EFFICIENCY AT MAXIMUM POWER AND OPTIMIZED EFFICIENCY: AN INFORMATION THEORY APPROACH

By

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Signature of Author

To my Sisters and Brothers.

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Abstract

We consider a model heat engine which is based on the concepts of information theory, attributing irreversible energy dissipation to the information transmission channels. The different communication channels, classical and quantum, are characterized by their channel capacity. We obtain that the model recovers the Carnot principle in the reversible limit and the efficiency at maximum power expression of non reversible thermodynamics. We use optimization criterion to derive the objective function which is a quantity that help us for the derivation of optimized efficiency. The criterion predicts the performance regime of the model heat engine lying between efficiency at maximum power and Carnot efficiency. Such regime should be considered as optimum performance regime of the model heat engine. We derive the optimized efficiency, which has the same linear term for the different information channels in the linear response regime, lying between efficiency at maximum power and Carnot efficiency. We expect the optimization method we used to be applicable to any finite-time thermodynamic heat engine which meets Hernandez et al [1] requirements. Besides it gives clue on the study of high performance thermodynamic heat engine.

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Chapter 1

Introduction

The Maxwell's demon puzzle, suggesting a violation of the second law of thermodynamics, has been exorcised using Landauer's memory erasure principle [7, 10]. Treating the demon's intelligence as information has manifested a fundamental connection between information and physics, revealing the specific role of memory in terms of the second law of thermodynamics [10]. The goal of this thesis is to study the performance of the minimal model of heat engine (demon) using information theory introduced previously by Yun Zhou and Dvira Segal [13]. In the engine irreversible energy dissipation is attributed to information transfer within the demon's channels, compensating entropy decrease in the system. The model helps us study the basic concepts in thermodynamics, like efficiency at maximum power, optimized efficiency and their relationship to the Carnot efficiency.

A basic topic in thermodynamics is the study of the efficiency of thermal engines. According to the Carnot result the efficiency (work output divided by heat input) of a cyclic thermal heat engine, operating between two heat baths temperatures T_c and T_h ($T_c < T_h$), is at most

$$\eta_c = 1 - \frac{T_c}{T_h}. \tag{1.0.1}$$

The upper limit is obtained for engines that work reversibly. However, as reversible processes occur infinitesimally slowly, the power produced is zero (i.e. we obtain the maximum possible output work in an infinite time). Operating away from equilibrium, a more practical questions are the efficiency at maximum power, η_{mp}, optimizing an engine cycle with respect to its power rather than Carnot efficiency and the optimized efficiency, η_{opt}, optimizing an engine cycle with respect to objective function, $\dot{\Omega}$. For a specific model of heat engine, Curzon and Ahlborn (CA) derived the efficiency at maximum power, η_{CA} [5],

$$\eta_{CA} = 1 - \left(\frac{T_c}{T_h}\right)^{\frac{1}{2}} = 1 - (1 - \eta_c)^{\frac{1}{2}}$$
$$\approx \frac{\eta_c}{2} + \frac{\eta_c^2}{8} + \frac{\eta_c^3}{16} + \bigcirc(\eta_c^4) \tag{1.0.2}$$

based on the endoreversible approximation in which the source of irreversibility is due to heat transfer processes [5]. In the linear response regime it has been recently proven that the efficiency at maximum power is upper bounded by Eq.(1.0.2), which in this regime is exactly half of the Carnot efficiency [4] and the maximum value for the optimized efficiency in the linear regime is reported by Mesfin A. and Mulugeta B. [9]. The upper limit is reached for a specific class of strongly coupled models for which the energy flux is directly proportional to the work-generating flux. In contrast, in the nonlinear regime general results are missing, and expressions deviating from the CA efficiency were reported [14]. In the present model irreversible loss of energy occurs at the communication channels, transmitting classical information (entropy) from a heat source (T_h) to the engine (T_c). We consider both classical and quantum channels, encoding classical information in quantum states, and show that in both cases Carnot efficiency can be achieved when the device works reversibly. We also demonstrate that in the linear regime (small temperature differences) the efficiency at maximum power is $\eta_c/2$ and the optimized efficiency is $7\eta_c/8$. We find that if a model obeys the Carnot limit for a reversible process, it must also follow the behavior, $\eta_{mp} = \eta_c/2$ and $\eta_{opt} = 7\eta_c/8$ for an irreversible process in the linear response regime.

1.1 Model

The model includes a finite subsystem immersed in a hot reservoir of temperature T_h and an engine (demon) in a cold reservoir of temperature T_c, as in fig(1.1). In particular, the subsystem may include two discrete states 0 and 1 of energies $\varepsilon_0 = 0$ and $\varepsilon_1 = \varepsilon$ respectively. The subsystem is assumed to be tightly attached to a heat bath at temperature T_h, thus in thermal equilibrium the levels population follows the Boltzmann distribution

$$P_0 = \frac{1}{1 + e^{-\varepsilon/T_h}}, \tag{1.1.1}$$

for state 0 and

$$P_1 = \frac{e^{-\varepsilon/T_h}}{1 + e^{-\varepsilon/T_h}}, \tag{1.1.2}$$

for state 1 by taking $k_B = 1$ (Boltzmann constant). And the average energy of the two level system (TLS) is

$$\bar{E} = P_0\varepsilon_0 + P_1\varepsilon_1. \tag{1.1.3}$$

As the two level system (TLS) is virtually a part of the hot reservoir (heat bath), from now onwards we refer to the combined object as a heat source. It is important to note that the TLS is held in isothermal conditions, it is tightly connected to a large thermal reservoir at T_h. Therefore, the engine absorbs energy from the TLS, yet the TLS does not cool down to zero, as its temperature is kept fixed. This is similar to the process of reversible isothermal expansion of ideal gases. Here the TLS model is introduced as a subsystem for simple demonstration but one may choose other model of subsystem. The right half of the figure, referred to as the heat engine. It consists four components; detector, communication channel, processor and receiver.

From Shannon general model for a communication system [2, 6], in a working cycle the detector detects the state of the heat source, encodes it, and sends this information through the transmission channel to the processor which will decode it. Based on this information, the processor sends an instruction to properly set the receiver, to accept the

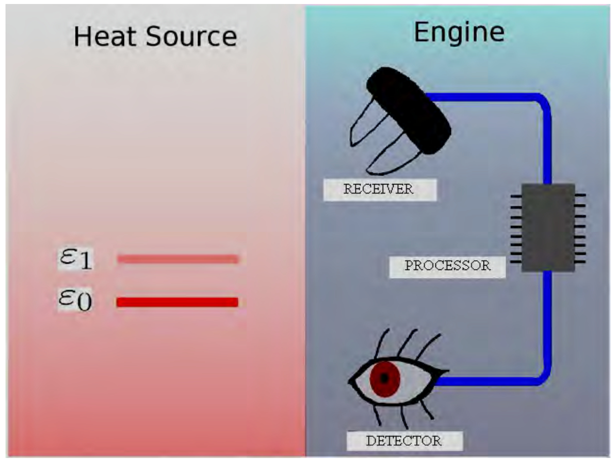

Figure 1.1: A schematic representation of the communication heat engine [1].

energy of the subsystem. While a measurement of the heat source state and receiving its energy can be done without energy consumption [7, 10], energy dissipation in the engine is attributed to the transmission of information in the channels and the resulting setup of the receiver, corresponding to the Landauer's erasure principle [7, 10]. The fundamental questions about heat engine are: how efficiently can we extract output energy out of the input energy? And how fast the process is to extract such useful energy? In the next chapter we are going to see the best compromise between useful energy and the fastest process of the model heat engine.

Chapter 2

Efficiency at maximum power

In this chapter we will try to see the expression for efficiency at maximum power of a nonreversible thermodynamic heat engine as done by Yun Zhou and Dvira Segal [13]. In each working cycle the engine needs to acquire information on the state of the finite subsystem. The total amount of information, I, that has to be absorbed during this process is given by

$$I = \frac{\bar{E}}{T_h}, \tag{2.0.1}$$

where T_h is the hot reservoir temperature. This information has to be transferred through the communication channel. The measure for a communication channel is the channel capacity $I_p(S, N_o)$ describing the maximum number of bits that a system can communicate reliably per channel use. Here S and N_o (a function of T_c) are the pulse (signal) and noise average energies, respectively. For the transfer of I on the status of the heat source to the engine we need to utilize an amount of energy Q given by

$$Q = \frac{I}{I_p} S, \tag{2.0.2}$$

which is the heat dissipated in the engine during a working cycle. The engine efficiency, η, is given by the amount of available work, W, divided by the input energy, $\bar{E}$. Since $W = \bar{E} - Q$ is the work extracted per cycle, the efficiency will then be

$$\eta = \frac{W}{\bar{E}} = 1 - \frac{S}{T_h I_p}. \tag{2.0.3}$$

We will optimize the engine cycle (i) with respect to its efficiency by minimizing Q, and (ii) with respect to its power $P = W/nB$, where n is the number of pulses that are needed to transfer the information, I, in a working cycle, which is proportional to the period (Π) of the heat engine where the proportionality constant is $1/B$, where B is bandwidth, which is constant and has no effect on the calculation. Therefore, Π becomes

$$\Pi = I/I_p. \tag{2.0.4}$$

Using Eqs.(2.0.1), (2.0.2) and (2.0.4), we get

$$P = \frac{W}{\Pi} = \frac{\bar{E} - Q}{\Pi} = T_h I_p - S. \tag{2.0.5}$$

Maximizing P with respect to the pulse power S and substituting the value of pulse power S_{mp} at which P is maximum into Eq.(2.0.3), gives the efficiency at maximum power

$$\eta_{mp} = 1 - \frac{S_{mp}}{T_h I_p(S_{mp}, N_o)}. \tag{2.0.6}$$

The period, Π_{mp}, taken at maximum power to transfer information I through the heat engine becomes

$$\Pi_{mp} = \frac{I}{I_p(S_{mp}, T_c)}. \tag{2.0.7}$$

In the following section, we will consider the different types of information communication channels and find their corresponding efficiency at maximum power, η_{mp}.

2.1 Additive White Gaussian Channel (AWGC)

Suppose we send information over a classical continuous memoryless channel of bandwidth B (in Hertz) subjected to an additive white Gaussian noise with a power spectral density N_o. Such channel is called Gaussian Channel. According to Shannon-Hartley [2, 6] its channel capacity, C, is given by

$$C = B \ln(1 + \frac{\mathbf{S}}{BN_o}), \tag{2.1.1}$$

where $\mathbf{S}$ and BN_o are the signal and the noise powers in a bandwidth B, respectively. Defining $S = \mathbf{S}/B$ as the average energy carried by the pulse and the dimensionless channel capacity as $I_p(S, N_o) = C/B$, we get

$$I_p(S, N_o) = \ln(1 + \frac{S}{N_o}). \tag{2.1.2}$$

We use $N_o = T_c$ as the thermal noise power spectral density. When $S \to 0$, $Q \to IT_c$, i.e., the engine works reversibly with minimal energy dissipation. Then the work extracted from the engine is

$$W = \bar{E} - Q = \bar{E} - IT_c = \bar{E}(1 - T_c/T_h). \tag{2.1.3}$$

And the efficiency at this reversible mode with minimum energy dissipation is nothing but the Carnot limit, i.e.

$$\eta_c = 1 - T_c/T_h. \tag{2.1.4}$$

We next find the efficiency of our engine when it operates at maximum power. Using Eqs.(2.1.2) into (2.0.5), the power P becomes

$$P = T_h \ln(1 + \frac{S}{T_c}) - S. \tag{2.1.5}$$

The engine power takes a maximum value when the condition

$$\frac{\partial P}{\partial S}\Big|_{S=S_{mp}} = 0, \tag{2.1.6}$$

is satisfied. The maximum power occurs at a finite value of S_{mp} which is

$$S_{mp} = T_h - T_c. \tag{2.1.7}$$

Substituting Eq.(2.1.7) into Eq.(2.1.2), we obtain

$$I_p(S_{mp}, T_c) = -\ln(1 - \eta_c). \tag{2.1.8}$$

Using Eqs.(2.1.7) and (2.1.8) into Eq.(2.0.6), we get

$$\eta_{mp} = 1 + \frac{\eta_c}{\ln(1 - \eta_c)}. \tag{2.1.9}$$

Expanding the above equation in powers of η_c, we have

$$\eta_{mp} = \frac{\eta_c}{2} + \frac{\eta_c^2}{12} + \frac{\eta_c^3}{24} + \bigcirc(\eta_c^4). \tag{2.1.10}$$

Note that the efficiency at maximum power converges to one when η_c approaches one and to zero when η_c goes to zero. In the linear regime $\eta_{mp} = \eta_c/2$. Here the quantities characterizing our results are T_c and T_h so we can scale T_h as $\tau = \frac{T_h}{T_c} - 1$. Therefore, in the following sections we use $\eta_c = \tau/(1+\tau)$ as free parameter for our plots.

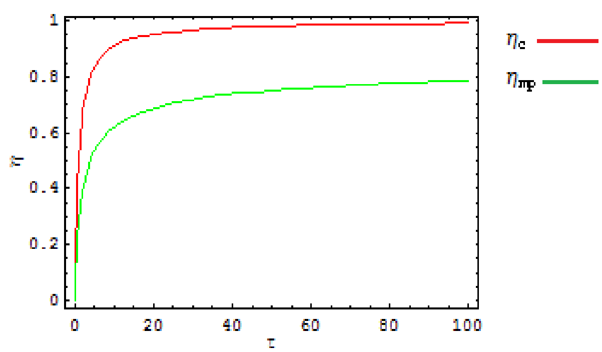

Figure 2.1: Plots of the efficiencies, η_{mp} and η_c, versus $\tau = (T_h/T_c - 1)$ for the case of Gaussian channel.

Plots of the efficiencies, η_{mp} and η_c, versus τ in Fig(2.1) shows that the efficiency is zero as τ goes to zero and one as τ goes to infinity. This is because as τ goes to zero the work extracted is zero so that the efficiency become zero. On other hand as τ goes to infinity we can extract the maximum possible work in each mode of operation so that the efficiencies goes to one. The amount of work (useful energy) extracted during Carnot cycle is larger than that of maximum power cycle but Carnot cycle is the slowest process ever whereas maximum power cycle is the fastest. Thus, maximum power cycle give the largest output power and the Carnot cycle give no power. As a result in terms of efficiency Carnot cycle is the best but it is unattainable because it is an infinite time process.

Using Eqs.(2.0.1) and (2.1.8) into Eq.(2.0.7) and expanding in powers of η_c, we obtain the period of the fastest process for Gaussian channel to be

$$\frac{\Pi_{mp}}{\bar{E}/T_h} = \frac{1}{\eta_c} - \frac{1}{2} - \frac{\eta_c}{12} - \frac{\eta_c^2}{24} + \bigcirc(\eta_c^3) \tag{2.1.11}$$

This is the period taken, in units of $\bar{E}/T_h$, at maximum power when we use Gaussian channel. Here we have Π_{mp} infinity when η_c approaches zero and zero when η_c goes to one where both are quasistatic limits.

2.2 Wideband Bosonic Channel

We send classical information, yet in the form of quantum states, over a memoryless quantum channel which is a bosonic field (e.g, electromagnetic radiation). The message information are encoded into modes of frequency ν and average photon number $n_B(\nu)$[3, 15]. The signal power is denoted by $\mathbf{S}$, noise power is $N = \pi T_c^2/12\hbar$ [11]. Under the constraint that the message-ensemble averaged energy of the channel is fixed, the dimensionless capacity of the channel [6] is given by

$$I_p = \frac{\pi}{6B\hbar}(T_e - T_c), \tag{2.2.1}$$

where T_e (the effective temperature of the decoder) is defined through the relation

$$\mathbf{S} + \frac{\pi T_c^2}{12\hbar} = \frac{\pi T_e^2}{12\hbar}, \tag{2.2.2}$$

so that T_e is given by

$$T_e = \left[\frac{12\hbar\mathbf{S}}{\pi} + T_c^2\right]^{\frac{1}{2}}. \tag{2.2.3}$$

Substituting Eq.(2.2.3) into Eq.(2.2.1), we obtain

$$I_p = \frac{T_c}{\lambda}\left[\left(\frac{2\lambda S}{T_c^2} + 1\right)^{\frac{1}{2}} - 1\right], \tag{2.2.4}$$

where $\lambda = 6B\hbar/\pi$ and $S = \mathbf{S}/B$. Minimizing the heat dissipated, $Q = (I/I_p)S$, in the channel, we find its minimum value to be

$$Q_{S\to 0} = \frac{\bar{E}T_c}{T_h}, \tag{2.2.5}$$

which gives us the efficiency in the reversible mode of operation,

$$\eta_c = \frac{\bar{E} - Q_{S\to 0}}{\bar{E}} = 1 - \frac{T_c}{T_h} \tag{2.2.6}$$

to be once again identical with Carnot efficiency. For the Wideband bosonic channel, the expression for power is

$$P = \frac{T_h T_c}{\lambda}\left[\left(\frac{2\lambda S}{T_c^2} + 1\right)^{\frac{1}{2}} - 1\right] - S. \tag{2.2.7}$$

Power is maximum when

$$\left.\frac{\partial P}{\partial S}\right|_{S=S_{mp}} = 0, \tag{2.2.8}$$

is satisfied. Maximum power occurs at finite value of S_{mp} which is

$$S_{mp} = \frac{T_c^2}{2\lambda}\left[\left(\frac{T_h^2}{T_c^2}\right) - 1\right]. \tag{2.2.9}$$

This is the value of the average signal energy which maximizes P. However, putting Eq.(3.2.9) into Eq.(3.2.4), we get

$$I_p(S_{mp}, T_c) = \frac{T_h}{\lambda}\left[1 - \frac{T_c}{T_h}\right]. \tag{2.2.10}$$

Using Eqs.(2.2.9) and (2.2.10) into Eq.(2.0.6), we finally get the efficiency at maximum power for the Wideband bosonic channel to be

$$\eta_{mp} = \frac{\eta_c}{2}. \tag{2.2.11}$$

As we can see the result reduces exactly to the linear term.

Plots of the efficiencies, η_{mp} and η_c, versus τ (Fig(2.2)) shows that as τ increase the efficiencies increase. This is because when τ increase the work extracted per cycle increases and the dissipated energy decreases as a result the efficiency increases. The upper most efficiency is η_c because the minimum possible dissipation of energy occurs during Carnot cycle whereas efficiency at maximum power, η_{mp}, is the smallest because there is high energy dissipation during operation. For τ infinity η_{mp} goes to 0.5 this means that the

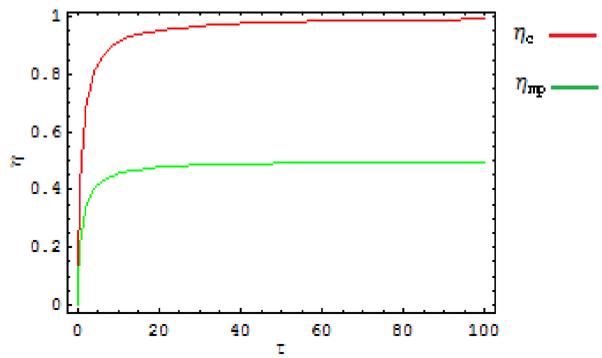

Figure 2.2: Plots of the efficiencies, η_{mp} and η_c, versus $\tau = (T_h/T_c - 1)$ for the case of Wideband bosonic channel.

maximum power cycle takes only 50 percent share of the input energy.

Let us compute the time taken, Π_{mp}, by the heat engine operating at maximum power. Using Eqs.(2.0.1) and (2.2.10), we get

$$\frac{\Pi_{mp}}{\lambda \bar{E}/T_h^2} = \frac{1}{\eta_c}. \tag{2.2.12}$$

This is the period taken to extract maximum power from the heat engine using Wideband bosonic channel. Here for η_c equal to unity Π_{mp} goes to one because for η_c unity, $I_p(S_{mp}, T_c)/(\bar{E}/T_h)$ equal to one.

2.3 Narrowband Photon Channel

In the case of Narrowband photon channel, $B \ll \nu$, where ν is the channel central frequency. Here the noise power (N) [3] is given by,

$$N = h\nu n_B(\nu, T_c)B \tag{2.3.1}$$

where $n_B = 1/(e^{h\nu/T_c} - 1)$ is the Bose-Einstein distribution. Based on the assumption that photon noise is additive, the dimensionless capacity (I_p) [3] becomes,

$$I_p(S, T_c) = \ln\big[1 + \frac{S}{h\nu(1+n_B)}\big] + \big(\frac{S}{h\nu} + n_B\big)\ln\big[1 + \frac{h\nu}{S + h\nu n_B}\big] - n_B \ln[1 + n_B^{-1}]. \tag{2.3.2}$$

Minimizing the heat dissipated, $Q = (I/I_p)S$ in the channel, we find its minimum value to be

$$Q_{S \to 0} = \frac{\bar{E} T_c}{T_h}, \tag{2.3.3}$$

which gives us the Carnot efficiency in the reversible mode of operation,

$$\eta_c = 1 - T_c/T_h. \tag{2.3.4}$$

For the Narrowband photon channel, the expression for power is

$$P = T_h \Big\{ \ln\big[1 + \frac{S}{h\nu(1+n_B)}\big] + \big(\frac{S}{h\nu} + n_B\big) \ln\big[1 + \frac{h\nu}{S + h\nu n_B}\big] - n_B \ln[1 + n_B^{-1}] \Big\} - S. \tag{2.3.5}$$

Power is maximum when

$$\frac{\partial P}{\partial S}\Big|_{S=S_{mp}} = 0, \tag{2.3.6}$$

is satisfied. Maximum power occurs for a particular value of S_{mp} which is

$$S_{mp} = h\nu \big[\frac{1}{e^{h\nu/T_h} - 1} - \frac{1}{e^{h\nu/T_c} - 1} \big]. \tag{2.3.7}$$

By using Eq.(2.3.7) into Eq.(2.3.2), we get

$$I_p(S_{mp}, T_c) = \frac{h\nu}{T_h}\big[1 + \frac{1}{e^{h\nu/T_h} - 1}\big] + \frac{h\nu}{T_c}\big[1 + \frac{1}{e^{h\nu/T_c} - 1}\big]. \tag{2.3.8}$$

Using Eqs.(2.3.7) and (2.3.8) into Eq.(2.0.6), and fixing $h = 1, T_c = 1$ and $\nu = 0.2$ and expanding in powers of η_c, the efficiency at maximum power for Narrowband photon channel takes the form

$$\eta_{mp} = 0.5\eta_c + 0.0827781\eta_c^2 + 0.0419439\eta_c^3 + \bigcirc(\eta_c^4), \tag{2.3.9}$$

which is $0.5\eta_c$ in the linear regime. Here we chose ν to be 0.2 but it is also possible to look at the nature of η_{mp} for some other values of ν. Even if η_{mp} is unaffected in the linear regime, η_{mp} decreases with increasing ν. We have seen that in the above three cases the efficiency at maximum power is exactly half of Carnot efficiency in the linear regime.

From fig(2.3) which is Plots of the efficiencies, η_{mp} and η_c, versus τ, we can see that the efficiency increases as τ increase. This is because as τ increase the dissipated energy decreases so that the work extracted increases which results in an increase in efficiency. From the Fig(2.3) energy dissipation is highest for η_{mp} cycle and lowest for η_c cycle. So that the work extracted for the case of η_c takes the highest share of the input energy compared to η_{mp} but this is unattainable within finite time. Therefore, the most practical efficiency is η_{mp}, which is the fastest process. Both efficiencies η_c and η_{mp} approach to unity for τ infinity.

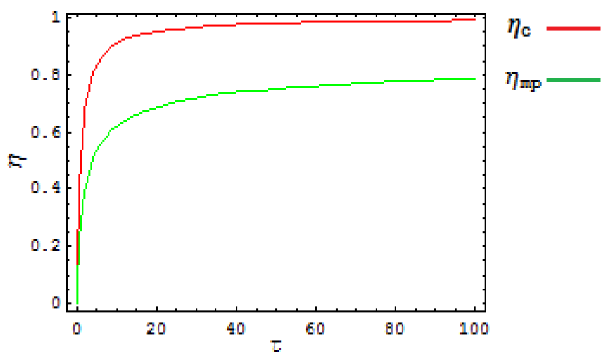

Figure 2.3: Plots of the efficiencies, η_{mp} and η_c, versus $\tau = (T_h/T_c - 1)$ for the case of Narrowband photon channel.

Using Eqs.(2.0.1) and (2.3.8) into Eq.(2.0.7) and expanding in powers of η_c, we get

$$\frac{\Pi_{mp}}{\bar{E}/T_c} = \frac{1}{\eta_c} - 1.50835 + 0.42363\eta_c + 0.039575\eta_c^2 + 0.0754026\eta_c^3 + \bigcirc(\eta_c^4), \qquad (2.3.10)$$

which is the time taken for one complete cycle, in units of $\bar{E}/T_c$, at maximum power mode of operation for the case of Narrowband photon channel. From the above equation, we can see that Π_{mp} goes to infinity when η_c approaches zero and zero when η_c goes to one. Note that in all the plots we use the expression of the efficiencies before expansion so that the plots include all non-linear terms.

Having the results of efficiency at maximum power and the time taken for one complete cycle of the three different communication channels, we proceed to find out the expressions for the optimized efficiencies and their respective period taken for a single cycle of these channels in the next chapter.

Chapter 3

Optimized Efficiency for a Minimal Model of Heat Engine

In this chapter we will find the expression for optimized efficiency of a nonreversible thermodynamic heat engine using optimization technique and the time taken to one complete cycle.

3.1 Optimization Using Objective Function

The subject of optimization of real devices has received continued attention in finite time thermodynamics. Optimization finds the pathway that yields optimum performance in a process operating at a non-zero rate. To achieve this, an objective function that depends on parameters of the problem must be optimized. Basically one has the freedom of choice of such objective function. A thermodynamic criterion devoted to analyze the optimum regime of operation in a real process should meet the requirements set by Hernandez et al [1]:

(i) its dependence on the parameters of the process should be a guidance to improve the performance of that process,

(ii) it should not depend on the parameters of the environment, and

(iii) the unavoidable dissipation of energy provoked by the process should be considered.

The optimization criterion which satisfy the above requirements can be applied to any energy converter. The importance of the proposed criterion is that it gives an optimized efficiency that lies between the maximum efficiency and the efficiency under maximum power condition.

Suppose we have an energy converter which can produce a useful energy $E_u(x;\{\alpha\})$, out of a given input energy $E_{in}(x;\{\alpha\})$, where x denotes free parameters and α denotes set of control parameters. The conventional efficiency of this energy converter, defined as the ratio between the useful and input energy

$$\eta = \frac{E_u(x;\{\alpha\})}{E_{in}(x;\{\alpha\})}, \tag{3.1.1}$$

satisfies the relation

$$\eta_{min}(\{\alpha\}) \leq \eta(x;\{\alpha\}) \leq \eta_{max}(\{\alpha\}), \tag{3.1.2}$$

where $\eta_{min}(\{\alpha\})$ and $\eta_{max}(\{\alpha\})$ are, respectively, the minimum and the maximum efficiencies in the allowed range of x for the given set of values of αs. Using the above two equations, we get

$$\eta_{min} E_{in}(x;\{\alpha\}) \leq E_u(x;\{\alpha\}) \leq \eta_{max} E_{in}(x;\{\alpha\}) \tag{3.1.3}$$

Having the above relation Hernandez et al [1] define an effective useful energy as

$$E_{u,eff}(x;\{\alpha\}) = E_u(x;\{\alpha\}) - \eta_{min} E_{in}(x;\{\alpha\}), \tag{3.1.4}$$

and a lost useful energy as

$$E_{u,lost}(x;\{\alpha\}) = \eta_{max} E_{in}(x;\{\alpha\}) - E_u(x;\{\alpha\}). \tag{3.1.5}$$

To obtain the best compromise between effective and lost useful energies Hernandez et al [1] propose Ω function (i.e. objective function) as the difference between the effective and lost useful energies,

$$\Omega(x,\{\alpha\}) = E_{u,eff}(x;\{\alpha\}) - E_{u,lost}(x;\{\alpha\}), \tag{3.1.6}$$

which can be expressed equivalently as

$$\Omega(x;\{\alpha\}) = [2\eta(x;\{\alpha\}) - \eta_{min}(\{\alpha\}) - \eta_{max}(\{\alpha\})]E_{in}(x;\{\alpha\}). \tag{3.1.7}$$

This quantity helps one to analyze the operation mode of any energy converter giving the best compromise between the maximum effective useful and the minimum lost useful energies for a specific job. In a heat engine the useful energy is the work delivered and the input energy is the heat supplied to the engine. We plan to use this criteria to optimize our minimal model heat engine.

3.2 Optimized Efficiency

The objective of this work is to obtain the best compromise between the maximum effective and the minimum lost useful energies of a heat engine, which we expect to be the optimum performance of the engine. This is possible by optimizing the objective function with respect to the free parameter S (average signal energy) of the system. The objective function is given as in Hernandez et al. [1] by

$$\Omega(S;\{\alpha\}) = (2\eta - \eta_{min} - \eta_{max})E_{in}. \tag{3.2.1}$$

For the system of heat engine we choose η_{min} as the efficiency at maximum power η_{mp} and η_{max} as the Carnot efficiency η_c, thus the objective function would take the form

$$\Omega(S;\{\alpha\}) = (2\eta - \eta_c - \eta_{mp})E_{in}, \tag{3.2.2}$$

where $E_{in} = W/\eta$. The objective function $\Omega(S;\{\alpha\})$ can be considered as the best compromise between maximum work performed and minimum lost work in a heat engine. For our system of heat engine we take the set of control parameters $\{\alpha\}$ as $\{T_c, T_h\}$. Taking the time derivative of $\Omega(S;\{T_c, T_h\})$, we obtain

$$\dot{\Omega}(S;\{T_c, T_h\}) = \frac{2\eta - \eta_c - \eta_{mp}}{\eta}\dot{W}. \tag{3.2.3}$$

We have work extracted per cycle, $W = \bar{E} - Q$. Here from Eq.(2.0.4) we have $I/I_p = \Pi$ which is the period of the heat engine to transfer the information I in a working cycle. So the engine efficiency, η, for the work extracted per cycle (W) divided by the input energy, E_{in}, is given as in Eq.(2.0.3) (for our case $E_{in}=\bar{E}$). Using Eqs.(2.0.3) and (2.0.5) into Eq.(3.2.3), we obtain the time rate of the objective function as

$$\dot{\Omega}(S;\{T_c,T_h\}) = (2-\eta_c-\eta_{mp})T_h I_p(S,N_o) - 2S. \tag{3.2.4}$$

Optimizing $\dot{\Omega}$ with respect to the pulse energy S and substituting the optimal value S_{opt} into Eq.(2.0.3), gives the optimized efficiency

$$\eta_{opt} = 1 - \frac{S_{opt}}{T_h I_p(S_{opt},N_o)}. \tag{3.2.5}$$

Substituting S_{opt} into Eq.(2.0.4), we obtain the corresponding optimized period, Π_{opt}, of a working cycle to be

$$\Pi_{opt} = \frac{I}{I_p(S_{opt},T_c)}. \tag{3.2.6}$$

Now let us consider the different types of information channel with their corresponding dimensionless channel capacity, I_p, and find the optimized efficiency by optimizing the objective function over the average pulse (signal) energy, S. In addition, we will compute the period to complete one cycle under optimized efficiency.

3.3 Additive White Gaussian Channel (AWGC)

Suppose we send information over a classical continuous memoryless channel subjected to an additive white Gaussian noise with a power spectral density N_o. Assuming a signal power BS, and taking BN_o as the total noise power in a bandwidth B, one obtains the Shannon-Hartley capacity [2, 6], in the form of Eq.(2.1.1). The expression for the dimensionless channel capacity is the same as Eq.(2.1.2) of section(2.1). Substituting Eq.(2.1.2) into Eq.(3.2.4), we get

$$\dot{\Omega}(S;\{T_c,T_h\}) = (2-\eta_c-\eta_{mp})T_h \ln(1+\frac{S}{N_o}) - 2S. \tag{3.3.1}$$

Optimizing $\dot{\Omega}$ over S,

$$\left.\frac{\partial\dot{\Omega}(S;\{T_c,T_h\})}{\partial S}\right|_{S=S_{opt}} = 0, \tag{3.3.2}$$

we obtain

$$S_{opt} = \frac{T_h}{2}(\eta_c - \eta_{mp}). \tag{3.3.3}$$

Substituting S_{opt} and $I(S_{opt}, T_c)$ into Eq.(3.2.5), we obtain

$$\eta_{opt} = 1 - \frac{\eta_c - \eta_{mp}}{2\ln[1 + \frac{\eta_c - \eta_{mp}}{2(1-\eta_c)}]}. \tag{3.3.4}$$

This is the expression for optimized efficiency of our heat engine for the case of Gaussian channel. Substituting η_{mp} from Eq.(2.1.9) into Eq.(3.3.4), we get

$$\eta_{opt} = 1 + \frac{1 - \eta_c + \frac{\eta_c}{\ln(1-\eta_c)}}{2\ln[\frac{1}{2} - \frac{\eta_c}{2(1-\eta_c)\ln(1-\eta_c)}]}. \tag{3.3.5}$$

Expanding η_{opt} up to fourth order in powers of η_c give us the optimized efficiency as a function of η_c

$$\eta_{opt} = \frac{7}{8}\eta_c + \frac{5}{192}\eta_c^2 + \frac{61}{4608}\eta_c^3 + \bigcirc(\eta_c^4) \tag{3.3.6}$$

This is the optimized efficiency of the heat engine when we use AWGC. In the linear regime $\eta_{opt} = (7/8)\eta_c$.

Here the quantities characterizing our results are T_c and T_h so we can scale T_h as $\tau = \frac{T_h}{T_c} - 1$. Therefore, from now on we use $\eta_c = \tau/(1+\tau)$ as free parameter for our plots.

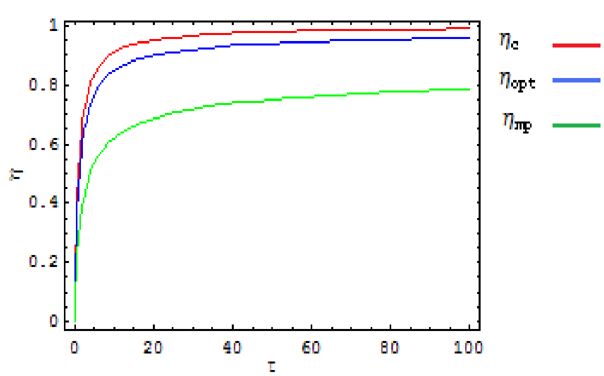

Figure 3.1: Plots of the efficiencies, η_c, η_{mp} and η_{opt}, versus $\tau = (T_h/T_c - 1)$ for the case of Gaussian channel.

Plots of the efficiencies, η_c, η_{mp} and η_{opt}, versus τ in Fig(3.1) shows that the efficiency is zero as τ goes to zero and one as τ goes to infinity. This is because as τ goes to zero the work extracted is zero so that the efficiency become zero. On other hand as τ goes to infinity we can extract the maximum possible work in each mode of operation so that the efficiencies goes to one. The amount of work extracted during Carnot cycle is the largest of all, optimized case second and efficiency at maximum power case give the least, this results in the order of efficiencies shown in Fig(3.1).

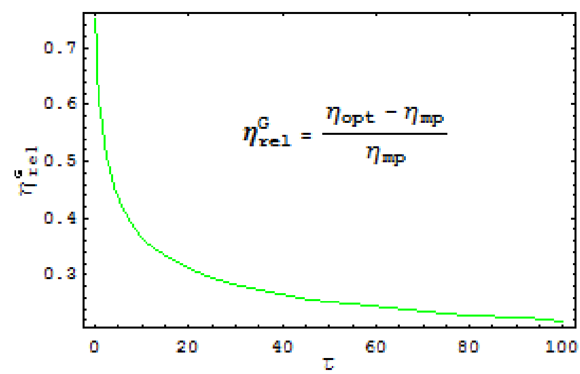

Figure 3.2: Plot of relative efficiency, η^G_{rel}, versus $\tau = (T_h/T_c - 1)$ for the case of Gaussian channel.

The plot of relative efficiency, η^G_{rel}, versus τ in Fig(3.2) shows that η^G_{rel} decreases as τ increases. This is because the difference between η_{opt} and η_{mp} gets smaller and smaller as τ increase. As we have seen in the Fig(3.1) η_{opt} and η_{mp} gets closer and closer and ultimately become equal to one for τ infinity. Note that the largest difference between η_{opt} and η_{mp} appears when τ is near zero which makes η^G_{rel} to take the largest value (i.e 0.75). The relative efficiency, η^G_{rel}, tells us η_{opt} is effective in the limit of τ small.

Now let us compute the time taken for one cycle (period) to extract optimum performance of the heat engine when we use AWGC. By use of Eqs.(2.0.1) and (2.1.2) into Eq.(3.2.6), we obtain

$$\Pi_{opt} = \frac{\bar{E}}{T_h \ln(1 + \frac{\eta_c - \eta_{mp}}{2(1-\eta_c)})}. \tag{3.3.7}$$

Using the expression for η_{mp} from Eq.(2.1.9), we get

$$\frac{\Pi_{opt}}{\bar{E}/T_h} = \frac{1}{\ln(\frac{1}{2} - \frac{\eta_c}{2(1-\eta_c)\ln(1-\eta_c)})}. \tag{3.3.8}$$

This is the time taken for one complete cycle, in units of $\bar{E}/T_h$, under optimized efficiency. Expanding the above equation about η_c, we obtain

$$\frac{\Pi_{opt}}{\bar{E}/T_h} = \frac{4}{\eta_c} - \frac{17}{6} - \frac{35}{144}\eta_c - \frac{2047}{17280}\eta_c^2 + \bigcirc(\eta_c^3) \tag{3.3.9}$$

This is the time taken for one full cycle, in units of $\bar{E}/T_h$, for the optimum performance cycle of the heat engine when we use Gaussian channel.

Figure(3.3) shows two plots of the period, Π, versus τ at optimum and maximum power. Π decreases as τ increases. As τ increases the amount of work extracted per cycle will increase due to decrease in heat dissipated. This is possible at the expense of time. For the case of efficiency at maximum power, η_{mp}, we extracted minimum work within the shortest time (Π_{mp}) possible but for the case of optimized efficiency, η_{opt}, we extract a higher amount of work at the expense of spending extra time, Π_{opt}, which is larger than Π_{mp}.

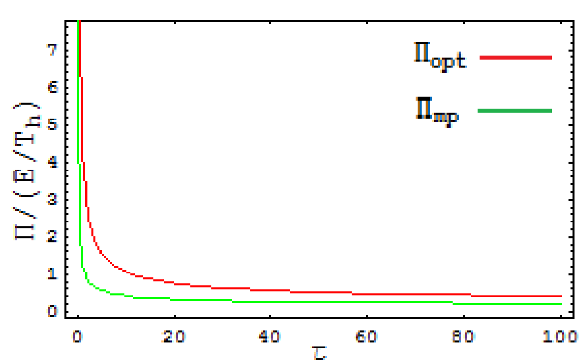

Figure 3.3: Plots of the periods, Π_{opt} and Π_{mp}, versus $\tau = (T_h/T_c - 1)$ for the case of Gaussian channel.

Figure(3.4) shows plot of Π_{rel} versus τ. The relative time, Π_{rel}, decreases as τ increase. This is because the difference between Π_{opt} and Π_{mp} is maximum for τ near zero and

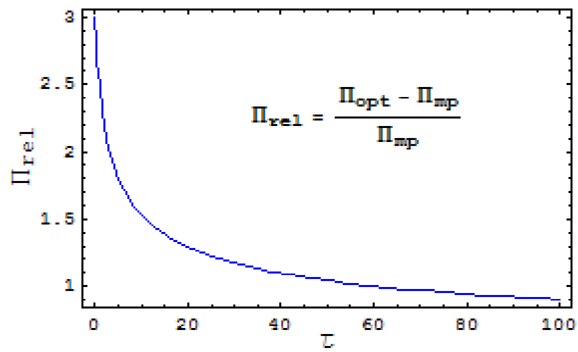

Figure 3.4: Plot of relative time, Π_{rel}, versus $\tau = (T_h/T_c - 1)$ for the case of Gaussian channel.

decreases down to zero as τ increases to infinity. Thus, we can see that the optimized cycle is the slowest process compared to maximum power cycle, the fastest process, near τ zero. Note that Π_{rel} equal to 3 as τ approaches zero and goes to zero for τ infinity.

3.4 Wideband Bosonic Channel

We send the same classical information (as in sect.(2.2)), yet in the form of quantum states, over a memoryless quantum channel which is a bosonic field (like an EM field). The message information is encoded into modes of frequency ν and average photon number $n_B(\nu)$. The signal power is denoted by $\mathbf{S}$, noise power is $\mathbf{N} = \pi T_c^2/12\hbar$ [11]. Under the constraint that the message-ensemble averaged energy of the channel is fixed, the capacity of the channel is given by $C = \frac{\pi}{6\hbar}(T_e - T_c)$, where T_e (the effective temperature of the decoder) defined through the relation in Eq.(2.2.2). Using the dimensionless channel capacity, I_p, of Wideband bosonic channel given as in Eq.(2.2.1) into Eq.(3.2.4), the time rate of our objective function becomes,

$$\dot{\Omega}(S;\{T_c,T_h\}) = (2-\eta_c-\eta_{mp})T_h\frac{T_c}{\lambda}\left[\left(\frac{2\lambda S}{T_c^2}+1\right)^{\frac{1}{2}}-1\right]-2S. \tag{3.4.1}$$

Optimizing $\dot{\Omega}$ with respect to S

$$\left.\frac{\partial\dot{\Omega}(S;\{T_c,T_h\})}{\partial S}\right|_{S=S_{opt}} = 0, \tag{3.4.2}$$

we find the value of S_{opt} to be

$$S_{opt} = \frac{T_c^2}{8\lambda}\left[(2-\eta_c-\eta_{mp})\frac{T_h}{T_c}^2 - 4\right]. \tag{3.4.3}$$

This is the value of the average signal energy which optimizes $\dot{\Omega}$. However, substituting the value of S_{opt} into Eq.(2.2.4), we get

$$I_p(S_{opt}, T_c) = \frac{T_c}{\lambda}\left[(2-\eta_c-\eta_{mp})\frac{T_h}{2T_c} - 1\right]. \tag{3.4.4}$$

Putting the above two equations into Eq.(3.2.5), we obtain

$$\eta_{opt} = 1 - \frac{T_c}{2T_h}\left[(2-\eta_c-\eta_{mp})\frac{T_h}{2T_c} + 1\right]. \tag{3.4.5}$$

From Carnot efficiency we have $T_h/T_c = 1/(1-\eta_c)$, therefore,

$$\eta_{opt} = \frac{3}{4}\eta_c + \frac{1}{4}\eta_{mp} \tag{3.4.6}$$

Substituting η_{mp} from Eq.(2.2.11), we get

$$\eta_{opt} = \frac{7}{8}\eta_c. \tag{3.4.7}$$

This is the optimized efficiency when we use a Wideband bosonic channel. We can see that it has a linear relation with Carnot efficiency with a slope of 0.875.

Plots of the efficiencies, η_c, η_{mp} and η_{opt}, versus τ (Fig(3.5)) shows that as τ increase the efficiencies increase. This is because when τ increase the work extracted per cycle increases and the dissipated energy decreases as a result the efficiency increases. The upper most efficiency is η_c, the optimized efficiency comes next η_{opt} and finally efficiency at maximum power η_{mp}. For τ infinity η_{mp} goes to 0.5 and η_{opt} goes to 0.875 this means that the maximum power cycle takes 50 percent share of the input energy where as the optimized cycle takes 87.5 percent share of the input energy for τ infinity. The maximum possible value for η_{opt} is 0.875 when τ is infinity, where as the minimum is zero when τ is zero.

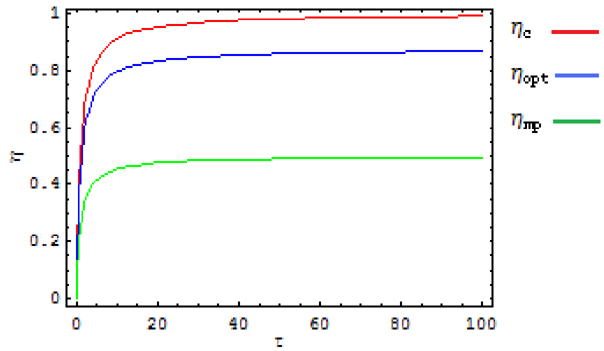

Figure 3.5: Plots of the efficiencies, η_c, η_{mp} and η_{opt}, versus $\tau = (T_h/T_c - 1)$ for the case of Wideband bosonic channel.

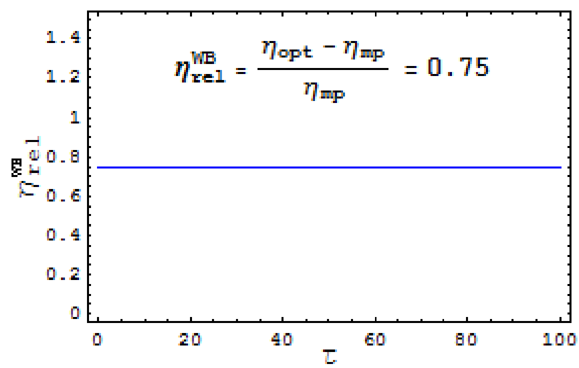

Figure 3.6: Plot of relative efficiency, η_{rel}^{WB}, versus $\tau = (T_h/T_c - 1)$ for the case of Wideband bosonic channel.

From Fig(3.6) which is a plot of relative efficiency, η_{rel}^{WB}, versus τ, we can see that η_{rel}^{WB} remains constant as τ increases. This is because for every τ (starting from 0.000001) η_{opt} and η_{mp} increases with equal interval so that the ratio of the difference between η_{opt} and η_{mp} divided by η_{mp} remain constant and have value equal to 0.75. Therefore, in terms of the extraction of useful energy the optimized cycle is 75 percent better than that of the maximum power cycle.

Let us compute the time taken for one complete cycle, Π_{opt}, when our information channel works at optimized efficiency. Using Eqs.(2.0.1) and (3.4.4), we get

$$\Pi_{opt} = \frac{\lambda \bar{E}}{T_h T_c\{(\frac{T_h(2-\eta_c-\eta_{mp})}{2T_c}) - 1\}}. \tag{3.4.8}$$

Simplifying and substituting the value of η_{mp} from Eq.(2.2.11), we get

$$\Pi_{opt} = \frac{4\lambda\bar{E}}{T_h^2\eta_c}. \quad (3.4.9)$$

Finally, we obtain

$$\frac{\Pi_{opt}}{\lambda\bar{E}/T_h^2} = \frac{4}{\eta_c}. \quad (3.4.10)$$

This is the time taken for one full cycle, in units of $\lambda\bar{E}/T_h^2$, under optimum mode of operation of the heat engine for Wideband bosonic channel.

From Fig(3.7) which is a plot of time taken for one full cycle, Π, versus τ, we observe that Π_{opt} and Π_{mp} decrease from infinity to four and to one, respectively, as τ goes from zero to infinity. For the case of efficiency at maximum power, η_{mp}, we extracted minimum work with in the shortest time (Π_{mp}) possible but for the case of optimized efficiency, η_{opt}, we extracted a higher amount of work with the expense of more extra time. That means the time of one complete cycle for the case of optimized efficiency, Π_{opt}, is greater than the time taken to complete one cycle in the case of efficiency at maximum power, Π_{mp}.

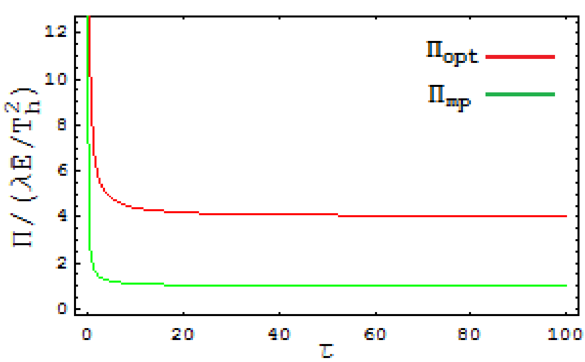

Figure 3.7: Plots of the periods, Π_{opt} and Π_{mp}, versus $\tau = (T_h/T_c - 1)$ for the case of Wideband bosonic channel.

The plot relative time, Π_{rel}, versus τ in Fig(3.8) shows that Π_{rel} is constant for the overall range of τ. This is because the difference between Π_{opt} and Π_{mp} is $3\Pi_{mp}$ for all values of τ. Therefore, Π_{rel} tells us the relation between Π_{opt} and Π_{mp} which is constant with respect to our parameter τ. Thus, we can clearly see that the process of the optimized cycle is three times slower than that of the maximum power cycle.

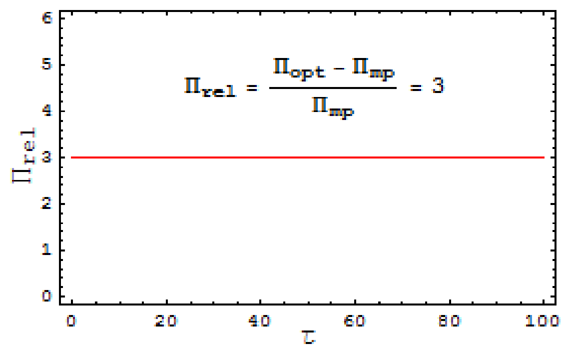

Figure 3.8: Plot of relative time, Π_{rel}, versus $\tau = (T_h/T_c - 1)$ for the case of Wideband bosonic channel.

3.5 Narrowband Photon Channel

In the case of narrowband photon channel, $B \ll \nu$, where ν is the channel central frequency. This time the noise power (N) is given by Eq.(2.3.1). The dimensionless capacity of Narrowband photon channel [3] is also given in the Eq.(2.3.2) . Therefore, using Eq.(2.3.2) and (2.3.1) the time rate of our objective function $\dot{\Omega}$ becomes,

$$\dot{\Omega} = T_h(2-\eta_c-\eta_{mp})\left\{ \ln\left[1+\frac{S}{h\nu(1+n_B)}\right]+\left(\frac{S}{h\nu}n_B\right)\ln\left[1+\frac{h\nu}{S+h\nu n_B}\right]-n_B\ln[1+n_B^{-1}]\right\}. \tag{3.5.1}$$

We have to optimize $\dot{\Omega}$ over S like the previous cases and obtain S_{opt}. So, the optimization relation

$$\left.\frac{\partial\dot{\Omega}(S;\{T_c,T_h\})}{\partial S}\right|_{S=S_{opt}} = 0, \tag{3.5.2}$$

gives S_{opt} to have a value

$$S_{opt} = h\nu\left[\frac{1}{e^{h\nu/T_h(2-\eta_c-\eta_{mp})} - n_B}\right]. \tag{3.5.3}$$

Using S_{opt}, we get

$$I_p(S_{opt}, T_c) = \ln\left[1 + \frac{h\nu\left\{\frac{1}{e^{h\nu/T_h(2-\eta_c-\eta_{mp})}-n_B}\right\}}{h\nu(1+n_B)}\right]$$
$$+\left[\frac{h\nu[\frac{1}{e^{h\nu/T_h(2-\eta_c-\eta_{mp})}-n_B}]}{h\nu} + n_B\right]\ln\left[1 + \frac{h\nu}{h\nu\left[\frac{1}{e^{h\nu(2-\eta_c-\eta_{mp})/T_h}-n_B}\right] + h\nu n_B}\right]$$
$$-n_B \ln[1 + n_B^{-1}]. \tag{3.5.4}$$

Using Eqs.(3.5.3) and (3.5.4) into Eq.(3.2.5), and fixing $h = 1, T_c = 1$ and $\nu = 0.2$ and expanding in powers of η_c, we once again get η_{opt}

$$\eta_{opt} = 0.875\eta_c + 0.0258682\eta_c^2 + 0.0133085\eta_c^3 + \bigcirc(\eta_c^4). \tag{3.5.5}$$

In the linear regime the optimized efficiency is $(7/8)\eta_c$, which is similar to the above two cases. This is a striking result, which we can see that in the linear regime $\eta_{opt} = (7/8)\eta_c$, that is uniform for the information channels we consider.

From fig(3.9) which is plots of the efficiencies, η_c, η_{mp} and η_{opt}, versus τ, we can see that

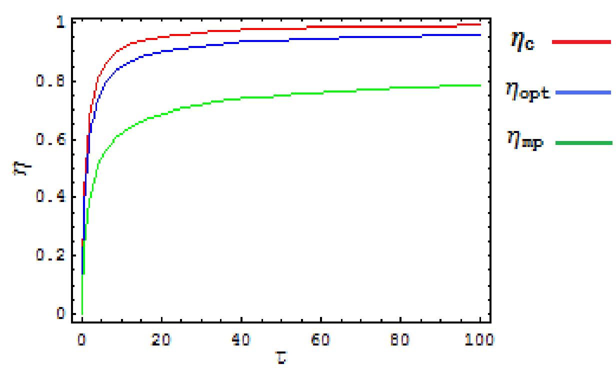

Figure 3.9: Plots of the efficiencies, η_c, η_{mp} and η_{opt}, versus $\tau = (T_h/T_c - 1)$ for the case of Narrowband photon channel.

the efficiency increases as τ increase. This is because as τ increase the dissipated energy decreases so that the work extracted increases which results in an increase in efficiency. Energy dissipation is highest for η_{mp} cycle followed by η_{opt} and η_c the least so that the work extracted for the case of η_{opt} takes the highest share of the input energy next to the case of Carnot cycle. All the efficiencies η_c, η_{opt} and η_{mp} approach to unity for τ infinity. As we can see η_{opt} is between η_c and η_{mp} which is above η_{mp} and more closer to η_c.

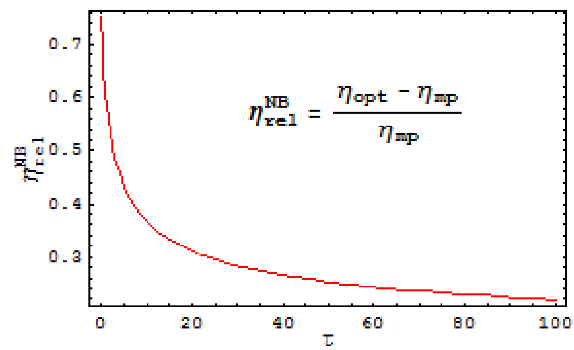

Figure 3.10: Plot of relative efficiency, η^{NB}_{rel}, versus $\tau = (T_h/T_c - 1)$ for the case of Narrowband photon channel.

Plot of relative efficiency, η^{NB}_{rel}, versus τ in Fig(3.10) shows that η^{NB}_{rel} decreases as τ increase. This is because as τ increases the difference between η_{opt} and η_{mp} decreases so that the the ratio of the difference between η_{opt} and η_{mp} divided by η_{mp} also decrease. The largest value of η^{NB}_{rel} is 0.75 for τ approaches zero and decreases to zero for τ infinity. Therefore, we can say that η_{opt} is effective for τ small.

The time taken for a working cycle under optimized efficiency in the case of Narrowband photon channel can be calculated using Eq.(3.2.6). Putting Eqs.(2.0.1) and (3.5.4) into Eq.(3.2.6), and expanding in powers of η_c, we get

$$\frac{\Pi_{opt}}{\bar{E}} = \frac{4}{\eta_c} - 6.86394 + 2.61248\eta_c + 0.118959\eta_c^2 + 0.0456113\eta_c^3 + \bigcirc(\eta_c^4) \qquad (3.5.6)$$

This is the period in units of $\bar{E}$ under optimum mode of operation of the heat engine for the case of Narrowband photon channel. From the above equation we can see that Π_{opt} goes to infinity for η_c zero and Π_{opt} is zero for η_c.

Plot of time taken for one complete cycle, Π, versus τ in Fig(3.11) shows that both Π_{opt} and Π_{mp} goes to infinity as τ approaches zero and to zero as τ goes to infinity. Again here Π_{opt} is greater than Π_{mp} (the shortest time) because we extract a better amount of

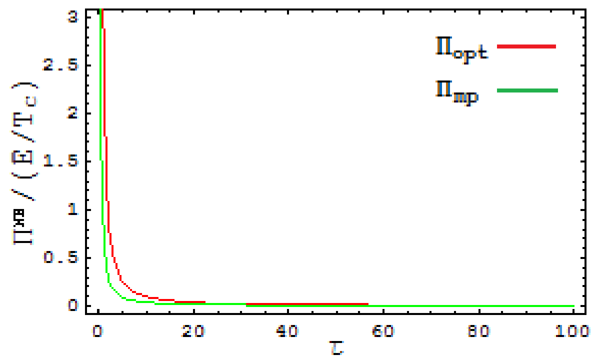

Figure 3.11: Plots of the periods, Π_{opt} and Π_{mp}, versus $\tau = (T_h/T_c - 1)$ for the case of Narrowband photon channel.

work in the case of optimized efficiency with the expense of time.

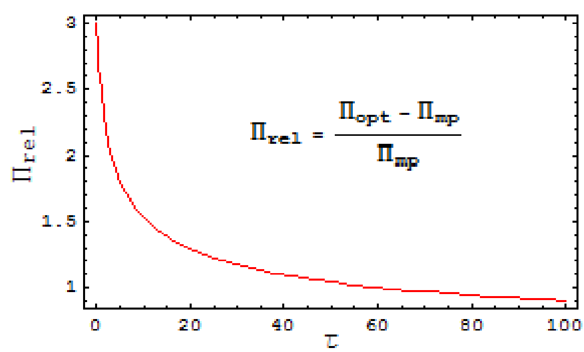

Figure 3.12: Plot of relative time, Π_{rel}, versus $\tau = (T_h/T_c - 1)$ for the case of Narrowband photon channel.

From Fig(3.12), which is a plot of relative time, Π_{rel}, versus τ, we can see that Π_{rel} decreases as τ increase. The reason is that the maximum possible difference difference between Π_{opt} and Π_{mp} appears for τ near zero. For τ infinity Π_{opt} is proportional to Π_{mp} which makes Π_{rel} zero. Note that Π_{rel} is large means the process in the optimized cycle is very slow compared to maximum power cycle and Π_{rel} is very small (almost zero) means that the speed of the process is comparably equal.

Note that in all the plots we use the expression of the efficiencies before expansion so that the plots include all non-linear terms.

Chapter 4

Summary and Conclusion

In this thesis we have considered a minimal model of finite-time thermodynamic heat engine (or a Maxwell Demon), attributing energy dissipation within the engine to irreversible loss of energy within the communication channels and explored its properties. Based on the model we obtained analytical expressions for work, objective function and different efficiencies of the heat engine for all the communication channel we considered. To analyze the best energy compromise between useful effective and useful loss energies, we derived the expression for the objective function $\dot{\Omega}$ using optimization method proposed by Hernandez et al [1]. We have used a condition which was originally tried by Yeneneh Yalew [16] in optimizing the engine's efficiency.

Analyzing both classical and quantum information channels, we derived the expression for efficiency at maximum power and optimized efficiency. For further comparison between the efficiency at maximum power and the optimized efficiency we derived the time (period) required for each mode of operation. We obtained the optimized efficiency lower bounded by efficiency at maximum power and upper bounded by Carnot efficiency. We have seen that the model heat engine satisfies the Carnot limit for reversible mode. Independently, it leads to the universal linear term in the maximum power efficiency as in [4] for finite-time thermodynamic process according to Yun Zhou and Divera Segal [13]. For finite-time process, we also obtain the same linear term in the optimized efficiency for the information channels we considered it is almost the same as the result of Mesfin A. and Mulugeta B. [9]. We have seen that the optimized efficiency in the linear response regime is independent of the type of information channel.

We expect this model to be useful for studying basic concepts in thermodynamics, like the model could serve as a cooling device or the performance could be analyzed for a subsystem away from the Boltzmann distribution. One could also consider other source of energy dissipation. One could also consider different conditions in optimizing the engine's efficiency. Finally, one could further advance the model by considering transmission of quantum information.

Appendix

Introduction to Information Transmission.

Although we seem to know the meaning of the word information, fundamentally that may not be the case. In reality information may be defined as related to usage. From the view point of mathematical formalism, entropy information is basically a probabilistic concept. In other words, without probability theory there would be no entropy information.

An information transmission has two basic disciplines: one developed by Wiener[6] and the other by Shannon[2, 6]. Although both share a common interest, there is a basic distinction between their ideas. The significance of Wiener's work is that, if a signal (information) is corrupted by some physical means (like, noise), it may be possible to recover the signal from the corrupted one. However, Shannon shows that the signal can be optimally transferred provided it is properly encoded; that is, the signal to be transferred can be processed before and after transmission. In other words, it is possible to combat the disturbances in a communication channel by properly encoding the signal. This is the reason that Shannon advocates the information measure of the signal and communication channel capacity. In fact the main objective of Shannon's theory is the efficient utilization of information communication channel.

Let us now define information measure, which is one of the vitally important aspect in the development of Shannon information theory. For simplicity, we consider discrete input and output message ensembles $A = \{a_i\}$ and $B = \{b_j\}$, respectively, as applied to a communication channel, as shown in Fig(4.1).

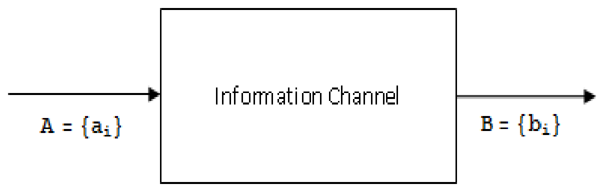

Figure 4.1: Input-output communication channel.

If a_i is an input event sent to the channel and b_j is the corresponding transmitted output event, then the information measure $I(a_i; b_j)$ (in bits) about the received event b_j specifies a_i and can be written as

$$I(a_i; b_j) = \log_2 \frac{p(a_i/b_j)}{p(a_i)} \tag{4.0.1}$$

where $P(a_i/b_j)$ is the conditional probability that input a_i depends on the output event b_j, $p(a_i)$ is a priori probability of input event a_i, $i = 1, 2, ..., M$ and $j = 1, 2, ..., N$. By symmetry property of the joint probability, we have

$$I(a_i; b_j) = I(b_j; a_i). \tag{4.0.2}$$

If the input and output events are statistically independent, that is, if $p(a_i, b_j) = p(a_i)p(b_j)$, then $I(a_i; b_j) = 0$.

The self-information of the input event and the output event are defined as

$$I(a_i) = \log_2 \frac{1}{p(a_i)} = -\log_2 p(a_i),$$
$$I(b_j) = \log_2 \frac{1}{p(b_j)} = -\log_2 p(b_j),$$

which represents the amount of information provided at the input and output of the channel respectively.

If $p(a_i/b_j) = 1$, then

$$I(a_i; b_j) = I(a_i). \tag{4.0.3}$$

When the above equation is true for all i, the communication channel is said to be *Noiseless*. However, if $p(b_j/a_i) = 1$, then

$$I(a_i; b_j) = I(b_j). \tag{4.0.4}$$

If this is true for all j, then the information channel is called *Deterministic*.

Note that the information measure can be easily extended to higher product spaces, such as

$$I(a_i; b_j/c_k) = \log_2 \frac{p(a_i/b_j c_k)}{p(a_i/c_k)}. \tag{4.0.5}$$

But measure of information is characterized by ensemble average; therefore, the average amount of information provided at the input end becomes

$$I(A) = -\sum_A p(a) \log_2 p(a) = H(A). \tag{4.0.6}$$

Similarly, the average amount of self-information provided at the output end can be written as

$$I(B) = -\sum_B p(b) \log_2 p(b) = H(B). \tag{4.0.7}$$

The above two equations have the same form as the entropy equation in statistical thermodynamics, in which $H(A)$ and $H(B)$ are used to describe information (Shannon) entropy. Note that H, from communication point of view, is a measure of uncertainty. However, from statistical thermodynamic point of view it is a measure of disorder.

Communication Channel

Information channels are usually described according to the type of input-output ensemble and are considered discrete or continuous. If both the input and output of the channel are discrete events, then the channel is called *Discrete Channel*. But if both the input and output of the channel are represented by continuous events, the channel is called *Continuous Channel*. However, if a channel have a discrete input and a continuous output or vice

versa, then the channel is called *Discrete-Continuous* or *Continuous-Discrete channel.*
A communication channel can have multiple inputs and multiple outputs. A channel is said to be one-way if it has one input terminal and one output terminal. However, if the channel possesses two input and two output terminals, it is two-way channel. One can also have a channel with n input and m output terminals.
Since communication channel is characterized by the input-output transitional probability distribution $P(B/A)$, if the transitional probability distribution remains the same for all successive input and output events, then the channel is a *memoryless channel.* However, if the transitional probability distribution changes with the preceding events, whether at the input or the output then the channel is a *memory channel.* Thus, if the memory is finite, the transitional probability depends on a finite number of preceding events, the channel is a *finite memory channel.*
It is trivial to extend the ensemble average to the conditional entropy:

$$I(B/A) = -\sum_B \sum_A p(a,b) \log_2 p(b/a) = H(B/A). \tag{4.0.8}$$

And the entropy of the product ensemble AB can be written as

$$H(AB) = -\sum_B \sum_A p(a,b) \log_2 p(a,b). \tag{4.0.9}$$

Thus, we obtain that

$$H(AB) = H(A) + H(B/A), H(BA) = H(B) + H(A/B). \tag{4.0.10}$$

Let us now turn to the definition of average mutual information. We consider the conditional average mutual information

$$I(A;b) = \sum_A p(a/b) I(a;b). \tag{4.0.11}$$

By taking the ensemble average of the above equation, the mutual information can be defined as

$$I(A;B) = \sum_B \sum_A p(a/b) \log_2 \frac{p(a/b)}{p(a)}$$
$$I(A;B) = \sum_A \sum_B p(a;b) I(a;b). \tag{4.0.12}$$

Although the mutual information between input event and output event can be negative, $I(a,b) < 0$, the average conditional mutual information can never be negative, $I(A,b) \geq 0$. Moreover, from symmetry property of $I(a,b)$, we have $I(A,B) = I(B,A)$.
Putting $p(a/b) = p(a,b)/p(b)$ into Eq.(4.0.12), we obtain

$$H(AB) = H(A) + H(B) - I(A;B). \tag{4.0.13}$$

Comparing the above equation with Eq.(4.0.10), we get

$$I(A;B) = H(A) - H(A/B), I(A;B) = H(B) - H(B/A), \tag{4.0.14}$$

where $H(A/B)$ is the amount of information loss due to noise or the equivocation of the channel, which is the average amount of information needed to specify the noise disturbance in the channel. A channel is said to be continuous if and only if the input and output ensembles are represented by continuous Euclidean spaces. For simplicity we only discuss one-dimensional case. A memoryless continuous channel is said to be distributed by an additive noise if and only if the transitional probability density $p(b/a)$ depend on the difference between the output and input random variables, $p(b/a) = p(b - a)$.
Let us evaluate one of the most useful channel, a memoryless, time continuous, band-limited, continuous channel. The channel is assumed to be affected by an additive white Gaussian noise, and a band-limited time-continuous signal, with an average power not to exceed a given value S, is applied at the input end of the channel.
Note that, if a random process is said to be a stationary Gaussian process, then the corresponding joint probability density distribution, assumed by a time function at any finite

time interval, is independent of time and it has a Gaussian distribution. If a stationary Gaussian process is said to be white, then the power spectral density must be uniform (constant) over the entire range of the frequency variable.

Let $c(t)$ be a white Gaussian noise. By the Karhunen-Loeve expansion theorem[6], $c(t)$ can be written over a time interval $-T/2 \leq t \leq T/2$,

$$c(t) = \sum_{i=-\infty}^{\infty} c_i \phi_i(t) \tag{4.0.15}$$

where the $\phi_i(t)$'s are orthogonal functions that can be represented by

$$\int_{-T/2}^{T/2} \phi_i(t)\phi_j(t)dt = \begin{cases} 1, i=j \\ 0, i \neq j, \end{cases} \tag{4.0.16}$$

and c_i are real coefficients commonly known as orthogonal expansion coefficients. Furthermore, the c_i's are statistically independent, and the individual probability densities have a stationary Gaussian distribution, with Zero mean and the variance equal to $N_o/2T$, where N_o is the corresponding power spectral density.

Now we consider an input $a(t)$ as applied to the communication channel, where the frequency spectrum is limited by the channel bandwidth $\Delta\nu$. Since the channel noise is assumed to be an additive white Gaussian noise, the output response of the channel is

$$b(t) = a(t) + c(t). \tag{4.0.17}$$

Such a channel is known as a *band-limited* channel with additive white Gaussian noise. Again by Karhunen-Loeve expansion theorem, the input and output events can be expanded

$$a(t) = \sum_{i=1}^{\infty} a_i \phi_i(t), \tag{4.0.18}$$

and

$$b(t) = \sum_{i=-\infty}^{\infty} b_i \phi_i(t). \tag{4.0.19}$$

Thus, we see that

$$b_i = a_i + c_i. \tag{4.0.20}$$

Since the input function $a(t)$ is band-limited by $\Delta\nu$, only $2T\Delta\nu$ coefficients a_i, $i = 1, 2, ..., 2T\Delta\nu$, within the passband are considered. In other words, the input signal ensemble can be represented by a $2T\Delta\nu$-order product ensemble over a; that is, $A^{2T\Delta\nu}$. This is similarly true for the output ensemble over b; that is, $B^{2T\Delta\nu}$. Thus, the average mutual information between the input and output ensembles is

$$I(A^{2T\Delta\nu}; B^{2T\Delta\nu}) = H(B^{2T\Delta\nu}) - H(B^{2T\Delta\nu}/A^{2T\Delta\nu}). \tag{4.0.21}$$

It is also clear that **a**, **b**, and **c** each form a $2T\Delta\nu$-dimensional vector space, for which we write

$$\mathbf{b} = \mathbf{a} + \mathbf{c}. \tag{4.0.22}$$

If we let $p(\mathbf{a})$ and $p(\mathbf{c})$ be the probability density of **a** and **c** respectively, then the transitional probability density of $p(\mathbf{b/a})$ is

$$p(\mathbf{b}) = p(\mathbf{b\text{-}a}) = p(\mathbf{c}). \tag{4.0.23}$$

where **a** and **b** are statistically independent. For simplicity, we let $X = A^{2T\Delta\nu}$ be the vector space (the product space) of **a**. The probability density distribution of **b** can be determined by

$$p(\mathbf{b}) = \int_X p(\mathbf{a})p(\mathbf{c})dX, \tag{4.0.24}$$

where the integral is over the entire vector space X. Similarly, $Y = B^{2T\Delta\nu}$ and $Z = C^{2T\Delta\nu}$ represent the vector space of **b** and **c**, respectively. The average mutual information can be therefore be written by

$$I(X;Y) = H(Y) - H(Z), \tag{4.0.25}$$

where $H(Y) = -\int_Y p(b)\log_2 p(b)dY$ and $H(Z) = H(Y/X) = -\int_Z p(c)\log_2 p(c)dZ$. The channel capacity can be determined by maximizing $I(X;Y)$, that is,

$$C = max_{T,p(a)}\frac{I(X;Y)}{T}, bits/time. \tag{4.0.26}$$

Under the constraint of the signal mean-square fluctuation that cannot exceed a specified value S,

$$\int_X |a|^2 p(\mathbf{a}) dX \leq S. \tag{4.0.27}$$

Since each of the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are represented by $2T\Delta\nu$ continuous variables, and each c_i is statistically independent Gaussian distribution with zero mean and has variance $\sigma_{c_i}^2 = N_o/2T$, we see that

$$I(X;Y) = I(A^{2T\Delta\nu}; B^{2T\Delta\nu}) = \sum_{i=1}^{2T\Delta\nu} I(A_i; B_i). \tag{4.0.28}$$

Thus, we have

$$H(Z) = 2T\Delta\nu H(C_i), \tag{4.0.29}$$

where

$$H(C_i) = \frac{1}{2}\log_2(2\pi e\sigma_{c_i}^2), \tag{4.0.30}$$

Then $H(Z)$ can be written as

$$H(Z) = T\Delta\nu \log_2 \frac{\pi e N_o}{T}. \tag{4.0.31}$$

Similarly, we have

$$H(B_i) = \frac{1}{2}\log_2(2\pi e\sigma_{b_i}^2), \tag{4.0.32}$$

when b_i has Gaussian distribution with zero mean and variance equal to $\sigma_{b_i}^2$, this is possible if and only if a_i has Gaussian distribution with zero mean. The average mutual information becomes

$$\begin{aligned} I(X;Y) &= \sum_{i=1}^{2T\Delta\nu} H(B_i) - H(Z), \\ I(X;Y) &= \frac{1}{2}[\log_2 \prod (2\pi e\sigma_{b_i}^2)] - T\Delta\nu \log_2 \frac{\pi e N_o}{T}, \end{aligned} \tag{4.0.33}$$

But the variance of $p(\mathbf{b})$ is given by

$$\sigma_{b_i}^2 = \sigma_{a_i}^2 + \sigma_{c_i}^2 = \sigma_{b_i}^2 + \frac{N_o}{2T}. \tag{4.0.34}$$

Using Eq.(4.0.13) we have

$$\sum_{i=1}^{2T\Delta\nu} \sigma_{b_i}^2 = \sum_{i=1}^{2T\Delta\nu} \sigma_{a_i}^2 + N_o\Delta\nu \leq S + N, \tag{4.0.35}$$

where $N = N_o\Delta\nu$. The equality holds for Eq.(4.0.21) when the input probability density distribution $p(\mathbf{a})$ has Gaussian distribution with zero mean and a variance equal to S. From Eq.(4.0.21), we get

$$\prod_{i=1}^{2T\Delta\nu} \sigma_{b_i}^2 \leq \left(\frac{S+N}{2T\Delta\nu}\right)^{2T\Delta\nu}, \tag{4.0.36}$$

where the equality holds for Eq.(4.0.22) if and only if $\sigma_{b_i}^2$ are all equal and the input probability density distribution $p(\mathbf{a})$ has Gaussian distribution with zero mean and a variance equal to S.

Therefore, the channel capacity becomes

$$C = max_{T,p(a)} \frac{I(X;Y)}{T} = \Delta\nu \log_2\left(1 + \frac{S}{N}\right) bits/sec \tag{4.0.37}$$

where S/N is the signal-to-noise ratio. We note that the above result is one of the most popular equation derived by Shannon [2] for a memoryless additive Gaussian channel. The channel capacity is derived under the assumption of additive white Gaussian noise regime, and the average input signal power cannot exceed a value S.

Capacity of Wideband photon channel

Let us denote the mean quantum number of the photon signal by $\bar{m}$, and a noise by $\bar{n}$. We assume an additive channel, thus, the signal plus noise is

$$\bar{f} = \bar{m} + \bar{n}. \tag{4.0.38}$$

Since the photon density (i.e. the mean number of photons per unit time per frequency) is the mean quantum number, the signal energy density per unit time can be written as

$$E_S(\nu) = \bar{m}h\nu, \tag{4.0.39}$$

where h is Planck's constant. Similarly, the noise energy density per unit time is

$$E_N(\nu) = \bar{n}h\nu. \tag{4.0.40}$$

Due to the fact that the mean quantum noise (blackbody radiation at temperature T) follows Planck's distribution (also known as Bose-Einstein distribution),

$$E_N(\nu) = \frac{h\nu}{e^{h\nu/k_BT} - 1}, \tag{4.0.41}$$

the noise energy per unit time (noise power) can be calculated by

$$N = \int_{\epsilon}^{\infty} E_N(\nu)d\nu = \int_{\epsilon}^{\infty} \frac{h\nu}{e^{h\nu/k_BT} - 1}d\nu = \frac{(\pi k_B T)^2}{6h}, \tag{4.0.42}$$

where ϵ is an arbitrary small positive constant. Thus, the minimum required entropy for the signal radiation is

$$\Delta S = \int_0^N \frac{dE_s(T')}{T'} = \int_0^T \frac{dE_s(T')}{dT'}\frac{dT'}{T'}, \tag{4.0.43}$$

where $E(T)$ is signal energy per unit time as a function of temperature T' and T is the temperature of the blackbody radiation. Thus, in the presence of a signal the output radiation energy per unit time (the power) can be written

$$P = S + N, \tag{4.0.44}$$

where S and N are the signal and the noise power, respectively. Since the signal is assumed to be deterministic (i.e. the microstate signal), the signal entropy can be considered zero. The validity of this assumption is based on the independent statistical nature between the signal and the noise, for which the photon statistics follow Bose-Einstein distribution. For the case of Bose-Einstein statistics, we get that the amount of entropy transfer by radiation remains unchanged,

$$H(B/A) = \frac{\Delta S}{k_B \ln 2} = \frac{1}{k_B \ln 2}\int_0^T \frac{dE_s(T')}{dT'}\frac{dT'}{T'}. \tag{4.0.45}$$

From the relation in Eq.(4.0.14), we see that $I(A; B)$ is maximum when $H(B)$ is maximum. Therefore, for maximum information transfer, the photon signal should be chosen randomly. But the maximum value of $H(B)$ occurs when the ensemble of the microstates of the total radiation corresponds to Gibbs's distribution, which reaches to the thermal equilibrium. Thus, the corresponding mean occupational quantum number of the total radiation also follows Bose-Einstein distribution at a given temperature $T_e \geq T$,

$$\bar{f}(\nu) = \frac{h\nu}{e^{h\nu/k_B T} - 1}, \tag{4.0.46}$$

where T_e is defined as the effective temperature. Whereas the output entropy can be determined by

$$H(B) = \frac{1}{k_B \ln 2} \int_0^{T_e} \frac{dE_N(T')}{dT'} \frac{dT'}{T'}, \tag{4.0.47}$$

the quantum mechanical channel capacity can be evaluated using Eqs.(4.0.31) and (4.0.33)

$$C = H(B) - H(B/A) = \frac{1}{k_B \ln 2} \int_T^{T_e} \frac{dE_N(T')}{dT'} \frac{dT'}{T'}, \tag{4.0.48}$$

where $E_N(T') = (\pi k_B T')^2/6h$. In view of output power

$$P = S + N = \frac{(\pi k_B T_e)^2}{6h} = S + \frac{(\pi k_B T)^2}{6h}, \tag{4.0.49}$$

the effective temperature becomes

$$T_e = \left[\frac{6hS}{(\pi k_B T_e)^2} + T^2\right]. \tag{4.0.50}$$

Substituting Eq.(4.0.36) into Eq.(4.0.34) and setting $k_B = 1$, we obtain the capacity of the photon channel to be

$$C = \frac{T}{\lambda \ln 2}\left[(1 + \frac{2\lambda S}{T^2}) - 1\right], \tag{4.0.51}$$

where $\lambda = \pi^2/3h$. Note that high signal-to-noise ratio corresponds to high frequency transmission ($h\nu \gg k_B T$), for which we have $6hS/(\pi k_B T)^2 \gg 1$. Then the photon channel capacity is limited by the quantum statistics, that is

$$C_{quant} = \frac{\pi}{\ln 2}\left(\frac{2S}{3h}\right)^{1/2}. \tag{4.0.52}$$

However, if the signal-to-noise ratio is low ($h\nu \gg k_B T$), the photon channel capacity reduces to the classical limit

$$C_{class} = \frac{S}{T \ln 2}. \tag{4.0.53}$$

Narrowband photon channel

In this section we consider a narrowband number-state channnel that operates at frequency, ν, with in a bandwidth $B \ll \nu$ as done by Caves and Drummond [3]. The input to the channel is are photon-number eigenstates $\mid n >$, thus the input alphabet is the set of non-negative integers, and a transmitted message is a sequence of integers. The input is described statistically by probabilities $p_N(n)$ to transmit n photon down the channel at each use. We let

$$\bar{n} = \sum_{n=0}^{\infty} n p_N(n), \tag{4.0.54}$$

which is the average signal photon transmitted per channel use. The average photon transmission rate is $B\bar{n}$. For a narrowband channel with fixed frequency and bandwidth, we can implement the power constraint as the constraint that the probabilities $p_N(n)$ have the well defined mean $\bar{n}$.

At the channel output the photons are counted by an ideal photodetector. Thus the output alphabet is also the set of non-negative integers, and a received message is a sequence of integers. Channel noise is characterized by a conditional probability to count m photon at the output when n photons are transmitted at the input. The unconditional probability to count m photons at the output is

$$p_M(m) = \sum_n p_{M/N}(m/n) p_N(n). \tag{4.0.55}$$

The information transmitted from input to output is quantified by mutual information $I(M, N)$,

$$I(M,N) = H(M,N) = H(M) - H(M/N) = \sum_{n,m} p_{M/N}(m/n) p_N(n) \log_2 \frac{p_{M/N}(m/n)}{p_M(m)}, \tag{4.0.56}$$

where $H(M)$ and $H(M/N)$ are obtained from Eq.(4.0.7) and Eq.(4.0.9) respectively.

$$H(M) = -\sum_n p_M(m) \log_2 p_M(m), \tag{4.0.57}$$

$$H(M/N) = -\sum_{n,m} p_{M/N}(m/n)p_N(n)\log_2 p_{M/N}(m/n), \tag{4.0.58}$$

are the total information available at the output and the conditional output information respectively.

The channel capacity, C, is the maximum mutual information, I, times bandwidth, B. The maximum is taken over the possible input probabilities $p_N(n)$, subject to the constraints on average photon number $\bar{n}$ and normalization $\sum_{n=0}^{\infty} p_N(n) = 1$. Therefore, channel capacity in bits/sec is

$$C = max_{p_N(n)} BI(M,N). \tag{4.0.59}$$

Suppose that the channel is affected by "thermal noise" at temperature T and if the thermal noise has probability

$$q(k) = \frac{1}{1+\bar{n}_T}\left(\frac{\bar{n}_T}{1+\bar{n}_T}\right)^k \tag{4.0.60}$$

to emit k photons, where $\bar{n}_T = (e^{h\nu/k_BT} - 1)^{-1}$ is the mean number of thermal photons emitted. The addition of n signal photons make change on Eq.(4.0.46). Since the photons, signal or thermal, are counted with unit efficiency at the output, we model the channel by a conditional probability

$$p_{M/N}(M/N) = \begin{cases} 0, m<n \\ q(m-n), m \geq n \end{cases}, \tag{4.0.61}$$

to count m photons at the output, given transmission of n signal photons. Using Eqs.(4.0.47) and (4.0.46) into Eq.(4.0.44) the conditional output information reduces to

$$\begin{aligned} H(M/N) &= -\sum_{k=0}^{\infty} q(k)\log_2 q(k) \\ &= \bar{n}_T \log_2(1+\bar{n}_T^{-1}) + \log_2(1+\bar{n}_T), \end{aligned} \tag{4.0.62}$$

independent of the input probabilities. So to maximize the mutual information $I(M,N) = H(M,N)$, we need to maximize the output information $H(M)$. This is possible by maximizing $H(M)$ with respect to the out put probabilities $p_M(m)$, subject to the constraints that the output distribution be normalized and its mean be $\bar{n}+\bar{n}_T$. Therefore, the output probability becomes

$$p_M(m) = \frac{1}{1+\bar{n}+\bar{n}_T}\left(\frac{\bar{n}+\bar{n}_T}{1+\bar{n}+\bar{n}_T}\right)^m, \tag{4.0.63}$$

which leads to maximum output information

$$\begin{aligned} H(M) &= -\sum_{m=0}^{\infty} p_M(m)\log_2 p_M(m) \\ &= (\bar{n}+\bar{n}_T)\log_2(1+\frac{1}{\bar{n}+\bar{n}_T}) + \log_2(1+\bar{n}+\bar{n}_T). \end{aligned} \tag{4.0.64}$$

Using Eq.(4.0.48) and (4.0.50) into Eq.(4.0.42), we obtain the mutual information $I(M, N)$ to be

$$I(M,N) = (\bar{n}+\bar{n}_T)\log_2(1+\frac{1}{\bar{n}+\bar{n}_T}) + \log_2(1+\frac{\bar{n}}{\bar{n}+\bar{n}_T}) - \bar{n}_T\log_2(1+\frac{1}{\bar{n}_T}), \tag{4.0.65}$$

which is the channel capacity, C, of narrowband photon channel contaminated by "thermal noise" per bandwidth B.

Now we hopefully believe that the introduction about information transmission give us enough background to deal with the model heat engine proposed by Yun Zhou and Devira Segal [13]. This heat engine is modeled based on the transmission of information through a communication channel, which is basically the Maxwell's demon puzzle that has been exorcised by Landauer's memory erasure principle[7, 10].

Bibliography

[1] A. C. Hernandez, A. medina, J. M. M. Roco, J. A. White, and S. Velasco, Phys. Rev. E, **63**, 037102 (2001).

[2] C. E. Shannon, A mathematical theory of Communication, Bell System Technical Journal, **27**, pp. 379-423, 623-656 (1948).

[3] C. M. Caves and P. D. Drummond, Rev. Mod. Phys., **66**,2 (1994).

[4] C. Van den Broeck, Phys. Rev. Lett. **95**, 190602 (2005).

[5] F. Curzon and B. Ahlborn, Am. J. Phys., **43**,22 (1975).

[6] F. T. S. Yu, S. Jutamulia, S. Yin, Introduction to Information Optics (Marcel-Dekker, New-York), 2000.

[7] K. Maruyama, F. Nori and V. Vedral, Rev. Mod. Phys., **81**, 1 (2009).

[8] Julius I. Bowen, IEEE transactions on information theory, vol. it-13, no. 2, 1967.

[9] Mesfin A. and Mulugeta Bekele, Eur. Phys. J. **38**, 457-461 (2004).

[10] M. B. Plenio and V. Vitelli, Contem. Phys., **42**, vol.1, pp.25-60 (2001).

[11] M. Blencowe, V. Vitelli, Phys. Rev. A, **62**, 052104(2000).

[12] M. Esposito, K. Lindenberg and C. V. den Broeck, Phys. Rev. Lett., **102**, 130602(2009).

[13] Y. Zhou and D. Segal, Minimal model of a heat engine: An information theory approach, arXiv:1002.2170v1. [cond-mat.stat-mech] (2010).

[14] T. Schmiedl and U. Seifert, Europhys. Lett. **81**, 20003 (2008).

[15] V. Vedral, Rev. Mod. Phys. **74**, 197 (2002).

[16] Yeneneh Yalew, Energetics and Efficiency Optimization of a Model Brownian Motor in Inhomogenous Medium with Spatially Varying Friction and Temperature, M.Sc. Thesis, unpublished (AAU), 2002.

Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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