STUDIES ON B-MESON DECAYS IN THE FRAMEWORK OF BETHE-SALPETER EQUATION(BSE)

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Abstract

In this thesis, we first study the lagrangian formulation of gauge theories, QED, Yang-Mills theory and the Glashow-Weinberg-Salam model for Electroweak interaction. We then, we use the framework of Bethe-Salpeter equation (BSE) under Covariant Instantaneous Ansatz (CIA) to calculate the leptonic decays of B-meson. The structure of hadron-quark vertex function $\Gamma(\hat{q})$ is obtained from BSE under CIA, while the structure of $q\bar{q}W^-$ vertex is obtained from GWS model for Electroweak interactions. We derive formulas for the decay constant f_B of B-meson. We obtain the numerical values of the decay constant, f_B of the B-meson as well as its decay width. Then the results are compared with existing experimental results.

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Chapter 1

Introduction

The main application of Quantum Field Theory is to describe the fundamental particles and their interactions using what scientists call the Standard Model. Standard Model, proposed by the Glashow, Weinberg and Salam [1,2,3], is used to describe electromagnetic and weak interactions. The interactions of these forces with quarks and leptons are also described by this model. The electroweak sector of the Standard Model which unifies electromagnetic and weak interactions is one of the most successful theory and this has provided plenty of successful predictions with an impressive level of precision. The elementary particles, Quarks and leptons are structureless and regarded as mathematical point like objects [4,5]. Quarks are fundamental particles that carry electrical charge and hence participate in the electromagnetic interaction. They also participate in the weak and strong interactions. But leptons do not participate in the strong interaction [5], that is they lack an attribute called color charge, which allow quarks to bind together[5]. After several decades of experimental and theoretical work, elementary particle physicists have found strong evidence that all hadrons are composite objects and consist of quarks and antiquarks. More specifically, the baryons consist of 3-bound quarks and the mesons consist of a bound quark-antiquark pair. As far as we know, the quarks are point like particles with fractional electric charge and like the leptons they are truly elementary. There are six different flavors of leptons and quarks. Hadrons can be further subdivided by their spin, Hadrons with half-integer spin are called baryons and with integer spin are referred to as mesons [6]. Charged mesons formed from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual $W^{\pm}$ boson. The mesons with a given quark-antiquark combination with $J^{pc} = 0^{-}$ are pseudoscalar mesons[7]. In this thesis, we deal with the decays of the B-meson, which happens to be a bound state of $b\bar{u}$ quarks. The quarks inside a meson stay together infinitely long and they can interact arbitrary often. It is clear that this situation can not described by the summation of a few Feynmen diagrams. This complex bound state interaction is described by the Bethe-Salpeter equation (BSE) [8], where BSE is an important tool for calculating bound state properties of hadrons such as mesons and baryons. In this thesis, we focus on the decays of the charged B^{-}-meson and is of current interest with experimental data, [9] on B-mesons only recently available.

The main goal of this thesis is to study the leptonic decays of a charged B-meson and calculation of its decay constant. We use two approaches: (**a**) where we consider B-meson as an elementary particle and (**b**) considering B-meson as a quark-antiquark composite. In this calculation while the hadron-quark vertex is derived from BSE, the quark-antiquark W^{-} vertex is obtained from GWS model for Electroweak interactions. Thus as run up to these studies, we would first discuss electroweak theory of Glashow-Weinberg-Salam which accounts for semi-leptonic decays of hadrons. We also discuss some developments in covariant instantaneous ansatz (CIA) framework of BSE.

Thsis thesis is organized as follows; In section 2-we will first discuss the abelian gauge theory QED which is then generalized to non-abelian,SU(2) Yang-Mills theory. We then discuss of GWS model of electroweak interactions and also discuss electroweak symmetry breaking due to Higgs mechanism in which there is a lot of current excitement due fact that LHC has recently become operational and data has started being analysed on Higgs searches where it is believed that Higgs particle as responsible for giving mass to all the particles in the universe due to thier coupling to Higgs field. Section 3 will discuss the

BSE and the structure of hadron-quark vertex for decay of charged B-meson under CIA. In section 4 we will calculate the analytical expressions as well as the numerical values of decay constant f_B in the framework of BSE under CIA and Bethe-Salpeter normalizer N_B of B-meson. Then, we will compare the numerical result of decay constant f_B with recent studies and experimental values. Finally we will write the conclusion.

Chapter 2

Gauge Theories

This chapter introduces the gauge theories. All fundamental interactions in physics are derived from a gauge principle. A gauge theory is a type of field theory in which the lagrangian is invariant under a continuous group of local transformations. The crux of gauge principle is that all fundamental interactions are derived from the local gauge invariance of free particle lagrangian. There are two types of gauge transformations. These are, the first is **global** gauge transformation, where the transformation which does not vary from point to point in space-time and the second is **local** gauge transformation which varies from point to point in space-time. The local invariance of atheory is a much more stringent condition to impose the global invariance.

2.1 Gauge Invariance of Quantum Electrodynamics (QED)

Quantum electrodynamics or QED was the first true quantum field theory that was developed and is regarded as the most successful and accurate theory in physics. As the first name implies, it is a quantum field theory that describes interactions of electrons with photons (i.e light with matter) [4,5,10]. QED is an Abelian gauge theory with the gauge group U(1). The gauge field which mediates the interaction between the charged spin-half particles, is the electromagnetic field. Let's consider the lagrangian describing a free Dirac particle,

$$\pounds_0 = -\bar{\psi}(\partial\!\!\!/ + m)\psi \tag{2.1.1}$$

Let U be a one parameter unitary transformation defined as;

$$U = \exp^{i\lambda} \tag{2.1.2}$$

where λ is a real number. These transformations form an Abelian group which is appropriately called a U(1) group. If λ is space-time independent then U is said to be a global transformation and if $\lambda = \lambda(x)$,U is said to be local transformation. Correspondingly, we call U(1) a global or a local group respectively. It can be easily seen that the lagrangian $\pounds_0$ is invariant under a global U(1) transformation. Since ψ and $\bar{\psi}$ fields transform as,

$$\psi \longrightarrow \psi' = \exp^{i\lambda} \psi = U\psi \tag{2.1.3}$$

$$\bar{\psi} \longrightarrow \bar{\psi}' = \bar{\psi} \exp^{-i\lambda} = \bar{\psi} U^{-1} \tag{2.1.4}$$

Thus $\pounds_0$ remains invariant provided λ is independent of x. However, the free Dirac lagrangian is no longer invariant if one allows the λ to depend on the space-time coordinate x. For instance lets now consider a local gauge transformation on ψ i.e.;

$$\psi \longrightarrow \psi' = \exp^{i\lambda(x)} \psi \tag{2.1.5}$$

$$\bar{\psi} \longrightarrow \bar{\psi}' = \bar{\psi} \exp^{-i\lambda(x)} \tag{2.1.6}$$

where $\lambda(x)$ is some arbitrarly function. The free lagrangian then transforms as,

$$\pounds_0' = \pounds_0 - i\bar{\psi}(x)\gamma_\mu(\partial_\mu\lambda(x))\psi(x) \tag{2.1.7}$$

Therefore, $\pounds_0$ is not invariant under local gauge transformations on ψ. We now want to see how one can restore the invariance. This is only possible if one adds some additional term to the lagrangian transforming in such a way as to cancel $\partial_\mu\lambda$ in the above equation. This is only possible if one introduces a carrier gauge field $A_\mu(x)$ which transforms as,

$$A_\mu \longrightarrow A_\mu' = A_\mu + \partial_\mu\lambda(x) \tag{2.1.8}$$

and define the covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu(x) \tag{2.1.9}$$

which has the required property of transforming like the field itself.

$$D_\mu\psi \rightarrow (D'_\mu\psi'(x)) = \exp^{i\lambda(x)}(D_\mu\psi) \tag{2.1.10}$$

Thus a D_μ defined by equation (2.1.9) with $A_\mu(x)$ constrained to transform under the U(1) gauge transformation according to equation (2.1.10) gives us a lagrangian density;

$$\pounds = \pounds_0 + ie\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x) \tag{2.1.11}$$

which can be easily seen to be invariant under local U(1) transformation. Identifying $A_\mu(x)$ with the electromagnetic field and e, can be identified as the charge of the Dirac field, which plays the role of a coupling constant between electromagnetic field and matter field. By demanding the gauge invariance of a free lagrangian, we have learnt that a four-vector field $A_\mu(x)$ must exist. This gauge field must couple in a mathematically self consistent manner with the matter fields. The constant e introduces infront of the field $A_\mu(x)$ in the definition of the covariant derivative appears as a coupling constant determining the strength of the interaction between the gauge field $A_\mu(x)$ and the matter field. The local gauge group involved is the abelian U(1) group due to the fact that the elements $\{U(\lambda) = \exp^{i\lambda(x)}\}$ commute. Further, the gauge field must be massless, for a mass term, $m^2A_\mu A^\mu$ is not invariant under the transformation law imposed upon the gauge field. Since we have introduced $A_\mu(x)$ as carrier gauge field, one needs to add a gauge invariant kinetic term in the lagrangian for $A_\mu(x)$ i.e,

$$\pounds_G = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{2.1.12}$$

where,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.1.13}$$

is called the electromagnetic field strength tensor. We can then construct the total lagrangian describing the electromagnetic field and its interactions with a Dirac field by writing the full QED lagrangian as;

$$\pounds_{QED} = -\bar{\psi}(\partial\!\!\!/ + m)\psi + ie\bar{\psi}\gamma_\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{2.1.14}$$

From a simple gauge symmetry requierment we have thus deduced the right **QED** lagrangian, which leads to a very successful quantum field theory describing a vast number of processes with a high degree of precision.

2.2 Gauge Invariance of Yang-Mills Theory

Yang-Mills theory is a gauge theory based on the weak isospin SU(2) gauge group. It was formulated by Yang and Mills in 1954 [11], in an effort to extend the original concept of gauge theory to the case of a non-Abelian SU(2) group with the intention to develop the gauge theory for electroweak interaction [10,11]. SU(2) being an exact symmetry the up state at one location may be the down state at another. Local symmetry leaves us free to choose the up and the down states at each point independently. Heisenberg, who introduced the concept of isospin(Heisenberg, 1932) regarded the fields for nucleon, in isospin space can be written as a two component isospinor ψ_N [10].

$$\psi_N = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \tag{2.2.1}$$

where ψ_p corresponds to the state $|I, I_3 >= |\frac{1}{2}, +\frac{1}{2}>$,while ψ_n to the state $|\frac{1}{2}, -\frac{1}{2}>$. Lets now consider that we have two Dirac fields. The lagrangian in the absence of any interaction is

$$\pounds = -\bar{\psi}_1(\partial\!\!\!/ + m_1)\psi_1 - \bar{\psi}_2(\partial\!\!\!/ + m_2)\psi_2 \tag{2.2.2}$$

It is just the sum of two Dirac lagrangians. But equation(2.2.2) can be written more compactly by combining ψ_1 and ψ_2 into a two component column matrix. Then, the lagrangian becomes,

$$\pounds = -\bar{\psi}(\partial\!\!\!/ + M)\psi \tag{2.2.3}$$

where $\bar{\psi}$ and ψ are nucleon fields and M is the mass matrix which is expressed as;

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2.2.4}$$

Lets transform the ψ and $\bar{\psi}$ under the SU(2) gauge transformations;

$$\psi(x) \rightarrow \psi'(x) = U\psi(x) \tag{2.2.5}$$

$$\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}U^{-1} \tag{2.2.6}$$

where, U is any 2×2 unitary matrix and hence the combination $\bar{\psi}\psi$ is invariant.

$$\bar{\psi}'\psi' = \bar{\psi}\psi \tag{2.2.7}$$

Moreover the most general 2×2 unitary matrix can be expressed as;

$$U = \exp^{\frac{i\vec{\tau}}{2}.\vec{\lambda}} \tag{2.2.8}$$

where, the three $\frac{\vec{\tau}}{2}$ matrices are just the familiar Pauli matrices. These are non-commuting matrix operators appearing in the exponent. The 2×2 matrices, $\frac{\vec{\tau}}{2}$ are called the generators of SU(2) in the two-dimensional defining representation. The ψ and $\partial_\mu\psi(x)$ transforms in the same manner under global gauge transformations. This makes $\pounds$ invariant under global SU(2) gauge transformations. Now lets extend the global to local gauge transformation. Under local gauge transformation of SU(2),

$$\psi \rightarrow \psi' = U(\lambda)\psi = \exp^{i\frac{\vec{\tau}}{2}.\vec{\lambda}(x)}\psi \tag{2.2.9}$$

$$\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}U^{-1} \tag{2.2.10}$$

where, $\lambda = \lambda(x)$ depends on spce-time. Then $\pounds$ is not invariant under such transformation, for the derivative picks up an extra term. To make the lagrangian locally invariant, lets introduce the covariant derivative as in the case of QED.

$$D_\mu = \partial_\mu - igA_\mu \tag{2.2.11}$$

where, g is the coupling constant. And for the isospin defining

$$A_\mu(x) = \frac{1}{2}\tau^a A^a_\mu(x), (a = 1, 2, 3) \tag{2.2.12}$$

where τ^a is 2×2 pauli matrices, A^a_μ is 1×1 matrix just a number and $A_\mu(x)$ must be 2×2 matrix. There are now three component gauge fields, each of which corresponds to a generator of SU(2) group. Then the lagrangian will be clearly invariant under the local gauge transformation of SU(2,

$$\psi(x) \longrightarrow \psi'(x) = \exp^{i\frac{\vec{\tau}}{2}.\vec{\lambda}(x)}\,\psi(x) \tag{2.2.13}$$

Under the above transformation, the derivative $D_\mu\psi(x)$ transforms as

$$(D_\mu\psi(x)) \longrightarrow (D_\mu\psi(x))' = \exp^{i\frac{\vec{\tau}}{2}.\vec{\lambda}(x)}(D_\mu\psi(x)) \tag{2.2.14}$$

Thus both $\psi(x)$ and $D_\mu\psi(x)$ transform in the same manner under local gauge transformation. The resulting lagrangian invariant under local gauge transformation is thus,

$$\pounds = -\bar{\psi}(\partial\!\!\!/ + m)\psi - ig\frac{1}{2}\bar{\psi}\gamma_\mu\tau^a\psi A^a_\mu \tag{2.2.15}$$

Let's investigate the behaviour of a $A_\mu(x)$, by combining equations (2.2.14) and (2.2.17), we find;

$$A'_\mu = UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1} \tag{2.2.16}$$

But we have been obliged to introduce three new vector fields, $\vec{A}_\mu(x)$, and they will require their own free lagrangian. Then the complete Yang- Mills lagrangian becomes;

$$\pounds_{SU(2)} = -\bar{\psi}(\partial\!\!\!/ + m)\psi - \frac{1}{2}Tr(F_{\mu\nu}F_{\mu\nu}) - i\frac{g}{2}\bar{\psi}\gamma_\mu\tau^a\psi A^a_\mu \tag{2.2.17}$$

where the gauge field tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \tag{2.2.18}$$

Then the Yang-Mills lagrangian is the sum of 3-parts; free matter field, interaction term and the free gauge field, and it is exactly invariant under local SU(2) gauge transformation. Yang-Mills theory describes two equal mass Dirac fields in interaction with three massless vector gauge fields. The gauge field must be massless, for a mass term desroys the gauge invariance of the lagrangian. In Yang-Mills theory corresponding to 3-gauge fields, $A^a_\mu (a = 1, 2, 3)$, we get three interaction lagrangian,

$$\pounds_{int} = \frac{1}{2} ig\bar{\psi}\gamma_\mu \tau^a \psi A^a_\mu \tag{2.2.19}$$

We know that in QED the photon has no electric charge and it does not couple to itself, and it couples only to charged particles. But the gauge fields of non-Abelian theories will have self-interactions due to the structure of $F_{\mu\nu}$ in (2.2.21). In the next section we will discuss the Glashow-Weinberg-Salam (GWS) theory which unifies the electromagnetic and weak interactions and is an extension of Yang-Mills theory.

2.3 Glashow-Weinberg-Salam Theory (GWS)

GWS theory was to unify the weak and electromagnetic interactions, to combine them into a single theoretical framework of one fundamental electroweak interactions [4]. The gauge group for the electroweak interactions is $SU(2) \times U(1)$. The weak interactions are mediated by the SU(2) gauge bosons, which includes the charged $W^\pm$ and neutral Z^0 massive bosons. The U(1) sector of the interaction is the electromagnetic interaction, which is mediated by the massless photon. We discuss now the lagrangian formulation of electroweak theory for leptons and quarks and then apply it to the study of weak decays of B-mesons.

2.3.1 Lagrangian Formulation Of Electroweak Interactions

In particle physics, the leptons can be broadly classified into three generations, namely e, μ and τ where [4,5,12,13]:

$$e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$
$$\mu = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$$
$$\tau = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \tag{2.3.1}$$

The e^- , μ^- and τ^- exist both as left-handed as well as right-handed particles, but ν_e , ν_μ and ν_τ exist as left-handed particles. There exist charged and neutral weak currents between leptons and charged currents couple only left-handed leptons.

We arrange leptons within a generation into left-handed isodoublets $\vec{T} = \vec{\frac{1}{2}}$ and right-handed isosinglet $\vec{T} = \vec{0}$. Lets form left-handed isodoublet (L_i) and right-handed isosinglet (R_i), where, $(i = e, \mu, \tau)$. Now for left-handed isodoublet $(\vec{T} = \vec{\frac{1}{2}})$

$$L_e = (\frac{1+\gamma_5}{2}) \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix}$$
$$L_\mu = (\frac{1+\gamma_5}{2}) \begin{pmatrix} \psi_{\nu_\mu} \\ \psi_\mu \end{pmatrix}$$
$$L_\tau = (\frac{1+\gamma_5}{2}) \begin{pmatrix} \psi_{\nu_\tau} \\ \psi_\tau \end{pmatrix} \tag{2.3.2}$$

Similarly for right-handed isosinglet $(\vec{T} = \vec{0})$,

$$R_e = (\frac{1-\gamma_5}{2})\psi e$$
$$R_\mu = (\frac{1-\gamma_5}{2})\psi \mu$$
$$R_\tau = (\frac{1-\gamma_5}{2})\psi \tau \tag{2.3.3}$$

Lets now focuss on the electron generation. The electron field and its associated neutrino field are combined together into a two-component object. We have been considering left-handed isospin doublets,

$$L_e = (\frac{1+\gamma_5}{2}) \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix} \tag{2.3.4}$$

and similarly right-handed isospin singlet,

$$R_e = (\frac{1-\gamma_5}{2})\psi e \tag{2.3.5}$$

The structure of charged weak current is described as [11,12,13];

$$\begin{aligned} J_\mu^{(e)} = \quad & \bar{\psi}_e(1-\gamma_5)\gamma_\mu\psi_{\nu_e} \\ & = \frac{1}{2}\bar{\psi}_e(1-\gamma_5)^2\gamma_\mu\psi_{\nu_e} \\ & = \frac{1}{2}\bar{\psi}_e(1-\gamma_5)\gamma_\mu(1+\gamma_5)\psi_{\nu_e} \\ & = 2\bar{\psi}_e^L\gamma_\mu\psi_{\nu_e}^L. \end{aligned} \tag{2.3.6}$$

Thus we see from equ.(2.3.6) that charged weak currents couple only left-handed leptons. We can write equ.(2.3.6) more compactly in the form;

$$J_\mu^{(e)} = 2\bar{L}_e\gamma_\mu T_- L_e \tag{2.3.7}$$

We know that electromagnetic current is described as,

$$\begin{aligned} J_\mu^{(e)e.m} = \quad & \bar{\psi}_e\gamma_\mu\psi_e \\ & = \bar{\psi}_e\frac{\gamma_\mu+\gamma_5\gamma_\mu-\gamma_5\gamma_\mu+\gamma_\mu}{2}\psi_e \\ & = \bar{\psi}_e(\frac{1+\gamma_5}{2})\gamma_\mu\psi_e + \bar{\psi}_e(\frac{1-\gamma_5}{2})\gamma_\mu\psi_e \\ & = \frac{1}{2}[\bar{\psi}_e(1+\gamma_5)\gamma_\mu\psi_e + \bar{\psi}_e(1-\gamma_5)\gamma_\mu\psi_e] \\ & = \frac{1}{4}[\bar{\psi}_e(1+\gamma_5)^2\gamma_\mu\psi_e + \bar{\psi}_e(1-\gamma_5)^2\gamma_\mu\psi_e] \\ & = [\bar{\psi}_e(\frac{1+\gamma_5}{2})\gamma_\mu(\frac{1-\gamma_5}{2})\psi_e + \bar{\psi}_e(\frac{1-\gamma_5}{2})\gamma_\mu(\frac{1+\gamma_5}{2})\psi_e] \\ & = \bar{R}_e\gamma_\mu R_e + \frac{1}{2}\bar{L}_e\gamma_\mu L_e - \bar{L}_e\gamma_\mu T_3 L_e \end{aligned} \tag{2.3.8}$$

The electromagnetic current contains a third component of an isotriplet vector and isosinglet part. Then, lets try to write the most general form of the total current for electroweak interactions.

$$J_\mu^{(e)} = \bar{R}_e\gamma_\mu R_e + \frac{1}{2}\bar{L}_e\gamma_\mu L_e - \bar{L}_e\gamma_\mu\vec{T}L_e \tag{2.3.9}$$

Lets write down the interaction lagrangian in case of QED;

$$\begin{aligned} \pounds_{int} = \quad & iej_\mu A_\mu \\ & = ie\bar{\psi}\gamma_\mu\psi A_\mu \end{aligned} \tag{2.3.10}$$

To write $\pounds_{int}^e$ we should have two gauge fields, one which is an isotriplet A_μ^a and other which is isosinglet B_μ. We can write down $\pounds_{int}^e$ as,

$$\pounds_{int}^e = -ig\bar{L}_e\gamma_\mu\vec{T}.\vec{A}_\mu L_e + ig'[\bar{R}_e\gamma_\mu R_e + \frac{1}{2}\bar{L}_e\gamma_\mu L_e]B_\mu \tag{2.3.11}$$

where, g and $g^{'}$ are coupling constants corresponding to $\vec{A}_\mu$ and B_μ respectively. If we identify B_μ as the electromagnetic gauge field interactions (for photon field), then B_μ should be couple only to isosinglet current which is, $(\bar{R}_e\gamma_\mu R_e + \frac{1}{2}\bar{L}_e\gamma_\mu L_e)$. But $J_\mu^{(e)e.m}$ involves not only $(\bar{R}_e\gamma_\mu R_e + \frac{1}{2}\bar{L}_e\gamma_\mu L_e)$ but also $(-\bar{L}_e\gamma_\mu T_3 L_e)$ in $J_\mu^{(e)e.m}$, since the electromagnetic field should couple to the full electromagnetic current $J_\mu^{(e)e.m}$ that means we can not identify B_μ as the physical photon field. This also implies that identification of $\vec{A}\mu$ field as W^+,W^- and Z^0 is incorrect. Therefore, W_μ^+,W_μ^- ,Z_μ^0 and A_μ should be mixtures of $\vec{A}\mu$ and B_μ fields. Lets write down A_μ as a mixture of B_μ and A_μ^3 fields[14],

$$A_\mu = \cos\theta B_\mu + \sin\theta A_\mu^3 \tag{2.3.12}$$

Similarly, let us consider a weak neutral current, if Z_μ which is orthogonal to A_μ,

$$Z_\mu = -\sin\theta B_\mu + \cos\theta A_\mu^3 \tag{2.3.13}$$

must describe the neutral intermidate boson of weak interaction. The inverse of equ.(2.3.12) and equ.(2.3.13) becomes;

$$B_\mu = \cos\theta A_\mu - \sin\theta Z_\mu$$
$$A^3_\mu = \sin\theta A_\mu + \cos\theta Z_\mu \tag{2.3.14}$$

where the mixing angle θ is called the Weinberg angle. Finally lets introduce charged vector fields A^+_μ and A^-_μ which are inturn expressible interms of the physical $W^\pm$ fields[14],

$$A^+_\mu = \frac{1}{\sqrt{2}}(A^1_\mu + iA^2_\mu) \equiv W^-_\mu$$
$$A^-_\mu = \frac{1}{\sqrt{2}}(A^1_\mu - iA^2_\mu) \equiv W^+_\mu \tag{2.3.15}$$

Glashow introduced the concept of weak isospin and weak hypercharge in analogy to strong interactions and suggested that Gell-Mann-Nishijima [5,10] relation;

$$Q = T^3 + \frac{Y}{2} \tag{2.3.16}$$

should be valid for leptons as well, where, electric charge is (Q), weak isospin is (T), and weak hypercharge is (Y). Here T^3 is the third component of weak isospin. Therefore, there should exist weak isospin and weak hypercharge for all leptons. for a left-handed spinor, $Y = -1$, while for a right-handed spinor, $Y = -2$.

We can always regard Y as generator of U(1) group and $\vec{T}$ can regarded as generator of SU(2) group. Then, they should commute, because they have different generators.

2.4 Gauge Invariance Of Electroweak Interactions:

Using gauge invariance, we have been able to determine the right QED and Yang-Mills lagrangians. Lets consider a brouder group of transformations of $SU(2) \times U(1)$ where SU(2) should correspond to three gauge bosons W^+_μ, W^-_μ, Z^0_μ and U(1) should correspond to photon gauge field, A_μ. As in the QED and Yang- Mills cases, let us consider the free Lagrangian for massless gauge fields and leptons; we can write a Dirac spinor as a two-component object,

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{2.4.1}$$

To keep matters simple, we begin with the free Dirac lagrangian, by setting the mass term to be zero. This gives;

$$\pounds_0 = -\bar{\psi}\partial\!\!\!/\psi \tag{2.4.2}$$

We wish to splitup the Lagrangian into two parts, one for the left-handed spinor and one for the right-handed spinor. Proceeding we have;

$$\begin{aligned} \pounds_0 = \quad & -\begin{pmatrix} \bar{\psi}_L & \bar{\psi}_R \end{pmatrix} \partial\!\!\!/ \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \\ & = -\bar{\psi}_L\partial\!\!\!/\psi_L - \bar{\psi}_R\partial\!\!\!/\psi_R \end{aligned} \tag{2.4.3}$$

and the Lagrangian separates nicely into left-handed and right-handed parts. $\pounds_0$ should be invariant under global gauge transformations of $SU(2) \times U(1)$,i.e.

$$\psi_L \to \psi_L' = \exp^{i\frac{\vec{\tau}}{2}.\vec{\lambda}}\psi_L = U_2\psi_L \tag{2.4.4}$$

$$\psi_R \to \psi_R' = \exp^{iY\beta}\psi_R = U_1\psi_R \tag{2.4.5}$$

where the SU(2) transformation,

$$U_2 = \quad \exp^{i\frac{\vec{\tau}}{2}.\vec{\lambda}} \tag{2.4.6}$$

The U (1) phase transformation is analogous to the QED one and the transformation U_2 is non-Abelian like in Yang-Mills theory. We now require the Lagrangian to be also invariant under local $SU(2)\times U(1)$ gauge transformations. The local gauge transformation on ψ_L and ψ_R are,

$$\psi_L \to \psi_L' = \exp^{i\frac{\vec{\tau}}{2}.\vec{\lambda}(x)}\psi_L = U_2(\lambda)\psi_L \tag{2.4.7}$$

$$\psi_R \rightarrow \psi'_R = \exp^{iY\beta(x)} \psi_R = U_1(\beta)\psi_R \tag{2.4.8}$$

In this transformation $\pounds_0$ is not invariant. In order to satisfy the invariance requirment, we need to introduce the covariant derivative as in case of QED and Yang-Mills theory,

$$D_\mu = \partial_\mu - ig\vec{T}.\vec{A}_\mu - \frac{ig'}{2} Y B_\mu \tag{2.4.9}$$

where, g and $g^{'}$ are coupling constants corresponding to $\vec{A}_\mu$ and B_μ respectively. We define

$$\vec{T}.\vec{A}_\mu = \frac{1}{2}\tau^a A^a_\mu \tag{2.4.10}$$

Thus, we have the correct number of gauge fields to describe the $W^\pm$, Z^0 and A_μ. We want $D_\mu\psi(x)$ to transform in exactly the same manner as the $\psi(x)$ fields. This fixes the transformation properties of the gauge fields:

$$B_\mu(x) \rightarrow B'_\mu(x) = B_\mu + \frac{i}{g'}\partial_\mu\beta(x) \tag{2.4.11}$$

$$A_\mu \rightarrow A'_\mu = U_2 A_\mu U_2^{-1} - \frac{i}{g}(\partial_\mu U_2)U_2^{-1} \tag{2.4.12}$$

The transformation of B_μ is identical to the one obtained in QED for the photon, while the SU(2) gauge field $\vec{A}_\mu$ transforms in a way analogous to the Yang-Mills theory. The Lagrangian

$$\pounds = -\bar{\psi}_L \not{D}\psi_L - \bar{\psi}_R \not{D}\psi_R \tag{2.4.13}$$

is invariant under local $SU(2) \times U(1)$ transformations. In order to build the gauge-invariant kinetic term for the gauge fields, we introduce the corresponding field strengths.

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.4.14}$$

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) - ig[A_\mu, A_\nu] \tag{2.4.15}$$

Then, the lagrangian for $SU(2) \times U(1)$ is given by:

$$\begin{aligned} \pounds = \quad & -\bar{\psi}_L \not{D}\psi_L - \bar{\psi}_R \not{D}\psi_R - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \frac{1}{2}Tr(F_{\mu\nu}F_{\mu\nu}) \\ & = -\bar{\psi}_L \not{D}\psi_L - \bar{\psi}_R \not{D}\psi_R - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \frac{1}{4}\vec{F_{\mu\nu}}.\vec{F_{\mu\nu}} \end{aligned} \tag{2.4.16}$$

From the lagrangian equ.(2.4.16), we obtain the interaction lagrangian,

$$\begin{aligned}
\pounds_{int} = \quad & -\bar{\psi}_L\gamma_\mu(-ig\vec{T}.\vec{A}_\mu - \frac{ig'}{2}YB_\mu)\psi_L \\
& -\bar{\psi}_R\gamma_\mu(-ig\vec{T}.\vec{A}_\mu - \frac{ig'}{2}YB_\mu)\psi_R \\
& = ig\bar{\psi}_L\gamma_\mu\vec{T}.\vec{A}_\mu\psi_L - i\frac{g'}{2}\bar{\psi}_L\gamma_\mu B_\mu\psi_L \\
& -ig'\bar{\psi}_R\gamma_\mu\psi_R B_\mu
\end{aligned} \tag{2.4.17}$$

Now, lets simplify equ.(2.4.16) by substituting equ.(2.4.9) and other ingredients to obtain,

$$\begin{aligned}
\pounds = \quad & -\bar{\psi}_L\gamma_\mu(\partial_\mu - ig\vec{T}.\vec{A}_\mu - \frac{ig'}{2}YB_\mu)\psi_L \\
& -\bar{\psi}_R\gamma_\mu(\partial_\mu - ig\vec{T}.\vec{A}_\mu - \frac{ig'}{2}YB_\mu)\psi_R - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \frac{1}{4}\vec{F_{\mu\nu}}.\vec{F_{\mu\nu}}
\end{aligned} \tag{2.4.18}$$

since the weak hypercharge left- and right-handed is given $Y = -1$ and $Y = -2$ respectively, then, equ.(2.4.18) becomes

$$\begin{aligned}
\pounds = - \quad & \bar{\psi}_L\partial\!\!\!/\psi_L + ig\bar{\psi}_L\gamma_\mu\vec{T}.\vec{A}_\mu\psi_L \\
& -\frac{ig'}{2}\bar{\psi}_L\gamma_\mu\psi_L B_\mu - \bar{\psi}_R\partial\!\!\!/\psi_R \\
& -ig'\bar{\psi}_R\gamma_\mu\psi_R B_\mu - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} - -\frac{1}{4}\vec{F_{\mu\nu}}.\vec{F_{\mu\nu}} \\
& = -\bar{\psi}_L\partial\!\!\!/\psi_L - \bar{\psi}_R\partial\!\!\!/\psi_R + ig\bar{\psi}_L\gamma_\mu\vec{T}.\vec{A}_\mu\psi_L \\
& -ig'[\frac{1}{2}\bar{\psi}_L\gamma_\mu\psi_L + \bar{\psi}_R\gamma_\mu\psi_R]B_\mu \\
& -\frac{1}{4}B_{\mu\nu}B_{\mu\nu} - -\frac{1}{4}\vec{F_{\mu\nu}}.\vec{F_{\mu\nu}}
\end{aligned} \tag{2.4.19}$$

But we know for left-handed isodoublet;

$$\psi_L = (\frac{1+\gamma_5}{2})\begin{pmatrix}\psi_{\nu_e} \\ \psi_e\end{pmatrix} \tag{2.4.20}$$

and similarly for right-handed isospin singlet;

$$\psi_R = (\frac{1-\gamma_5}{2})\psi e \tag{2.4.21}$$

and from equ.(2.3.14) and equ.(2.3.15), the $SU(2) \times U(1)$ gauge invariant lagrangian interms of $W_\mu^\pm$,Z_μ and A_μ becomes;

$$\begin{aligned}
\pounds = \quad & \bar{\psi}_{\nu_e}(\frac{1-\gamma_5}{2})\partial\!\!\!/ \psi_{\nu_e} - \bar{\psi}_e \partial\!\!\!/ \psi_e \\
& + \frac{ig}{2\sqrt{2}}\bar{\psi}_e(1-\gamma_5)\gamma_\mu \psi_{\nu_e} W_\mu^- \\
& + \frac{ig}{2\sqrt{2}}\bar{\psi}_{\nu_e}(1-\gamma_5)\gamma_\mu \psi_e W_\mu^+ \\
& + \frac{ig}{4cos\theta}[\bar{\psi}_{\nu_e}(1-\gamma_5)\gamma_\mu\psi_{\nu_e} - \bar{\psi}_e(g'_v - \gamma_5)\psi_e]Z_\mu \\
& - ie\bar{\psi}_e\gamma_\mu\psi_e A_\mu \\
& - \frac{1}{4}\vec{F}.\vec{F} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} \qquad (2.4.22)
\end{aligned}$$

where, $g'_v = 1 - 4\sin^2\theta$.

Equ(2.4.22) contains both electromagnetic and weak interactions. However the weak gauge bosons and the leptons are considering as massless. But we know they are massive. We shall achive this by applying the Higgs mechanism [15], to give masses to these fields and at the same time without sacrificing the gauge invariance. Higgs field should form isodoublet to interact with the isodoublet leptons. Lets choose the scalar Higgs field to be a complex weak isodoublet, with $\vec{T} = \frac{\vec{1}}{2}$ and weak hypercharge $Y = +1$ [12,13]

$$\phi = \begin{pmatrix} \phi^a \\ \phi^b \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad (2.4.23)$$

with the scalar field lagrangian written down under local gauge invariant form as;

$$\pounds_\phi = -(D_\mu\phi)^+(D_\mu\phi) - \mu^2\phi^+\phi - \lambda^2(\phi^+\phi)^2 \qquad (2.4.24)$$

where the covariant derivative:

$$D_\mu = \partial_\mu - ig\vec{T}.\vec{A}_\mu - \frac{ig'}{2}YB_\mu \qquad (2.4.25)$$

After the unitary gauge transformation the Higgs field simply becomes:

$$\phi = \begin{pmatrix} 0 \\ H + \nu \end{pmatrix} \qquad (2.4.26)$$

This choice of vacuum breaks the symmetry, where H is the Higgs field and is the vacuum expectation value of Higgs field.

Then, the Higgs lagrangian interms of $W^{\pm}$ and Z^0 can be written as in[14,15],

$$\begin{aligned}\pounds_{Higgs} = - \quad & (\partial_\mu H)^2 - \frac{g^2}{2}(H+\nu)^2 W_\mu^+ W_\mu^- \\ & -\frac{g^2}{4\cos^2\theta}(H+\nu)^2 Z_\mu Z_\mu \\ & -\lambda^2\nu^2 H^2 - \frac{1}{4}\lambda^2 H^4 - \frac{1}{4}\lambda^2\nu^4 \\ & -\lambda^2 H^3 \nu - f_e(H+\nu)\bar{\psi}_e\psi_e \end{aligned} \tag{2.4.27}$$

This choice of vacuum breaks the symmetry, where, H is the Higgs field and ν is the vacuum expectation value of Higg's field. The lagrangian for complex scalar field is expressed in general as[14,15];

$$\pounds_\phi = -(\partial_\mu \phi^*)(\partial_\mu \phi) - \frac{m^2}{2}\phi^*\phi. \tag{2.4.28}$$

Comparing coefficients of mass terms in the above two equations (2.4.27) and (2.4.28) we immediately deduce that the mass of $W_\mu^{\pm}$ and Z_μ bosons as;

$$\begin{aligned} M_{w_\mu^\pm} &\simeq g\nu \\ M_{z_\mu} &\simeq \frac{g\nu}{\sqrt{2}\cos\theta} \end{aligned} \tag{2.4.29}$$

while the electron mass is expressed as,

$$m_e \simeq f_e \nu \tag{2.4.30}$$

which are in very good agreement with the experimentally determined values. But the Higgs field leaves the photon, A_μ and neutrino massless, since the lagrangian does not possess terms representing the couplings of these fields with Higg's fields.

Finally, the complete lagrangian for electroweak theory for electrons can be written as;

$$
\begin{aligned}
\pounds = - \quad & \bar{\psi}_{\nu_e}(\frac{1-\gamma_5}{2})\partial\!\!\!/\psi_{\nu_e} - \bar{\psi}_e\partial\!\!\!/\psi_e \\
& + \frac{ig}{2\sqrt{2}}\bar{\psi}_e(1-\gamma_5)\gamma_\mu\psi_{\nu_e}W_\mu^- \\
& + \frac{ig}{2\sqrt{2}}\bar{\psi}_{\nu_e}(1-\gamma_5)\gamma_\mu\psi_e W_\mu^+ \\
& + \frac{ig}{4\cos\theta}[\bar{\psi}_{\nu_e}(1-\gamma_5)\gamma_\mu\psi_{\nu_e} - \bar{\psi}_e(g_v' - \gamma_5)\psi_e]Z_\mu \\
& - ie\bar{\psi}_e\gamma_\mu\psi_e A_\mu \\
& - \frac{1}{4}\vec{F}_{\mu\nu}.\vec{F}_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} \\
& - (\partial_\mu H)^2 - \frac{g^2}{2}(H+\nu)^2 W_\mu^+ W_\mu^- \\
& - \frac{g^2}{4\cos^2\theta}(H+\nu)^2 Z_\mu Z_\mu \\
& - \lambda^2\nu^2H^2 - \frac{1}{4}\lambda^2H^4 - \frac{1}{4}\lambda^2\nu^4 \\
& - \lambda^2H^3\nu - f_e(H+\nu)\bar{\psi}_e\psi_e \qquad (2.4.31)
\end{aligned}
$$

Now, we have to rewrite the free field parts $\vec{F}_{\mu\nu}.\vec{F}_{\mu\nu}$ and $B_{\mu\nu}B_{\mu\nu}$ interms of the physical fields $W_\mu^\pm$,Z_μ^0 and A_μ.

$$
\begin{aligned}
\vec{F}_{\mu\nu}.\vec{F}_{\mu\nu} = \quad & 2[\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig\cos\theta(W_\nu^- Z_\nu - W_\mu^- Z_\mu) - ig(W_\mu^- A_\nu - W_\nu^- A_\mu)] \\
& \times[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig\cos\theta(W_\nu^+ Z_\nu - W_\mu^+ Z_\mu) + ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu)] \\
& + [\cos\theta(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \sin\theta(\partial_\mu A_\nu - \partial_\nu A_\mu) + ig(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)] \\
& \times[\cos\theta(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \cos\theta(\partial_\mu A_\nu - \partial_\nu A_\mu) \\
& + ig(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)] \qquad (2.4.32)
\end{aligned}
$$

and the free part of the isosinglet field is;

$$\begin{aligned} B_{\mu\nu}B_{\mu\nu} = \quad & (\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial_\mu B_\nu - \partial_\nu B_\mu) \\ & = \cos^2\theta(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ & + \sin^2(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial_\mu Z_\nu - \partial_\nu Z_\mu) \\ & -2\sin\theta\cos\theta(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu Z_\nu - \partial_\nu Z_\mu) \end{aligned} \tag{2.4.33}$$

The lagrangian equ.(2.4.31) contains interactions of the electron fields with the gauge bosons, electromagnetic fields. Then,we can represent by the Feynman diagram, where the vertices are, $\frac{ig}{2\sqrt{2}}(1-\gamma_5)\gamma_\mu$, $\frac{ig}{4\cos\theta}(g'_v-\gamma_5)$ and $ie\gamma_\mu$ respectively for the charged, neutral gauge bosons and eletromagnetic couplings.

The gauge bosons $W^\pm_\mu$ and Z^0 have self interactions due to the coupling terms in the $\vec{F}_{\mu\nu}.\vec{F}_{\mu\nu}$ terms in the lagrangian due to equations (2.4.32) but A_μ does not interact.

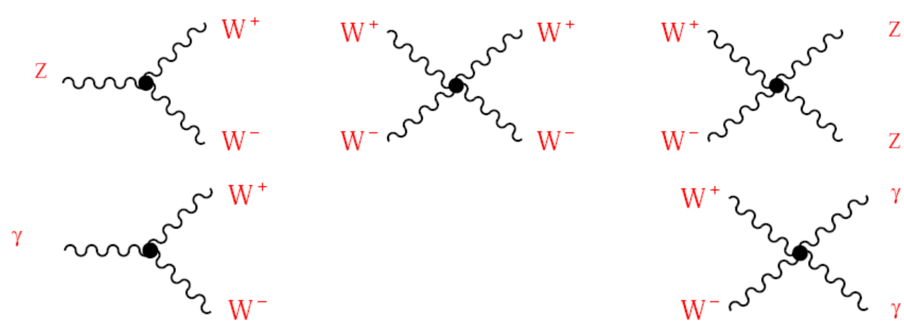

Figure 2.1: Gauge boson self-interaction vertices

Also we draw the interaction vertices for the Higgs field H coupling to W, Z bosons as in figure 2.2, where the coupling constants $g_{H-W^\pm}$ and g_{H-Z^0} determining the couplings of Higgs to $W^\pm$ and Z^0 are $\frac{g^2}{2}$ and $\frac{g^2}{4\cos^2}$ respectively.

We can generalize the electroweak lagrangian to include all lepton generations by changing the index $(e \rightarrow l)$ lepton where l runs over all the lepton generations

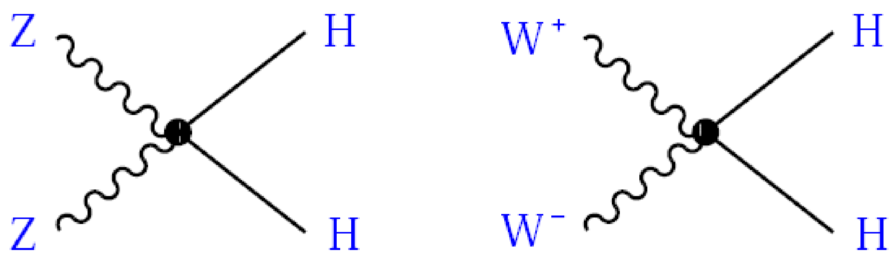

Figure 2.2: The Higgs field H coupling to W, Z bosons

$(l = e^-, \mu^-, \tau^-)$.Then the complete lagrangian for GWS becomes.

$$
\begin{aligned}
\pounds_{GWS} = - \quad & \bar{\psi}_{\nu_l}(\frac{1-\gamma_5}{2})\not{\partial}\psi_{\nu_l} - \bar{\psi}_l \not{\partial}\psi_l \\
& + \frac{ig}{2\sqrt{2}}\bar{\psi}_l(1-\gamma_5)\gamma_\mu\psi_{\nu_l}W_\mu^- \\
& + \frac{ig}{2\sqrt{2}}\bar{\psi}_{\nu_l}(1-\gamma_5)\gamma_\mu\psi_l W_\mu^+ \\
& + \frac{ig}{4\cos\theta}[\bar{\psi}_{\nu_l}(1-\gamma_5)\gamma_\mu\psi_{\nu_l} - \bar{\psi}_l(g_v' - \gamma_5)\psi_l]Z_\mu \\
& - ie\bar{\psi}_l\gamma_\mu\psi_l A_\mu \\
& - \frac{1}{4}\vec{F}_{\mu\nu}.\vec{F}\mu\nu - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} \\
& - (\partial_\mu H)^2 - \frac{g^2}{2}(H+\nu)^2 W_\mu^+ W_\mu^- \\
& - \frac{g^2}{4\cos^2\theta}(H+\nu)^2 Z_\mu Z_\mu \\
& - \lambda^2\nu^2 H^2 - \frac{1}{4}\lambda^2 H^4 - \frac{1}{4}\lambda^2\nu^4 \\
& - \lambda^2 H^3\nu - f_l(H+\nu)\bar{\psi}_l\psi_l
\end{aligned}
\tag{2.4.34}
$$

Thus we have determined all terms of the GWS lagrangian describing the electroweak interaction of the leptons.

Now, lets generalize the electroweak interactions to include quarks. The generation structure is similar to the leptons;

$$
\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}
\tag{2.4.35}
$$

Leptons are couple only within the same generation. However in case of quarks, coupling occurs within the same as well as different generation. To take account of this cross generational coupling, Cabibbo-Kobayashi-Maskawa [16], modfied the weak coupling of quarks.

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix} \tag{2.4.36}$$

We arrange the quarks into left-handed isodoublets corresponding to isospin $\vec{t} = \frac{\vec{1}}{2}$ and weak hypercharge $Y = \frac{1}{3}$;

$$\begin{aligned} L_u &= (\frac{1+\gamma_5}{2}) \begin{pmatrix} u \\ d' \end{pmatrix} \\ L_c &= (\frac{1+\gamma_5}{2}) \begin{pmatrix} c \\ s' \end{pmatrix} \\ L_t &= (\frac{1+\gamma_5}{2}) \begin{pmatrix} t \\ b' \end{pmatrix} \end{aligned} \tag{2.4.37}$$

The lower component of the isodoublet is less than by one unit of electric charge from the upper component. Lets write down the right-handed isosinglet corresponds to isospin $\vec{t} = \vec{0}$ and the hypercharge for all the upper part $Y = \frac{4}{3}$ and for all the lower part $Y = -\frac{2}{3}$

$$\begin{aligned} R_u &= (\frac{1-\gamma_5}{2}) \begin{pmatrix} u \\ d' \end{pmatrix} \\ R_c &= (\frac{1-\gamma_5}{2}) \begin{pmatrix} c \\ s' \end{pmatrix} \\ R_t &= (\frac{1-\gamma_5}{2}) \begin{pmatrix} t \\ b' \end{pmatrix} \end{aligned} \tag{2.4.38}$$

where, $d^{'}$, $s^{'}$, $b^{'}$ taking into account the hadron sector [4,16-19]:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2.4.39}$$

where, V_{CKM} is called the Cabibbo-Kobayashi-Maskawa matrix. When the top quark was discovered and B-meson decays observed, there by validating couplings of b quarks the CKM matrix is given as in [17-19];

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.4.40}$$

$$V_{CKM} = \begin{pmatrix} 0.975 & 0.221 & 0.005 \\ 0.221 & 0.97 & 0.04 \\ 0.01 & 0.041 & 0.999 \end{pmatrix} \tag{2.4.41}$$

where we have assumed that the off-diagonal elements are small, and V_{ud} specifies the coupling of $d \rightarrow u + W^{-}$, and similar to the other matrix elements.

In the same manner with leptons, we generalize the full lagrangian for quarks as;

$$\pounds = \Sigma_{i=u,c,t}[-\bar{L}_i \not{D} L_i - \bar{R}_i \not{D} R_i - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu}.\vec{F}_{\mu\nu} + \pounds_{Higgs} \tag{2.4.42}$$

where, D_μ is the usual covariant derivative given as;

$$D_\mu = \partial_\mu - \frac{ig}{2} \vec{T}.\vec{A}_\mu - ig' Y B_\mu \tag{2.4.43}$$

we obtain the interaction lagrangian for quarks from the above equ.(2.4.41);

$$\begin{aligned} \pounds_{int}^{quarks} = \quad & i\Sigma_i \{ \frac{g}{\sqrt{2}} \bar{L}_i \gamma_\mu (T^- W_\mu^- + T^+ W_\mu^+ + T_3 A_\mu^3) L_i \\ & + g' [\frac{1}{2} \bar{L}_i Y \gamma_\mu L_i + \bar{R}_i Y \gamma_\mu R_i] B_\mu \} \end{aligned} \tag{2.4.44}$$

Now, lets focuss only on the first generation quarks,

$$\begin{aligned}
\pounds_{int}^{quarks} = \quad & \frac{ig}{\sqrt{2}}[\bar{L_u}\gamma_\mu(T^- W_\mu^- + T^+ W_\mu^+ + T_3 A_\mu^3)L_u] \\
& + g'[\frac{1}{2}\bar{L_u} Y \gamma_\mu L_u + \bar{R}_u Y \gamma_\mu R_u]B_\mu\} \\
& = \frac{ig}{\sqrt{2}}[\bar{L_u}\gamma_\mu \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} L_u + \bar{L_u}\gamma_\mu T_3 A_\mu^3 L_u] \\
& + \frac{ig'}{6}\bar{L_u}\gamma_\mu L_u B_\mu + ig'\bar{R}_u Y \gamma_\mu R_u B_\mu
\end{aligned} \tag{2.4.45}$$

where, $T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and the weak hypercharge for all left-handed isodoublets is $Y = +\frac{1}{3}$ and the weak hypercharge for right-handed isosinglet, the upper component $Y = +\frac{4}{3}$ and for the lower component $Y = -\frac{2}{3}$ and we know;

$$\begin{aligned}
L_u &= (\frac{1+\gamma_5}{2}) \begin{pmatrix} u \\ d' \end{pmatrix} \\
R_u &= (\frac{1-\gamma_5}{2}) \begin{pmatrix} u \\ d' \end{pmatrix} \\
B_\mu &= \cos\theta A_\mu - \sin\theta Z_\mu \\
A_\mu^3 &= \sin\theta A_\mu + \cos\theta Z_\mu
\end{aligned} \tag{2.4.46}$$

After substituting equ.(2.4.45) into equ.(2.4.44) we get the following relation;

$$\begin{aligned}
\pounds_{int}^{quarks} = \quad & \frac{ig}{2\sqrt{2}}[\bar{u}(1-\gamma_5)\gamma_\mu d^{'}]W_\mu^- \\
& + \frac{ig}{2\sqrt{2}}[\bar{d}^{'}(1-\gamma_5)\gamma_\mu u]W_\mu^+ \\
& + \frac{ig\cos\theta}{2}[\bar{u}(1-\gamma_5)\gamma_\mu u - \bar{d}^{'}(1-\gamma_5)\gamma_\mu d^{'}]Z_\mu \\
& - \frac{ig\sin^2\theta}{12\cos\theta}[\bar{u}(1-\gamma_5)\gamma_\mu u + \bar{d}^{'}(1-\gamma_5)\gamma_\mu d^{'}]Z_\mu \\
& - \frac{ig\sin^2\theta}{3\cos\theta}[\bar{u}(1+\gamma_5)\gamma_\mu u - \frac{1}{2}\bar{d}^{'}(1+\gamma_5)\gamma_\mu d^{'}]Z_\mu \\
& + \frac{ie}{12}[\bar{u}(1-\gamma_5)\gamma_\mu u + \bar{d}^{'}(1-\gamma_5)\gamma_\mu d^{'}]A_\mu \\
& + \frac{ie}{3}[\bar{u}(1+\gamma_5)\gamma_\mu u - \frac{1}{2}\bar{d}^{'}(1+\gamma_5)\gamma_\mu d^{'}]A_\mu \\
& + \frac{ie}{2}[\bar{u}(1-\gamma_5)\gamma_\mu u - \bar{d}^{'}(1-\gamma_5)\gamma_\mu d^{'}]A_\mu
\end{aligned} \tag{2.4.47}$$

if we see the first term from the above equ.(2.4.46);

$$\pounds^1_{int} = \frac{ig}{2\sqrt{2}}[\bar{u}(1-\gamma_5)\gamma_\mu d^{'}]W^-_\mu \tag{2.4.48}$$

but we know that the $d^{'}$ from CKM matrix relation which is given as;

$$d^{'} = dV_{ud} + sV_{us} + bV_{ub} \tag{2.4.49}$$

Then, equ.(2.4.47) can be expressed as;

$$\begin{aligned}\pounds^1_{int} = \quad & \frac{ig}{2\sqrt{2}}[\bar{\psi}_u(1-\gamma_5)\gamma_\mu(dV_{ud} + sV_{us} + bV_{ub})]W^-_\mu \\ & = \frac{ig}{2\sqrt{2}}V_{ud}\bar{\psi}_u(1-\gamma_5)\gamma_\mu\psi_d W^-_\mu \\ & + \frac{ig}{2\sqrt{2}}V_{us}\bar{\psi}_u(1-\gamma_5)\gamma_\mu\psi_s W^-_\mu \\ & + \frac{ig}{2\sqrt{2}}V_{ub}\bar{\psi}_u(1-\gamma_5)\gamma_\mu\psi_b W^-_\mu\end{aligned} \tag{2.4.50}$$

if we represent equ.(2.4.49) by Feynman diagram in the same case with leptons;

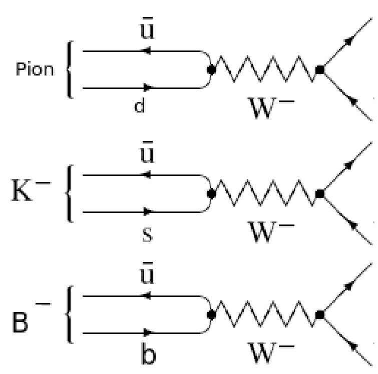

Figure 2.3: Feynman diagram for the charged weak decays

and we can check for the other terms by the same method. The charged weak interactions of qarks depicted in figure.2.3. above carries a vertex $\frac{ig}{2\sqrt{2}}V_{ud}(1-\gamma_5)\gamma_\mu$, $\frac{ig}{2\sqrt{2}}V_{us}(1-\gamma_5)\gamma_\mu$, $\frac{ig}{2\sqrt{2}}V_{ub}(1-\gamma_5)\gamma_\mu$ for weak decays of pion, kaon and B-meson respectively.

The quark fields also acquire mass through Higgs mechanism similar to the case of lepton and gauge fields ($W^{\pm}$, Z^0)

Now, we makke use of the $b\bar{u}W^-$ vertex derived from GWS model in equ.(2.4.50) to calculate leptonic decays of B^--meson at quark level in chapter 4. But before that we study decays of W^- boson in next section which is a such process in leptonic decays of B-mesons.

2.5 Decay Of The Charged boson:W^-

As a mathematical preliminary before stuyind the decays of B^--meson, we now employing the lagrangian formulation of electroweak theory discussed above to first, discuss the decay of the negatively charged boson, $W^- \rightarrow e^- + \bar{\nu}_e$. The Feynman diagram for this process is represented by figure 2.4.

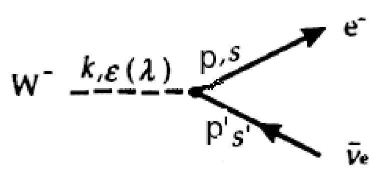

Figure 2.4: leptonic decay intermediate boson

where (k, ϵ) is the initial four momentum and the polarization state of W^- and (p,s), ($p^{'}$, $s^{'}$) are the final four momenta and spin states of electron and $\bar{\nu}_e$ respectively. The lowest order scattering matrix element for this process $S(^1)_{fi}$ is expressed as;

$$S(^1)_{fi} = -i\int d^4x\bar{\psi}_e(x)(\frac{ig}{2\sqrt{2}})(1-\gamma_5)\gamma_\mu\psi_{\nu_e}(x)W^-_\mu(x) \tag{2.5.1}$$

where the wave functions of the incoming and outgoing particles are;

$$\bar{\psi}_e(x) = \sqrt{\frac{m_e}{EV}}\bar{U}(\vec{p}, s)\exp^{-ip.x}$$
$$\psi_{\nu_e}(x) = \sqrt{\frac{1}{2\omega' V}}V(\vec{p}', s')\exp^{-ip'.x}$$
$$W_\mu^-(x) = \sqrt{\frac{M_w}{\omega V}}\epsilon_\mu(\vec{k}, \lambda)\exp^{ik.x} \quad (2.5.2)$$

By inserting equ.(2.5.2) into equ.(2.5.1) becomes;

$$\begin{aligned} S(^1)_{fi} = \quad & (-i)\sqrt{\frac{m_e}{EV}}\sqrt{\frac{1}{2\omega' V}}\sqrt{\frac{M_w}{\omega V}} \\ & \times(\frac{ig}{2\sqrt{2}})\int d^4x \exp^{i(k-p-p').x}\bar{U}(\vec{p}, s)(1-\gamma_5)\gamma_\mu V(\vec{p}', s')\epsilon_\mu(\vec{k}, \lambda) \\ & = (\frac{g}{2\sqrt{2}})\sqrt{\frac{m_e}{EV}}\sqrt{\frac{1}{2\omega' V}}\sqrt{\frac{M_w}{\omega V}} \\ & \times(2\pi)^4\delta^4(k-p-p')M_{fi} \end{aligned} \quad (2.5.3)$$

where,

$$M_{fi} = \bar{U}(\vec{p}, s)(1-\gamma_5)\gamma_\mu V(\vec{p}', s')\epsilon_\mu(\vec{k}, \lambda) \quad (2.5.4)$$

and

$$(2\pi)^4\delta^4(k-p-p') = \int d^4x \exp^{i(k-p-p').x} \quad (2.5.5)$$

The transition probability from initial to final state is;

$$\left|S^1_{fi}\right|^2 = \frac{g^2}{8}\frac{m_e M_w}{2E\omega\omega' V^3}(2\pi)^4 VT\delta^4(k-p-p')\left|M_{fi}\right|^2 \quad (2.5.6)$$

where,

$$[(2\pi)^4\delta^4(k-p-p')]^2 = (2\pi)^4 VT\delta^4(k-p-p') \quad (2.5.7)$$

By summing or integrating over the final states of two particles, the total transition probability, W, is expressed as [4,14];

$$\begin{aligned} W = \quad & \int \left|S^1_{fi}\right|^2 \frac{Vd^3\vec{p}}{(2\pi)^3}.\frac{Vd^3\vec{p}'}{(2\pi)^3} \\ & = \frac{g^2}{8}\frac{m_e M_w T}{2E\omega\omega'}\int\frac{\delta^4(k-p-p')}{(2\pi)^2}\left|M_{fi}\right|^2 d^3\vec{p}d^3\vec{p}' \end{aligned} \quad (2.5.8)$$

Then, the decay rate Γ, is expressed as the total transition per unit time [4,14];

$$\begin{aligned}\Gamma = & \frac{W}{T} \\ & = \frac{g^2}{8}\frac{m_e M_w}{2E\omega\omega'}\int \frac{\delta^4(k-p-p')}{(2\pi)^2}\left|M_{fi}\right|^2 d^3\vec{p}d^3\vec{p}' \end{aligned} \quad (2.5.9)$$

Using the theorems for calculating traces of Dirac spinors and γ-matrices, we get the following result;

$$\begin{aligned}\left|M_{fi}\right|^2 = & \Sigma_\lambda \Sigma_{s,s'}\left|M_{fi}\right|^2 \\ & = \Sigma_\lambda \epsilon_\mu(\vec{k},\lambda)\epsilon_\nu(\vec{k},\lambda) \\ & \times \Sigma_{s,s'}[\bar{U}(\vec{p},s)(1-\gamma_5)\gamma_\mu V(\vec{p}',s')] \\ & \times[\bar{V}(\vec{p}',s')(1-\gamma_5)\gamma_\nu U(\vec{p},s)] \\ & = (\delta_{\mu\nu}+\frac{K_\mu K_\nu}{K^2})Tr\{(\frac{\not{p}+m_e}{2m_e})(1-\gamma_5)\gamma_\mu \not{p}'(1-\gamma_5)\gamma_\nu\} \\ & = (\delta_{\mu\nu}+\frac{K_\mu K_\nu}{K^2})\frac{4}{m_e}[p_\mu p'_\nu + p_\nu p'_\mu - (p.p')\delta_{\mu\nu} + i\epsilon_{\alpha\mu\beta\nu}p_\alpha p'_\beta] \end{aligned} \quad (2.5.10)$$

where,

$$\Sigma_\lambda \epsilon_\mu(\vec{k},\lambda)\epsilon_\nu(\vec{k},\lambda) = (\delta_{\mu\nu}+\frac{K_\mu K_\nu}{K^2}) \quad (2.5.11)$$

But $\epsilon_{\alpha\mu\beta\nu}(\delta_{\mu\nu}+\frac{K_\mu K_\nu}{K^2}) = 0$, because $\epsilon_{\alpha\mu\beta\nu}$ is antisymmetric tensor, whereas $(\delta_{\mu\nu}+\frac{K_\mu K_\nu}{K^2})$ is symmetric. Then, equ.(2.5.11) becomes;

$$\left|M_{fi}\right|^2 = \frac{M_w^2}{3m_e}(1-\frac{m_e^2}{M_w^2})(1-\frac{m_e^2}{2M_w^2}) \quad (2.5.12)$$

Now, after substituting equ.(2.5.12) into equ.(2.5.9) and integrating over the final states, the decay rate is expressed as;

$$\Gamma = \frac{g^2}{48}\frac{M_w^2}{4\pi}(1-\frac{m_e^2}{M_w^2})^2(1-\frac{m_e^2}{2M_w^2}) \quad (2.5.13)$$

which is in complete agreement with experiment. We will now discuss the decays of B^--mesons, $(B^- \rightarrow e^- + \bar{\nu}_e)$ at quark level in the framework of Bethe-Salpeter equation as an another application of lagrangian formulation of electroweak theory discussed earlier. As a run up to this study, we first study the Bethe-Salpeter equation in chapter 3.

Chapter 3

The Bethe-Salpeter Equation

The Bethe-salpeter equation (BSE) is describes the bound states of a two-particles quantum mechanically in a relativistically covariant formalism. The BSE is an important tool for calculating bound states in the realm of elementary particle physics and it is an application of the Feynman and a resummation of the infinite set of diagrams by using integral equation. When we deal with bound states the particles stay together infinitely long and they can therefore interact arbitrarily often [8]. It is clear that this situation can not be described by the summation of a few Feynman diagrams. Then we employ the framework of BSE under covariant instantaneous ansatz(CIA) to study the leptonic decays of B-mesons. The BSE provides a field theoretical starting point to describe hadrons as relativistic bound states of quarks and antiquarks. Inorder to study the behaviour of bound states, we consider two particle and we generalize the definition of the single particle propagators to 2-particle propagators with particles in mutual interaction. Let us start the present analysis with a brief exposition of the essential features of the Bethe-Salpeter equation for bound states composed of a fermion and an antifermions.

3.1 Two particle propagators

Inorder to study the behaviour of two partticles, we generalize the definition of the single particle propagator to two particle propagator with particles in mutual interaction. First consider two particles propagating without interaction with eachother, so that we can write down the free propagator of two particles, that propagates from space-time point (3,4) to (1,2) as [8].

$$S^0(12;34) = S_F^0(1,3)S_F^0(2,4) \tag{3.1.1}$$

where, $1 \rightarrow x_1$,$2 \rightarrow x_2$, $3 \rightarrow x_3$, $4 \rightarrow x_4$, etc. Going beyond this trivial case we have to consider what happens, if these two particles interact electromagnetically. Lets consider the simplest possibility that is Moller interaction, which is by the exchange of single photon[8].

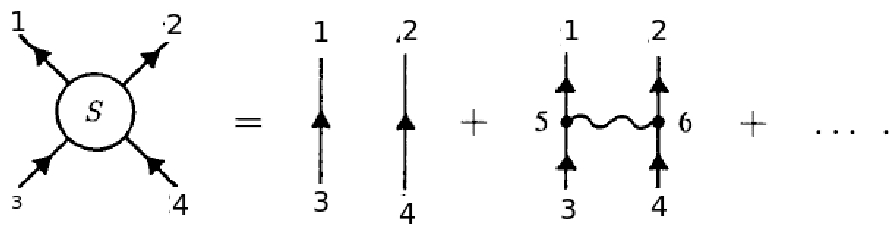

Figure 3.1: propagation of two particles non-interacting and interacting by the exchange of single photon

The bubble denoted by the circle represents the full two particle propagator where the unconnected fermion lines represents the free two particle propagator S^0 and the second term on right-side represents the propagation of two charged particles, propagating from space-time points (3,4) to(1,2) and interacting through a photon exchange connecting them at space-time vertices,5 and 6. given in equ.(3.1.1). Now using the Feynman rules we can represent figure 3.1 by:

$$\begin{aligned} S(12;34) = \quad & S_F^0(1,3)S_F^0(2,4) \\ & + \int d^45 \int d^46 S_F(1,5)S_F(2,6) \\ & \times[(-ie\gamma_\mu)D_F^{\mu\nu}(5,6)(-ie\gamma_\nu)]S_F(5,3)S_F(6,4) + ... \end{aligned} \tag{3.1.2}$$

where $D_F^{\mu\nu}(5,6)$ is the fourier transform of photon propagator in 4-momentum space and given as:

$$D_F^{\mu\nu}(5,6) = \frac{-i\delta_{\mu\nu}}{(2\pi)^4} \int d^4k \frac{\exp^{ik.(x_5,x_6)}}{k^2} \tag{3.1.3}$$

In equ.(3.1.2) we have written down only the first term of an infinite series. The result of the infinite series can be represented by a function $K(12;34)$ which is called the interaction Kernel [8]. The exact form of equ.(3.1.2) reads;

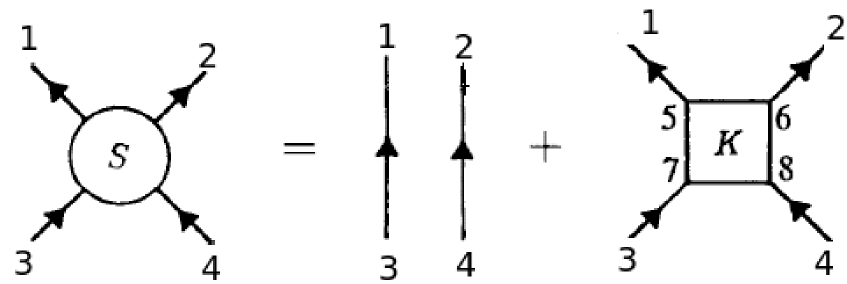

Figure 3.2: irreducible interaction Kernel

Then equ.(3.1.2) can be written out explicitly:

$$\begin{aligned} S(12;34) = \quad & S_F^0(1,3)S_F^0(2,4) + \int d^45 \int d^46 \int d^47 \int d^48 \\ & S_F(1,5)S_F(2,6)K(56;78)S_F(7,3)S_F(8,4) \end{aligned} \quad (3.1.4)$$

We have not gained much by doing this, since K is an extremly complicated 4-point function. Only in first order perturbation theory does the kernel become very simple, that is according to equ.(3.1.2)

$$K(56;78) = (-ie\gamma\mu))D_F^{\mu\nu}(5,6)(-ie\gamma_\nu)\delta^4(5,7)\delta^4(6,8) \quad (3.1.5)$$

The complete function K is a sum over infinitely many graphs of arbitrarily high order. The interaction kernel K contains an infinite number of Feynman graphs as well as and can not be calculated exactly. As we have discussed at the beginning of this section the inclusion of an infinite number of interactions is necessary if one is interested in bound systems. For many practical purposes one restricts oneself to the lowest order of the interaction kernel K as in [8],to the single photon exchange. This prescription is called the ladder approximation. This name suggests itself from the below figure 3.3

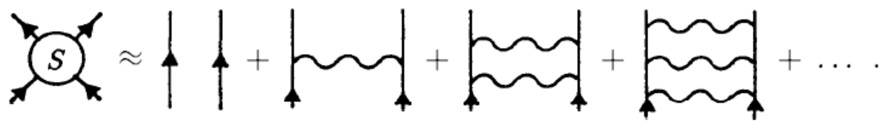

Figure 3.3: ladder approximation

This diagram shows us summation over infinite ladders of one quantum exchange Moller interactions between two particles. Then equ.(3.1.4) can be expressed as;

$$S(12;34) = S_F^0(1,3)S_F^0(2,4) + \int d^45 \int d^46 \int d^47 \int d^48$$
$$S_F(1,5)S_F(2,6)K(56;78)S_F(7,3)S_F(8,4) \quad (3.1.6)$$

This equation is called the full two particle propagator.

If we know the initial wave function $\psi(3,4)$, we can write the full wave function of two particles as;

$$\psi(1,2) = \int d^33 \int d^34 S_F(12,34)\gamma_4^a\gamma_4^b\psi(3,4) \quad (3.1.7)$$

Now, substitute equ.(3.1.8) into (3.1.9)

$$\psi(1,2) = \int d^33 \int d^34\{S_F^0(1,3)S_F^0(2,4) + \int d^45 \int d^46 \int d^47 \int d^48$$
$$S_F(1,5)S_F(2,6)K(56;78)S_F(7,3)S_F(8,4)\}\gamma_4^a\gamma_4^b\psi(3,4) \quad (3.1.8)$$

$$\psi(1,2) = \psi_0(1,2) + \int d^45 \int d^46 \int d^47 \int d^48 S_F(1,5)S_F(2,6)K(56;78)\psi(7,8) \quad (3.1.9)$$

where,$\psi_0(1,2)$ is the free two-particle wave function which is given as;

$$\psi_0(1,2) = \int d^33 \int d^34 S_F^0(1,3)S_F^0(2,4)\gamma_4^a\gamma_4^b\psi(3,4) \quad (3.1.10)$$

and $\psi(7,8)$ is according to equ.(3.1.9) expressed as,

$$\psi(7,8) = \int d^33 \int d^34 S_F(7,3)S_F(8,4)\gamma_4^a\gamma_4^b\psi(3,4) \quad (3.1.11)$$

If one is interested in bound states ,then $\psi_0(1,2)$ will be vanish. Then the full wave function for bound state particles expressed as;

$$\psi(1,2) = \int d^43 \int d^44 \int d^45 \int d^46 S_F(1,5)S_F(2,6)K(56;34)\psi(3,4) \quad (3.1.12)$$

3.2 Derivation of Bethe-Salpeter Equation

We want to derive the BSE from equ.(3.1.12). Lets operate equ.(3.1.12) both sides by the free Dirac operators $(\not\partial_1 + m_1)$ and $(\not\partial_2 + m_2)$. Since the one particle propagators obey the relations;

$$(\not\partial_1 + m_1)S_F(1,5) = -i\delta^4(1,5)$$
$$(\not\partial_2 + m_2)S_F(2,6) = -i\delta^4(2,6) \tag{3.2.1}$$

Because the interactions are on space-time (5,6), we can carry out differentiations of $\partial_{1\mu}$ and $\partial_{2\nu}$ under the integration sign. Then equ.(3.1.14) becomes;

$$(\not\partial_1 + m_1)(\not\partial_2 + m_2)\psi(1,2) = -\int d^4 3 \int d^4 4 K(12;34)\psi(3,4) \tag{3.2.2}$$

In this form the BSE is an integro-differential equation. For practical purposes it is easier to work with BSE in momentum space. If we define the wave function in momentum space as;

$$\psi(p_1,p_2) = \frac{1}{(2\pi)^4}\int d^4x_1 \int d^4x_2 \exp^{i(p_1.x_1+p_2.x_2)}\psi(1,2) \tag{3.2.3}$$

Then the fourier transform of equ.(3.2.2) becomes

$$\frac{1}{(2\pi)^4}\int d^4x_1 \int d^4x_2 \exp^{i(p_1.x_1+p_2.x_2)} (\not\partial_1 + m_1)(\not\partial_2 + m_2)\psi(1,2) = \frac{-1}{(2\pi)^4}\int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \exp^{i(p_1.x_1+p_2.x_2)} K(12;34)\psi(3,4) \tag{3.2.4}$$

On the left-hand side we integrate by parts so that the differential operator acts on the exponential function and on the right-hand side we insert two delta functions, $\delta^4(x'_3 - x_3)$ and $\delta^4(x'_4 - x_4)$;

$$\begin{aligned}
&(i\not{p}_1 + m_1)(i\not{p}_2 + m_2)\psi(p_1, p_2) \\
&= \frac{-1}{(2\pi)^4} \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x'_3 \\
&\int d^4x_4 \int d^4x'_4 \delta^4(x'_3 - x_3)\delta^4(x'_4 - x_4) \exp^{i(p_1.x_1+p_2.x_2)} \\
&K(x_1, x_2; x_3, x_4)\psi(x'_3, x'_4)
\end{aligned} \tag{3.2.5}$$

By using the integral representation of the delta functions:

$$\delta^4(x'_3 - x_3) = \frac{1}{(2\pi)^4} \int d^4p'_1 \exp^{ip'_1.(x'_3 - x_3)} \tag{3.2.6}$$

$$\delta^4(x'_4 - x_4) = \frac{1}{(2\pi)^4} \int d^4p'_2 \exp^{ip'_2.(x'_4 - x_4)} \tag{3.2.7}$$

After equ.(3.2.6) and (3.2.7) substitute into equ.(3.2.5) then it becomes;

$$\begin{aligned}
&(i\not{p}_1 + m_1)(i\not{p}_2 + m_2)\psi(p_1, p_2) \\
&= \frac{-1}{(2\pi)^{12}} \int d^4x_1 \int d^4x_2 \int d^4x_3 \\
&\int d^4x_4 \int d^4x'_3 \int d^4x'_4 \int d^4p'_1 \int d^4p'_2 \\
&\exp^{ip'_1.(x'_3 - x_3) + ip'_2.(x'_4 - x_4)} \\
&\exp^{i(p_1.x_1+p_2.x_2)} K(12; 34)\psi(x'_3, x'_4)
\end{aligned} \tag{3.2.8}$$

one can express the right-hand side of equ.(3.2.8) as a product of momentum-space wave functions and the interaction kernel in momentum-space can be written as;

$$\begin{aligned}
K(p_1, p_2; p_3, p_4) = \quad & \frac{i}{(2\pi)^4} \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \\
& \exp^{i(p_1.x_1+p_2.x_2+p_3.x_3+p_4.x_4)} K(12; 34)
\end{aligned} \tag{3.2.9}$$

and we express;

$$\Psi(p_1', p_2') = \frac{1}{(2\pi)^4} \int d^4x_3' \int d^4x_4' \exp^{i(p_1'.x_3' + p_2'.x_4')} \psi(x_3', x_4') \tag{3.2.10}$$

Then equ.(3.2.8) becomes;

$$(i\not{p}_1 + m_1)(i\not{p}_2 + m_2)\Psi(p_1, p_2) = \frac{-i}{(2\pi)^4} \int d^4p_1' \int d^4p_2' K(p_1, p_2; p_1', p_2')\Psi(p_1', p_2') \tag{3.2.11}$$

This equation is BSE in momentum-space. When we treat a two particle system, it is always advantageous to transform to absolute and relative co-ordinates. For simplicity we assume that both particles have the same mass $m_1 = m_2 = m$ and we define; $P = p_1 + p_2$ and $q = \frac{1}{2}(p_1 - p_2)$ where P is the total momentum of two particles and q is the relative momentum of the particles. Since the interaction described by the kernel K must conserve momentum, i.e.

$$p_1 + p_2 = p_1' + p_2' \tag{3.2.12}$$

By using the Ansatz [21,22],

$$K(p_1, p_2; p_1', p_2') = \delta^4(P - P')K(q, q'; P') \tag{3.2.13}$$

Now, by substituting equ.(3.2.13) into equ.(3.2.11) becomes;

$$(i\frac{1}{2}\not{P} + \not{q} + m)(i\frac{1}{2}\not{P} - \not{q} + m)\Psi(q, P) = \frac{-i}{(2\pi)^4} \int d^4q' d^4P' \left| \frac{\partial(p_1', p_2')}{\partial(q', P')} \right| K(q, q'; P')\delta^4(P - P')\Psi(q', P') \tag{3.2.14}$$

Since the Jacobian determinant for the transformation of the volume element in equation(3.2.14) is equal to one, then equ.(3.2.14) it becomes;

$$(i\not{p}_1 + m)(i\not{p}_2 + m)\Psi(q, P) = \frac{-i}{(2\pi)^4}\int d^4q' K(q, q'; P)\Psi(q', P) \qquad (3.2.15)$$

where, $\not{p}_1 = \frac{1}{2}\not{P} + \not{q}$ and $\not{p}_1 = \frac{1}{2}\not{P} - \not{q}$. In equ.(3.2.15) P plays only the role of a parameter. The structure of equ.(3.2.15) is still so complicated that one will not succeed in finding exact solutions. Equ.(3.2.15) is the BSE in momentum-space for the inverse propagators of two spin-half particles or fermion propagator.

$$S_F(p) = \frac{1}{i\not{p} + m} \qquad (3.2.16)$$

By substituting equ.(3.2.16) into equ.(3.2.15) it becomes;

$$S_F^{-1}(p_1)S_F^{-1}(p_2)\Psi(q, P) = \frac{-i}{(2\pi)^4}\int d^4q' K(q, q'; P)\Psi(q', P) \qquad (3.2.17)$$

Now, we call equ.(3.2.17) the BSE for fermion particles or spin-half particles. For a quark and anti-quark forming a meson, the total momentum of the hadron is $P = p_1 + p_2$ and the relative momentum of quark and anti-quark is given by $q = \frac{1}{2}(p_1 - p_2)$. Now lets consider the BSE for a boson or spinless particle in the analogy of the spin-$\frac{1}{2}$ particle;

$$i\Delta_1\Delta_2\Phi(q, P) = \frac{1}{(2\pi)^4}\int d^4q' K(q, q'; P)\Phi(q', P) \qquad (3.2.18)$$

where Δ_1 and Δ_2 are the inverse propagators of spinless (spin-0) particles, which are given by

$$\Delta_1 = p_1^2 + m_1^2 \qquad (3.2.19)$$

and

$$\Delta_2 = p_2^2 + m_2^2 \qquad (3.2.20)$$

and $\Phi(q, P)$ is the 4D wave function of spinless particles. Therefore, equ.(3.2.18) is called the BSE for a boson or spinless particles.

3.3 The Bethe-Salpeter (BS) vertex function $\Gamma(\hat{q})$ for pseudo-scalar mesons

To discuss the generalized hadron-quark vertex in BS wave function under CIA, we start with a 4D BSE for scalar quark-antiquark system with an effective kernel K and 4D wave function $\Phi(P, q)$. Then from equ.(3.2.18)

$$i(2\pi)^4 \Delta_1 \Delta_2 \Phi(q, P) = \int d^4q' K(q, q') \Phi(q', P) \tag{3.3.1}$$

where $\Delta_{1,2}$, is the inverse propagators of two scalar quarks, give as:

$$\Delta_1 = m_1^2 + p_1^2 \tag{3.3.2}$$

and

$$\Delta_2 = m_2^2 + p_2^2 \tag{3.3.3}$$

here m_1 , m_2 are constituent masses of quarks. The 4-momenta of the quark and anti-quark, p_1 and p_2, are related to the internal 4-momentum q_μ and total momentum P of hadron of mass M as;

$$p_{1\mu} = \hat{m}_1 P_\mu + q_\mu \tag{3.3.4}$$

and

$$p_{2\mu} = \hat{m}_2 P_\mu - q_\mu \tag{3.3.5}$$

where $\hat{m}_{1,2} = \frac{1}{2}[1 \pm \frac{m_1^2 - m_2^2}{M^2}]$ are the Wightman-Garding (WG) definitions as given in [21,23,24] of masses of individual quarks. By using the CIA Ansatz on the BS kernel K in eq.(3.3.1) can be written as [23,30],

$$K(q, q') = K(\hat{q}, \hat{q}') \tag{3.3.6}$$

where, we can write the internal momentum as $q_\mu = (\hat{q}_\mu, iM\sigma)$, the transverse component is expressed as [21,22,23,24],

$$\hat{q}_\mu = q_\mu - \frac{q.P}{P^2} P_\mu \tag{3.3.7}$$

is observed to be orthogonal to the total 4-momentum P (i.e., $\hat{q}.P = 0$), [21,22,23,24]. The longitudinal component of q_μ

$$M\sigma = M\frac{q.P}{P^2} \tag{3.3.8}$$

does not appear in the form $K(\hat{q}, \hat{q}')$ of the kernel. For reducing eq.(3.3.1) to the 3D form, one can define a 3D wave function $\phi(\hat{q})$ as

$$\phi(\hat{q}) = \int_{-\infty}^{+\infty} Md\sigma\Phi(q, P) \tag{3.3.9}$$

The 4D volume element in momentum-space can be expressed as;

$$d^4q = d^3\hat{q}Md\sigma \tag{3.3.10}$$

we get a covariant version of the salpeter equation as given in [23,24],

$$i(2\pi)^4\Delta_1\Delta_2\Phi(q, P) = \int d^3\hat{q}'K(\hat{q}, \hat{q}') \int_{-\infty}^{+\infty} Md\sigma'\Phi(q', P) \tag{3.3.11}$$

Let us postulate, if one integrate over q', we get

$$\phi(\hat{q}') = \int_{-\infty}^{+\infty} Md\sigma'\Phi(q', P) \tag{3.3.12}$$

Then equ.(3.3.11) becomes;

$$i(2\pi)^4\Delta_1\Delta_2\Phi(q, P) = \int d^3\hat{q}'K(\hat{q}, \hat{q}')\phi(\hat{q}') \tag{3.3.13}$$

from equ.(3.3.1) we can write

$$\Phi(q, P) = \frac{1}{(2\pi)^4 i\Delta_1\Delta_2} \int d^4q'K(q, q')\Phi(q', P) \tag{3.3.14}$$

Lets integrate equ.(3.3.14) both sides with respect to $Md\sigma$;

$$\int_{-\infty}^{+\infty} \Phi(P, q)Md\sigma = \frac{1}{(2\pi)^3} \int d^3\hat{q}'K(\hat{q}, \hat{q}')\phi(\hat{q}') \int_{-\infty}^{+\infty} \frac{Md\sigma}{2\pi i\Delta_1\Delta_2} \tag{3.3.15}$$

Then equ.(3.3.15) becomes;

$$(2\pi)^3D(\hat{q})\phi(\hat{q}) = \int d^3K(\hat{q}, \hat{q}')\phi(\hat{q}') \tag{3.3.16}$$

where $D(\hat{q})$ is a 3D denominator function whose value can be easily worked out by contour integration by noting the positions of the poles in the complex σ-plane as in [21,22,23,24], which is given as;

$$\frac{1}{D(\hat{q})} = \int_{-\infty}^{+\infty} \frac{M d\sigma}{2\pi i \Delta_1 \Delta_2} \tag{3.3.17}$$

when we compare equations (3.3.13) and (3.3.16) the interaction is only on the right-hand sides, then we can simply equalise with the interaction vertex as in [23,24].

$$\begin{aligned} i(2\pi)^4 \Delta_1 \Delta_2 \Phi(q,P) &= (2\pi)^3 D(\hat{q})\phi(\hat{q}) \\ \Delta_1 \Delta_2 \Phi(q,P) &= \frac{D(\hat{q})\phi(\hat{q})}{2\pi i} \end{aligned} \tag{3.3.18}$$

This leads to the structure of hadron-quark vertex for scalar quarks as,

$$\Gamma(\hat{q}) \equiv \frac{D(\hat{q})\phi(\hat{q})}{2\pi i} \tag{3.3.19}$$

where, $\Gamma(\hat{q})$ is the Hadron-quark BS vertex function under CIA. Now there is an exact interconnection between 3D wave function $\phi(\hat{q})$ and the 4D wave function $\Phi(q,P)$. To apply the above simplified discussions to the case of fermionic quarks constituting a particular meson we proceed in the same manner as [20,21,22,23,24,30]. The scalar propagators $\Delta_{1,2}$ in the above equations are replaced by the proper fermionic propagators S_F. Then, on observing the vertex $\Gamma(\hat{q})$ now is a 4×4 matrix in the spinor space, we should incorporate its relevant Dirac structures. This is taken as the leading covariant γ_5. Then the vertex function for pseudoscalar is expressed as in [23,24],

$$\Gamma(\hat{q}) = N_P \gamma_5 \frac{D(\hat{q})\phi(\hat{q})}{2\pi i} \tag{3.3.20}$$

where, the quantity N_P is the standard BS normalization factor which goes with the vertex function (3.2.20).

Now lets comes to the problem of the 3D BS wave function, $\phi(\hat{q})$ the corresponding wave function which satisfy a Lorentz-covariant, representing the 3D reduction of the 4D

BSE as a result of the above ansatz. In particular, a gaussian form[23,24,25]

$$\phi(\hat{q}) \approx \exp -\frac{\hat{q}^2}{2\beta^2} \tag{3.3.21}$$

emerges (for harmonic confinement) as a solution of the 3D BSE, with β^2 obtained analytically from the input structure of the BS-kernel and is given in detail in Reference[23,24]. The structure of the parameter ($\beta = 0.44728 GeV$) in $\phi(\hat{q})$ appearing in equ(4.2.24) is taken from Ref. [23,24,25],(for details see Ref. [23,24]).

Chapter 4

Leptonic Decays Of B-meson At Quark Level

Charged mesons formed from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual $W^{\pm}$ boson [26]. The quark-antiquark annihilations via a virtual $W^{+}(W^{-})$ to the $l^{+} + \nu_l$ ($l^{-} + \bar{\nu}_l$) final states occur for the $\pi^{\pm}$, $K^{\pm}$, and $B^{\pm}$ mesons etc.

4.1 Decay of the B-meson at macroscopic level considering it as elementary particle

The study of pure leptonic decay of B-meson is the most natural process to determine the decay constant f_B [16,27]. The decay proceeds via quark annihilation into a $W^{\pm}$ boson, and the decay rate is directly proportional to f_B^2. According to the quark model, the decay of a charged B-meson is really a scattering event in which the incident quarks happen to be bound together. Because the B-meson is composite, we may not know how the W couples to the B-meson, but we do know how it couples to the leptons, so it is simpler to proceed as follows. We first redraw the Feynman diagram in case of elementary particle without considering the quark compositeness of B-meson with a blob to represent the coupling of B^{-} to W^{-} [4],

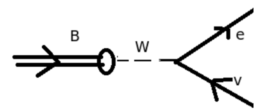

Figure 4.1: coupling of B^--meson.

The amplitude must have the general form;

$$M = G[\bar{\psi}_e(1-\gamma_5)\gamma_\mu\psi_{\nu_e}]F_\mu \tag{4.1.1}$$

where, F_μ is a form factor describing the $B^- \to W^-$ blob and G is the coupling constant of B-meson. F_μ has to be a 4-vector to contract with γ_μ in the lepton. So F_μ has to be some scalar quantity times the four- momentum P_μ of B-meson, i.e.

$$F_\mu = f_B P_\mu \tag{4.1.2}$$

where, f_B is the B-meson decay constant [4]. The decay constant f_B is a measure of the probability amplitude for the quarks to have zero separation, which is necessary for them to annihilate. Therefore,equ.(4.1.1) can be expressed as;

$$M = Gf_B[\bar{\psi}_e(p_1)(1-\gamma_5)\gamma_\mu\psi_{\nu_e}(p_2)]P_\mu \tag{4.1.3}$$

where, P, p_1, p_2 are four momenta of the B-meson, charged lepton and the associated lepton neutrino respectively. By the law of conservation of momentum, we have

$$P_\mu = p_{\mu 1} + p_{\mu 2} \tag{4.1.4}$$

Using Dirac equation and also substituting $m_{\nu_e} = 0$, equ.(4.1.3) becomes

$$M = -iGf_B m_e[\bar{\psi}_e(p_1)(1+\gamma_5)\psi_{\nu_e}(p_2)] \tag{4.1.5}$$

and its complex conjugate is;

$$M^+ = iGf_B m_e[\bar{\psi}_{\nu_e}(p_2)(1-\gamma_5)\psi_e(p_1)] \tag{4.1.6}$$

Then;

$$\sum_{s,s'} \Big|M\Big|^2 = \sum_{s,s'} M^+M = \sum_{s,s'} G^2 f_B^2 m_e^2 [\bar{\psi}_{\nu_e}(p_1)(1-\gamma_5)\psi_e(p_2) \times \bar{\psi}_e(p_1)(1+\gamma_5)\psi_{\nu_e}(p_2)]$$
$$= G^2 f_B^2 m_e^2 Tr[(1-\gamma_5)(-i\not{p}_1 + m_1)(1+\gamma_5)(-i\not{p}_2)]$$
$$= G^2 f_B^2 m_e^2 8 p_1.p_2 \tag{4.1.7}$$

The differential decay rate is defined as in [17];

$$d\Gamma = \frac{1}{M_B} \sum_{s,s'} \Big|M\Big|^2 (2\pi)^4 \delta^4(P - p_1 - p_2)$$
$$\frac{d^3p_1}{(2\pi)^3 2E_1} \times \frac{d^3p_2}{(2\pi)^3 2E_2} \tag{4.1.8}$$

where, M_B is the mass of B-meson. Hence, integrating over lepton momenta, we get;

$$\Gamma(B^- \to e^- + \bar{\nu}_e) = \frac{1}{8\pi} G^2 f_B^2 m_e^2 M_B (1 - \frac{m_e^2}{M_B^2})^2 \tag{4.1.9}$$

All quantities in this decay rate are well established.

4.2 Calculation Of Decay Constant For Charged B-meson

Now we do the calculation of decay rate of, $B^- \to e^- + \bar{\nu}_e$, by invoking the quark compositeness $(b\bar{u})$ of B^--meson. To calculate the decay constant f_B for charged B-meson we make use of $b\bar{u}W^-$ vertex from equ.(2.4.50) and the Hadron-quark vertex function for B^--meson from BSE under CIA.

$$f_B P_\mu = < 0 \Big| \bar{q} i(1-\gamma_5)\gamma_\mu q \Big| B(P) > \tag{4.2.1}$$

The decay constant f_B can be evaluated through the quark loop diagram shown in Figure 4.2 which gives the coupling of the two-quark loop to the charged weak current and can be expressed as a loop integtal:

$$f_B P_\mu = \sqrt{3} \int d^4q Tr[\psi(P,q) i(1-\gamma_5)\gamma_\mu] \tag{4.2.2}$$

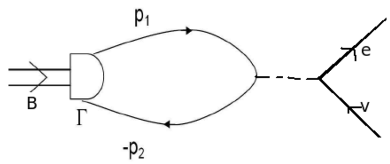

Figure 4.2: loop diagram coupling of the two-quarks

where, V is the weak interaction vertex which is given as, $V = (1 - \gamma_5)\gamma_\mu$. The complete Bethe-Salpeter wave function $\Psi(P, q)$ for a B-meson is expressed as

$$\Psi(P, q) = S_F(p_1)\Gamma(\hat{q})S_F(-p_2) \tag{4.2.3}$$

where $S_F(p)$ is the usual fermionic propagator of the hadron(i.e.B-meson), expressed as;

$$\begin{aligned} S_F(p_1) = \quad & -i\frac{-i\not{p}_1 + m_1}{\Delta_1} \\ & S_F(-p_2) = -i\frac{i\not{p}_2 + m_2}{\Delta_2} \end{aligned} \tag{4.2.4}$$

$$\Gamma(\hat{q}) = N_P\gamma_5\frac{D(\hat{q})\phi(\hat{q})}{2\pi i} \tag{4.2.5}$$

By using substitutions of equ.(4.2.3),(4.2.4),(4.2.5) and (3.2.20) into equ.(4.2.2), we get

$$\begin{aligned} f_B P_\mu = \quad & -\sqrt{3}N_B\int d^4q\frac{D(\hat{q})\phi(\hat{q})}{2\pi\Delta_1\Delta_2}\{Tr[(-i\not{p}_1 + m_1)\gamma_5(i\not{p}_2 + m_2)\gamma_5\gamma_\mu] \\ & -Tr[(-i\not{p}_1 + m_1)\gamma_5(i\not{p}_2 + m_2)\gamma_\mu]\} \end{aligned} \tag{4.2.6}$$

Now we can evaluate the trace over γ-matrices by using the trace theorems which is given as in the Appendix (B).

$$\begin{aligned} & Tr[(-i\not{p}_1 + m_1)\gamma_5(i\not{p}_2 + m_2)\gamma_\mu\gamma_5] = -4i(m_1p_{2\mu} + m_2p_{1\mu}) \\ & Tr[(-i\not{p}_1 + m_1)\gamma_5(i\not{p}_2 + m_2)\gamma_\mu] = 0 \end{aligned} \tag{4.2.7}$$

The decay constant for B-meson f_B can be expressed as;

$$f_B P_\mu = -4\sqrt{3}N_B\int d^4q\frac{D(\hat{q})\phi(\hat{q})}{2\pi i\Delta_1\Delta_2}(m_1p_{2\mu} + m_2p_{1\mu}) \tag{4.2.8}$$

Lets multiply equ.(4.2.8) both sides by $(\frac{P_\mu}{M^2})$ and the 4D volume element momentum space can be expressed as $d^4q = d^3\hat{q}Md\sigma$, then equ.(4.2.8) becomes;

$$f_B = \sqrt{3}N_B \int d^3\hat{q}D(\hat{q})\phi(\hat{q})I \tag{4.2.9}$$

where,

$$I = \frac{4}{M^2}\int_{-\infty}^{+\infty} \frac{Md\sigma}{2\pi i\Delta_1\Delta_2}(m_1p_2.P + m_2p_1.P) \tag{4.2.10}$$

The dot product of momentum of constituent quarks p_1 and p_2 with hadron momentum, P, can be expressed as;

$$p_1.P = -M^2(\hat{m}_1 + \sigma) \tag{4.2.11}$$

$$p_2.P = -M^2(\hat{m}_2 - \sigma) \tag{4.2.12}$$

Then equ.(4.2.10) can be expressed as;

$$I = 2\int_{-\infty}^{+\infty} \frac{Md\sigma}{2\pi i\Delta_1\Delta_2}[(m_1 + m_2)(1 - \frac{\delta m^2}{M^2}) + 2\delta m\sigma] \tag{4.2.13}$$

where, $\delta m = m_1 - m_2$. Here we have employed unequal mass kinematics when the hadron constituents have different masses. Now we can evaluate the integrals by using the contour integration. When we closed the contour from above the real axis or from below the real axis the results are equal [28]. If we can shift the poles either above the real axis or below the real axis, as shown in figure 4.3.,

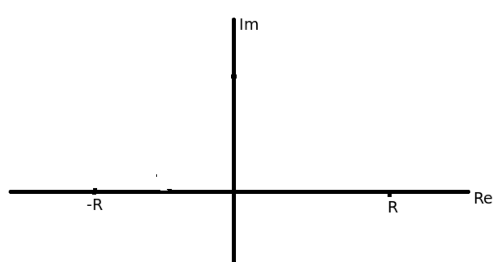

Figure 4.3: contour integral

we can calculate the contour integral by cauchy residue theorem [28];

$$I = \pm 2\pi i \sum Residue \tag{4.2.14}$$

where,

$$\sum R = lim_{\sigma \to \sigma_{1,2}} f(\sigma)(\sigma - \sigma_{1,2}) \tag{4.2.15}$$

and

$$f(\sigma) = \frac{M}{2\pi i \Delta_1 \Delta_2}\sigma \tag{4.2.16}$$

The inverse propagators of the two constituent quarks forming the B-meson, can inturn be expressed interms of the off-shell parameters as;

$$\Delta_1 = \omega_1^2 - M^2(\hat{m}_1 + \sigma)^2, \omega_1^2 = m_1^2 + \hat{q}^2 \tag{4.2.17}$$

$$\Delta_2 = \omega_2^2 - M^2(\hat{m}_2 - \sigma)^2, \omega_2^2 = m_2^2 + \hat{q}^2 \tag{4.2.18}$$

In equ.(4.2.13), carrying out integration over dσ by noting the pole positions in the complex σ-plane;

$$\Delta_1 = 0 \Rightarrow \sigma_1^{\pm} = \pm\frac{\omega_1}{M} - \hat{m}_1 \tag{4.2.19}$$

$$\Delta_2 = 0 \Rightarrow \sigma_2^{\pm} = \pm\frac{\omega_2}{M} + \hat{m}_2 \tag{4.2.20}$$

The denominator function $D(\hat{q})$ obtained from carrying out pole integrations in σ-plane, after the contour integration ,can be expressed as;

$$D(\hat{q}) = \frac{(\omega_1 + \omega_2)^2 - M^2}{\frac{1}{2\omega_1} + \frac{1}{2\omega_2}} \tag{4.2.21}$$

Lets evaluate the contour integral for equa.(4.2.13)

$$\begin{aligned} R_1 = & \int_{-\infty}^{+\infty} \frac{M d\sigma}{2\pi i \Delta_1 \Delta_2}\sigma \\ & = \frac{[M^2(\omega_2 - \omega_1) + (m_1^2 - m_2^2)(\omega_1 + \omega_2)]}{4M^2\omega_1\omega_2[(\omega_1 + \omega_2)^2 - M^2]} \end{aligned} \tag{4.2.22}$$

The result of σ-contour integration of equ.(4.2.13) becomes;

$$\begin{aligned} I = & 2\{(m_1 + m_2)(1 - \frac{\delta m^2}{M^2})\frac{1}{D(\hat{q})} \\ & + \frac{2\delta m[M^2(\omega_2 - \omega_1) + (m_1^2 - m_2^2)(\omega_1 + \omega_2)]}{4M^2\omega_1\omega_2[(\omega_1 + \omega_2)^2 - M^2]}\} \end{aligned} \tag{4.2.23}$$

The decay constant can be expressed as;

$$\begin{aligned} f_B = & \sqrt{3}N_B \int d^3\hat{q} D(\hat{q})\phi(\hat{q}) \\ & 2\{(m_1+m_2)(1-\frac{\delta m^2}{M^2})\frac{1}{D(\hat{q}} \\ & +\frac{2\delta m[M^2(\omega_2-\omega_1)+(m_1^2-m_2^2)(\omega_1+\omega_2)]}{4M^2\omega_1\omega_2[(\omega_1+\omega_2)^2-M^2]}\} \end{aligned} \tag{4.2.24}$$

To calculate the normalization factor, N_B which enters into the expressions for f_B, we use the current conservation condition [22,23].

$$2iP_\mu = (2\pi)^4 \int d^4q Tr[\bar{\Psi}(P,q)(\frac{\partial}{\partial P_\mu} S_F^{-1}(p_1))\Psi(P,q)S_F^{-1}(-p_2)] + 1 \Leftrightarrow 2 \tag{4.2.25}$$

where,

$$\bar{\Psi}(P,q) = \gamma_4 \Psi(P,q)^+ \gamma_4 \tag{4.2.26}$$

$$\Psi(P,q) = S_F(p_1)\Gamma(\hat{q})S_F(-p_2) \tag{4.2.27}$$

$$\bar{\Psi}(P,q) = S_F(-p_2)\Gamma(\hat{q})S_F(p_1) \tag{4.2.28}$$

When we substitute the values of $\bar{\Psi}(P,q)$,$\Psi(P,q)$,$S_F(p_1)$ and $S_F(-p_2)$ into equ.(4.3.25) and evaluate trace over γ-matrices, the Bethe-Salpeter Normalization can be expressed as;

$$N_B^{-2} = (2\pi)^2 i \int d^3\hat{q} D^2(\hat{q})\phi^2(\hat{q}) I \tag{4.2.29}$$

where,

$$I = \frac{-4}{M^2}\int_{-\infty}^{+\infty} \frac{M d\sigma}{\Delta_1^2 \Delta_2}\{2(p_1.p_2)(p_1.P) - \Delta_1(p_2.P) - 2m_1m_2(p_1.P)\} \tag{4.2.30}$$

where the scalar dot products, $p_1.p_2$, $p_1.P$ and $p_2.P$, can be expressed as,

$$p_1.P = -M^2(\hat{m}_1 + \sigma) \tag{4.2.31}$$

$$p_2.P = -M^2(\hat{m}_2 - \sigma) \tag{4.2.32}$$

$$p_1.p_2 = -M^2(\hat{m}_2 - \sigma)(\hat{m}_1 + \sigma) - \hat{q}^2 \tag{4.2.33}$$

After we substitute the above scalar products into equ.(4.3.30), it expressed as;

$$\begin{aligned} I = \quad & 2\pi i\{\frac{2}{M^2}(M^2 - m_1^2 + m_2^2)\frac{1}{D(\hat{q})} - \frac{M^2(\omega_2 - \omega_1) + (m_1^2 - m_2^2)(\omega_2 + \omega_1)}{M^2\omega_1\omega_2(M^2 - (\omega_2 + \omega_1)^2)}\} \\ & + \frac{1}{M^4}(M^2 + m_1^2 - m_2^2)\{(M^2 + m_1^2 - m_2^2)(M^2 - m_1^2 + m_2^2) + 4M^4\hat{q}^2\}I_1 \\ & + \frac{4}{M^2}\{(M^2 + m_1^2 - m_2^2)(m_2^2 - m_1^2) - M^2\hat{q}^2\}I_2 \\ & + 4\{3(m_2^2 - m_1^2) - M^2\}I_3 - 8M^2 I_4 \end{aligned} \tag{4.2.34}$$

where the integrals I_1,I_2,I_3 and I_4 over dσ becomes;

$$\begin{aligned} I_1 = \quad & \int_{-\infty}^{+\infty} \frac{M d\sigma}{\Delta_1^2 \Delta_2} \\ & = 2\pi i\{\frac{2\omega_1^3 - \omega_2(M^2 + 5\omega_1^2 + 4\omega_1\omega_2 + \omega_2^2)}{4\omega_1^3\omega_2((\omega_1 + \omega_2)^2 - M^2)^2}\} \end{aligned} \tag{4.2.35}$$

$$\begin{aligned} I_2 = \quad & \int_{-\infty}^{+\infty} \frac{M d\sigma}{\Delta_1^2 \Delta_2}\sigma \\ & = 2\pi i\{\frac{(m_1^2 - m_2^2)(\omega_1 + \omega_2)^2(2\omega_1 + \omega_2) - M^4\omega_2}{8M^2\omega_1^3\omega_2((\omega_1 + \omega_2)^2 - M^2)^2} \\ & + \frac{M^2(6\omega_1^3 + 9\omega_1^2\omega_2 + 4\omega_1\omega_2^2 + \omega_2((m_1^2 - m_2^2) + \omega_2^2))}{8M^2 w_1^3\omega_2((\omega_1 + \omega_2)^2 - M^2)^2}\} \end{aligned} \tag{4.2.36}$$

$$\begin{aligned} I_3 = \quad & \int_{-\infty}^{+\infty} \frac{M d\sigma}{\Delta_1^2 \Delta_2}\sigma^2 \\ & = \frac{2\pi i}{16M^4}\{\frac{2(M^2 + (m_1^2 - m_2^2)2M\omega_2)^2}{\omega_2(\omega_1 + \omega_2 + M)^2(\omega_2 - \omega_1 + M)^2} \\ & + \frac{2M(M - m_1)(M^2 - (m_1^2 - m_2^2) - 2M\omega_1)^2}{\omega_1^2(\omega_2^2 - (M - \omega_1)^2)^2} \\ & - \frac{4M^2(M^2 - (m_1^2 - m_2^2) - 2M\omega_1)}{\omega_1^2(\omega_2^2 - (M - \omega_1)^2)^2} \\ & - \frac{M(M^2 - (m_1^2 - m_2^2) - 2M\omega_1)^2}{\omega_1^3(\omega_2^2 - (M - \omega_1)^2)}\} \end{aligned} \tag{4.2.37}$$

and,

$$
\begin{aligned}
I_4 = \quad & \int_{-\infty}^{+\infty} \frac{Md\sigma}{\Delta_1^2\Delta_2}\sigma^3 \\
& = \frac{2\pi i}{16M^6}\{\frac{2(M^2+(m_1^2-m_2^2)+2M\omega_2)^3}{\omega_2(\omega_2^2+M^2-\omega_1^2 2M\omega_2)^2} \\
& + \frac{(M^2-(m_1^2-m_2^2)-2M\omega_1)^2(M^4-M^2((m_1^2-m_2^2)+\omega_1^2+\omega_2^2)}{\omega_1^3(\omega_1^2+M^2-\omega_2^2-2M\omega_2)^2} \\
& - \frac{\delta m^2(3\omega_1^2-\omega_2^2)-4M\omega_1(\omega_2^2-(m_1^2-m_2^2))}{\omega_1^3(\omega_1^2+M^2-\omega_2^2-2M\omega_2)^2}\}
\end{aligned} \tag{4.2.38}
$$

We have thus evaluated the general expressions for f_B and N_B in the framework of BSE under CIA, with Dirac structure, γ_5 introduced in the hadron-quark vertex function. We see that so far the results are independent of any model for $\phi(\hat{q})$. Now, to calculate the numerical values of the decay constant f_B, first we obtain the BS normalizer N_B, by substituting all the constant values. The input parameters are given as in reference [31], $M = 5271MeV$, $m_u = 300MeV$, $m_b = 4500MeV$, where M is mass of hadron, $m_{u,b}$ are the masses of constituent quarks. We substituting input parameters into equ.(4.3.34) and perform 3-D integrate over $d^3\hat{q}$. The numerical calculation for N_B and f_B have been done using Mathematica. We get the B-meson BS normalizer,

$$
N_B = 0.0209GeV^{-3} \tag{4.2.39}
$$

Putting this value of N_B obtained above into equ.(4.2.24) and again performing 3-D integrate over $d^3\hat{q}$, we get the B-meson decay constant f_B as,

$$
f_B = 0.2189GeV \tag{4.2.40}
$$

The result for f_B obtained in this framework is using close to the experimental data for f_B obtained recently by at[9],

$$
f_B(exp.) = 0.200 \pm 0.04GeV \tag{4.2.41}
$$

We see that our theoretical result for f_B calculated lies within the error bars of experimental data. The decay width, Γ_B for B-meson in this BSE framework comes out to be,

$$\Gamma_B = 3.3338 \times 10^{-27} GeV \tag{4.2.42}$$

which is again comparable to the experimental decay width of B-meson at,

$$\Gamma_B(exp.) = 2.8943 \times 10^{-27} GeV \tag{4.2.43}$$

The accuracy of the results for f_B and Γ_B in this BSE framework is a validation of this framework.

Chapter 5

Conclusion

In this thesis, chapter 1 deals with the introduction. In chapter 2 we have discuss the lagrangian formulation of U(1) gauge theory QED, SU(2) Yang-Mills gauge theory and the $SU(2) \times U(1)$ Glashow-Weinberg-Salam (GWS) Model for Electroweak interactions. GWS model has been discussed both for the leptonic sector as well as for quark sector. We have also discussed the Higgs mechanism which is responsible for generating masses for fermions as well as the gauge bosons $W^{\pm}$ and Z^0 due to coupling of these fields with the Higgs field.

We have also derived the Feynman rules for QED as well as the electroweak interactions. The interaction vertices have been explicitly worked out for QED as well as for elctroweak theory. These interaction vertices are then used for the study of the process,$B^- \to e^- + \bar{\nu}_e$ in the framework of Bethe-Salpeter equation in chapter 4.

As a preliminary exercise, we have disussed the decay of the charged, $W^- \to e^- + \bar{\nu}_e$ boson which enables us to formlate its decay width as an application of GWS theory. This decay of charged,W^- boson is a subprocess in the decay, $B^- \to e^- + \bar{\nu}_e$ which is studied next in chapter 4.

In chapter 3 we have disussed that, the Bethe-Salpeter equation (BSE) and the generalized structure of hadron-quark vertex for B-mesons. Since hadrons are not point like particles, they are composites of quark and antiquark (i.e. meson); the hadron-quark vertex is an extended vertices and carries all the non-perturbative ingredients of QCD. This hadron-quark vertex for B-mesons is derived explicitly in chapter 3.

In chapter 4 we have discussed the weak leptonic decays of B-meson first considering it as elementary particle in the macroscopic approch and we have calculated the decay width of B-meson. We then disussed the microscopic approch and studied the decays of B-meson at quark level in the framework of Bethe-Salpeter equation by invoking the quark compositeness ($b\bar{u}$) of B-meson and using the non-perturbative hadron-quark vertex derived in chapter 3. We derive formulas of the decay constant, f_B for B-meson with different mass quarks in the framework of BSE under covariant instantaneous ansatz(CIA), using the non-perturbative hadron-quark vertex function,$\Gamma(\hat{q})$ of B-meson. We obtain the numerical values of the decay constant, f_B and decay width Γ_B of the B-meson in the framework of BSE. Then the results are compared with other recent calculations and existing experimental results.

The decay constantf_B and decay width Γ_B for B-meson predicted in our framework is

$$\begin{aligned} f_B = \quad & 0.2189 GeV \\ & \Gamma_B = 3.3338 \times 10^{-27} GeV \end{aligned} \tag{5.0.1}$$

The experimental data for these quantities is [9]:

$$\begin{aligned} f_B(exp.) = \quad & 0.200 \pm 0.04 GeV[9] \\ & \Gamma_B(exp.) = 2.8943 \times 10^{-27} GeV \end{aligned} \tag{5.0.2}$$

We can see that our results are within the error bars of experimental data for B-mesons. This validates our framrwork of BSE.

Appendix A

Appendices

A.1 Basic Inputs From Quantum Fields Theory

A.1.1 Wave equations

In quantum mechanics energy and momentum correspond to operators acting on the particle wave function. The substitutions $E = i\hbar\frac{\partial}{\partial t}$ and $\vec{P} = -i\hbar\vec{\nabla}$ lead then to the Schrodinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(x) = -\frac{\hbar^2}{2m}\vec{\nabla}^2 \tag{A.1.1}$$

We can write the energy and momentum operators in a relativistic covariant way as

$$P_\mu = -i\partial_\mu \tag{A.1.2}$$

where we have adopted the usual natural units convention $\hbar = c = 1$. The relation

$$E^2 = \vec{P}^2 + m^2 \tag{A.1.3}$$

determines the Klein-Gordon equation for a relativistic free particle:

$$(\Box^2 + m^2)\phi = 0, \Box^2 = \partial_\mu\partial_\mu \tag{A.1.4}$$

The Klein-Gordon equation is quadratic on the time derivative because relativity puts the space and time coordinates on an equal footing. Let us investigate whether an equation linear in derivatives could exist. Relativistic covariance and dimensional analysis restrict its possible form to

$$(\gamma_\mu \partial_\mu + m)\psi(x) = 0 \tag{A.1.5}$$

Notice that γ_μ should transform as a Lorentz four-vector. provided the coefficients γ_μ satisfy the algebraic relation

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \tag{A.1.6}$$

which defines the so-called Dirac algebra. Obviously the components of the four-vector γ_μ cannot simply be numbers. The three 2×2 pauli matrices satisfy $\{\sigma_i, \sigma_j\} = 2\ \delta_{ij}$. An explicit representation is given by:

$$\gamma_4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{A.1.7}$$

where, $(i = 1 - 4)$

Thus, the wave function $\psi(x)$ is a column vector with four- components in the Dirac space. The presence of the Pauli matrices strongly suggests that it contains two spin-$\frac{1}{2}$ components. A proper physical analysis of its solutions shows that the Dirac equation describes simultaneously a fermion of spin-$\frac{1}{2}$ and its own antiparticle . It turns useful to define the following combinations of gamma matrices:

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 \tag{A.1.8}$$

Some important properties are:

$$\{\gamma_\mu, \gamma_5\} = 0$$

$$(\gamma_5)^2 = \mathbf{1}$$

$$\gamma_4\gamma_5\gamma_4 = -\gamma_5$$

$$\gamma_4\gamma_\mu\gamma_4 = -\gamma_\mu$$

(A.1.9)

A.1.2 Lagrangian formalism

The Lagrangian formulation of a physical system provides a compact dynamical description and makes easier to discuss the underlying symmetries. Similarly to classical mechanics, the dynamics is encoded in the action

$$S = \int d^4x \pounds(\phi_i, \partial_\mu \phi_i) \tag{A.1.10}$$

The integration is over the four space-time coordinates to preserve relativistic invariance. The Lagrangian density L is a Lorentz-invariant functional of the fields $\phi_i(x)$ and their derivatives. The space integral $L = \int d^3x \pounds$ would correspond to the usual non-relativistic Lagrangian. The principle of stationary action requires the variation δS of the action to be zero under small fluctuations $\delta\phi_i$ of the fields. Assuming that the variations $\delta\phi_i$ are differentiable and vanish outside some bounded region of space-time (which allows an integration by parts), the condition $\delta S = 0$ determines the Euler-Lagrange equations of motion for the fields:

$$\frac{\partial \pounds}{\partial \phi_i} - \partial_\mu \left(\frac{\partial \pounds}{\partial(\partial_\mu \phi_i)}\right) = 0 \tag{A.1.11}$$

One can easily find appropriate Lagrangians to generate the Klein-Gordon and Dirac equations. The Lagrangian

$$\pounds = (\partial_\mu \phi^*)(\partial_\mu \phi) - m^2 \phi^* \phi \tag{A.1.12}$$

describes a complex scalar field without interactions. Both the field ϕ and its complex conjugate ϕ^* satisfy the Klein-Gordon equation. Thus, $\phi(x)$ describes a particle of mass m without spin and its antiparticle. Particles which are their own antiparticles (i.e. with no internal charges) have only one degree of freedom and are described through a real scalar field. The appropriate Klein-Gordon Lagrangian is then

$$\pounds = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 \tag{A.1.13}$$

The Dirac equation can be derived from the Lagrangian density

$$\pounds_D = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi \tag{A.1.14}$$

The adjoint spinor

$$\bar{\psi}(x) = \psi^{+}\gamma_4 \tag{A.1.15}$$

closes the Dirac indices. The matrix γ_4 is included to guarantee the proper behaviour under Lorentz transformations.

A.1.3 SU(N) Algebra

SU (N) is the group of $N \times N$ unitary matrices, $UU^{+} = U^{+}U = 1$, with $detU = 1$. Any SU (N) matrix can be written in the form

$$U = \exp^{iT^{a}\alpha^{a}} \tag{A.1.16}$$

where $a = 1, 2, 3, ..., N^2 - 1$ with $T^a = \frac{\tau^a}{2}$ hermitian, traceless matrices. Their commutation relations

$$[T^a, T^b] = i\epsilon_{abc}T^c \tag{A.1.17}$$

define the SU (N) algebra. The $N \times N$ matrices $\frac{\tau^a}{2}$ generate the fundamental representation of the SU (N) algebra. The basis of generators $\frac{\tau^a}{2}$ can be chosen so that the structure constants ϵ_{abc} are real and totally antisymmetric. For $N = 2$, τ^a are the usual Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.1.18}$$

which satisfy the commutation relation

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \tag{A.1.19}$$

Other useful properties are: $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$ and $Tr(\sigma_i\sigma_j) = 2\delta_{ij}$

Appendix B

Appendices

B.1 γ - Matrices

The fundamental

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \tag{B.1.1}$$

may be used to prove the following results:

$$\gamma_\mu \gamma_\nu = 4\delta_{\mu\nu} \tag{B.1.2}$$

$$\gamma_\mu \not{a} \gamma_\mu = -2\not{a} \tag{B.1.3}$$

$$\gamma_\mu \not{a}\not{b}\not{c}\not{a} \gamma_\mu = -2a.b \tag{B.1.4}$$

$$\gamma_\mu \not{a}\not{b}\not{c} \gamma_\mu = -2\not{c}\not{b}\not{a} \tag{B.1.5}$$

$$\gamma_\mu p_\mu = \not{p}, \gamma_\mu p_\mu = \gamma.p \tag{B.1.6}$$

B.2 Trace theorems

$$1.Tr(1) = 4 \tag{B.2.1}$$

$$2.Tr(\gamma_5) = 0 \tag{B.2.2}$$

$$3. Tr(odd no. of \gamma's) = 0 \tag{B.2.3}$$

$$4. Tr(\not{a}\not{b}) = 4a.b \tag{B.2.4}$$

$$5. Tr(\not{a}\not{b}\not{c}\not{d}) = 4[(a.b)(c.d) - (a.c)(b.d) + (a.d)(b.c)] \tag{B.2.5}$$

$$6. Tr(\gamma_5 \not{a}) = 0 \tag{B.2.6}$$

$$7. Tr(\gamma_5 \not{a}\not{b}) = 0 \tag{B.2.7}$$

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