ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

ADDIS ABABA INSTITUTE OF TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

APPROXIMATE UNIAXIAL INTERACTION DIAGRAM FOR SLENDER COLUMN USING SECOND ORDER FORMULA FROM EBCS2, 1995

A thesis submitted to the school of Graduate Studies in Partial fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering (Structures)

By

Kabtamu Getachew

Advisor: **Dr.Ing. Adil Zekaria**

April 2012

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Approved by Board of Examiners

Dr.Ing. Adil Zekaria__	__________________	______________
Advisor	Signature	Date
Dr.Ing. Girma Zerayohannes	__________________	______________
External Examiner	Signature	Date
Dr. Esayas Gebreyohannes	__________________	______________
Internal Examiner	Signature	Date
Ato Michael Asrat	__________________	______________
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DECLARATION

I, the undersigned, declare that this thesis is my work and all sources of materials used for the thesis have been duly acknowledged.

Name: Kabtamu Getachew

Signature: ____________________

Place: Addis Ababa Institute of Technology

Addis Ababa University

Date of submission: April 2012

Advisor: Dr.Ing.Adil Zekaria

Signature: ____________________

This work is dedicated to:

My Family

And

My Advisor Dr.Ing.Adil Zekaria

ACKNOWLEDGEMENT

Primarily I would like to thank the Almighty God for his unending blessings. Secondly, I am very grateful to express my deepest gratitude to my advisor Dr.Ing.Adil Zekaria for his unreserved assistance, constructive and timely comments at all stages of my work.

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LIST OF SYMBOLS AND NOTATIONS

The following are list of notations used in this thesis work

RC Reinforced Concrete

SRCC Slender Reinforced Concrete Colum

EBCS Ethiopian Building Code Standard

ACI American Concrete Institute

ETABS Extended Three Dimensional Analysis of Building Systems

C_c Compressive force developed in the concrete

C_{s2} Compressive force developed in the in the compression reinforcement

C_{s1} Tensile force or compression force developed in the bottom reinforcement of column cross section

P_u Ultimate axial load capacity of column

P_{sd} Design values of internal axial load

P_{sc} Slender column axial load capacity

M_u Ultimate moment capacity of column in uniaxial bending

M_{sc} Slender column cross section ultimate moment capacity of column in uniaxial bending

M_0 First order moment of column in uniaxial bending

M_{bal} Balanced moment capacity of column in uniaxial bending

M_{sd} Design moment at the critical section including second order effect

A_c Gross area of concrete section

$A_{s,tot}$ Total area of reinforcement in columns

L Clear height of column

L_e Effective buckling length

λ Slenderness ratio

I Moment of inertia

I_g Gross moment of inertia of section about centroid

i Radius of gyration

b, h Dimensions of rectangular section (width, height)respectively

d Effective depth of rectangular section

h' Concrete cover to the centroid of the reinforcement

e Eccentricity

e_o Equivalent uniform first order eccentricity

e_a Additional eccentricity

e_2 Second order eccentricity

e_{tot} Total eccentricity

f_{cu} Cube compression strength of concrete

f_{cd} Design compressive strength of concrete

f_{ck} Characteristic compressive strength of concrete

f_{yd} Design yield strength of reinforcement

k_x Relative depth of neutral axis

r Radius of curvature

α_c Relative compressive force in concrete

β_c Relative distance of point of application of the compressive force in the concrete ,*Cc* from the outermost concrete fiber under compression

ε_{cm} Compressive strain in the outer most fiber

ε_o Strain at the point on the parabolic _rectangular stress diagram where the parabolic section joins the linear section

Ec Tangent Modulus of Elasticity of Concrete at stress σ=0 and 28 days

ε_{sy} Strain reinforcement at the yield point

ε_{yd} Design of yield strain of steel

ε_{s1} Strain in tensile reinforcement

ε_{s2} Strain in compressive reinforcement

$\nu_u = \frac{Pu}{f_{cd}A_c}$, Design value of the ultimate relative axial load

$\mu_u = \frac{Mu}{f_{cd}A_c h}$, Design moment capacity of columns with uniaxial bending

$\nu_{sd} = \frac{Psd}{f_{cd}A_c}$, Design value of the ultimate relative axial load

$\mu_{sd} = \frac{Msd}{f_{cd}A_c h}$, Design moment of columns with uniaxial bending respectively

ABSTRACT

In concrete buildings, recently the design of column is increasingly determined by architectural, aesthetic, and economic criteria leading to slender cross-sections of columns. However, slender column design requires rigorous analysis to account second order effect due to deflection of the column. Because the deflection provides additional eccentricity to axial load that induces additional second order moment. If second order effect is not considered adequately, it can cause stability failure, which is catastrophic. Since exact rigorous analysis demands more computational effort, simplified methods have been proposed in building codes such as EBCS_2, 1995, CEB_FIP1990 and ACI code still that involves iterative procedure. Moreover, even recently developed commercial computer programmes such as ETABS and SAP2000 do not consider second order effects due to the deflection of the column between its ends adequately as compared to relative deflection between ends of columns.

In this thesis, approximate uniaxial P_M interaction diagram for non-sway slender RC rectangular column is presented based on the simplified method of EBCS_2, 1995-second order eccentricity formula (deflection between two ends of a column) which is rather simple and unsophisticated for design of slender column. For the preparation of the interaction diagram, cross section interaction diagram and column slenderness is used. Since the slender column capacity is smaller than short column capacity, the cross section interaction diagram is modified so that it would have a room for the slenderness effect. This is made by deducting additional moment due to second order and geometric imperfection eccentricity from cross section interaction diagram.

Finally, the approximate interaction diagram is checked for its validity and satisfactory result is obtained. Therefore, the interaction diagram can be used as a design aid as well as preliminary cross section capacity estimation since it is very easy and quick method. Moreover, it can be used for checking of computer output.

CHAPTER ONE

INTRODUCTION

1.1 Background

"*Young engineers are often perfect in using computers, but they don't know where to put the comma. It is not the computer that produces ideas. Let us not throw away the pencil and the slide rule.*" Anton Tedesko (1967), shell designer and builder.

In building structures, a reinforced concrete (RC) column, which is a primary structural member, is subjected to the axial force and bending moment which may be due to end restraint arising from the monolithic placement of floor beams and columns or due to eccentricity from imperfect alignment. Therefore, column section should be designed to resist the combined action of axial load and bending moment. Recently, because of architectural aesthetics and efficiency in use of space, relatively slender columns have frequently been used in many building structures, either throughout an entire building or in some parts of a structure, e.g., the exterior of buildings and the interior of lobbies. Moreover, the use of high strength steel and concrete has led to an increased use of slender members.

However, as slender RC columns may fail due to not only material failure in a section but also instability of a structure, which require more rigorous numerical analyses to consider secondary effects such as the P–delta effect and creep deformation of concrete in order to preserve their strength and serviceability. In this thesis, secondary effect due to P_delta effect is considered. Building codes normally recognize two types of P-delta effect as shown in figure (1_1).The first type is the deflection of the column between its two ends denoted by (δ), which is common in braced or non-sway columns. Another one is relative deflection of one end of column to the other end of column denoted by (Δ), which is common in sway frames. Eventhougn both types of deflections do occur in a frame as shown in figure (1_1) only first type of second order effect is considered in this thesis.

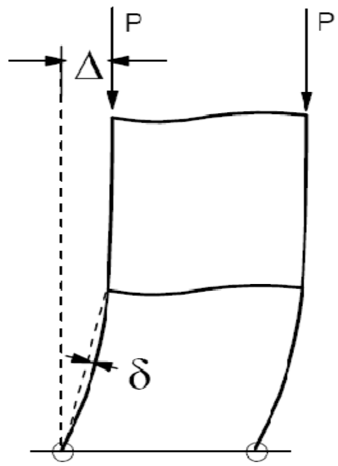


Figure 1-1 P-Δ and P-δ Effects [After, S.L. Chan, 2004]

For calculation of second order effect in slender columns, it is required to formulate equilibrium equation on the deformed geometry of the column, which is non-linear analysis. Non-linear analysis involves both geometric and material non-linearity. However, here in this thesis only geometric non-linearity (P_delta effect) is the point of discussion. Compared to non-linear analysis, linear analysis assumes the applied load is directly proportional to displacement, stresses, reactions, etc and it does not consider second order effect. In non-linear analysis, to account geometric non-linearity (P_delta effect) iteration is required since the deformed geometry is not known before the first linear analysis.

However, non-linear analysis is very difficult and sometimes impossible. So far different simplified methods have been prepared such as ACI magnification factors, second order eccentricity equations recommended in many building codes including EBCS2, 1995 with some iteration using uniaxial P_M interaction diagram design charts , which have been used for many years to assist in the proportioning and detailing of reinforced concrete columns. Moreover, in recent days better computer programming is being developed for faster and easier design of structures such as ETABS, SAP2000, and STAADPro etc. Nevertheless, design charts and tables will remain useful. Because they can serve as providing data for those who are engaged in developing computer programmes. On the one hand, an aid for checking the validity and correctness of programmes coming from different sources on the other hand [EBCS2: Part 2, 1995].

Yet, still there is no such graphical design aids (chart) like that of steel columns for slender RC column in EBCS_2,995. Moreover, if there had been such design aid so far, it could be used for checking of computer outputs. Because, now a day's, most designs are carried out using commercial software, which could give erroneous result due to various reasons. For example, there is lack of understanding a software limitation. To support this sentence, in the commercial software user manual of ETABS and SAP2000 it is recommended to use some magnification factors for the first type (δ) of second order deflection ,which similar to non sway column design procedure of EBCS2,1995 or ACI moment magnification factor for non sway column.

Until now, EBCS_2, 1995 recommends simplified second order eccentricity equations for the design of slender non-sway column, which is iterative in nature to account axial force effect on the curvature of the column. Therefore, the main purpose of this thesis work is to prepare approximate interaction diagram for slender RC columns, which is graphical summary of slender column capacity, based on EBCS2, 1995-second order eccentricity formula and cross section interaction diagram. For the calculation of the interaction diagram Ms Excel sheet is employed. Finally, the interaction diagram developed is checked for its validity by comparing with the chart of Schneider, 2006 and second order eccentricity calculation formula of EBCS2, 1995 and satisfactory result is obtained. Therefore, the charts can be used as alternative design method for slender RC column (See the Appendices).

1.2 Objectives

General Objective

- To prepare design aid for slender RC column

Specific Objective

- To prepare approximate interaction diagram for slender RC column
- To show direction for future works

1.3 Statement of the problem

For design of non-sway slender RC column, EBCS2, 1995 recommends second order eccentricity formula to account the deflection between two column ends. The second order eccentricity formula involves iteration to account effect of axial force on the curvature of the column, which is calculated based column slenderness. In this thesis, to avoid the iteration involved approximate uniaxial slender RC column interaction diagram is derived which is graphical summary of various slender column capacity for different slenderness ratio.

1.4 Scope and limitation of the study

This thesis is limited to the preparation of approximate uniaxial interaction diagram for slender rectangular RC columns and symmetrically reinforced section.

1.5 Methodology

In the preparation of the approximate slender column interaction diagram, ultimate limit state of EBCS2, 1995, the simplified second order eccentricity formula of EBCS2, 1995 and cross section interaction diagram is used. For calculations of numerical values Ms Excel sheet, is employed throughout the thesis work

1.6 Organization of the thesis

The thesis is organized into four chapters. Chapter 1 introduces the background, the objectives, and the scope of the thesis work. Chapter 2 describes previous works done on the design of slender column. Chapter 3, which is the main body of the thesis work, shows the procedures followed in the calculations and verification of slender column design aid. Finally, Chapter 4 describes conclusion and recommendation of this work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Behavior of Slender RC column

2.1.1 Buckling of Axially Loaded Elastic Columns

Normally when structural members are in compression, it is a good thing. They will not fail except by crushing (exceeding their compressive yield strength), and fatigue does not occur for elements in compression. However if the geometry of the member is such that it is a "column" then buckling can occur. Buckling is particularly dangerous because it is a catastrophic failure that gives no warning. That is, the structural system collapses often resulting in total destruction of the system and unlike yielding failures, there may be no signs that the collapse is about to occur. Thus, design engineers must be constantly on vigil against buckling failure.

For the illustration of elastic buckling three states of equilibrium are shown in figure 2_1 by J.MacGregor *et.al*.(2005). If the ball in figure 2_1 (a) is displaced laterally and released, it will return to its original position. This is stable equilibrium. If the ball in fig 2_1(c) is displaced laterally and released, it will roll off the hill. This is unstable equilibrium. The transition between stable and unstable equilibrium is neutral equilibrium, shown in fig 2_1 (b). Here the ball will remain in the displaced position. Similar states of equilibrium exist for the axially loaded column in figure 2_2(a). If the column is returns to its original position when it is pushed laterally at mid height and released it is stable and so on.

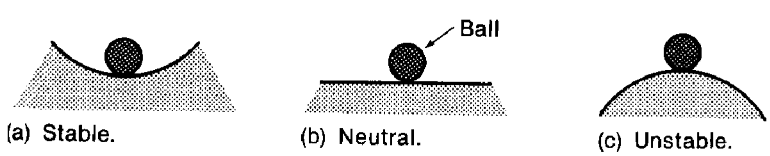


Figure 2_1 States of Equilibrium [After, MacGregor *et al*, 2005]

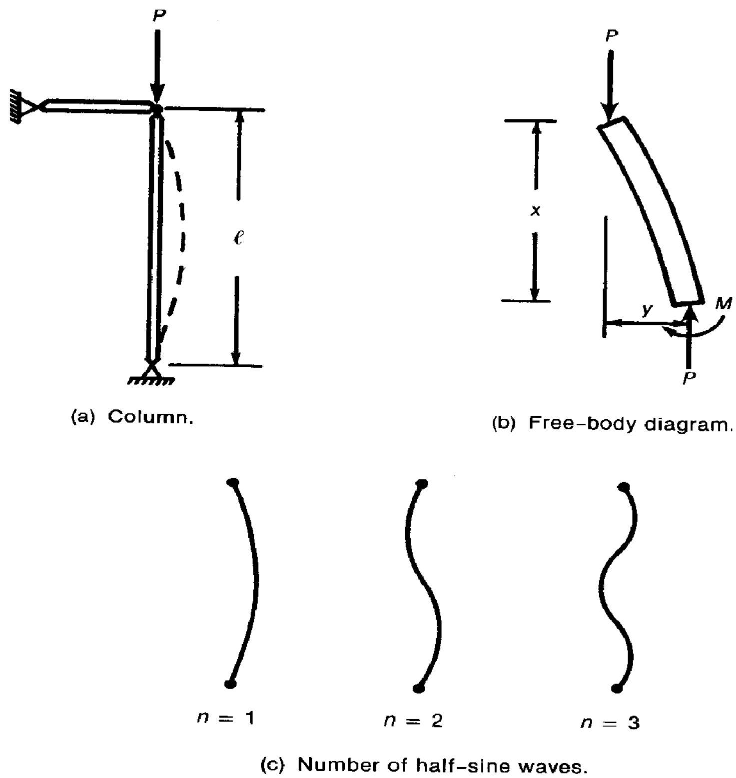


Figure 2_2 buckling of a pin-ended column [After, MacGregor *etal*, 2005]

Figure 2_2 (b) shows a portion of a column that is in a state of neutral equilibrium. The differential equation for this column is

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI}$$ Eqn. 2_1, MacGregor *etal* (12_3)

In 1744, Leonhard Euler solved equation 2_1 and its solution is

$$P_c = \frac{n^2\pi^2EI}{L^2}$$ Eqn. 2_2, MacGregor *etal* (12_4)

Where

EI =flexural rigidity of the column cross section,

L= length of the column, n=number of half sine waves in the deformed shape of the column. Cases with n=1, 2 and 3 are shown in figure 2_2(c). The lowest value of P_c will occur with n=1.0 this gives what is called **Euler buckling Load.**

$$P_c = \frac{\pi^2 EI}{L^2}$$ Eqn. 2_3, MacGregor *etal* (12_5)

Such a column is shown figure (2_2 a). If this column were unable to move sideways at the mid height, as shown in figure 2_2b, it would buckle with n=2 and the buckling would be

$$P_c = \frac{2^2\pi^2 EI}{L^2}$$

which is four times the critical load of the same column without mid height brace. Another way of looking at this involves the concept of the effective length of the column. The effective length of the column is the length of a pin-ended column having the same buckling load. Thus, the column in figure 2_3 (b) has the same load as that of figure 2_3 (c). The effective length of is, L/2 in this case, where L/2 is the length of each of the half sine waves in the deflected shape of the column in figure 2_3(b). The effective length, kL is equal to l/n. The effective length factor is k=1/n. Generally, equation 2.2 can be rewritten as

$$P_c = \frac{\pi^2 EI}{(kL)^2}$$ Eqn. 2_4, MacGregor *etal* (12_6)

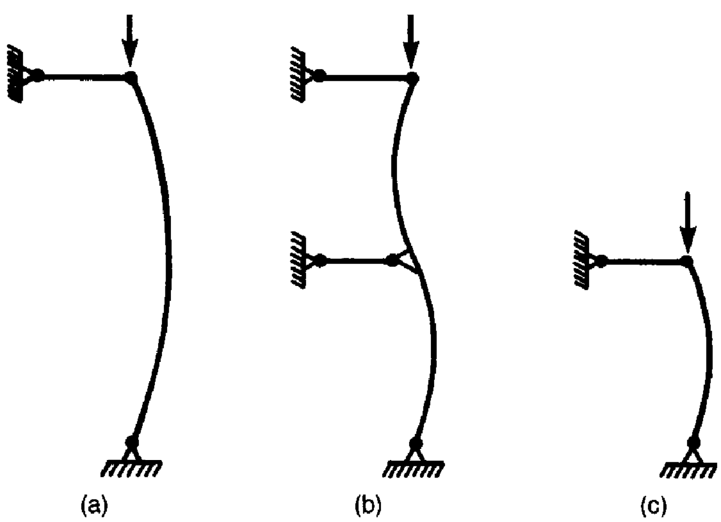


Figure 2_ 3 Effective lengths of columns [After, MacGregor *etal*, 2005]

In equation 2_4, the critical axial load is derived to analyze the stability of the column using elastic buckling analysis for elastic columns. In fact, elastic columns are too ideal, but with many assumptions, steel columns can behave elastically and hence their stability can be determined. So far, buckling curves have been produced for steel columns to assist steel column stability analysis such as European Buckling Curves. However, unlike steel columns the conditions for elastic buckling analysis are never satisfied in RC columns. Because concrete is inelastic, section cracks, loading is eccentric, columns are not perfectly straight (imperfection). Hence, the stability analysis of RC columns is carried out using second order theory.

2.1.2 Second Order Theory

In the first order theory, the equations of equilibrium are formulated on the undeformed structure. First order analysis assumes small deflection behavior; the resulting forces and moments take no account of the additional effect due to the deformation of the structure under load. Strictly speaking, this is not accurate. Actually, the conditions of equilibrium are satisfied on the deformed structure, which is second order theory. First order theory is sufficiently accurate for most structures such as beam and slab. However, the additional moment may not be negligible for columns in the presence of axial load especially for slender columns [Girma Zerayohannes, 2010 and White *e tal*.1991]

Therefore, second order theory should be used in the analysis and design of a column. For example an eccentrically loaded, pinned end column is shown in figure 2_4 the moments at the ends of the column are $M = P*e$ .When the load **P** is applied the column deflects laterally by an amount equal to δ as shown in figure 2_4 (a). For equilibrium, the internal moment at mid height must be $M_{mid} = P*(e+ \delta)$, in figure 2_4 (b), $M_c = P *(e+ \delta)$, i.e. the deflection increases the moments for which the columns must be designed. In figure 2_5, the load moment curve is drawn by including the deformation effect.

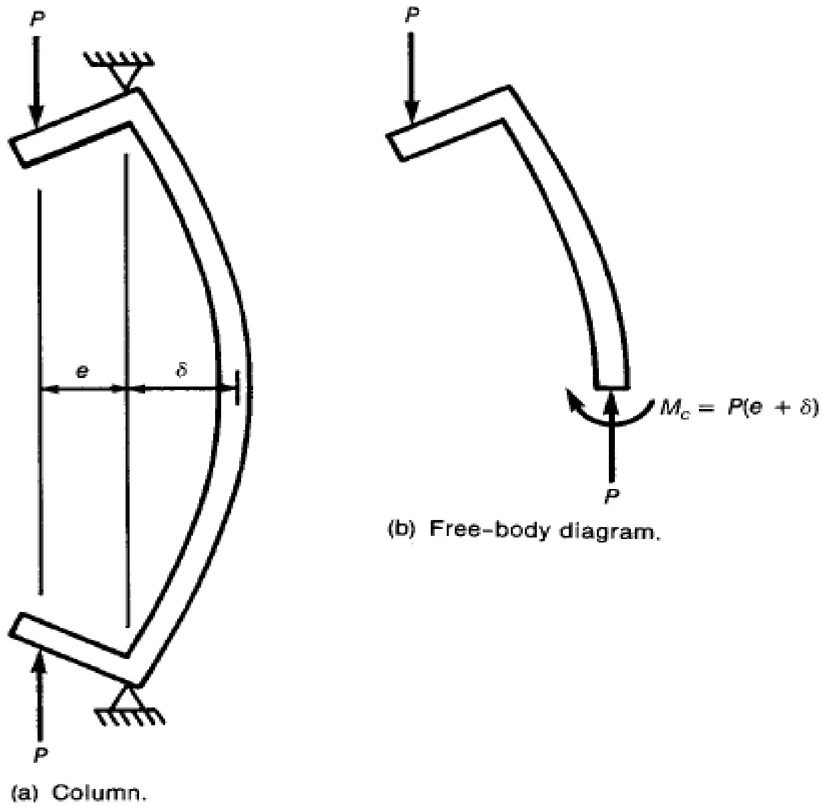


Figure 2_ 4 Forces in a deflected column [After, MacGregor *et al*, 2005]

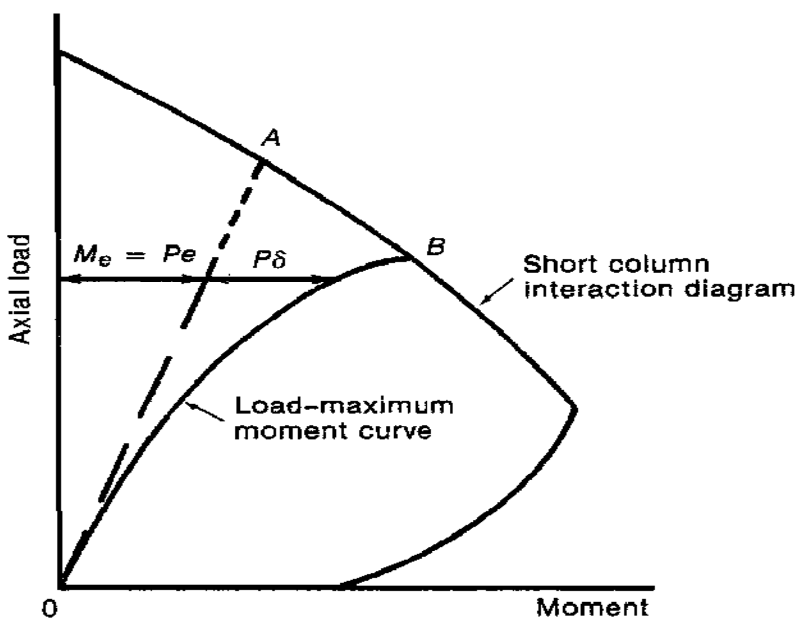


Figure 2_5 Load and moment in a column [After, J.MacGregor *et al*, 2005]

2.1.3 Column failure Mechanism

According to Bazant, *et al.* (2003), there are two types of failure mechanisms for a column. The first is the failure of the material. It is governed by the value of the material strength or yield limit, which is independent of column geometry and size. By contrast, the load at which a column becomes unstable regarded as independent of material strength or yield limit: it depends on structural stiffness of the material, characterized by, for example by elastic modulus. Failures of elastic structures due to structural instability have their primary cause in geometric effects: the geometry of deformation introduces non-linearity that amplifies the stresses calculated on the bases of the initial undeformed configuration of the structure. This is special problem of slender column.

In the figure 2_6, failure mechanisms are shown. Curve OB is the load-moment curve for the maximum column moment. Curve OA is the load-moment curve for the end moment. Failure occurs when the load moment curve OB for the critical section intersects the interaction diagram for the cross-section. Thus, the load and moment at failure are denoted by point B in figure 2_6.Because of the increase in the maximum moment due to deflections; the axial load capacity is reduced from A to B. This reduction in axial load capacity results from slenderness of the column.. As the slenderness increases the value of $\frac{\partial P}{\partial M}$ approaches zero and hence stability failure initiated rather than cross section failure as in point C.

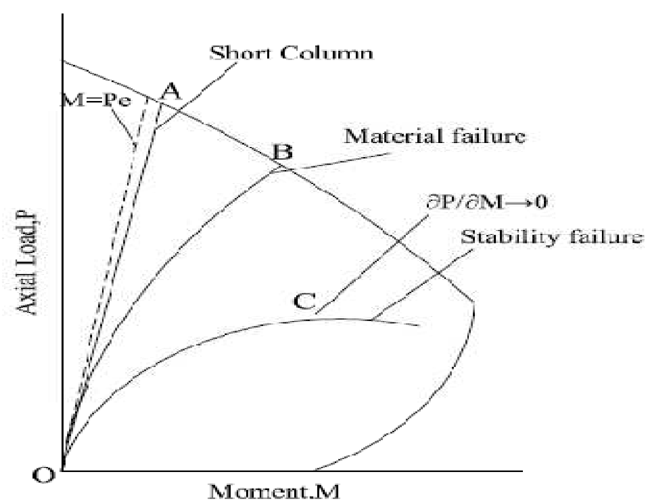


Figure 2_6 Material and stability failure [After, J.MacGregor *et al*, 2005]

2.2 Design methods for Slender RC columns

So far, different methods have been proposed and used in the design of slender RC columns. This includes, design curves or charts, approximate equations for equivalent second order eccentricity and moment magnifier to account second order effect. Each of the methods will be discussed in the following subsections.

2.2.1 Slender RC column interaction curves

In the past, different authors have derived slender interaction diagram. One of the pioneer MacGregor *et al.* (1970) stated that interaction diagram for slender column can be prepared considering section capacity and length to width ration or slenderness ratio as shown in figure 2_7(a) and (b) to account second order effect with constant value of ω.

Bazant *et al.* (1991) also showed reduced column failure envelope in figure 2_8 based on slenderness ration. On the other hand, Schneider (2006) derived interaction diagram for slender columns with constant value of λ as shown in figure 2_9. Similarly, Josip Galič *et al.* (2005) derived interaction diagrams as in figure 2_10 and 2_11 for constant value of ω and λ respectively.

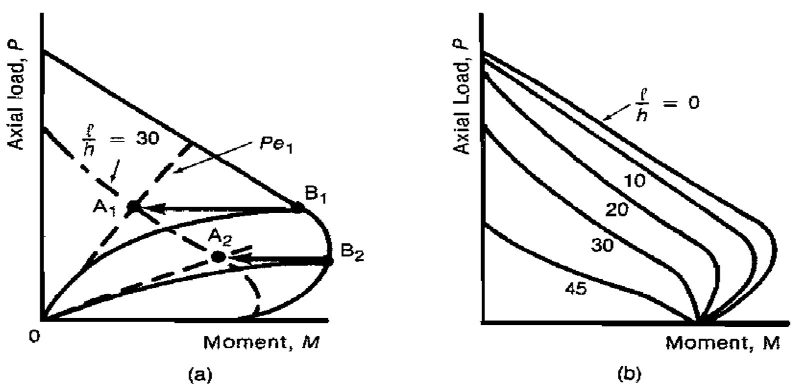


Figure 2_7 Slender Column interaction curve [After, MacGregor *et al* ,2005]

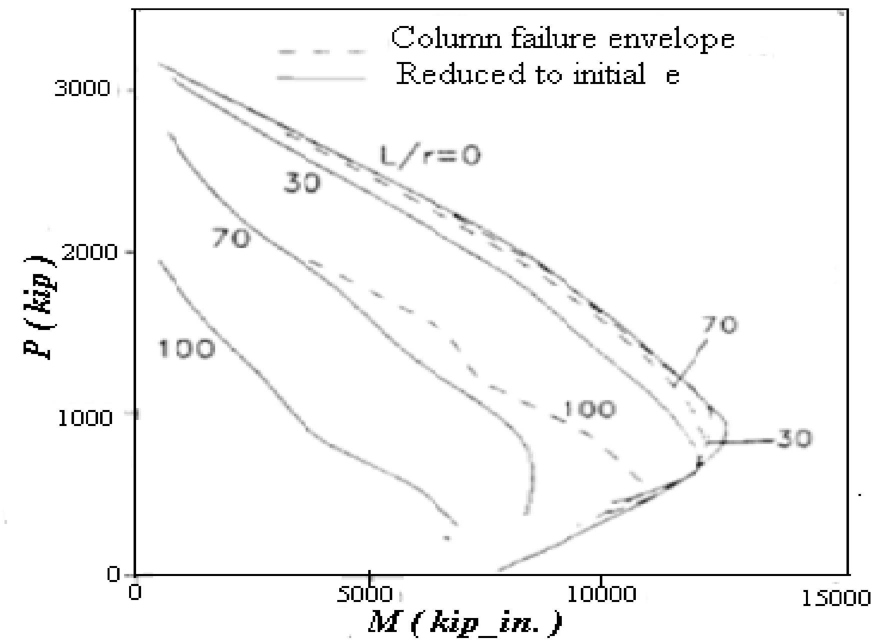


Figure 2_8 Column failure envelopes and reduced failure envelopes [After, Bazant *et al*, 1991]

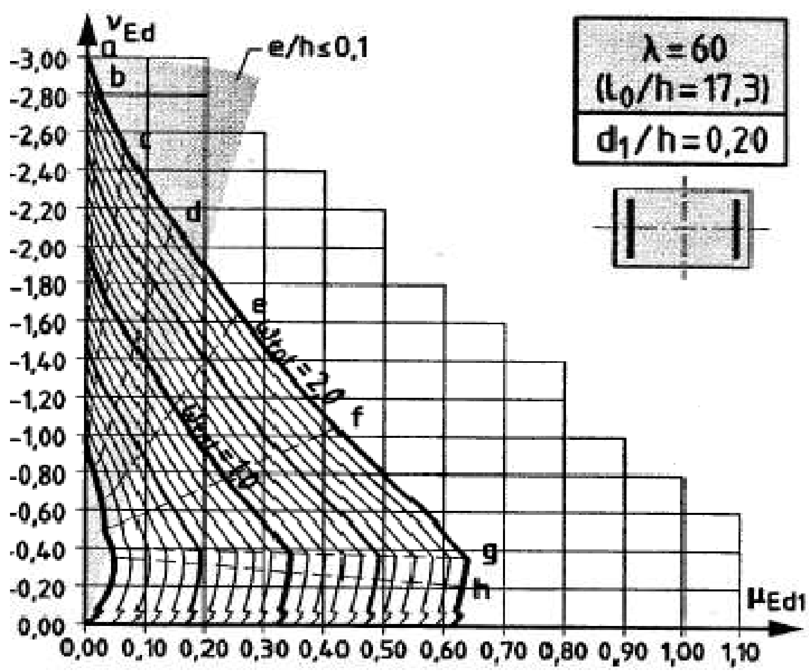


Figure 2_9 Interaction diagram for slender column [After, Schneider, 2006]

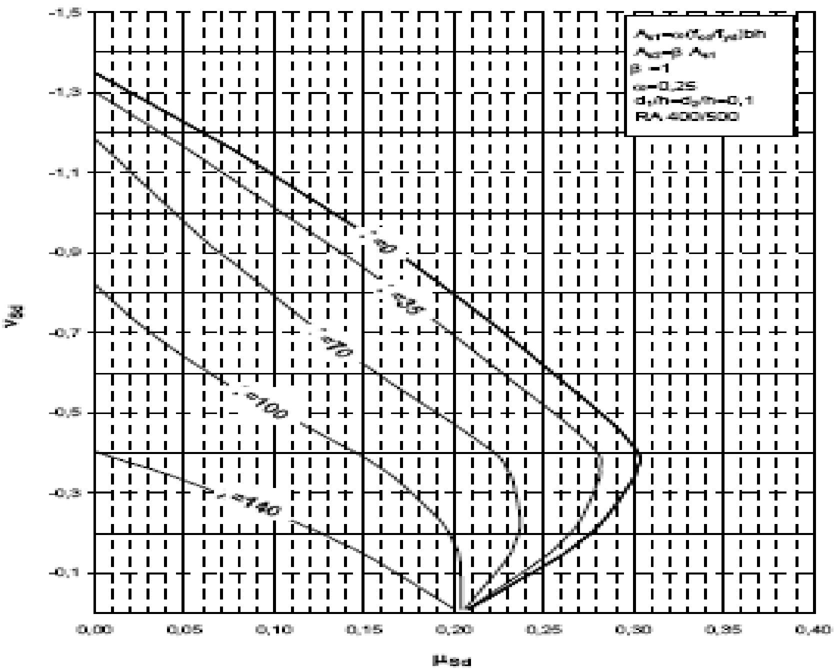


Figure 2_10 Interaction diagram for slender column with constant ω[After, Josip Galič *et al* ,2005]

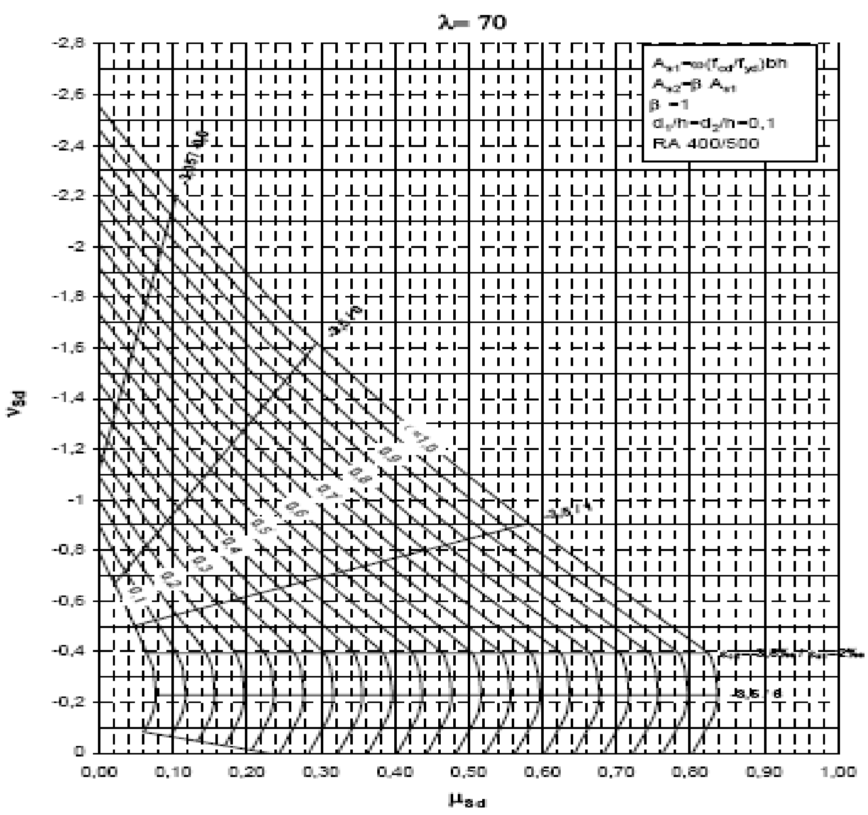


Figure 2_11 Interaction diagram for slender column with constant λ [After, Josip Galič *etal* ,2005]

2.2.2 Approximate methods of Slender RC Column design

2.2.2.1 EBCS 2, 1995 Recomendation

According to EBCS 2, 1995 recommendation, articles 4.4.4.4, and 4.10 described columns may be considered as isolated columns when they are isolated compression members. Such as individual isolated columns and columns with articulations in a non-sway structure or compression members which are integral parts of a structure which are considered to be isolated for design purposes (such as slender bracing elements considered as isolated columns, and columns with restrained ends in a non-sway structure)

For buildings, a design method may be used which assumes the compression members to be isolated and adopts a simplified shape for the deformed axis of the column. The additional eccentricity induced in the column by its deflection is then calculated as a function of slenderness ratio. Accordingly, the deflection of the column can be expressed as in equation (2.6).

1) The total eccentricity to be used for the design of columns of constant cross-section at the critical section is given $\mathrm{e_{tot}} = \mathrm{e_a} + \mathrm{e_o} + \mathrm{e_2}$ Eqn.2_5, (EBCS2, 1995 Eqn.4_14)

Where

e_{tot}= total design eccentricity

e_a=additional equivalent geometric imperfection eccentricity given by $\mathrm{e_a} = \frac{\mathrm{Le}}{300} \geq 20\mathrm{mm}$

e_o= is equivalent constant first order eccentricity of the design axial load

e_2=second order eccentricity

For non-sway frames, the second-order eccentricity, e_2 , of an isolated column may be obtained as

$$\mathrm{e_2} = \frac{\mathrm{k_1 le^2}}{10} * \frac{1}{\mathrm{r}}$$ Eqn.2_6, (EBCS2, 1995 Eqn.4_18)

Where $k_1 = \frac{\lambda}{20} - 0.75$ for $15 \leq \lambda \leq 35$

k_1=1.0, for $\lambda > 35$

$\frac{1}{r}$ is the curvature at the critical section

For isolated columns the slenderness ratio is defined by

$$\lambda = \frac{le}{i}$$ Eqn. 2_7, (EBCS2, 1995 Eqn.4_4)

i is the minimum radius of gyration of the concrete section only.

L_e is effective buckling length of the column

(1) The curvature is generally a non-linear function of the axial load and bending moment in the critical section, but the following approximate value may be used in the absence of more accurate methods:

$$\frac{1}{r} = k_2 * \left(\frac{5}{d}\right) * 10^{-3}$$ Eqn. 2_8, (EBCS2, 1995 Eqn.4_19)

Where **d** is the column dimension in the buckling plane less the cover to the center of the longitudinal reinforcement

$$k_2 = \frac{M_d}{M_{bal}}$$

M_d= is the design moment at the critical section including second-order effects

M_{bal}=is the balanced moment capacity of the column.

(2) The appropriate value of k_2 may be found iteratively taking an initial value corresponding to first order actions

2.2.2.2 CEB_FIP 1990 Simplified Design Methods for Isolated members

A design method may be used which adopts a simplified shape for the deformed axis of the member. The second order deflection e_2 is then calculated as function of the member length (l), the eccenricities e_{o1} and e_{o2} of the axial force at the endes of the member and the curvature $1/r_{tot}$ in the critical section with the total eccentricity e_{tot} according to equation Eqn.2_9

$$e_{tot} = e_0 + e_a + e_2$$ Eqn.2_9 , (CEB Eqn.6.6_16)

Where

e_o denotes first oreder eccentriciy e_0=$M_{sd,1}/N_{sd}$

$M_{sd,1}$ denotes the maximum design bending moment

N_{sd} denotes the applied design axial force

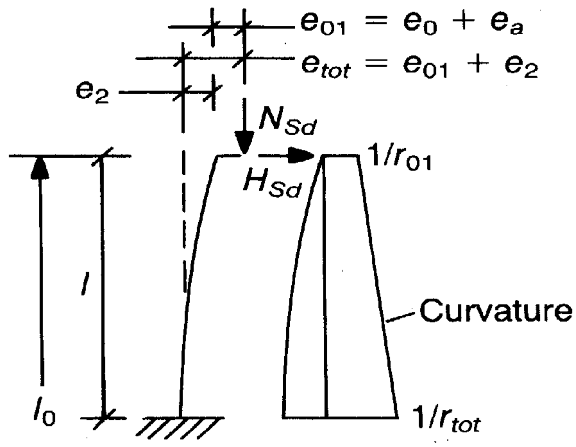


Figure 2_12 Model colum [After,CEB-FIP1990]

The' model column' as shown in figure 2_12 is a cantilever column with constant cross section , fixed at the base and free at the top. It is being bent in single curvature under loads and moments which give maximum moment at the base.In the case of constant reinforcement,the maximum delection is assumed to be:

$$e_2 = 0.1k_1 l^2 \left(\frac{4}{r_{tot}} + \frac{1}{r_{01}}\right) \quad \text{Eqn.2\_10, (CEB Eqn.6.6\_17)}$$

$$e_2 = 0.1k_1 l^2 \left(4 + \frac{e_{01}}{e_{tot}}\right) \frac{1}{r_{tot}} \quad \text{Eqn.2\_11, (CEB Eqn.6.6\_18)}$$

and when the reinforcement is curtailed in accordance with the bending moment diagram.It may be assumed to be

$$e_2 = 0.5k_1 l^2 \frac{1}{r_{tot}} \quad \text{Eqn.2\_12, (CEB Eqn.6.6\_19)}$$

where $1/r_{tot}$ =the curvature associated with eccentricity e tot

$1/r_{01}$= the curvature associated with eccentricity e_{01} and which may be assumed to be $(e_{01}/e_{tot})1/r_{tot}$

k_1 = a coefficient which is introduced in order to avoid discontinuty of the function describing the design bearing capacity when the slenderness bound (λ_1) is exceeded; it is obtained from Eqn.(2.13)

$$k_1 = 2\left(\frac{\lambda}{\lambda_1} - 1\right) \text{ for } \lambda_1 \le \lambda \le 1.5\lambda_1 \quad \text{Eqn.2\_13, (CEB Eqn.6.6\_20)}$$

k_1 =1 for $\lambda > \lambda_1$ Eqn.2_14, (CEB Eqn.6.6_21)

The curvature $1/r_{tot}$ is derived from the equilibrium of the internal and external forces

For pin ended column with constant cross_section and reinforcement and subjected tpo first order moments varying linearly along their lenghth and $|e_{02}| > |e_{01}|$ an equivalent eccentricity e_c may be taken as

$$e_c = 0.6e_{02} + 0.4e_{01}$$ Eqn. 2_15, (CEB Eqn.6.6_22)

The stability verificatiion may then be done as for a ‘model column’ but having only half length (l) of the real column.The cross_section design with e_{02} is also necessary. A fictitious curvature $1/r_{tot}$ in eqns.(6.6_17,_18 and_19) may be derived for rectangular cross_section with symmetrically arranged reinforcement in a top and bottom layer from

$$\frac{1}{r_{tot}} = 2k_2 \frac{\varepsilon_{yd}}{z_s}$$ Eqn.2_16 , (CEB Eqn.6.6_23)

Where

$\varepsilon_{yd}=f_{yd}/E_s$ is the design yield strain of steel reinforcement

z_s is the distance between compression and tension reinforcement,approximately z_s=0.9d

k_2 is coefficient, taking ito account the decrease of the curvature with increasing axial force as defined by Eqn.(6.6-24)

$$k_2 = \frac{N_{ud}-N_{sd}}{N_{ud}-N_{bal}} \leq 1$$ Eqn.2_17, (CEB Eqn. 6.6_24)

where

N_{ud} is the design ultimate capacity of the section subjected to axial load only,it may be taken as $0.85f_{cd}Ac+f_{yd}A_s$

N_{sd} is the actual design axial force

N_{bal} is the design axial load,when the applied to concrete section maximamizes its ultimate moment capacity;for symmetrically reinforced rectangular sections it may be taken as $0.4f_{cd}A_c$.It will always be conservative to assume k_2=1.For columns with coss_sections other than rectangular or with distiributed reinforcement equivalent values may be used for z_s

2.2.2.3 Euro code 2, Commentary Simplified methods and their common basis

In a simplified calculation method, one can use the difference between cross section resistance and first order moment M_u-M_o, in figure 2_13, as a nominal second order moment. When this moment is added to the first order moment, a design moment is obtained for which the cross section can be designed with regard to its ultimate resistance. As pointed out above, this nominal second order moment is sometimes greater than the" true" second order moment. However, it can give correct results, even in cases where the load capacity is governed by a stability failure before reaching the cross section resistance, if given appropriate values.

For practical design, there are two principal methods to calculate this nominal second order moment:

(1) Estimation of the flexural stiffness EI to be used in a linear second order analysis (i.e. considering geometrical non-linearity but assuming linear material behavior); this method is here called stiffness method

(2) Estimation of the curvature 1/r corresponding to a second order deflection for which the second order moment is calculated; this method is here called curvature method

The total moment including second order moment for a simple isolated member is:

$$M = M_o + M_2 = M_o + N * y = M_o + N\frac{1}{r} * \frac{l^2}{c} \qquad \text{Eqn. 2\_18}$$

Where (see figure 2_13)

M = total moment, M_0 = first order moment, M_2= second order moment, N=axial force,

y = deflection corresponding to curvature $(1/r)$, $1/r$ = curvature corresponding to y,

l = length, c = factor for curvature distribution

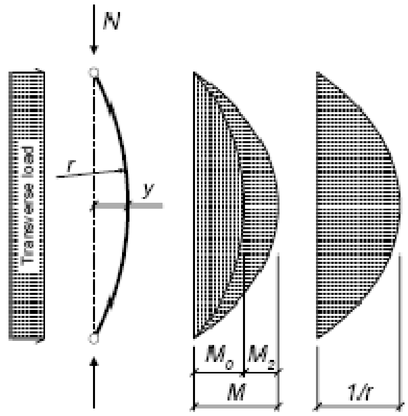


Figure 2_13 Illustration of deformations and moments in a pin-ended column [After, Euro Code Commentary, 2008]

In the figure 2_13, first order moment is exemplified as the effect of a transverse load. First, order moment could also be given by eccentricity of the axial load. The difference between the two methods differs in the formulation of the curvature, $\frac{1}{r}$.In the stiffness method; $\frac{1}{r}$ is expressed in terms of an estimated nominal flexural stiffness EI in Eqn.2_19

$$\frac{1}{r} = \frac{M}{EI} \qquad \text{Eqn.2\_19}$$

The stiffness EI should be defined in such a way that ultimate limit state (ULS) cross section design for the total moment M will give an acceptable result in comparison with the general method. This includes, among other things, taking account of cracking, creep, and non-linear material properties.

In the curvature method, the curvature 1/r is estimated directly defined by Eqn. (2.20), based on assuming yield strain in tensile and compressive reinforcement

$$\frac{1}{r} = \frac{2\varepsilon_{yd}}{0.9d} \qquad \text{Eqn.2\_20}$$

This model overestimates the curvature in those cases where yielding is not reached, giving a too conservative result. The typical example is where the ultimate load is governed by stability failure, before reaching the cross section resistance. The model may also underestimate the curvature in some cases, since it does not take into account creep. However, various corrections can be introduced to improve the result.

2.2.2.4 ACI code Recommendation

ACI_318_08 recommends slenderness effects neglected in the following cases:

(a) For compression members not braced against side sway when: $\frac{klu}{r} \leq 22$

(b) For compression members braced against side sway when: $\frac{klu}{r} \leq 34 - 12(\frac{M1}{M2}) \leq 40$

Where $\frac{M1}{M2}$ positive if the column is bent in single curvature and negative if the member is bent in double curvature.

ACI section10.10.6 — Moment magnification procedure for non sway

Compression members shall be designed for factored axial force P_u and the factored moment amplified for the effects of member curvature M_c as

$M_c = \delta_{ns} M_2$ Eqn. 2_21, ACI Eqn. (10_11)

Where $\delta_{ns} = \frac{C_m}{1 - \frac{Pu}{0.75Pc}} \geq 1.0,$ Eqn.2_22, ACI Eqn. (10_12)

$Pc = \frac{\pi^2 EI}{(klu)^2}$ Eqn.2_23, ACI Eqn. (10_13)

The values of EI shall be taken as

$EI = \frac{0.2EcIg + EsI_{se}}{1 + \beta_{dns}}$, Eqn. 2_24, ACI Eqn. (10_14)

Or $EI = \frac{0.4EcIg}{1 + \beta_{dns}}$, Eqn. 2_25, ACI Eqn. (10_15)

The term β_{dns} shall be taken as the ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination, but shall not be taken greater than 1.0.The effective length factor, k, shall be permitted to be taken as 1.0**.**For members without transverse loads between supports, C_m shall be taken as in Eqn.2_26

$c_m = 0.6 + 0.4\frac{M1}{M2}$ Eqn.2_26, ACI Eqn. (10_16)

Where $\frac{M1}{M2}$ positive if the column is is bent in single curvature, and negative if the member is bent in double curvature. For members with transverse loads between supports, C_m shall be taken as 1.0.Factored moment, M_2, in Eq. 2_27 shall not be taken less than about each axis separately, where 0.6, and h are in inches.

$$M_{2,min} = P_u(0.6 + 0.03h) \qquad \text{Eqn.2\_27, ACI Eqn. (10\_17)}$$

For members in which $M_{2,\,min}$ exceeds M_2, the value of C_m in Eqn.2_23 shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments,$\frac{M1}{M2}$ .In ACI 318_08 the design of slender RC column directly depends on exact determination of EI for moment magnification factor.

2.2.3 Summary of the literature review

So far, in the literatures reviewed including EBCS2, 1995, it is observed that the common bases of calculating second order eccentricity is almost the same for CEB1990 and EBCS_2, 1995. The deflected shape is assumed sinusoidal except the factors used in connection with factors relating second order eccentricity with effective length but the equations are simplified. Now, the focus of this thesis work is to use the simplified equation indicated in EBCS2, 1995 and to convert it in to approximate slender column interaction diagram design charts.

CHAPTER THREE

APPROXIMATE SLENDER RC COLUMN INTERACTION DIAGRAM

3.1 Reinforced Concrete Section Analysis

In the preparation of approximate interaction diagrams for slender column, section analysis is the first step followed by inclusion of column slenderness effect. According to Bill Mosley *et al.* (2007), in the analysis of the column section, the manipulation and juggling with equations should never be allowed to obscure the fundamental principles that unite the analysis. The three most important principles are

1. The stresses and strains are related by the material properties, including the stress strain curves for concrete and steel.
2. The distribution of strains must be compatible with the distorted shape of the cross section
3. The resultant forces developed by the sections must balance the applied loads for static equilibrium.

These principles are true irrespective of how the stresses and strains are distributed, or how the member is loaded, or whatever the shape of the cross section. This section describes analyses of a member section under load. It derives the basic equations used in the design and those equations required for the preparation of design charts. Emphasis has been placed mostly on the analysis associated with ultimate limit state.

3.1.1 Basic assumptions in the Analysis of Sections in the Ultimate Limit state

The strength of a cross section depends on the dimension of the cross section, the relative configuration and amount of the steel and concrete components and the material properties of the steel and concrete. In particular, stresses strain relationships for concrete and steel. Material properties are described in EBCS2, 1995 as shown in the figure (3_2) and (3_5), concrete and steel respectively. For a cross section capacity determination, subjected to bending moment and /or axial force, there are a number of assumptions. According to EBCS_2, 1995 the calculation of the ultimate resistance of members for flexure and axial loads shall be based on the following assumptions.

1) The strain assumed to distribute linearly over the cross section i.e. that is plane section remain plane after loading.
2) The reinforcement steel bar is fully bonded to the concrete such that there is no slip between concrete and steel bar(i.e. strain compatibility, perfect bond)
3) The tensile strength of the concrete is ignored in cross section strength analysis .(In ultimate limit state, it makes no noticeable difference)
4) The maximum compressive strain in the concrete is taken to be: 0.0035 in bending (simple or compound) 0.002 in axial compression,
5) The maximum tensile strain in the reinforcement is taken to be 0.01

3.1.2 Strain distribution of RC Section according to EBCS-2,1995

In figure 3_1, the strain diagram shall be assumed to pass through one of the three points A, B or C. In region 1 axial tension and tension plus small eccentricity in which entire cross-section is under tension, therefore reinforcement yields. In region 2 pure bending and bending plus axial load. This is the case of "*steel failure*." The neutral axis lies within the cross-section. The steel reinforcement reaches its maximum strain (ε_{s1}= 10 ‰), but the concrete has reserve with $|\varepsilon_c|$< 3.5‰. This is often the case with small reinforcement ratio.

In region 3 pure bending and bending plus axial load. This is case of "*concrete failure*." The concrete is fully exploited (ε_c = -3.5‰). The strain in the steel lies between the yield strain (ε_{yd} = 2.0 ‰) and the ultimate strain (10‰). In region 4 pure bending and bending plus axial load. The concrete is fully exploited. The steel strength is not yet fully exploited. The steel strain lies between 0 and ε_{yd}. The neutral axis lies deep in the cross-section.

Region 5 is a case with axial compression with small eccentricity. The entire cross-section is under compression. The strain in the most compressed concrete fiber lies between -2 and -3.5‰ on the more compressed edge of the cross-section and between 0 and -2‰ on the other edge. In this region, all strain profiles pass through point C, which is located at 3/7 h from the more compressed edge of the cross-section. The loadings in columns often result in strain distribution in this region.

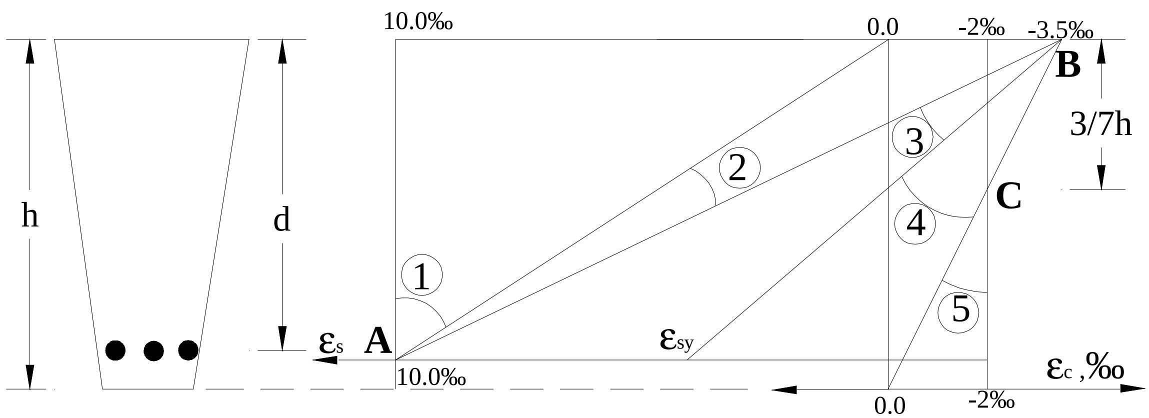


Figure 3_1 Strain Diagram in the Ultimate Limit State [After, EBCS_2, 1995]

3.1.3 Stress _Strain Relations

Short-term stress strain curves for concrete and steel are presented in EBCS2, 1995.These curves are in an idealized from which can be used in the analysis of member sections.

3.1.3.1 Concrete

The behavior of structural concrete is represented by a parabolic stress strain relationship up to ε_o, from which the strain increases while the stress remains constant as shown in figure 3_2. The ultimate design stress is given by

$$f_{cd} = \frac{0.85 f_{ck}}{\gamma_c}$$

Where the factor of 0.85 allows for the difference between the bending strength and the cylinder crushing strength of the concrete and γ_c=1.5 is the usual partial safety factor for the strength of concrete. The ultimate strain of ε=0.0035 is typical for classes of concrete<C50/60 [Bill Mosley, *et al*, 2007].Concrete classes <C50/60 are considered throughout this thesis work.

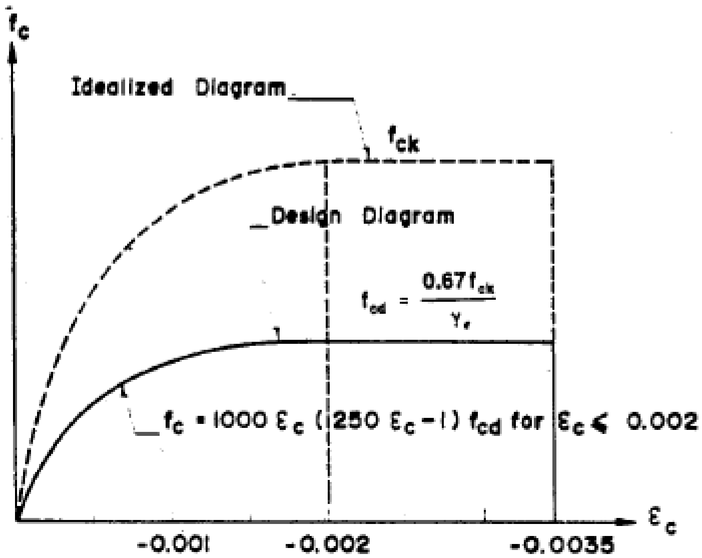


Figure 3_2 Parabolic _rectangular stress strain diagram for concrete in compression [After, EBCS_2, 1995]

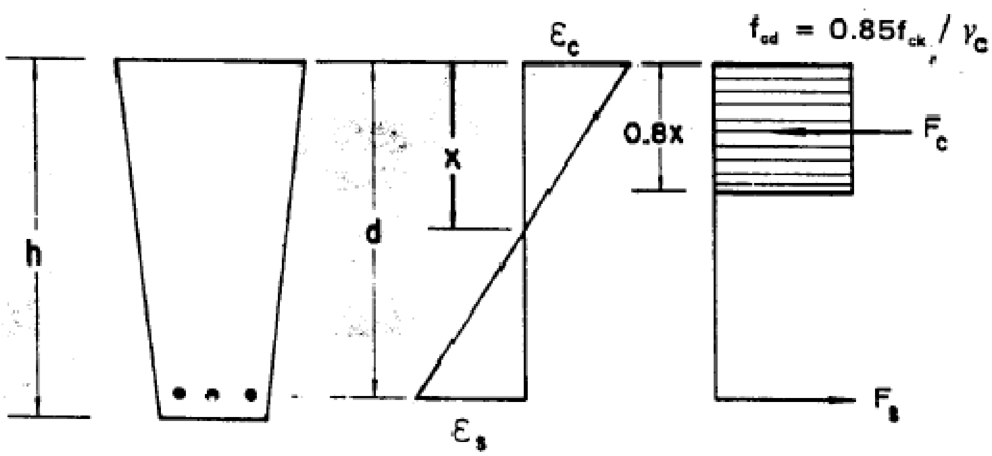


Figure 3_3 Rectangular Stress Diagram [After, EBCS_2, 1995]

3.1.3.2 Reinforcing Steel

The representative short-term design, stress strain curve for reinforcement is shown in figure (3_4). The behavior of steel is identical in tension and in compression, being linear in elastic range to the design yield stress of f_{yk}/γ_s, where f_{yk} is the characteristic yielding stress and γ_s is the partial factor of safety. Within the elastic range, the relationship between the stress and strain is;

Stress=elastic modulus * strain= $E_s*\varepsilon_s$, so that the design yield strain is at the ultimate limit

$$\varepsilon_y = \frac{(f_{yk/}\gamma_s)}{E_s}$$

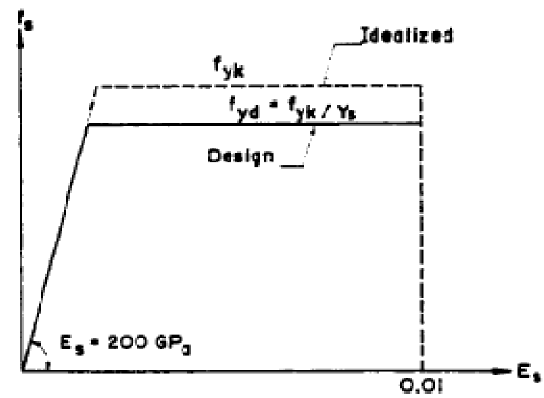


Figure 3_4 Design stress strain curve for reinforcement [After, EBCS_2, 1995]

3.1.4 Reinforced Concrete Column Section Stress Resultant

The determination of stress resultants with the locations of the point of application is one of the important single steps in the analysis of reinforced concrete sections. Each strain profile in the five zones in figure 3_1 corresponds to a particular combination of ultimate internal forces (axial force and bending moment) which are determined as the stress resultants in the concrete and reinforcement steel. Analysis of sections according to EBCS_2, 1995 follows the integration of stress strain diagram and equilibrium equations are shown below for different cases. Using the different cases the total compressive force on concrete and its point of application is determined, the next step is to compute the forces in the reinforcement steel, which are easily determined by using the strain distribution diagram and idealized stress strain relationship for steel.

Once the internal forces in the steel and concrete with their locations are calculated, the capacity of the given cross section at the ultimate limit states can be determined. The ultimate axial load capacity P_u for the assumed strain distribution is found by summing all internal forces in the axial direction .Therefore, the axial load capacity will be

$$P_u = C_c + C_{s2} + C_{s1}$$

The ultimate moment capacity M_u for the assumed strain distribution is found by summing moments of all internal forces about the centroid of the column section. Because this is the axis, about which moments are computed in conventional structural analysis [MacGregor *et al*, 2005]. Therefore, moment section capacity is calculated as follows

$$M_u = Cc\left(\frac{h}{2} - \beta_c d\right) + C_{s2}\left(\frac{h}{2} - h'\right) + C_{s1}\left(\frac{h}{2} - h'\right)$$

Where: $C_c=\alpha_c f_{cd} bd$ .The value of coefficients α_c and β_c is determined using the strain profiles indicated in figure 3-1.According to ε_{cm} value and depth of neutral axis x, the calculation of the coefficients α_c and β_c is made in the following three cases.

Case I: The neutral axis lies within the cross section i.e. $x \leq h$ and $\varepsilon_{cm} \leq \varepsilon_o = 2‰$

For the strain distribution in figure 3_6 (i) b, the strain at a distance z from the neutral axis can be expressed as:

$$\varepsilon = \frac{\varepsilon_{cm}}{x} y \qquad \text{Eqn.3\_1}$$

The stress in the concrete at the corresponding point can be determined from the idealized stress _strain relationship for concrete

$$f_c = \varepsilon\left(1 - \frac{\varepsilon}{4}\right) f_{cd} = \frac{\varepsilon_{cm}}{x} y \left(1 - \frac{\varepsilon_{cm}}{4x} y\right) f_{cd} \qquad \text{Eqn.3\_2}$$

The resultant normal force on the compressed concrete zone can be obtained by integrating the stress distribution over the compressed area

$$C_c = \int_0^{y=x} f_c b_y \, dy \qquad \text{Eqn.3\_3}$$

Where ,b_y is the width of the cross section

Inserting Eqn. 3_2 into Eqn. 3_3 and performing the integration over the given limits, the total compressive force on the compressed zone of the concrete sections with constant width b can be expressed as follows after rearranging terms

$$C_c = \alpha_c b d f_{cd} \qquad \text{Eqn.3\_4a}$$

Where

$$\alpha_c = \frac{\varepsilon_c}{12}(6 - \varepsilon_c)\frac{x}{d} = \frac{\varepsilon_c}{12}(6 - \varepsilon_c)k_x \qquad \text{Eqn.3\_4 b}$$

$$\text{And} \qquad k_x = \frac{x}{d}$$

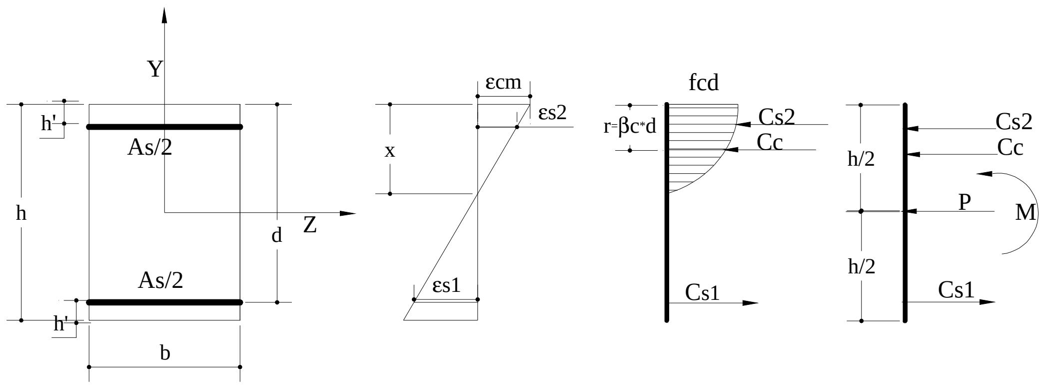


Figure 3_5 Stress strain diagram (i)[x≤h and $\varepsilon_{cm} \leq \varepsilon_o = 2‰$]

The location of Cc from the most compressed edge,$\beta c * d$, can be determined by finding the centroid of the stress distribution with the reference to the same point. From Engineering Mechanics Statics Centroid of a volume can be defined as

$$\hat{y} = \frac{\int y dv}{V}$$

The same analogy can be made for the stress volume:

$$\hat{y} = \frac{\int_0^{y=x} f_c b_y \, y dy}{Cc}$$

Where ŷ,is measure up ward from the neutral axis, b_y deducting from the neutral axis depth we can get: $\quad r = x - \hat{y}$

$$r = x - \frac{\int_0^{y=x} f_c b_y y dy}{Cc} \qquad \text{Eqn.3\_5}$$

Substituting the value of Cc from equation (3.4) and an expression for f_c from Eqn.3_2 into Eqn.3_5 and performing the integration over the given limits, the location of Cc from the most compressed edge for sections with constant width b can be expressed as follows

$$r = a * x \qquad \text{Eqn.3\_6a}$$

Where, $$a = \frac{8-\varepsilon_{cm}}{4(6-\varepsilon_{cm})} \qquad \text{Eqn.3\_6b}$$

By dividing Eqn. 3_6a by effective depth, d, we can get another coefficient(βc) of effective depth

$$\beta c = \left[\frac{8-\varepsilon_{cm}}{4(6-\varepsilon_{cm})}\right]\frac{x}{d} = \left[\frac{8-\varepsilon_{cm}}{4(6-\varepsilon_{cm})}\right]k_x = \frac{r}{d} \qquad \text{Eqn. 3\_6c}$$

$$\alpha_c = \frac{\varepsilon_{cm}(6-\varepsilon_{cm})}{12}k_x = \frac{\varepsilon_{cm}(6-\varepsilon_{cm})}{12}\frac{x}{d} = \frac{Cc}{f_{cd}bd} \qquad \text{From Eqn.3\_4b}$$

$$\beta c = \left[\frac{8-\varepsilon_{cm}}{4(6-\varepsilon_{cm})}\right]k_x = \left[\frac{8-\varepsilon_{cm}}{4(6-\varepsilon_{cm})}\right]\frac{x}{d} = \frac{r}{d} \qquad \text{From Eqn.3\_6c}$$

In similar way for concrete extreme fiber strain (ε_{cm})> ε_o=2‰ and neutral axis x≤h or x>h the value of α_c and β_c is derived as it is indicated in EBCS-2,1995, Part_2,1995 in Case II and Case III in the following section

Case II: The neutral axis lies within the cross section i.e $x \leq h$ and $\varepsilon_{cm} > \varepsilon_o = 2‰$

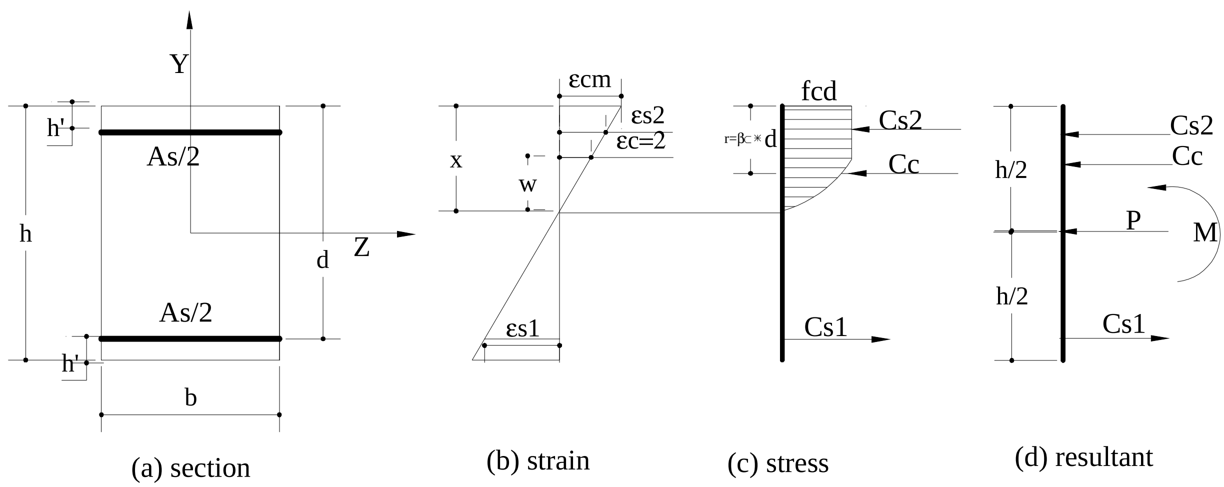


Figure 3_5 Stress strain diagram (ii) [$x \leq h$ and $\varepsilon_{cm} > \varepsilon_o = 2‰$]

$$\alpha_c = \frac{3\varepsilon_{cm}-2}{3\varepsilon_{cm}} k_x = \frac{3\varepsilon_{cm}-2}{3\varepsilon_{cm}} \frac{x}{d} = \frac{Cc}{f_{cd} bd} \qquad \text{Eqn.3\_7}$$

$$\beta c = \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm}-4)+2}{3\varepsilon_{cm}-2}\right] k_x = \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm}-4)+2}{3\varepsilon_{cm}-2}\right] \frac{x}{d} = \frac{r}{d} \qquad \text{Eqn.3\_8}$$

Case III: The neutral axis lies outside the cross section i.e $x > h$ and $\varepsilon_{cm} > \varepsilon_o = 2‰$

$$\alpha_c = \frac{1}{189} * [125 + 64\varepsilon_{cm} - 16{\varepsilon_{cm}}^2] = \frac{C_c}{f_{cd}\, bd} \qquad \text{Eqn.3\_9}$$

$$\beta_c = 0.5 - \frac{40}{7} * \left[\frac{(\varepsilon_{cm}-2)^2}{125+64\varepsilon_{cm}-16{\varepsilon_{cm}}^2}\right] = \frac{r}{d} \qquad \text{Eqn.3\_10}$$

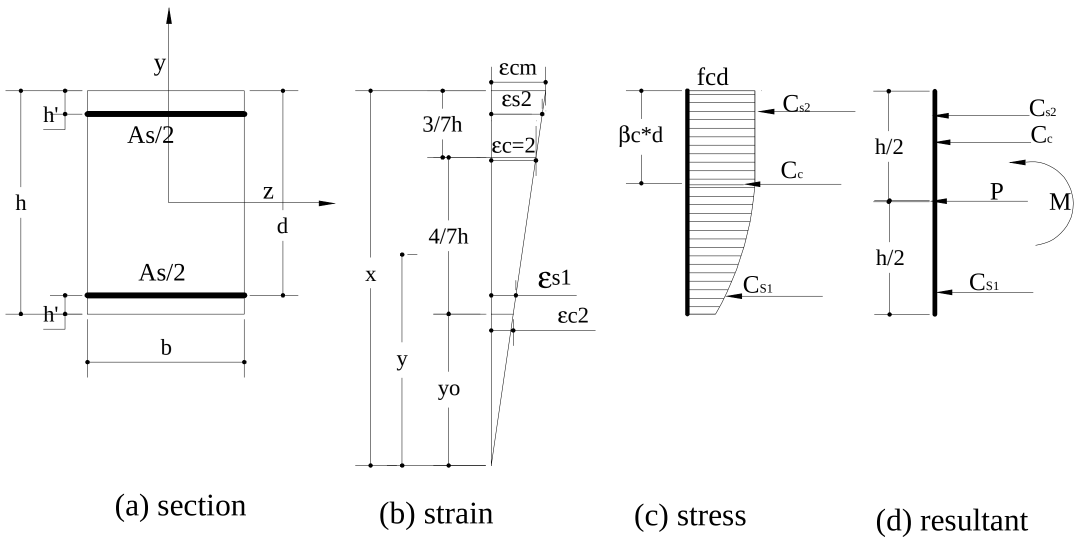


Figure 3_5 Stress strain diagram (iii)[x>h and $\varepsilon_{cm}>\varepsilon_o=2‰$]

3.2 Uniaxial Interaction Diagram

The design of RC columns is more difficult than the design of RC beams. Because the ultimate section capacity varies with neutral axis depth and strain, profile and cannot be generalized in single equation. For determination of section capacity of a given compression member, it is required to iterate and balance the capacity with external load.

In practice, the longitudinal steel in an RC column is usually chosen with the aid of an interaction diagram. An interaction diagram is a graphical summary of the ultimate bending capacity of a range of RC columns with different dimensions and areas of longitudinal reinforcement.

Interaction diagrams for reinforced concrete sections can in general be plotted by assuming a series of strain distribution at failure in different zones of the strain profile as shown figure 3_1 and computing the corresponding values of P and M as it is shown in section 3.1.4. One pair of such values represents the coordinate of a particular point on the interaction diagram. Figure 3-6 shows an interaction diagram for a rectangular reinforced concrete cross section having ω=1.0 ,C_30 and

f_{yk}=460 Mpa, in non dimensionalized form, which can be plotted by choosing sufficient number of strain distribution in the ultimate limit state and determining the corresponding stress resultants as described in section 3.1.4

In this approach, the strains could be main controlling variable manner; however, the corresponding points on the interaction diagram are not evenly spaced because internal force is not linearly related to strain. Therefore, the points could be clustered together at some regions while dispersed at other regions, which make it unsuitable for symmetric generation of interaction diagrams. A preferred method is therefore, is to vary the normal force P, in a controlled manner and determine the associated ultimate bending moment; a much more difficult task requiring iteration. The points obtained are not evenly spaced. Therefore, result in a smooth or best-fit plot of interaction diagram [Tefera Desta, 1999]. This can be done better in a computer program [Girma Zerayohannes, 1995]. Nevertheless, here in this thesis work the interaction diagram is plotted by choosing a number of strain distribution calculating corresponding moment and axial capacity using MS Excel sheet.

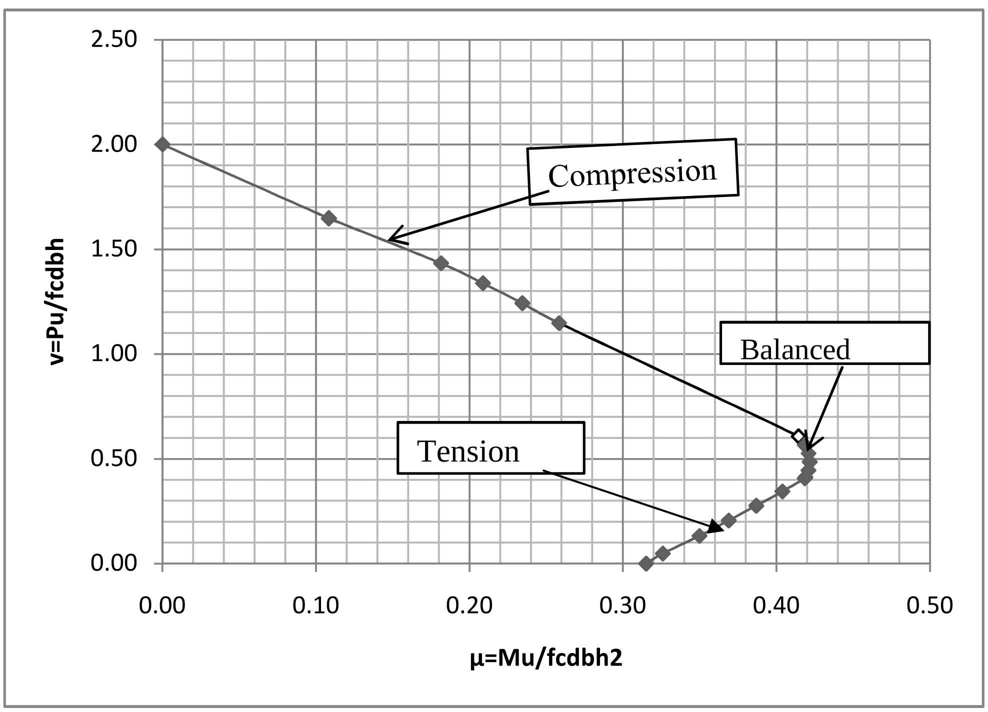


Figure 3_6 Uniaxial P-M Interaction diagram failure zones

3.2.1 Basic Equations for Calculation of Interaction Diagram

The equations derived in section 3.1.4 are used again here. The forces developed within the cross section, therefore must balance the applied force (P).

$$P_u = C_c + C_{s2} + C_{s1}$$

In this equation, C_{s1} will be negative whenever the position of the neutral axis is such that the reinforcement A_{s1} is in tension.

$$P_u = \alpha_c f_{cd} bd + f_{s2} A_{s2} + f_{s1} A_{s1} \qquad \text{Eqn.3\_11}$$

$$M_u = \alpha_c f_{cd} bd\left(\frac{h}{2} - \beta_c d\right) + f_{s2} A_{s2}\left(\frac{h}{2} - h'\right) - f_{s1} A_{s1}\left(\frac{h}{2} - h'\right) \qquad \text{Eqn.3\_12}$$

Where

Cc: be the compressive force developed in the concrete and acting through the centroid of the stress block

C_{s2}: be the compressive force in the reinforcement area A_{s2} and acting through its centroid.

C_{s1}: be the tensile or compressive force in the reinforcement area A_{s1} and acting through its centroid.

f_{s2} is the compressive stress in reinforcement A_{s2} and

f_{s1} is the tensile or compressive stress in reinforcement A_{s1}

For a symmetrical arrangement of reinforcement (A_{s2}=A_{s1}=A_s/2) and h'=h-d) Eqn.3_11 and 3_12 can be rewritten in non dimensional form as follows

$$\nu = \frac{P}{f_{cd} A_c} = \frac{\alpha_c f_{cd} bd}{f_{cd} A_c} + \frac{f_{s2} A_s}{2 f_{cd} A_c} + \frac{f_{s1} As}{2 f_{cd} A_c} \qquad \text{Eqn.3\_13}$$

$$\mu = \frac{M_u}{f_{cd} A_c h} = \frac{\alpha_c f_{cd} bd\left(\frac{h}{2} - \beta_c d\right)}{f_{cd} A_c h} + \frac{f_{s2} As\left(\frac{h}{2} - h'\right)}{2 f_{cd} A_c h} + \frac{-f_{s1} As\left(\frac{h}{2} - h'\right)}{2 f_{cd} A_c h} \qquad \text{Eqn.3\_14}$$

In these equations the steel strains , and hence the stresses f_{s2} and f_{s1} vary with depth of the neutral axis (x).Thus ν and μ can be calculated for specified ratios of $\frac{As}{bh}$ and $\frac{x}{h}$ so that the column design charts for a symmetrical arrangement of reinforcement such as the one shown in figure 3_6 and 3_8 can be plotted.

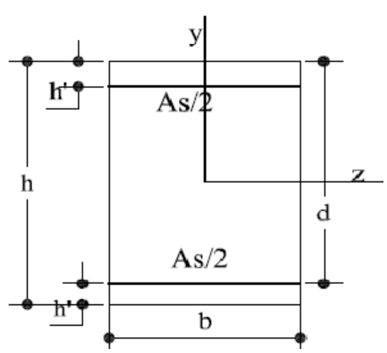
3.2.2 Parameters for the Cross section Interaction Diagram Calculation

Parameters in the Interaction diagram calculation by ***MS Excel Sheets*** for rectangular column cross section in figure 3.8

1. Class work I, safety factor (γ): Concrete γ_c=1.5, Steel γ_s=1.15
2. Concrete class C _25 to C-60Mpa , $f_{cd} = \frac{0.85f_{ck}}{\gamma c} = \frac{0.85f_{cu}/1.25}{\gamma c} = \frac{0.68f_{cu}}{\gamma c}$
3. Reinforcement steel class: S-300 to S-500 MPa, $f_{yd} = \frac{f_{yk}}{\gamma_s}$, Es = 200 GPa
4. Rectangular Column section : **b/h** and Concrete cover ratio h'/h=0.1
5. The column reinforcements are symmetrical
6. Mechanical steel reinforcement ratio, $\boldsymbol{\omega} = \frac{A_s f_{yd}}{A_c f_{cd}}$
7. EBCS 2, 1995 Mechanical steel reinforcement ratio limits: $A_{s,tot} = \left[\frac{\omega * f_{cd}}{f_{yd}}\right] * A_c = \rho * A_c$

$A_{s,\,min}$=0.04A_c and $A_{s,max}$=0.008A_c,

For C-60 and S-300,$\omega_{min} = 0.08$ and C-25 and S-500 $\omega_{max} = 1.53$.However, in recent days the maximum value of mechanical reinforcement ratio, ω=2 is used [Schneider K-J,2006]. In addition, for easier concrete compaction during construction reinforcements in the column are lapped alternatively rather than lapping all reinforcement at floor level. Therefore, in this thesis work the maximum value of ω is 2.



Figur3_7 Rectangular symmetrically RC column section

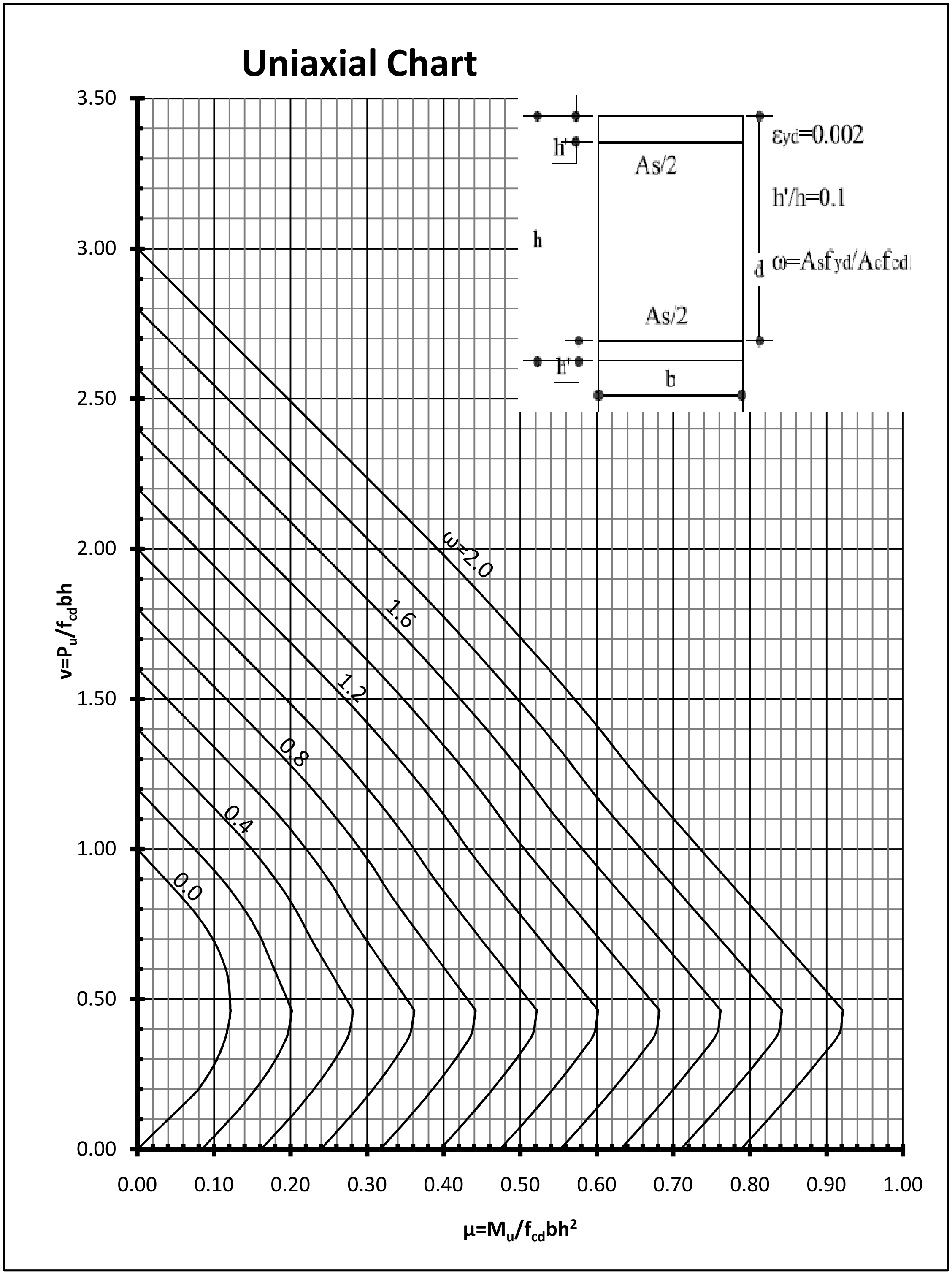


Figure 3_8 uniaxial interaction diagrams

3.3 Approximate Slender RC Column Interaction Diagrams Preparation

In the analysis of RC columns, it is obvious that the capacity of slender column is less than short column of the same cross section and boundary condition. For the derivation of slender interaction diagram, different authors have different approach using cross section diagram and slenderness ratio (λ).MacGregor *et.al.* (2005) and Bazant *et al.* (1991) used load moment curves to derive slender column interaction diagram rigorously. On the other hand, Hyo-Gyoung Kwak *et al.* (2005) and Thomas C.Edwards *et.al.* (1965) used reduction factor developed from cross section interaction diagram and then derived slender interaction diagram

However, in this thesis, approximate uniaxail slender column interaction diagram is derived which is based on cross section interaction diagram and second order eccentricity formula of EBCS_2, 1995. The method is similar to that of Josip Galič *et al.* (2005).In the method additional moment (M_{add}) due to second order and geometric imperfection eccentricity is deducted from cross section moment capacity (M_u) of interaction diagram is so that it would have a room for the slenderness effect as it is shown in figure 3_9. This can be done as in Eqn.3_15 and 3_16.

$$\mu_{sc} = \frac{M_{sc}}{f_{cd}bh^2} = \frac{M_u - M_{add}}{f_{cd}bh^2} = \mu_u - \nu_u * \left(\frac{e_2 + e_a}{h}\right) \qquad \text{Eqn. 3\_15}$$

$$\nu_{sc} = \frac{P_{sc}}{f_{cd}bh} = \frac{P_u}{f_{cd}bh} \qquad \text{Eqn. 3\_16}$$

Where

μ_{sc}=normalized slender moment capacity

μ_u=normalized ultimate cross section moment capacity and

$$\mu_u = \frac{M_u}{f_{cd}A_c h} = \frac{\alpha_c f_{cd} bd\left(\frac{h}{2} - \beta_c d\right)}{f_{cd}A_c h} + \frac{f_{s2}As\left(\frac{h}{2} - h'\right)}{2f_{cd}A_c h} + \frac{-f_{s1}As\left(\frac{h}{2} - h'\right)}{2f_{cd}A_c h} \qquad \text{From Eqn.3\_14}$$

μ_{add}=normalized additional moment due to slenderness

e_2 = second order eccentricity Eqn.3_20a of section 3.3.1

e_a= geometric imperfection eccentricity Eqn.3_19 of section 3.3.1

ν_u= normalized ultimate cross section axial load capacity

ν_{sc}= normalized ultimate slender column axial load capacity

$$\nu_u = \frac{P_u}{f_{cd}A_c} = \frac{\alpha_c f_{cd} bd}{f_{cd}A_c} + \frac{f_{s2}A_s}{2f_{cd}A_c} + \frac{f_{s1}As}{2f_{cd}A_c}$$ From Eqn.3_13

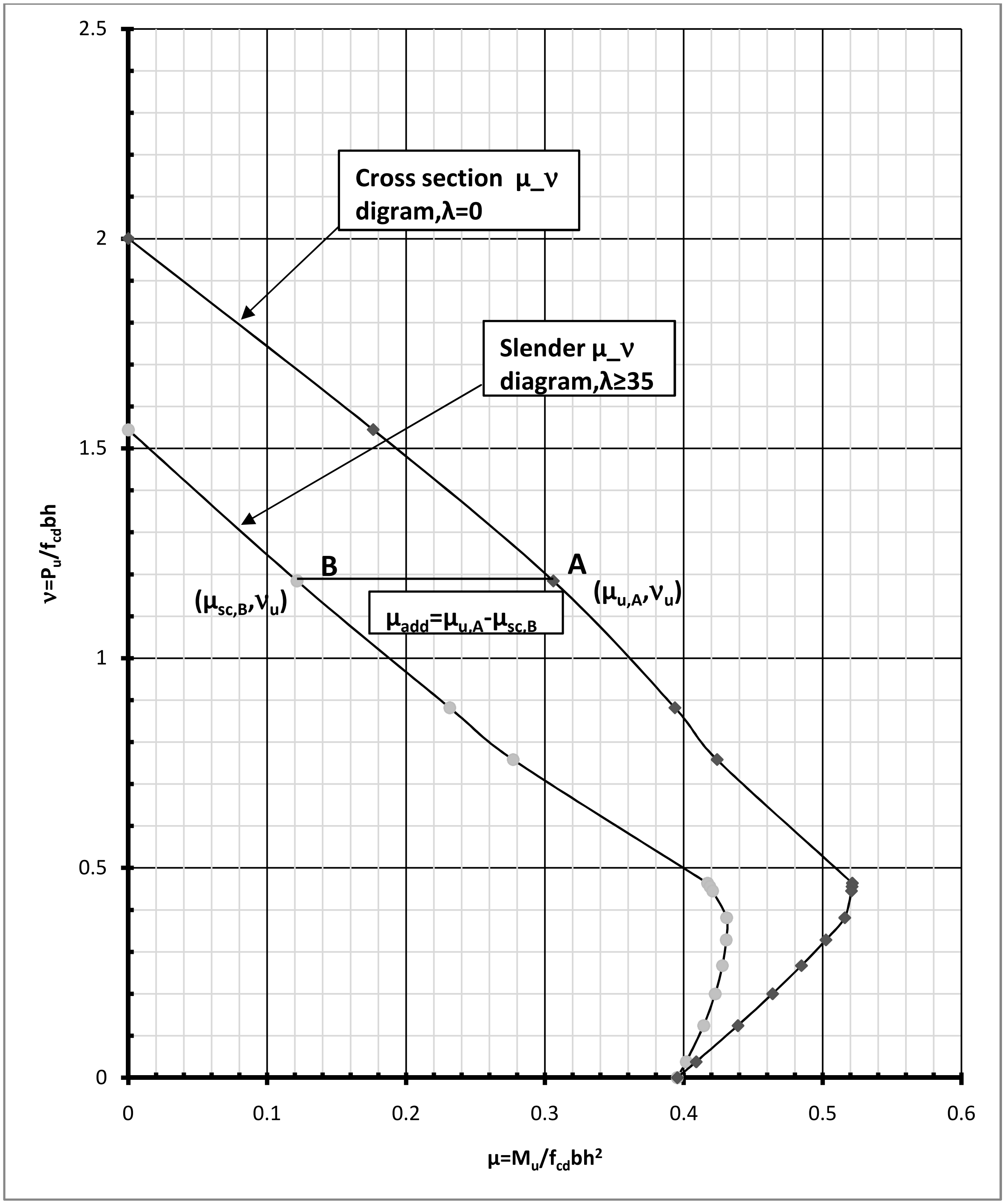


Figure 3_9 Slender Column interaction diagram derivation

3.3.1 Second order and geometric imperfection eccentricity

According to EBCS 2, 1995, the total eccentricity of a non-sway isolated column is given Eqn.3_18. However, in the preparation of slender column interaction diagram only second order and geometric imperfection eccentricities are used because by their nature these are the main causes for reduction of column capacity. The first order eccentricity is equivalent eccentricity in short and perfectly aligned column, which is vey ideal. Therefore, throughout this thesis work additional moment is calculated due second order and geometric imperfection eccentricity.

$$e_{tot} = e_o + e_a + e_2 \qquad \text{Eqn.3\_18}$$

Where

e_{tot}= total design eccentricity

e_a=additional equivalent geometric imperfection eccentricity given by

$$e_a = \frac{Le}{300} \geq 20mm \qquad \text{Eqn.3\_19}$$

e_o= is equivalent constant first order eccentricity[EBCS_2,1995 Eqn4.15,4.16a and4.16b]

e_2=second order eccentricity

$$e_2 = \frac{k_1 le^2}{10} * \frac{1}{r} \qquad \text{Eqn.3\_20a}$$

Where

$$k_1 = \frac{\lambda}{20} - 0.75 \qquad \text{for } 15 \leq \lambda \leq 35 \qquad \text{Eqn.3\_20b}$$

k_1=1.0, for $\lambda > 35$

For isolated columns the slenderness ratio is defined by

$$\lambda = \frac{le}{i} \qquad \text{Eqn. 3\_20c}$$

Where

i is the minimum radius of gyration of the concrete section only.

l_e is effective buckling length of the column (section 4.4.7 of EBCS_2,1995).

$\frac{1}{r}$ is curvature at the critical section

The curvature is generally a non-linear function of the axial load and bending moment in the critical section, but the following approximate value may be used in the absence of more accurate methods:

$$\frac{1}{r} = k_2 * \left(\frac{5}{d}\right) * 10^{-3} \qquad \text{Eqn. 3\_20d}$$

Where d is the column dimension in the buckling plane less the cover to the center of the longitudinal reinforcement

$$k_2 = \frac{M_d}{M_{bal}} \leq 1.0 \qquad \text{Eqn. 3\_20e}$$

M_d= is the design moment at the critical section including second-order effects

M_{bal}=is the balanced moment capacity of the column.

By substituting the values of k_2 and k_1 in to Eqn. 3_20a we can get

$$e_2 = \left(\frac{\lambda}{20} - 0.75\right) \frac{Msd}{M_{bal}} \frac{le^2}{10} \left(\frac{5}{d} * 10^{-3}\right) \qquad \text{Eqn.3\_21}$$

From the above paragraph, it can be clearly observed that some parameters are can be taken as unity. For instance, $k_1 = \frac{\lambda}{20} - 0.75$ will be 1.0 for λ>35.This shows that smaller slenderness ration tends to reduce the values of second order eccentricity. It is clear that the effective length le, k_2 and effective cross section depth (d) affect second order eccentricity. Therefore, the simplified second order eccentricity Eqn.3_21 can be expressed as in Eqn.3_22

$$e_2 = \frac{k_2 \; le^2}{10} \left(\frac{5}{d} * 10^{-3}\right) \qquad \text{Eqn.3\_22}$$

This is second order eccentricity due the deformation of the column caused by eccentric loading or lateral loading. In addition, due to accidental eccentricity (e_a) the column deviation from centerline is indicated by Eqn. 3_19 which is also dependent on the effective length of the column. Therefore, it is intended to add both eccentricities together and expressed Eqn.3_23.

$$e_{2+a} = \frac{k_2 \; le^2}{10} \left(\frac{5}{d} * 10^{-3}\right) + \max\left[20, \frac{le}{300}\right] mm \qquad \text{Eqn.3\_23}$$

Here, Eqn. 3_23 is backbone of this thesis work. Now we can observe that from Eqn.3_23 the second order eccentricity is part of total eccentricity .i.e.

$$e_{tot} = \quad e_o + e_{2+a} = \quad e_o + \frac{k_2 \, le^2}{10} \left(\frac{5}{d} * 10^{-3}\right) + \max\left[20, \frac{le}{300}\right] \qquad \text{Eqn.3\_24}$$

In Eqn.3_24 we can observe that the column length (le) is involved in both second order eccentricity and additional eccentricity, however, in this thesis work, the slender column interaction diagrams are drawn by varying the slenderness ratio rather than length. Therefore, the length of column is expressed in terms of slenderness ratio and column depth as in Eqn. 3_25d.

Slenderness Ratio (λ) defined by Eqn. 3_20c

$$\lambda = \frac{le}{i} \qquad \text{Eqn.3\_20c}$$

and Radius of gyration (i)

$$i = \sqrt{\frac{I_g}{A_g}} \qquad \text{Eqn.3\_25a}$$

For rectangular column section b/h, $I_g = \frac{bh^3}{12}$ and $A_g = bh$ Eqn.3_25b

Where

I_g is gross moment of inertia of the section bending is about the major axis

A_g is gross cross sectional area

From Eqn.3_20 a ,3_25a and 3_25b

$$\lambda \frac{h}{\sqrt{12}} = le \qquad \text{Eqn.3\_25c}$$

L_e is effective buckling length of the column.

By substituting Eqn.3_25c into Eqn.3_24, we can get second order eccentricity in terms of slenderness ration and section depth.

$$e_{2+a} = \frac{k_2(\lambda * h)^2}{12}\left(\frac{5*10^{-4}}{d}\right) + \max\left[20, \frac{\lambda * h}{300\sqrt{12}}\right] \qquad \text{Eqn.3\_26a}$$

Let $\frac{d}{h} = m$ and hence $d = m * h$ and again equation (3.23a), can be expressed as

$$e_{2+a} = \left[\left(\frac{k_2 * 5*10^{-4}}{12}\right)\frac{h(\lambda)^2}{m}\right] + \max\left[20, \frac{\lambda * h}{300\sqrt{12}}\right] \qquad \text{Eqn.3\_26b}$$

The appropriate value of k_2 in equation 3_20e and 3_26 may be found iteratively taking an initial value corresponding to first order actions. In this thesis work, the value of k_2 is calculated by taking cross section moment as a design moment and the corresponding balanced moment. In equation 3_26, the slenderness ratio (λ) ranges from 35 to 140.The, lower limit of slenderness is 35 because below this value it has no significant second order effect as in equation 3_20b.

3.3.2 Approximate Uniaxial SRCC charts

According to section 3.3, Approximate Uniaxial SRCC (slender reinforced concrete column) charts are derived by deducting additional moment from cross section interaction diagram coordinates as in Eqn.3_15 and 3_16 using excel sheet. For each mechanical reinforcement ratios, corresponding additional eccentricity is calculated. The additional eccentricity considered shown as in equation 3_26b, which is expressed in terms of column slenderness.

In the calculation of slender column interaction diagram, two alternative procedures may be followed which depend mechanical reinforcement ratio (ω) and slenderness ratio. The first is by keeping reinforcement ratio constant and varying slenderness ration as shown in figure (3_14) which is very similar to the diagrams described in the literature part, section 2.2.1

The second is varying the reinforcement ratio and keeping the slenderness ratio constant as shown in figure (3_15) and (3_16). In this thesis work, the second method is preferred, since it is suitable for direct calculation of reinforcement for a given slenderness ratio, moment and axial load calculated by first order analysis. In the same way, the interaction diagram for slender RC column is plotted for different mechanical reinforcement ratio (ω) and slenderness ratio (λ) for concrete cover ratio (h'/h) of 0.1.

For example for slenderness ratio λ=35 and 40 the interaction diagrams are plotted in figure (3_14) and (3_15) and the remaining diagrams are shown in the Appendix A. It is also possible to plot SRCC chart concrete cover ratio. As an example for λ=60 and concrete cover ratio of h'/h=0.05,0.15,0.20 and0.25 SRCC chart is drawn in Appendix B. Finally, for end users of the chart, user guideline is prepared and given in Appendix C.

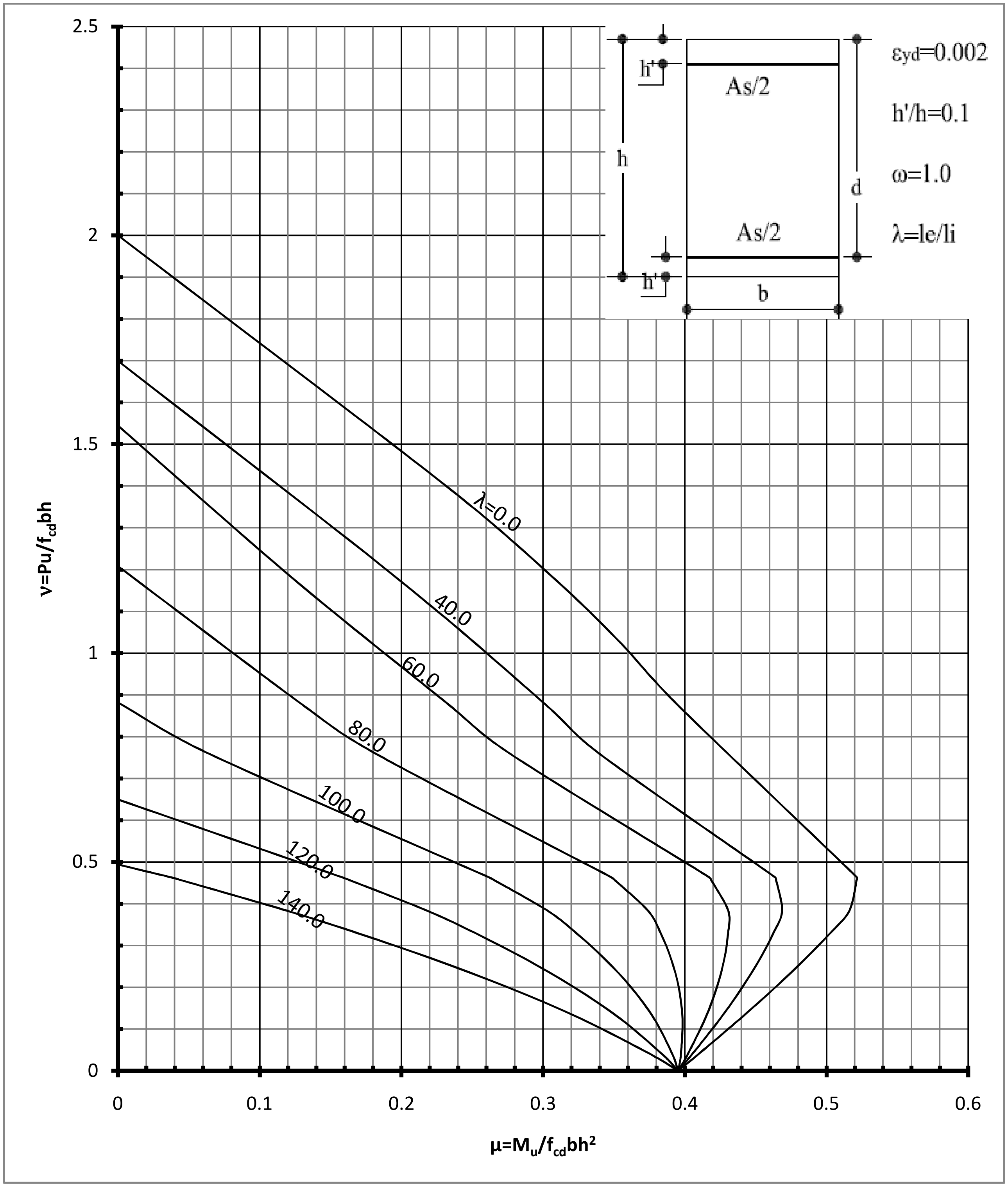


Figure 3_10 Uniaxial slender interaction diagram for ω=1.0

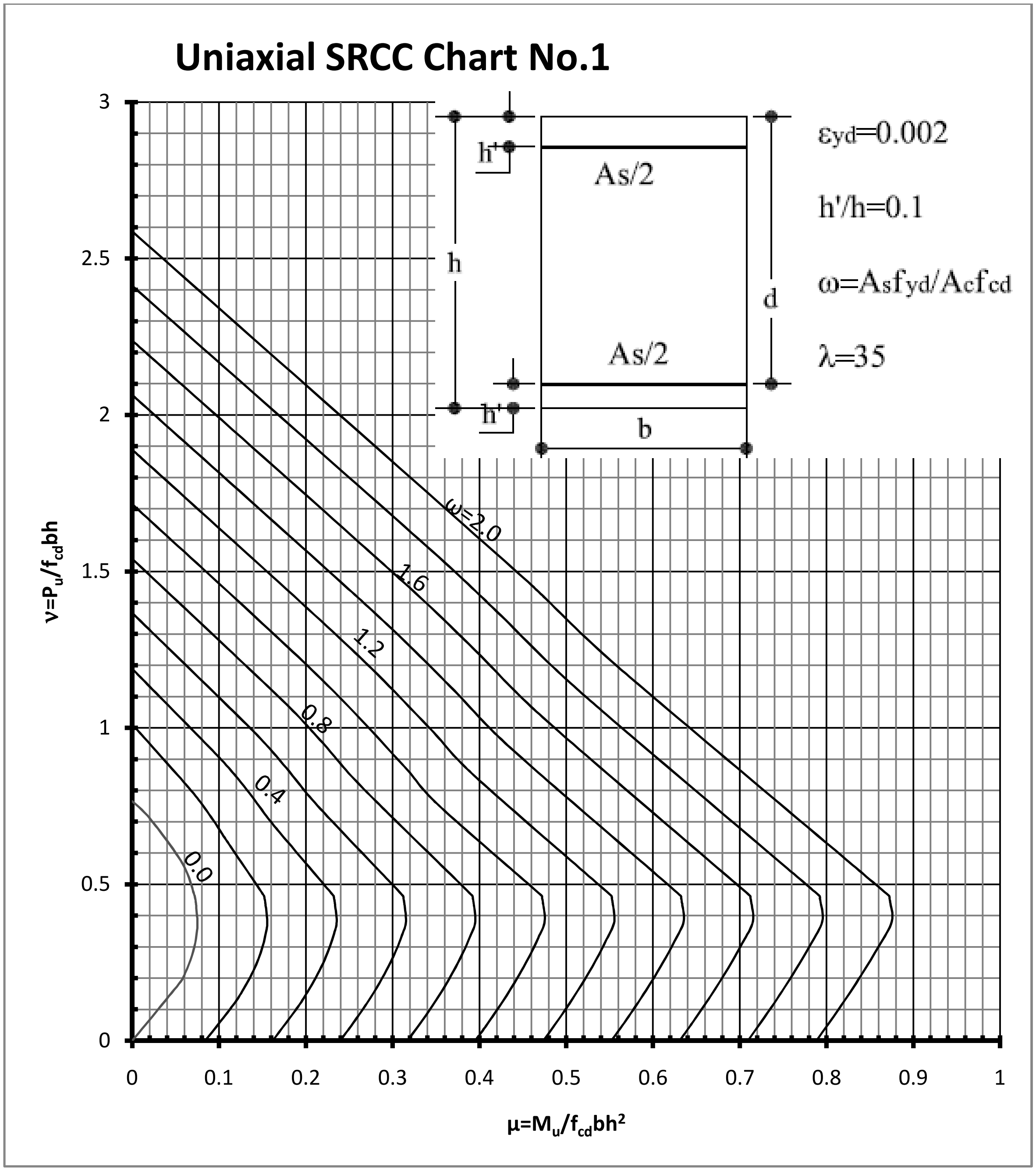


Figure 3_11 Interaction diagram for slender RC column λ=35

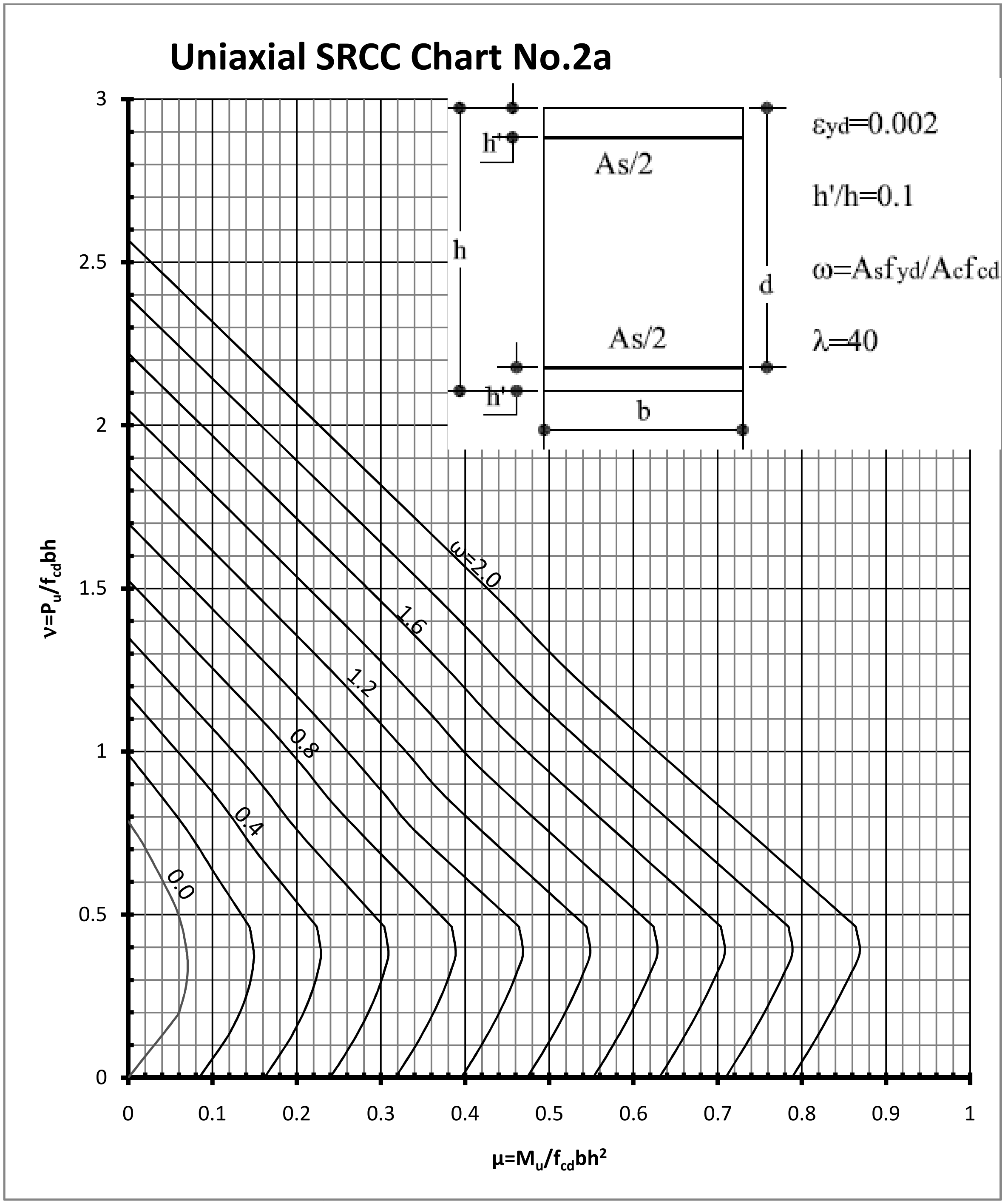


Figure 3_12 Interaction diagram for slender RC column λ=40

3.3.3 Verification of the Approximate SRCC design charts

In order for the approximate slender column uniaxial interaction diagrams developed in thesis work to be used as design aids it is necessary to check its validity with other existing methods. The SRCC chart is compared with iterative procedure of EBCS2, 1995.In the absence of such chart in EBCS2, 1995 Schneider K-J Charts [16] also used for verification, which is very similar to CEB-FIP1990. As it is pointed out in the literature part, the bases of EBCS_2, 1995 and CEB_FIP1990 second order eccentricity calculation method is similar, but not the same. The verification is done using examples shown below. The main verification parameter for comparison is mechanical reinforcement ratio (ω). For clarity, the first example is done in detail and the remaining ones are summarized in table form in section 3.3.5

Example 1: Taken from EBCS2, 1995 Part 2 Design aid for concrete sections

Design of slender braced Column Subjected to uniaxial bending

Given

Action effects:

- Factored axial load=1650kN
- Factored first order moment equivalent constant moment=130kN.m

Geometry: L=7.0m, le=k*L=0.7*L=4.90m

Material Data: Concrete C30 and Steel Grade 400

Required: Quantity of Reinforcement. A_s=?

***Solution 1.* Using SRCC chart developed in this thesis**

1) First order moment and axial load given
2) Design material strength.

Concrete $$f_{cd} = \frac{0.85 f_{ck}}{\gamma c} = \frac{0.85 * 30/1.25}{1.5} = 13.6\text{Mpa}$$

Steel $$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400\text{Mpa}$$

3) Column cross section:

Assume column size, b/h=400/400mm with cover ratio to reinforcement =20mm.

Reinforcement bar diameter: ϕ20 and Ties ϕ=10mm d'=20+10+0.5x20=40mm and d'/h=40/400=0.1

Calculate the relative first order axial force and moment

$\nu_{sd} = \frac{P_{sd}}{f_{cd}A_c} = \frac{1650*10^3}{13.6*400^2} = 0.76$ and $\mu_o = \frac{M_o}{f_{cd}A_c h} = \frac{(130)*10^6}{13.6*400*400*400} = 0.15$

4) Calculate slenderness ratio $\lambda = \frac{le}{i} = \frac{le\sqrt{12}}{h} = \frac{4900\sqrt{12}}{400} = 42.4$

5) Reading ω from Uniaxial SRCC Chart in figure (3_13) below

***Solution 2*: Schneider K-J Charts [16]**

Taking the same cross section properties as *Solution 1*

$$\nu_{sd} = \frac{P_{sd}}{f_{cd}A_c} = \frac{1650 * 10^3}{13.6 * 400^2} = 0.76$$

$$\mu_{o+a} = \frac{M_{sd} + e_a * P_{sd}}{f_{cd}A_c h} = \frac{(130 + 0.02 * 1650) * 10^6}{13.6 * 400 * 400 * 400} = 0.19$$

$$\lambda = \frac{le}{i} = \frac{le\sqrt{12}}{h} = \frac{4900\sqrt{12}}{400} = 42.4$$

Using Tafel 10a, Interpolation is required.

For λ=40,ω=0.44 and λ=50,ω=0.48

$$:\omega_{42.4} = \omega_{50} - (\omega_{50} - \omega_{40})\left(\frac{50-42.4}{50-40}\right) = 0.48 - (0.48 - 0.44)\left(\frac{50-42.4}{50-40}\right) = 0.45$$

$$\text{SRCC Difference with Schineider } \% = \left(\frac{\omega1-\omega2}{\omega1}\right) = \left(\frac{0.48-0.45}{0.48}\right) * 100\% = 6.3\%$$

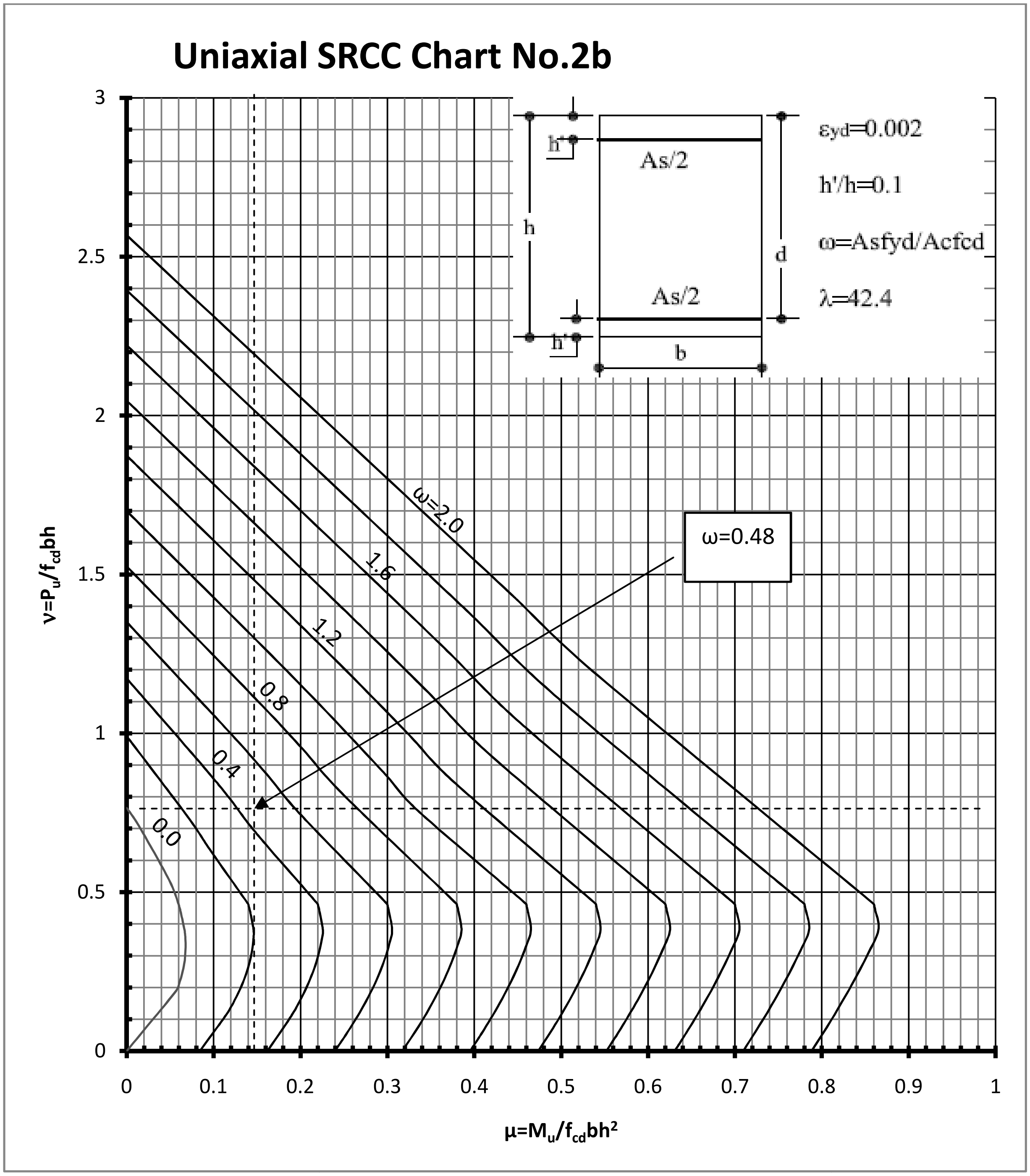


Figure 3_13 Interaction diagram for slender RC column λ=42.4

Solution 3: **Iterative Method of EBCS_2, 1995 Part 2 Design Aids for Reinforced Concrete Sections on the Basis of EBCS2-2,1995**

Taking the same section properties as *Solution 1* and initial eccentricity value of k_2 corresponding to the first order actions (including accidental eccentricity, e_a) i.e. k_2=1.0

$$e_a = \frac{le}{300} \geq 20mm \qquad \text{Equation (4\_1) in EBCS\_ 2, 1995}$$

$$e_a = \max\left(\frac{le}{300}, 20mm\right) = \max(16.33mm, 20mm) = 20mm$$

$$\nu_{sd} = \frac{P_{sd}}{f_{cd}A_c} = \frac{1650 * 10^3}{13.6 * 400^2} = 0.76$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}A_c h} = \frac{(M_o + P_{sd} * ea)}{f_{cd}A_c h} = \frac{(130 + 1650 * 20 * 10^{-3}) * 10^6}{13.6 * 400 * 400 * 400} = 0.19$$

Using the Uniaxial Chart No.2 EBCS_2, 1995 Part 2 $\quad \omega = 0.32$

$$\mu_{bal} = 0.25 \quad \text{and} \quad k_2 = \frac{\mu_{sd}}{\mu_{bal}} = 0.75$$

$$\frac{1}{r} = k_2\left(\frac{5}{d}\right) * 10^{-3} \qquad \text{Equation (4\_19) in EBCS \_2, 1995}$$

$$\frac{1}{r} = 10.42 * 10^{-6}\, mm^{-1}$$

$$e_2 = \frac{k1Le^2}{10}\frac{1}{r} \qquad \text{Equation (4\_18) in EBCS \_2, 1995}$$

$$\lambda = \frac{Le}{0.289h} = 42.4 \geq 35. \Rightarrow k_1 = 1.0$$

$$\Rightarrow e_2 = 25.0mm$$

$$e_{tot} = e_o + e_a + e_2 = \frac{M_o}{P_{sd}} + 20mm + 25mm = 123.8mm$$

$$M_{sd} = P_{sd} * e_{tot} = 204.3kN.m$$

μ_{sd}=0.23,ν_{sd}=0.76, then ω=0.45, Uniaxail Chart No. 2 ,EBCS_2,1995 Part2

Recalculate k_2, based on this value of reinforcement: and μ_{bal}=0.3,k_2=0.23/0.3=0.77 iteration may be stopped.

Alternatively, using Excel Sheet Iteration can be made as in table 3.1 For λ=42.4, e_o=78.80mm and e_a=20mm and f_{cd}=13.6Mpa and f_{yd}=400Mpa using Uniaxial Chart No.2, EBCS_2,1995 Part2.

Table 3.1 Iteration Using Excel sheet for Design of Slender Column Based on EBCS_2, 1995

Iteration No	Msd (kN.m)	Psd (kN)	e_2 (mm)	e_{tot} (mm)	b (mm)	h (mm)	k_2	μ_{bal}	μ_{sd}	ν	ω
1	163.00	1650.00	0.00	98.79	400.00	400.00	1.00	0.25	0.19	0.76	0.32
2	204.22	1650.00	24.98	123.77	400.00	400.00	0.75	0.30	0.23	0.76	0.45
3	206.03	1650.00	26.08	124.87	400.00	400.00	0.78	0.31	0.24	0.76	0.47

$$\text{SRCC Difference with Iterative EBCS\_2,1995 \%} = \left(\frac{\omega 1 - \omega 3}{\omega 1}\right) = \left(\frac{0.48 - 0.47}{0.48}\right) * 100\% = 2.0\%$$

3.3.4 Summary of Comparison

For different values of first order moment and axial load the mechanical reinforcement is calculated iteratively using excel sheet as in table 3.1 using Unixial Chart No.2 of EBCS_2,1995 Part 2 and Uniaxial Chart figure and its value is compared with SRCC chart and Schneider charts and summarized as in table 3.2 to 3.4

Table 3.2 Verification for λ=60

S.No	u_o	μ_{o+a}	v_{sd}	ω EBCS_2 1995 Iterative	ω SRCC chart	ω Schneider Chart	SRCC D/nce with Iterative%	SRCC D/nce with Schneider%
1	0.02	0.12	1.80	1.37	1.40	1.30	2.14	7.14
2	0.02	0.11	1.60	1.14	1.15	1.10	0.87	4.35
3	0.05	0.15	1.80	1.47	1.47	1.40	0.00	4.76
4	0.05	0.12	1.20	0.78	0.80	0.75	3.13	6.25
5	0.02	0.09	1.20	0.67	0.68	0.60	0.74	11.11
6	0.05	0.10	0.80	0.35	0.35	0.30	0.00	14.29
7	0.10	0.15	0.80	0.53	0.53	0.50	1.50	6.19
8	0.14	0.19	0.80	0.64	0.64	0.60	0.00	6.25
9	0.30	0.35	0.80	1.10	1.10	1.10	0.00	0.00
10	0.14	0.16	0.40	0.28	0.28	0.27	0.00	0.00
11	0.14	0.15	0.20	0.25	0.25	0.25	0.00	0.00

Table 3.3 Verification for λ=100

S. No	u_o	u_{o+a}	v_{sd}	ω EBCS2,1995 Part2 Iterative	ω SRCC chart	ω Schneider Chart	SRCC D/nce with Iterative %	SRCC D/nce with Schneider %
1	0.02	0.15	1.40	1.90	1.93	1.80	1.55	6.74
2	0.05	0.17	1.20	1.70	1.70	1.60	0.00	5.88
3	0.02	0.14	1.20	1.60	1.60	1.50	0.00	6.25
4	0.05	0.13	0.80	1.03	1.05	0.95	1.90	9.52
5	0.10	0.18	0.80	1.15	1.15	1.10	0.00	4.35
6	0.14	0.22	0.80	1.28	1.28	1.20	0.00	5.51
7	0.30	0.38	0.80	1.70	1.70	1.65	0.00	2.94
8	0.20	0.26	0.60	1.13	1.13	1.10	0.00	2.65
9	0.14	0.18	0.40	0.60	0.60	0.60	0.00	0.00
10	0.14	0.16	0.20	0.40	0.40	0.4	0.00	0.00

Table 3.4 Verification for different slenderness

S.No	μ_o	μ_{o+a}	ν	λ	ω EBCS_2 1995 Iterative	ω SRCC chart	ω Schneider chart	SRCC D/nce with Iterative %	SRCC D/nce with Schneider %
1	0.15	0.19	0.76	42.40	0.47	0.48	0.45	0.64	4.26
2	0.14	0.18	0.76	60.00	0.59	0.60	0.55	1.67	8.33
3	0.08	0.16	1.22	66.80	1.01	1.04	0.95	2.88	8.65
4	0.10	0.12	0.19	80.00	0.22	0.22	0.24	0.00	-9.09
5	0.10	0.15	0.10	100.00	0.35	0.36	0.38	3.00	-6.00

3.3.5 Discussion

The uniaxial interaction diagram prepared for slender column is comparison with iterative method and SRCC chart is shown in table 3.2 to 3.4 in section3.3.4 above, which shows closer agreement with a difference of 0% to 3.13%. This indicates that SRCC can be used as design for slender RC column. Whereas comparison of SRCC chart with Schneider Charts shows bigger difference with range of -9.09 % to 14%. The reason may be difference in the value of k_2 used for second order eccentricity calculation in EBCS_2, 1995 and CEB_FIP1990, which is the base of Eurocode_2, 1992 for second order eccentricity calculation because Amdetsion (2003) showed that the difference between the two methods, namely EBCS2,1995 and EC-2,1992.

In Amdetsion thesis, the value of k_2 in EBCS_2, 1995 is smaller than in EC_2, 1992 for axial loads greater than balanced load, lower amount of mechanical reinforcement(ω<0.5) and concrete cover ratio(h'/h=0.1) which agrees with the above result obtained in table 3.4. On the other hand, he also showed that for larger amount of mechanical reinforcement ratio the value of k_2 of EBCS2, 1995 is greater than EC2, results obtained in table 3.2, 3.3 and 3.4 proves the same thing.

CHAPTER FOUR

CONCLUSION AND RECOMMENDATION

4.1 Conclusion

In general, compared to iterative method slender column design, the approximate interaction diagram gives us summary of slender RC column capacity for wide range of slenderness ratio. It is very quick and easy. Therefore, the graphical design aid can be used as an alternative design aid; preliminary section capacity estimation and checking of computer outputs at hand in the slender column design.

4.2 Recommendation

The approximate interaction diagram for slender RC column prepared in this thesis work is only for uniaxially loaded rectangular column with symmetrical reinforcement and concrete cover ratio of h'/h=0.1. For illustration of the method, it is shown for different cover ratio h'/h=0.05,0.15,02,0.25 in the Appendix B. In the rectangular column, the bending is assumed to occur about the major axis of the cross section. Since SRCC charts is first in its kind in the context of EBCS_2, 1995, the author recommends for future to extend the dimension of the chart development in terms of:

- Uniaxial column with uniformly distributed reinforcement on all column sides that can be used for approximate design procedure of biaxial slender column according to [EBCS_2, 1995 and Girma Zerayohannes *et al*.2001] recommendations.
- Non-rectangular RC sections column
- Biaxial loaded slender RC column
- Creep and shrinkage factors through time, therefore this kind of factors reduce the capacity of column through time that needs further investigation.
- Development of computer programmes for automatic and integrated design
- Apply rigorous method for the derivation of slender column interaction diagram

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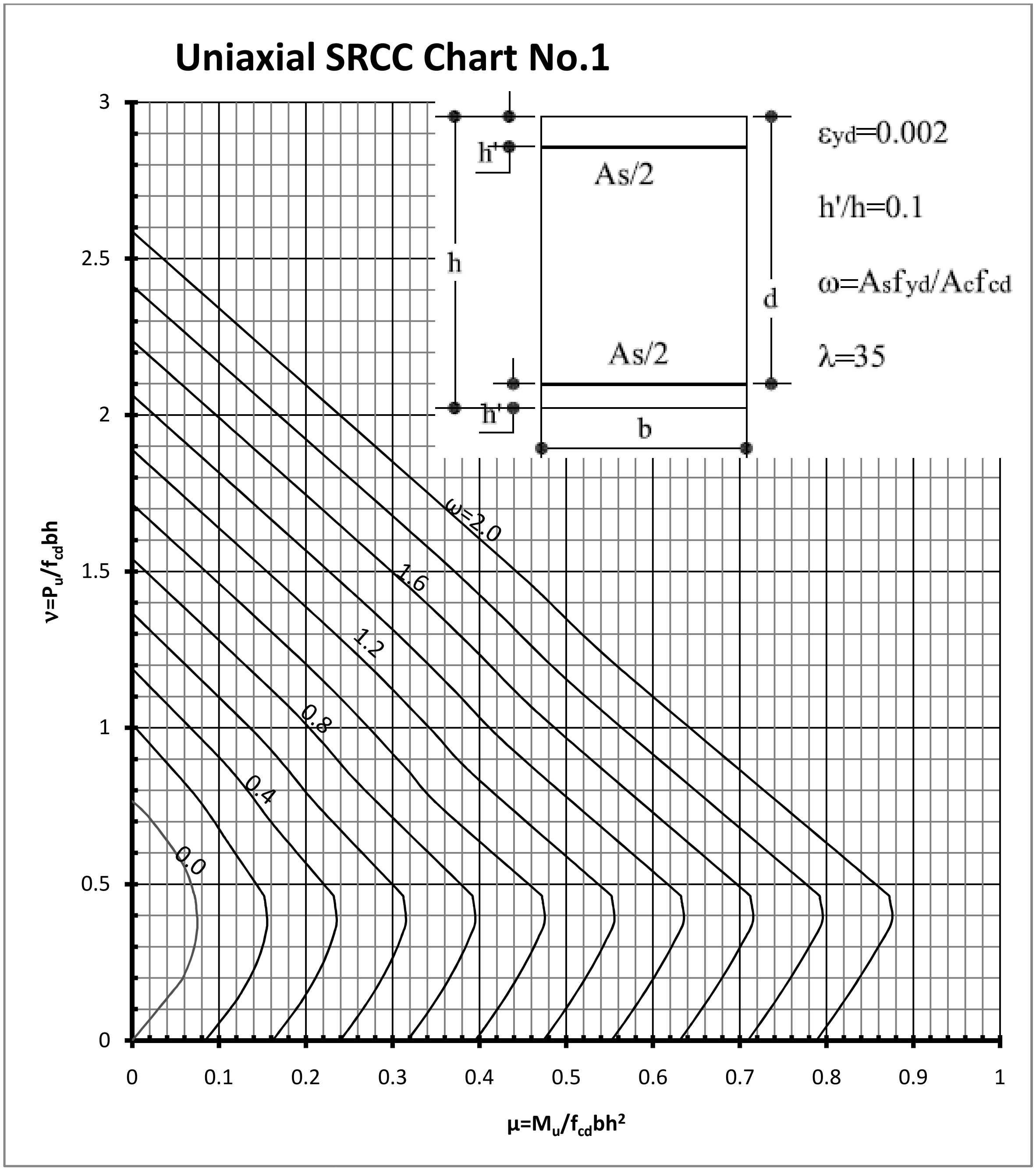
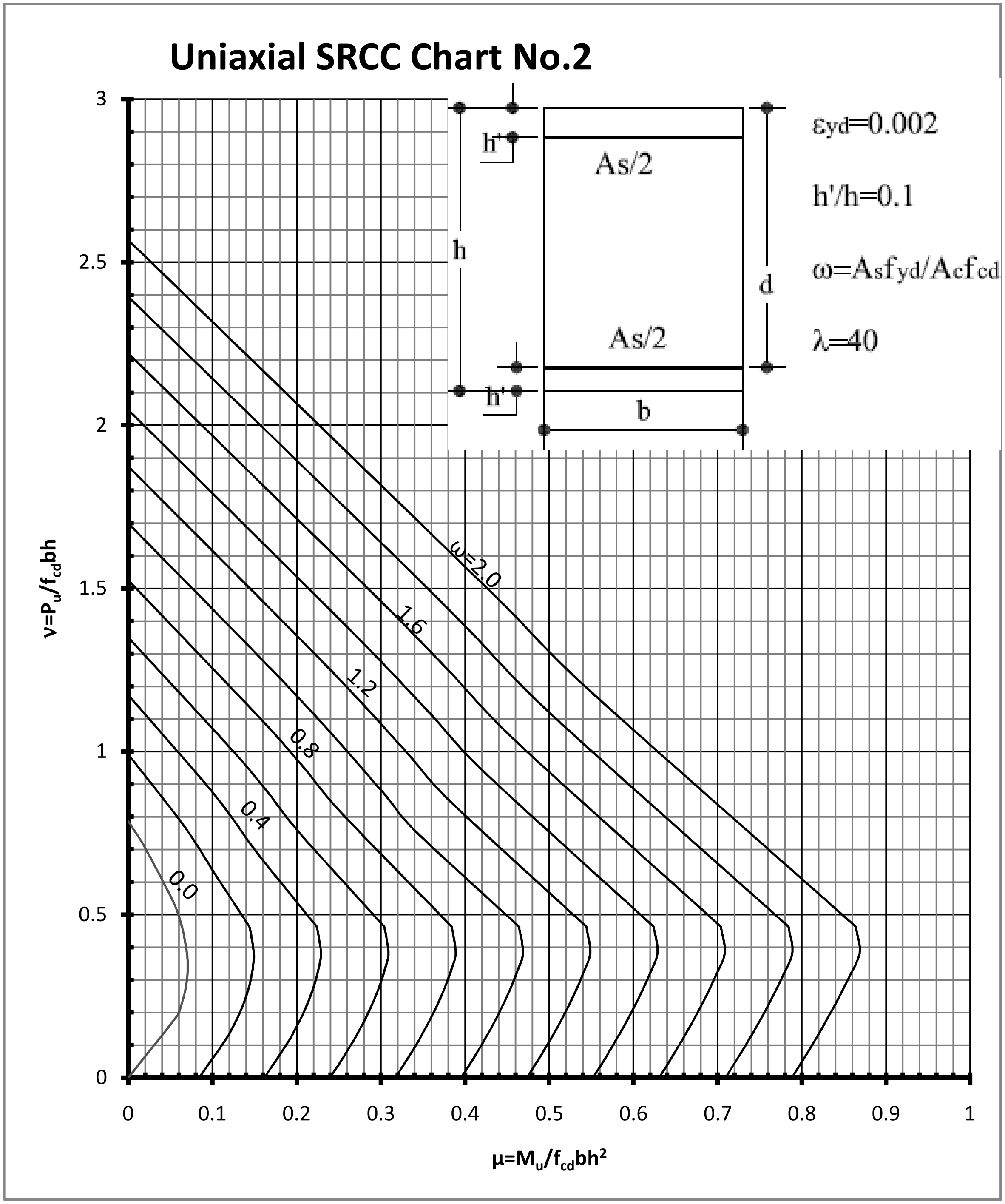
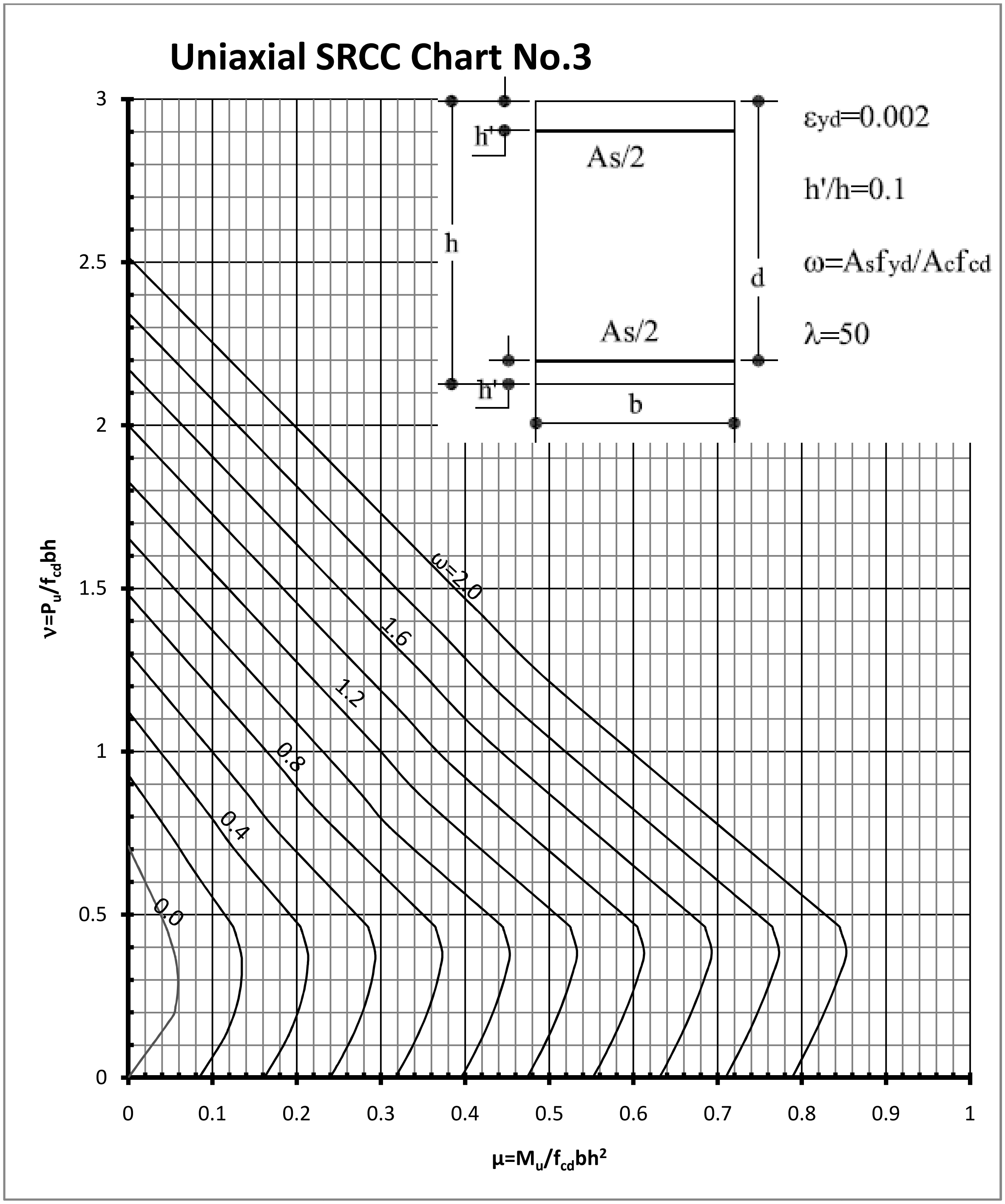
APPENDICES

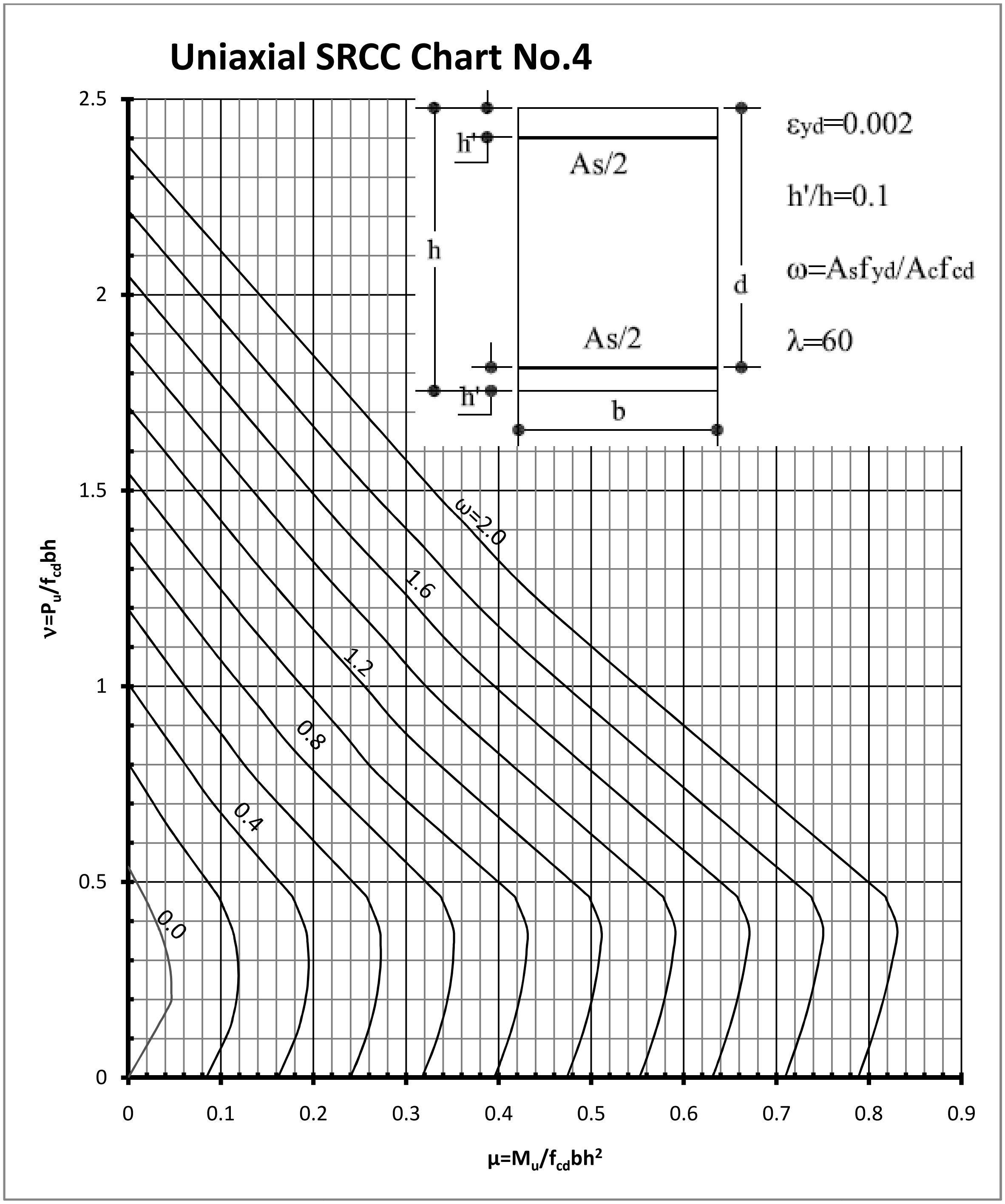
Appendix A: SRCC Charts for 0.0 ≤ ω≤ 2.0 and 35 ≤λ≤140

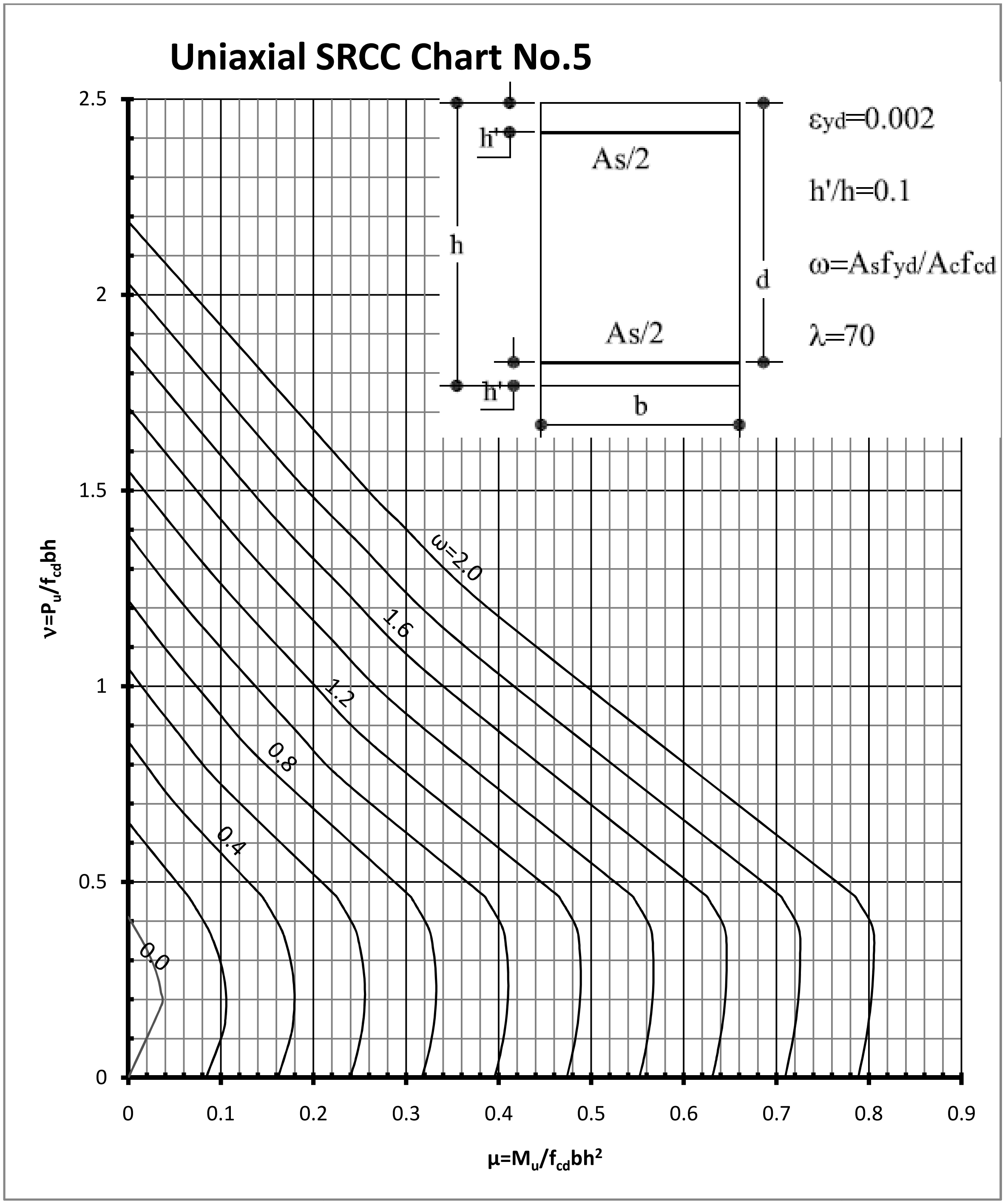


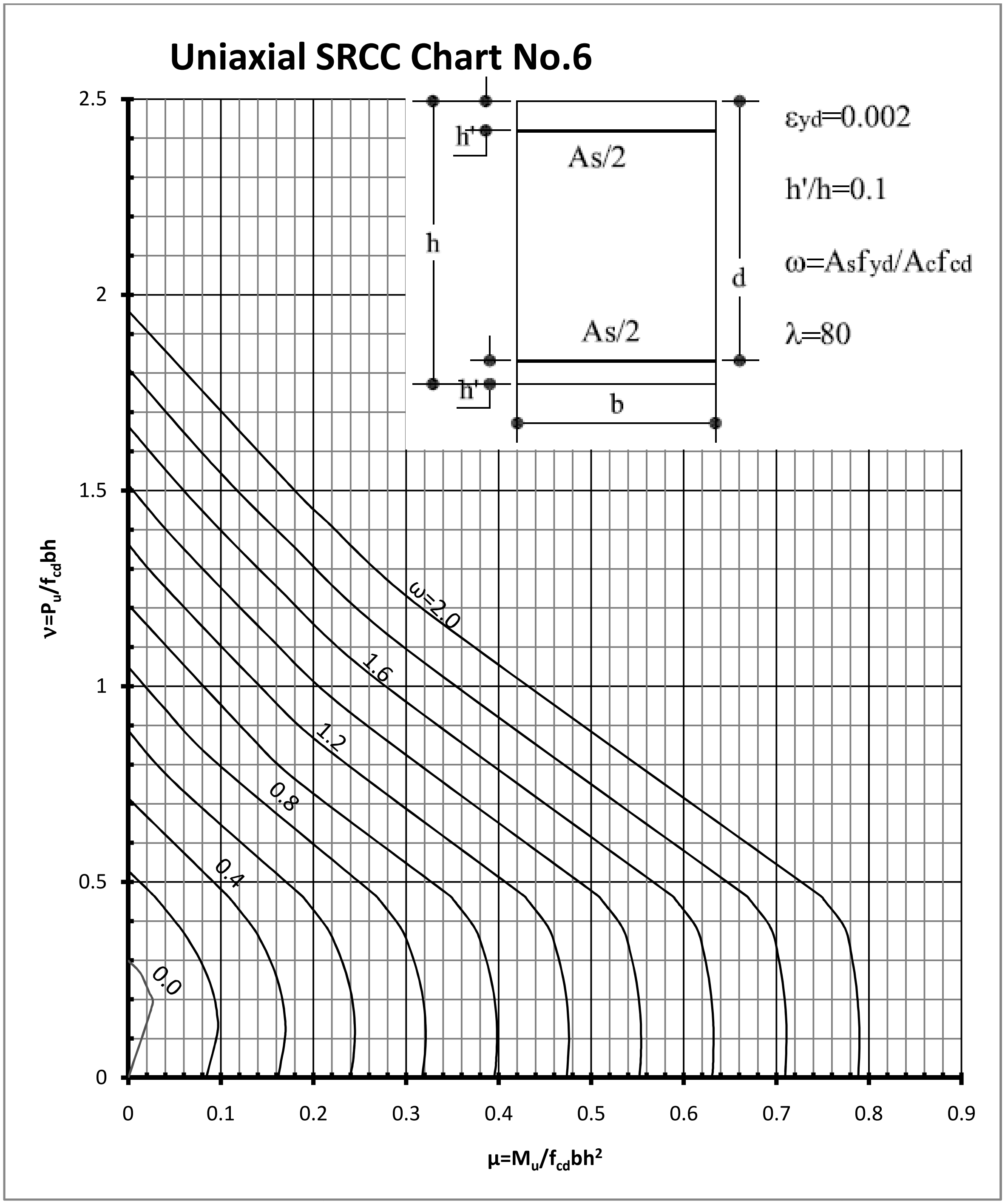


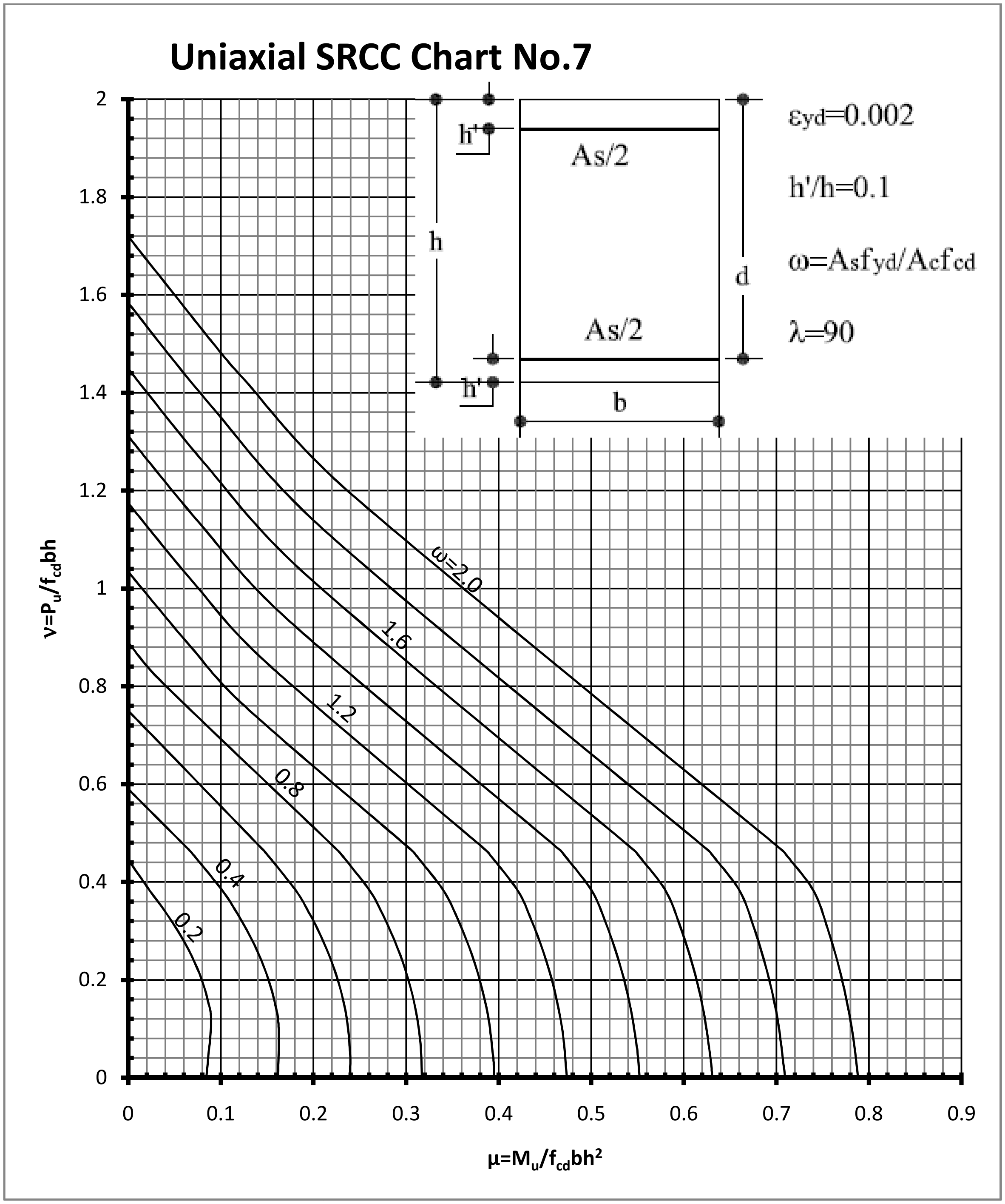
Uniaxial SRCC Chart No.2
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=40
As/2
As/2
h
d
h'
h'
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²


Uniaxial SRCC Chart No.3
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=50
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.5
1
1.5
2
2.5
3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

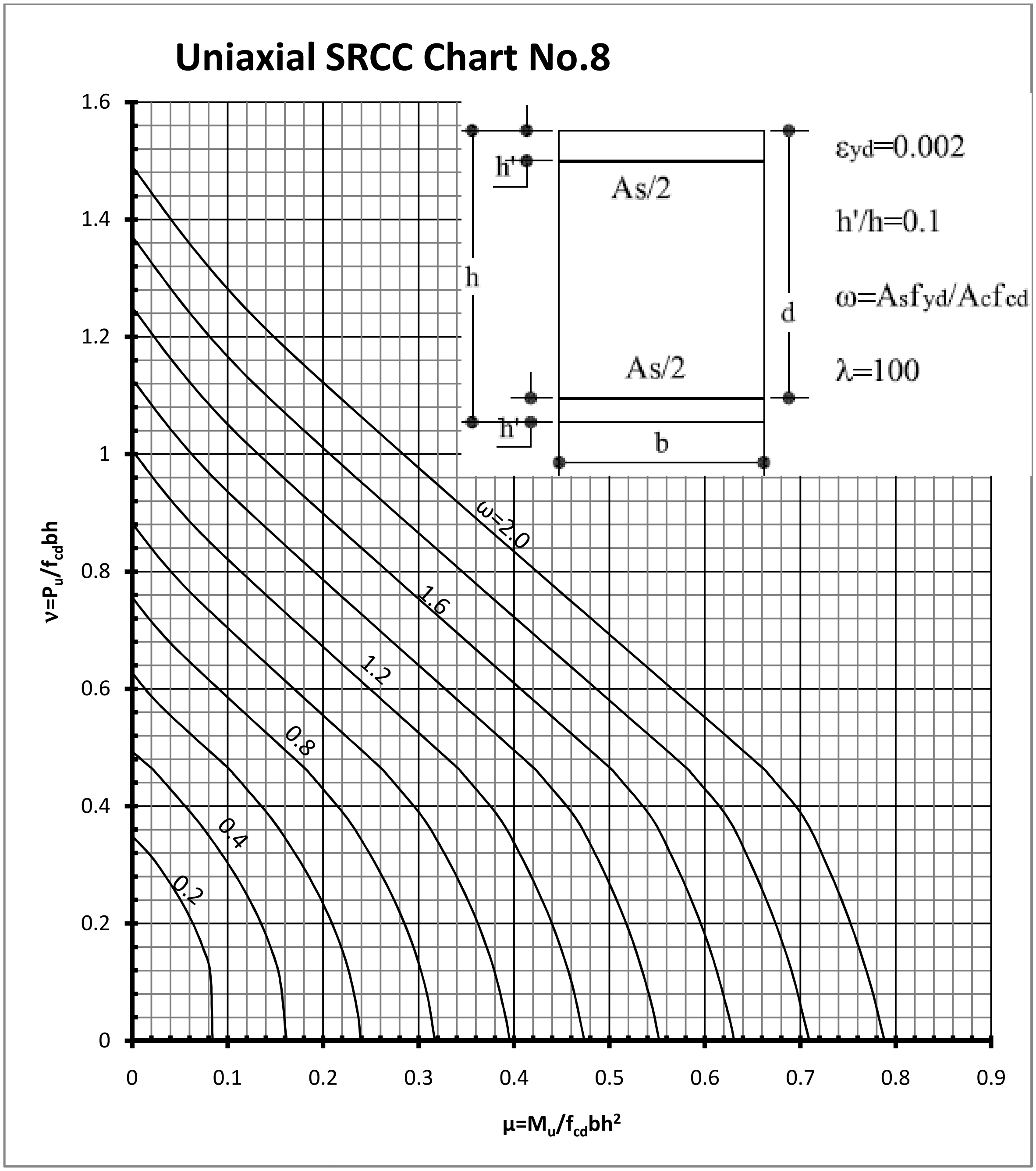

Uniaxial SRCC Chart No.4
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=60
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

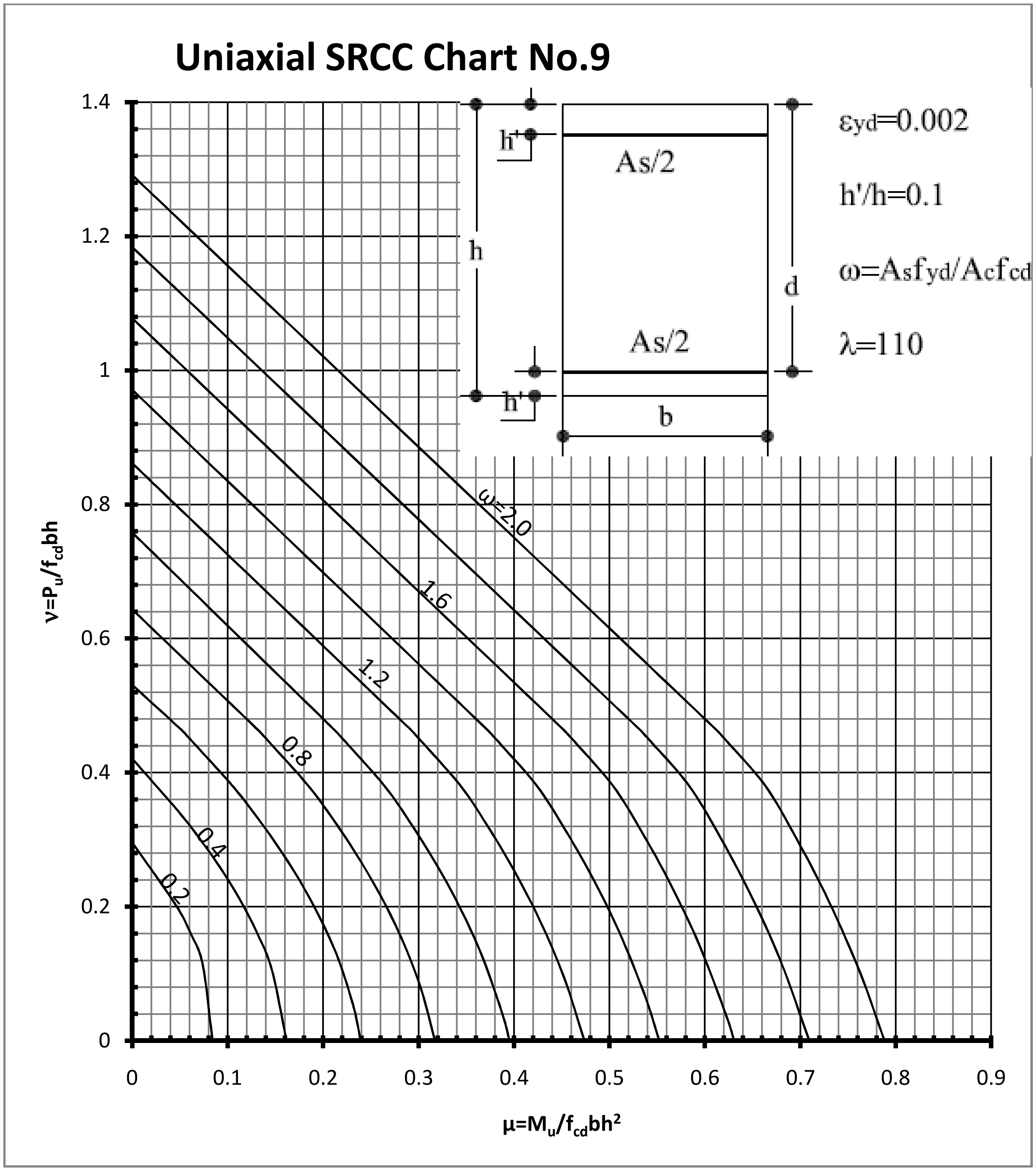

Uniaxial SRCC Chart No.5
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=70
h'
As/2
h
d
As/2
h'
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Uniaxial SRCC Chart No.6
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=80
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

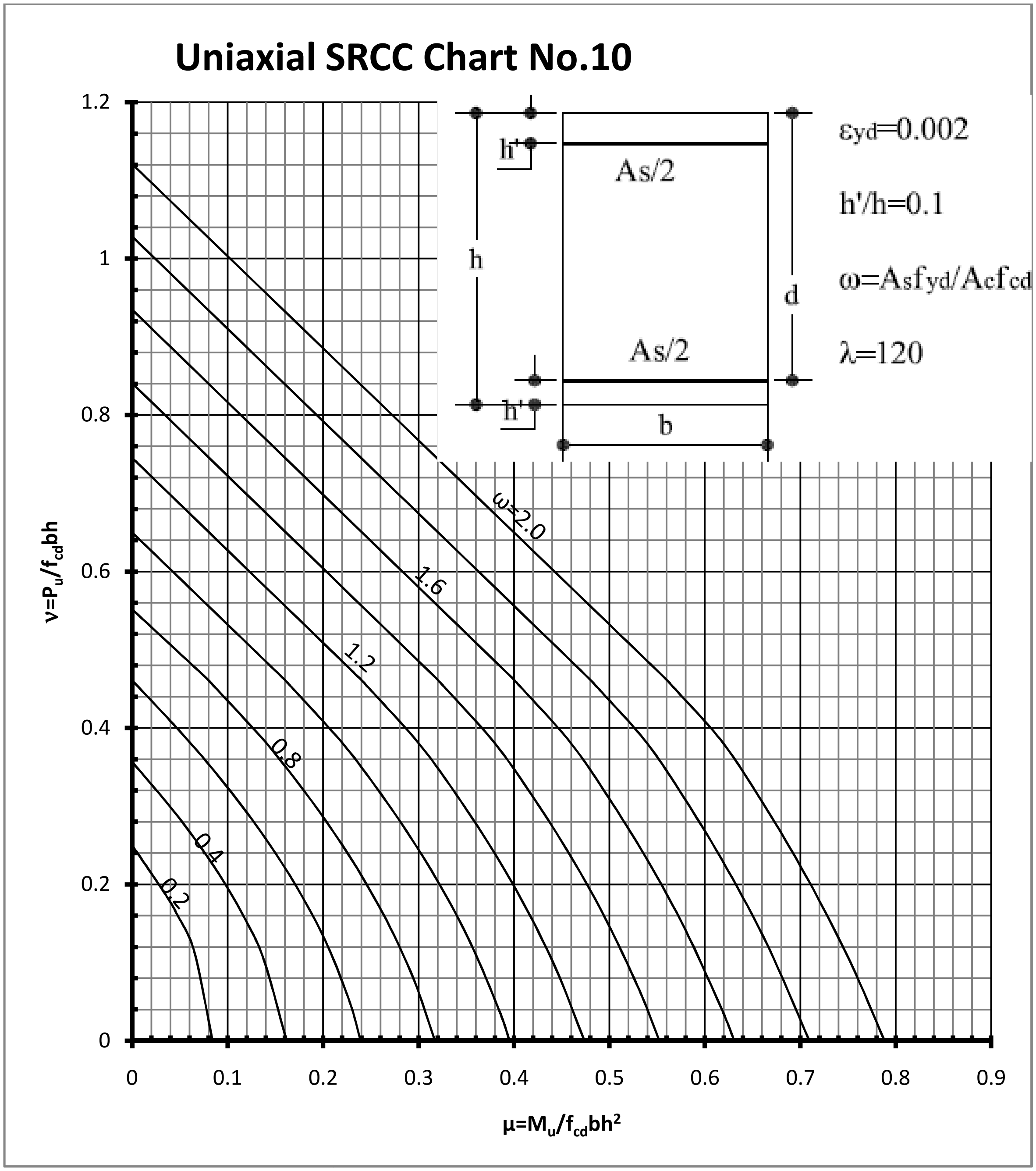


Uniaxial SRCC Chart No.7
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=90
h'
As/2
h
d
As/2
h'
b
ω=2.0
1.6
1.2
0.8
0.4
0.2
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.1
0.3
0.5
0.7
0.9

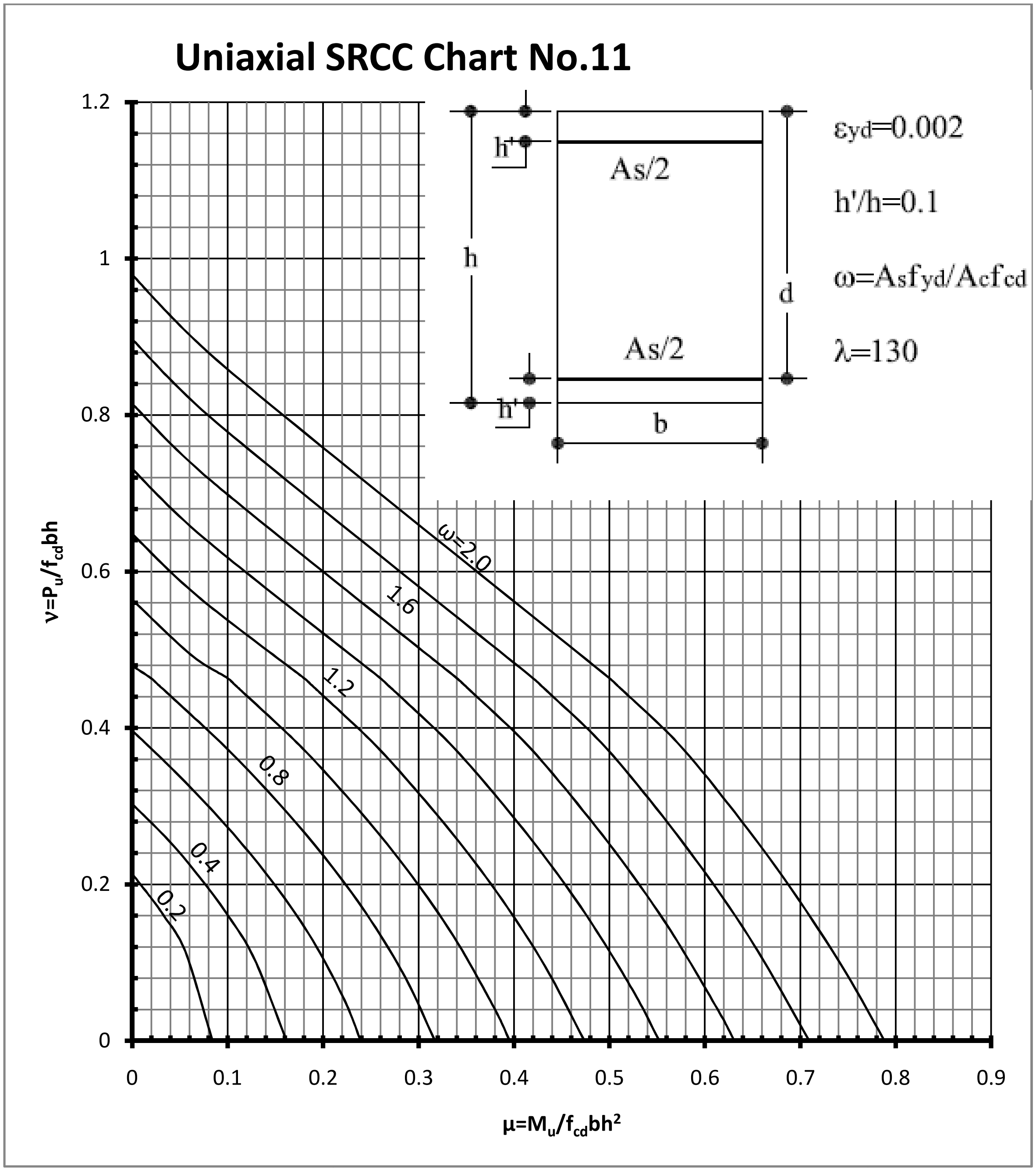

Uniaxial SRCC Chart No.8
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=100
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.2
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.1
0.3
0.5
0.6
0.7
0.8
0.9



Uniaxial SRCC Chart No.9
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=110
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.2
ν=Pu/fcdbh
μ=Mu/fcdbh²
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

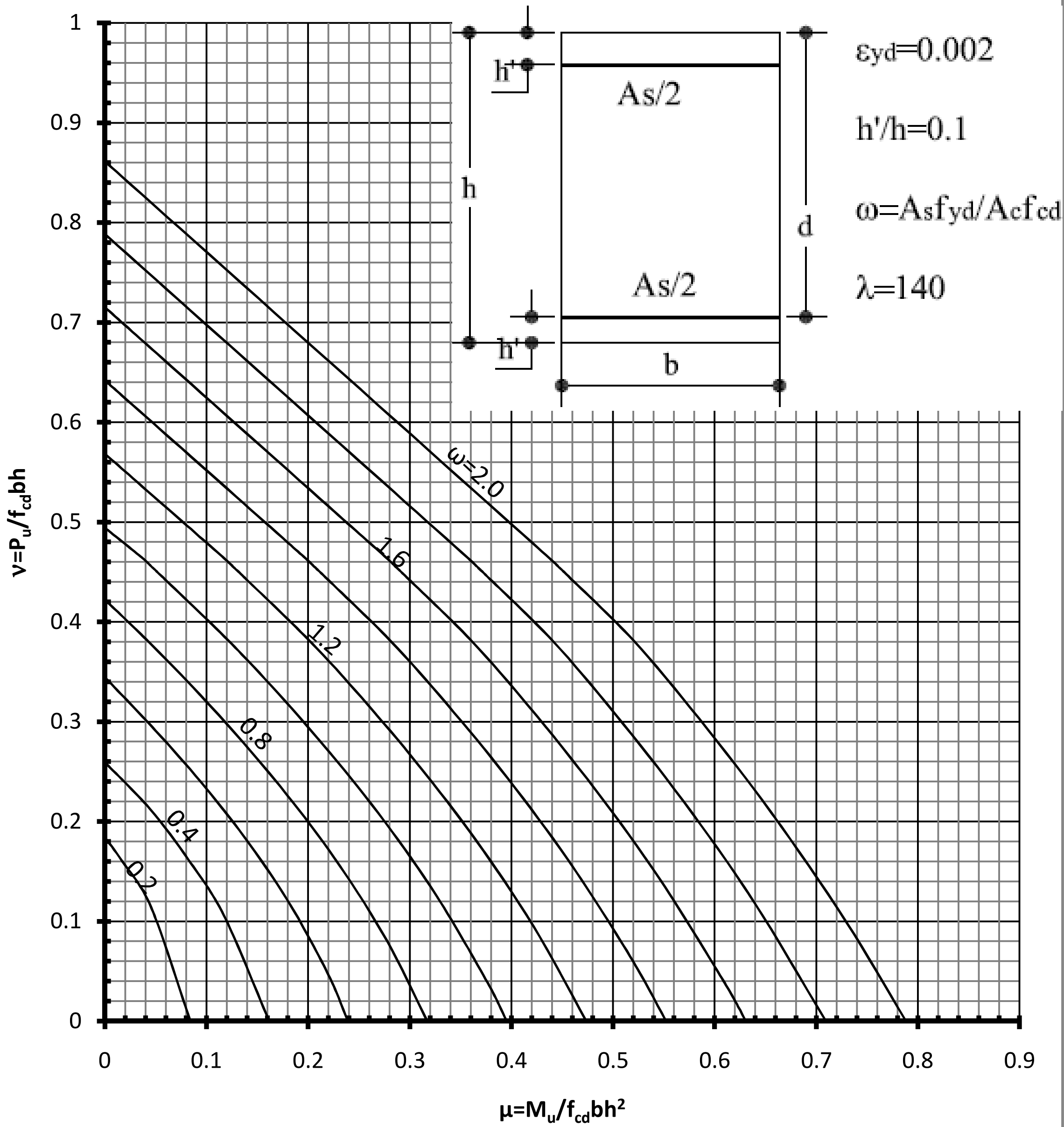


Uniaxial SRCC Chart No.10
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=120
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.2
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.2
0.4
0.6
0.8
1
1.2
0.1
0.3
0.5
0.7
0.9

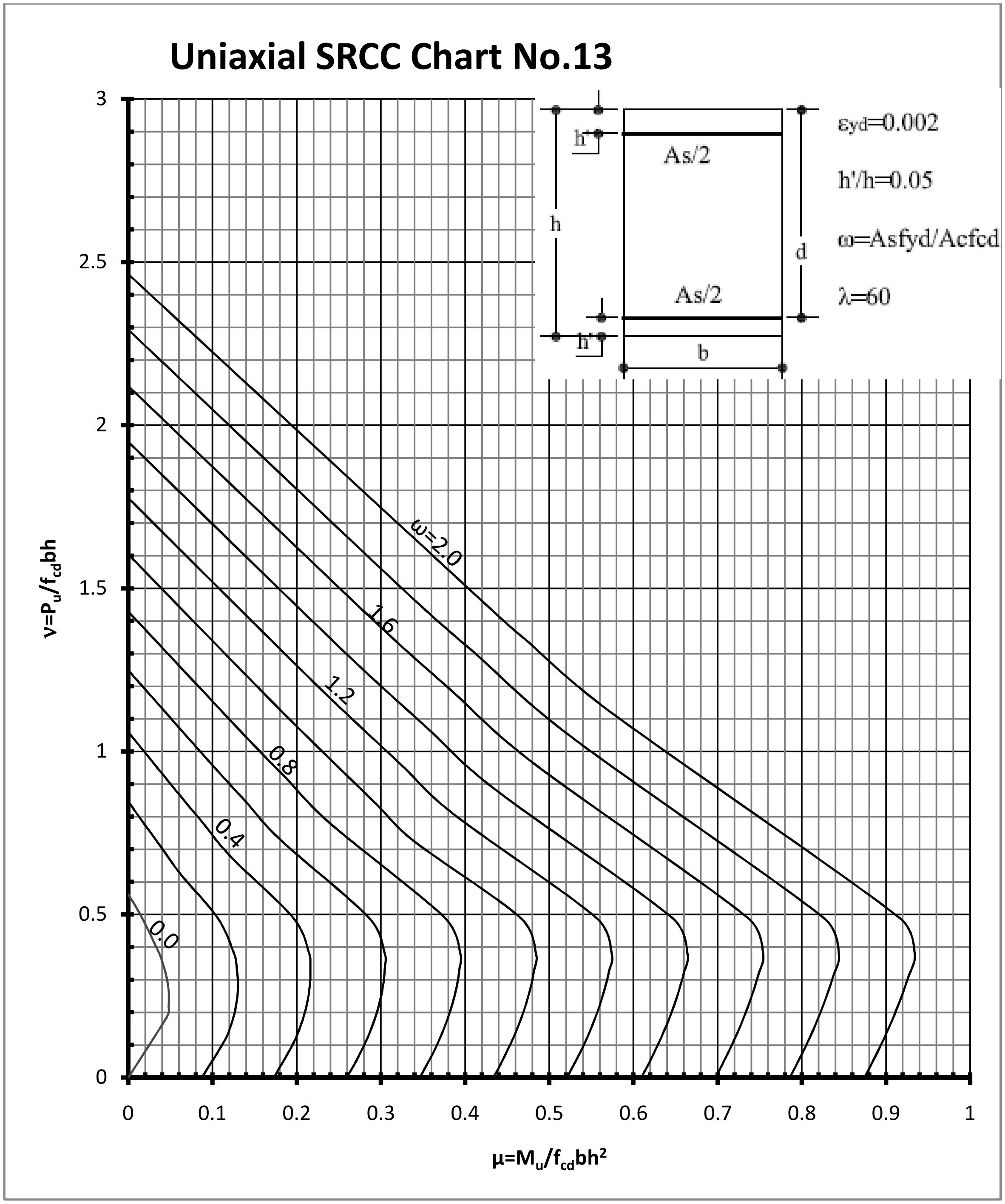


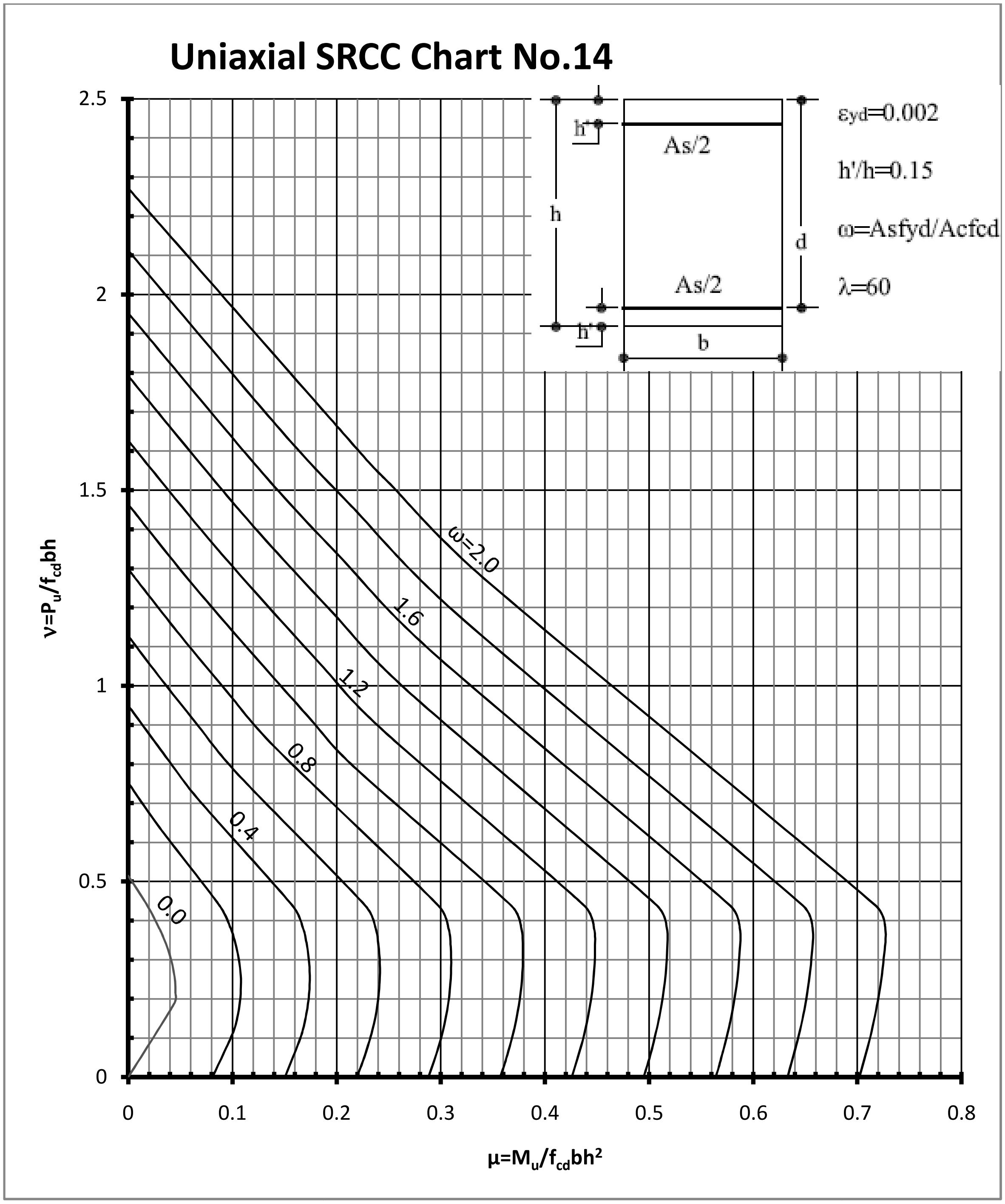
Uniaxial SRCC Chart No.11
εyd=0.002
h'/h=0.1
ω=Asfyd/Acfcd
λ=130
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.2
ν=Pu/fcdbh
μ=Mu/fcdbh²
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

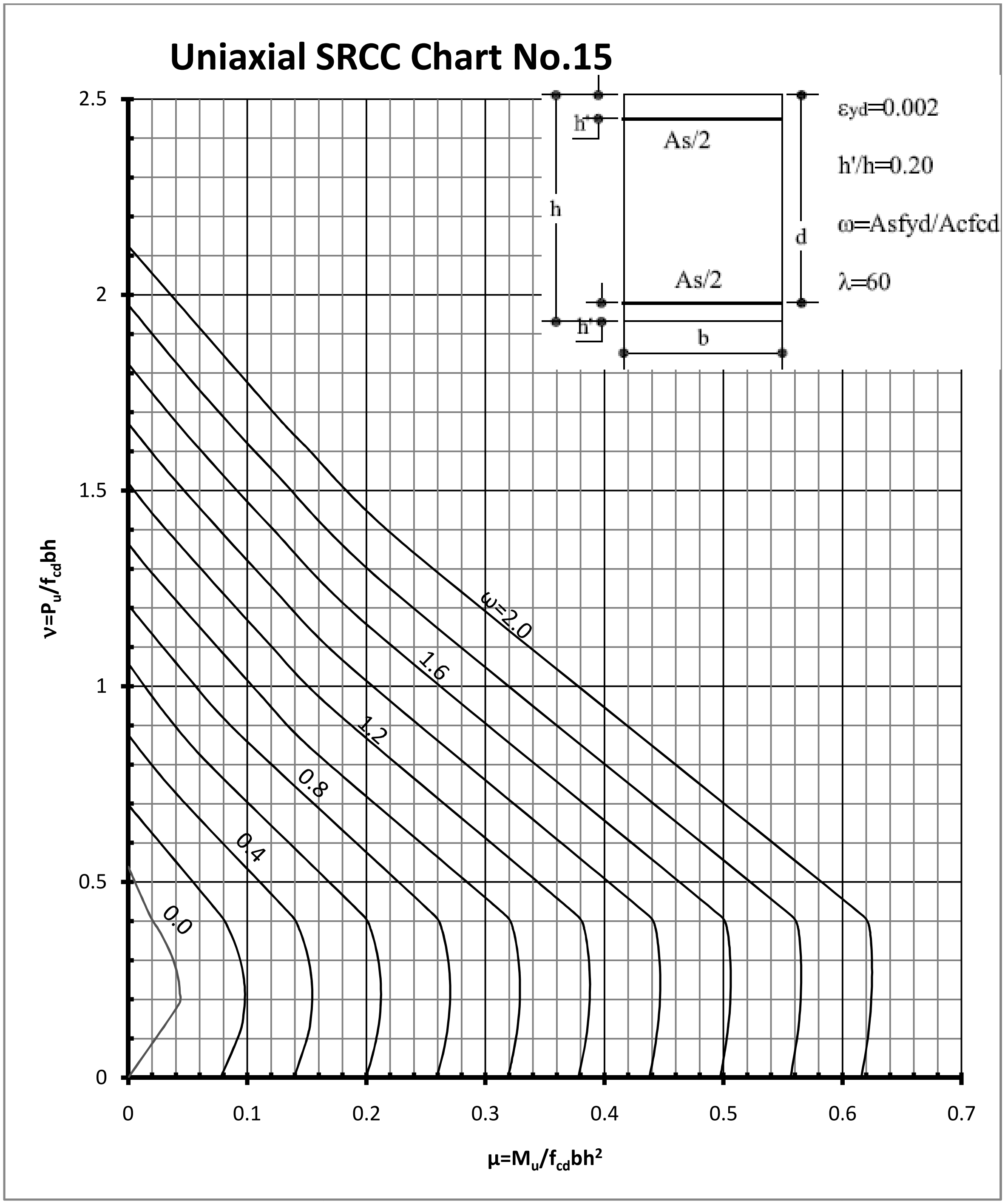
Uniaxial SRCC Chart No.12



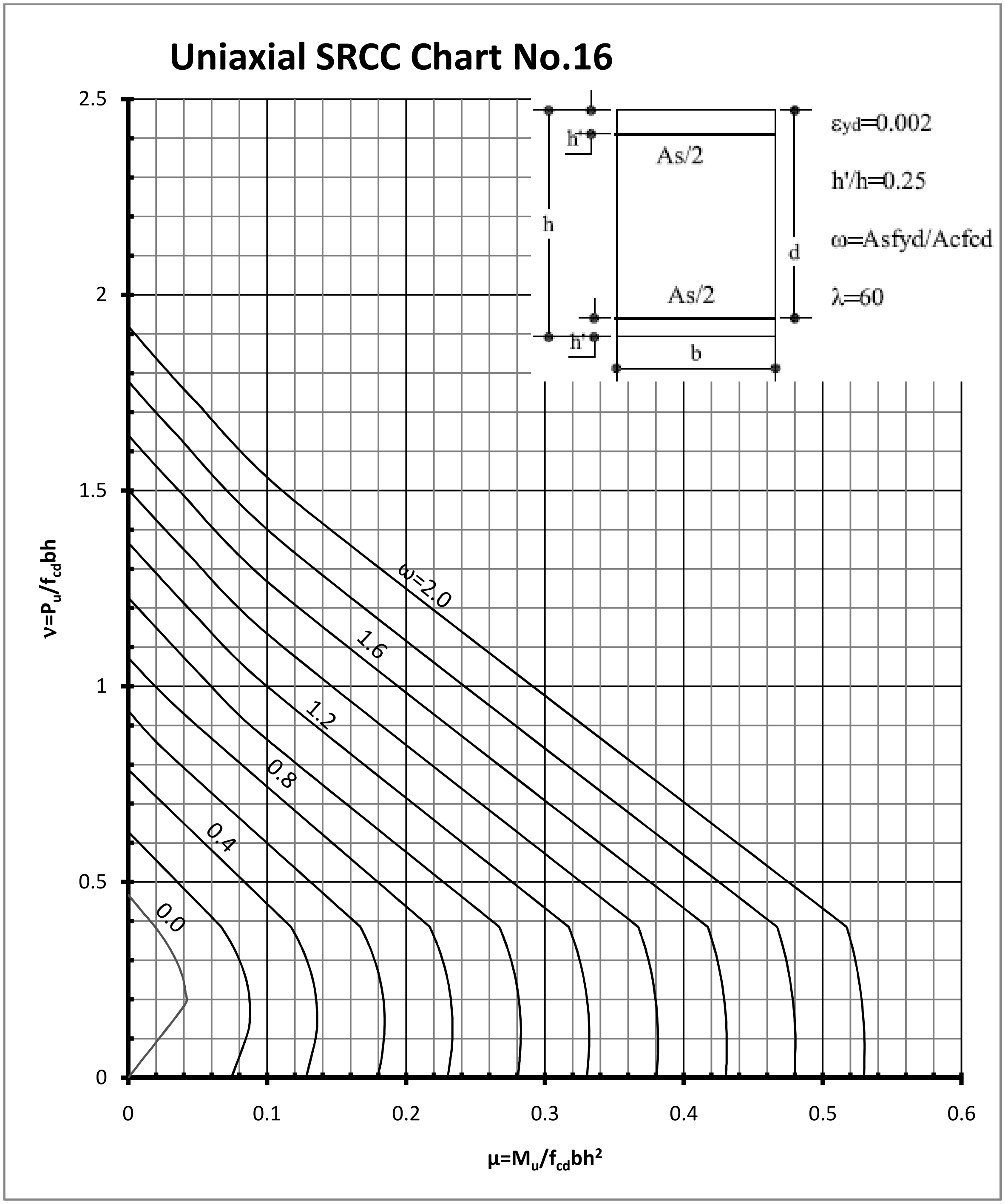
Appendix B: SRCC charts for λ=60 and h'/h=0.05,0.15,0.20,0.25




Uniaxial SRCC Chart No.14
εyd=0.002
h'/h=0.15
ω=Asfyd/Acfcd
λ=60
h'
h
d
As/2
As/2
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.5
1
1.5
2
2.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



Uniaxial SRCC Chart No.15
εyd=0.002
h'/h=0.20
ω=Asfyd/Acfcd
λ=60
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.5
1
1.5
2
2.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


Uniaxial SRCC Chart No.16
εyd=0.002
h'/h=0.25
ω=Asfyd/Acfcd
λ=60
As/2
As/2
h'
h'
h
d
b
ω=2.0
1.6
1.2
0.8
0.4
0.0
ν=Pu/fcdbh
μ=Mu/fcdbh²
0
0.1
0.2
0.3
0.4
0.5
0.6
0.5
1
1.5
2
2.5

Appendix C: User guideline for Approximate Slender RC column interaction diagram

To use uniaxial SRCC charts the following procedure should be followed.

1. Calculate first order moment ,M_o and axial load ,P_{sd} of a column of non sway column
2. Select material grade concrete C_ and steel reinforcement S_ and calculate design strength

   Concrete C _: design concrete strength, $f_{cd} = \frac{0.85f_{ck}}{\gamma c}$

   Reinforcement steel S_: design steel reinforcement strength, $f_{yd} = \frac{f_{yk}}{\gamma_s}$
3. Select rectangular column section ,width b, depth h and concrete cover ratio h'/h

   Calculate the non-dimensional moment, u and axial load ,v

   $\mu_o = \frac{M_o}{f_{cd}A_c h}$ and $\nu_{sd} = \frac{P_{sd}}{f_{cd}A_c}$
4. Calculate the effective length ,le, and slenderness ratio,λ

$$\lambda = \frac{le\sqrt{12}}{h}$$

5. Read mechanical reinforcement ratio,ω from the chart and calculate total reinforcement quantity, symmetric arrangement of the reinforcement half in top and the remaining on the bottom of the column section.

$$A_{s,tot} = \frac{\omega f_{cd} A_c}{f_{yd}}$$

.