ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOGY

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

ANALYTICAL AND FINITE ELEMENT ANALYSIS OF RAIL DEFLECTION AND BENDING STRESS UNDER DIFFERENT SLEEPER SPACING

A THESIS IN RAILWAY ENGINEERING, CIVIL STREAM.

BY LEMMI GURMESSA TESSO

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A Thesis

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science

The undersigned have examined the thesis entitled 'ANALYTICAL AND FINITE ELEMENT ANALYSIS OF RAIL DEFLECTION AND BENDING STRESS UNDER DIFFERENT SLEEPER SPACING' presented by **Lemmi Gurmessa**, a candidate for the degree of **Master of Science** and hereby certify that it is worthy of acceptance.

Abdulsetar Siraj	________________	__________
Advisor	Signature	Date
Matias Kabtamu	________________	__________
Internal Examiner	Signature	Date
Mequanent Mulugeta	________________	__________
External Examiner	Signature	Date
Dr. Birhanu Besha	________________	__________
Chair person	Signature	Date

UNDERTAKING

I certify that research work titled "ANALYTICAL AND FINITE ELEMENT ANALYSIS OF RAIL DEFLECTION AND BENDING STRESS UNDER DIFFERENT SLEEPER SPACING" is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources it has been properly acknowledged / referred.

Lemmi Gurmessa

ABSTRACT

The amount of wheel load that distributed through each sleeper depends on the track bed quality, sleeper type and sleeper space. The relationship between applied loads, track stresses, and track deformations are factors to be considered in proper track design and maintenance. Any defects and changes in track components will produce defects in rails in turn. Among these results rails bending stress, rails deflection and rails support loads are from the most parameters to be considered during the design of rail track.

This research has investigated the effects of sleeper spacing, particularly the rails deflection, the induced rail bending stresses and the rail supporting force under several concentrated (Wheel) moving loads and single wheel load. Finite element method model and beam on elastic foundation (BOEF) track model have been used to get numerical and analytical solutions respectively. The Finite element method analysis was carried out in ANSYS work bench. Results from both analyses were analyzed by quantitative approach and compared together. Train axle arrangements and wheel static load have been taken from rolling stock proposed for Sabata- Mieso-Djibute Line which is identified as "HXD1C high power AC drive six-axle (7200kw) freight electric locomotive".

The Analysis shows numerical analysis gives higher values of rail deflection; lower values of rail supporting force and rail bending stress than analytical analysis. On the other hand the analytical analysis shows that multiple wheel loads gives higher values of rail deflection and rail supporting force but lower values of rail bending stress than single wheel load while numerical analysis shows single wheel load gives higher values (rail deflection, rail supporting force and rail bending stress) than multiple wheel loads. This one occurred due to the fact that the effect of one axle load gives positive value while the other gives negative values. The summation of the positive and negative values will be lower than the single positive value obtained from single wheel load. The differences of these values depend on the sleeper space and rail track bed quality.

Key Words: - Rail deflection, Rail bending stress, Rail supporting force, Sleeper space, Beam on elastic foundation (BOEF), Finite element method (FEM)

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ABBREVIATIONS AND NOTATIONS

ABBREVIATIONS

AAIT	Addis Ababa Institute of Technology
AALRT	Addis Ababa Light Rail Transport
BOEF	Beam On Elastic Foundation
D	Wheel diameter (mm)
EF	Elastic foundation
ERC	Ethiopian Railway Corporation
FEA	Finite Element Analysis
FEM	Finite Element Method
M _ i	Model _ i
WMATA	Washington Metropolitan Transit Authority

NOTATIONS

$\boldsymbol{\alpha}$	Coefficients for deflection
μ	Track modulus(the stiffness of the spring per unit length of track)
a	Sleeper spacing (m)
C	Ballast coefficient (N/m^3)
E_b	Modules Elasticity
EI	Beam bending stiffness
γ	Coefficients for rail support force

K	Track stiffness
ø	Dynamic impact factor
P or Po	Static wheel load
Pd	Dynamic wheel load
β	Coefficients for bending stress
θ	Friction angle of ballast/sub ballast

CHAPTER 1 INTRODUCTION

1.1 Background

The analysis of bending of beams on an elastic foundation is developed on the assumption that the reaction forces of the foundation are proportional at every point to the deflection of the beam at that point. The vertical deformation characteristics of the foundation are defined by means of identical, independent, closely spaced, discrete and linearly elastic springs. The constant of proportionality of these springs is known as the modulus of sub-grade reaction. This simple and relatively crude mechanical representation of soil foundation was firstly introduced by Winkler, in 1867[1] [2], [3].

The Winkler model, which has been originally developed for the analysis of railroad tracks, is very simple but does not accurately represents the characteristics of many practical foundations. One of the most important deficiencies of the Winkler model is that a displacement discontinuity appears between the loaded and the unloaded part of the foundation surface. In reality, the soil surface does not show any discontinuity (Fig. 1-1). Winkler's theory are categorized as one-dimensional analysis of a railway structure and are simplification of a beam laid on a continuous support (soil's sub grade or foundation)[4].

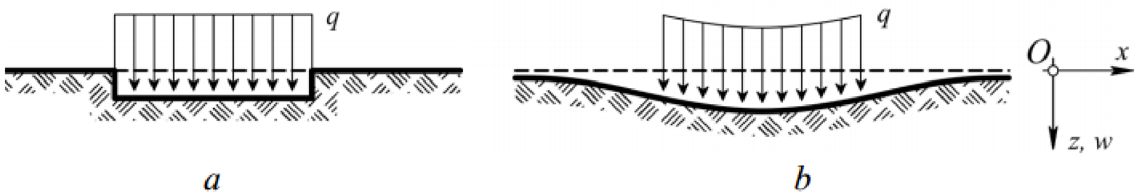


Figure 1-1: Deflections of elastic foundations under uniform pressure [2]: a– Winkler foundation's– practical soil foundations.

The classical concepts of railway track analysis, using beam on elastic foundation (BOEF) methods are still very useful for analyzing a simple design and analysis of railway track systems. Howe ever, for doing a complex analysis of a railway track, these methods have lack of capabilities, since they only take into account one-dimensional system and neglect the actual discrete support provided by sleeper, ballast, sub ballast and sub-grade. Nowadays, the use of computer software for doing Finite element method (FEM) or

Finite element analysis (FEA) of a structure is very common for engineers. FEA consists of a huge amount of complex calculations; therefore, a manual calculation by hand is almost impossible to be done. Hence, the use of computer software will be very useful in this manner[4].

The railway network plays an important role in the transport system of world. Ethiopia is going to construct about 5000km railway lines in which most of the lines track have been designed. Thus appropriate design of Railway track provides an economical and safe transportation system for passenger and freight traffic. So, knowing the appropriate design approaches for track design is great factor in economical and safe design.

Sleeper type and spacing are among the parameters for varying the track modulus. In this thesis the properties of rails bending stresses, rails deflection and rails support force with respect to sleeper space have been analyzed for numerical and analytical analysis.

1.2 Objectives

1.2.1 General objectives

Some analytical equations were developed based on beam on elastic foundation theory. These analytical equations were derived from some scientific assumption with some mathematical derivation for simplifying the model and easy for calculation. However, it is important to analyses weather these analytical equations can represent the real track model or not and comparing with numerical modeling of the same track. As it is too vast to consider all parameters at the same time, the effects of sleeper spaces (independent variable) on rail deflection, rails bending stress and rail support force; (Dependent variables); have been analyzed and checked with the allowable components.

1.2.2 Specific objectives

- Compares the analytical and numerical investigation of rail's deflection ,rail support force and bending stress
- To know the effect of sleeper spacing on the rail foundation stiffness, rail bending stress, rail deflection and rail supporting force
- Correlate numerical investigation with analytical analysis with the following:-
  - Sleeper spaces Vs Rails bending stress

- Sleeper spaces Vs Rail support force; and
- Sleeper spaces Vs Rail deflection, with graphical representation and summarizing tables.

1.3 Scope of the Thesis

The thesis deals with only sleeper spaces as an independent variable for the study of rail bending stress; rail deflection and rail supporting force for three types of track beds. Other supporting system such as rail fastening system, ballast and sub grade have been keep constant (except for three track beds which are proposed) even though they should have been investigated further.

1.4 Thesis Organization

The Thesis has organized into five different chapters. A brief outline of each chapter is given below:

Chapter 1 is an introductory part which contains the general background, statement of the problem, objectives of the research and structure of the thesis.

Chapter 2 is the literature review begins with a general description of various track components and its function. Under this chapter track components and their characteristics, railway track stuffiness, track loading (forces), types of track modeling and track design methods are discussed in details.

Chapter 3 is the Methodology which starts by describing the methodology through which the thesis was carried out. Under methodology, each step in collecting data, modeling of tracks and analyzing were specified. The chapter continues with summarizing the flow chart of the work process.

Chapter 4 is the portion where the main objectives of the thesis are tried to be answered. Under this chapter the track is modeled in ANSYS (FEM) and beam on elastic foundation (BOEF) and analyzed numerically and analytically. The results of the analysis, i.e. rail bending stress, rail deflection and rail supporting force are analyzed separately for each analysis then compared together. Sensitivity of rail deflection, rail supporting force and rail bending stress with respect to sleeper space are analyzed for both analysis and compared together.

The chapter continues by introducing some factors which correlate the numerical analysis with analytical analysis. Lastly using these correlating factors the results that will be obtained from analytical analysis are converted from the analytical analysis and compared with the actual numerical analysis results.

Last chapter, Chapter 5 summarizes the main findings of this research study; conclusions have been made based on the result of the FEM and BOEF analysis.

Tables, graphs, charts and figures were included in the report for more illustration. Finally the thesis provides conclusions and recommendations for future work.

1.5 Significance of the Thesis

1. Asses the difference between analytical and numerical Solutions based on rail bending stress, rail deflection and rail supporting force.
2. Give correlation between BOEF and numerical. Thus, the Thesis provides coefficients(**α, β** and γ) for BOEF to correlate with numerical, where, "**α**" is coefficients for deflection, "**β**" is coefficients for bending stress and "γ" is coefficients for rail support force. These coefficients will vary with the range of sleeper spacing.

CHAPTER 2 LITRETURE REVIEW

2.1 Track Components and their Characteristics

2.1.1 Rail

Rail as the most important track element which is in direct contact with the rolling stock and subjected to wheel loads that must be able to securely sustain these loads applied in vertical, lateral, and longitudinal directions and subsequently transfer them to the underlying supports. It is therefore very necessary, in particular from a safety point of view, to ensure the proper functioning of rails in the track system [5]

Hence, some journals summarize [5], [13] the function of rails as below:-

- Rails provide a hard, smooth and unchanging surface for passage of heavy moving loads with a minimum friction between the steel rails and steel wheels.
- Rails bear the stresses developed due to heavy vertical loads, lateral and braking forces and thermal stresses.
- The rails act as girders to transmit the wheel load to the sleepers and consequently reduce pressure on ballast and formation below.

All modern rails now are made by hot rolled steel of specific cross-section (profile) which approximates to I-beam but is asymmetric about a horizontal axis. It is composed of three parts; rail head, web and Foot, and overall cross-sectional dimensions are always made of rail height (A), foot width (B) and head width(C). Together with the web thickness (D), the 4 sizes are overall dimensions for rail type classifications.

Rails are made in a large number of different sizes and weights. Different countries have their own rail weights and sizes. AGICO GROUP (a rails manufacturing industry) and JFE Steel Corporation classify the rails for production based on the interesting standards

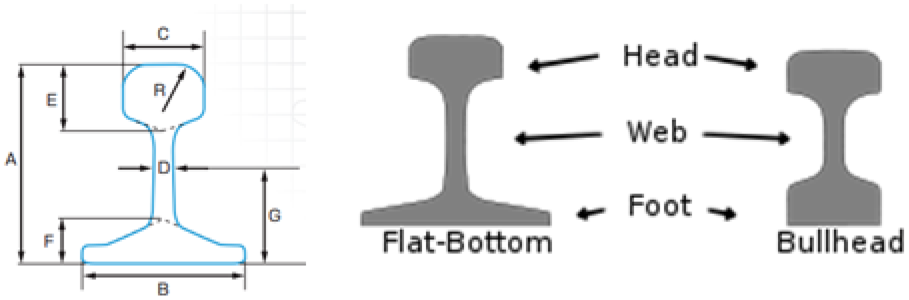
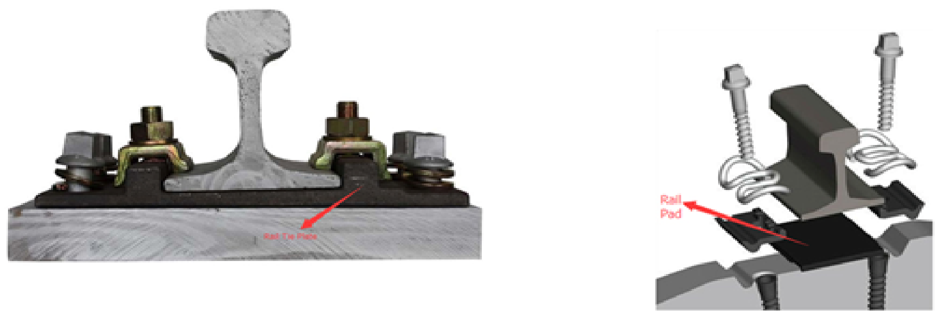


Figure 2-1: Rail Profile [6]

2.1.2 Rail Fastening System

Rail fastenings are one of the major elements of the top structure of the track. Traffic safety of the trains depends on their reliable work. They carry out the function of connection between the rail and the sleeper. They carry out function of the primary amortization for dynamic interaction between a track and a rolling stock. Rail fastenings carry out also the important function of fixation long welded tracks at the temperature influence on the rails. And, last, rail fastenings carry out the function of the electric isolation between track ways (rails) [9].

The fastening system consists of rail pad (Tie Plate), Rail Clamp, Anchor (shoulder) and insulator.



A) B)

Figure 2-2: Rail Fastening System

After assembled [7] (A), before assembled [8] (B)

Various types of fastenings have been developed to meet the requirements of railway development.

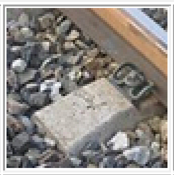
						
Rail spike with base plate above the tie	E-Clip' fastening	Pandrol 'fastclip' fastening	Tension clamp fastening	Bolt clamped fastening	Track joint and chairs fastening	Steel spring keyed rail in chair

Figure 2-3: Types of rail fastening system [10]

2.1.3 Sleeper

Sleepers are one of the superstructure components of the railway track. Over hundreds years ago, Wooden sleepers were introduced followed by a limited number of steel sleeper generation. Now days, Concrete sleepers are common type of sleepers in different Countries due to the fact that concrete sleeper has longer life, much stabilizes and needs less maintenance than other types of sleepers[11],[12].

Different literatures lists the function of sleeper in which sleepers are used to transfer and distribute forces from the rails to the ballast, fix the track gauge, maintain adequate rail inclination, resist rail movements in all directions (horizontal, vertical and longitudinal)[12],[13],[14].

There are two types of common concrete sleepers, Mono block and twin-block sleepers as shown in figure 2.4. Twin-block sleepers were originally developed in France and used in Europe (Belgium, Denmark, Netherlands, and Greece), India, Brazil, and Mexico. Increased lateral resistance and lower weight are the main advantages of this sleeper type. It can be used under various loading conditions and its service life is about 50 years. Mono block sleepers first came from the UK and have been adopted in countries such as Australia, Canada, China, Japan, the UK, the USA, and the former USSR, now days, Ethiopia. It can be used for high speed railway and heavy loading. And the service life would be also about 50 years[11],[15].

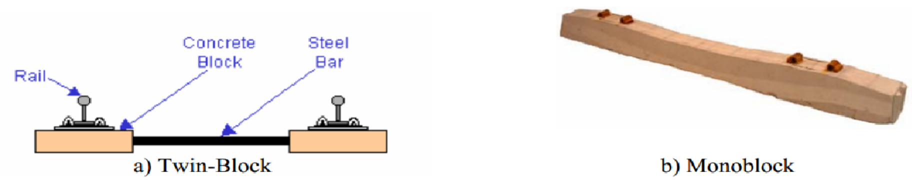


Figure 2-4 : Types of Concrete sleepers

Sleepers may be modeled as rigid beams or beams with flexural and shear stiffness or rigid bodies. At higher frequencies, the mass of a sleeper becomes increasingly important and it is essential to consider the sleeper as a dynamic component that has both mass and stiffness. Because of its distributed mass and stiffness, the sleeper resonates at a series of frequencies, the most significant for typical sleepers being around 200 Hz and 650 Hz [16].

Some researchers had been conducted on the dynamic and static analysis, design and modeling of sleepers. Shan Li, in her master degree project, studied a numerical static and dynamic analysis for a pre-stressed concrete mono-block railway sleeper. Using four types of support condition under deterministic analysis and neural network methodology under stochastic support condition, she observed the worst support condition, Variations of vertical displacement, tensile stress at rail seat and in center of sleeper.

Hence, Shan Li, identified that the worst support condition is where there is sleeper-ballast interaction only underneath the rail seat, while there is no ballast underlying in the Centre of the sleeper or on both sides(shown in fig 2.5 ,case 4)[11].

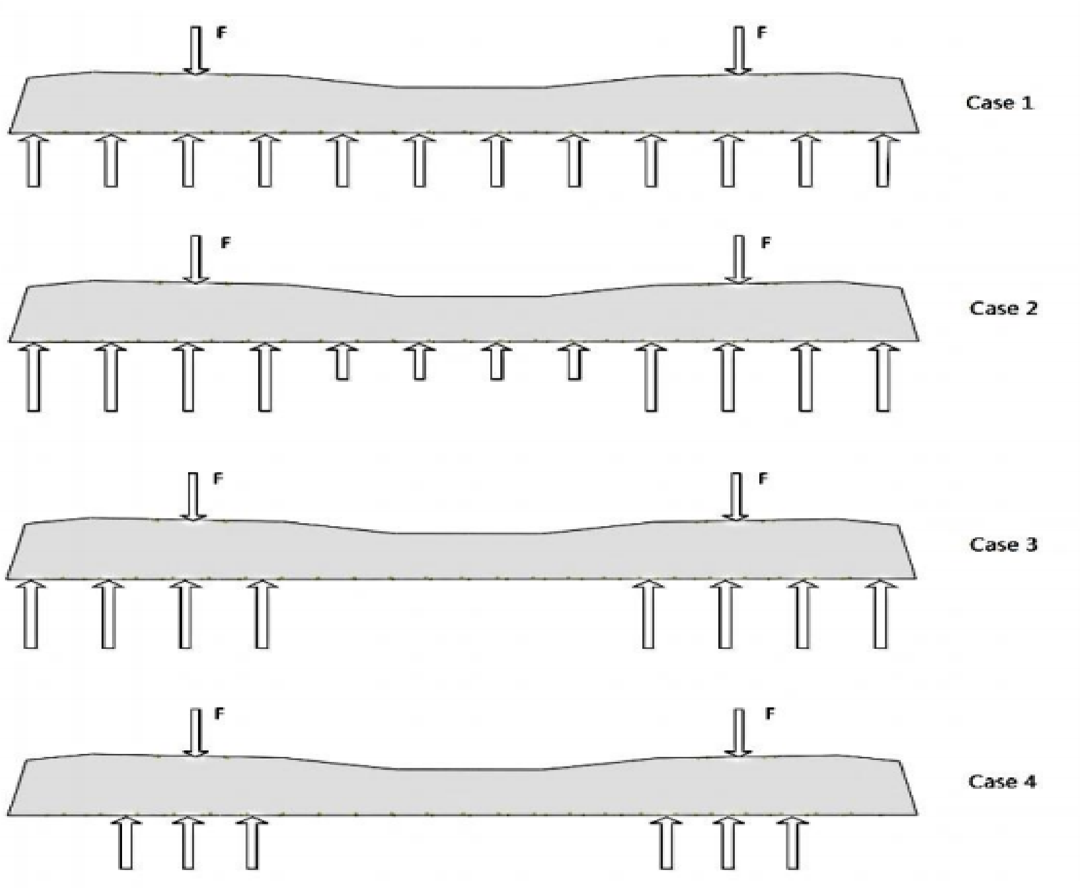


Figure 2-5: Different cases of sleeper support conditions [11]

2.1.4 Ballast and Sub ballast

Ballast is the select crushed granular material placed as the top layer of the substructure in which the sleepers are embedded. A wide variety of materials have been used for ballast such as crushed granite, basalt, limestone, slag and gravel. Thus, availability and economic considerations have been the prime factors considered in the selection of ballast materials [17].

The most important functions of ballast are [13], [17], [18:-

- ✓ Resist vertical, lateral and longitudinal forces applied to the sleepers to retain track in its required position
- ✓ Provide some of the resiliency and energy absorption for the track
- ✓ Provide large voids for storage of fouling material in the ballast, and movement of particles though the ballast
- ✓ Facilitate maintenance surfacing and lining operations

- ✓ Provide immediate drainage of water falling onto the track
- ✓ Reduce pressures from the sleeper bearing area to acceptable stress levels for the underlying material
- ✓ To facilitate track maintenance operations, such as the correction of track surface and alignment errors
- ✓ Prevent vegetation growth in the track

The mechanical properties of ballast result from a combination of the physical properties of the individual ballast material and its in-situ. Physical state can be defined by the in-situ density, while the physical properties of the material can be described by various indices such as particle size, shape, angularity, hardness, surface texture and durability.

In service the ballast gradation changes as a result of [17]:-

- o Mechanical particle degradation during construction and maintenance work, and under traffic loading
- o Chemical and mechanical weathering degradation from environmental changes, and
- o Migration of fine particles from the surface and underlying layers.

Ballast and sub-ballast is typically modeled as a load distributing material, which usually modeled by discrete or distributed linear springs and viscous dampers in the vertical directions [13].

2.2 Railway Track Stiffens

The function of a sleeper depends on the support conditions and these are directly related to the track stiffness. Thus, track stiffness is an important track property. It is constituted from the properties of the substructure and superstructure [14] [16].

The most general understanding defines track stiffness as the elastic rail deflection that takes place under a wheel loading. Fundamental analysis of track stiffness has typically been approached by means of the beam on elastic foundation (EF) approach [16], [18].

This analysis technique is known to be reasonably representative, but introduces the concept of the track modulus, i.e. the stiffness of the spring μ per unit length of track. The differences between track stiffness and track modulus have been described in [16] as shown in table 2.3 below.

Table 2-1: Differences between track stiffness and track modulus [1]

Track stiffness	Track modulus
➢ **Track stiffness refers to the rails.** **It is** ✓ The elastic rail deflection that takes place under a wheel loading. ✓ the ratio of the load applied to the rail to vertical rail	➢ **Track modulus refers to the track.** **It is** ✓ The stiffness of the spring μ per unit length of track. ✓ A measure of the vertical stiffness of the track foundation

Having in mind, the above definition of the track stiffness, it can be also given in terms of formula for the analytical computation as below.

$$\mathbf{k} = \frac{\mathbf{P}}{\mathbf{Y}} \qquad \textbf{2-1}$$

Where, K is track stiffness (N/m)

- P is Vertical force (N)
- Y is vertical rail displacement (m)

Each components of the track has its own effect in the determination of track stiffness. Each one are connected in serial (rail, rail fastener, sleeper and ballast & sub grade) or parallel (Clips and rail pad).

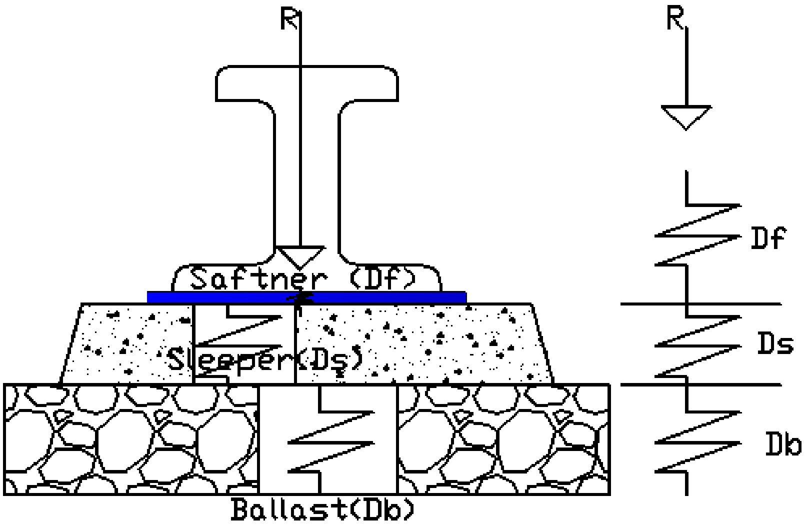


Figure 2-6: Serial connection of track stiffener

For stiffness arranged in series the representative (Resultant, D) will be computed as below.

$$\frac{1}{D} = \frac{1}{Df} + \frac{1}{Ds} + \frac{1}{Db} \quad \text{2-2}$$

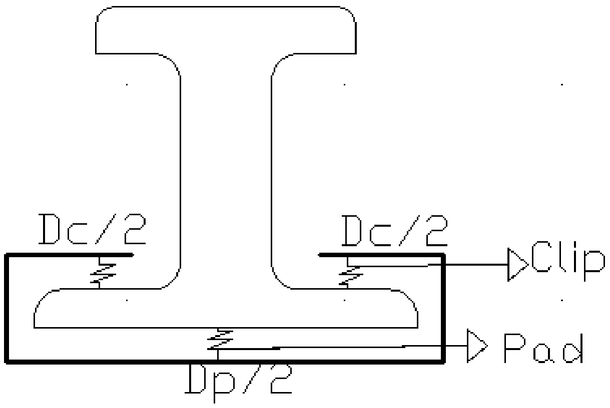


Figure 2-7 : parallel Connection of track Stiffener

For Parallel Connection,

$$D_f = D_p + D_c \quad \text{2-3}$$

The distribution of the ballast stiffness depends on the types of sleeper (wood or concrete), effective contact area of sleeper-ballast interface and the ballast coefficient. It has been given in [19] that the ballast stiffener as below in formula

$$\mathbf{K_b}=\frac{\alpha Clb}{2} \qquad \mathbf{2\text{-}4}$$

Where,

- $\alpha = \begin{cases} 0.81 - 0.92, \text{for wooden sleeper} \\ 1, \text{ for concrete sleeper} \end{cases}$
- C=ballast coefficient (N/m^3) is the pressure acting on the top surface of the ballast to make the ballast top surface produce one unit deformation , characterized the resilient characteristic of ballast and sub grade
- lb/2 effective area of sleeper (m^2)
- K_b is ballast Stiffness (N/m)

Additionally, a ballast pyramid model was developed to determine the stiffness of the ballast. It was developed using the experimental data for a load on a single wood plate 7” X 9” X 8”. Typical values for ballast properties were obtained from various sources and these data were included in a simplified analysis of the ballast pressure variation as a function of ballast depth and effective tie bearing area [20].

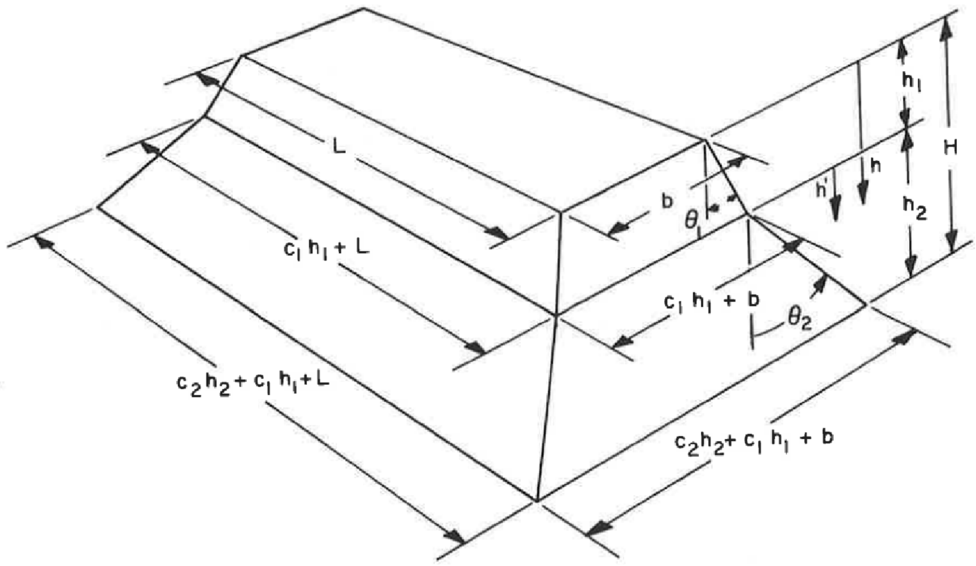


Figure 2-8: pyramid ballast model [24]

The ballast has been divided into two sections each having different friction angle in order to include the effects of sub ballast. The equations for the effective area then given by:

$A(h) = (C_1h + b)(C_1h+L)$-----$0<=h=<h_1$..2-5

$A(h) = (C_2h'+C_1h_1+b)(C_2h'+C_1h_1+L)$, $h_1=<h=<H$,…………………………….……2-6

Where,

- ✓ h_1 is ballast depth
- ✓ h_2 is sub ballast depth
- ✓ $H=h_1+h_2$
- ✓ $C_i= 2 \tan\theta_i$, θ is friction angle of ballast/sub ballast
- ✓ b is the Tie width
- ✓ L is the effective length of the tie

The effective of the total pyramid will be the effective stiffness of the two springs in series [13].

$$\mathbf{K\,b} = \frac{\mathbf{K1K2}}{\mathbf{k1+k2}} \quad \text{.......................... 2-7}$$

The stiffness of each spring can be determined as from the following formulae [21].

$$K_1 = \frac{c_1\,(L - b)\,E_b}{\ln_e \left[\dfrac{L(c_1h_1 + b)}{b(c_1h_1 + L)}\right]} \quad \text{……………2-8}$$

$$K_2 = \frac{c_2\,(L - b)\,E_b}{\ln_e \left[\dfrac{(c_1h_1 + L)(c_2h' + c_1h_1 + b)}{(c_1h_1 + b)(c_2h' + c_1h_1 + L)}\right]} \quad \text{……………2-9}$$

Where, E_b is the Modules Elasticity of the ballast

Having known the stiffeners of all track components (fastener, sleeper, and ballast and sub grade), the representative of the all in one will be computed using formula (springs in series) as given below.

$$\frac{1}{k} = \sum\frac{1}{Ki} = \frac{1}{k,fastner} + \frac{1}{k,sleepr} + \frac{1}{k,ballast} + \frac{1}{k,substructure} \quad \text{……………2-10}$$

This constant k is which have been obtained by applying a uniformly distributed load along the whole track and recording the deflection. The modulus, therefore, represents the equivalent series stiffness of all the components of the track [16].

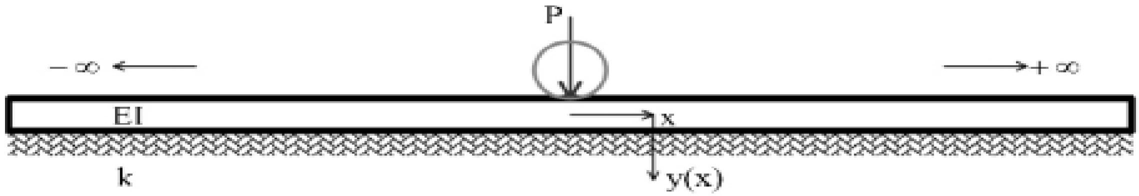


Figure 2-9: Beam on elastic foundation [5]

Sleeper spacing also has its own effects on the foundation stiffness. Some literatures [19] give effects of sleeper spacing mathematically as below.

$$\mu = \frac{K}{a} \quad \text{..........} \quad 2\text{-}11$$

Where,

- "μ" is Rail foundation stiffness (N/m^2)
- "K" is Rail support stiffness (N/m); and "a" is sleeper spaces (m)

This simplification of the multiple track components into a single representative foundation is a powerful method for the design of rail, especially for the computation of rail deflection, rail moment and rail support force analytically.

Hence, the rail foundation stiffness (μ) is one of the most powerful parameter in the computation of rail deflection, rail bending moment and rail support force under the beam on elastic foundation (BEOF) track model [16] [22].

Table 2.4 lists the various quantities used to express the elasticity of the rail supporting structure together with their units. Moreover, global values are given which correspond to the qualification 'poor' and 'good' to characterize the condition of the foundation [22].

Table 2-2: Order of magnitude of elasticity constants [22]

Quality of track support	**Unit**	**Poor**	**Good**
Foundation modulus(C)	[N/mm^3]	0.02	0.20
Spring constant(k)	[N/mm]	5.5	55
Foundation coefficient (μ)	[N/mm^2]	9	90
Characteristic length(L)	[m]	1.3	0.70

2.3 Track Loading (Forces)

In rail analysis, the usual starting point for determining the suitability of a particular rail to carry out its function of withstanding the applied vehicular loading is to calculate the design wheel load. Having calculated the design wheel load, the beam on elastic foundation model(analytical model) or/and computer model(Numerical analysis) are then used to calculate the rail bending stresses , rail deflection and rail supporting forces caused by this loading[13].

The forces acting on the track result from the running train and thermal (expansion and contraction) loads .These Loads can be classified based on their nature and/or their applied direction [13] [16] as listed in table 2.5.

Table 2-3: Forces on Railway track

Based on their applied direction	Based on their Nature
- Vertical Loads , - Lateral Loads; and - Longitudinal Loads	- Static Loads, - Quasi-static Loads; and - Dynamic Loads

Table 2-4: Fundamental concept of Statics and Dynamics [19]

Loads type		Track Structure analogy	Response of track/Vehicles	Remark
Magnitude, Direction, Action point	Static Loads	constraint Stiffness,	Displacement, Internal forces, Stresses	Unchanged with time or change slowly
	Dynamic load	Damping , Mass, Mode shape Stiffness, constraint,	Displacement, Internal forces, Stresses, Velocity, Acceleration	Chang with time

2.3.1 The Design Vertical Wheel load

The nominal vehicle axle load is usually measured for the static condition, but in the design of railway track the actual stress in the various components of the track structure and in the rolling stock must be determined from the dynamic vertical and lateral forces imposed by the design vehicle moving at speed. The dynamic wheel loads causes increases in the rail stress values above those of the static condition due to the following facts [22]:-

- Lateral bending of the rail
- Eccentric vertical loading
- Transfer of the wheel loads due to the rolling action of the vehicles
- Vertical impact of wheel on rail due to speed
- Irregularities and non-uniformities in the track and the wheel and rail profiles

The general method used in the determination of the design vertical wheel load is to empirically express it as a function of the static wheel load as given as below [22]:-

P = ø.P……………………………………………………………………………2-12

Where,

- P = design wheel load (kN),
- P_o = static wheel load (kN), and
- Ø = dimensionless impact factor (always >l).

The expression used for the calculation of the impact factor is determined empirically and is always expressed in terms of train speed. When developing expressions of the impact factor, the number of above mentioned factors considered depend upon the amount and quality of the track instrumentation used, and the assumptions used in relating the parameter [22].

Some empirical formulae were developed for the impact factors are of this form. The main criticism of this type of empirical formulae is that they neglect any vertical track elasticity which absorbs some of the impact blow on the rail.

Table 2.7 lists the summary of expressions that have been used to determine the value of the impact factor (Ø) [22].

Table 2-5: Mathematical expression of impact factor [22]

Origin (Name) of the formulae	**Mathematical expression of the impact**
1) Indian Formula (Agarwal 1974)	$ø = 1 + \frac{v}{58.14(k)^{0.5}}$
2) German Formula (Schramm 1961): a. For speeds up to 100 km/h	$ø = 1 + \frac{v^2}{3 * 10^7}$
b. For speeds above 100 km/h	$ø = 1 + \frac{4.5v^2}{10^5} - \frac{1.5v^2}{10^7}$
3) South African Formula (Lombard 1974), (Narrow gauge track)	$ø = 1 + 4.92\frac{v}{D}$
4) Clarke Formula (Clarke 1957)	$ø = 1 + \frac{19.65v}{D(k)^{0.5}}$
5) WMATA Formula (Prause et al. 1974):	$ø = [1 + 3.86 * 10^{-5}v^2]^{0.67}$

Where ,

- V = vehicle speed (km/h),
- k = track modulus (MPa) , and
- D = wheel diameter (mm).

WMATA is the abbreviation of the Washington Metropolitan Transit Authority

2.4 Track Modeling

2.4.1 Beam on Elastic Foundation Modeling (BOEF)

The first Considerable development in modeling of the track system was made by Winkler [23]. In the railway application, in the concept of Winkler support model, the elements of conventional track are basically modeled as two parallel continuous beams (the rails), which are constrained at regular intervals (space) of sleepers. Some papers summarize the Winkler assumptions as below:

- ✓ The reaction forces of the foundation are proportional at every point to the deflection of the beam at that point[2][3];
- ✓ Supporting sleepers fastened tightly to the rail would rest against rail bending through their rotational stiffness[4];
- ✓ At each point of support the compressive stress is proportional to the local compression[4].
- ✓ Track support is considered as a one-layer component
- ✓ Sleepers have no deformation as they are supported from below and from the sides by ballast bed.

Winkler theory neglects some actual conditions of railway tracks such as; [2], [3], [4]

1. Actual discrete support provided by sleepers
2. Interaction between track components(ballast sub-ballast; and sub grade)
3. Different track supporting layers
4. Predicting accurately the displacement of some solids, such as soil.

Based on Winkler's theory of elasticity and strength, Zimmermann (In the 1880s) developed a method to determine the forces and deflections which occur in a single supported track. The basic idea in Zimmermann method is to transform the single supported beam by transferring the bearing areas into a continuously supported beam. In the Zimmermann method, the single value of C (N/mm^3) or modulus sub-grade reaction or ballast module is used. Howe ever, in the reality, in ballasted track systems, the components of rail-pad, ballast, sub-ballast and sub soil have different C values. Hence, the material properties of those components should be combined into single C_{tot} value by using this correlation[4]:

$$\frac{1}{C,tot} = \frac{1}{C,rail-pad} + \frac{1}{C,ballast} + \frac{1}{C,subballast} + \frac{1}{C,subgrade} \qquad \textbf{2-13}$$

If the property of material is presented by k value (spring coefficient), then into Zimmermann, k can be converted to C by using this correlation [4]:

$$C = \frac{K}{ab} \qquad \textbf{2-14}$$

(a=sleeper space and b=sleeper width)

In this Model, the rail is modeled as a beam (with bending stiffness EI) which rests on a continuous elastic foundation (Figure 2.10). The elastic foundation represents all track components and is modeled by evenly distributed linear spring stiffness. The distributed force supporting the beam is then proportional to the beam deflection (Figure 2.11).

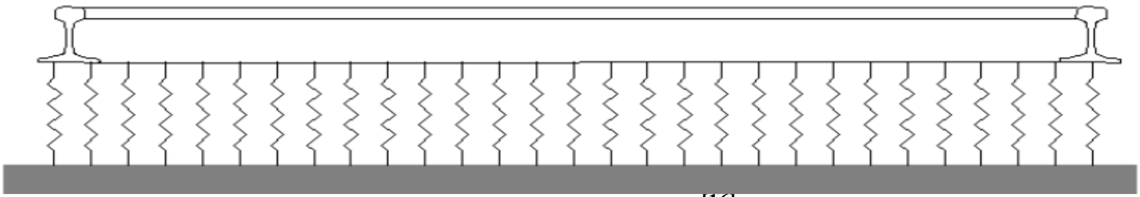


Figure 2-10: Beam on elastic foundation (unloaded) [16]

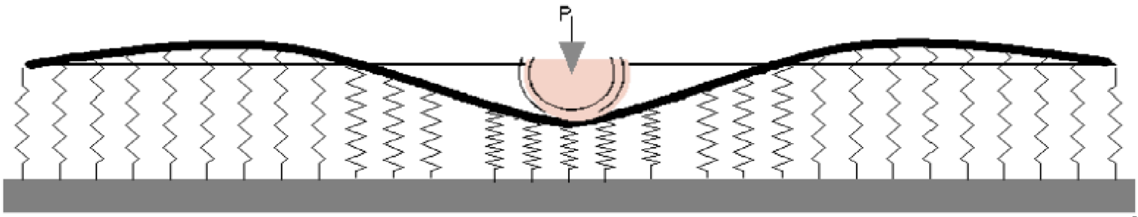


Figure 2-11: Beam on elastic foundation loaded with a point force P from the wheel [16].

Based on this model, the deflection, rail supporting force and bending moment of the rail beam due to the moving load can be determined [16].

$$Y(x) = -\frac{P.\beta}{2\mu}.e^{-\beta.x}.\ [\cos(\beta x) + \sin(\beta x)] \qquad \textbf{2-15}$$

$$M(X) = -\frac{P}{4\beta}.e^{-\beta.x}.\ [\cos(\beta x) - \sin(\beta x \qquad \textbf{2-16}$$

$$R(X) = \frac{P.\beta.a}{2}.e^{-\beta.x}.\ [\cos(\beta x) + \sin(\beta x)] \qquad \textbf{2-17}$$

Where, $\beta = \sqrt[4]{(\frac{\mu}{4EI})} \quad m^{-1}$

- P is the load

- x is the distance along the track
- E is the Young's modulus of elasticity of the rail steel (beam) and
- I is the second moment of area of the beam.
- μ is the estimate of the track modulus (rail foundation stiffness)

This is the simplest track model and is still in use for easy and quick track deflection calculations for track design and analysis purposes [13], [17].

It is known that the above three equations is used for a single wheel load. An extension has been given for multiple concentrated loads based on their position from the point of interest (where to compute) by the means of superposition [22].

$$\mathbf{Y(x) = \frac{\beta}{2\mu}\sum_{i=1}^{n} P\eta(\beta xi)} \quad \text{2-18}$$

$$\mathbf{M(x) = \frac{1}{4\beta}\sum_{i=1}^{n} P\boldsymbol{u}(\beta xi)} \quad \text{2-19}$$

$$\mathbf{R(x) = \frac{\beta a}{2}\sum_{i=1}^{n} P\eta(\beta xi)} \quad \text{2-20}$$

Where,

- ✓ $\eta(\beta x_i) = e^{-\beta xi}. [\cos(\beta x_i) + \sin(\beta x_i)]$; and
- ✓ $\boldsymbol{u}(\beta x_i) = e^{-\beta xi}. [\cos(\beta x_i) - \sin(\beta x_i)]$
- ✓ X_i is the distance from the wheel load to the point of computation along the track
- ✓ Other parameters are as specified above

2.4.2 Discrete Modeling

Modern railway tracks are characterized by accurately positioned continuous welded rails and sleepers. This makes the mechanical modeling of railway track rather simple: two straight parallel beams supported by sleepers at equal distances.

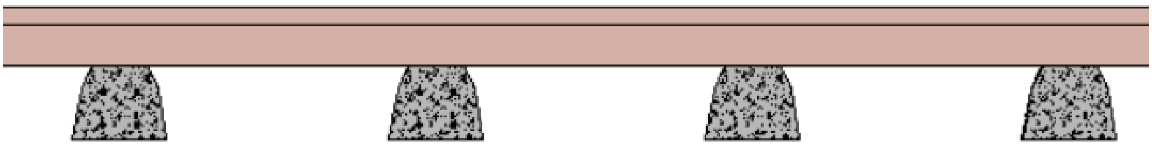


Figure 2-12: Discrete rail supporting system [16]

Discretely supported models are similar to the continuously supported models but

- They consider the discrete spacing of sleepers.

- They often have multiple layers representing the Rail pads, sleepers, ballast, and sub ballast and sub grade.

According to this presentation, a model for the entire railway system can be developed consisting of masses, springs and dampers.

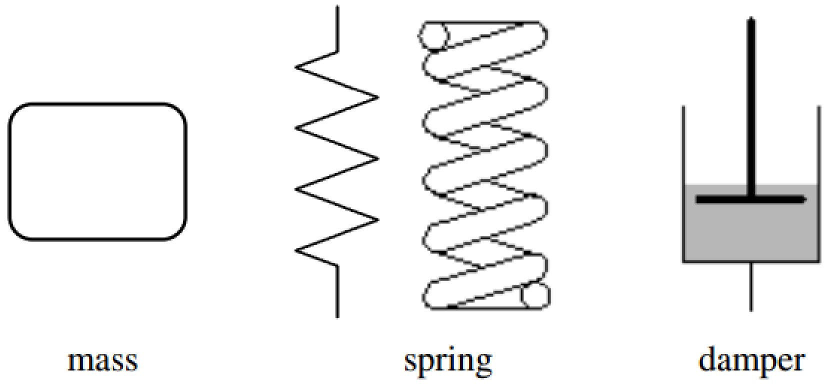


Figure 2-13: Symbols used for the constituents of the model for the railway system [16]

The commonly used to model discretely supported model is to place the rail (beam) on a spring and a damper in parallel. Generally, the discrete model is shown in figures below.

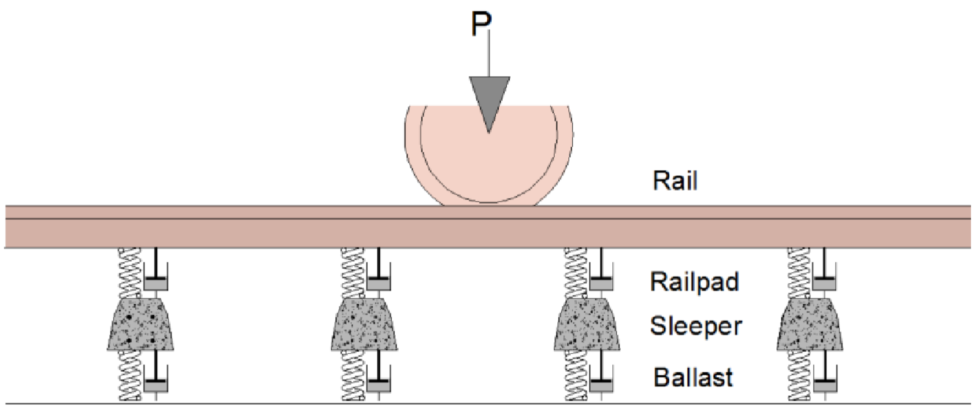


Figure 2-14: Rail on discrete supports with rail pad, sleeper and ballast [16]

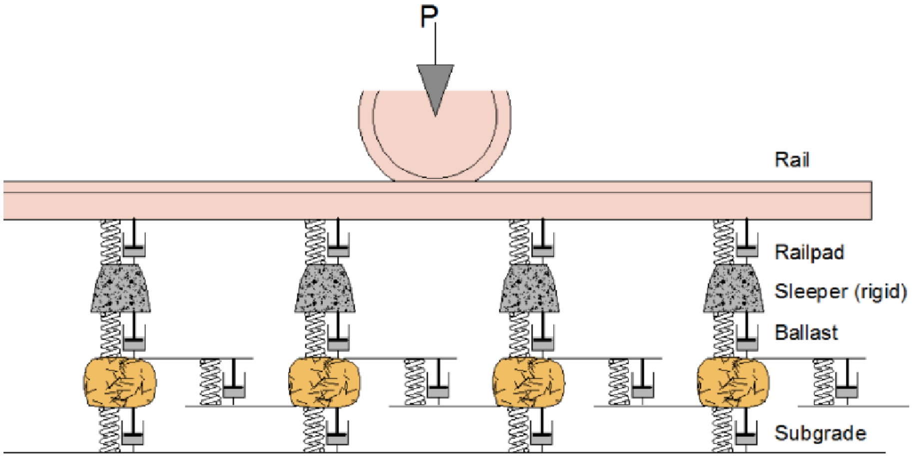


Figure 2-15: discretely Supported Track Including Ballast and Sub grade [16]

In discretely supported models, the railway structure components are modeled based on their response behavior of the loading and material properties [16].

Hence:

- rail is modeled as a beam,
- the rail pads are modeled by spring – damper systems,
- the sleepers are rigid masses,
- the ballasts modeled by spring – damper systems
- the sub grade is modeled by spring – damper systems

2.4.3 Finite Element Method (FEM)

Nowadays, it becomes a powerful computational method to approximate solutions of a variety of "real-world" practical engineering problems, which have complex domains subjected to general boundary conditions. The basis of FEA relies on the division of the problem domain into a finite number of sub domains (elements) [27]. Then, known physical laws are applied to each element, which usually has a very simple geometry. As the result, FEA reduces the problem complexity by solving matrix equations (also so called interpolation functions) of each element by iteration at specific points, referred to as nodes. With respect to the further development and wider area of application of this method, in a complex and detail analysis, the amount of the equations to be solved is usually so large, so that obtaining solution without using computer is practically almost impossible. Therefore, the need of using FEM software packages is necessary [3], [4]

2.5 Track Design Methods

2.5.1 Rail Design Methods

The Rail design criteria are mainly divided into two categories [1]. **Structural strength criteria** include wheel-rail contact stresses and rail bending stresses. Having satisfied the structural strength criteria, **the serviceability requirements** should be completely met for a specific rail section to ensure its proper structural and operational performance as shown in fig 2.16.

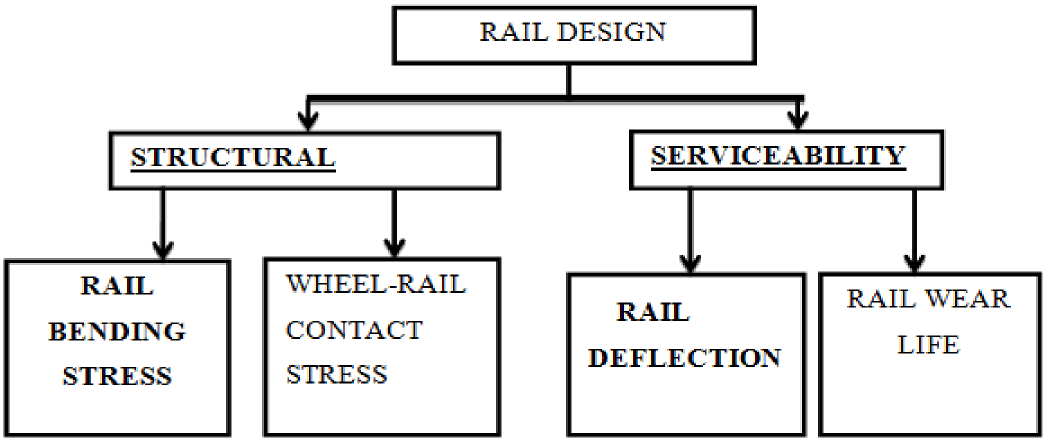


Figure 2-16: Recommended rail design criteria[5]

Current practices in the calculation of rail bending moments and vertical deflection are mainly based on “beam on elastic foundation”model[5].

2.6 Previous Related Works/Findings

Javad Sadaghi[24] in his Doctoral Thesis Titled “Investigation of characteristics and Modeling of Railway Track System”’ in Wollongong University had investigate the characteristics of track components.

Under this thesis, a three dimensional model of track was developed using a finite element Computer package. This model comprises of all track components in three dimensions. Optimization of track models and track system was investigated through sensitivity analysis of the model.

Through a sensitivity analysis of the model, the minimum required dimensions of track models were investigated. It was found that [24]:

1. 20 sleepers are sufficient to be considered in track models based on the results obtained there is only 2% reduction in the maximum ballast vertical stress a % reduction in the maximum sub grade contact pressure, and a negligible reduction in rail deflection and rail bending stress when increasing the number of sleepers from 20 to 72.Therefore , 20 sleepers in a track model are adequate to remove the dependence of the deflection and stresses in the track components on the number of sleeper,
2. There are slight changes in maximum rail deflection obtained from the analysis when increasing the sub grade layers width more than 7m;any increase in the depth of the sub grade layers after 3.5m does not also considerably influence the analysis results. therefore, 3.5m depth and 7m width are adequate sub grade dimensions in track models,
3. Consideration of tensile stress for ballast n track models does not have considerable effect on the results of static analysis of the track system, but it has notable influence on the results obtained from dynamic analysis.

Using the computer model, a parametric study was conducted on track system. Several important findings were made for the optimization of the track system. These findings include the following conclusions [24].

1. An increase in ballast depth from 100mm to 300mm results in a sharp decrease in the sub grade contact pressure and any increase in the ballast depth after 300mm does not significantly decrease the maximum ballast vertical stress and the ballast-sub grade contact pressure. The result indicates that any increase in ballast depth cause less settlement in the sub grade and higher settlement in the ballast.
2. Result obtained from analysis of track with concrete sleeper shows less rail deflection, a smoother deflection curve, and fewer rails bending stress in compression with those obtained from analysis of the same track with timber sleeper.

Table 2-6: Results obtained from analysis of models with different sleeper types [24]

	Model with timber sleeper	Model with concrete sleeper
Maximum ballast stress (KPa)	615	588
Maximum subgrade contact pressure (KPa)	151	146
Maximum rail bendng stress(MPa)	117.81	103.44

3. There is considerable increase in rail deflection when increasing sleeper spacing. Rail deflection is more sensitive to changes in sleeper spacing for lower values than for higher values of sleeper spacing. The results for maximum bending stress in the rail show a slight increase in the bending stress caused by an increase in sleeper spacing. By increasing the sleeper spacing by 100mm maximum sub-grade contact pressure increased 6% while maximum vertical stress in the ballast increased between 15% and 30%. The results for bending stress at the center of sleepers showed a 15% increase for 100mm increase in sleeper spacing.

Table 2-7: Results obtained from analysis of models with different sleeper Spacing[24]

Sleeper spacing (mm)	Bending stress at the centre of sleeper (Mpa)	Maximum ballast vertical stress (KPa)	Sub grade contact pressure (KPa)	Maximum rail bending stress (MPa)	Maximum Rail deflection (mm)
400	6.21	426	137	109.16	3.01
500	7.34	556	145	114.87	3.70
550	7.85	615	151	117.81	4.01
600	8.26	666	159	120.51	4.26
650	8.66	712	166	123.05	4.47
700	8.98	759	173	125.65	5.09
800	9.59	838	185	129.04	5.65

4. 2.5m is an optimum length for sleepers based on maximum stress in sleepers. For lengths of sleepers smaller than 205m the value of sleepers stress at the centre of

sleepers is higher than the stress at the rail seat while for the length of sleepers higher than 2.5m it is vice versa. An increase in sleeper length from 2.3m to 2.54m causes a 9% and a 3% decrease in maximum rail deflection and maximum rail bending stress respectively.

Table 2-8: Results obtained from analysis of models with different sleeper Length [24]

Sleeper length (mm)	Sleeper bending stress at the rail seat (MPa)	Sleeper stress at the centre (MPa)	Maximum rail bending stress (MPa)	Maximum rail deflection (mm)
2.3	3.29	8.89	119.22	4.17
2.4	3.98	7.85	117.81	4.01
2.5	4.84	7.00	116.06	3.82
2.6	5.92	6.17	114.38	3.61
2.7	6.95	5.80	112.20	3.42

5. Increasing the sleeper width resulted in negligible reductions in rail deflection and rail bending stress, but a considerable reduction in ballast contact pressure. Results shows that increasing the sleeper width from 200mm to 300mm results in a 43% reduction in a maximum rail bending stress, and a 6% reduction in maximum rail deflection.

Table 2-9: Results obtained from analysis of models with different sleeper width [24]

sleeper Width (mm)	Ballast-sleeper contact pressure (KPa)	Sleeper Bending stress at the centre (Mpa)	Maximum rail bending stress (MPa)	Maximum rail deflection (mm)
200	459	8.24	118.71	4.08
229	410	7.85	117.81	4.01
254	3725	7.51	116.81	3.95
300	320	7.00	114.98	3.85

6. Any increase in the sleeper depth causes a negligible reduction in rail bending stress, some reduction in rail deflection and ballast contact pressure, but a significant reduction in sleeper bending stress. that is, an increase in the sleeper

depth from 130mm to 150mm (15%) results in a 5% reduction in ballast contact pressure, a 56% reduction in sleeper bending stress at the centre, a 1% reduction in maximum rail bending stress, and a 3%reduction in maximum rail deflection.

Table 2-10: Results obtained from analysis of models with different sleeper depth [24]

Sleeper Depth (mm)	Ballast-sleeper contact pressure (KPa)	Sleeper Bending stress at the centre (Mpa)	Maximum rail bending stress (MPa)	Maximum rail deflection (mm)
100	438	16.11	119.93	4.19
130	410	7.85	117.81	4.10
150	391	5.03	116.34	3.90
170	380	3.55	114.81	3.83

7. Increasing elastic modulus of ballast and sub-grade resulted in a significant increase in rail deflection. the results indicate almost linear correlation between ballast modulus and maximum rail deflection, but there was a linear relationship between sub-grade modulus and maximum rail deflection only for higher values (more than 30 Mpa)

In order to evaluate current practices in analysis of the railway track system, the analysis results of the computer models were discussed and compared with those obtained from current practices. It was found that there are some differences between the results obtained from both analyses. While maximum rail deflection and maximum bending stress obtained from current practices were higher than those obtained from the computer analysis, ballast contact pressure and sub-grade contact pressure obtained from current practices were less than those obtained from computer analysis. The analysis was made under sleeper spaces of 550mm.

Table 2-11: Compression Between analysis result of current practices and computer model [24]

Types of analysiis	Maximum real deflection (mm)	Maximum bending stress (MPa)	Subgrade contact pressure (kPa)	Ballast contact pressure (kPa)
Current Practice	4.69	127.23	133	387
Computer modelling	4.01	117.81	151	410

Other Guy, Puguh B. Prakoso[4] , published a paper which is titled as “The Basic Concepts of Modeling Railway Track Systems using Conventional and Finite Element Methods”. under this paper he proposed to discuss the basic theories behind the conventional and advanced ways of modeling of railway track system, to show the basic concepts of modeling railway track systems using FEM, to present two- and three dimensional FEM models of railway superstructures which are built using software ANSYS, and to demonstrate the way of doing the verification of the results using Zimmermann method.

Accordingly he modeled 2D and 3D track in ANSYS software with along the conventional method (Zimmermann method) analyzed and compared together for rail deflection.

He observed that in the two dimensional FEM model, the result of deflection line of FEM ANSYS is very similar with that of Zimmermann method. It is also proved by using t-student test to check the similarity o f both results, which is shown by t-student test value of 99.73%.z

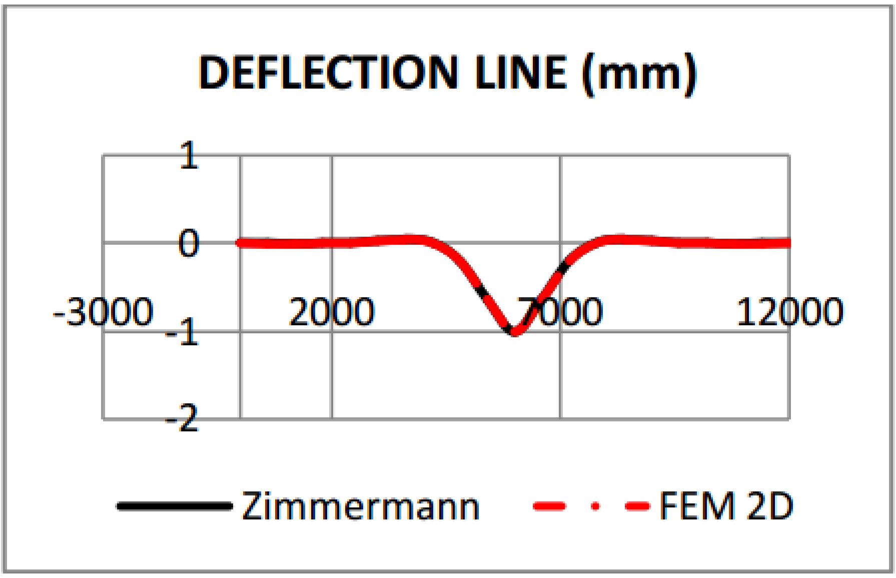


Figure 2-17:- Deflection results comparison between basic model FEM 2D and Zimmermann method [4].

Meanwhile, in the three-dimensional FEM model, there is slightly different result of deflections between FEM and the manual calculation using Zimmermann method. For the comparison between 3D FEM basic model and Zimmermann, the t-student test value is 99.39%.

According to Puguh B. Prakoso Some factors caused different result of deflections between 3D-FEM and the manual calculation using Zimmermann method these are:

[1] the different of geometry of element and the boundary systems between one- (Zimmermann) and three- dimensional (FEM) systems;

[2] the different concepts between a rail on a continuous support (Zimmermann) and 3D rail lies on discrete support (FEM);

[3] the Poisson’s ratio of material is taken into account in FEM; and

[4] The behaviors of contact elements used in 3D FEM model.

CHAPTER 3 METHODOLOGY

3.1 Introduction

To achieve the objective of the thesis the appropriate models and methodology have been used. Discrete track model and beam on elastic foundation (BOEF) track Model were used to get numerical and analytical solutions respectively. FEM software's (ANSYS) was the tools used for numerical solutions while available analytical equations were used for the analytical analysis. Additionally quantitative approach has been used as the method for analysis the solution (data). Concentrated several moving loads (Multiple wheel loads) and single wheel load of the train were modeled as the cause for the effects and the results of multiple wheel loads were taken to analysis in both types of developed track models.

Mono Block types of sleeper were used during the model of the track in numerical analysis and the analogous dimension of the sleeper were taken in analytical analysis. Generally the appropriate steps are can be summarized as below.

1) Track model types, dimensions, track loadings and track properties were investigated and parameters that have been used for the model were selected from literatures.
2) The track Structures were modeled for different sleeper spaces while all other track components kept constant including their material properties and dimensions. Sleeper space was taken to be varying from 500mm to 700mm for better illustration. Totally one track model for analytical analysis and 9 track models in ANSYS for numerical analysis were included.
3) For each track model (each sleeper space) track coefficients (track stiffness and track modulus) were computed for the next step.
4) Having track coefficients, track dimensions and track loadings in steps 1-3, rail deflection, rail support force and rail bending stress were analyzed both numerically and analytically.
5) Using quantitative approaches the solution obtained from both analyses were analyzed and compared.

Generally, the following flow chart has been developed to show the progress of the work.

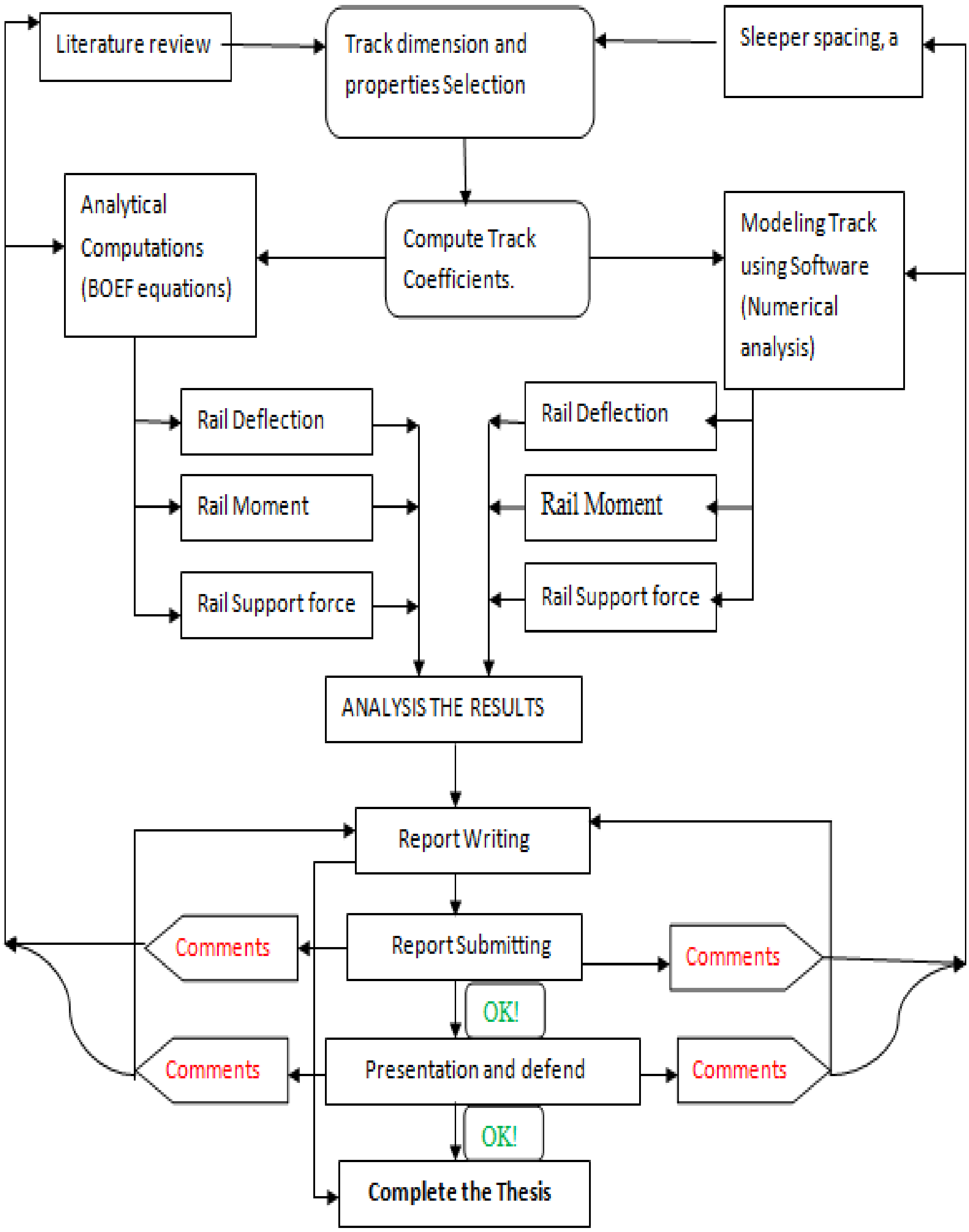


Figure 3-1: Flow Chart

3.2 Track model

3.2.1 Track Model for Analysis

Static analysis has been selected for the analysis, while dynamic design load is used. In static analysis the position of load has its factor for determination of rail deflection, rail supporting force and rail bending stress. That is the position at which maximum deflection is found, maximum rail supporting force may not found. Thus to find maximum rail supporting force and maximum rail deflection the track is modeled separately.

Normally maximum rail deflection is occurred when the resultant of wheel loads path between two sleepers; while maximum rail supporting force is happened when resultant of wheel loads is on the sleeper. Generally 4 types of models have been modeled from which 1 model is analytical model (BOEF Model) while the rest 3 are numerical models (FEM). Each model was analyzed for single and multiple wheel loads to know the effects of multiple wheel load with respect to single wheel load for which BOEF Created.

3.2.2 Analytical Model (BOEF)

A beam on elastic foundation model was prepared for different sleeper space (0 .50m - 0.70m).Using analytical equations (equations 2:15-2:20), a railway track with three types of track properties (Soft track bed, Typical Track bed and Stiff track bed) have been modeled analytically.

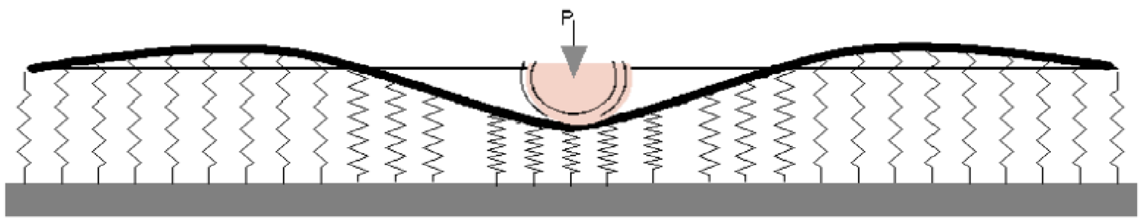


Figure 3-2: BOEF Model [16]

Table 3-1: Material properties for BOEF (Analytical Model) [13][17][27]

Track Components	Material Properties		Units	Soft track bed	Typical track bed	Stiff track bed
Rail	E		pa	2.07e11	2.07e11	2.07e11
	I		m^4	3.22e-5	3.22e-5	3.22e-5
	Section Modulus	at head	m^3	0.000339	0.000339	0.000339
		at base	m^3	0.000396	0.000396	0.000396
Rail supporting components stiffness	**Fastener stiffness**	Clip stiffness	MN/m	100	100	100
		rail pad stiffness	MN/m	146	146	146
		Resultant (kf)	MN/m	246	246	246
	ballast stiffness (kb)		MN/m	106	141	212
	sub -ballast stiffness (ksb)		MN/m	43	57	86
Total Supporting stiffness (k)			***MN/m***	***13.6***	***17.4***	***24.5***

3.2.3 Finite element Model (FEM)

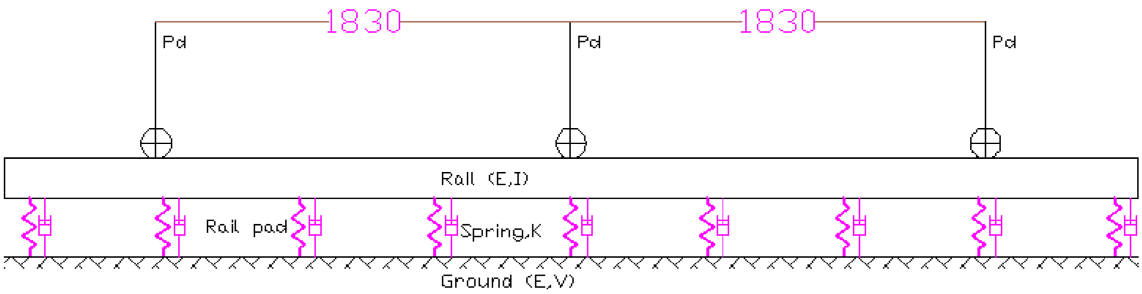
A 2D-track have been modeled in ANSYS for different track length (20m, 10m and 5m), Track properties (Soft track bed, typical track bed, stiff track bed);and at different sleeper spacing (0.50m, 0.55m, 0.60m, 0.6m and 0.70m). Three types of FEM have been developed for analysis in which each of the models is described in the following topics.

3.2.3.1 Model – 1

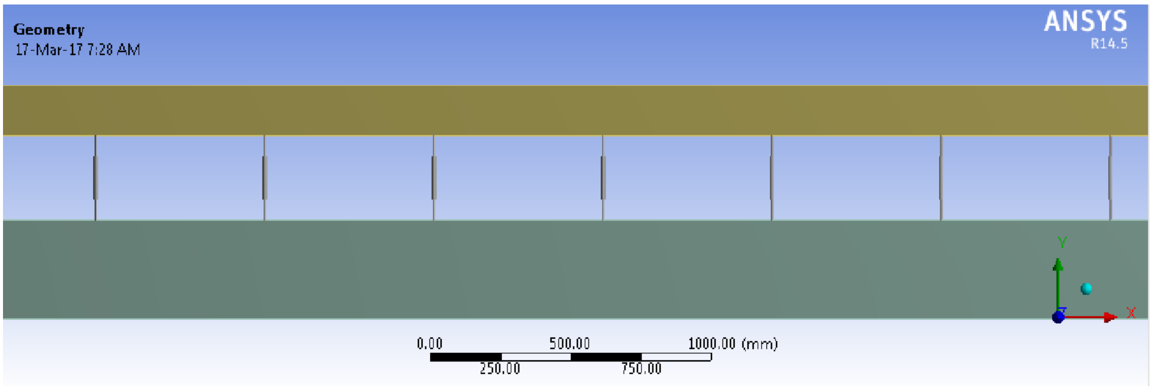
This model is analogues with BOEF. In this model the rail tack is modeled as a flexible beam rested on a spring spaced the same as sleeper space. In this model the substructure track components represented by a spring using equation 2-13.

Table 3-2: Spring Value for Track model-1

Track components stiffness(MN/m)	Track Type		
	Soft Track bed	**Typical Track bed**	**Stiff Track bed**
Rail pad stiffness	246	246	246
Ballast stiffness	106	141	212
Sub-ballast stiffness	43	57	86
Total stiffness	13.604	17.421	24.498



a) Schematic model



b) Model in ANSYS

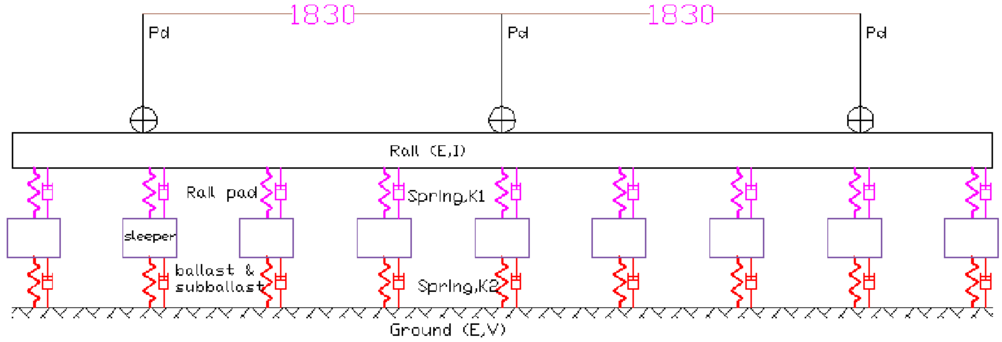
Figure 3-3: Track Model - 1

3.2.3.2 Model – 2 (Desecrate Model)

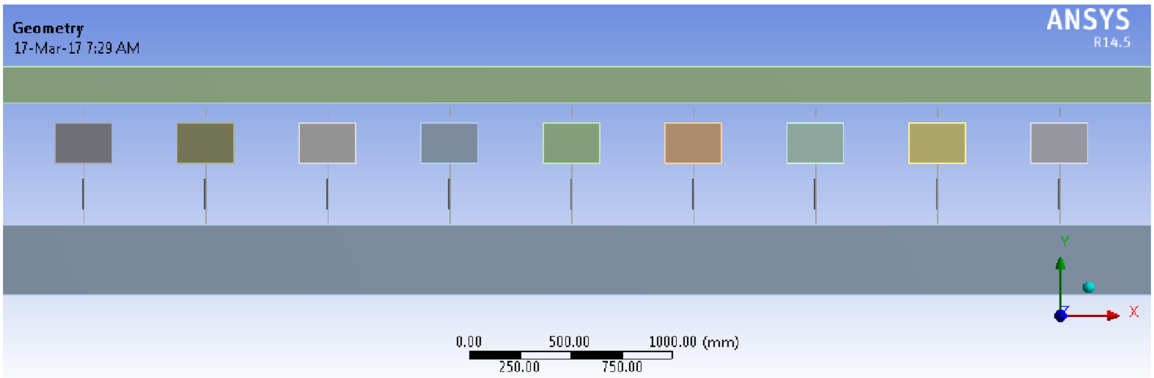
A rail track consists of rail, rail pads (represented by spring & damper), sleeper, ballast and sub-ballasted was created in ANSYS 14.5 as shown in figure 3-3. A rail is supported by rail pads (spring and damper) which in turn supported by sleeper arranged at constant space. Ballast and sub-ballast are represented by single spring and damper for simplification of the model and are supported by the ground. Table 3-3: lists the properties of these springs.

Table 3-3: Spring Value for Track model-2

Track Component	Soft Track Bed	Typical Track Bed	Stiff Track Bed
	Spring Value (MN/m)	Spring Value (MN/m)	Spring Value (MN/m)
Rail Pad (k1)	246	246	246
Ballast and sub-ballast (k2)	30.6	40.6	61.2



a) Schematic model

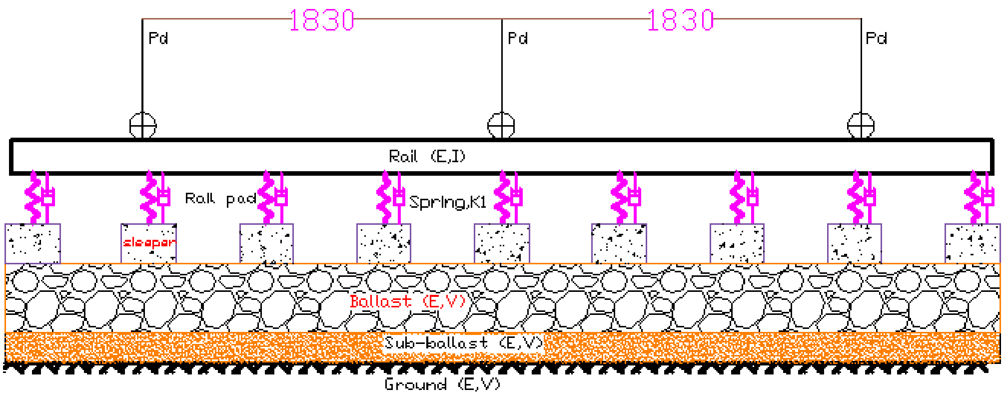


b) Model in ANSYS

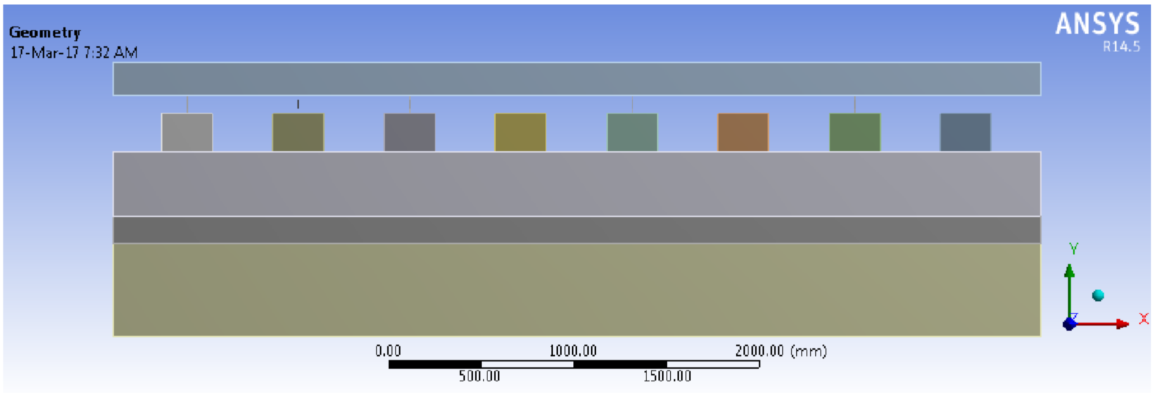
Figure 3-4: Model-2

3.2.3.3 Model-3

A rail on discrete supports model was created in ANSYS as shown in Figure 3.5 below. The rail was placed on a spring and damper in parallel. This spring-damper system models the rail pad. The Rail pads connect the rail with the sleepers. Sleepers are rested on ballast; and under ballast, sub-ballast and Sub-grade have been modeled respectively. Contact between each track components was defined to be bounded contact.



a) Schematic model



a) Model in ANSYS

Figure 3-5: Model-3

3.2.4 2D and 3DMmodel Selection

As describing on figure 3-6; there are 135 FEM models in which each model will be analyzed for both single wheel load and multiple wheel load cases. Thus, totally we do have 270 times analysis. It is obvious that 3D model will take much longer time for analysis and needs high RAM computers. Considering the time allowed for this paper, the number of models and analysis; and the capacity of computers to be used, 2D-model and analysis has been selected for FEM analysis.

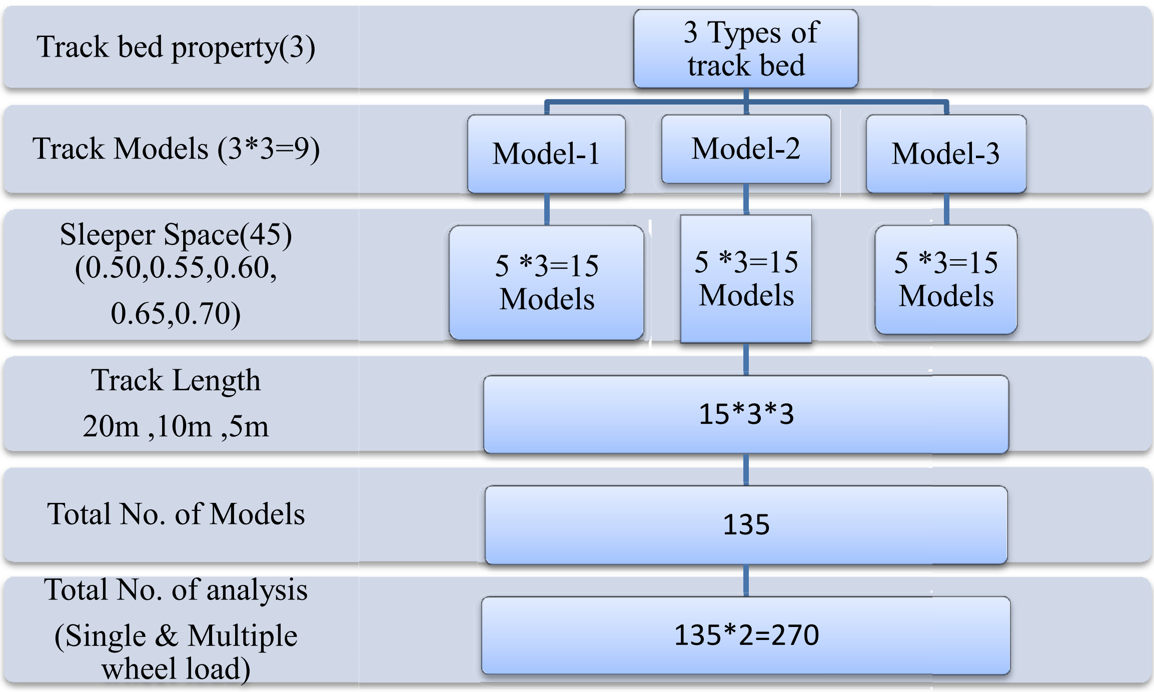


Figure 3-6: Total number of FEM model

3.2.5 Meshing

The models have been meshed with different sizes. The rail is meshed with 1mm mesh size while other track components are meshed with 10mm mesh sizes for all track models. Figures below show the meshed body of track models.

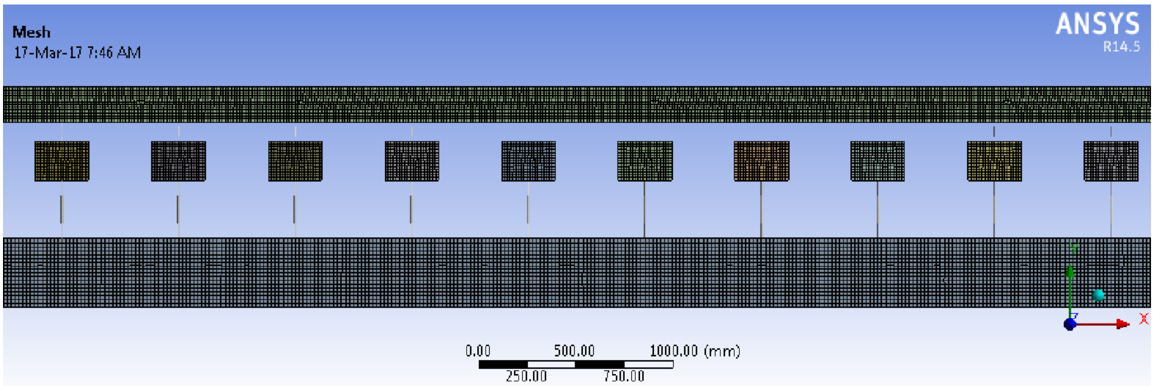


Figure 3-7: Meshed model-2

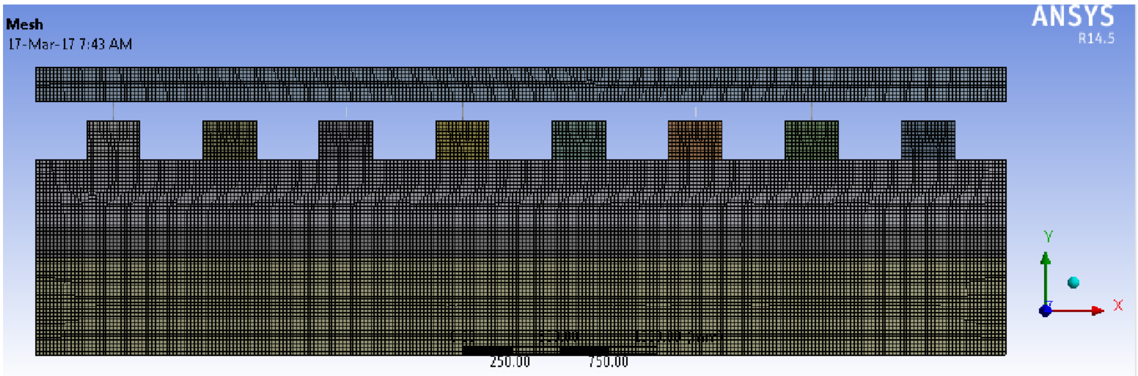


Figure 3-8: Meshed model-2

3.2.6 Track Properties

Technical and general characteristics of track system constituents were specified based on international standards so that they can be used as input data for modeling.

Material and Physical Properties of the track components used in the analysis are listed below.

1) RAIL
   - Standard-GBS 2585-81
   - Mass/length-60kg/m
   - Density=7850Kg/m^3
   - I=3217cm^4=3.217E-5m^4
   - Section Modulus=396cm^3=0.000396m^3

- Poisson's ratio=0.3
- Elastic Modulus=$2.07*10^6$kg/cm^2=$2.07*10^{11}$pa

2) Sleeper
   - Type: mono block
   - Spacing:-0.4m-0.9m with increasing of 0.05m
   - Density=2400kg/m^3
   - Dimension Properties:-
     - Mono-block, L=2.5m, b=0.275m, d=0.203m, I=$1.917*10^{-4}$m^4
   - Stiffness:- Ks=∞, For concrete sleeper(Rigid body)
   - Elastic Modulus=7000Mpa
   - Poisson=0.3
3) Fasteners:-
   - Clip:- E-type rail fastening systems, With stiffness 100kN/mm
   - Rail pad:- Rail rubber pads used for E-type rail fastening systems with stiffness ranges from 40-450KN/mm and 146KN/mm is used for this thesis.
   - Faster Stiffness=K_{clip}+K_{pad}=(100+146)N/mm=246N/mm=246MN/m, parallel
4) Ballast, Sub-ballast and Sub-grade

Table 3-4: Sub structural Material Properties

Sub structural Material Properties							
Track Type	Track Components	Elastic Modulus (Mpa)	Poisson ratio (v)	Density (KN/m^3)	Thickness (m)	angle of friction (deg.)	Stiffness ,K (MN/m)
Track-1 (soft track bed)	Ballast	150	0.3	20	0.35	45	106
	Sub-ballast	75	0.35	19	0.15	35	43
	Sub-grade	20	0.4	16	-	27	9
Track-2 (Typical track bed)	Ballast	200	0.3	20	0.35	45	141
	Sub-ballast	100	0.35	19	0.15	35	57
	Sub-grade	40	0.4	16	-	27	18
Track-3 (stiff track bed)	Ballast	300	0.3	20	0.35	45	212
	Sub-ballast	150	0.35	19	0.15	35	86
	Sub-grade	60	0.4	16	-	27	27

3.2.7 Train Loads

From rolling stock specification [17] "HXD1C High Power AC Drive Six-Axle (7200kw) Freight Electric Locomotive" have been selected for Train Load.

i) Rolling Stocks Specifications [25]:

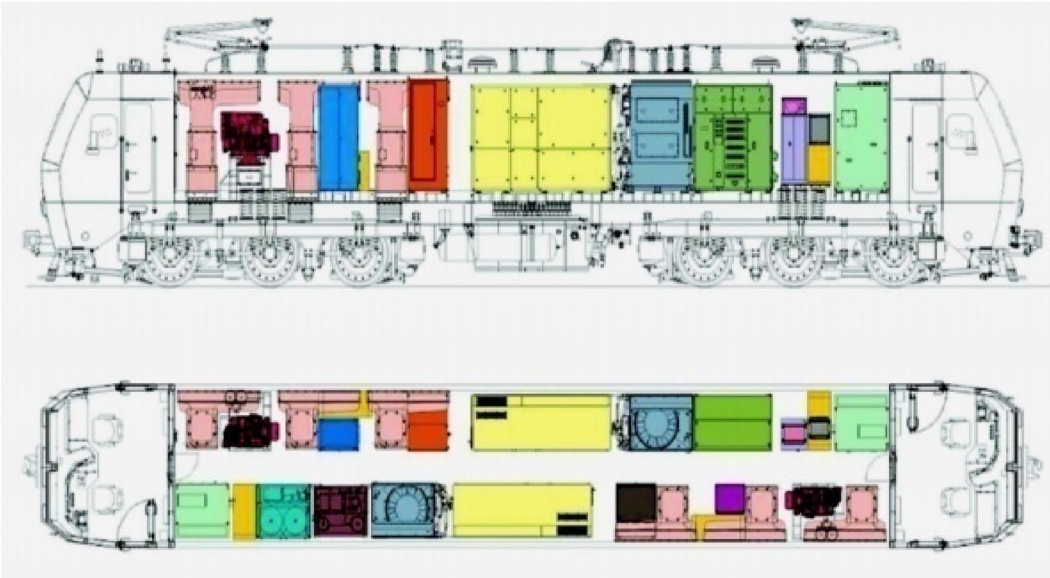


Figure 3-9: HXD1C High Power AC Drive Six-Axle (7200kw) Freight Electric Locomotive [25]

Table 3-5: Rolling Stocks Specification [25

Axle Type	CO-CO
Locomotive total weight in working order	150(+3)% t(25t axle load) 138(-1))% t(23t axle load
Wheel rim power	7200kw
Startup traction effort	570kN(25t axle load 520kN(23t axle load)
Continuous traction effort	400kN(25t axle load) 370kN(23t axle load)
Continuous speed	65 km/h(25t axle load) 70 km/h(23t axle load)
Max. electric braking effort	400kN(25t axle load) 370kN(23t axle load)

Table 3-6: Rolling Stocks Key Parameters and Dimensions [25]:

parameters	Dimension/Value
Loading Capacity	70t
Dead weight	≤25t
Volume	$145m^3$
Minimum Curve Radius	145m
Vehicle Length (at Coupler Connection Point)	≤1740mm
Height from Center of Coupler to Top of Rail	880mm

Table 3-7: Technical parameters of the bogie [25]:

Bogie Parameters	Dimension/Value
Fixed Wheel Base	1830mm
Side Bearing Center Distance	1520mm
Height from Cored Disk to Rail Surface	80mm
Wheel Diameter	840mm
Railway Gauge	1435mm
Axle Load	25t
Design speed	120km

Normally Axle Load of Train 25 tone =25*1000Kg*9.8m/s^2=245000N=245KN, each axle has two wheels, hence, Static load under wheel is 245KN/2=122.5KN.

Actually the designed wheel load should not be the static one as the train has speed, and naturally any moving body generates dynamic effects on the structures.

In the design of railway track scholars and researchers had investigated the impact factors which used to change the static wheel load to dynamic design load. Some of these impact factors are listed under chapter 2.

In this Thesis the "Clarke formula [22]" has been used, since it accounts the effects of Train speed (V), Wheel diameter (D) and Foundation modulus (μ) of the track for determining the "Wheel design load".

$$ø = 1 + \frac{19.65v}{D(\mu)^{0.5}} \qquad 3\text{-}1$$

$$\mathbf{Pd = øPo} \qquad 3\text{-}2$$

Where,

- V = vehicle speed (km/h),
- μ = track modulus (MPa) , and
- D = wheel diameter (mm).
- P = design wheel load (kN),
- P_o = static wheel load (kN), and
- Ø = dimensionless impact factor (always >l).

Having the design train speed of 120km/h , 840mm of wheel diameter , foundation modulus (it depends on sleeper space) and static wheel Load of 122.5KN, the value of impact factors and design wheel load have been calculated as below in table 3.6.

Table 3-8: Impact factors and wheel Design Loads

sleeper Spacing (m)	**Soft Track bed**			**Typical Track bed**			**Stiff Track bed**		
	Track Modulus (Mpa)	Impact factors (ø)	Wheel Design Load Pd (N)	Track Modulus (Mpa)	Impact factors (ø)	Wheel Design Load Pd (N)	Track Modulus (Mpa)	Impact factors (ø)	Wheel Design Load Pd (N)
0.50	27.207	1.538	188426	34.842	1.476	180757	48.996	1.401	171627
0.55	24.734	1.564	191644	31.674	1.499	183601	44.542	1.421	174025
0.60	22.673	1.590	194718	29.035	1.521	186318	40.83	1.439	176316
0.65	20.929	1.614	197667	26.801	1.542	188923	37.689	1.457	178514
0.70	19.434	1.637	200505	24.887	1.563	191431	34.997	1.475	180628

3.3 Sensitiveness analysis (Elasticity)

Elasticity is the sensitiveness or responsiveness of something (dependent Variables) for changes of independent variable. It is used to measure the change in quantity of dependent variable when independent variable changes other thing being equal (constant).

In order to check the response of rail deflection, rail supporting force and rail bending stress to the change of sleeper space their elasticity have been defined as below.

$$\boldsymbol{Elasticity} = \frac{\%\ \boldsymbol{of change\ \ dependent\ variable}\ (\boldsymbol{Y})}{\%\ \boldsymbol{of change\ \ independent\ variable}\ (\boldsymbol{X})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3\text{-}3$$

Where;

- Y= Rail deflection, Rail supporting force and Rail bending separately
- X=Sleeper space

CHAPTER 4 ANALYSIS

4.1 Analytical (BOEF) Analysis

The track was modeled as BOEF and computed for rail bending stress, rail deflection and rails supporting force for different sleeper spaces and different track beds. The analytical Solutions obtained from analysis of BOEF are illustrated in table 4.1; and figures 4-1, 4-2 and 4-3 below.

Table 4-1: Results from Analytical Analysis

Sleeper space	Rail Deflection (mm)			Rail supporting force (KN)			Bending Stress (Mpa)		
	Soft Track bed	Typical Track bed	Stiff Track bed	Soft Track bed	Typical Track bed	Stiff Track bed	Soft Track bed	Typical Track bed	Stiff Track bed
0.50	4.254	3.205	2.192	65.512	61.827	56.949	109.88	101.54	91.5
0.55	4.751	3.572	2.435	66.96	63.234	58.286	113.38	104.63	94.16
0.60	5.258	3.946	2.681	68.295	64.533	59.526	116.73	107.59	96.68
0.65	5.775	4.328	2.933	69.533	65.739	60.681	119.75	110.42	99.08
0.70	6.301	4.717	3.189	70.688	66.866	61.762	123.05	113.15	101.4

4.1.1 Parameter Study Based on Analytical Analysis

The BOEF analysis shows that with increasing the sleeper spaces the rail deflection, rail supporting force and rail bending Stress also increases linearly as given in figures 4-1, 4-2, and 4-3 respectively.

From the material quality properties, it is observed that as quality of track bed decreases the rail deflection, rail supporting force and rail bending stress increases. This one is occurred due to the degree of distribution of wheel load from rail to sub-grade. Soft track bed has the lowest degree distribution of wheel load to the sub-grade that is why it shows highest deflection. Only few numbers of sleepers in effective length shares the highest percent of wheel load for transferring it to sub-grade. Thus the critical sleeper which shares higher values of the load than that of others will be leading to higher deflection. In another hand, stiff track bed has the highest degree of distribution of wheel load to the sub-grade that is why it shows lowest deflection. A larger numbers of sleepers participate in transferring the wheel load almost uniformly to sub-grade than that of soft and typical track beds. Thus, the critical sleeper shares lower values of the load than that of others (Soft and typical track beds) which leads to lower deflection and lower rail bending stress.

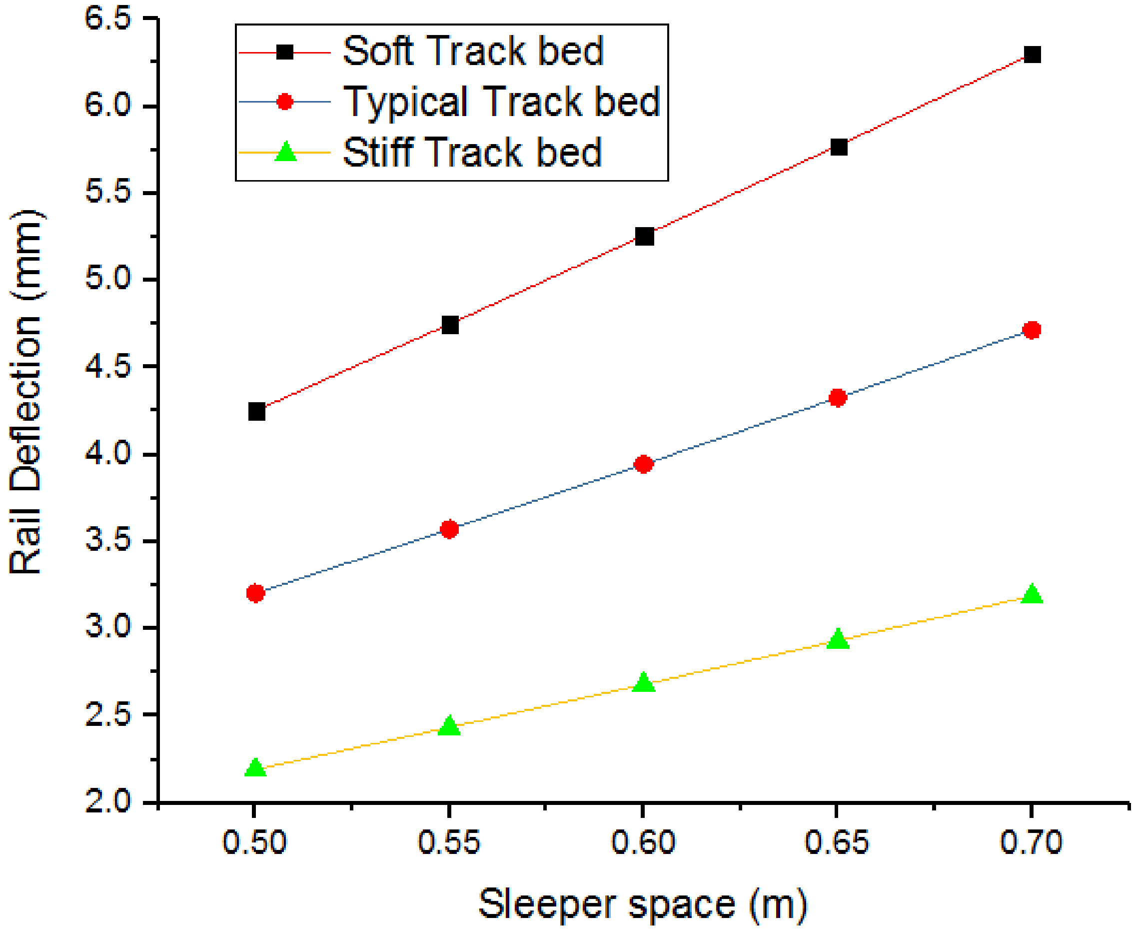


Figure 4-1: Rail deflection analysis (BOEF)

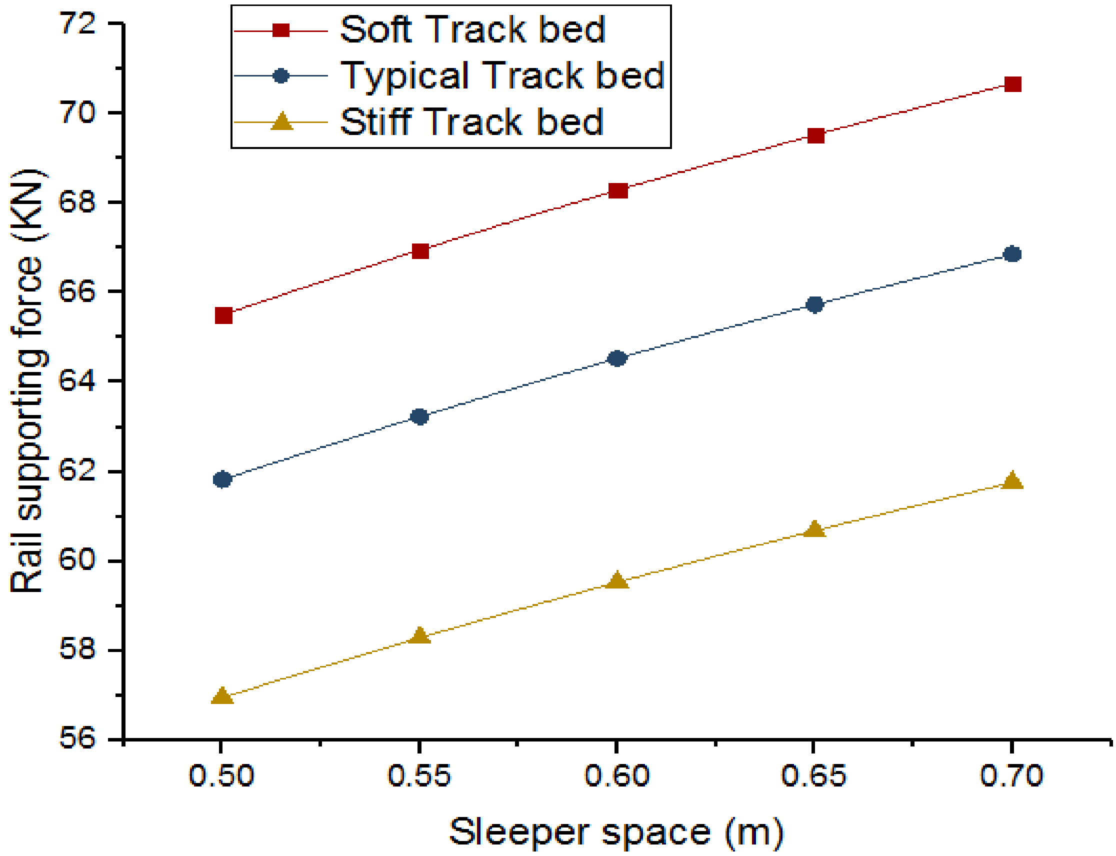


Figure 4-2: Rail Supporting Force analysis (BOEF)

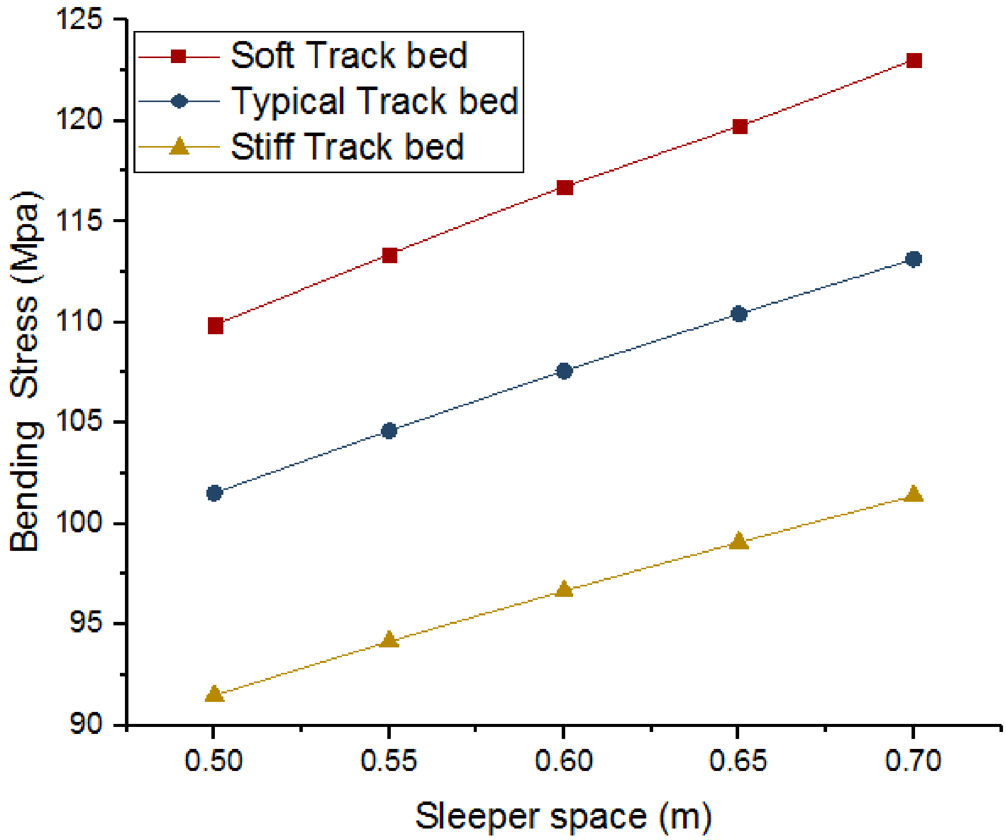


Figure 4-3: Rail bending stress analysis (BOEF)

The Sensitive analysis of rail deflection decreases with increasing of track bed quality and more sensitive at higher sleeper space. On other hand the rail deflection is more sensitive under soft track bed than typical and stiff track beds. Table 4-2 shows sensitiveness of rail deflection to sleeper space.

Table 4-2: Sensitiveness of Rail deflection to sleeper space (BOEF output)

Sleeper space(m)	Sensitiveness of Rail Deflection to sleeper space		
	Soft Track bed	Typical Track bed	Stiff Track bed
0.50	-	-	-
0.55	1.151	1.130	1.098
0.60	1.157	1.137	1.101
0.65	1.164	1.147	1.117
0.70	1.169	1.155	1.124
Average	**1.160**	**1.142**	**1.110**

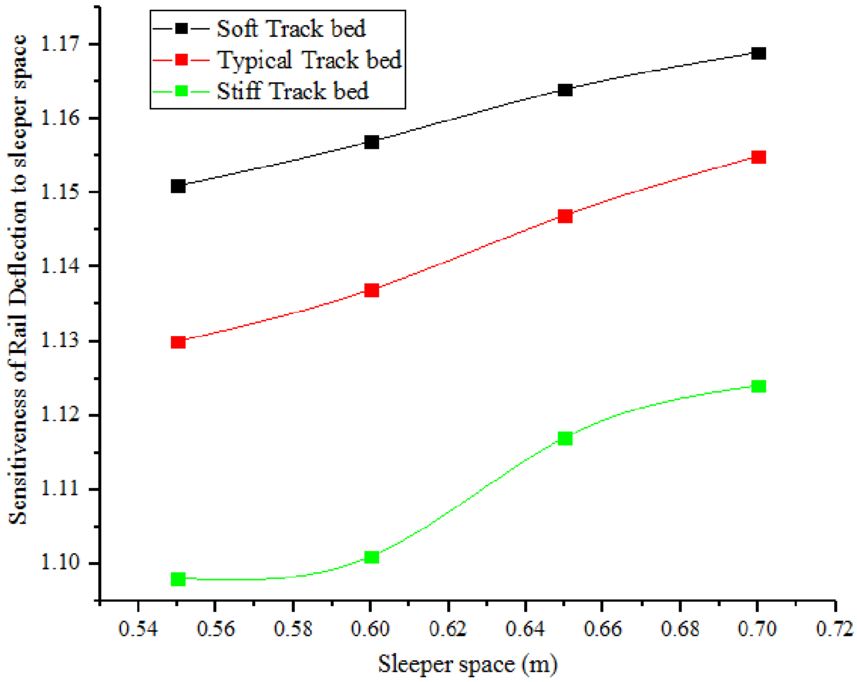


Figure 4-4: Sensitiveness of Rail deflection to sleeper space (BOEF output)

Like rail deflection as listed in table 4-3 the sensitiveness of rail bending stress also decreases with increasing of track bed quality and increase with increasing of sleeper space. On other hand the sensitiveness of rail supporting force increase with increasing of track bed quality and decrease with increasing of sleeper space.

Table 4-3: Sensitiveness of Rail supporting force and rail bending stress to sleeper space (BOEF output)

Sleeper space(m)	Sensitiveness of Rail supporting force to sleeper space			Sensitiveness of Bending Moment Stress		
	Soft Track bed	Typical Track bed	Stiff Track bed	Soft Track bed	Typical Track bed	Stiff Track bed
0.50	-	-	-	-	-	-
0.55	0.238	0.245	0.252	0.340	0.325	0.311
0.60	0.235	0.242	0.250	0.344	0.330	0.313
0.65	0.231	0.238	0.247	0.360	0.333	0.315
0.70	0.229	0.236	0.245	0.375	0.338	0.320
Average	**0.233**	**0.240**	**0.249**	**0.355**	**0.331**	**0.315**

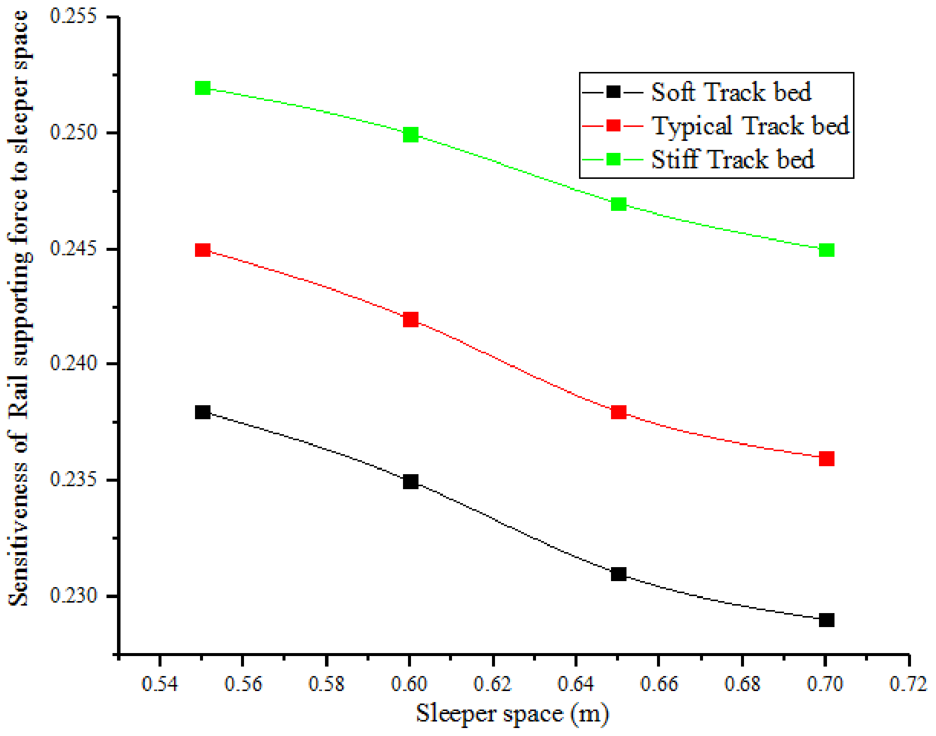


Figure 4-5: Sensitiveness of Rail supporting force to sleeper space (BOEF output)

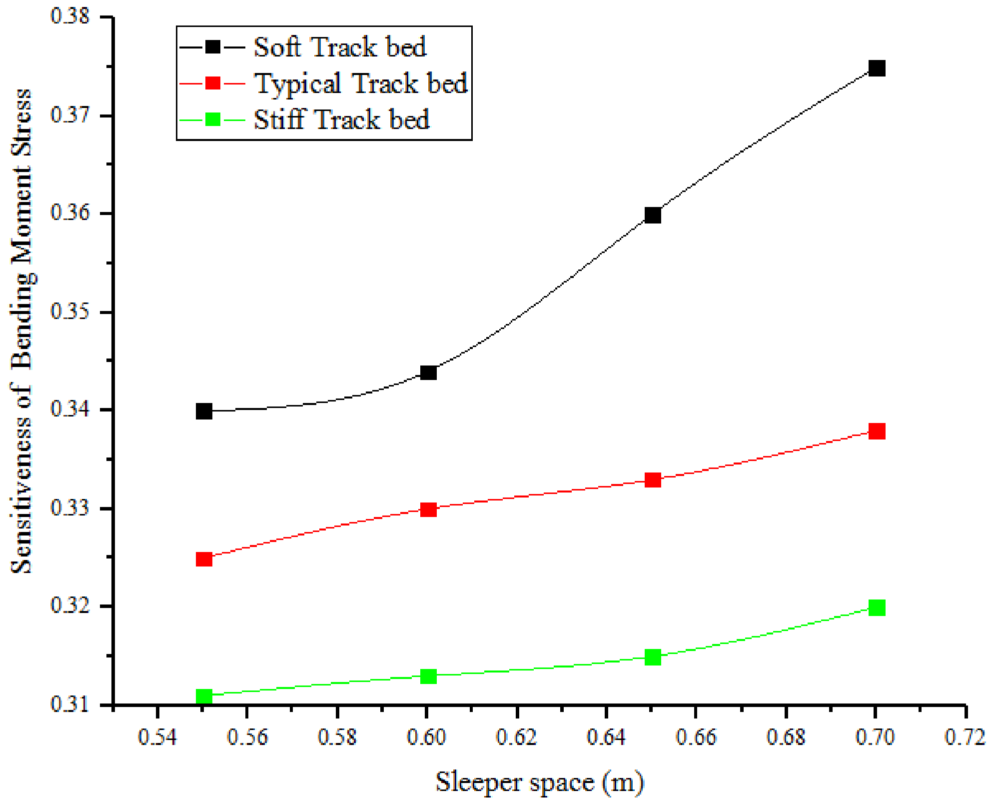


Figure 4-6: Sensitiveness of Rail bending stress to sleeper space (BOEF output)

4.1.2 Summary of Analytical (BOEF) Analysis

In general Analytical analysis shows that:-

[1] With increasing of Track bed materials quality (from soft track bed to stiff track bed) the rail deflection, rail supporting force and rail bending stress decreases.

[2] With increasing of Sleeper space, rail supporting force and rail bending stress also increases.

[3] The rail deflection, rail supporting force and rail bending stress are more sensitive to sleeper space under soft track bed than typical and stiff track beds.

[4] Rail deflection and rail bending stress are less sensitive at lower sleeper space than at higher Sleeper space while rail supporting force is more sensitive at lower sleeper space.

4.2 FEM Analysis

Static analysis has been selected for the analysis, while dynamic design load is used. In static analysis the position of load has its factor for determination of rail deflection, rail supporting force and rail bending stress. That is the position at which maximum deflection is found, maximum rail supporting force may not found. Thus to find maximum rail supporting force and maximum rail deflection the track is modeled separately

A 2D-track have been modeled in ANSYS for different track length(20m, 10m and 5m), track properties (Soft track bed, typical track bed, stiff track bed);and at different sleeper spacing (0.50m, 0.55m, 0.60m, 0.6m and 0.70m).

From the three track length, the optimum track length has been selected for FEM analysis. To select the optimum rail track two types of boundary condition were set. The length which gives almost the same output under both boundary conditions is the optimum length of rail track.

Boundary Condition-1:

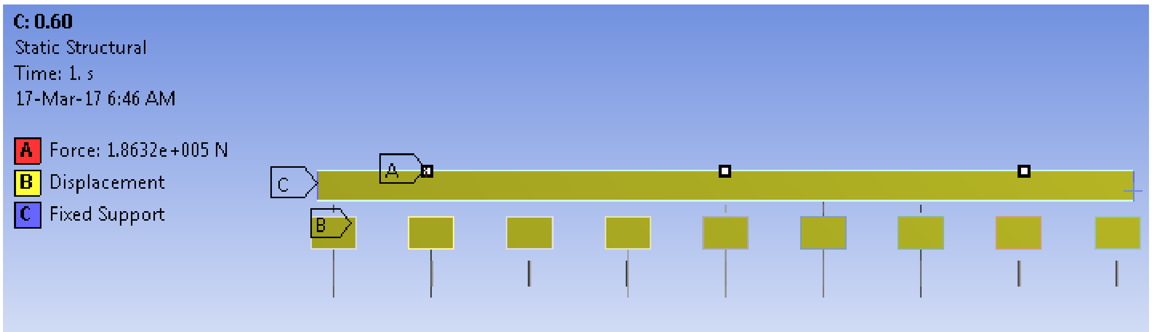


Figure 4-7: Boundary Condition-1

- Fixing the rail at the end
- Sleeper, ballast, sub-ballast and sub-grade are restricted from displacing into “x” directions while they are allowed free in “y” direction.

As known the rail has infinite length. Most of the designers used to fix the rail at the ends for analysis. The drawback of this boundary condition is; for rail track length lower than the optimum length, the wheel load which should have to transferred to the sleeper will be supported by the end support as reaction force, thus the seating load will be reduced from the real.

Boundary Condition-2:

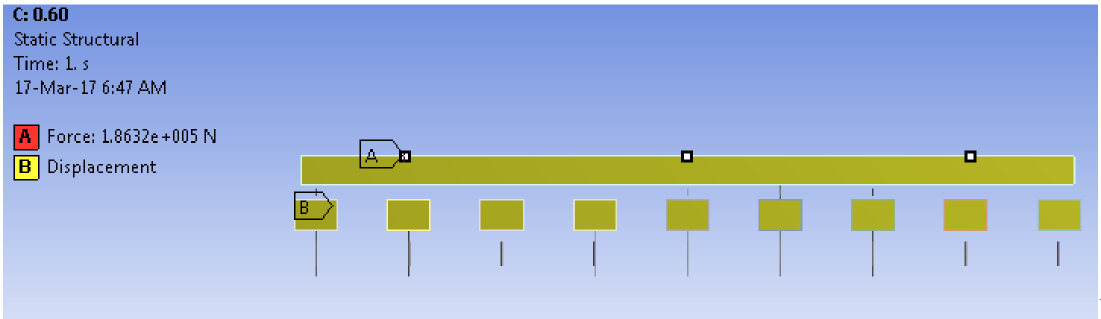


Figure 4-8: Boundary condtion-2

- Sleeper, ballast, sub-ballast and sub-grade are restricted from displacing into "x" directions while they are allowed free in "y" direction.

This type of boundary condition gives higher burden to the sleeper as all wheel loads directly transferred to it. For lower length of track, there will be few numbers of sleepers under the rail. Under this condition it will give high value of deflection and rail supporting force. As the length of the track increase the burden on sleeper will decreases.

As described on figure 4-9 as the length of rail track increases the analysis of output converges to the real result for both boundary conditions.

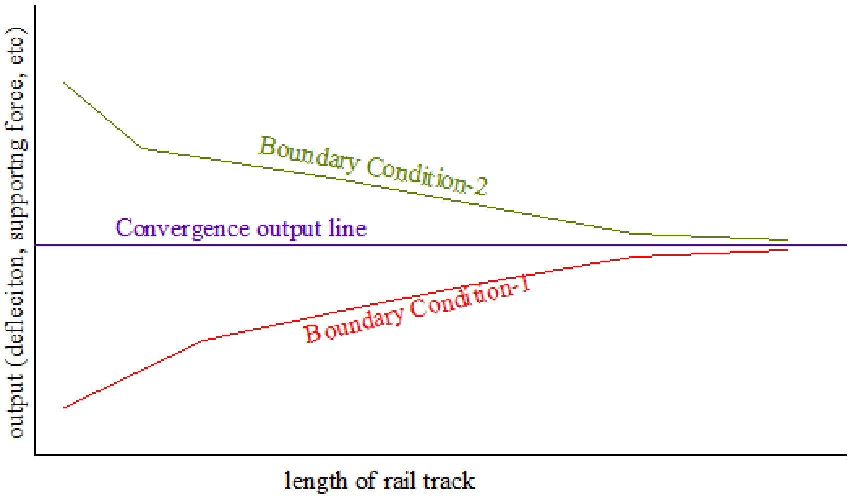


Figure 4-9: Rail length versus boundary conditions (General)

The tracks have been analyzed under both boundary conditions for track lengths of 5m, 10m and 20m. The output from both boundary conditions was compared and the optimum track length has been selected.

Figure 4-11 and 4-12 shows the total deformation of rail track length of 20m for typical track bed and at sleeper space of 0.6m. It is observed that both boundary conditions give almost the same total deformation at 20m track length.

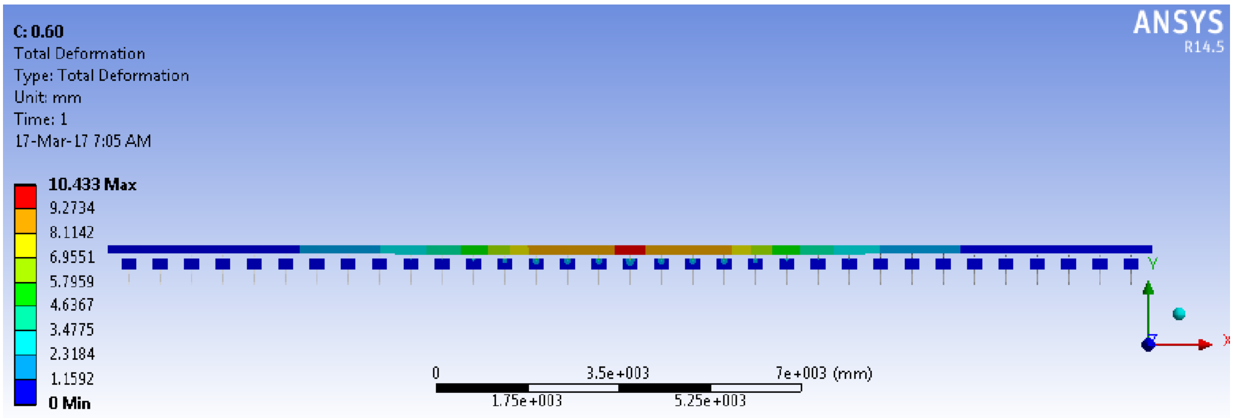


Figure 4-10: Boundary condtion-1 result (20m track length, Model-2, Typical track bed)

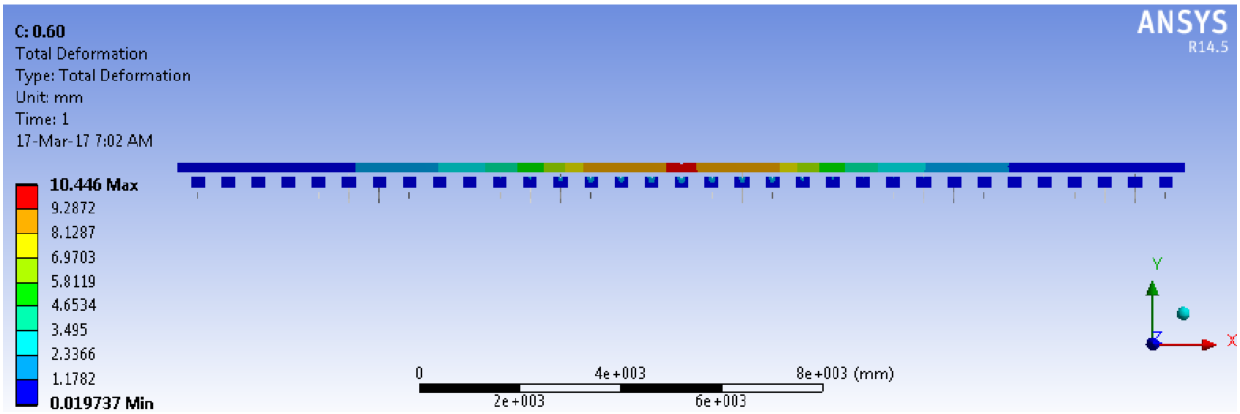


Figure 4-11: Boundary condtion-2 results (20m track length, Model-2, Typical track bed)

Generally, Figures 4-12, 4-13 and 4-14 show the output of FEM models analysis under both boundary conditions for typical track bed at 0.60m sleeper space.

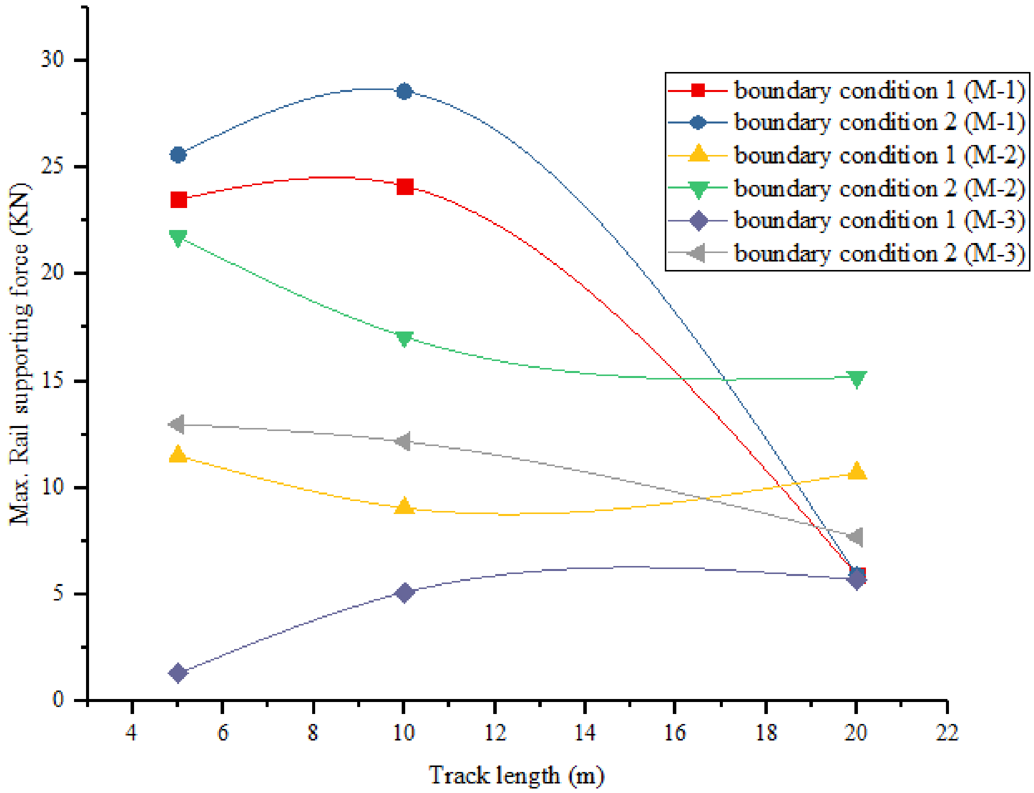


Figure 4-12: FEM output based on Track Length (Rail supporting force)

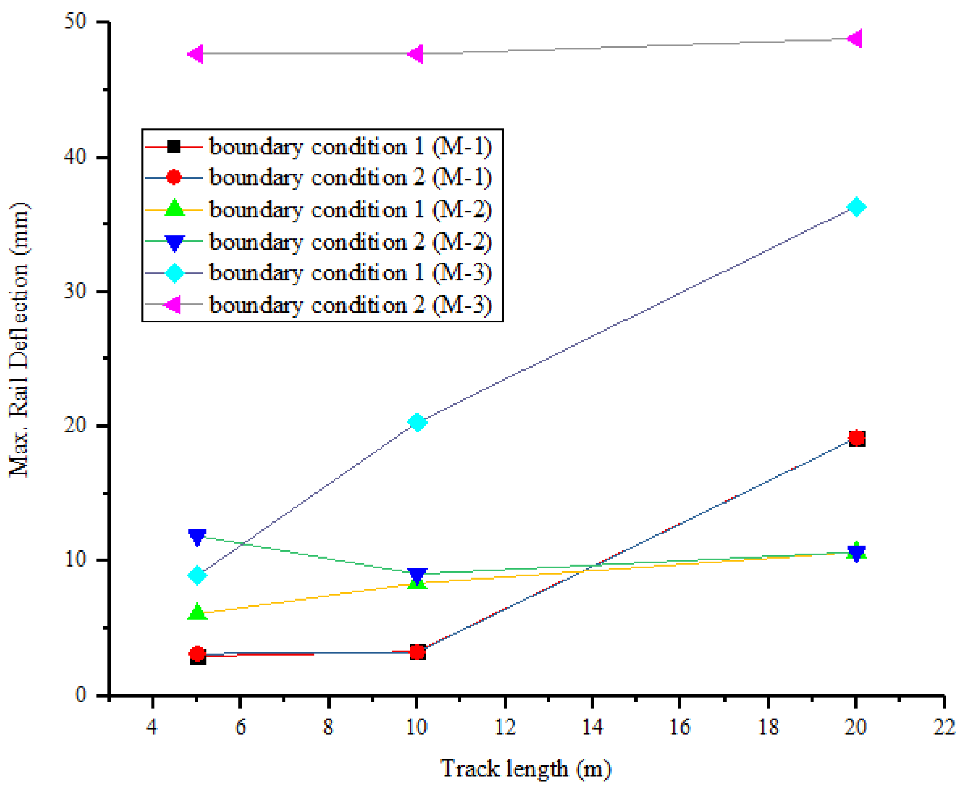


Figure 4-13: FEM output based on Track Length (Rail deflection)

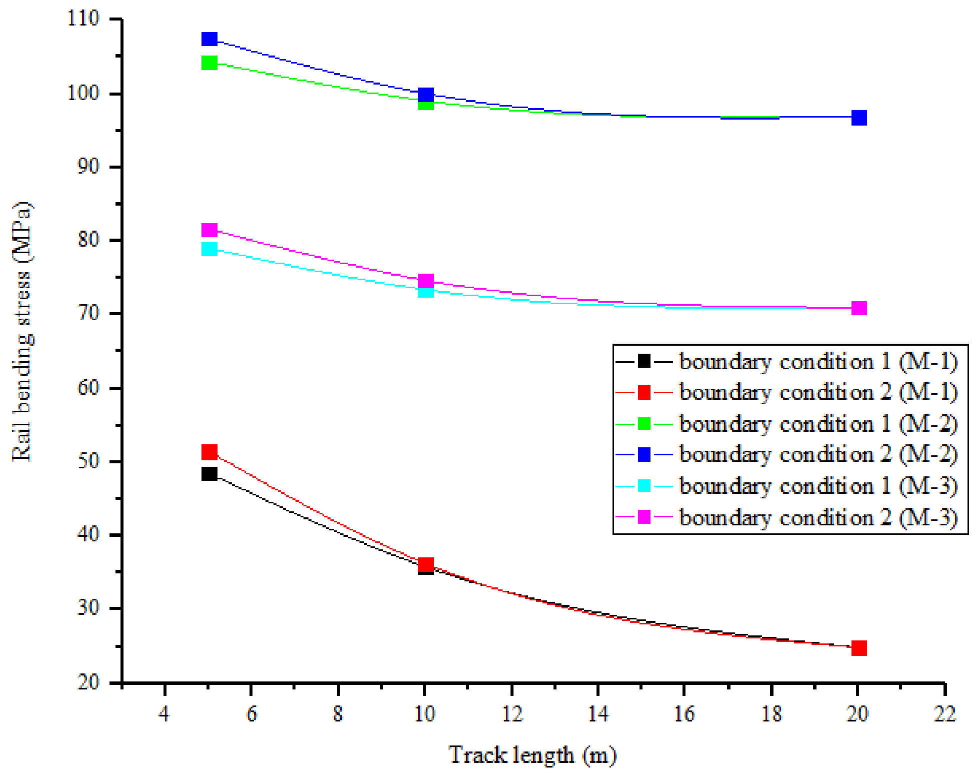


Figure 4-14: FEM output based on Track Length (Rail bending stress)

Based on figures 4-12, 4-13 and 4-14 results of both boundary conditions; a track length of 20m gives almost the same results under both boundary conditions. Thus, a 20m track length has been selected as optimum track length for analysis for all proposed FEM models.

The analysis was done for a single axle load and multiple wheel loads where as results from multiple wheel loads are recommended for design even though single wheel load gives higher value of deflection and rail supporting force. It is known that there is no single wheel rail truck that used the line; that is why single wheel load analysis results are not recommended for design from FEM analysis.

Results of each analysis are shown graphically on figures 4-15, 4-16 and 4-17 for rail deflection, rail supporting force and rail bending stress respectively of 20m track length under multiple wheel loads analysis.

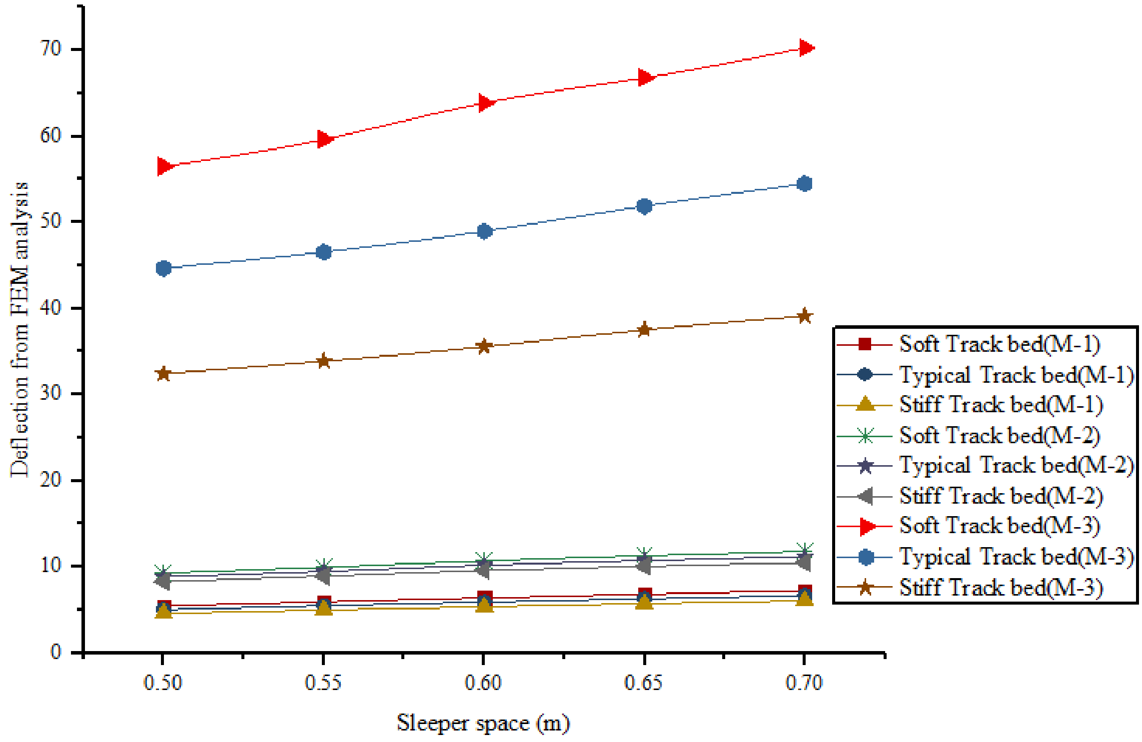


Figure 4-15: Deflection from FEM analysis (mm), 20m track length

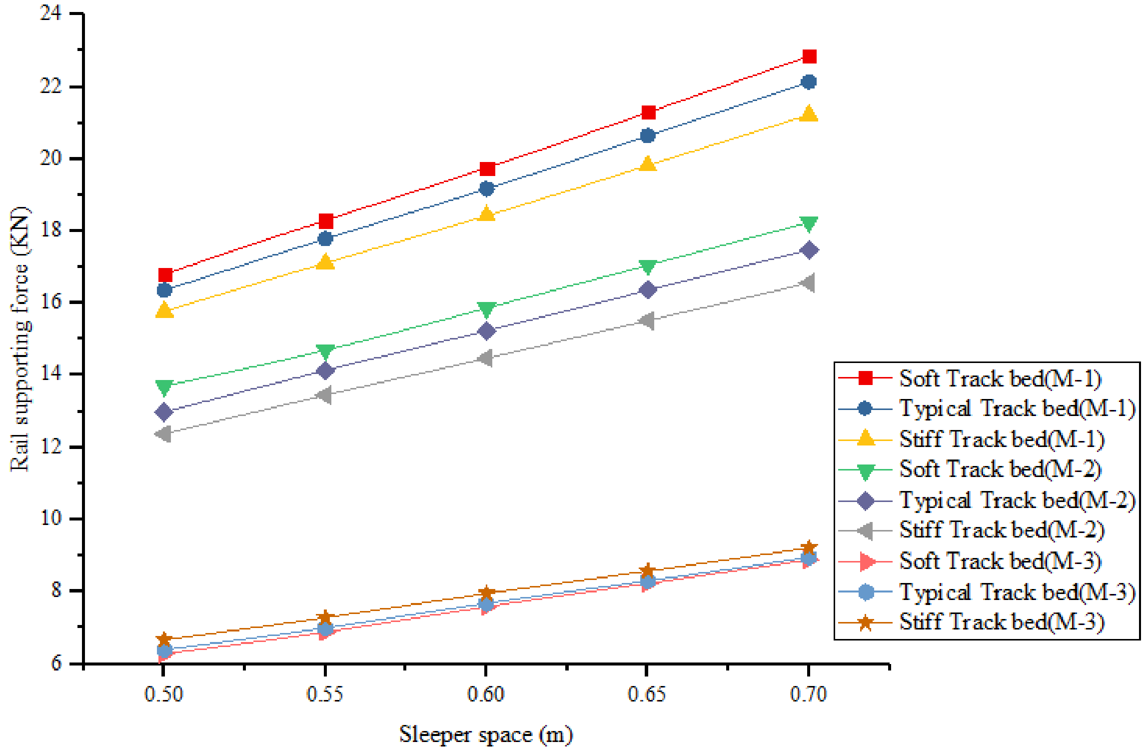


Figure 4-16: Rail supporting force from FEM analysis (KN), 20m track length

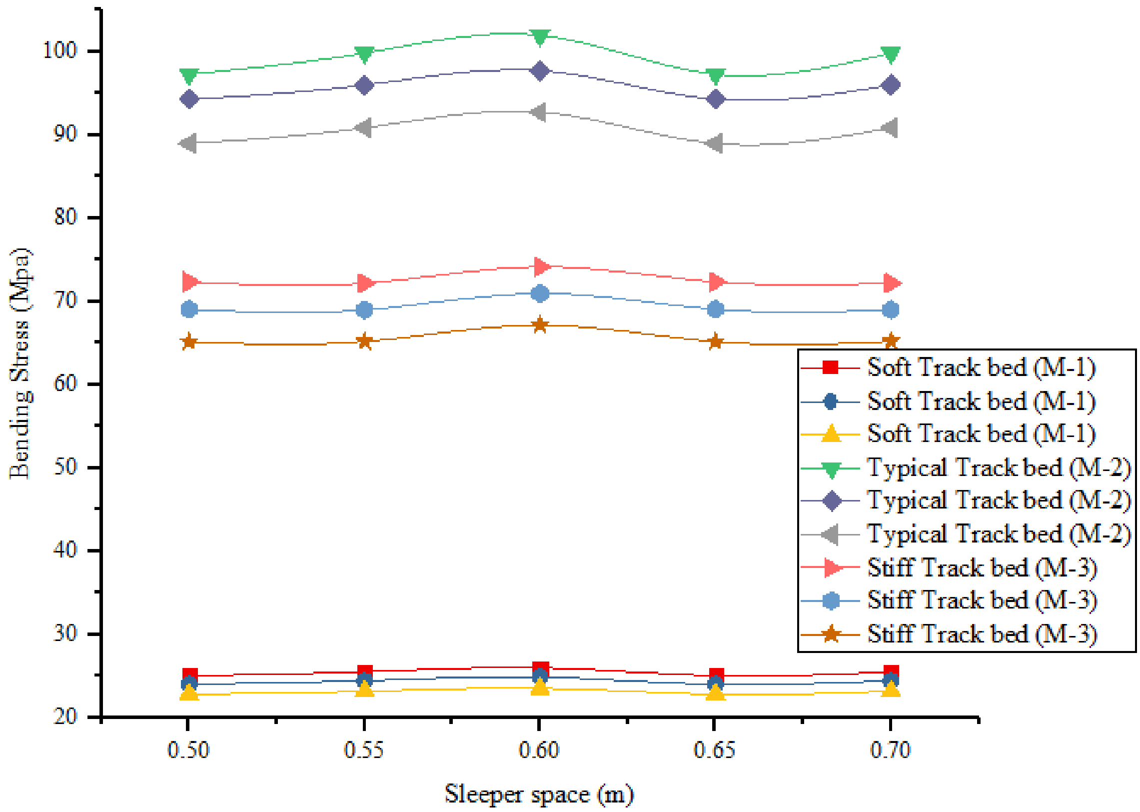


Figure 4-17: Bending Stress from FEM analysis (Mpa), 20m track length

From the above figures it is observed that each model gives different values for the same track properties and loading. Model-3 gives the highest deflection and lowest values of rail supporting force while model-1 gives the lowest values of deflection and highest values of rail supporting force when compared together. The question that might be rise here is: “which model is true or which values shall be taken for the next analysis/Design?”

To answer this question a lot work have been done. But lastly, it was arrived that model-1 and model-2 are selected for next analysis based on the purpose of the use.

It is remembered for selecting the rail track length; two types of boundary condition were set. On that analysis, Model-3 gives much different values for both boundary conditions while model-1 and model-2 gives relatively the same values for both boundary conditions at all proposed track lengths (refer figures 4-5, 4-6 and 4-7). One can easily see that model-3 is the poorest model of the three models.

Table 4-4: Recommended Models

FEM Model to be used	Purpose of the Analysis
Model-1	❖ For Sleeper stress analysis/ for sleeper design ❖ For the analysis of rail track components deflection
Model-2	❖ For the analysis of rail track components deflection ❖ For rail bending stress analysis

4.2.1 Parameter Study Based on FEM Analysis

Under section 4-2, it is discussed that results of model-1 and model-2 are selected for the next analysis while results of model-3 are eliminated since it gives exaggerated results.

Results from FEM analysis shows that with increasing of sleeper space the rail deflection, rail supporting force and rail bending stress also increased linearly (Refer Figure 4-18, 4-19 and 4-20 below).

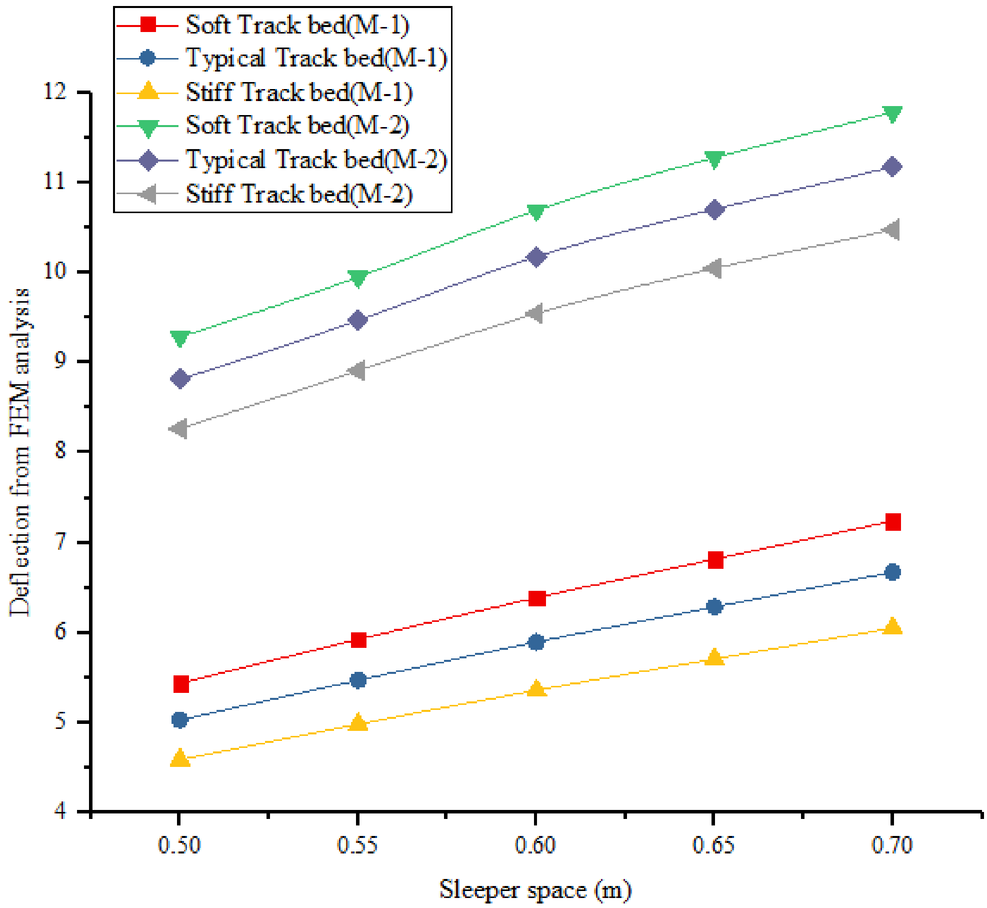


Figure 4-18: Rail deflection, FEM output

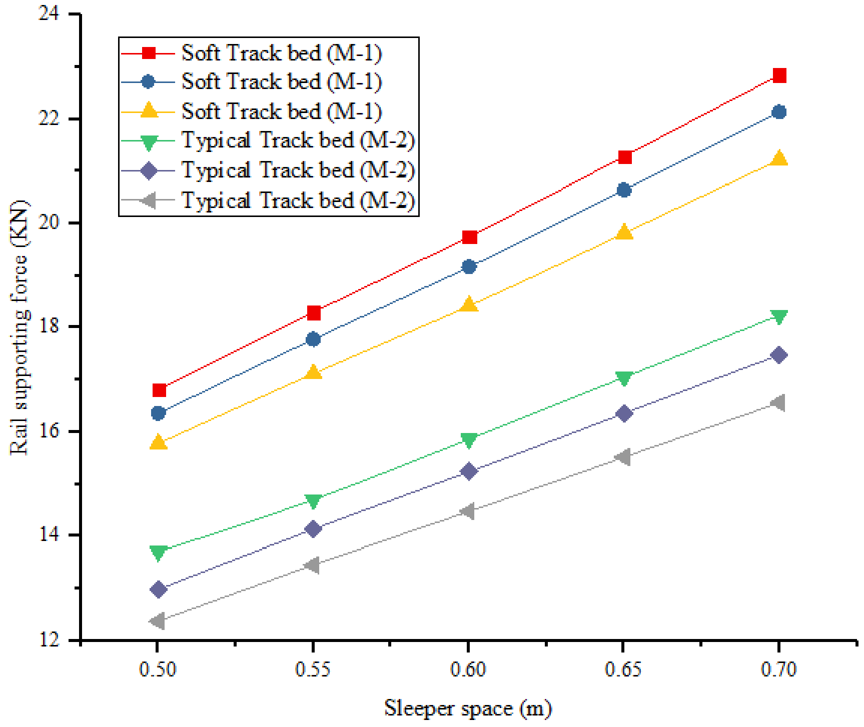


Figure 4-19: Rail Supporting Force, FEM output

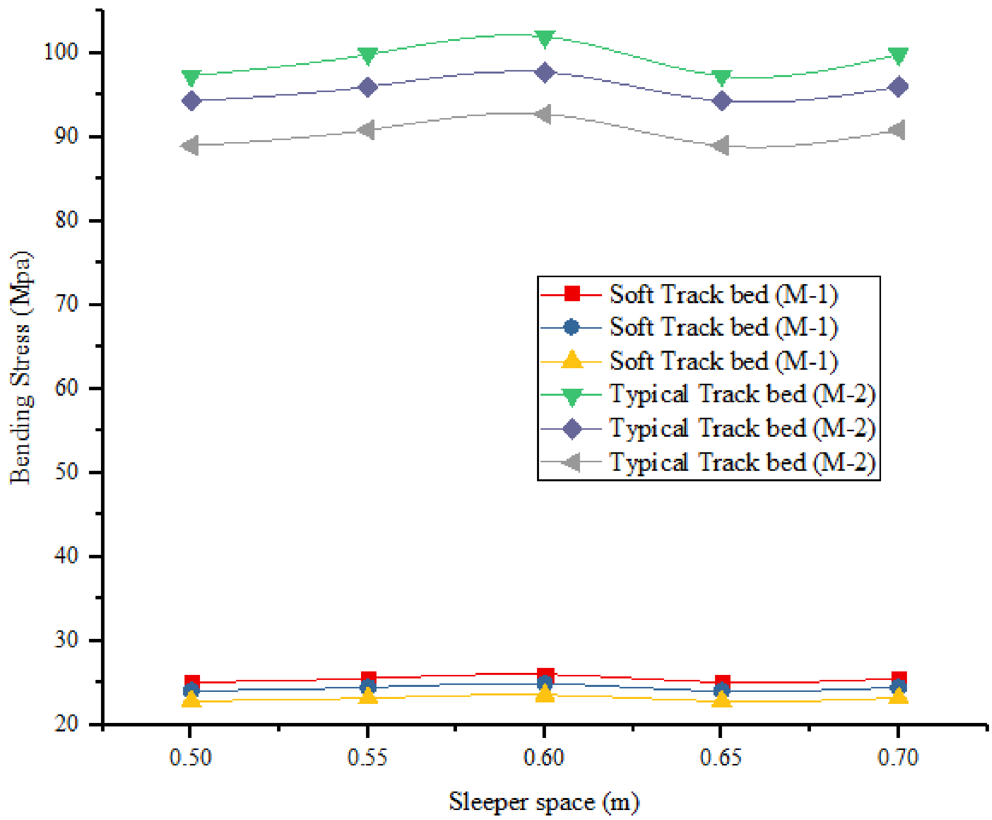


Figure 4-20: Rail Bending Stress, FEM output

Sensitive analysis shows that rail deflection is high sensitive to low quality of track beds and lower sleeper space. With increasing of sleeper space and track bed quality rail deflection sensitiveness will be decreased. Table 4-5 lists the average value of rail deflection increased for each increase of sleeper space by 0.05m.

Table 4-5: Sensitiveness of Rail deflection, FEM output

Sensitiveness of Rail Deflection to sleeper space						
Sleeper space (m)	Model-1			Model-2		
	Soft Track bed	Typical Track bed	Stiff Track bed	Soft Track bed	Typical Track bed	Stiff Track bed
0.50	-	-	-	-	-	-
0.55	0.903	0.891	0.875	0.739	0.760	0.796
0.60	0.876	0.865	0.851	0.831	0.835	0.799
0.65	0.818	0.807	0.791	0.675	0.644	0.652
0.70	0.822	0.814	0.803	0.603	0.589	0.572
Average	**0.855**	**0.844**	**0.830**	**0.712**	**0.707**	**0.705**

Generally, Sensitive analysis shows that rail supporting force is high sensitive to low quality of track beds and higher sleeper space. With increasing of sleeper space rail supporting force sensitiveness will be increased while with increasing track bed quality it will be decreased. Table 4-6 lists the average value of rail supporting force increased for each increase of sleeper space by 0.05m.

Table 4-6: Sensitiveness of Rail supporting force, FEM output

Sensitiveness of Rail supporting force to sleeper space, KN						
Sleeper space (m)	Model-1			Model-2		
	Soft Track bed	Typical Track bed	Stiff Track bed	Soft Track bed	Typical Track bed	Stiff Track bed
0.50	-	-	-	-	-	-
0.55	0.894	0.880	0.863	0.744	0.898	0.880

0.60	0.882	0.868	0.850	0.880	0.866	0.849
0.65	0.939	0.928	0.913	0.906	0.856	0.842
0.70	0.955	0.944	0.928	0.912	0.810	0.798
Average	**0.917**	**0.905**	**0.888**	**0.861**	**0.858**	**0.842**

4.2.2 Summary of FEM Analysis

In general Numerical analysis shows that:-

a) With increasing of Track bed materials quality (from soft track bed to stiff track bed) the rail deflection, rail supporting force and rail bending stress decreases.
b) With increasing of Sleeper space, rail deflection, rail supporting force and rail bending stress also increases.
c) The rail deflection, rail supporting force and rail bending stress are more sensitive to sleeper space under soft track bed than typical and stiff track beds.
d) Rail deflection is high sensitive at lower sleeper space and lower track quality.
e) With increasing of sleeper space rail supporting force sensitiveness will be increased while with increasing track bed quality it will be decreased.

4.3 Comparison of Analytical and FEM models results

Under sections 4-1 and 4-2, the tracks were modeled and analyzed for both BOEF and numerical (FEM) of the same track components. It is observed that BOEF and FEM analysis gave different values of maximum rail deflection, maximum rail bending stress and maximum rail supporting force for the same track components and properties. Each of the differences has been discussed below.

a) Differences on deflection

It is observed that the FEM (Numerical analysis) gives higher values of rail deflection than BOEF (Analytical analysis). The differences depend on the type of FEM model and the track bed quality. Generally as quality of track bed increases the differences of rail deflection between numerical analysis and analytical analysis increases. Figures 4 -21 and 4-22 below show the ratio of deflection results from Analytical analysis and numerical analysis.

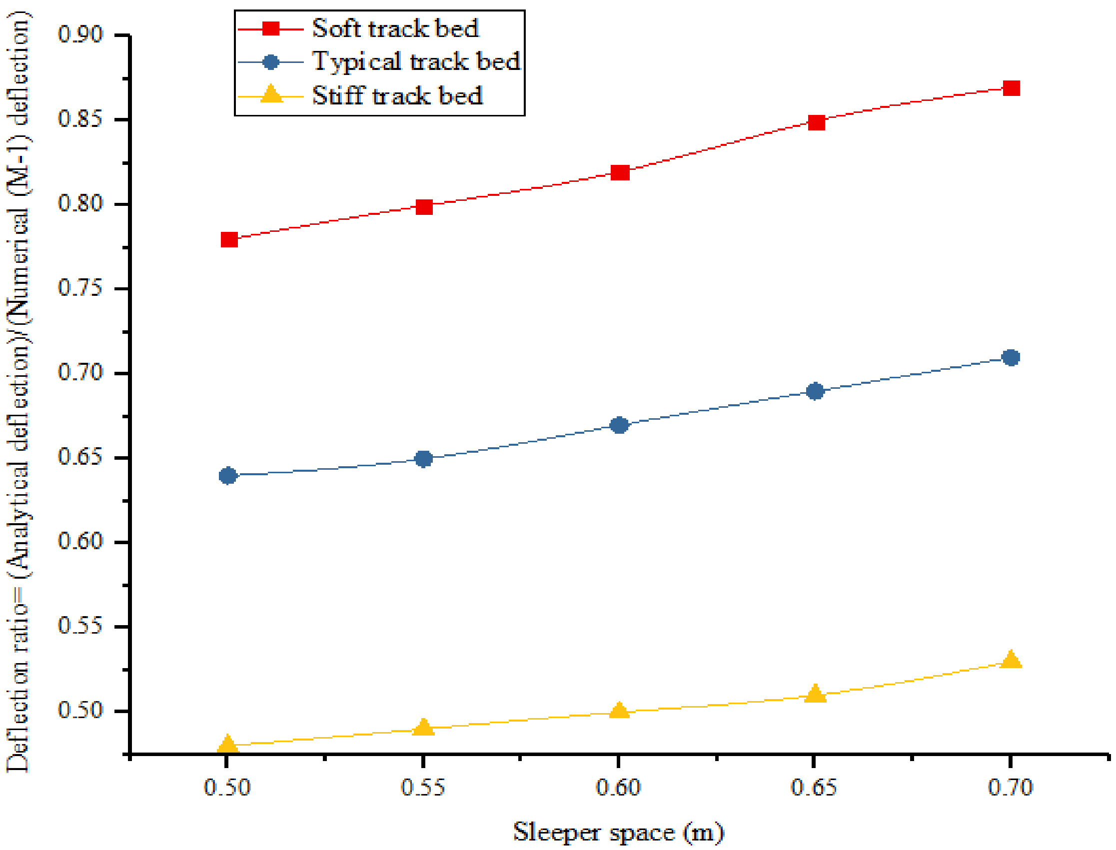


Figure 4-21: Rail deflection ratio (analytical VS numerical (Model-1))

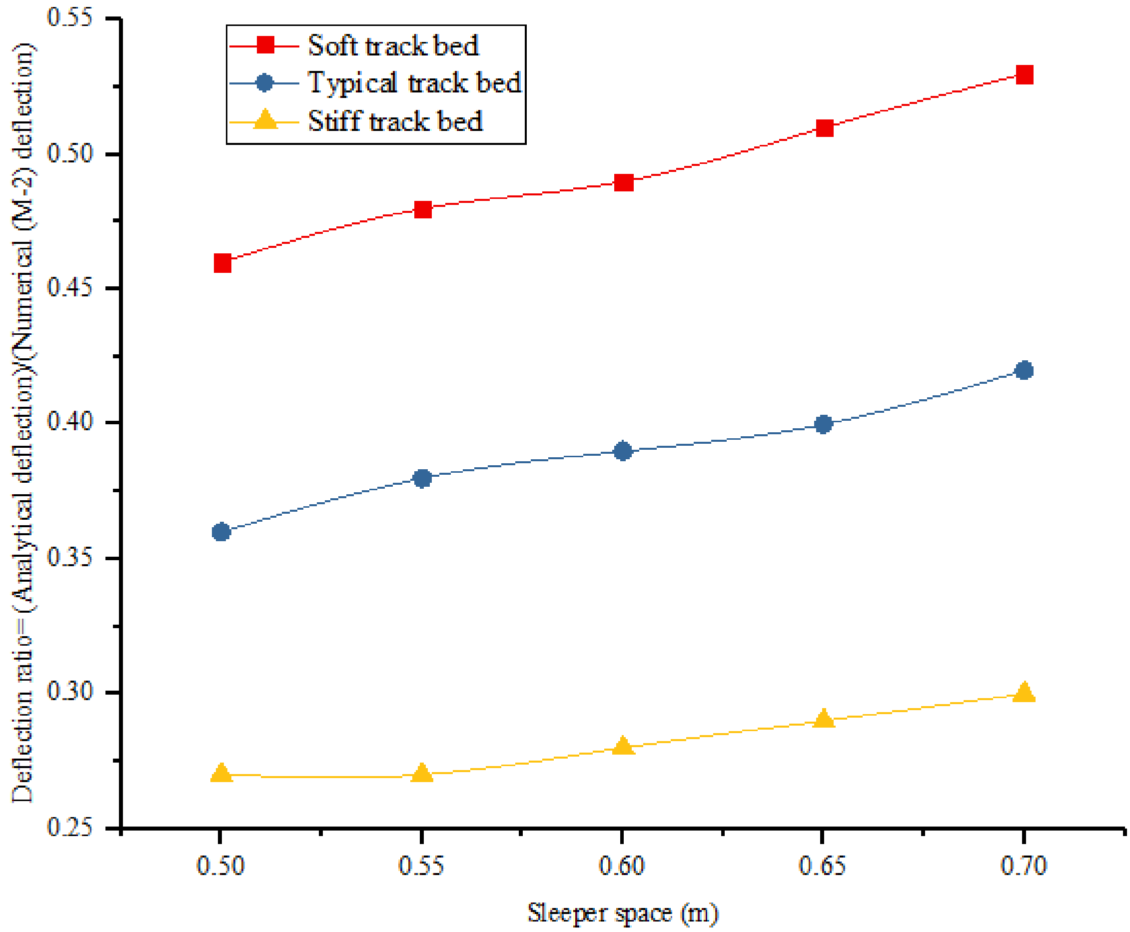


Figure 4-22: Rail deflection ratio (analytical VS numerical (Model-2))

a) Differences On Rail Supporting Force

Unlike Rail deflection, it is observed that the FEM (Numerical Analysis) gives lower values of rail supporting force than BOEF (Analytical analysis). However like rail deflection, the differences of rail supporting forces depend on the type of FEM model and the track bed quality.

Generally as quality of track bed increases the differences of rail Supporting Force between numerical analysis and analytical analysis decreases. Figures 4-23 and 4-24 below show the ratio of Rail supporting force results from analytical analysis and numerical analysis.

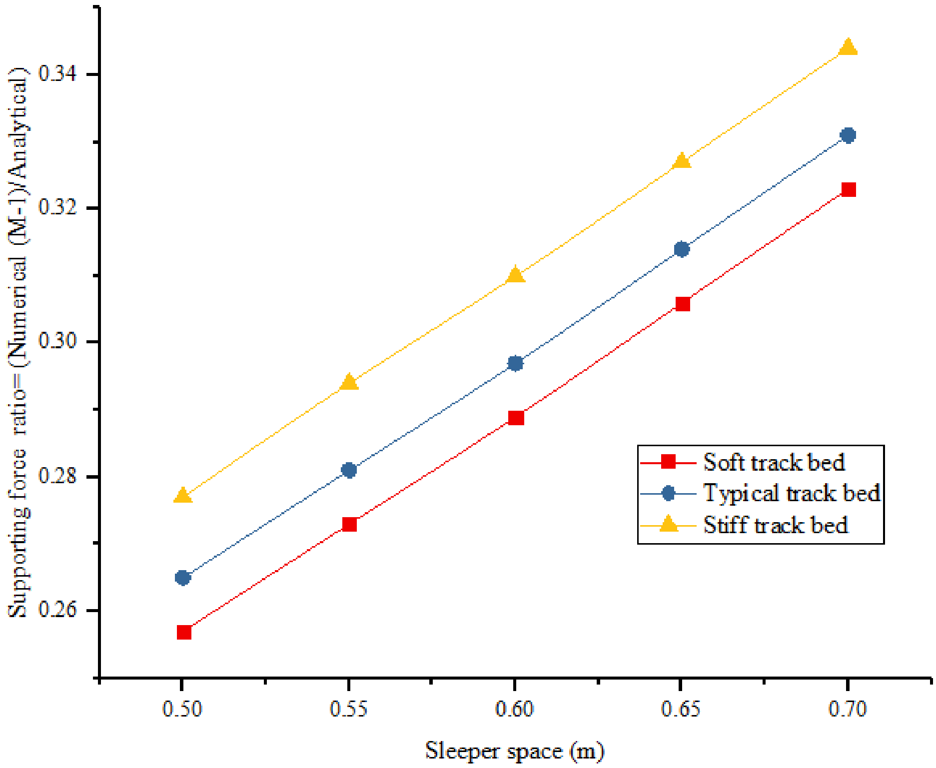


Figure 4-23: Rail Supporting Force ratio (analytical VS numerical (Model-1))

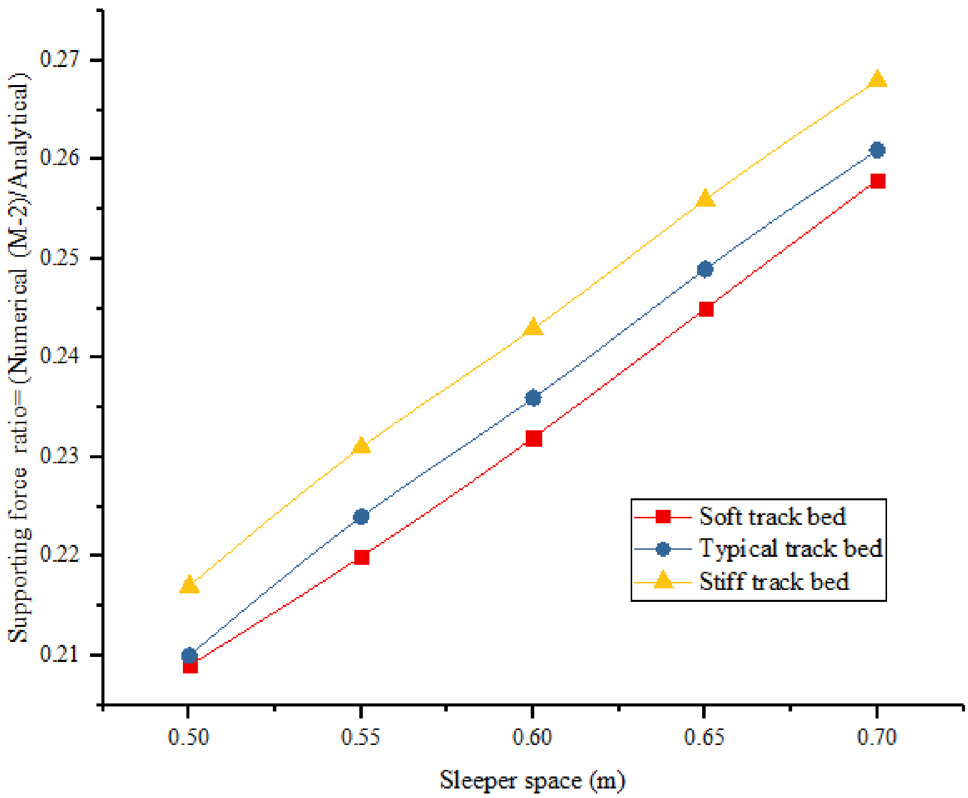


Figure 4-24: Rail Supporting Force ratio (analytical VS numerical (Model-2))

b) Differences on Bending Stresses

Model_1 Gives the lowest rail bending stress and analytical analysis gives the highest bending stress while Model_2 gives moderate rail bending stress when compared together. On other hand model-2 gives almost the same rail bending stress as analytical analysis. Generally analytical analysis gives higher values of rail bending stresses than numerical analysis.

Figure 4-25 below concludes the differences on bending stresses of analytical and numerical analysis.

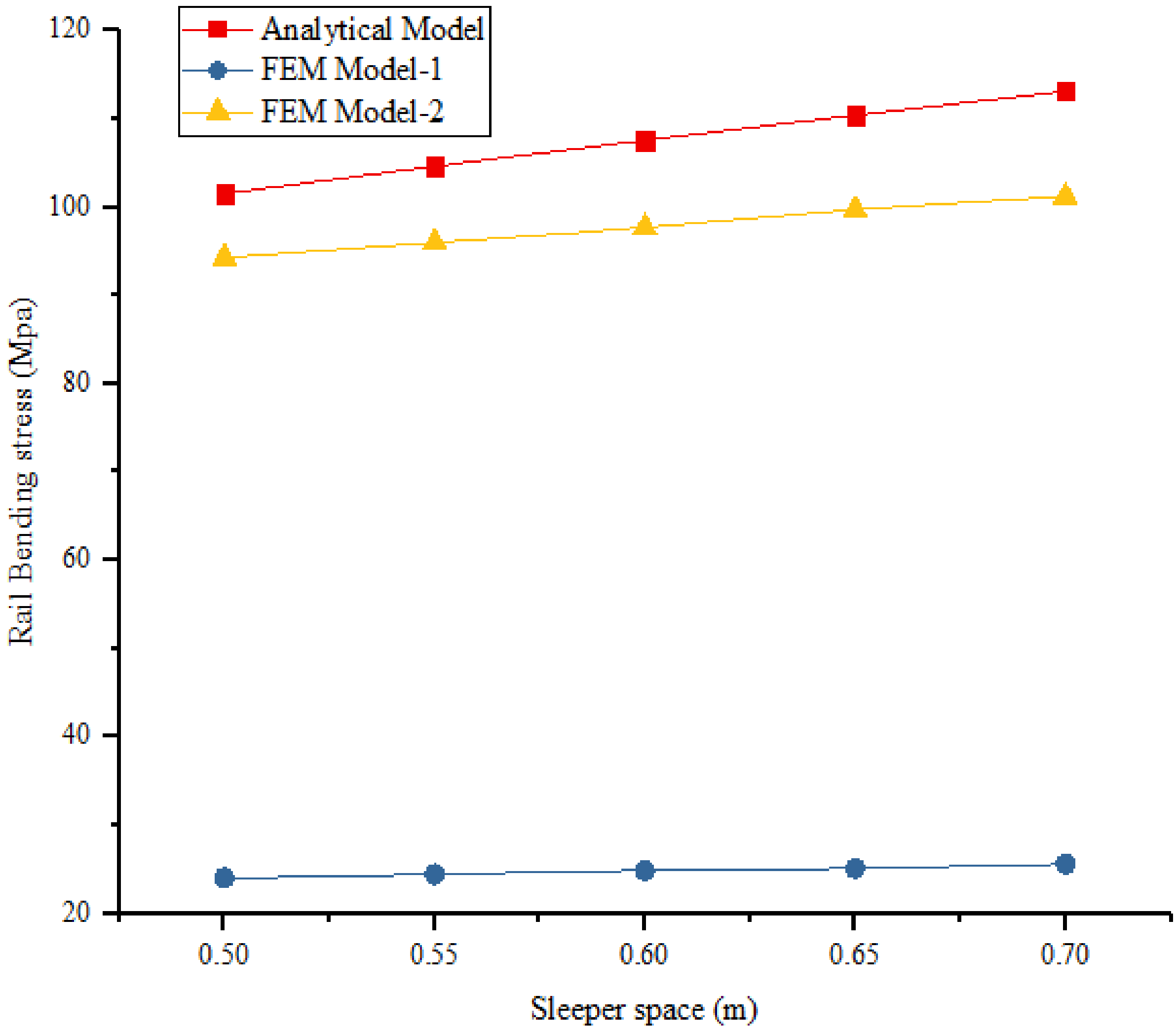


Figure 4-25: Rail bending stress of analytical VS numerical Analysis (for typical track bed)

Table 4-7: Rail bending stress of analytical VS numerical Analysis

Sleeper space (m)	Rail Bending stress from different model analysis(Mpa)								
	Soft track bed			Typical track bed			Stiff track bed		
	Analytical Model	*Numerical model*		Analytical Model	*Numerical model*		Analytical Model	*Numerical model*	
		model-1	model-2		model-1	model-2		model-1	model-2
0.50	109.88	25.042	97.300	101.54	24.0230	94.304	91.50	22.810	88.983
0.55	113.38	25.515	99.858	104.63	24.445	96.016	94.16	23.171	90.860
0.60	116.73	25.987	101.964	107.59	24.8660	97.728	96.68	23.531	92.737
0.65	119.75	26.185	104.071	110.42	25.1280	99.703	99.08	23.743	94.502
0.70	123.05	26.291	105.497	113.15	25.6010	101.202	101.40	24.156	96.091

c) Sensitive Analysis

The Sensitive Analysis has been carried out for both numerical and analytical analysis with respect to sleeper space and rail track bed quality. The result shows that each model has its own sensitive property; thus do not have the same sensitiveness property.

Table 4-8 lists the differences and comparison of sensitive analysis for different track models.

Table 4-8: Sensitiveness comparison

Sensitiveness of	BOEF MODEL (Analytical analysis)		FEM MODELS			
			MODEL-1		MODEL-2	
	With increasing of sleeper space	With increasing of track bed quality	With increasing of sleeper space	With increasing of track bed quality	With increasing of sleeper space	With increasing of track bed quality
Rail deflection	Increases	Decreases	Decreases	Decreases	Decreases	Decreases
Rail supporting	Decreases	Increases	Increases	Decreases	Increases	Decreases
Rail bending stress	Increases	Decreases	It is not suitable to generalize as either decreases or increases as no flow of results for these models. The result (data) flow shows randomly.			

d) Single Wheel Load Vs Multiple Wheel Load analysis

After the track has been analyzed both under single wheel load and multiple wheel load they have been taken for selection. The analysis shows that multiple wheel loads gives higher values of rail deflection and rail supporting force but lower values of rail bending stress than single wheel load analysis under analytical analysis; Howe ever, numerical analysis shows that single wheel load gives higher values (rail deflection, rail supporting force and rail bending stress) than multiple wheel loads. Table 4-9 shows the analysis rail track under single and multiple wheel loads of rail track with typical track bed and 0.6m sleeper space.

Multiple wheel loads gives lower values under numerical analysis. This one occurred duet the fact that the effect of one axle load gives positive value while the other gives negative values. The summation of the positive and negative values will be lower than the single positive value obtained from single wheel load.

Table 4-9: Single wheel Vs Multiple wheel loads (BOEF)

Analytical Output	single wheel load	Multiple wheel loads
Rail deflection (mm)	3.28	3.946
Rail supporting force (KN)	46.579	64.533
Rail bending stress(MPa)	134.31	107.59

Table 4-10: Single wheel Vs Multiple wheel loads (FEM)

Numerical out put	model-1		model-2	
	single wheel load	Multiple wheel loads	single wheel load	Multiple wheel loads
Rail deflection (mm)	11.605	5.893	17.595	10.184
Rail supporting force (KN)	35.627	19.171	23.804	15.237
Rail bending stress(MPa)	5498.6	24.866	18873	96.876

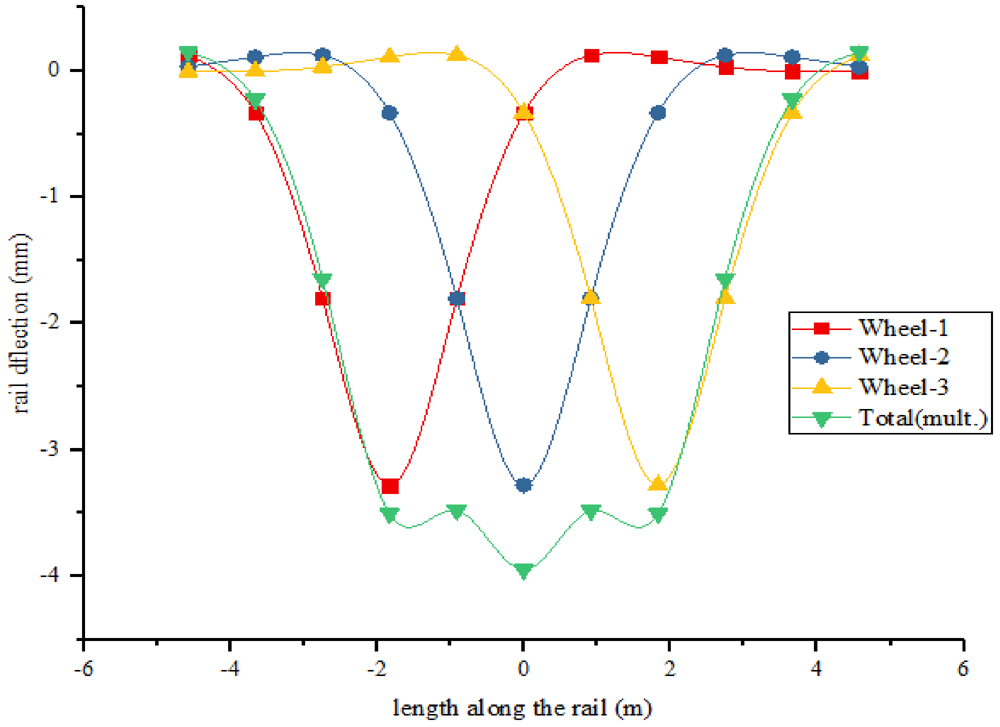


Figure 4-26: Single Vs Multiple wheel loads (BOEF deflection)

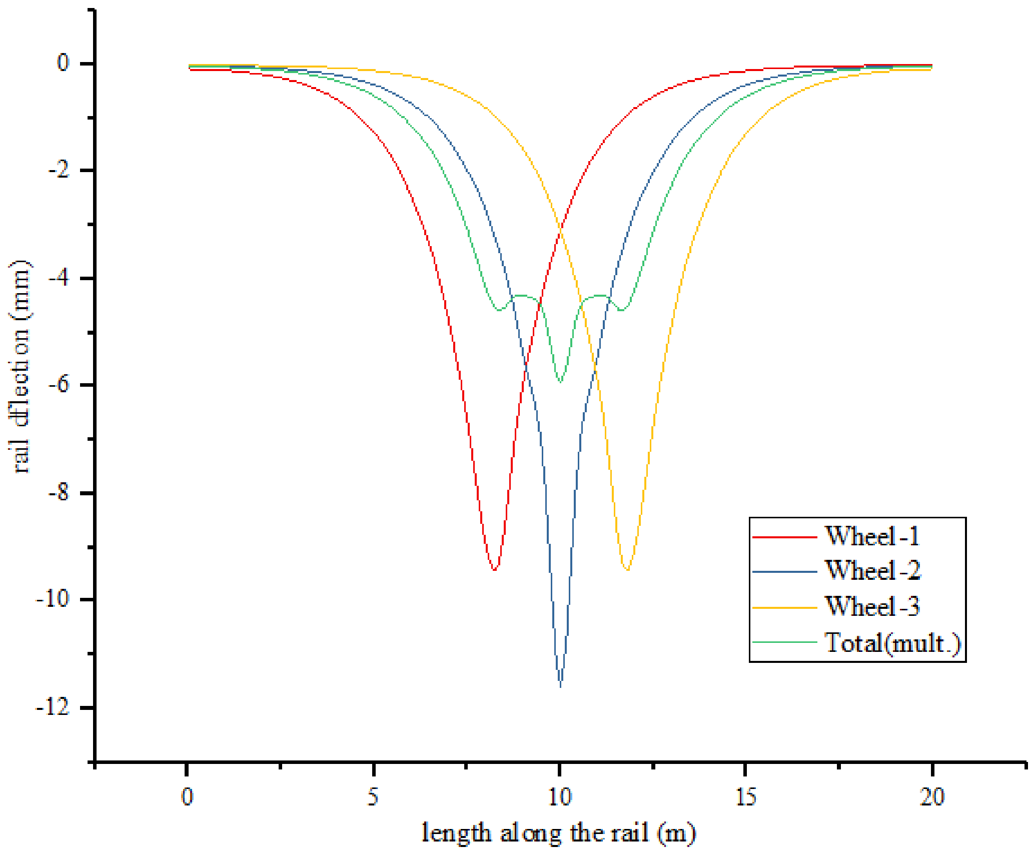


Figure 4-27 : Single Vs Multiple wheel loads (FEM deflection)

e) Superposition of the Load

BOEF considers the multiple wheel loads through the superposition principles. Each single wheel load is positioned at different positions at different times and analyzed. The effects of each single wheel load will be recorded and summed together to be accounted as multiple wheel load effects. This paper also check whether the superposition principles is true or not for numerical models.

In order to check it the following steps have been passed:

- Single wheel load is positioned at different positions along the rail and analyzed separately.
- Each effects of single wheel load are recorded separately and summed together by superposition principles then accounted as multiple wheel loads effects
- All wheel loads are positioned at the same time at different locations along the rail and analyzed and the effects recorded as multiple wheel loads effects.
- The effects from both analysis(from the superposition and multiple wheel load) were compared together

Figures 4-28 and 4-29 below show the final results obtained from the analysis of 20m track length. As observed the effects obtained from super position is much higher than that obtained from multiple wheel loads directly.

Hence it is possible to conclude that the principles of superposition are only true for analytical (BOEF) analysis and cannot work for numerical (FEM) analysis.

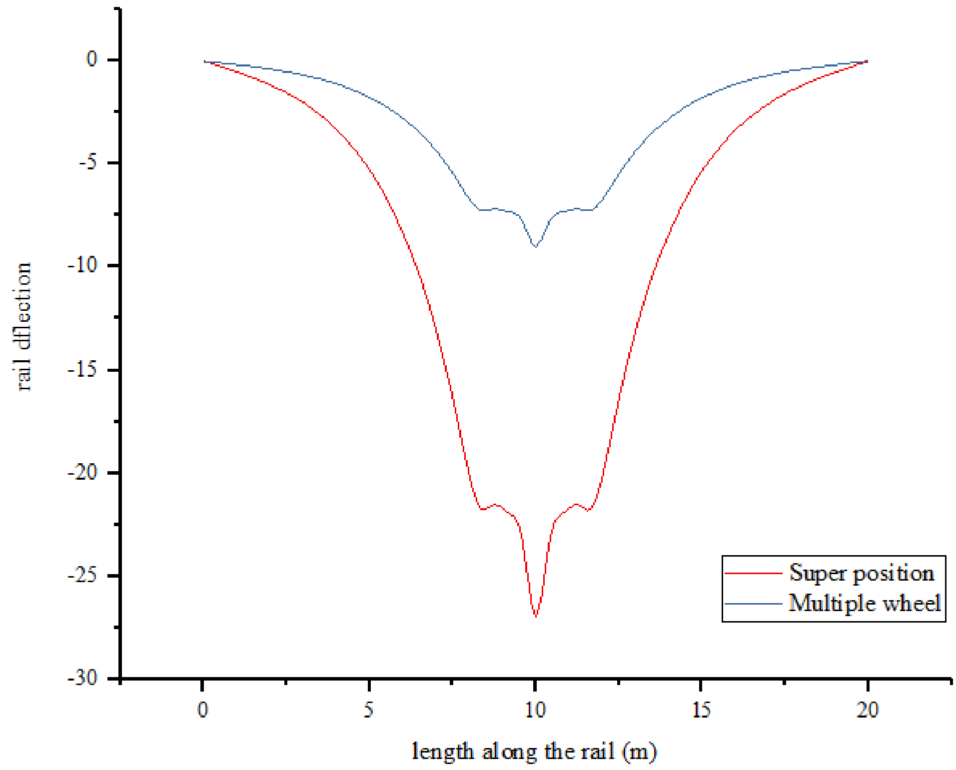


Figure 4-28: Superposition effects (FEM)

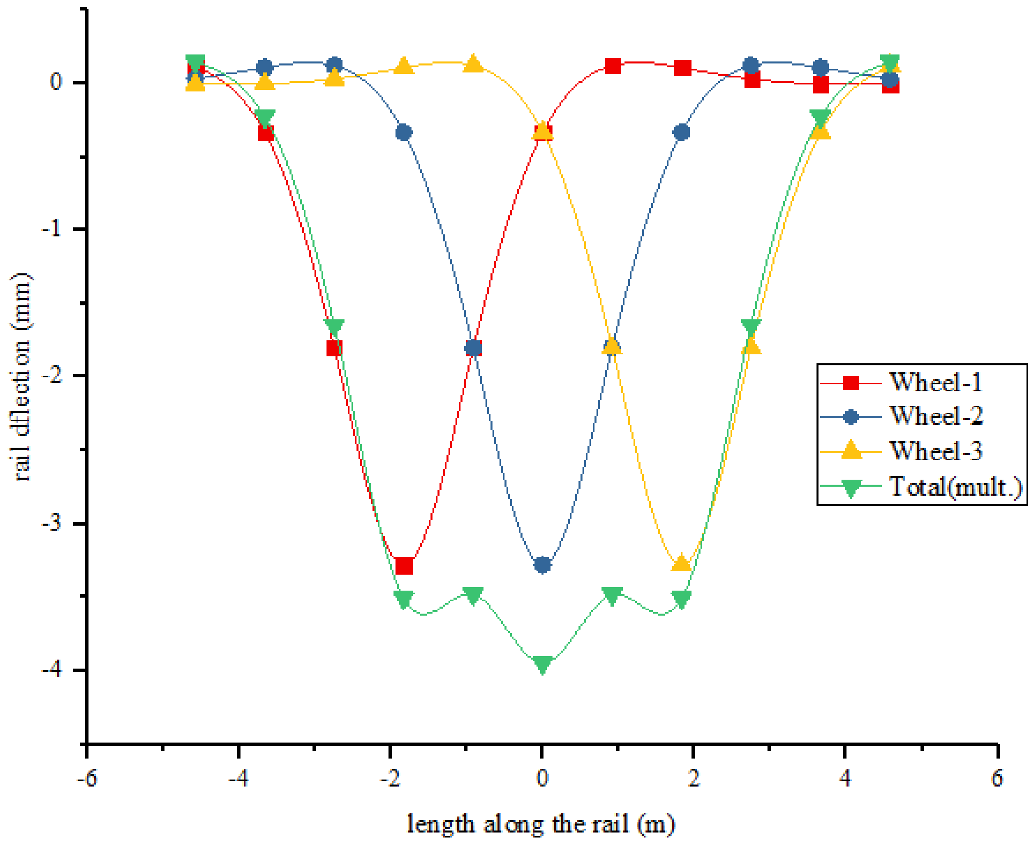


Figure 4-29: Superposition effects (BOEF)

4.4 Development of Correlating factor

In the above sections, rail track were modeled both numerically and analytically. The analysis was made both under single wheel load and multiple wheel loads. The results of analysis were given for each model in section 4.4 and 4.5; and compared together in section 4.6.

Having in mind the differences of these analysis values, some correlating factors have been introduced. These correlating factors are useful factors in which they help to change the result of analytical analysis into numerical analysis. For someone who cannot run FEM software nor have no high ram computers may use this average factor to get the probability result from FEM software's once he/she get the analytical values from BOEF Model.

The developed correlating factors depend on track bed property, type of numerical model (Model_1or Model_2) and Sleeper space. It should be understand that these factors have been introduced only for multiple wheel load and static analysis of numerical models. On another hand, the track bed property should be converted to spring type before use of these factors.

Generally there are six correlating factors were introduced based on the results discussed on the above sections in the form of charts as given in figure 4-30 up to figure 4-35.

Correlating factors	Numerical Model Type	
	Model_1	Model_2
Deflection correlating factors (**α**)	α_1	α_2
Rail supporting force correlating factors (γ)	γ_1	γ_2
Rail bending stress correlating factors (**β**)	β_1	β_2

Table 4-11: Types of correlating factors

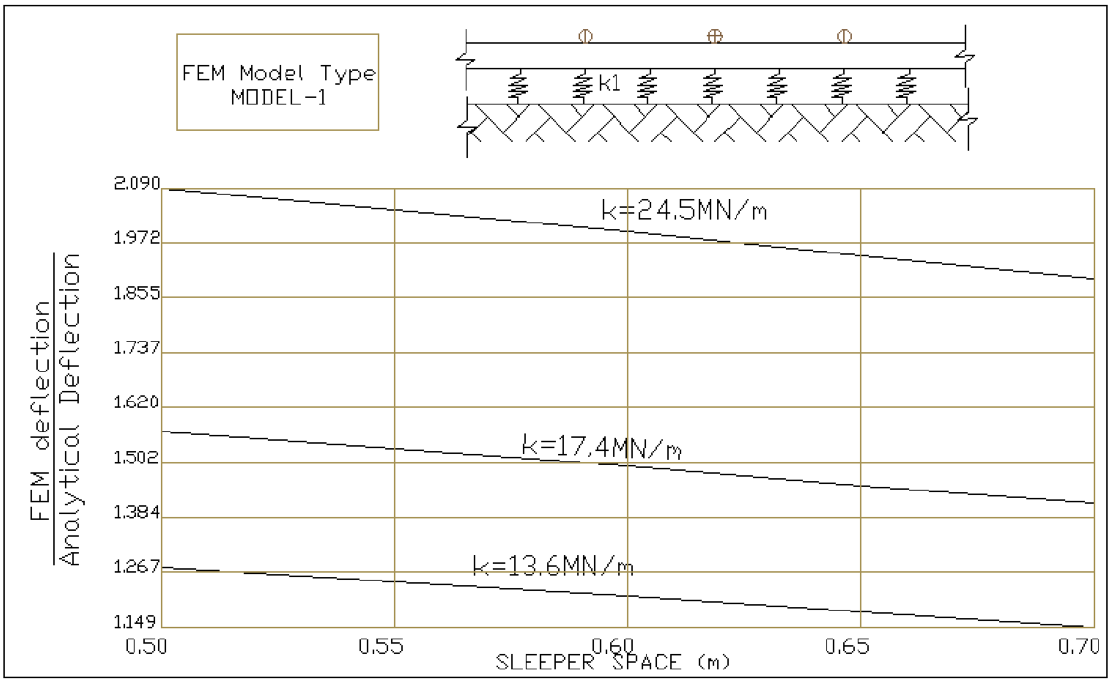


Figure 4-30: Deflection correlating factor-1 (α_1)

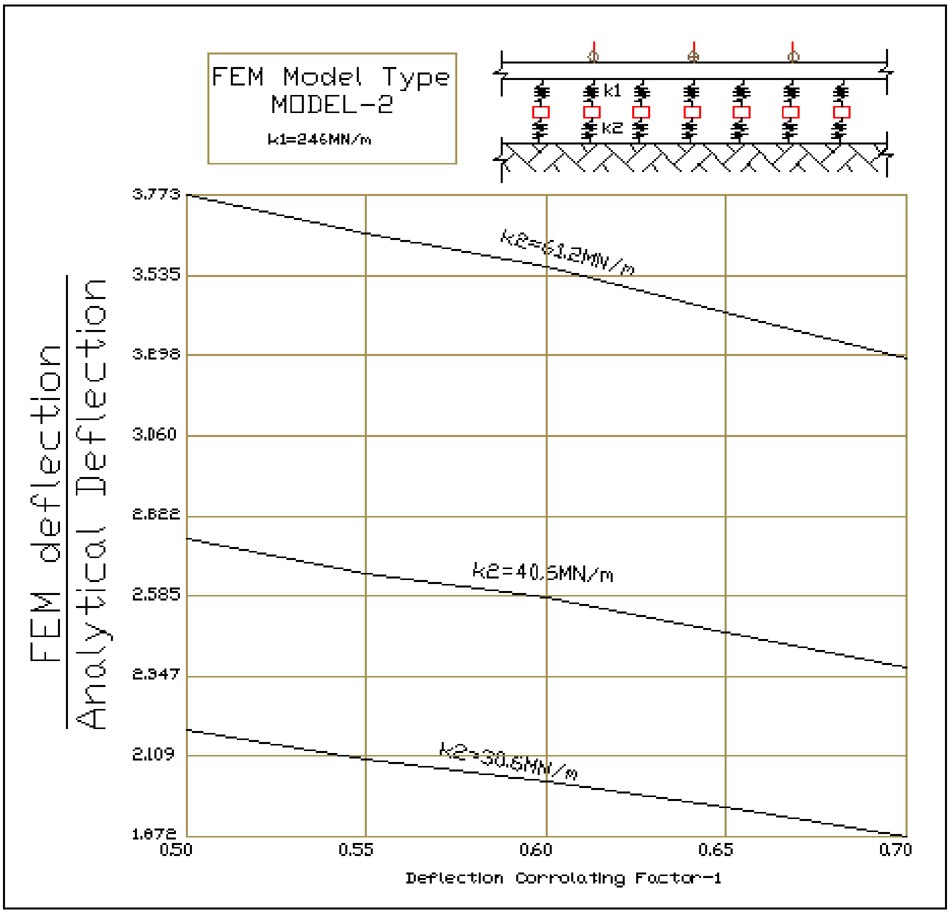


Figure 4-31: Deflection correlating factor-2 (α_2)

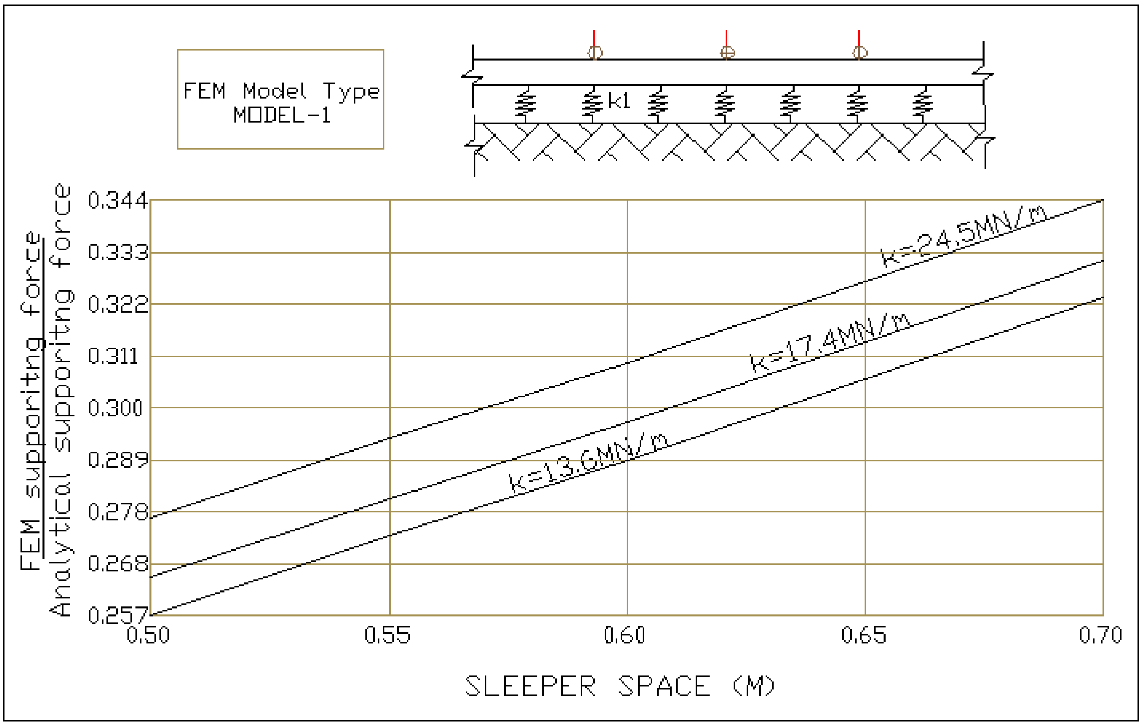


Figure 4-32: Rail support correlating factor-1 (γ_1)

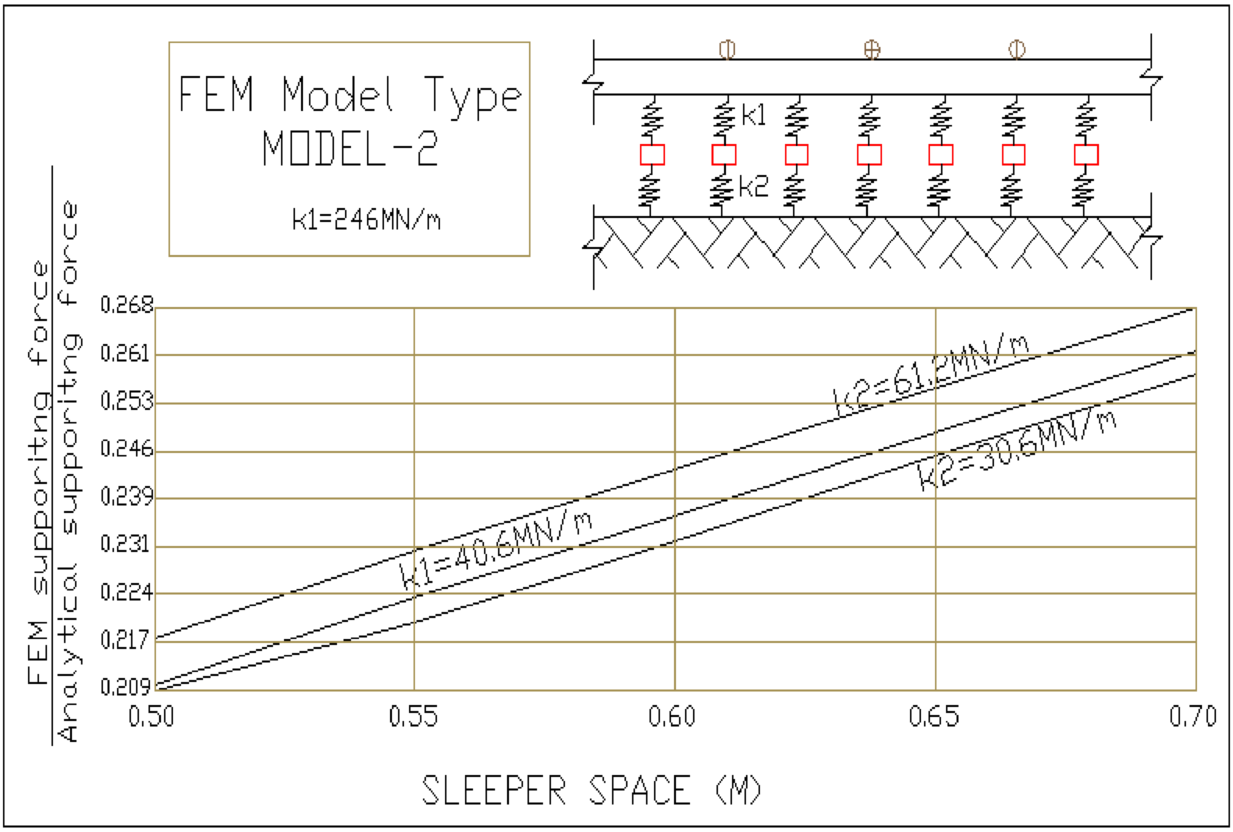


Figure 4-33: Rail support correlating factor-2 (γ_2)

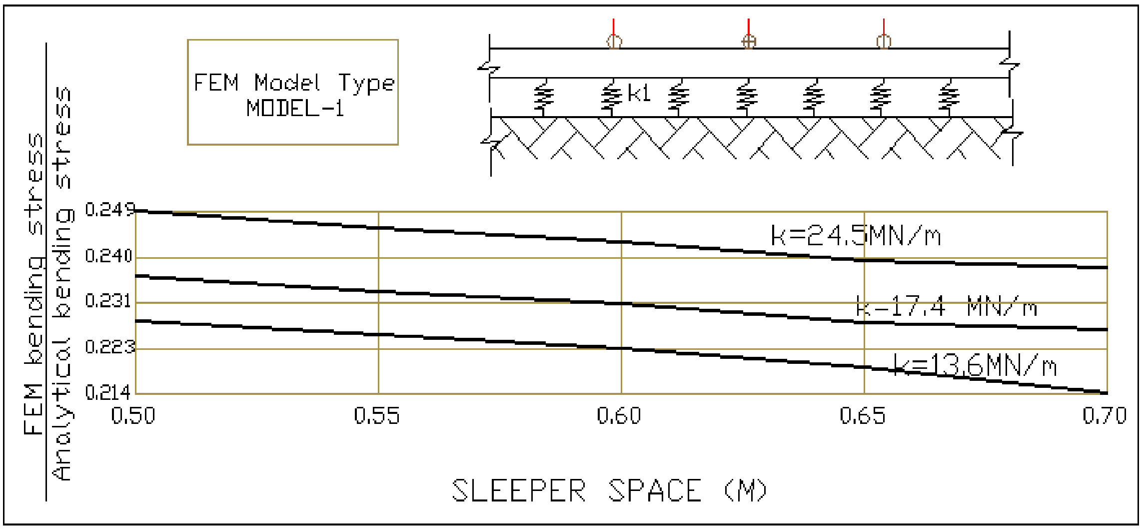


Figure 4-34: Rail bending stress correlating factor-1 (β_1)

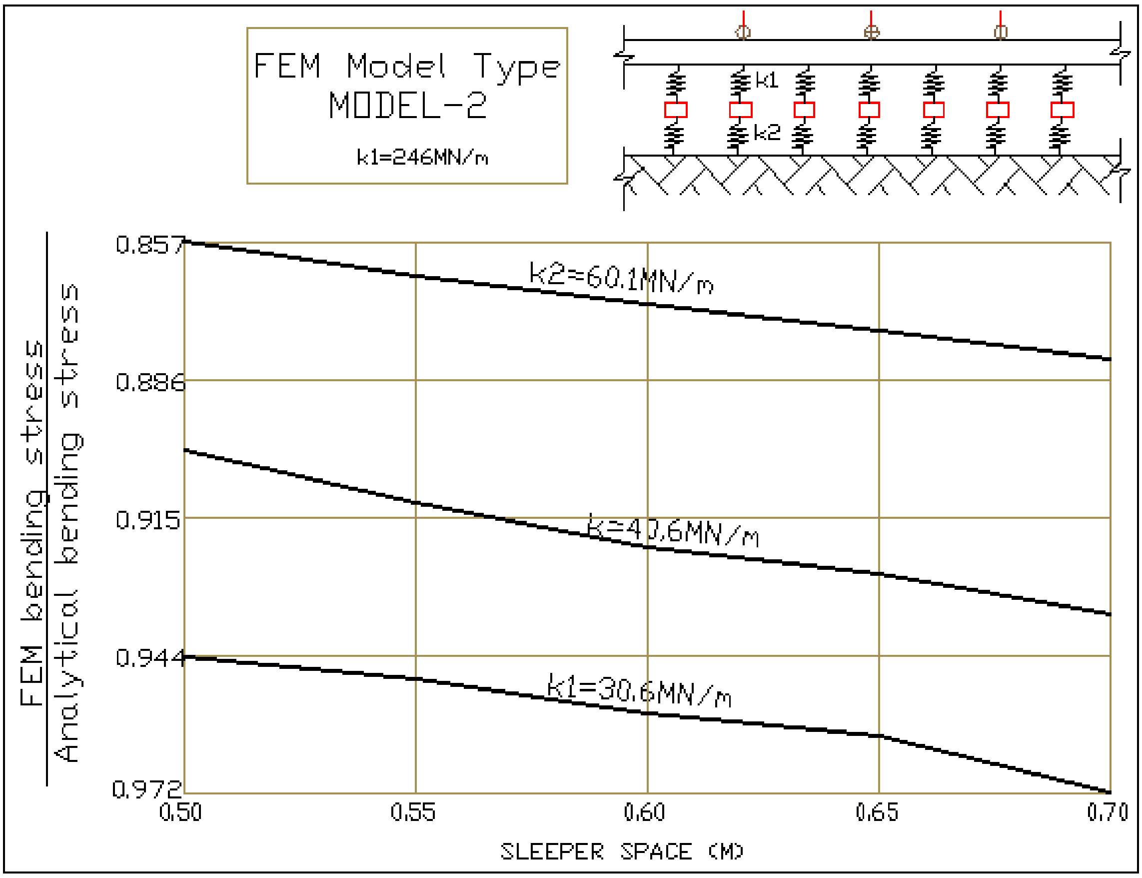


Figure 4-35: Rail bending stress correlating factor-2 (β_2)

4.4.1 How to Use the Correlating Factors

1) Select track bed properties, wheel load and sleeper space
2) Model the track analytically (BOEF) and analyze at specified sleeper space
3) Take the maximum value of the result to be known (rail deflection, rail supporting force and rail bending stress) from analytical analysis
4) Select the numerical model type to be analyzed (Model_1 or Model_2)
5) Find the track bed Stiffness
    i. Model_1----k
    ii. Model_2----K_1 and K_2
6) Read the correlating factors from the graph (charts) with respect to sleeper space and track bed stiffness
7) Use this correlating factors to get the probably of the numerical solution of the proposed numerical model
    a) Numerical Rail deflection= α*Analytical rail deflection
    b) Numerical Rail supporting force = γ *Analytical Rail supporting force
    c) Numerical Rail Bending Stress = β* Analytical Rail Bending Stress
8) Interpolate for intermediate sleeper spaces and track bed stiffness's

One example has done to check the accuracy of these correlating factors. First the model was analyzed analytically and the numerical results obtained using these correlating factors from analytical solution.

Secondly the same track has been modeled numerically (Model-1 and model-2); and analyzed. The solution converting from analytical solution and from numerical analysis are compared together as below in tables.

- Track bed Stiffness:
    - Model-1,K=50.4MN/m
    - Model-2, k1=246MN/m and K2=20.92MN/m
- Sleeper space=0.6m
- Wheel Load(Dynamic)=180744N

From the charts provided for corralling factors and interpolate for k=20.92MN/m (Model_1); and K2=50.4MN/m (Model_2) the following correlated factors obtained (Table 4-19).

Table 4-12: Correlating factors (Example)

Correlating factors	Numerical Model Type	
	Model_1	Model_2
Rail Deflection correlating factors (**α**)	1.74	30.67
Rail supporting force correlating factors (γ)	0.3034	0.239
Rail bending stress correlating factors (**β**)	0.236	0.934

Using these Correlating factors the solution that might be obtained from numerical analysis are forecasted and compared with the actual one as expressed table 4-13

Table 4-13: Analytical Vs FEM forecasted (Example)

Output(Required results)	Analytical value	Model-1		Model-2	
		Converted from Analytical	Direct from Modeling	Converted from Analytical	Direct from Modeling
Rail Deflection(mm)	3.202	5.571	5.6187	9.821	10.247
Rail supporting force(KN)	61.816	18.755	18.764	14.774	14.814
Rail bending stress (MPa)	101.510	23.935	24.313	94.773	95.066

Table 4-13 shows that the actual analysis from numerical model gives almost the same result as the results these are converted from analytical analysis using Correlating factors. The accuracy of these factors based on the solved example is listed in table 4-14 below.

Table 4-14:- Accuracy of correlating factors (Example)

Correlating factors	Error of Correlating factors	
	Model-1	Model-2
Rail Deflection correlating factors (**α**)	0.849%	4.157%
Rail supporting force correlating factors (**g**)	0.048%	0.27%
Rail bending stress correlating factors (**β**)	1.55%	0.31%

CHAPTER 5 CONCLUSIONS AND RECCOMENDATIONS

5.1 CONCLUSIONS

Finite Element Analysis and Beam on elastic Foundation (BOEF) track Model have been modeled from the same track properties to get numerical and analytical solutions respectively. The numerical (Finite element method) analysis was carried out in ANSYS work bench. Results from both analysis (Numerical and Analytical) were analyzed and compared together. Generally, the following conclusions are developed through serious analysis of the railway track.

1) With increasing sleeper space the track foundation modulus decreases while rail supporting force, rail deflection and rail bending stresses increases linearly.
2) Numerical analysis gives higher values of rail deflection than analytical analysis while analytical analysis gives higher values of rail supporting force and rail bending stress even though the differences depends on the sleeper space and rail track bed quality.
3) The analytical analysis shows that multiple wheel loads gives higher values of rail deflection and rail supporting force but lower values of rail bending stress than single wheel load; Howe ever, numerical analysis shows that single wheel load gives higher values (rail deflection, rail supporting force and rail bending stress) than multiple wheel loads.
4) Multiple wheel loads gives lower values under numerical analysis. This one occurred due to the fact that the effect of one axle load gives positive value while the other gives negative values. The summation of the positive and negative values will be lower than the single positive value obtained from single wheel load.
5) It is tried to check whether the super position principles also possible for FEM analysis. However, finally it is arrived that the principles of superposition are only true for analytical (BOEF) analysis and cannot work for numerical (FEM) analysis
6) Some correlating factors of numerical analysis with analytical analysis are introduced. These are: "γ", "β" and "α "are rail supporting force, rail bending stress and rail deflection correlating factors respectively. The developed correlating factors depend on track bed property, type of numerical model and Sleeper space. These factors have

been introduced only from multiple wheel load and static analysis of numerical Models.

5.2 RECCOMENDATIONS

2D-analysis was selected for carry out the results of FEM. On other hand, there is a need to improve and investigate the consistency of this analytical and numerical relationship by consideration 3D models of FEM. Additionally, Numerical analysis have been analyzed statically which is not the same as the natural one, thus one can do dynamically and compares with the statically results.

Numerical analysis was carried out On ANSYS 14.5 work bench; again one can do with another FEM Software such as ABAQUS and strengths the relationship between analytical and numerical analysis.

The contacts between each components of track are governed by frictional contact except contact between sleeper and rails. In this Thesis all contacts between sleeper and ballast, ballast and sub-ballast were considered to be bounded contact. One can study this relationship by using the contacts between each track element considering the true contact i.e. fractional contact.

The correlating factors were provided in chaper-4 were developed based on a limited no of sleeper space and rail track bed properties. Hence, there is a need to improve and investigate the consistency (validation) of these correlating factors by consideration of wider range of sleeper space and rail track properties.

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