Variation Theory Based GeoGebra Assisted Problem Solving Instructional Approach: Advancing Learning in Geometry

Belete Abebaw Kassa

A Dissertation Submitted to Department of Teacher Education Presented in Fulfillment of the Requirements for the Degree of the Doctor of Philosophy in Mathematics Education

Advisor: Mulugeta Atnafu (Prof)
Co-advisor: Aweke Shishigu (PhD)

Addis Ababa University
Addis Ababa, Ethiopia
May, 2026

Declaration

I, Belete Abebaw, declare that this dissertation entitled **—*Variation Theory Based GeoGebra assisted problem solving instructional Approach: Advancing Learning in Geometry*** submitted in partial fulfillment of the requirement for the Degree of the Doctor of Philosophy in Mathematics Education Submitted to Department of Teacher Education, Addis Ababa University, is my original work and all the sources that I have used or quoted have been acknowledge and referenced.

Belete Abebaw

Signature_____________________________

Date 17/04/2026

This dissertation has been submitted for examination with my approval as an advisor:

Advisor: Prof. Mulugeta Atnafu

Signature____________________

Date 17/04/2026_________

Co- Advisor: Dr. Aweke Shishigu

Signature ___________

Date 17/04/202

Dissertation Approval Sheet

This is to certify that the thesis prepared by Belete Abebaw entitled: **Variation Theory Based GeoGebra assisted problem solving instructional Approach: Advancing Learning in Geometry** and submitted in the partial fulfillment of the requirements for the degree of the Doctor of philosophy in Mathematics Education compiles with the regulation of the University and meets the accepted standards with respect to originality and quality.

Signed by the examining board committee

Advisor ‘s name	Signature	Date
Co-Advisor ‘s name	Signature	Date
Internal Examiner ‘s name	Signature	Date
Prof. Solomon Melese (PhD)		May 2, 2026
External Examiner ‘s name	Signature	Date
Chair Persons ‘Name	Signature	Date

Abstract

Variation Theory Based GeoGebra assisted problem solving instructional Approach: Advancing Learning in Geometry

Belete Abebaw

Addis Ababa University, 2026

The study aimed to examine the effect of Variation Theory Based GeoGebra assisted problem-solving instructional Approach on students' learning in Geometry. The study was conducted in Addis Ababa government secondary schools. A total of 125 students divided into 4 intact groups participated in the study. The study employed quasi-experimental research design. Three intact groups were randomly assigned as a treatment group, while one group was taken as a comparison group. Each of the three treatment groups took specific instructional approach, while the comparison group proceeded without any changes to the instructional process for all sessions. Pre and post conceptual understanding test, mathematics motivational questionnaire and problem-solving test were administered to all groups. To analyze the data and examine the differences (if any) in each group, Pearson correlation, ANCOVA and Paired samples t-test were employed. There was a significant mean difference between student's pre-test and post-test in their conceptual understanding, problem solving skills and motivation of each treatment group and no such difference observed on the control group. Furthermore, the variation theory based GeoGebra assisted problem solving instructional approaches had a large mean difference than the control group. The result also showed that Variation theory based GeoGebra assisted problem solving approach treatment had an essentially higher mean difference on student 's conceptual understanding on their problem-solving skills and motivations too. Furthermore, the variation theory based GeoGebra assisted problem solving approach had big mean difference on their motivation. When an instruction was led by variation theory based GeoGebra assisted problem solving approach, students 'conceptual understanding and their motivation significantly affect students 'problem solving skill. It was recommended that teachers give attention to using variation theory-based GeoGebra assisted geometry problem-solving approaches for students 'better understanding, motivation to learn and problem solving. Since GeoGebra assisted problem solving instructional approaches motivate students to learn geometry administrators would emphasis on launching GeoGebra software through IT labs in schools, and government officials on the educational sectors should appreciate the implementation of technology in schools. School principals and mathematics teachers would appreciate the implementation of technology in schools.

Key words: Conceptual understanding, GeoGebra, Learning Geometry, Problem-Solving Approaches and Variation theory.

Acknowledgements

I want to thank God first and foremost for making it possible for me to be at this level. I would want to express my gratitude to my advisors, Professor Mulugeta Atnafu and Dr. Aweke Shishigu. Without the constant direction, encouragement, support, and tolerance that Prof. Mulugeta Atnafu and Dr. Aweke Shishigu offered during the course of this research, this thesis would not have been able to be completed. At every stage of the investigation, they provided the researcher with useful comments. Department of Teacher Education staff members of Addis Ababa University are also acknowledged by the researcher for their continuous support in reviewing the paper, counsel, and helpful criticism. In particular, Dr. Abera Abate and Dr. Kassa Micheal have contributed significantly by offering their insights and critiques on my dissertation work. They deserve my thanks and my appreciation for their unwavering support. I would like to thank Federal Technical and Vocational Training Institute for helping to make this research project financially possible. Many thanks also go to the principals, teachers, and grade nine students of Hidasei Lideta, W/ro Kelemwore, Bulbula, and Misrak Goh Secondary schools for their kind cooperation, patience, and unreserved provision of the data.

Special thanks also go to W/ro Aynegida Awoke, my wife, for her continuous encouragement during the course of this study. I extend my thanks to my children Mekidesemariam Belete, Mebatsion Belete and Meron Belete. My thanks also extend to Dr. Teshome Feten and his family for their unlimited support and appreciation at the time of the study. My father Ato Abebaw Kassa, my mother W/ro Yideneku Shimelis and my brother Betremariam Abebaw, Yitayal Abebaw and Tena Abebaw were also contributing a continuous encouragement during the course of this study. Last but not least I want to thank my brothers and sisters who deserve special thanks for their moral support.

Table of content that

List of Figures

List of Tables

List of Acronyms and Abbreviations

VT	Variation theory
PSS	Problem Solving Skill
CIA	Conventional Instructional Approach
CU	Conceptual understanding
PSS	Problem Solving Skills
TG_1	Treatment Group 1
TG_2	Treatment Group 2
TG_3	Treatment Group 3
CG	Comparison Group
GG	GeoGebra
PS	Problem Solving

CHAPTER ONE: INTRODUCTION

1.1 Background of the Study

All educational levels, primary, secondary, and higher education, require students to take mathematics. Because mathematics fosters higher-level thinking in pupils, it is imperative that they learn it (Laurens et al., 2018; Ramya et al., 2023). Understanding the world without mathematics is becoming difficult. Mathematics helps us to explain hidden patterns in a day-to-day activity, and it is paramount to an individual's growth and to a nation's economic growth (Narayan, 2015; Sunita, 2019). Furthermore, the foundation of nation-building is mathematics since the degree of proficiency in the subject greatly influences the degree of science and technological components, which are essential to any country's progress (Jayanthi, 2019; Nitisha et al., 2018; Weng, 2017). A human with a growth mindset can be formed through mathematics. The development of human nature can be influenced by the acquisition of mathematical knowledge, which plays a meaningful role by promoting emotional intelligence and facilitating integration with indigenous knowledge and religious beliefs (Irfan, 2016; Xu, 2019).

It is important to place such a broad emphasis on mathematics education given that it is still arguably one of the most internationally oriented topics in postsecondary education. With many national leaders worried about better results in international performance measures like the Programmed for International Student Assessment (PISA) or Trends in International Mathematics and Science Study (TIMSS), this global focus paves the way for economic success (Cipriano & Cipriano, 2023). Focusing on mathematics is due to its extensive employment in computer-aided design, geometric modeling, robotics, medical imaging, computer animation, and visual presentation, in addition to being utilized in many other mathematical areas to solve challenging and complex problems (Methkal & Algani, 2022; Oktaviana & Putri, 2020).

Due to its vital role for the economical, sociological, and political wellbeing of the country's development from the starting point of modern education up to today, policymakers in Ethiopia give a vital and fundamental emphasis to the components of the mathematics curriculum at all grade levels (MOE, 2009). This is because many occupations require the ability to apply mathematical concepts. Furthermore, studying mathematics allows students to improve their conceptual understanding, problem-solving ability, mathematical creativity, and attitude toward mathematics. Because mathematical notions are closely related to one another, it follows that certain mathematical connections contain other mathematical connections, or vice versa. In order to learn and consider the links between mathematical concepts, it is crucial to have an overall perspective of mathematics. As a result, before teaching idea B, for instance, a teacher needs to introduce or focus on concept A (Hidayati et al., 2019; Simon, 2011). For example, to teach similarities of triangles, a teacher should teach about congruent angles and sides.

According to Battista (1999), as cited by Abate et al. (2022), it should be noted that there are several ways to view the significance of mathematical concepts. (i) Practically speaking, mastering these ideas enhances one's capacity to comprehend and manage situations and issues in the real world. (i) Psychologically speaking, mathematics offers a wide range of opportunities for students to build and expand the mental structures necessary for ongoing intellectual development. (iii) Mathematically speaking, knowing the fundamentals of mathematics serves as the foundation for all other mathematical ideas. Students must therefore acquire suitable mathematical abilities in addition to a strong understanding of mathematical concepts.

Mathematics demands a high level of conceptual comprehension from its students. It involves methods that could appear abstract and unapplied to daily life. In order to develop problem-solving skills and to acquire knowledge, it is important to understand the concepts of a content, which are fundamentals of learning (Ningrum et al., 2023). A theory of conceptual understanding is useful for mathematics education (Godino, 1953). Strong conceptual information is essential for the advancement of solid numerical capacities, including problem-solving, communication, association, thinking, and representation capacities. Learning results in solving scientific issues that are profoundly impacted by students' comprehension of mathematical thoughts (Amin & Mulyono, 2022). If students miss an understanding of the concepts, misconceptions among them might result from grasping the incorrect idea (Thanheiser, 2023; Yuliarni & Hidayati, 2023).

In learning mathematics, educators hold the opinion that procedural knowledge is not as significant as conceptual understanding. Unless students understand mathematical concepts, they are unable to understand and apply what they learn in their day-to-day real-life problems. When students are co-opted and integrate their knowledge into the existing real world, then it is possible to say they are developing a good understanding of the mathematical concepts. Furthermore, it helps them in order to familiarize themselves with a mathematical idea by interpreting, analyzing, and transforming it into models and forms of mathematical equations (Hussein & Csíkos, 2023; Mahendra & Rini Purwati, 2019).

Students are observed straining to reach high degrees of difficulty in learning mathematics due to the fact that they lack the focus on conceptual understanding that is needed to be more effective in mathematics (Bangalan et al., 2023). Students find their studies in mathematics to be problematic and fruitless. One of the main reasons for students' reluctance toward mathematics is their lack of understanding (Aguilar, 2021; Akhter, 2018). Students' lack of conceptual understanding in learning mathematics is basically a result of well-known weaknesses in conventional methods of teaching mathematics and negative perceptions of mathematics subjects (Busaka et al., 2022).

n learning geometry, many students had great difficulties handling basic concepts. From many basic reasons, teachers' methods of education, students' lack of proof, their lack of prior knowledge, and their weak geometric reasoning skills are some of the variables that contribute to students' struggles with learning geometry (Aziz, 2021; Chand et al., 2021).

The curriculum of mathematics has pointed to supplying students with the basic scientific aptitudes required for encouraging instruction and getting scientific concepts; creating their claim numerical considerations and problem-solving forms; using these abilities both in genuine life and within the classroom; methodically progressing their aptitudes; and carrying on mindfully. Pedagogical policies in mathematics should focus on graduating mathematical thinking students who can use mathematical language and techniques effectively both in and outside of the classroom. Teachers may have individual styles reflecting their distinct personalities and curriculums, but it’s crucial that they remain focused on their teaching objectives. It is also recommended that, before teaching classes or working in schools, teachers receive formal training (Ünal, 2017).

Ethiopia's mathematics teaching policy focuses on improving learning outcomes through the General Education Quality Improvement Program (GEQIP) and the Education Sector Development Program (ESDP VI). Key measures include standardized time allocations (e.g., 40–45 minutes per period for primary/secondary), increasing teacher content knowledge through in-service training, and promoting culturally relevant pedagogy. The Ministry of Education (MoE) emphasizes STEM literacy and digital technology to meet modern workforce demands (Tiruneh et al., 2021).

As was previously indicated, secondary school students in Ethiopia and Africa face the same worldwide mathematical issues. Research indicates that a number of factors, including students' poor attitude toward the study of mathematics, lack of learning strategies, difficulty understanding mathematical concepts, difficulty recalling previously taught material, and fast forgetting of the material, all contribute to low mathematics performance at the individual and national levels (Alemu, 2010; Tesfamicael & Ayalew, 2021; Tiruneh, Hoddinott, et al., 2021). Most of misconception on students understanding was emanated from teachers' wrong understanding which is a critical problem for teaching mathematics (Takele, 2023; Tiruneh, Sabates, et al., 2021).

To address poor learning outcomes, in 2018, Ethiopia released an educational roadmap with the 1994 education and training policy. It was developed for linking knowledge with better skills other than the development of concept acquisition, problem-solving skills, mathematical creativity, and culture in the curriculum, structure, and method of instruction with a focus on the acquisition of scientific knowledge by amending the 1994 educational policy, which is modified by the educational roadmap (MoE, 2013; Tefera et al., 2018). Furthermore, in 2023, the Ethiopian educational policy also stresses the expansion of problem-solving abilities and building of innovation among students for the development of the country (MOE, 2023).

As demonstrated by the national learning assessment series, which is administered every four years (Atherton & Outhred, 2020), there is a periodic decline in Ethiopian middle school students' mathematical achievement. According to Ethiopia's national learning evaluation, most students did not meet the minimal learning competencies of each grade cycle, which are listed under each syllabus.

Furthermore, students were faced with fulfilling the minimum learning competence at each grade as reported by the national learning assessment. The mathematics average result of students was 35.2, 33.14, and 32.6 for grade 8 according to the fifth, sixth, and seventh NLA reports, respectively. This indicated that students are unable to attain the required minimum competencies (NAE, 2016; NAE, 2020; NAE, 2022). The decline of students' achievement was not limited to middle school; rather, there was a series of declines in results in secondary school, as reported by the Ministry of Education. Only 3.9% and below scores were recorded, and mathematics was the lowest from this result, which was reported in the consecutive academic calendars in 2022 and 2023. This demonstrated that students are increasingly failing to achieve the minimum proficiency level of 50% required by the Ethiopian Education and Training Policy (MOE, 1994). The developer of the education roadmap also indicated students' learning outcomes were very low in a deteriorating trend (Tefera et al., 2018). These show common difficulties in learning mathematics.

There are different reasons that contribute to the decline of students' results in mathematics. Some of the basic factors are students' lack of motivation to learn mathematics, teachers' employment of inadequate teaching methods, and the learning environment being unappealing to students (Aguilar, J. J., 2021). Additionally, math teachers are not well suited to teach mathematics (Demssie, 2019; Mekuria et al., 2018; Walde, 2019). The instructional approaches used are a core issue in increasing students' mathematics achievement because they improve students' attitudes and mathematical achievement.

Mathematics teachers should use effective teaching strategies for students' better understanding of concepts. Teachers must inspire students and establish the necessary environments to succeed in mathematics classrooms (Mekuria et al., 2018). The limitation on teaching strategies starts from the admission of unqualified applicants, the curriculum's poor quality, and the absence of an appropriate program for ongoing professional development at the teacher education institution, which constituted the main contextual issues facing Ethiopia's teacher education program (Ayalew, 2021; Gemechu & Aba-Oli, 2024).

Variation theory as a method of teaching and learning mathematics can be a powerful approach to building understanding within a mathematics classroom. It supports pupils in utilizing their own experiences and perceiving learning from various angles, and it enables them to enhance their capacity to extrapolate or apply what they have learned to new contexts (Cheng, 2016). Using variation theory-based learning as a pedagogical design tool will help students become more knowledgeable, proficient in procedures, and able to solve problems. It will also help close the gap that exists between the planned and actual learning object objectives by providing a structured, design-based approach to teaching that focuses student attention on the critical aspects of a concept rather than mere repetition or content coverage (Mendoza & Lapinid, 2024).

At all educational levels, from primary to tertiary, there are many reasons to use variation theory in school mathematics. Through appropriate depiction of the model representation of the problem by discernment, students were taught how to solve problems correctly on triangles (Mårtensson & Ekdahl, 2021; Mendoza & Lapinid, 2024). Guiding classroom instruction with variation theory greatly increases students' performance (Jing et al., 2017a). Variation theory can improve pupils' comprehension of mathematics while simultaneously fostering their own acquisition of mathematical content, and students are better able to integrate mathematical concepts and methods outside of the classroom when they are exposed to key components of the subject (Alkalah, 2016; Mendoza & Lapinid, 2024). Therefore, problem solving, conceptual understanding, and inspiration in learning geometry of students can be fostered when guiding teaching strategies with variation theory (Baskoro, 2021; B. Kassa & Ding, 2019; Mårtensson & Ekdahl, 2021). Comparing a traditional teaching approach, the variation theory as an instructional approach can help students improve their performance and motivation for learning mathematics (Baskoro, 2021; Jing et al., 2017a; Save et al., 2018).

Problem-solving instruction, in which students receive more individualized mathematical instruction through in-person interactions with teachers and classmates, is another teaching approach that helps students who struggle in learning mathematics (Aydogdu & Kesan, 2014; Pehkonen et al., 2004). A student's poor mathematics performance in general and geometry in particular is due to low emphasis on problem-solving instruction (Cooper, 2011; Pardimin & Widodo, 2021). Problem-solving instruction is the appropriate method of mathematics teaching and learning in the classroom to improve students' learning (Ramdjid et al., 2022; Sulistyowati et al., 2017).

Therefore, while a teacher is thinking about what will be my effective teaching method, he/she must prioritize problem-solving instructional approaches (Abakah & Brijlall, 2022). An instructional approach led by problem-solving influences students' attitudes toward geometry. The basic aspect of the problem-solving teaching method is that it gives priority to students' formation of mathematical habits that include using an inquiry method to make hypotheses, motivating working independently, searching for alternative approaches and representations for problems, and supporting and explaining findings (Santos-Trigo, 2024; Schettino, 2011). Thus, a problem-solving instructional approach should be included in mathematics instruction by math teachers in the geometry class (Ahamad et al., 2018; Garba & Sidi, 2020).

The use of GeoGebra can minimize students' difficulties in learning mathematics. It can facilitate pupils' understanding of geometry. Additionally, students who use GeoGebra, which operates questions using a computer, report being happier when learning geometry since it helps them to visualize concepts (Dahal et al., 2022; Tamam & Dasari, 2021). When students learn concepts using GeoGebra, they take an increasingly active role in creating their mathematical understanding of the circle (Simbolon & Siahaan, 2021; Thapa et al., 2022).

Visualization of geometric figures and related calculations can be shown using GeoGebra, which helps students in learning mathematics in order to visualize concepts and is used to develop their mathematical reasoning and understanding level (Belgheis & Kamalludeen, 2018; Leelavardhini, 2018; Narh-Kert & Sabtiwu, 2022; Simbolon & Siahaan, 2021). Students had good scores, and they were motivated more in the geometry class while the teacher led the lesson using GeoGebra (Hosseini et al., 2022; Tamam & Dasari, 2021).

In the classroom, mathematics teachers should integrate different instructional approaches. Integrating one instructional approach with other teaching approaches will enhance students' learning in mathematics, as the saying goes, 'There is no one best teaching method’ in the pedagogy literature. It helps students to think and reason at a higher level, and it resembles a student-centered approach, which prioritizes collaboration and critical thinking (Cambridge, 2022; Patiga, n.d.; Pilani et al., 2024). In line with this, several theories have been put up independently to explain the functions and uses of variation theory and GeoGebra-assisted problem-based learning strategies in the study of mathematics (Arbain & Shukor, 2015a; Ekawati & Lin, n.d.; Hidayat et al., 2023; Jing et al., 2017a). There is a gap in linking the effect of GeoGebra with learning theories.

Investigating how variation theory-based GeoGebra-assisted problem-solving techniques affect students' motivation, problem-solving skills, and conceptual understanding in mathematics learning is crucial. The variation theory-based GeoGebra-aided problem-solving methodologies should help increase students' performance in learning mathematics from different points of view, such as problem-solving skills, motivation, and conceptual understanding. Thus, this study put its primary goal on investigating the impact of applying variation theory-based GeoGebra-assisted problem-solving approaches, variation theory-based problem-solving approaches, and GeoGebra-assisted problem-solving approaches on early high school students' problem-solving skills, conceptual understanding, and motivation toward learning geometry.

1.2 Statement of the Problem

The utilization of essential numerical operations such as division, proportions, rates, expansion, subtraction, word issue interpretation, and image utilization all through arithmetic talk can be utilized to distinguish the connections between numerical concepts. Meanwhile, the interconnected elements of mathematics include pattern recognition and analysis, logical reasoning applied to systems, and the identification and justification of the underlying links between these systems (Koji et al., 2016; Ogan & George, 2015; Pokhrel, 2023), and many students are unable to defend their methods for solving mathematical problems, and they exhibit minimal indications of conceptual knowledge in these situations.

Geometry holds an essential thought in the mathematics curriculum due to the abundance of concepts it touches. From a psychological perspective, the way abstractions from visual and spatial experiences are the focus of geometry, such as fields, patterns, measurements, and mapping, is presented. On the other hand, geometry offers methods for solving problems when it comes to, say, coordinate systems, vectors, pictures, diagrams, and transformations (Aziz, 2021; Siagian et al., 2020). Students face different challenges in understanding different geometric theorems and the properties and relations of each geometric figure, such as lines, angles, and circles, from different interactions of circles and lines. The lack of instructional tools, the complexity of the themes, and the teachers' failure to fully explain geometry concepts to their students are the main causes of the difficulty students have when learning geometry (Kpotosu et al., 2024; Silmi Juman et al., 2022; Ubi et al., 2018).

High school students need to master geometry, but it has been influenced by different factors, including learning style (LA), gender, degree of fundamental geometry skills (BGC), mathematical self-efficacy (MSE), and the teacher's teaching style or the instructional method he/she used (Alghadari et al., 2020; MIFETU et al., 2019; Parre & Marpa, 2019). Showing a low performance of students in learning geometry is basically associated with the unprofessionalism of the teachers, like using inefficient teaching methods (Abiam et al., 2016; D & Olotu, 2022). Selective use of instructional approaches other than the conventional approaches affect students' learning in geometry (Ca et al., 2024; Gambari, 2010; State & State, 2024).

In the developing world, achieving learning objectives has long been a problem for educational systems, with serious ramifications for economic growth (WB, 2022). Ethiopian mathematics learning appears to be in the same situation as the world. Due to the complexity of the issues, finding solutions will take a team effort from all parties involved. There are different parameters counted, including instructional methods, teacher competency, student and teacher attitudes, and social, economic, administrative, and policy elements (Bekele, 2007; Tibebu et al., 2021). Teachers were using authoritarian teaching methods, and pupils were not studying mathematics according to their preferred learning styles and tactics (Geche, 2009). There are weak pedagogical instructional approaches in Ethiopia that are undermining the impact of mathematics for innovation and creativity in technology (Belay et al., n.d.; Alkalah, 2016).

Even though it might be challenging to get students to conceptualize challenging and complex mathematical ideas, variation theory problem-solving techniques can keep the students interested since variation is needed in learning geometry in order to discern new features of an object of learning for which the variation theory of learning is stressed (Ekdahl, 2021; Kullberg et al., 2017). When the learning objectives are linked with the perimeter and area of rectangles, students should understand key ideas of rectangles. This can be fostered as a result of the use of problem-based tasks with patterns of variation and invariance (Kassa & Ding, 2019; Kassa et al., 2023).

In Ethiopia, teachers perceived that the ultimate goal of education is to acquire knowledge. Most teachers used traditional teaching and learning approaches due to perception. Furthermore, teachers did not practice authentic instruction, and the real-life situation in the mathematics class cannot be addressed effectively (Abate, 2023; Gizaw, 2023). Teachers have poor quality in their instruction; it may arise from their training or less attention to pedagogical knowledge (Somantri et al., 2019). The quality of mathematics education in Ethiopia is currently declining, and this is turning into a major issue for the educational system in the nation (Ahmed et al., 2020; Assefa & Rogers, 2006). It has also been known that only 4% and fewer students passed in the 2022 and 2023 academic calendars.

According to the seventh NLA report, from a mean score of 32.6 for the mathematics subject, students' mean scores on number systems, geometry, data and probability, and algebra were 33.6, 30.9, 37.1, and 32.2, respectively (NAE, 2022). This shows that students' performance on geometry is the lowest from other subfields of mathematics.

Teachers need to provide knowledge and skills that are worthwhile in order to help students create an internal motivation for learning. Put otherwise, it is imperative that students comprehend the value of the mathematical education they receive, not just in the short term but also in terms of equipping them with knowledge about the subject and other domains where mathematics finds application, such as physics, engineering, business, etc. (Atnafu, 2012). Thus, using mathematical software in the teaching-learning process has additional benefits and significance that could improve student motivation to learn and enhance mathematical comprehension.

It is also a more recent requirement, as revealed by the education roadmap's conclusions, which highlight the significance of placing a strong emphasis on technology-enhanced learning at all levels. This attempt to include new technology advancements could help solve some instructional challenges, encourage the development of conceptual knowledge and problem-solving abilities, and boost students' enthusiasm (Tefera et al., 2018).

Students often struggle with understanding geometric concepts due to the static nature of diagrams in textbooks. This difficulty is compounded by the abstract nature of many geometric concepts, which can lead students to find the subject rigorous and challenging. To bring motivation, a collaborative approach integrated with GeoGebra could improve students' motivation to learn geometry relative to other instructional methods (Bache et al., 2024). These research reports make it clear that more research is needed to determine how secondary school students' conceptual understanding, skill to solve problems, and motivation are affected by variation theory problem-based approaches and GeoGebra-assisted problem-solving approaches, which were primarily conducted separately and in an integrated way.

Therefore, observing the impact of applying integrated different teaching strategies, such as variation theory-assisted problem-solving, the GeoGebra problem-solving learning approach, and variation theory, GeoGebra-assisted problem-solving approaches are fundamental to secondary school students' mathematics learning in terms of conceptual comprehension, skills of problem-solving, and motivation in learning geometry and to develop an efficient learning model for the purpose.

1.3 Objectives of the Study

The purpose of this study was to investigate the effect of using variation theory-based GeoGebra assisted problem- solving approach on students' conceptual understanding, problem-solving skill, and motivation for studying geometry in secondary school. Therefore, the study was conducted with the following precise specific objectives:

1. To investigate whether there are significant differences in students 'conceptual understanding (CU) in mathematics across the three interventions and the control groups.
2. To investigate whether there are significant differences in students 'problem-solving skills (PS) in mathematics across the three interventions and the comparison groups.

3. To investigate whether there are significant differences in students ‘motivation (M) in mathematics across the three interventions group and the control groups.
4. To examine whether there are significance relationships between the mean gains of the variables problem-solving skills, motivation and conceptual understanding in geometry in the intervention groups
5. To analyze the contributions of motivation and conceptual understanding on problem-solving skills in geometry in the intervention groups.

1.4 Research Questions

The study attempted the following research questions based on the objectives stated above.

1. Are there significant differences in students ‘conceptual understanding (CU) in learning geometry across the three interventions and the comparison groups? How do students develop conceptual understanding in learning Geometry through variation theory based GeoGebra assisted problem-solving approaches?
2. Are there significant differences in students ‘problem-solving skills (PS) in geometry across the three interventions and the comparison groups? How do students develop problem solving skill in learning Geometry through variation theory based GeoGebra assisted problem-solving approaches?
3. Are there significant differences in students ‘motivation (M) in geometry across the three interventions and the comparison groups?
4. Are there any relationships between the mean gain of the variables problem-solving skills, motivation and conceptual understanding in geometry in the intervention groups?
5. What are the contributions of motivation and conceptual understanding on problem-solving skills in learning Geometry in the intervention groups?

1.5 Significance of the Study

Students should give great emphasis to mathematics in their academic successes. In many fields of both natural and social science, it has different applications (Benbow, 1992; Oco et al., 2023). Higher mathematics performance is correlated with students' enthusiasm for the topic, which is shown in other courses as well. Additionally, the development of reasonable thinking in all subjects emanates from students' effectiveness in mathematics (Braak et al., 2022: Leal et al., 2022).

The minimum learning competency of students in mathematics was below standards, as studied by researchers. Studies relate these problems to the selection of better instructional strategies in mathematics class. Thus, variation theory on GeoGebra problem-solving approaches in teaching mathematics will help students to improve their learning in geometry as a new learning approach.

It is anticipated that by performing this, this study will be important in the following ways:

- It provides information to the school directors, educational leaders from each level from woreda to national level, researchers, curriculum designers, and other bodies about students' conceptual knowledge, students' skills of problem solving, and motivation of the students for studying geometry.

- It gives teachers, researchers, curriculum designers, and other bodies concerned with teaching and learning geometry insight into how the variation theory problem-solving approach, GeoGebra-assisted problem-solving approaches, and the variation theory- based GeoGebra-assisted problem-solving approach affect students' learning with respect to problem-solving skills, conceptual understanding, and motivation toward learning mathematics in general and geometry in particular.

- It prompts math teachers to take corrective action on their teaching methods by considering the significant impact that GeoGebra assisted problem solving approaches, variation theory based GeoGebra assisted problem solving approaches, and GeoGebra assisted problem solving approaches have on students' conceptual understanding, problem-solving skills, and motivation to learn geometry.

- Teachers, trainers, and educators involved in teaching and training programs will find it useful to assess and prioritize their training concerning students' conceptual acquisition, problem-solving skills, mathematical creativity, and attitude toward mathematics. Above all, since research is an ongoing phenomenon that requires further investigation to find better work; this study may initiate and serve as a basis for others to conduct similar research in this area. This research could be the starting point and model for future studies in mathematics education.

1.6 Scope of the Study

In this study learning geometry was delimited to students' conceptual knowledge, skills for problem-solving, and motivation for geometry. Although mathematics is a larger field, learning geometry was restricted to grade 9 students. The research covers the geometric figures and measurements found in the grade 9 mathematics syllabus. Since this subject is covered in the secondary school mathematics curriculum, it was taken into consideration. Furthermore, multiple treatments with pretest-posttest and a nonequivalent quasi-experiment research design framed the study. Students of a government school in grade 9 from Ethiopia's Addis Ababa City Administration participated in the study.

1.7 Limitation of the Study

This investigation was constrained in some respects. First it would have been preferable if raters other than the researcher had evaluated the students' written answers to the problem. However, for various reasons, the researcher was the only one who graded the students' written answers. The teachers and the schools may affect the intervention because it is difficult to control all teachers and school related variables, even though the researcher tried to select similar schools and teachers from different perspectives, gave trainings on the instructional approaches and ways of implementation and conducting classroom observations and discussions with the teachers during implementation to ensure the research protocols were adhered to. On the other hand, the time of conducting this study was during the end of the old curriculum for which the study was undertaken and the pilot of the new curriculum, which may be a limitation.

1.8 Operational Definition of Terms

Learning: refers students 'outcome on geometry with respect to their motivation, skills on solving problems and Conceptual understanding to learn mathematics using the treatments of the study as measured by tests and motivation.

Motivation: the initiation of students towards the geometry learning like direction and intensively as measured by by Intrinsic motivation, Self- efficacy, Utility value, Test anxiety, Extrinsic motivation, Attention and Satisfaction ….

Conventional instruction: refers to the usual teaching practice in in the control group

Conceptual understanding: refers to a means of functionalizing an integrated idea about geometrical concepts to learn new ideas by connecting those ideas to what students already know as measured by two tier questions

Problem solving approach: A means of instruction which focus on identifying and determining a solution to a problem with consecutive steps by posing a problem.

Variation theory: variation theory is an instructional that enables teachers to teach by discerning learning from different perspectives.

GeoGebra assisted instructional approaches: refers to instruction or remediation presented using a GeoGebra software.

Problem solving Skill: the capacity or vital competence appeared by students in understanding, selecting approaches and understanding methodologies and show to solve a problem.

Constructivism: a learning philosophy which gives great emphasis to experience of learners to generate knowledge.

Social constructivism; is a means of a learning with valuing social interaction for creation of knowledge.

1.9 Organization of Chapters

There are five chapters in this research report. The foundation of the study was clarified within the first chapter. This chapter contains the rationale of the study, the objective of the research, and the research questions. It also includes operational definitions of terminology along with the study's importance, delimitation, and limitations.

Chapter two described the theoretical and conceptual framework related to the research questions. In this chapter, literature reviews were organized with respect to students' learning in geometry, such as skills in solving problems, motivations, and conceptual understanding through the variation theory problem-solving approach and the GeoGebra-assisted problem-solving approach.

The third chapter establishes the study's methodology and justifies the use of a concurrent embedded mixed-methods design. The instruments for data collecting, ways of analysis methods, and research ethics are all discussed.

The fourth chapter shows the findings and discussions from the study that was detailed in Chapter Three. In light of this, this chapter presents and analyzes students' conceptual knowledge, problem-solving ability, and mathematical creativity based on quantitative and qualitative student outcomes on exams measuring these three skills. The chapter also shows and discusses student attitudes toward mathematics based on quantitative attitude survey data. It also involves classroom observation discussions based on students' work and interviews with data from their work. Lastly, it gives an outline of the common discoveries from both subjective and quantitative information with regard to the inquiries about questions of the study.

Chapter five evaluates the extent to which the theoretical and methodological goals have been attained as well as the degree to which the research questions have been addressed. In order to solve the issue, the study then proceeds to a general discussion of the key discovery. The chapter ends with a summary, conclusions, suggestions, and recommendations for additional research.

CHAPTER TWO: REVIEW OF RELATED LITERATURE

Introduction

To understand previous studies about a particular area and to justify a new study, it is important to review literature and ensure it is a meaningful synopsis. When we review literature, different studies should be assessed and organized in a systematic way so as to give meaning to a particular idea. A literature review needs to provide a theoretical framework to assist the researcher (Kumar Das, 2022). It helps to make comparisons and identify similarities in theories and findings, which is different from stabilizing single ideas. To bring themes of organized review, there should be a particular focus on some facts. Rather than summarizing each theory, finding, etc. separately, it should compare and relate them. Additionally, a certain subject or focus should be used to structure the evaluation (Harvey, 2010).

The knowledge of prior researchers regarding what is known and what is unknown and untested is quite good. Effective research is predicated on existing information; thus, this process prevents duplication and offers practical and beneficial recommendations for more research (Chavan, Rajand & Pratibha, 2014; Marume et al., 2016). Reviewing different studies about a particular field can help in order to evaluate the body of knowledge that is available for the scholars with a specific academic topic to search out whether there exists a gap or not and to forward suggestions for future studies (Chigbu et al., 2023). As a result, the study will offer opinions and findings put forth by academics regarding the topic the researcher is looking into in this chapter.

As a result, the study will offer opinions and findings put forth by academics regarding the topic the researcher is looking into in this chapter. The researcher tries to perform an exhaustive analysis of pertinent reviewed works. For this study the researcher tries to search earlier findings related to how variation theory problem-solving approaches and GeoGebra-assisted problem-solving approaches affect students' skills in their problem-solving, understanding concepts in geometry, and motivation in learning geometry. The review begins with fundamental ideas that serve as the basis for the research to aid with task tracking.

2.1 Students' Learning in Mathematics

It is difficult to bring the importance of mathematics through the world unless students should be encouraged while learning mathematics, which yields better student achievement. But it is not simple to bring such better achievements smoothly. Students' results in mathematics are affected by the contribution of different factors. These factors have different contributions to mathematics achievement. Some have a positive impact; others have a negative impact. For instance, it is known that students who have a positive attitude towards mathematics can score higher on their results in mathematics (Apriliyanto et al., 2018). Different studies indicated that a student who is good in mathematics showed a better reading skill and had better problem-solving ability.

According to Kusmaryono and Wijayanti (2023), learning of mathematics may take place inside or outside the classroom. Creating a good learning atmosphere in mathematics class can enhance students' engagement in all domains of their experience. For instance, their cognitive domain increases by showing better reasoning, critical thinking, and ability to solve problems. Similarly, students' affective domain with respect to their positive beliefs about mathematics also increases. Students also develop better cooperation by developing social communication skills in mathematics learning. The ultimate goal of school mathematics is to enhance students' creative and imaginative thinking abilities (Ulusoy & Argun, 2019).

A teacher should focus on his/her students, how their learning in mathematics is taken over, and how he/she is responsible for the selection of the instructional strategies employed. Since the strategies may be affected by different mechanisms like students' engagement, students' perception of the relevance of the subject, and their habit of applying it in real-life activities (Tañola & Lomibao, 2025). A student's motivational skills also enhance their mathematics achievement. Studying students' achievements in mathematics is associated with the importance of mathematics in daily lives and formal education too (Mitchell & George, 2022).

2.2 Students Learning in Geometry

In the development of curriculum in mathematics, experts' emphasis on the instructional approaches to learning and teaching of geometry brings students' results so as to be good. There are different daily life problems that are associated with geometry. Thus, geometry has a vital role in all levels of education (J. Alex and Mammen, 2018; Silmi Juman et al., 2022). For many students, geometric concepts are not easily understood, and they give students unpleasant experiences while learning. That is why great emphasis on the curriculum is placed in the 21st century. Further, Sunzuma & Maharaj (2019) rely on students' skills of visualization, reasoning on a logical argument, and skills in solving problems and forming a proof generated when students learn geometry contextually and meaningfully.

In Ethiopia the mathematics curriculum modules incorporate the teaching of geometrical concepts. These are displayed in course readings through sayings, definitions, hypotheses, and proofs. Regularly, the learning practices comprise the recitation of definitions and equations and tackling scheduled work. Understudies are rarely energized to consider the forms in which concepts and equations are inferred. Instead, the equations are memorized with the point of applying them straightforwardly to unravel ordinary works. Inside the existing curriculum practice, one cannot anticipate a great execution from the students' work in which they get to justify the validity of given equations or to solve conceptual issues (Sebsibe & Feza, 2019).

A conceptual understanding of geometry, which is an understanding based on understanding, calls for mental creative ability since the proofs and inductions of equations are based on the adaptability and generalizability of figures or shapes. Course readings cannot visualize the energetic nature of geometrical figures on paper. As a result, students are forced to rationally explore the conceivable properties of geometrical objects without an outside way to extend understanding of the related concepts, and so students frequently come up short in creating experiences with the instructed concepts. In this way, internalizing the geometrical representations may be a psychological challenge to understudies within the pencil-and-paper medium, which makes learning geometry troublesome to numerous understudies (Denbel, 2015).

Learning geometry emphasizes examining particular representations such as virtual manipulatives, composed math conditions, and verbal clarifications, which help students develop logical concepts and create fundamental thinking (Jablonski & Ludwig, 2023; Silmi Juman et al., 2022).

Some content of high school geometry requires learning media in order to encourage students' motivation, particularly when learning about solid figures (Rohendi et al., 2025). It is possible to bring students a positive perception and to think of geometry as not a difficult subject by supporting the lesson using different software, which can have the power to motivate them and to yield them a better understanding of each concept. For example, the Geometric Sketchpad can be used to foster students' creativity in problematic situations or teaching activities (Samphantakul & Thinwiangthong, 2019).

2.2.1 Students' Conceptual Understanding in Geometry

Fundamental learning will occur while students possess good conceptual understanding skills in order to solve problems and to generate knowledge from what they have learned (Ningrum et al., 2023). According to Kakoma (2016) and Middleton and Spanias (1999), the comprehension of mathematical concepts, operations, and relations will be captured if the student is good in conceptual understanding of the lesson. In order to acquire mathematical ideas in a meaningful way, conceptual understanding of mathematical concepts is the core issue. Skills with respect to solving emanate from conceptual understanding and procedural knowledge of mathematical concepts. Conceptual understanding enables learners in mathematics to know more facts and methods in computing problems. It helps students to contextualize every mathematical idea with its importance in daily life activities.

A student may face a problem demonstrating mathematical concepts if he/she cannot recognize, tag, and generate examples of concepts; varied representation of concepts; and applying facts and definitions to compare and contrast ideas (Lord & McGee, 2001). Conceptual understanding of mathematical concepts helps students to communicate on ideas easily (Kilpatrick et al., 2001). Students who have better conceptual understanding in mathematics are enabled to make choices and apply their understanding through active engagement. According to Capraro and Joffrion (2006), in order to complete an algebra problem successfully, students must develop both procedural and conceptual understanding.

Students should develop understanding of geometry concepts to justify the relationship between each content, as the aim of learning mathematics underlies such conditions. It is a critical issue in getting a conceptual understanding of concepts in geometry. These ways of teaching the learning process are affected by the approach to geometry teaching and learning. According to Alex J. K. (2019), in day-to-day activities to solve problems, it is important to enhance conceptual understanding in geometric thoughts. Geometric objects and their image can help students to build their skills to observe, analyze, and reason by developing good conceptual understanding about the lesson. The effect of learning geometry is vital within the by and large authority of arithmetic and encourages clarity as to why geometry assumes a prevailing position within the school arithmetic educational program of numerous nations (J. Alex & Mammen, 2018). Teachers should develop a meaningful teaching approach to mathematics in general and geometry in particular in order to increase students' conceptual understanding of every concept.

A teacher develops visual and verbal pictures in order to enhance a student's conceptual understanding in teaching geometry. Thus, wise participation in the emergence of a new geometry curriculum can help students to have expressive learning of different concepts (J. Alex & Mammen, 2018). Students learning geometry is not simple. A student's habit misconception during the critical aspects of cognitive development enhances interruption in understanding concepts. The root cause for such a case is the lack of conceptual understanding in geometry concepts (Dutta & Boruah, 2025).

The classroom teacher should incorporate different methods that actively engage students. Without implementation of appropriate approaches, it is difficult for students to learn geometry by developing understanding concepts and thinking critically to make them self-directed learners (Gurmu et al., 2024). Rather than teaching students only procedures for finding the solution to a problem, it is important to measure the area to have better conceptual understanding abilities interactively and creatively. Thus, students should focus on understanding geometric concepts for enhancing their ability to solve problems related to geometry (Idrus et al., 2022).

From much mathematical content, students face a challenge or difficulty in understanding the overall language of what it has, especially on geometry. This is because students experienced low conceptual understanding and had poor strategic competence. In order to overcome the difficulties in students learning geometry on their conceptual understanding, a teacher should incorporate integrated teaching approaches through mediation, such as visual marking and supporting memorizing their prior knowledge and engaging in finding the solution for each problem in geometry (Kivkovich, 2015; Nugraheni et al., 2018).).

2.2.2 Students' Problem-Solving Skills in Geometry

Merriam-Webster and Britannica dictionaries define the concept of problem-solving as the process or act of finding a solution to a problem. Accumulating and storing previous experience from an organized mental model helps students to remember a problem from their last stored memory. This organized mental model is called a schema (Gurat, 2018). The pattern of arrangement sets tasks with basic connections to find the unknown. People's day-to-day life problems can be eliminated while students develop problem-solving skills, self-confidence, and free masterminds. This showed that when a person is good in problem-solving skills, he will be a good, reasonable thinker (Özreçberoğlu & Çağanağa, 2018). Without developing prior knowledge in learning, a student will never develop a skill for solving non-routine problem activities at a school. (Wun et al., 2017).

In the long-term and short-term educational endeavor, students' ability in problem-solving can help them succeed in their learning and schooling. But the culture of learning with skills on solving problems is not optimal because students perceive that problem-solving is observed as the most difficult thing. This feeling is also the teacher's attitude while teaching mathematics (Tambychik et al., 2010).

Teachers are expected to have a good understanding about problem-solving instructional approaches so that their students develop a process for solving problems and build an ability to identify necessary conditions and concepts for solving mathematical problems and to make an inference based on the solved problems from their own previously organized skills. Students can integrate different concepts and learn problem-solving skills when a teacher comes to class with a problem-solving instructional approach (Khatimah & Sugiman, 2019). From it crucially, different researchers are interested in conducting research on how students improve problem-solving in mathematics. Scholars like Polya put forth four steps of solving problems in mathematics. These are a clear understanding of the issue, determining the procedure, and executing the chosen technique and assessment (Jahudin & Siew, 2024). Furthermore, problem-solving approaches also contribute a positive effect to mathematics teachers' skills on solving (Ersoy, 2016).

Computing geometric problems using procedural steps is the difficulty of many students. They may answer geometric questions in incomplete detail, falsely, or totally not answer due to inability to understand the problems, formulate, or think of the solution plan. Furthermore, students did not reexamine the procedures to complete the solution to make any generalization. On the other hand, when a student understands the procedures of how the problem should be solved, they have the capacity to reexamine the geometric concepts (Ramdjid et al., 2022; Rejeki et al., 2021).

It is not only learners' task to be effective in problem-solving geometric questions; rather, organizing and developing different mechanisms of skills on how to solve problems is also a teacher's duty in the geometry class in order to encourage their students. Furthermore, teachers and teacher candidates must teach their students the solving methods. Developing such a method is not left at elementary schools but also at the secondary schools too (Osman et al., 2018; Aydogdu & Kesan, 2014).

A mathematics teacher should practice different strategies in finding the solution of geometry problems, like making a drawing to visualize the problem with respect to lines and geometric shapes; using an intelligent guessing and testing strategy that goes on until the correct answer is found; simplifying the problem that goes on until all the sub-problems are solved easily and then these separated parts are recombined for the solution of the original problem; and using known information by using the formula or correlation or relationship we know beforehand (Aydogdu & Kesan, 2014).

2.2.3 Students' Motivation in Learning Geometry

The power of reasoning, interpretation, and problem-solving skills emanates from students who are motivated highly while they are learning mathematics. It also helps us in order to examine the relationship between good academic performance and motivation in learning mathematics. Regarding this, there is a great positive impact of motivation on students' achievement (Middleton & Spanias, 2016). Affective factors in mathematics classes have an impact on students' learning behavior. The ongoing increment of students' academic achievement and the reflection of good behavior in students' learning are associated with indications of better motivation in learning mathematics (Pantziara & Philippou, 2015).

Motivation influences achievement (Hammoudi & Grira, 2023; Saadati & Celis, 2023). There is a remarkable correlation between students' motivation in learning geometry and students' results. Students who are highly motivated can respond effectively and efficiently to questions that seem more difficult than students who are passive in the geometry lesson. Furthermore, failures of students in geometry were associated with a lack of their motivation. According to Halat (2007), in middle school and secondary levels, the poor performance and perceived difficulty of geometry were a result of students' low motivation in learning mathematics in general and geometry in particular. Therefore, a student who performs better shows a better motivation than the lower performer in geometry (Asanre, 2024).

In the learning competency of mathematics in general and geometry in particular, motivation of students, the instructional approach used by teachers, and proficiency of the mathematics teacher in the content are three basic factors in students' learning competencies. The impact of motivation on students' mathematics and geometry achievement is significant (Gunaseelan & Pazhanivelu, 2016). To bring up students' motivation while they are learning geometry content, the teacher should use different mechanisms other than the conventional method of teaching (Pambudi, 2022).

The motivation of students learning geometry can be enhanced through various games that display two-dimensional concepts. This approach increases both intrinsic and extrinsic motivation by facilitating interactions among students, between students and teachers, and among teachers themselves (Mukmin et al., 2020). These games not only make the learning process more engaging but also foster a collaborative environment where students can explore geometric concept ideas clearly. When students solve problems in groups, they can demonstrate ideas in a deeper understanding, and their ability to think critically also improves.

Students' outcomes in learning and their motivation depend on the learning strategy, instructional media, and teaching material employed in the classroom. Especially in the context of geometry, transformation has not increased students' motivation due to its difficulty. Because of this, students' achievements in geometry are low. Thus, the mathematics teacher should incorporate teaching material that fosters the student's motivation in learning geometry (Febriana et al., 2018).

2.3 Instructional Approaches in Teaching Mathematics

2.3.1 Variation Theory-based Instructional Approaches

The demand for teaching and learning processes in mathematics throughout the globe is ever-increasing from time to time. In order to search for marginal and effective instructional methods from the previously known instructions, different researchers are struggling to announce a new approach to enhance students' learning in mathematics too. The classroom teachers are responsible for addressing the single concepts from different points of understanding, and hence they are often perplexed, realizing students end up with different understandings (Kullberg et al., 2024; Mendoza & Lapinid, 2024).

Ference Marton and his colleagues developed a new approach to learning known as the variation theory of learning. (Ference Marton & Booth, 1997). This theory of learning emanated from the concept of phenomenographic research. Searching the critical aspect of the object of learning is the baseline for the founder of the theory. This critical aspect is essential and highly important for success in learning mathematics (Cheng, 2016; Marton & Pang, 2013; Mendoza & Lapinid, 2024).

A theory of learning and involvement associated with variation theory shows how a student might come to see or connect a given concept in a wonderful way. There are core views of points of a given concept at which a student gives attention in variation theory that learners must, at the same time, be mindful of and center on to get a good experience about the phenomenon in a definite way (Kullberg et al., 2017). Learners can build their memorizing ability of mathematical concepts when they are experiencing a shifting of certain basic angles invariantly till a focal awareness toward the critical aspect of object learning is captured (Marton & Throb, 2013).

Does variation theory emphasize the "what" and the "how" angles of learning? The subject matter or the content to be learned by the learner is termed as the object of learning, and this is the main substance in the existence of teaching and learning. On the other hand, there is an interaction of the content with the student, which determines the quality of the learning known as the indirect object of learning and created by the learner. (Ference Marton & Booth, 1997).

Variation theory emphasized two aspects in mathematics learning and teaching: procedural variation and conceptual variation (Lo, 2012; Pogorelova, 2023). Conceptual variation gives multiple perspectives of the concepts of a specific point for students. On the other hand, procedural variation gives the formation of concepts with process, which helps students to experience their problem-solving skills by manipulating varying problems with each step.

The core issue of the variation theory of learning is that mathematical learning should be meaningful while a learner is experiencing the critical aspects of an object of learning (Ference Marton & Booth, 1997; Kullberg et al., 2017). In variation theory, a description to students of what to learn is given by the object of learning, which is basic for learning to occur. This means that learning is observing as an alteration of students' means of understanding the content. Students should consider the object of learning in an observable manner. An excellent way of acting emanated from the great way of seeing. Thus, it is possible to conclude that in a learning atmosphere some ways of observation are more powerful and more effective than the others (Åkerlind, 2015).

The founder of variation theory, Marton, and his colleagues list four patterns of variation for which the concepts of 'critical aspect' will be discerned. These are generalization, contrast, fusion, and separation (Marton et al., 2004).).

- **Contrast** – There should be an experience in order to compare a new experience about something.
- **Generalization** – To separate immaterial features and to create a full understanding of something experiencing in various form is important.
- **Separation-** in arrange to encounter a given perspective of something, and in arrange to isolated this viewpoint from other perspectives, it must shift whereas

other angles stay invariant

- **Fusion** – when there are several critical aspects that the learner needs to consider at the same time, they must be simultaneously experiencing multiple critical aspects of the object of learning.

While implementing the variation theory of learning and teaching with variation, students learning in mathematics should improve while the teacher critically observes the student’s ability to experience and explore the critical aspect of learning in a serious way (Berie Kassa & Ding, 2019). To have a good understanding of the critical aspect of learning objects, students must experience possible alternatives (variations). This is the baseline of variation theory in order to learn a particular concept by creating a certain situation with variation and invariants (Marton et al., 2004), and developing and patterning such situations can bring differences in learning (Kullberg et al., 2017).

2.3.1.1 Variation Theory-based Instruction and Student’s Conceptual Understanding in Learning Geometry

Students can understand concepts better; the teacher's method focuses on how what they're learning by employing and using variation theory in math lessons can greatly help connect to their own experiences, helping them build a meaningful way of thinking that makes learning relevant and useful. Research by Mun. Lingo Lo. and Marton (2012) shows that students learn more effectively when they can relate the lesson to their everyday lives. This link helps them understand better and respond more effectively to challenges. A key part of variation theory is the pertinence structure, which connects students' experiences to what they're learning (Mendoza & Lapinid, 2024). It helps clarify how the learning process affects their ideas and understanding of topics like geometry. For learning to be successful, students need to focus on the important parts of the problem they are working on.

Looking at shapes repeatedly helps students understand geometry better. During the lesson, students were surprised by how plane shapes are formed. Huang and Leung (2005) explain that procedural variation helps learners build knowledge gradually, improve their skill in solving problems, and form well-organized knowledge. Students also give a great emphasis on the main features of what they are learning. To notice something, they have to see it in the background. A study by Jing, Tarmizi, Bakar, et al. (2017b) supports the effectiveness of variation theory in improving student achievement.

Variation theory significantly enhances better conceptual understanding in learning solid geometry content in Ethiopian government secondary schools at grade ten. Students' understanding of geometry concepts can foster while a teacher is using potential teaching approaches like the variation theory of teaching. A learner's motivation, their determination, and scoring better results are a result of a teacher's implementation of variation theory instructional approaches (Yeshanew et al., 2024). When the variation theory of instruction is implemented in the class, most students can show good progress in easily identifying basic concepts in computing the perimeter and area of a rectangle in consecutive activities. This was happening when a student identified each critical aspect of the content significantly (Berie, G. Kassa & Ding, 2019).

2.3.1.2 Variation Theory-based Instruction and Student's Problem-Solving Skill in Geometry

Solving a problem in mathematics learning is considered the basic component for which students should build their thinking abilities. To address everyday challenges, students need to enhance their problem-solving skills, as noted by Simbolon and Siahaan (2021). When dealing with a geometry problem, similar to mathematical word problems, the person solving it must thoroughly read, understand, and interpret the information provided. They then need to follow the correct steps in a meaningful way to arrive at the accurate solution. After solving the problem, students are expected to explain their answers clearly and logically (Guven & Cabakcor, 2013).

For students eager to explore new elements of a subject, teachers should implement learning approaches based on the theory of variation and highlight the significance of variation in the learning process. Using variation theory in mathematics education can serve as a new method to boost higher-order thinking skills. This approach also shows that variation plays a vital role in learning mathematics (Yuan et al., 2021). Problem-based activities have the potential to integrate patterns of variation and invariant that can help students to know basic aspects of the content in learning geometry, like perimeter and area, with their relations (Kassa & Ding, 2019).

Students can better understand how to present a problem when they are taught to differentiate between examples and non-examples of various trigonometric concepts, such as angle of elevation and angle of depression. This indicates that using variation carefully had a potential for bringing the development of new ideas in mathematical thinking and strong problem-solving abilities (Mendoza & Lapinid, 2024).

2.3.1.3 Variation Theory-based Instruction and Student's Motivation in Learning Geometry

It is possible to assess a student's motivation in mathematics from various perspectives. For instance, Fiorella et al. (2021) proposed a practical measure to evaluate the motivation of secondary students in learning mathematics. This measure has strong validity and reliability and shows that motivation varies within a multidimensional context. These dimensions include students' intrinsic motivation, utility value, self-regulation of test anxiety, and self-efficacy. Similarly, extrinsic motivation involves students engaging in academic activities for external reasons, such as earning a degree in mathematics to improve job prospects in fields like actuary science or engineering (Miller et al., 1988). Therefore, components such as a student's intrinsic motivation, utility value, self-efficacy, test anxiety, attention, extrinsic motivation, and a student's satisfaction are all factors that help in measuring motivation in different ways.

Recently, it has been found that using a variation theory teaching approach can help boost students' motivation in learning geometry. Variation theory takes a different approach to motivation compared to social motivational theories. It focuses on the connection between the learning content and how students perceive its relevance. When integrating everyday experience with the topic of each mathematics lesson, students can understand better and develop a positive attitude toward the subject (Lo, 2012).

Students can show higher motivation in terms of satisfaction and attention when an instruction is led by the variation theory of learning. Students' motivation was positively affected by the variation theory-assisted instruction strategy significantly (Yeshanew et al., 2024). This result conflicts with the findings of 2010. According to Wong et al., students in Hong Kong (an urban area) indicated a negative response to variation theory-based instruction.

Variation theory can positively influence student motivation when it is integrated with other instructional approaches. Combining the theory of variation in mathematics learning with guided inquiry-based approaches effectively improves self-belief of students in learning geometry, in particular solid figures, thereby helping to overcome educational challenges faced by students in Ethiopia (Yeshanew et al., 2024). According to Ahlquist and Gynther (2020), variation theory supports students in engaging with tasks in a meaningful and constructive way. When an instruction is delivered by variation theory in teaching mathematical concepts, it encourages student motivation in the classroom and enhances their problem-solving abilities (Jing, Tarmizi, Bakar, et al., 2017a).

Variation theory grasped a distinctive idea of inspiration from a hypothesis of social motivation. The central idea of the theory is the linking of the object of learning, which enables students to attend the lesson significantly by establishing structure in learning mathematics. Observing the day-to-day activity with the object of learning can help students to have better understanding favorable to the content (Lo, 2012).

Developing a good layout that connects learners' experiences with the object of learning has a potential influence on their thinking and seeing ability of a concept. The critical features of the object of learning emanated from students' focus of attention on their learning (Lo, 2012; Mendoza & Lapinid, 2024). Discerning the main idea from the background helps students to be aware of the overall concept of the lesson.

2.3.2 GeoGebra Assisted instructional Approach

Recently, the quality of instructional approaches in mathematics depends on the application of technologies and innovations. It influences the quality of the learning environment and helps the teacher to increase his/her thinking and imagination abilities about the content. The engagement of students in technology-oriented instruction helps them regulate student-centered learning. To maximize students' mathematics learning outcomes, mathematics technological instruction should be employed (Arbain & Shukor, 2015b). In mathematics learning, different web-based applications can be employed. A lot of studies were conducted on learning geometry with the help of software. In learning geometry in primary and secondary school, a teacher can deliver the lesson by visualizing geometric figures. It helps students to understand geometry concepts easily. Furthermore, different research showed that teachers who use dynamic geometric software while teaching secondary-level mathematics develop good investigation and consolidation ability in teaching geometry (Rohendi et al., 2025; Tamam & Dasari, 2021).

Basically, integrating a lesson with geometric software brings an energetic environment to the class by creating dynamic learning for the learners. (Bache et al., 2024). The teacher should identify different computer programs to be utilized in mathematics class with adequate information on how to access the software for him and his students too.

Whereas utilizing the GeoGebra computer program in educating geometry, the theoretical geometric objects can be visualized and controlled rapidly, precisely, and proficiently (Sunzuma, 2023). The GeoGebra program is a free computer program that can be easily downloaded on a personal computer from its official website; that is why it is the foremost suggested program.

2.3.2.1 GeoGebra Assisted Instruction and Students Conceptual Understanding in Geometry

According to Gurmu et al. (2024), conceptual understanding of students in geometry class can be enhanced when the teacher supports his/her instruction with GeoGebra. The integration of GeoGebra as instructional technology in the class significantly improved students' understanding of the concepts in mathematics generally and geometric concepts particularly. Using GeoGebra yields different positive impacts on learning geometry (Tamam & Dasari, 2021). Students' and teachers' abilities of proofing, reasoning, and solving problems in mathematics can be improved by using GeoGebra wisely.

GeoGebra can improve students' skills in creative thinking by linking the geometry idea with the algebraic one by discerning it with a three-dimensional view, which is more graspable than a two-dimensional view. According to Wassie and Zergaw (2018), the Pythagorean theorem is easily understood by the learner when the lesson is supported by GeoGebra by demonstrating the triangles in an automated way. A student's attention will be grasped differently while visualizing geometric figures displayed by GeoGebra, and it creates a driving force to clarify the hypothesis (Bhagat & Chang, 2015).

Herceg and Herceg (2010) found that students learning numerical activity can be diminished when a lesson should be consolidated with the help of computer-based learning with better understanding of the concepts. According to them, students who were assisted by GeoGebra showed better understanding than those who did not apply it. It is highly recommended that utilizing GeoGebra can comfort students to use their time effectively. Students' mathematical understanding improved by using dynamic programs, especially in geometry content (Erhan; 2013).

Rohendi et al. (2025), in their findings, showed that students' ability to understand concepts with procedures increased when a teacher used GeoGebra. Compared to students who used GeoGebra, who received a conventional approach, they develop better conceptual information. In any case, both conceptual and procedural information is critical for understudies to pick up a way better understanding in their scholarly accomplishments. In any case, both procedural and conceptual information is critical for learners to pick up a way better understanding in their scholarly accomplishments. It can be said that GeoGebra made a difference for understudies within the accomplishments and understanding of mathematics (Gebremeskel, 2025).

The use of GeoGebra appears to have a higher impact on the increment in students' conceptual and procedural information and their accomplishment. This result is caused by the truth that GeoGebra can give simple steps for understudies to secure concepts and strategies in understanding an issue (Batiibwe, 2024; Rittle-Johnson et al., 2009).

2.3.2.2 GeoGebra assisted Instruction and Students Problem-Solving Skills in Geometry

A student's skill of solving mathematics problems can be improved by using GeoGebra software in the class. Many students are interested in developing and improving their ability to solve problems since it is important to practicing reasonable thinking (Jaberi & Gheith, 2016).

Students who studied geometry lessons without GeoGebra showed less problem-solving skill than those who were supported with GeoGebra-assisted problem-solving learning approaches (Septian et al., 2020). According to Schaver (2019), students' problem-solving skills increase when the lesson is delivered by GeoGebra. Using GeoGebra is not only fostering high school students' skills in solving mathematics problems but also helps to develop 21st-century skills. Furthermore, to bring better problem-solving skills to students, a teacher can integrate his lesson with GeoGebra technology (Rosyidi et al., 2024).

According to Sofyan et al. (2021), if two curves are given, calculating the area between them can be easily manipulated using GeoGebra. Determination of the volume of the rotated object is also generated using this software. Students' different abilities in learning mathematics, like skills of problem-solving, reasoning ability, and skills of proving mathematical argumentation, can be enhanced using GeoGebra (Murni et al., 2017).

Students' skills of solving problems and conceptual understanding can be improved by exploring using GeoGebra in an interactive and winning way. It can simplify abstract ideas, making them more accessible and enhancing critical thinking. Furthermore, critical thinking ability generated from using GeoGebra promotes deeper understanding of trigonometric ratios in particular and bridges gaps in mathematical proficiency in general with better results and engagement. This showed that students with positive perceptions about GeoGebra and their academic performance were highly correlated. It helps also to implement a student-centered approach in the classroom. In general, using GeoGebra in school mathematics showed a powerful approach to student learning by simplifying challenged topics in an easier way (Trejo, 2024).

Integrating GeoGebra in learning geometry is more valuable in improving students' skills in solving problems than the conventional model (Nugroho et al., 2025). When an instruction was supported by GeoGebra, students' deduction level, which involves understanding and creating deductive proofs, differentiating between definitions and theorems, and grasping the logical relationships between geometric concepts, also increased. Similarly, a student's rigor skill, which involves understanding how mathematical systems are established, using all types of proofs, and comprehending the impact of axioms on a geometric system, including the study of non-Euclidean geometries, is significantly enhanced (Ansah & Kabutey, 2022).

2.3.2.3 GeoGebra assisted Instruction and Student’s Motivation in Learning Geometry

Inquire about the findings, which suggest that innovation can serve as a powerful motivational tool. As students' confidence grew when both GeoGebra and learning recordings were used to enhance their learning process, this was particularly beneficial for students with lower ability levels. Technology functioned as a scaffold, helping learners to reach their zone of proximal development (Palincsar, 1998). This result is supported by the study conducted by Dogan (2011), which found that computer-based activities promoted higher-order thinking skills and had a positive impact on students' motivation to learn. Technology acted as a scaffold, which enabled learners to reach their zone of proximal development (Palincsar, 1998). This finding is supported by Dogan's (2011) study, whereby it was observed that computer-based activities encouraged higher-order thinking skills and had a positive effect on motivating students toward learning.

Students can easily understand the nature of circles by using GeoGebra, which can play a part in filling up the gap by visualizing an increment and decrement of their radius. A review of writing also appears that utilizing GeoGebra has an effect on students' understanding of geometry. A study conducted by Dogan (2011) showed students' motivation will increase while learning geometry is supported by GeoGebra and contributes to better achievements gradually.

When students were asked how the software affected them, they had many positive things to say, such as it made them more engaged in the learning and enabled them to think at higher levels. Furthermore, a study by Shadaan & Eu (2013) on form six students in a Malaysian secondary school discovered that using Geometer's Sketchpad software had positive effects on students' achievement and attitude toward mathematics.

The use of GeoGebra in learning has received a positive response from most teachers at any education level. As a result, it is one of the foremost suggested programs connected as an imaginative way of instructing science supported by technology (Zakaria & Lee, 2012). In line with these discoveries, another investigation, moreover, found that numerous understudies have positive discernments of utilizing the GeoGebra program, improving students' learning outcomes (Shadaan & Eu, 2013). Students were more motivated in learning geometry while the teacher was utilizing GeoGebra software since it promotes their understanding of the taught concepts (Sudihartinih & Purniati, 2019). Not only that, but it also enhances students' mathematical communication skills on geometry material (Kusumah et al., 2020).

2.3.3 Problem Solving Instructional Approach

One of the logical aptitudes that students must have and fulfill is problem-solving. Problem-solving is exceptionally near to mathematical characteristics. In order to reach a logical conclusion based on facts and various principles, a student should develop good problem-solving skills. Problem-solving activities must be done by students; if they do not do the activity of thinking while learning, then what they get is just memorization, and they do not understand the core or concept of the material that has been learned. With the existence of problem-solving activities when learning, students will arrive at the correct conclusions about the material being studied because it has gone through a logical thinking process when learning (Manalu et al., 2018).

Some research results on problem-solving skills and student attitudes toward mathematics: stretch-related activities made a more positive contribution to the logical mathematical-thinking skill, academic achievement, and problem-solving ability of students than the educational traditional teaching activities; this contribution was especially significant for the mathematical relationship factor (Awofala, 2014).

Strong positive mentality and confidence among mathematics education students, leading to students feeling good, thinking hard, and actively participating in their own mathematics learning and students' learning of the geometry through problem-solving approaches (In'am & Hajar, 2017). However, many teachers and students assume that the process of teaching mathematical problem solving is a very complicated activity to do. Problem solving is one of the important mathematical skills that every student must have as a skill for better achievements beyond schooling. The difficulties in solving problems are due to the lack of skills developed while learning mathematics.

2.3.3.1 Krulik and Rudnik Models of Problem-Solving Aproaches in Mathematics

At the end of the decade of the 80s, a paradigm shift in the study of mathematics was initiated by the National Council of Teachers of Mathematics. Problem-solving and reasoning become the main objectives in school mathematics (Hekimoglu & Sloan, 2005). In the world, including America, in 1989, with the introduction of Curriculum and

Evaluation Standards for School Mathematics, great emphasis was given to problem-solving skills.

Concurring with Polya (Carson, 2007), the issue of solving problems can be exercised with the help of four steps, to be specific: (1) understanding of the problem (see), (2) planning, (3) doing based on the plan, and (4) checking. For detailed problem-solving skills, running the four steps is the base for good understanding of the concepts. Along with the steps above, another step of problem-solving has also been proposed, namely reading, analyzing, exploration, planning/implementation, and verification (Goos et al., 2002).

Meanwhile, in 1995, Krulik & Rudnik introduced the five stages of problem solving as a heuristic. A heuristic is a step in getting things done without having to do them sequentially (Kusdinar et al., 2017). The steps are: (A) Read and Think: Identify the facts, identify the questions, visualize the situation, explain the settings, and determine the next action. (B) Explore and Plan: Organize information; find out if there is information appropriate/necessary; discover whether there's data that's not required; draw/illustrate the demonstration of the issue; as well as make charts, tables, or pictures. (C) Select a Strategy: Find/create a pattern, work backwards, try and do simulations or experiments, simplify or expand, create a list of sequential logical deductions, or divide or categorize the problem to become a simple matter. (D) Find and Answer: predicting, using numeracy, using algebra capabilities, using geometric capabilities, and using a calculator if necessary; (E) Reflect and Extend: recheck the answers, determine alternative solutions, develop answers to other situations, develop a response (generalization or conceptualization), and discuss the answers, as well as create a variety of problems that are the origin of the problem.

2.3.3.2 Problem-Solving Instruction and Students Learning in Geometry

There are different benefits of the problem-solving approach for students and teachers. It contributes to students' having a significantly greater ability to simplify problems in mathematics. Problem-solving approaches also help teachers to run an active learning interaction due to students' attention to the lesson (Khatimah & Sugiman, 2019). If the classroom instruction is trivial, the ability of students in solving geometry is difficult in learning mathematics, which is observed in their answers, which are incomplete in detail and incorrect did not answer, so that they tended to be unable to understand the problem, formulate or think of the solution plans, and were unable to solve the problem according to the plan. In addition, students could not reexamine the procedure and the complete results (Ramdjid et al., 2022).

Problem-solving instructional methods have shown a statistically significant positive effect on students' problem-solving and creativity abilities. The approach also establishes strong positive connections, especially between critical thinking and problem-solving, critical thinking and logical reasoning, logical reasoning and problem-solving, and logical reasoning and creativity. Additionally, creativity itself has been found to have a significant positive influence on problem-solving (Tursynkulova et al., 2023). According to Ali et al. (2010), the use of the problem-solving instructional approach enhanced the achievement of the students in mathematics in general and geometry in particular. Employing problem-solving methods in teaching mathematical concepts like sets, information handling, geometry, etc. has a great impact in the classroom for active learning interaction.

Critical concepts in mathematics can be easily understood by students when the lesson is delivered by the problem-solving approach. Knowing the methods can be best instructed through problem-fathoming tasks or exercises that lock in students in considering the critical concepts and abilities they got to learn (Albay, 2019). Students learning in geometry can be improved by using problem-solving instructional strategy. Problem-solving instruction is not only enhancing students' academic achievement, but also it contributes to students showing good motivation and attitude towards geometry.

Teachers are recommended to incorporate problem-solving strategies in teaching geometry, as it improves the academic achievement of students, which helps to decrease the failure of students in geometry in particular and mathematics in general (Garba & Sidi, 2020).

2.3.4 Integrated Instructional Approaches and Students Learning in Geometry

Integrated instructional approaches in learning geometry can enhance the successful implementation of geometry Education presents a promising avenue for educational advancement, offering an interactive, engaging, and effective approach to learning that aligns with the needs and interests of today's digital-native learners. In order to shape future learning in geometry, it is difficult to take into account to have a paramount educational paradigm without the effective alignment of advanced educational technology in geometry (Dildabayeva & Zhaydakbayeva, 2024). Integration of different teaching and learning approaches is powerful in enhancing students' overall engagement and its entire component in learning data handling (Tesfaw et al., 2024).

Using the combined learning strategy can significantly enhance students' performance in mathematics, particularly in geometry, and help them retain the material more effectively. Additionally, there is a notable difference in the achievement scores between male and female students who were taught math using the blended learning approach compared to those taught with traditional methods. This study contributes meaningfully to the field of mathematics education (Egara, 2024). Students can exhibit a big mean score on geometry when they were instructed by mixed instructional strategies than those who were taught using conventional method (Voráčová, 2024).

As technology advances rapidly, the methods of teaching and learning are also changing in similar ways. Technology has transformed how people interact, communicate, study, and conduct research (Sarah & Batiibwe, 2024; Shadaan & Eu, 2013). Students are becoming more involved in class, moving from passive to active learning, with teachers using technology to support their teaching and it increases student’s ability of solving problem and their critical thinking too (Yurniwati & Soleh, 2020). Similarly, Clough et al. (2013) discussed the use of technology in learning from a constructivist viewpoint. They suggest that computers can be used as tools to help develop higher-level thinking abilities.

Hohenwarter and Fuchs (2005) argued that GeoGebra is a powerful tool for mathematics education because: 1) it supports demonstration and visualization, 2) it has the ability to create graphics, 2D and 3D shapes, and more, and 3) knowledge and understanding grow over time with the help of GeoGebra.

Students can explore and visualize circles when they are taught geometry by integrating GeoGebra, which makes geometry concepts easier to understand and has a significant impact. Dogan (2011) found that GeoGebra positively influenced students' learning and achievement, as well as increased their motivation. Shadaan and Eu (2013) also showed student performance improved from time to time when the lesson was delivered by mixed instruction using dynamic geometric software.

To minimize those challenges of understanding abstract concepts, a mathematics teacher who teaches geometry ought to design an integrated geometry lesson with technology which improves students' learning. When an integrated technology–rich environment is created, students can investigate, solve, and explain geometrical concepts in different forms (Sunzuma, 2023). Schools should encircle mixed teaching instructions as an integral part of their pedagogical concern. It boosts students' understanding in overall academic performance in geometry with confidence (Baidoo & Luneta, 2024).

2.4 Theoretical Framework

The theoretical framework of this study is based on Variation theory and social constructivist learning theory. Variation theory allows students to become active in receiving and generating concepts by focusing on how differences in experiences or exposure to phenomena lead to different understandings and allowing them to discern critical aspects, understand deeper patterns, and apply concepts across different contexts (Baskoro, 2021 ; Kullberg et al., 2024). On the other hand, social constructivist learning theory allows students to construct knowledge while making constructs and analyzing figures, images, and diagrams collectively (Hanggara et al., 2023)**.**

2.4.1 Variation Theory

Variation theory starts by highlighting how individuals become captured by concepts when new issues are introduced, either through differences from the existing environment or through variations in the internal parts of a concept (internal horizon) (Cheng, 2016). When analyzing variation, there are some types that appear obvious on the surface and others that emphasize important details.

The main idea in variation theory is that the object of learning. The object of learning is the specific knowledge or capability that students are intended to gain by experiencing specific aspects of a phenomenon or concept. This shows that without taking the object of learning into account, teaching and learning should not be considered. According to Marton & Pang (2013), students are expected to develop skills or abilities which are specific insights of the object of learning. This object is divided into two types: the direct object of learning, which refers to the content to be learned, and the indirect object of learning, which refer to the purpose or outcome of the learning. Furthermore, from the viewed point of object of learning, the object of learning is termed as the intended, the enacted, and the lived object of learning (Häggström, 2008).

The intended object of learning refers to what teachers aim to have students learn, based on their understanding of what students should be able to do with the subject matter in the classroom. This is a second-order perspective, based on the teacher's intentions.

The enacted object of learning describes how teachers implement their intentions during classroom instruction to make the learning objective achievable. According to Marton and Pang, (2013), variation theory emphasizes that critical features can only be noticed when students experience variation in their learning environment. This type of learning focuses on what changes and what remains constant in the learning space. These changes occur through interactions between students and teachers during lessons. Language plays an essential role in this process. The enacted object of learning provides opportunities for learning, which is why it is also referred to as the space of variation or the space of learning. This type of learning focuses on the possibilities of learning through experienced variation, not just the potential for learning in an environment.

The lived object of learning refers to what students actually experience and learn in the classroom. This is often compared with the enacted object of learning to determine which learning possibilities are taken advantage of by students. In research, the question "What was learned?" is answered from the learners' perspectives.

Variations can help in identifying a phenomenon, but there is also variation in the ways individuals perceive or become aware of a phenomenon. These variations in awareness are used in phonomyography studies to form categories of description. It has been found that there are only a limited number of ways people become aware of a phenomenon (Cheng, 2016; Kullberg et al., 2024). According to the theory, learning happens when a person becomes aware of a phenomenon in a new and different way conceptually and procedurally. The conceptual and procedural variation are explained below.

1) Conceptual Variation

This type of variation involves examining a concept from different perspectives, which helps students gain a deeper understanding. Teachers provide examples of a concept in different contexts or representations. For instance, the number three could be understood as a quantity, or as a collection of different items, sounds, movements, or arrangements, or as a number related to two and four. These varied experiences contribute to a better understanding of the mathematical structure of the concept (Jing et al., 2017a).

In order to investigate the effect of Variation theory and mixed instruction on students' learning geometry in general and students' conceptual understanding in particular, the theory of conceptual variation was used.

2) Procedural Variation

Procedural variation is a teaching method that helps students grasp mathematical concepts by using sets of problems. Sun (2011) classified these problem sets into three phases. The first phase involves changing one condition of a problem to help students identify both changing and constant relationships, known as one problem with multiple changes (OPMC). The second phase involves exploring several methods to solve the same problem, known as one problem with multiple solutions (OPMS). Finally, students should use a single approach in order to solve different problems, which are termed as multiple problems with one solution (MPOS).

Using both conceptual and procedural variation offers learners a 'space of relations' in which they can generalize, leading to a more comprehensive understanding of mathematical concepts, as noted by Sun. In this study, the focus is specifically on the use of problems involving procedural variation (Jacques, 2018).

Therefore, in this research, conceptual variation is applied for teaching concepts in a variety of ways using GeoGebra-assisted instruction; procedural variation is applied for solving problems using GeoGebra-assisted instruction.

2.4.2 Constructivism Learning Theory

In education, the concept of constructivism indicates a huge idea with respect to how the teacher teaches and how he/she understands to lead his/her students, which is enormous. The contribution of constructivism is basically for focusing the instruction on learner-centered learning and it helps students to acquire knowledge by constructing their meaning

(Bada & Olusegun, 2015 ; Prakash Chand, 2023).

Constructivism is an approach to learning that holds that individuals effectively build or make their own knowledge, which is decided by the experiences of the learners. This shows that Constructivism could be a logic of learning established on the introducing of what learners understand, by reflecting on their encounters, we build our claim understanding of the world we live in (Allen, 2022 ; McCray, 2007).

In the twenty-first century there is a growing shift of education from teacher-centered to student-centered by facilitating student knowledge building. Constructivism turned the nature of education from filling one ‘s mind to creating knowledge products by students themselves other than concentrating on root memorization and forcing for examination (Golder& Bengal,2018). Hence social and radical constructivists are related to this study and are discussed as follows:

A. Social constructivism

Social constructivism theory emphasizes on that students can generate knowledge with integration and sharing of ideas among with their classmates (Akpan et al., 2020). Social constructivism sees each learner as a special person with interesting needs and foundations. The learner is additionally seen as complex and multidimensional. Social constructivism not as it were recognizing the uniqueness and complexity of the learner, but really energizes, utilizes and rewards it as a fundamentally portion of the learning prepare (Amna Saleem et al., 2021).

The center thought of social constructivism in science is that scientific substances are social builds that exist in ethicalness of social hones (Rytilä, 2021). Subject interaction in the environment can construct the human mind. The subjects may emanate from the physical world, culture and value of the society or any artifacts (Rahman, 2024). The background and culture of the learner affect the truth which is emphasized by social constructivist. Historical improvements and symbol frameworks, such as language, rationale, and numerical frameworks, are acquired by the learner as a part of a specific culture which can be learned in learner life (Amna Saleem et al., 2021 ; Palincsar, 1998).

Without the social interaction with other more learned individuals, it is outlandish to procure social meaning of vital image frameworks and learn how to utilize them. Youthful children create their considering capacities by connection with other children, grown-ups and the physical world. From the social constructivist perspective, it is in this way imperative to require under consideration the foundation and culture of the learner all through the learning prepare, as this foundation too makes a difference to shape the information and truth that the learner makes, finds and achieves within the learning prepare (Miia Rannikmäe, Jack Holbrook, 2016; Nithideechaiwarachok & Chano, 2024).

In this study, social constructivism will be applied by promoting student autonomy and initiative, and by incorporating discussions even during lectures. The teacher will encourage this by asking open-ended questions and allowing sufficient time for students to respond. Social constructivism was applied by providing the socio-culturalframeworkwithinwhichlearnersconstructmeaningthroughinteractionandshared experiences, enhancing the pedagogical effectiveness of variation theory's and GeoGebra Which is serving as a tool to facilitate group exploration, visualization, and problem-solving and focus on presenting critical aspects of geometric concepts with varied examples. This means that the teachers designed lessons using variation theory to foster deeper understanding by creating scaffolding and collaborative environments where students learn from each other.

B. Radical Constructivism

Radical constructivism is a means of creating one ‘s own experiential reality in a subjective approach which is not a representation of objective truth given by the teacher (Glasersfeld, 1996). This may be occurring from the observer's point of view, not criticizing the other. This theory emphasizes that the individuals are the last resort for creating meaning and developing knowledge by experience. Hence taking a cognitive or mental point of view is a radical constructivist aspect. In spite of the fact that social interaction is seen as a critical setting for learning, the center is on the coming about reorganization of person cognition (Glasersfeld, 1996 ; Haryadi et al., 2016 ; Liang, 2019).

Under radical constructivism, knowledge can be developed within a continuous development of individual experience which does not depend on others' realities. It is an approach to understanding how knowledge is formed, which suggests that individuals create knowledge actively and continually based on their personal experiences and values.

According to this view, knowledge is meaningful and valid within the context of an individual's own experiences, rather than being measured against an external or objective reality (Cheli, 2018). In radical constructivism, there are no set of principles to guide a learner to have a certain knowledge; rather it is an instrument for discerning about knowing with personal experience (Thompson, 2021).

In this study the researcher was trained as a participant teacher to focus on students' active participation and they should facilitate those students who create their meaning on the concepts other than passive receivers of facts. In order to apply this, the teacher should invite students to interpret their own meaning based on their past experience with the help of GeoGebra by discerning the critical aspect of geometric ideas keeping variation theory as a tool.

2.5 Gap from the Literature Review

There is a lot of research conducted on the effect of variation theory on mathematics achievement, the role of computer-assisted learning on students' achievement, and variation theory on problem-solving skills. For example, Jing et al. (2017b) examine the effect of Variation Theory Based Strategy (VTBS) on students' algebraic achievement and motivation of learning among students in Malaysia.

Kassa and Ding (2019) also point out the usefulness of the variation theory as a theoretical framework in guiding the researcher to design and implement problem-based tasks to improve students' learning in mathematics. In order to answer basic questions (1) to what extent does the theoretical framework help to design the instruction of problem-based tasks? (2) To what extent does the theoretical framework help to analyze the implementation of such design in the classroom?

Thente (2019) in his research project has shown that there can be a link between working with problem-solving tasks in mathematics and motivation. The pupils who have taken part in the project have in different ways experienced motivation, something which suggests that working with problem-solving tasks can have a positive effect on pupils' motivation. In his study he didn’t show what happens if he supports the instruction with computer-aided instruction.

In any of the above researchers' studies, teaching with a variation theory-based GeoGebra-assisted problem-solving approach with students learning in geometry was not conducted. Similarly, there were no studies on the effect of student conceptual understanding and motivation on their problem-solving skills on mixed instruction of variation theory-based GeoGebra-assisted problem-solving approaches on problem-solving approaches in grade 9 students learning in geometry and measurement with the support of GeoGebra. Thus, in this study, the researcher tried to investigate the impact of Variation theory and integrated instruction on students' conceptual understanding, motivation and problem-solving skills with grade 9 students learning in geometry.

2.6 Conceptual Framework

In this study the Conceptual framework should demonstrate an understanding of what variable influences what. Variation theory based GeoGebra assisted problem solving approaches may affect students ‘problem-solving skill, conceptual understanding and their motivation. Similarly, Variation theory-based problem- solving approach affects student ‘s problem-solving skill, conceptual understanding and their motivation. On the other hand, GeoGebra assisted problem solving approach also affects student ‘s problem-solving skill, conceptual understanding and their motivation.

In general, the following diagram represents the model of the conceptual frame work.

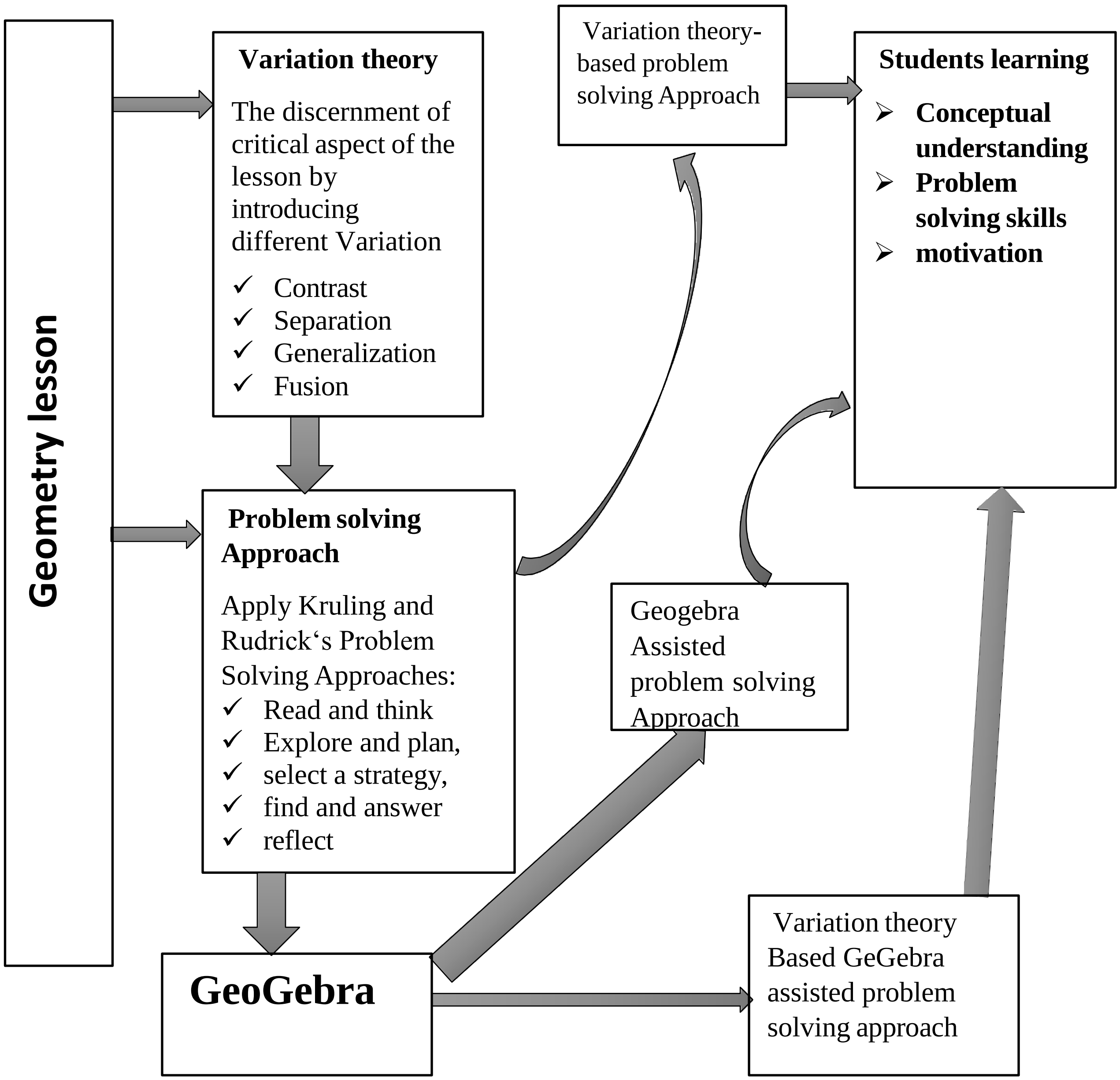


__Figure 2.1__: Conceptual Framework for the study

CHAPTER THREE: RESEARCH METHODOLOGY

Introduction

The purpose of this study is to explore the effect of a variation theory-based GeoGebra-assisted problem-solving approach on advancing learning in geometry for 9th-grade students learning geometry. This chapter presents the philosophical view guiding this research, inquiries about the plan and strategies utilized to conduct the study with the utilization of the conceptual system indicated above the effects of the instructional approaches of variation theory-based GeoGebra-assisted problem-solving approach, variation theory problem-solving approaches, and GeoGebra problem-solving approach. on students' conceptual understanding, problem-solving skills and motivation in their learning of geometry. A quasi-experimental investigation plan as well as a case study plan in a pragmatism paradigm underpinned was utilized in this study. A concurrent embedded design of blended strategies (objective approach, subjective subjective) was utilized.

3.1 Philosophical Orientation of the Study

In order to adjust both objective and subjective strategy of data collection and examination, the study's plan was guided by the theoretical presumptions of pragmatism philosophy, which emphasizes positivist and interpretive standards. Where pragmatism research paradigm seems to be the most appropriate in social science inquiry, the exploratory and explanatory nature of research in education involves the application of both quantitative and qualitative methodologies. This permits the accomplishment of investigative goals within the most effective and commonsense way, as the single use of one technique or logic may as it was delivered fractional knowledge. Based on addressed perspectives, pragmatist philosophers see social reality as a multidimensional one which requires analysis from varied paradigms or methodologies. The Pragmatist philosophy therefore seems to be one which adequately captures the multiple perspectives of social science for interpretation and inference making (Fauzia Tanko, 2015).

The philosophical preface of this study is positivism and interpretivism constructivism. The conception of positivism is following the judges see that as it were real information picked up through perception the faculties counting estimation is reliable after the treatment of the two bunch in this suppose about Positivism limits the critic to information collection and restatement in an objective way which implies the information collection and restatement will be grounded on the particular destinations expressed. Positivism discharges the use of subjective experience and no attention on valuing research participants' experience at the time of data collection. The researcher does not interact with the respondent's experience during the study, rather stays objective (Park et al., 2020).

Furthermore, on the qualitative part of the study constructivism was strengthening. That's because the conception those scholars construct knowledge on computer-supported instruction needed through their intelligence, gestures and relations with the classroom and the intervention too. According to constructivism, reality is a mortal construct; therefore, reality is always private. Interpretivism or Constructivism approach is one of the leading philosophical approaches that makes a difference analyst in understanding the complexities and variety of phenomena especially in this 21st century where innovative considerations are undertaking and innovations marking novelty is genuinely looked for through our investigate in education (Adom et al., 2016). Constructivism relates to colorful propositions, including literalism and pragmatism. Unlike positivism, constructivism rejects that scientific styles can induce or corroborate knowledge.

In general, positivism and constructivism are both used in this exploration as long as the researcher uses a mixed system. That means positivist leads to experimental design (quantitative) and constructivist leads with the qualitative interpretation of scholars' learning. Thus, pragmatism is the best paradigm that guided this study. With this perspective, the researcher was able to select the methodologies, strategies, and procedures for this study that best met the research questions.

In support of this, Creswell (2007) contends that people who have a pragmatism worldview utilized a variety of information collection methods to best reply to inquiries about questions. Both objective and subjective information sources were emphasized in the esteem of conducting inquiries about that specifically addressing the research's commonsense suggestions Logic does not accept absolute solidarity, and blended strategies analysts see an assortment of ways to information collecting and examination instead of sticking to a single strategy (objective or subjective).

3.2 Research Method

In order to run this, consider the blended inquiry that was utilized. A mixed investigative strategy may be a strategy for collecting, analyzing, and blending both quantitative and qualitative strategies in a single ponder or an arrangement of ponders to understand an investigative issue (Memon & Syed, 2017). The essential assumption is that the uses of both quantitative and qualitative strategies in combination give a much better understanding of the issue and address it than either strategy by itself. For this study, both qualitative and quantitative approaches were used in order to explore students' learning in geometry in line with the level of understanding of concepts within a full investigation in an objective and subjective view. The in-depth understanding includes looking at concepts, terms, and factors and developing definitions, declarations, speculations, and hypotheses for clarity and coherence, basically scrutinizing their consistent relations and distinguishing presumptions and suggestions.

The finding from subjective information makes a difference to bolster or fight the finding of the quantitative information so as to have solid proof. Supporting the quantitative information gotten from the arithmetic conceptual understanding test and survey on inspiration can evaluate understudies' learning, but it can't help understand the level of explaining a concept and their feeling To support this, the information should be backed by qualitative data obtained from semi-structured interviews, and the course observation checklist and problem-solving questions will be used to provide strong evidence for the conclusions through triangulation. Additionally, subjective information approximates the real classroom, hones understudies, and instructor discernment towards the intercession that the analysts collected through classroom observation and interviews. In general, the flow of data collection, analysis and interpretation was according to the following framework, which describes how mixed approaches were implemented.

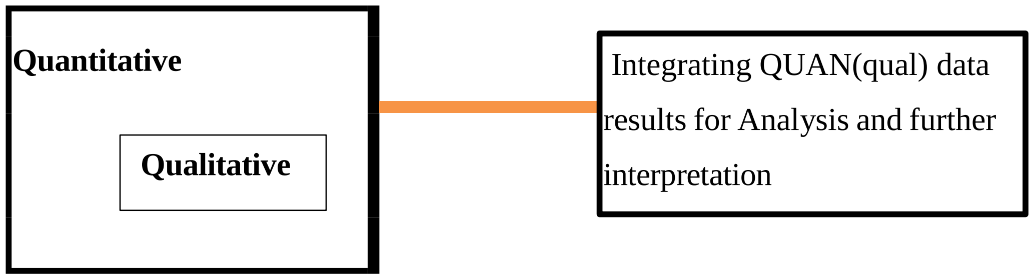


Figure 3.1: Mixed Embedded research designs adapted from Creswell and Plano Clark, 2007

3.3 Research Design

It is known that in order to investigate and conduct instructive investigations, it is troublesome to consider genuine tests. Randomizing is difficult as long as there are variables that cannot be totally controlled. Due to this fact an alternative research method termed a "quasi-experiment" emerged. For this investigation, a nonequivalent pretest-posttest control group semi-tested investigation was used, and different treatments were utilized.

A nonequivalent quasi-experimental design was used to manipulate the pretest and posttest results of respondents who are not randomly arranged in a group (Best & Kahn, 2006). For this study, students were assigned by the school officials at the beginning of the year according to their own criteria without random allocation of their section. According to Vernon and Gage (1965), a quasi-experimental research design will be implemented when there is a difficulty to manage the experimental stimuli perfectly. Thus, a nonequivalent controls group quasi-experiment was used for this study.

Three intervention groups and one comparison group were given a pretest and posttest. The intervention groups were given instruction with variation theory-based GeoGebra-assisted problem-solving instructional approaches and variation theory-based problem-solving instructions and GeoGebra-assisted problem-solving instructional approaches, and the comparison group continued as usual (conventional instruction).

__Table 3.1:__ classification of Groups for the intervention

Groups	Pre-test	Treatment	Post test
Treatment group1 (TG1)	T_1	X_1	T_2
Treatment group2 (TG2)	T_1	X_2	T_2
Treatment group3 (TG3)	T_1	X_3	T_2
Comparison Group (CG)	T_1		T_2

To generate an in-depth understanding of students' skills in solving problems and the level of understanding of geometric concepts, an instrumental case study research design was implemented to support the results of data collected quantitatively. According to Shorten and Smith (2017), when a researcher is interested in organizing a bounded idea with a clear insight about the issue, an instrumental case study will be used. This helps the researcher to get more relevant information about the study.

Mathematics teachers who had been teaching grade 9 in the academic year with almost the same experience led the treatments for the groups. A training about the aim of the research and how to apply variation theory on the GeoGebra-assisted problem-solving approach (treatment group 1), the variation-theory-based problem-solving approach (treatment group 2), and the GeoGebra-assisted problem-solving approach (treatment group 3) was given.

3.4 Sampling Techniques and Participants

Grade 9 students of the Addis Ababa administration were the population of the study. The reason behind selecting this grade is that secondary education starts from grade nine, and this level serves as a foundation for the next grade levels. The Addis Ababa City Administration was chosen since it is amazingly troublesome to recognize four schools in Ethiopia with respect to time and cost.

Four schools were selected using purposive sampling after selecting the sub-city by lottery method for the intervention group (Hidasei Lideta, W/ro Kelemwork, and Bulbula secondary school) and comparison group (Misrak Goh secondary school). This is because, according to Louis Cohen and Morrison (2007), the researcher should have to choose similar groups as much as feasible, even when differences between groups inevitably occur in nonequivalent quasi-experimental study designs. In order to prevent information contamination during intervention and data collection, multiple schools are used.

Next, using simple random sampling (lottery method), the four schools were divided into treatment and comparison groups. The sampled sections of Hidasei Lideta Secondary School, W/ro Kelemwork Secondary School, Bulbul Bulbula Secondary School, and Misrak Goh Secondary School were randomly assigned to intervention group 1, intervention group 2, intervention group 3, and the comparison group using a lottery method before administering the instruments as a pretest, respectively.

One section of ninth-grade students and one math teacher from each of the four schools took part in the study. Teachers that were involved in the study should teach grade nine mathematics in the academic year. Moreover, they should have almost similar qualifications and experience in teaching grade 9 mathematic.

The intervention covers all the three topics of geometry under unit five of grade nine textbooks. Regular polygons, further on congruency and similarity, circles, and measurement are in the unit. The syllabus states that the unit must be finished in 36 periods. As a result, the intervention took roughly seven weeks and one day (5 periods per week, 45 minutes each). The reason for the selection of this unit is that geometry is a very important concept in many fields. They are utilized broadly in arithmetic as well as in real life.

3.5 Variables

In this study there are three independent variables and three dependent variables were manipulated for this study. The dependent variables are conceptual understanding, problem-solving skills and motivation. The independent variables of the study are variation theory-based GeoGebra-assisted instructional approaches, variation theory-based problem-solving approaches and GeoGebra-assisted problem-solving approach.

Table 3.2: *Variables of the Study*

Variable	Type	Nature
Variation theory based GeoGebra assisted Problem Solving approaches	Independent variable	Categorical
Variation theory-based Problem-Solving approaches	Independent variable	Categorical
GeoGebra assisted Problem Solving approaches	Independent variable	Categorical
Geometry Conceptual understanding	Dependent variable	Continuous
Geometry Problem Solving Skills	Dependent variable	Continuous
Motivation to learn geometry	Dependent variable	Continuous

3.6 Data Gathering Tools

One of the most useful stages in an inquiry about study is information collection that empowers the analyst to discover answers to investigative questions. Data collection is the method of collecting information pointing to picking up bits of knowledge with respect to the investigated subject. There are diverse sorts of information and distinctive information collection strategies accordingly. However, it may be challenging for analysts to choose the foremost appropriate information collection instrument based on the sort of information that's utilized within the research (Taherdoost, 2021).

The researcher used several data collection methods to address the research topics outlined in unit one for this project. The data gathering tools of this research were geometry conceptual understanding test, open-ended geometry problem-solving questions, and learning motivation questionnaire on geometry, semi-structured interview and classroom observation.

3.6.1 Geometry Conceptual Understanding Test

The researcher developed ten two-tier tests (pre-test and post-test). The two-tier test is a highly important tool that is widely used in education. According to Sıbıç et al. (2022), two-tier assessments are utilized for several learning objectives, including misconception. The responses to the tests and their corresponding reasons were administered before and after intervention to obtain information on geometry conceptual understanding. To understand the conceptual understanding of students, multiple representations of tasks, like two-tier tests, play a vital role (Chand et al., 2021).

As indicated in Appendix-5, after arranging four multiple-choice questions, there is an equivalent reasoning response choice of the correct answer for the first tier. This helps students to answer the correct response with the appropriate reason when they fully understand the concept of the question. The questions were adopted from the grade-8 regional exam of Addis Ababa and the national exam of grade-10. The mathematics teacher of the selected school validated the accuracy of both tests of the study.

3.6.2 Geometry Problem Solving Skill Test

The development of abilities in solving problems in our day-to-day activities is associated with one's problem-solving skills in mathematics. Hence, problem-solving skill tests are very crucial to students' academic development (Yilmaz, 2007). The assessment of core skills in a real-life context and the development of problem-solving skill tests should be critically considered.

For this study, five open-ended pre- and post-problem-solving ability test items were delivered to students to help identify and evaluate students' problem-solving skills. Furthermore, the problem-solving experiences in daily life are typically open-ended (Yu et al., 2015). Based on the syllabus of grade nine on the unit containing geometry, the problem-solving questions were organized. The pre-test questions were adopted from the Primary Leaving Entrance Exam of grade eight, and the post-test was from the grade-10 national exam by making the multiple choices open-ended, which was given before 2011 E.C. for five consecutive years. Furthermore, the accuracy of the question was also validated by the teacher who participated in the study.

3.6.3 Motivation Questionnaire in Learning Geometry (MQLG)

The motivation for learning mathematics is a fundamental issue in foreseeing the performance of secondary school students. Students who are intrinsically or externally driven to learn mathematics typically perform better than those who are not. In any case, an appropriately designed instrument on motivation questionnaire in mathematics should matter to assess effectively (Zakariya & Massimiliano, 2021).

It is possible to determine a student's level of motivation in learning geometry by developing or adopting a motivational questionnaire. These questionnaires help to understand the classroom condition associated with learners' motivation and its effect on their academic achievement too. For this study, the questionnaire was given two times, before and after the treatment.

According to Wigfield et al. (2016), assessing student motivation in learning mathematics at secondary school is a core task from other assessments because this stage is a turning point of beliefs and perceptions that determine future learning of mathematics. For this study, the motivational questionnaires on learning mathematics were responded to from 1 to 5. Weighted 1 = Strongly Disagree to 5 = Strongly Agree. Some of the items were adopted from Fiorella et al. (2021), which contains 19 items; others are developed and validated by the researcher.

3.6.4 Interview Questions

For an effective and investigative result, the interview plan or guide ought to be decided by the interview's arrangement, the sort of investigative questions, and the reason for the study, as well as the study's destinations. It is additionally basic for the interviewer to permit the respondents to talk openly and at their own pace, whereas advertising-related comments and testing questions are essential in order to conduct effective interviews in a relaxed environment. The analyst ought to also, beyond any doubt, make the members feel at ease and unthreatened amid the interview preparation. data collection with successful interviews is iterative, which guarantees the analyst goes back to the information they have collected as often as possible to guarantee the data is exact and useful (Kwesi, 2023).

To examine students' learning in geometry, a semi-structured interview was conducted to raise the validity and credibility of the study at which the students gained from the interventions. Interviews were also conducted with teachers to triangulate and have a better understanding of how the experimental procedure is perceived by students and teachers. Based on their results scored on the level of understanding the concepts of geometry and their problem-solving skill, a total of 18 students and 6 individuals from each treatment group were interviewed. Two students from the top scorer, middle scorer, and bottom of the scores of each treatment group participated. Ten to twenty minutes were spent with each student. To do so, 20 structured interviews for teachers and 18 questions were employed. The researcher participates in collecting data from these interviews.

3.6.5 Classroom Observation Checklist

To examine the desired implementation of the intervention for each group of this study, the researchers were attaining a classroom observation with an appropriate checklist to collect well-organized, in-depth data. According to Bruns et al. (2016), without observing the real interaction of students and teachers in the learning-teaching process, it is difficult to generalize about students' learning. The influence of the instruction on learners could be understood from classroom observations. When investigating mathematics instruction, classroom observation is vital. Bostic et al. (2021), in their study, indicate that for each scientific achievement of students, the instruction delivered in the classroom matters.

For five weeks, three observations per week each were attended, one for each intervention group, with an observation checklist prepared by the researcher. The checklists were developed based on variation theory-based GeoGebra-assisted problem-solving approaches. Variation theory problem-solving approaches and GeoGebra-assisted problem-solving approaches. A checklist containing 14 items with a ‘yes’ or ‘no’ response and respondent comments about the overall learning on geometry by the end of the checklist was used. To handle data that are out of the checklist, a writing pad was used to write the observed phenomenon. The classroom observations were also conducted after actual implementation of the treatment.

3.7 Tools Validity

Validity with respect to content and face for the instrument was checked using expert opinion from mathematics teachers, PhD students (researcher classmates), and scholars in different fields. Instruments of data collection like problem-solving test and conceptual understanding on learning geometry were given to mathematics teacher and my PhD classmates to check the consistency of items with the designed objectives of the textbook. Then valuable recommendations and corrections were done accordingly. The researcher invited an Amharic language expert and psychology professionals for translating questionnaires on students' learning of geometry. And necessary comments were included in the final version. After the development of all tools, a pilot study was used in order to see the reliability of items.

3.8 Pilot Study and Reliability of Instrument

The research tools must be profoundly solid to guarantee reliable results. The reliability of inquiry about tools appears how reliable and solid the instrument is in measuring the factors under a study (Saputra, 2025). The degree to which independent administration of the same instrument consistently yields the same results under comparable settings is referred to as reliability (DeVos, 2002).

The reliability difficulty and discrimination level of each test item was conducted to see instruments practicability by piloting at a selected school other than the study groups on the same administration of 40 students of grade nine students. Motivational questionnaire internal consistency was estimated by Cronbach coefficient alpha and Kuder-Richardson test will be employed for the tests as shown in table 3.3.

Item Analysis

It is possible to take an analysis on the examination of student's pilot result of each item to get information on the consistency and validity of each item given. This helps the researcher skills in test item construction and identifying specific areas of the construct about the issue to be measured which lead a big clarity on the study (Date et al., 2019). Analyzing the items also strengthens the availability of questions for future study. It helps for modification of the items to achieve the ultimate goals (Namdeo & Sahoo, 2016).

Pilot study of conceptual understanding and problem-solving skill question results were employed for analyzing the item. In order to modify and improve the quality of an item, the researcher must check the difficulty of an item and its discriminating power (Matlock-Hetzel, 1997).

Discrimination index is the capacity of the item to discriminate the higher achiever from the lower achiever. Items difficulty index calculated by subtracting the number of students in the higher (33% of the upper class) from the lower group (33% of the lower class) then divided by the number of students. If an item's discrimination index counts 0.35 or less, it is subject to rejection or modification. On the other hand, a value of 0.35 and above indicates that the items had good discrimination power. If the discrimination index is above 0.35, then the item had good discrimination power of students from higher achiever to lower achiever (Mahjabeen et al., 2018).

On the other hand, Item difficulty is the portion of students that properly answer an item. Items difficulty index calculated by adding the number of students who answer correctly divided by the total number of students. If an item is difficult its index counts 0.3 or less is and subject to reject or modified it. On the other hand, a value of 0.7 and above indicates that the items are too easy and should be improved. According to Mahjabeen et al. (2018), the optimal value for difficulty index is between 0.3 and 0.7.

Table 3.3*: Difficulty index of the pilot study conceptual understanding*

	Difficulty index		**Discrimination Index**	
ITEM	**pre-test**	**post test**	**pre-test**	**post test**
1	0.4875	0.525	0.425	0.6
2	0.525	0.575	0.4	0.525
3	0.4875	0.5	0.45	0.575
4	0.5125	0.525	0.475	0.5
5	0.475	0.525	0.4	0.525
6	0.525	0.475	0.6	0.4
7	0.5125	0.475	0.55	0.5
8	0.5375	0.4875	0.45	0.525
9	0.4375	0.4625	0.525	0.6
10	0.4875	0.4625	0.475	0.45

Table 3.3 indicated that the item difficulty for all conceptual understanding questions for both the pretest and posttest items was between 0.4 and 0.6. This shows that the items level of difficulty was optimal and acceptable for use. Similarly, the discrimination index of conceptual understanding question lies between 0.4 and 0, which shows that the items had good discrimination power. Validation of item with respect to item analysis about the item difficulty level and the power of the item to discriminate the lower achiever to the higher achiever is mandatory and most of the time the range is lying between 0.4-0.6 (Mahjabeen et al., 2018; Yakout et al., 2024).

__Table 3.4:__ Difficult index and discrimination index of Problem-Solving Skill

Item	**Difficulty index**		**Discrimination Index**	
	pre-test	**post test**	**pre-test**	**post test**
1	0.45	0.4875	0.475	0.45
2	0.4625	0.55	0.425	0.425
3	0.5	0.4375	0.5	0.5
4	0.475	0.4375	0.525	0.525
5	0.45	0.4875	0.45	0.475

Table 3.4 Indicated that the item difficulty for all problem-solving skill items for both the pretest and posttest was observed between 0.4and 0.6. This shows that the items level of difficulty was optimal and acceptable for use. Similarly, the discrimination index of conceptual understanding items lies between 0.4 and 0 which show that the items had good discrimination power. Every scholar uses different ranges but for most of scholars agreed on the range of validation of item with respect to item analysis about the item difficulty level and the power of the item to discriminate the lower achiever to the higher achiever is mandatory and most of the time the range is lying between 0.4-0.6 (Wanna, 2023; Yakout et al., 2024).

Table 3.5*: Reliability of instruments on the pilot study*

Variables	N of Items	Cronbach's Alpha	Kuder Richardson-20 (KR-20)
CU- pre	10		0.816
CU-post	5		0.935
PSS-Pre	10		0.712
PSS-Post	5		0.803
Intrinsic motivation	5	0.974	
Self-efficacy	5	0.912	
Utility value	5	0.929	
Test Anxiety	5	0.907	
Extrinsic motivation	5	0.883	
Attention	5	0.930	
Satisfaction	5	0.918	
Over all Motivation	35	0.922	

Using SPSS version 20, the coefficient of Cronbach alpha was obtained for each subscale and overall motivation of the pilot study to validate the consistency of mathematics motivational questionnaires. The internal reliability of an instrument is estimated by the Alpha Cronbach value (Taber, 2018; Tavakol & Dennick, 2011). As indicated in Table 3.8, the reliability coefficients for the Intrinsic motivation, Self-efficacy, Utility value, Test Anxiety, Extrinsic motivation, Attention, Satisfaction and Overall Motivation scale were 0.974, 0.912, 0.929, 0.907, 0.883, 0.930, 0.918 and 0.922 respectively, in the pilot phase. It shows that Cronbach’s alpha coefficient suggested that the test was internally consistent. After that, the test was administered to the actual information sources.

The reliability of conceptual understanding and problem-solving skills was tested by Kuder-Richardson Formula 20, which is a well-known assessment tool for estimating the reliability of items that are recorded (scored) as correct and incorrect (Ntumi et al., 2023). Most of the time in the event that the esteem of KR20 is 0.7 and over at that point the thing is said to be dependable. From the over information Pre-CU, Post-CU, Pre-PSS and Post-PSSS includes a KR20 value of 0.816, 0.935, 0.712 and 0.803 respectively It shows that the items are reliable and suggested to use for this study.

In order to confirm the reliability of qualitative tools items, persistent observations, triangulation, referential adequacy, peer or advisor debriefing, and member checks were also used.

The final version of the instrument was administered after checking that all the necessary steps had been passed Student's motivation on learning geometry, their ability on understanding concepts and their problem-solving skills were administered for each group before and after the treatments.

3.9 Data Collections Procedure

Collection of information about a study is a means of gathering perceptions or measurements on certain issues Whether the researcher is performing research for governmental or academic purposes, data collection allows gaining first-hand knowledge and original insights into the research problem.

Before the main data (data used for the study) piloting on the instruments was done on a school which is different from the study area. Then after, an appropriate lesson was designed for the three experimental groups in the selected schools and sections. In order to have the desired data training was given to the teachers who teach the intervention groups about the implementations of the interventions as designed. Before starting the intervention, a pre-test was given to the four groups. Then interventions were implemented in the schools for 7 weeks and 1 day. At the time of the interventions, qualitative data were collected using interview and classroom observation checklists.

A post-test results were generated from a test given to students on problem-solving skills and understanding of concepts while learning geometry with the assistance of their mathematics teacher of the selected section for this study. Some questions pointed by students were handled by the assistance of the data collectors since they were well oriented about the overall data collection processes and how to respond to the questions raised by students. The teachers clearly explained the instructions of the tools and the confidentiality of their information. On the same day respondents were answered the questionnaire and their responses were coded with the test. An interview was also taken after administering tests with carful coding of the respondents.

Four groups with three intervention groups and one comparison group in this study were participated. First, one section for each intervention from different schools was assigned for variation theory-based GeoGebra-assisted problem-solving approaches, variation theory-based problem-solving approaches, GeoGebra-based problem-solving approaches and one section for comparison group which received conventional instructional teaching.

Second, Teachers in intervention groups received training on how to implement the instructional approach they were assigned to perform in their class. The researcher conducted the training, which lasted for six days and four hours per day. A brief overview of each instructional approach was included in the training manual of this study. The training covered how teachers use variation theory-based GeoGebra-assisted problem-solving approaches, variation theory-based problem-solving approaches and GeoGebra-based problem-solving approaches.

Third, before the intervention, students' conceptual understanding, problem-solving ability, and mathematical creativity, as well as their attitude toward mathematics, were assessed. The interventions began after these tasks were completed. The school's regular class schedule was used during the intervention because there was no need to change their regular school schedules. All students in the four groups were given a post-test about mathematics conceptual understanding, problem-solving ability, mathematical creativity, and attitude toward mathematics at the end of the interventions. Furthermore, the lesson was observed during the intervention, interviews were conducted before and after intervention, and focused group discussions were conducted after the intervention.

Table 3.6: *Summary of the treatments and data collection process*

Group	Pre-test	Training	Trainees	Treatment	Post Test
CG	Yes	No	No		Yes
TG1	Yes	Yes (Variation Theory Based GeoGebra Assisted Problem solving Approaches)	Teacher	Variation Theory Based GeoGebra Assisted Problem solving Approaches	Yes
TG2	Ye s	Yes (Variation Theory Based Problem-solving Approaches)	Teacher	Variation Theory Based Problem-solving Approaches	Yes
TG3	Yes	Yes (G e o G e b r a Assisted Problem solving Approaches)	Teacher	GeoGebra Assisted Problem solving Approaches	Yes

3.10 Analysis Methods

For this study, Statistical Package for Social Sciences (SPSS) software version 20 was used to analyze data statistically using appropriate data analysis methodologies and by assimilating different ideas for qualitative information acquired from various sources.

3.10.1 Quantitative Data Analysis

Before and after the intervention, all the teachers of the intervention groups and comparison group were made a code record of all the students who took the pretest and posttest of motivation and motivational scales questionnaire, problem-solving skill test and conceptual understanding test. The score of students' results was recorded corresponding to their code.

In order to analyze students' conceptual understanding, a classification category was adopted and modified from Coştu et al. (2007), as shown below. The test was scored from both the content and the reasoning side to analyze the students' CU in learning Geometry. Thus, the highest score that a student would achieve using the criteria was 20 (10 content and 10 reasoning).

Table 3.7*: Criteria for analyzing the two-tier items of CU*

Categories’		
Content	**Reasoning**	**Allotted Mark**
True response	True reason	2
True response	False reason	1
False response	True reason	1
False response	False reason	0

Source: Adopted and modified from Coştu et al. (2007)

In addition to the fixed reasoning option listed on the second tier, students are given freedom to write their own reasoning if they believe the reason for the content is not stated. Therefore, students' answers were also analyzed to characterize their conceptions based on pre- and post-test data.

For scoring students' written PSS test solutions, adapted scoring guidelines for mathematical problem-solving abilities were utilized from Charles et al. (1991). According to these scholars, three PSS Holistic indicators are included in the scoring guidelines.

Table 3.8: *Holistic Scale*

Score	Reason for scoring
2	Students who was answered with complete step and correct result
1	Given for students gave a partially correct answer, or task
0	Given for those who did not address task, unresponsive, unrelated or inappropriate.

Once a respondent's demographic data with ID number was recorded, then each pre-test and post-test result was recorded. And data cleaning for missing data was performed before beginning the analysis of the instruments. That means data were deleted for those respondents who were not present either on the pre-test or post-test and students who were carelessly responding to the instruments. Thus, a new sheet with original data for each group was obtained for analysis. That was utilized for both expressive and inferential measurable examination.

Descriptive statistics such as skewness and Kurtosis were used for each group to ensure that parametric statistics assumptions were not violated. The parametric statistics' assumptions were not violated by the descriptive statistics. Parametric tests such as one-way ANOVA, Pearson correlation analysis and multiple regressions were applied depending on the nature of the data. Furthermore, due to covariates such as students' Pre-PSS, Pre-CU and Pre-motivation and its subscales scores, ANCOVA was employed to conduct an inferential statistic.

The information collected during the classroom observation was parallelly analyzed in order to pick up a common understanding of the different angles on variation theory-based GeoGebra-assisted problem-solving approaches, variation theory-based problem-solving approaches by collecting similar themes which give similar meanings and percentage reflection of classroom observation checklist and GeoGebra-based problem-solving approaches. Teacher-student interaction; student involvement in lessons; and school environment were also analyzed.

3.10.2 Qualitative Data Analysis

Those students within each group were given codes in order to recognize the sources of the transcripts. Subsequently, semi-structured interviews were recorded into composed writings, and each transcript was composed against the distinguishing proof of their particular groups. Each semi-structured interview theme, counting numerous documented sections, was completely inspected to decide the students' and teachers' common discernments. The study's discoveries were examined utilizing these essential impressions and comparisons as a system.

3.10.3 Triangulation

The results were effectively combined to respond, ask about the address, and provide a more compelling and meaningful picture of the problem under investigation. This may be fulfilled by organizing them into major think-about subjects by incorporating both objective and subjective issues on geometry learning. Thus, students CU, PSS and their motivation score were triangulated with the interview collected from them and the classroom observation environment of the students on each treatment group Therefore, the qualitative data obtained from semi-structured interviews were categorized under themes. Students' responses to interview questions were compared among the four groups. Similarly, data obtained from conceptual understanding tests were also categorized, analyzed and compared among the four groups in relation to sound understanding, partial understanding and no understanding.

3.11 Ethical Issue

In this study there was no possible harm to the participant, organization and the educational society at all. After the proposal of this study was approved by the postgraduate committee of the Department of Science and Mathematics Education, an official letter was submitted to the Educational Office of Addis Ababa City to get a permission by explaining the objectives of the study. Hence the assigned officials approved and wrote a letter to the targeted group for which the study was free from harm on anything and they underline the study was ethical.

A permission was given from schools and students of the study area since the study site's cooperation was essential for the study. Since the study was a normal instruction approach, there was no parental consent. Furthermore, all study participants were briefed on the purposes and content of the study in a non-threatening and culturally acceptable manner and opportunities were given to them to ask any questions if they had. Respondents' confidentiality was kept and the researcher was hiding their privacy. Voluntary participation was also part of the study and they had the right to withdraw at any time for any reason. The researcher told the participants that there were no benefits or incentives obtained for participating in the research.

Furthermore, honesty and confidentiality were taken in to consideration while conducting research by giving a code for each respondent which was understandable only by the researcher. There were regular reports about the process of the research to advisers, files on the computer were protected with password and documents will be locked in a locker. Access to information was only with the advisors and personally identifiable information will be protected.

CHAPTER FOUR: PRESENTATION OF THE RESULTS, ANALYSIS AND DISCUSSION ON THE FINDINGS

4.1 Introduction

Result of the data analysis and its discussion were indicated in this chapter. The ultimate objective of this study was to examine the effect of variation theory based GeoGebra assisted problem solving instructional approach on 9th grade students' learning geometry to their level of understanding concepts, in solving problems and their motivation. Hence the qualitative and quantitative data were analysis after the presentation of the data then the findings were discussed. Thus, the populations in this study were all Addis Ababa City administration secondary school students and mathematics teachers. This study included four schools; one section from each school was selected.

4.2 Respondents' Characteristics

In this study the number of students and teachers and their distribution to each treatment and comparison group were indicated on the following table as respondent characteristics.

Table 4.1 *Respondents Characteristics*

Variables	**Components**	**Frequency**	**Percent (%)**
Students	Intervention Group1(IG1)	34	27.2
	Intervention Group2(IG2)	35	28
	Intervention Group3(IG3)	29	23.2
	Comparison Group (CG)	27	21.6
	Total	**125**	**100.0**
Teachers	Intervention Group1(IG1)	1	25
	Intervention Group2(IG2)	1	25
	Intervention Group3(IG3)	1	25
	Comparison Group (CG)	1	25
	Total	**4**	**100**

As can be seen from Table 4.1, a total of 125 ninth grade students were involved in this study. Among which 34 (27.2%) were in Intervention Group 1, 35 (28%) were in

Intervention Group 2, 29 (23.2%) were in Intervention Group 3, and 27 (21.6%) were comparison group students. Moreover, one teacher (25%) from each group also participated in this study.

4.3 Results

The analysis of the data for the study is presented on this section. SPSS Version 20 was used for analyzing the quantitative data in order to check the assumptions of each parametric test of both pretest and posttest results. Manual transcription and categorizing in to themes were employed for the analysis of qualitative data.

The study was conducted on four different groups. Three of them were given a separate treatment and one of the groups was a control group. Before implementation of treatments; a motivation questionnaire on Mathematics, conceptual understanding and problem-solving question on learning geometry was administered to all groups to see their status on motivation, conceptual understanding and problem-solving ability. After assumptions for normality was checked One Way ANOVA was run to check their status of mathematics motivation, conceptual understanding and problem-solving ability.

4.3.1 Students' conceptual understanding in Geometry

1. ***Are there significant differences in students' Conceptual Understanding (CU) in geometry across the three interventions and comparison groups? How do students develop conceptual understanding and reasoning in learning Geometry through variation theory based GeoGebra assisted problem-solving approaches?***

In order to answer the question above, first check whether there are significant differences in the mean of pre-tests of students' Conceptual in learning geometry among the three interventions (Variation theory based GeoGebra problem-solving instructional approaches; variation theory-based problem-solving instruction; GeoGebra assisted instructional approach) and control groups. To check this ANOVA was used. To do so the assumption of ANOVA must be fulfilled.

1. Since the data is continuous its normality can be checked by graphically and checked by skewness as indicated by Appendix-1 Therefore, the variable is distributed relatively normally, allowing parametric tests (ANOVA) to be used

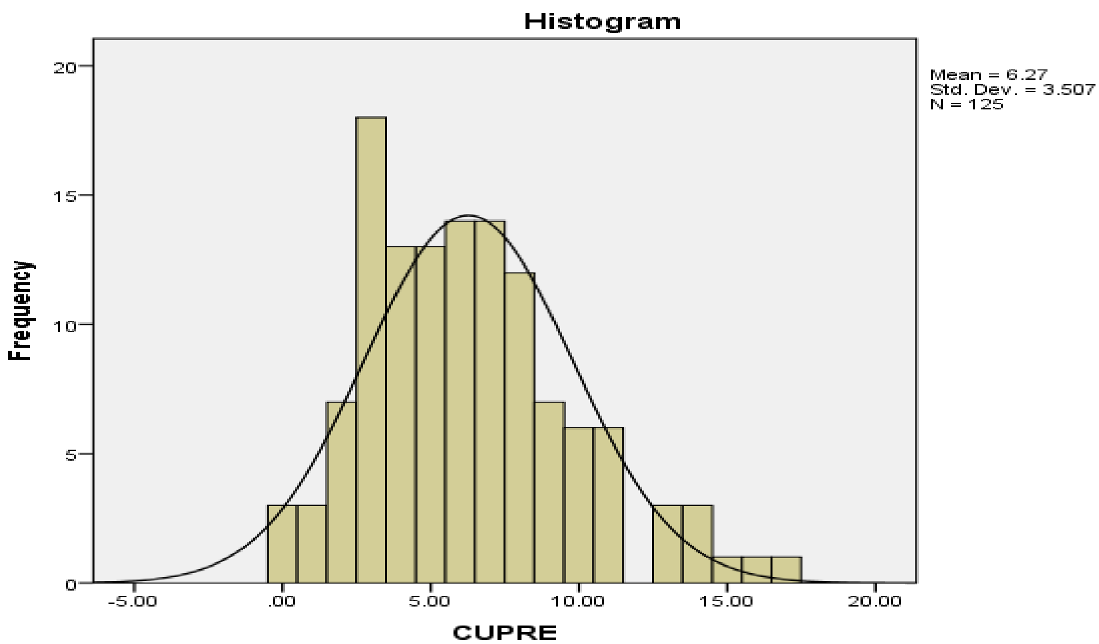


Figure 4.1: *Normality of pretest of student's CU pre- test*

2. There is no reason to suppose that one participant's score is systematically related to the scores of others, hence the data are independent since the intervention had been conducted in different school in which there is no chance for the respondent to communicate each other.

3. The dependent variable has the same variance across groups. The sample sizes for intervention group 1, intervention group 2, intervention group 3, and comparison group were respectively 34, 35, 29 and 27. This shows that the dependent variable's variations are nearly identical for each group.

4. In the following table the assessment of a statistical homogeneity of variance were indicated to the four groups.

__Table 4.2:__ Test of Homogeneity of Variances

Dependent Variables	**Levene Statistic**	**df1**	**df2**	**p**
Conceptual Understanding pretest	4.271	3	121	.007

This assumption is also checked using Levene's equality of variances test. It is expected to obtain a non-significant result for the dependent variable. The result showed that F (3, 121) = 4.271, P =.007 for the pre-CU score, as shown in table 4.2. This indicates that the variances of the groups were not equal. When the group's size are more or less equal violation of the assumption is not a problem (Pallant, 2007). Thus, the expected assumptions are not violated and in the following table student's pretest result on conceptual understanding of their geometry learning were showed as below.

***Table 4.3:** The descriptive statistics of pre- test of conceptual understanding in learning geometry scores by groups*

Dependent Variable	**Groups**											
	TG1			**TG2**			**TG3**			**CG**		
	N	M	SD	N	M	SD	N	M	SD	N	M	SD
Pre- intervention geometry CU	34	6.82	4.35	35	5.77	3.47	29	7.03	2.71	27	5.41	2.99

Table 4.3 show that the mean Per-CU scores for intervention group1, intervention group2, and intervention group3 were M = 6.82, M = 5.77, and M = 7.03, respectively, and M =5.41 for the comparison group. In addition to the mean score results on pre -geometry CU the following table shows the result of One-way ANOVA whether there exists a significance difference on students pretest result or not.

***Table 4.4**: ANOVA Test on pre-test of students conceptual Understanding (CU) in learning geometry scores by groups*

Variables		**Sum of Squares**	**df**	**Mean Square**	**F**	**p**
Pre- intervention geometry CU	Between Groups	56.155	3	18.718	1.542	.207
	Within Groups	1468.597	121	12.137		
	Total	1524.752	124			

As can be seen in Table 4.4, the result of the study showed F (3, 121) = 1.542, p = 0.207 for Pre- intervention CU in learning geometry scores between the groups. This shows that there was no statistically significant mean difference in the four groups on their pretest Conceptual understanding result since p value >0.05

Now it is time to check, whether there are significant differences in the mean gains of pre-test and post-test of students' problem-solving skills in learning geometry among the three interventions and control groups.

Assumptions on Paired Samples T-test

1. The independent variable is dichotomous and its levels (or groups) are paired, or matched, in some way
2. The dependent variable is normally distributed as shown figure 4.1.
3. The dependent variable (Conceptual understanding result) was continuous.
4. There is no outlier across either group as shown Appendix 4.3 the z-score is between -3and and3.

Since all the assumption were fit, the following table helped us to in order to examine the significance difference of pre-test to post-test paired samples of each groups,

Table- 4.5: *Paired Samples T-test on mean gains of the pre-test and post-test of students' conceptual understanding in learning geometry scores by groups*

Mean gain	Group	Paired Differences M	Paired Differences SD	T	df	P
Post-CU—Pre-CU	Variation theory based GeoGebra problem-solving instructional approaches;	7.70588	4.30313	10.442	33	.000
	Variation theory-based problem-solving instruction;	2.60000	3.13613	4.905	34	.000
	GeoGebra assisted instructional approach	1.86207	3.52262	2.847	28	.008
	Comparison Group	.70370	2.61379	1.399	26	.174

The above data shows that the mean difference in students' conceptual understanding from pretest to posttest in the group given a treatment of variation theory-based GeoGebra problem-solving instructional approaches is 7.70588, with a standard deviation of 4.30313, a t-value of 10.442, a df of 33, and a p-value of .000. On the other hand, the mean difference in students' conceptual understanding from pretest to posttest in the group given a treatment of variation theory-based problem-solving instruction is 2.6, with a standard deviation, t-value, df, and p-value of 3.13613, 4.905, 34, and .000, respectively.

The GeoGebra-assisted problem-solving instructional approach shows a 1.86207 mean difference between students' conceptual understanding on the pretest and posttest in the group, with standard deviation, t-value, df, and P-value being 3.52262, 2.847, 28, and .008, respectively. Finally, the mean difference in students' conceptual understanding from pretest to posttest in the comparison group is 0.70370, with a standard deviation, t- value, df, and p-value of 2.61379, 1.3992, 26, and .1742, respectively.

From the data, the mean difference from students’ pre-test to post-test of their conceptual understanding indicates the existence of a significant difference in each treatment group since the p-value < 0.005, but there is no significant difference in the comparison group‘s pre- to post-test result of students' conceptual understanding since p = 0.174 (which is > 0.05).

The above analysis shows only the existence of paired difference, but it cannot show which group significantly has a good difference. To do so, we checked whether there were significant differences in students' conceptual understanding in learning geometry among the three interventions and control groups. To do so, ANCOVA was manipulated after the assumption was verified.

The covariates are measured before the intervention manipulation to avoid scores on the covariates being changed by intervention, and they are not substantially connected with one another, as previously mentioned. Furthermore, the covariates are also measured rather consistently. These findings back up some of ANCOVA's assumptions. The other assumptions, such as observation independence, normality, homogeneity of variance, and linearity between covariates and dependent variables at each level of the independent variables, are discussed at length in the preceding sections.

1. *Observation Independence*

Since the students were selected from various Addis Ababa city administration sub-cities government school, the data are independent, and there is no reason to infer that one participant's score is consistently related to the scores of the other students.

2. *Normality of Dependent Variables*

As indicated in figure below the data could be treated as normal for this study and the data was also skewed as indicated in Apendeix-1. These data also show that the variable is fairly normally distributed, allowing ANCOVA to be used.

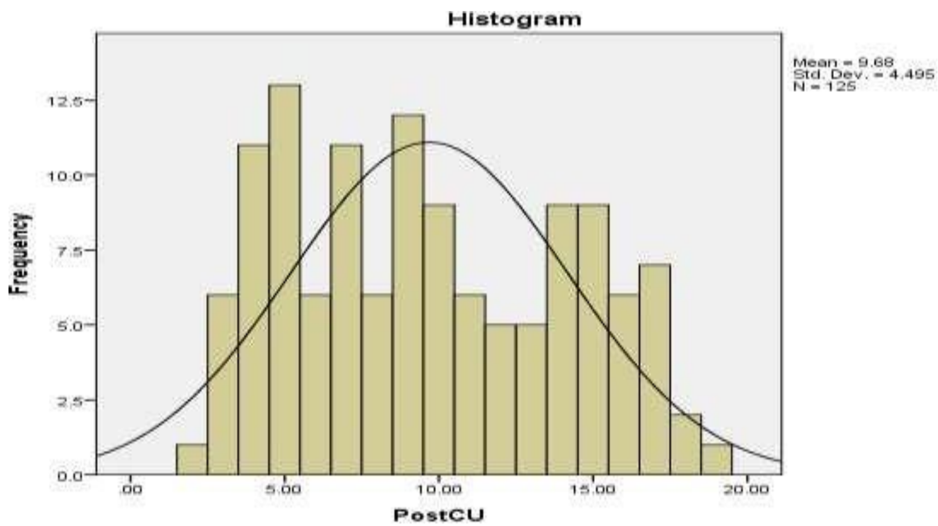


Figure 4.2: *Normality of posttest of students' CU post-test*

3. *Homogeneity of Variances*

ANCOVA is predicated on the assumption that samples are drawn from populations with similar variances. This demonstrates that the scores of the variables are similar across all of the groups. This assumption is tested using Levene‘s equality of variances test. It is expected to obtain a non- significant result for the dependent variable. In the following table an assessment of a statistical homogeneity of variance were indicated to the four groups on their post conceptual understanding.

Table 4.6: *Levene's Test of homogeneity of Variances*

Dependent Variables	***F***	***df1***	***df2***	***p***
POST Conceptual understanding	.654	3	121	.582

The result showed that F (3, 121) = .654, P =.582 for the post-CU score, as seen in Table 4.7. This indicates that the variances of the groups were not similar. There is no violation of the Levene‘s test assumption since P =.582 which is greater than 0.05 (Pallant, 2007). The dependent variable has the same variance across groups.

4. *Linearity between the covariates and the dependent variables at each level of the independent variables.*

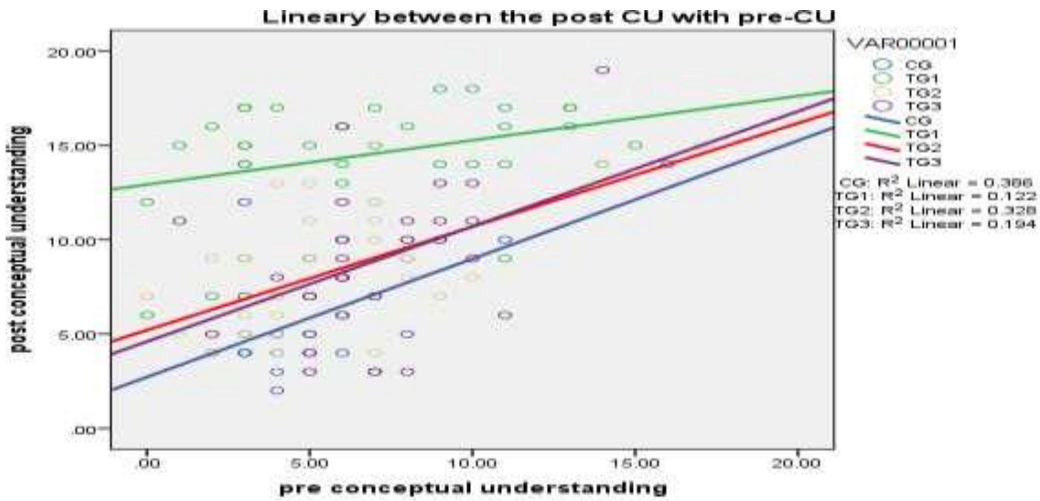


Figure 4.3: *Scatter plot of Covariates and dependent variables*

At each level of the independent variable, the assumption of linearity requires a direct relationship between the covariate and the dependent variable (Pallant, 2011). A scatterplot between the covariate and dependent variable at each level of the independent variable, as illustrated in Figure 4.3, can be used to confirm this. Because the scatter plot shows no deviations from linearity, ANCOVA assumptions appear to be met for this study since the slope of the lines had similar signs.

The following table describes the posttest result of students' conceptual understanding before any statistical adjustments and after students' pretest result on conceptual understanding has been statistically controlled or accounted for.

***Table 4.7**: Adjusted and Unadjusted Means and Variability on the four Groups for conceptual understanding in learning geometry Using Pre-test as a Covariate*

Groups		**Unadjusted**		**Adjusted**	
	N	**M**	**SD**	**M**	**SD**
Variation theory based GeoGebra problem-solving instructional approaches;	34	14.529	2.915	14.400	.495
Variation theory-based problem-solving instruction;	35	8.371	3.317	8.646	.489
GeoGebra assisted instructional approach	29	8.897	3.745	8.431	.553
Comparison Group	27	6.111	3.017	6.653	.574

Table 4.7 shows the mean of posttest results on student's conceptual understanding in learning geometry for both the unadjusted and adjusted means. With this information, the group that received an intervention; the variation theory-based GeoGebra problem- solving instructional approaches group, the variation theory-based problem-solving instruction group, the GeoGebra-assisted instructional approach group, and the comparison group had unadjusted mean results of 14.529, 8.371, 8.897, and 6.111, respectively. On the other hand, the Variation theory-based GeoGebra problem-solving instructional approaches group, the Variation theory-based problem-solving instruction group, the GeoGebra-assisted instructional approach group, and the Comparison group had adjusted mean results of 14.400, 8.646, 8.431, and 6.653, respectively.

The above data shows that there is no such significant mean difference between the adjusted and unadjusted mean of students' conceptual understanding in learning geometry. To see its difference, the following graph shows clearly.

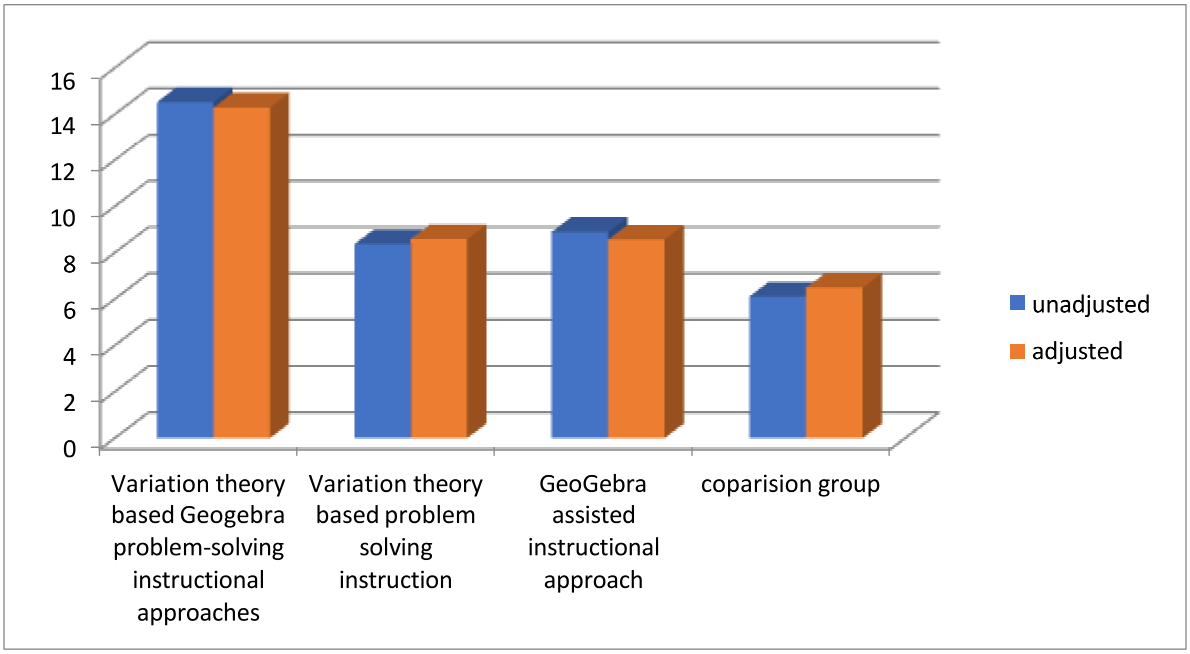


Figure 4.4: *Unadjusted and Adjusted mean for student's CU post-test*

Figure 4.4 shows that the estimated marginal means for all of the groups were statistically adjusted to allow comparisons between the pre-test and post-test, as well as among the groups in their post-test. The adjusted post-CU mean scores of the students similarly demonstrate a significant difference in post-test scores in favor of the intervention and comparison groups, as shown in table 4.7 and figure 4.4. They indicate that mean scores changed after using the variation theory-based GeoGebra problem-solving instructional approaches, the variation theory-based problem-solving instruction group, and the comparison groups. Yet, it does not confirm pairwise differences or identify better-performing groups.

Because of the existence of more than two groups, the Bonferroni test, which is a type of post hoc test, was used on the adjusted post-test means to determine which group did better. Pallant (2011) defines ―adjusted‖ as the impact of a covariate that has been statistically removed. The results of the Bonferroni test were reported in Table 4.10 to confirm the source of the observed.

The following table shows results Analysis of Covariance for conceptual understanding in learning geometry as a Function of the four Groups, Pre-test as a Covariate

Table 4.8: *Tests of Between-Subjects Effects*

Source	df	MS	F	P	Partial eta^2
Pre-CU	1	279.803	33.436	.000	.218
Group	3	353.023	42.186	.000	.513
Error	117	8.197			

When the pretest result on student's conceptual understanding was controlled table4.8 showed that students post-CU among the group differ significantly with F (3,117) = 42.186, P = .000, and η^2 = .513. It showed that this difference exists due to the treatments given to the groups. And their pretest results have no significant effect on students' post-conceptual understanding.

In this study, the partial eta-squared value is .513, which equates to a large effect size according to Cohen's guidelines (1988). Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. It is suggested the instructional approaches were responsible for 51.3% of the variability in the dependent variable. These indicated that the teaching approaches used had a moderate amount effect on the groups' post-test mean scores.

The table also indicates that the pretest results on students' conceptual understanding in learning geometry have a significant impact, with an effect size of 0.218 (21.8%), which is larger than small but not medium effect. In order to check between which groups the difference occurred, the following table will show the Bonferroni comparison effect.

***Table 4.9**: Group comparison on students post test result*

(I) group	(J) group	Mean Difference (I-J)	P
TG1	CG	7.800*	.000
	TG2	5.699*	.000
	TG3	5.725*	.000
TG2	CG	2.101*	.032
	TG3	.026	1.000
TG3	CG	2.075	.055

Dependent Variable: Problem solving skill post
*The mean difference is significant at the .05 level.
. Adjustment for multiple comparisons: Bonferroni.

From the adjusted post-test score results in the Bonferroni test show that the mean difference between TG1 and CG is 7.8 with a p-value of 0.000; the mean difference between TG1 and TG2 is 5.669 with a p-value of 0.000; and the mean difference between TG1 and TG3 is 5.752 with a p-value of 0.000. Similarly, the adjusted post-test score results in the Bonferroni test show that the mean difference between TG2 and CG is 2.101 with a p-value of 0.032; the mean difference between TG2 and TG3 is 0.026 with a p-value of 1.00. On the other hand, the mean difference between TG3 and CG is 2.075 with a p-value of 0.055.

The data indicate that there was a statistically significant difference in students' conceptual understanding between treatment group-1 and the other three groups and between treatment group-2 and the comparison group. This means that treatment group 1 shows better conceptual understanding while learning geometry than the other group. Whereas there was no statistically significant difference in students' conceptual understanding between students in intervention group 2 and intervention group 3 and intervention group -3 and the comparison group since the p-value was greater than 0.05.

From classroom observation, I have seen that students within the intervention-1 lesson can review geometric facts related to the subject and, moreover, clarify the ―why‖ behind those truths, interface concepts over distinctive circumstances, apply information to unused issues on geometry, and express their thinking in a clear and coherent way, showing a profound understanding of the basic standards included in geometry. Students who have a conceptual understanding can see the relationship between concepts and strategies and can allow a contention to clarify why fewer truths are the result of reality than others. Similarly, the teacher who led intervention-1 also agreed that when students supported a variation theory and mixed instructional approaches, they could easily understand the concepts.

Students in intervention 1 and 2 respond to a positive feeling, and they become more confident in order to visualize and easily understand geometric concepts due to the flexible manipulation features of GeoGebra software, which guide them to a deeper grasp of geometric relationships and their properties. This is important for those who struggle with traditional static representations of geometric shapes.

How do students develop conceptual understanding and reasoning in learning Geometry through variation theory based GeoGebra assisted problem-solving approaches?

From classroom observation in treatment group-1, the researcher understands that students work together with peers to understand geometry, the content objectives were achieved in a better way, students get the opportunity to further grasp the lessons, and the presentation of content and arrangement is well organized in accordance with Variation theory GeoGebra based problem-solving approach. Furthermore, students were comfortable reflecting on their understanding, and got an opportunity to regulate one's progress, allowing them to express themselves in a written form. In this treatment group, the classroom teacher gave time limitations to assist students while for better understanding of the concepts and there was a limitation of resources for one-to-one computer access. Students appeared comfortable, and they got a chance to pose questions and forward their views.

Under treatment group-2, students work together with peers to understand geometry conceptually while the teacher leads the instruction using a variation theory-based problem-solving approach and the teacher's listed content objectives were achieved in a better way. In a classroom setting, students get the opportunity to further grasp the lessons by discerning the critical object of the learning. The presentation of contents and arrangement is well organized in accordance with a variation theory-based problem-solving approach, and students were comfortable reflecting their understanding of the concepts and allowing students to express themselves in a written form. The researcher understands that the classroom teacher is well aware of the purpose of the variation theory-based problem-solving approach, and he allowed students to get a chance to pose questions and forward their views.

Students work together with peers to understand geometry conceptually when the teacher delivers the lesson using a GeoGebra-based problem-solving approach. Students were interested in the environment by repeatedly sketching geometric figures in order to enrich their conceptual understanding by using the software freely without anxiety. Students were trying to solve problems after understanding the geometric figures on the sketch field. Furthermore, by using GeoGebra, the teacher addressed the content objectives in a better way. Students were comfortable reflecting on their understanding since GeoGebra- based problem-solving instruction provided an opportunity to regulate their progress and to express themselves in a written form. As long as the classroom teacher was well aware of the purpose of GeoGebra and showed the purpose of it properly to his students, thus, the researcher understands that GeoGebra problem-solving approaches have contributed a lot to the learning process.

In order to understand the impact of the treatments on students' conceptual understanding, a student should answer correctly and choose the correct reason for the corresponding question. The following table shows the percentage of correct answers and correct corresponding reasons for the conceptual understanding questions between each treatment group and the comparison group.

Table 4.10 *students understanding and reasoning capacity across the group for each item*

Item	Comparison group		Treatment group-1		Treatment group-2		Treatment group-3	
	No. of students	%	No. of students	%	No. of students	%	No. of students	%
1	5	18.52	25	73.53	7	20	6	20.69
2	1	3.70	9	26.47	4	11.43	2	6.89
3	2	7.41	21	61.76	2	5.71	5	17.24
4	1	3.70	25	73.52	7	20	4	13.79
5	4	14.81	22	64.71	10	28.57	6	20.69
6	2	7.41	24	70.59	7	20	4	13.79
7	1	3.70	23	67.65	9	25.71	6	20.69
8	2	7.41	21	61.76	7	20	7	24.14
9	1	3.73	12	35.29	8	22.86	7	24.14
10	3	11.11	10	29.41	7	20	4	13.79
Total	**22**	**8.15**	**192**	**56.47**	**68**	**19.43**	**51**	**17.59**

From the above table 4.10 we can observe that students' conceptual understanding of item 1 count at 18.52%, 73.53%, 20%, and 20.69% for CG, TG1, TG2, and TG3, respectively. The student's conceptual understanding for item 2 counts 3.7%, 26.47%, 11.43%, and 6.89% for CG, TG1, TG2, and TG3, respectively. The result showed that the student's conceptual understanding for item 3 counts 7.41%, 61.76%, 5.71%, and 17.24% for CG, TG1, TG2, and TG3, respectively. For item 4, students responded with the correct reason with 3.7%, 73.52%, 20%, and 13.79% for CG, TG1, TG2, and TG3, respectively. For item 5, students responded with the correct reason, with 14.81%, 64.71%, 28.57%, and 20.69% for CG, TG1, TG2, and TG3, respectively.

For item 6, students responded with the correct reason, with 7.41%, 70.59%, 20%, and 13.79% for CG, TG1, TG2, and TG3, respectively. The student's conceptual understanding for item 7 counts 3.7%, 67.65%, 25.71%, and 20.69% for CG, TG1, TG2, and TG3, respectively. For item 8, students responded with the correct reason with

7.41%, 61.76%, 20%, and 24.14% for CG, TG1, TG2, and TG3, respectively. Similarly, the student's percentage response for the correct reason counts to 3.73%, 35.29%, 22.86%, and 24.14% for CG, TG1, TG2, and TG3, respectively, for item 9. For item 10, students responded with the correct reason, with 11.11%, 29.41%, 20%, and 13.79% for CG, TG1, TG2, and TG3, respectively.

From the above data, the percentages of students who answered the correct response with the correct corresponding reason were 8.1%, 56.4%, 19.28%, and 17.5% for CG, TG1, TG2, and TG3, respectively.

Students in each intervention group showed better conceptual understanding than the comparison group. Moreover, students who are guided by variation theory based on the GeoGebra-assisted problem-solving approach scored the best conceptual understanding by responding to the correct answer with the appropriate corresponding reason for the overall item. The intervention group-3 students, who are guided by a GeoGebra-assisted problem-solving approach, scored better results on their conceptual understanding than the intervention group-2, which is guided by a variation-based problem-solving approach.

From the student's interview in each intervention group, it was generalized that students believed that geometry can help them to observe different geometric shapes and their real relationship with the physical world. Respondents from the variation theory-based GeoGebra-assisted problem-solving approaches were responding that they could easily understand geometric concepts while the instruction was leading with this approach. The reason behind their agreement was that GeoGebra can help them to visualize and realize the concepts. Visualization helps us to put accurate reason for our response to geometric tasks.

Students in each intervention group underlined that the basic challenge in giving explanations of geometrical concepts was finding the relationships between geometric facts and shapes. For example, students in each intervention group were perceived to know the necessary conditions for similarity of a triangle; they were discerning the critical aspect with respect to the length and sides of a triangle by exploring GeoGebra with multiple examples and changing the direction of each triangle with a triangle that will be similar. Furthermore, students in each intervention group highly recommended that to learn geometry by realizing the concepts, the instruction should be organized with variation theory-based problem-solving approaches, a Geogebra-based problem-solving approach, and mixed instruction with technology.

4.3.2 Students' Geometry Problem-Solving Skills

2. Are there significant differences in students' geometry problem-solving skills (PS) across the three interventions and comparison groups? How do they solve problems and develop Geometry problem solving skill?

In order to answer the question above first check whether there are significant differences in the mean of pre-tests of students' geometry problem-solving skills among the three interventions (Variation theory based GeoGebra problem-solving instructional approaches; variation theory-based problem-solving instruction; GeoGebra assisted instructional approach) and control groups. To check this ANOVA test was used. To do so the assumption of ANOVA must be fulfilled.

ANOVA Assumptions

1. Since the data is continuous its normality can be checked by graphically and it was skewed as indicated in appendix-1. The following graph shows to suggests that the variable is distributed relatively normally, allowing parametric tests (ANOVA) to be used.

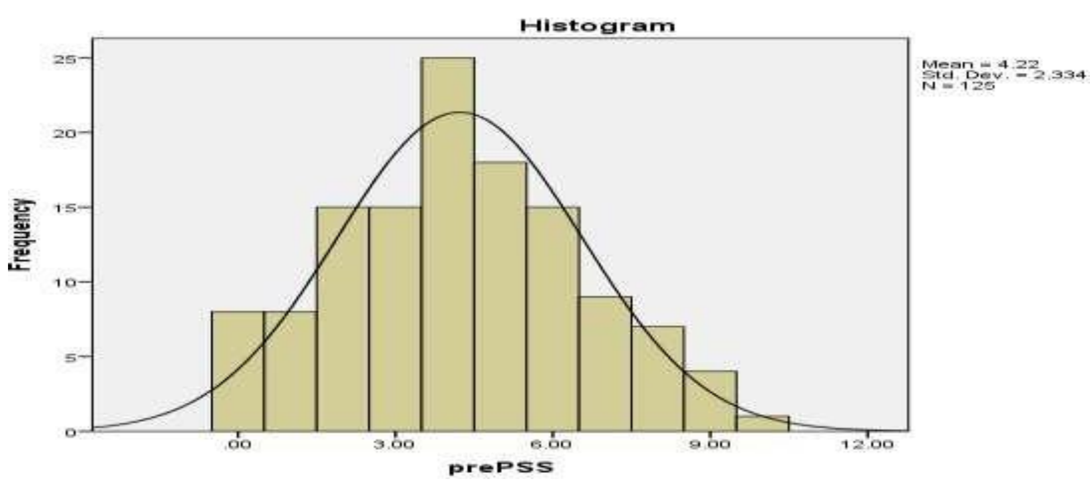


Figure 4.5: *Normality of pretest of students PSS*

2. There is no reason to suppose that one participant's score is systematically related to the scores of others, hence the data are independent since the intervention had been conducted in different school in which there is no chance for the respondent to communicate each other.

3. The dependent variable has the same variance across groups. The assumption indicated that samples are drawn from populations with similar variances. This demonstrates that the scores of the variables are similar across all of the groups. This assumption is tested using Levene's equality of variances test. It is expected to obtain a non- significant result for the dependent variable. In the following table the assessment of a statistical homogeneity of variance were indicated to the four groups.

Table 4.11: *Levene's Test of homogeneity of Variances*

Dependent variable	F	df1	df2	P
Pre-PSS in geometry	4.355	3	121	.006

The result showed that F (3, 121) = 4.355, P =.006 for the pre-PSS score, as seen in Table 4.11. This indicates that the variances of the groups were not similar. The violation of the result showed that F (3, 121) = 4.355, P = .006 for the pre-PSS score, as seen in Table 4.11. This indicates that the variances of the groups were not similar. This indicates that the variances of the groups were not equal. When the group's size is more or less equal violation of the assumption is not a problem (Pallant, 2007). The sample sizes for intervention group 1, intervention group 2, intervention group 3, and the comparison group were 34, 35, 29, and 27, respectively. This shows that the dependent variable's variations are nearly identical for each group.

The following table shows the summarized pretest mean result of the student's problem-solving skills on learning geometry, which can help to understand the main features of pretest results on their PSS.

Table 4.12: *The descriptive statistics of pre- test of geometry problem-solving skills (PSS) scores by groups*

Dependent Variable	Groups											
	TG1			TG2			TG3			CG		
	N	**M**	**SD**	**N**	**M**	**SD**	**N**	**M**	**SD**	**N**	**M**	**SD**
Pre- intervention geometry PSS	34	4.32	2.67	35	4.32	2.67	29	3.51	2.69	27	4. 1	34

Table 4.12 shows the mean of student's problem-solving skill (PSS) for pre-test results. Students in intervention group 1 (variation theory-based GeoGebra-assisted problem-solving instructional approaches) scored a mean of 4.32. Students in intervention group 2 (variation theory-based problem-solving instructional approaches) shows a mean of 4.32 on students' problem-solving skill (PSS). The mean of students' problem-solving skill (PSS) for pre-test was 3.51 for students in intervention group 3 (GeoGebra-assisted instructional approaches). Students in the comparison group scored a mean 4.1. To support the fact that there is no significant difference between them, the following ANOVA results would be confirmed.

The following table indicates what looks like the existence of the student's pre-geometry problem-solving skills difference between each group before the intervention.

Table 4.13: *ANOVA Test on pre-test of geometry problem-solving skills (PSS) scores by groups*

Variables		**Sum of Squares**	**df**	**Mean Square**	**F**	**p**
Pre- geometry PSS	Between Groups	28.222	3	9.407	1.758	.159
	Within Groups	647.506	121	5.351		
	Total	675.728	124			

As can be seen in Table 4.13, the results of the study showed that there was no statistically significant mean difference in pre-intervention geometry PSS scores between the groups, with $F(3, 121) = 1.758$, $p = 0.159$. This suggests that before intervention, the groups seemed similar in terms of problem-solving skills. To compensate for the non-equivalence of groups, an analysis that controls the initial differences in pre-test mean scores should be utilized, allowing the post-test scores to be analyzed against equivalent pre-test scores.

Hence, ANCOVA was utilized to control the effect of pre-test scores on post-test results statistically. The following tasks were completed in order to achieve this: First, the inferential statistics' covariates must be determined. Second, the steps utilized to verify the ANCOVA assumptions were discussed.

Assumptions on Paired T-test

1. The independent variable is dichotomous and its levels (or groups) are paired, or matched, in some way
2. The dependent variable is continuous.
3. There was no outlier across both variables as indicated by Apendix-4.
4. There is normal distribution of both variables

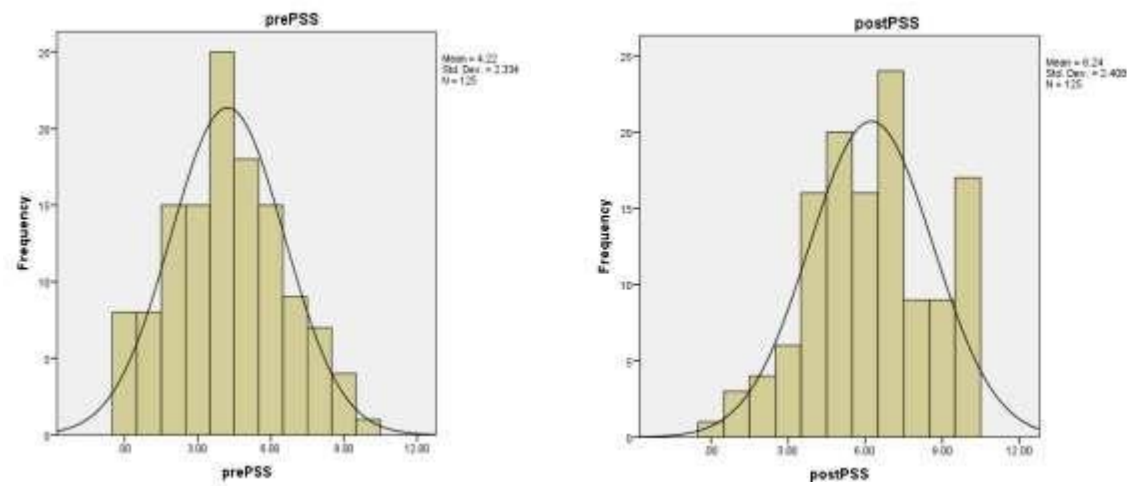


Figure 4.6: *Normality of protest and post-test of students PSS*

From figure 4.6, the graph is normal, with a symmetrical and bell-shaped curve. The graph shows that most data points cluster around the students' mean result of their problem-solving skills.

The following table appears to detail approximately whether the pre-test and post-test results of the understudies on their problem-solving skills were essentially diverse or not. In order to watch in case, the observed contrast is likely due to a genuine impact instead of arbitrary chance.

Table 4.14*: Paired Samples T-test on mean gains of the pre-test and post-test of students' geometry problem-solving skills scores by groups*

Mean gain	**Group**	**Paired Differences**		**t**	**df**	**P**
		M	**SD**			
Pre-PSS to Post PSS	Variation theory based GeoGebra problem-solving instructional approaches;	4.23529	2.64069	9.352	33	.000
	Variation theory-based problem-solving instruction;	1.05714	2.11358	2.959	34	.006
	GeoGebra assisted instructional approach	2.51724	2.11492	6.410	28	.000
	Comparison Group	-.07407	1.61545	-.238	26	.814

The data in table 4.14 shows that the mean difference in students' geometry problem-solving skills from pretest to posttest in the group given a treatment of variation theory-based GeoGebra problem-solving instructional approaches is 4.23529, with a standard deviation of 2.64069, a t-value of 9.352, a df of 33, and a p-value of .000. On the other hand, the mean difference in students' problem-solving skills from pretest to posttest in the group given a treatment of variation theory-based problem-solving instruction is -1.0571, with a standard deviation of 2.11258, a t-value of 2.959, a df of 34, and a p-value of .006. Similarly, the mean difference in students' problem-solving skills from pretest to posttest in the group given a treatment with the GeoGebra-assisted instructional approach is -2.51724, with a standard deviation of 2.11492, a t-value of 6.410, a df of 28, and a p-value of .000. Finally, the mean difference in students' problem-solving skills from pretest to posttest in the comparison group is -.07407, with a standard deviation of 1.61545, a t-value of -.238, a df of 26, and a p-value of .814.

From the data, the mean difference between students' pre-test and post-test of their problem-solving skills indicates there is a significant difference in each treatment group since the p-value < 0.005, but there is no significant difference in the comparison group between its pre- and post-test results of students' problem-solving skills in learning geometry since p = 0.814 (which is > 0.05).

Thirdly, check whether there are significant differences in students' problem-solving skills in learning geometry among the three interventions and control groups.

Now it is time to check whether there are significant differences in students' problem-solving skills in learning geometry among the three interventions and control groups. To do so, ANCOVA was manipulated after the assumption was verified.

The covariates are measured before the intervention manipulation to avoid scores on the covariates being changed by intervention, and they are not substantially connected with one another, as previously mentioned. Furthermore, the covariates are also measured rather consistently. These findings back up some of ANCOVA's assumptions. The other assumptions, such as observation independence, normality, homogeneity of variance, and linearity between covariates and dependent variables at each level of the independent variables, are discussed at length in the preceding sections.

1. *Observation Independence*

   Since the students were selected from various Addis Ababa city administration sub-cities government school, the data are independent, and there is no reason to infer that one participant 's score is consistently related to the scores of the other students.

2. *Normality of Dependent Variables*

   As indicated in Appendix-1 the data was skewed and it was also treated as normal for this study. These data also show that the variable is fairly normally distributed, allowing ANCOVA to be used as shown above figure 4.7.

3. *Homogeneity of Variances*

   ANCOVA is predicated on the assumption that samples are drawn from populations with similar variances. This demonstrates that the scores of the variables are similar across all of the groups. This assumption is tested using Levene's equality of variances test. It is expected to obtain a non- significant result for the dependent variable.

__Table 4.15__ Levene's Test of homogeneity of Variances

Dependent Variables	***F***	***df1***	***df2***	***P***
Post-CU	4.298	3	121	.006

The result showed that F (3, 121) = 4.298, P = .006 for the post-PSS score, as seen in Table 4.15. This indicates that the variances of the groups were not similar. This indicates that the variances of the groups were not equal. When the groups size are more or less equal violation of the assumption is not a problem (Pallant, 2007). The dependent variable has the same variance across groups. The sample sizes for intervention group 1, intervention group 2, intervention group 3, and the comparison group were 34, 35, 29, and 27, respectively.

4. *Linearity between the covariates and the dependent variables at each level of the independent variables.*

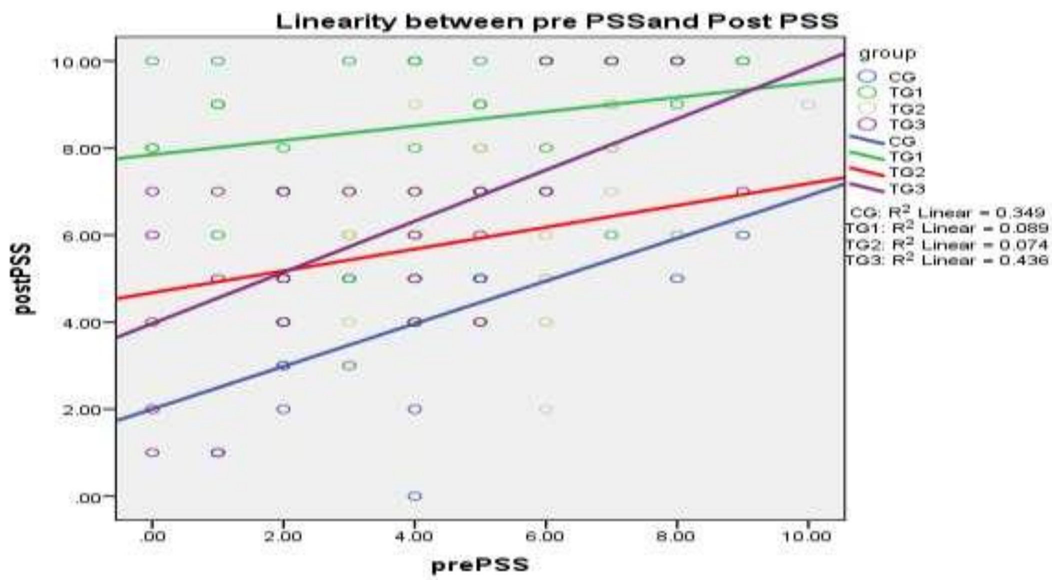


Figure 4.7: *Scatter plot of Covariates and dependent variables*

At each level of the independent variable, the assumption of linearity requires a direct relationship between the covariate and the dependent variable (Pallant, 2011). A scatterplot between the covariate and dependent variable at each level of the independent variable, as illustrated in Figure 4.8, can be used to confirm this. Because the scatter plot shows no deviations from linearity, ANCOVA assumptions appear to be met for this study since the lines had linear shapes.

The following table describes the posttest result of students' motivation and subscales of motivation before any statistical adjustments and after students' pretest result on their motivation and subscales has been statistically controlled or accounted for.

Table 4.16: *Adjusted and Unadjusted Means and Variability on the four Groups for problem-solving skill in learning geometry Using Pre-test as a Covariate*

Groups		**Unadjusted**		**Adjusted**	
	N	**M**	**SD**	**M**	**SD**
Variation theory based GeoGebra problem-solving instructional approaches;	34	8.5588	1.48	8.52	.276
Variation theory-based problem-solving instruction;	35	5.8857	1.67	5.67	.275
GeoGebra assisted instructional approach	29	6.0345	2.39	6.29	.302
Comparison Group	27	4.0000	1.59	4.05	.310

The above table can be presented by the following bar graph

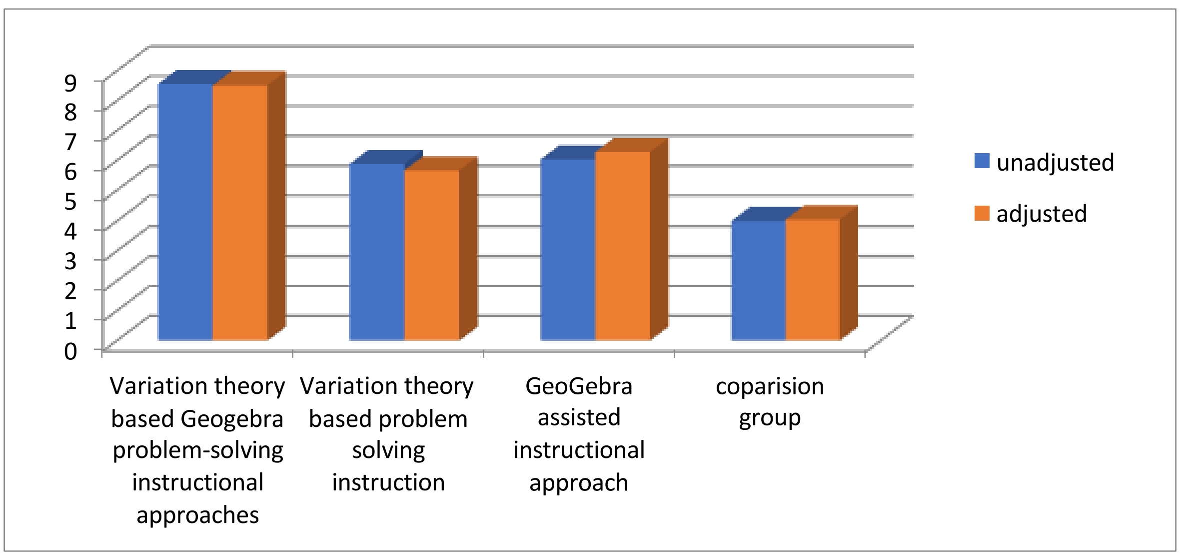


Figure 4.8: *Adjusted and unadjusted mean of the posttest result on students PSS*

Figure 4.8 showed the means of unadjusted and adjusted problem-solving skills of students in each group. On the post-test, the estimated marginal means for all of the groups were statistically adjusted to allow comparisons between the pre-test and post-test, as well as among the groups in their post-test. The adjusted post-PSS mean scores of the students demonstrate a significant difference in post-test scores in favor of the intervention and comparison groups, as shown in table 4.16 and figure 4.8. The result indicates that the mean score changed after using variation theory-based GeoGebra problem-solving instructional approaches, variation theory-based problem-solving instruction, the GeoGebra-assisted problem-solving instructional approach, and traditional instructional approaches. Yet, it does not confirm pairwise differences or identify better-performing groups. Because of the existence of more than two groups, the Bonferroni test, which is a type of post hoc test, was used on the adjusted post-test means to determine which group did better. Pallant (2011) defines as the impact of a covariate that has been statistically removed. The results of the Bonferroni test were reported in Table 4.16 to confirm the source of the observed differences.

The following table used for the Analysis of Covariance for Problem-Solving Skill in learning geometry as a function of the four Groups, Pre-test as a Covariate

Table 4.17: *Tests of Between-Subjects Effects*

Source	**Df**	**MS**	**F**	**P**	$\boldsymbol{\eta 2}$
Pre-PSS	1	84.312	32.576	.000	.214
Group	3	105.910	40.921	.000	.506
Error	120	2.588			

When the effect of the students' pre-PSS was controlled, there was a statistically significant difference between the groups' post-PSS scores with F (3,117) = 32.576, P = .000, and η^2 = 0.506. This revealed that the students' Post-PSS scores were associated with the instructional methods used. In this study, the partial eta-squared value is .506, which equates to a big effect size according to Cohen's guidelines (1988). Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. It was suggested the instructional approaches were responsible for 50.6% of the variability in the dependent variable. The teaching approaches used had a moderate effect on the groups' post-test mean scores. To check between which groups the difference occurred, the following table will show the Bonferroni comparison effect.

Table 4.18 Mean difference between the groups

(I) group	(J) group	Mean Difference (I-J)	P
TG1	CG	4.469*	.000
	TG2	2.855*	.000
	TG3	2.233	.000
TG2	CG	1.1613*	.001
	TG3	-.627	.805
TG3	CG	2.2235*	.000

Dependent Variable: Problem solving skill post
*. The mean difference is significant at the .05 level.
. Adjustment for multiple comparisons: Bonferroni.

The adjusted post-test score results in the Bonferroni test show that the mean difference between TG1 and CG is 4.469 with a p-value of 0.000; the mean difference between TG1 and TG2 is 2.855 with a p-value of 0.000; and the mean difference between TG1 and TG3 is 2.233 with a p-value of 0.000. Similarly, the adjusted post-test score results in the Bonferroni test show that the mean difference between TG2 and CG is 1.1613 with a p-value of 0.001; the mean difference between TG2 and TG3 is -0.627 with a p-value of 0.805. On the other hand, the mean difference between TG3 and CG is 2.2235 with a p-value of 0.000.

The data indicate that there was a statistically significant difference in students' problem-solving skills between treatment group-1 and treatment group-2, treatment group-3, and the comparison group. This means that treatment group 1 shows better problem-solving while learning geometry than the other group. Similarly, treatment group -2 and treatment group -3 show better problem-solving skills in learning geometry than the comparison group since the p-value is less than 0.05. Whereas there was no statistically significant difference in students' problem-solving skills between students in intervention group 2 and intervention group 3 since the p-value was greater than 0.05.

From classroom observation I have seen that students in the intervention -1, 2, and 3 classes can not only recall geometric formulas and facts related to the topic but also explain step-by-step the ―why behind those facts, connect concepts across different situations, apply knowledge to new problems in geometry, and articulate their reasoning in a clear and logical manner, showing a deep understanding of the underlying principles involved in geometry. Students who have a good problem-solving ability can see the relationship between concepts and procedures and can give an argument to explain why some facts are the result of other facts. Similarly, the teacher who led interventions -1, 2, and 3 also agreed that when students are supported by mixed instruction, they can easily understand the concepts and solve problems step by step.

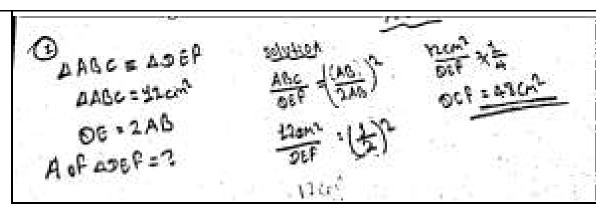
Students in intervention 1 and 2 respond to the positive feeling, and they become more confident in order to solve problems and easily manipulate geometric concepts due to the flexible manipulation features of GeoGebra software, which guide them to a deeper grasp of geometric relationships and their properties. This is important for those who struggle with traditional static representations of geometric shapes.

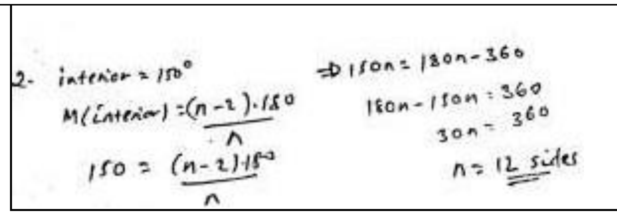
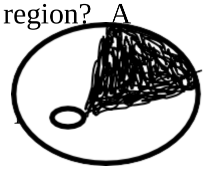
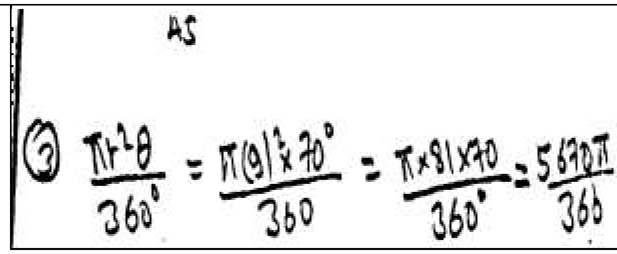
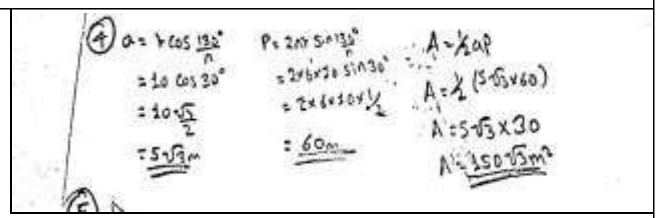
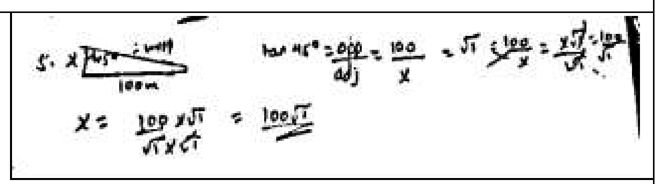
Furthermore, the results below are descriptions of the student problem solving result. The description is based how students manipulate their task.

A. *High-Achiever students*

The following table is about of how higher achiever students manipulated their problem-solving questions from beginning to end in learning geometry.

***Table 4.19**: - problem solving question and students answer*

	Question	Students answer
1	If ΔABC and ΔDEF are similar and the area of ΔABC =12 square units. .If DE is twice of AB ,then the area of ΔDEF =___________	

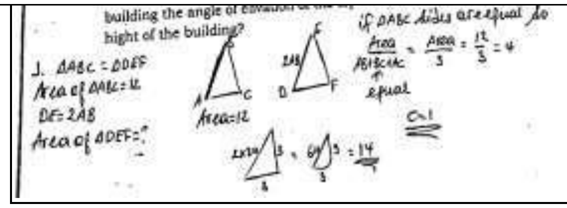
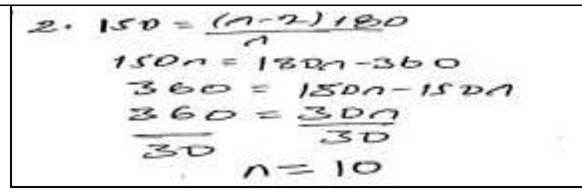
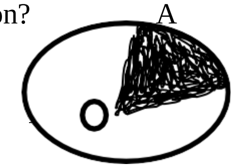
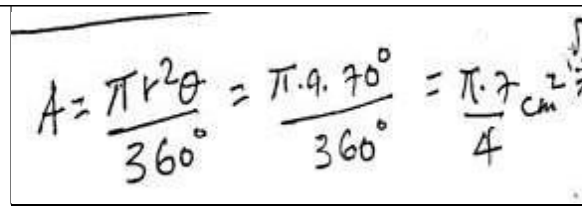
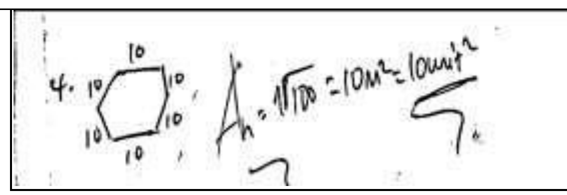
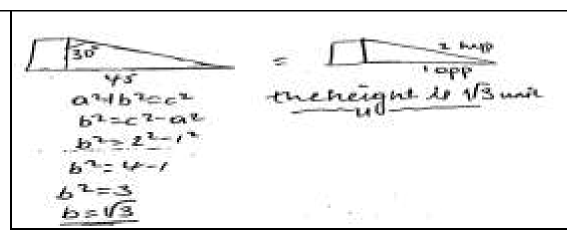
2	Find the number of sides of a regular polygon, if the measure of each of its interior angle is 150^0	
3	In the circle shaded below; O is the center, the radius measures 9cm and (<AOB) =70^0. What is the area of the shaded region? A 	
4	Each sides of a regular hexagon is 10 units long. What is the area of this hexagon?	
5	From a point on the ground, which is 100 meters away from the wall of a building the angle of elevation of the top of the building is 45^0.what is the height of the building?	

From table 4.19 it can be seen that the work of the students was very attractive with necessary steps. Students show good understanding on the problem, students did not write down what is given what is required. At the stage of devising a plan, students showed which appropriate formula was used. Finally, it is possible to observe students presented a representation of the solution in the form of a sketch and provide the appropriate solution.

B. Medium achiever students

The following table is about of how medium achiever students manipulated their problem-solving questions from beginning to end in learning geometry

***Table 4.20**: - problem solving question and students answer*

	Question	Students Answer
1	If ΔABC and ΔDEF are similar and the area of ΔABC =12 square units. If DE is twice of AB , then the area of ΔDEF =__________	
2	Find the number of sides of a regular polygon, if the measure of each of its interior angle is 150^0	
3	In the circle shaded below; O is the center, the radius measures 9cm and (<AOB) =70^0. What is the area of the shaded region? A B 	
4	Each sides of a regular hexagon is 10 units long. What is the area of this hexagon?	
5	From a point on the ground, which is 100 meters away from the wall of a building the angle of elevation of the top of the building is 45^0.what is the height of the building?	

From the above table 4.20, it is possible to observe that students who are medium achievers understand the problem stage but did not write down the steps and the problem's main idea. These students cannot use systematic steps and plan. Students did not possess the conclusions, even though they presented a representation of the solution in the form of a sketch.

C. Low Achiever students

The following table is about of how lower achiever students manipulated their problem-solving questions from beginning to end in learning geometry

Table 4.21: *problem solving question and students answer*

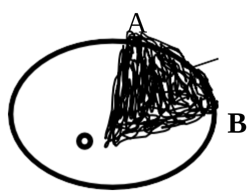
	Question	
1	If ΔABC and ΔDEF are similar and the area of ΔABC =12 square units. if DE is twice of AB ,then the area of ΔDEF =___________	1. ABC ≅ DEF Area of ABC = 12^2 $\frac{AB}{DE} = \frac{BC}{EF} = $ [illegible]
2	Find the number of sides of a regular polygon, if the measure of each of its interior angle is 150^0	2. $150^\circ = \frac{360^\circ}{n}$ $150n = 360^\circ$ $n = \frac{36}{15}$
3	In the circle shaded below; O is the center, the radius measures 9cm and (<AOB) =70^0. What is the area of the shaded region? 	3. $A = \frac{70^\circ}{360^\circ}(9cm)^2$ $= \frac{7(81)}{36} = \frac{567}{36}$ $= 15.7 cm^2$
4	Each sides of a regular hexagon is 10 units long. What is the area of this hexagon?	4, A = L x W = 10 X 6 = 60 unit
5	From a point on the ground, which is 100 meters away from the wall of a building the angle of elevation of the top of the building is 45^0.what is the height of the building?	(5)

Table 4.21 shows that in understanding the problem, the important things about the problem were not presented. The information on the problem was also not presented. In devising a plan, there was no systematic process that took place that might conduct.

From classroom observation of treatment group-1, the researcher understands that students work together with peers in order to solve geometric problem questions, the content objectives were achieved in a better way, students get the opportunity to further grasp the lessons, and the presentation of contents and arrangement is well organized in accordance with the variation theory and the GeoGebra-based problem-solving approach. Furthermore, students were comfortable with reflecting on their problem-solving skills, and getting an opportunity to regulate their progress allowed them to express themselves in a written form. In this treatment group the classroom teacher had time limitations to assist students while looking for better procedures for getting the solution for each open- ended question, and there were limitations of resources for one-to-one computer access. Students appeared comfortable, and they got a chance to pose questions and forward their views.

Under treatment group-2, students work together with peers to solve problems on geometry questions while the teacher leads the instruction using a variation theory-based problem-solving approach, and the teacher-listed content objectives are achieved in a better way. During classroom settings, students get the opportunity to further grasp the lessons by discerning the critical object of the learning. Students felt comfortable commenting on their grasp of the concepts and were given the opportunity to express themselves in writing because of the well-organized content presentation and layout, which adheres to a variation theory-based approach to problem solving. According to the researcher, the instructor in the classroom is fully aware of the goal of the variation theory-based approach to problem solving, and he gave the students an opportunity to voice their opinions and ask questions.

Following their comprehension of the geometric form on the sketch field, students attempted to solve problems. Additionally, the teacher was able to effectively address the content objectives by using GeoGebra. Because GeoGebra-based problem-solving training allowed students to control their development and express themselves in writing, they felt at ease reflecting on what they had learned. As long as the instructor in the classroom understood the goal of GeoGebra and effectively demonstrated it to his students. As a result, the researcher recognizes that GeoGebra's problem-solving techniques greatly aided the learning process.

In order to understand the impact of the treatments on students' problem-solving skills, a student should answer correctly with the necessary steps to be given a full mark (2 points); partial steps with an answer (some understanding of the problem) but not exactly (1 point); and a zero mark if he understands wrongly and does not write anything for the corresponding question. The following table shows the percentage of correct answers and correct procedures for the problem-solving questions along each treatment group and the comparison group.

Table 4.22*: Students post problem solving skills (Post PSS)*

		Treatment-1		Treatment-2		Treatment-3		comparison Group	
Item	Score	students	%	Number	%	students	%	students	%
	Full (2)	27	79.41	11	31.43	6	20.69	2	7.41
	Partial (1)	7	20.59	21	60	15	51.72	18	66. 67
1	Null	0	0	3	8.57	8	27.58	7	25.93
	Full (2)	24	70.59	9	25.72	12	41.38	2	7.41
	Partial (1)	9	26.47	24	68.57	13	44.82	20	74.07
2	Null	1	2.9	2	5.71	4	13.79	5	18.52
	Full (2)	22	64.71	9	25.71	11	37.93	3	11.11
	Partial (1)	12	35.29	23	65.71	12	41.38	16	59.25
3	Null	0	0	3	8.57	6	20.68	8	29.63
	Full (2)	25	73.53	11	31.42	13	44.83	1	3.7
	Partial (1)	9	26.47	20	57.14	13	44.83	20	74.07
4	Null	0	0	4	11.42	3	10.34	6	22.22
	Full (2)	24	70.59	7	20	12	41.38	0	0
	Partial (1)	9	26.47	21	60	15	51.72	18	66. 67
5	Null	1	2.94	7	20	2	6.9	9	33.33

From the above table 4.22, students on item 1 understand the problem and provide the correct answer with the necessary steps, with counts of 79.41%, 31.43%, 20.69%, and 7.41% for TG1, TG2, TG3, and CG, respectively. On this item, the percentage of students who understand the problem partially or those who answer the question without the necessary steps counts 20.59%, 60%, 51.72%, and 66.67% for TG1, TG2, TG3, and CG, respectively. On the other hand, there are students who didn't 't get either the correctanswer or the necessary steps, which counts 0%, 8.5%, 27.58%, and 25.93% for TG1, TG2, TG3, and CG, respectively.

The table indicated that student's problem-solving skills for item-2, understanding the problem and providing the correct answer with the necessary steps, count 70.59%, 25.72%, 41.38%, and 7.41% for TG1, TG2, TG3, and CG, respectively. On this item, the percentage portion of students who understand the problem partially or those who answer the question without the necessary steps counts 26.47%, 68.57%, 44.82%, and 74.07% for TG1, TG2, TG3, and CG, respectively. On the other hand, there are students who didn't get either the correct answer or the necessary steps, which counts 2.9%, 5.71%, 13.79%, and 18.52% for TG1, TG2, TG3, and CG, respectively.

From the above table 4.22, we can observe the solving skill on item-3, which understands the problem and provides the correct answer with the necessary steps, counts 64.71%, 25.71%, 37.93%, and 11.11% for TG1, TG2, TG3, and CG, respectively. On this item, the percentage portion of students who understand the problem partially or those who answer the question without the necessary steps counts 35.29%, 65.71%, 41.38%, and 59.25% for TG1, TG2, TG3, and CG, respectively. On the other hand, there are students who didn't get either the correct answer or the necessary steps, which counts for 0%, 8.57%, 20.68%, and 29.63% for TG1, TG2, TG3, and CG, respectively.

The table indicated that student 's problem-solving skills for item 4, understanding the problem and providing the correct answer with the necessary steps, count 73.53%, 31.42%, 44.83%, and 3.7% for TG1, TG2, TG3, and CG, respectively. On this item, the percentage portion of students who understand the problem partially or those who answer the question without the necessary steps counts 26.47%, 57.14%, 44.83%, and 74.07% for TG1, TG2, TG3, and CG, respectively. On the other hand, there are students who didn't get either the correct answer or the necessary steps, which counts for 0%, 11.42%, 10.34%, and 22.22% for TG1, TG2, TG3, and CG, respectively.

The table indicated that student ‘s problem-solving skills for item 5, understanding the problem and providing the correct answer with the necessary steps, count 70.59%, 20%, 41.38%, and 0% for TG1, TG2, TG3, and CG, respectively. On this item, the percentage portion of students who understand the problem partially or those who answer the question without the necessary steps counts 26.47%, 60%, 51.72%, and 66.67% for TG1, TG2, TG3, and CG, respectively. On the other hand, there are students who didn’t get either the correct answer or the necessary steps, which counts for 2.94%, 20%, 6.9%, and 33.33% for TG1, TG2, TG3, and CG, respectively.

Students in each intervention group showed better problem-solving skills than the comparison group. Moreover, students who are guided by the variation theory-based GeoGebra-assisted problem-solving approach scored the best problem-solving skills by responding with the correct answer with the appropriate corresponding steps for the overall items. The intervention group—3 students who are guided by the GeoGebra- assisted problem-solving approach—scored better results on their problem-solving skills than intervention group 2, which is guided by the variation-based problem-solving approach.

From student interviews in each intervention group, it was generalized that students believed that geometry can help them to observe different geometric shapes and their real relation with the physical world. Respondents from the variation theory-based GeoGebra-assisted problem-solving approaches were responding that they could easily understand geometric concepts while the instruction was leading with this approach. The reason behind their agreement was that GeoGebra can help them to visualize and realize the concepts. Visualization helps us to put accurate reasoning for our response on geometric tasks.

Students in each intervention group underline that the basic challenge in giving explanations of geometrical concepts was finding the relationships between geometric facts and shapes. For example, students in each intervention group perceived that to know the necessary condition for similarity of triangles, they were discerning the critical aspect with respect to the length and sides of the triangle by exploring GeoGebra with multiple examples and changing the direction of each triangle with the triangle that will be similar.

Furthermore, each student in the intervention group highly recommended learning geometry with realizing the concepts; the instruction should be organized with variation theory-based problem-solving approaches, the GeoGebra-based problem-solving approach, and mixed instruction with technology.

4.3.3 Students' motivation in learning Geometry

3. ***Are there significant differences in students' motivation (M) in geometry across the three interventions and comparison groups?***

Even though the mathematics motivational questionnaires were adopted and modified from previous works, factor analysis was used to explore the relationship between the variables either they are on the same category or not according to the pilot study.

Assumptions

1. Since the instrument was an ordinal scale and the skewness and kurtosis of the distribution for all the items and each component were between -1 and +1, the data could be considered as normal. This result also suggests that the variable is distributed relatively normally, allowing parametric tests to be used. (Appendix 1).

2. At least many of the variables should be correlated at a moderate level. Bartlett 's test of Sphericity addresses this assumption. The Kaiser-Meyer-Oikin (KMO) measure should be greater than .70, and is inadequate if less than .50. The KMO test tells one whether or not enough items are predicted by each factor. The Bartlett test should be significant (i.e., a significance value of less than .05); this means that the variables are correlated highly enough to provide a reasonable basis for factor analysis. As shown in the following table below the assumption fulfil.

Table 4.23: *KMO and Bartlett 's Test result*

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.761
Bartlett's Test of Sphericity	Approx. Chi-Square	71.059
	Df	21
	Sig	.000

3. Assumption of linearity can be assumed with matrix scatterplot. The determinant (located under the correlation matrix) should be more than .00001 as shown below it met the assumption.

In the following table a Pearson correlation were used in order to show a statistical measure of the strength and direction of the linear relationships between motivation and motivational subscales in learning geometry at each group

Table 4.24: *Correlation Matrix between the motivation scales*

Correlation	**Intrinsic**	**Self-efficacy**	**Utility value**	**Test anxiety**	**Extrinsic**	**Attention**	**Satisfaction**
Intrinsic	1.000	.528	.434	-.312	.449	.492	.530
Self- efficacy	.528	1.000	.162	-.119	.317	.212	.428
Utility value	.434	.162	1.000	-.005	.376	.499	.469
Test anxiety	-.312	-.119	-.005	1.000	-.059	-.060	-.257
Extrinsic	.449	.317	.376	-.059	1.000	.311	.453
Attention	.492	.212	.499	-.060	.311	1.000	.353
Satisfaction	.530	.428	.469	-.257	.453	.353	1.000
p-value (1-tailed)							
Intrinsic		.000	.003	.025	.002	.001	.000
Self- efficacy	.000		.159	.233	.023	.095	.003
Utility value	.003	.159		.488	.008	.001	.001
Test anxiety	.025	.233	.488		.359	.356	.054
Extrinsic	.002	.023	.008	.359		.025	.002
Attention	.001	.095	.001	.356	.025		.013
Satisfaction	.000	.003	.001	.054	.002	.013	

Keeping the above assumption as indicated at **Appendex-2** the factor loading of the data shows similar item with the same load.

Secondly, the significant differences in the mean of pre-test of students' motivation and components of motivation in learning geometry among the three interventions (Variation theory based GeoGebra problem-solving instructional approaches; variation theory-based problem-solving instruction; GeoGebra assisted instructional approach) and control groups were checked.

In order to answer whether there is significance difference between the groups on their pretest result I ANOVA since its assumption fit as indicated below.

ANOVA Assumptions

1. Since the data is continuous its normality can be checked skewness and it was skewed as indicated in appendix-1. Pre-test motivation and the subscales were distributed relatively normally, allowing parametric tests (ANOVA) to be used.

2. There is no reason to suppose that one participant ‘s score is systematically related to the scores of others; hence the data are independent since the intervention had been conducted in different school in which there is no chance for the respondent to communicate each other.

3. The dependent variable has the same variance across groups. The assumption hat samples are drawn from populations with similar variances. This demonstrates that the scores of the variables are similar across all of the groups. This assumption is tested using Levene‘s equality of variances test using the following table. It is expected to obtain a non- significant result for the dependent variable.

Table 4.25: *Levene's Test of homogeneity of Variances*

Dependent variable	F	df1	df2	Sig.
Pre-Motivation	1.403	3	121	.245
Pre- Intrinsic	3.260	3	121	.024
Pre- Self- efficacy	2.550	3	121	.059
Pre- Utility value	2.194	3	121	.092
Pre- Test anxiety	5.887	3	121	.001
Pre- Extrinsic	3.941	3	121	.010
Pre- Attention	1.448	3	121	.232
Pre- Satisfaction	2.425	3	121	.069

The results show that $F(3, 121) = 1.403$, $P = .245$ for the pre-motivation score; $F(3, 121) = 3.260$, $P = .024$ for pre-intrinsic motivation; $F(3, 121) = 2.55$, $P = .059$ for pre-self-efficacy; $F(3, 121) = 2.194$, $P = .092$ for pre-utility value; $F(3, 121) = 5.887$, $P = .001$ for pre-test anxiety; $F(3, 121) = 3.941$, $P = .01$ for pre-extrinsic motivation; $F(3, 121) = 1.448$, $P = .232$ for pre-attention; and $F(3, 121) = 2.425$, $P = .069$ for pre-satisfaction, as seen in Table 4.25.

From the pre-intrinsic table, anxiety and extrinsic motivation scales indicate that the variances of the groups were not similar. This indicates that the variances of the groups were not equal. When the groups' size is more or less equal violation of the assumption is not a problem (Pallant, 2007). The sample sizes for intervention group 1, intervention group 2, intervention group 3, and the comparison group were 34, 35, 29, and 27, respectively. This shows that the dependent variable's variations are nearly identical for each group.

The following table shows the summarized pretest mean result of the student's motivation and the subscales on learning geometry, which can help to understand the main features of pretest results on their motivation and subscales.

***Table 4.26**: The descriptive statistics of pre- test of students' motivation and components of motivation in learning geometry by groups*

Dependent Variable	Variation theory based GeoGebra assisted problem Solving Approach			Variation theory problem Solving Approach			GeoGebra assisted problem Solving Approach			Conventional Approach		
	N	M	**SD**	**N**	**M**	**SD**	**N**	**M**	**SD**	**N**	**M**	**SD**
Motivation	34	4.158	0.5160	35	3.849	0.5857	29	3.723	0.6795	27	3.358	0.623
Intrinsic	34	4.482	0.6274	35	4.171	0.7098	29	3.862	0.8833	27	3.244	0.8064
Self- efficacy	34	4.247	0.677	35	4.028	0.7517	29	3.883	0.900	27	3.193	0.8494
Utility value	34	4.435	0.6353	35	4.131	0.7745	29	4.069	0.8041	27	3.4	0.7421
Test anxiety	34	3.106	0.9109	35	2.76	0.8565	29	2.8	0.6908	27	3.667	0.903
Extrinsic	34	4.264	0.5608	35	3.794	1.0341	29	3.862	0.8994	27	3.333	1.0466
Attention	34	4.3	0.5914	35	4.063	0.8875	29	3.869	0.8251	27	3.326	0.7828
Satisfaction	34	4.371	0.5595	35	3.994	0.9399	29	3.717	0.9599	27	3.341	0.7292

(Group spans all four approach columns.)

Table 2.26 shows that mean and standard deviation of the interventions and comparison groups result of motivation and its scales. For visibility of the data can be support by the following the graph

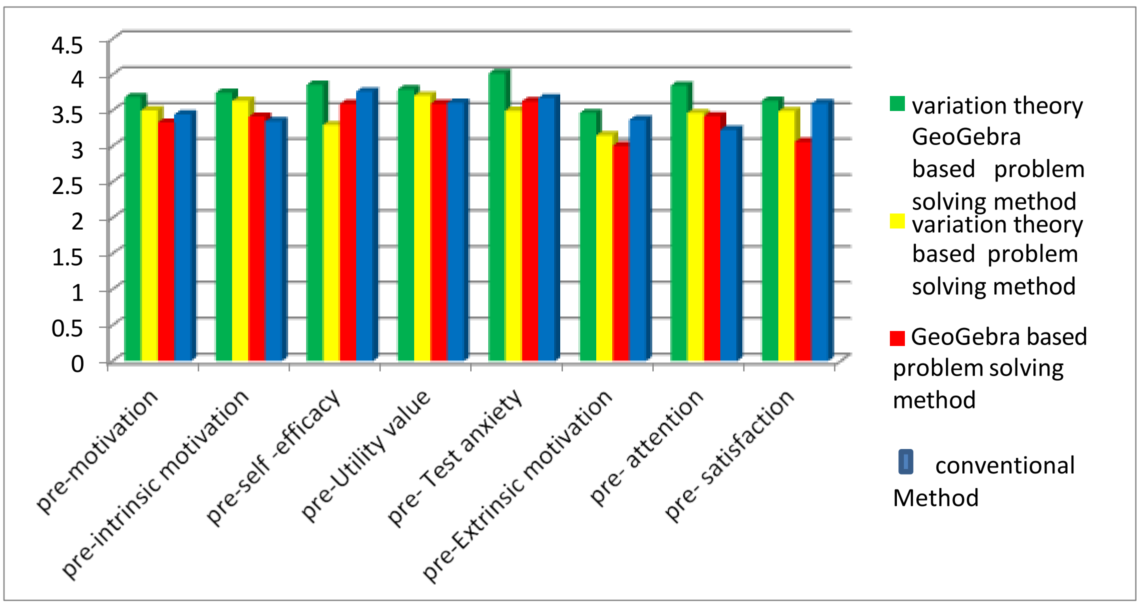


***Figure 4.9:** Pre-test result of students' motivation in learning geometry*

From table 4.26 and figure 4.9, we can see that students under the variation theory GeoGebra-based problem-solving method had scored mean values of 4.158, 4.382, 4.247, 4.435, 3.106, 4.264, 4.3, and 4.371 on their overall motivation, pre-intrinsic motivation, pre-self-efficacy, pre-utility value, pre-test anxiety, pre-extrinsic motivation, pre- attention, and pre-satisfaction respectively.

Similarly, students who use the variation theory-based problem-solving method had a mean value of 3.849, 4.171, 4.028, 4.131, 2.76, 3.794, 4.063, and 3.994 for overall pre-motivation, pre-intrinsic motivation, pre-self-efficacy, pre-utility value, pre-test anxiety, pre-extrinsic motivation, pre-attention, and pre-satisfaction, respectively.

The GeoGebra-based problem-solving instruction treatment group scored a mean value of 3.723, 3.862, 3.883, 4.069, 2.8, 3.862, 3.869, and 3.717 for overall pre-motivation, pre-intrinsic motivation, pre-self-efficacy, pre-utility value, pre-test anxiety, pre-extrinsic motivation, pre-attention, and pre-satisfaction, respectively. Similarly, students under the comparison group had a mean value of 3.358, 3.244, 3.193, 3.4, 3.667, 3.333, 3.326, and 3.341 for overall pre-motivation, pre-intrinsic motivation, pre-self-efficacy, pre-utility value, pre-test anxiety, pre-extrinsic motivation, pre-attention, and pre-satisfaction, respectively.

After descriptive statistics, it is time to check the existence of a significant difference in students' pre-test results between their motivations in learning geometry. To test the existence, the following table describes this. Since the assumption of ANOVA is met, we can use it.

Table 4.27: *ANOVA Test on pre-test of motivation and its components in learning geometry by group*s

Variables		**Sum of Squares**	**Df**	**Mean Square**	**F**	**p**
Pre-Motivation	Between Groups	2.096	3	.699	1.986	.120
	Within Groups	42.573	121	.352		
	Total	44.668	124			
Pre-Intrinsic value	Between Groups	3.207	3	1.069	1.423	.239
	Within Groups	90.905	121	.751		
	Total	94.112	124			
Pre-Self-efficacy	Between Groups	4.835	3	1.612	2.384	.073
	Within Groups	81.819	121	.676		
	Total	86.654	124			
Pre-Utility value	Between Groups	1.281	3	.427	.638	.592
	Within Groups	80.991	121	.669		
	Total	82.272	124			
Pre-Test anxiety	Between Groups	6.135	3	2.045	2.438	.068
	Within Groups	101.493	121	.839		
	Total	107.628	124			
Pre-Extrinsic motivation	Between Groups	3.993	3	1.331	1.553	.204
	Within Groups	103.679	121	.857		
	Total	107.672	124			
Pre-Attention	Between Groups	4.028	3	1.343	1.867	.139
	Within Groups	87.030	121	.719		
	Total	91.058	124			
Pre-satisfaction	Between Groups	6.197	3	2.066	2.394	.072
	Within Groups	104.393	121	.863		
	Total	110.590	124			

As can be seen in Table 4.27, the result of the study showed that there was no statistically significant mean difference in Pre-Motivation scores between the groups, with F (3, 121) = 1.986, p = 0.120 , pre-intrinsic motivation scores between the groups, with F (3, 121) = 1.423, p = 0.239. , pre-self –efficacy scores between the groups, with F (3, 121) = 2.384, p = 0.073. , pre-Utility value scores between the groups, with F (3, 121) = 6.38, p = 0.592., pre- test anxiety scores between the groups, with F (3, 121) = 2.438, p = 0.068, pre-extrinsic motivation scores between the groups, with F (3, 121) = 1.553, p = 0.204, pre-attention scores between the groups, with F (3, 121) = 1.867, p = 0.139 and, pre-satisfaction scores between the groups, with F (3, 121) = 2.394, p=0.072 0621.

This suggests that before intervention, the groups seemed to be similar in terms of their motivation in learning geometry, even though the mean difference between the groups seems to be a slight difference. To compensate for the non-equivalence of groups, an analysis that controls the initial differences in pre-test mean scores should be utilized, allowing the post-test scores to be analyzed against equivalent pre-test scores. Hence, ANCOVA was utilized to control the effect of pre-test scores on post-test results statistically.

Now, it is time to check whether there are significant differences in the mean gains of pre-test and post-test of students' motivation and components of motivation in learning geometry among the three interventions and control groups.

Assumptions on Paired T-test

1. The independent variable is dichotomous and its levels (or groups) are paired, or matched, in some way

2. The dependent variable is normally distributed in as shown in Appendix 1 which is skewed between ± 1.

3. There were no outliers across the groups since the Z-score as indicated Appendex-4 between ±3

The following table appears a detail approximately whether the pretest and posttest come about of understudies on their motivation and subscales were essentially diverse or not. In arrange to watch in case to watched contrast is likely due to a genuine impact instead of arbitrary chance

Table 4.28: *Paired Samples T-test on mean gains of the pre-test and post-test of students' motivation and components of motivation in learning geometry by groups*

Mean gain	Group	Paired Differences		t	Df	p
		M	SD			
Motivation: Post – Pre	TG1	0.47143	0.57657	4.768	33	.000
	TG2	0.36245	0.67877	3.159	34	.003
	TG3	0.39212	0.39197	5.387	28	.000
	CG	-0.08783	0.49879	-0.915	26	.369
Intrinsic value: Post – Pre	TG1	0.63529	0.85631	4.326	33	.000
	TG2	0.53714	1.15379	2.754	34	.009
	TG3	0.44828	0.89707	2.691	28	.012
	CG	-0.1037	0.9944	-0.542	26	.593
Self-efficacy: Post – Pre	TG1	0.67059	0.77519	5.044	33	.000
	TG2	0.42286	1.11437	2.245	34	.031
	TG3	0.68966	0.91275	4.069	28	.000
	CG	-0.00741	0.83479	-0.046	26	.964
Utility value: Post – Pre	TG1	0.64118	0.77347	4.834	33	.000
	TG2	0.32	0.91677	2.065	34	.047
	TG3	0.47586	0.73371	3.493	28	.002
	CG	-0.20741	0.9089	-1.186	26	.246
Test anxiety: Post – Pre	TG1	-0.75294	0.84575	-5.191	33	.000
	TG2	-0.53714	1.33617	-2.378	34	.023
	TG3	-0.82759	1.13606	-3.923	28	.001
	CG	-0.0963	0.46862	-1.068	26	.295
Extrinsic motivation: Post – Pre	TG1	0.8	0.77538	6.016	33	.000
	TG2	0.64	1.15382	3.282	34	.002
	TG3	0.85517	0.89586	5.141	28	.000
	CG	-0.03704	0.82655	-0.233	26	.818
Attention: Post – Pre	TG1	0.57059	0.84586	3.933	33	.000
	TG2	0.65143	1.0785	3.573	34	.001
	TG3	0.44828	0.87449	2.761	28	.010
	CG	0.0963	0.83826	0.597	26	.556
Satisfaction: Post – Pre	TG1	0.73529	0.8609	4.98	33	.000
	TG2	0.50286	1.21207	2.454	34	.019
	TG3	0.65517	0.85506	4.126	28	.000
	CG	-0.25926	0.89411	-1.507	26	.144

The above data shows the mean difference on students' motivation and motivation scales on learning geometry from pretest to posttest in the group. Students given a treatment with variation theory-based GeoGebra problem-solving instructional approaches had a mean difference of motivation, intrinsic value, self-efficacy, utility value, test anxiety, extrinsic motivation, and attention satisfaction of .47143, .63529, .67059, .64118, -.75294, .80000, .57059, and .73529, respectively. The standard deviation for this group was (.57657, .85631, .77519, .77347, .84575, .77538, .84586, and .86090), respectively, on each scale. The t-values for this group were 4.768, 4.326, 5.044, 4.834, -5.191, 6.016, 3.933, and 4.980, respectively. The group has df = 33, and its p-value was 0.000 for each motivation scale.

Variation theory-based problem- solving instructional approaches treatment can help students to score a mean difference of motivation, intrinsic value, self-efficacy, utility value, test anxiety, extrinsic motivation, and attention satisfaction of .36245, .53714, .42286, .32000, -.53714, .64000, .65143, and .50286, respectively. The standard deviation for this group was (.67877, 1.15379, 1.11437, .91677, 1.33617, 1.15382, 1.07850, and 1.21207) on each scale, respectively. The t-values for this group were 3.159, 2.754, 2.245, 2.065, -2.378, 3.282, 3.573, and 2.454, respectively. The group has df=34, and its p-values were .003, .009, .031, .047, .023, .002, .001, and .019, respectively.

The data also shows that students given a treatment with GeoGebra-assisted problem-solving instructional approaches had a mean difference of motivation, intrinsic value, self-efficacy, utility value, test anxiety, extrinsic motivation, and attention satisfaction of .39212, .44828, .68966, .47586, -.82759, .85517, .44828, and .65517, respectively. The standard deviation for this group was (.39197, .89707, .91275, .73371, 1.13606, .89586, .87449, and .85506) on each scale, respectively. The t-values for this group were 5.387, 2.691, 4.069, 3.493, -3.923, 5.141, 2.761, and 4.126, respectively. The group has df=28, and its p-values were .000, .012, .000, .002, .001, .000, .010, and .000, respectively.

Students under the comparison group (without any treatment) had a mean difference of motivation, intrinsic value, self-efficacy, utility value, test anxiety, extrinsic motivation, and attention satisfaction of (-.08783, -.10370, -.00741, -.20741, -.09630, -.03704, .09630, and -.25926), respectively. The standard deviations for this group were .49879, .99440, .83479, .90890, .46862, .82655, .83826, and .89411, respectively, on each scale. The t-values for this group were (-.915, -.542, -.046, -1.186, -1.068, -.233, .597, and -

1.507), respectively. The group had df=26, values were (0.369, .593, .964, .246, .295, .818, .556, .144), respectively.

From the data it is possible to say that there is a significant difference between students' pretest and posttest results on the overall motivation and each motivation scale on learning geometry for each intervention group since the p-value < 0.05. But there is no significant difference in their overall motivation and each motivation scale for the comparison group since the p-value > 0.05.

The above data shows only the existence of difference before the interventions and after the intervention of students' overall motivation and the subscales. Now it is time to check whether there are significant differences in students' motivation and components of motivation in learning geometry among the three interventions and control groups. To do so, ANCOVA was manipulated after the assumption was verified. The covariates are measured before the intervention manipulation to avoid scores on the covariates being changed by intervention, and they are not substantially connected with one another, as previously mentioned. Such things as observation independence, normality, homogeneity of variance, and linearity between covariates and dependent variables at each level of the independent variables are discussed at length in the preceding sections.

1. *Observation Independence*

   Since the students were selected from various Addis Ababa city administration sub-cities government school, the data are independent, and there is no reason to infer that one participant's score is consistently related to the scores of the other students.

2. *Normality of Dependent Variables*

   Since the instruments for MM tests were on an ordinal scale and the skewness of the distribution for all dependent variables and each component was between -1 and +1 (Pallant, 2007) as indicated in appendix-1, the data could be treated as normal for this study. These data also show that the variable is fairly normally distributed, allowing ANCOVA to be used.

3. Homogeneity of Variances

ANCOVA is predicated on the assumption that samples are drawn from populations with similar variances. This demonstrates that the scores of the variables are similar across all of the groups. This assumption is tested using Levene’s equality of variances test. It is expected to obtain a non- significant result for the dependent variable.

Table 4.29: - *Levene‘s Test of homogeneity of Variances for motivation and its scales*

Dependent Variables	***F***	***df1***	***df2***	***p***
Motivation	.789	3	121	.502
Intrinsic motivation	1.773	3	121	.156
Self-efficacy	1.338	3	121	.265
Utility value	.741	3	121	.530
Test anxiety	1.169	3	121	.325
Extrinsic motivation	4.032	3	121	.009
Attention	1.454	3	121	.230
Satisfaction	3.308	3	121	.023

The result showed that F (3, 121) = .789, P = .502 for the overall motivation; F (3, 121) = 1.773, P = .156 for intrinsic motivation; F (3, 121) = 1.338, P = .65 for self-efficacy; F (3, 121) = .741, P = .530 for utility value; F (3, 121) = 1.169, P = .3251 for test anxiety; F (3, 121) = 4.032, P = .009 for extrinsic motivation; F (3, 121) = 1.454, P = .230 for attention; and F (3, 121) = 3.308, P = .023 for the satisfaction score, as seen in table 4.29. This indicates that the variances of the groups were similar except for the result on extrinsic motivation and satisfaction since the P-value for all except the two subscales was greater than 0.05.

On the non-similar one, the violation of the Levene‘s test assumption is somewhat robust in instances where the groups‘ sizes are reasonably similar (Pallant, 2007). As a result, the violation of this assumption is not a concern in this study because the groups’ sizes were more or less similar since the groups’ samples were 27, 29, 34, and 35. This shows that the ratio of the highest group to the slowest is below 0.5. Thus, the assumption is valid.

4. *Linearity between the covariates and the dependent variables at each level of the independent variables.*

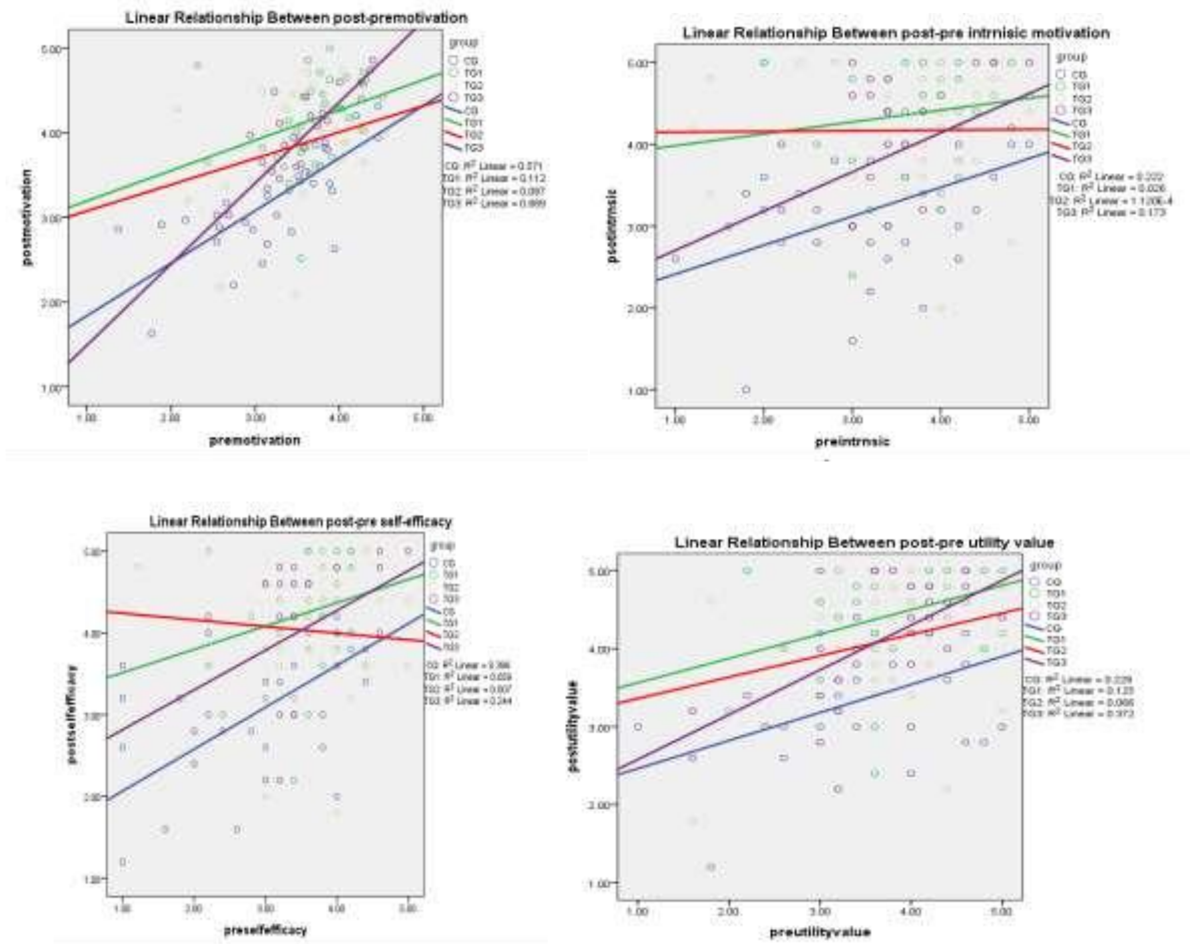


Figure 4.10a: *Scatter plot of Covariates and dependent variables*

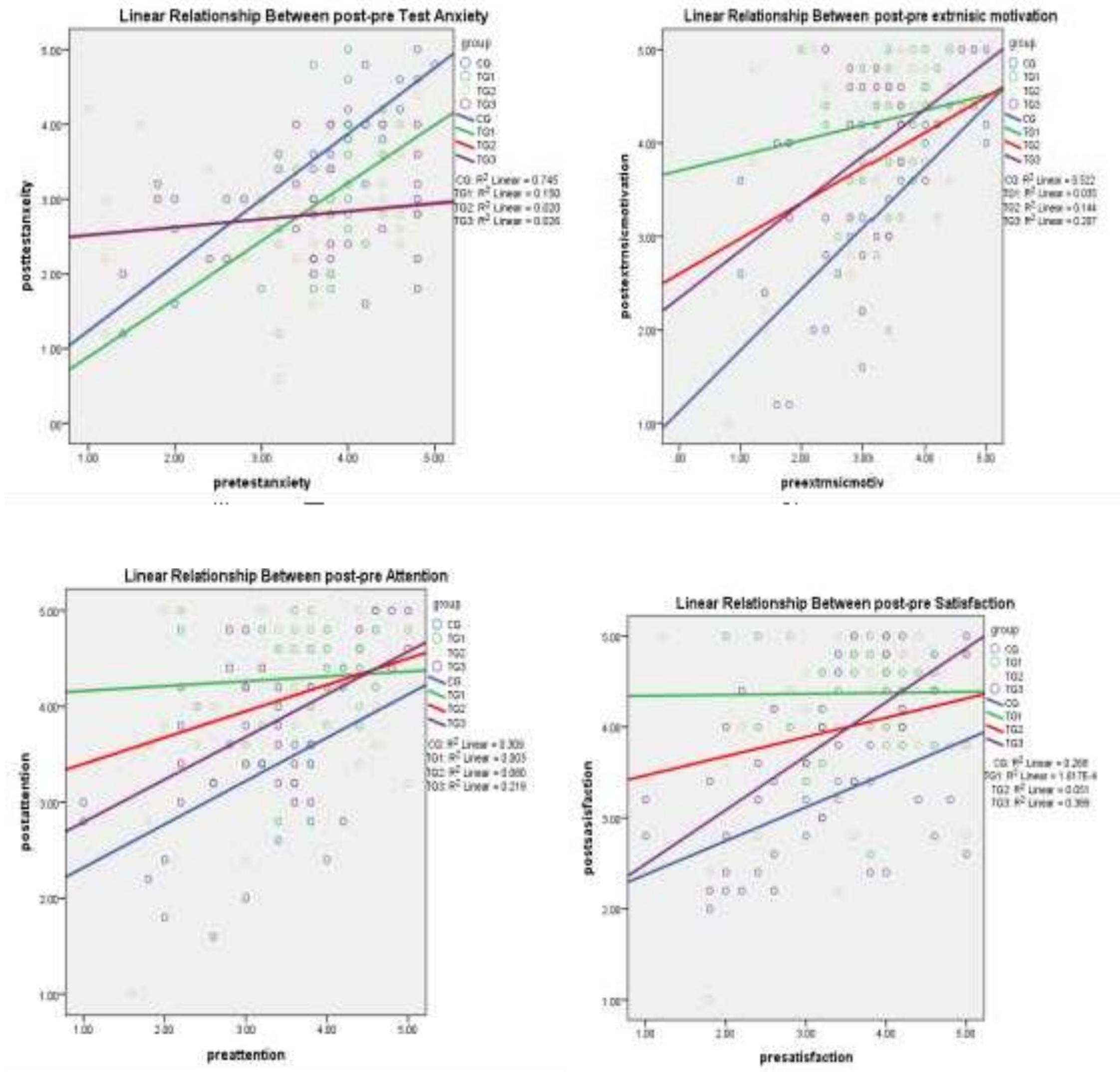


***Figure 4.10b**: Scatter plot of Covariates and dependent variables*

This assumption refers to the presence of a straight-line relationship between each pair of your dependent and independent variables (Pallant, 2011). A scatterplot between covariate and dependent variable at each level of the independent variable, as illustrated in Figures 4.10a and 4.10b, can be used to confirm this. Because the scatter plot shows no deviations from linearity, ANCOVA assumptions appear to be met for this study.

The following table describes the posttest result of students' motivation and subscales of motivation before any statistical adjustments and after students' pretest result on their motivation and subscales has been statistically controlled or accounted for.

Table 4.30: *Adjusted and Unadjusted Means and Variability on the four Groups for students' motivation and components of motivation in learning geometry Using Pre-test as a Covariate*

Variable	**Groups**		**Unadjusted**		**Adjusted**	
		N	**M**	**SD**	**M**	**SE**
Motivation	TG1	34	4.1580	.51607	4.049	.086
	TG2;	35	3.8490	.58569	3.854	.084
	TG3	29	3.7232	.67948	3.817	.093
	CG	27	3.3577	.62297	3.387	.096
Intrinsic value	TG1	34	4.3824	.62739	4.337	.126
	TG2;	35	4.1714	.70983	4.152	.124
	TG3	29	3.8621	.88334	3.894	.136
	CG	27	3.2444	.80638	3.292	.141
Self-efficacy	TG1	34	4.2471	.67700	4.193	.129
	TG2;	35	4.0286	.75169	3.965	.127
	TG3	29	3.8828	.90003	3.956	.140
	CG	27	3.1926	.84940	3.263	.145
Utility value	TG1	34	4.4353	.63527	4.404	.115
	TG2;	35	4.1314	.77451	4.093	.114
	TG3	29	4.0690	.80405	4.115	.125
	CG	27	3.4000	.74214	3.440	.130
Test Anxiety	TG1	34	3.1059	.91086	3.035	.139
	TG2;	35	2.7600	.85653	2.861	.138
	TG3	29	2.8000	.69076	2.800	.149
	CG	27	3.6667	.90299	3.625	.155
Extrinsic motivation	TG1	34	4.2647	.56078	4.166	.137
	TG2;	35	3.7943	1.03411	3.839	.134
	TG3	29	3.8621	.89937	3.975	.149
	CG	27	3.3333	1.04661	3.278	.153
Attention	TG1	34	4.3000	.59135	4.211	.127
	TG2;	35	4.0629	.88752	4.079	.123
	TG3	29	3.8690	.82510	3.882	.135
	CG	27	3.3259	.78279	3.402	.142
Satisfaction	TG1	34	4.3706	.55951	4.311	.130
	TG2;	35	3.9943	.93994	3.982	.128
	TG3	29	3.7172	.95994	3.846	.143
	CG	27	3.3407	.72920	3.293	.146

The above table shows that the adjusted and unadjusted mean of posttest results in students' overall motivation and motivation scales for each group. In order to see clearly the mean difference between adjusted to unadjusted the following bar graph was used.

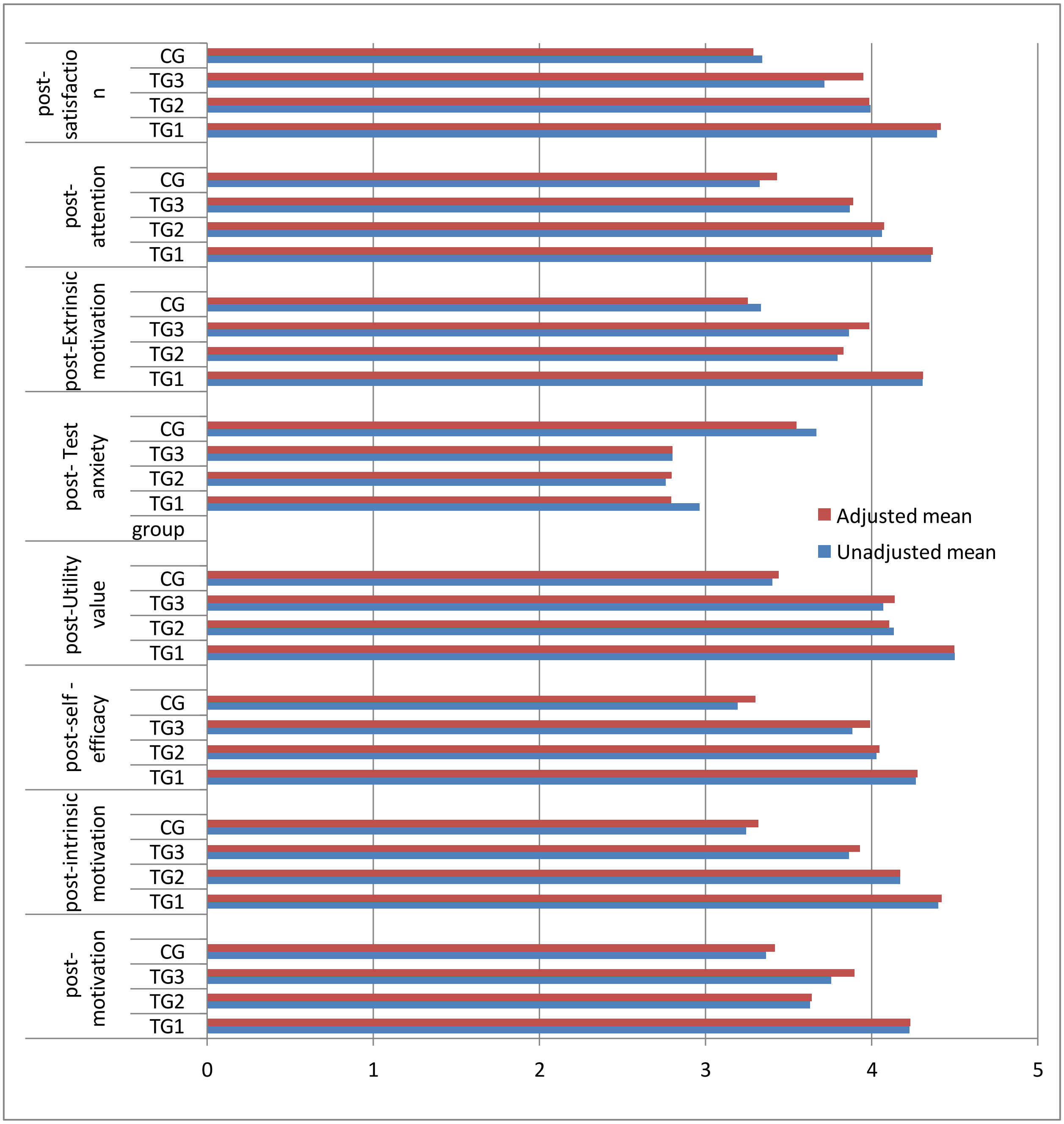


Figure 4.11 *adjusted and unadjusted mean of motivation and motivational scales*

From table 2.30 and figure 4.11, we can observe that whether we adjust the post result or not, the mean value of students' motivation and motivational scales are not changed. In order to see the effect of the pretest result on the outcome of the posttest, the following table shows clearly.

The following table indicates what looks like the effect of instructional approaches used on students' post-motivation and subscales of motivation, where their pre-motivation and subscales were controlled in order to answer the above research question.

Table 4.31: *Analysis of Covariance for students' motivation and components of motivation in learning geometry as a Function of the Three Groups, Pre- test as a Covariate*

Variable	Source	df	Ms	F	P	Eta^2
Motivation	*Pre-test*	1	9.213	41.359	.000	.261
	Groups	3	1.792	8.046	.000	.171
	Error	117	.223			
Intrinsic value	*Pre-test*	3	2.979	6.041	.018	.047
	Groups	1	2.844	5.767	.001	.134
	Error	117	.493			
Self-efficacy	*Pre-test*	1	3.231	6.440	.012	.052
	Groups	3	3.367	6.711	.000	.147
	Error	117	.502			
Utility value	*Pre-test*	1	7.935	18.461	.000	.136
	Groups	3	1.461	3.399	.020	.080
	Error	117	.430			
Test anxiety	*Pre-test*	1	13.35	27.485	.000	.190
	Groups	3	2.079	4.279	.007	.099
	Error	117	.486			
	Pre-test	1	12.74	20.169	.000	.147
	Groups	3	2.941	4.655	.004	.107
	Error	117	.632			
Attention	*Pre-test*	1	6.472	12.320	.001	.095
	Groups	3	1.707	3.250	.024	.077
	Error	117	.525			
Satisfaction	*Pre-test*	1	6.559	12.241	.001	.095
	Groups	3	2.760	5.151	.002	.117
	Error	117	.536			

As can be seen in Table 4.31, when the effect of the students' pre-motivation was controlled, there was a statistically significant difference between the groups' post-motivation and motivation scales. The above data shows that overall motivation, intrinsic value, self-efficacy, utility value, test anxiety, extrinsic motivation, attention, and satisfaction of students in learning geometry have F (3,117) = 41.359, 6.041, 6.44, 18.461, 27.485, 20.169, 12.32, and 12.241, with p=0.000, 0.018, 0.012, 0.000, 0.000, 0.000, 0.000, 0.001, and 0.001, respectively, on the pretest. This shows that the previous test for all scales has a significant effect on the posttest result since the p-value is >0.05.

As can be seen in Table 4.31, when the effect the intervention after the students' pre-motivation was controlled, there was a statistically significant difference between the groups' post-motivation scores when F (3,117) = 8.046, P = .000, and η^2 = .171. This revealed that the students' post-motivational scores were associated with the instructional methods used. In this study, the partial eta-squared value is .171, which equates to a small effect size according to Cohen's guidelines (1988). Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 17.1% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post-test mean scores.

When the effect of the students' pre-intrinsic motivation was controlled, there was a statistically significant difference between the groups' post-PSS scores when F (3,117) = 5.7600, P = .001, and η^2 = .134. This revealed that the students' post-intrinsic motivation scores were associated with the instructional methods used. In this study, the partial eta-squared value is .134, which equates to a small effect size according to Cohen's guidelines (1988). Furthermore, the value of η^2indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 13.1% of the variability in the dependent variable. This indicates that the teaching approaches used had a small effect on the groups' post-test mean scores.

Table 4.31also indicates that, when the effect of the students' pre-self-efficacy was controlled, there was a statistically significant difference between the groups' post-self-efficacy scores when F (3,117) = 6.711, P = .000, and η^2 = .147. This revealed that the students' post-self-efficacy scores were associated with the instructional methods used. In this study, the partial eta-squared value is .147, which equates to a small effect size (Louis Cohen & Morrison, 2007). Furthermore, the value of η^2 indicates that mean post- test scores differed across groups. This suggests the instructional approaches were responsible for 14.7% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post-test mean scores.

When the effect of the students' pre-utility value was controlled, there was a statistically significant difference between the groups' post-utility value scores when F (3,117) = 3.399, P = .000, and η^2 = .080. This revealed that the students' post-utility value scores were associated with the instructional methods used. In this study, the partial eta-squared value is .080, which equates to a small effect size. Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 8% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post- test mean scores.

Similarly, when the effect of the students' pre-test anxiety was controlled, there was a statistically significant difference between the groups' post-test anxiety scores when F (3,117) = 4.279, P = .007, and η^2 = .099. This revealed that the students' post-test anxiety scores were associated with the instructional methods used. In this study, the partial eta-squared value is 0.099, which equates to a small effect size. Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 9.9% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post-test mean scores.

On the other hand, as can be seen in Table 4.31, when the effect of the student' pre-extrinsic motivation was controlled, there was a statistically significant difference between the groups' post-extrinsic motivation scores when F (3,117) = 4.655, P = .004, and η2 = .107. This revealed that the students' post-extrinsic motivation scores were associated with the instructional methods used. In this study, the partial eta-squared value is .107, which equates to a small effect size. Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 10.7% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post-test mean scores.

When the effect of the students' pre-attention was controlled, there was a statistically significant difference between the groups' post-attention scores when F (3,117) = 3.25, P = .024, and η^2 = .077. This revealed that the students' post-attention scores were associated with the instructional methods used. In this study, the partial eta-squared value is .077, which equates to a small effect size. Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 7.7% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post-test mean scores.

If the effect of the students' pre-satisfaction was controlled, there was a statistically significant difference between the groups' post-satisfaction scores when F (3,117) = 5.151, P = .002, and η^2 = .117. This revealed that the students' post-satisfaction scores were associated with the instructional methods used. In this study, the partial eta-squared value is .117, which equates to a small effect size. Furthermore, the value of η^2 indicates that mean post-test scores differed across groups. This suggests the instructional approaches were responsible for 11.7% of the variability in the dependent variable. This indicates that the teaching approaches used had a considerable effect on the groups' post- test mean scores.

Since the data on table 4.31 didn't show between which groups a significant difference exists, the following data show among which groups the difference exists, keeping their pretest result as a covariate on the motivational scales. Then it is time to show between which groups the difference exists.

As indicated in Appendix-3 from the adjusted post-test score results in the Bonferroni test show that the mean difference on the post-overall motivation, Post-intrinsic, Post-self-efficacy, Post utility Value, Post-test anxiety, Post-extrinsic motivation, Post- Attention , Post-satisfaction betweenTG1 with CG was 0.776,1.103, 0.974, 1.058, -0.754, 1.053, 0.938 and 1.129 with p-value 0.000, 0.000, 0.000,0.001, 0.000, 0.000 and 0.000 respectively. the mean difference on the post-overall motivation, Post-intrinsic, Post- self-efficacy, Post utility Value, Post-test anxiety, Post-extrinsic motivation, Post- attention, Post-satisfaction between TG1 with TG2 was 0.313, 0.249, 0.232,0.391, - 0.001, 0.478,0.292 and 0.429 with p-value 0.055, 0.915 ,1.00, 0.093, 1.00, 0.101, 0.672 and 0.111 respectively. The mean difference on the post-overall motivation, Post- intrinsic, Post-self-efficacy, Post utility Value, Post-test anxiety, Post-extrinsic motivation, Post-attention, Post-satisfaction between TG1 with TG3 was 0.284, 0.491, 0.286, 0.358, -0.007, 0.323, 0.482 and 0.465 with p-value 0.159, 0.049, 0.752, 0.200, 1.00 0.767, 0.077 and 0.114 respectively.

Similarly, the adjusted post-test score results in the Bonferroni test show that the mean difference on the post-overall motivation, Post-intrinsic, Post-self-efficacy, Post utility Value, Post-test anxiety, Post-extrinsic motivation, Post- Attention, Post- satisfaction betweenTG2 with CG was0.463, 0.854, 0.742, 0.667, 0.-753, 0.576, 0.646 and 0.700 with p-value 0.001, 0.000, 0.001, 0.001, 0.000 ,0.035 0.005 and 0.002 respectively. The mean difference on the post-overall motivation, Post-intrinsic, Post- self-efficacy, Post utility Value, Post-test anxiety, Post-extrinsic motivation, Post- Attention, Post-satisfaction between TG2 with TG3 was -0.029, 0.242, 0.054, -0.033, - 0.006, -0.154, 0.190 and 0.036 respectively with p-value 1.000 for all scales.

On the other hand, the mean difference on the post-overall motivation, post-intrinsic, post-self-efficacy, post-utility value, post-test anxiety, post-extrinsic motivation, post- attention, and post-satisfaction between TG3 and CG was 0.492, 0.612, 0.700, 0.747, 0.730, 0.456, and 0.663, with p-values of 0.001, 0.011, 0.003, 0.001, 0.001, 0.007, 0.133, and 0.009, respectively.

The data indicate that there was a statistically significant difference in students' overall motivation and each motivational scale between treatment group-1 and the comparison group; treatment group-2 and the comparison group; and treatment group-3 and the comparison group since the p-value was <0.005 except for attention between TG3 and CG, where p=0.133. This means that each treatment group shows better overall motivation and motivation scales. Furthermore, the mean difference between treatment group 1 and the comparison group was the highest of all the other categories (i.e., from TG2 with CG and TG3 with CG).

Similarly, the data revealed that there is no significant difference between each treatment group on motivation and motivational scales since the p-value is >0.05, except treatment group 1 significantly differs on students' intrinsic motivation with a p-value of 0.049 differs from treatment group 3.

From classroom observation I have seen that students in the intervention-1, 2, and 3 classes can not only recall geometric formulas and facts related to the topic, but also they were highly motivated in order to explain step-by-step the ―why‖ behind those facts, connect concepts across different situations, apply knowledge to new problems in geometry, and articulate their reasoning in a clear and logical manner, showing a good attitude in order to understand and underlay principles involved in geometry. Students who have a good motivation in learning geometry can see the relationship between concepts and procedures and can give an argument to explain why some facts are the result of other facts. Similarly, the teacher who led interventions 1, 2, and 3 also agreed that when students are supported by mixed instruction, they can easily understand the concepts.

Students in intervention 1 and 2 respond to the positive feeling, and they become more motivated in order to solve problems and easily manipulate geometric concepts due to the flexible manipulation features of GeoGebra software, which guide them to a deeper grasp of geometric relationships and their properties. This is important for those who struggle with traditional static representations of geometric shapes.

4.3.3 Contributions of students' Conceptual Understanding and Motivation on Problem-Solving Skills in Learning Geometry

4. ***Are there any relationships between the mean gain of the variables problem-solving skills motivation and conceptual understanding in geometry in the intervention groups?***

In order to answer the question above first, let as check whether there are relationships between the mean gain of the variables conceptual understanding, problem-solving skills and motivation in geometry in the intervention groups Pearson correlation will employed after all of it assumption fit as shown below correlation.

Assumptions of Pearson's correlation

1. Each variable is continuous
2. Each observation have equal pairs of values
3. The data was normally distributed as shown before

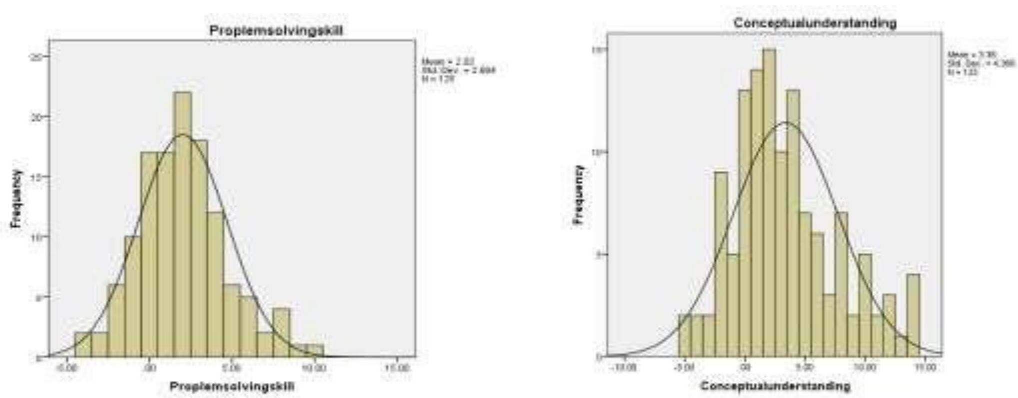


Figure 4.12a: *Normal curve of the mean gain of PSS and CU.*

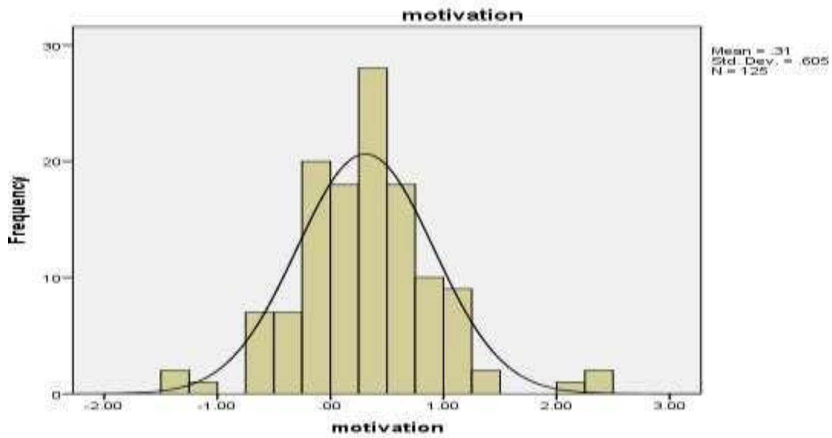


__Figure 4.12b:__ Normal curve of the mean gain of motivation

4. There is no outlier since the Z- score for all groups for each variable were less than which is accepted as no outlier (Sakti & Kartikaningdyah, 2018).as indicated on **Appendex-4**, above data shows that there is no outlier for each data.

5. Linearity between the variables shown below

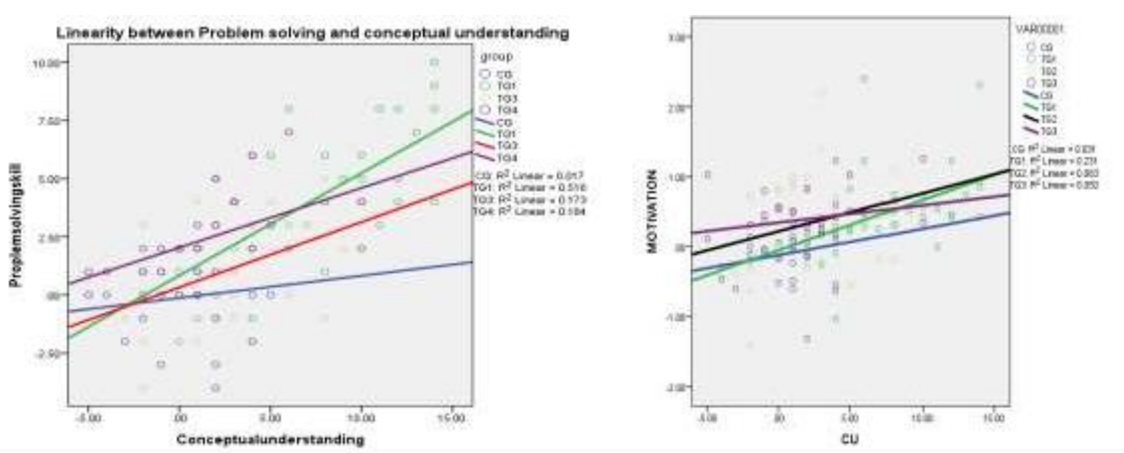


__Figure 4.13a:__ linearity between variables

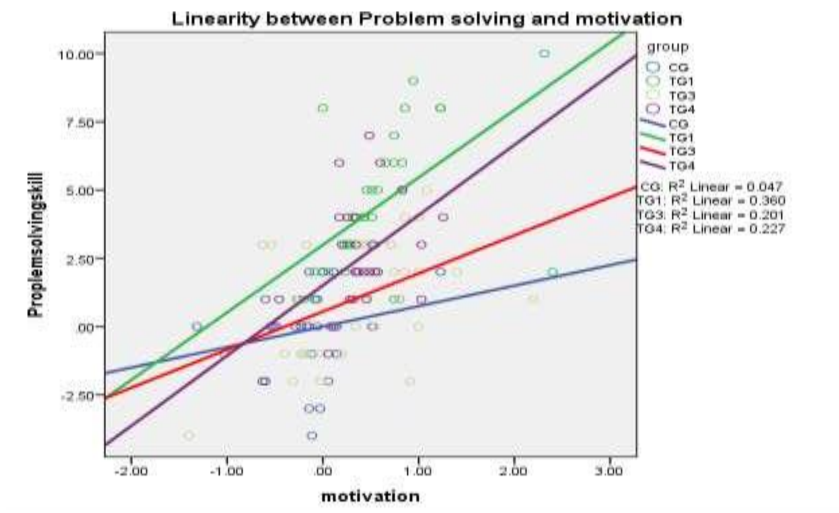


Figure 4.13b: *linearity between variables*

In the following table a Pearson correlation were used in order to show a statistical measure of the strength and direction of the linear Relationships between conceptual understanding, problem-solving skills, and motivation in learning geometry at each group.

Table 4.32: Relationships between conceptual understanding, problem-solving skills, and motivation in learning geometry at each group

		Pearson Correlations I				
GROUP	**Variable**	**PSS**	**CU**	**Motivation**	**Mean**	**SD**
CG	PSS	1	.131	.218	-.0741	1.61545
	CU	.131	1	.176	.5556	2.20721
	Motivation	.218	.176	1	-.0974	.46916
TG1	PSS	1	.698**	.600**	4.2353	2.64069
	CU	.698**	1	.480**	7.7059	4.30313
	Motivation	.600**	.480**	1	.5042	.64146
TG2	PSS	1	.416*	.448**	1.0571	2.11358
	CU	.416*	1	.251	2.6000	3.13613
	Motivation	.448**	.251	1	.3624	.67877
TG3	PSS	1	.429*	.477**	2.5172	2.11492
	CU	.429*	1	.223	1.8621	3.54284
	Motivation	.477**	.223	1	.3921	.39197

** Correlation is significant at the 0.01 level (2-tailed).

*Correlation is significant at the 0.05 level (2-tailed).

The above data shows that the relation between PSS and motivation for the comparison group, the variation theory-based GeoGebra problem-solving instructional approaches group, the variation theory-based problem-solving instruction group, and the GeoGebra-assisted instructional approach group was 0.218, 0.600, 0.448, and 0.477, respectively. Similarly, the data shows that the relation between PSS and conceptual understanding in learning geometry for the comparison group, the variation theory-based GeoGebra problem-solving instructional approaches group, the variation theory-based problem-solving instruction group, and the GeoGebra-assisted instructional approach group was 0.131, 0.698, 0.416, and 0.429, respectively.

Furthermore, the data shows that the relation between motivation and conceptual understanding in learning geometry for the comparison group, the variation theory-based GeoGebra problem-solving instructional approaches group, the variation theory-based problem-solving instruction group, and the GeoGebra-assisted instructional approach group was 176, 0.480, 0.251, and 0.223.

5. ***What are the contributions of motivation and conceptual understanding on problem-solving skills in learning Geometry in the intervention groups?***

Now it is time to find the contributions of the mean gain of conceptual understanding, and motivation and on problem solving skills in learning Geometry in the intervention groups. To do so multiple regression analysis was used since multiple regressions can be used to address a variety of research questions. It can tell you how well a set of variables is able to predict a particular outcome. To run the statistics first le as check the assumptions

Assumptions of Multiple Linear Regressions

Preliminary analyses were also conducted in order to ensure no violation of the assumptions of multiple regressions.

1. *Sample size and independence of Observation*

For this research, the sample size was enough to conduct the analysis for each variable and students completed the residuals individually to assure independence of observation assumption. Barbar.G & Linda.S, (1983) give a formula for calculating sample size requirements, considering the number of independent variables that you wish to use: $N > 50 + 8m$ (where m = number of independent variables). In this study $125>50+8*4=$ (82). Hence sample size and independence of observation assumption was not violated to run multiple regressions.

2. *Multiple collinearity*

Multiple regression works best when the dependent variables are moderately correlated (Pallant, 2011). Correlation was computed for all dependent variables (mean gain of –CU, mean gain PSS, mean gain motivation) to check multi-collinearity assumptions. Thus, there was no strong relationship between them ($r < .7$) as indicated table 2.27 it fit to the assumption to run multiple regressions.

3. *Linearity*

To run a multiple regression, the residuals should have a straight-line relationship with predicted dependent variable scores In scatter Plot, the residuals have a straight-line relationship with predicted dependent variable scores (See Figure: 4.15a and 4.15b). Accordingly, the residuals were linear with the predicted dependent variable scores and hence linearity assumption was not violated to run multiple regressions.

4. *Absence of outlier*

To run a multiple regression, another assumption is that there should be absence of in the residuals. Outliers can be checked using visual inspection of the scatter plots of the cases with standardized residuals falling outside the range of ± 3 z-score units on the dependent variables should the presence of outliers (Pituch and Stevens,2015). As observer Appendex-4 standardized residuals falling inside the range of ± 3 z-score units on the dependent variables. Thus, there were no outliers for residuals.

5. Normality

To run a multiple regression, the residuals should be normally distributed about the predicted dependent variable scores. Thus, the normal distribution about the predicted dependent variable scores can be checked by visual inspection of the Normal Probability plot(P-P) of the Regression Standardized Residual. In Normal P-P Plot, the score points lie in a reasonably straight diagonal line from bottom left to top right (See Figure: 4.14). This would suggest no major deviations from normality (Barbar.G & Linda.S, 1983). Accordingly, the residuals were approximately normally distributed about the predicted dependent variable scores and hence normality assumption was not violated to run multiple regressions**.**

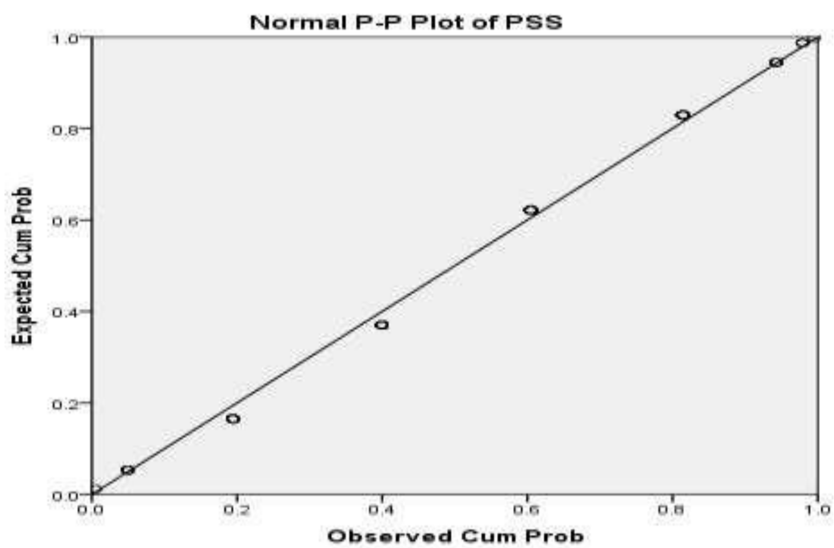


Figure 4.14*: Normal P-P Plot Regression Standardized Residual for mean gain PSS*

6. homoscedasticity:

The variance of the residuals about predicted DV scores should be the same for all predicted scores. This assumption can be checked from the residuals scatterplots which are generated as part of the multiple regression procedure. As shown below there are points equally distributed above and below zero on the X axis, and to the left and right of zero on the Y axis. Thus, it met the assumption.

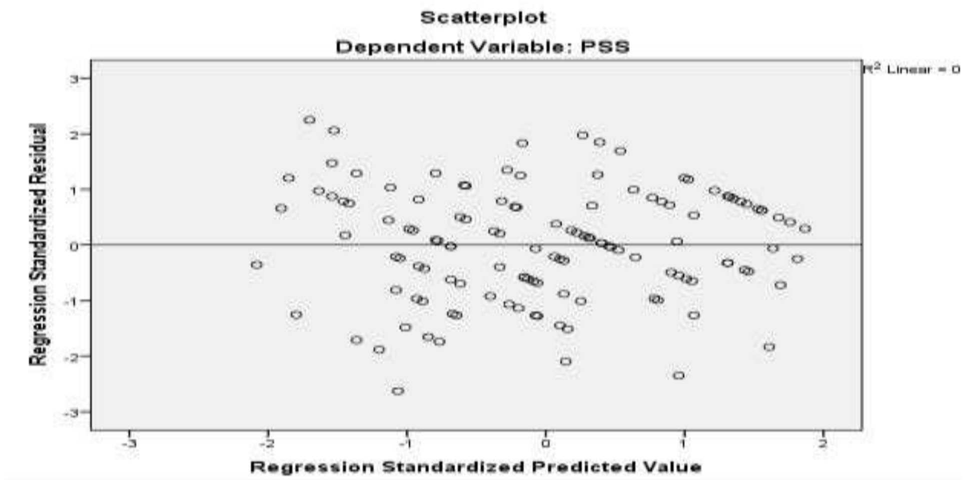


Figure 4.15: *scatter plot output for homoscedasticity test*

The following table summarizes the regression result how students conceptual understanding and their motivation in learning geometry affect students Problem solving skills and the factual noteworthiness of these connections exist or not under the group.

***Table 4.33**: Multiple regression analysis for Variation theory based GeoGebra problem- solving instructional approaches group*

Multiple R = 0.823			R^2 = 0.5958		
	Sum of Squares	**df**	**Mean Square**	**F**	**P**
Regression	138.171	2	69.085	23.292	.000
Residual	91.947	31	2.966		
Total	230.118	33			

Variables in the Equation

Variables	**R**	**B**	**Std. Error**	**β**	**T**	**P**
Constant		.903	.613		1.473	.151
Conceptual Understanding	0.698	.343	.079	.569	4.319	.000
Motivation (Mot)	0.600	1.364	.533	.331	2.560	.016

In Table 4.33, the ANOVA table F value indicates that the multiple correlations R were significant. That is, the contribution of variables CU and motivation collectively had a significant effect on students' problem-solving skills in learning geometry since p=0.000. Similarly, the multiple linear regression analysis indicates that the contribution of conceptual understanding had a significant effect on the students' problem-solving skills ($\beta = .569$, $p = .000$, < 0.05) and the contribution of students' motivation had a significant effect on the students' problem-solving skills in learning geometry ($\beta = 0.331$, $p = .016$, $< .05$). This indicates that both variable conceptual understanding and students' motivation could predict the students' problem-solving skills in learning geometry while the intervention was given (variation theory-based GeoGebra problem-solving instructional approaches).

From the model summary, since $R^2 = 0.600$ for PSS, it can explain 60% of the variance in CU; motivation had an effect on students' problem-solving skills. The percent effect or contribution of the variable's CU and motivation on students' problem-solving skills in learning geometry is given by:

$$R^2 = (\beta \text{cu} \ r \text{cu} + \beta \text{Mot} \ r \text{Mot}) \times 100\%$$

$$59.58\% = (.569 \times .698 + 0.331 \times 0.6) \times 100\%$$

$$= 39.72\% + 19.86\%$$

Therefore, the contribution of students' conceptual understanding enhanced their problem solving by 39.72%, and the contribution of students' motivation in learning geometry enhanced their problem-solving skills by 19.86%. This showed that, almost, students' CU and their motivation in learning geometry have a big contribution to students' PSS while the intervention was given.

The following table summarizes the regression result of how students' conceptual understanding and their motivation in learning geometry affect students. Problem-solving skills and the factual noteworthiness of these connections exist or not under the group.

Table 4.34: *Multiple regression analysis for variation theory-based problem-solving instruction group*

Multiple R = 0.547					R^2 = 0.299	
ANOVA Table	**Sum of Squares**	**Df**	**Mean Square**	**F**	**P**	
Regression	45.431	2	22.715	6.828	.003	
Residual	106.455	32	3.327			
Total	151.886	34				
Variables in the Equation						
Variables	**R**	**B**	**Std. Error**	**β**	**t**	**P**
Constant		.075	.416		.181	.858
Conceptual Understanding	0.416*	.219	.103	.324	2.121	.042
Motivation	.448**	1.141	.476	.367	2.397	.023

In Table 4.34, the ANOVA table F value indicates that the multiple correlations R were significant. That is, the contribution of variables CU and motivation collectively had a significant effect on students' attitude towards students' problem-solving skills in learning geometry since p=0.003, which is <.05. Similarly, the multiple linear regression analysis indicates that the contribution of conceptual understanding had a significant effect on the students' problem-solving skills (β = .324, p = .042, < 0.05) and the contribution of students' motivation had a significant effect on the students' problem- solving skills towards students' problem-solving skills in learning geometry (β = 0.367, p = .023, < .05). This indicates that both variable conceptual understanding and students' motivation could predict the students' problem-solving skills in learning geometry while the intervention was being given (variation theory-based problem-solving instruction approaches).

From the model summary, since R^2 = 0.299 for PSS, it can explain 29.9% of the variance in CU; motivation had an effect on students' problem-solving skills collectively. The percent effect or contribution of the variable's CU and motivation on students' problem-solving skills in learning geometry is by:

$$R^2 = (\beta_{cu}\, r_{cu} + \beta_{Mot}\, r_{Mot}) \times 100\%$$

$$29.9\% \cong (0.342 \times 0.416 + 0.364 \times 0.448) \times 100\%$$

$$\cong 13.48\% + 16.44\%$$

Therefore, the contribution of students' conceptual understanding enhanced their problem solving by 13.48%, and the contribution of students' motivation in learning geometry enhanced their problem -solving skills by 16.44% This showed that almost, students' motivation had better contribution for students PSS than their CU in learning geometry. While variation theory-based problem-solving instructional approaches was giving.

The following table summarizes the regression result how students conceptual understanding and their motivation in learning geometry affect students Problem solving skills and the factual noteworthiness of these connections exist or not under the group.

Table 4.35*: Multiple regression analysis for GeoGebra based problem solving instructional approach group*

Multiple R = 0.581					$R^2 = 0.337$	
ANOVA Table	**Sum of Squares**	**df**	**Mean Square**	**F**	**P**	
Regression	42.209	2	21.104	6.608	.005	
Residual	83.032	26	3.194			
Total	125.241	28				
Variables in the Equation						
Variables	**R**	**B**	**Std. Error**	**β**	**t**	**p**
Constant		1.291	.485		2.662	.013
Conceptual Understanding (CU)	.429*	.203	.098	.340	2.076	.048
Motivation (Mot)	.477**	2.163	.884	.401	2.447	.021

**. Correlation is significant at the 0.01 level (2-tailed

In Table 4.35, the ANOVA table F value indicates that the multiple correlations R was significant. That is, the contribution of variables CU, motivation collectively had a significantly effect on students' problem-solving skills in learning geometry since p=0.000 which is <.05. Similarly, the multiple linear regression analysis indicate that the contribution of conceptual understanding had a significant effect on the students' problem-solving skills (β = .340, p = .048, <0.05). on the other hand, the contribution of student's motivation had no a significant effect on the students' problem-solving skill towards students problem-solving skills in learning geometry (β = 0.401, p = .021, < .05). This indicates that both variable (conceptual understanding and motivation) could predict the students' problem-solving skills in learning geometry while the intervention was giving (GeoGebra based problem solving instruction approaches).

From the model summary, since $R^2 = 0.337$ for PSS, it can explain 33.7 % of the variance. CU and motivation had an effect on students' problem-solving skills collectively. The percent effect or contribution of the variable's CU and Motivation on students' problem-solving skills in learning geometry is

$$R^2 = (\beta_{\mathrm{CU}} * r_{\mathrm{cu}} + \beta_{\mathrm{Mot}} \; r_{\mathrm{Mot}}) \text{ x } 100\%$$

$$33.7\ \% \cong (0.340*.429 + 0.401*0.477) \text{ x}100\%$$

$$\cong 14.59\% + 19.13\%$$

Therefore, the contribution of students' conceptual understanding enhanced their problem solving by 14.59 %, and the contribution of students' motivation in learning geometry enhanced their Problem -solving skills by 19.13 %. This showed that students' motivation had better contribution for students PSS than their CU in learning geometry while GeoGebra based problem solving instruction approaches was giving.

The following table summarizes the regression result how students conceptual understanding and their motivation in learning geometry affect students Problem solving skills and the factual noteworthiness of these connections exist or not under the group.

Table 4.36: *Multiple regression analysis for comparison groups group*

Multiple R = 0.237					R^2 = 0.056	
ANOVA Table	**Sum of Squares**	**Df**	**Mean Square**	**F**	**P**	
Regression	3.813	2	1.907	.715	.500	
Residual	64.038	24	2.668			
Total	67.852	26				
Variables in the Equation						
Variables	**R**	**B**	**Std. Error**	**β**	**t**	**p**
Constant		−.045	.335		−.136	.893
Conceptual Understanding	0.131	.070	.147	.095	.473	.641
Motivation	0.218	.692	.694	.201	.998	.328

In Table 4.36, the ANOVA table F value indicates that the multiple correlations R were not significant. That is, the contribution of variables CU, motivation collectively had not a significantly effect on students' problem-solving skills in learning geometry since p=0.500 which is >.05. Similarly, the multiple linear regression analysis indicate that the contribution of conceptual understanding had not a significant effect on the students' problem-solving skills (β = 0.095, p = 0.641, >0.05). on the other hand, the contribution of students' motivation had no a significant effect on the students' problem-solving skill towards students' problem- solving skills in learning geometry (β = 0. 201, p = .0328, > .05). This indicates that one both variable (conceptual understanding and students' motivation) couldn't predict the students' problem-solving skills in learning geometry while no intervention was giving (comparison groups).

From the model summary, since R^2 = 0.056 for PSS, it can explain 5.6 % of the variance CU, motivation had an effect on students' problem-solving skills collectively. The percent effect or contribution of the variable's CU and Motivation on students' problem- solving skills in learning geometry is

$$R^2 = (\beta_{\text{CU}} * r_{\text{cu}} + \beta_{\text{Mot}}\ r_{\text{Mot}}) \times 100\%$$

$$5.6\% \cong (0.095*0.131 + 0.201*0.218) \times 100\%$$

$$\cong 1.24\% + 4.38\%$$

Therefore, the contribution of students' conceptual understanding enhanced their problem solving by 1.24 %, and the contribution of students' motivation in learning geometry enhanced their Problem -solving skills by 4.38 % This showed that students' CU and their motivation in learning geometry had almost the same contribution for students PSS While no treatment were given.

4.4 Discussions on the Findings

The quantitative and qualitative results were reported in previous sections, and these two types of data were triangulated together to gain a thorough picture of the subject under investigation. The effects of variation theory-based GeoGebra-assisted problem-solving approaches on learning geometry are discussed one by one in the following sections.

4.4.1 Students Geometry Conceptual Understanding

After the intervention, it was discovered that the students' conceptual understanding had improved in all three groups. This increase in student scores from before to after teaching was an expected result of the study, given each of the three groups practiced using a specific instructional approach, and it was statistically significant for all three groups.

The results of an ANCOVA analysis of the students' post-intervention CU scores show that there is a statistically significant mean difference between the groups when adjusted for pre-intervention CU scores. For this section of the study, the eta-squared value for the groups was 0.513, or 51.3%. This indicated that the instructional methods used had a significant effect on the learners' conceptual understanding mean score.

Except for intervention groups 1, 2, and 3, the results of the Bonferroni test adjusted for pre-test mean scores revealed statistically significant differences between each of the groups. This research suggested that teaching geometric figures and measurement utilizing variation theory problem-based, GeoGebra problem-based, and variation theory-based GeoGebra-assisted problem-solving approaches are a better method than using the conventional instructional approaches.

Furthermore, the variation theory-based GeoGebra-assisted problem-solving approaches, or intervention 1, revealed significant differences between each of the other two interventions.

Furthermore, students taught with variation theory-based GeoGebra-assisted problem-solving approaches (Ma = 22.35) had higher adjusted post-mean scores than students taught with a problem-based learning approach (Ma = 19.56). Furthermore, students taught with visualization techniques (Ma = 14.4) had higher adjusted post-mean scores than students taught with a variation theory problem-based learning approach and GeoGebra problem-solving approaches with Ma = 8.642 and Ma = 8.431, respectively. The study findings revealed a significant difference in students' conceptual understanding in favor of the group who were taught using integrated instruction rather than a variation of the theory problem-based learning method or the GeoGebra problem-based method. This result corroborated the findings of treatment group 2 (variation theory-assessed instruction); in line with this finding, some scholars also support its significance and effect on students' conceptual understanding.

Visualizing geometric figures repeatedly helps students have good conceptual understanding. At the time of the lesson delivery, students were surprised at the formation of the plane figures. Huang and Leung (2005) point out that the function of procedural variation is to help learners acquire knowledge stepwise, develop learners' experience in problem-solving progressively, and form well-structured knowledge. Furthermore, the focus of students' attention should always be on the critical features of the object of learning (Lapinid & Mendoza, 2024). To become aware of something, the students have to show it in the background. Similarly, the Jing et al. (2017b) study confirmed the effectiveness of variation theory as a means for better achievement.

In Tamam and Dasari (2021), GeoGebra will provide students with a virtual display that simulates a real visual environment, making geometric concepts easy to understand. In addition, compared to learning without geometry, GeoGebra helps students comprehend geometry concepts more fully (Jelatu, 2018). This obviousness was further supported by other research, which found that students' mathematical comprehension was greatly impacted by visualizing and critical thinking (Suciati et al., 2022).

The study conducted by Kassa & Ding (2019) also revealed that students' understanding of the key concepts of the learning object—the relationships between rectangles' area and perimeter—improved significantly when problem-based tasks featuring patterns of variation and invariance were used.

In this study, students' conceptual understanding improved while the lesson was run with variation theory and geometry-assisted problem-solving approaches, and more results were shown when the hybrid approaches were delivered. The students' conceptual understanding was also good when the lesson was delivered by integrating variation theory with geometry. Since both GeoGebra and variation theory enhance students' conceptual understanding when the teacher implements them effectively, at the time of intervention, the teacher becomes confused for the first time. But for the next class, he was familiar, and I have observed that students also participated actively and were eager to learn.

In a similar view, Cheng (2016) contends that variation theory directs educators toward appropriate pedagogical design that can assist students in identifying the learning objective and producing successful outcomes.

One of the reasons why the variation theory problem-based, GeoGebra problem-based, and variation theory-based GeoGebra-assisted problem-solving approaches score higher than the conventional method in that students are seen as more engaged when they have the opportunity to learn utilizing the variation theory problem-based, GeoGebra problem-based, and variation theory-based methods. GeoGebra-assisted problem-solving approaches. Students can more actively participate in the activities presented to them during the implementation of this instructional approach, which is projected to improve the students' conceptual understanding. The differences in performance between the three groups after the intervention are expected to be derived from the teaching approaches used in the lesson of geometric figures and measurement.

Participants in the three intervention groups said that using the variation theory problem-based, GeoGebra problem-based, and variation theory-based GeoGebra-assisted problem-solving approaches, as well as connecting the content of the lesson to real-life applications, were key factors in their better performance.

Other aspects of the teaching materials and approaches that may have benefited students' performance were linking existing knowledge to lesson content and relating content to practical application. For effective instruction, the dual approach suggested connecting previously taught concepts to new geometrical concepts or registers. These instructional approaches have an effect on the start of classes with cognitive activities such as treatment and conversion that students must understand. These basics have provided students with chances to clarify and explore the applicability of mathematical concepts.

Students can use variation theory-based GeoGebra-assisted problem-solving approaches to help them translate abstract mathematical concepts into more concrete forms and connect new information to previous knowledge. The concrete representations of abstract problems in geometry, as revealed in the interview, were an important part of this method, in addition to continuously engaging students in a difficult task and increasing opportunities for sharing their practical experience and discourse, as well as developing social and thinking abilities. Such a setting offers higher understanding and engagement. As a result, students have more opportunities to get insightful learning and understand the instruction of geometry (Baye et al., 2021; Mendoza & Lapinid, 2024). This may help students understand the concepts of geometry in a better way.

Another challenge is that students may have low conceptual understanding due to the predetermined content delivery in the curriculum, which may have prevented high-achieving students from receiving enrichment. This means that even in the students' textbook, the importance of concepts was undervalued, as the book begins the presentation of the lesson with a definition rather than a real-life application of the notion. As I observed the students' textbook, it was not written in a way that would help them enhance their visualizing skills. They should not simply have the students repeat things again and over, as this will only result in superficial learning.

In Ethiopia, on the other hand, students were regularly evaluated utilizing standard quantitative accomplishment appraisals in lesson exams. As a result, students are new to the two-tiered tests that are utilized to evaluate students' CU. The CU test was troublesome for a few groups of students in three intervention groups during the classroom observation. The types of conceptual questions might have affected students' securing of concepts, as they were commonly evaluated with accomplishment tests that center on control of conditions instead of concepts. As a result, students may have had trouble working with actual get for the primary time and so may not have given adequate callings to bolster their first-tier reactions. As evidence for this, the students scored the highest on the first-tier items but not on the second tier. This suggests that students may memorize the correct response for the content item without understanding the underlying reasons. These findings are supported by Indratno et al. (2023): there is a limitation in answering the question and selecting a reason behind the justification of the concept.

Furthermore, conventional instructional techniques of teaching geometry do not connect geometrical concepts to real-world applications. Thus, the low learning performance in geometry of the comparison groups might be explained by conventional instructional techniques of teaching geometry, which typically consist of the transfer of abstract notions and occasionally use variation theory-based GeoGebra-assisted problem-solving instruction, variation theory problem-solving instruction, and GeoGebra problem-solving instruction. To internalize curriculum materials and textbook concepts, many students focus on memorization of concepts without understanding them in detail with the intention of passing exams, as indicated (Prabha, 2020).

4.4.2 Students Geometry Problem Solving Skills (PSS)

The effects of variation theory-based GeoGebra-assisted problem-solving instruction, variation theory problem-solving instruction, and GeoGebra problem-solving instruction on students' PSS in learning geometry were also investigated in this study. The group that was taught using variation theory-based GeoGebra-assisted problem-solving instruction, variation theory problem-solving instruction, and GeoGebra problem-solving instruction had higher adjusted post-PSS mean scores than the group that was taught with the conventional instruction. Thus, variation theory-based GeoGebra-assisted problem-solving instruction improved students' PSS. the result of the study is supported by Baye et al.'s (2021) finding that describing instruction with integrated instruction is more successful than conventional teaching. The teacher's comments also suggest that groups of students who were taught geometric figures and measurement and dealt with variables with the variation theory-based GeoGebra-assisted problem-solving instruction had better PSS than conventional approaches because they were able to apply it to real-life applications.

The implementation of problem-based tasks with patterns of variation and invariance helped students' ability to understand the critical aspects of the object of learning (the relationships between the area and perimeter of rectangles). Kassa and Ding (2019) and Series (2020) also support the above finding that the problem-solving ability and attitude toward mathematics of students who were taught by the discovery learning model assisted with GeoGebra media were better than the problem-solving ability and attitude toward mathematics of students who were taught by conventional learning.

Similarly, Sofyan et al. (2021) show that the use of GeoGebra can improve most of the students' problem-solving ability. For example, in determining the area between two curves and determining the volume of rotating objects and creating simulation objects. It enhances students' mathematical abilities, such as mathematical proof abilities, mathematical reasoning abilities, and mathematical problem-solving abilities (Murni et al., 2017).

Generally, the adjusted mean of the students in each of the three intervention groups was higher than the scores of the students in the comparison group. This variance was also found to be statistically significant. In line with the finding, the teachers who took part in the interview acknowledged the importance of variation theory-based GeoGebra-assisted problem-solving instruction, variation theory problem-solving instruction, and GeoGebra problem-solving instruction in developing students' problem-solving skills.

Moreover, the study findings revealed that the majority of students did not do better on the post-PSS test and had difficulty in understanding mathematical problems that affect the problem-solving process as well as the CU of learning geometry. Consequently, the students do not understand the problems' keywords and are unable to translate them into mathematical sentences; they are unable to visualize what to assume and what concepts from the problem are required to solve the problem; they tend to guess the answer without any thought process; they are impatient and dislike reading mathematical problems, particularly long ones; and they are impatient and dislike reading long problems. The findings of this study are consistent with Kusuma et al.'s (2023) findings, which found that students have trouble understanding mathematical problems, which affects the problem-solving process. This is also observed in the students' post-mean scores for each of the four phases of the problem-solving indicators employed in the study. This may explain why most students did not adequately prepare for the post-PSS test.

The conventional instructional approach of teaching and learning geometry follows a procedure of activities and demonstrations to enhance students' ability to solve problems. For improving the math teaching and learning processes at the middle schools, Silmi Juman et al. (2022) pointed out how the methodology of teaching that is comprised of the use of problem-solving should be restructured. Furthermore, it forces students to create an abstract representation of the problem without a thorough understanding of the concepts. This forces the students to focus solely on the final response. This could possibly be the reason why the majority of students did not prepare adequately for the post-PSS test. Hence, students employ tactics that allow them to identify the solution path by manipulating basic equations without understanding the concepts.

In Ethiopia, students are taught to solve mathematics problems using conventional problem-solving techniques (Brhane, 2012). Problems are solved using conventional problem-solving strategies in the currently in-use student textbook.

Furthermore, most students in Ethiopia had difficulty applying the phases of problem-solving in mathematical problems because they considered getting the final answers to be the most important issue. As a result, most students did not attempt to re-examine the final result and give it meaning or interpretation once they had finished solving the issue. Thus, finding and reporting solutions to mathematical problems simply served to exacerbate the students' low performance (Ayele & Dad, 2016). This could possibly be the reason why the majority of students did not prepare adequately for the post-PSS test.

Despite the fact that much research has utilized various approaches and drawn attention to several crucial issues in order to boost students' PSS, it is essential to inform students about the role of each of the problem-solving phases. Thus, students must be able to solve a variety of problems in order to increase their ability to solve problems through variation theory-based GeoGebra-assisted problem-solving instruction, variation theory problem-solving instruction, and GeoGebra problem-solving instruction. Teachers and students, in particular, should be trained in order to improve their understanding of the phases of the problem-solving process. The study carried out by Aydogdu & Kesan (2014), Bal & Or (2022), and Ramdjid et al. (2022) indicated that students' understanding of the phases and roles of the indicators of PSS used for solving problems would assist them to tackle complicated, hard, and poorly structured problems in an appropriate manner in the future.

The findings obtained from this study also confirm this fact. Because when students came across simple problems, they were observed while solving the problems using the conventional strategies of problem-solving without any difficulty. Thus, when the problem required them to apply the concepts they had learned, the majority of the students, particularly those in the comparison group, were unable to solve it. Thus, students were seen having difficulty when solving problems that required conceptual understanding in addition to carrying out the equation, as they failed to apply the concept they had learned while solving the problem. Therefore, the findings of this study highlight the importance of emphasizing the understanding of concepts needed to solve problems in order to prevent students from viewing geometric problem-solving as a hunt for formulas that are difficult to recall after class.

As a result, both students and teachers need to be familiar with the phases utilized to solve problems, as the phases focus on the conceptual nature of the problems. In general, the findings of this study showed that using single instructional approaches alone had no discernible effect on students' PSS without first teaching them about the phases of problem-solving.

4.4.3 Students Motivation in Learning Geometry

The study's other major focus was on how the variation theory problem-based approaches, the GeoGebra problem-based approaches, and the variation theory GeoGebra-assisted problem-based approaches improved students' motivation toward geometry learning.

The study indicated students' overall motivation, which means that the variation theory-based GeoGebra problem-solving instructional approach (TG1), the variation theory-based problem-solving instructional approach (TG2), and the GeoGebra problem-solving instructional approach (TG3) had a more significant impact on students' motivation for learning geometry than the conventional method. Moreover, the variation theory-based

GeoGebra problem-solving instructional approach (mixed approaches) contributes a higher mean difference than the other two intervention groups to students' motivation. On the other hand, the GeoGebra problem-solving instructional approach (TG3) contributes a higher mean difference to student motivation than the variation theory-based problem-solving instructional approach (TG2). The result shows that integrating technology with other approaches yields students who have better motivation in learning geometry.

Students under all treatment groups had better motivation components—these are intrinsic motivation, self-efficacy, utility value, test anxiety, extrinsic motivation, attention, and satisfaction in learning geometry—than the comparison group students. Moreover, TG1 contributes the highest mean difference compared to the other two intervention groups (TG2 and TG3) on students' intrinsic motivation, self-efficacy, utility value, test anxiety, extrinsic motivation, attention, and satisfaction motivation.

On the other hand, the GeoGebra problem-solving instructional approach contributes to a higher mean difference in students' intrinsic motivation, self-efficacy, utility value, test anxiety, extrinsic motivation, attention, and satisfaction than the variation theory-based problem-solving instructional approach. The study shows that integration of technology with other approaches contributes to students having good components of motivation. Students learning geometry with integrated instructional approaches improved more than with the separate instructional approaches.

In line with the above findings, there is a lot of research that has arrived at similar results with respect to the effects of variation theory and GeoGebra on motivation in general. For example, the above findings were consistent with other reported studies, such as Jing et al. (2017b), which indicated that utilization of variation theory-based instruction yields students with higher motivation, attention, and satisfaction. A multivariate analysis of variance (MANOVA) was conducted to explore the impact of instructional strategy on students' overall motivation and its subscales. The result indicated that there was a statistically significant effect of instructional strategy (group) on students' overall motivation (Cheng, 2016; Jing, Tarmizi, Bakar, et al., 2017b). Furthermore, some variation theory integrated with other instructional approaches significantly enhances Ethiopian grade ten students' self-belief in learning solid geometry. The observational evidence underpins the potential of this directional approach to address the determined instructive challenges in Ethiopia, especially in learning mathematics (Jing, Tarmizi, Bakar, et al., 2017a; Yeshanew et al., 2024).

Similarly, Shadaan and Eu (2013) observed that GeoGebra-based activities encouraged higher-order thinking skills and had a positive effect on motivating students toward learning. Students enjoyed learning mathematics much more when using GeoGebra and were able to form better connections between previous learning and new learning. However, some students reported they were not so confident when using the GeoGebra software.

When the components of pre-motivation and pre-overall motivation were controlled, those taught using a variation theory-based GeoGebra-assisted problem-solving approach had better overall motivation and its components toward learning geometry than those taught with a conventional instructional method.

The interview results also indicated an improvement in the students' post-motivation components and post-motivation for learning geometry. Group 3 students' better motivation toward learning geometry was due to the effective formation of the teaching approach through GeoGebra-assisted problem-solving teaching. This study's findings are consistent with those of Ahlquist & Gynther (2020). Furthermore, Murni et al. (2017) found that using GeoGebra-assisted problem-based learning can help students to solve problems and improve their attitudes toward mathematics. This idea is supported by the results of the teacher interview protocol. Thus, it emphasizes the significance of adding structured variation theory and a mixed instructional approach used in class. As a result, the integrated approaches can help students feel more liberated and positive about learning geometry.

In general, variation theory and GeoGebra-assisted problem-based learning could be directly attributed to the study's instructional methods. Integrating them encouraged the students to examine the existing knowledge that they had in their minds. The significance of geometrical concepts to what the students are familiar with is emphasized in each of the intervention groups, which encourages them to learn geometry.

Depending on the content of the instructions, students in all intervention groups engaged with attending in a motivated way through frequent engagement with their classmates. As a result, rather than receiving the concepts inertly, they had the opportunity to explore the concepts of geometry, as well as the relationships between the concepts of geometry. All of these events are designed to boost students' interest in geometry, as well as their motivation toward learning geometry.

The results of the teacher interviews also revealed that students' motivation toward geometry had improved. The use of variation theory-based GeoGebra-assisted problem-solving approaches, variation theory problem-solving approaches, and GeoGebra-assisted problem-solving approaches could have made the importance of learning geometry more explicit to students. It could also have enhanced students' motivation components toward learning geometry and motivated them to learn more, allowing them to excel in the topic. The intervention group students' motivation and components of motivation were improved, and the variation theory and GeoGebra problem-solving approaches were applied separately and in mixed instructions, according to the teachers who taught the three intervention groups. As a result, students appear to like geometry more since they have the ability to learn realistically by applying it to their daily lives.

4.4.4 Contribution of CU and Motivation to PSS

The findings of this study revealed that the variables' CU and students' motivation towards learning geometry under treatment group-1 had a significant overall effect on students' conceptual understanding, $F(2, 33) = 23.292$, $p = .000$, adjusted $R^2 = .5958$, or 59.58% for PSS, with all two variables found to be significant predictors, namely CU and motivation. Other variables, which were not included in this study, influenced the remaining 40.42% of the students' PSS. That is, they may be influenced by cognitive variables like intellectual ability and perceived competence.

Therefore, the contribution of students' conceptual understanding enhanced their problem-solving by 39.72%, and the contribution of students' motivation in learning geometry enhanced their problem-solving skills by 19.86%. This showed that almost all students' CU and their motivation in learning geometry have the same contribution to their PSS while the intervention was given. Furthermore, educational environment factors like attitudes of teachers, textbooks, co-educational school organization, classroom environment, the association between teacher behavior and student learning, qualification of teachers, class size, inspiration to teachers by the head, feedback, attention to difficult topics, student characteristics like use of learning strategies, home environment like occupation of parents, and school context like quality of instruction, etc. are important in determining students' conceptual understanding in mathematics (Alghadari et al., 2020; Rejeki et al., 2021).

The study findings also revealed that two predictor variables contributed significantly to students' PSS in treatment group-2. Since the F value indicates that the multiple correlations R were significant. That is, the contribution of variables CU and motivation collectively had a significant effect on students' attitude towards students' problem-solving skills in learning geometry since p=0.000, which is <.05. Similarly, the multiple linear regression analysis indicates that the contribution of conceptual understanding had a significant effect on the students' problem-solving skills (β = .324, p = .000, < 0.05) and the contribution of students' motivation had a significant effect on the students' problem-solving skills towards students' problem-solving skills in learning geometry (β = .367, p = .026, < .05). This indicates that both variable conceptual understanding and student motivation could predict the students' problem-solving skills in learning geometry while the intervention was being given (variation theory-based problem-solving instruction approaches).

Furthermore, the R^2 = 0.299 for PSS; it can explain 29.9% of the variance in CU. Motivation had an effect on students 'problem-solving skills collectively, and the contribution of students' conceptual understanding enhanced their problem-solving by 13.48%, and the contribution of students' motivation in learning geometry enhanced their problem-solving skills by 16.44%. This showed that almost all students were motivated a better contribution for students' PSS than their CU in learning geometry while variation theory-based problem-solving instructional approaches were given.

The findings of this study also revealed that the variables' CU and students' motivation had a significant overall effect on students' problem-solving skills under treatment group-3. That is, the contribution of variables CU and motivation collectively had a significant effect on students' problem-solving skills in learning geometry since p=0.005, which is

<.05. Similarly, the multiple linear regression analysis indicates that the contribution of conceptual understanding had a significant effect on the students' problem-solving skills ($\beta = 0.34$, $p = .048$, <0.05). On the other hand, the contribution of student motivation had significant effect on the students' problem-solving skills in learning geometry ($\beta = 0.41$, $p = 0.021$, $> .05$).

This indicates that one variable (conceptual understanding) could predict the students' problem-solving skills in learning geometry while the intervention was being given (variation theory-based problem-solving instruction approaches). And student motivation couldn't 't predicts the students' problem-solving skills in learning geometry while the intervention (GeoGebra-based problem-solving instruction approaches) was being given. $R^2 = 0.337$ for PSS; it can explain 33.7% of the variance in CU. Motivation had an effect on students' problem-solving skills collectively. In order to compare the contribution of the variable's CU and Motivation PSS, that is, the highest contribution for Students' PSS was by conceptual understanding of students in learning geometry. Therefore, the contribution of students' conceptual understanding enhanced their problem-solving by 14.58%, and the contribution of students' motivation in learning geometry enhanced their problem-solving skills by 19.12%. This showed that students' motivation had a better contribution for students' PSS than their CU in learning geometry. While GeoGebra-based problem-solving instructional approaches were given.

CHAPTER FIVE: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study was to see the effect of a variation theory-based GeoGebra-assisted problem-solving instructional approach on 9th graders' learning geometry, their level of understanding concepts, their problem-solving skills, and their motivation. The chapter begins with a summary of the whole study, followed by a presentation of the study's fundamental conclusions that are drawn from the discoveries. The chapter concludes with a discussion of suggestions and recommendations for assisting the investigation.

5.1 Summary

The primary purpose of this research is to investigate the effect of the variation theory-based GeoGebra-assisted problem-solving instructional approach on 9th graders' learning geometry, their level of understanding concepts, their problem-solving skills, and their motivation. Students from Kelemework, BulBula, Hidas Lideta, and Shimelis Habtei secondary schools in Addis Ababa City Administration are among the study's participants. A quasi-experimental research design was employed, comprising multiple treatments with a pretest-posttest non-equivalent control group design, as well as a case study design, to answer the following research question.

1. Are there significant differences in students' geometry conceptual understanding (CU) across the three interventions and the comparison groups? How do students develop geometry conceptual understanding and reasoning when taught through variation theory-based GeoGebra-assisted problem-solving approaches?

2. Are there significant differences in students' geometry problem-solving skills (PS)across the three interventions and the comparison groups? How do students solve geometry problems and develop problem-solving skills when taught through variation theory-based GeoGebra-assisted problem-solving approaches?

3. Are there significant differences in students' motivation (M) in geometry across the three interventions and the comparison groups?

4. Are there any relationships between the mean gains of the variable's conceptual understanding, problem-solving skills, and motivation in geometry in the intervention groups?

5. What are the contributions of conceptual understanding, motivation to problem-solving skills in learning geometry in the intervention groups?

Before the intervention, students in the three intervention groups and the comparison group took the conceptual understanding test, problem-solving test, and geometry motivation questionnaires test as a pre-test to determine the groups' equivalence at the beginning of the study and to identify potential variations that could be used as covariates in subsequent inferential analyses. During the intervention period, students of intervention group 1 were taught with variation theory-based GeoGebra-assisted problem-solving approaches. Students in intervention group 2 were taught with variation theory-based problem-solving approaches. On the other hand, students in intervention group 3 were taught with GeoGebra-assisted problem-solving approaches. The conventional instructional approaches, on the other hand, were employed to teach comparison groups.

The conceptual understanding test, problem-solving skill test, and motivational questionnaire were administered as post-tests after the intervention was completed. During the seven-week intervention period, the researcher observed each of the three intervention groups in the classroom. Because the teacher who led each intervention group had an instructional manual to deliver the expected lesson accordingly, teachers were offered a well-defined order of instruction; it was ensured that the teachers executed the interventions as predicted by the researcher.

Classroom observation and interviews were used in addition to the test and questionnaires. It is important to strengthen the quantitative aspect to the qualitative. Pre-intervention conceptual understanding, pre-intervention problem-solving skills, and pre-intervention motivation of students toward learning geometry were assigned as covariates to compensate for group non-equivalence. The post-test scores were compared to equivalent pre-test scores using an analysis that controlled for initial differences in the pre-test mean scores. As a result, ANCOVA was employed on post-test scores to statistically control the effect of pre-test scores on post-test results, such as post-conceptual understanding, post-problem-solving skills, and post-motivation of students toward learning geometry, which were assigned as dependent variables. Additionally, data from classroom observation and interviews were analyzed. The data gathered from classroom observation of students' interaction and teachers' interviews in the intervention groups were used to give answers to the final research question.

According to the findings, there was a statistically significant mean difference between the three groups in terms of post-intervention conceptual understanding, post-intervention problem-solving skills, and post-intervention motivation. The conceptual understanding, problem-solving skills, and student motivation scores in the variation theory-based GeoGebra-assisted problem-solving approaches group were higher than those of students in the variation theory-based problem-solving approaches and GeoGebra-assisted problem-solving approaches groups. Furthermore, the students taught with variation theory-based GeoGebra-assisted problem-solving approaches, GeoGebra-assisted problem-solving approaches, and variation theory-based problem-solving approaches had significantly higher conceptual understanding and problem-solving skills, as well as student motivation scores toward geometry, than the comparison group.

In terms of learners' conceptual understanding, problem-solving skills, and students' motivation scores, it was found that students' performances in the variation theory-based GeoGebra-assisted problem-solving approaches, GeoGebra-assisted problem-solving approaches, and variation theory-based problem-solving approaches were better than the conventional instruction of teaching. Additionally, motivation components such as post-intervention intrinsic motivation, post-intervention self-efficacy, post-intervention utility value, post-test anxiety, post-intervention extrinsic motivation, post-intervention attention, and post-intervention satisfaction have a considerable effect on students' geometry learning.

The students' sole concentration was on selecting a formula and substituting the specified quantities into the formula to arrive at the final solution. This showed a focus on carrying out the plan and devising a plan, rather than understanding the problem and revisiting the problem-solving skills indicator's result section. Furthermore, students determine the required quantity and then solve the problem by substituting the given numbers into the formula. As a result, even though students solve problems quantitatively by substituting values into equations, they do not develop the necessary ability to transfer their understanding to new problems, which in turn allows them to solve more complicated problems that require more than one concept to solve. As a result, students may struggle to connect the results to other concepts, despite simply solving problems in textbooks.

Generally, the disparities in the students' performances of the three groups following the intervention are expected to have been derived from the teaching approaches used to teach geometry. According to the results of the study, students from the three intervention groups acknowledge that the use of variation theory, GeoGebra-based problem-solving approaches, GeoGebra-assisted problem-solving approaches, and variation theory problem-solving approaches that the students could practically associate with geometric concepts are possible factors that improved the intervention group students' performance.

Moreover, the contribution of conceptual understanding and motivation had the greatest impact on their problem-solving skills in learning geometry. Students' motivation was found to be a strong independent predictor of students' problem-solving ability, and their conceptual understanding was also a predictor of thier problem-solving ability.

5.2 Conclusions

The following conclusions were drawn based on the study's analysis and findings: The study brought into the discussion that the use of variation theory-based GeoGebra-assisted problem-solving instruction, variation theory problem-solving approaches, and GeoGebra problem-solving approaches accounted for improved conceptual understanding, problem-solving skills, and motivation in learning geometry. The use of variation theory-based GeoGebra problem-solving instruction was found to significantly affect better performance in terms of conceptual understanding. Therefore, the findings showed that variation theory GeoGebra-based problem-solving instruction was superior to variation theory problem-solving approaches and GeoGebra problem-solving approaches alone, which in turn were found to be better than the conventional instructional approaches.

Nonetheless, the three types of intervention were demonstrated to have significant contributions to one another, as explained by the regression models. It is, hence, concluded that schools need to devise ways to regularly implement variation theory. GeoGebra-based problem-solving instruction, variation theory problem-solving approaches, and GeoGebra problem-solving approaches improve the students' conceptual understanding, problem-solving skills, and motivation towards learning geometry.

Students employ carrying out the plan and designing a plan to learn geometry themes instead of paying attention to understanding the problem and re-examination of the result section. Despite the fact that the study found that variation theory, GeoGebra-based problem-solving instruction, variation theory problem-solving approaches, and GeoGebra problem-solving approaches have significant effects on students' problem-solving skills, the majority of the students solved problems using conventional problem-solving procedures. To improve their problem-solving skills, students should practice how to use phases of problem-solving in the geometry classroom.

The study found that students who were taught with the variation theory GeoGebra-based problem-solving approach instruction provided more distinctive and varied solutions to the problem than students who were taught with variation theory problem-solving approaches and GeoGebra problem-solving approaches, who had remarkably similar performance in mathematical creativity.

Moreover, the findings of this study also revealed that in conceptual understanding tasks, students taught with variation theory GeoGebra-based problem-solving instruction, variation theory problem-solving approaches, and GeoGebra problem-solving approaches all used different strategies. The study found that students taught with variation theory GeoGebra-based problem-solving instruction were able to flexibly apply the concepts of geometry to fit specific concepts. These abilities appear to have helped them score higher than students who were taught using variation theory problem-solving approaches and GeoGebra problem-solving approaches, a conventional instructional method. It may be claimed that students who received variation theory GeoGebra-based problem-solving instruction had a number of similarities with students who were taught using GeoGebra problem-solving approaches.

Moreover, when compared to the students in the comparison group, the students who were taught with variation theory GeoGebra-based problem-solving instruction, variation theory problem-solving approaches, and GeoGebra problem-solving approaches exhibited better motivation toward learning geometry. Hence, variation theory-based GeoGebra problem-solving instruction, variation theory problem-solving approaches, and GeoGebra problem-solving approaches can be used in the classroom to encourage and develop motivation toward learning geometry among students.

Regression analysis shows a student’s conceptual understanding and motivation towards learning geometry in the subject are the best predictors of their problem-solving skills. This suggests that the greater a student's problem-solving ability towards learning geometry is, the better their conceptual understanding of geometric concepts and motivation too.

5.3 Recommendations

The findings of this study revealed that variation theory-based GeoGebra problem-solving instructional approaches, variation theory problem-solving approaches, and the GeoGebra problem-solving-based learning approach were all beneficial in improving students’ CU, PSS, and motivations in learning geometry. Thus, the three interventions—variation theory GeoGebra-based problem-solving instructional approaches, variation theory problem-solving approaches, and GeoGebra problem-solving-based learning approaches—can be used to teach geometry effectively by increasing students' interest and motivation.

The following recommendations are made based on the conclusions indicated above:

- The findings of this study support those found in the literature by indicating that students understand geometrical concepts. It is feasible to achieve a significantly better acquisition of geometrical concepts by appropriately using teaching approaches. Through training, teachers should be thoroughly knowledgeable about different teaching approaches, such as variation theory, problem-based learning approaches, using GeoGebra, and integrated instruction.

- The VT-GG-PS, VT-PS, and GG-PS instructional approaches be used as the foundation for preparing and developing new curricula. In order to generate well-educated students, curriculum developers should consider adapting school mathematics to meet these teaching approaches. Furthermore, textbook writers should also give emphasis to making mathematical concepts concrete or real by adding students’ misconceptions found in the literature.

- Two-tier tests are regarded as a valid and reliable technique for better assessing student concepts since they consider students' clarification for their responses. As a result, it is advised that mathematics teachers employ two-tier tests to assess their students' conceptual understanding of the content that they are teaching.

- Students and teachers should be familiar with appropriate VT-GG-PS, VT-PS, and GG-PS instructional approaches as alternatives to the conventional instructional approaches; all should be included in the curricula of teacher training institutions of mathematics for secondary schools.

- Strategies can develop students' mathematical creativity as well as their ability to apply geometrical concepts in particular and mathematical concepts in general to practical applications in their daily lives and solve various types of problems. Thus, math textbook writers and curriculum developers should reconsider incorporating different approaches like variation theory-based GeoGebra-assisted problem-solving instructional approaches, variation theory-based problem-solving approaches, and GeoGebra-assisted problem-solving learning approaches in the mathematics curriculum packages.

- VT-GG-PS, VT-PS, and GG-PS instructional approaches were an indicator of effective instruction at the secondary school level in teaching geometry. Primary mathematics teachers can also implement them for their lesson delivery.

- Development of CU, PSS, and motivation toward mathematics can be achieved over a long period of time. Thus, mathematics teachers must be deliberate in choosing appropriate VT-GG-PS, VT-PS, and GG-PS for levels of students' cognition. Mathematics teachers in charge are encouraged to inculcate conceptual understanding in mathematics, as it can increase positively their problem-solving ability in that subject, since it was found that conceptual understanding was a predictor of problem-solving ability in mathematics. It is also suggested that teachers check their students' conceptual understanding of learning geometry.

- Mathematics teachers are advised to keep track of how to get the inner concepts in geometry class. Because they can motivate students to a deep understanding and improve problem-solving in geometry. The researcher strongly recommends all participants in this study maintain a conceptual knowledge of mathematics in general and geometry in particular as a result of this considerable shift in motivation toward mathematics. To ensure the best problem-solving ability in the field of mathematics, students' conceptual understanding and motivation are critical. Teachers, school administrators, and parents will work together to ensure that everyone, especially the students, achieves exceptional success.

- Mathematics teachers play a critical role in developing students' problem-solving dispositions. They should select problems that will engage students. They need to create an environment for students' confidence to discover, take risks, share failures and successes, and challenge one another. Students develop the confidence they need to examine problems and the ability to alter their problem-solving strategies in such supportive environments.

5.4 Recommendations for Further Studies

- Future studies can be done at other grade levels other than grade nine to see the implications of the VT-GG-PS, VT-PS, and GG-PS; integrating and separating them on other topics can be specified and investigated in future studies.

- Widening the target population: this research was conducted in four government schools in the capital city, Addis Ababa for about 7 weeks. Therefore, research can be conducted in different government and private schools and grade levels with a larger sample size at different regions for a long period of time to increase generalizability of conclusions.

- Widening the content areas: this research was conducted on mathematics, specifically on geometry contents. Further research, however, can be conducted to investigate the effect of VT-GG-PS, VT-PS, and GG-PS on students' conceptual understanding, problem-solving skills and their motivation, other than geometry concept. Such research may be able to provide evidence about the effectiveness of the method over different mathematics concepts

References

Ababa, A. (2020). *General Education Curriculum Framework*. *December*.

Abakah, F., & Brijlall, D. (2022). *Conjecturing an Effective Problem-Solving Heuristic Instructional Approach for Circle Geometry*. *6*(6), 8933–8946.

Abate, A., Atnafu, M., & Michael, K. (2022). Visualization and Problem-based Learning Approaches and Students 'Attitude toward Learning Mathematics. *Pedagogical Research*, *7*(2), em0119. https://doi.org/10.29333/pr/11725

Abate, T. M. (2023). *Examining practices and challenges of authentic learning in Mathematics lessons in upper primary schools*. *23*(3), 122–145.

Abiam, P. O., Abonyi, O. S., & Ugama, J. O. (2016). *Effects of Ethnomathematics-based Instructional Approach on Primary School Pupils' Achievement in Geometry Effects of Ethnomathematics-based Instructional Approach on Primary School Pupils' Achievement in Geometry*. *January*. https://doi.org/10.9734/JSRR/2016/19079

Adom, D., Attah, A. Y., & Ankrah, K. (2016). Constructivism Philosophical Paradigm: Implication for Research, Teaching and Learning. *Global Journal of Arts Humanities and Social Sciences*, *4*(10), 1–9.

Aguilar, J. J. (2021). High School Students' Reasons for disliking Mathematics: The Intersection Between Teacher's Role and Student's Emotions, Belief and Self-efficacy. *International Electronic Journal of Mathematics Education*, *16*(3), em0658. https://doi.org/10.29333/iejme/11294

Ahamad, S. N. S. H., Li, H. C., Shahrill, M., & Prahmana, R. C. I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, *943*(1). https://doi.org/10.1088/1742-6596/943/1/012008

Ahmed, A., Melesse, S., & Wondimuneh, T. (2020). Students' perception of the application of cooperative problem-solving method and its effect on mathematics performance: The case of secondary schools in Awi-zone, Ethiopia. *Research in Pedagogy, 10*(1), 1–12. https://doi.org/10.5937/istrped2001001a

Åkerlind, G. (2015). From Phenomenography to Variation Theory: A review of the development of the Variation Theory of Learning and implications for pedagogical design in higher education. *HERDSA Review of Higher Education*, *2*(January 2015), 5–26. http://www.herdsa.org.au/?page_id=4085

Akhter, N. (2018). Learning in Mathematics: Difficulties and Perceptions of Students. *Journal of Educational Research*, *21*(1), 147–163.

Akpan, V. I., Igwe, U. A., Blessing, I., Mpamah, I., & Okoro, C. O. (2020). *SOCIAL CONSTRUCTIVISM: IMPLICATIONS ON TEACHING AND LEARNING*. *8*(8), 49–56.

Albay, E. M. (2019). Analyzing the effects of the problem-solving approach to the performance and attitude of first year university students. *Social Sciences and Humanities Open*, *1*(1), 100006. https://doi.org/10.1016/j.ssaho.2019.100006

Aldridge, D. (2020). Special issue: Ethical questions in educational research. *British Educational Research Journal*, *46*(4), 725–727. https://doi.org/10.1002/berj.3676

Alemu, B. M. (2010). *Active Learning Approaches in Mathematics Education At Universities in Oromia, Ethiopia*. *November*.

Alex, J. K. (2019). The preparation of secondary school mathematics teachers in South Africa: Prospective teachers‘student level disciplinary content knowledge. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12). https://doi.org/10.29333/ejmste/105782

Alex, J., & Mammen, K. J. (2018). Students ‘understanding of geometry terminology through the lens of Van Hiele theory. *Pythagoras*, *39*(1), 1–8. https://doi.org/10.4102/pythagoras.v39i1.376

Alghadari, F., Herman, T., & Prabawanto, S. (2020). Factors Affecting Senior High School Students to Solve Three-Dimensional Geometry Problems. *International Electronic Journal of Mathematics Education*, *15*(3), 1–11. https://doi.org/10.29333/iejme/8234

Ali, R., Hukamdad, D., Akhter, A., & Khan, A. (2010). Effect of Using Problem Solving Method in Teaching Mathematics on the Achievement of Mathematics Students. *Asian Social Science*, *6*(2), 67–72. https://doi.org/10.5539/ass.v6n2p67

Alkalah, C. (2016). *Students' difficulties in solving geometry problems*. *19*(5), 1–23.

Allen, A. (2022). *An Introduction to Constructivism: Its Theoretical Roots and Impact* *on Contemporary Education*. *1*(1), 1–11.

Amin, R., & Mulyono, B. (2022). Mathematical Concepts Understanding Skill of Class VIII Students on Relation and Function Using Problem Based Learning Models. *Proceedings of the 2nd National Conference on Mathematics Education 2021 (NaCoME 2021)*, *656*(NaCoME 2021), 171–176. https://doi.org/10.2991/assehr.k.220403.024

Amna Saleem, Huma Kausar, & Farah Deeba. (2021). Social Constructivism: A New Paradigm in Teaching and Learning Environment. *Perennial Journal of History*, *2*(2), 403–421. https://doi.org/10.52700/pjh.v2i2.86

Ansah, S., & Kabutey, S. K. A.-A. D. T. (2022). Investigating the Effect of Using GeoGebra as an Instructional Tool on van Hiele‘s Geometric Thinking Levels of Senior High Technical School Students‘. *International Journal of Mathematics and Statistics Studies*, *10*(1), 31–39. https://doi.org/10.37745/ijmss.13/vol10no1pp.31-39

Arbain, N., & Shukor, N. A. (2015a). The Effects of GeoGebra on Students Achievement. *Procedia - Social and Behavioral Sciences*, *172*(January), 208–214. https://doi.org/10.1016/j.sbspro.2015.01.356

Arbain, N., & Shukor, N. A. (2015b). The Effects of GeoGebra on Students Achievement. *Procedia - Social and Behavioral Sciences*, *172*(2007), 208–214. https://doi.org/10.1016/j.sbspro.2015.01.356

Asanre, A. A. (2024). Motivational factors as determinant of mathematics achievement of students in senior secondary schools. *Interdisciplinary Journal of Education Research*, *6*, 1–9. https://doi.org/10.38140/ijer-2024.vol6.19

Assefa, T., & Rogers, P. C. (2006). *Integration of ICT in Education: the Plasma Mode of Education in Ethiopian High Schools ICTs Application in Education*. 1–4.

Atherton, P., & Outhred, R. (2020). *Prepared by GPE 2020 Country-level Prospective Evaluations FIRST ANNUAL REPORT: ETHIOPIA (DECEMBER 2018)*. *December 2018*.

Atnafu, M. (2012). Motivation, Social Support, Alienation from the School and their Impact on Students’ Achievement in Mathematics: The Case of Tenth Grade Students. *Ethiopian Journal of Education and Sciences*, *8*(1), 53–74. http://www.ajol.info/index.php/ejesc/article/view/88370

Awofala, A. O. A. (2014). Examining Personalization of Instruction, Attitudes toward and Achievement in Mathematics Word Problems among Nigerian Senior Secondary School Students. *International Journal of Education in Mathematics, Science and Technology*, *2*(4), 273. https://doi.org/10.18404/ijemst.91464

Ayalew, Y. (2021). A Five Circles Model for Designing Mathematics Teacher Education Programs and Framing Common Standards for Educators. *International Electronic Journal of Mathematics Education*, *12*(3), 265–280. https://doi.org/10.29333/iejme/613

Aydogdu, M. Z., & Kesan, C. (2014). A Research on Geometry Problem Solving Strategies Used by Elementary Mathematics Teacher Candidates. *Journal of Educational and Instructional Studies in the World*, *4*(1), 53–62.

Aziz, A. (2021). Understanding of Students in the Concepts of Geometry at Secondary Level District Lasbela, Baluchistan: An Evaluative Study. *Pakistan Social Sciences Review*, *5*(IV), 242–257. https://doi.org/10.35484/pssr.2021(5-iv)19

Bache, F. G., Deressa, C. T., & Hunde, A. B. (2024). GeoGebra-supported learning and secondary school students ‘motivation for learning geometry. *Ethiop. J. Educ. & Sci.*, *Vol 19*(No. 2), 11–27.

Bada, & Olusegun, S. (2015). Constructivism Learning Theory: A Paradigm for Teaching and Learning. *Journal of Research and Method in Education*, *5*(6), 23–34. https://doi.org/10.9790/7388-05616670

Baidoo, J., & Luneta, K. (2024). Implementing blended learning to enhance the teaching of 3-dimensional trigonometry. *Journal of Education and E-Learning Research*, *11*(2), 332–344. https://doi.org/10.20448/jeelr.v11i2.5565

Bangalan, R. C., Hipona, J. B., Bangalan, R. C., Hipona, J. B., Guzman, M. T., Journal, I., Education, I., Sevier, J., Morris, E., Bossé, M., Cadorna, E. A., Riboroso, R. A., Restituto, M., Frankina, M., Luzano, J. F. P., Texas, A., Texas, M. U., Lamichhane, B. R., Campus, S. M., … Situmeang, T. (2023). Promoting Student conceptual understanding of mathematics in elementary classrooms. *Psychology and Education: A Multidisciplinary Journal*, *6*(2), 22–39. https://www.researchgate.net/publication/364813755

Barbar.G, T., & Linda.S, F. (1983). Review of Using Multivariate Statistics. In

Contemporary Psychology: A Journal of Reviews (Vol. 28, Issue 8). https://doi.org/10.1037/022267

Baskoro, I. (2021). Variation theory-based mathematics teaching: The new method in improving higher order thinking skills. *Journal of Physics: Conference Series*, *1957*(1). https://doi.org/10.1088/1742-6596/1957/1/012016

Batiibwe, M. S. K. (2024). Application of interactive software in classrooms: a case of GeoGebra in learning geometry in secondary schools in Uganda. *Discover Education*, *3*(1). https://doi.org/10.1007/s44217-024-00291-8

Bekele, Y. (2007). Barriers to teaching and learning mathematics in grade four:University of Oslo, Norway. *Thesis*.

Belay, S., Atnafu, M., Michael, K., & Ermias, A. (n.d.). *Strategic Policy for National Science , Technology and Mathematics Education*.

Belgheis, S., & Kamalludeen, R. (2018). *The Intention to Use GeoGebra in the Teaching of Mathematics Among Malaysian Teachers*. *6*(1), 109–115.

Benbow, C. P. (1992). Academic Achievement in Mathematics and Science of Students Between Ages 13 and 23: Are There Differences Among Students in the Top One Percent of Mathematical Ability? *Journal of Educational Psychology*, *84*(1), 51–61. https://doi.org/10.1037/0022-0663.84.1.51

Bhagat, K. K., & Chang, C. Y. (2015). Incorporating GeoGebra into geometry learning-A lesson from India. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(1), 77–86. https://doi.org/10.12973/eurasia.2015.1307a

Busaka, C., Umugiraneza, O., & Kitta, S. R. (2022). Mathematics teachers' conceptual understanding of soft skills in secondary schools in Zambia. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(7). https://doi.org/10.29333/ejmste/12160

Ca, A., Ogunrinade, S. O., & Awe, I. A. (2024). *Effectiveness of geometrical instructional teaching strategy on junior secondary school students' Achievement, Gender and Attitude in Plane Geometry in Odeda Local Government, Ogun state, Nigeria*. *2002*.

Calder, N. (2011). *processing mathematics through degital technology*.

Cambridge, T. (2022). 3. Approaches to teaching and learning. *Teaching and Learning*, 40–47.

Canque, M. S., Trinidad, G. A., Jane, M., & Cortes, P. (2021). *Differentiated Instruction through Tiered Activities in Teaching Geometry to the Junior High School students*. *12*(6), 5373–5380.

Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, *27*(2–3), 147–164. https://doi.org/10.1080/02702710600642467

Carson, J. (2007). A problem with problem solving: Teaching thinking without teaching knowledge. *The Mathematics Educator, 17*(2), 7–14.

Carter, N., Bryant-Lukosius, D., Dicenso, A., Blythe, J., & Neville, A. J. (2014). The use of triangulation in qualitative research. *Oncology Nursing Forum*, *41*(5), 545–547. https://doi.org/10.1188/14.ONF.545-547

Chand, S., Chaudhary, K., Prasad, A., & Chand, V. (2021). Perceived Causes of Students' Poor Performance in Mathematics: A Case Study at Ba and Tavua Secondary Schools. *Frontiers in Applied Mathematics and Statistics*, *7*(April), 1–13. https://doi.org/10.3389/fams.2021.614408

Charles, S., Daffer, O., & How, P. (1991). *201 Math Problem Solving*.

Cheli, S. (2018). Radical constructivism: Historical roots and contemporary debate. *Historical Roots and Contemporary Debate*, *July*. https://doi.org/10.31235/osf.io/69ne8

Cheng, E. W. L. (2016). Learning through the Variation Theory: A Case Study. *International Journal of Teaching and Learning in Higher Education*, *28*(2), 283–292. http://www.isetl.org/ijtlhe/

Cipriano, N. P., & Cipriano, N. P. (2023). *Exploring Mathematics Education In The 21st Century Exploring Mathematics Education In The 21st Century*. *14*(May), 749–758. https://doi.org/10.47750/jett.2023.14.03.086

Cohen, J. (1988). Statistical Power Analysis for the Behavioral Sciences (2nd ed.). Routledge.https://doi.org/10.4324/9780203771587

Cooper, P. (2011). *problem-solving in geometry in collaborative small GROUP SETTINGS: How learners' appropriate mathematical tools while working in small groups. November*, 98. https://etd.uwc.ac.za/handle/11394/4248

Coştu, B., Ayas, A., Niaz, M., Ünal, S., & Çalik, M. (2007). Facilitating conceptual change in students' understanding of boiling concept. *Journal of Science Education and Technology*, *16*(6), 524–536. https://doi.org/10.1007/s10956-007-9079-x

D, Toinpere & Olotu, A. R. (2022). *areas and reasons for students' difficulties in learning geometry by gender in senior*. *2*(6), 55–65.

Dahal, N., Pant, B. P., Shrestha, I. M., & Manandhar, N. K. (2022). Use of GeoGebra in Teaching and Learning Geometric Transformation in School Mathematics. *International Journal of Interactive Mobile Technologies*, *16*(8), 65–78. https://doi.org/10.3991/ijim.v16i08.29575

Dawadi, S., Shrestha, S., & Giri, R. A. (2021). Mixed-Methods Research: A Discussion on its Types, Challenges, and Criticisms. *Journal of Practical Studies in Education*, *2*(2), 25–36. https://doi.org/10.46809/jpse.v2i2.20

Demssie, M. (2019). Perceived factors affecting students 'mathematics performance in North Showa zone secondary schools in Ethiopia. *International Journal of Advanced Research in Engineering and Applied Sciences, 8*(5), 21–36. www.garph.co.uk

Denzin, N. K. (2012). Triangulation in Mixed Method. *Journal of Mixed Methods Research*, *6*(2), 80–88.

Dibekulu, D. (2022). *An Overview of Data Analysis and Interpretations in*. *November*. https://doi.org/10.14662/IJARER2020.015

Dildabayeva, M., & Zhaydakbayeva, L. (2024). Enhancing Geometry Teaching in STEAM Education with Interactive Learning Environments. *International Journal of Information and Education Technology*, *14*(9), 1239–1251. https://doi.org/10.18178/ijiet.2024.14.9.2153

Dogan, M. (2011). The role of dynamic geometry software in the process of learning: GeoGebra example about triangles. Retrieved from: http://www.time2010.uma.es/Proceedings/Papers/A026_Paper.pdf. *International Journal of Human Sciences, 8*(1), 1441–1458.

Donkoh, S. (2023). Application of triangulation in qualitative research. *Journal of Applied Biotechnology & Bioengineering*, *10*(1), 6–9. https://doi.org/10.15406/jabb.2023.10.00319

Dutta, B., & Boruah, P. J. (2025). To Study Middle-Grade Students'Conceptual Understanding in Geometry. *International Research Journal of Modernization in*

Engineering Technology and Science, *12*(02). https://doi.org/10.56726/irjmets70214

Ederio, N. T., P. Inocian, E., I. Calaca, N., & M. Espiritu. (2023). Ethical Research Practices in Educational Institutions: A Literature Review. *International Journal of Current Science Research and Review*, *06*(05), 2709–2724. https://doi.org/10.47191/ijcsrr/v6-i5-02

Egara, F. O. (2024). *Learners' Mathematics Achievement and Retention*. 19863–19888.

Ekawati, R., & Lin, F. (n.d.). *DESIGNING TEACHER PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHING WITH VARIATION THEORY*. 127–137.

Ekdahl, A. (2021). Variation theory and teaching experiences as tools to generate knowledge about teaching and learning mathematics – the case of pre-service teachers. . . *Nordic Studies in Mathematics Education*, *26*(2010), 91–112.

ENLA. (2017). *Technical Working Group:*

Ersoy, E. (2016). Problem Solving and Its Teaching in Mathematics. *The Online Journal of New Horizons in Education*, *6*(2), 79–87.

Febriana. R, Haryono. Y, & Yusri. R. (2018). Improving Motivation and Student Learning Outcome on Geometry Transformationsubject With Using Modules Based on Guided Discovery. *International Journal of Advanced Research*, *6*(6), 745–747. https://doi.org/10.21474/ijar01/7271

Ference Marton, & Booth, S. (1997). *Learning and Awareness*. 6.

Gambari, B. (2010). *Effects of Instructional Models on The Performance of Junior Secondary School Students in Geometry in Minna, Niger State*. *9*(1), 54–61.

Garba, S., & Sidi, S. B. (2020). Effects of Problem-Solving Instructional Strategy on Students 'Academic Achievement in Geometry in Gwale Local Government Area secondary schools. *Abacus (Mathematics Education Series)*, *45*(1), 32–44.

Gebremeskel, A. A. (2025). *Student Engagement, Conceptual-Understanding, And Problem-Solving Ability In Learning Plane Geometry Through An Integrated Instructional Approach*. *21*(5).

Geche, T. J. (2009). Learning Styles and Strategies of Ethiopian Secondary School Students in Learning Mathematics. *Thesis*, *October*.

Gemechu, E. &, & Aba-Oli, Z. (2024). Ethiopian College of Teacher Education Program:

A Tension between Selection, Curriculum and Professional Development. *International Journal of Progressive Education*, *20*(20), 1–11. https://doi.org/10.29329/ijpe.2024.664.1

Gemechu, E., Mogiso, A., & Husen, Y. (2021). *Ethiopian Public Universities' Mathematics Curriculum: a Change Needed to Technology Supported*. *15*(1), 58–85.

Giri, C. K. (2024). Variables in Educational Research. *The International Journal of Indian Psychology*, *12*(3), 1086–1095. https://doi.org/10.25215/1203.104

Gizaw, A. K. (2023). *Factors Affecting Secondary School Mathematics Teachers' Teaching Methodology At Western Oromiya & Snnp Regions, Research Article Factors Affecting Secondary School Mathematics Teachers' Teaching Methodology at Western Oromiya & Snnp Regions, Ethiopia*. *August*.

Glasersfeld, E. von. (1996). *Radical constructivism: a way of knowing and learning* (Issue 9). http://books.google.com/books?id=XgXgRaG50xoC&pgis=1

Godino, J. D. (1953). Mathematical Concepts, Their Meanings, and. *Proceedings of XX Conference of the International Group for the Psychology of Mathematics Education*, *2*, 417–425.

Golder, J., & Bengal, W. (2018). Constructivism: a Paradigm for. *International Journal of Research and Analytical Reviews (IJRAR)*, *5*(3), 678–686.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Costs of secondary parasitism in the facultative hyperparasitoid Pachycrepoideus dubius: Does host size matter? In *Entomologia Experimentalis et Applicata* (Vol. 103, Issue 3). https://doi.org/10.1023/A

Gunaseelan, & Pazhanivelu, B. (2016). Identifying Factors Affecting the Mathematics Achievement Through Review Analysis. *International Journal of Development Research*, *06*(07), 8804–8809. http://www.journalijdr.com

Gurmu, F., Tuge, C., & Hunde, A. B. (2024). Effects of GeoGebra-assisted instructional methods on students 'conceptual understanding of geometry. *Cogent Education*, *11*(1). https://doi.org/10.1080/2331186X.2024.2379745

Häggström, J. (2008). *Häggström, Johan (2008). Teaching systems of linear equations in Sweden and China: what is made possible to learn? Diss. Göteborg : Göteborgs* *universitet, 2008 E-published version available at http://hdl.handle.net/2077/17286*.

Halat, E. (2007). Reform-based curriculum & acquisition of the levels. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(1), 41–49. https://doi.org/10.12973/ejmste/75373

Hammoudi, M. M., & Grira, S. (2023). Improving students ‘motivation in calculus courses at institutions of higher education: Evidence from graph-based visualization of two models. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(1). https://doi.org/10.29333/ejmste/12771

Hanggara, Y., Sa‘dijah, C., & Qohar, A. (2023). Theory and practice of constructive approaches in mathematics learning. *Cahaya Pendidikan*, *9*(1), 27–34. https://doi.org/10.33373/chypend.v9i1.5127

Haryadi, H., Iskandar, I., & Nofriansyah, D. (2016). The Constructivist Approach: Radical and Social Constructivism in the Relationship by Using the Implementation Career Level on the Vocational Education. *Innovation of Vocational Technology Education*, *12*(1), 16–21. https://doi.org/10.17509/invotec.v12i1.4499

Hekimoglu, S., & Sloan, M. (2005). A compendium of views on the NCTM Standards. *Mathematics Educator*, *15*(1), 35–43.

Herceg, Đ., & Herceg, D. (2010). *Numerical Integration with GeoGebra in High School , The International Journal for Technology in Mathematics Education – Numerical Integration with GeoGebra in High School*.
October 2020, 205–210.

Hidayat, R., Kamarazan, N. A., Nasir, N., & Ayub, A. F. M. (2023). The Effect of GeoGebra Software on Achievement and Engagement Among Secondary School Students. *Malaysian Journal of Mathematical Sciences*, *17*(4), 611–627. https://doi.org/10.47836/MJMS.17.4.06

Hidayati, V. R., Maulyda, M. A., Hasbi, M., Lukito, A., & Sulaiman, R. (2019). *Mathematical connection ability: teacher' s perception and experience in learning Mathematical connection ability: teacher' s perception and experience in learning*. https://doi.org/10.1088/1742-6596/1315/1/012041

Hohenwarter, M., & Fuchs, K. (2005). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. *Computer Algebra Systems and Dynamic*

Geometry Systems in Mathematics Teaching Conference 2004, 2002(July), 1–6. http://www.geogebratube.org/material/show/id/747

Hosseini, Z., Mehdizadeh, M., & Sadeghi, M. (2022). Using GeoGebra in Teaching Geometry to Enhance Students Academic Achievement and Motivation. *Innovare Journal of Education*, *10*(3), 34–38. https://doi.org/10.22159/ijoe.2022v10i3.44792

Howe, K. R., & Moses, M. S. (1999). Chapter 2: Ethics in Educational Research. *Review of Research in Education*, *24*(1), 21–59. https://doi.org/10.3102/0091732x024001021

Huang, R., & Leung, F. K. S. (2005). Deconstructing Teacher-Centeredness and Student-Centeredness Dichotomy: A Case Study of a Shanghai Mathematics Lesson. *The Mathematics Educator*, *15*(2), 35–41.

Hussein, Y. F., & Csíkos, C. (2023). The effect of teaching conceptual knowledge on students ‘achievement, anxiety about, and attitude toward mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(2). https://doi.org/10.29333/ejmste/12938

Idrus, H., Rahim, S. S. A., & Zulnaidi, H. (2022). Conceptual knowledge in area measurement for primary school students: A systematic review. *STEM Education, 2*(1), 47–58. https://doi.org/10.3934/steme.2022003

In‘am, A., & Hajar, S. (2017). Learning geometry through discovery learning using a scientific approach. *International Journal of Instruction*, *10*(1), 55–70. https://doi.org/10.12973/iji.2017.1014a

Irfan, M. (2016). *Role of Learning Mathematics in the Character Building*. 599–604.

Jaberi, N., & Gheith, E. (2016). *Pre-Service Class Teacher’ Ability in Solving Mathematical Problems and Skills in Solving Daily Problems Pre-Service Class Teacher’ Ability in Solving Mathematical Problems and Skills in Solving Daily Problems*. *July*. https://doi.org/10.5539/hes.v6n3p32

Jacques, L. (2018). *What is teaching with variation and is it relevant to teaching and learning mathematics in England? April*, 104–110.

Jahudin, J., & Siew, N. M. (2024). the Effects of Polya‘S Problem Solving With Digital Bar Model on the Algebraic Thinking Skills of Seventh Graders. *Problems of Education in the 21st Century*, *82*(3), 390–409.

https://doi.org/10.33225/pec/24.82.390

Jatiariska, I. G. A. (2021). *The use of Geogebra software in teaching mathematics the use of Geogebra software in teaching mathematics*. https://doi.org/10.1088/1742-6596/1882/1/012042

Jayanthi. (2019). Mathematics in society development - A Study. *Iconic Research and Engineering Journals*, *3*(3), 59–64.

Jelatu, S. (2018). *Effect of GeoGebra - Aided REACT Strategy on Understanding of Geometry Concepts*. *11*(4), 325–336.

Jing, T. J., Tarmizi, R. A., Bakar, K. A., & Aralas, D. (2017a). The Adoption of variation theory in the classroom: Effect on students‘algebraic achievement and motivation to learn. *Electronic Journal of Research in Educational Psychology*, *15*(2), 307–325. https://doi.org/10.14204/ejrep.42.16070

Jing, T. J., Tarmizi, R. A., Bakar, K. A., & Aralas, D. (2017b). Utilization of variation theory in the classroom: Effect on students‘algebraic achievement and motivation. *AIP Conference Proceedings*, *1795*. https://doi.org/10.1063/1.4972172

Kakoma, L. (2016). Mathematical Cognition: Understanding How Children Acquire Mathematical Knowledge and Skills. *Journal Klagenfurt, Austria*, *23*(4 April), 292–296.

Kassa, B., & Ding, L. (2019). *The use of variation theory in a problem-based task design study: HAL Id: hal-02423413*.

Kassa, B. G., & Ding, L. (2019). *The use of variation theory in a problem-based task design study To cite this version: HAL Id: hal-02423413*.

Kassa, B. G., Ding, L., & Tamiru, A. B. (2023). Describing and Interpreting the Space of Classroom Learning in Problem-Solving-Based Mathematics Instruction: Variation as an Analytical Lens. *Education Sciences*, *13*(2). https://doi.org/10.3390/educsci13020111

Khatimah, H., & Sugiman, S. (2019). The effect of problem-solving approach to mathematics problem solving ability in fifth grade. *Journal of Physics: Conference Series*, *1157*(4). https://doi.org/10.1088/1742-6596/1157/4/042104

Kilpatrick, J., Swafford, J., & Findell, and B. (2001). *Adding it up: Helping Children Learn Mathematics*. National Academy Press. http://www.wakamonoup.jp/top/pdf/Third-party_evaluation_2013_points.pdf

Kivkovich, N. (2015). A Tool for Solving Geometric Problems Using Mediated Mathematical Discourse (for Teachers and Pupils). *Procedia - Social and Behavioral Sciences*, *209*(July), 519–525. https://doi.org/10.1016/j.sbspro.2015.11.282

Koji, S., Mulenga, H. M., & Mukuka, A. (2016). An Investigation into Challenges Faced by Secondary School Teachers and Pupils in Algebraic Linear Equations: A Case of Mufulira District, Zambia. *Journal of Education and Practice*, *7*(26), 99–106.

Kpotosu, C. K., Amegbor, S., Mifetu, B., Bessa, R., & Ezah, K. (2024). *Senior High School Students' Difficulties with Geometry Topics*. *May*. https://doi.org/10.20944/preprints202405.0126.v1

Kullberg, A., Ingerman, Å., & Marton, F. (2024). The Variation Theory of Learning. In *Planning and Analyzing Teaching* (Issue January). https://doi.org/10.4324/9781003194903-3

Kullberg, A., Runesson, U., & Marton, F. (2017). *What is made possible to learn when using the variation theory of learning in teaching mathematics? 49*, 559–569. https://doi.org/10.1007/s11858-017-0858-4

Kusdinar, U., Sukestiyarno, S., Isnarto, I., & Istiandaru, A. (2017). Krulik and Rudnik Model Heuristic Strategy in Mathematics Problem Solving. *International Journal on Emerging Mathematics Education*, *1*(2), 205. https://doi.org/10.12928/ijeme.v1i2.5708

Kusumah, Y. S., Kustiawati, D., & Herman, T. (2020). The effect of geogebra in three-dimensional geometry learning on students'mathematical communication ability. *International Journal of Instruction*, *13*(2), 895–908. https://doi.org/10.29333/iji.2020.13260a

Kwesi, E. (2023). *Effectiveness of Interviews as a Tool for Data Collection*. *X*(12), 4235–4241. www.euacademic.org

L, C. R., & Pratibha, P. (2014). *Significance of Review of Related Literature in Social Sciences Presented by Chintamanrao College of Commerce, Sangli*.

Lapinid, M. R. C., & Mendoza, J. B. (2024). The Use of Variation Theory of Learning in Teaching Solving Right Triangles. *Mathematics Teaching-Research Journal*, *16*(3), 58–79.

Laurens, T., Batlolona, F. A., Batlolona, J. R., & Leasa, M. (2018). How does realistic mathematics education (RME) improve students 'mathematics cognitive achievement? *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(2), 569–578. https://doi.org/10.12973/ejmste/76959

Leal, R., Fernández-Cézar, R., Prada Núñez, R., & Augusto Hernández Suárez, C. (2022). Academic Performance and Beliefs About Mathematics In College Students. *Journal of Language and Linguistic Studies*, *18*(3), 148–160.

Leech, N., Barrett, K., & Morgan, G. A. (2013). SPSS for Intermediate Statistics. In *SPSS for Intermediate Statistics* (Issue January). https://doi.org/10.4324/9781410616739

Leelavardhini, D. (2018). *the Impact of Geogebra in Teaching Mathematics and in Math Teachers Professional Development*. *6*(2), 2320–2882. www.ijcrt.org

Liang, B. (2019). A radical constructivist model of teachers 'mathematical learning through student-teacher interaction. *Proceedings of the 41st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *November*, 1814–1819.

Lo, M. L. (2012). Variation theory and the improvement of teaching and learning. *Educational Action Reseach*.

Lord, C., & McGee, J. P. (2001). Educating Children with Autism: Committee on educational interventions for children with autism. In *Division of behavioural and social sciences and education*.

Mahendra, I. W. E., & Rini Purwati, N. K. (2019). Factor Analysis at Item Test Conceptual Understanding of Numerical Method. *JPI (Jurnal Pendidikan Indonesia)*, *8*(1), 77. https://doi.org/10.23887/jpi-undiksha.v8i1.18289

Mahjabeen, W., Alam, S., Hassan, U., Zafar, T., Butt, R., Konain, S., & Rizvi, M. (2018). *Difficulty Index, Discrimination Index and Distractor Efficiency in Multiple Choice Questions*. *March*.

Manalu, R., Napitupulu, E. E., Manullang, M., Simanjuntak, D., & H. Sinambela, J. (2018). The Difference of Mathematical Problem-Solving Ability through the Scientific Approach and the Scientific Approach Assisted by Software Autograph. *American Journal of Educational Research*, *6*(12), 1693–1701.

https://doi.org/10.12691/education-6-12-16

Mårtensson, P., & Ekdahl, A.-L. (2021). Variation theory and teaching experiences as tools to generate knowledge about teaching and learning mathematics-the case of pre-service teachers. *Nordisk Matematikkdidaktikk*, *26*(3–4), 91–112.

Martinez, A. R. (2017). *Digital Commons @ CSUMB The Effects of Using GeoGebra on Student Achievement in Secondary Mathematics the Effects of Using GeoGebra on Student Achievement in Secondary Mathematics*.

Marton, F., & Pang, M. F. (2013). Meanings are acquired from experiencing differences against a background of sameness, rather than from experiencing sameness against a background of difference. *Frontline Learning Research*, *1*, 24–41. http://dx.doi.org/10.14786/flr.v1i1.16

Marton, F., Tsui, A. B. M., Chik, P. R. M., YukKo, P., Lo, M. L., Mok, I. A. C., RPNg, D., Pang, M. F., Pong, W. Y., & Runesson, U. (2004). Classroom Discourse and the Space of Learning. In *Sustainability (Switzerland)* (Vol. 11, Issue 1). y Lawrence Erlbaum Associates, Inc.

Masanja, V. G. (2016). Mathematics and Other Disciplines The Impact of Modern Mathematics in other Disciplines The Impact of Modern Mathematics in other Disciplines University of Dar es Salaam. *University of Dar Es Salaam Mathematics Department*, *July 2008*.

Matlock-hetzel, S. (1997). *I ' Basic Concepts in Item and Test Analysis*.

McCray, K. (2007). Constructivist approach: Improving social studies skills. *Online Submission*, 1–21. https://files.eric.ed.gov/fulltext/ED499380.pdf

Mekuria, Y. S., Alemu, S. D., & Tadesse, H. (2018). Analysis of Students' Commitment and Attitude towards Mathematics in Secondary School of Dega Woreda, Oromia National Regional State, Ethiopia. *Humaniora*, *9*(2), 187. https://doi.org/10.21512/humaniora.v9i2.4738

Mendoza, J. B., & Lapinid, M. R. C. (2024). The Use of Variation Theory of Learning in

Teaching Solving Right Triangles. *Mathematics Teaching-Research Journal*, *16*(3), 58–79.

Methkal, Y., & Algani, A. (2022). Role, need and benefits of mathematics in the development of society. *Journal for the Mathematics Education and Teaching Practices*, *3(1), 23-29, June 2022*, *3*(June), 23–29.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, *30*(1), 65–88. https://doi.org/10.2307/749630

Middleton, J. A., & Spanias, P. A. (2016). *the Research Linked references are available on JSTOR for this article: Findings, Generalizations, and Criticisms of the Research*. *30*(1), 65–88.

Mifetu, B., Kpotosu, C. K., Ezah, R. B. K., & Amegbor, S. (2019). Geometry Topics In Mathematics Perceived Difficult To Study By Senior High School Students In The Cape Coast Metropolis. *University of Cape Coast*, *May*.

Miia Rannikmäe, Jack Holbrook, and R. S. (2016). *Social Constructivism — Jerome Bruner*. 259–275.

MoE. (2013). *Ministry of Education Education Policy and Contemporary Issues in Ethiopia September 2013 Addis Ababa Ministry of Education*. *September*.

MOE. (1994). *Federal Democratic Republic Government of Ethiopia.*

MOE. (2009). Curriculum Framework for Ethiopian Education (KG – Grade 12). Federal Democratic Republic of Ethiopia Ministry of Education).

MOE. (2023). *Ethiopia Education Transformation Programme (EETP)*. *August*.

Moleko, M. M. (2022). *Using universal design for instruction principles to guide flexible , inclusive and accessible teaching of geometry*. *32*(2), 613–633.

Moukhliss, M. (2023). *The Impact of GeoGebra on Algebraic Modeling Problem-Solving in Moroccan Middle School Students*. 57–70.

Murni, V., Sariyasa, S., & Ardana, I. M. (2017). GeoGebra Assist Discovery Learning Model for Problem Solving Ability and Attitude toward Mathematics. *Journal of Physics: Conference Series*, *895*(1). https://doi.org/10.1088/1742-6596/895/1/012049

Namdeo, S., & Sahoo, B. (2016). Item analysis of multiple-choice questions from an assessment of medical students in Bhubaneswar, India. *International Journal of*

Research in Medical Sciences, *4*(5), 1716–1719. https://doi.org/10.18203/23206012.ijrms20161256

Narayan, N. A. (2015). *Role of mathematics for an individual's and national growth. November 2015*.

Narh-Kert, M., & Sabtiwu, R. (2022). the Creative Commons Attributions License [CC BY-NC-ND 4.0] Use of GeoGebra to improve Performance in Geometry. *African Journal of Educational Studies in Mathematics and Sciences*, *18*(1). http://creativecommons.org/licenses/by-nc- nd/4.0.DOI: https://dx.doi.org/10.4314/ajesms.v18i1.3

NEAEA. (2020). *NATIONAL EDUCATIONAL ASSESSMENT AND EXAMINATIONS AGENCY (NEAEA)*.

Ningrum, D. P. N., Usodo, B., & Subanti, S. (2023). Students 'Mathematical Conceptual Understanding: What. *AIP Converence Proceedings*, *2566*(1), 020017.

Nithideechaiwarachok, B., & Chano, J. (2024). Socio-cultural and Social Constructivist Theories and Its Application in EFL Classroom for Thai Pre-service Teachers: A Review for Further Research. *International Journal of Language Education*, *8*(3), 564–572. https://doi.org/10.26858/ijole.v8i3.66499

Nitisha, M., Vincent, S., & College, P. (2018). Role of Mathematics Education in Sustainable Rural Development. *International Journal for Research in Engineering Application & Management (IJREAM)*, 51–53.

Ntumi, S., Agbenyo, S., & Bulala, T. (2023). Estimating the Psychometric Properties (Item Difficulty, Discrimination and Reliability Indices) of Test Items using Kuder-Richardson Approach (KR-20). *Shanlax International Journal of Education*, *11*(3), 18–28. https://doi.org/10.34293/education.v11i3.6081

Nugraheni, Z., Budiyono, B., & Slamet, I. (2018). Upgrading geometry conceptual understanding and strategic competence through implementing rigorous mathematical thinking (RMT). *Journal of Physics: Conference Series*, *983*(1). https://doi.org/10.1088/1742-6596/983/1/012121

Nugroho, H., Desti, D., Kismiantini, K., Setyaningrum, L., Laili, R. N., & Sugiman, S. (2025). *Integrating GeoGebra into Project-Based Learning to Improve Students' Problem-Solving Skill on Circle Topic* (Issue Ismei 2024). Atlantis Press SARL. https://doi.org/10.2991/978-2-38476-434-1_4

Oco, R., Jade, S., Sabasaje, N., & Oco, R. M. (2023). Students ‘Mathematical Skills and Performance. *International Journal of Multidisciplinary Research and Publications (IJMRAP)*, *6*(2), 328–332. https://www.researchgate.net/publication/373110709

Ogan, G. C., & George, N. R. (2015). Investigating difficult concepts in senior secondary school mathematics curriculum as perceived by students. *International Journal of Academic Research and Reflection*, *3*(6), 67–74. www.idpublications.org

Oktaviana, R., & Putri, E. (2020). *A STUDY ON THE JUNIOR HIGH SCHOOL STUDENTS' SPATIAL ABILITY AND MATHEMATICS PROBLEM*. *6*(1), 775–783.

Osman, S., Che Yang, C. N. A., Abu, M. S., Ismail, N., Jambari, H., & Kumar, J. A. (2018). Enhancing Students‘Mathematical Problem-Solving Skills through Bar Model Visualisation Technique. *International Electronic Journal of Mathematics Education*, *13*(3), 273–279. https://doi.org/10.12973/iejme/3919

Palincsar, A. S. (1998). Social constructivist perspectives on teaching and learning. *Annual Review of Psychology*, *49*(January), 345–375. https://doi.org/10.1146/annurev.psych.49.1.345

Pallant, J. (2011). *SPSS Survival Manual - 6th Edition 2017.pdf*. Open University Press.

Pambudi, D. S. (2022). The Effect of Outdoor Learning Method on Elementary Students Motivation and Achievement in Geometry. *International Journal of Instruction*, *15*(1), 747–764. https://doi.org/10.29333/iji.2022.15143a

Pantziara, M., & Philippou, G. N. (2015). Students ‘Motivation in the Mathematics Classroom. Revealing Causes and Consequences. *International Journal of Science and Mathematics Education*, *13*(January 2014), 385–411. https://doi.org/10.1007/s10763-013-9502-0

Pardimin, P., & Widodo, S. A. (2021). Development Comic Based Problem Solving in Geometry. *International Electronic Journal of Mathematics Education*, *12*(3), 233–241. https://doi.org/10.29333/iejme/611

Park, Y. S., Konge, L., & Artino, A. R. (2020). The Positivism Paradigm of Research. *Academic Medicine*, *95*(5), 690–694. https://doi.org/10.1097/ACM.0000000000003093

Parre, R. L., & Marpa, E. P. (2019). *A Qualitative Analysis of Grade Nine Students Difficulties in Geometry*. *7*(2), 389–398.

Patiga, J. L. (n.d.). *Integrative Teaching Strategies (ITS).*

Pehkonen, E., Choe, Y. H., Cai, J., Ishida, J., Stacey, K., Yabe, T., & Nohda, N. (2004). TSG 11: Problem Solving in Mathematics Education. In *Proceedings of the Ninth International Congress on Mathematical Education*. https://doi.org/10.1007/978-94-010-9046-9_81

Pilani, S., Mahesh, J., & Pilani, S. (2024). *Exploring Integrated Teaching Approaches, Methods, and Strategies: Relevance for Educators in the Contemporary Classroom Exploring Integrated Teaching Approaches, Methods, and Strategies: Relevance for Educators in the Contemporary Classroom*. *April*.

Pokhrel, M. (2023). Exploring Challenges Towards Learning Mathematics Among Secondary School Students in Nepal. *Academic Journal of Mathematics Education*, *6*(1), 20–32. https://doi.org/10.3126/ajme.v6i1.63784

Prakash Chand, S. (2023). Constructivism in Education: Exploring the Contributions of Piaget, Vygotsky, and Bruner. *International Journal of Science and Research (IJSR)*, *12*(7), 274–278. https://doi.org/10.21275/sr23630021800

Rahman, L. (2024). Vygotsky 's Zone of Proximal Development of Teaching and Learning in STEM Education. *International Journal of Engineering Research & Technology*, *13*(8), 389–394. https://www.ijert.org/vygotskys-zone-of-proximal-development-of-teaching-and-learning-in-stem-education

Ramdjid, N. R., Sukestiyarno, S., Rochmad, R., & Mulyono, M. (2022). Students'difficulties in solving geometry problems. *Cypriot Journal of Educational Sciences*, *17*(12), 4628–4640. https://doi.org/10.18844/cjes.v17i12.7039

Ramya, K., Mani, N., & Karpagam, K. (2023). A study on 'learning mathematics' and its impact on society a study on_ learning mathematics' and its impact on society especially in education. *Eur. Chem, Bull*, *12*(5), 7148–7156.

Rejeki, S., Riyadi, & Siswanto. (2021). Analysis of students 'problem-solving ability in solving geometry problem. *Journal of Physics: Conference Series*, *1918*(4). https://doi.org/10.1088/1742-6596/1918/4/042075

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The Importance of Prior Knowledge When Comparing Examples: Influences on Conceptual and Procedural Knowledge of Equation Solving. *Journal of Educational Psychology*, *101*(4), 836–852.

https://doi.org/10.1037/a0016026

Rohendi, D., Ramadhan, M. O., Rahim, S. S. A., & Zulnaidi, H. (2025). Enhancing student ‘s interactivity and responses in learning geometry by using augmented reality. *Eurasia Journal of Mathematics, Science and Technology Education*, *21*(1), em2559–em2559. https://doi.org/10.29333/ejmste/15796

Rosyidi, A. H., Sari, Y. M., Fardah, D. K., Masriyah, M., Info, A., & Classroom, G. (2024). *Designing mathematics problem-solving assessment with GeoGebra Classroom: proving the instrument validity*. *18*(3), 1038–1046. https://doi.org/10.11591/edulearn.v18i3.21191

Rytilä, J. (2021). Social constructivism in mathematics? The promise and shortcomings of Julian Cole ‘s institutional account. *Synthese*, *199*(3), 11517–11540. https://doi.org/10.1007/s11229-021-03300-7

Saadati, F., & Celis, S. (2023). Student Motivation in Learning Mathematics in Technical and Vocational Higher Education: Development of an Instrument. *International Journal of Education in Mathematics, Science and Technology*, *11*(1), 156–178. https://doi.org/10.46328/ijemst.2194

Sakti, B. H., & Kartikaningdyah, E. (2018). Testing Bhandari Z-Score Model Bankruptcy Prediction Using Cash Flow. *Journal of Applied Managerial Accounting*, *2*(1), 121–128. https://doi.org/10.30871/jama.v2i1.728

Santos-Trigo, M. (2024). Problem solving in mathematics education: tracing its foundations and current research-practice trends. *ZDM - Mathematics Education*, *56*(2), 211–222. https://doi.org/10.1007/s11858-024-01578-8

Saputra, F. (2025). Instrument Reliability Analysis. *Jurnal Indonesia Sosial Sains*, *6*(3), 791–806. https://doi.org/10.59141/jiss.v6i3.1669

Sarah, M., & Batiibwe, K. (2024). *East African Journal of Education Studies Integration of GeoGebra in Learning Mathematics: Benefits and Challenges*. *7*(4), 684–697. https://doi.org/10.37284/eajes.7.4.2454

Save, T. O., Maria, T. H. E., Victorina, D., & Today, O. F. (2018). *Using Variation Theory to enhance the learning of ―Fitting Shapes‖ of primary four students in Hong Kong*. *May*, 1–33.

Schaver, Z. (2019). *NWCommons The Effects of GeoGebra on Student Achievements,* *Critical Thinking / Problem-Solving Skills, and Engagement / Motivation in High School Mathematics*.

Schettino, C. (2011). Teaching Geometry through Problem-Based Learning. *The Mathematics Teacher*, *105*(5), 346–351. https://doi.org/10.5951/mathteacher.105.5.0346

Selvy, Y., Ikhsan1, M., Johar1, R., & Saminan, A. (2020). *Improving students' mathematical creative thinking and motivation through GeoGebra assisted problem-based learning Improving students' mathematical creative thinking and motivation through GeoGebra assisted problem-based learning*. https://doi.org/10.1088/1742-6596/1460/1/012004

Septian, A., Inayah, S., Suwarman, R. F., & Nugraha, R. (2020). *GeoGebra-Assisted Problem Based Learning to Improve Mathematical Problem-Solving Ability*. *467*(Semantik), 67–71. https://doi.org/10.2991/assehr.k.200827.119

Shadaan, P., & Eu, L. K. (2013). *Effectiveness of Using Geogebra on Students' Understanding in Learning Circles*. *1*(4), 1–11.

Shorten, A., & Smith, J. (2017). *Mixed methods research: expanding the evidence base*. *20*(3), 74–75.

Siagian, P. P. G., Sinaga, B., & Mulyono, M. (2020). Analysis of Difficulty of Student 's Geometry Problem Solving Based on Van Hiele's Thinking Theory with Macromedia Flash. *International Journal on Emerging Mathematics Education*, *3*(2), 162. https://doi.org/10.12928/ijeme.v3i2.13658

Silmi Juman, Z. A. M., Mathavan, M., Ambegedara, A. S., & Udagedara, I. G. K. (2022). Difficulties in Learning Geometry Component in Mathematics and Active-Based Learning Methods to Overcome the Difficulties. *Shanlax International Journal of Education*, *10*(2), 41–58. https://doi.org/10.34293/education.v10i2.4299

Simbolon, A. K. A. P., & Siahaan, L. M. (2021). *The Use of Geogebra Software in Improving Student's Mathematical Abilities in Learning Geometry*. *Cesit 2020*, 352–360. https://doi.org/10.5220/0010311803520360

Simon, M. A. (2011). *PME-NA 2011 Proceedings 31*. 31–43.

Sofyan, D., Puspitasari, N., Maryati, I., Basuki, B., & Madio, S. S. (2021). *The Effects of GeoGebra on Problems Solving Skill in the Integral Calculus*.

https://doi.org/10.4108/eai.11-7-2019.2297424

Somantri, N. L., Yusof, A. A., Hamid, N. H. A., Othman, Z. S., & Spector, M. (2019). *The actual status of general secondary school mathematics teaching and learning: The case of Ethiopia The actual status of general secondary school mathematics teaching and learning : The case of Ethiopia*. https://doi.org/10.1088/1742-6596/1176/4/042097

Stadtländer, C. T. K.-H. (2009). Qualitative, Quantitative, and Mixed-Methods Research. *Microbe Magazine*, *4*(11), 485–485. https://doi.org/10.1128/microbe.4.485.1

State, T., & State, D. (2024). *Effects Of Conceptual Instructional Strategy On Students' Achievement And Retention In Geometry In Delta State , Nigeria*. *12*(2), 115–126.

Sudihartinih, E., & Purniati, T. (2019). Using geogebra to develop students understanding on circle concept. *Journal of Physics: Conference Series*, *1157*(4). https://doi.org/10.1088/1742-6596/1157/4/042090

Sulistyowati, F., Budiyono, B., & Slamet, I. (2017). Problem Solving Reasoning and Problem Based Instruction in Geometry Learning. *Journal of Physics: Conference Series*, *895*(1). https://doi.org/10.1088/1742-6596/895/1/012045

Sun, X. (2011). ―Variation problems‖ and their roles in the topic of fraction division in Chinese mathematics textbook examples. *Educational Studies in Mathematics*, *76*(1), 65–85. https://doi.org/10.1007/s10649-010-9263-4

Sunita, Y. (2019). Role of Mathematics in the. *International Journal of Research and Analytical Reviews (IJRAR)*, *6*(4), 295–298.

Sunzuma, G. (2023). Technology integration in geometry teaching and learning: A systematic review (2010-2022). *Lumat*, *11*(3). https://doi.org/10.31129/LUMAT.11.3.1938

Taber, K. S. (2018). The Use of Cronbach 's Alpha When Developing and Reporting Research Instruments in Science Education. *Research in Science Education*, *48*(6), 1273–1296. https://doi.org/10.1007/s11165-016-9602-2

Taherdoost, H. (2020). Different Types of Data Analysis; Data Analysis Methods and Techniques in Research Projects Authors Hamed Taherdoost To cite this version: HAL Id : hal-03741837 Different Types of Data Analysis ; Data Analysis Methods and Techniques in Research Projects. *International Journal of Academic Research* *in Management (IJARM)*, *9*(1), 1–9.

Taherdoost, H. (2021). Data Collection Methods and Tools for Research; A Step-by-Step Guide to Choose Data Collection Technique for Academic and Business Research Projects. *International Journal of Academic Research in Management (IJARM)*, *10*(1), 10–38. https://hal.science/hal-03741847%0Ahttps://hal.science/hal-03741847/document

Takele, M. (2023). *An Assessment of Prospective Mathematics Teachers' Capability in Performing Basic Operations , Difficulties and Misconceptions on Fractions in Jimma College of Teachers ' Education ,... An Assessment of Prospective Mathematics Teachers' Capability in Pe*. *June*.

Tamam, B., & Dasari, D. (2021). The use of Geogebra software in teaching mathematics. *Journal of Physics: Conference Series*, *1882*(1). https://doi.org/10.1088/1742-6596/1882/1/012042

Tavakol, M., & Dennick, R. (2011). Making sense of Cronbach 's alpha. *International Journal of Medical Education*, *2*, 53–55. https://doi.org/10.5116/ijme.4dfb.8dfd

Tefera, T., Asgidom, A., Omer, J., W/Hanna, T., Dalelo, A., & Assefa, B. (2018). Ethiopian education development roadmap (2018-30). *MoE*, *July*, 1–101. https://5y1.org/download/f80344e9046fa0346fc402c29dd76288.pdf

ten Braak, D., Lenes, R., Purpura, D. J., Schmitt, S. A., & Størksen, I. (2022). Why do early mathematics skills predict later mathematics and reading achievement? The role of executive function. *Journal of Experimental Child Psychology*, *214*, 105306. https://doi.org/10.1016/j.jecp.2021.105306

Tesfamicael, S. A., & Ayalew, Y. (2021). Mathematics Education in Ethiopia in the Era of COVID-19: Boosting Equitable Access for All Learners via Opportunity to Learning. *Contemporary Mathematics and Science Education*, *2*(1), ep21005. https://doi.org/10.30935/conmaths/9680

Tesfaw, B. K., Ayele, M. A., & Wondimuneh, T. E. (2024). Context-based problem-posing and solving instructional approach and students 'engagement in learning data handling. *Cogent Education*, *11*(1). https://doi.org/10.1080/2331186X.2024.2389486

Tesfaye, A. (2021). *Ethiopian Early Grade Mathematics Curriculum Organization*.

12(25), 18–27. https://doi.org/10.7176/JEP/12-25-03

Thanheiser, E. (2023). What is the Mathematics in Mathematics Education? *Journal of Mathematical Behavior*, *70*(January), 101033. https://doi.org/10.1016/j.jmathb.2023.101033

Thapa, R., Dahal, N., & Pant, B. P. (2022). *GeoGebra Integration in High School Mathematics: An Experiential Exploration on Concepts of Circle*. *14*(5), 16–33.

Thente, P. (2019). *Relationship between problem solving and motivation in mathematics: A qualitative study on how problem-solving influences pupils' motivation in mathematics. June*. https://uis.brage.unit.no/uis-xmlui/handle/11250/2623286%0Ahttps://uis.brage.unit.no/uis-xmlui/bitstream/handle/11250/2623286/Thente_Pontus.pdf?sequence=1

Thompson, P. W. (2021). Radical Constructivism: Reflections and Directions. *Radical Constructivism in Action, June*, 307–331. https://doi.org/10.4324/9780203487518-27

Tibebu, D., Sabates, R., Rolleston, C., & Hoddinott, J. (2021). *Trends in Mathematics Learning in Ethiopia: 2012 - 2019 Educational Reforms in Ethiopia: The General Education Quality Improvement Program (GEQIP)*. *21*(1), 2012–2019.

Tiruneh, D. T., Hoddinott, J., Rolleston, C., Sabates, R., & Woldehanna, T. (2021). *Understanding Achievement in Numeracy Among Primary School Children in Ethiopia: Evidence from RISE Ethiopia Study*. *May*, 1–34.

Tiruneh, D. T., Sabates, R., Rolleston, C., & Hoddinott, J. (2021). Trends in Mathematics Learning in Ethiopia: 2012 - 2019. *Bahir Dar Journal of Education*, *21*(1), 26–45.

Trejo, R. C. (2024). *The Impacts of Using Geogebra on Students' Perceptions and Achievement in Learning Trigonometric Ratios*. 1–177.

Tuge, C. (n.d.). *Mathematics Curriculum, the Philosophy of Mathematics and its Implications on Ethiopian Schools Mathematics Curriculum*.

Ubi, E. E., Odiong, A. U., & Igiri, O. I. (2018). Geometry viewed as a difficult mathematics. *International Journal of Innovative Science and Research Technology*, *3*(11), 251–255.

Ünal, M. (2017). Preferences of Teaching Methods and Techniques Universal Journal of Educational Research 5(2): 194-202, 2017 http://www.hrpub.org DOI: 10.13189/ujer.2017.050204

Unamba, E. C., & Ugochukwu, N. J. (2021). *Using blended-learning strategy in mathematics to strengthen the teaching of geometric 2D shapes and their properties to primary 6 learners 1*. *16*(April), 0–2.

Vivek, R., Nanthagopan, Y., & Piriyatharshan, S. (2023). Beyond Methods: Theoretical Underpinnings of Triangulation in Qualitative and Multi-Method Studies‖. *SEEU Review*, *18*(2), 105–122. https://doi.org/10.2478/seeur-2023-0088

Voráčová, Š. (2024). Research on the Use of Dynamic Geometry via Blended Learning. *Proceedings of the Asian Technology Conference in Mathematics*, 334–343.

Walde, G. S. (2019). The actual status of general secondary school mathematics teaching and learning: The case of Ethiopia. *Journal of Physics: Conference Series*, *1176*(4). https://doi.org/10.1088/1742-6596/1176/4/042097

Wanna, W. (2023). *of Culture Language Studies Proposing Difficulty and Discrimination Level Indices to Validate Teacher- Made Multiple-Choice Test Items: A Remedy to Variations Observed in Scholars' Propositions*. 42–62.

Wassie, Y. A., & Zergaw, G. A. (2018). Capabilities and contributions of the dynamic math software, GeoGebra. *North American GeoGebra Journal*, *7*(1), 68–86. https://mathed.miamioh.edu/index.php/ggbj/article/view/149

Wassie, Y. A., & Zergaw, G. A. (2019). *Some of the Potential Affordances, Challenges and Limitations of Using GeoGebra in Mathematics Education*. *15*(8).

WB. (2022). *Confronting the Learning Crisis: Lessons from World Bank Support for Basic Education, 2012 – 22*. 1–41.

Weng, T.-S. (2017). The Importance of Mathematics and Science Education in the Context of Digital Technology on Industrial Innovation. *Research Journal of Applied Sciences, Engineering and Technology*, *14*(11), 418–426. https://doi.org/10.19026/rjaset.14.5142

Xu, L. (2019). Relating Teaching and Learning of Science Through the Lens of Variation Theory. *Scandinavian Journal of Educational Research*, *63*(1), 145–162. https://doi.org/10.1080/00313831.2017.1357143

Yakout, S. M., Hadi, K., Alolayan, M., Alasmari, M. M., Alhaji, K. M., Alobaid, L. M., Alonazi, M. A., & Rn, M. K. A. P. (2024). Analysis of Difficulty Index, Discrimination Index and

Distractor Efficiency in Multiple Choice Questions.*Journal of Advanced Zoology*, *2*(1), 15–28. https://doi.org/10.53555/jaz.v45i3.4138

Yeshanew, A., Belachew, T., Gezahegn, H., & Tesfa, T. (2024). *Effects of Variation Theory Integrated Guided Inquiry-Based Instruction on Grade Ten Students Self-Belief in Learning Solid Geometry. 12(3), 56–67.*

Yuliarni, R. W., & Hidayati, Y. M. (2023). *Analysis of Students' Misconseptions in Addition and Subtraction Material*. Atlantis Press SARL. https://doi.org/10.2991/978-2-38476-086-2_148

Yurniwati, & Soleh, D. A. (2020). The effectiveness of computer-based problem solving to improve higher order thinking skills on prospective teachers. *International Journal of Instruction*, *13*(2), 393–406. https://doi.org/10.29333/iji.2020.13227a

Zakaria, E., & Lee, L. S. (2012). Teachers‘perceptions toward the use of geogebra in the teaching and learning of mathematics. *Journal of Mathematics and Statistics, 8*(2), 253–257. https://doi.org/10.3844/jmssp.2012.253.257

Zakariya, Y. F., & Massimiliano, B. (2021). Development of mathematics motivation scale: A preliminary exploratory study with a focus on secondary school students. *International Journal of Progressive Education*, *17*(1), 314–324. https://doi.org/10.29329/ijpe.2021.329.20

Żukowska, J., & Strelau, R. (2025). *The challenges of mixed methods research in studies on 21 st century skills The challenges of mixed methods research in studies on 21 st century skills*. *January*. https://doi.org/10.24427/az-2025-0011

APPENDEX

Appndex1: Normality Test

1.1 Normality test Motivation pretest scales

	Descriptive Statistics							
Variable	Questions	N	Minimum	Maximum	Mean	Std. Deviation	Skewness	
		Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error
INTERSINIC MOTIVATION	Q1	125	1.00	5.00	3.4960	1.10442	-.318	.217
	Q2	125	1.00	5.00	3.4800	1.12594	-.449	.217
	Q3	125	1.00	5.00	3.6480	1.13063	-.596	.217
	Q4	125	1.00	5.00	3.4720	1.17484	-.447	.217
	Q5	125	1.00	5.00	3.7040	1.10734	-.801	.217
SELFEFFICACY	Q6	125	1.00	5.00	3.4160	1.08646	-.279	.217
	Q7	125	1.00	5.00	3.2320	1.10085	-.290	.217
	Q8	125	1.00	5.00	3.5200	1.15423	-.449	.217
	Q9	125	1.00	5.00	3.3920	1.19066	-.336	.217
	Q10	125	1.00	5.00	3.5120	1.12606	-.530	.217
UTILITY VALUE	Q11	125	1.00	5.00	3.8160	1.18719	-.929	.217
	Q12	125	1.00	5.00	3.7840	1.08942	-.700	.217
	Q13	125	1.00	5.00	3.6560	1.19882	-.761	.217
	Q14	125	1.00	5.00	3.5360	1.18147	-.550	.217
	Q15	125	1.00	5.00	3.7360	1.17874	-.642	.217
TEST ANXIETY	Q16	125	1.00	5.00	3.6640	1.26328	-.804	.217
	Q17	125	1.00	5.00	3.7520	1.08253	-.769	.217
	Q18	125	1.00	5.00	3.6240	1.16857	-.675	.217
	Q19	125	1.00	5.00	3.4000	1.19137	-.587	.217
	Q20	125	1.00	5.00	3.6960	1.21962	-.751	.217
EXTERNISIC MOTIVATION	Q21	125	1.00	5.00	3.0400	1.27886	-.029	.217
	Q22	125	1.00	5.00	3.1760	1.19180	-.115	.217
	Q23	125	1.00	5.00	3.0560	1.25919	-.008	.217
	Q24	125	1.00	5.00	3.5280	1.22195	-.565	.217

	Q25	125	1.00	5.00	3.4640	1.25431	-.515	.217
ATTENTION	Q26	125	1.00	5.00	3.4560	1.25405	-.422	.217
	Q25	125	1.00	5.00	3.5680	1.07265	-.618	.217
	Q28	125	1.00	5.00	3.2400	1.13166	-.113	.217
	Q29	125	1.00	5.00	3.4800	1.17501	-.345	.217
	Q30	125	1.00	5.00	3.5600	1.13166	-.287	.217
SATISFACTION	Q31	125	1.00	5.00	3.5520	1.18768	-.537	.217
	Q32	125	1.00	5.00	3.4480	1.19445	-.265	.217
	Q33	125	1.00	5.00	3.3760	1.22911	-.172	.217
	Q34	125	1.00	5.00	3.3680	1.22169	-.281	.217
	Q35	125	1.00	5.00	3.5120	1.18876	-.322	.217

1.2 Normality test Motivation scales post test

		N	Minimum	Maximum	Mean	Std. Deviation	Skewness	
		Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error
INTERSINIC MOTIVATION	Q1	125	1.00	5.00	3.9520	1.09143	-1.076	.217
	Q2	125	1.00	5.00	3.9760	1.00373	-.924	.217
	Q3	125	1.00	5.00	3.8480	1.09291	-.747	.217
	Q4	125	1.00	5.00	3.9440	1.07990	-.785	.217
	Q5	125	1.00	5.00	4.0400	.96219	-.854	.217
SELFEFFICACY	Q6	125	1.00	5.00	3.7840	1.09680	-.678	.217
	Q7	125	1.00	5.00	3.8880	.99366	-.774	.217
	Q8	125	1.00	5.00	3.9440	1.05726	-.927	.217
	Q9	125	1.00	5.00	3.8960	1.04591	-.649	.217
	Q10	125	1.00	5.00	3.8400	1.06559	-.731	.217
UTILITY VALUE	Q11	125	1.00	5.00	4.1040	.96574	-.976	.217
	Q12	125	1.00	5.00	4.0320	.98321	-.996	.217
	Q13	125	1.00	5.00	4.0880	.95896	-.903	.217
	Q14	125	1.00	5.00	3.9440	1.04961	-.907	.217

	Q15	125	1.00	5.00	4.0640	.93966	-.899	.217
	Q16	125	1.00	5.00	3.1280	1.13575	-.121	.217
	Q17	125	1.00	5.00	3.1760	1.12209	-.041	.217
	Q18	125	1.00	5.00	2.8320	1.13405	-.101	.217
	Q19	125	1.00	5.00	3.0800	1.18866	-.098	.217
	Q20	125	1.00	5.00	3.0400	1.08806	-.157	.217
EXTERNISIC MOTIVATION	Q21	125	1.00	5.00	3.7600	1.20081	-.746	.217
	Q22	125	1.00	5.00	3.8480	1.10026	-.727	.217
	Q23	125	1.00	5.00	3.8800	1.14723	-.804	.217
	Q24	125	1.00	5.00	3.8320	1.11973	-.748	.217
	Q25	125	1.00	5.00	3.8880	1.10873	-.821	.217
ATTENTION	Q26	125	1.00	5.00	3.9920	1.11801	-.899	.217
	Q25	125	1.00	5.00	3.9520	.97432	-.754	.217
	Q28	125	1.00	5.00	3.9120	1.09997	-.636	.217
	Q29	125	1.00	5.00	3.8640	1.10237	-.754	.217
	Q30	125	1.00	5.00	3.9520	1.09143	-.887	.217
SATISFACTION	Q31	125	1.00	5.00	3.9520	1.00688	-.770	.217
	Q32	125	1.00	5.00	3.8480	1.08551	-.807	.217
	Q33	125	1.00	5.00	3.9760	1.01968	-.833	.217
	Q34	125	1.00	5.00	3.8320	1.11973	-.748	.217
	Q35	125	1.00	5.00	3.8880	1.15155	-.937	.217

1.3 Normality test conceptual understanding and problem-solving skills

Variable	N	Minimum	Maximum	Mean	SD	Skewness	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
Pre-pss	125	0.00	10	4.224	2.334	0.146	.217
Pre-CU	125	00	17	6.2720	3.50662	0.717	.217
Post-cu	125	2	19	9.68	4.495	0.219	.217
Post-pss	125	00	10	6.2400	2.40765	-0.146	.217

Appendix -2: Factor Loadings for the Rotated Factors

Item	Factor Loading							Com munali ty
	1	2	3	4	5	6	7	
Intrinsic value								
1	.702							.829
2	.820							.914
3	.885							.960
4	.835							.922
5	.816							.929
Self-efficacy								
1						.779		.689
2						.806		.867
3						.804		.751
4						.820		.909
5						.841		.823
Utility value								
1			.917					.937
2			.710					.773
3			.746					.730
4			.819					.861
5			.846					.852
Test anxiety								
1				.830				.821
2				.723				.580
3				.901				.853
4				.892				.873
5				.848				.822
Extrinsic Motivation								
1					.851			.773
2					.792			.776
3					.792			.768
4					.725			.680
5					.675			.709
Attention								
1							.851	.920
2							.792	.798
3							.792	.665
4							.725	.942
5							.675	.846
Satisfaction								
1		.900						.764

2		.772						.847
3		.671						.870
4		.943						.861
5		.822						.653

Note. Loadings < .40 are omitted. Factors may be more than three.

Appendix -3: Bonferroni test result

The Results of Bonferroni test based on the adjusted components of post-attitude and post- overall attitude mean by Groups

Variable	(I) group	(J) group	Mean Difference (I-J)	p
Post-intrinsic	TG1	CC	1.103*	.000
		TG2	.249	.915
		TG3	.491*	.049
	TG2	CC	.854*	.000
		TG3	.242	1.000
	TG3	CC	.612*	.011
Post-self-efficacy	TG1	CC	.974*	.000
		TG2	.232	1.000
		TG3	.286	.752
	TG2	CC	.742*	.001
		TG3	.054	1.000
	TG3	CC	.688*	.003
Post utility Value	TG1	CC	1.058*	.000
		TG2	.391	.093
		TG3	.358	.206
	TG2	CC	.667*	.001
		TG3	-.033	1.000
	TG3	CC	.700*	.001
Post-test anxiety	TG1	CC	-.754*	.001
		TG2	-.001	1.000
		TG3	-.007	1.000
	TG2	CC	-.753*	.000
		TG3	-.006	1.000
	TG3	CC	-.747*	.001
Post-extrinsic motivation	TG1	CC	1.053*	.000
		TG2	.478	.101
		TG3	.323	.767
	TG2	CC	.576*	.035
		TG3	-.154	1.000

	TG3	CC	.730*	.007

Variable	(I) group	(J) group	Mean Difference (I-J)	Sig
Post- Attention	TG1	CC	.938*	.000
		TG2	.292	.672
		TG3	.482	.077
	TG2	CC	.646*	.005
		TG3	.190	1.000
	TG3	CC	.456	.133
Post-satisfaction	TG1	CC	1.129*	.000
		TG2	.429	.111
		TG3	.465	.114
	TG2	CC	.700*	.002
		TG3	.036	1.000
	TG3	CC	.663*	.009
Post- motivation	TG1	CC	.776*	.000
		TG2	.313	.055
		TG3	.284	.159
	TG2	CC	.463*	.001
		TG3	-.029	1.000
	TG3	CC	.492*	.001

*The mean difference is significant at the .05 level.

Adjustment for multiple comparisons: Bonferroni.

Appendex-4: Different Z-score Values

APPENDEX-4.1: Z-score Value of the mean gain

	Z-score Value of the mean gain						
group		Z-CU	Z-MOT	group	Z-PSS	Z-CU	Z-MOT
1		-0.77331	-0.41588	3	-0.00594	0.14293	0.10397
1		-1.23143	-0.2741	3	-0.74832	-0.31519	-0.36862
1		-0.31519	-0.74669	3	-1.4907	-0.77331	-0.55766
1		-1.00237	-0.60492	3	-2.23308	-1.23143	-2.8261
1		-0.77331	-0.65218	3	0.36525	0.14293	0.67108
1		-0.54425	-0.5104	3	-1.11951	-0.31519	-0.55766
1		-0.31519	-0.69944	3	1.10763	1.05917	1.28545
1		-0.54425	-0.60492	3	0.36525	0.14293	0.24575
1		-0.54425	-0.88847	3	0.36525	0.37199	-1.40832
1	-1.11951	-1.23143	-0.69944	3	-0.74832	-0.31519	1.14367
1	-0.00594	0.14293	-0.32136	3	-0.37713	-0.08613	0.19849
1	-1.86189	-1.00237	-0.55766	3	-0.74832	-0.08613	0.05671
1		-1.68954	-0.98299	3	-0.37713	-0.08613	3.00123
1	-1.86189	-0.31519	-0.74669	3	-1.11951	-0.31519	-0.88847
1		-1.23143	-0.5104	3	-1.4907	-0.54425	1.00189
1		-0.77331	-0.79395	3	0.36525	0.60105	0.15123
1		-0.77331	-0.74669	3	-0.00594	1.28823	0.71834
1		-1.00237	0.81286	3	-0.37713	-0.77331	0.81286
1		0.14293	-1.40832	3	-1.11951	1.05917	-0.79395
1		-0.54425	-1.50284	3	-0.00594	-0.08613	1.8053
1		-0.77331	-1.36106	3	-0.00594	-0.77331	0.34027
1	-1.11951	-0.54425	-0.2741	3	-0.00594	-0.77331	0.90738
1	-0.74832	-0.54425	-1.3138	3	0.36525	0.83011	0.24575
1	-0.74832	0.37199	0.34027	3	-0.74832	0.60105	0.38753
1		-1.46049	-1.50284	3	0.73644	-0.54425	0.90738
1	-0.74832	-0.31519	-2.68432	3	0.36525	0.83011	-0.79395
1	-0.00594	0.14293	1.52175	3	0.36525	-1.23143	-1.5501
2		1.05917	0.71834	3	0.36525	-1.23143	0.67108
2	2.22121	0.60105	1.52175	3	-0.37713	-0.31519	-0.98299
2		0.60105	3.00000	3	0.73644	0.37199	1.14367
2	1.10763	1.97541	0.24575	3	-1.11951	-0.08613	-0.84121
2	2.22121	1.74635	-0.5104	3	-1.11951	-0.54425	-0.17958
2		1.97541	1.52175	3	-1.11951	-1.46049	-1.17203
2		1.05917	0.57656	3	-1.4907	-1.23143	-1.03025
2		1.74635	0.38753	3	-0.00594	-0.54425	1.14367
2	1.47882	0.37199	0.86012	4	-0.00594	-1.00237	0.10397
2	0.73644	2.43353	0.19849	4	-0.00594	-0.54425	0.05671

2		2.43353	3.0000	4	-0.00594	1.51729	0.05671
2	-0.00594	-0.77331	-0.41588	4	-1.4907	0.14293	-1.5501
2		1.51729	0.19849	4	-1.11951	-0.31519	-0.41588
2		0.14293	-2.21173	4	-0.37713	-0.31519	-0.03781
2	0.73644	1.97541	0.19849	4	-0.00594	0.14293	0.19849
2		2.43353	1.04915	4	-0.74832	-1.9186	-0.32136
2		-1.23143	-0.84121	4	-0.00594	-0.77331	0.43478
2		2.20447	0.71834	4	-0.74832	-0.54425	-0.36862
2		1.28823	0.34027	4	-0.37713	-1.9186	1.19093
2		0.14293	-0.03781	4	0.73644	-0.08613	0.05671
2		-0.77331	-0.65218	4	0.36525	-0.54425	0.34027
2		0.37199	-0.08507	4	1.47882	0.14293	0.48204
2		-0.08613	-0.93573	4	1.47882	0.14293	-0.22684
2	0.36525	0.37199	-0.13233	4	1.85002	0.60105	0.29301
2		-0.08613	-0.13233	4	0.36525	-0.31519	0.05671
2	1.47882	1.51729	0.71834	4	0.73644	-0.08613	-0.08507
2	0.73644	1.05917	0.34027	4	-0.37713	-1.68954	-1.26655
2	0.36525	0.60105	-0.03781	4	0.73644	-0.08613	-0.22684
2	-0.00594	0.14293	-0.65218	4	-0.37713	-1.23143	0.00945
2	0.73644	1.05917	0.00945	4	0.36525	-0.31519	-0.17958
2		0.83011	-0.08507	4	-0.00594	-0.77331	0.38753
2		2.43353	0.90738	4	-0.00594	-0.54425	0.29301
2	0.73644	1.05917	0.00945	4	-0.37713	-1.00237	0.24575
2		1.51729	0.43478	4	0.36525	0.37199	1.19093
3		0.14293	0.10397	4	1.10763	-0.31519	0.86012
				4	0.73644	1.51729	1.569
				4	1.10763	-0.31519	0.86012

Appendex-4.2 Z-score of Conceptual understanding

group	z-score post	Z-score pre	group	z-score post	Z-score pre
1	-1.0412	-0.36274	3	-0.81872	-0.64792
1	-1.26368	-0.07757	3	-0.59624	0.20761
1	-1.0412	-0.93309	3	-1.48616	-0.36274
1	-1.26368	-0.36274	3	-0.15129	-0.36274
1	-0.81872	-0.07757	3	0.07119	0.49278
1	-1.0412	-0.64792	3	0.73863	-0.36274
1	-1.0412	-0.93309	3	0.29367	0.20761
1	-1.26368	-0.93309	3	0.51615	0.20761
1	-1.26368	-0.93309	3	-1.26368	-1.21827

1	-1.70864	-0.64792	3	-0.59624	-0.64792
1	0.07119	-0.07757	3	-1.0412	-1.21827
1	-1.48616	-0.64792	3	-0.59624	-0.64792
1	-1.48616	0.20761	3	-1.0412	-0.93309
1	0.07119	0.49278	3	-0.15129	0.49278
1	0.96111	2.77418	3	0.29367	-0.36274
1	-1.0412	-0.36274	3	0.73863	-0.64792
1	-0.59624	0.20761	3	-1.26368	-0.64792
1	0.07119	1.34831	3	1.18359	0.20761
1	-0.15129	-0.36274	3	-0.81872	-0.93309
1	-1.26368	-0.93309	3	0.96111	2.20383
1	-1.26368	-0.64792	3	-0.37377	0.49278
1	-0.15129	0.49278	3	-0.59624	-1.78862
1	-1.0412	-0.64792	3	-0.15129	-0.93309
1	0.51615	-0.93309	3	-0.15129	0.49278
1	-1.0412	0.49278	3	-0.15129	-1.21827
1	-1.26368	-1.21827	3	-0.59624	0.77796
1	-0.59624	-0.93309	3	-0.37377	1.06313
2	1.18359	0.20761	3	-1.0412	-0.93309
2	-0.15129	-0.93309	3	0.51615	0.20761
2	-0.81872	-1.78862	3	0.07119	0.20761
2	0.51615	-1.78862	3	-1.0412	-0.64792
2	0.96111	-0.93309	3	-1.26368	0.20761
2	1.18359	-0.93309	3	1.18359	3.05936
2	1.18359	0.20761	3	-0.15129	0.49278
2	0.96111	-0.93309	4	-0.15129	1.06313
2	-0.59624	-1.21827	4	0.07119	0.77796
2	1.40607	-1.21827	4	0.29367	-1.50344
2	1.62855	-0.93309	4	-1.0412	-1.21827
2	0.96111	2.20383	4	-0.59624	-0.36274
2	1.62855	0.20761	4	-0.37377	-0.07757
2	1.62855	1.91866	4	0.07119	-0.07757
2	1.18359	-0.93309	4	-1.48616	0.49278
2	1.18359	-1.50344	4	-0.81872	-0.07757
2	-0.15129	1.34831	4	-0.37377	-0.07757
2	1.62855	-0.64792	4	-0.81872	1.34831
2	1.85103	0.77796	4	-0.15129	-0.07757
2	0.96111	1.06313	4	0.29367	1.06313
2	1.18359	2.48901	4	-0.37377	-0.64792
2	1.40607	1.34831	4	0.73863	0.77796
2	1.40607	1.91866	4	0.51615	-0.07757

2	0.96111	0.77796	4	-0.37377	-0.07757
2	0.96111	1.34831	4	0.73863	1.06313
2	1.18359	-0.36274	4	-1.48616	0.20761
2	1.40607	0.49278	4	0.29367	0.49278
2	1.62855	1.34831	4	-1.48616	-0.36274
2	1.62855	1.91866	4	0.07119	0.49278
2	1.85103	1.06313	4	-0.59624	0.20761
2	0.73863	-0.07757	4	0.07119	0.77796
2	1.62855	-0.93309	4	-1.26368	-0.36274
2	0.96111	-0.07757	4	2.07351	2.20383
2	1.40607	-0.07757	4	0.29367	0.77796
3	-1.0412	-1.50344	4	1.40607	-0.07757
			4	-0.59624	-0.36274

Appendix 4.3 z-score of Problem solving

Group	z-score post	Z-score pre	Group	z-score post	Z-score pre
1	-1.76105	-0.09596	3	0.31566	1.18917
1	-0.93037	-0.09596	3	-0.93037	0.7608
1	-1.34571	-0.52433	3	-1.76105	0.7608
1	-2.1764	-1.38108	3	-0.09968	-0.52433
1	-1.34571	-0.95271	3	-0.51503	0.7608
1	-0.93037	-0.95271	3	1.14635	-0.09596
1	-2.59174	-0.09596	3	0.731	0.33242
1	-1.34571	-0.95271	3	0.31566	-0.09596
1	-0.93037	-0.09596	3	-0.09968	0.7608
1	-0.93037	0.33242	3	-0.93037	-0.52433
1	-0.93037	-0.95271	3	-1.34571	-0.52433
1	-0.51503	1.61755	3	0.731	1.18917
1	-0.51503	0.33242	3	-0.51503	0.7608
1	-0.09968	2.04592	3	-0.09968	1.61755
1	0.31566	0.33242	3	0.31566	-0.09596
1	-0.51503	0.33242	3	1.14635	1.18917
1	-0.09968	-0.09596	3	0.31566	0.7608
1	-0.51503	-0.09596	3	1.14635	2.4743
1	-1.34571	-0.52433	3	-0.09968	-0.09596
1	-0.51503	-0.09596	3	-0.09968	-0.09596
1	-1.76105	-0.95271	3	0.31566	0.33242
1	-0.93037	0.33242	3	-0.51503	-0.95271
1	-0.93037	-0.09596	3	-0.93037	-0.09596
1	-1.34571	-0.52433	3	0.31566	-0.52433
1	-0.09968	1.61755	3	-0.51503	-0.95271

1	-0.51503	0.33242	3	-0.09968	-0.52433
1	-0.51503	-0.52433	3	-0.51503	-0.95271
2	1.56169	2.04592	3	-0.51503	-0.09596
2	1.14635	-1.38108	3	0.31566	-0.52433
2	-0.51503	-0.52433	3	-0.93037	0.33242
2	0.31566	-0.95271	3	-0.51503	0.7608
2	1.14635	-1.38108	3	-0.51503	0.7608
2	0.731	-1.80946	3	-0.93037	0.7608
2	1.56169	-0.09596	3	0.31566	0.33242
2	0.31566	-0.09596	4	0.31566	0.33242
2	0.731	-0.95271	4	-0.93037	-0.95271
2	1.14635	0.33242	4	1.56169	1.61755
2	1.56169	-1.80946	4	0.31566	2.04592
2	1.56169	1.61755	4	-0.93037	0.33242
2	1.56169	0.7608	4	-0.51503	-0.09596
2	-0.09968	1.18917	4	1.56169	1.61755
2	0.31566	-0.52433	4	-2.1764	-1.38108
2	1.56169	-1.38108	4	-1.76105	-1.80946
2	1.14635	1.61755	4	-0.93037	-0.09596
2	1.56169	-0.52433	4	-0.09968	0.33242
2	1.56169	0.33242	4	-0.51503	-1.38108
2	0.31566	-0.09596	4	0.31566	-0.09596
2	1.56169	2.04592	4	-0.09968	-1.80946
2	0.731	0.33242	4	0.31566	-1.38108
2	0.731	1.18917	4	0.31566	-1.80946
2	-0.09968	-0.52433	4	-0.51503	-0.95271
2	1.14635	1.18917	4	0.31566	-0.52433
2	1.56169	-0.09596	4	-2.1764	-1.80946
2	0.731	-0.09596	4	-0.93037	-1.80946
2	1.56169	1.18917	4	0.31566	0.7608
2	0.731	0.7608	4	-0.51503	-0.95271
2	1.14635	0.33242	4	0.31566	0.33242
2	1.56169	1.18917	4	-0.09968	-0.09596
2	0.731	-1.80946	4	0.31566	0.7608
2	1.56169	0.7608	4	1.56169	1.18917
2	-0.09968	-1.38108	4	0.31566	-0.95271
3	-0.09968	-0.09596	4	1.56169	0.7608
			4	0.31566	-0.95271

Appendex-5: Questionnaire

Addis Ababa University

School of Graduate Study

College of Education and Behavioral studies

Department of Mathematics Education

Objectives: -the main objectives of this questionnaire are to investigate the impact of Variation theory GeoGebra based problem solving approaches on student‘s motivation in learning geometry. Hence you are kindly requested to respond for each item through critical reading.

Instruction: please read each item carefully and then put your own level of agreement or disagreement, with the items circle the number that best matches your attitudes or feelings about each motivation scales to math. I respect your responds confidentiality Ethically

1 = (SD) Strongly Disagree 2 = (D) Just Disagree

3 = (U) Unsure or no feelings one way or the other **4** = (A) Just Agree

5= (SA) Strongly Agree

Mathematic motivation questionnaire

No		**1**	**2**	**3**	**4**	**5**
Intrinsic value						
1	I enjoy learning geometry	**1**	**2**	**3**	**4**	**5**
2	I find learning geometry is interesting	**1**	**2**	**3**	**4**	**5**
3	I like geometry that challenges me	**1**	**2**	**3**	**4**	**5**
4	When I have the opportunity, I choose geometry past questions and solve on my own Item	**1**	**2**	**3**	**4**	**5**
5	Understanding the topics of geometry is very important to me	**1**	**2**	**3**	**4**	**5**
Self-efficacy						
6	I am confident I will do well on geometry assignments and projects	**1**	**2**	**3**	**4**	**5**

7	I am confident I will do well on geometry tests	**1**	**2**	**3**	**4**	**5**
8	I believe I can master the knowledge and skills in the geometry lesson	**1**	**2**	**3**	**4**	**5**
9	I believe I can earn a grade of A in the geometry topics	**1**	**2**	**3**	**4**	**5**
10	I am confident I can understand the basic concepts taught in geometry	**1**	**2**	**3**	**4**	**5**
Utility value						
11	I think about how the geometry I learn will be helpful to me	**1**	**2**	**3**	**4**	**5**
12	I think about how I will use geometry I learn	**1**	**2**	**3**	**4**	**5**
13	I think about how learning geometry can help me get a good job	**1**	**2**	**3**	**4**	**5**
14	I think about how learning geometry can help my career	**1**	**2**	**3**	**4**	**5**
15	I think that learning geometry is important because it stimulates my thinking	**1**	**2**	**3**	**4**	**5**
Test anxiety						
16	I become anxious when it is time to take a geometry test	**1**	**2**	**3**	**4**	**5**
17	I am nervous about how I will do on the geometry tests	**1**	**2**	**3**	**4**	**5**
18	I worry about failing geometry tests	**1**	**2**	**3**	**4**	**5**
19	I am concerned that the other students are better in geometry	**1**	**2**	**3**	**4**	**5**
20	I think about how my geometry grade will affect me overall grade point average	**1**	**2**	**3**	**4**	**5**
Extrinsic Motivation						
21	I learn geometry to gain prestige	**1**	**2**	**3**	**4**	**5**
22	I learn geometry because it will increase me social acceptance.	**1**	**2**	**3**	**4**	**5**
23	I learn geometry to widen my environment.	**1**	**2**	**3**	**4**	**5**
24	I learn geometry to reach upper level.	**1**	**2**	**3**	**4**	**5**
25	I want to do well in geometry because it is	**1**	**2**	**3**	**4**	**5**

	important to show my ability to my family, friends, employer, or others					
Attention						
26	There was something interesting at the beginning of geometry class that got my attention	**1**	**2**	**3**	**4**	**5**
27	geometry is eye catching subject	**1**	**2**	**3**	**4**	**5**
28	The quality of the writing in geometry class holds my attention	**1**	**2**	**3**	**4**	**5**
29	The way the information is arranged on the geometry class helped keep my attention	**1**	**2**	**3**	**4**	**5**
30	The variety of reading passages, exercises, diagrams, etc. helped keep my attention on geometry class	**1**	**2**	**3**	**4**	**5**
Satisfaction						
31	I enjoyed geometry so much that I would like to know more about it.	**1**	**2**	**3**	**4**	**5**
32	I really enjoyed learning with geometry	**1**	**2**	**3**	**4**	**5**
33	The working of the feedback and/or the links to go back and review material after the exercises made me feel rewarded for my effort	**1**	**2**	**3**	**4**	**5**
34	It was a pleasure to work with geometry	**1**	**2**	**3**	**4**	**5**
35	Practice exams of the geometry lesson helped me to find and define the problems and issues in my learning	**1**	**2**	**3**	**4**	**5**

Appendex-6: መጠይቅ

አዲስ አበባ ዩኒቨርሲቲ

የትምህርትና ስነ-ባህርይ ኮላጅ

የሳይንስና ሂሳብ ትምህርት ት/ክፍል

ዓሊማ ፡- የዚህ መጠይቅ ዋና ዓሊማ —to investigate the impact of Variation theory GeoGebra based problem solving approaches on student ‘s motivation in learning geometry ስ-ሆዚህ ሃእያንዳንዱ ነገር በትኩረት አንብበው ምሊሽ እንዲሰጡ በአክብሮት ተጠይቀዋሌ።

መመሪያ፡ እባክዎን እያንዳንዱን ጥያቄ በጥንቃቄ ያንብቡ እና የራስዎን የስምምነት ደረጃ ወይም ያሇመስማማት በተቀመጠው መመዘኛ ያስቀምጡ

1 = በፍጹም አልስማማም

2 = አልስማማም።

3 = እርግጠኛ አይደለሁም

4 = እስማማለሁ

5= በጣም እስማማለሁ

Mathematic motivation questionnaire

No.	Construct	*1*	*2*	*3*	*4*	*5*
	ውስጣዊ እሴት (Intrinsic value)					
1	ጂኦሜትሪ ስማር ደስ ይለኛል።	1	2	3	4	5
2	ጂኦሜትሪ መማር ደስ የሚያሰኝ ሆኖ አግኝቼዋልሁ።	1	2	3	4	5
3	የሚፈትነኝን ጂኦሜትሪ መስራት አወዳልሁ።	1	2	3	4	5
4	አጋጣሚዉን ሳገኝ የበፊት ጂኦሜትሪ ጥያቄዎችን በራሴ መስራት እመርጣለሁ።	1	2	3	4	5
5	የ ጂኦሜትሪ ርዕሶችን መረዳት ለእኔ በጣም አስፈላጊ ነው።	1	2	3	4	5
	ራሰን መቻል (Self-efficacy)					
6	በጂኦሜትሪ ስራዎች እና ፕሮጀክቶች ሊይ ጥሩ እንደምሰራ እርግጠኛ ነኝ።	1	2	3	4	5
7	የጂኦሜትሪ ፈተናዎችን በሚገባ እንደምሰራ እተማመናልሁ።	1	2	3	4	5
8	የጂኦሜትሪ እዉቀትንና ችልታን ማሳደግ እንደምችሌ አምናለሁ።	1	2	3	4	5
9	በጂኦሜትሪ ትምህርት አንደኛ ደረጃ ዉጤት እንደማመጣ አምናለሁ።	1	2	3	4	5
10	በጂኦሜትሪ ውስጥ የምማረውን መሰረታዊ ፅንሰ ሀሳቦች መረዳት እንደምችል እርግጠኛ ነኝ።	1	2	3	4	5
	የሂሳብ ትምህርት ጠቃሚነት (Utility value)					
11	ጂኦሜትሪ መማር እንዴት ሉጠቅመኝ እንደሚችሌ አስባለሁ።	1	2	3	4	5
12	የምማረዉን ጂኦሜትሪ እንዴት ልጠቀምበት እንደምችል አስባለሁ።	1	2	3	4	5

13	ጂኦሜትሪ መማር እንዴት ጥሩ ስራ ለማግኘት እንደሚረዳኝ አስባለሁ።	1	2	3	4	5
14	ጂኦሜትሪ መማር እንዴት ሙያዬን ሆማሳደግ እንደሚረዳኝ አስባሆሁ።	1	2	3	4	5
15	ጂኦሜትሪ መማር ጠቃሚ ነዉ ብዬ አስባሆሁ ምክንያቱም አስተሳሰቤን ያነቃቃዋሌ።	1	2	3	4	5
የፈተና ፍርሀት (Test anxiety)						
16	ጂኦሜትሪ በምፈተንበት ጊዜ ጭንቀት ዉስጥ እገባለሁ።	1	2	3	4	5
17	የ ጂኦሜትሪ ፈተናን እንዴት ልሰራዉ እንደምችል ሳስብ እጨነቃለሁ።	1	2	3	4	5
18	የ ጂኦሜትሪ ፈተናዎችን እወድቃሆሁ ብዬ እጨነቃለሁ።	1	2	3	4	5
19	ተማሪዎች በጂኦሜትሪ ትምህርት ከእኔ እንደሚሻለ አስባለሁ።	1	2	3	4	5
20	በ ጂኦሜትሪ ትምህርት የማገኘዉ ዉጤት የሁሉንም ትምህርት አማካኝ ዉጤት ላይ እንዴት ተጽዕኖ እንደሚፈጥርብኝ አስባልሁ።	1	2	3	4	5
ውጫዊ ተነሳሽነት (Extrinsic Motivation)						
21	ጂኦሜትሪ የምማረው ክብር ለማግኘት።	1	2	3	4	5
22	ጂኦሜትሪ የምማረው ማህበራዊ ተቀባይነትን ስለሚጨምርልኝ ነው።	1	2	3	4	5
23	ጂኦሜትሪን የምማረዉ በማኅበረሰቡ ዉስጥ ያለኝን ተቀባይነት ለማስፋት ነዉ።	1	2	3	4	5
24	ጂኦሜትሪን የምማረዉ ትለቅ ደረጃ ለመድረስ ነዉ።	1	2	3	4	5
25	ለቤተሰቦቼ ፣ ለጓደኞቼ ፣ ለቀጣሪዎችና ያለኝን አቅም ለማሳየት በጂኦሜትሪ ጥሩ ዉጤት ማምጣት አፈልጋለሁ።	1	2	3	4	5
ትኩረት (Attention)						
26	ለመጀመሪያ ጊዜ በነበረዉ የሂሳብ ክፍለ-ጊዜ ትኩረቴን የሳበዉ ደስ የሚያሰኝ ነገር ነበረ።	1	2	3	4	5
27	ጂኦሜትሪ በትኩረት ማዬትን የሚፈጥር ትምህርት ነዉ ።	1	2	3	4	5
28	በጂኦሜትሪ ትምህርት ወቅት የጽሁፍ ጥራት ትኩረቴን ይስበዋሌ።	1	2	3	4	5
29	በጂኦሜትሪ ትምህርት ወቅት መረጃዎች የሚቀመጡበት መንገድ ትኩረቴን ሰብስቤ እንድማር ያግዘኛል።	1	2	3	4	5
30	የተለያዩ ምንባቦች፣ መልመጃዎች፣ ሥዕላዊ መግለጫዎች፣ ወዘተ ትኩረቴን በ ጂኦሜትሪ ክፍል ላይ እንዲቀይ ረድተውኛል።	1	2	3	4	5
እርካታ (Satisfaction)						
31	ጂኦሜትሪ መስራት በጣም ያዝናናኛለ ከፍ ያለ እዉቀት እንዲኖረኝም እፈልጋለሁ።	1	2	3	4	5
32	ጂኦሜትሪ መማር በጣም ያዝናናኛሌ።	1	2	3	4	5
33	የሚሰጠኝ ግብረ መለስ ወደ ኋላ ተመልሼ ስራዬን ለመገምገም እና ለጥረቴ ዋጋ እንድሰጥ አድርጎኛለ።	1	2	3	4	5

34	ጂኦሜትሪ መስራት በጣም አስደሳች ነው።	1	2	3	4	5
35	የ ጂኦሜትሪ ትምህርት የመለማመጃ ፈተናዎች በትምህርቱ ላይ ያጋጠሙኝን ችግሮች ለመፍታት ረድቶኛል።	1	2	3	4	5

Appendex7: Conceptual Understanding and Problem-Solving posttest Questions

Addis Ababa University

College of Education and Behavioral Studies

Department of Science and Mathematics Education

Part- 1 Geometry and Measurement Conceptual Understanding Posttest

Instruction: This test consists of 10 questions from grade 9 Geometry topics. Each question has two parts. The first part is a multiple-choice response and the second part is reason. Therefore, choose the best answer from the given alternatives and choose your reason from the given alternatives

Background Information

1. Name of School ____
2. Gender: Put a tick mark (√)

   Male ☐ Female ☐
3. Grade Level _____________Section _________

. **Conceptual understanding**

1. In the following figure $\overline{AD}$ and $\overline{BE}$ bisect each other at C. The appropriate theorem or postulate used to prove that $\Delta ABC \equiv \Delta DEC$

   A. Side-Side-Side Theorem

   B. Side-Angle-Side Theorem

   C. Angle-Side-Angle Theorem

   D. Angle-Angle-Side Theorem

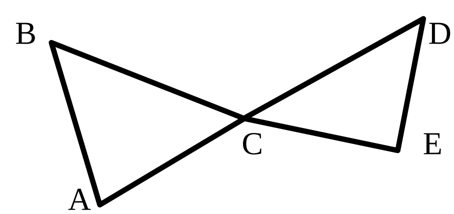


Figure-1

Reason: What was the reason for the above answer?

1) The corresponding two sides and one angle of the two triangles are congruent
2) The corresponding three sides are congruent
3) It is possible to observe that the two angles and one of the sides of the two triangles are congruent
4) The two triangles are right angled triangle with congruent acute angle
5) _____________________________

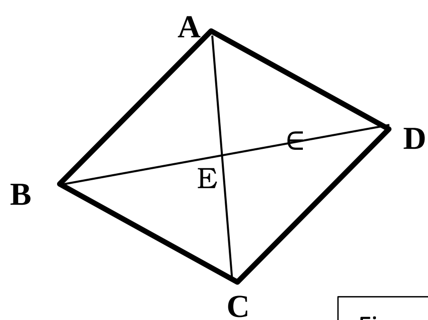
2. The measure of an angle is 583^0 in clockwise direction, and then in which quadrant does its terminal side fall?

A. III B.IV C. I D.II

Reason: What was the reason for the above answer?

1. The initial position of an angle is coinciding to the positive Y-axis then staring 583^0 in standard position. Clockwise direction means positive angle.
2. The initial position of an angle is coinciding to the positive x-axis then staring 583^0 in standard position. Clockwise direction means positive angle.
3. The initial position of an angle is coinciding to the positive x-axis then staring 583^0 in standard position. Clockwise direction means negative angle.
4. The initial position of an angle is coinciding to the positive Y-axis then staring 583^0 in standard position. Clockwise direction means negative angle.
5. ____

3. Given $\overline{AC}$ and $\overline{BD}$ are perpendicular bisector of each other, then which one of the following congruence of triangle holds true



A .$\Delta ABC \equiv \Delta ADE$

B.$\Delta ADE \equiv \Delta CBE$

C. $\Delta ABD \equiv \Delta ABC$

D.$\Delta ABE \equiv \Delta ADE$

Figure-2

Reason: What was the reason for the above answer?

1. The congruence is occurred due to Angle-Side-Angle Theorem
2. The congruence is occurred due to Side -Angle-Side Theorem
3. The congruence is occurred due to Side -Side-Side Theorem
4. The congruence is occurred due to Angle -Angle-Side Theorem

____

4. Which one of the following is an angle property of a circle?

A. All angles inscribed in a semicircle are congruent

B. The opposite angles of a cyclic quadrilateral are congruent

C. If a central angle measures 180^0, then it is subtended by a major arc

D. The angle inscribed in a minor arc of a circle is an acute angle

Reason: What was the reason for the above answer?

1. Opposite angles of a cyclic quadrilateral are complementary
2. All angles inscribed by the same arc are equal
3. Opposite angles of a cyclic quadrilateral are supplementary
4. The inscribed angle is twice to the arc that inscribe.
5. ____

5.A regular polygon in which the measure of each interior angle is equal to measure of each of its central angle is:

A. regular hexagon C. regular pentagon

B. square D. equilateral triangle

Reason: What was the reason for the above answer?

1. The degree measure of each central angle of a regular polygon is $(\frac{360}{n})^0$ and its interior angle is equal to (n-2) x180^0
2. The degree measure of each central angle of a regular polygon is $(\frac{360}{n})^0$ and its interior angle is equal to $\frac{(n-2)180^0}{n}$
3. The degree measure of each central angle of a regular polygon is and its interior angle is equal to (n-2) x180^0
4. The degree measure of each central angle of a regular polygon is and its interior angle is equal to $(\frac{360}{n})^0$
5. ____

6. If the $\cos\theta = 0.425$ and $\sin\theta$>0 then at which quadrant is θ found?

A. IV B.II C. I D.III

Reason: What was the reason for the above answer?

1. The angle is obtuse angle
2. $\sin\theta$ and $\cos\theta$ are negative in the 3rd quadrant
3. $\sin\theta$ and $\cos\theta$ are positive in the 1st quadrant
4. $\sin\theta$ is negative and $\cos\theta$ is positive in the 4th quadrant
5. ____

7. Which one of the following is correct?

A. Rhombus is square C. Square is a rectangle

B. Parallelogram is rectangle D. Rectangle is Rhombus

Reason: What was the reason for the above answer?

1. A rectangle fulfills all properties of a square
2. A square fulfills all properties of a rectangle
3. A rhombus fulfills all properties of square
4. A parallelogram fulfills all properties rectangle
5. ____

8. If sine of the angle is equal to cosine of the other angles then which one of the following is true?

A. The two angles are complementary

B. The two angles are supplementary

C. The two angles are obtuse

D. The sum of the two angle is 180^0

Reason: What was the reason for the above answer?

1. If the sum of the two angle is 180 then the cosine of one of the angles is equal to the sine of the other
2. If the sum of the two angle is 360 then the cosine of one of the angles is equal to the sine of the other
3. If the angle is equal then
4. If the sum of the two angle is 90 then the cosine of one of the angles is equal to the sine of the other
5. ____

9. If the ratio of the length of the corresponding sides of two similar triangles is K then what will be the ratio of their area?

A. Twice of k C. square of K

B. Half of K D.it is also K

Reason: What was the reason for the above answer?

1. Their perimeter is K
2. The ratio of their perimeters equals the ratio of their corresponding sides

3. The ratio of their areas is the square of the ratio of their corresponding sides.
4. The ratio of their area is 4k
5. ____

10. Which one of the following is true about the relation between following figures

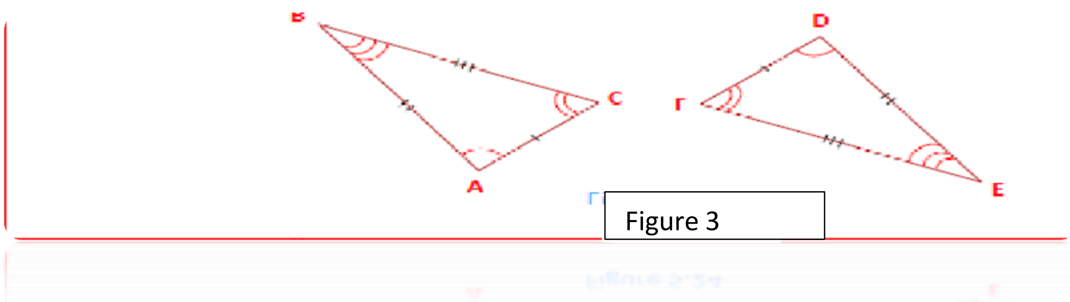


Figure 3

A. $\Delta ABC \equiv \Delta FDE$ B. $\Delta ABC \equiv \Delta$DEF C. $\Delta ABC \equiv \Delta EFD$ D. $\Delta ABC \equiv \Delta FED$

Reason: What was the reason for the above answer?

1. The corresponding angles are congruent
2. The corresponding sides are similar
3. The corresponding sides and angles are congruent
4. They are congruent since their corresponding angles are right angle.
5. ____

Part- 2 Geometry and Measurement Problem solving Test

Instruction: This test consists of 5 questions from grade 9 Geometry and Measurement topics. You have to solve the problem by showing all the necessary steps.

1. If $\Delta ABC and\ \Delta DEF$ are similar and the area of ΔABC =12 square units. if DE is twice of AB ,then the area of ΔDEF =___________
2. Suppose a building cast a shadow of length 45 meters when the angle of elevation of the sun is 30^0. How long high is the building.
3. In the circle shaded below; O is the center, the radius measures 9cm and (<AOB) $=70^0$. What is the area of the shaded region?

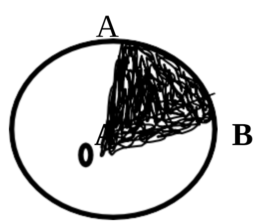


Figure-4

4. Each sides of a regular hexagon are 10 units long. What is the area of this hexagon?
5. From a point on the ground, which is 100 meters away from the wall of a building the angle of elevation of the top of the building is 45^0.what is the height of the building?

Appendex-8: Geometry Conceptual Understanding and problem-solving Pretest question

Addis Ababa University

College of Education and Behavioral Studies

Department of Science and Mathematics Education

Part-1 Geometry Conceptual Understanding Pretest

Instruction: This test consists of 10 questions from grade 8 Geometry topics. Each question has two parts. The first part is a multiple-choice response and the second part is reason. Therefore, choose the best answer from the given alternatives and choose your reason from the given alternatives

Background Information

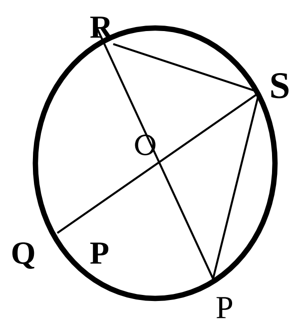
4. Name of School ____

5. Gender: Put a tick mark (√)

   Male ☐ Female ☐

6. Grade Level _____________Section _________

1. In the figure below O is the center of the circle which of the following statement if **not true**



$A.\ \widehat{PSR}$ is a semi-circle C. <PSR right angle?

$B.\ \widehat{PQ}$ is a Mirro D.<PSQ is obtuse angle?

Reason: - What is the reason for your answer?

1. The arc which inscribe by semicircle is right angle
2. An angle inscribed by semicircle is 180^0
3. The arc is below semicircle
4. The arc is above semicircle
5. ____

2. Which of the following families of solid figures are not similar?

A. Cubes B.rectangular tetrahedrons

B. Cones D. Spheres

Reason: - What is the reason for your answer?

1. The shape every Cubes are not similar
2. The shape every cone is not similar
3. The shape every sphere is not similar
4. The shape every rectangular tetrahedrons are not similar
5. ____

3. If a quadrilateral ABCD is a trapezium, then which one of the following statements is true about the trapezium?

A. Two pair of its opposite sides are parallel

B. The diagonals are parallel

C. The opposite sides are congruent

D. One paire of its opposite sides are parallel

Reason: - What is the reason for your answer?

1. The opposite sides are perpendicular
2. Only one pair of its opposite sides are parallel
3. The diagonals never intersect
4. The opposite sides are parallel
5. ____

4. Which one of the following quadrilateral can always be inscribed in a circle

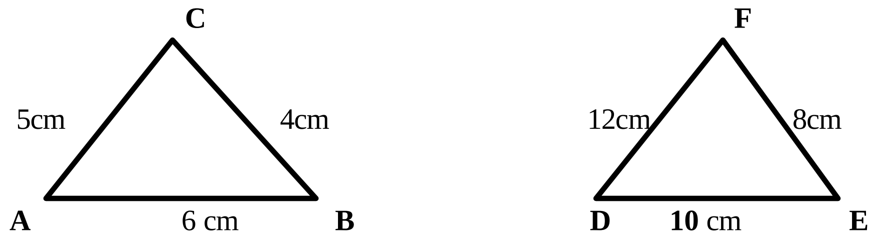
A. Rectangle C. Rhombus

B. Trapezium D. Parallelogram

Reason: - What is the reason for your answer?

1. The opposite angles are supplementary
2. The adjacent angles are supplementary
3. The opposite angles are complementary
4. The adjacent angles are complementary
5. ____

5. In the following figures, the two triangles are similar by SSS theorem. Which one of the following describes the given information?



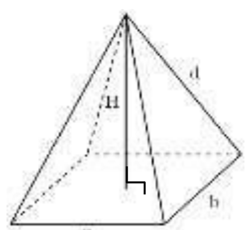
A. $\Delta ABC \sim \Delta DEF$ C. $\Delta ABC \sim \Delta\, EDF$

B. $\Delta ABC \sim \Delta DFE$ $D. \Delta ABC \sim \Delta FDE$

Reason: What was the reason for the above answer?

1. The corresponding two sides and one angle of the two triangles are congruent
2. The corresponding three sides are congruent
3. It is possible to observe that the two angles and one of the sides of the two triangles are congruent
4. The corresponding three sides are similar
5. ____

6. The figure below is a regular pyramid. which of the following is not true about the regular pyramid?



A. The lateral edges are all equal in length

B. The pyramid must be right pyramid

C. The lateral faces are equilateral triangles

D. The foot of the altitude must be at the center of the base

Reason: What was the reason for the above answer?

1. The altitude of the pyramid is perpendicular to the base
2. The lateral edges are different in length
3. The lateral face may not be equilateral
4. The foot of the altitude may not be at the center of the base.
5. ____

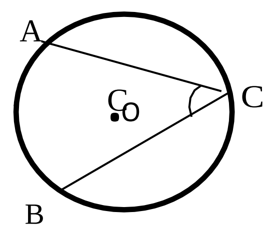
7. In the figure below, O is the center of the circle. If m(<ACB)=80^0, what is the measure of $\widehat{ACB}$?

   A. 160^0 B. 200^0

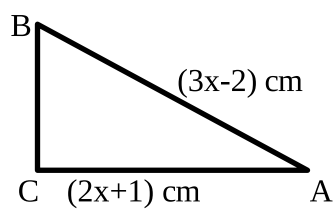
   C. 180^0 D. 80^0

Reason: What was the reason for the above answer?

1. m(<ACB) = $\frac{1}{2}$ m (< $\hat{AB}$)
2. m(<ACB) = m (< $\widehat{ACB}$)
3. m(<ACB) = 2 m (< $\widehat{ACB}$)
4. 2 m(<ACB) = m (< $\widehat{ACB}$)
5. ____



8. In the figure below, ABC is a right angled at C. if AB=(3x-2) cm, AC=(2x+1)cm and cosA= $\frac{3}{4}$, then what is the value of x?



A. 8cm C.10cm

B. 6cm D.12cm

Reason: What was the reason for the above answer?

1. sinA= $\frac{BC}{AB}$
2. sinA= $\frac{AB}{AC}$
3. cosA= $\frac{AC}{AB}$
4. cosA= $\frac{AB}{AC}$
5. ____

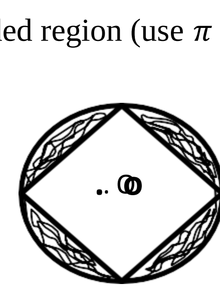
9. A square is cut from a circle as shown in the figure below. If the radius of circle is 7cm, what is the total area of the shaded region (use $\pi = \frac{22}{7}$)

A. 154 cm^2

B. 98 cm^2

C. 56 cm^2

D. 105 cm^2



Reason: What was the reason for the above answer?

1. The sides of the square are twice the radius of the circle
2. The sides of the square are half of the radius of the circle
3. The radius of the circle is equal to the sides of the square
4. The sides of the square are twice the square of radius of the circle
5. ____

10. If measures of the interior angles of a triangle are $\frac{1}{3}\alpha$, $\frac{5}{3}\alpha$ and 2α, then what is the value of?

A. 14 B.30 C.45 D.75

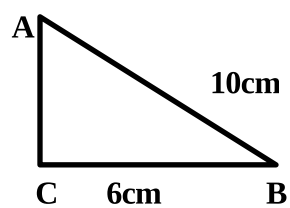
Reason: What was the reason for the above answer?

1. The sum of the interior angles is 360^o
2. The sum of the interior angles is 180^0
3. The interior angles of a triangles are always equal
4. One of the interior angles of a triangle is 45^o
5. ____

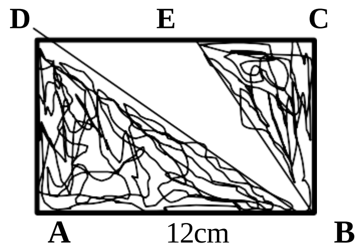
Part- 2 Geometry and Measurement Problem solving Test

Instruction: This test consists of 5 questions from grade 8 Geometry and Measurement topics. You have to solve the problem by showing all the necessary steps.

1. In the figure below ΔABC is right angled at C. If $\overline{BC}$ =6cm and $\overline{BC}$=10cm, then find $\sin B$



2. In the figure below ABCD is a square of side length 12 cm. if E is midpoint on$\overline{CD}$, what is the area of the shaded region?



3. A ladder of 12m leans against a wall so that the angle between the ladder and the wall is 60^0. How far is the base of the ladder from the wall?
4. If the measure of each interior angles of a rectangular polygon is 135^0, then how many sides does the polygon have?
5. From one vertex of an n-sided polygon 5 different diagonals can be drawn. What is the degree measure of all interior angles of this polygon?

Appendex-9: Classroom Observation Check List (for intervention -1)

School __________________________ Teacher Name __________________________

Grade Level ________ Duration of observation ______

Date ________________________ Observer __________________________

Subject ______________

Lesson topic ____

		Yes	No	Remark
1	Students work together with peers to understand geometry conceptually			
2	The Variation theory Geogebra based problem solving approach is rich in resource and it is user friendly for students			
3	Using Variation theory Geogebra based problem solving approach content objectives are achieved in a better way			
4	During classroom setting (Variation theory Geogebra based problem solving approach) students get Opportunity to further grasp the lessons.			
5	The presentation of contents and arrangement is well organized in accordance with Variation theory Geogebra based problem solving approach			
6	Students are comfortable with reflecting their understanding			
7	Variation theory GeoGebra based problem solving approach provides an opportunity to regulator one's progress			
8	Variation theory GeoGebra based problem solving approach allows students to express themselves in a written form			
9	The classroom teacher is well aware of the purpose of Variation theory GeoGebra based problem solving approach			
10	The classroom teacher gives time to assist students while he teaches using Variation theory GeoGebra based problem solving approach for better understanding of the concepts			
11	Students appear comfortable with the Variation theory GeoGebra based problem solving approach			
12	The Variation theory GeoGebra based problem solving approach addresses the diverse nature of students			
13	The learning resources attached to the GeoGebra problem solving approaches contribute little in the learning process			
14	During the Variation theory GeoGebra based problem solving approach , students get a chance to pose question and forward their views.			

Comment:1. What were the challenges observed during the implementation of Variation theory Geogebra based problem solving approaches for geometry class?

Appendex-10: Classroom Observation Check List (for intervention -2)

School __________________________ Teacher Name _____________________

Grade level _________ Duration of observation ____

Date ________________________ Observer __________________________

Subject ______________

Lesson topic ____

		Yes	**No**	**Remark**
1	Students work together with peers to understand geometry conceptually			
2	The Variation theory-based problem-solving approach is rich in resource and it is user friendly for students			
3	Using Variation theory-based problem-solving approach content objectives are achieved in a better way			
4	During classroom setting (Variation theory- b a s e d problem solving approach) students get Opportunity to further grasp the lessons.			
5	The presentation of contents and arrangement is well organized in accordance with Variation theory- b a s e d problem-solving approach			
6	Students are comfortable with reflecting their understanding			
7	Variation theory- b a s e d problem-solving approach provides an opportunity to regulator one's progress			
8	Variation theory- b a s e d problem-solving approach allows students to express themselves in a written form			
9	The classroom teacher is well aware of the purpose of Variation theory based problem solving approach			
10	The classroom teacher gives time to assist students while he teaches using Variation theory- b a s e d problem-solving approach for better understanding of the concepts			
11	Students appear comfortable with the Variation theory- b a s e d problem solving approach approaches			
12	The Variation theory- b a s e d problem-solving approach addresses the diverse nature of students			
14	During the Variation theory- b a s e d problem-solving approach, students get a chance to pose question and forward their views.			

Comment:

1. What were the challenges observed during the implementation of Variation theory-based problem-solving approaches for geometry class?

Appendex-11: Classroom Observation Check List (for intervention -3)

School ________________________ Teacher Name ___________________

Grade Level ________ Duration of observation _____

Date ______________________ Observer ________________________

Subject _____________

Lesson topic ____

		Yes	No	Remark
1	Students work together with peers to understand geometry conceptually			
2	The GeoGebra environment is rich in resource and it is user friendly for students			
3	Using GeoGebra, content objectives are achieved in a better way			
4	During classroom setting, students get Opportunity to further grasp the lessons.			
5	The presentation of contents and arrangement is well organized in accordance with GeoGebra software			
6	Students are comfortable with reflecting their understanding			
7	GeoGebra provides an opportunity to regulator one 's progress			
8	GeoGebra allows students to express themselves in a written form			
9	The classroom teacher is well aware of the purpose of GeoGebra			
10	The classroom teacher gives time to assist students while he teaches using GeoGebra for better understanding of the concepts			
11	Students appear comfortable with the GeoGebra problem solving approaches approach			
12	The GeoGebra Based problem solving approach addresses the diverse nature of students			
13	The learning resources attached to the GeoGebra problem solving approaches contribute little in the learning process			
14	During the GeoGebra problem solving approaches, students get a chance to pose question and forward their views.			

Comment:

1. What were the challenges observed during the implementation of GeoGebra problem solving approaches for geometry class?______________________

Appendex-12: Interview Questions for students

Addis Ababa University

College of Education and Behavioral Studies

Department of Science and Mathematics Education

Interview Questions for students

The main objective of this interview is to gatherer data with regards to students 'conceptual understanding, problem solving skills and their motivation in learning mathematics with variation theory Geogebra based approach in the School. Your correct and complete answer to the following questions will have great value for this study. Therefore, you are kindly requested to answer all the questions when you are asked.

Part-1 about conceptual understanding

1. What is your belief about geometry?
2. Do you easily understand geometric concepts?
3. When you answer question on geometry do you have a culture of putting your reason behind your answer?
4. What challenge do you face for defining and giving explanation of geometrical concepts?
5. What problems do you face in order to give one or more examples in geometry concepts?
6. What are the challenges for you to give a counter example for proofing geometry questions?
7. What condition (s) is (are) necessary condition for congruence of triangles?
8. Do your belief that geometry can describe mathematical idea in various contexts? Can you give me examples?

Part-2 about problem solving

1. Do you understand all the words used in stating the geometric problems problem?
2. Before you start to solve problems on geometry can you restate the problem in your own words?
3. Do you easily identify what are you asked to find or show in problems related to geometry? What is your difficulty?

4. How do you solve problems on geometry? Can you tell me your strategies?
5. Can you think of a picture or diagram that might help you understand the problem on geometry and measurement? How?
6. How do you know that whether the given information is sufficient or not for solving problems on geometry and measurement?
7. Do you understand and apply the different problem-solving approaches can you give me some examples?
8. If you cannot solve the proposed problem on geometry? Could you think of other data appropriate to determine the unknown?
9. While computing problems on geometry and measurement, once the problem has been understood and strategy has been selected. Can you see clearly that the step is correct? And can you prove that it is correct?
10. While computing problems on geometry and measurement, once you get the answer can you check the result? Can you check the argument? Or can you derive the solution differently?

Appendex-13: Interview Questions for teachers

Addis Ababa University

College of Education and Behavioral Studies

Department of Science and Mathematics Education

Interview Questions for teachers

The main objective of this interview is to gatherer data with regards to students' conceptual understanding, problem solving skills and their motivation in learning mathematics with variation theory GeoGebra based approach in the School. Your correct and complete answer to the following questions will have great value for this study. Therefore, you are kindly requested to answer all the questions when you are asked.

Part-1 about conceptual understanding

1. What is your belief about teaching geometry?
2. Do you easily understand and teach geometric concepts using GeoGebra?
3. While you are teaching and when you are showing examples on question on geometry do you have a culture of putting your reason behind your answer?
4. What challenge do you face for defining and giving explanation of geometrical concepts when you teach geometry? How do you manage the problems?
5. What problems do you face in order to give one or more examples in geometry concepts?
6. What is the challenge for you to give a counter example for proofing geometry questions for your students?
7. Can you take comparison of objects while teaching geometric concepts?
8. Did using GeoGebra software use to identify condition those should be satisfied to say two triangles are similar, congruent?
9. How do you show that geometry can describe mathematical idea in various contexts? Can you give me examples?
10. How do you teach your students to increase their proficiency in geometry and measurement?

Part-2 about problem solving

1. Do your students understand all the words used in stating the geometric problems problem? if not what problem do you observe?
2. Before you start to solve problems on geometry did your students restate the problem in their own words?
3. How do you teach to solve problems on geometry? Can you tell me your strategies?
4. What is the roll of GeoGebra in teaching geometry?
5. Can you think of a picture or diagram that might help your students understanding for the problem on geometry? How?
6. How your students do know that whether the given information is sufficient or not for solving problems on geometry?
7. Do you understand and apply the different problem-solving approaches can you give me some examples?
8. If your students cannot solve the proposed problem on geometry, could you give to think of other data appropriate to determine the unknown?
9. While computing problems on geometry in teaching geometry, once the problem has been understood and strategy has been selected. Can you see clearly that the step is correct? And can you prove for students that it is correct?
10. Can you describe the use of GeoGebra While computing problems on geometry; once you get the answer can you check the result? Can you check the argument? Or can you derive the solution differently?

Appendex-14: Permission Letters

A. From the department

አዲስ አበባ ዩኒቨርሲቲ
ሥነ-ትምህርትና ባሕሪይ ጥናት ኮሌጅ
የሳይንስና ሒሳብ ትምህርት ክፍል
አዲስ አበባ፣ኢትዮጵያ

Addis Ababa University
College of Education and Behavioral Studies
Department of Science and Mathematics Education
Addis Ababa, Ethiopia

Date: February 6, 2023

Ref. No.: SMED/914/2015-23

To: Addis Ababa City Education Bureau
Addis Ababa

Subject:- Support

As part of the PhD studies, Mr. Mr. Belete Abebaw has selected some schools from Addis Ababa City Administration for his research entitild by "***Variation theory Based GoGebra Assisted Problem-Solving Approaches and Students' Learning of Geometry***". The sub cities and schools he has selected are:

1. Lideta Sub city
   a) Hidase Lideta Secondary School
   b) Africa Hibret Secondary School
2. Arada Sub city
   a) W/ro Kelemwork secondary School
3. Kirkos Sub City
   a) Abyot kirs Secondary School
   b) Misrak Goh Secondary School
4. Akaki Kality Sub city
   a) Bulbula Secondary School

During his stay in these schools, he will be taking a number of engagements such as data collection through various approaches and interventions. Hence, our Department would like Addis Ababa City Education Bureau to support him in executing data for his PhD research through the provisions.

We would like to thank in advance for the support rendered.

With regards,

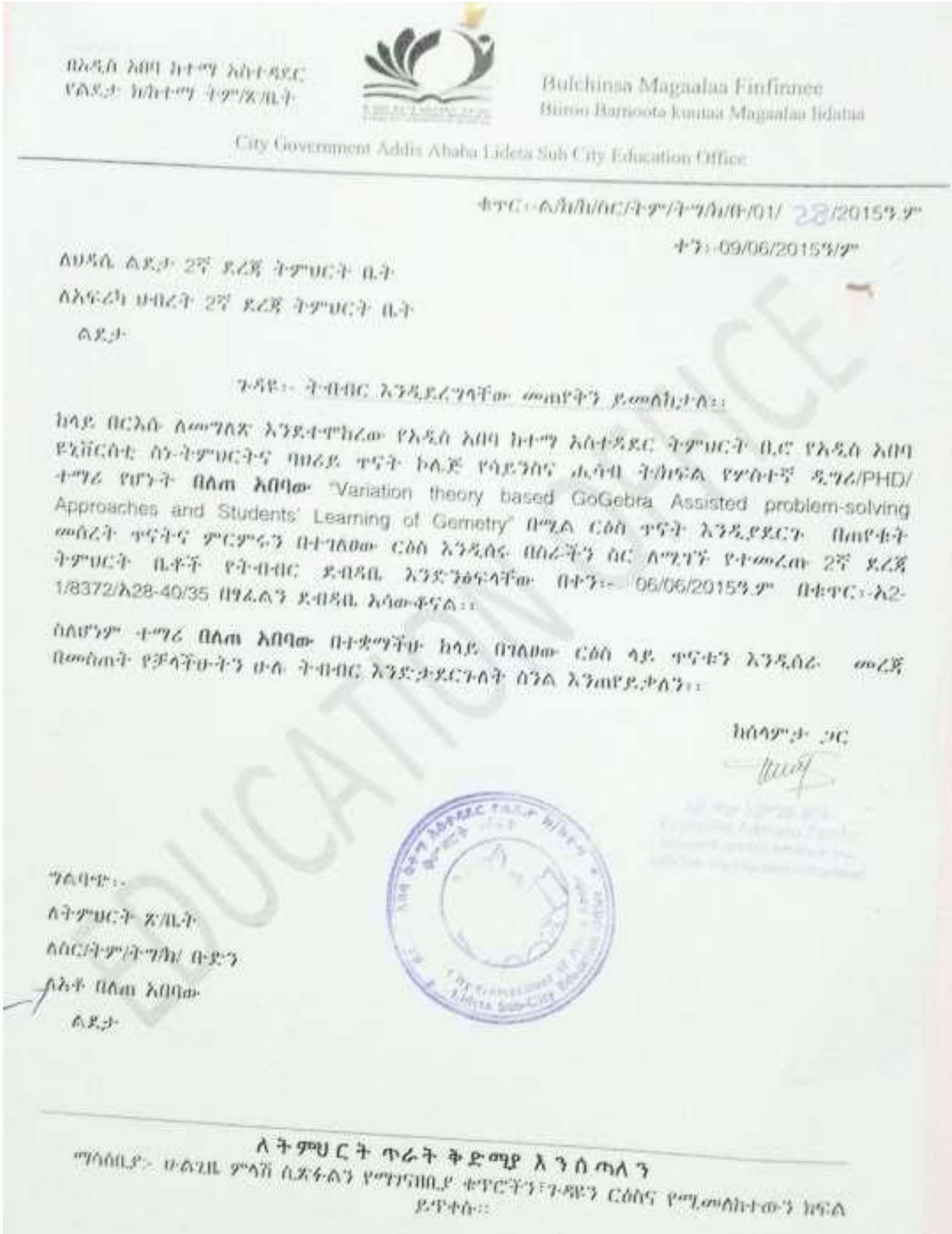
Habtamu Wodaj (PhD)

Chairman, Department of Science and Mathematics Education

☎ 011 1 22 07 67

✉ 1176

B. **From Sub-Cities**



በአዲስ አበባ ከተማ አስተዳደር
የልደታ ክ/ከተማ ትም/ጽ/ቤት

Bulchinsa Magaalaa Finfinnee
Biiroo Barnoota kutaa Magaalaa lidataa

City Government Addis Ababa Lideta Sub City Education Office

ቁጥር፡-ል/ክ/ከ/ስር/ትም/ትግ/ክ/ዑ/01/ 28/2015ዓ.ም

ቀን፡-09/06/2015ዓ/ም

ለሀዲሴ ልደታ 2ኛ ደረጃ ትምህርት ቤት
ለአፍሪካ ህብረት 2ኛ ደረጃ ትምህርት ቤት
ልደታ

ጉዳዩ፡- ትብብር እንዲደረግላቸው መጠየቅን ይመለከታል፡፡

ከላይ በርእሱ ለመግለጽ እንደተሞከረው የአዲስ አበባ ከተማ አስተዳደር ትምህርት ቢሮ የአዲስ አበባ ዩኒቨርስቲ ስነ-ትምህርትና ባህሪይ ጥናት ኮሌጅ የሳይንስና ሒሳብ ት/ክፍል የሦስተኛ ዲግሪ/PHD/ ተማሪ የሆኑት **በለጠ አበባው** "Variation theory based GoGebra Assisted problem-solving Approaches and Students' Learning of Gemetry" በሚል ርዕስ ጥናት እንዲያደርጉ በጠየቁት መሰረት ጥናትና ምርምሩን በተገለፀው ርዕስ እንዲሰሩ በስራችን ስር ለሚገኙ የተመረጡ 2ኛ ደረጃ ትምህርት ቤቶች የትብብር ደብዳቤ እንድንፅፍላቸው በቀን፡- 06/06/2015ዓ.ም በቁጥር፡-አ2-1/8372/አ28-40/35 በፃፈልን ደብዳቤ አሳውቀናል፡፡

ስለሆነም ተማሪ **በለጠ አበባው** በተቋማችሁ ከላይ በገለፀው ርዕስ ላይ ጥናቱን እንዲሰራ መረጃ በመስጠት የቻላችሁትን ሁሉ ትብብር እንድታደርጉለት ስንል እንጠየቃለን፡፡

ከሰላምታ ጋር

ግልባጭ፡-
ለትምህርት ጽ/ቤት
ለስር/ትም/ትግ/ክ/ ዑድን
ለአቶ በለጠ አበባው
ልደታ

ለ ት ምህ ር ት ጥራ ት ቅ ድ ሚያ እ ን ሰ ጣለ ን
ማሳሰቢያ፡- ሁልጊዜ ምላሽ ሲጽፉልን የማገናበቢያ ቁጥራችንን፣ጉዳዩን ርዕስና የሚመለከተውን ክፍል ይጥቀሱ፡፡

በአራዳ ክ/ከ/አስተ/ትምህርት
ጽ/ቤት

አዲስ አበባ

WAAJJIRA BARNOOTAKUTAA

MAGAALAA ARAADAA

FINFINNEE

ARADA-SUB-CITY ADMINISTRATION EDUCATION OFFICE

ADDIS ABABA

☎ 0111-57-18-64/0111 265029

ቁጥር፡ አ/ክ/ከ/ት/ጽ/መ/ት/አ/ል/616/2015 ዓ.ም

ቀን10/6/2015 ዓም

ለወ/ሮ ቀለመወርቅ 2ኛ ደረጃ ት/ቤት

ጉዳዩ፡ትብብር እንዲደረግላቸው ስለመጠየቅ

ከላይ በርዕሱ ለመጥቀስ እንደተሞከረው **አቶ በሰጠ አበባው** የሶስተኛ ዲግሪያቸውን በአዲስ አበባ ዩኒቨርሲቲ በመስራት ላይ ይገኛሉ ::በመሆኑም ለዚህ ትምህርታቸው ማሟያ ጥናት እየሰሩ ስለሆነ ለጥናታቸው መረጃ ከት/ቤታችሁ ለማግኘት ትብብር ደብዳቤ እንዲጻፍላቸው ጠይቀዋል::ስለዚህ አስፈላጊውን ትብብር ታደርጉላቸው ዘንድ እንጠይቃለን::

ከሰላምታ ጋር

ግልባጭ፣

ለአራዳ ክ/ከተማ ት/ት ጽ/ቤት መም/የትም/አ/ል/ቡድን

ለአቶ በሰጠ አበባው

አቃቂ ቃሊቲ ክፍለ ከተማ አስተዳደር
ትምህርት ጽ/ቤት የትምህርት አስተዳደርና
መረጃ ስርዓት ቡድን

Waajjira Barnootaa Kutaa
Magaalaa Aqaaqii Qaallittii
bulchiinsa barnoota fi garee sirna
ragaa

AKakiKaliti Sub-City Administration Education Office
EDUCATION MANAGEMENT INFORMATION SYSTEM TEAM

ቁጥር፡-አቃ/ቃ/ክ/ከ//የት/አስ/መ/ስ/ቡ/ ፩፮ /15
Ref.No
ቀን፡- 10/06/2015 ዓ.ም
Date

በአቃቂ ቃሊቲ ክፍለ ከተማ ትምህርት ፅ/ቤት
- ለ ቡልቡላ 2ኛ ደረጃ ት/ቤት

አዲስ አበባ

ጉዳዩ፡- ትብብር ስለመጠየቅ ይሆናል

ከላይ በጉዳዩ ለመግለጽ እንደተሞከረው የአዲስ አበባ ዩኒቨርስቲ ስነ ትምህርትና ባህሪ ጥናት ኮሌጅ የሳይንስና ሒሳብ ት/ክፍል የሦስተኛ ዲግሪ /PHD/ የሆኑት በለጠ አበባው Variation theory Based GoGebre Assisted problem-Solving Approaches and Students Learning of Geometry በሚል ርዕስ ጥናት እንዲያደርጉ በጠየቁት መሰረት ጥናትና ምርምሩን በተገለፀው ርዕስ በተቋማችሁ እንዲሰሩ ትብብር እንድታደርጉላቸው እንጠይቃለን።

ከሰላምታ ጋር!

ግልባጭ፡-

- ለት/ጽ/ቤት
- ለአቶ በለጠ አበባው

አዲስ አበባ

"ጥራት ያለው ትምህርት ለሁሉም" " Barnota Qulqullina Qabu Hundaaf "
አድራሻ፡-ቃሊቲ መነሀሪያ ፊትለፊት ያለው የክ/ከተማው ህንጻ 7ኛ ፎቅ

በአዲስ አበባ ከተማ አስተዳደር
የቂርቆስ ክፍለ ከተማ
ትምህርት ጽ/ቤት

Biirroo Barnootaa Bulchiinsa Magaalaa Finfinneetti Waajjira Barnootaa Kutaa Magaalaa Qiirqoos

Addis Ababa City Administration Kirkos Education Office

ቁጥር-ቂ/ክ/ት/ጽ/ቤት-7111-/06/15

ቀን 07/06/2015 ዓ.ም

በቂርቆስ ክ/ከተማ አስተዳደር ት/ት/ጽ/ቤት
ለምስራቅ ጎህ 2ኛ ደረጃ ት/ቤት
ለአብዮት ቅርስ 2ኛ ደረጃ ት/ቤት
አዲስ አበባ

ጉዳዩ፡-ትብብር መጠየቅን ይመለከታል

ከላይ በርዕሱ እንደተገለጸው የአዲስ አበባ ዩኒቨርስቲ ስነ-ትምህርት ባህሪይ ጥናት ኮሌጅ የሳይንስና ሒሳብ ት/ክፍል የሦስተኛ ዲግሪ/PHD/ ተማሪ የሆኑት በለጠ አበባው "Variation theory Based GoGebra Assisted Problem-Solving Approches and Students' Learning of Gemetry" በሚል ርዕስ ጥናት እንዲያደርጉ በጠየቁት መሰረት ጥናትና ምርምሩን በተገለፀው ርዕስ እንዲሰሩ እንድትተባበሯቸው እንጠይቃለን፡፡

ከሰላምታ ጋር!

Mulu Andargie Belachew
Office Head

ግልባጭ፡-
በቂርቆስ ክፍለ ከተማ ት/ጽ/ቤት
ለትምህርት ቤት መሻሻልና ሱፐርቪዥን ቡድን
አዲስ አበባ

011-558-42-10 011558-19-94 011-558-10-34 011-558-11-06 011-558-19-94
ማሳሰቢያ ፡- ሁል ጊዜ ማጣቀሻ ቁጥሮችን ፣ ቀን ፣ ጉዳዩን ፣
REMINDER - PLEASE ALWAYS PROVIDE REF. NO. SUBJECT AND ATTENTION TO.

Appendix15: Ethical Clearance

Ref: CEBS_IRC_SMED_002/2024
Date: February 14, 2024

COLLEGE OF EDUCATION AND BEHAVIORAL STUDIES
ADDIS ABABA UNIVERSITY
CEBS's INSTITUTIONAL REVIEW BOARD (IRB)
Ethical Approval Certificate
Graduate program students

Form: CEBS_IRC_SMED_Template_Form_002

IRC Approval Reference No. CEBS_IERC_SMED_002/2024

To: Belete Abebaw
Principal Investigator: Variation Theory Based GeoGebra Assisted Problem-solving Approaches and Students' Learning of Geometry

Advisor(s): Prof. Mulugeta Atnafu and Dr Aweke Shishigu

From: Dr. Abera Abate
Department of Science and Mathematics Education (SMED), Head)

Re: Approval of a Research Ethics

Protocol Title	PhD Dissertation Project: Variation Theory Based GeoGebra Assisted Problem-solving Approaches and Students' Learning of Geometry
Protocol Number	CEBS_IERC_SMED_002/2024
Principal Investigator	Belete Abebaw
Institute	*Department of Science and Mathematics Education*
Study Site(s)	Secondary Schools in Addis Ababa
Decision	*All ethical standards are found to be met. Hence, the SMED Ethical Review Committee has approved this research project.*
Date the approval was issued	February 14, 2024
Expiry of this approved certificate (*Not exceeding 2 years*)	February 13, 2026

Additional notes (If any) to the PI.
SMED has established that this research fulfills all ethical requirements to be conducted in Secondary Schools in Addis Ababa.

Remark:

- Data collection will be conducted in Secondary Schools in Addis Ababa.
- ____

Signature: ________________

Date: February 15, 2024

For Academic Unit Head/Graduate program coordinator

Signature: ________________

Date: March 07, 2024

For CEBS-IRC