ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES
FACULTY OF TECHNOLOGY
DEPARTMENT OF
ELECTRICAL AND COMPUTER ENGINEERING

Performance Comparison of Spatial Multiplexing and Spatial Diversity in Rate Adaptive Wireless MIMO Systems

A thesis submitted to the school of graduate studies of Addis Ababa University in partial fulfillment of the requirements for the degree of
Masters of Science in Electrical Engineering

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List of Abbreviations

AWGN Additive White Gaussian Noise

BER Bit Error Rate

BLAST Bell Lab Layered Space Time

DBLAST Diagonal BLAST

EGC Equal Gain Combining

LST Layered Space Time

MIMO Multiple Input Multiple Output

MISO Multiple Input Single Output

ML Maximum Likelihood

MMSE Minimum Mean Square Error

MMSE-SIC Minimum Mean Square Error with Successive Interference Cancellation

MRC Maximum Ratio Combing

SC Selection Combining

SD Spatial Diversity

SER Symbol Error Rate

SIC Successive Interference Cancellation

SIMO Single Input Multiple Output

SM Spatial Multiplexing

SNR Signal to Noise Ratio

STBC Space Time Block Code

SVD Singular Value Decomposition

VBLAST Vertical BLAST

ZF Zero Forcing

ZF-SIC Zero Forcing with Successive Interference Cancellation

Abstract

Multiple antenna systems are known to provide spatial diversity and spatial multiplexing advantages in constructing a reliable and spectrally efficient communication link. In spatial diversity scheme the same message or replica of the same message is transmitted from the number of transmit antennas in such a way that at least one of transmitted messages reaches the receiver with out error. In this scheme the main objective is reduction in the probability of error. Unlike spatial diversity, in spatial multiplexing scheme different messages are transmitted from each of the transmit antennas so that the rate of data transfer between the transmitter and receiver is increased. As far as practical system is concerned it needs to have both advantages in order to have a high data rate link with low probability of error.

In this thesis by adaptively varying the modulation order of the spatial diversity and spatial multiplexing systems both advantages are obtained for an application with a certain error rate performance requirement. The simulation result has shown that the spatial multiplexing system has a higher throughput at high SNR values as compared to the spatial diversity system. At low SNR values, it is the spatial diversity system that results in high throughput. The ML detection of spatial diversity system has less computational complexity as compared to ML detection spatial multiplexing scheme.

Keywords: Spatial Diversity, Spatial Multiplexing, Maximum Likelihood Detection, Space Time Coding, D-Blast and V-Blast.

5 CHAPTER ONE

6 INTRODUCTION

1.1 MIMO channel

Realizing reliable and efficient communication over a wireless channel has been a challenge due to the channel's rapid time variation and multipath fading. On one occasion the transmission may experience less fading and the error probability will be low. On the other hand the transmission may experience severe fading and the error rate will be high. One other key challenge in wireless communication is that, while data applications require much higher data rates, the resources in wireless systems (bandwidth and power) are scarce and expensive, hence one needs to maximize the rate of communication with in a given bandwidth and power, in other words, maximize the spectral and power efficiencies.

A very promising recent approach that addresses these challenging problems is a multiple antenna technology, where the transmitter is equipped with n_T transmit antenna and a receiver with n_R receive antennas. This leads to transforming the channel into multiple input multiple output (MIMO) wireless fading channel. Meaning, multiple antennas create extra dimensions in the spatial domain and these dimensions can be used in different ways.

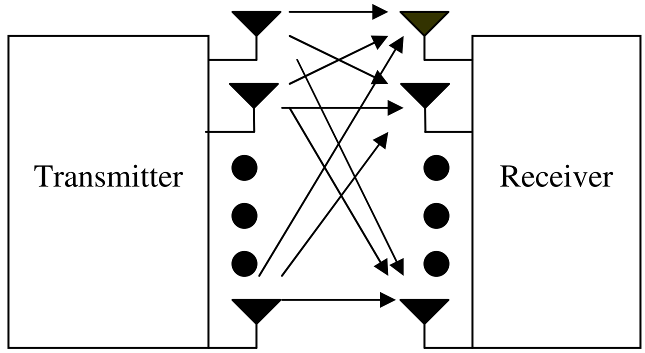

Figure 1.1 Illustration of MIMO channel

One way is to send independent information streams on these spatial channels to realize increased transmission rate. i.e. different information sequence of bits are transmitted form each transmit antennas. This effect is called Spatial Multiplexing(SM) and is particularity important in high SNR region. In this scheme there is a linear increase in channel capacity with the minimum number of transmit (n_T) and receive (n_r) antennas. i.e. the spectral efficiency grows linearly with number of parallel n_T and n_R antennas at a fixed power.

On the other hand, one can deliver the same information bits over the multiple spatial channels to exploit the spatial diversity so as to enhance the reliability of the transmission. The idea is just to supply the receiver with multiple independently faded replicas of the same information symbol so that the probability that all the signal components fade simultaneously is reduced. The scheme that exploits this diversity advantage is called Spatial Diversity (SD).

MIMO systems are thus used to increase either the reliability through spatial diversity or data rate through spatial multiplexing schemes. Let's say a certain particular application requires a certain level of error rate for acceptable performance. This error rate can be met with reasonable data rate by:

1. Using the spatial diversity advantage of MIMO system. But there are instances that the transmission experience less fading and the error rate performance improvement of the spatial diversity scheme is far more than the required level. At these instances, one can trade this improved error rate performance of MIMO diversity for more data rate indirectly through allowing higher order modulation. Thus by making the modulation order to depend on the channel condition one can achieve the required error rate with reasonable data rate.
2. Using spatial multiplexing system. In this scheme the transmission is made from constellation with large Euclidean distance. This increased constellation distance guarantees the minimum error rate requirement of the particular application. As before, there are instances that the transmission experiences less fading and doesn't require that much constellation distance between symbols or vectors. At these instances one can increase the modulation order in order to support high data rate. Again by varying the modulation order with the channel conditions we can meet the application's requirement with reasonable data rate.

The question then will be which of the two schemes results in higher throughput while satisfying the minimum error rate requirement of the application at hand? Determining this is the aim of this thesis.

6.1 **1.2 Objectives**

The objective of this thesis is to study the rate performance of the two schemes mentioned in rate adaptive system while satisfying a certain error rate requirement. In doing so the capacity of MIMO channels and its variations in slow and flat Rayliegh fading channels and the capacity relation with the number of antennas is investigated. The error rate performance of the different detection methods of spatial multiplexing scheme is also given. The antenna separation is assumed to be large enough to insure independent fading between sub-channels. In addition, full channel state information at the receiver is assumed.

6.2 **1.3 Literature Review**

Most researches in MIMO were focused on making use of only one approach, either to maximize the rate through spatial multiplexing or reliability through spatial diversity. But maximizing one scheme may not necessarily mean maximizing the other. Current researches, however, try to combine both schemes trading one for the other based on the channel state. To this end [14] has shown that there is an optimum trade off that can be made between spatial diversity and spatial multiplexing schemes. With the assumption that transmission block length $T \geq n_R + n_T - 1$, the optimal tradeoff curve is given by:

$$d(k) = (n_R - k)(n_T - k) \qquad \text{.......................................} \; (1.1)$$

$$\begin{aligned} &where: d(k)\, is\, the\, diversity\, advantage \\ &\qquad k = is\, the\, multiplexing\, advantage \end{aligned}$$

What the above expression states is that the maximum diversity advantage that one can get form n_T by n_R MIMO channel is n_T*n_R while the multiplexing advantage is zero and the maximum possible multiplexing advantage is $K=\min(n_R,n_T)$ while the diversity advantage is $(n_T-K)*(n_R-K)$. The optimum tradeoff curve for a 4 by 4 MIMO channel is shown in figure 1.2.

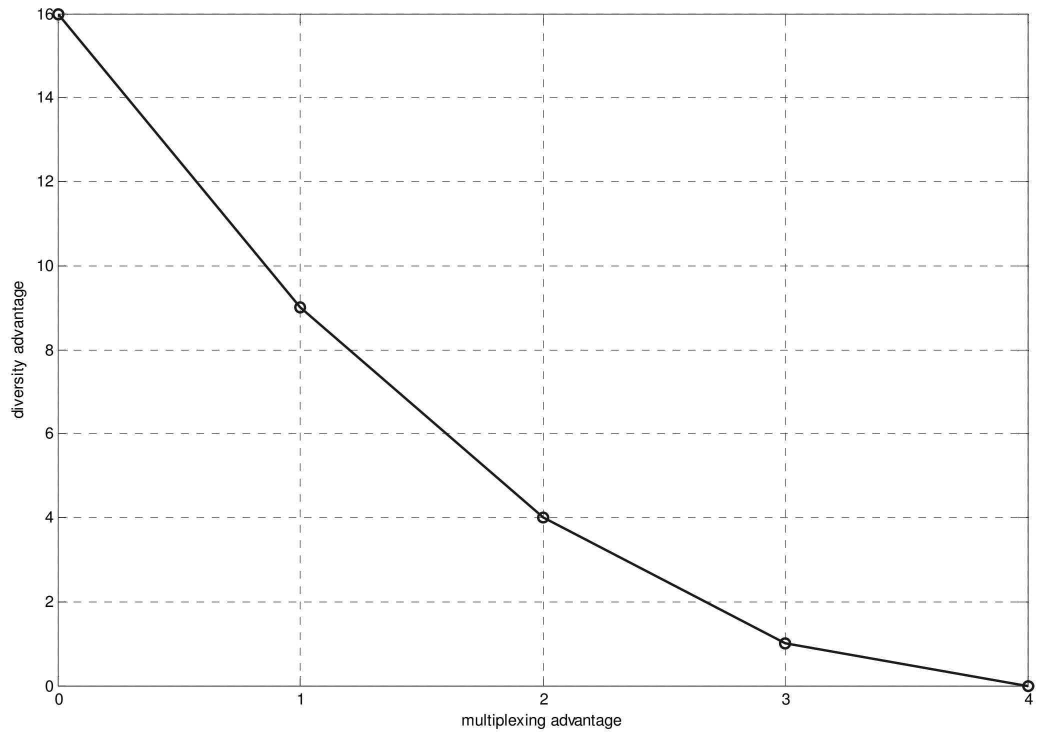

Fig 1.2, optimum tradeoff curve for 4 by 4 antenna

From the graph it can be seen that one needs to dedicate k transmit and k receive antennas for a multiplexing advantage of k, the rest (n_R-k) receive and (n_T-k) transmit antennas for diversity advantage. Lizhong Zheng has used this optimum trade off curve to evaluate the performance of different schemes [14].

Robert Wendell Heath has used a system that switches between spatial multiplexing and spatial diversity to reduce the probability of symbol error for a fixed data rate [3]. It has made the switching when the worst minimum constellation distance of spatial multiplexing is better than the best minimum distance for the diversity transmission.

6.3 1.4 Outline of the thesis

In Chapter 2 different diversity techniques are introduced. The capacity of MIMO channels as well as its variations is given. The two common performance measures of wireless channels are also discussed.

Chapter 3 introduces the different diversity combining techniques along with their performance comparison. The space time coding that is used in connection with the spatial diversity scheme and its detection procedure is also discussed.

Chapter 4 introduces spatial multiplexing scheme along with the space time coding used. The various spatial multiplexing detection methods and their performance comparison is given.

Chapter 5 introduces the different adaptation techniques. The performance of the rate adaptive spatial diversity and spatial multiplexing schemes are given. Finally the computational complexity that is evolved in the detection of the two schemes is discussed.

In Chapter 6 performance comparison of the spatial diversity and spatial multiplexing schemes and some idea of future work are given.

7 CHAPTER TWO

8 CAPACITY AND PERFORMANCE OF MIMO CHANNEL

8.1 2.1 Introduction

The capacity of a channel is defined as the maximum data rate that can be supported by the channel under arbitrarily small probability of error rate. Multiple input multiple out put wireless channels can offer high capacity if the multi path fading is properly exploited. In a system with n_R receive antenna and n_T transmit antenna there can at most be min(n_R,n_T) independent paths (sub-channels) through which independent information stream can be sent. The sum of channel capacities of these independent sub-channels is what we call the capacity of MIMO system. The scheme that exploits this capacity advantage is the spatial multiplexing system. The other scheme that exploits the diversity advantage of MIMO system is a spatial diversity system where the objective is reduction in error rate. Before we go to the details of these techniques, let's first see the different diversity techniques.

8.2 2.2 Diversity Techniques

Diversity techniques are used to reduce the effects of multipath fading and improve the reliability of transmission. It is all about delivering the same or replicas of the same message to the receiver through channels that have no or small correlation in fading statistics, where by the probability of all the message signals being simultaneously blow some given level is much smaller than the probability of any individual samples being below a given level. Depending on the domain in which diversity is implemented, diversity techniques are classified into time, frequency and space.

Time diversity is achieved by transmitting identical message signals in time slots that are uncorrelated in fading statistics. To achieve this, the time separation between these time slots should at least be equal to the coherence time of the channel. The coherence time is a statistical measure in which the channel's fading statistics is constant. This time separation is obtained by time interleaving the sequence of bits or symbols. Since this time interleaving results in decoding delay, this technique is effective for delay insensitive applications such as data transmission or fast fading channels where the channels coherence time is small. For slow fading channels, the large interleaving distance can lead to significant delay which is untolerable for delay sensitive applications such as voice. In addition, time diversity results in some data rate lose as the same message is sent in different time periods which could have been used for new message.

In frequency diversity, replicas of the same message are transmitted in different frequency slots that are separated enough to ensure independent fading. The frequency separation between these slots should at least be equal to the coherence bandwidth of the channel which is the frequency band in which the fading statistics is correlated. Similar to time diversity, frequency diversity results in the reduction of spectral efficiency as different frequency bands are used to transmit the same message.

Spatial diversity is implemented using multiple antennas or antenna arrays that are separated in distance at least equal to the coherence distance of the channel. Coherence distance is a distance over which the channel fading statistics is correlated. Depending on the antenna height, propagation environment and frequency, the channel's coherence distance vary. Through the number of paths that are created due to these multiple antennas, the same information signal can be sent such that each message from the respective antenna sees different fading statistics. Unlike time and frequency diversity, space diversity does not induce much delay and any loss in bandwidth efficiency and it is this property of it that makes spatial diversity attractive for high data rate wireless communication[5].

8.3 **2.3 Capacity of MIMO Channels**

Diversity techniques like space, time and frequency are well known techniques to improve reliability of wireless communication systems. Among them, spatial diversity technique is the most promising one, because it does not require any additional bandwidth and does not introduce additional delays in signal transmission. In addition to reduction in fading, spatial diversity can exploit multipath fading to increase data communication rate. Foschini and Telatrar have shown that the capacity of flat Rayleigh fading channel increases almost linearly with the minimum number of transmit and receive antennas[7]. Capacity of a channel is defined as the maximum data rate that can be transmitted over the channel with arbitrarily small probability of error.

The output of a general MIMO channel is modeled as:

$$Y = HX + n \qquad \text{.............................. ... (2.1)}$$

Where: X is the transmitted signal

H channel transition matrix and

n is additive Gaussian noise

The capacity of MIMO channel is an extension of SISO (single input single output) channel capacity to a matrix form. i.e.

$$C = \max_{p(x)} I(X,Y) \qquad \text{.................... (2.2)}$$

The maximization of I(X,Y) for full channel state information (CSI) only at the receiver with uniform power allocation yields the capacity expression [1]

$$C = w\log_2 \det(I_m + \frac{P}{n_T\delta^2}Q) \qquad \text{........................(2.3)}$$

$$where\, Q = \begin{cases} HH^*, & n_R < n_T \\ H^*H, & n_R \geq n_T \end{cases}$$

$$P - is\, total\, transmitted\;\; power$$

$$w - is\, the\, bandwidth$$

Using singular value decomposition (SVD) we can write H as:

$$H = UDV^* \qquad \text{............................... (2.4)}$$

Where D is $n_R \times n_T$ diagonal matrix whose entries are the non-negative square roots of the eigenvalues(λ) of matrix HH^*. U and V are $n_R \times n_R$ and $n_T \times n_T$ unitary matrices respectively. The number of non-zero eigenvalues of a matrix HH^* is equal to the rank(r) of a matrix H[5]. For $n_R \times n_T$ matrix H, the rank is at most equals to m = min(n_R,n_T). This implies that there are at most m non-zero eigenvalues.

The above capacity expression then reduces to:

$$C = w\sum_{i=1}^{r}\log_2(1+\frac{p_{ri}}{\delta^2}), \qquad p_{ri} = \frac{\lambda_i P}{n_T}$$
$$= w\log_2\prod_{i=1}^{r}(1+\frac{\lambda_i P}{n_T\delta^2}) \qquad \text{.......................... (2.5)}$$

Where as SISO's channel capacity is given by:

$$C = w\log(1+\frac{P}{\delta^2}|h|^2) \qquad \text{...................... (2.6)}$$
$$where\,|h|^2\; is\, the\, path\, gain$$

Examination of equation (2.5) and (2.6) indicates that MIMO's channel capacity can be interpreted as the sum of the channel capacities of the sub-channels with channel path gain λ_i. i = 1,2,.....r. This implies that, MIMO channel has a number of sub-channels whose received signal take the form:

$$\begin{bmatrix} r_1 \\ r_2 \\ . \\ . \\ r_{n_R} \end{bmatrix} = \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0....0 \\ 0 & \sqrt{\lambda_2} & 0....0 \\ . & & \\ . & & \\ 0 & 0 & 0....0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_{n_R} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ . \\ . \\ n_{n_R} \end{bmatrix}$$
$$r_i = \sqrt{\lambda_i}x_i + n_i, \quad i = 1,2,...r \qquad \text{...................... (2.7)}$$
$$r_i = n_i,\, i = r+1, \quad i = r+1,....n_R$$

This analysis shows that, there exists at most m = min(n_T,n_R) sub-channels through which independent information streams can be sent. i.e. each sub-channel can be taken as an independent information path and independent information streams can be sent. Hence the total channel capacity is the sum of the individual sub-channel's capacity. This is how spatial multiplexing is achieved in MIMO systems so as to increase the throughput of a channel.

At the receiver, with full channel sate matrix, the individual bit streams are separated and estimated. The separation is possible only if the individual antennas sees a sufficiently different (independent) or uncorrelated path. This is some thing similar to solving a simultaneous equation with n unknowns. The equation is solvable if we have n independent equations. In matrix form it is all the same as saying, the rank of the coefficient matrix should be n. Since for $n_R \times n_T$ channel matrix, the rank is at most equal to $\min(n_R,n_T)$, implying that we can have a maximum of $\min(n_R,n_T)$ independent paths through which independent information can be sent.

Some authors use to take analogy between this scheme and that of CDMA system[8]. In CDMA multiple users share the same time/frequency channel by use of pseudo random codes that are quasi-orthogonal (independent). At the receiver these unique code signatures are used to separate each user. Unlike CDMA's, MIMO's unique signature is provided by the independent multipaths, hence no need for frequency spreading that result in spectrum inefficiency.

If the channel coefficients are random variables, the above channel capacity expressions give instantaneous capacity values. By averaging over all channel realization one can find the mean (ergodic) channel capacity. i.e.

$$C = E[w \log_2 \det(I_m + \frac{P}{n_T \delta^2} Q)] \qquad \text{.................. (2.8)}$$

Where: E is an expectation operator

Simulated result of the above channel capacity for different number of transmit and receive antennas is shown in figures below.

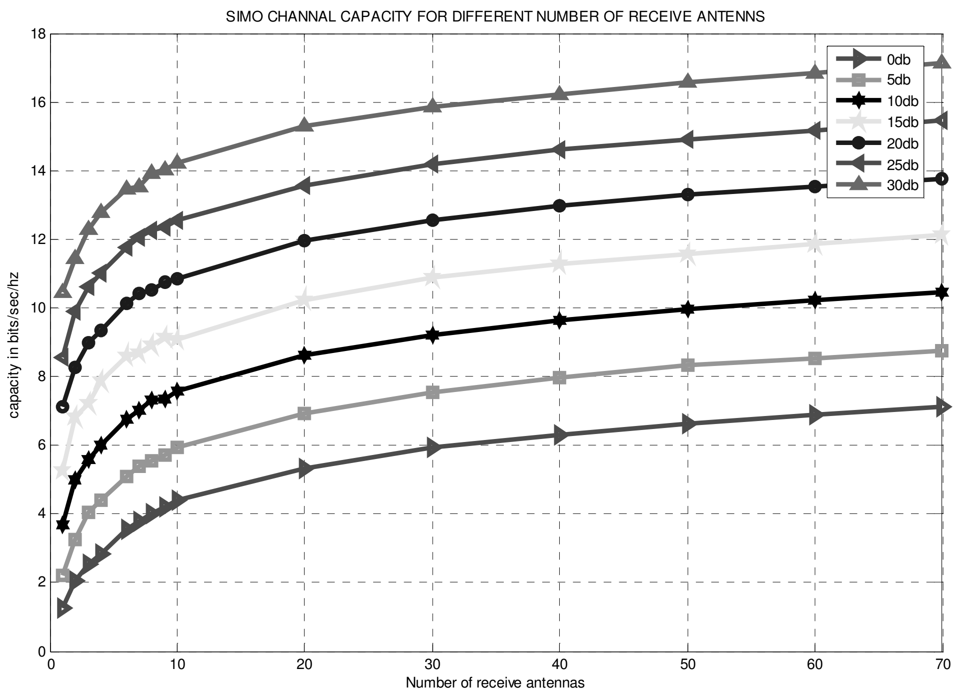

Fig 2.1 Capacity of receive diversity for different values SNR

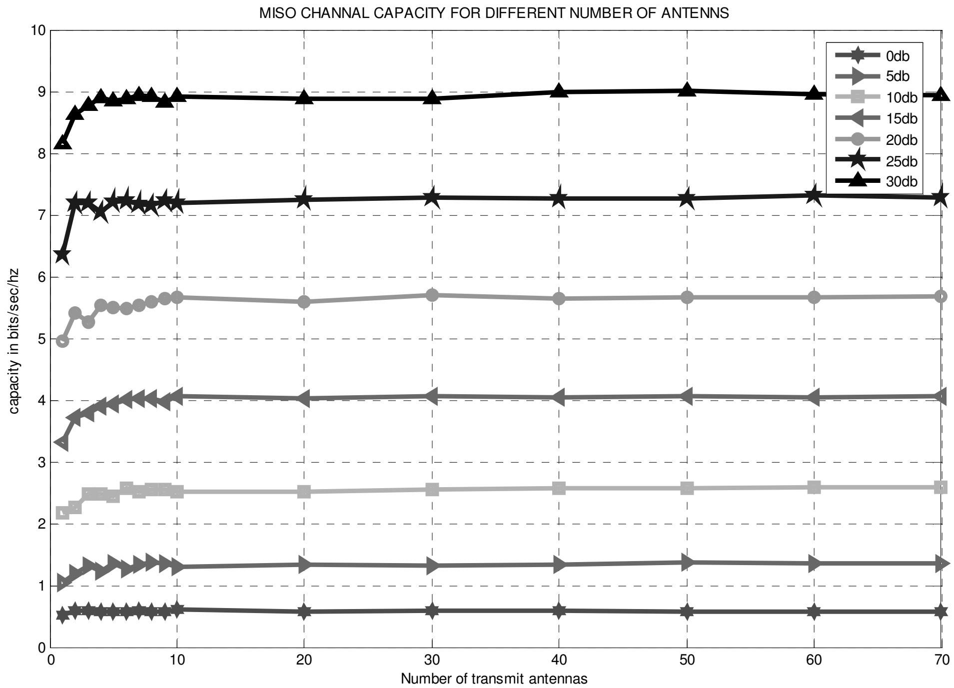

Fig 2.2 Capacity of transmit diversity for different values SNR

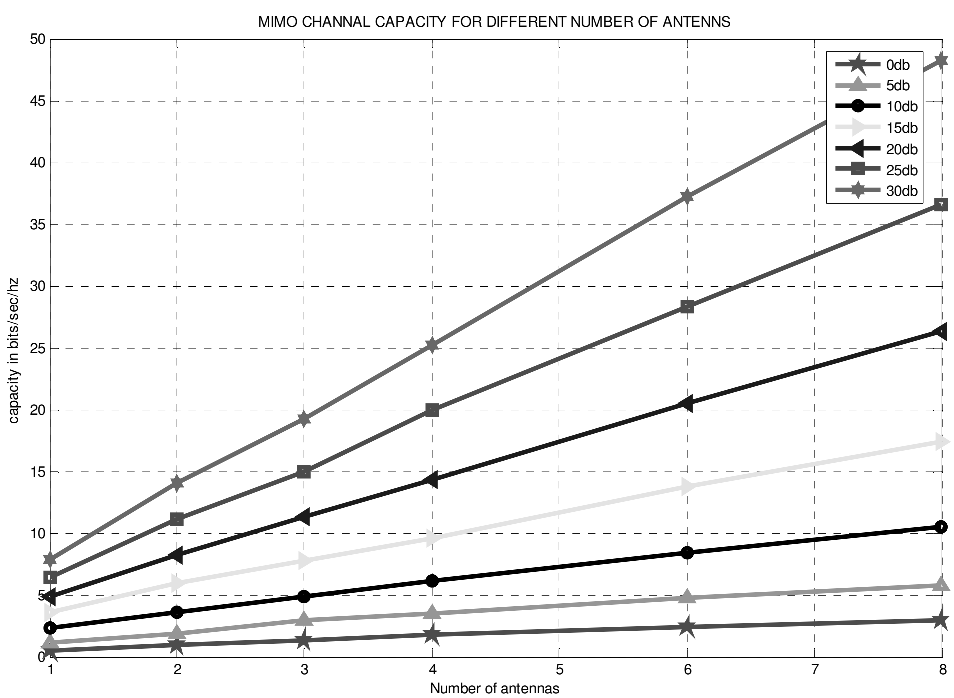

Fig 2.3 Capacity of MIMO channel for different values of SNR

From figure 2.1 we can see that for receive diversity or single input multiple output (SIMO) system, the channel capacity increases almost logarithmically with the number of receive antennas for a given signal to noise ratio. This is in agreement with the channel capacity expression for SIMO system which is given by:

$$C_n = \log_2(1 + \frac{P}{\delta^2}\sum_{i=1}^{n_R}|h_i|^2)\ bits/\sec/Hz \qquad \text{................ (2.9)}$$

Where C_n is normalized capacity with channel bandwidth. Expression (2.9) shows that increasing the number of receive antennas result in increasing the array gain through the summation, which in turn is related to the capacity logarithmically. But figure 2.2 shows that for transmit diversity or multiple input single output (MISO) system, the capacity doesn't increase that much as the number of transmit antennas increase. The maximum capacity that can be achieved in transmit diversity is that of AWGN's channel capacity. This is also implied in the capacity expression of MISO systems.

$$C_n = \log_2(1 + \frac{P}{n_T\delta^2}\sum_{i=1}^{n_T}|h_i|^2)\ bits/\sec/Hz \qquad \text{................ (2.10)}$$

Since $|h_i|^2$ has values between 0 and 1, the summation will have values between 0 and n_T. Hence the maximum capacity would be $\log_2(1+\frac{p}{\delta^2})\ bits/\sec/Hz$, which is the capacity of AWGN channel. Figure 2.4 shows the capacity versus SNR of AWGN and MISO channels.

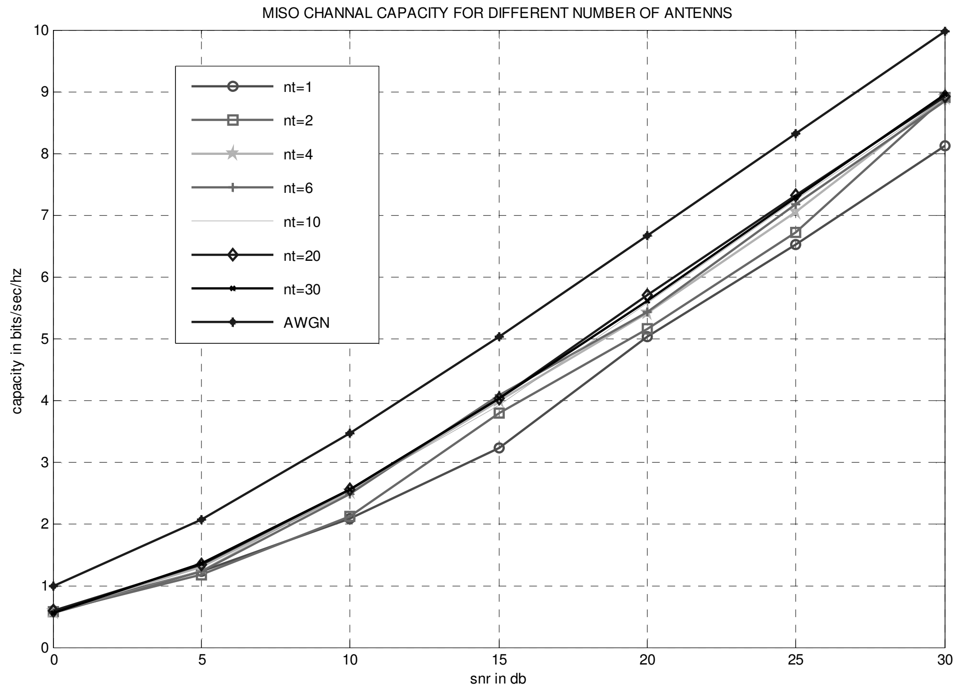

Fig 2.4 capacity of AWGN and MISO Rayleigh fading channel

Unlike figure 2.1 and 2.2, figure 2.3 shows that there exists almost a linear increase in capacity with the number of transmit and receive antennas in MIMO system. In this plot the number of transmit antennas is assumed to be equal to the number of receive antennas. All in all the above analysis shows that it is advantageous to transmit signals through different sub-channels with reduced power to enhance the capacity rather than sending the total power through a single channel.

8.4 **2.4 Performance Analysis**

For fading channels there are two performance criteria

1. the outage probability
2. the average error probability

Outage probability is the probability that the system (channel) fails to support a certain acceptable performance(data rate). Since the data rate is related to SNR (γ), the outage probability is also expressed in terms of γ. Outage probability is expressed as

$$p_{out} = p(\gamma < \gamma_0) = \int_0^{\gamma_0} p(\gamma) d\gamma \qquad \text{......................... (2.11)}$$

Where γ_0 specifies the minimum SNR required for acceptable performance. The signal to noise ratio for Rayleigh fading channel is given by $\gamma = h^2 \frac{E_s}{N_0}$, where h is the Rayleigh fading coefficient. Then it is easy to show that the PDF of γ is given by [2]

$$\begin{aligned} p(\gamma) &= p(h^2 \frac{E_s}{N_0}) = \frac{N_0}{2\delta^2 E_s} e^{\frac{-\gamma N_0}{2\delta^2 E_s}} \\ &= \frac{1}{\bar{\gamma}} e^{\frac{-\gamma}{\bar{\gamma}}} \quad where\,\bar{\gamma} = \frac{2\delta^2 E_s}{N_0} \end{aligned} \qquad \text{................... (2.12)}$$

E_s is the energy per symbol. Hence the outage probability becomes

$$p_{out} = \int_0^{\gamma_0} \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} d\gamma = 1 - e^{-\gamma_0/\bar{\gamma}} \qquad \text{.................... (2.13)}$$

The average probability of error is computed by integrating the error probability over fading distribution. i.e.

$$\bar{p}_e = \int_0^{\infty} p_e(\gamma) p(\gamma) d\gamma \qquad \text{...................... (2.14)}$$

Where $p_e(\gamma)$ is symbol error probability as a function of SNR and $p(\gamma)$ is the pdf of γ. For Rayleigh fading channel this reduces to

$$\bar{p}_e = \int_0^\infty p_e(\gamma) \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} d\gamma \qquad \text{......................} (2.15)$$

In this thesis maximum likelihood (ML) code word detection is used and therefore the metric of interest is the probability of symbol (codeword) error. Since the exact probability of vector symbol error doesn't have a closed form expression we resort to computing an upper bound on the probability of vector symbol error. The key to computing this bound is to obtain a closed form expression for a pair wise error probability. ML detection error occurs if $\|r - s_j\| < \|r - s_i\|$, where r is the received symbol vector, $s_i = hx_i$ is the vector symbol sent after channel fading and $s_j = hx_j$ is any vector symbol in the constellation set after passing through the channel, where h is the fading coefficient. Error then occurs if the distance between the received vector symbol and the symbol vector sent is greater than the distance between the received symbol vector and any other vector symbol. For equiprobable symbol vectors the decision region is symmetric with respect to each vector symbols. The decision region for a 4PSK signal is shown figure 2.5 as an example.

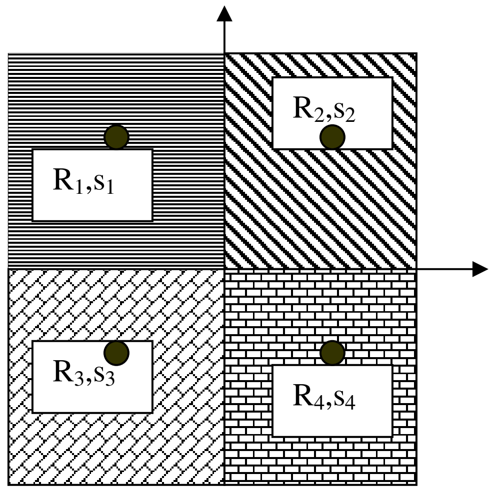

Fig 2.5 Decision regions for a 4PSK signaling

An error then will occur if the received symbol vector lies in a region different from the decision region of the vector symbol sent. The probability of vector symbol error can then be written as

$$p_e = \sum_{i=1}^{M} p(r \notin R_i / s_i) p(s_i) \qquad \text{.................} \; (2.16)$$

Where: s_i is the vector symbol sent and

R_i is its decision region.

For equiprobable vector symbols, it reduces to [1]

$$\begin{aligned} p_e &= \frac{1}{M} \sum_{i=1}^{M} p(r \notin R_i / s_i) \\ &= 1 - \frac{1}{M} \sum_{i=1}^{M} p(r \in R_i / s_i) \\ &= 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{R_i} p(r / s_i) dr \end{aligned} \qquad \text{.......................} \; (2.17)$$

The above equation gives the exact vector symbol error probability but is difficult to solve in a closed form. Hence we need to switch to the union bound on error probability which yields a closed form expression that is a function of the distance between signal constellation points.

Let A_{ij} denote the probability that $\|r - s_j\| < \|r - s_i\|$, given that the symbol vector s_i is sent. If the event A_{ij} occurs, then the symbols will be decoded in error. Thus

$$\begin{aligned} p_e = p(\bigcup_{\substack{i=1 \\ i \neq j}}^{M} A_{ij}) &\leq \sum_{\substack{i=1 \\ i \neq j}} p(A_{ij}) \\ &\leq \sum_{\substack{i=1 \\ i \neq j}}^{M} p(\|r - s_j\| < \|r - s_i\| / s_i) \end{aligned} \qquad \text{.............. (2.18)}$$

The probability of decoding s_j given that s_i is transmitted is known as the pairwise error probability and is given by the expression [3]

$$p(s_i \longrightarrow s_j) = Q\left(\sqrt{\frac{E_s}{2N_0} d_{ij}^2}\right) \qquad \text{..................... (2.19)}$$

Where $d_{ij}{}^2 = \|s_i - s_j\|^2$

There for,

$$\begin{aligned} p_e &\leq \sum_{\substack{i=1 \\ i \neq j}}^{M} p(s_i \longrightarrow s_j) \\ &\leq \sum_{\substack{i=1 \\ i \neq j}}^{M} Q\left(\sqrt{\frac{E_s}{2N_0} d_{ij}^2}\right) \end{aligned}$$

The average probability of error over the M possible symbol vectors is then given by:

$$\overline{p}_e \leq \frac{1}{M}\sum_{j=1}^{M}\sum_{\substack{i=1\\ i\neq j}}^{M} Q\left(\sqrt{\frac{E_s}{2N_0}d_{ij}^2}\right) \qquad \text{.....................} \ (2.20)$$

Which is the average union bound for symbol vector error. We can further bound $\overline{p}_e$ by defining d_{min}=min d_{ij}, for i,j = 1,2,…M, then

$$\overline{p}_e \leq \frac{1}{M}\sum_{j=1}^{M}\sum_{\substack{i=1\\ i\neq j}}^{M} Q\left(\sqrt{\frac{E_s}{2N_0}d_{ij}^{\ 2}}\right) \leq (M-1)Q\left(\sqrt{\frac{E_s}{2N_0}d_{\min}^{\ 2}}\right) \qquad \text{...........} \ (2.21)$$

It is obvious that this bound is some what looser as there may exist some d_{ij}s that are greater than d_{min}. Some times $\overline{p}_e$ is approximated as the probability of error associated with constellations at the minimum distance d_{min} multiplied by the number of neighbors (N_{min}) at this distance. i.e. $\overline{p}_e \approx N_{\min} Q\left(\sqrt{\frac{E_s}{2N_0}d_{\min}^2}\right)$. This is called the nearest neighbor approximation which is always less than the union bound since this approximation does not include the error associated with constellations further apart than the minimum distance.

9 CHAPTER THREE

10 SPATIAL DIVERSITY

10.1 3.1 Introduction

Spatial diversity is one of the diversity techniques that do not require any additional bandwidth and does not introduce much decoding delay. A spatial diversity system equipped with n_T transmit and n_R receive antennas is shown in figure 3.1

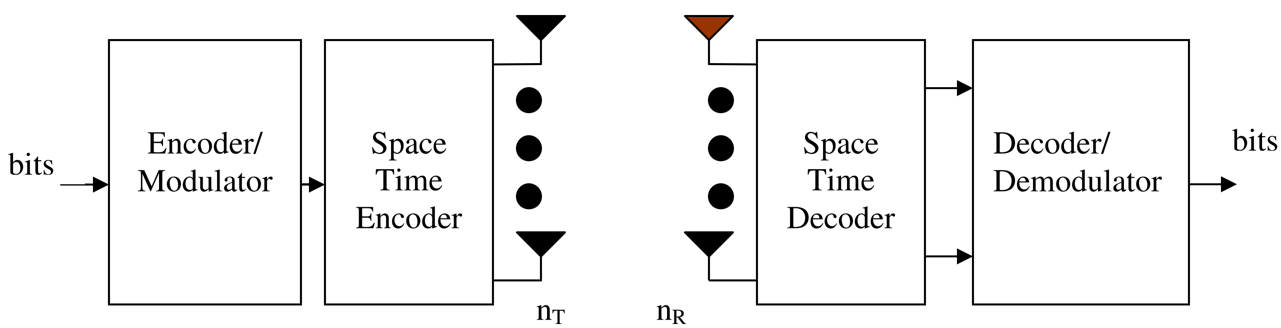

Fig 3.1 spatial diversity system

In MIMO diversity, the same information is spread across n_T multiple antennas to maximize the diversity advantage. The sequence of bits after being encoded and modulated is mapped to the respective antennas by the space time encoder. The receiver equipped with n_R receive antennas is supplied with n_R independent copies of the transmitted signal, then the probability that all n_R signals fade simultaneously is p^{n_R}, where p is the probability that any one signal is in fade.

Diversity transmission due to its inherent redundancy across the transmit antennas is typically a high reliability system. For Rayleigh matrix channel optimum MIMO diversity techniques obtain a diversity advantage as large as the product of the number of transmit and receive antennas ($n_T \times n_R$). If at least one of these possible paths is not fading, successful communication is possible.

The concept of diversity reception and combining techniques are introduced in the next section.

10.2 3.2 Diversity Combining Techniques

Diversity combing methods enable us to exploit the advantage of the independently faded signals. There are four main types of combining techniques: switched combining, selection combining, equal gain combining (EGC) and maximal ratio combining (MRC). The signal to noise ratio (SNR) at the output of these different techniques is a function of the number of diversity paths and fading distribution of each path.

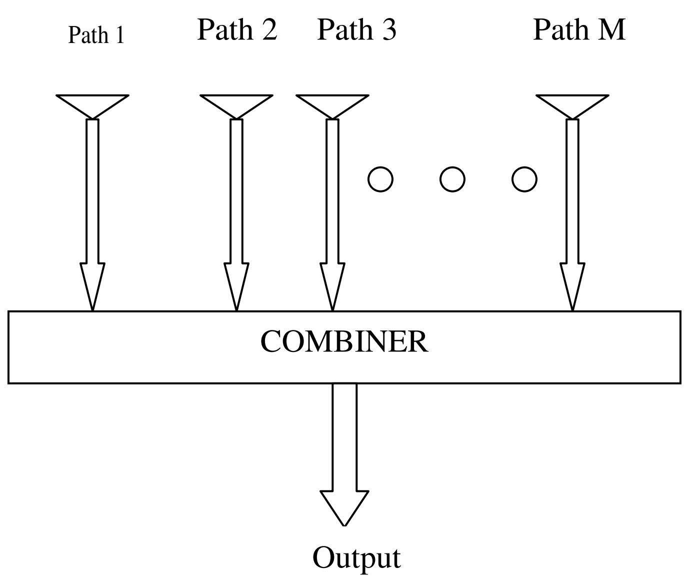

Fig 3.2 diversity combining schemes

In switched combining the receiver scans for a path with SNR above a certain predetermined threshold. This path is selected as an output until its SNR drops below the specified threshold. When this happens, the receiver starts scanning and switch to another path with SNR above the threshold. Hence only one receiver is required that switch into the active path (channel).

In selection combining a path with the largest instantaneous SNR is selected as an output. Hence the output SNR is equal to that of the best incoming signal. Compared to the switched combining, selection combining do have better performance as it continuously pick up the best instantaneous path. However it is some what difficult to implement as it requires continuous and simultaneous monitoring of all the diversity paths. Meaning more number of receivers are required. Both selection and switched combing methods do not require any knowledge of the channel state information, hence they can be used in conjunction with coherent and non coherent detection. Let's try to see the outage probability of selection combining.

For M-path, the cumulative distribution function (CDF) for SNR (γ) is given by [1]

$$p_{out}(\gamma_0) = p(\gamma < \gamma_0) = p[\max(\gamma_1, \gamma_2, ...\gamma_M) < \gamma_0] \qquad \text{........ (3.1)}$$

Note that equation 3.1 is the same as equation 2.11, implying that the above equation is the outage probability of selection combining. For independently distributed γ, this reduces to

$$
\begin{aligned}
p_{out}(\gamma_0) &= \prod_{i=1}^{M} p(\gamma_i < \gamma_0) \\
&= \prod_{i=1}^{M} \int_0^{\gamma_0} p(\gamma_i)
\end{aligned}
\qquad \text{............... (3.2)}
$$

For Rayleigh fading channel $p(\gamma_i)$ is given by

$$
\begin{aligned}
& p(\gamma_i) = \frac{1}{\bar{\gamma}_i} e^{-\gamma/\bar{\gamma}_i} \; and \\
& \int_0^{\gamma_0} p(\gamma_i) = \int_0^{\gamma_0} \frac{1}{\bar{\gamma}_i} e^{-\gamma/\bar{\gamma}_i} d\gamma = 1 - e^{-\gamma_0/\bar{\gamma}_i}
\end{aligned}
\qquad \text{............ (3.3)}
$$

Where $\bar{\gamma}_i$ is the average signal to noise ratio of the ith path and the result of the integral being the outage probability ith path. Substituting this in the above equation yields the outage probability (p_{out}) of selection combining.

$$
p_{out}(\gamma_0) = \prod_{i=1}^{M} (1 - e^{-\gamma_0/\bar{\gamma}_i}) \qquad \text{.................. (3.4)}
$$

If all the channels (paths) do have same $\bar{\gamma}$, then this reduces to

$$
p_{out}(\gamma_0) = [1 - e^{-\gamma_0/\bar{\gamma}}]^M \qquad \text{.................. (3.5)}
$$

Figure 3.3 shows outage probability (P_{out}) of selection combining for 1,2,4,6,10,20 independent channel paths at different average SNRs.

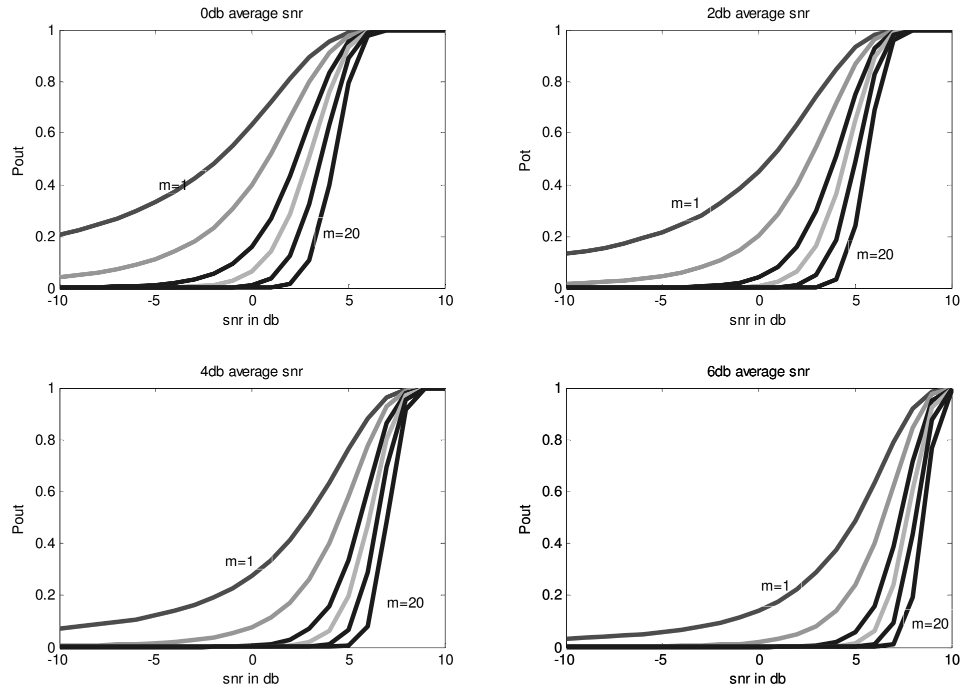

Fig 3.3 outage probability of selection combining

From the plot we can see that increasing the number of independent paths result in reduction of outage probability at lower threshold values. There is big reduction in outage probability when number of path is increased from one path to two than from two to four or from ten to twenty, indicating that the return in reduction in outage probability is small as the number of paths is increased. This reduction no longer even be possible for threshold SNRs above certain values.

Replacing γ_0 by the random variable γ and differentiating this with respect to γ yields the probability density function (pdf) of γ.

$$p(\gamma) = \frac{M}{\bar{\gamma}}[1 - e^{-\gamma/\bar{\gamma}}]^{M-1} e^{-\gamma/\bar{\gamma}} \qquad \text{...................... (3.6)}$$

Equal gain combining is a linear combining method where the various inputs are equally weighted and added together to get an output signal. The output signal is given by $r = \sum \alpha_i r_i$ where $\alpha_i = e^{-j\phi_i}$ and ϕ_i is the phase of i[th]'s path (channel) received signal. But in maximum ratio combining, the weighting factor of each channel is chosen to be in proportion to its own signal magnitude. i.e. $\alpha_i = |r_i| e^{-j\phi_i}$. The combiner output then will be equal to $r = \sum_{i=1}^{M} |r_i|^2$. Thus, the output SNR of the MRC is the sum of the SNR's of each path resulting in a linear increase of SNR with the number of paths. MRC is an optimum combining method since it can maximize the output SNR [4]. Each individual signals is co-phased, weighted with its corresponding amplitude and then summed. Both EGC and MRC are used in conjunction with coherent combining but are not practical for non-coherent combining.

To obtain the pdf of γ for MRC, [2] has taken the product of the characteristics functions assuming i.i.d Rayleigh fading channel on each channel with equal average SNR ($\bar{\gamma}$) and has found the following expression

$$p(\gamma) = \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}(M-1)!} \;, \gamma > 0 \qquad \text{.................... (3.7)}$$

The corresponding outage probability is given by the expression [1]

$$p_{out} = p(\gamma < \gamma_0) = \int_0^{\gamma_0} p(\gamma) d\gamma = 1 - e^{-\gamma/\bar{\gamma}} \sum_{k=1}^{M} \frac{(\gamma_0 / \bar{\gamma})^{k-1}}{(k-1)!} \qquad \text{.................. (3.8)}$$

Figure 3.4 shows the outage probability of MRC for 1,2,4,6,10,20 paths at different average SNR.

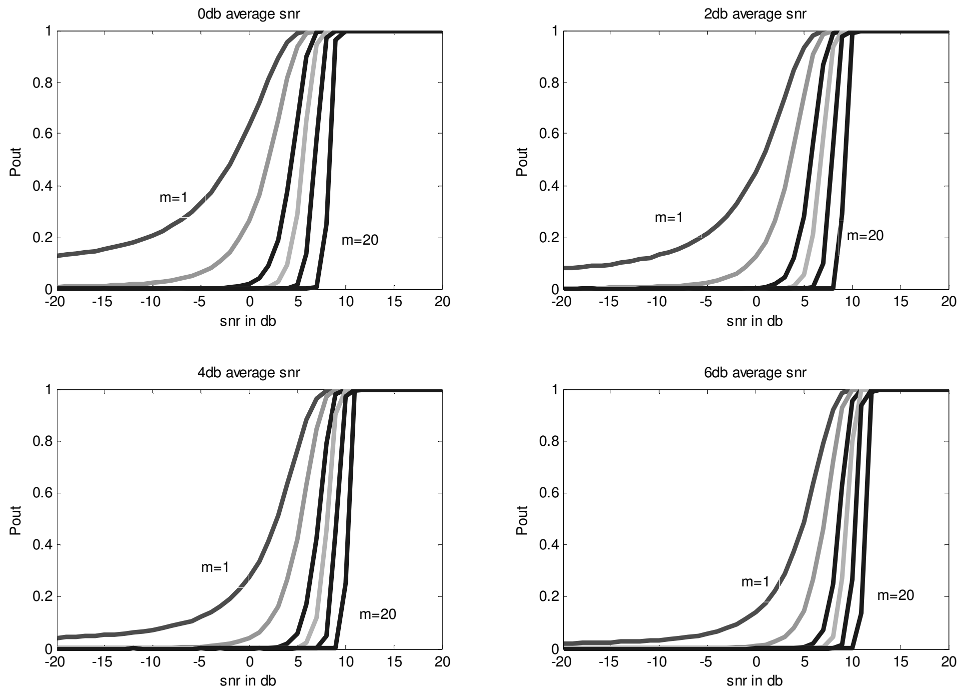

Fig 3.4 outage probability of MRC

Compared to selection combining, maximum ratio combining pushes the SNR rage over which outage probability of less than one is possible. This indicates that MRC outperform SC.

10.3 **3.3 Space Time Coding**

MIMO wireless systems are used in conjunction with space time coding. There are different types of space time coding including space time block code (STBC), space time trellis code (STTC), space time turbo trellis code and layered space time (LST) code. In this thesis STBCs, in particular Alamouti's scheme is used. A space time encoder in the figure 3.1 above maps the sequence of symbols in to a sequence of matrix symbols of dimension $n_T \times L$, where L is STC's frame length. For Alamouti's scheme the encoder takes two symbols x_1 and x_2 at a time and maps them to the two transmit antenna according to code matrix

$$X = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \qquad \text{.............................. (3.9)}$$

The encoder outputs are thus transmitted in two consecutive transmission periods where the column of the code matrix indicates symbols transmitted simultaneously and the rows indicate symbols transmitted from respective antennas. It is clear that the encoding is done in both time and space domains.

Lets assume that two receive antennas are used at the receiver and also assume that the fading coefficient be constant for at least two consecutive symbol periods. Then the received signals over the two symbol interval then is given by

$$\begin{bmatrix} r_1^1 \\ r_2^1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1^1 \\ n_2^1 \end{bmatrix} \text{and} \begin{bmatrix} r_1^2 \\ r_2^2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + \begin{bmatrix} n_1^2 \\ n_2^2 \end{bmatrix} \quad \text{......(3,10)}$$

Where r_j^i is the received signal at time i by receive antenna j and n_j^i is the additive noise of receive antenna j at time i. With the assumption of full channel information, the receiver chooses $x_1 \, and \, x_2$ based on Maximum Likelihood (ML) criteria that minimizes the distance metric

$$\begin{aligned} &d^2(r_1^1, h_{11}x_1 + h_{12}x_2) + d^2(r_2^1, h_{21}x_1 + h_{22}x_2) + d^2(r_1^2, -h_{11}x_2^* + h_{12}x_1^*) + \\ &d^2(r_2^2, -h_{21}x_2^* + h_{22}x_1^*) \end{aligned} \qquad \text{..... (3.11)}$$

Substituting for respective r_j^i, the ML decoding can be expressed as [5]

$$(x_1, x_2) = \min_{(x_1,x_2)\in S} \begin{bmatrix} (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 - 1)(|x_1|^2 + |x_2|^2) + \\ d^2(x_1', x_1) + d^2(x_2', x_2) \end{bmatrix} \quad \text{...... (3.12)}$$

Where

$$\begin{aligned} x_1' &= h_{11}^* r_1^1 + h_{12}(r_1^2)^* + h_{21}^* r_2^1 + h_{22}(r_2^2)^* \\ x_2' &= h_{12}^* r_1^1 - h_{11}(r_1^2)^* + h_{22}^* r_2^1 - h_{21}(r_2^2)^* \end{aligned} \quad \text{............. (3.13)}$$

After substitution of r_j^is

$$\begin{aligned} x_1' &= (|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2)x_1 + h_{11}^* n_1^1 + h_{12}(n_1^2)^* + h_{21}^* n_2^1 + h_{22}(n_2^2)^* \\ x_2' &= (|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2)x_2 + h_{12}^* n_1^1 - h_{11}(n_1^2)^* + h_{22}^* n_2^1 - h_{21}(n_2^2)^* \end{aligned} \quad \text{..... (3.14)}$$

As can be seen, the decision statistics for x_j', j =1,2, is a function of only x_j, thus the ML decoding rule of equation 3.12 can be separated in to two independent decoding rules

$$\begin{aligned} x_1 &= \min_{x_1\in S}\left[(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 - 1)|x_1|^2 + d^2(x_1', x_1)\right] \\ x_2 &= \min_{x_2\in S}\left[(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 - 1)|x_2|^2 + d^2(x_2', x_2)\right] \end{aligned} \quad \text{......... (3.15)}$$

For MPSK modulation, all the symbols in the constellation set S have equal energy, hence the ML decoding rule then becomes

$$\begin{aligned} x_1 &= \min_{x_1\in S} d^2(x_1', x_1) \\ x_2 &= \min_{x_2\in S} d^2(x_2', x_2) \end{aligned} \quad \text{................ (3.16)}$$

The ML detection of the code matrix X at the receiver then reduces to ML detection of symbols that set up the code matrix. Hence the computational complexity of ML detection of the Alamuti's STBC is reduced by far.

11 CHAPTER FOUR

12 SPATIAL MULTIPLEXING

12.1 4.1 Introduction

Information theory research has shown that the rich scattering environment of wireless channel is capable of enormous theoretical capacities if the channel is properly exploited. Especially techniques that use arrays of multiple transmit and receive antennas may offer high capacity to present and future wireless communications systems. Multiple input multiple output (MIMO) systems provide a linear increase in capacity with the number of antenna elements, affording significant increase in capacity over single input single output systems. Spatial Multiplexing (SM) is a way of space time coding in which independent information streams are transmitted from each antenna in MIMO wireless system. Unlike spatial diversity, the main objective in this scheme is to increase the throughput of communication channel. The high level block diagram of spatial multiplexing system is shown in the figure 1.

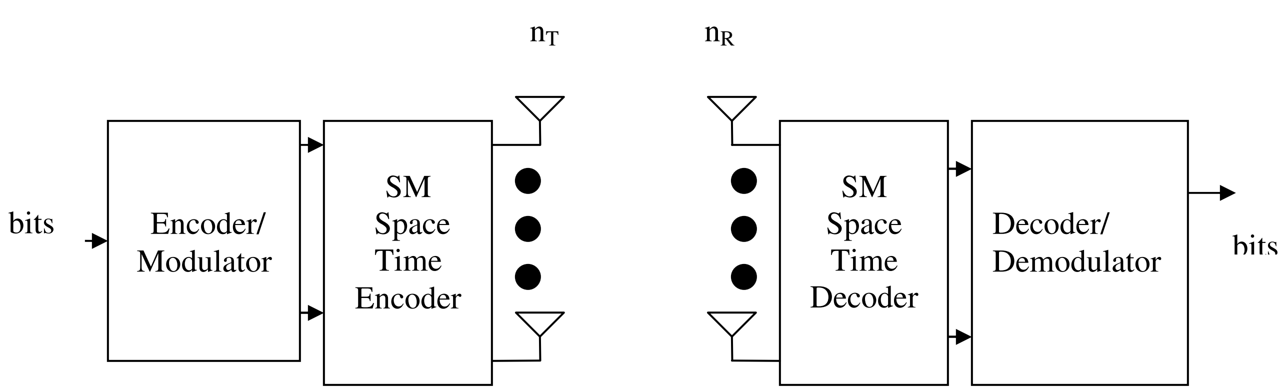

Fig 4.1, block diagram of spatial multiplexing system

In the figure 4.1, the single data stream is encoded, modulated and demultiplexed in to n_T branches by a space time encoder and transmitted by each of the n_T transmit antennas. The mapping of symbols to each of the transmit antenna can be done in to two common schemes: Diagonal Blast (D-BLAST) and Vertical Blast (V-BLAST).

In D-BLAST architecture the modulated symbol of each encoder is distributed among the n_T transmit antennas along the diagonal of the transmission array. For a three by three antenna array this encoding process is shown below.

$$\begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & x_4^1 \ldots. \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 \ldots \\ x_1^3 & x_2^3 & x_3^3 & x_4^3 \ldots \end{bmatrix} \Rightarrow \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & x_4^1 \ldots & & \\ 0 & x_1^2 & x_2^2 & x_3^2 & x_4^2 \ldots & \\ 0 & 0 & x_1^3 & x_2^3 & x_3^3 & x_4^3 \ldots \end{bmatrix} \Rightarrow \begin{bmatrix} x_1^1 & x_1^2 & x_1^3 & x_4^1 \ldots. \\ 0 & x_2^1 & x_2^2 & x_2^3 \ldots. \\ 0 & 0 & x_3^1 & x_3^2 \ldots. \end{bmatrix} \quad \ldots. (4.1)$$

where x_i^j is the symbol transmitted from antenna j at time i. In the first transmission matrix, the output of each modulator is supposed to be mapped to each of the respective transmit antennas. But in D-BLAST architecture the lower rows are padded with zeros that introduce delays as is shown in the second transmission matrix. Finally the symbols in the first diagonal are transmitted from antenna one, symbols in the second diagonal are transmitted from antenna two and so on. So, the rows of the last transmission matrix indicate symbols transmitted from the respective antennas. As we can see from this last transmission matrix, in the first transmitting time slot only the first antenna will transmit and at the second transmitting time slot only antenna one and two will transmit and antenna 3 will start transmitting at the third time slot.

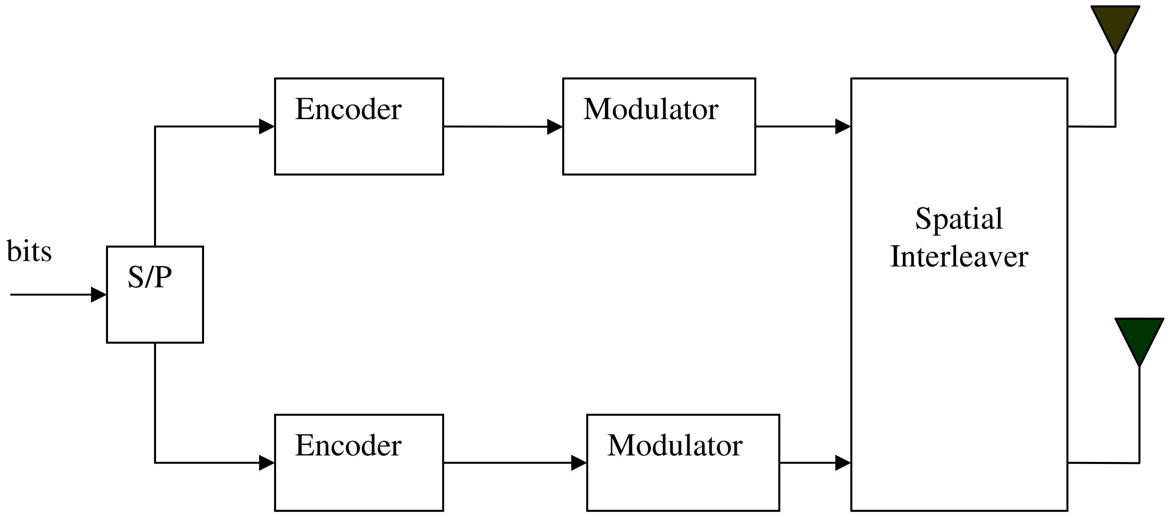

Fig 4.2 block diagram of a two layer DBLAST

The diagonal layering introduces space diversity and thus achieves a better performance even though there is some rate loss since the portion of the transmission matrix on the left is padded with zeros. The added complexity in D-BLAST architecture makes its initial implementation some what difficult and led to V-BLAST architecture.

In V-BLAST the information sequence is modulated by an M level modulation and then demultiplexed in to n_T substreams for transmission by the n_T antenna arrays. i.e. the encoding process is simply a demultiplexing operation, no inter-substream coding or interleaving of any kind is used. The signal processing related to an individual sub-stream is referred to as layer.

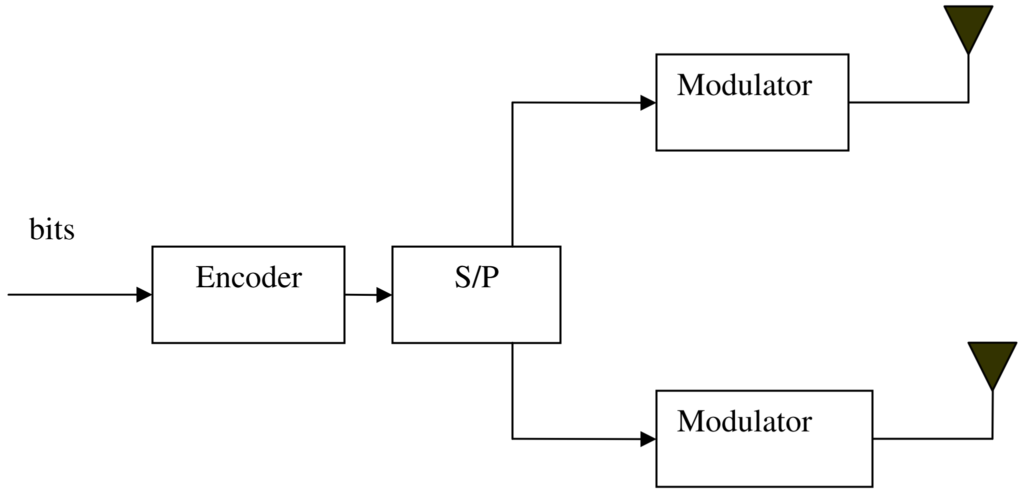

Fig 4.3 block diagram of a two layer VBLAST

This scheme is implemented in real time in laboratory and has demonstrated to have spectral efficiency of 20-40 bits/Hz at an average SNR of 24-34db in an indoor environment with 8 transmit and 12 receive antennas [11]. The V-BLAST transmission matrix for a two transmit antenna would like:

$$\begin{bmatrix} s_1 & s_3 & s_5 & \ldots \\ s_2 & s_4 & s_6 & \ldots \end{bmatrix} \qquad \text{.......................} \ (4.2)$$

Where the first row is symbols transmitted from antenna one and the second row is symbols transmitted from the second antenna. Had the number of transmit antenna be greater than two, the number of rows of the transmission matrix which is equal to the number of transmit antennas would be greater than two. Columns of the transmission matrix indicate symbols transmitted at time instant t, t+1 etc. Unlike the transmission matrix of 3.9, four different symbols are transmitted in two symbol periods and hence added rate of transmission.

Receive antennas n_1-n_R receives signals radiated from all transmit antennas, hence the need arise to remove the mixing operation of the channel. Depending on how this is done, there are different methods of receiver implementation, the most common being Maximum Likelihood (ML), Zero Forcing (ZF) and Minimum Mean Square Error (MMSE).

Maximum likelihood (ML) is the optimum but computationally intensive receiver implementation where the distance between possible code vectors is the parameter of interest. In this scheme, the receiver first finds the hypothesized received symbol vectors for the given channel realization and then selects code vector that do have a minimum distance with the received symbol vector. i.e.

$$\hat{X} = \min_{x \varepsilon S} \left\| Y - HX \right\|^2 \qquad \text{......................... (4.3)}$$

where Y is the received vector and the minimization is done over all set of possible code vectors S.

12.2 **4.2 Zero Forcing Method**

Let the received signal at time t be expressed as:

$$Y = Hx + n \qquad \text{.................................. (4.4)}$$

Where: Y is n_R by one column matrix.
H is n_R by n_T transition matrix, and
n is the additive noise vector

This can be also be written in another form as:

$$Y = x_1h_1 + x_2h_2 + x_3h_3 + \ldots + x_{nt}h_{nt} + n \qquad \text{.................... (4.5)}$$

were x_i is the symbol transmitted from the i^{th} antenna and h_i is the i^{th} column of the transition matrix. To perform the unmixing operation the received signal is multiplied by vector w such that:

$$w_i\, h_j = \begin{Bmatrix} 0, & i \neq j \\ 1, & i = j \end{Bmatrix} \qquad \text{.......................... (4.6)}$$

The decision statistics for the i^{th} transmitted symbol can then be formulated as:

$$\begin{aligned} d_i &= w_i\; Y \\ &= w_i\, h_1 x_1 + w_i\, h_2 x_2 + \ldots + w_i\, h_i x_i + \ldots + w_i\, h_{nt} x_{nt} + w_i\, n \\ &= 0 + 0 + \ldots + x_i + \ldots 0 + w_i\, n \end{aligned} \qquad \text{............. (4.7)}$$

Then hard decision on the decision statistics d_i will result in the symbol transmitted from the i^{th} transmit antenna. The multiplication of the received vector signal by w_i is called nulling. i.e. nulling out the effect of the other symbols on the current symbol under detection. The above process where the effects of all the non current symbols are multiplied by zero is called Zero Forcing (ZF). [24] has shown that the nulling matrix w for the ZF method is given by:

$$w \;= (H^* H)^{-1} H^* \qquad \text{.............................. (4.8)}$$

Zero forcing method is a detection method where the effect of the additive noise is not considered rather its effect is enhanced through multiplication by w. Nulling operation is usually followed by canceling operation where the effect of the already detected symbol is removed from the received signal. This generally improves the performance of the scheme. The procedure is as follows: the decision statistics for the first symbol to be decoded is given by:

$$d_1 = w_1 Y \qquad \text{.....................} \ (4.9)$$

The hard decision on d_1 will result in the symbol x_1. The effect of x_1 on other symbols yet to be detected is removed (canceled) through the operation

$$Y_2 = Y - h_1 x_1 \qquad \text{................} \ (4.10)$$

After finding w_2, x_2 can be decoded in similar way using Y_2 as a received signal and the process continues till the last symbol.

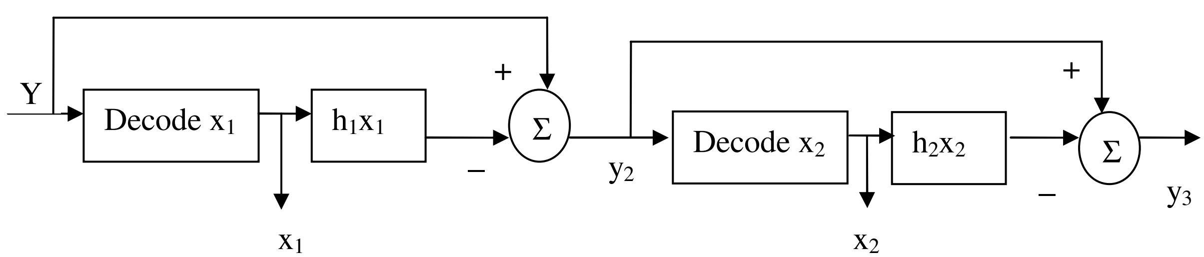

Fig 4.4 block diagram of SIC detection of VBLAST

This operation is in general termed as Successive Interference Cancellation (SIC). When combined with ZF method it is called Zero Forcing with Successive Interference Cancellation (ZF-SIC).

12.3 **4.3 Minimum Mean Square Error Method**

In Minimum Mean Square Error (MMSE) method the vector w (nulling matrix) is selected in such a way that the mean square error between the transmitted vector and the linear combination of the received vectors wY is minimized. i.e. w selected in such that $E[(x-w\ Y)^2]$ is minimized, where E is an expectation operator. [24] has shown that w satisfying this is given by:

$$w=[H^*H+\frac{I_{n_T}}{\gamma}]^{-1}H^* \qquad \text{.......................... (4.11)}$$

where γ is the SNR value. Similar to the ZF method, the nulling operation of MMSE is followed by cancellation. The general procedure for MMSE-SIC is as follows [5]:

$$
\begin{aligned}
&set\ i=n_T\\
&\quad y_{n_T}=Y\\
&\quad while\ i\geq 1\\
&\qquad \{\\
&\qquad\quad w=[H^*H+\frac{I_{n_T}}{\gamma}]^{-1}H^*\\
&\qquad\quad d_i=w_i y_i\\
&\qquad\quad x_i=q(d_i) \qquad \%hard\ decision\ on\ y_i\\
&\qquad\quad y_{i-1}=y_i-x_i'h_i \qquad \%h_i\ is\ i^{th}\ column\ of\ H\\
&\qquad\quad H=H_d^i\\
&\qquad\quad i=i-1\\
&\qquad \}
\end{aligned}
$$

Where x_i is the ith layer transmitted symbol, x_i' is the estimated ith layer symbol and y_i is the received signal after $(n_T\text{-}i)^{th}$ cancellation step. H_d^i is a deflated channel matrix formed by deleting the i^{th} and higher columns or a column corresponding to the already detected (recovered) symbol.

The simulated bit error performance with the assumption $p_b \approx \dfrac{p_s}{\log_2 M}$ for ML, MMSE-SIC and ZF-SIC methods is shown in figure 4.5 for a 4 by 4 antenna system with QPSK modulation.

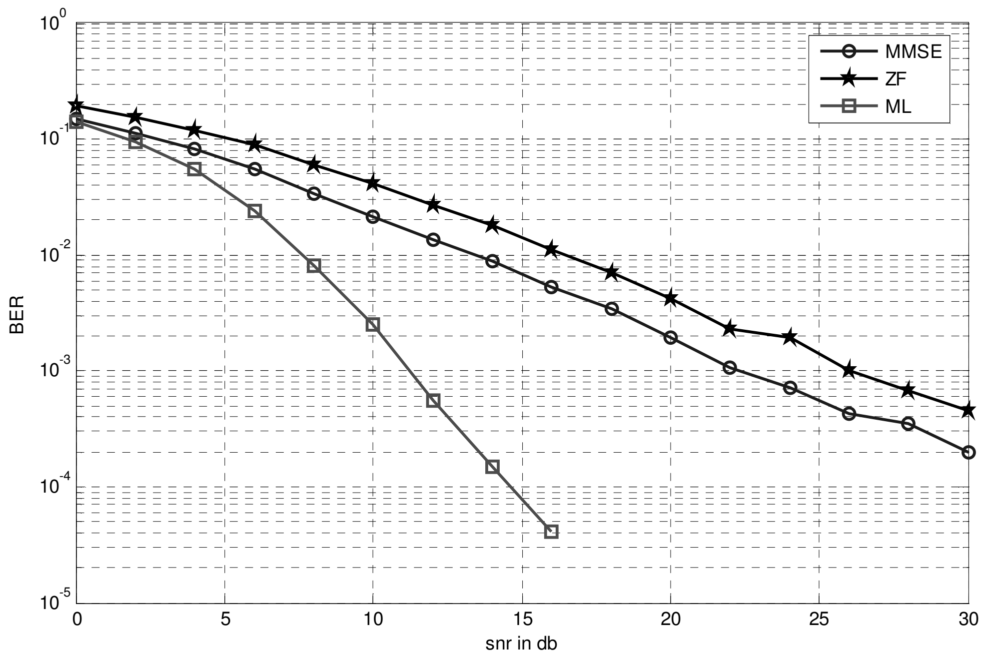

Fig 4.5, Bit error performance of ML, MMSE-SIC and ZF-SIC receivers

In the SIC scheme correct detection of symbols is assumed in each layer cancellation, but this may not be the case and results in error at one layer to be propagated to other layers.

$$r^i = h_i x_i + \sum_{j=i+1}^{n_T} h_j (x_j - x'_j) + \sum_{j=1}^{j=i-1} h_j x_j + n_i \qquad \text{.................. (4.12)}$$

Where the first term represents the desired layer, the second term represents the error propagating from higher layers and the third term is the uncancelled layers and the last term is AWGN. Under the assumption of ideal cancellation, the second term is zero as $x_j = x_j'$ and error propagation wouldn't occur.

The effect of this error propagation can be minimized by proper ordering of the detection process. i.e. first recover a symbol corresponding to a signal with high SNR in such a way that the probability of committing an error in its detection is minimized. Subtract the effect of this symbol from the received signal and then recover the symbol corresponding to a signal with the next higher SNR and keep on till the end. Some times this approach is called best first cancellation and [11] has shown the mathematical proof of the optimality gain obtained by this proper ordering. The other method of hindering error propagation is through the use of external channel coding, not covered in this thesis.

The performance of MMSE-SIC and ZF-SIC with ordering is shown in figure 4.6 for a 4 by 4 antenna array with QPSK modulation.

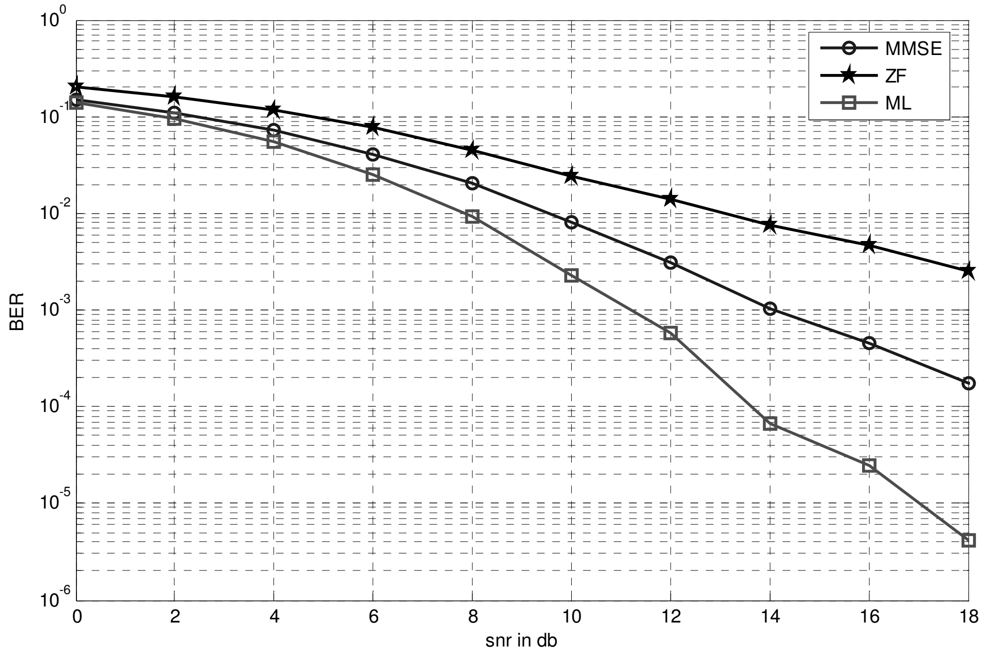

Fig 4.6, Bit error performance of ML and ordered MMSE-SIC and ZF-SIC

Both figure 4.5 and 4.6 shows that the performance of ML detection method outperforms MMSE-SIC and ZF-SIC detection methods and hence used in this thesis. There is a huge performance improvement in MMSE-SIC with optimum ordering but is still poor compared to ML.

CHAPTER FIVE

ADAPTIVE MODULATION

12.4 5.1 Introduction

Adaptive modulation is a way of constructing spectrally efficient and reliable communication system in a wireless channel. It is based on the idea of estimating the channel at the receiver and feeding this estimate back to the transmitter. The transmitter can then adapt its transmission parameters according to the channel fading statistics. Modulation that do not adapt to the channel condition must be designed for the worst fading condition for acceptable performance. For example, Rayleigh fading channel can cause a signal power loss up to 30db [24], designing for this fading statistics will result in system that is power and spectral inefficient. Adapting to the channel fading statistics can increase the average throughput, reduce transmit power or reduce the average probability of error.

Adaptive systems require a feedback path through which the estimate of a channel parameters is sent. And this path needs to be error free which can be achieved by using strong signal and/or strong error correction coding. Adaptive systems would perform poorly if the channel fading statistics changes at a rate faster than it can reliably estimated and feedback to the transmitter, implying that adaptive modulation should be used in conjunction with slow fading channels. And it is important to make the amount of feedback information as small as possible. The other point that needs to be considered in adaptive system is the comparison of the advantage provided by adaptive scheme (in terms of power or rate) to that of the cost it needs in terms of feedback bandwidth.

5.2 Types of adaptive modulation

Depending on the parameter the transmitter varies, adaptive schemes could be power adaptive, code adaptive or rate adaptive. SISO system power adaptation scheme generally used to compensate for SNR variation due to fading. The goal is to maintain constant SNR so that the channel appears as an AWGN channel to the demodulator. This power inversion is given by [1]

$$\frac{s(\gamma)}{\bar{s}} = \frac{\sigma}{\gamma} \qquad \text{.................................. (5.1)}$$

Where: $s(\gamma)$ transmitted power as a function of SNR

$\bar{s}$ is average transmitted power

σ required constant SNR

γ instantaneous SNR value

For a certain constant average power value of $\bar{s}$, the achievable σ is governed by the equation [1]:

$$\int \frac{s(\gamma)}{\bar{s}} p(\gamma) d\gamma = \int \frac{\sigma}{\gamma} p(\gamma) d\gamma = 1 \qquad \text{......................... (5.2)}$$

From which $\sigma = 1/E[1/\gamma]$. So if the amount of σ required to meet the application requirement is greater than $1/E[1/\gamma]$, then it can't be met. For Rayleigh fading channel $E[1/\gamma] = \infty$, implying that any targeted performance with zero outage can't be met. Fading can also be inverted above a certain cutoff value γ_0 leading to a truncated channel inversion which is given by:

$$\frac{s(\gamma)}{\bar{s}} = \begin{cases} \dfrac{\sigma}{\gamma} & ,\gamma \geq \gamma_0 \\ 0 & ,\gamma < \gamma_0 \end{cases} \qquad \text{(5.3)}$$

The other adaptation technique is water filling where the main objective is to take advantage of the good channel condition. When the channel is in good condition (large SNR) more power and higher data rate is used, but when it is in bad condition (small SNR) less power and small data rate is used. The amount of power allocated for a given SNR γ is given by[1]:

$$\frac{s(\gamma)}{\bar{s}} = \begin{cases} \dfrac{1}{\gamma_0} - \dfrac{1}{\gamma} & ,\gamma \geq \gamma_0 \\ 0 & ,\gamma \leq \gamma_0 \end{cases} \qquad \text{(5.4)}$$

The power adaptation seen above is done in time domain. In MIMO system the adaptation is done in time as well as spatial domain (antenna selection) which make the adaptation some what more complex. In addition to that this adaptation technique needs full channel state information at the transmitter which would require high capacity feedback channel that makes its cost higher.

In adaptive coding different codes that do have different coding gain are used. Strong error correction coding is used under sever fading and small or no coding is used under small fading. This can be done by multiplexing codes with different correction capabilities or by using rate compatible punctured convolutional (RCPC) codes.

In variable rate adaptive modulation, the rate R is varied according to the channel fading statistics. This can be done by fixing the symbol rate R_s of the modulation and using variable constellation size or by fixing the modulation order and varying the symbol rate. Symbol rate variation is difficult to implement in practice since a varying signal bandwidth is impractical and complicates bandwidth sharing [1]. In contrast, changing the constellation size with a fixed symbol rate is fairly easy and is used in this thesis. This scheme, in addition to being simple to implement, it requires two or three bits of feedback information to the transmitter enabling selection between four up to eight states of transmission. Different order modulation allows sending more bits per symbol period under favorable channel conditions and thus achieving higher throughputs or better spectral efficiencies. The receiver after estimating the channel fading coefficients it will try to determine the optimum modulation order that result in maximum throughput for the current channel state. The code corresponding to this order is sent to the transmitter through the low rate feedback path.

The proposed rate adaptive system block diagram is shown in the figure 5.1. The optimum modulation order evaluator selects one MPSK modulation order based on the application's error performance requirement.

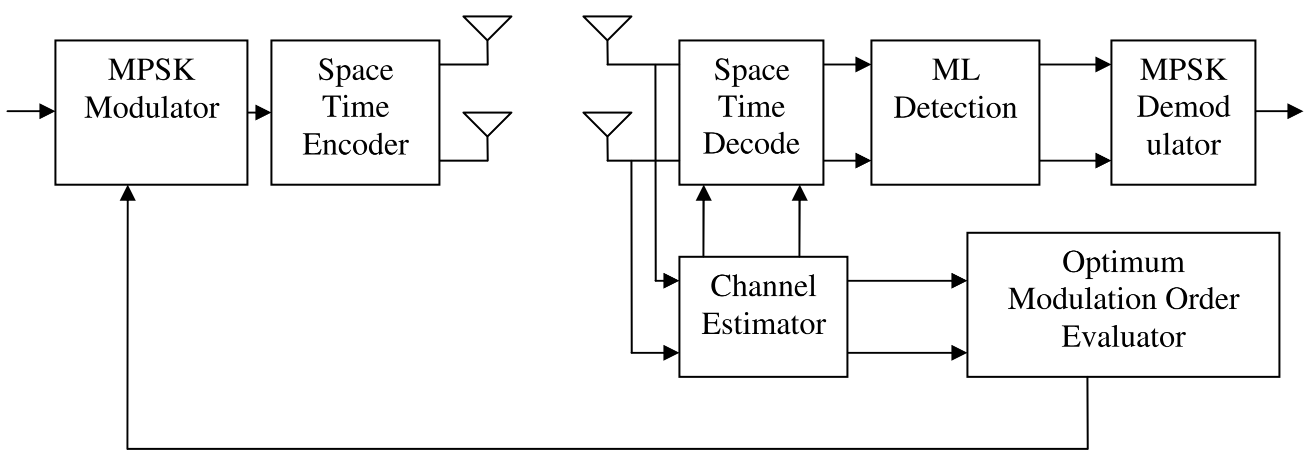

Fig 5.1 proposed rate adaptive system

The average upper union pound vector error probability can be used as a parameter based on which the modulation order at a transmitter is varied. i.e

$$p_e \leq \frac{1}{M}\sum_{i}\sum_{\substack{j \\ i\neq j}} Q\left(\sqrt{\frac{E_s}{2N_0}d_{ij}^2}\right) \qquad \text{.................... (5.5)}$$

Where M is the number of possible code vectors, i,j =1,2…M, and

$$d_{ij}^2 = \left\|s_i - s_j\right\|^2 = \left\|H(X_1 - X_2)\right\|^2 \qquad \text{................ (5.6)}$$

is the distance between vector symbols at the receiver.

5.3 Simulation results

The simulated average bit error performance of the two by two Alamouti's spatial diversity scheme for different order MPSK modulation is shown in the figure 5.2. It is assumed that the bit to symbol mapping is done in such a way that a symbol error associated with decision to adjacent symbol, which is the most likely way to make error, corresponds only to one bit error and assuming that mistaking a signal constellation for constellation other than it's nearest neighbors has a very low probability, we can make the approximation

$$p_b \approx \frac{p_s}{\log M} \qquad \text{........ .... (5.7)}$$

Where p_b is bit error and p_s is symbol error

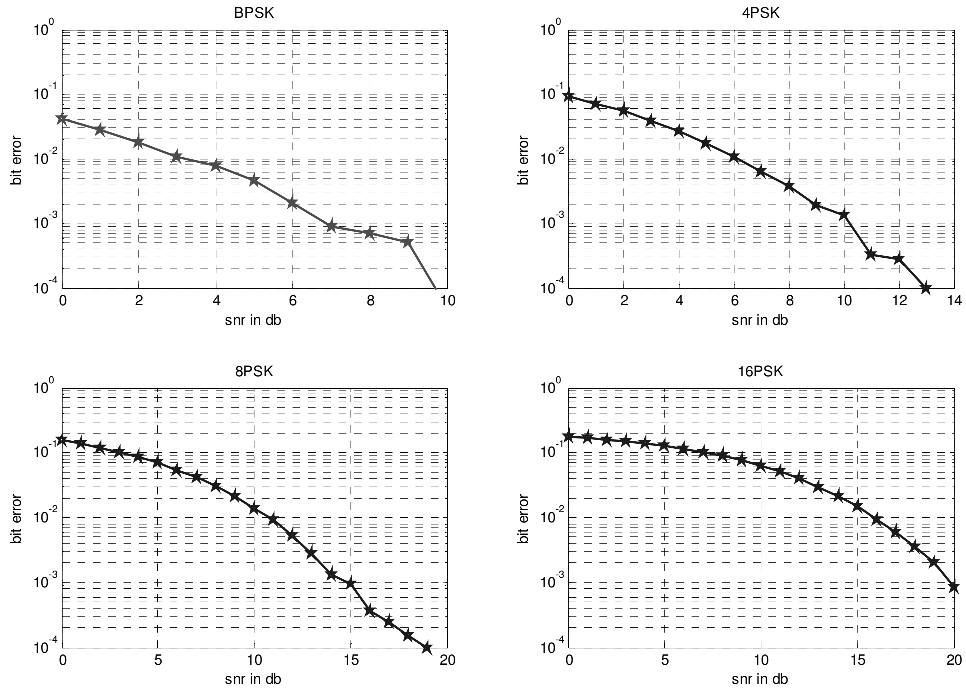

Fig 5.2 BER for MPSK signaling.

From the figure 5.2, for an application that require 10^{-3} bit error rate, it is no more advantageous two use BPSK for SNR above around 11db or QPSK for SNR above 15db . So by observing points where the application requirement is met we can adjust the modulation order that enable us to increase the throughput of a system.

The corresponding simulated bit error performance of spatial multiplexing scheme for a two by two antenna system with ML detection is shown in the figure 5.3 for different order MPSK modulation.

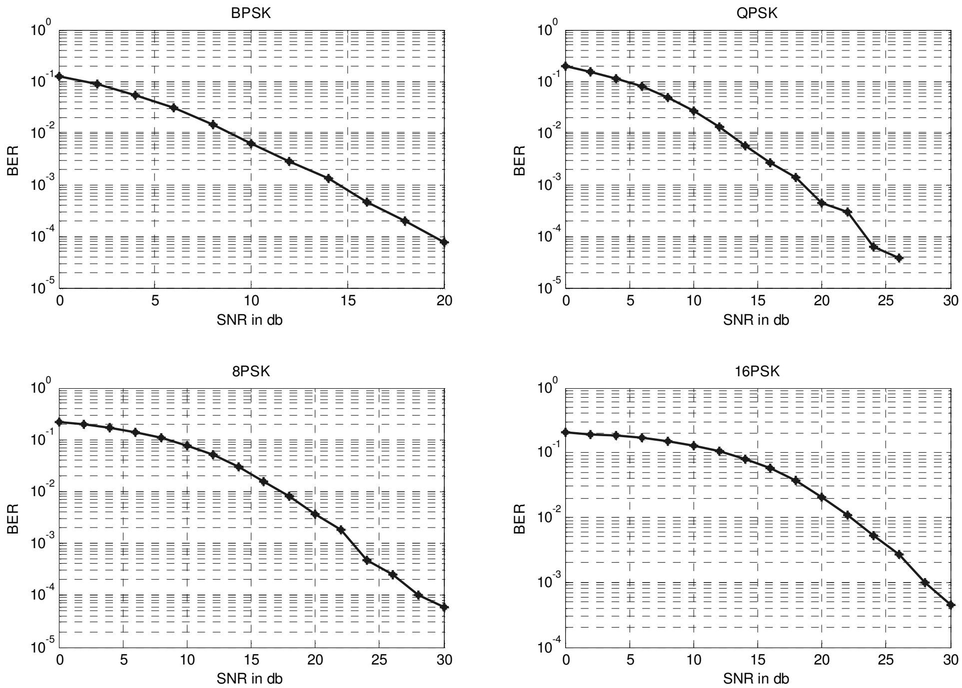

Fig 5.3, Bit error performance of 2 by 2 SM system

Similarly, for 10^{-3} BER requirement BPSK a reasonable selection in SNR range of 15-19db, QPSK in the range 19-24db, 8PSK in the range 24-28db and 16PSK for SNR grater than 28db. Note that, if the system uses only one order, then number of bits transferred per channel use will be $2(\log_2 M)*p_d$, where p_d is the probability that MPSK modulation satisfy the error rate requirement of the application and M is the modulation order. The factor 2 is due to the fact that two symbols are transmitted per channel use.

For adaptation parameter of equation 5.5, the simulated modulation order (bits per symbol) used versus SNR is shown in the figures 5.4 for symbol vector error of 10^{-2} for the spatial diversity systems. The system first finds the upper union bound error probability for a higher order modulation under a given channel realization using equation 5.5. If this error rate is smaller than the required rate, then the selected modulation order is used for the rest of the channels coherence time. But if it is greater than the required error rate, the next lower level modulation order is checked. If all modulation orders fail to satisfy the error rate requirement, no data will be sent.

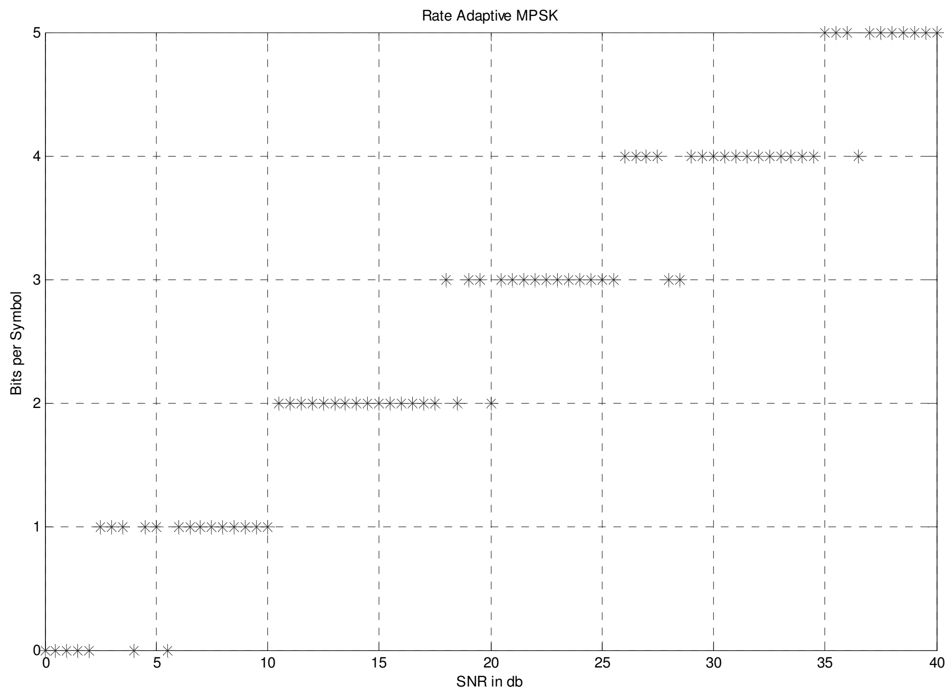

Fig 5.4 modulation order used in rate adaptive SD system

The average number of bits transferred while satisfying the applications requirement in the adaptive as well as the non adaptive scheme of spatial diversity system is shown in the table 5.1. Spatial diversity average number of bits/channel use = $\Sigma(n_i/N)*(b_i)$ where n_i is the number of times modulation order i can used, b_i is the number of bits per symbol in order i and N is the total number of transmission.

Modulation type	Average number of bits/per channel use
BPSK	0.913
QPSK	1.48
8PSK	1.59
16PSK	1.33
32PSK	0.62
Adaptive	2.64

Table 5.1, average number of bits/channel use for adaptive and non adaptive schemes

From the table we can see that the average maximum reliable rate that would be possible over the range of 0-40db without adaptation is 1.59bits/channel use through 8PSK modulation. But this rate increases to 2.64bits/channel use when the system is rate adaptive, which is about 66.04% increase relative to the non-adaptive scheme.

The corresponding result for the spatial multiplexing system is shown in figure 5.5.

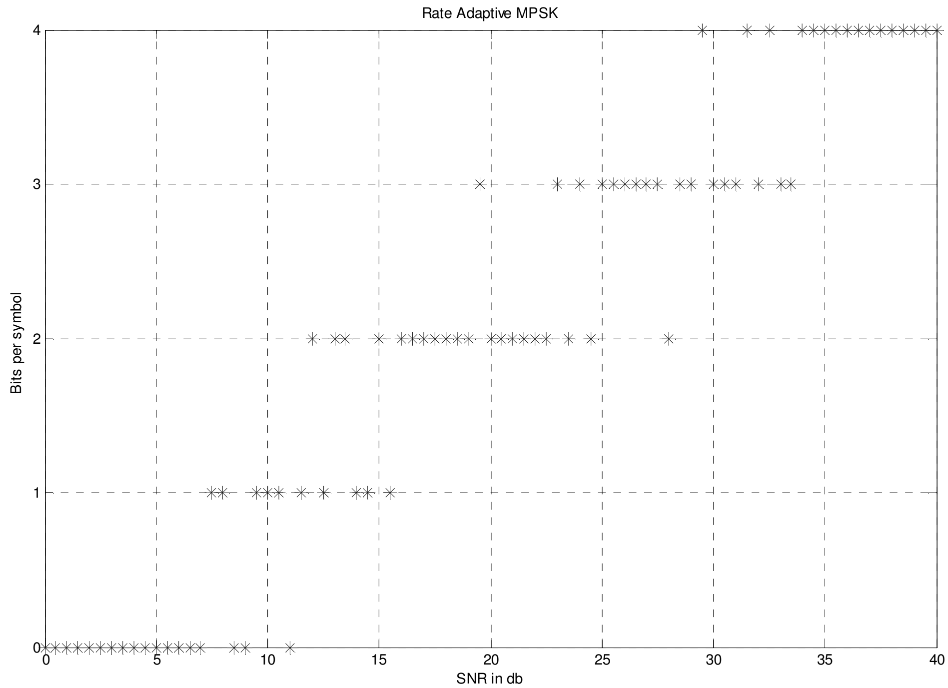

Fig 5.5 Modulation order used in rate adaptive SM system

The average number of bits transferred per channel use while satisfying the application requirement for this scheme is shown in table 5.2 for adaptive as well as non adaptive systems over a range of 0-40db SNR.

Modulation type	Average number of bits/per channel use
BPSK	1.56
QPSK	2.62
8PSK	2.44
16PSK	1.58
Adaptive	4.07

Table 5.2, average number of bits per channel use

There is 55.43 % rate increase in the adaptive system relative to the maximum possible rate of the non adaptive scheme.

The computation of upper union bound using equation 5.5 requires M(M-1)/2 number of Q function evaluation that would make its implementation computationally costly. A less costly way of implementing the above adaptation would be the use of the loser bound equation 2.21 where only one Q function inversion is evolved.

$$p_e \leq (M-1)Q\left(\sqrt{\frac{E_s}{2N_0} d_{\min}^2}\right) \qquad \text{..................... (5.8)}$$

Where $d_{\min}^2 = \min\limits_{\substack{i,j \\ i \neq j}} \left\| s_i - s_j \right\|^2$. The simulated result of this looser bound for the spatial diversity system is shown in fig 5.6 and table 5.3 shows the average bits/channel use.

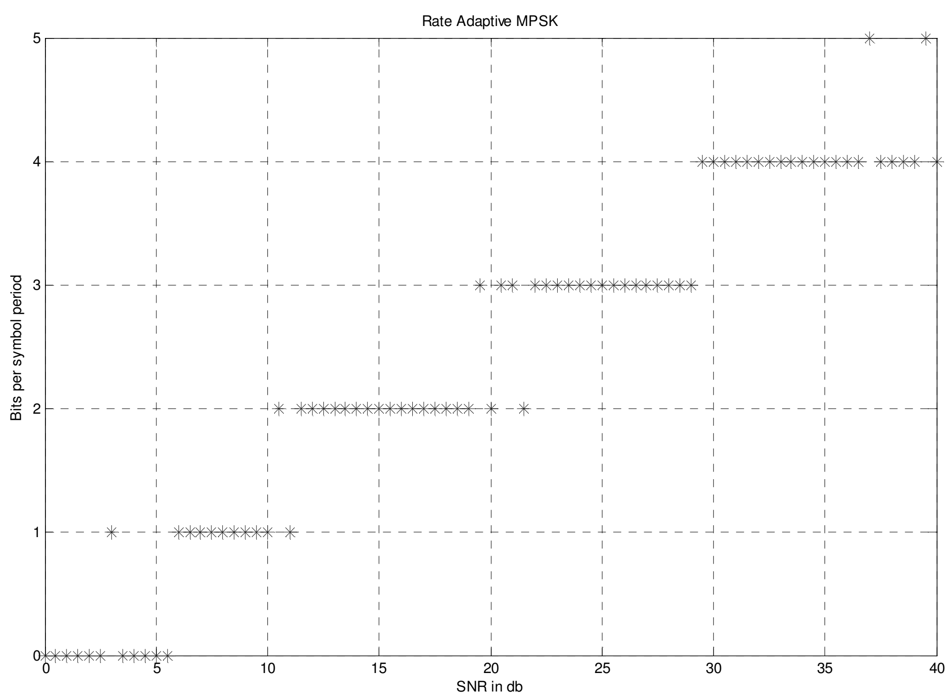

Fig 5.6, modulation order used for looser bound

In this scheme the system first evaluates the minimum distance required for symbol vector error of 10^{-2} for the highest order MPSK. If this distance is less than the actual minimum distance between the hypothesized matrix symbols, then this order will be used till the end of the coherence time of the channel. But if the distance required is grater than the hypothesized minimum distance, it will check for the next lower order. If all orders fail to satisfy the distance requirement, again no data will be sent.

Modulation type	Average number of bits/per channel use
BPSK	0.86
QPSK	1.46
8PSK	1.48
16PSK	1.09
32PSK	0.12
Adaptive	2.38

Table 5.3, average number of bits/channel use for looser bound

Reduction in computation achieved in this scheme costs a 9.85% reduction in the rate performance. The computational complexity of this looser bound scheme which persists in the distance computation between code vectors in the search for the minimum distance can further be reduced by using upper and lower bounds for the minimum distance between code vectors [3]. Besides to this some sort of lookup table can also be developed that can get rid off the Q function inversion required.

The number of bits per symbol used for the spatial multiplexing system in connection with the looser bound is shown in fig 5.7

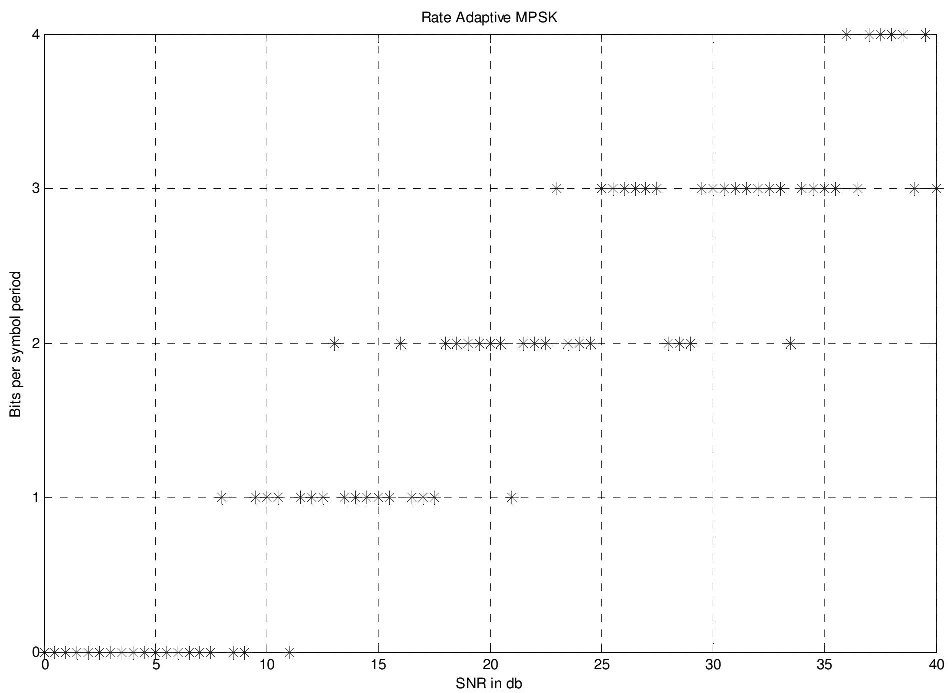

Fig 5.7, Modulation order used for looser bound in SM system

The corresponding average number of bits transferred per channel use over the range of 0-40db SNR for adaptive and nonadaptive schemes is shown in table 5.4.

Modulation order	Average number of Bits per channel use
BPSK	1.53
QPSK	2.27
8PSK	2.07
16PSK	0.59
Adaptive	3.51

Table 5.4, average number of bits per channel use

Table 5.4 shows that, for fixed order modulation the maximum average rate possible in SNR rage 0-40db in step of 0.5db is 2.272bits/channel_use through QPSK modulation. But the adaptive scheme can achieve a rate of 3.507 bits/channel_use, which is 54.36% increase as compared to that of QPSK. The reductions in the computational complexity of equation 5.8 result in a 13.76% performance lose as compared to the performance using equation 5.5.

12.5 5.4 Computational complexity comparison

The computational complexity of the rate adaptive spatial diversity system discussed in the previous section mainly lies in the search for either the upper bound vector error probability or minimum distance between hypothesized code vectors. As far as the ML vector detection is concerned, it boils down to ML symbol detection due to the inherent space time encoding and decoding procedure of the Alamuti's scheme. The search for the minimum code word distance between the hypothesized code vectors can further be reduced by using the bound [3]

$$\left\|E_{i,j}\right\|^2 \lambda_{\min}^2(H) \le d_{i,j}^2(H) \le \left\|E_{i,j}\right\|^2 \lambda_{\max}^2(H) \qquad \text{.................. (5.9)}$$

Where $E_{i,j} = x_i - x_j$ is the code vector distance before transmission and $d_{i,j}^2$ is the actual distance of the received vectors.

In the computational complexity comparison of the spatial diversity and spatial multiplexing systems we will consider only operation that are specific to the particular system. The analysis considers the number of complex number addition, multiplication and subtraction as an operation. Since the adaptation process for the two schemes is almost the same, we will not have any complexity comparison in this regard. The comparison that will be made is thus depend mainly on the detection algorithm of the two schemes. Complex number multiplication and addition are considered separately and the respective figures are given. And complex number addition and subtraction are regarded as equivalent.

The Alamuti's MPSK detection starts by calculating the decision statistics for the two symbols transmitted using equation 3.13. In this equation there exist four multiplications and four additions of complex numbers per symbol resulting in a total of 8 of each operation per vector symbol. The rest operation involves distance computation between this decision symbols and the hypothesized constellation symbols. For an MPSK modulation this involves M complex number subtraction per symbol, hence a total of 2M per vector and 2M multiplication in the operation of norm finding. The total number of the operations that are peculiar to the Alamuti's space time decoding is thus shown in table 5.5.

Operation	Number of operation
multiplication	2M+8
Addition	2M+8

Table 5.5, number of complex number operation in SD system

The computational complexity of spatial multiplexing system depends on the type of detection algorithm used. The algorithm used for ML detection is quite simple conceptually but a bit involved computationally. It calculates distances between received code vector and hypothesized code vectors unlike of spatial diversity system where distance between symbols is calculated. This would require $2M^2$ subtraction and $2M^2$

multiplication. i.e. each received vector symbol has a form $\begin{bmatrix} s_1' \\ s_2' \end{bmatrix}$. The distance of this vector from each of the M^2 set of vector constellations require $2M^2$ subtraction operation. The magnitude calculations of M^2 subtraction results require $2M^2$ multiplication. The total number of the operation that is peculiar to this detection process is shown in table 5.6.

Operation	Number of operation
multiplication	$2M^2$
Addition	$2M^2$

Table 5.6, number of complex number operation in SM system

In this analysis we have considered only those operations that are uncommon to the spatial multiplexing and diversity systems for comparison purpose. As far as the complexity of the other detection methods is concerned [12] has given detailed analysis.

13 CHAPTER SIX

14 Conclusions

In chapter 2 the capacity of MIMO channels was studied and it is implied that this capacity is optimally achieved if the number of transmit and receive antennas are equal. And chapter 4 showed that the performance of the different detection methods for the spatial multiplexing system. Out of these techniques the ML detection method is optimum in regard to bit error performance.

Coming to the performance of the SD and SM systems, one can see from table 5.1 and 5.2 or from 5.3 and 5.4 that the average rate of spatial multiplexing system is better than the spatial diversity system over the range of 0-40db SNR. i.e the average data rates of those tables is over 0-40db SNR. But there are range of values for which the spatial diversity scheme outperforms the spatial multiplexing system. To illustrate this, results of data plots of fig 5.6 and 5.7 are used in calculating the average rate of the two systems in the range specified in table 6.1

SNR range	Spatial diversity	Spatial multiplexing
0-10	0.48	0.29
10-20	2	2.6
20-30	3.05	4.8
30-40	4.1	6.5

Table 6.1, bits/channel use of the two schemes

As can be seen from the table the spatial diversity scheme has higher rate in the range of SNR 0-10db implying that the average rate comparison and percentage increase in rate performance of spatial multiplexing over the spatial diversity is rage dependent. What can be said about the performance of the two schemes in regard to rate performance is that, SD is a preferred chose at low SNR operation and SM at higher SNR region. It would be in general wrong to say SM outperforms SD.

As far as the computational complexity of the two schemes is concerned, table 5.5 and 5.6 showed that the number of operations that are involved in the detection of spatial multiplexing system is higher than that of the spatial diversity system. So the high rate performance of the spatial multiplexing scheme comes at a cost of high computational complexity.

The other point that needs mentioning from the plots of the two schemes is that spatial diversity's system performance is some what stable for certain region of SNRs. i.e. the modulation order that is used is roughly constant for certain regions of SNRs forming a certain boundary between SNRs that use different modulation order. This implies that it is possible to have an adaptation that is based on the SNR value of the received signal that compromise error rate and complexity. Spatial multiplexing scheme on the other hand doesn't show such characteristics, rather there is randomness (instability) in the modulation order that is used over adjacent SNR values.

15 Future work

The adaptation process we have followed in this thesis has more of theoretical value only because the search for the minimum Euclidean distance or upper bound vector error probability is some what involved computationally, at least for the current systems. A method that compromise rate, computational complexity and error performance for the adaptation process in connection with equation 5.9 or others is left as future work. Besides, the performance analysis of the rate adaptive spatial multiplexing system that uses sub-optimum detection methods with low computational complexity is another window for future work. Another most interesting thing to do is to investigate the rate performance of a scheme that switch between rate adaptive spatial diversity and multiplexing schemes for a given error rate performance. And finally, the investigation of the above systems in correlated fading channel.

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