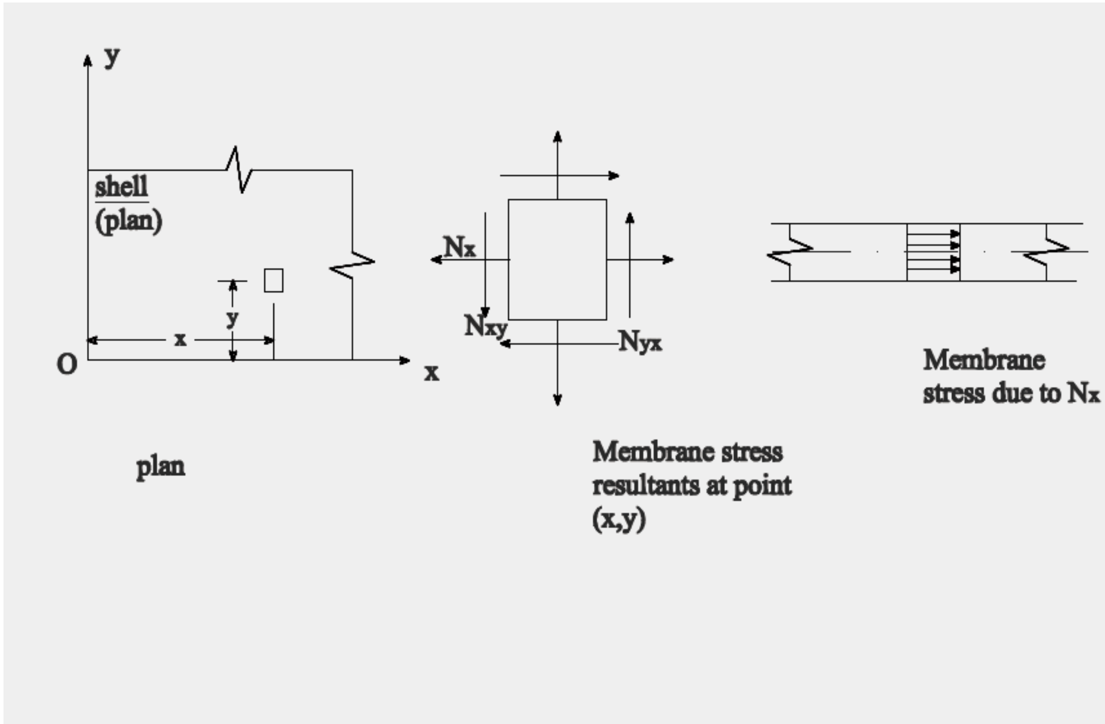


Fig. 4.2 Membrane Stress Resultants in Shells

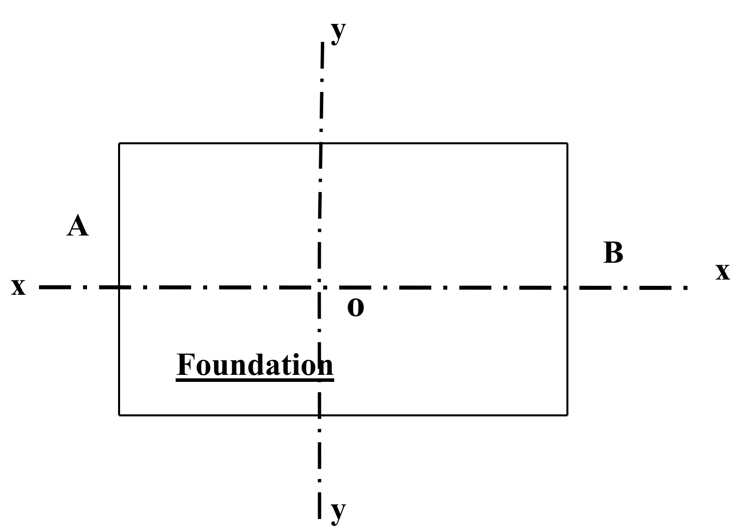
These in – plane effect are appropriately termed membrane stress resultants due to their analogy with thin membranes, which, on account of their negligible stiffness, resist all applied loads purely by membrane forces. This analogy, however, does not hold too far since while compressive and tensile stresses can exist in a shell membrane, only tensile stresses develop in an actual membrane like a stretched rubber sheet or a rope (one dimensional) for that matter.

4.2 General system of loads on shell foundations

Foundations, whether plain or shell, are designed essentially for the soil reactions or contact pressures produced by the superimposed loads. In the conventional design of shell foundations, which is based on membrane theory, the soil reactions are assumed to be uniform or uniformly varying depending upon whether the resultant applied load on the foundation is concentric or eccentric. This is also the same as what is done in respect of the conventional design of plain foundations. This contact pressure distribution is statically determinate, and is consistent with the membrane theory for the shell, in that the latter is also statically determinate.

We shall now examine the transmission of general system of loads acting on a foundation and see how the same can be reduced to a form suitable for the determination of the contact pressures for the design of shell foundations.

Figure 4.3a shows a plain foundation acted upon by a general system of loading consisting of vertical loads, horizontal loads and moments. For simplicity we shall assume a two-dimensional situation, in which all the loads, viz., vertical loads like V_1, horizontal loads like H_1, and moments like M_1, act on the vertical plane passing through the axis XOX. Our aim is to determine the effect of this system of loads at point O, which is the centroid of the area of contact between the foundation and the soil. Let v_1 and h_1 be the distances from point O to the lines of action of the loads V_1 and H_1, respectively. Just as the forces V_1 and H_1, we have forces V_2 acting at v_2 and V_3 acting at v_3, and also H_2 acting at h_2 and H_3 at h_3 and so on, which are not shown in the figure.



Plan

,

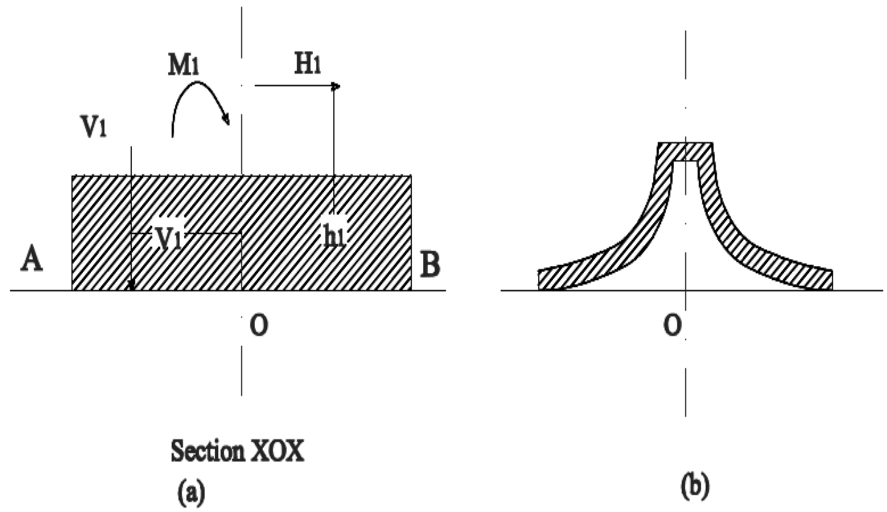


Fig.4.3 Foundation Acted Upon by General System Of Loads

At point O, the effect of V_1 acting at distance v_1(lever arm) is a vertical load V_1 acting at O+ a moment $V_1 * v_1$ which is counterclockwise . Similarly the effect of H_1,a horizontal load acting at 0+ a moment $H_1 \times h_1$ which is clockwise . The effect of M_1 is the same wherever it acts on the plane XOX.

If we now call :

$$V = \sum V_1$$

$$H = \sum H_1 \qquad (4.1)$$

$$M = \sum M_1 \text{ ,and}$$

$$\overline{M} = \sum (V_1 \times v_1) \pm \sum (H_1 \times h_1) \pm \sum (M_1)$$

The effect of the above system of loads, as far as point O is concerned is, a vertical load V, a horizontal load H, and a moment $\overline{M}$. Of these, H is resisted by the frictional force {which is equal to $(V+W)\mu$} , where W is the self weight of the foundation, and μ,the coefficient of friction), mobilized at the foundation soil- interface. Note that the load for structural design does not include the self weight of the foundation.

Of the remaining effects, V produces a uniform soil pressure as shown in (Fig 4.4),

$$P = \frac{V}{A} \tag{4.2a}$$

While $\overline{M}$ produces a uniformly varying soil pressure,

$$p' = \frac{\overline{M} * x}{I_{yy}} \tag{4.2b}$$

Where x is the distance of the section from the axis YOY and I_{yy}, the moment of inertia of the area of contact about YOY. The resultant contact pressure distribution is the sum of these two effects, as shown in Fig 4.4b.

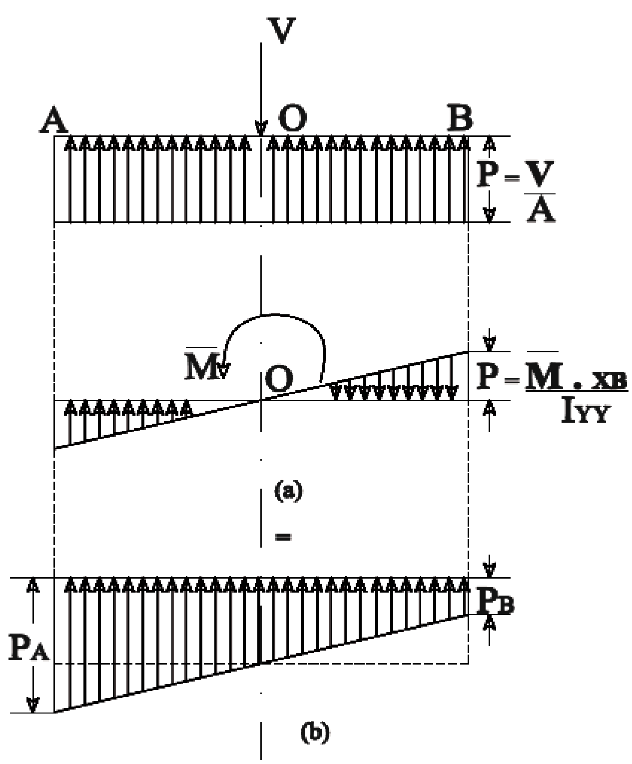


Fig.4.4 Soil Reactions

Sometimes it is convenient to isolate H, and consider the combined effect of V and $\overline{M}$ as a V only, acting at an eccentricity,

$$e = \frac{\overline{M}}{V} \tag{4.3}$$

Using which the net extreme values of soil pressure P_A and P_B (Fig 4.4b) can be obtained as:

$$P_{A,B} = \frac{V}{A}\left(1 \pm \frac{6e}{L}\right) \tag{4.4}$$

From the above expression we notice that, if V acts outside the mid-third points, the resultant soil pressure is tensile over a stretch of the contact area (Fig 4.5a). Since this cannot occur and the footing can only get partially lifted out of contact with the soil , a redistribution of contact pressure must take place such that the soil pressure is zero over the stretch of no contact as shown in (Fig 4.5b). The two unknowns p'_A and L',defining the new redistributed contact pressure diagram, (Note that L' in Fig 4.5b) is not the same as the distance AC in Fig 4.5a) can be easily determined by invoking the two conditions of static equilibrium, which are :

(i) $\sum V = 0$, which means:

$$V = \frac{1}{2} P' \times L' \times B \tag{4.5a}$$

And
(ii) V and the resultant of the contact pressure must be collinear (so that $\sum \overline{M} = 0$) from which :

$$\left(\frac{L}{2} - e\right) = \frac{L'}{3} \tag{4.5b}$$

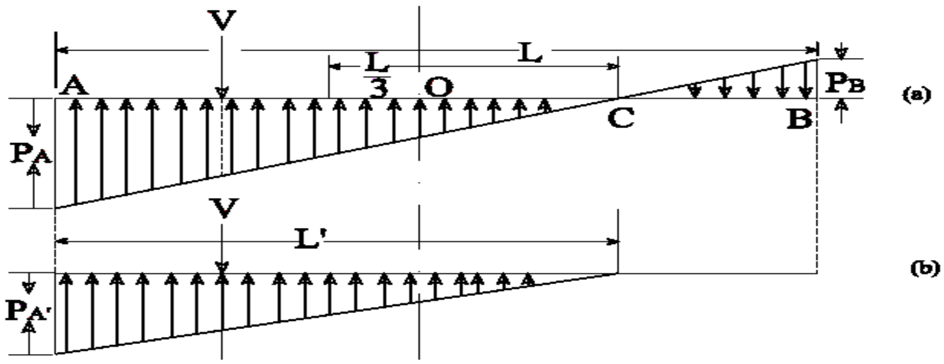


Fig.4.5 Redistribution of Soil Pressures

In the case of shell foundation also, the above condition applies , if we consider O as the centroid of the horizontal projection A_p of the curvilinear area of contact between the shell and the soil as shown in Fig 4.3b .

In a general case, where the system reduces as a central vertical load V plus moments M $\overline{M}_x$, and $\overline{M}_y$ about both the axes, the resultant soil pressure *p* at any point (x,y) is obtained as (Fig.4.6):

$$P = \frac{V}{A} \pm \frac{\overline{M}_x * x}{I_{yy}} \pm \frac{\overline{M}_y * y}{I_{xx}} \tag{4.6}$$

However, if *p* at A,B,C and D alone are required, it will be convenient to recompose the above system as a single vertical load V acting at eccentricities e_x, and e_y ,where

$$e_x = \frac{\overline{M}_x}{V}$$

and

$$e_y = \frac{\overline{M}_y}{V}$$

using which:

$$p_{A,B,C,D} = \frac{V}{A}\left(1 \pm \frac{6e_x}{L} \pm \frac{6e_y}{B}\right) \qquad (4.7)$$

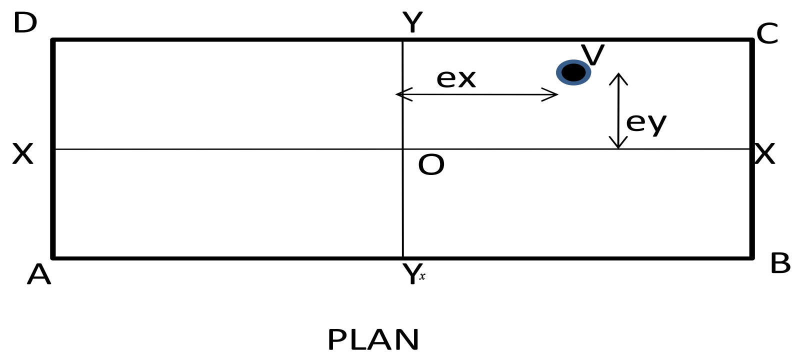


Fig.4.6 Resultant contact pressure distribution

One can easily visualize that the resultant contact pressure distribution is depicted by an inclined plane (inclined to both axes) passing through A', B', C' and D' where, AA', etc. represents the resultant contact pressure at the respective corners. The redistributed contact pressure plane resulting from load of high eccentricities, which will intersect the base of the footing along an inclined line (in plan) is easily visualized, but its actual determination, would indeed be a complex exercise.

4.3 General remarks on membrane theory

If a shell may be regarded as a highly inextensible, but at the same time a perfectly flexible membrane that is to say, it has enormous membrane rigidity, but negligible stiffness against transverse bending, it must resist applied loads predominantly by membrane stresses .

The membrane analysis is based on equilibrium of forces (only), that is to say, it seeks to establish equilibrium between the applied loads and the three membrane stress resultants

N_x, N_y, and N_{xy} (or N_r ,N_θ ,and $N_{r\theta}$, if the co-ordinates are polar). Membrane theory is applicable only when the boundary conditions are compatible with the conditions of equilibrium ,and as such one should note the important fact that membrane solutions are not available for all arbitrary loading and boundary conditions. In other words, one is not free to choose the boundary conditions in all cases if the load is to be equilibrated by membrane stresses alone.

The boundary conditions that will develop a state of pure membrane stresses in the shell can be described in terms of (i) compatibility of forces (ii) compatibility of deformation. Whenever either or both of these requirements are not met, bending stresses must necessarily develop in the shell.

The membrane theory is statically determinate and as such one does not have to invoke the cross-sectional and material properties of the shell for obtaining membrane solutions, unlike the bending theory which is statically indeterminate. In assessing the merits and recognizing the limitations of the membrane theory, one should clearly bear in mind that, while in many cases membrane theory may be sufficient, in many others, particularly those involving arbitrary, and even realistic, loading, boundary and geometric conditions, it may be entirely inadequate. In such cases resort to bending as inevitable, if what is sought are refined and realistic solutions.

4.4 Membrane stresses in conical foundation shells

Whether as footings for columns or as rafts for tower shafts it is not the full cone, but practically the frustum of a cone that is used as foundation. Since it has a circular plan the cone can be used in individual units only, not in combinations as the hypar foundation. Conical shell subjected to both vertical and normal uniform soil pressure, develops stress resultants N_s, N_θ and $N_{s\theta}$ as given in (Fig.4.7) below Kurian (2006).

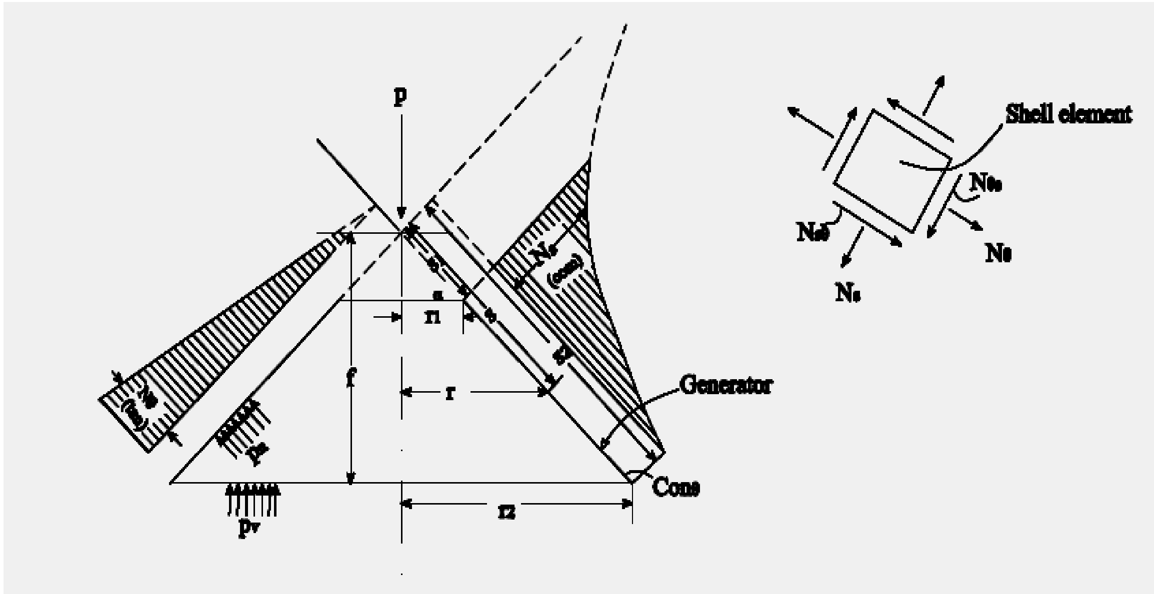


Fig.4.7Membrane Stress Resultants in Cone

Stress resultants under vertical soil pressure:

$$N_s = \frac{p_v \tan\alpha(s_2^2 - s_1^2)}{2s} \tag{4.8}$$

$$N\theta = \frac{p_v s \sin^3\alpha}{\cos\alpha} \tag{4.8}$$

$$N_{s\theta} = 0 \tag{4.10}$$

Stress resultants under normal vertical soil pressure:

$$N_s = \frac{p_n \tan\alpha(s_2^2 - s_1^2)}{2s} \tag{4.11}$$

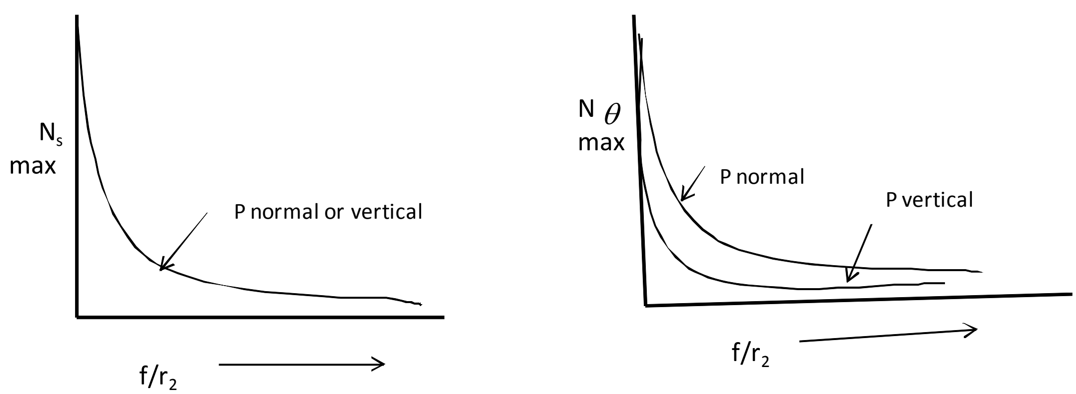
$$N\theta = p_n s \tan\alpha \tag{4.12}$$

$$N_{s\theta} = 0 \tag{4.13}$$

Both the above stress resultants apply under the boundary condition $N_s = 0$ at the bottom.

Figure 4.7 also shows the variation of the stress resultants N_s, and N_θ along the shell generator for uniform normal soil pressure. From the figure we find that under a uniform normal contact pressure, the conical shell is subjected to a meridian compression with intensity decreasing downwards from a maximum at the top to zero at the bottom (the latter is the boundary condition), and a hoop tension decreasing linearly upwards from a maximum at the base, representing a seeming balance between the two effects.

It is seen from Eqs. (4.8) and (4.11) that N_s is the same, whether the loading is assumed to be vertical or normal. Figure 4.8 shows the variation of the maximum values of N_s and N_θ against the rise- to –base radius ratio (f/r_2), for both conditions of loading. It is seen that both $N_{s\ max}$ and $N_{\theta\ max}$ vary inversely with the rise; with $N_{\theta\ max}$ showing greater decrease at increasing values of rise.



(a) (b)

Fig.4.8 Variation Of $N_{\theta\ max}$ and $N_{s\ max}$ with (f/r_2) Ratio: (a) $N_{s\ max}$ (b) $N_{\theta\ max}$

It may be noted when f=0 (Fig.4.7) the cone reduces to a flat circular plate,and when $f = \infty$, i.e.$\alpha = 0$, we have the extreme case of an infinitely long cylinder. If we examine the variation of $N_{\theta\ max}$, with f, it is seen that as f approaches to 0.i.e.as $\alpha \rightarrow 90^{o}$, both $N_{\theta\ max}$ for P_v and $N_{\theta\ max}$ for P_n approaches $\rightarrow\infty$.

Therefore what could happen is that the actual value of $N_{\theta\ max}$ takes a turn towards zero at some definite value of the (f/r_2) ratio. Kurian et.al (2006) reports a set of tests on models of conical footings to illustrate this aspect, the result against the theoretical variation as shown in Fig 4.9.

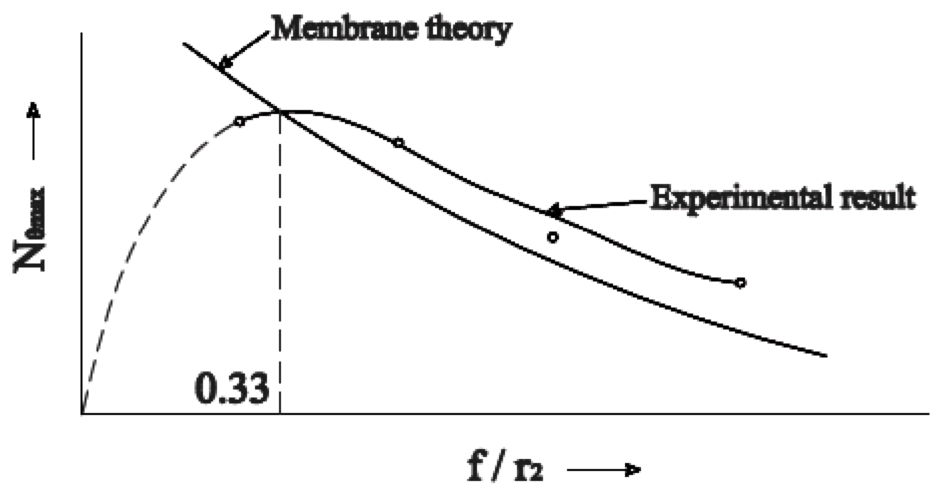


Fig.4.9 Conical Footing –Range of Applicability of Membrane Theory Kurian (2006).

A ring beam at the bottom of the cone is theoretically not indicated, since $N_s = N_{s\theta} = 0$ and N_θ is self-balancing without external aid. Therefore its provision is a matter of choice. But when provided, it contributes to stiffness at lower rise, delays the onset of creaking and leads to higher values of ultimate strength.

4.4.1 Stress resultants in conical shell under moment

Referring to (Fig.4.10) the stress resultants in the full cone subjected to anti-symmetrical normal soil pressure, given by Kurian and Datta Roy (2006) are:

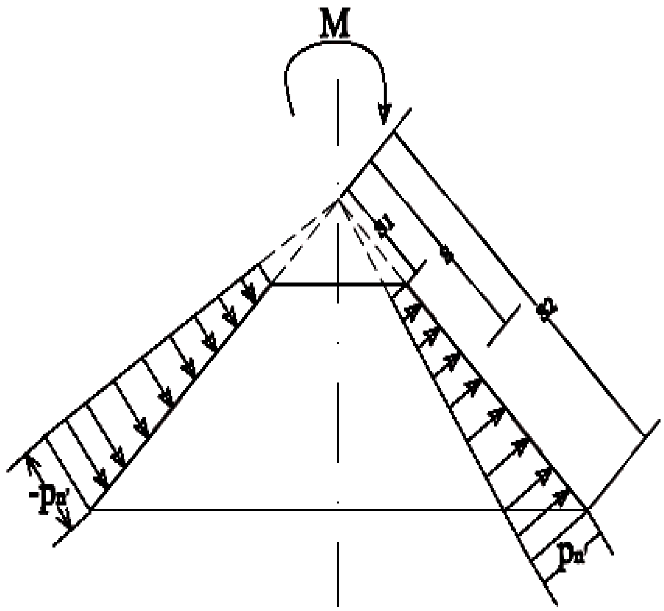


Fig. 4.10 Frustrum of Cone (normal soil pressure)

$$P'_n = \frac{My}{I} = \frac{4M}{\pi\ r^3} \tag{4.14}$$

$$N'_r = \frac{2p'_n}{s_2 \sin 2\alpha}\left(\frac{s_2^4 - s^4}{4s^2} - \frac{s_2^4 - s^3}{3s}\cos^2\alpha\right)\cos\theta \tag{4.15}$$

$$N'\theta = \frac{p'_n s^2}{s_2}\tan\alpha\cos\theta \tag{4.16}$$

$$N'_{r\theta} = p'_n \frac{s_2^4 - s^4}{4s_2 s^2 \cos\alpha} sin\,\theta \tag{4.17}$$

These are non-dimensionalised for the convenience of plotting, by defining;

$$\overline{N_s}' = \frac{N'_s \sin 2\alpha}{2p'_n s_2} \tag{4.18}$$

$$\overline{N_\theta}' = \frac{N'_\theta}{p'_n s_2 \tan\alpha} \tag{4.19}$$

And

$$\overline{N_{s\theta}}' = \frac{N'_{s\theta} 4\cos\alpha}{p'_n s_2} \tag{4.20}$$

Where,

$$\overline{N_s}' = (-)\ \frac{1}{s_2^2} \left(\frac{s_2^4 - s^4}{4s^2} - \frac{s_2^3 - s^3}{3s} \cos^2\alpha \right) \cos\theta \tag{4.21}$$

$$\overline{N_\theta}' = \left(\frac{s}{s_2} \right)^2 \cos\theta \tag{4.22}$$

$$\overline{N_{s\theta}}' = \left(\frac{s_2^4 - s^4}{s_2^2 s^2} \right) \sin\theta \tag{4.23}$$

Denoting the maximum positive values by the asterisk (*) we can now state.

$$\overline{N_s}^* = (-)\ \frac{1}{s_2^2} \left(\frac{s_2^4 - s^4}{4s^2} - \frac{s_2^3 - s^3}{3s} \cos^2\alpha \right) \tag{4.24}$$

$$\overline{N_\theta}^* = \left(\frac{s}{s_2} \right)^2 \tag{4.25}$$

$$\overline{N_{s\theta}}^* = \left(\frac{s_2^4 - s^4}{s_2^2 s^2} \right) \tag{4.26}$$

5.0 Ultimate strength analysis of conical shell foundation

5.1 Introduction

By ultimate strength of a foundation we mean the maximum load it can sustain structurally under a given set of conditions. In a design, based on working load conditions, ultimate strength analysis helps us to estimate the load factors involved in the design.

The ultimate strength analysis has the advantage of relative simplicity, particularly when considering the fact that aspects like depth and thickness of the shell present no serious problem in this analysis. Another aspect that can be cited as an advantage in respect of foundations is that where the design is governed by bearing capacity the ultimate strength approach should be deemed as more appropriate as it is theoretically consistent with bearing capacity, which itself is an ultimate concept on the case of the soil.

The basic assumption involved is that in the ultimate load stage the structure attains a plastic state and eventually reduces to a mechanism, and the ultimate analysis consists in analyzing this state. The ultimate capacities of the sections of the structure, which may be in flexure, or axial (i.e., membrane) tension or compression, or both, eventually leads to the determination of the ultimate strength of the structure.

The ultimate load analysis of a structure takes two courses: 1) based on kinematics (deformations), and 2) based on static equilibrium. In the first method the kinematic method also known as work method, we give the mechanism a small deformation and arrive at the value of ultimate load by applying the 'principle of virtual work", which equates the external work done on the mechanism to the internal work done by the mechanism. In the second method, the equilibrium method, we write equations of static equilibrium in forces and moments in the plastic state and arrive at the value of the unknown, which is the ultimate load. In an ideal case both approaches must lead to the same result for ultimate strength.

Our aim is to determine the maximum value of the column load that can be resisted by a reinforced concrete, conical shell footing with a central column subjected to a central vertical load only. For the development of the ultimate theory we are not employing any shell theory but are only invoking the shell geometry.

In deriving the expression for the ultimate load , P_u we shall apply the work method (principle of virtual work) on a mechanism of collapse which is based on a failure hypothesis arrived at from a large number of tests conducted on models of actual conical footings, and which was found to remain consistent over a wide range of parameters. We shall,however,ignore the interaction between the effect of membrane capacity (tension or compression) and flexural capacity along the yield lines, and consider only the effect which is predominant.

An important assumption that we make is with regard to the contact pressure distribution, the soil reaction is assumed to be normal in all cases and what the expressions generally give are the ultimate values of the normal soil reactions form which the corresponding values of the ultimate column loads may be obtained.

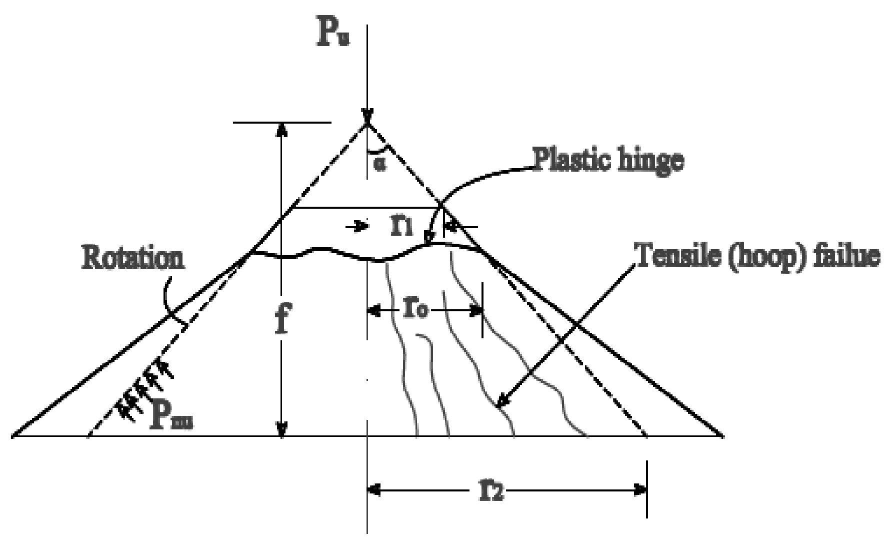


Fig 5.1 Ultimate Failure of Conical Footing (Kurian,2006)

Kurian and Kalisky (2006) derived the ultimate uniform normal soil pressure (eqn. 5.1) By assuming the footing to be fixed at the top and free at the bottom, and considering failure by circumferential (hoop) tension and radial bending as shown in Fig. 5.1

$$p_{nu} = 6\left[\frac{\frac{\cos\alpha}{2}\left(1-R_o^2\right)+\frac{M\sin^2\alpha}{Nr^2}}{R_o^3-3R_o+2}\right]\frac{N}{r_2} \qquad (5.1)$$

Where,

$$R_o = \frac{r_o}{r_2}$$

r_o = the radius corresponding to the location of the plastic hinge (r_o may be taken as r_1 for all practical purposes) according to Kurian (2006).

r_2 = the radius at base.

N = ultimate capacity of the shell section per unit width in direct tension in the hoop direction (assumed constant) and,

M = moment capacity of the plastic hinge, per unit width

The actual value of Ro consistent with the minimum value of p_{nu} is to be obtained by setting:

$$\frac{\partial p_{nu}}{\partial R_o} = 0$$

The mode of failure observed could be idealized into one of circumferential splitting and widening of the tensile cracks (Fig5.2) along an infinite number of radial yield lines, starting form the bottom and progressing in an upward direction towards the column base. The radial sectors were ultimately found to collapse by bending up wards about a circumferential plastic hinge, which in all cases were found to form at the foot of the column base (i.e., $r_o = r_1$)as shown in Fig. 5.1. The ultimate strength is seen to be mainly governed by the tensile strength of the shell section.

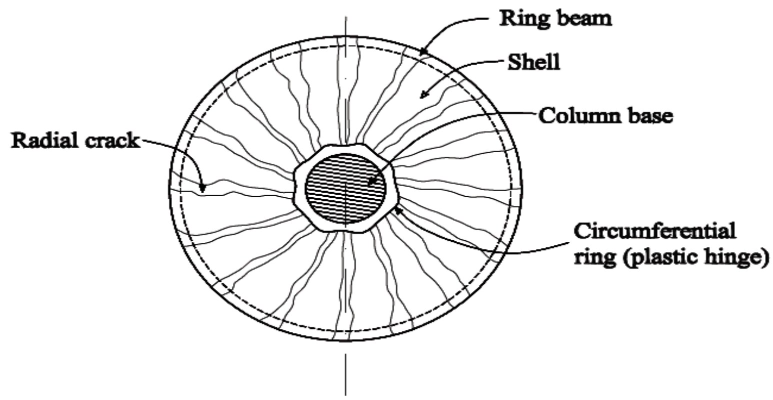


Fig.5.2 Mechanism of Failure of Conical Footing (Kurian,2006)

Based on the above failure mechanism, expressions have been derived for the kinematically satisfactory ultimate normal soil pressure corresponding to the assumed kinematically satisfactory deformation pattern, covering the following cases:

Case 1: The capacity of the shell sections in the circumferential direction, constant, or triangular (i.e., hoop steel is spaced consistent with the variation in hoop tension as per the membrane theory) as shown in Fig.5.3.

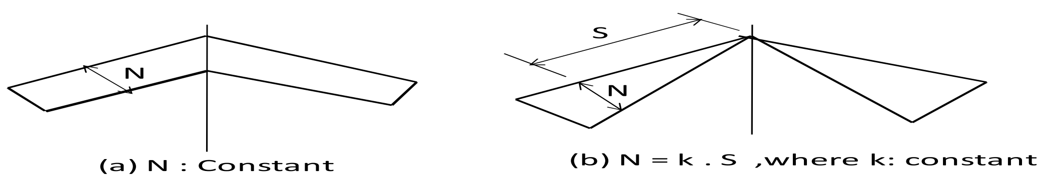


Fig.5.3 Variation Of N

Case 2: The distribution of the normal soil pressure uniform ,or triangular (consistent with its tendency for concentration towards the centre in the ultimate stage) as shown in Fig.5.4.

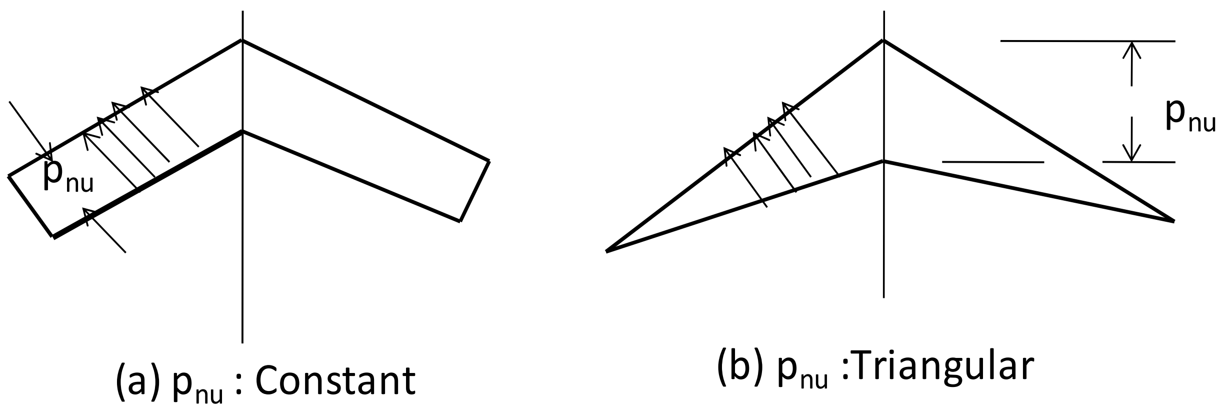


Fig.5.4 Variation of p_{nu}

Case 3: The bottom of the shell free or with edge (ring) beam, as shown in Fig.5.5.

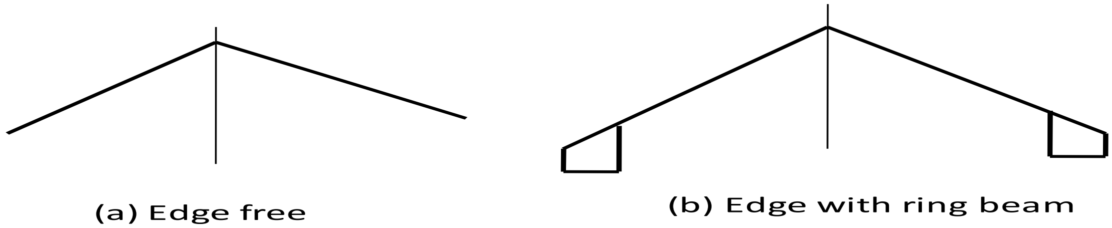


Fig.5.5 Bottom Edge (a) free (b) with ring beam

The following assumptions have been made in the derivation of the expressions for ultimate soil pressure.

1) The full cone is considered ignoring the dimension of the column base. Further the edge beam (bottom ring beam) steel is assumed to be concentrated at the centre of the shell edge.

2) The interaction between tension and circumferential moment along the radial yield lines and between radial moment and compression at the plastic hinge has been neglected.

3) The hoop and radial reinforcements in the shell have no influence on each other.

The expressions for ultimate strength derived by Kurian and Vijayan (1969) covering the cases listed above are given below:

Case 1: Hoop tension triangular, soil pressure uniform, and edge free :

$$p_{nu} = \left[k \cot\alpha + \frac{6M \sin^2\alpha}{r_2^2} \right] \frac{R_o}{R_o^3 - 3R_o + 2} \tag{5.2}$$

$$P_U = p_{nu} * A_p \tag{5.3}$$

It is seen that p_{nu} decreases as α increases or as f decreases. From the calculation of ultimate capacity, it is also found that detailing of steel at varying intensity (spacing) in accordance with the variation in hoop tension given by the membrane theory, is more efficient than spacing the same quantity of steel uniformly.

Case 2: Hoop tension constant, soil pressure uniform, and edge with ring beam :

$$p_{nu} = 6\left[\frac{N\cos\alpha\left(1-R^{2}\right)}{2r_{2}\left(R_{o}^{3}-3R_{o}+2\right)}+\frac{M\sin^{2}\alpha}{r_{2}^{2}}*\frac{R_{o}}{\left(R_{o}^{3}-3R_{o}+2\right)}+\frac{N_{b}\cos\alpha\sin\alpha\left(1-R_{o}\right)}{r_{2}^{2}\left(R_{o}^{3}-3R_{o}+2\right)}\right] \quad (5.4)$$

Case 3: Hoop tension constant, soil pressure triangular, and edge free :

$$p_{nu} = \frac{6}{\left(1-2R_{o}-R_{o}^{4}+R_{o}^{3}\right)}\left[\frac{N\cos\alpha\left(1-Ro2\right)}{r_{2}}+\frac{2M\sin^{2}\alpha*R_{o}}{r_{2}^{2}}\right] \quad (5.5)$$

$$P_{U} = p_{nu}\left[\frac{r_{2}^{3}}{3}\left(2R_{1}^{3}-3R_{1}^{2}+1\right)\right] \quad (5.6)$$

Case 4: Hoop tension triangulate, soil pressure trilingual, and edge free :

$$p_{nu} = \frac{1}{\left(1-2R_o+2R_o^3+R_o^4\right)}\left[2k\cot\alpha\left(R_o^3-3R_o+2\right)+\frac{12MR_o}{r_2^2}\sin^2\alpha\right] \qquad (5.7)$$

Where k is defined such that :

$$N = k * s \qquad \textit{(see Fig. 5.3b)}$$

Case 5: Hoop Tension constant, soil pressure triangular, and edge with ring beam :

$$p_{nu} = \frac{12}{\left(1-2R_o+2R_o^3+R_0^4\right)}\left[\frac{N\cos\alpha\left(1-R_o^2\right)}{2r_2}+\frac{2M\sin^2\alpha * R_o}{r_2^2}+\frac{N_b\left(1-R_o\right)\sin\alpha\cos\alpha}{r_2^2}\right] \qquad (5.8)$$

P_U Under cases 4 and 5 may be obtained by Eq. (5.6).

It may be recalled that the provision of the ring beam at the bottom of the cone is optional, but when provided, it contributes to stiffness at lower rises, delays cracking and contributes to higher ultimate strengths.

We shall now attempt a generalization of the conical case by invoking moment in addition to vertical load. Being an axisymmetric case, we consider the resulting eccentricity e, which applies on the radius along which moments acts.

5.2 Analysis for the ultimate strength of an upright conical shell foundation subjected to vertical load and moment

The method essentially consists of the kinematic analysis of the mechanism of failure of these shell footings idealized from their mode of failure observed from tests conducted on models designed in accordance with the membrane theory, which was found to remain consistent over a wide range of parameters.

This mechanism essentially consists of radial yield lines across which the shell fails in hoop tension and a plastic hinge forming around the column base about which the radial strips rotate (Fig5.6). In the analysis this mechanism is given virtual deformation at the bottom edges allowing them to rotate about the plastic hinge at the top.

The effect of moment in addition to vertical load is accommodated by assigning deformations along the bottom edge consistent with the resultant trapezoidal soil pressure distribution due to vertical load and moment.

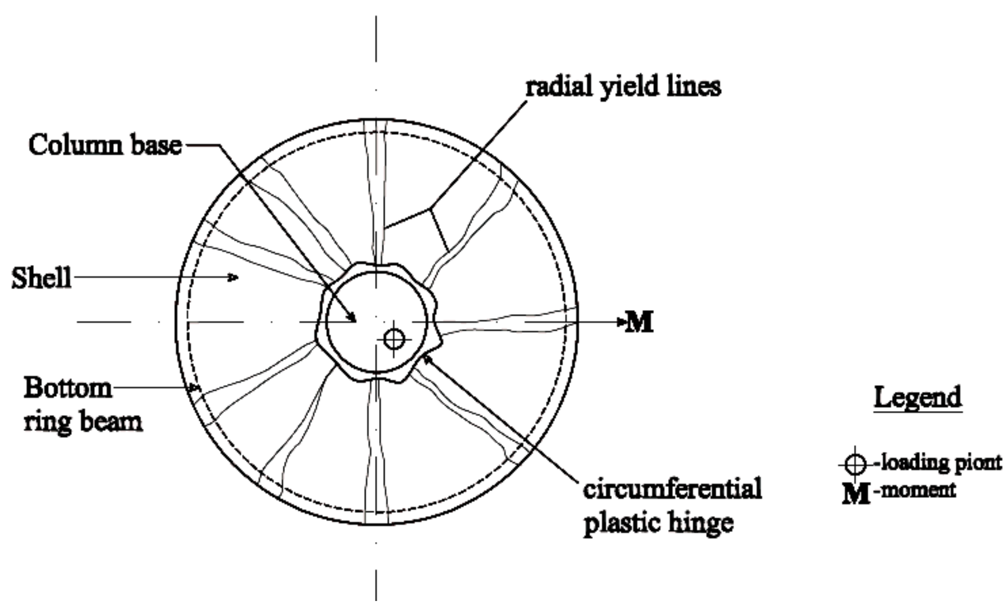


Fig.5.6 Mode of Failure of Conical Footing Under Eccentric Load (Kurian,2006)

Computation of Pressure and deformations :

Referring to Fig .5.7 the resultant pressure at any point is,

$$p_n = p_v = \frac{P}{A} \pm \frac{M * y}{I} \tag{5.9}$$

At point (r, θ), the pressure,(Fig.5.7b)

$$P_A = \frac{p_{un}}{\pi r_2^2} + \frac{p_{un} e r \cos\theta}{\frac{\pi}{4} r_2^2} \tag{5.10}$$

$$P_A = \frac{p_{un}}{\pi r_2^2}\left[1 + \frac{4 e r \cos\theta}{r_2^2}\right] \tag{5.11}$$

Where p_{un} is the ultimate strength of the footing due to normal soil pressure .Since the vertical deformations at the bottom edge are proportional to the resultant soil pressure at the points concerned, if δv is the vertical deformation assigned at point F, the vertical deformation at point B is given by:

$$\delta \upsilon_B = \frac{p_B}{p_F} \delta \upsilon$$

Rotation at point D, (Fig. 5.7c)

$$\delta \psi = \frac{\delta \upsilon}{(r_2 - r_1)}$$

Therefore horizontal deformation at F,

$$\delta h = \delta \psi \; (r_2 - r_1) \cot \alpha = \delta \upsilon \cot \alpha$$

Similarly, rotation at the point M, (Fig. 5.7d)

$$\delta\psi_M = \frac{\delta\upsilon_B}{(r_2 - r_1)} = \frac{p_B}{p_F} \frac{\delta\upsilon}{(r_2 - r_1)}$$

Therefore,

$$\delta\upsilon_A = \delta\psi_M (r - r_1) = \frac{p_B}{p_F} \frac{\delta\upsilon}{(r_2 - r_1)} (r - r_1) \tag{5.12}$$

Similarly,

$$\delta h_A = \delta\psi_M (r - r_1) \cot \alpha$$

$$= \frac{p_B}{p_F} \frac{\delta\upsilon}{(r_2 - r_1)} (r - r_1) \cot \alpha \tag{5.13}$$

Circumferential strain at A,

$$\varepsilon\theta_A = \frac{2\pi (r + \delta h_A) - 2\pi r}{2\pi r} = \frac{\delta h_A}{r}$$

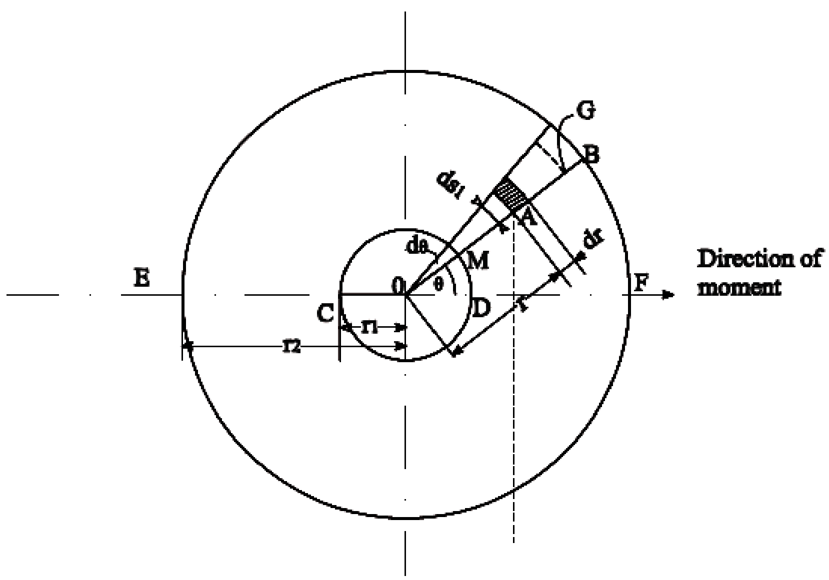
$$= \frac{p_B}{p_F} \frac{\delta\upsilon}{(r_2 - r_1)} (r - r_1) \frac{\cot\alpha}{r} \tag{5.14}$$

The dimensions of the element at A in Fig. 5.7a, and Fig 5.7d respectively are:

$$ds_1 = rd\theta$$

and

$$ds_2 = \frac{dr}{\sin\alpha}$$



(a) Plan showing dimension and co-ordinates

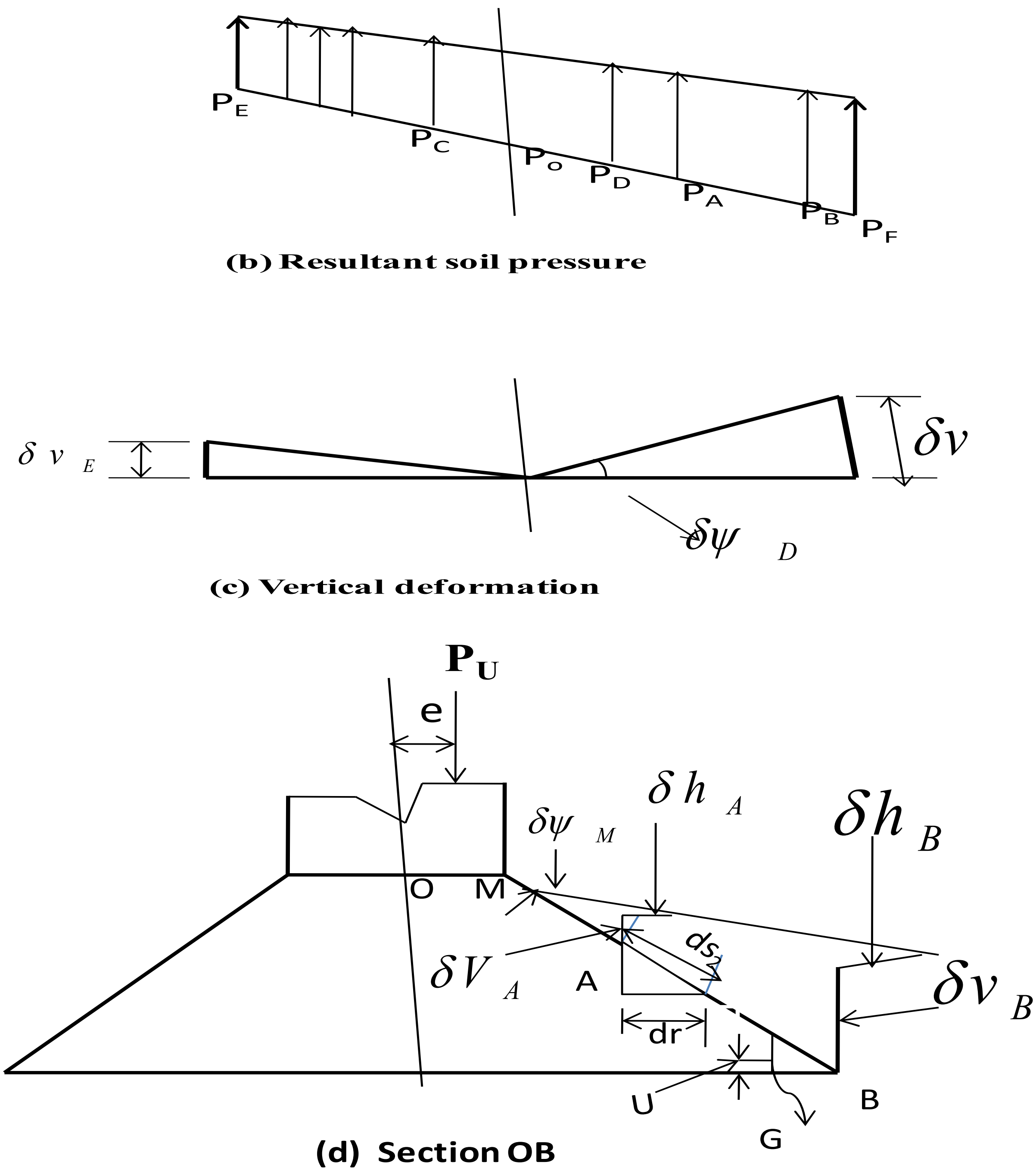


Fig.5.7 Failure mechanism under deformation

Internal work:

The total internal work done by the footing consists of work done by:

1) The varying tensile capacity of the shell sections along the generator in the circumferential direction,
2) The constant flexural capacity of the shell section at the plastic hinge, and

3) The constant tensile capacity of the ring beam in the hoop direction.

The work done by the above are determined individually in the following

(1) In a conical shell footing designed for vertical load and moment, hoop steel is provided against the stress resultants N_θ due to vertical load, N'due to moment and the principal diagonal tension produced by $N'_{s\theta}$ due to moment. Hence in the following sections the work done by the tensile capacity of the shell under each of these effects will be determined.

Designating the tensile capacity of the shell section due to steel provided for N_θ as n_θ we write

$$n_\theta = N_\theta \frac{\sigma_{sy}}{\sigma_{st}}$$

Where σ_{st} is the permissible stress in tension in the steel, and σ_{sy}, its yield strength.

Internal work done by n_θ ,

$$IW n_\theta = \int_{r=r_1}^{r_2} \int_{\theta=0}^{2\pi} (n_\theta ds_2)(\varepsilon_{\theta A} ds_1)$$

Solving the integration we obtain:

$$IW\,n_{\theta} = \frac{2\pi\delta\upsilon}{r_2 - r_1}\left(\frac{p_n}{\sigma_{st}}\sigma_{sy}\right)\frac{1}{6\sin^2\alpha}\left(r_1^3 + 2r_2^3 - 3r_1 r_2^2\right) \tag{5.15}$$

In the same manner, designating the capacity of the section due to N'_{θ} as n'_{θ}, and the same due to the principal tension due to $N'_{s\theta}$ as n''_{θ},we obtain:

$$IW\,n'_{\theta} = \frac{2\pi\delta\upsilon}{p_F\left(r_2 - r_1\right)}\,\frac{p_{un}}{A}\left(\frac{p'_n}{\sigma_{st}}\sigma_{sy}\right)\frac{1}{12r_2\sin^2\alpha}\left(r_1^4 3r_2^4 - 4r_1 r_2^3\right) \tag{5.16}$$

And,

$$IWn''_{\theta} = \frac{2\pi\delta\upsilon}{p_F\left(r_2 - r_1\right)}\,\frac{p_{un}}{A}\left(\frac{p'_n}{\sigma_{st}}\sigma_{sy}\right)\frac{1}{4r_2\sin^3\alpha}\,*$$

$$\left[r_2^4\log_e\left(r_2 - r_1\right) - \frac{1}{4}\left(r_2^4 - r_1^4\right) + r_2^3\left(r_1 - r_2\right) + \frac{r_1}{3}\left(r_2^3 - r_1^3\right)\right] \tag{5.17}$$

(2) The radial steel at the column face is composed of steel provided for N_s ,N'_s and the principal effect to $N'_{s\theta}$.If m is the flexural capacity of the resulting section, the corresponding internal work done is,

$$IW_m = \int_{\theta=0}^{2\pi} m\,\left(rd\,\theta\right)d\psi_M$$

If the total area of steel required due to the above effects is designated as A_{st} per unit width, the flexural capacity of the section can be written as:

$$A_{st}\ \sigma_{sy} L$$

Where L is the lever arm, using the above, one obtains the final capacity as:

$$IW_m = \frac{2\pi\delta\upsilon}{p_F(r_2 - r_1)} \frac{p_{un}}{A} A_{st}\ \sigma_{sy} L\, r_1 \tag{5.18}$$

(3) if A_s is the area of steel provided in the section of the bottom ring beam, which is of arbitrary design, the capacity of the section is,

$$N = A_s\, \sigma_{sy}$$

The corresponding internal work done is,

$$IW_N = \int_{\theta=0}^{2\pi} N\,(\varepsilon\theta_G r_2 d\theta)$$

Where G is the centroid of the ring beam located at a height u above B.(The horizontal distance of G from B is ignored).The final result is obtained as:

$$IW_N = \frac{2\pi\delta\upsilon}{p_F(r_2 - r_1)} \frac{p_{un}}{A} N\,[(r_2 - r_1)\cot\alpha - u] \tag{5.19}$$

External work:

The total external work done by the soil reaction is determined as the sum of the work done by its vertical and horizontal components of identical intensity.

To determine the external work done by vertical soil reaction ,we consider the elemental area at A (Fig.5.7a) ,from which,

$$Ew_v = \int_{r=r_1}^{r_2} \int_{\theta=0}^{2\pi} p_{vA} \left(ds_1 dr\right) \delta\upsilon_A$$

This is reduced as:

$$Ew_v = \frac{2\pi\delta\upsilon}{p_F\left(r_2 - r_1\right)} \left(\frac{p_{un}}{A}\right)^2 \frac{1}{6}\left[\left(r_1^3 + 2r_2^3 - 3r_1 r_2^2\right) + \frac{4e^2}{r_2^3}\left(r_1^4 + 3r_2^4 - 4r_1 r_2^3\right)\right]$$

(5.20)

In the same manner the external work done by the horizontal soil pressure at A (Fig. 5.7d),

$$EW_h = \int_{r=r_1}^{r_2} \int_{\theta=0}^{2\pi} p_{hA} \left(ds_1 ds_2 \cos\alpha\right)$$

This is reduced as:

$$EW_h = \frac{2\pi\delta\upsilon}{p_F\left(r_2 - r_1\right)} \left(\frac{p_{un}}{A}\right)^2 \frac{1}{6} cot^2 \alpha \left[\left(r_1^3 + 2r_2^3 - 3r_1r_2^2\right) + \frac{4e^2}{r_2^3}\left(r_1^4 + 3r_2^4 - 4r_1r_2^3\right)\right] \tag{5.21}$$

Equating the total internal work obtained by adding Eqs.(5.15),(5.16),(5.17).(5.18) and (5.19) and the total of external work obtained by adding Eqs.(5.20),(and(5.21) and simplifying. one gets the following final expression for P_{un} as a function of e.

$$P_{un} = \frac{\left[\pi r_2^2 \left[\begin{array}{l} p_n \dfrac{\sigma_{sy}}{\sigma_{st}} \dfrac{1}{6\sin^2\alpha}\left(r_1^3 + 2r_2^3 - 3r_1r_2^2\right)\left(1 + \dfrac{4e}{r_2}\right) + p'_n \dfrac{\sigma_{sy}}{\sigma_{st}} \dfrac{1}{r_2} \dfrac{1}{12\sin^2\alpha}\left(r_1^4 3r_2^4 - 4r_1r_2^3\right) \\ + p'_n \dfrac{\sigma_{sy}}{\sigma_{st}} \dfrac{1}{r_2} \dfrac{1}{4\sin^3\alpha} \left[\begin{array}{l} r_2^4 \log_e\left(r_2 - r_1\right) - \dfrac{1}{4}\left(r_2^4 - r_1^4\right) + r_2^3\left(r_1 - r_2\right) \\ + \dfrac{r_1}{3}\left(r_2^3 - r_1^3\right) + r_1 A_{st}\sigma_{sy} L + N\left(r_2 - r_1\right)\cot\alpha - u \end{array}\right] \end{array}\right]\right]}{\left[\dfrac{1}{6\sin^2\alpha}\left(r_1^3 + 2r_2^3 - 3r_1r_2^2\right)\dfrac{4e^2}{r_2^3}\left(r_1^4 + 2r_2^4 - 4r_1r_2^3\right)\right]} \tag{5.22}$$

It may be noted that the above expression is capable of giving the ultimate strength of a conical footing designed for one value of eccentricity of load when subjected to load at another value of eccentricity.

The above being the most general case, expression pertaining to simpler cases can be obtained by suitable substitutions in the above result. For example, the ultimate strength of a footing designed for and subjected to, central vertical load only and with out ring beam, is obtained as:

$$P_{un} = \frac{\left[\pi r_2^2 \left[p_n \frac{\sigma_{sy}}{\sigma_{st}} \frac{1}{6\sin^2\alpha} \left(r_1^3 + 2r_2^3 - 3r_1 r_2^2\right) + \left[r_1 A_{st} \sigma_{sy} L \right]\right]\right]}{\left[\frac{1}{6\sin^2\alpha} \left(r_1^3 + 2r_2^3 - 3r_1 r_2^2\right)\right]}$$

(5.23)

On the other had , if one wants to determine the ultimate strength of a centrally load conical footing designed for and subjected to vertical soil pressure, the external work done is modified only to the extent of deletion of the external work done by the horizontal component of soil pressure.

However,as far as the internal work is concerned , even though the contributions from the plastic hinge retains the same form as earlier ,$IW_{n\theta}$ differs on account of the difference between N_θ due to normal and vertical soil pressures, which affects the design.
The final results is,

$$P_{un} = \frac{\left[\pi r_2^2 \left[p_{nv} \frac{\sigma_{sy}}{\sigma_{st}} \frac{1}{6} \left(r_1^3 + 2r_2^3 - 3r_1 r_2^2\right) + \left[r_1 A_{st} \sigma_{sy} L \right]\right]\right]}{\left[\frac{1}{6} \left(r_1^3 + 2r_2^3 - 3r_1 r_2^2\right)\right]}$$

(5.24)

Where, p_{uv} stands for ultimate strength due to vertical soil pressure.

It is seen from Eqs.(5.23) and (5.24) that if the contribution of the hoop tensile capacity of the shell section alone is considered $P_{un} = P_{uv}$. On the other hand, if the design of the footing is for the same soil pressure conditions one obtains the following result,

$$\frac{p_{uv}}{p_{un}} = \frac{1}{\sin^2\alpha} \qquad (5.25)$$

Since external work done due to vertical soil pressure is less than that due to normal soil pressure, it is obvious that ultimate strength under vertical soil pressure will be higher than under normal soil pressure. The difference between the two is a function of the slope of the footing. The higher the value of α, the lesser the work done by horizontal pressure and the lesser the difference between the two ultimate strengths, the limit being α =90^{0,} where the difference vanishes.

The ratio $\frac{p_{uv}}{p_{un}}$ is shown plotted against α in Fig.5.8.

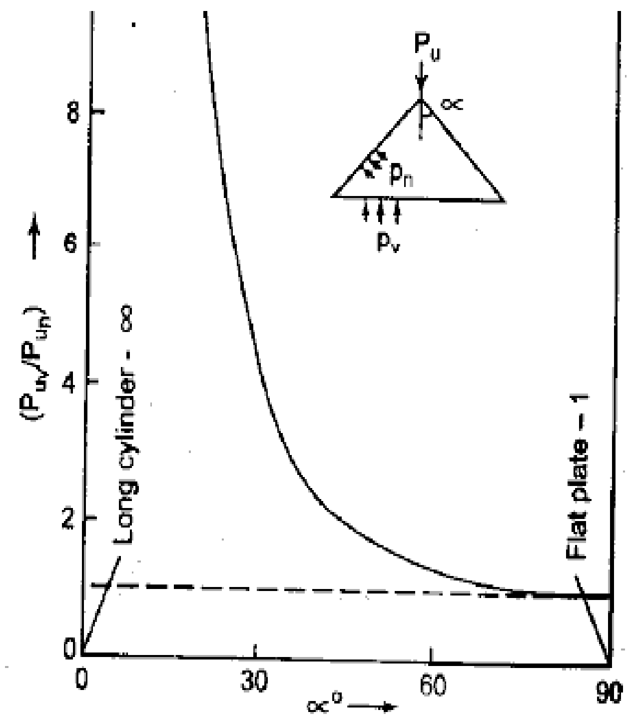


Fig.5.8 Variation of $\frac{p_{uv}}{p_{un}}$ with α

6.0 Finite element analysis of conical shell foundation

6.1 Finite element model

The Conical shell footing and the soil were modeled and analyzed using the finite element soft ware PLAXIS. The program 'PLAXIS' uses the incremental tangent stiffness approach in the analysis, in which the load is divided into a number of small increments, which are applied simultaneously. During each load increment, the stiffness properties appropriate for the current stress level are employed in the numerical analysis.

For comparison purpose two types of footing models: Conical shell and plain circular footings are selected for the analysis as shown in Fig. 6.1. Foundation composed of locally available red clay soil ,whose shear strength parameters were obtained from laboratory CU triaxial tests have been used in the analysis as shown on Table 6.1.

The Mohr-Coulomb model have been used to represent the behavior of the soil in the finite element analysis.The model involves fifve parameters ,namely young's modulus,E, poisson's ratio, ν ,the cohesion,c,the friction angle, φ ,and the dilatancy angle, ψ .

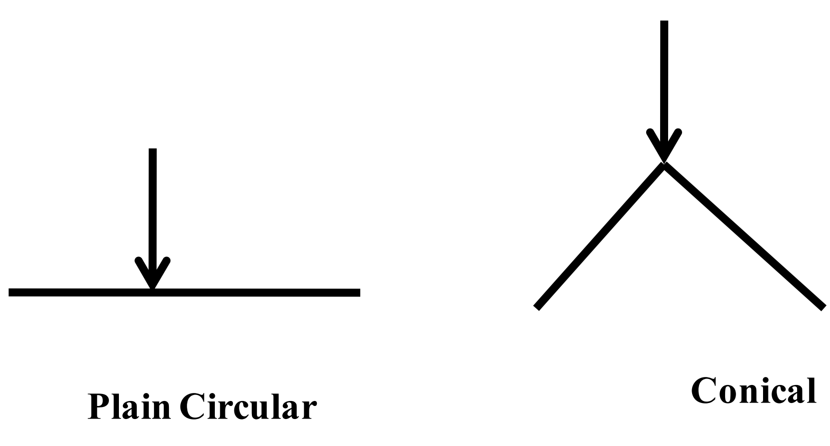


Fig. 6.1 Conical and Plain circular footing models used for the analysis

Table 6.1 Soil properties of the Finite Element model

Properties	Type	Unit	Value
Unsaturated unit weight	γ_{unsat}	KN/M^3	18
Saturated unit weight	γ_{sat}	KN/M^3	19
Poisson Ratio	ν		0.3
Cohesion	c	KN/M^2	15
Young's modulus	E	KN/M^2	4200
Friction angle	ϕ	$(^{O})$	24
Dilatancy angle	φ	$(^{0})$	0
Material Model	Mohr -Coulomb		

The geometry of the mesh for plain strain condition is symmetrical about the centerline, therefore only one half of the cross section passing through the axis of symmetry of the footing is considered.

The soil and the footing were modeled using 15-noded triangular element.Smaller size element, for the soil were selected in the vicinity of the footing where the variations of stresses and strains are expected to be more significant.

Figure 6.2 shows the typical generated mesh for plain circular and conical shell footing.

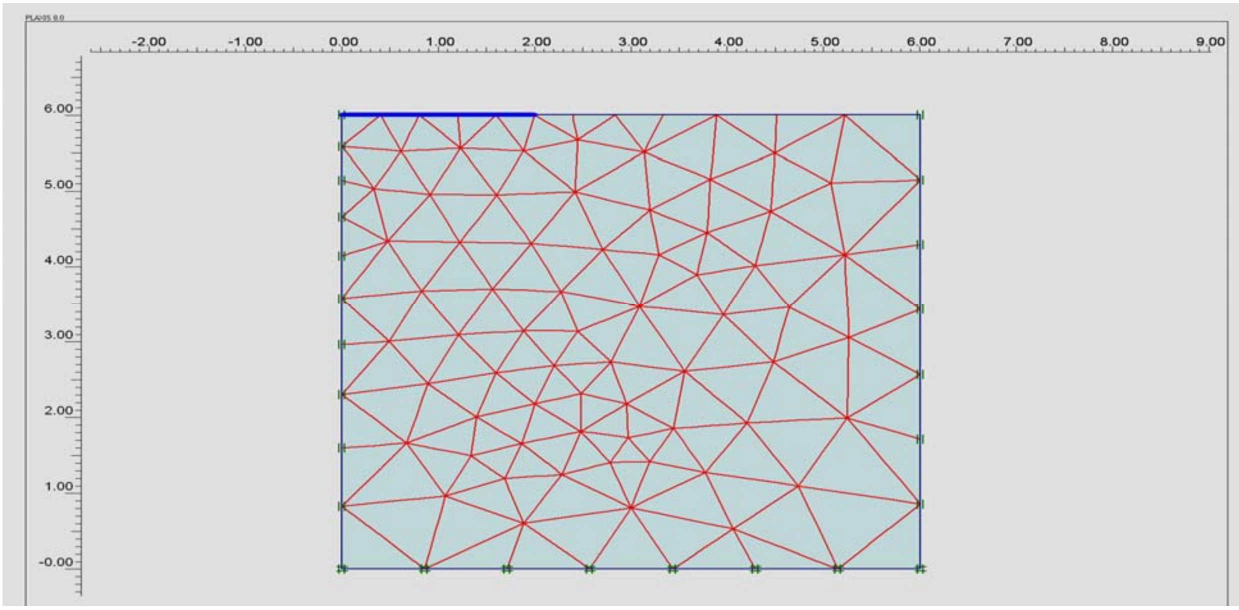


Fig .6.2a Generated mesh for plain circular footing

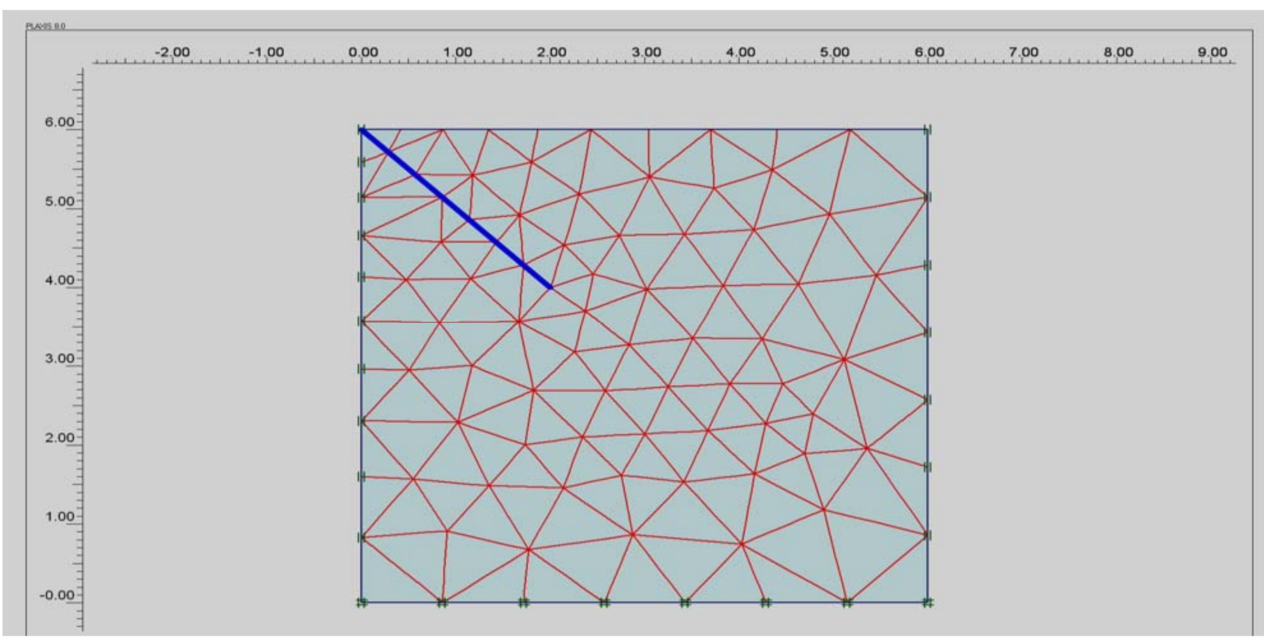


Fig .6.2b Generated mesh for conical footing

Figure 6.3 and 6.4 shows typical deformed mesh and vertical displacement pattern for conical shell footing respectively.

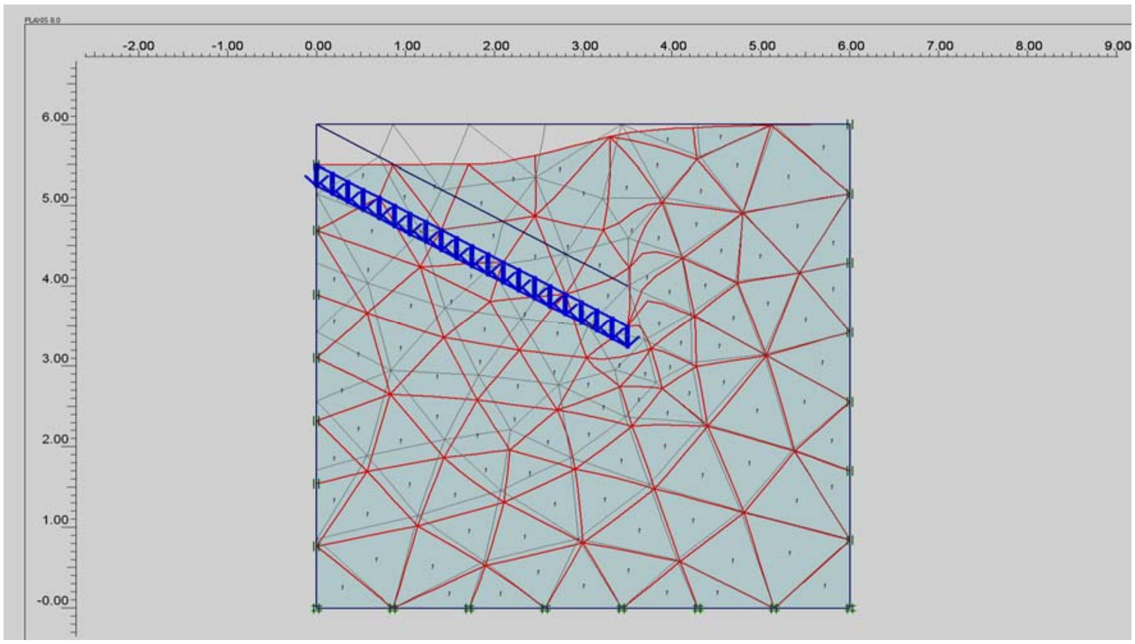


Fig .6.3 Deformed mesh for conical footing

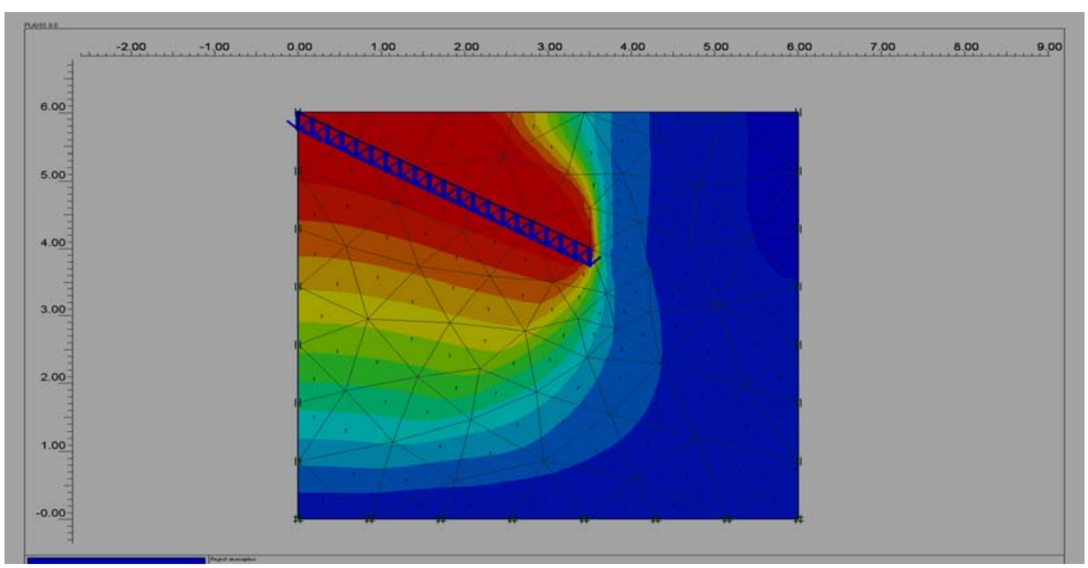


Fig.6.4 Vertical displacement pattern for conical footing

The results of the finite element analysis for circular and conical shell footings are given on table 6.2.

Table 6.2 Finite element analysis result –Ultimate load and vertical displacement

	Ultimate Load (KN)		Ultimate load
Displacement (m)	Circular	Conical shell	Difference (KN)
0.00	0	0	0
0.01	148.01	263.24	115.23
0.02	409.82	755.60	345.78
0.03	856.38	987.01	130.63
0.04	1245.33	1356.82	111.49
0.05	1386.90	1496.56	109.65

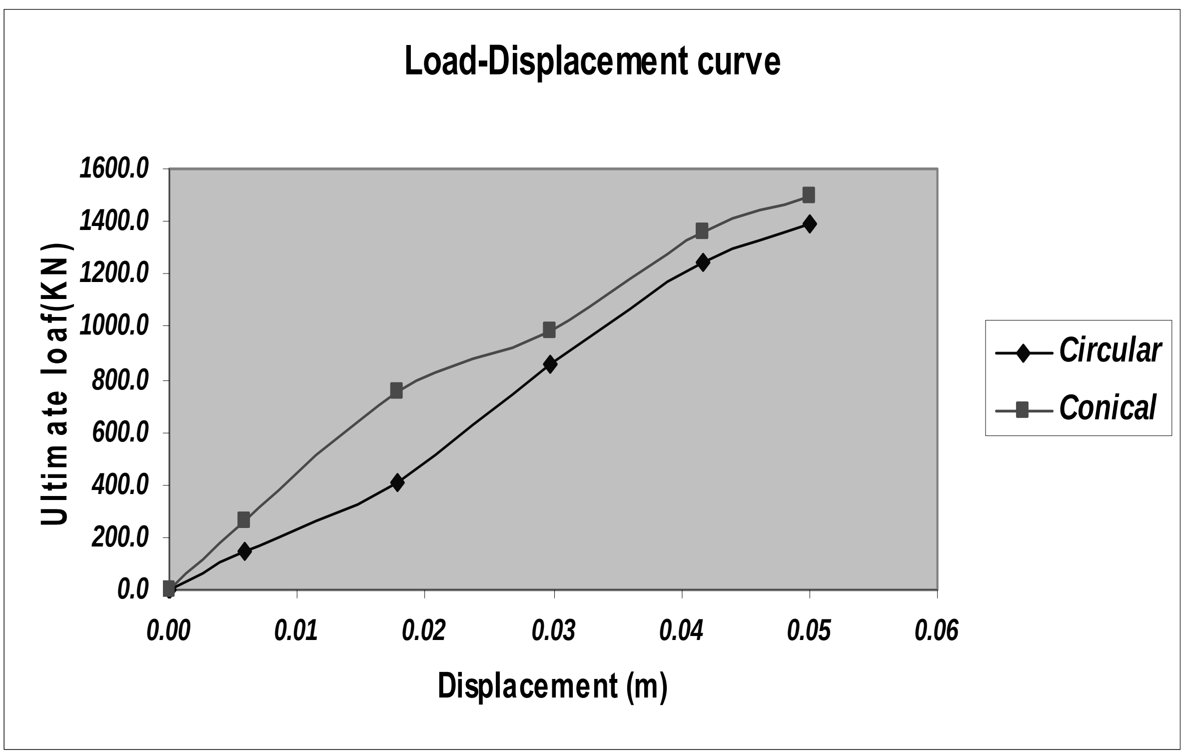


Fig.6.5 Load displacement curve for conical and circular footing

The results of the finite element model clearly indicates that the load carrying capacity of conical shell footing is higher than plain circular footing under same soil properties. The conical shell footings shows better settlement characteristic under the same ultimate load.

The finite element analysis have been further carried out to study the effect of the cone slope angle on the load carring capacity of conical shell foundation.Three types of conical shell foundation have been used for the analysis. The data used is as shown on table 6.3.

Table 6.3 Cone footing parameters used for the finite element analysis

	Conical type 1	Conical type 2	Conica type 3
Cone angle ,α	30	45	60
Cone rise ,f (m)	2.0	2	2.0
Radius ,r (m)	1.15	2	3.5

The results of the analysis is as shown in Fig.6.6 .

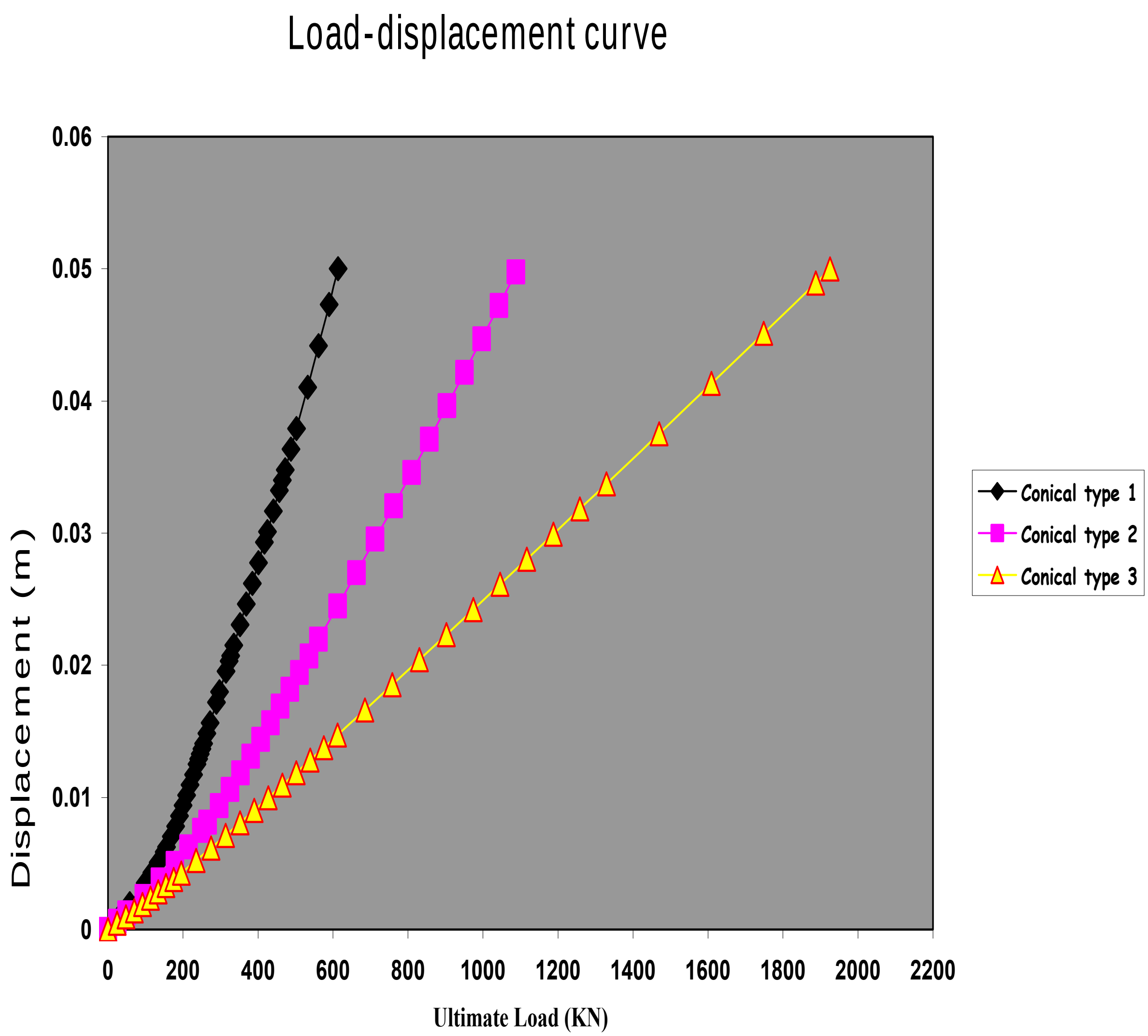


Fig. 6.6 Comparison curves for three types of conical footing.

From the results of the finite element analysis ,the cone footing with cone slope angle of 60^0 was found carrying much more load as compared to the other two types of conical shells.This indicates that flater slopes gives the conical shell to be more stable,besides the advantage of pouring concrete during insitu construction.

7.0 Structural design of conical shell foundations

7.1 Introduction

The structural design of conical shell foundation by the Limit state Method is essentially based on the membrane theory, but we shall make use of the ultimate theory to compute the ultimate strength of our designs, wherever possible, to be able to estimate the load factors involved in the design.

Our approach to design, based on the membrane theory, should be deemed as conventional ,since we assume the soil reaction to be uniformly varying depending upon whether the resultant of the applied system of loads and moments is concentric or eccentric with respect to the centroid of the plan of the contact between the foundation and the soil. As far as the direction of the resultant soil pressure is concerned, we shall take to be normal under cases like soft clay where the existence of tangential components is doubtful due to the low contact friction mobilized, and vertical ,where we are certain of the tangential component of substantial magnitudes.

However the best way would be to design for the condition leading to the higher values of stress resultants, which is generally the former. From the point of view of membrane theory, it is preferable to adopt the higher rise-to-base ratios of the shell, but this should be limited to values which,nevertheless,should not pose any special problem of construction. For example if the foundation is too steep ,concreting the shell will be difficult with out shuttering at top.

7.2 Limit state design of conical shell foundation

Like any reinforced concrete structure ,limit state design of reinforced concrete shell foundations must be carried out for factored loads obtained by multiplying characteristic loads by appropriate safety factors, the latter depending upon the type of load and the limit state being considered. The membrane stress resultants needed for membrane design are determined from these factored loads. The properties of the materials (concrete and steel) needed for design called design strengths are obtained by dividing the characteristic strengths of the material by appropriate safety factors.

The limit state design envisages that the design satisfies the limit state of collapse as well as serviceability. Collapse in the membrane modes involves direct compression or tension. The serviceability limit state applies to deflection and cracking the latter being of special importance to foundation, for chemically aggressive ground water and the consequent chances of corrosion of reinforcement and disruption of concrete.

As regards to compression, whether in the shell section or in an edge beam, the following short column formula can be used (Kurian, 2006).

$$C_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_s \tag{7.1}$$

Where,

C_u; is the design compression force, the subscript indicating that is the ultimate or limit state value,

f_{ck}; the characteristic strength of concrete,

f_y; the yield strength of steel,

A_c; the area of concrete (alone), and,

A_s; the area of steel.

If, on the other hand ,we designate A as the gross cross section, and specify the percentage of steel as ρ,Eq. (7.1) can be restated as:

$$C_u = A\left(1 - \frac{\rho}{100}\right) 0.4\ f_{ck} + A\,.\frac{\rho}{100}\ 0.67\ f_y \tag{7.2}$$

Eq.(7.2) can give the section A and the area of steel.

As far as tension is concerned, the full tension is to be taken by steel ,whether the deign is by limit state or working stress method, concrete serving as a protective cover for the steel. This however causes a possibility of cracking of the surrounding concrete, particularly if the width of cracks are likely to exceed acceptable limits. This exigency is overcome in working stress design for members in direct tension, which places a limit on the tensile stress in the equivalent concrete section. The corresponding compression is,

$$\frac{T}{A_c + mA_s} = \sigma_t \tag{7.3}$$

Where ,

T; is the tension acting on the section ,and

σ_t ; the value of permissible (limiting) tensile stress,

m ; the modular ratio given as $\frac{280}{3\sigma_{cbc}}$,

where,

σ_{cbc} is the permissible compressive stress in concrete in bending. In terms of the gross section A Eq.(7.3) can be restated as:

$$\frac{T}{A + (m-1)A_s} = \sigma_t \tag{7.4}$$

Membrane design would prefer the steel in the shell to be detailed in the middle plane. When it comes to placing steel in perpendicular direction, the right practice would be to place them on either side of the centerline. In so doing it would be preferable, where bending is anticipated, to place one layer on side of tension to take advantage of increased lever arm depending up on the direction of bending. The steel detailed in both the shell and beam should insure adequate cover (on all sides) and spacing as per the relevant codes.

From the point of view of cover, we shall keep the minimum thickness of the shell as150 mm in in-situ construction and 120 mm for precast work. We shall also maintain the percentage of steel between the limits of 0.5% (minimum) and 5% (maximum) ,whether it is in the shell or beam for practical purposes (Kurian, 2006).

7.3 Essential steps in the design of conical shell foundation

The shell of the conical footing is to be designed for the meridonal compression and hoop tension developing due to a uniform soil reaction assumed to act in the vertical or normal direction, depending upon the type of soil on which the footing rests.

The first essential step given by Kurian (2006) in the design of conical shell footing is the fixing the center-line triangle The dimensions of the column determine the position the cone is truncated, from which we know the value of s_1 as shown in (Fig.7.1).

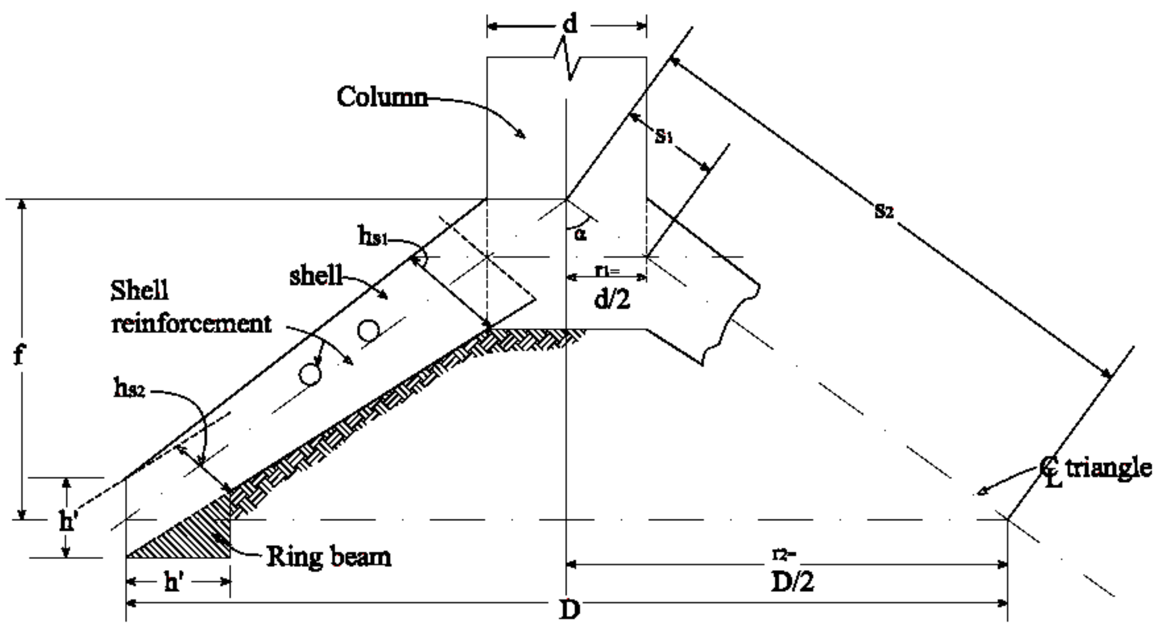


Fig. 7.1 Dimensioning of the Conical Footing (Kurian,2006).

1. Meridian compression

Since the meridian compression is highest at s_1,the thickness of the shell at this point should be sufficient to resist this compression at a maximum of 5% steel, but not less than 150mm or 120mm depending upon whether the construction is in-situ,or precast. Since the meridonal compression N_s vanishes at the base, both concrete and steel can be theoretically nil there, and hence the minimum thickness of 150 mm or 120 mm may be provided at this point from the practical point of view. The steel designed at the top may be fanned out radially till the bottom, but because of the very fast reduction in N_s with s ,curtailment of the radial steel can be effected after ensuring that all sections are provided with the minimum nominal steel of 0.5% of the gross section.

2.Hoop tension

The hoop tension is to be fully resisted by steel placed in the form of rings in the circumferential direction. Since the hoop tension is maximum at the base, and linearly decreases to a minimum at s_1 ,the spacing of the hoop steel may be determined at these points, and uniformly varied along the generator, subject to the requirement regarding nominal steel, and after ensuring that there are enough hoops to tie the radial steel

.

The minimum thickness of concrete at the top and bottom for hoop tension, should be taken as the maximum of:

a) The thickness satisfying the requirement regarding tension in the equivalent concrete section as per Eq.(7.4).

b) Thickness at which the hoop steel does not exceed 5% of the section, and

c) 150 mm or 120 mm,depending upon the method of construction.

The design value of h at s_1 and s_2 is the higher of the values determined on the basis of meridonal compression and hoop tension, as above, at the corresponding section. The footing may now be dimensioned in the following manner.

- Set out the thickness h_{s1} and h_{s2} obtained as above centrally along perpendiculars drawn to the center line at s_1 and s_2 .

- Vary the thickness uniformly between s_1 and s_2.

- Drop verticals at s_1 and s_2 and finalize the profile of the footing.

- A triangular ring beam may be provided inwards so that the plan of the loaded area of the foundation (i.e., soil reaction) remains intact as a circle of diameter D, as per the geotechnical design.

7.4 Design examples

A design examples shown on APPENDIX A-1 and APPENDIX A-2 have been worked out to illustrate how one can approach the design of Conical shell foundation. In the previous chapters the fundamental principles and analysis of the forces involved on conical shell foundations have been discussed .

In these examples we will use the basic principles and analysis for designing purposes. These design examples have also been used for comparison purposes of materials required for plain circular footing and conical shell footing.

Terzaghi's bearing capacity factors and soil properties from a thesis submitted by T.Samuel (1989) "Investigation in to some of the engineering properties of Addis Ababa clay red soils" has been used for computing bearing capacity . The results are presented in tabular form as shown on Table 7.1.

The bearing capacity is obtained using the following analytical equation .

$$Q_{ult} = CN_c S_c + qN_q S_q + \frac{1}{2} \gamma B N_\gamma S_\gamma \qquad (7.5)$$

$$q_{all} = \frac{q_{ult}}{F.S}$$

A factor of safety F.S =3 was used for computation of allowable bearing capacity.

Table 7.1 Soil properties and bearing capacity used for design

Foundation depth (m)	Diameter of footing (m)	Soil unit weight (KN/M^3)	c	ϕ	Allowable bearing capacity (KN/M^2)
2	4..5	18	15	24	315

The soil properties indicated on Table 7.1 have been used on the design examples on APPEDIX A-1 and APPENDIX A-2. The critical section for punching and wide beam shear for the design of circular footings (Fig. 7.2) is the same as square or rectangular footings. One can employs the following formulas for the design of circular footing.

Moment :

$$M_U = q_U * \frac{\pi}{3} \left(2r_2^3 + r_1 - 3r_1 r_2^2\right) \qquad (7.6)$$

Depth from punching shear (V_{rd}) , $\frac{d}{2}$ from the face of column:

$$\pi * \left(r + \frac{d}{2}\right) d * V_{rd} = q_U * \left(r_1 + \frac{d}{2}\right)^2 \qquad (7.7)$$

Wide beam shear (V_w) ,d from the face of column:

$$2*\pi\left(r_1 + d\right)*d*V_W = \pi*q_U\left[r_2^2-\left(r_1+d\right)^2\right] \tag{7.9}$$

Where,

M_U ; is the bending moment

q_U ;is soil contact pressure

r_1; is the radius of the column

r_2; is the radius of the footing

d; is the effective depth of the footing

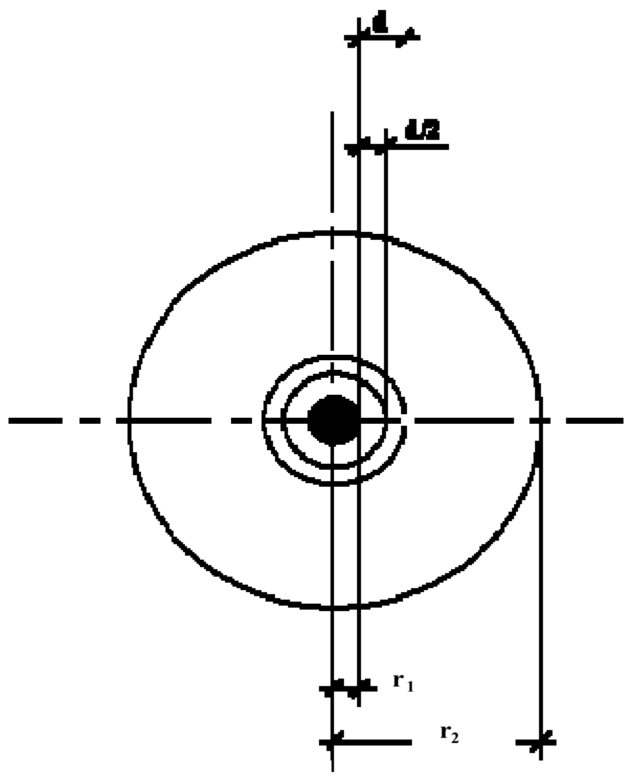


Fig. 7.2 Critical Section for Circular footing design

The results of the design examples are summarized in terms of steel area and concrete volume which are the basic that affects the cost of the project.

Table 7.2 Summary of steel area and concrete volume

Steel area ,As (mm^2)		Concrete volume (m3)	
Cone	Circular	Cone	Circular
5312.64	*5800.18*	2.14	5.17

The examples have been worked out for the purpose of showing how one can approach the design of conical shell foundations and to indicate how shells can be used as an alternative material saving technique in the design of foundations. Therefore the main advantage of using shell as a structural foundation is to minimize the cost of materials.

The construction of shell foundation might be facilitated by making the slope of the cone flatter so that pouring of concrete will be easy. Making use of precast concrete shell foundation is an alternative under different site conditions to minimize some difficulties during in-situ construction.

8.0 Conclusion and Recommendation

1. This study introduces cone,inverted dome,hyperbolic paraboloid,folded plate,and combined shell foundations that could be employed as a structural foundation as an alternative to plain foundations with further developments in the fields of this study in Ethiopia

2. The Finite element analysis shows that conical shells have high load carrying capacity than circular footings under the same soil properties.

3. The results of this study show that conical shell foundations are found out to use less amount of material in comparison to plain circular footings. The advantage in material saving makes conical shells preferable than circular footings.

4. The construction of conical shell foundation can be facilitated by using flatter slopes or by using precast foundation shells under different site conditions.

Recommendations

From the results of this study, the following points are recommended.

1. This study introduces the design and advantage of conical shell foundation with the objective of introducing as an alternative foundation in Ethiopia, therefore it is recommended that more detail research is required before practical implementations.

2. Because of the geometry of the footing the soil interaction is very complex ,therefore detailed investigation using model shell footings is required in order to analyze the failure mechanism and settlement under different soil condition.

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APPENDIX A-1

Structural Design of Conical Shell Footing

Data

Colunmn load for structural desin , P=1000KN

Diameter of the cone ,D = 4.5m

Diameter of the column base,r_1 = 400 mm

Soil reaction assumed to be normal and uniform

Concrete and Steel

Concrete : C-25

Steel : S 300

$p_n = p_v = P/A_p \quad = 62.91 KN/m^2$

Where,A_p=the projected area of the cone

The center line is fixed with $f/r_2 = 1/2$

$f = 1/2x2.25 \qquad =1.125m$

$s_1 = \sqrt{0.2^2 + 0.1^2} = 0.224m$

$s_1 = \sqrt{0.2^2 + 0.1^2} = 2.516m$

$\tan \alpha = 2$

$\alpha = 63.435$

$\sin \alpha = 0.894$

$\cos \alpha = 0.448$

Design of meridonal compression:

At $s = s_1$

$$N_s = \frac{p_n \tan\alpha(s_2^2 - s_1^2)}{2s}$$

Ns =62.91x2_(2.516^2 -0.224^2)/(2x0.224)=1763.75KN/m =(N/mm)

Thickness h at 5% steel is obtained from:

0.95h x 0.4 x 20 +.67 x415 x0.05h =1763.75, from which

h =82.0 mm, raise to 120 mm for adequate, cover

Area os steel required ,A_{s1} =82x.05 =4100 mm^2/m

Providing Φ 12mm bars

Spacing =113.04/4.1=27.6,use 50 mm at the circle s_1

No of bars at the circle s_1 =π x400 /50=25.12,say 26

As1 provided =26x113.04/π x400 =2340.00 mm^2 /m

Percentage area of steel =2.34/140 x100 =1.67, O.K.

Design of hoop tension:

At $s = s_2$

Nθ =62.91 x2.516 x2 =316.50KN/m =(N/mm)

*Area of steel required A_{s2}=*316.5/.87x300=1212.641 mm^2/m

Providing Φ12 mm bars, as=113.04

Spacing =113.04/1.213 =93.2,say 90mm

As_2 provided =113.04/90=1256.000>1212.64mm^2/m

At $s = s_1$

Nθ =62.91 x0.224 x2 =28.13KN/m =(N/mm)

*Area of steel required A_{s3}=*28.13/.87x300=107.79mm^2/m

Providing Φ12mm bars, as=113.04

Spacing =113.04/.108 =1048.7,say 1050mm

From practical considerations ,use spacing of 250 mm

As_3 provided =113.04/250=452.16mm^2/m

Ultimate Load

N, assuming the average spacing of 200 mm of the hoop steel to be constant

$$N=\frac{113.04}{200x300} =169.56KN/m =(N/mm)$$

$Nb=314.2x300 =94.26KN$

Column radius $r_1 =0.2$ m *Footing radius* $,r_2 =2.25$ m

Plan area of the cone$=\pi$ x $(4.5/2)^2 15.90$ m^2 $R_o=r_1/r_2 =0.089$

Hoop tension:

$$Pu \text{ (by Equ, 5.4)} = \frac{15.90x6[169.56x0.448(1.089)^2]}{2x2.25(0.089^3-3x0.089+2)} =917.83KN$$

Meridional flexure:

$$\frac{15.90x6[17.659x0.894^2x.089)}{2.25^2 x (0.089^3-3x0.089+2)}= 13.66 KN$$

Ring beam tension:

$$\frac{15.90x6[94.26x0.447x0.894(1-.089)]}{2.25^2 x (0.089^3-3x0.089+2)} =373.13KN$$

P_u. =917.83+13.66+373.13 =1304.62KN

P_u. > P, 1304.62 > 1000 KN O.K.

APPENDIX A-2

Structural Design of Circular Footing

Take rho = r_{min} = 0.002

(fyk) S = 300 *fyd=fyk/1.15=260.87*

(fc) C = 25 *fck=0.8*c=20.00*

fctk=0.21(fck)^(2/3)= 1.547* *fcd=0.85fck/1.5=11.33*

f_{ctd} = fctk/1.5= 1.0315

Allowable bearing pressure, (Kpa)= 315

Foundation depth, f_d =(m) 2.00

Unit weight of soil, γ_s (KN/m^3) 18.00

Column diameter, r (m) 0.40

Super-structure load, P_s (KN) 1000.00

Footing Dimension, Diameter (m) 4.50,radius of footing = 2.25 m

Footing Area, A(m^2) 15.90

Maximum stress from load,qmax = P/A = 62.91KN/M^2

If qmax<qallow , the diameter provided (D) is enough!

Depth determination using punching shear:

Assumed bar diameter: 12mm a_s =113.04mm^2

d(mm)= 300.00 , u=1.57

k_1=(1+50r)$\leq$2= 1.1 k2=1.6-d $\geq$1=1.30

*Punching shear resistance(KN): Vrd= 0.25*fctd*k1*k2*u*d =173.91 KN*

*Acting Punching Shear(KN): Vp=q*3.14*((r/2+d)^2)=49.38 KN*

, the depth considered is SAFE!

Check depth for wide beam shear:

bw=3.78

*Wide beam shear resistance(KN): Vw=0.25*fctd*k1*k2*L*d = 347.38 KN*

*Acting wide beam shear(KN): Va=q*3.14/4*(R^2-(r+d)^2) =237.65 KN*

,the depth considered is SAFE!

Use ,depth of footing=360 mm

Reinforcement calculation:

*$A_{s,min}$=0.5*b*d/fyk = 2250.00 mm^2*

*M_{sd}(KN.m)=qmax*1.05(2R^2+r^3-3rR^2) =217.89KN-M*

A_s=5800.18 mm^2

$A_{s,provided}$=5800.18 mm^2

spacing : dia.12mm c/c 200 mm

Use, dia. 12 mm c/c 200mm

DECLARATION

I, The undersigned, declare that this thesis is my original work performed under the supervision of my research advisor Dr. Hadush Seged and has not been presented as a thesis for a degree in any other university, All sources of materials used for this thesis have also been acknowledged.

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