A SIMPLE STRUCTURAL SYSTEM SYNTHESIS FROM MODAL ANALYSIS RESULTS

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ABSTRACT

In this paper presented a simple structural system synthesized from modal analysis results. A method for determining the eigenvalues of a synthesized system by using the frequency response function (FRF). This method first used matrix Auto-Regressive Moving Average (ARMA) model in the Laplace domain to describe each subsystem. Then a modal force method by ARMA model could be established. Only the FRF at the connecting joints is needed in the analysis to form a matrix named modal force matrix (FRF matrix). From this matrix, both synthesized system modes and substructure modes could be extracted simultaneously. Since the inverse operation is not required to form modal force matrix, the computation is reduced drastically. The eigensolution of the system in any frequency range could be determined independently.

ANSYS and MATLAB software’s used to extract the modal parameters of synthesized structural system and each subsystem separately from this matrix (modal force matrix). Finally compared their results.

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CHAPTER ONE

INTRODUCTION

1.1 Background

Synthesizing of structures, such as bridge, building and industrial structures, has emerged in recent years as a technological possibility. The subject of structural system identification lies at the heart of condition assessment and residual life assessment of existing structure. These problems constitute an important class of inverse problems in structural engineering. The difficulties lie in the development of algorithms that use measured data from the system to characterize it without significant prior knowledge of the system. The developments in this area have received significant boost in recent years due to advances in sensor technology and due to availability of cheap and faster computational power. Tools such as experimental modal analysis is being used in mechanicals, automotive and aerospace industries for quite some times now, with their application in civil infrastructure in the world, it has become important that engineers in the world are exposed to the principle of structural system synthesis[1,2].

For the engineers, eigenvalues/vectors are useful because they represent the shape that the structure will vibrate in free motion. These some shapes tend to dominate the motion during operation conditions. By understanding the modes of vibration, we can better identify the structure and if needed modification of that [3].

It is important to synthesize the systems from dynamic characteristics to up date the existing structural design and also to extract uncertain parameters [4].

The understanding of the physical nature of vibration phenomena has always been important for researchers and engineers, even more as today as structures are becoming lighter and more flexible due to increased demands for efficiency, speed, safety and comfort. When any structure vibrates, it causes major problems and operating limitations ranging from discomfort (including noise), malfunction, reduced performance and fatigue. Two approaches may be considered to resolve the vibration problem: first, prevention, through proper design, and second, cure, by modification of structure or a vibration control design. In any case, a thorough understanding of vibration of the structure is essential. Hence, accurate mathematical models are required to describe the vibration characteristics of the structure. For simple structures, such as beams and plates, good analytical predictions using closed form solutions can be easily found in various reference books and tables [20, 21]. However, for more complex structures, more powerful tools are needed. Today, analytical tools and experimental tools are used to model the dynamic behavior of the structures.

The most widely used analytical tool is the Finite Element Method (FEM) [20, 22], while the experimental counterparts are largely based on modal testing and analysis [24]. Due to different built-in limitations, assumptions and choices, each approach has its own advantages and disadvantages.

1.2 Objectives

The general objective is:

- To synthesize a simple structural system from the modal analysis results.

The specific objectives are;

- Carry out a literature survey on the subjects of Theory of Vibration, modal testing, modal analysis and FE updating.
- To highlight the role of structural system identification in the broad areas of structural health monitoring/design.
- To device methods of synthesizing the structure from the modal data.
- By using modal force matrix (FRF matrix) determine the synthesized and substructures modes from this matrix.
- To highlight the modal testing technique.
- To introduce the modified modal force technique.

- Develop the ANSYS and MATLAB model of the synthesized structure and analyze it.

1.3 Methodology

This thesis organized from six chapters. Chapter one gives detailed information about the background of the thesis, literature review and brief notes on finite element method (FEM) and modal testing method, finally the objectives and the scope of the thesis. Chapter two discussed theory of vibration in detailed. Chapter three presented the mathematical model of the synthesized simple structural system. In Chapter four create a solid modeling of the synthesized system model by using AutoCAD 2007 and imported the model to ANYS software and analyze it, and perform MATLAB modal analysis of it. In chapter five compare ANSYS and MATLAB results and discussed the results. Finally chapter six provides the conclusion obtained from this work and suggest future work in this area.

1.4 Literature Review

Many modal synthesis techniques have been presented in the past by using subsystem FRF. This technique may be divided in to two categories. The first type of methods use Ritz based transformation matrix to reduce the order of the substructure. Goldman [8] introduces the free-free interface method, in which only rigid body modes and free-free normal modes are employed in reducing the order of the substructure. Poor accuracy limits the use of this method. Inertia relief modes are introduced by MacNeal [9] to offset the static contribution of the truncated the higher order modes of the component. Rubin [10] extends MacNeal’s approach in approximating the residual inertia and flexibility by proposing higher order corrections to offset the contribution of higher order truncated modes. The accuracy of these methods depends on how to construct the transformation matrix and how they are coupled together. Besides, the parameter estimation algorithm used to extract the needed information from substructure FRF is also important.

The second type of methods finds the dynamical behavior of the synthesized system directly through the use of the subsystem FRF. The dynamic stiffness approach was initially developed by Klosteman [11]. The primary concept of this technique is superposition. Thus, the phrase building block approach was used by this technique. In 1984 another technique which was named as SMURF (Structural Modifications Using Response Functions) was published and implemented by Crowley and Klosterman [11, 12]. It is primarily a trouble shooting technique.

Recently, a direct substructure modal synthesis method named modal Force Technique was developed by Yee and Tsuel [13, 14]. It directly uses the substructure FRF at the connecting joints to form a matrix named Modal Force Matrix. This matrix contains both synthesized system and substructure information. A Modified Modal Force Technique is developed in this paper. It first uses ARMA model to describe each substructure. Then a modal force method by ARMA model can be established. Only single parameter estimation algorithm is needed to extract both synthesized system modes and substructure modes simultaneously.

1.5 Finite Element Method (FEM)

The main assumption in Finite Element Method (FEM) is that a continuous structure can be discretized by describing it as an assembly of finite (discrete) elements, each with a number of boundary points which are commonly referred to as nodes. For structural dynamic analysis, element mass, stiffness and damping matrices are generated first and then assembled into global system matrices. Dynamic analysis of the produced model gives the modal properties; the natural frequencies (Eigenvalues) and corresponding mode shapes (Eigenvectors). The modal solution can subsequently be used to calculate forced vibration response levels for the structure under study. Element system matrices have been developed for many simple structures, such as beams, plates, shells and bricks. Most general-purpose FE programs have a wide range of choice of element types, and the user must select the appropriate elements for the structure under investigation and its particular application. Further theoretical background and practical implementation of the FE method are given in various text books, [18, 20, 22]

The FE method is extensively used in industry as it can produce a good representation of a true structure. However, for complicated structures, due to limitations in the method and application, FE model can lead to errors.

The sources of errors in Finite Element models are:

1. Inaccuracy in estimation of the physical properties of the structure.
2. Poor quality of mesh generation and selection of individual shape functions.
3. Poor approximation of boundary conditions.
4. Poor estimation or omission of damping properties of the system.
5. Computational errors which are mainly due to rounding off. (ill conditioned matrices)
6. Linear Modeling of highly nonlinear structures. (Geometric and material non-linearities)

The result of a finite element analysis is mainly dependent on the judgment and experience of the operator and the software package used.

1.6 Modal Testing Method

The experimental approach to modeling the dynamic behavior of structures (modal testing) relies mostly on extracting the vibration characteristics of a structure from measurements.

The procedure consists of three steps:

1. Acquiring the modal data.
2. Analyzing the measured modal data.
3. Constructing the dynamic model behavior by using extracted modal parameters from the analyzed data.

Vibration measurements are taken directly from a physical structure, without any assumptions about the structure and that is the reason why modal testing models are considered to be more reliable than Finite Element models. However, due to a number of limitations and errors, the model created from the measured data may not represent the actual behavior of the structure as closely as desired.

The theoretical background of modal testing and practical aspects of vibration measurement techniques are discussed [24].

In general, limitations and errors of modal testing are:

- Random errors due to noise.
- Loose attachment of transducers to the structure.
- Non-linear behavior of the structure or attached mechanical devices
- Poor modal analysis of experimental data (user experience).
- Limited number of measured degrees of freedom.
- Not all modes being excited due to excitation at a node.
- Difficulty in measuring rotational degrees of freedom.

1.7 Applications of Modal Test Models

It is generally believed that more confidence can be placed in experimental data since measurements are taken on the true structure. Therefore, the mathematical models, which have been created as a result of modal testing, can be used in various ways to avoid or to cure the problems encountered in structural dynamics. In this Section, the applications of modal testing methods for improving the structural dynamics will be considered.

1.7.1 Updating of the Analytical Models (Calibration)

One of the applications of the result of a modal test is the updating of an analytical model (usually a model derived using finite element method). Model updating can be defined as adjustment of an existing analytical model which represents the structure under study, using experimental data [19, 23, 25, 28]. Therefore, updated FE model more accurately reflects the dynamic behavior of that structure.

Model updating can be divided into three steps:

1. Comparison of FE model and modal testing results.
2. Modifying FE model in order to correlate FEM and modal testing results.
3. Analyzing updated FE model and return to step 1 until convergence is achieved.

Comparison can be defined as the initial step to assess the quality of the analytical model. If the difference between analytical and experimental data is within some preset tolerances, the analytical model can be judged to be adequate and no updating is necessary.

Most difficulties are encountered in the second step. The difficulties in locating the errors in a theoretical model are mostly due to measurement process and can be summarized as:

1. Insufficient experimental modes;
2. Insufficient experimental coordinates;
3. Size and mesh incompatibility of the experimental and FE models;
4. Experimental random and systematic errors.
5. Absence of damping in the FE model

In spite of extensive research over the last two decades, model updating is still far from mature and no reliable and general applicable procedures have been formulated so far.

1.7.2 Structural Dynamic Modification

Structural dynamic modification can be defined as the study of changes (in natural frequencies and mode shapes) of measured dynamic properties of the test structure due to modified mass or stiffness or damping of the structure. In principle, the modification process is a form of optimization of the structure to bring its modal properties of the structures to some desired condition [30, 32].

For example a bridge with certain vibration problems should be modified. Sometimes, the effect of an addition to a structure must be known. For instance, addition of a storey to an existing building or addition of a missile to a fighter jet plane. Modeling the existing structure might be too costly and unnecessary. The existing structure can be modal tested and effect of the new addition (i.e. additional floor or missile) can be evaluated using structural dynamic modification.

This method saves large amounts of redesign time as it reduces the cycle time in the test, analysis, redesign, shop drawings, install redesign, and retest cycle. In practice, the measured properties of existing structure at the boundary level are only translational degrees of freedom (DOF.). However for structural dynamic modification, rotational DOFs are also needed for proper assembly. Deficiency in rotational degrees of freedom has problems in dealing with real-world problems such as addition of a plate, beam, and rotor or other structural elements with bending resistance.

Moreover, there is a need for sufficient modal vector information to carry out real world structural modification. This demands that more data be taken, but the number of data points is limited by the time available from the highly-trained modal staff. Generally, most modal tests are limited to 50-60 x-y-z degrees of freedom. This is usually too few for the structural modification to be implemented without excessive retesting. Sometimes it is desired to construct a mathematical model of a complete structural assembly formed by the assembly of several individual substructures. There are a number of methods for assembling such a model which are extensions of modification methods and called "structural assembly methods". The essential difference is that here the modifications are themselves dynamic systems, rather than simple mass or stiffness elements. It is possible to combine subsystem or component models derived from different sources or analyses for example from a mixture of analytical and experimental studies. Again the same problems encountered with modification methods are encountered.

There are other quantitative applications of the modal test models, which demand a high degree of both accuracy and completeness (enough points and enough DOFs on the test structure) of the test data.

These applications are:

- Response predictions for the test structure if it is subjected to other excitations.
- Force determination, from measured responses.
- Damage detection. (Operating curves).

1.8 Sources of Lack of Precision in Modal Testing

Laboratory experiments and practical measurements serve several purposes, some of which do not demand high accuracy. Some experiments are exploratory in the sense of looking for the existence and direction of some effect before trying to establish its magnitude; others are chiefly instructional, to demonstrate theoretical principles. Some industrial measurements are needed only to control and repeat process in accordance with previously established values. Errors in such cases may be harmless. However, engineering applications of some experiments demand test data of high quality. In these cases, large errors may arise if test data with poor quality are used. In recent years there has been a strong demand for modal testing with high quality suitable for advanced applications such as structural modification and model updating.

In Section 1.4, it is explained in general terms how the mathematical models which have been created as a result of modal tests, can be used in various ways to apply in vibration-related problems encountered in theory and practice, and how these applications are hindered from a lack of precision in modal testing. In this Section, the problem of the sources of the lack of precision in modal testing is studied more systematically. In the following paragraphs attention is drawn to the reasons why the experimental modal test data can depart from the true values it purports to measure.

The sources of a lack of precision in modal testing procedure can be categorized in three groups:

(i) Experimental data acquisition errors

(ii) Signal processing errors and

(iii) Modal analysis errors, each of them has been categorized itself in below [18].

Figure 1-1 Three stages of the modal testing

(i) Experimental data acquisition errors:

a) Quality:

1) Mechanical errors:

- Mass loading effect of transducers
- Shaker-structure interaction
- Supporting of the structure

2) Measurement noise

3) Nonlinearity

b) Quantity:

- Measuring enough points on the structure
- Measuring enough Degrees of Freedom (i.e. Rotational DOFs)

(ii) Signal processing errors:

- Leakage
- Aliasing
- Effect of window functions
- Effect of Discrete Fourier Transform
- Effect of averaging

(iii) Modal analysis errors:

- Circle-Fit Modal Analysis
- Line-Fit Modal Analysis
- Global Modal Analysis

CHAPTER TWO

THEORY OF VIBRATION

2.1 Introduction

One who wants to study modal testing and modal analysis should briefly have background about dynamics but specially ”Theory of vibration”. The following Section briefly explains theory of vibration, starting with single degree of freedom systems and continues to multi degree of freedom systems [17, 21, 24].

2.2 Single Degree of Freedom Systems

In order to understand modal analysis, single degree of freedom systems must be understood. In particular, the complete familiarity with single degree of freedom systems (as presented and evaluated in the time, frequency (Fourier), and Laplace domains) serves as the basis for many of the models that are used in modal parameter estimation. The single degree of freedom approach is obviously trivial for the modal analysis case. The importance of this approach results from the fact that the multiple degree of freedom case can be viewed as a linear superposition of single degrees of freedom systems. Single degree of freedom system is described in Figure 2-1.

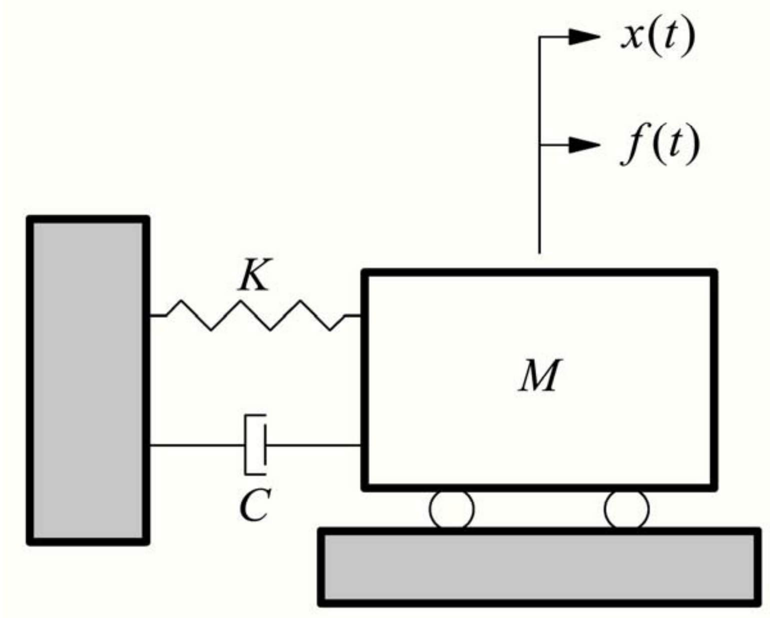

Figure 2-1 Single Degree of Freedom System

Free body diagram of Figure 2-1 is shown in Figure 2-2.

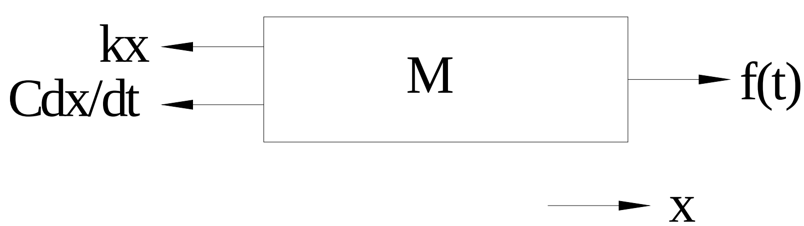

Figure 2-2 Free Body Diagram of SDOF system

The general mathematical representation of a single degree of freedom system is obtained from Newton's Law of Motion and expressed in Equation 2-1,where total forces acting on the system is equaled to mass (M) times acceleration.

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=f(t) \qquad \textbf{Equation 2.1}$$

Where;

- M = Mass of the system
- C = Damping of the system
- K = Stiffness of the system
- f(t) = General force function

By setting f(t)= 0, the homogeneous form of Equation 2-2 can be solved.

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=0 \qquad \textbf{Equation 2-2}$$

From differential equation theory, the solution can be assumed to be of the form $x(t)=Xe^{st}$, where s is a complex valued number to be determined. Taking appropriate derivatives and substituting into Equation 2-2 yields:

$$(Ms^2+Cs+K)Xe^{st}=0 \qquad \textbf{Equation 2-3}$$

Thus, for a non-trivial solution:

$$Ms^2+Cs+K=0 \qquad \textbf{Equation 2-4}$$

Where:

s = Complex-valued frequency variable (Laplace variable)

Equation 2-4 is the characteristic equation of the system, whose roots λ_1 and λ_2 are:

$$\lambda_{1,2}=-\frac{C}{2M}\pm\left\{\left(\frac{C}{2M}\right)^2-\left(\frac{K}{M}\right)\right\}^{\frac{1}{2}} \qquad \textbf{Equation 2-5}$$

Thus the general solution of Equation 2-2 is:

$$x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$ **Equation 2-6**

A and B are constants determined from the initial conditions imposed on the system at time t = 0.

For most real structures, unless active damping systems are present, the damping ratio is rarely greater than ten percent. For this reason, all further discussion is restricted to under damped systems (ζ < 1). With reference to Equation 2-6, this means that the two roots $\lambda_{1,2}$, are always complex conjugates. Also, the two coefficients (A and B) are complex conjugates of one another (A and A*). For an under damped system, the roots of the characteristic equation can be written as:

$$\lambda_1 = \sigma_1 + j\omega_1 \qquad \lambda_2 = \sigma_1 - j\omega_1$$ **Equation 2-7**

Where;

σ_1 = Damping Factor

ω_1 = Damped Natural Frequency

The roots of characteristic Equation 2-4 can also be written as:

$$\lambda_1, \lambda_1^* = -\zeta_1 \Omega_1 \pm j\Omega_1 \sqrt{1-\zeta_1^2}$$ **Equation 2-8**

Where;

Ω_1 = Undamped Natural Frequency

ζ_1 = Percent damping with respect to critical damping.

The damping factor σ_1, is defined as the real part of a root of the characteristic equation. This parameter has the same units as the imaginary part of the root of the characteristic equation, radians per second. The damping factor describes the exponential decay or growth of the oscillation. In real-world structures energy of the system is dissipated through damping mechanism. Therefore there is always exponential decay in oscillation. Exponential growth of the oscillation is theoretical and is not valid for real world structures.

Critical damping (C_c), is defined as being the damping which reduces the radical in the solution of the characteristic equation to zero. This form of damping representation is a physical approach and therefore involves the appropriate units for equivalent viscous damping.

$$\left(\frac{C_c}{2M}\right)^2 - \left(\frac{K}{M}\right) = 0 \rightarrow \frac{C_c}{2M} = \sqrt{\frac{K}{M}}$$
$$C_c = 2M\sqrt{\frac{K}{M}} = 2M\Omega_1$$

Equation 2-9

The damping ratio ζ , is the ratio of the actual system damping to the critical system damping. The damping ratio is dimensionless since the units are normalized.

$$\zeta_1 = \frac{C}{C_c} = \frac{-\sigma_1}{\Omega_1}$$

Equation 2-10

2.2.1 Time Domain: Impulse Response Function

The impulse response function of a single degree of freedom system can be determined from Equation 2-6 assuming that the initial conditions are zero and that the system excitation, f (t), is a unit impulse. The response of the system, x(t), to such a unit impulse is known as the impulse response function, h(t), of the system.
Therefore:

$$h(t) = Ae^{\lambda_1 t} + A^* e^{\lambda_1^* t}$$

Equation 2-11

$$h(t) = e^{\sigma_1 t}\left[Ae^{(+j\omega_1 t)} + A^* e^{(j\omega_1 t)}\right]$$

Equation 2-12

Thus, the coefficients (A and A^*) control the amplitude of the impulse response, the real part of the pole is the decay rate and the imaginary part of the pole is the frequency of oscillation.

2.2.2 Frequency Domain: Frequency Response Function

An equivalent equation of motion for Equation 2-1 is determined for the Fourier or frequency (ω) domain. This representation has the advantage of converting a differential equation to an algebraic equation. This is accomplished by taking the Fourier transform [26] of Equation 2-1.

Letting the forcing function of Equation 2-1 be in the form of

$$F(t) = F(\omega)e^{j\omega t} \qquad \textbf{Equation 2-13}$$

.

Therefore the solution of Equation 2-1 will be in the form of;

$$X(t) = X(\omega)e^{j\omega t} \qquad \textbf{Equation 2-14}$$

Thus;

$$\dot{X}(t) = j\omega X(\omega)e^{j\omega t} \qquad \textbf{Equation 2-15}$$

$$\ddot{X}(t) = -\omega^2 X(\omega)e^{j\omega t} \qquad \textbf{Equation 2-16}$$

Substituting Equation 2-14, Equation 2-15 and Equation 2-16 into Equation 2-1 yields;

$$\left[-M\omega^2 X(\omega)e^{j\omega t} + j\omega X(\omega)e^{j\omega t} + kX(\omega)e^{j\omega t}\right] = F(\omega)e^{j\omega t} \qquad \textbf{Equation 2-17}$$

After rearranging the common terms in Equation 2-17;

$$\left[-Mw^2 \;\; + jCw + K\right]X(w)e^{jwt} \; = F(w)e^{jwt}$$ **Equation 2-18**

Simplifying Equation 2-18 yields;

$$\left[-M\omega^2 \;\; + jC\omega + K\right]X(\omega) = F(\omega)$$ **Equation 2-19**

Restating Equation 2-19 yields:
Where;

$$B(\omega) = -M\omega^2 \, + \, jC\omega + K$$ **Equation 2-20**

Equation 2-20 states that the system response X(ω) is directly related to the system forcing function F(ω) through the quantity B(ω) , the impedance function. If the system forcing function F(ω) and its response X (ω) are known, B(ω) can be calculated. That is:

$$B(\omega) = \frac{F(\omega)}{X(\omega)}$$ **Equation 2-21**

More frequently, the system response, X (ω) due to a known input F(ω) is of interest.

$$X(\omega) = \frac{F(\omega)}{B(\omega)}$$ **Equation 2-22**

Equation 2-22 becomes:

$$X(\omega) \, = \, H(\omega)F(\omega)$$ **Equation 2-23**

Where:

$$H(\omega) = \frac{1}{-M\omega^2 + jC\omega + K}$$

The quantity H(ω) is known as the Frequency Response Function of the system. The frequency response function relates the Fourier transform of the system input to the Fourier transform of the system response. From Equation 2-23, the frequency response function can be defined as:

$$H(\omega) = \frac{X(\omega)}{F(\omega)}$$ **Equation 2-24**

Going back to Equation 2-19, the frequency response function can be written as,

$$H(\omega) = \frac{1}{-M\omega^2 + jC\omega + K} = \frac{\frac{1}{M}}{-\omega^2 + j\left(\frac{C}{M}\right)\omega + \frac{K}{M}}$$ **Equation 2-25**

The denominator of Equation 2-25 is known as the "characteristic equation" of the system and is in the same form as Equation 2-4. The characteristic values of this complex equation are in general complex even though the equation is a function of a real valued independent variable "ω". The characteristic values of this equation are known as the complex roots of the characteristic equation or the complex poles of the system. These characteristic values are also called the "modal frequencies".

The frequency response function H(ω) can also be written as a function of the complex poles as follows:

$$H(\omega) = \frac{\frac{1}{M}}{(j\omega - \lambda_1)(j\omega - \lambda_1^*)} = \frac{A}{(j\omega - \lambda_1)} + \frac{A^*}{(j\omega - \lambda_1^*)}$$ **Equation 2-26**

Where;

λ_1 = Complex Pole

$\lambda_1 = \sigma_1 + j\omega_1$

$\lambda_1^* = \sigma_1 - j\omega_1$

$\sigma_1 = -\zeta_1 \Omega_1$

$\omega_1 = \Omega_1 \sqrt{1-\zeta_1^2}$

Since the frequency response function is a complex valued function of a real valued independent variable (ω), the frequency response function, real-imaginary, magnitude-phase, and log magnitude-phase graphs.

2.2.3 Laplace Domain: Transfer Function

Just as in Section 2.2.2 for the frequency domain, the equivalent information can be represented in the Laplace domain by way of the Laplace transform. The only significant difference between the two domains is that the Fourier transform is defined from negative infinity to positive infinity while the Laplace transform is defined from zero to positive infinity with initial conditions. The Laplace representation, also, has the advantage of converting a differential equation to an algebraic equation. Theory behind Laplace transform is shown in almost every classical text concerning vibrations [26]. The development using Laplace transforms begins by taking the Laplace transform of Equation 2-1. Thus, Equation 2-1 becomes:

$$\left[Ms^2 + Cs + K\right]X(s) = F(s) + \left[Ms + C\right]X(0) + M\,\dot{X}(0)$$ **Equation 2-27**

X(0) and $\dot{X}(0)$ are the initial displacements and velocities at time t = 0, respectively If the initial conditions are taken as zero, Equation 2-27 becomes:

$$\left[Ms^2 + Cs + K\right]X(s) = F(s)$$ **Equation 2-28**

Then Equation 2-28 becomes:

$$F(s) = B(s)X(s) \qquad \textbf{Equation 2-29}$$

Where:

$B(s) = Ms^2 + Cs + K$

Therefore, using the same logic as in the frequency domain case, the transfer function can be defined in the same way that the frequency response function was defined previously.

$$X(s) = H(s)F(s) \qquad \textbf{Equation 2-30}$$

Where:

$$H(s) = \frac{1}{Ms^2 + Cs + K}\cdot$$

The quantity H(s) is defined as the ”transfer function” of the system. In other words, a transfer function relates the Laplace transform of the system input to the Laplace transform of the system response. From Equation 2-30, the transfer function can be defined as:

$$H(s) = \frac{X(s)}{F(s)} \qquad \textbf{Equation 2-31}$$

Going back to Equation (2.25), the transfer function can be written:

$$H(s) = \frac{1}{Ms^2 + Cs + K} = \frac{\frac{1}{M}}{s^2 + \left(\frac{C}{M}\right)s + \frac{K}{M}} \qquad \textbf{Equation 2-32}$$

Note that Equation 2-32 is valid under the assumption that the initial conditions are zero. The denominator term is once again referred to as the characteristic equation of the system. As noted in the previous two cases, the roots of the characteristic equation are given in Equation 2-5.The transfer function, H(s), can now be rewritten, just as in the frequency response function case, as:

$$H(s) = \frac{\frac{1}{M}}{(s-\lambda_1)(s-\lambda_1^*)} = \frac{A}{(s-\lambda_1)} + \frac{A^*}{(s-\lambda_1^*)} \qquad \textbf{Equation 2-33}$$

Since the transfer function is a complex valued function of a complex independent variable (s), the transfer function is represented. Remember that the variable s in Equation 2-27 is a complex variable, that is, it has a real part and an imaginary part. Therefore, it can be viewed as a function of two variables which represent a surface.

The definition of undamped natural frequency, damped natural frequency, damping factor, percent of critical damping, and residue are all relative to the information represented. The projection of this information onto the plane of zero amplitude yields the information as shown in Figure2-3.

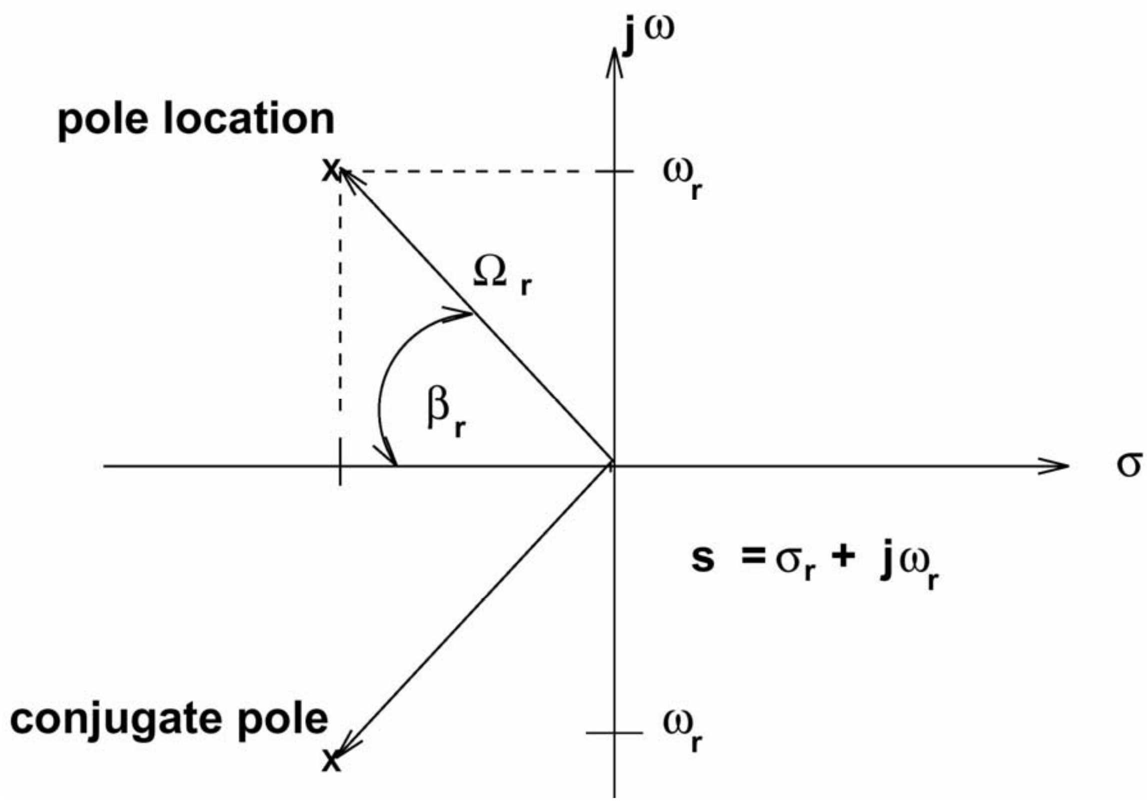

Figure 2-3 Transfer Function - Laplace Plane Projection

Where;

σ_r = Damping coefficient

ω_r = Damped natural frequency

Ω_r = Undamped natural frequency

$\zeta_r = \cos\beta_r$ = Damping factor (percent of critical damping)

The concept of residues is now defined in terms of the partial fraction expansion of the transfer function equation. Equation 2-33 can be expressed in terms of partial fractions as follows:

$$H(s) = \frac{\frac{1}{M}}{(s-\lambda_1)(s-\lambda_1^*)} = \frac{A}{(s-\lambda_1)} + \frac{A^*}{(s-\lambda_1^*)}$$ **Equation 2-34**

The residues of the transfer function are defined as being the constants A and A*. The terminology and development of residues comes from the evaluation of analytic functions in complex analysis. The residues of the transfer function are directly related to the amplitude of the impulse response function. In general, the residue A can be a complex quantity.

It can be noted that the Laplace transform formulation is simply the general case of the Fourier transform development if the initial conditions are zero. The Frequency Response Function is the part of the transfer function evaluated along the $s = j\omega$ axis.

From an experimental point of view, the transfer function is not estimated from measured input-output data (modal testing). Instead, the Frequency Response Function is actually estimated via the discrete Fourier transform.

2.3 Multiple Degree of Freedom Systems

The real applications of modal analysis concepts begin when a continuous, non-homogeneous structure is described as a lumped mass, multiple degree-of-freedom systems. At this point, the modal frequencies, the modal damping, and the modal vectors, or relative patterns of motion, can be found via an estimate of the mass, damping, and stiffness matrices or via the measurement of the associated frequency response functions. The two-degree of freedom system, shown in Figure 2-4, is the most basic example of a multiple degree of freedom system. This example is useful for discussing modal analysis concepts since a theoretical solution can be formulated in terms of the mass, stiffness and damping matrices or in terms of the frequency response functions.

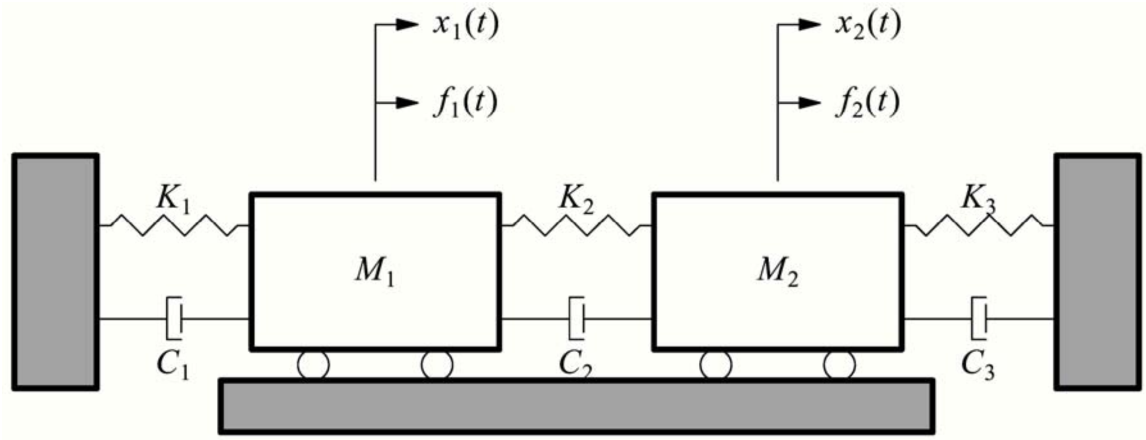

Figure2-4Multi-Degree of Freedom System

The equations of motion for the system in Figure 2-4, using matrix notation, are as follows:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (C_1 + C_2) & -C_2 \\ -C_2 & (C_2 + C_3) \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} (K_1 + K_2) & -K_2 \\ -K_2 & (K_2 + K_3) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Equation 2-35

The process of solving Equation 2-35 when the mass, damping, and stiffness matrices are known is shown in almost every classical text concerning vibrations and [20, 22, 26].

The development of the frequency response function solution for the multiple degree of freedom case is similar to the single degree-of-freedom case, which relates the mass, damping, and stiffness matrices to a transfer function model involving multiple degrees of freedom. Just as in the analytical case, where the ultimate solution can be described in terms of one degree of freedom systems, the frequency response functions between any input and response degree of freedom can be represented as a linear superposition of the single degree of freedom models derived previously.

As a result of the linear superposition concept, the equations for the impulse response function, the frequency response function, and the transfer function for the multiple degree of freedom system are defined as follows:

Impulse Response Function (Time domain):

$$[h(t)] = \sum_{r=1}^{N} [A_r] e^{\lambda_r t} + [A_r^*] e^{\lambda_r^* t} = \sum_{r=1}^{2N} [A_r] e^{\lambda_r t}$$ **Equation 2-36**

Frequency Response Function (Frequency Domain):

$$[H(\omega)] = \sum_{r=1}^{N} \frac{[A_r]}{j\omega - \lambda_r} + \frac{[A_r^*]}{j\omega - \lambda_r} = \sum_{r=1}^{2N} \frac{[A_r]}{j\omega - \lambda_r}$$ **Equation 2-37**

Transfer Function (Laplace Domain):

$$[H(s)] = \sum_{r=1}^{N} \frac{[A_r]}{s - \lambda_r} + \frac{[A_r^*]}{s - \lambda_r} = \sum_{r=1}^{2N} \frac{[A_r]}{s - \lambda_r}$$ **Equation 2-38**

Where:

t= Time variable

ω = Frequency variable

s = Laplace variable

p = Measured degree of freedom (output)

q = Measured degree of freedom (input)

r = Modal vector number

A_{pqr} = Residue

$A_{pqr} = Q_r \psi_{pr} \psi_{qr}$

Q_r = Modal scaling factor

ψ_{pr} = Modal coefficient

λ_r = System pole

N= Number of positive modal frequencies

It is important to note that the residue, A_{pqr} , in Equation 2-36 through Equation 2-38 is the product of the modal deformations at the input q and response p degrees of freedom and a modal scaling factor for mode r. Therefore, the product of these three terms is unique but each of the three terms by themselves is not unique. This is consistent with the arbitrary normalization of the modal vectors. Modal scaling, Q_r, refers to the relationship between the normalized modal vectors and the absolute scaling of the mass matrix (analytical case) and/or the absolute scaling of the residue information (experimental case). Modal scaling is normally presented as modal mass or modal A.

The driving point residue, A_{qqr}, is particularly important in deriving the modal scaling.

$$A_{pqr} = Q_r \psi_{qr} \psi_{qr} = Q_r \psi_{qr}^2 \qquad \textbf{Equation 2-39}$$

For undamped and proportionally damped systems, the r-th modal mass of a multi degree of freedom system can be defined as:

$$M_r = \frac{1}{j2Q_r \omega_r} = \frac{\psi_{pr} \psi_{qr}}{j2A_{pqr} \omega_r} \qquad \textbf{Equation 2-40}$$

Where:

M_r = Modal mass

Q_r = Modal scaling constant

ω_r = Damped natural frequency

If the largest scaled modal coefficient is equal to unity, Equation 2-40 will also compute a quantity of modal mass that has physical significance. The physical significance is that the quantity of modal mass computed under these conditions will be a number between zero and the total mass of the system. Therefore, under this scaling condition, the modal mass can be viewed as the amount of mass that is participating in each mode of vibration.

Obviously, for a translational rigid body mode of vibration, the modal mass should be equal to the total mass of the system.

The modal mass defined in Equation 2-40 is developed in terms of displacement over force units. If measurements and therefore residues, are developed in terms of any other units (velocity over force or acceleration over force), Equation 2-40 will have to be altered accordingly.

Once the modal mass is known, the modal damping and modal stiffness can be obtained through the following single degree of freedom equations:

Modal Damping

$$C_r = 2\sigma_r M_r \qquad \textbf{Equation 2-41}$$

Modal Stiffness

$$K_r = \left(\sigma_r^2 + \omega_r^2\right) M_r = \Omega_r^2 M_r \qquad \textbf{Equation 2-42}$$

There are several references in the literature in which modal mass, stiffness, and damping matrices are calculated from estimated modal parameters [27, 29, 31].

For systems with non-proportional damping, modal mass cannot be used for modal scaling. For non-proportional case, and increasingly for undamped and proportionally damped cases as well, the modal A scaling factor is used as the basis for the relationship between the scaled modal vectors and the residues determined from the measured frequency response functions. This relationship is as follows:

Modal A

$$M_{A_r} = \frac{\psi_{pr}\psi_{qr}}{A_{pqr}} = \frac{1}{Q_r} \qquad \textbf{Equation 2-43}$$

This definition of modal A is also developed in terms of displacement over force units. Once modal A is known, modal B can be obtained through the following single degree of freedom equation:

Modal B

$$M_{B_r} = -\lambda_r M_{A_R} \qquad \textbf{Equation 2-44}$$

For undamped and proportionally damped systems, the relationship between modal mass and modal A scaling factors can be stated.

$$M_{A_r} = \pm 2 M_r \omega_r \qquad \textbf{Equation 2-45}$$

In general, modal vectors are considered to be dimensionless since they represent relative patterns of motion. Therefore, the modal mass or modal A scaling terms carry the units of the respective measurement. For example, the development of the frequency response is based upon displacement over force units. The residue must therefore, have units of length over force-seconds. Since the modal A scaling coefficient is inversely related to the residue, modal A will have units of force-seconds over length. This unit combination is the same as mass over seconds. Likewise, since modal mass is related to modal A, for proportionally damped systems, through a direct relationship involving the damped natural frequency, the units on modal mass are mass units as expected.

2.4 Damping Mechanisms

In order to evaluate multiple degree of freedom systems that are present in the real world, the effect of damping on the complex frequencies and modal vectors must be considered.

Many physical mechanisms are needed to describe all of the possible forms of damping that may be present in a particular structure or system.

Some of the classical types are:
1) Structural Damping;
2) Viscous Damping; and
3) Coulomb Damping.

It is generally difficult to ascertain which type of damping is present in any particular structure. Indeed most structures exhibit damping characteristics that result from a combination of all the above. Rather than consider the many different physical mechanisms, the probable location of each mechanism, and the particular mathematical representation of the mechanism of damping that is needed to describe the dissipative energy of the system, a model will be used that is only concerned with the resultant mathematical form. This model will represent a hypothetical form of damping, which is proportional to the system mass or stiffness matrix. Therefore:

$$[C] = \alpha[M] + \beta[K]$$ **Equation 2-46**

Under this assumption, proportional damping is the case where the equivalent damping matrix is equal to a linear combination of the mass and stiffness matrices. For this mathematical form of damping, the coordinate transformation that diagonalizes the system mass and stiffness matrices also diagonalizes the system damping matrix. Non-proportional damping is the case where this linear combination does not exist. Therefore when a system with proportional damping exists, that system of coupled equations of motion can be transformed to a system of equations that represent an uncoupled system of single degree-of-freedom systems that are easily solved. With respect to modal parameters, a system with proportional damping has real-valued modal vectors (real or normal modes) while a system with non-proportional damping has complex-valued modal vectors (complex modes).

CHAPTER THREE

MATHEMATICAL MODEL OF SYNTHESIZED SIMPLE STRUCTURAL SYSTEM

3.1 Introduction

A method for determining the eigenvalues of a synthesized system from modal analysis results, by using the frequency response function (FRF). This method first uses matrix Auto-Regressive Moving Average (ARMA) model in the Laplace domain to describe each subsystem. Then a modal force method by ARMA model can be established. Only the FRF at the connecting joints is needed in the analysis to form a matrix named modal force matrix (FRF matrix). From this matrix, both synthesized system modes and substructure modes can be extracted simultaneously. Since the inverse operation is not required to form modal force matrix, the computation is reduced drastically. The eigensolution of the system in any frequency range can be determined independently. Numerical study suggests that good results can be achieved by this method.

3.2 Modal Analysis for Substructures

A complex structure can be divided into substructures. Each substructure is connected to the other substructures through joints. The equation of motion for undamped substructure can be expressed as

$$M \ddot{x}(t) + KX(t) = f(t) \qquad \textbf{Equation 3-1}$$

The natural frequencies and mode shapes of the substructure can be obtained by solving equation 3-1 with f(t) equals zero. For harmonic excitation at a frequency of ω, the response function for the substructure can be expressed as

$$X = \Phi(\lambda - \omega^2 I)\Phi^T f$$ **Equation 3-2**

$$X = H(\omega) f$$ **Equation 3-3**

Where

$f(t) = fe^{i\omega t}$

$X(t) = Xe^{i\omega t}$

λ is a diagonal eigenvalue matrix

Φ is modal matrix

It is noted that the vector X is function of frequency. Since $\lambda - \omega^2 I$ is a diagonal matrix the frequency response function (FRF) matrix H(ω) can be obtained by a simple multiplication when the eigensolution Φ of the substructure is provided.

3.3 Modified Modal Force Technique

The degrees of freedom of each substructure are further partitioned in two sets. One set is the internal degrees of freedom, and the other set is the boundary degrees of freedom. While the internal degrees of freedom do not connect to other substructures, the boundary degrees of freedom do. The structure shown in figure 3-1 can be considered as synthesized from two substructures A and B that are rigidly connected at their boundary α .

In modal force technique (MFT), only the displacement response equations of substructure at connecting joints are considered. Let superscript A and B indicate substructure A and B respectively, then the displacement response equation of substructures are

$$X^A = H^A f^A$$
$$X^B = H^B f^B$$ **Equation 3-4**

The geometric compatibility and the force equilibrium equations are expressed as

$$f^{A} + f^{B} = 0$$
$$X^{A} - X^{B} = 0$$

Equation 3-5

To derive the synthesized system equation, the Receptance method deletes the forces and retains the physical coordinates at joints, while the MFT eliminates the physical coordinates at joints. Thus the equation for the synthesized system has fewer inversions of matrices than that by the Receptance method. Thus applying equation 3-4 and 3-5, the modal force equation can be expressed

$$(H^{A} + H^{B}) f^{A} = 0$$

Equation3-6

$$\overline{H}\,\overline{f} = 0$$

Equation 3-7

Where

$\overline{H}$ is defined as modal force matrix (FRF matrix) and

$\overline{f}$ is modal force vector

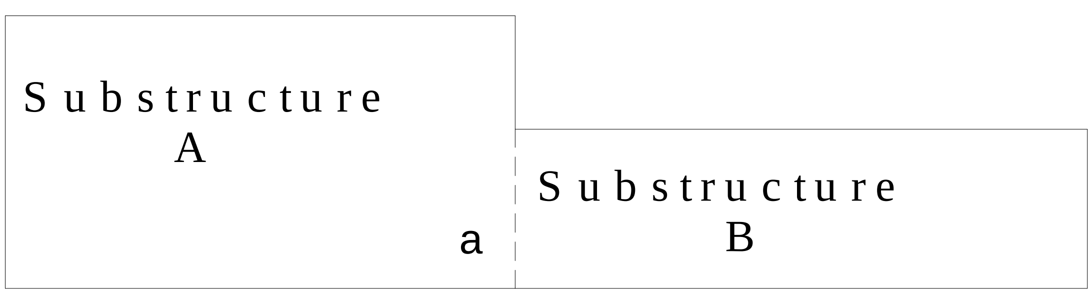

Fig. 3-1 A structure synthesized by substructure A and B.

3-4 Modified Modal Force Equation

A general ARMA model for each linear subsystem at connecting joints can be formulated as follow

$$\left(\sum_{k=0}^{p} A(k)s^{k}\right)X(s)=\left(\sum_{k=0}^{q} B(k)s^{k}\right)f(s)$$

Equation 3- 8

Where p is the order of matrix polynomial chosen in the AR part and q is the order of matrix polynomial chosen in the MA part. A(k) and B(k) are the matrix polynomial coefficient of AR and MA part respectively.

By applying a unit impulse force at each of the N_i connecting points, the corresponding response vector yields the column of the FRF matrix associated with the respective excitation point. When all force vectors and response vectors are brought together, this yields,

$$\left(\sum_{k=0}^{p} A(k)s^{k}\right)H(s)=\left(\sum_{k=0}^{q} B(k)s^{k}\right)I=\sum_{k=0}^{q} B(k)s^{k}$$

Equation 3- 9

Or

$$H(s)=\left(\sum_{k=0}^{p} A(k)s^{k}\right)^{-1}\left(\sum_{k=0}^{q} B(k)s^{k}\right)$$

Equation 3-10

Apply equation 3-10 in equation 3-6 the modified modal force equation will be generated.

$$\left[\left(\sum_{k=0}^{p^A} A^A(k)s^{k}\right)^{-1}\left(\sum_{k=0}^{q^A} B^A(k)s^{k}\right)+\left(\sum_{k=0}^{p^B} A^B(k)s^{k}\right)^{-1}\left(\sum_{k=0}^{q^B} B^B(k)s^{k}\right)\right]f^{A}(s)=0$$

Equation 3- 11

It can also be expressed as

$$\left[\left(\sum_{k=0}^{p}\bar{A}(k)s^{k}\right)^{-1}\left(\sum_{k=0}^{q}\bar{B}(k)s^{k}\right)\right]f^{A}(s)=0 \qquad \textbf{Equation 3-12}$$

Where

$$p = p^{A} + p^{B}$$

$$q = \max(p^{A} + q^{B}, p^{B} + q^{A})$$

$$\bar{A}(k) = \sum A^{A}(i)A^{B}(k-i)$$

and

$$\bar{B}(k) = \sum_{i=0}^{k}\left[A^{B}(i)B^{A}(k-i) + A^{A}(i)B^{B}(i-k)\right]$$

3.5 The ARMA Model for Modal Force Technique

Comparing equation 3-6 with equation 3-12 the ARMA model for modal force technique can be established.

$$\bar{H} = H^{A} + H^{B} = \left(\sum_{k=0}^{p}\bar{A}(k)s^{k}\right)^{-1}\left(\sum_{k=0}^{q}\bar{B}(k)s^{k}\right) \qquad \textbf{Equation 3- 13}$$

Or

$$\left(\sum_{k=0}^{p}\bar{A}(k)s^{k}\right)\bar{H} = \left(\sum_{k=0}^{q}\bar{B}(k)s^{k}\right) \qquad \textbf{Equation 3-14}$$

The polynomial of AR part determines the poles and the MA part define the zeros. From equation 3-6, the determinant of the modal force matrix equals to zero will decide the synthesized system modes. Also from equation 3-12 the polynomial of the AR part can be expressed as a product of the characteristic polynomial of each substructure. Thus the matrix polynomial coefficients of the AR part contain the information regarding substructure modes, and the MA part contains the information regarding the synthesized system modes. The synthesized system modes and the substructure modes can be identified simultaneously.

3.6 The Order of the ARMA Model

The matrix coefficient A(k) and B(k) have dimension of $N_i x N_i$, where N_i is equal to the number of degrees of freedom at the connecting joints. Note that $p = p^A + p^B$. In order to choose the correct order of the AR part, $p^A x N_i$ should be larger than the number of modes in substructure A and $p^B x N_i$ should be larger than the number of modes in substructure B. In order to account for the residual effects, the order of the MA part should be larger than the order of the AR part plus 2, that is q=p+2. When noise contaminated data are used in the analysis, the orders of the AR and MA parts may be raised higher than the effective number of degrees of freedom. The extra degrees of freedom will results in computational modes that need to be identified later.

3.7 Synthesized System Modes and Subsystem Modes

Let the coefficient matrix $\bar{A}(p) = I$, the identify matrix. By rearranging equation 3-13, taking the transpose, evaluating the results at several points, then rearranging the results, the following equation can be obtained.

$$\begin{bmatrix} \bar{H}(s_1)^T s_1^0 \ldots & \bar{H}(s)^T s_1^{p-1} & -Is_1^0 \ldots & -Is_1^q \\ \bar{H}(s_2)^T s_2^0 \ldots & \bar{H}(s_2)^T s_2^{p-2} & -Is_2^0 \ldots & -Is_2^q \\ & . & & \\ & . & & \\ & . & & \\ \bar{H}(s_1)^T s_l^0 \ldots & \bar{H}(s_l)^T s_l^{p-1} & -Is_l^0 \ldots & -Is_l^q \end{bmatrix} \begin{bmatrix} \bar{A}(0)^T \\ \\ \bar{A}(p-1)^T \\ \bar{B}(0)^T \\ \\ \bar{B}(q)^T \end{bmatrix} = \begin{bmatrix} -\bar{H}(s_1)^T s_1^p \\ -\bar{H}(s_1)^T s_2^p \\ . \\ . \\ . \\ -\bar{H}(s_1)^T s_1^p \end{bmatrix}$$

Equation 3-15

The coefficient matrices $\bar{A}(k)$ and $\bar{B}(k)$ can be obtained by standard Pseudo-inverse techniques, if the number of spectrum lines l , is chosen that $l \geq p+q+1$. The noise effect can be reduced in this process due to minimizing the squared error between the measured data and the estimation model.

The synthesized system modes and substructure modes can be determined respectively through

$$\det\left|\sum_{k=0}^{q} \bar{B}(k)s^k\right| = 0$$ **Equation 3-16**

And

$$\det\left|\sum_{k=0}^{p} \bar{A}(k)s^k\right| = 0$$ **Equation 3-17**

The companion matrix method is used to extract the needed information from the matrix power.

CHAPTER FOUR

SOLID MODELING OF THE SYNTHESIZED STRUCTURAL SYSTEM AND FEM PACKAGES

4.1 Introduction

In this chapter solid modeling and FEM packages of the synthesized simple structural system is created and list the results. Two rectangular identical beam substructures of size 381mmx38mmx19mm made from mild steel are rigidly connected to form a free-free structure as shown in figure 4.1. This model is created by AutoCAD 2007 and exported to ANSYS.

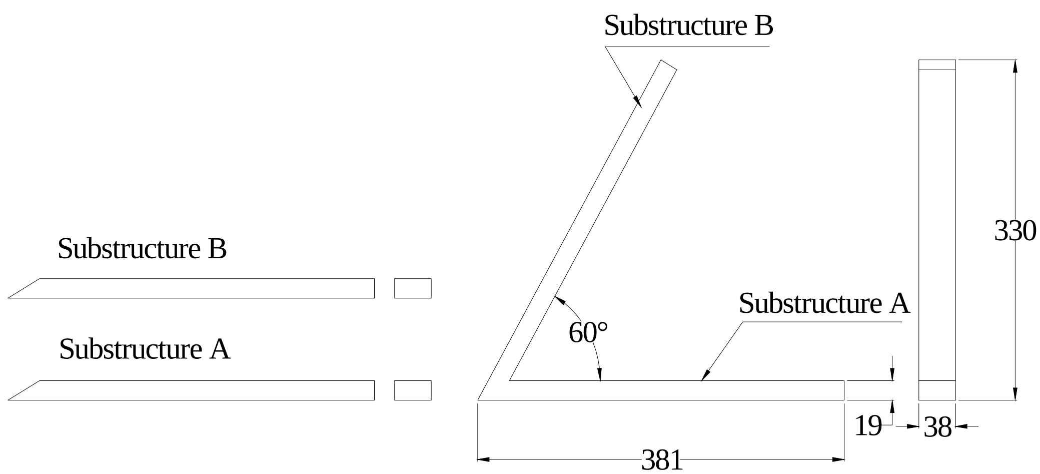

Figure 4-1 Structure synthesized by two beams.

4.2 Finite Element Formulation

For three dimensional modal analyses solid elements are useful for the solution of problems. These solid elements are broadly grouped under tetrahedral, triangular and hexahedral family elements.

In this thesis, the three dimensional eight nodded solid element is chosen from tetrahedral family for the element representation that uses for three dimensional analysis based isoparametric formulation.

The concept of isoparametric element is based on the transformation of parent element in local or natural coordinate to system to an arbitrary shape in Cartesian coordinate system.

4.3 Three-Dimensional Solid Element Formulation

The three dimensional eight nodded solid element shown in Figure 4.2 has eight nodes located at the corners and three translational degrees of freedom at each node. The shape function defining the geometry and the variation of displacement is given by

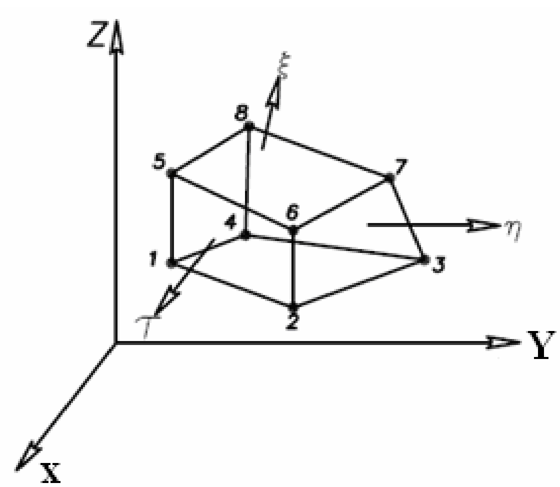

Figure 4.2a Cartesian coordinate system

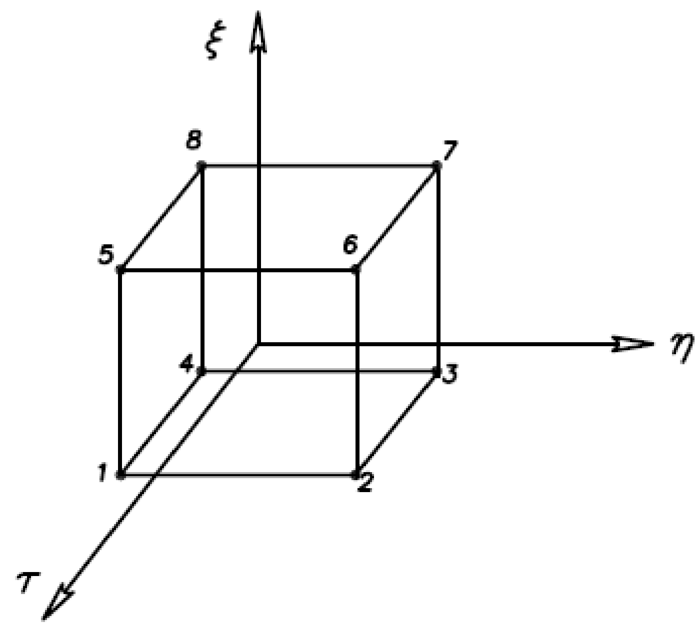

Figure 4.2b Local Coordinate system

$$N = \frac{1}{8}(1+\tau\tau_i)(1+\eta\eta_i)(1+\xi\xi_i)$$
$$i = 1,2,3,\ldots 8$$

Equation 4-1

Where τ, η, ξ are natural coordinates and τ_i, η_i, ξ_i are the values of the coordinates for node i.

Thus, the geometry of the element can be represented by a matrix as,

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{8} N_i(\tau, \eta_i \xi) \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} \qquad \textbf{Equation 4-2}$$

Where $x_i, y_i and z_i$ are the global coordinates of node i.

The variation of displacement inside the element is expressed using the same functions as,

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i(\tau, \eta_i \xi) \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} \qquad \textbf{Equation 4-3}$$

Where $u_i, v_i and w_i$ are nodal displacements of node i, in the Cartesian coordinate system.

4.4 Solid Modeling

Solid Modeling is geometrical representation of a real object without losing information the real object would have. It has volume and therefore, if some one provides a value for density of the material, it will have mass and inertia. Unlike the surface model, if one makes a hole or cut in a solid model, a new surface is automatically created and the model recognizes which side of the surface is solid material. The most useful thing about solid modeling is that it is impossible to create a computer model that is ambiguous or physically non-realizable [33].

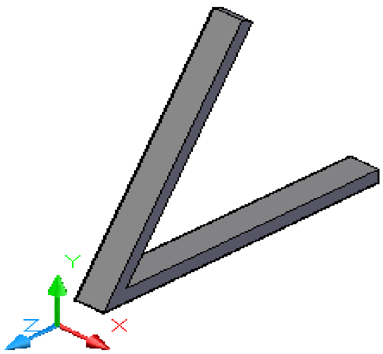

Figure 4.3 Solid modeling of the synthesized structure

4.5 FEM Package

ANSYS is the name commonly used for ANSYS mechanical, general-purpose finite element analysis (FEA) computer aided engineering software tools developed by ANSYS Inc. ANSYS mechanical is a self contained analysis tool incorporating pre-processing such as creation of geometry and meshing, solver and post processing modules in a unified graphical user interface. ANSYS is a general-purpose finite element-modeling package for numerically solving a wide variety of mechanical and other engineering problems. These problems include linear structural and contact analysis that is non-linear. Among the various FEM packages, in this work ANSYS is used to perform the analysis [33].

The following steps are used in the solution procedure using ANSYS

1. The geometry of the synthesized structural system model to be analyzed is imported from AutoCAD 2007; this is compatible with the ANSYS.
2. The element type and materials properties such as Young's modulus and Poisson's ratio are specified.
3. Meshing the three-dimensional synthesized structural system model. Figure 4.4 shows the meshed 3D solid model of it.
4. The boundary conditions and external loads are applied.
5. The solution is generated based on the previous input parameters.

6. Finally, the solution is viewed in a variety of displays.

4.6 Vibration Characteristics of the Synthesized Structural System and Subsystems

This section presents modal analysis of the synthesized system and subsystems. As mentioned in section 4.1 the model is made from mild steel as shown in figure 4.1 arranged the substructures 60 degree each other.

4.6.1 Modal Analysis of the Synthesized System

To carry out the modal analysis in ANSYS the model dimension and material properties specifies as follow;

The model dimension: Two rectangular identical beam substructures of size 381mmx38mmx19mm made from mild steel are rigidly connected to form a free-free structure as shown in figure 4.1.

Material: Mild steel

Material properties:

Young's modules 200e9 Pa.

Poisson's ratio 0.3

Density 7800 kg/m3.

Boundary condition: constrained at the lower free end (substructure A).

Loading: Unloading

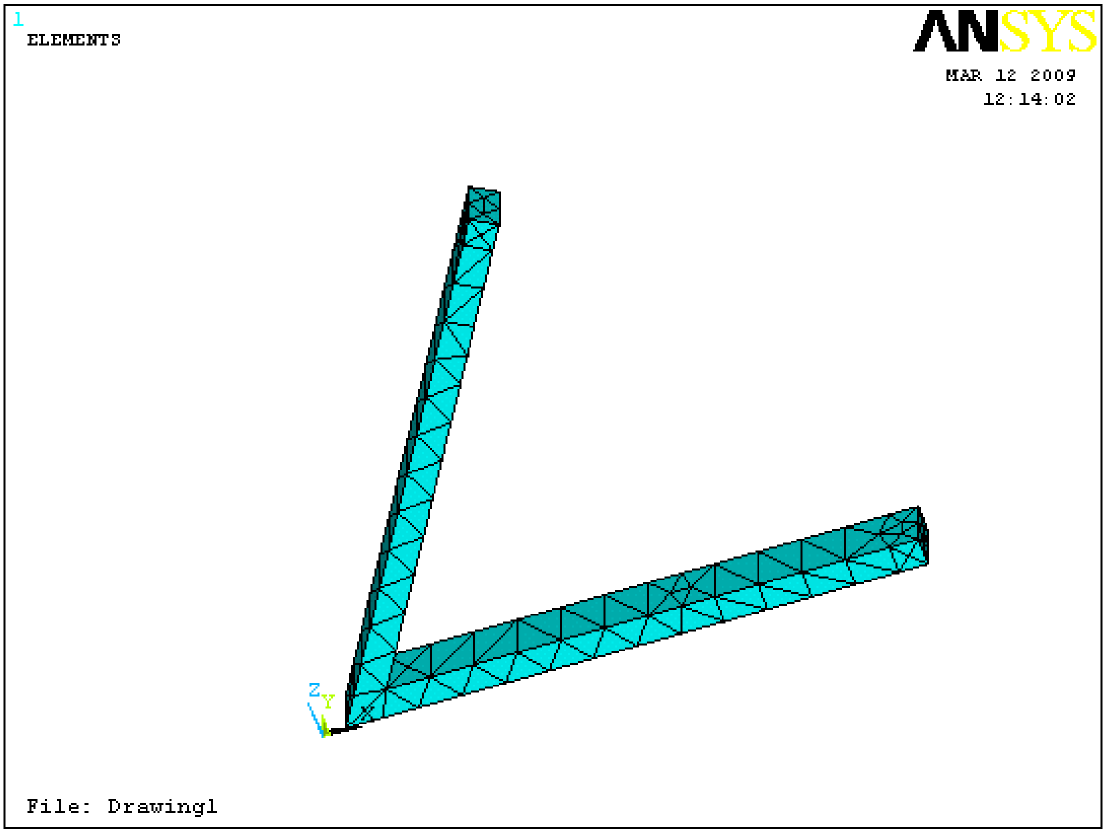

Figure 4.4 meshed model of the synthesized system

The five natural frequencies of the synthesized system presented in table 4.1

Number of modes	Natural frequency in Hz
1st	125
2nd	580
3rd	710
4th	850
5th	960

Table 4-1 five natural frequencies of the synthesized system (ANSYS result)

And the corresponding five mode shapes of the synthesized system shown below;

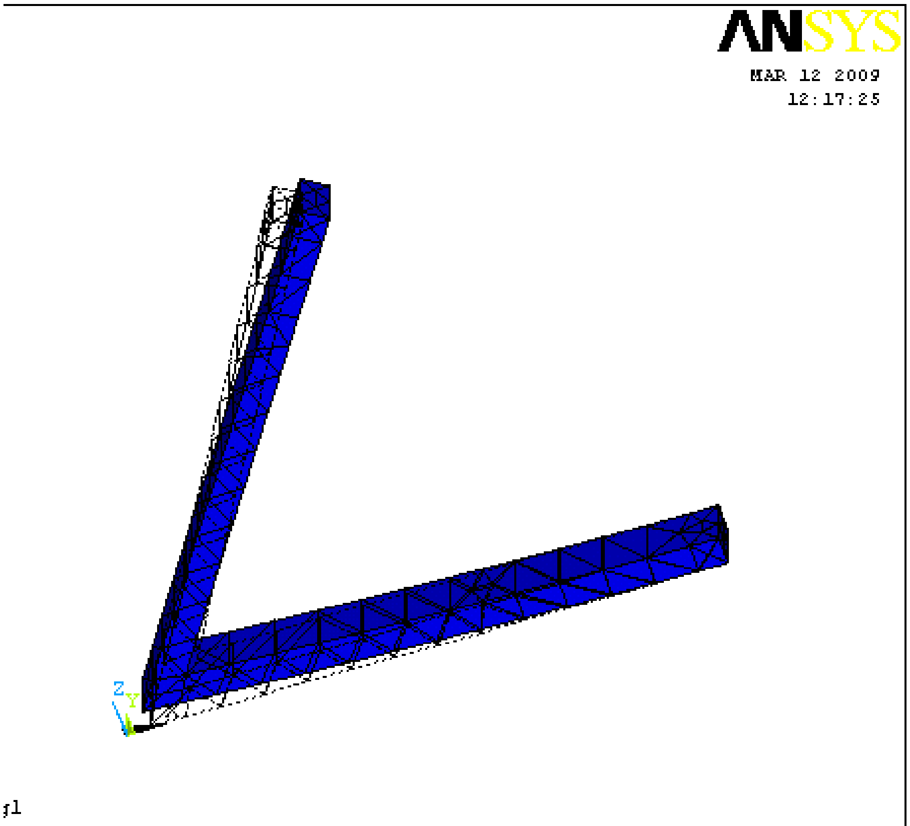

Figure 4.5 the first mode shape

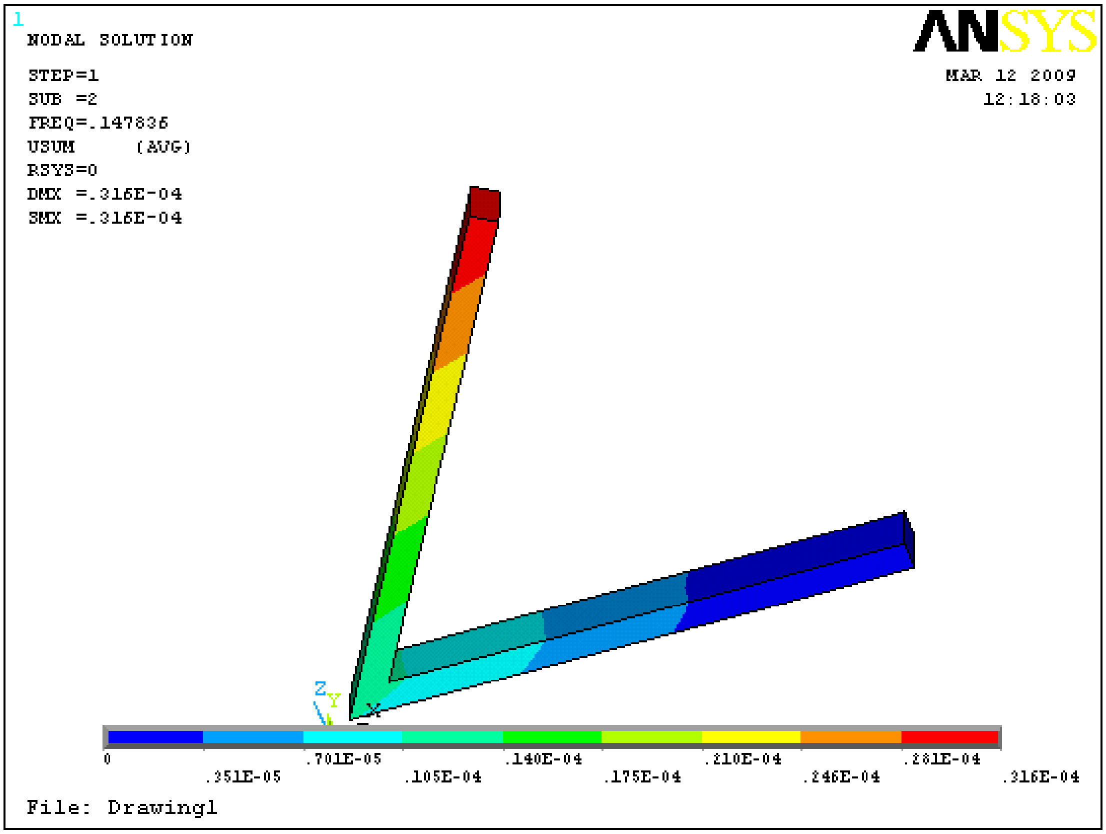

Figure 4.6 the second mode shape

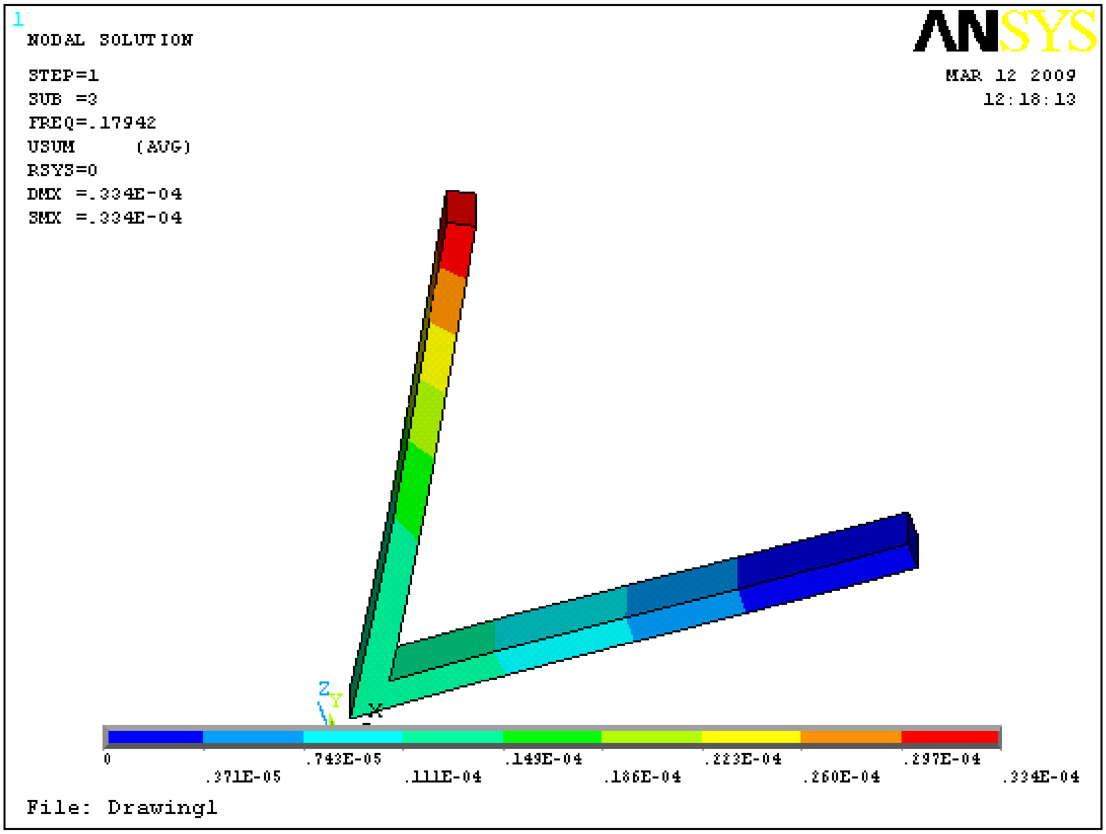

Figure4.7 the third mode shape

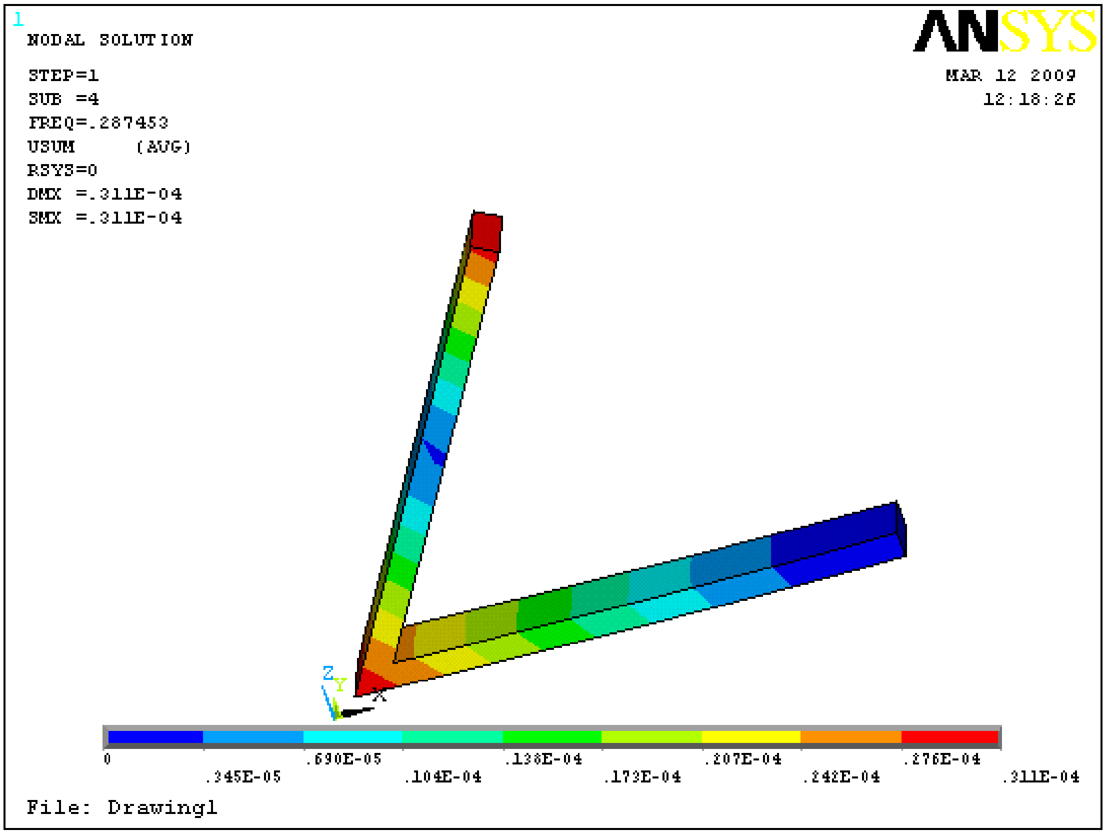

Figure 4.8 the fourth mode shape

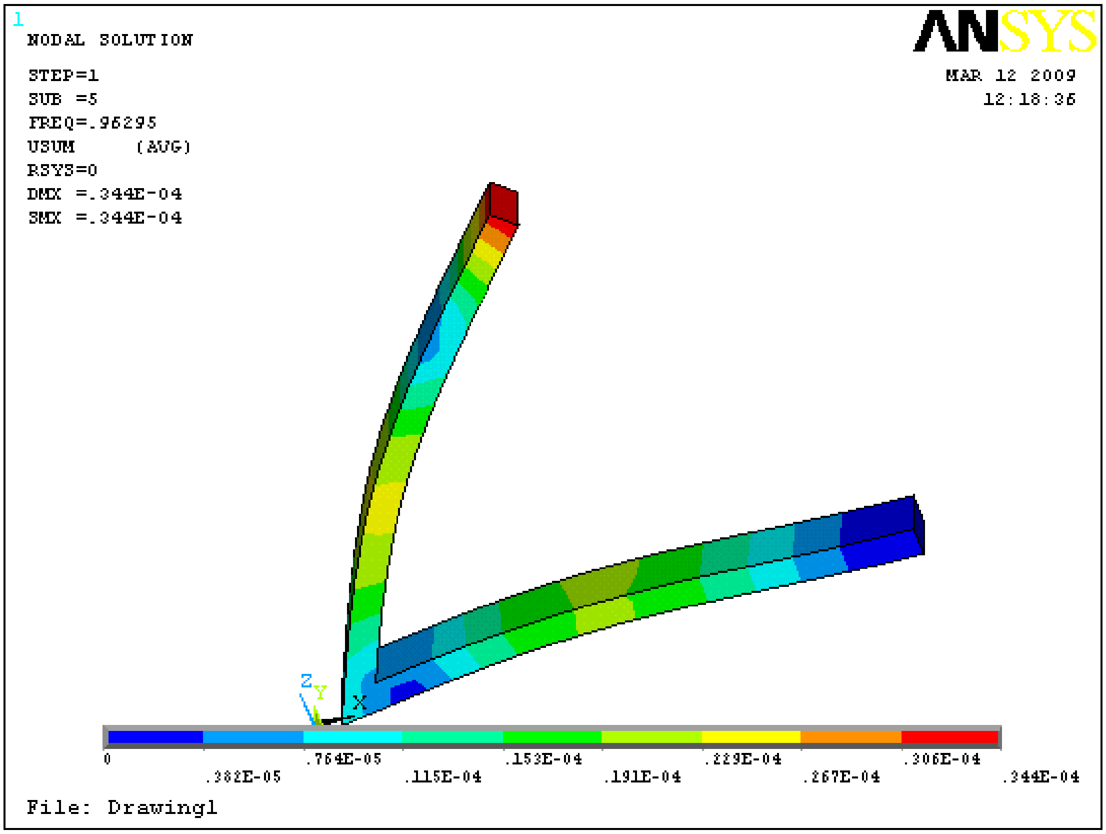

Figure 4.9 the fifth mode shape

4.6.2 Harmonic (forced) Analysis of both Synthesized System and Substructures

To carry out the harmonic analysis in ANSYS the model dimension and material properties specifies as follow;

The model dimension: Two rectangular identical beam substructures of size 381mmx38mmx19mm made from mild steel are rigidly connected to form a free-free structure as shown in figure 4.1.

Material: Mild steel

Material properties:

Young's modules 200e9 Pa.

Poisson's ratio 0.3

Density 7800 kg/m3.

Boundary condition: constrained at the lower free end (on substructure A).

Loading: 2 MPa compressive force is applied on the other (upper) free end of the model (on substructure B).

Frequency range: From 0 to 1000 Hz

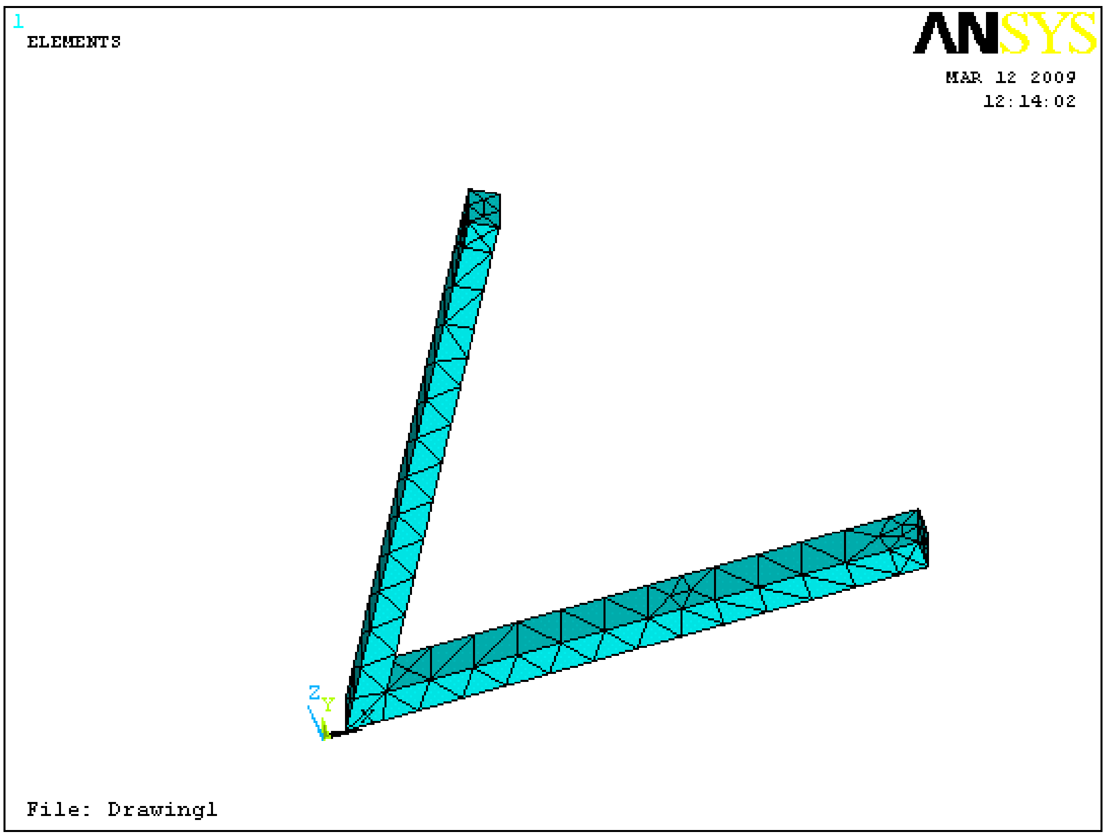

Figure 4.10 2 MPa compressive force is applied on the other (upper) free end of the model.

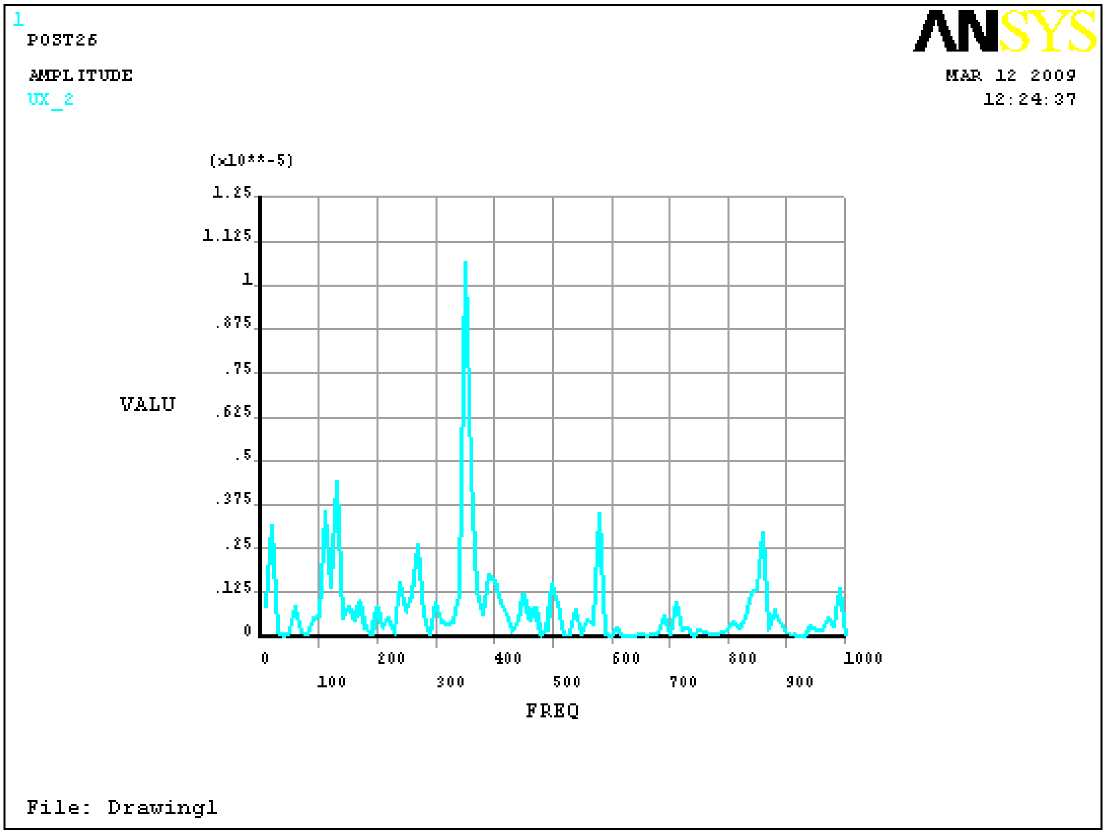

Figure 4.11 Frequency Responses of substructure A

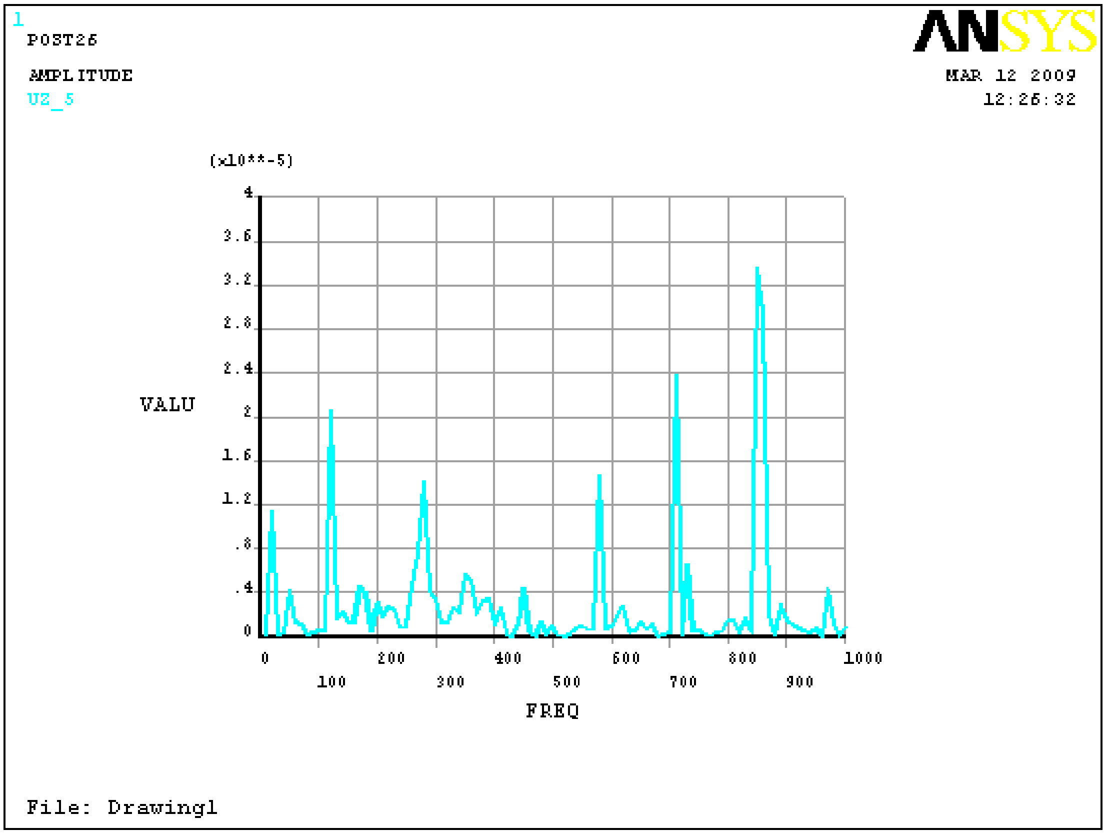

Figure 4.12 Frequency Responses of synthesized system at connecting points.

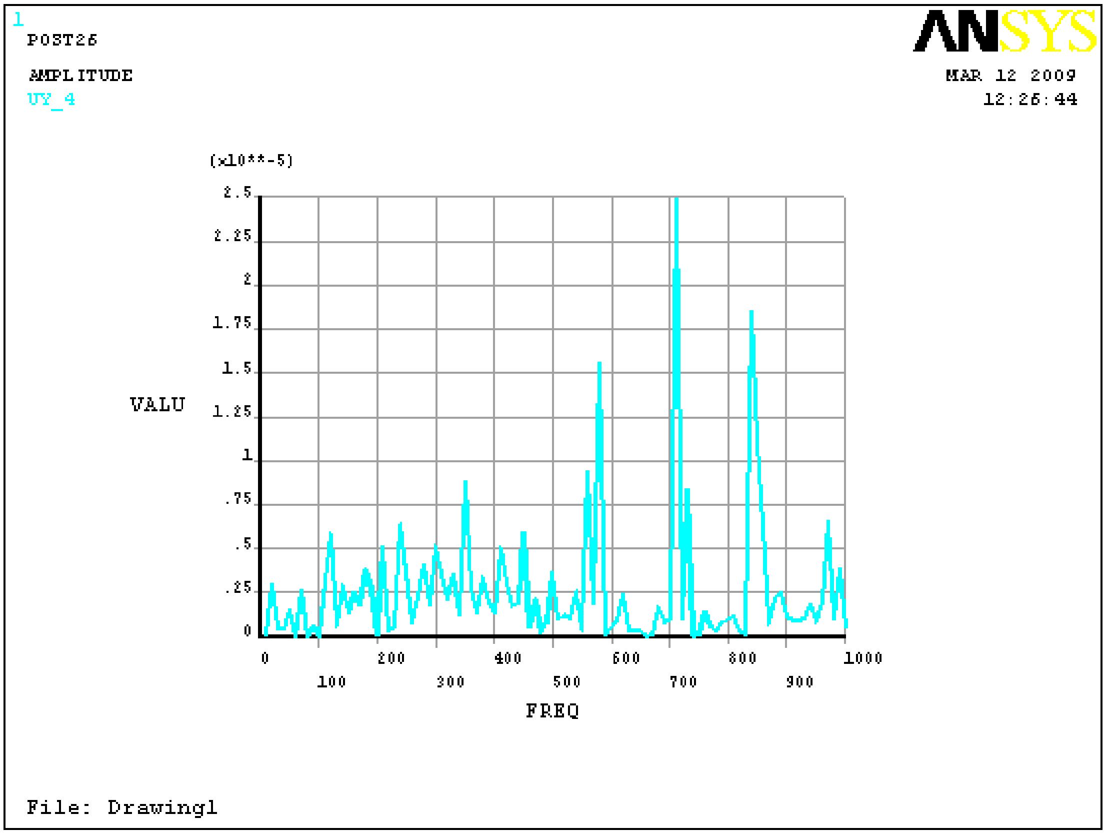

Figure 4.13 Frequency Responses of substructure B

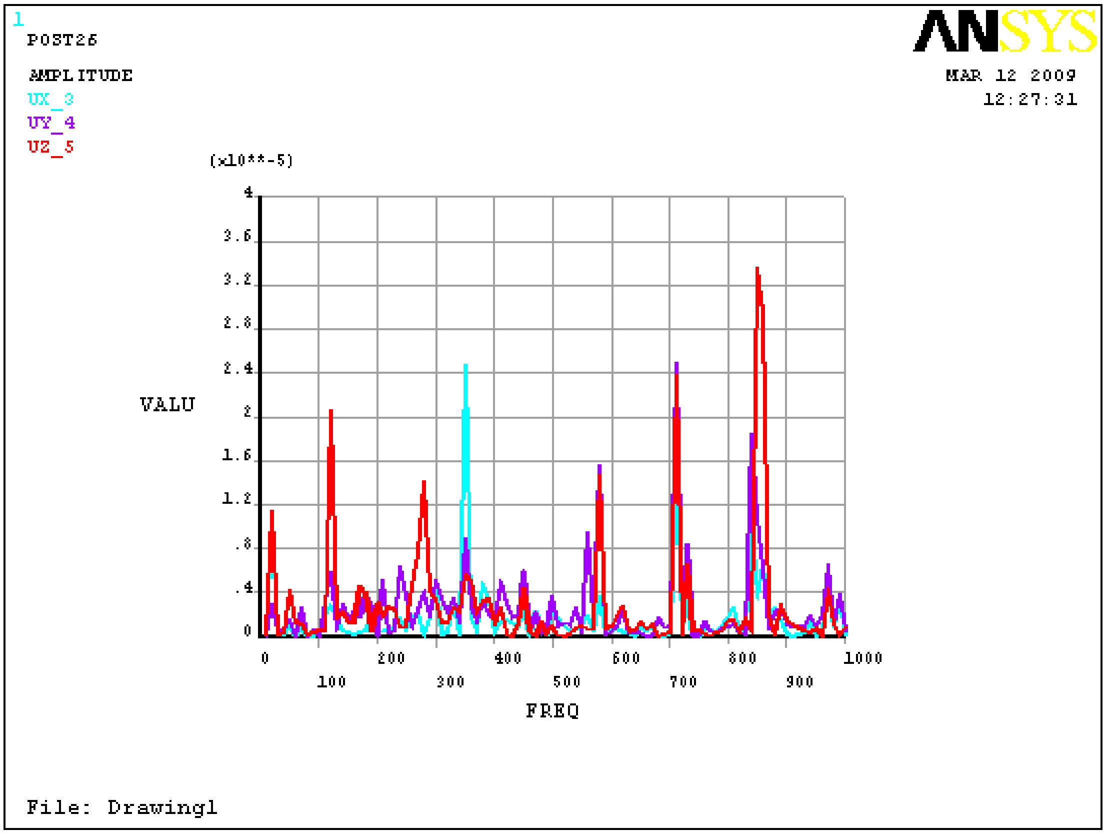

Figure 4.14 Combined Frequency Responses of both synthesized system and substructures

4.7 Modal Analysis of the Synthesized Structural System and Substructures by MATLAB 7

Two identical beam substructures of size 380mmx38mmx19mm are rigidly connected to form a free-free structure as shown in figure 4.1. As discussed in chapter four for each substructure, only the FRFs at the connecting joints are needed to form the modal force matrix, which is contained both synthesized and substructures modes.

By using MATLAB 7 software the required FRF for synthesized system and substructures are generated. Using equation 3.16 and equation 3.17 and physical geometry of the model the MATLAB program generates the natural frequency of the synthesized structure and plot a graph magnitude vs. frequency.

Let the order of AR part be equal to 4 and the order MA part be equal to 10, and the frequency range between 0 and 1000 Hz. Using these parameter, geometrical properties and material properties develop a MATLAB program to predict the synthesized system natural frequencies as shown in table 4.2 and magnitude of modal force matrix vs. frequency as shown in figure 4.15.

Number of modes	Natural frequency in Hz
1^{st}	117.5
2^{nd}	537.75
3^{rd}	688.94
4^{th}	873.18
5^{th}	977.45

Table 4.2 Natural frequencies of the synthesized system (MATLAB result).

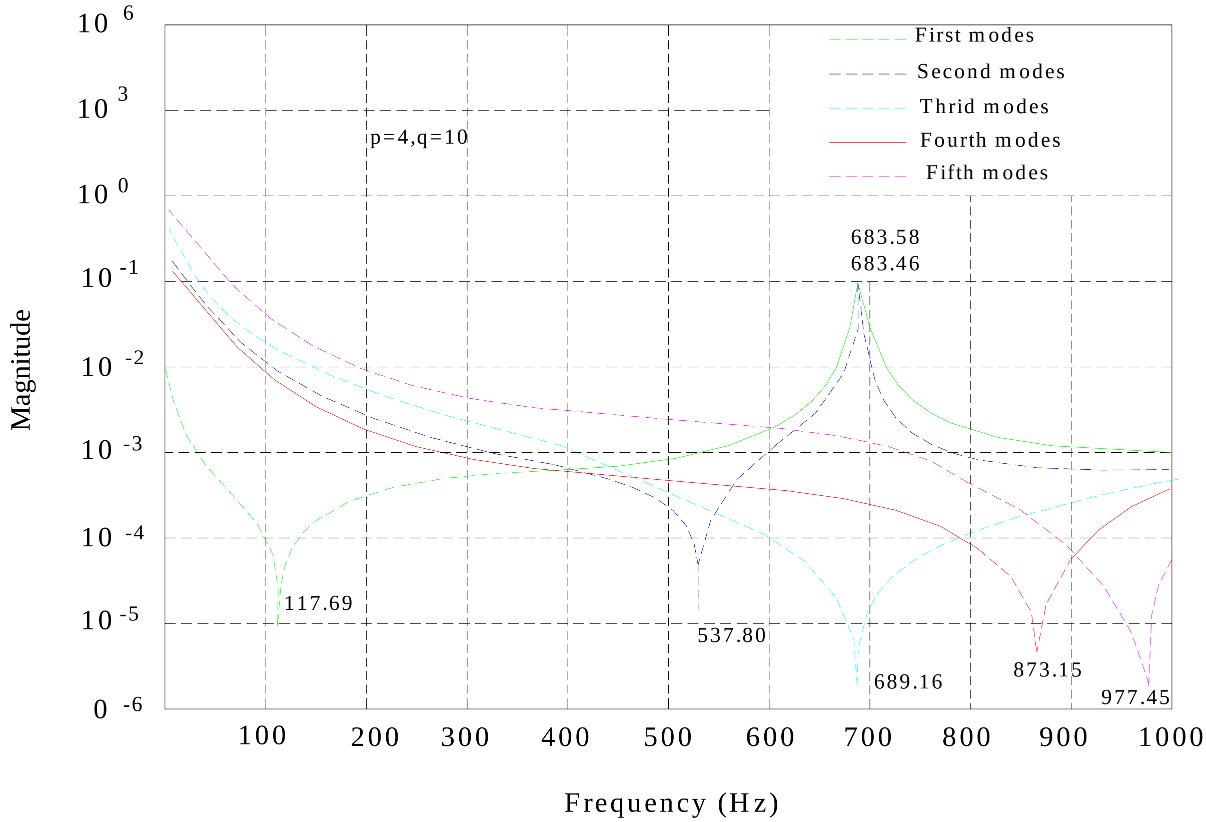

Figure 4.15 the generated plot of the synthesized structure modes

Figure 4.15 shows as the peaks will indicate the location of the substructure modes and the anti-peaks will indicate the location of the synthesized system modes.

CHAPTER FIVE

DISCUSSION OF RESULTS

5.1 Introduction

In this chapter discussed results of the two finite element based soft ware, i.e ANSYS 10 and MATLAB 7 families. First compare results obtained by the two software's and secondly perform discussion on results being gained by the two software independently.

5.2 Compare ANSYS and MATLAB results

The major work in this thesis study was synthesizing a simple structural system from modal analysis results. This is done first synthesize a modal force matrix by using FRF method as discussed in chapter four. Secondly analyze the modal analysis of this mathematical model (modal force matrix or FRF matrix) by using the two finite element based software as discussed in chapter four. Now discuss results gained by these software's.

Table 5.1 shows that natural frequencies obtained by these software's and their percentage difference of the synthesized system. If their difference is below ten percent, the results will be reasonable [34]. From the table the maximum difference is 7.86% and minimum is 1.82%. From this the one conclude the analysis is acceptable.

No of modes	Natural frequency (Hz) By ANSYS	Natural frequency (Hz) by MATLAB	Percentage difference (%)
1^{st}	125	117.5	6.4
2^{nd}	580	537.57	7.86
3^{rd}	710	688.94	3.1
4^{th}	850	873.18	2.73
5^{th}	960	977.45	1.82

Table 5.1 Comparison between results obtained by ANSYS and MATLAB and percentage difference.

5.3 Discussion of Results of ANSYS 10 Software

In section 4.6.1 shows the results gained by ANSYS i.e the synthesized system natural frequencies and their corresponding mode shape.

Figure 4.5 through figure 4.9 shows the deformation pattern will result when the excitation coincides with one of the natural frequencies, one would see a deformation pattern that exist in the structure.

The one see that in figure 4.5, first natural frequency, there is a first bending deformation on the structure. When one sees that in figure 4.6, second natural frequency there is a first little twisting deformation is shown in the model. When one see in figure 4.7, third natural frequency, there is first bending and first twisting deformation is shown. When one in figure 4.8, fourth natural frequency, there is also first bending and first twisting deformation is shown. And one see in figure 4.9, fifth natural frequency, there is second bending and first twisting deformation is shown.

In section 4.6.2 display results on harmonic (forced) analysis of both synthesized system and subsystem responses by applying a constant compressive load (2 Mpa) on the upper free end of the synthesized system, by varying the frequency from 0 to 1000 Hz.

Figure 4.11 shows a substructure A responses applying a constant force and varying frequencies from 0 to 1000 Hz. The one sees from the graph around a frequency 360 Hz the substructure A has maximum response.

In figure 4.12 shows that a synthesize system responses at the connecting points. The one sees the structure has maximum responses at the following frequencies. 125 Hz, 580 Hz, 710Hz, 850 Hz and 960 Hz.

And figure 4.13 shows substructure B responses. As shown in the figure maximum deflection is shown specially at the upper free end of substructure (substructure B). This maximum deflection is occur at a frequency of 710 Hz and some relatively high responses are register at frequencies 580 Hz, 840 Hz and soon.

From this the one conclude that resonances are occurred at frequencies mentioned above.

5.4 Discussion of Results of MATLAB software

In section 4.7 present the results obtained by MATLAB i.e modal analysis of the synthesized system and substructures.

Figure 4.15 shows as the peaks will indicate the location of the synthesized system modes. This implies the synthesized modes are as follows; 117.69 Hz, 537.8 Hz, 689.16 Hz, 873.15 Hz and 977.45 Hz.

And the substructure modes are 683.58 Hz and 683.46 Hz, this is close result to results obtained from ANSYS i.e 710 Hz substructure B mode.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

A method of using ARMA model in the Laplace domain for modal synthesis is presented. Then a modal force method by ARMA model can be established. Only the FRF at the connecting joints is needed in the analysis to form a matrix named modal force matrix (FRF matrix). From this matrix, both synthesized system modes and substructure modes can be extracted simultaneously by using ANSYS and MATLAB.

The AR part contains the information regarding substructures modes and the MA part contains the information regarding synthesized system modes. Both synthesized system modes and subsystem modes can be extracted simultaneously. This method works well in narrow frequency bands.

The eigensolusion of the system in any frequency range can be determined independently. The data with any frequency range can be determined independently. The data with any frequency spacing can be used by this method. Since only one parameter estimation algorithm is needed to extract the synthesized system modes and substructures modes, the computation is reduced drastically.

ANSYS and MATLAB results are reasonable good. Their maximum percentage difference is 7.86 %, it may be due to selection of material property, physical geometry, boundary condition, un proper mesh generation of the model, working area and soon.

6.1 Recommendations

In the future work, it will do for highly complicated structure synthesized (like bridge, industrial structure, aircraft bodies, automotive bodies) by this method, using a little modified of it.

It will be done, by using experimental and analytical modal analysis techniques. Compare their results and if there is a difference update the analytical one, in order to correlate each other.

The experimental approach is done by modeling the dynamic behavior of structures (modal testing) relies mostly on extracting the vibration characteristics of a structure from measurements. The procedure consists of three steps:

1. Acquiring the modal data.
2. Analyzing the measured modal data.
3. Constructing the dynamic model behavior by using extracted modal parameter from the analyzed data.

Vibration measurements are taken directly from a physical structure, without any assumptions about the structure and that is the reason why modal testing models are considered to be more reliable than Finite Element models.

Due to these reasons and other unique advantages explained in chapter 0ne, it used for updating of the analytical models (calibration) and structural modifications.

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