THE CONTRIBBUTION OF COULOMBIC PRESSURE TO THE STABILITY OF THIN KEPLERIAN ACCRETION DISCS AROUND A NEUTRON STAR WITH AXISYMMETRIC MAGNETIC DIPOLE

By

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This is to certify that the thesis prepared by **MUKTAR JEMAL**, entitled "**The Contribbution Of Coulombic Pressure To The Stability Of Thin Keplerian Accretion Discs Around a neutron Star with axisymmetric Magnetic Dipole** " and submitted in partial fulfillment of the requirements for the degree of **Master of Science**. compiles with the regulations of the University and meets the accepted standards with respect to originality.

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Abstract

Based on the contribution of intraction between various charged components of the system the stability analysis of an axisymmetric geometrically thin and optically thick accretion disk around a magnetized neutron star is presented.The disk we consider is modified by including the effect of coulombic pressure on the total pressure.We consider three region of the disks outer, middle and inner region.The outer and middle region of the disk is dominated by gas pressure and the inner region is dominated by radiation pressure. The opacity in the middle and inner region is mainly due to electron scattering whereas that in the outer region is mainly due to free-free emission. Starting from the vertically integrated non-relativistic hydrodynamics equations we set up the basic equations which govern the structure of the disc and for the stability analysis of the disc model we have kept the time dependencies in the equations. Even if we include coulombic pressure We find that the gas pressure dominated region is thermally stable.The graph of stability parameter as afunction of the disk radius shows that the inner region of the disc is viscously unstable.

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Introduction

An accretion disk is likely to be formed when a compact object ,aneutron star in this case, is a member of a close binary system and matter transferred from a giant type companion onto the compact star with high angular momentum. Shakura and sunyaev (1973) initiated this discussion considering a very simplistic but effective standard model of a geometrically thin, optically thick accretion disk. They were able to obtain an analytical solution of height integrated hydrodynamical equations by using Newtonian gravitational potential.The study of stability of the accretion disc is one of the important criteria in this context. The stability of geometrically thin accretion discs has been studied extensively after the construction of standard α-discs. According to the standard theory of accretion discs (Shakura and Sunyaev 1973), the middle and outer parts of the disc are dominated by the gas pressure and the inner region is dominated by radiation pressure. Those regions have been found to be stable to the thermal and viscous modes but pulsationary unstable to the acoustic modes (Blumenthal,Yang, and Lin 1984). Some recent research work about the isothermal accretion disc also obtained similar results (Wallinder 1990; Wu et al.1995b; Wu, Yang, and Yang 1994).

Some early analyses about the stability of gas pressure dominated discs have incorporated azimuthal perturbations (Livio and Shaviv 1977,1981; Van Hon, Wesemael, and Winger 1980.). However, the radial perturbations were neglected in all the studies . McKee (1991) has investigated the contribution of gas pressure to the stability of a standard alpha-disc. He found that the disc is stable when $\beta < 0.6$(β is the ratio of the dominant pressure to the total pressure). This implies that a gas pressure dominated disc is more stable. It has been also found that the disc is thermally and viscously unstable if it is optically thick and radiation pressure dominated (Pringle,Rees and Pocholczyk, 1973; Lightman and Eardely, 1976; Shahura and Sunyaev, 1976). There is also a possible mode of pulsational overstability. In this case, one looks for instabilities in which oscillations on the orbital timescale grow in amplitude because of the effects of viscosity (Lin and Paploizou, 1996). Kato (1978) considered the evolution of small perturbations of all three components of velocity as well as T and S. He found that the disc experience pulsational instability besides the viscous instabilities and thermal instabilities. If a geometrically thin disc is optically thin, it has been found also that it is viscously stable but thermally unstable (Piran, 1978). Those instabilities are believed to be relevant to some light variation observed in many systems such as X-ray binaries. In the standard α model, the viscous heating is balanced by radiative cooling. However, if the radiative cooling is not efficient, the advection will be non negligible. Particularly in an optically thin disc, the radiative cooling rate is so slow that most of the viscous generated energy is advected radially. Recently, the accretion disc models with gas pressure dominated with either electron scattering or free-free opacity have been studied(Abramowicz etal., 1988; Kato, Honma and Matsumaoto, 1988;Narayan and Popham, 1993; Narayan and Yi, 1994, 1995a , 1996b; Abramowicz et al., 1995; Chen et al., 1995; Chen, 1995). The gas dominated disk model with electron scatering opacity has also been adopted successfully to explain the observations of low and high luminosity systems (Nayan, Yi and Mahadeven, 1995; Narayan, Mcclintock and YI, 1996).

The possibility of steady and stable disc formation by incoming matter toward a neutron star is allowed only for a certain sets of initial parameters. Abramowicz Zurek (1981) studied the effect of angular momentum on the accretion and the corresponding stability of the transonic nature of the in falling matter on to the star. The gas elements in the disc lose angular momentum, due to the interaction or friction between adjacent layers and spiral inwards. Part of the released gravitational energy increases the kinetic energy of the rotation and the other part is converted in to thermal energy which is radiated from the disk surface. Thus, viscosity converts gravitational potential energy in efficient manner in to radiation Accretion disk around a compact star has been thought to play an important role in various X-ray sources. The interaction between a magnetized star and a surrounding accretion disc is one of the most poorly understood aspects of accretion.The magnetic field of the star penetrates the surrounding accretion disc and couples the two. According to the Ghosh and Lamb (1979) model, the part of the accretion disc that is located inside the co rotation radius provides a spin up torque on the star, since it is rotating faster than the star, while the more slowly rotating outer part of the accretion disc brakes the star. The net torque is determined by the location of the inner edge of the disc, which moves inwards as the accretion rate increases, thereby increasing the spin up-torque on the star. In this respect Shapiro et al. (1976) gave a detailed two-temperature disk model which might be promising if radiation pressure dominated inner region of the disk was secularly unstable. On the other hand, Bisnovatyi-Kogan and Blinnikov (1977) proposed a corona disk model where the accretion process would not be steady, by investigation of particle motions taking account of the radiation from the disk and gravitational field of a black hole.

In all the studies of stability mentioned above, we see that although a gas pressure dominated region with either an electron scattering dominated or free-free absorbation opacity is studied, consideration of the effect of coulombic pressure is not made. The aim of this work is then to expound on the effect of coulombic pressure on the stability of thin Keplerian accretion discs.

The structure of this thesis is as follows,The first chapter is devoted to the discussion of the coulombic pressure. In chapter two the equation of state and the basic equations of thin accretions disks for non-relativistic case, such as the equation of conservation of mass,angular and radial momentum,energy and equations for the vertical structure are briefly discussed.Accretion onto campact objects is fully dealt with in chapter three.The local structure of the disc will be cosiderd in chapter four. The stability analysis of the thin accretion disks thermal instaility, viscous instability, and the stability parameter β in the inner, middle and outer regions are given in chapter five. Finally conclusion is made.

Chapter 1

COULOMBIC PRESSURE

Since we are concerned with real gases ,because they have direct applications in astrophysics as compared to ideal gases .So we have to include the effect of interactions between the various charged components of the system.Coulombic pressure is the result of this effect. To get the total pressure we include coulombic potential on top of gravitational potential,Ω_0.

Recall that from kinetic molecular theory of gases ,gas pressure is given by

$$P = \frac{2}{3}\frac{K}{v} \tag{1.0.1}$$

where K is average kinetic energy of the gases and its is dtermined from Virial theorem.To know coulombic pressure the following subtopics should be taken in to account.

1.1 Virial Theorem

The virial theorem states the that total kinetic enery of the star is equals to half of its gravitational potential energy.If a star contracts , half of its gravitational energy is transformed in to kinetic energy, the other half is lost, i.e radiated away(A1.2).If we denote the kinetic energy with K gravitational potential with Ω_0 we have

$$K = \frac{-\Omega_0}{2} \tag{1.1.1}$$

thus equation 1.0.1 becomes

$$P = \frac{-1}{3}\frac{\Omega_0}{v} \tag{1.1.2}$$

where $\Omega_0 < 0$

In the presence of coulombic potential U_c , we expect the total pressure to carry additional term$P_{coul}.Since\ \ U_c > 0$, We expect

$$P_{coul} = \frac{1}{3}\frac{U_{coul}}{v} \tag{1.1.3}$$

1.2 Plasma Concepts

A plasma is an ionised gas that is in state of electrical quasi-neutrality, the behaviour of which is governed by collective effects due to long-range electromagnetic interactions between the charged particles .So to find Pcoul one has to calculate first U_c . Since our system is a plasma that is, polarizable, the potential of every ion is shielded and its long range effect is cut short. Whenever we want to consider the behavior of a gas on a length scales comparable to the mean free path between collisions, we must use the idea of plasma physics that will be important to our study of accretion. A plasma differs from an atomic gas or molecular gas in that it consists a mixture of two gases of electrically charged particles, an electron gas and an ion gas, with very different particle masses m_e and m_i. The electrons and ions interact with each other through their electrostatic coulombic attractions and repulsions. These coulomb forces decrease only slowly ($\propto 1/r^2$) with distance and do not have a characteristic length scale. Thus, a plasma particle interacts with many others at any one instant, and this makes the description of the collisions more complicated than in atomic or molecular gases, where the inter particle forces are very short range. A further complication arises from the great differences in particle masses m_e and m_i . Since collisions between particles of very different masses can transfer only a small fraction of the kinetic energy of order $m_e/m_i \ll 1$, it is possible for electrons and ions to have significantly different temperatures over appreciable time scales. These two properties- the long range nature of the coulomb force and the disparity in electron and ion masses give the physics of plasmas its particular character. A further series of complex phenomena occurs when the plasma is permeated by a large scale magnetic field, this is particularly relevant for the study of gas accreting on highly magnetized neutron stars.

Formally the shielded potential is given by

$$V \propto \frac{e^{-(const)r}}{r} \tag{1.2.1}$$

where r is the distance between the two interacting particles. It is calculated from the requirement that,

$$\rho_q = \rho_o e^{-\frac{zeV}{k_B T}} \tag{1.2.2}$$

where

$$\rho_q = e \sum_z (z n_z) \tag{1.2.3}$$

is the charge density and

$$n_z = n_o e^{-\frac{zeV}{k_B T}} \tag{1.2.4}$$

is the number of ions per unit volume, for our system to be nearly perfect we expect, $n_z << 1$. Since ρ_q satisfies Poissons equation,

$$\bigtriangledown^2 V = -4\pi\rho_q = -4\pi e \sum_z (z n_z) \tag{1.2.5}$$

Or

$$\bigtriangledown^2 V = -4\pi\rho_q = -4\pi e \sum_z (z n_o) e^{-\frac{zeV}{k_B T}} \tag{1.2.6}$$

For slightly real system zeV<< $k_B T$, thus

$$\bigtriangledown^2 V \approx -4\pi e (1 - \frac{-zeV}{k_B T}) \sum_z (z n_o) \tag{1.2.7}$$

using the fact that the number densities of ions and electrons at any point must be approximately equal, and therefore a plasma must always be close to charge neutrality: even a small charge imbalance would result in very large electric fields which would act to move the plasma particles so as to restore neutrality very quickly. Hence,$\sum_z (zn_o)$ =0, we have

$$\bigtriangledown^2 V \approx \frac{4\pi e^2}{k_B T} \sum_z (z^2 n_o V) \tag{1.2.8}$$

Or

$$\bigtriangledown^2 V \approx {k_D}^2 V \tag{1.2.9}$$

where

$${k_D}^2 = \frac{4\pi e^2}{k_B T} \sum_z (z^2 n_o) \tag{1.2.10}$$

From the fact that $\frac{(ze)^2}{K_B T}$ has the dimension of length and n_o has the dimension of inverse length cube we notice that K_D is an inverse length.The solution to Poissons equation is easily determined by inspection based on the facts

i)$\rho_q = \rho_o e^{-\frac{zeV}{k_B T}}$

ii)$V \propto \frac{1}{r}$

We therefore expect the solution to be of the form

$$V = \Upsilon \frac{e^{-k_D r}}{r} \tag{1.2.11}$$

This can be checked by direct substitution in to $\bigtriangledown^2 V \approx {k_D}^2 V$,and to find the constant Υ we use the usual potential for small value of r i,e lim$V_{r\to 0}$=$\frac{ez}{r}$,implying const =ez

Hence

$$V = ez \frac{e^{-k_D r}}{r} \tag{1.2.12}$$

The fact that we have weak interaction is telling us,$K_D r << 1$

In this case $e^{-k_D r}$ may be written as

$$e^{-k_D r} \approx 1 - K_D r \tag{1.2.13}$$

and it gives, $V(r) = \frac{ez}{r} + \phi_z$,where $\phi_z = -ezK_D$,is the potential due to other charges at the ions.

The coulomb energy per unit volume is commonly given as

$$\frac{U_c}{v} = \frac{1}{2}\sum_z (zen_o\phi_z), where \tag{1.2.14}$$

$$\phi_z = -ezK_D, K_D = (\frac{4\pi f e^2\rho}{k_B m_p T})^{\frac{1}{2}} \tag{1.2.15}$$

Here we use the parameter $f = \sum(z^2)\frac{x_z}{A_z}$, where $x_z$is the mass abundance of the gas and A_z is atomic weight. Since we are considering the disk system (whirling gas) as a hydrogen gas for which the atomic number, $z = 1$ and its abundance $x_z = 1$ and its atomic weight$A_z = 1.0079$, $\frac{u_c}{v} = -e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}}$.Thus the coulomb pressure, $p_{coul} = \frac{1}{3}\frac{U_c}{v}$,following from this will be

$$P_{coul} = -\frac{1}{3}e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}} \tag{1.2.16}$$

,

Chapter 2

STRUCTURE EQUATION

In this section we set up the basic equations which govern the structure of the thin accretion disc around a neutron star with a magnetic dipole field. These equations are basically derivable from the equations of non-relativistic magneto-hydrodynamics. The disc structure we consider was first investigated by Shakura and Sunyaev but is modified. It is modified since we include the effect of coulombic pressure on the total pressure. To specify a disc model we need to give a viscosity prescription and a relation for the opacity equation.In addition, we are interested in three regions:the outer and middle region of the disk dominated by gas pressure, and the inner region dominated by radiation pressure.In the portion of the disc where gas pressure is dominant we shall drop the radiation pressure term and in the portion of the disc where the radiation pressure is dominant we shall drop the gas pressure term from the equation of state.

2.1 Assumptions

The simplest accretion disc model to construct is that of the thin disc. We obtain a simplified set of equations by integrating over the z-dimension and by assuming that the flow is steady and is axisymmetric in the mean.Since We consider an axisymmetric disk structure of accreting matter and we employ a cylindrical coordinate system (R, φ, z) with the z-axis chosen as the axis of rotation. The basic structure of a disc is the same in all systems. A disc rotates around a central object,and usually the central object provides all of the gravitational force on the disc.In some cases there are additional sources of gravity, such as the self-gravity of the disc it self.The orbit of the accreted material is close to circular,but the rotation of the disc is generally differential, so that the rotation velocity and the rotation period change with distance from the center. If all of the gravitational force is provided by the central object, the disc is aKeplerian disc, and the rotation period depends on radius according to the Keplerian laws of orbital motion: T$\propto R^3$, where T is the orbital period and R is the distance from the center of the disc. Therefore, throughout the analysis of this paper the following assumptions will made:

1. The time derivatives of the mean flow variables are zero.
2. Relativistic effects and self-gravity effects, and hence gravitational instabilities are therefore neglected. This assumption will be valid for disks around magnetized neutron stars.
3. The disc is assumed to consist of fully ionized hydrogen gas.
4. The viscosity may be adequately represented by an α- disc model such that $f_{R\varphi} = \alpha P$, where $f_{R\varphi}$ is the only appreciable component of the stress tensor and P is the total pressure. the viscosity parameter,α, will be assumed constant within the interval $10^{-3} < \alpha \leq 1.0$
5. The disk is axisymmetric, i.e. there is no dependence of the mean flow on the azimuthal angle φ
6. The disc is thin (disc half thickness= $H << R$ =disc radius) and all equations will be averaged over the vertical structure.This means that we can construct useful equations by vertical averaging.
7. The velocity in the disk is dominated by the Keplerian velocity.This means all of the gravitational force is provided by the central object i,e $v_\varphi = \sqrt{\frac{Gm}{R}}$

8. Vertical hydrostatic equilibrium is maintained as the instabilities develop. This is certainly true in the case of the viscous instability. However, the vertical time scale t_z , becomes comparable to the thermal time scale, t_{th} , for alpha near unity $t_{th} \approx \alpha^{-1} t_z$. As pointed out by Pringle(1976) , the growth rate for the thermal instability may be an underestimate due to this assumption.

9. The kinetic energy of the star is equals to half of its gravitational potential energy ,this leads to the velocity in excess of keplerian velocity

2.2 The Equation of State

The equation of state is the function that relates the pressure to the density, molecular weight, and temperature at any place in the star.In this case the total pressure in the presence of coulombic pressure is the sum of gas pressure(P_g) ,radiation pressure(P_r) and coulombic pressure(P_{coul}) where $P_g = \frac{N_o k_B T \rho}{\mu}$ and $P_r = \frac{1}{3} a T^4$,where a is the radiation constant

$$P = P_g + P_r + P_{coul} = \frac{N_o k_B T \rho}{\mu} + \frac{1}{3} a T^4 - \frac{1}{3} e^3 (\frac{\pi}{k_B T})^{\frac{1}{2}} (f \rho N_o)^{\frac{3}{2}} \quad (2.2.1)$$

2.3 Mass Conservation

The continuity equation that describes the mass conservation for a fluid flow of density ρ flowing at velocity v = (v_R, v_φ, v_z) in unit time from (A2.6) of the appendix two is given by

$$\frac{\partial \rho}{\partial t} + \triangledown.(\rho v) = 0 \quad (2.3.1)$$

since the disk is steady in the mean, i.e. time derivatives of the mean flow variables are zero

$$\triangledown.(\rho v) = 0 \quad (2.3.2)$$

For thin cylindrically symmetric disc $Eq.(2.3.1)$ can be expressed as

$$\frac{1}{R}\left[\partial_R R(\rho v_R)\right] + \partial_\varphi(\rho v_\varphi) + \partial_z(\rho v_z) = 0 \tag{2.3.3}$$

But from the assumption there is no dependence of the mean flow on the azimuthal angle φ .Dropping the φ derivatives, thus we get

$$\frac{1}{R}\partial_R R(\rho v_R) + \partial_z(\rho v_z) = 0 \tag{2.3.4}$$

Multiplying equation (2.3.4) by 2πR and integrating over z :

$$2\pi \int_{-H}^{H} \partial_R(\rho R v_R)dz + 2\pi R \int_{-H}^{H} \partial_z(\rho r v_z)dz = 0 \tag{2.3.5}$$

So for a thin axisymmetric disc after neglecting the vertical out flow from the disc, the gas will have a radial component velocity only i,e there is no wind from the top and bottom surfaces of the disc. Therefore $v_z = 0 \;\; at \;\; z = \pm H$ .This implies
$2\pi\partial_z(\rho R v_z)|_{-H}^{H}\rho dz = 0$
thus we have

$$2\pi \int_{-H}^{H} \partial_R(\rho R v_R)dz = 2\pi\partial_R R v_R \int_{-H}^{H} \rho dz = 0 \tag{2.3.6}$$

Introducing a useful variable called surface density
$S(R,t) = \int_{-H}^{H} \rho dz$
where ρ is the vertically average density and H is half thickness of the disc.We have

$$\partial_R R v_R S \; = 0 \tag{2.3.7}$$

Defining also the mass accretion rate by:

$$\dot{M} = -2\pi R v_R S \tag{2.3.8}$$

finally for steady state we have

$$\partial_R \dot{M} = 0 \Rightarrow \dot{M} = constant \tag{2.3.9}$$

i.e. the accretion rate is constant with radius.

2.4 Viscosity

Viscosity is a microscopic property of fluids and decribes the inner friction of the fluid.In real fluid the transport of momentum occurs in part by the transport of fluid volumes having different velocities. But additional transfer is caused by the internal friction between particles moving with adjacent layers of the fluid having different velocities. Thus angular momentum has been transported outward as a result of collisions, the effect of collisions in this case is to introduce a positive co-relation between azimuthal velocity fluc tuations and radial velocity fluctuations, which in the language of turbulent fluids means that the Reynolds stress $\langle \rho v_R v_\varphi \rangle$ transports angular momentum outwards. Because the chaotic motion takes place in an equilibrium flow, the exchange of fluid elements cannot result in the net transfer of any matter between the two rings(fig 2.1). Therefore, mass crosses the surface R=constant at equal rates in both directions, of the order H$\rho\tilde{v}$ per unit arc ,so the average upward and downward mass fluxes are the same.

If the elements are not interacting with the streaming fluid, and subject only to external forces (e.g. gravity), the appropriate assumption is that angular momentum is conserved. If the effects of surrounding fluid (e.g. pressure gradients) must be included, the external forces are canceled by bulk rotation and pressure gradients in a steady state, so stream-wise momentum is conserved.

The net upward φ-momentum flux density is following approximately equals to

$$\rho\tilde{v}[(R + \frac{\lambda}{2})v_\varphi(R - \frac{\lambda}{2}) - (R - \frac{\lambda}{2})v_\varphi(R + \frac{\lambda}{2})] \approx \rho\tilde{v}\lambda R^2 \Omega' \tag{2.4.1}$$

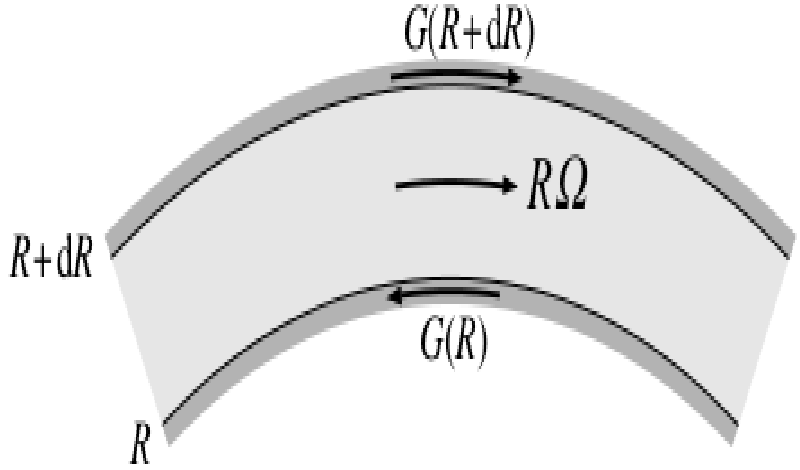

Figure 2.1: Differential viscous torque

where $\Omega' = \frac{d\Omega}{dR}$ *and* λ is the spatial scale or characteristic wave length of the turbulence .There is a transport of angular momentum due to the chaotic process, i.e, a viscous torque exerted on the outer stream by the inner stream and an equal and opposite torque exerted by the outer stream on the inner. The torque exerted on the outer ring by the inner ring is given by the net outward angular momentum flux. Since the mass flux due to chaotic motions is the same in both directions, one obtains to first order in λ the torque per unit arc length as

$$-\rho\tilde{v}H\lambda R^2\Omega' \tag{2.4.2}$$

and we assume that the angular velocity changes slowly over the length scale of chaotic motions. The non-vanishing component of the stress in this case is the force in the φ direction per unit area, and is given by

$$f_{r\varphi} = -\eta R\Omega' \sim -\rho\dot{v}\lambda R^2\Omega' \tag{2.4.3}$$

yielding a positive kinematic viscosity $\nu \sim \lambda \dot{v}$. But generally in an accretion disc, the total torque is obtained simply by multiplying by the length $2\pi R$ of the circular boundary. Setting $\rho H = S$(the surface density), we can write the torque exerted by the outer ring on the inner is equals to the negative the torque of the inner on the outer as

$$G(R) = 2\pi R \nu S R^2 \Omega' \tag{2.4.4}$$

The sign of this equation tells us for a rotation law in which $\Omega(R)$ decreases outwards, hence G(R) is negative, in this case the inner rings lose angular momentum to the outer ones and the gas slowly spirals in. Now let us consider the net torque on a ring of gas between R and R + dR. As this has both an inner and an outer edge, it is subject to competing torques (Fig. 2.1); the net torque (trying to speed up) is

$$G(R + dR) - G(R) = \frac{\partial G}{\partial R} dR \tag{2.4.5}$$

Because this torque is acting in the sense of angular velocity Ω, there is a rate of working

$$\Omega \frac{\partial G}{\partial R} dR = [\frac{\partial}{\partial R}(G\Omega) - G\Omega'] dR \tag{2.4.6}$$

by the torque. But the term

$$\frac{\partial}{\partial R}(G\Omega) dR$$

is just the rate of convection of rotational energy through the gas by the torques and the term $-G\Omega'$dR represents a local rate of mechanical energy to the gas. This lost energy must go in to internal (heat) energy. The viscous torques therefore cause viscous dissipation energy with in the gas at a rate $G\Omega'$dR per ring width dR. Ultimately, this energy will be radiated over the upper and lower faces of the disc.

2.5 Momentum Conservation

Accretion discs are formed due to the angular momentum of the incoming gas.In afluid the transferring material has high specific angular momentum, so that it cannot accrete directly onto the star.In real fluid the transport of momentum is expressed by the advective term in Eulers (momentum) equation and the derivation of the Navier-Stockes equation begins with an application of Newtons second law, for a fluid of density ρ, flowing with a velocity v .Eulers equation is given by

$$\frac{du}{dt} = - \bigtriangledown P - \rho \bigtriangledown \Phi \tag{2.5.1}$$

where P is the pressure and Φ is the gravitational potential. If dissipative terms repre senting the action of viscous forces are included on the right hand side of Eq.(2.5.1) it becomes the Navier-Stockes equation .i,e what makes Navier stokes equation different from eulers eqaution is that, the existance of stress tensor in later.

$$\frac{du}{dt} = - \bigtriangledown P - \rho \bigtriangledown \Phi + \bigtriangledown . f \tag{2.5.2}$$

where f represents the stress tenser.At the interface between adjacent cells of the resulting chaotic field, the gradients become so strong that magnetic field line reconnection occurs. Clearly, we are not able to handle these complicated phenomena in quantitative manner.A reasonable parametrization has, however, been given by ,following Norbert Straumann,

Using

$$D_t = \partial_t + v_R \partial_R + v_\varphi \frac{1}{R} \partial_\varphi + v_z \partial_z \tag{2.5.3}$$

Using the cylindrical coordinates(R, φ, z), and considering first each term separately then after a set of mathematical manipulations connecting the expansion terms according to their components, we have the following results as shown in different subsections.

2.5.1 The Azimuthal Component

Here we consider the φ component of the momentum equation.From (A2.11) of the Appendix page it will be

$$\rho(D_t v_\varphi + \frac{v_\varphi v_R}{R}) = -\frac{1}{R}\partial_\varphi P - \rho\frac{1}{R}\partial_\varphi \Phi + \frac{1}{R}\partial_R(Rf_{\varphi R}) + \partial_z(Rf_{\varphi z}) + \frac{1}{R}f_{R\varphi}$$

now for steady flow $\partial_t = 0$ by symmetry the fluid variables are independent of φ thus,

$$\rho(D_t v_\varphi + \frac{v_\varphi v_R}{R}) = \frac{1}{R}\partial_R(Rf_{\varphi R}) + \partial_z(Rf_{\varphi z}) + \frac{1}{R}f_{\varphi R}$$

Multiplying this equation with R then for φ independent functions we obtain

$$\rho(D_t v_\varphi) = \frac{1}{R}\partial_R(R^2 f_{R\varphi}) + \partial_z(Rf_{R\varphi}) \qquad (2.5.4)$$

Now we multiply the continuity equation

$$\frac{1}{R}\partial_R R(\rho v_R) + \partial_z(\rho v_z) = 0$$

with Rv_φ and add the resulting equation to Eq.(2.5.4) and obtain

$$\frac{1}{R}\partial_R R(\rho v_R R v_\varphi) + \partial_z(\rho v_z r v_\varphi) = \frac{1}{R}\partial_R(R^2 f_{R\varphi})$$

Integrating over z gives

$$\frac{1}{R}\partial_R \int R^2(\rho v_R v_\varphi)dz = \frac{1}{R}\partial_R(R^2 W_{R\varphi}) \qquad (2.5.5)$$

where

$$W_{R\varphi} = \int f_{R\varphi}dz$$

for thin disc Eq.(2.5.5) is approximately

$$\frac{1}{R}\partial_R(v_R S R^2 v_\varphi) = \frac{1}{R}\partial_R(R^2 W_{R\varphi})$$

but for steady state

$$\frac{1}{R}\partial_R(v_R S R) = 0$$

thus we have

$$S v_R \partial_R(R v_\varphi) = \frac{1}{R}\partial_R(R^2 W_{R\varphi}) \tag{2.5.6}$$

where Rv_φ is the specific angular momentum and the component $W_{R\varphi}$ has the form $w_{R\varphi} = \int \eta R \partial_R(\frac{v_\varphi}{R} dz)$,where η is the dynamic vioscosity.

2.5.2 The Radial Component

The radial component of the momentum equation is

$$\rho(v_R\partial_R v_R + \frac{v_\varphi}{R}\partial_\varphi v_R + v_z\partial_z v_R - \frac{v_\varphi^2}{R}) = -\partial_R P - \rho(\partial_R\Phi) + \frac{1}{R}\partial_R(R f_{RR}) + \partial_z f_{rz} - \frac{1}{R} f_{\varphi\varphi} \tag{2.5.7}$$

Similarly $\partial_\varphi = 0$ (by symmetry). Except for $f_{R\varphi}$, all the viscous stresses can be neglected and ignoring the vertical variation (out flow), we have to sufficient accuracy

$$\rho(v_R\partial_R v_R) = -\partial_R P + \rho(\frac{v_\varphi^2}{R} - \partial_R\Phi) \tag{2.5.8}$$

Since we use the fact that

$$\Phi = \frac{GM}{(R^2+z^2)^{\frac{1}{2}}}$$

thus

$$\partial_R\Phi = -\frac{1}{2}2R\frac{GM}{(R^2+z^2)^{\frac{3}{2}}} = -R\frac{GM}{(R^2+z^2)^{\frac{3}{2}}}$$

since $R >> z$

$$\Longrightarrow \partial_R\Phi \approx -\frac{GM}{R^2}$$

hence after integration over z Eq.(2.5.8) can be rewritten as

$$S v_R \partial_R v_R = S(\frac{v_\varphi^2}{R} - \frac{GM}{R^2}) - \partial_R W$$

where

$$W = \int P dz$$

2.5.3 The Vertical Component

Since the z component of the momentum equation contains only the small components of the viscosity, then ignoring these and the φ derivative we have,

$$\rho(v_R \partial_R v_z + v_z \partial_z v_z) = -\partial_z P - \rho(\partial_z \Phi)$$

we assume that the motion in the z direction are subsonic. Thus, the LHS is a factor of $(\frac{z}{R})^2$. Now neglecting the vertical out flows, the equation reduces to the equation of hydrostatic equilibrium in the z- direction, i.e,

$$\partial_z P = -\rho \frac{GM}{R^2} \frac{z}{R} \tag{2.5.9}$$

Using H as half thickness of the disc, the pressure at the mid plane of the disc is,

$$P = \int_0^H \frac{GM}{R^3} \frac{z}{R} dz = \frac{1}{2} \rho H \frac{GM}{R^3} H = \frac{1}{2} H S \frac{GM}{R^3}$$

Making slight approximation over Eq.(2.5.8), i.e, letting

$$\frac{1}{\rho} \partial_z P \approx \frac{1}{\rho} \frac{P}{z} = \frac{GM}{R^3} z$$

since the Keplerian velocity

$$v_k^2 = \frac{GM}{R}$$

and

$$c_s^2 = \frac{P}{\rho}$$

we can rewrite the hydrostatic equilibrium equation as

$$c_s^2 = v_k^2 (\frac{H}{R})^2$$

This implies

$$\frac{H}{R} = \frac{c_s}{v_k} \tag{2.5.10}$$

In this approximation we have used,

$$v_k = v_\varphi$$

The thin disc requirement thus says that the circular flow velocity is highly supersonic.

2.6 Conservation Of Energy

Viscosity in the gas disc converts the free energy of differential rotation in to thermal energy, which is then radiated away. As the potential energy is released, the gas slowly spirals inward, completing many revolutions around the neutron star before significantly changing its distance from the central source. The amount of gravitational potential energy release by the gas in the disc increases as the gas draws closer to the central object. Our stating point is Eq.(A2.24) of the Appendix. Since the dominant part of the scalar product of the stress tenser and the velocity is the radial component of magnitude $f_{R\varphi}v_\varphi$ and thus

$$\bigtriangledown.(fv) = \frac{1}{R}\partial_R(Rf_{R\varphi}) \tag{2.6.1}$$

Ignoring again the z-component of the velocity and in addition the radial component of the energy flux vector, we have

$$\rho(v_R\partial_R)(\frac{1}{2}v_R^2 + \frac{1}{2}v_\varphi^2 + \Phi) = \frac{1}{R}\partial_R(Rf_{r\varphi}) - \partial_z F \tag{2.6.2}$$

where F is the vertical energy flux density (F = qz). If we denote the energy flux per unit area emitted at the disc surface by Q^- =2F and the dissipation function $q^+ = f_{R\varphi}R\partial_R(\frac{v_\varphi}{R})$then the energy produced per unit area Q^+ is given by then the energy produced per unit area Q^+ is given by

$$Q^+ = \int q^+ dz = W_{R\varphi}R\partial_R(\frac{v_\varphi}{R})$$

Usually it is assumed that the energy dissipated in to heat is radiated on the spot in the vertical direction. Then we have

$$\partial_z F = q^+ = f_{R\varphi}R\partial_R(\frac{v_\varphi}{R})$$

Since we approximate the azimuthal component of velocity by the circular Keplerian velocity, i.e, $v_\varphi \approx \Omega R$,where $\Omega = \frac{Gm}{R^3}^{\frac{1}{2}}$

Then we obtain

$$\frac{\dot{M}\Omega R}{2} = -2\pi\partial_R(W_{R\varphi}R^2)$$

where

$$W_{R\varphi} = R\frac{d\Omega}{dR}\int \eta dz$$

and thus we have

$$\dot{M}\Omega R^2 + 2\pi R^3\frac{d\Omega}{dR}\int \eta dz = I$$

where η is the turbulent viscosity and I is independent of R .

The Keplerian approximation follows if inertia and pressure gradient terms are neglected. Corrections are of order $(\frac{H}{R})^2 \ll 1$

From the above relations we find

$$\partial_z F = \frac{9}{4}\eta\frac{GM}{R^3}$$

then the energy produced per unit area will be given by

$$Q^+ = \frac{9}{4}\frac{GM}{R^3}\int \eta dz$$

The angular momentum conservation implies

$$\dot{M}\Omega R^2 = -2\pi R^2 W_{r\varphi} + I$$

Here the constant I is the net rapidly inward flux of angular momentum, whose value is usually assumed to be of the order of$\dot{M}\Omega(R_A)R_A^2$, where R_A(Alfven) is the inner edge of the disc. Then we have for specific angular momentum $l = R^2\Omega$

$$\dot{M}[l(R) - l(R_A)] = -2\pi R^2 W_{R\varphi} \qquad (2.6.3)$$

The torque $2\pi R^2 W_{R\varphi}$ is on the other hand determined from $W_{R\varphi} = -\frac{3}{2}\Omega \int dz$

In the energy equation above we neglect derivatives of W, S, and v_R; $\frac{d}{dR}[\dot{M}\frac{1}{2}v_\varphi^2 - \frac{GM}{R}) + 2\pi R^2 W_{R\varphi}\Omega] = -2\pi R Q^-$.Finally we express the cooling rate Q^-independent of η

$$Q^- = \frac{3}{4\pi}\dot{M}\frac{GM}{R^3}[1 - (\frac{R_A}{R})^{\frac{1}{2}}] \tag{2.6.4}$$

Chapter 3

ACCRETION ON TO COMPACT OBJECT

The accretion of matter includes the release of gravitational energy.Because of this accretion discs are considered an efficient machines for extracting gravitational potential energy and converting it into radiation.They convert about half of the gravitational energy to rotational keinetic enrgy and the rest to radiation energy. Neutron stars arise as a result of gravitational collapse of cores of massive stars after exhaustion of all thermonuclear energy sources.A black hole is an object, which has so strong gravitational field, that it does not radiate either electromagnetic or gravitational waves (all radiation and matter are confined inside the horizon of events the effective boundary around the black hole, from which no information can escape to the entire universe).The angular velocity $\Omega(R)$ in the disc remains very close to the. Keplerian value $\Omega_k(R) = (\frac{GM}{R^3})^{1/2}$ until the accreting matter enters a boundary layer of radial extent b just outside the surface $R = R_*$ of the accreting star. Within this boundary layer Ω must decrease from a value $\Omega(R_* + b) \cong \Omega_K(R_* + b)$ to the surface angular velocity $\Omega_* < \Omega_k(R_*)$. Thus we envisage Ω_R as a function having the form shown in Fig. 3.1. We can see how much luminosity can be radiated from the boundary using the following argument. At first sight one might think that this is simply given by the specific energy difference between a Kepler orbit at R and a particle with angular velocity Ω_* at this radius, i.e.

$$E = \frac{1}{2}\dot{M}R_*^2(\Omega_k^2 - \Omega_*^2) \tag{3.0.1}$$

and indeed an incorrect expression of this form was current in the literature for many years. However, this neglects the edge term GΩ acting on the star at its surface

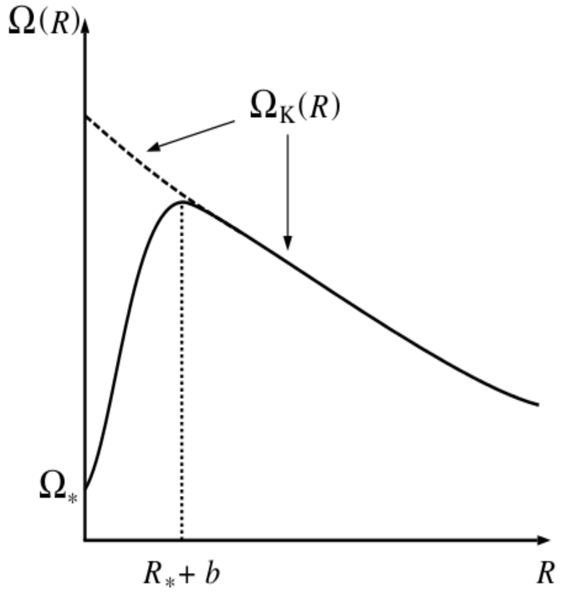

Figure 3.1: Distribution $\Omega(R)$ of angular velocity near the inner edge of an accretion disc around a star with surface angular velocity Ω_* less than the break-up value $\Omega_K(R)$. The R_* , is the boundary layer. region $R_* < R < R_* +$ b, with b<< R_* is the boundary layer

We therefore need an expression for the viscous torque on the star. The stationary angular momentum conservation equation

$$-\nu S\Omega' = S(-v_R)\Omega + \frac{C}{2\pi R^3} \tag{3.0.2}$$

Now by definition the outer limit of the boundary layer is the point $R = R_* + b$ where $\Omega' = 0$. Then we find within the boundary layer that

$$C = -[2\pi R^3 S(-v_R)\Omega]_{R=R_*+b} \simeq \dot{M}R_*^2\Omega_k(R_*) \tag{3.0.3}$$

where we have assumed that b << R_* and that Ω is very close to the Kepler value at $R = R_* + b$. From (2.5.4) we can deduce that the viscous torque is

$$G = 2\pi R^3 S\nu\Omega' = -\dot{M}R^2\Omega + \dot{M}R_*\Omega_k \tag{3.0.4}$$

which therefore takes the value

$$G_* = \dot{M}R_*^2(\Omega_k - \Omega_*) \tag{3.0.5}$$

at the stellar surface, where Ω_k is now understood as evaluated at $R = R$. Subtracting $G_*\Omega_*$ from E given by (3.0.1) we finally get the luminosity available to be radiated by the boundary layer as

$$L_{BL} = \frac{1}{2}\dot{M}R_*^2[\Omega_k^2 - \Omega_*^2 + 2\Omega_*^2 - 2\Omega_*\Omega_k] = \frac{GM\dot{M}}{2R_*}\left[1 - \frac{\Omega_k}{\Omega_*}\right]^2 \quad (3.0.6)$$

Our derivation of the inner boundary condition on the disc, as well as the R_* . Let us now justify this assumption. We discussion of L_{BL} above, we assumed b≪R. In the thin disc approximation the radial component of the Euler equation is given by :

$$v_R\frac{\partial v_R}{\partial R} - \frac{v_\varphi^2}{R} + \frac{1}{\rho}\frac{\partial P}{\partial R} + \frac{GM}{R^2} = 0 \quad (3.0.7)$$

In a thin Keplerian disc this equation is dominated by the centrifugal ($\frac{v_\varphi^2}{R}$) and gravity ($\frac{GM}{R^2}$) terms . However, the boundary layer is by definition that region of the disc in which $v_\varphi < v_k = \frac{GM}{R}$. Thus the gravity term must be balanced either by $v_R \partial vR/\partial R \sim v_R^2/b$or the pressure gradient $\rho^{-1}\partial P/\partial R \sim c^2/b$ where as usual we have taken $P = \rho c_s^2$ and set $\partial/\partial R \sim b^1$ in

the boundary layer. But we can infer that $c_s^2 > v_R^2$, since otherwise the inflow to the stellar surface would be supersonic and the news of the presence of the stellar surface at R = R_* could not be communicated outwards to the disc. Hence in the boundary layer

$$\frac{c_s^2}{b} \sim \frac{GM}{R_*^2} \quad (3.0.8)$$

The boundary-layer size b is now given by noting that just outside R_* + b the standard disc relation implies a scaleheight (assuming radiation pressure can be neglected)

$$H \sim c_s\left(\frac{R_*}{GM}\right)^{\frac{1}{2}} R_* \quad (3.0.9)$$

Assuming that H and cs$justinside$R$_*$ + b are similar to their values just outside, eq(3.0.7) and (3.0.8) can be combined to give

$$b \sim \frac{H^2}{R_*} \quad (3.0.10)$$

This justifies our assumption $b \ll R_*$,for

$b \sim \frac{H^2}{R_*} \ll H \ll R_*$ If the accretion rate, and therefore the density, in this region is high enough it will be optically thick and radiate roughly as a blackbody of area $\sim 2R_*Hx2$. We already know that the luminosity emitted by this area must be $\frac{1}{2}L_{acc} = GMM/2R_*$. Thus there is a characteristic boundary layer blackbody temperature T_{BL} given by

$$4\pi R_* H \sigma T_{BL}^4 \sim \frac{GMM}{2R_*}$$

where σ is the StefanBoltzmann constant. For the characteristic disc blackbody temperature T_* shows that

$$T_{BL} \sim \left(\frac{R_*}{H}\right)^{1/4} T_* \tag{3.0.11}$$

Using (3.0.9) to eliminate H and remembering that $c_s^2 \sim kT_*/\mu mH$ in this region, we get

$$T_{BL} \sim T_*(T_s T_*)^{1/8} \tag{3.0.12}$$

Chapter 4

LOCAL DISC VARIABLE

If the thin disc approximation hold, the task of computing the detailed disc structure is enormously simplified. Both the pressure and temperature gradients are essentially vertical, so that the vertical and radial structures are largely decoupled. From the work of Kippenhahn and weigert (1990) we have the equations of hydrostatic equilibrium and energy transport to solve with the radial disc structure only entering the calculations in the fixing of the local energy generation rate.

The temperature T_c must it self be given by an energy equation relating the energy flux in the vertical direction to the rate of generation of energy by viscous dissipation. The vertical energy transport mechanism may be either radiative or convective, depending on whether or not the temperature gradient required for radiative transport is smaller or greater than the gradient given by the adiabatic assumption. Then here we are assuming that the transport is radiative, this is indeed true in many important cases. Because of the thin disc approximation, the disc medium is essentially plane-parallel at each radius, so that the temperature gradient is effectively in the z-direction, as we pointed out in chapter two. Under these circumstances, the flux energy though a surface z=constant for gas pressure dominated region is

$$F(z) = -(\frac{4caT^3}{\chi\rho}\frac{\partial T}{\partial z}) \qquad (4.0.1)$$

where χ is the Rosseland mean opacity. It is implicitly assumed in writing that the disc is optically thick in the sense that

$$\tau = \rho H \chi = S\chi >> 1$$

For the radiation pressure dominated region the flux energy is given by

$$F = \frac{c}{\chi}\frac{Gm}{R^2}\frac{H}{R} \qquad if\ the\ disk\ is\ optically\ thick \tag{4.0.2}$$

So that the radiation field is locally very close to the black-body value, once the optical depth given above becomes less than(or equal) to unity, the expression for radiative cooling given above breaks down as the radiation can escape directly.

4.1 Central Density and Central Temperature With the Presence of Coulombic Pressure

from hydrostatic equilibrium in -z direction we have

$$\partial_z P = -\rho \frac{GM}{R^2}\frac{z}{R} \tag{4.1.1}$$

and from the energy conservation equation we derived that the volume rate of energy generated by viscous iteractions isq^+ is

$$\partial_z F = \frac{9}{4}\eta \frac{GM}{R^3} \tag{4.1.2}$$

We also have the equation of state (total pressure equation)

$$P = \frac{N_o k_B T \rho}{\mu} + \frac{1}{3}aT^4 - \frac{1}{3}e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}} \tag{4.1.3}$$

We can treat the hydrodynamical equitions integrated exactly over the thickness of the disc. As for the viscous stress $f_{R\varphi}$ we have

$$f_{R\varphi} = -\eta R\Omega' \sim -\rho\dot{v}\lambda R^2\Omega'$$

where $\Omega' = \partial_R \frac{v_\varphi}{R}$,hence the above equation becomes

$$f_{R\varphi} = -\eta R \partial_R \frac{v_\varphi}{R} = -\eta R \partial_R \Omega \tag{4.1.4}$$

If we assume the turbulent viscosity to be a linear function of the turbulent velocity and the Keplerian angular velocity the viscous stress is reduced to the familiar Shakura and Sunyaev α-model: $f_{R\varphi} = -\alpha P$

The parameter $\alpha \sim \frac{v_t}{c_s}$ where c_s and v_t are the sound and turbulent velocities respectively. and also

$$\partial_R \frac{v_\varphi}{R} = -\frac{3}{2}(\frac{GM}{R^3})^{\frac{1}{2}} \tag{4.1.5}$$

Thus the above expression for the viscous stress gives us the equation:

$$-\frac{3}{2}\eta(\frac{GM}{R^3})^{\frac{1}{2}} = -\alpha P$$

finally we get

$$\eta = \frac{2}{3}\alpha P(\frac{GM}{R^3})^{-\frac{1}{2}}$$

Once a formula for the viscosity is given, we can determine the local structure of the disc. We assume a polytropic equation of state for fixed R:

$$p(z) = K\rho(z)^{1+\frac{1}{N}} \tag{4.1.6}$$

Eq.(4.1.1) can then immediately be solved with the result:

$K(1+N)\rho^{\frac{1}{N}} = \frac{1}{2}\frac{GM}{R}[(\frac{H}{R})^2 - (\frac{z}{R})^2]$

For the values in the central plane making z= 0 we obtain

$$\frac{P_c}{\rho_c} = \frac{N}{2(1+N)}\frac{GM}{R}(\frac{H}{R})^2 \tag{4.1.7}$$

$$S = 2\rho_c H I(N) \tag{4.1.8}$$

$$W = 2P_cHI(N+1) \tag{4.1.9}$$

where $I(N) = \frac{(2^N N!)^2}{(2N+1)!}$

Since we are dealing with the spirit of the modified α- prescription of Shakura and Sunyaev(1973) that we are modeling the disk as geometrically thin the energy transport equation is radiation transport dominated.

For optically thick parts of the disc, we use the expression

$$\frac{dT}{dz} = -\frac{3\chi\rho F}{4acT^3} \tag{4.1.10}$$

The outer and middle region of the disk is dominated by gas pressure, where as the inner regin is dominated by radiation pressure,hence

for middle and outer region we have

$$P = \frac{N_o k_B T\rho}{\mu} - \frac{1}{3}e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}}$$

for inner region we have

$$P = \frac{1}{3}aT^4 - \frac{1}{3}e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}}$$

with $Q^- = 2F$ for the middle and outer region we have

$$Q^- = -2(\frac{4acT^3}{3\chi\rho}\frac{dT}{dz})surface \tag{4.1.11}$$

If the radiation pressure dominates, we obtain from the equation of state and hydrostaic equilibrium in the z-direction

$$Q^- = 2\frac{c}{\chi}\frac{Gm}{R^2}\frac{H}{R} \qquad if\ the\ disk\ is\ optically\ thick \tag{4.1.12}$$

For the region dominated by gas pressure

If we let $A = \frac{N_o k_B T\rho}{\mu}$ and B=$\frac{1}{3}e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}}$ then Eq.(4.1.3) and (4.1.7) give us:

$$K\rho(z)^{1+\frac{1}{N}} = A\rho T - B(\frac{\rho^3}{T})^{\frac{1}{2}}$$

$$K\rho(z)^{\frac{1}{N}} = AT - B(\frac{\rho}{T})^{\frac{1}{2}}$$

but $P_c = K\rho_c^{1+\frac{1}{N}}$ we get

$$\rho_c^{\frac{1}{N}} = \frac{1}{2K(1+N)}\frac{GM}{R}(\frac{H}{R})^2$$

but for z≠0 we have

$$\rho(z)^{\frac{1}{N}} = \frac{1}{2K(1+N)}\frac{GM}{R}\left((\frac{H}{R})^2 - (\frac{z}{R})^2\right)$$

thus

$$K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}} = AT^{\frac{3}{2}} - B\rho^{\frac{1}{2}} \quad or \quad AT^{\frac{3}{2}} - B\rho^{\frac{1}{2}} - K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}} = 0$$

let $x = T^{\frac{1}{2}}$ then the above equation becomes

$$x^3 - ax + b = 0, where \quad a = \frac{K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}}}{A} \quad and \quad b = \frac{B}{A}\rho^{\frac{1}{2}}$$

from $(A3.6)$ we have

$$T(z) = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho(z)^{\frac{1}{6}} = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}}\left[1 - (\frac{z}{H})^2\right]^{\frac{N}{6}}$$

$$T_c = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}}(4.1.13)$$

where

$$\rho_c = \left[\frac{1}{2K(1+N)}\frac{GM}{r}(\frac{H}{r})^2\right]^N$$

Since the general form of the opacity is given by

$$\chi = \chi_o\rho^n T^{-s} \tag{4.1.14}$$

where n = 1, s = 3.5 Kramers law which is particularly good representation of the opacity when it is dominated by free-free absorption. n = .75, s = 3.5 Schwarzschilds opacity which yields somewhat better results if bound-free opacity makes animportant contribution, and n = 0, s = 0 is used when electron scattering freeopacitydominated $\chi_o = 5x10^{23}m^2/kg$ and for electron-scattering opacity dominated χ_o=0.04m$^2/kg$

Thus

$\chi\rho = \chi_o\rho^{n+1}T^{-s}$

hence from $(A3.7)$ of appendix two

$$\frac{dT}{dz} = -\frac{N}{3}3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}}\frac{z}{H^2}\left[1-(\frac{z}{H})^2\right]^{\frac{N}{6}-1} \tag{4.1.15}$$

thus

$$Q^- = \frac{32acT^{3+s}}{3\chi_o}\rho^{-n-1}\frac{N}{3}3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}}\frac{z}{H^2}\left[1-(\frac{z}{H})^2\right]^{\frac{N}{6}-1}$$

$$= \frac{32Nac}{9\chi_o}\left[\rho_c{}^{(13-6n+6s)/6}\right]\left[3^{(20+5s)/6}\right]\left[\frac{B}{A}\right]^{(4+s)/3}\frac{z}{H^2}\left[1-(\frac{z}{H})^2\right]^{\frac{N(4+s)}{6}-n-2}$$

the limit z$\longrightarrow$H exists only if $\frac{N(4+s)}{6}-n-2$=0 or N=$\frac{6n+2}{4+s}$,

then we have

$$Q^- = \frac{32Nac}{9H\chi_o}\left[\rho_c{}^{(13-6n+6s)/6}\right]\left[3^{(20+5s)/6}\right]\left[\frac{B}{A}\right]^{(4+s)/3} \tag{4.1.16}$$

Comparing these two Eqs. 2.6.4 and 4.1.27 we get

$$H = \left(\frac{128\pi ac}{27\chi_o}\right)\frac{R^3}{Gm}\frac{1}{\dot{M}}\left(1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right)^{-1}\left[\rho_c{}^{(13-6n+6s)/6}\right]\left[3^{(20+5s)/6}\right]\left[\frac{B}{A}\right]^{(4+s)/3}$$

The vertical structure described in the previous paragraph depends on the parameters H, ρ_c which are all functions of the radius. The radial equations (chapter 2) together with the α-model allow us to determine these functions for a given (m, $\dot{M}$) From Eq.(2.5.3) and $W_{R\varphi} = \alpha W$ we have

$$W(r) = \frac{\dot{M}}{2\pi R^2}[l(R)-l(R_o)] \tag{4.1.17}$$

with l(R)=$(Gmr)^{\frac{1}{2}}$ .On the left hand side, we insert Eq.(4.1.8) to obtain

$$P_c(R) = \left(\frac{1}{I(N+1)}\right)\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)\left(\frac{Gm}{R}\right)^{\frac{1}{2}}\left(\frac{H}{R}\right)^{-1}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right] \tag{4.1.18}$$

This equation contains still the parameter H. Using the relation (4.1.7) between P_c and ρ_c gives

$$\rho_c(R) = \left(\frac{2(N+1)}{I(N+1)}\right)\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)\left(\frac{Gm}{R}\right)^{-\frac{1}{2}}\left(\frac{H}{R}\right)^{-3}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right] \quad (4.1.19)$$

$$H = \left(\frac{128N\pi ac}{27\chi_o}\right)\frac{R^3}{Gm\dot{M}}\left(1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right)^{-1}\left[{\rho_c}^{(13-6n+6s)/6}\right]\left[3^{(20+5s)/6}\right]\left[\frac{B}{A}\right]^{(4+s)/3}$$

4.1.1 Outer Region (free-free absorption dominated opacity)

We said that the radiative cooling converges as z⟶H only if N=$\frac{6n+2}{4+s}$ here n = 1 and s = 7/2 , from the condition for convergence of the limit of radiative cooling we set N =16/15

$$Q^- = \frac{12ac}{135H\chi_o}\left[{\rho_c}^{(14/3)}\right]\left[3^{(85/12)}\right]\left[\frac{B}{A}\right]^{(5/2)}$$

and

$$H = \left(\frac{512\pi ac}{405\chi_o}\right)\frac{R^3}{Gm\dot{M}}\left(1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right)^{-1}\left[{\rho_c}^{(14/3)}\right]\left[3^{(85/12)}\right]\left[\frac{B}{A}\right]^{(5/2)}$$

we have also from (A2.27)

$$T_c = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}{\rho_c}^{\frac{1}{6}} \quad (4.1.20)$$

with the central density given by

$$\rho_c^{(14/3)}(R) = \left(\frac{(62/15)}{I(31/15)}\right)^{(14/3)}\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)^{(14/3)}\left(\frac{Gm}{r}\right)^{-7/3}\left(\frac{H}{R}\right)^{-(14/)}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]^{(14/3)} \quad (4.1.21)$$

hence

$$H = \left[\frac{512\pi ac}{405\chi_o}\right]^{(1/15)}\left[\frac{(62/15)}{4\pi\alpha I(31/15)}\right]^{(14/45)}\dot{M}^{-(11/45)}r^{(2/5)}$$

$$*\left[Gm\right]^{-(2/9)}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]^{(11/45)}\left[3^{(17/36)}\right]\left[\frac{B}{A}\right]^{(1/6)} \quad (4.1.22)$$

and approximating I(31/15)∼ I(2)= 8/15

$$H = \left[\frac{512\pi ac}{405\chi_o}\right]^{(1/15)}\left[\frac{31}{16\pi\alpha}\right]^{(14/45)}\dot{M}^{-(11/45)}R^{(2/5)}$$

$$*\left[Gm\right]^{-(2/9)}\left[3^{(17/36)}\right]\left[\frac{B}{A}\right]^{(1/6)}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]^{(11/45)} \tag{4.1.23}$$

Up on inserting the constants (the used constants are given at the back of this material), we find the approximated value for the half thickness of the disc for this particular region:

$$H = 4.6\times 10^{-5}R^{0.4}[R^{0.5}-10^{3}]^{11/45} \tag{4.1.24}$$

4.1.2 Middle Region (Electron-scattering opacity dominated)

now n=0, and s= 0 we get N=$\frac{1}{2}$ Hence E.(4.1.11) can be written as

$$Q^{-} = \frac{16ac}{9H\chi_o}\left[\rho_c{}^{(13/6)}\right]\left[3^{(10/3)}\right]\left[\frac{B}{A}\right]^{(4/3)}$$

hence

$$T_c = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}} \tag{4.1.25}$$

the central density in this region is

$$\rho_c(R) = \left(\frac{3}{I(\frac{3}{2})}\right)\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)\left(\frac{Gm}{R}\right)^{-\frac{1}{2}}\left(\frac{H}{R}\right)^{-3}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right] \tag{4.1.26}$$

where

$$H = \left(\frac{64\pi\sigma}{27c\chi_o}\right)\frac{R^3}{Gm\dot{M}}\left(1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right)^{-1}\left[\rho_c{}^{(13/6)}\right]\left[3^{(10/3)}\right]\left[\frac{B}{A}\right]^{(4/3)} \tag{4.1.27}$$

equating Eq.(4.1.25 and 4.1.26) we have

$$H = \left[\frac{3}{I(3/2)}\right]^{(2/15)}\left[3^{(4/9)}\right]\left[\frac{16\pi ac}{27\chi_o}\right]\left[Gm^{(-5/18)}r^{(5/6)}\dot{M}^{(7/45)}\right]$$

$$*\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]^{(7/45)}\left[\frac{1}{4\pi\alpha}\right]^{(13/45)}\Bigg]\left[\frac{B}{A}\right]^{(8/45)} \tag{4.1.28}$$

For simplicity we approximate I(3/2)=$\frac{I(1)+I(2)}{2}$)= $(\frac{2}{3}+\frac{8}{15})/2$=$\frac{3}{5}$

Then we find the value for the half thickness as a function of the radial variable R as

$$H = \left[\frac{3}{6/5}\right]^{(2/15)}\left[3^{(4/9)}\right]\left[\frac{64\pi ac}{27\chi_o}\right]^{2/15}\left[Gm^{(-5/18)}\dot{M}^{(7/45)}\right]$$

$$*\left[\frac{1}{4\pi\alpha}\right]^{(13/45)}\Bigg]\left[\frac{B}{A}\right]^{(8/45)}R^{(5/6)}\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]^{(7/45)} \tag{4.1.29}$$

Up on inserting the constants (the used constants are given at the back of this material), we find the approximated value for the half thickness of the disc for this particular region:

$$H = 1.27 \times R^{1.2} [R^{0.5} - 10^3]^{7/45} \tag{4.1.30}$$

4.1.3 Inner Region (Electron-scattering opacity dominated)

Since the inner region of the disc is given doinated by radiation pressure, then the total pressure with the presence of coulombic pressure is given by

$$P = P_r + P_{coul} = \frac{1}{3} a T^4 - B(\frac{\rho^3}{T})^{1/2} \tag{4.1.31}$$

Equating the equation above with Eq.(4.1.6) of polyprotic equation we get

$$K\rho(z)^{1+\frac{1}{N}} = \frac{1}{3} a T^4 - B(\frac{\rho^3}{T})^{1/2}$$

solving for T(z) finally we get

$$T(z) = \left(\frac{3B\rho^{3/2}(z)}{a}\right)^{2/9}$$

hence the central temprature T_c is given by

$$T_c = \left(\frac{3B\rho_c^{3/2}}{a}\right)^{2/9} \tag{4.1.32}$$

where

$$\rho_c(R) = \left(\frac{2(N+1)}{I(N+1)}\right)\left(\frac{\dot{M}}{4\pi R^2 \alpha}\right)\left(\frac{Gm}{R}\right)^{-\frac{1}{2}}\left(\frac{H}{R}\right)^{-3}\left[1 - \left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]$$

to get the half thickness of the disk we have to equate the coolling rate given in Eq.(4.1.12) with Eq.(2.6.4)which is

$$Q^{-} = 2\frac{c}{\chi}\frac{Gm}{R^2}\frac{H}{R} = \frac{3}{4\pi}\dot{M}\frac{GM}{R^3}[1 - (\frac{R_A}{R})^{\frac{1}{2}}]$$

finally the half thickness of the disk for the inner region becomes

$$H = \frac{3\chi_o \dot{M}}{8\pi c}[1 - (\frac{R_A}{R})^{\frac{1}{2}}] \tag{4.1.33}$$

4.2 Central Density and Central Temperature with out Coulombic Pressure

We shall now determine the disc variables in the absence of coulombic pressure in the region where the pressure is dominated by the gas pressure. This will allow us to check if the coulombic pressure in the thin disc approximation has a real contribution to the stability of the accretion disc by comparing those equations derived in Section 4.1 and here below.

Now the equation of state for gas pressure dominated region becomes

$$P = \frac{N_o K_\beta \rho T}{\mu} = A\rho T$$

Using still the polytropic relation $P(z) = K\rho(z)^{\frac{1}{N}+1} = A\rho T$ Putting the expression for $\rho(z)^{\frac{1}{N}}$ given above we find

$$T(z) = \frac{1}{2A(1+N)}\frac{GM}{R}\left[(\frac{H}{R})^2 - (\frac{z}{R})^2\right]$$

And the central temperature is

$$T(z=0) = \frac{1}{2A(1+N)}\frac{GM}{R}\left[(\frac{H}{R})^2\right] \tag{4.2.1}$$

Thus we have

$$\frac{T}{T_c} = \left(1 - (\frac{z}{H})^2\right) \tag{4.2.2}$$

$$T^3\frac{dT}{dz} = -2T_c^4\left(1 - (\frac{z}{H})^2\right)^3 \tag{4.2.3}$$

$$\chi = \chi_o \rho^n T^{-s}$$

$$= \chi_o \rho^n T_c^{-s}\left(1 - (\frac{z}{H})^2\right)^{N(N+1)-s}$$

The equation

$$Q^- = -\frac{8\sigma T^3}{3\chi_o \rho^n T^{-s}}\frac{dT}{dz}$$

becomes

$$Q^- = \left(\frac{4aT^4}{3\chi\rho}\right)_c\left(\frac{z}{H}\right)\left(1 - (\frac{z}{H^2})^2\right)^{3+s-N(n+1)}$$

Using the fact that $(Q^-)surface = lim_{z \longrightarrow H} Q^-$, and the limit exists only if $3 +$ s-N (N + 1) = 0 or if $N = \frac{3+s}{n+1}$ becomes

$$Q^- = \left(\frac{8aT^4}{3\chi\rho} \right)_c \left(\frac{1}{H} \right)$$

Comparing this with Eq.(2.6.4) we get

$$H = \left(\frac{8aT^4}{3\chi\rho} \right)_c \frac{4\pi}{3} \frac{R^3}{Gm} \frac{1}{\dot{M}} \left(1 - \left(\frac{R_A}{R} \right)^{\frac{1}{2}} \right)^{-1}$$

or

$$H = \left(\frac{8aT^{4+s}}{3\chi\rho^{n+1}} \right)_c \frac{4\pi}{3} \frac{R^3}{Gm} \frac{1}{\dot{M}} \left(1 - \left(\frac{R_A}{R} \right)^{\frac{1}{2}} \right)^{-1} \tag{4.2.4}$$

Since T_c and ρ_c are still functions of H, using Eq.(4.1.16) for ρ_c that is

$$\rho_c^{n+1}(R) = \left(\frac{2(N+1)}{I(N+1)} \right)^{n+1} \left(\frac{\dot{M}}{4\pi R^2 \alpha} \right)^{n+1} \left(\frac{Gm}{R} \right)^{-\frac{n+1}{2}} \left(\frac{H}{R} \right)^{(n+1)} \left[1 - \left(\frac{R_A}{R} \right)^{\frac{1}{2}} \right]^{n+1} \tag{4.2.5}$$

4.2.1 Outer Region (free-free absorption dominated opacity)

for n = 1, s = 3.5 we have N $= \frac{13}{4}$ Eq.(4.2.5) becomes

$$\rho_c^2(R) = \left(\frac{17}{I(\frac{17}{4})} \right)^2 \left(\frac{\dot{M}}{4\pi R^2 \alpha} \right)^2 \left(\frac{Gm}{R} \right)^{-1} \left(\frac{H}{R} \right)^{(-6)} \left[1 - \left(\frac{R_A}{R} \right)^{\frac{1}{2}} \right]^2$$

I($\frac{17}{4}$)is approximated to I(4) $= \frac{2^7}{315}$ and also for $T_c^{\frac{15}{2}}$ imply

$$T_c^{\frac{15}{2}} = \left(\frac{2Gm}{17A} \right)^{\frac{15}{2}} H^{15} R^{-\frac{45}{2}} \tag{4.2.6}$$

Thus Eq.(4.2.4) gives

$$H = \left(\frac{7.5x10^5 \chi_o \dot{M}^3}{I(4)\sigma c \alpha^2 \pi^3} \right)^{\frac{1}{20}} \left(\frac{A^{\frac{3}{2}}}{Gm} \right)^{\frac{1}{4}} R^{1.92} \left(1 - \left(\frac{R_A}{R} \right)^{\frac{1}{2}} \right)^{\frac{1}{10}} \tag{4.2.7}$$

4.2.2 Middle Region (Electron-scattering opacity dominated)

now for n = 0, s = 0 we have N = 3, Eq.(4.2.5) becomes

$$\rho_c(R) = \left(\frac{8}{I(4)}\right)\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)\left(\frac{Gm}{R}\right)^{-\frac{1}{2}}\left(\frac{H}{R}\right)\left[1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right]$$

And also for T_c^4 imply

$$T_c^4 = \left(\frac{1}{8A}\right)^4\left(\frac{Gm}{R}\right)^4\left(\frac{H}{R}\right)^8$$

$$H = \left(\frac{1152\chi_o A^4\dot{M}^2}{I(4)\sigma c\pi^2(Gm)^{\frac{7}{2}}}\right)^{\frac{1}{10}} R^{\frac{21}{20}}\left(1-\left(\frac{R_A}{R}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}}$$

since I(4)=$\frac{2^7}{315}$ then we have

$$H = \left(\frac{2.84\times 10^3\chi_o A^4\dot{M}^2}{I(4)\sigma c\pi^2(Gm)^{\frac{7}{2}}}\right)^{\frac{1}{10}} r^{\frac{21}{20}}\left(1-\left(\frac{r_A}{r}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}} \qquad (4.2.8)$$

Up on inserting the constants (the used constants are given at the back of this material), we find the approximated value for the half thickness of the disc for this particular region:

$$H \simeq 1.41x10^{-3.05}R^{\frac{19}{20}}\left[R^{\frac{1}{2}} - 10^3\right]^{\frac{1}{5}} \quad (4.2.9)$$

The the other disc variables will be expressed as purely in terms of the radial variable r.

$$\rho_c = 2.624\times 10^{13.15}R^{\frac{-37}{20}}\left[R^{\frac{1}{2}} - 10^3\right]^{\frac{2}{5}}$$

$$T_c = 3.34\times 10^{12.1}R^{\frac{-11}{20}}\left[R^{\frac{1}{2}} - 10^3\right]^{\frac{2}{5}}$$

4.2.3 Inner Region (Electron-scattering opacity dominated)

We shall now determine the disc variables in the absence of coulombic pressure in the region where the pressure is dominated by the radiation pressure. This will allow us to check if the coulombic pressure in the thin disc approximation has a real contribution to the stability of the accretion disc by comparing those equations derived in Section 4.1 and here below.

Now the equation of state becomes

$$P = \frac{1}{3}aT^4 \tag{4.2.10}$$

Using still the polytropic relation $P(z) = K\rho(z)^{\frac{1}{N}+1} = \frac{1}{3}aT^4$. Putting the expression for $\rho(z)^{\frac{1}{N}+1}$ given above we find

$$T(z) = \frac{3}{a}K\rho(z)^{\frac{1}{4N}+\frac{1}{4}} \tag{4.2.11}$$

hence the central temprature $T_c = T(z = 0)$ in the inner of the disk as afunction of radius is given by [5]

$$T_c = (2.5x10^7K)\tilde{I}\tilde{\alpha}^{-14}\left(\frac{3R_g}{R_A}\right)^{3/8}\left(\frac{M}{M_\odot}\right)^{-1/4}\left(\frac{R}{R_A}\right)^{-3/8}$$

Where

$$\check{I} = \frac{3}{2}I(N+1)$$

$$I(N) = I(N) = \frac{(2^N N!)^2}{(2N+1)!}$$

Since N=3 for electron oppacity dominated region, then

$$\check{I} \approx 0.61 \tag{4.2.12}$$

The gravitational radiusR_g is defined by,

$$R_g = \frac{2GM}{c^2} \tag{4.2.13}$$

Using the fudicial model, we take a neutron star of M $= 1.4M_\odot$, and the dimensionless parameter $\alpha = 0.01$ to rewrite T_c as

$$T_c = 3.47{M_\odot}^{3/8}R^{-3/8} \tag{4.2.14}$$

where $M_\odot$ is Solar mass ($M_\odot = 1.989{\times}10^{30}$ K.g).

To get the central density using Eq.(4.2.10) then we have

$$K\rho_c^{1/3} = \frac{1}{3}aT_c^4$$

finally the central density as afunction of radias becomes

$$\rho_c = \frac{1}{3K}3.47a{M_\odot}^{3/8}R^{-9/8} \tag{4.2.15}$$

Chapter 5

STABILITY ANALYSIS OF THIN ACCRETION DISC

5.1 Time dependence stability

There are several reasons for extending this study to time-dependent discs. One is that we must check that the steady-state models are stable against small perturbations: if not, we have probably made some assumption in the course of constructing these discs which is not compatible with the further assumption of steadiness. Another reason is that the observable properties of the steady-state,are largely independent of viscosity; this is a fortunate occurrence for the purpose of showing that such discs do indeed exist, but it means that observations of steady discs are unlikely to give much information about the viscosity. The time dependence of disc flow is, on the other hand, controlled by the size of the viscosity .Hence observations of time-dependent disc behaviour offer one of the few sources of quantitative information about disc viscosity. In view of our present ignorance of the basic physical processes involved, such a semi-empirical approach to the problem seems the most reasonable.We begin by identifying the typical timescales on which the disc structure may vary.

1. Dynamical timescale, the timescale on which inhomogeneities on the disk surface rotate, or hydrostatic equilibrium in the vertical direction is established.

$$t_\varphi = \frac{R}{v_\varphi} \approx \Omega_k^{-1}$$

2. Viscous timescale, the timescale on which matter diffuses through the disk under the effect of viscous torques.

$$t_{visc} = \frac{R^2}{\nu} \approx \frac{R}{v_R}$$

3. Thermal timescale, the timescale for re-adjustment to thermal equilibrium.

$$t_{th} = \frac{Sc_s^2}{Q^+(R)} \approx \frac{H}{R}^2 \approx t_{visc}$$

5.2 Thermal Instabilities

In the last chapters, we have dealt in some detail with the theory of steady thin ac cretion discs. We investigate now the work of Shakura and Sunyaev, the stability of the steady discs described in chapter two. Only stable models have a chance to be physically relevant. It will turn out that possible instabilities depend strongly on the assumed vis cosity. There are several reasons for extending this study to time dependent discs. One is that we must check that the steady-state models are stable against small perturbations if not, we have probably made some assumption in the course of constructing these discs which is not compatible with the further assumption of steadiness. The time dependence of disc flow is controlled by the size of the viscosity then they offer one of the few sources of qualitative information. We begin by identifying the typical timescales on which the disc structure may vary by encountering the viscous timescale

$$t_{visc} = \frac{R^2}{\nu} \approx \frac{R}{v_R}$$

In equilibrium we must have $Q^+ = Q^-$. But if, when the central temperature T_c is increased by a small perturbation δT_c , Q^+ increases faster than Q^- , T_c will rise further because the cooling rate is inadequate. In other words, a steady state is impossible in a parameter regime where the instability would grow, despite the fact that formally an equilibrium solution can be found. If the energy balance is disturbed in the disc, any stability will grow on a timescale t_{th} given by the ratio of heat content per unit disc area to the dissipation rate per unit disc area. It is the timescale for readjustment to the thermal equilibrium, if, say, the dissipation rate is altered. Since the heat content per unit volume of a gas is

$\frac{\rho k_\beta T}{\mu m_p} \sim \rho {c_s}^2$

Suppose now that a small perturbation is made to a putative equilibrium solution and that this perturbation continues to grow rather than being damped. The difference we have found in these timescales means we can distinguish different types of instabilities. If for example the energy balance is disturbed in the disc, any instability will grow on a timescale t_{th} , which is much less than t_{visc}

.If $\nu = \frac{\eta}{\rho}$ denotes the kinematic viscosity

$Q^+ = \frac{9}{4}\nu S \Omega^2$

And for the α-model $\nu = \alpha c_s H = \frac{2\alpha}{3}\frac{W}{S}\frac{1}{\Omega}$

Since t_{visc} is the time scale for significant changes in the surface density S to occur, we can assume that S is fixed during the growth time t_{th} we refer to this as a thermal instability and such instabilities arise when the local (volume) cooling rate, q^- with in the disc can no longer cope with the volume heating rate, q^+ .

In general the customary way of writing the instability criterion is

$$\frac{dq^-}{dT_c} < \frac{dq^+}{dT_c}$$

Or

$$\frac{dlnq^-}{dlnT_c} < \frac{dlnq^+}{dlnT_c}$$

Now in our case since we assume H << R ,and also T_c and ρ_c have direct relation ship ,then Eq.(4.1.16) can be approximated to give volume cooling rate

$q^- = \frac{Q^-}{H} \sim \frac{\sigma T_c^4}{\chi \rho H^2}$

For free-free opacity it will be (omitting the constraints)

$q^- = \frac{Q^-}{H} \sim \rho_c^{20/6} \sim T_c^{20}$

The volume heating rate q^+ is also given by

$q^+ \sim \frac{Q^+}{H} \sim \frac{\nu}{H} \sim \alpha c_s \sim \alpha T_c^{\frac{1}{2}}$

where we have used the α-parametrization $\nu = \alpha c_s H$

Comparing these two expression, we see that the thermal instability will grow ifq^- in creases less rapidly with T_c than does q^+ Similarly from Eq.(4.1.25)for the electron-scattering opacity we have

$q^- = \frac{Q^-}{H} \sim \rho_c^{25/6} \sim T_c^{25}$

In both regions

$$\frac{dq^-}{dT_c} > \frac{dq^+}{dT_c}$$

Thus for gas pressure dominated region,the disc will be thermally stable.

Following the same procedure for the radiaton dominated region dividing Eq.(4.1.12)) by H,$q^- = 2\frac{c}{\chi}\frac{Gm}{R^3}$ and $T_c \sim R^{1/2}\left(1 - (R_A/R)\right)^{-2/3}$ we see that

$$\frac{dq^-}{dT_c} < \frac{dq^+}{dT_c}$$

hence the radiation dominated region is tharmally unstable.

5.3 Viscous Instabilities

Let us now consider changes in the disc structure which take place on the viscous timescale. This includes viscous instabilities and the evolution of discs in response to changes in external conditions, such as the mass transfer rate . As $t_{th} >> t_z \sim t_{visc}$ we assume that the disc adjusts so rapidly that it always maintains both thermal and hydrostatic equilibrium. Hence, some of equations describing steady discs apply also to time-dependent discs.

Defining the mass transfer rate $\dot{M} = \dot{M}(R,t)$ at each radius in which S and v_R are now allowed to be functions of t, we can rewrite the mass conservation equation with S and v_t be as functions of r, i.e,

$$\dot{M} = -2\pi R v_R S$$

but $v_R = \frac{\partial R}{\partial t}$,the above eqaution becomes

$$\frac{\partial \dot{M}}{dt} = 2\pi R \frac{\partial S}{\partial t}$$

Suppose that the the surface density in a steady disc is perturbed axsiymmetrically at each R, so that

$$S = S_o + \delta S$$

where So is the steady state distribution. In the Keplerian approximation for v_φ we get from equation of conservation of momentum equation considering the azimutal component and dividing both sides with $\partial_R R v_\varphi$ since v_φ is time independent we have

$$RSv_R = \frac{\partial_R R^2 W_{R\varphi}}{\partial_R R v_\varphi} \tag{5.3.1}$$

using $\nu = \frac{\eta}{\rho}$ from section 2.5

$$W_{R\varphi} = R\frac{d\Omega}{dR}\int \nu\rho dz = R\frac{d\Omega}{dR}\nu S \tag{5.3.2}$$

Hence we obtain from Eq (5.3.1)

$$RSv_R = -\frac{3}{R\Omega}\partial_R[\nu R^2 \Omega S] \tag{5.3.3}$$

Applying on this equation the operator $R_1 \partial R$ and using the mass conservation equation gives

$$\frac{\partial S}{\partial t} = \frac{3}{R}\partial_R[\nu R^2 \Omega S] \tag{5.3.4}$$

Denoting $\mu = \nu S$ there will be a corresponding perturbation μ, since $\nu = \nu(R, S)$ thus $\mu = \mu(R, S)$ so that

$$\frac{\delta\mu}{\delta S} = \frac{\partial \mu}{\partial S}$$

$$\frac{\partial}{\partial t}(\delta S) = \frac{3}{R}\partial_R[\nu R^2 \Omega(\delta S)]$$

eliminating δS we obtain the equation governing the growth of the perturbation

$$\frac{\partial}{\partial t}(\delta\mu) = \frac{3}{R}\partial_R[\nu R^2\Omega(\delta\mu)]$$

$\delta\mu$obeys a diffusion equation having the diffusion coefficient proportional to $\frac{\partial\mu}{\partial S}$ if $\frac{\partial\mu}{\partial S}$ is positive the perturbation decays on a viscous timescale. However, if it is negative, more material will be fed into those regions of the disc that are denser than their surroundings and material will be removed from those regions that are less dense, so that the disc will tend to breakup in to rings. This breakup of the disc on a timescale tvisc constitutes the viscous instabilities, more precisely stated; steady disc flow is only possible provided

$$\frac{\partial\mu}{\partial S} > 0$$

5.4 The Stability Parameter β

Since the task of approximating we did above (viscous instability) may lead to wrong conclusion. Thus the right way of determining the instability is using the stability parameter, which is derived basically from the two dominant (radiation and gas) pressures then we apply the general condition to our case. We consider only axially symmetric perturbations of wave length Λ satisfying H $<< \Lambda << R$ and which change little on the dynamical time scale Ω^{-1}($\Omega^{-1}$is also roughly the time it takes for a sound wave to cross the disc in the transverse direction). For a linear stability analysis, we to linearize the basic time dependent equations around the equilibrium solutions. For the type of perturbations,which we want to consider, we can still use in the vertical direction the hydrostatic equation (neglecting terms of order $\frac{H}{R}^2$.Furthermore,$v_\varphi$is still Keplerian, up to terms of order $\frac{H}{R}^2$, $\frac{H^2}{R\Lambda}$.

Using the continuity equation we define another form of the energy equation given in Chapter two

$$\partial_t(\epsilon\rho) + div[(\epsilon\rho + P)v] - \bigtriangledown_v P = q^+ - divq$$

thus

$$\partial_t(\epsilon\rho) + \frac{1}{R}\partial_R[Rv_R(\epsilon\rho + P)v] + \partial_z[v_z(\rho\epsilon + P)]) - v_R\partial_r P - v_z\partial_z P = q^+ - divq^-$$

Integrating over z gives

$$\partial_t \int(\epsilon\rho)dz + \frac{1}{R}\int \partial_R[Rv_R(\epsilon\rho + P)v]dz - \int v_R\partial_R P dz - \int v_z\partial_z P dz = Q^+ - Q^- \tag{5.4.1}$$

Where ϵ is the internal specific energy given by

$$\epsilon = c_v T + \frac{\alpha T^4}{\rho}$$

Let $\gamma = \frac{c_p}{c_v}$ and $P_g = \beta P$.Using $c_p = C_v + A$ we get

$$\epsilon = \frac{LP}{\rho}$$

In the thin disc approximation and using the hydrostatic equation for $\partial_z P$ such that

$$W = \int P dz$$

we obtain

$$\partial_t(LW) + \frac{1}{R}\partial_R[Rv_R(L+1)W] - v_R\partial_R W - \Omega^2 R\int v_z\partial_z P dz = Q^+ - Q^- \tag{5.4.2}$$

From now on, we choose for simplicity a constant density in the z-direction and assume that the perturbations in the z- direction preserve this property. Then the hydrostatic equation in the z-direction gives

$$P(z) = P_c[1 - (\frac{z}{H})^2], P_c = \frac{1}{4}S\Omega^2 H$$

and thus the average pressure is

$$P = \frac{1}{6}S\Omega^2 H$$

we also have

$$W = \frac{4}{3}P_c H = \frac{1}{3}S\Omega^2 H^2$$

and thus for α-model

$$\nu = \frac{2}{3}\alpha\Omega^2 H^2 \tag{5.4.3}$$

Furthermore, since $v_z = \frac{z}{H}\partial_t H$

$$\int \rho v_z z dz = \frac{1}{3}SH\partial_t H$$

Inserting these expressions into Eq.(5.4.2) gives

$$\frac{1}{3}\partial_t[LS\Omega^2 H^2]+\frac{1}{3}\frac{1}{R}\partial_R[Rv_R(L+1)S\Omega^2 H^2]-\frac{1}{3}v_R\partial_R(S\Omega^2 H^2))+\frac{1}{3}S\Omega^2 H\partial_t H = Q^+ - Q^-$$

In the second term on the left we use Eq.(5.3.2) to get the second basic equation:

$$\frac{1}{3}\partial_t[LS\Omega^2 H^2]-\frac{1}{R}\partial_R[Rv_R(L+1)S\Omega^2 H^2\partial_R(\nu\Omega R^2 S)]-\frac{1}{3}v_R\partial_R(S\Omega^2 H^2))+\frac{1}{3}S\Omega^2 H\partial_t H = Q^+ - Q^- \tag{5.4.4}$$

Eqs.(5.3.3) and (5.4.4) have to be linearized now about the equilibrium. Let us introduce the following notations for the changes of S and H from their equilibrium values

$$\frac{\delta S}{S} = u, \frac{\delta H}{H} = h, |u|, |h| << 1 \tag{5.4.5}$$

we set

$$\frac{\delta\nu}{\nu} = nu + mh + ... \tag{5.4.6}$$

if we use the α law Eq.(5.4.3),then

$$n = 0, m = 2 \tag{5.4.7}$$

Inserting Eq.(5.4.5) and Eq.(5.4.6) gives for the linearizion of Eq.(5.3.3)

$$S\partial_t u = 3\nu S\partial_R^2[(n+1)u + mh] \tag{5.4.8}$$

The linearizion of Eq.(5.3.4) is a bit more complicated.

Let

$$\frac{\delta Q^-}{Q^-} = lu + kh + ... \tag{5.4.9}$$

where the expansion coefficients depend on the opacity χ One must also include variations of L. For definiteness we choose $\gamma = \frac{5}{3}$.Then

$$L = \frac{3}{2}(1+\dot{\beta}), \dot{\beta} = 1 - \beta \tag{5.4.10}$$

First order changes of are obtained from the equation of state

$$P = \dot{\beta}P + A\frac{S}{2H}(\frac{3\dot{\beta}P}{\alpha})^{\frac{1}{4}}$$

and from the expression P= $\frac{1}{6}S\Omega^2 H$.Computing the variations of these two equations gives

$$\frac{\delta\dot{\beta}}{\dot{\beta}} = \frac{1-\dot{\beta}}{1+3\dot{\beta}}(7h-u) \tag{5.4.11}$$

Now the linearizion of Eq.(5.4.4) is straightforward. Using the equilibrium conditions (in particular $Q^+ = Q^-$), one finds for the -model, if only the determinant terms of order$(\frac{H}{\Lambda})^2$ are kept

$$3(1+3\dot{\beta}+4\dot{\beta}^2)\partial_t u+(8+51\dot{\beta}-3\dot{\beta}^2)\partial_t h-3(1+3\dot{\beta})\alpha\Omega[(n+1-l)u+(m-k)h] = \frac{2}{3}\alpha\Omega H^2(5+18\dot{\beta}+9\dot{\beta}^2)\partial \tag{5.4.12}$$

For an ionized gas at T¿ $10^4 K$ the major competitive opacity is electron scattering. Thus if the opacity is dominated by the electron scattering opacity ($\chi \sim \frac{\sigma T}{m_p} \sim 0.4 cm^2/g$),using $P_c = \frac{1}{4} S \Omega^2 H$

$$Q^- = e_s \frac{8 m_p c}{\sigma T} \frac{\dot{\beta} P_c}{S} = 2 e_s \frac{m_p c}{\sigma T} \dot{\beta} H \Omega^2 \tag{5.4.13}$$

Thus

$$\frac{\delta Q^-}{Q^-} = \frac{\delta \dot{\beta}}{\dot{\beta}} + h \tag{5.4.14}$$

Using Eq.(5.4.11) this gives

$$k = \frac{8 - 4\dot{\beta}}{1 + 3\dot{\beta}}, l = \frac{\dot{\beta} - 1}{1 + 3\dot{\beta}} \tag{5.4.15}$$

All perturbations are fully described by Eqs.(5.4.8) and (5.4.12) since all quantities of interest, in particular $\dot{M}$, can be expressed in terms of u and h. We consider harmonic perturbations

$$u(r,t) = u(R) e^{wt}, \qquad h(R,t) = h(R) e^{wt}$$

and write a single equation for the combination

$$\varPsi = (n+1) u + m h \tag{5.4.16}$$

This amplitude describes the viscous perturbations, as can be seen from $Q^+ = \frac{9}{4} \nu S \Omega^2$ Eqs.(4.4.5), and (4.4.6).

From (5.3.8) we get for the α-law

$$u = \frac{2}{3} \alpha \frac{\Omega}{w} H^2 \partial_R^2 \psi \tag{5.4.17}$$

Using Eqs.(5.4.16),(5.4.17) in Eq.(5.4.12) gives

$$w \frac{C w - 3(1 + 3\dot{\beta}) \alpha \Omega (m - k)}{D w - 3(1 + 3\dot{\beta}) \alpha \Omega [m l - k(n+1)]} \psi = u = \frac{2}{3} \alpha \frac{\Omega}{w} H^2 \partial_r^2 \psi \tag{5.4.18}$$

whre $C(\dot{\beta}) = 8 + 51\dot{\beta} - 3\dot{\beta}^2$

and $D(\dot{\beta}) = (N+1)C(\dot{\beta}) + M(2+9\dot{\beta} - 3\dot{\beta}^2)$ For solutions proportional to $sin(\frac{r}{\Lambda})$

we obtain the dissipation relation

$$C(\frac{w}{3\alpha\Omega})^2 + [2D(\frac{H}{3\Lambda})^2 - (1+3\dot{\beta})(m-k)]\frac{w}{3\alpha\Omega} - 2(\frac{H}{3\Lambda})^2(1+3\dot{\beta}[ml-(n+1)k] = 0 \tag{5.4.19}$$

for the special values of Eqs.(5.4.7) and (5.4.15) we obtain

$$\frac{w}{\alpha\Omega} = \frac{3}{C}[-(D(\frac{H}{3\Lambda})^2 + (1+3\dot{\beta})\pm[D(\frac{H}{3\Lambda})^2 + (3-5\dot{\beta})^2 - 4C(5-3\dot{\beta}(\frac{H}{3\Lambda})^2]^{\frac{1}{2}}] \tag{5.4.20}$$

with

$$C(\dot{\beta}) = 8 + 51\dot{\beta}^2 > 0.D(\dot{\beta}) = (N+1)C(\dot{\beta}) + M(2+9\dot{\beta}) - 3\dot{\beta}^2) > 0$$

Obviously, $Re\omega < 0$,if $3 - 5\dot{\beta} < \frac{3}{5}$

which is the case if the gas pressure dominates.

In our case for the region dominated by gas the total pressure is given by

$$P = A\rho T - B(\frac{\rho^3}{T})^{\frac{1}{2}}$$

thus if the disc is stable in the region the condition

$$\beta = \frac{P_g}{P_g + P_c} < 0.4$$

should be fulifilled

Using Eq.(4.1.15) the expression for the central pressure with H given by Eq.(4.1.17) in the case of total pressure and in the case of gas pressure only H is given by Eq.(4.2.4)

$$\frac{P_g}{P_g + P_{coul}} = \frac{\left(\frac{1}{I(N+1)_{total}}\right)\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)\left(\frac{Gm}{R}\right)^{-\frac{1}{2}}\left(\frac{H_{gas}}{R}\right)^{-1}\left[1-\left(\frac{R_o}{R}\right)^{\frac{1}{2}}\right]}{\left(\frac{1}{I(N+1)_{gas}}\right)\left(\frac{\dot{M}}{4\pi R^2\alpha}\right)\left(\frac{Gm}{R}\right)^{-\frac{1}{2}}\left(\frac{H_{total}}{R}\right)^{-1}\left[1-\left(\frac{R_o}{R}\right)^{\frac{1}{2}}\right]} \tag{5.4.21}$$

$$= \frac{[I(N+1)_{total} H_{total}]}{[I(N+1)_{gas} H_{gas}]} \tag{5.4.22}$$

$I(N+1)_{gas} = I(4) = \frac{2^7}{315}$ for outer region

$I(N+1)_{gas} \sim I(4) = \frac{2^7}{315}$ for middle region

$I(N+1)_{total} \sim \frac{I(1)+I(2)}{2} = 6/5$ for middle region

$I(N+1)_{total} \sim I(2) = 8/15$ for outer region

Inserting all the required constants, we determine the approximate values of the half thickness in the two regions.

Outer Region

Eq.(4.1.24) and (4.2.7) respectively gives

$$H_{total} = 4.6 \times 10^{-5} \times R^{0.4} \left[R^{0.5} - 10^3 \right]^{11/45}$$

$$H_{gas} \sim 3.75 \times 10^{-1.3} R^{1.87} \left[R^{0.5} - 10^3 \right]^{1/10}$$

Then

$$\beta(R) = 1.22 \times 10^{-3.7} \times R^{-1.47} \left[R^{0.5} - 10^3 \right]^{13/90}$$

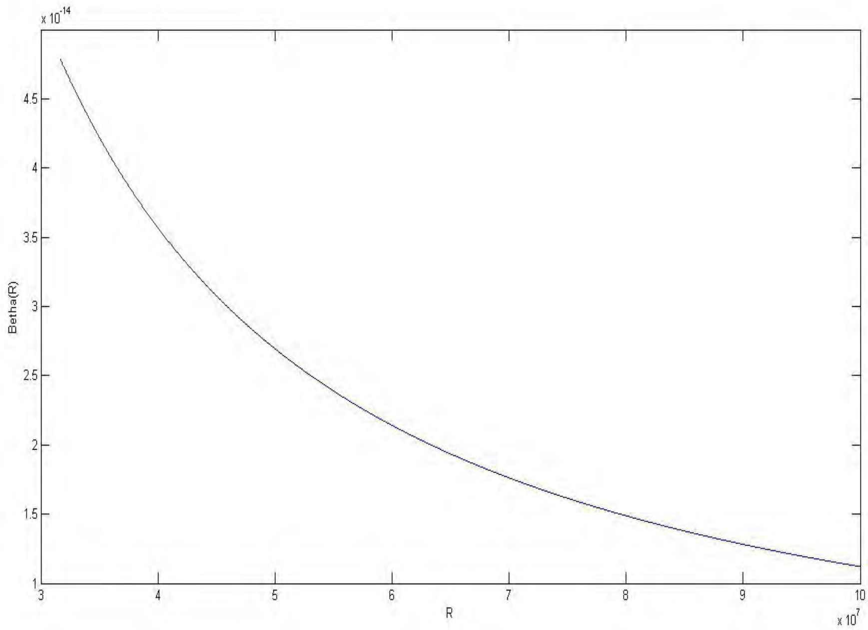

Figure 5.1: The stability parameter for outer region (R> $10^{7.5}$)

Middle Region

Eq.(4.1.29)

$$H_{total} = 1.27 \times 10^{-1} \times R^{1.2}[R^{0.5} - 10^3]^{7/45}$$

and from Eq.(4.2.9) we find

$$H_{gas} \simeq 1.41 \times 10^{-3.05} \times R^{\frac{19}{20}} \left[R^{\frac{1}{2}} - 10^3 \right]^{\frac{1}{5}}$$

Thus

$$\beta(R) = 0.9 \times R^{-0.25} [R^{0.5} - 10^3]^{-2/45}$$

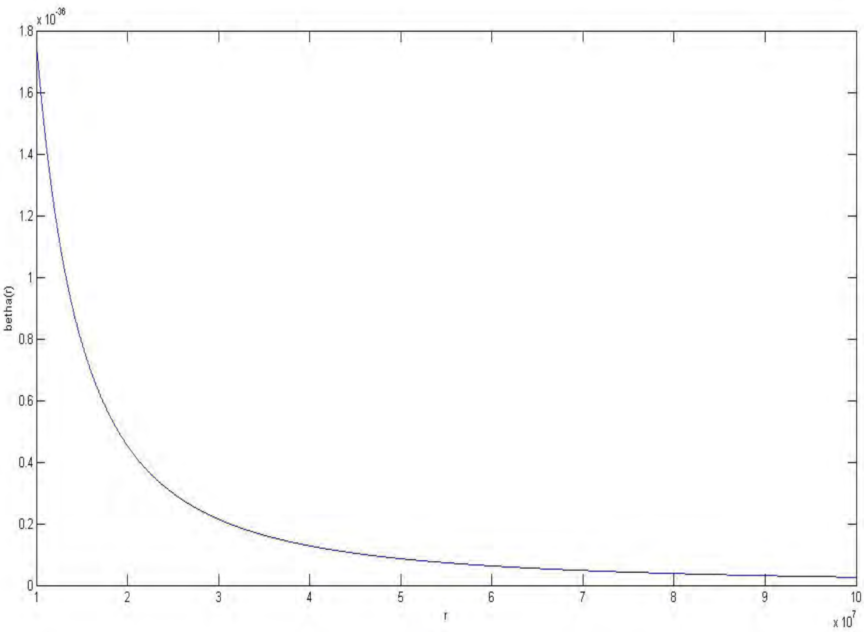

Figure 5.2: The stability parameter for middle region (R$\sim 10^7 \quad to 10^{7.5}$)

Inner Region

The inner region of the disc is dominated by radiation pressure,hence the condition

$$\beta = \frac{P_R}{P_R + P_c} < 0.4$$

should be fulifilled

deviding Eq.(4.2.9) by Eq.(4.1.18) and using the half thickness of the disk given in Eq.(4.1.31)

$$\beta(R) = \frac{aT_c^4 I(N+1)\chi_o \alpha\sqrt{Gm}}{2cR} \approx \frac{1.05\times10^6}{R} < 0.4 \qquad (5.4.23)$$

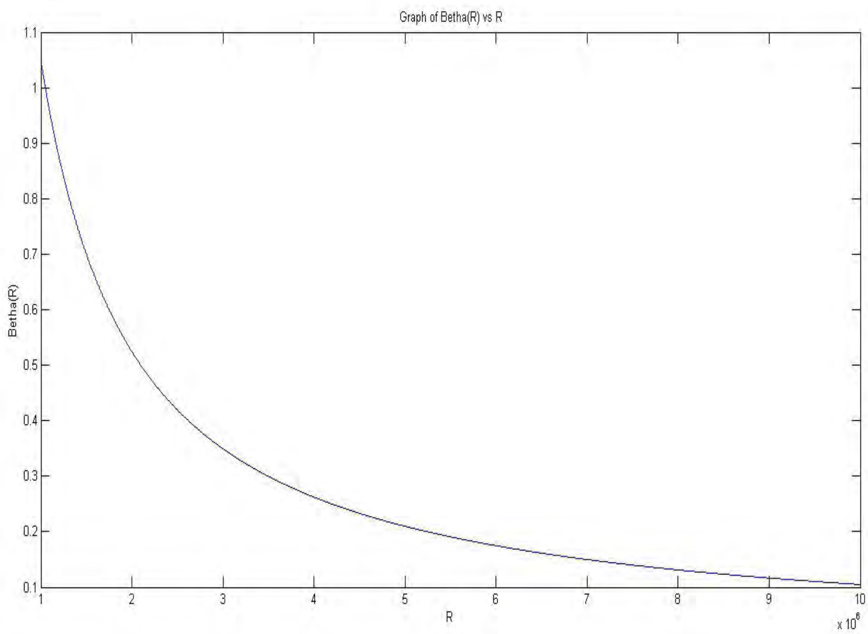

Figure 5.3: The stability parameter for inner region (R$\sim 10^{6.3}$ *to* 10^7)

Chapter 6

CONCLUSION

In this thesis ,the stability of gas pressure-dominated and radiation pressure-dominated accretion disc with the addition of coulombic pressure on equation of state(total pressure)has been studied.The model we considered is geometrically thin with electron- scattering opacity dominated (middle region and inner region) and free-free absorption opacity dominated (outer region).The gas pressure dominated region is thermally stable but the radiation presure dominated region is thermally ustable.The stability parameter graph shows that the disc is stable for the outer region for $R > 10^{7.5}$. Similarly for the case of middle region the disc is stable for R$\sim 10^{7} \quad to \quad 10^{7.5}$ .And also stable for the inner region for R$\sim 10^{6.3} \quad to \quad 10^{7}$ .But not stable for the region R$\sim 10^{6}$ to $10^{6.3}$. The instability shown in the stability parameter graph is mainly caused due to the viscosity.Where as the thermal instability for the inner region is caused by the faster increase of the disipation rate than that of the coolling rate. Thus with a set of such assumptions and with the result we obtain, it could be concluded that the middle and outer region of the disc is thermally and viscously stable.Where as the inner region is both viscously and thermally unstable.

Appendix One

Virial theorem

Starting from the hydrostatical equilibrium equation: Multiply by $4\pi r^3$ and integrate :

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = -\int_0^R 4\pi r^3 G \frac{\rho M}{r^2} dr \qquad (A1.1)$$

Left term :

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = -3 \int_0^M 4\pi r^3 \frac{P}{\rho} dM$$

using $dM(r) = 4r\pi^2 \rho dr$

(assuming P at the surface is completely negligible)

Right term :

$$-\int_0^R 4\pi r^3 G \frac{\rho M}{r^2} dr = -\int_0^R (4\pi r^2 \rho dr) \frac{GM}{r} = \int_0^M \frac{GM(r)dM}{r} = \Omega_0$$

= total gravitational energy

$$\Longrightarrow -3 \int_0^M 4\pi r^3 \frac{P}{\rho} dM = \Omega_0$$

Kinetic theory of gas relates the pressure to the average (translational) kinetic energy:

$$PV = P/\rho = 2/3 E_{kin} (nonrelativisticcase)$$

If K is the total kinetic energy of a star:

$$K = \int_0^M E_{kin} dM$$

$$\Longrightarrow K = \int_0^M \frac{3}{2}\frac{P}{\rho} dM$$

$$\Longrightarrow -2K = \Omega_0$$

or

$$K = -\frac{\Omega_0}{2} \qquad (A1.2)$$

The total kinetic energy of a star is equal to half its total gravitational energy if a star contracts (release of gravitational energy), half of its gravitational energy is transformed in kinetic energy, the other half is lost, i.e radiated away

Appendix Two

Non Relativistic Hydrodynamics of Viscous Fluids

We develop here briefly the principal equations of fluid dynamics from a phenomenological (continuum) viewpoint.

Let v(x, t) be the velocity field and ρ(x, t) the matter density. The material (or substantial) derivative of a function f is defined by

$$D_t f = \partial_t f \quad + L_v f \qquad A2.1$$

We decompose the velocity-gradient tensor (in Euclidean coordinates) as

$$v_{i,k} = \Theta_{i,k} + w_{i,k} \qquad A2.2$$

where

$$\Theta_{i,k} = \frac{1}{2}(v_{i,k} + v_{k,i}) \qquad A2.3$$

is the rate of deformation tensor and

$$w_{i,k} = \frac{1}{2}(v_{i,k} + v_{k,i}) \qquad A2.4$$

Denote by$\varphi_t(x) = \varphi(x, t)$ the trajectory of a fluid particle that is at position x at time t = 0.

The conservation of mass says that for nice domain D C R^3

$$\int_{\varphi t(D)} \rho\eta = \int D\rho\eta \qquad A2.5$$

where η is the volume of R^3 (as a three-dimensional Riemannian manifold) using the change-of-variable formula and the definition of the Lie Derivative,(A.5)for any D is equivalent to

$$\frac{\partial \rho}{dt}\eta + L_v \rho\eta = 0$$

But

$$L_v(\rho\eta) = (L_v\rho)\eta + \rho(L_v\eta) = (L_v\rho + \rho divv)\eta$$

and thus we have the continuity equation

$$D_t\rho + \rho divv = 0$$

or equivalently,

$$\partial_t\rho + \bigtriangledown.(\rho v) = 0 \qquad A2.6$$

As a corollary, we obtain the transport theorem :

$$\frac{d}{dt}\int_{\varphi t(D)} \rho f dV = \frac{d}{dt}\int_{\varphi t(D)} D_t\rho f dV \qquad A2.7$$

Indeed, as before we first get

$$\frac{d}{dt}\int_{\varphi t(D)} \rho f dV = \int_{\varphi t(D)} [\partial_t\rho f + L_v\rho f + \rho f divv]dV \qquad A2.8$$

But the bracket is, with (A2.6),equals to $D_t f$. Next we formulate the balance of momentum in integral form. we consider again a comoving fluid element in $\varphi_t(D)$. The forces which act on it are of two types. The first kind are external, or body forces, such as gravity or magnetic field which exert a force per unit volume on the continuum. The second kind of force consists of a surface force, which represents the action of the rest of the continuum through the surface of a fluid element. These stress forces are represented by the last term of the following momentum balance equation (G is the body force density):

$$\frac{d}{dt}\int_{\varphi t(D)} \rho v dV = \int_{\varphi t(D)} [\partial_t\rho G dV + \int_{\varphi t(D)} [\partial_t T(n)dS$$

where n is the out ward unit normal. one can show that the Cauchy traction vector T(n) depends linearly on n (Cauchy Lemma):

$$T_{in} = T_{ik} n_k \qquad A2.9$$

with the transport theorem (A2.7) and Gauss theorem we find from (A.8)

$$\rho D_t v_i = \rho G_i + T_{ik,k} \qquad A2.10$$

This holds in Cartesian coordinate, for which $L_v v_i = (\bigtriangledown_v v)_i$.Thus the invariant form of (A2.10) reads

$$\partial_t v + (\bigtriangledown_v v) = G + \frac{1}{\rho} divT \qquad (A2.11)$$

one can show easily that these equations of motion are compati- ble with angular momentum conservation for the fluid element in $\varphi_t(D)$ If and only if T_{ik} is symmetric.(we exclude strongly polar media.) We decompose T_{ik} into an isotropic pressure term and a viscous part tik which is due to velocity gradients

$$T_{ik} = -P\delta_{ik} + f_{ik} \qquad (A2.12)$$

since $_{ik}$ represents a rigid rotation, the viscous-tenser T_{ik} will be a linear function of φ_{ik} . If We consider only isotropic media, we have the following decomposition into irreducible parts :

$$f_{ik} = 2\eta\sigma_{ik} + \zeta\Theta\delta_{ik} \qquad (A2.13)$$

Where

$$\sigma_{ik} = \Theta_{ik} - \frac{2}{3}\delta_{ik} \qquad (A2.14)$$

and

$$\Theta = \Theta_{kk} = div \qquad (A2.15)$$

In the stress law (A2.13) η is the shear viscosity and ζ the bulk viscosity. Finally we consider various equivalent formulations of energy conservation. The rate of energy increase for a material volumeφ_t(D) is equal to the rate at which is energy is transfered to the volume via work and heat

$$\frac{d}{dt}\int_{\varphi t(D)}\rho(\epsilon+\frac{1}{2}v^2)dV = \int_{\varphi t(D)}\rho G.vdV+\int_{\varphi t(D)}\rho T(n).vdS-\int_{\varphi t(D)}\rho q.ndS \qquad A.16$$

Here,ϵ is the specific internal energy and q in the last term is the heat flux. Using again the transport theorem, the differential formulation of (A2.16) reads,with Gauss theorem,

$$\rho D_t(\frac{1}{2}v^2+\epsilon) = \rho G.v + div(T.v) - div(q) \qquad A2.16$$

For another form of this energy equation we write the second term on the right-hand side with the help of the equation of motion (A.10) as follows

$$div(T.v) = \partial_k(v_{ik}T_{ik}) = v_{i,k}T_{ik} + v_iT_{ik,k}$$

$$= \frac{1}{2}\rho D_t v^2 + v_{i,k}T_{ik} - \rho G_i v_i$$

Using this gives

$$\rho D_t\epsilon = T_{ik}\Theta_{ik} - divq \qquad A2.17$$

or,with the decomposition

$$\rho D_t\epsilon = -P - divq + q^+ \qquad A2.18$$

where the dissipation function Υ is given by

$$q^+ = Tr(t\Theta) = 2\eta r\sigma^2 + \zeta\Theta^2 \geq 0 \qquad A2.19$$

This represents the part of the viscous work going into the deformation of a fluid particle. With Eq.(A2.6) we can also write Eq.(A2.19) in the form

$$\rho\left[D_t\epsilon + PD_t\frac{1}{\rho}\right] = -divq + q^+ \qquad A2.20$$

We now introduce the Gibbs equation

$$Tds = d\epsilon + Pd(\frac{1}{\rho}) \qquad A2.21$$

Which allows us to write as

$$T\rho D_t S = divq + q^+ \qquad A2.22$$

We next derive still another alternative form of the energy equation. we start from (A2.17) and write this time

$$div(T.v) = -v_i P_i + \frac{P}{\rho} + D_t \frac{1}{\rho} + (f_{ik} v_k)_{,k}$$

After a few manipulations we obtain from (A2.17)

$$\rho D_t S(\epsilon + \frac{P}{\rho}\frac{1}{2}v^2) = \partial_t P + (f_{ik} v_i)_{,k} + \rho G.v - divq \qquad A2.23$$

If furthermore, G =gradφ and φ is stationary, then

$$\rho D_t S(\frac{1}{2}v^2 + h + \varphi) = \partial_t P + (f_{ik} v_{iK})_{,i} + \rho G.v - divq \qquad A2.24$$

Here ,h = $\epsilon + P/\rho$ is the specific enthalpy. Equation (A2.24) contains all the various equations which are called Bernoullis equation . For example,if the follow is steady and inviscid (t=o,q=0)then (A2.24) implies that $\frac{1}{2}v^2 + h + \varphi$ is constant on any given streamline. Finally we write down the constitutive relation between heat flux and temperature gradient,

Appendix Three

Central Density and Central Temperature With the Presence of Coulombic Pressure for middle and outer region of the disc

If we let $A = \frac{N_o k_B T \rho}{\mu}$ and B=$\frac{1}{3}e^3(\frac{\pi}{k_B T})^{\frac{1}{2}}(f\rho N_o)^{\frac{3}{2}}$

$$K\rho(z)^{1+\frac{1}{N}} = A\rho T - B(\frac{\rho^3}{T})^{\frac{1}{2}}$$

$$K\rho(z)^{\frac{1}{N}} = AT - B(\frac{\rho}{T})^{\frac{1}{2}} \qquad A3.1$$

but $P_c = K\rho_c^{1+\frac{1}{N}}$ we get

$$\rho_c^{\frac{1}{N}} = \frac{1}{2K(1+N)}\frac{GM}{r}(\frac{H}{r})^2 \qquad A3.2$$

but for z≠0 we have

$$\rho(z)^{\frac{1}{N}} = \frac{1}{2K(1+N)}\frac{GM}{r}\left((\frac{H}{r})^2 - (\frac{z}{r})^2\right) \qquad A3.4$$

thus

$$K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}} = AT^{\frac{3}{2}} - B\rho^{\frac{1}{2}} \quad or \quad AT^{\frac{3}{2}} - B\rho^{\frac{1}{2}} - K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}} = 0$$

let $x = T^{\frac{1}{2}}$ then the above equation becomes

$$x^3 - ax + b = 0, where \quad a = \frac{K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}}}{A} \quad and \quad b = \frac{B}{A}\rho^{\frac{1}{2}}$$

solving for x gives

$$x = \frac{-(2/3)^{\frac{1}{3}}\frac{K\rho_c^{\frac{1}{N}}T^{\frac{1}{2}}}{A}}{\sqrt{3\sqrt{27b^2-4a^3}-9b}^{\frac{1}{3}}} - \frac{\sqrt{3\sqrt{27b^2-4a^3}-9c}^{\frac{1}{3}}}{(18)^{\frac{1}{3}}}$$

$x = T^{\frac{1}{2}}$ solving for T we get $T = x^2$ and thus we have from(wolframalpha.com)

$$T = \left(\frac{-(2/3)^{\frac{1}{3}}a}{\sqrt{3\sqrt{27b^2-4a^3}-9b}^{\frac{1}{3}}} - \frac{\sqrt{3\sqrt{27b^2-4a^3}-9b}^{\frac{1}{3}}}{(18)^{\frac{1}{3}}}\right)^2$$

$$T(z) = \left[\frac{-(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A} - \sqrt{3\sqrt{27\left[(\frac{B}{A})\rho(z)^{\frac{1}{2}}\right]^2 - 4\left[(\frac{K\rho(z)^{\frac{1}{N}}}{A})\right]^3} - 9(\frac{B}{A})\rho(z)^{\frac{1}{2}}}^{\frac{2}{3}}}{\sqrt{972\sqrt{27\left[(\frac{B}{A})\rho(z)^{\frac{1}{2}}\right]^2 - 4\left[(\frac{K\rho(z)^{\frac{1}{N}}}{A})\right]^3} - 9(\frac{B}{A})\rho(z)^{\frac{1}{2}}}^{\frac{1}{3}}}\right]^2$$

$$\rho(z) = \left[\frac{1}{2K(1+N)}\frac{GM}{r}\left[(\frac{H}{r})^2 - (\frac{z}{r})^2\right]\right]^N$$

finally the central temprature becomes

$$T_c = T(z=0) = \left[\frac{-(12)^{\frac{1}{3}}\frac{K\rho_c^{\frac{1}{N}}}{A} - \sqrt{3\sqrt{27\left[(\frac{B}{A})\rho_c^{\frac{1}{2}}\right]^2 - 4\left[(\frac{K\rho_c^{\frac{1}{N}}}{A})\right]^3} - 9(\frac{B}{A})\rho^{\frac{1}{2}}}^{\frac{2}{3}}}{\sqrt{972\sqrt{27\left[(\frac{B}{A})\rho^{\frac{1}{2}}\right]^2 - 4\left[(\frac{K\rho_c^{\frac{1}{N}}}{A})\right]^3} - 9(\frac{B}{A})\rho_c^{\frac{1}{2}}}^{\frac{1}{3}}}\right]^2$$

let Y= $\sqrt{3\sqrt{27\left[(\frac{B}{A})\rho_c^{\frac{1}{2}}\right]^2 - 4\left[(\frac{K\rho_c^{\frac{1}{N}}}{A})\right]^3} - 9(\frac{B}{A})\rho_c^{\frac{1}{2}}}^{\frac{1}{3}}$ where

$$\rho_c^{\frac{1}{N}} = \rho_c^{\frac{1}{N}} = \frac{1}{2K(1+N)}\frac{GM}{r}(\frac{H}{r})^2$$

and

$$\rho_c = \left[\frac{1}{2K(1+N)}\frac{GM}{r}(\frac{H}{r})^2\right]^N$$

$$T_c = \left[(-(12)^{\frac{1}{3}}\frac{K\rho_c^{\frac{1}{N}}}{A} - Y)/Y\right]^2 \qquad (A3.5)$$

to get $\frac{dT}{dz}$we have to determine first $\frac{d\rho}{dz}$

$$\frac{d\rho}{dz} = -N\left[\frac{1}{2K(1+N)}\frac{GM}{r^3}\right]\left[(\frac{H}{r})^2 - (\frac{z}{r})^2\right]^{N-1} z$$

let

$$M = \sqrt{3\sqrt{27\left[(\frac{B}{A})\rho(z)^{\frac{1}{2}}\right]^2 - 4\left[(\frac{K\rho(z)^{\frac{1}{N}}}{A})\right]^3} - 9(\frac{B}{A})\rho(z)^{\frac{1}{2}}}^{\frac{1}{3}}$$

thus

$$\frac{dT}{dz} = 2\left[\frac{-(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A} - M^2}{(18M)^3}\right]\left[-18M(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A}\left(\frac{\rho(z)^{-1}}{N}\frac{d\rho}{dz} - \frac{dM}{dz}\right) - 18M^2\frac{dM}{dz}\right]$$

$$Q^- = -\frac{32\sigma c c T^3}{3\chi\rho}\left[\frac{-(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A} - M^2}{(18M)^3}\right]\left[-18M(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A}\left(\frac{\rho(z)^{-1}}{N}\frac{d\rho}{dz} - \frac{dM}{dz}\right) - 18M^2\frac{dM}{dz}\right.$$

Since the general form of the opacity is given by

$$\chi = \chi_o\rho^n T^{-s}$$

where

n = 1, s = 3.5 Kramers law which is particularly good representation of the opacity when it is dominated by free-free absorption.

n = .75, s = 3.5 Schwarzschilds opacity which yields somewhat better results if bound- free opacity mak$\beta esanimportantcontribution, andn = 0, s = 0isusedwhenelectronscatte$ $freeopacitydominated\chi_o = 5x10^{23}m/kg$ and for electron-scattering opac ity dominated we have $\chi_o = 0.04m^2/kg$.

Thus

$$\chi\rho = \chi_o\rho^{n+1}T^{-s} = \chi_o\rho^{n+1}\left(\left[\frac{-(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A} - M^2}{18M}\right]^2\right)^{-s}$$

$$Q^- = -\frac{32\sigma c T^{3+s}}{3\chi_o}\rho^{-n-1}\left[\frac{-(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A} - M^2}{(18M)^3}\right]\left[-18M(12)^{\frac{1}{3}}\frac{K\rho(Z)^{\frac{1}{N}}}{A}\left(\frac{\rho(z)^{-1}}{N}\frac{d\rho}{dz} - \frac{dM}{dz}\right) - 18\right.$$

The energy flux per unit area emitted at the disc surface is

$$(Q^-)_{surface} = limQ^-_{z\rightarrow H}$$

The above limit exists only if the limit of T(z) as z$\longrightarrow$ H exists.Since the limit of all of the expressions doesn't exist at the same time.Bcause when we multply $\frac{dT}{dz}$ with $T^{3+s}3\chi_o\rho^{-n-1}$,$\rho(z)$ have different exponents which are afunction of (N,s,n).When one of the limits exists the others doesn't exist.The limit of T(z) exists only if the epressin $\left[(\frac{B}{A})\rho(z)^{\frac{1}{2}}\right]^2$ exists.Eliminating the other expressions we get

$$T(z) = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho(z)^{\frac{1}{6}} = 3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}}\left[1-(\frac{z}{H})^2\right]^{\frac{N}{6}} \qquad (A3.6)$$

$$\frac{dT}{dz} = -\frac{N}{3}3^{\frac{5}{6}}\left[\frac{B}{A}\right]^{\frac{1}{3}}\rho_c{}^{\frac{1}{6}}\frac{z}{H^2}\left[1-(\frac{z}{H})^2\right]^{\frac{N}{6}-1} \qquad (A3.7)$$

Used Constants

Speed of light	$c = 3x10^8 m/s$
Constant of gravity	$G = 6.67X10^{-11} m^3/kgs$
Avogadros number	$N_o 6.02X10^{23} part/mol$
Gas constant	$R = 8.31J/Kmol$
Boltzmann	$k_\beta = R/N_o = 1.38X10^{-23} J/K$
Proton mass	$m_p = 1.67x10{-}27kg$
Electron mass	$m_e = 9.11x10^{-31} kg$
Elementary charge	$e = 1.602x10^{-19}$
Atomic weight for hydrogen	$A_z = 1.0071$
Mean molecular weight	$\mu = 0.6$
Stefan-Boltzmann constant	$\sigma = 5.67x10^{-5} erg/cm^2 SK^4$
Radiation constant	$a = 7.57x10^{16} Jm^3 K^4$
Mass of the sun	$M_m 1.99x10^{30} kg$
Mass of the Neutron star	$m = 1.4M_m$
Accretion rate	$\dot{M} = 10^{-19} M_m/year$
Alfven radius	$R_A = 10^6$
The free-viscous parameter	$\alpha = 0.1$
Electron-scattering Opacity	$\chi_o = 0.04m^2/kg$
Free-free absorption opacity	$\chi_o = 5x10^{23} m^2/kg$
Prameter f	f=0.9267

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Declaration

I here by declare that this thesis is my original work and has not been presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

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