INVESTIGATION OF NONLINEAR EFFECTS IN OPTICAL FIBERS

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By

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The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance a thesis entitled "**INVESTIGATION OF NONLINEAR EFFECTS IN OPTICAL FIBERS**" by **Desta Regassa** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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Abstract

Nonlinear effects in optical fiber occur either due to intensity dependence of refractive index of the medium and the effects are an important effects originated due to anharmonic motion of bound electrons under the influence of an applied field. The purpose of this thesis is to investigate nonlinearity due to higher order such as third and fifth order harmonic polarization. Another aim is to find out frequency chirp due to nonlinear effect of self phase modulation. Finally, some parameters which are very important to plot the figure for frequency chirp and nonlinear polarization have to be calculated. In order to investigate nonlinearity in optical fibers, we verify types of fiber and its structure, loss mechanism in fibers, some properties of light wave when its propagate through the fibers like the relation between wave vector and frequency of the light, types of nonlinear effects and threshold values in which self phase modulation of nonlinear effect, frequency chirp and third order polarization. Generally we calculate third order nonlinear susceptibility which is the proper substitution for other odd order nonlinear polarization.

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Chapter 1

Introduction

There has always been a demand for increased capacity of information. Scientist and engineers continuously pursue technological routes for achieving this goal. The technological advances ever since the invention of the laser by Townes et.al in 1960 have indeed revolutionalized the area of telecommunication and network [1]. The beginning of the field of non linear optics is often taken to be the discovery of second harmonic generation by Franken et al.in 1961 after demonstration of first working Laser [2]. Terhune et.al.(1962) first observation of third harmonic generation, Woodburry and NG(1962) first demonstration of stimulated Raman scattering[4], Armstrong et.al.(1962) formulation of general permutation symmetry relation in nonlinear optics[5], Hasegawa and Tappert(1973) first theoretical prediction of soliton generation in optical fibers [6]. Gibbs et.al.(1976) first demonstration and explanation of optical bistability [7], Mollenues et.al(1980), established confirmation of soliton generation in optical fibers[8,9]. Recently, many advances in nonlinear optics have been made, with alot of effort with fields of Bose-Einstein condensation and Laser cooling. At the heart of light wave communication system is the optical fiber, which is very simple in structure. They consist of two sections: the glass core and the cladding layer. The core is a cylindrical structure, and the cladding is a cylinder without a core. Core and cladding have different refractive indices, with the core having a refractive index, n_1, which is slightly higher than that of the cladding, n_2. It is this difference in refractive indices that enables the fiber to guide the light. Because of this guiding property, the fiber is also referred to as an optical waveguide. As a minimum there is also a further layer known as the secondary cladding that does not participate in the propagation but gives the fiber a minimum level of protection. This second layer is referred to as a coating[9]. Although a variaty of optical fibers in most use today are the so called single mode fibers with core diameter of $10\mu m$, optical fibers with typical losses in the range of 0.2 $\frac{dB}{km}$ at 1550nm and capable of transmission at 2-10$\frac{Gb}{sec}$ are now commercially available. But, most current installed systems are based on communication at 1300nm optical window of transmission. The choice of this wavelength was dictated by the fact that the optical pulse propagates through conventional single mode fibers with almost no broadening. Because silica has lowest loss in the 1550nm wavelength band, specially fibers known as dispersion shifted fibers have been developed in the 1550nm band, thus providing us with fibers having lowest loss and almost negligible dispersion. This is not the same for different types of fibers. Example, in step index fiber the refractive of index of the core is uniform throughout and undergoes step change at the core cladding boundary. The light waves propagating through the fiber are in the form of meridional rays which will cross the the fiber axis during every reflection at the core cladding boundary and propagate in a zig zag manner. But, in the graded index fiber, the refractive index of the core is made to vary in parabolic manner such that the maximum value of refractive index is at the center of the core. The index rays propagating through it in the form of skew rays or helical rays which will not cross the fiber axis at any time and are propagating around fiber axis in helical or spiral manner. Based on the number of modes propagating inthe fiber, there are multimode fibers and single mode fibers. Mode means here a mathematical concept of describing the nature of propagation of electromagnetic waves in the waveguide. It is the nature of electromagnetic field pattern or configuration along the light path inside the fiber. In single mode fibers only one mode LP_{01} can propagate through the fiber. Normally the number of modes propagating through the fiber is proportional to parameter connected with the cutoff condition defined by V_{number}. Where

$V_{number} = \frac{2\pi}{\lambda} n_1 a\sqrt{2\triangle}$, where **a** is radius of the core of the fiber, $\mathbf{n_1}$ is refractive index of the core, λ is wavelength of the light propagating through the fiber and $\triangle$ is relative refractive index $= \frac{n_1^2 - n_2^2}{2n_1} \approx n_1 - n_2$. This is a dimensionless number that determines how many modes a fiber can support. In the case of single mode fibers $V_{number} \leqslant 2.405$. Single mode fiber has smaller core diameter ($10\mu m$) and the difference between refractive index of the core and cladding is very small. Fabrication of single mode fiber is difficult and so fiber is expensive. Further the launching of light into a single mode fibers is also difficult. Generally in the single mode fibers transmission loss and dispersions are very small. So single mode fibers are very useful in long distance communication. Multimode fibers allow a large number of modes for the light rays traveling through it. Here $V_{number} >$ 2.405. The total number of modes M propagating through a given multimode step index fiber is $M = \frac{V}{2} = 4.9\left[\frac{dn_1\sqrt{2\triangle}}{\lambda}\right]^2$ Where **d** is diameter of the core. For multimode graded index having parabolic refractive index profile core $M = \frac{V^2}{4}$ which is half of the number supported by multimode step index fibers. Generally in multimode fibers the core diameter and relative refractive index difference are larger than in single mode fibers. In a case of multimode graded index fiber, signal distortion is very low because of self focusing effects. Here the light ray travel at different speeds in different paths of the fiber because of parabolic variation of refractive index of the core. In fact light rays are continuously refocused as they travel down the fiber and almost all the rays reach the exit end of the fiber at the time due to the helical path of the light propagation. Launching of light into the fiber and fabrication of the fiber are easy. When Laser light propagate through the the fibers because the intensity of the beam is very high it generates nonlinearity in fibers. This is the study of the interaction of intense laser light with matter. It is the study of phenomena that occur as a consequence of the modification of the optical properties of a material system by the presence of light. Nonlinear optical phenomena are nonlinear in the sense that they occur when the response of a material system to an applied optical field depends in a nonlinear manner on the strength of the optical field. For example, second-harmonic generation occurs as a result of the part of the atomic response that scales quadratically with the strength of the applied optical field. Consequently, the intensity of the light generated at the second-harmonic frequency tends to increase as the square of the intensity of the applied laser light. On the other hand the terms linear and nonlinear in optics mean intensity independent and intensity dependent phenomena respectively. Nonlinear effects in optical fibers occur due to change in the refractive index of the medium with optical intensity and inelastic-scattering phenomenon. The power dependence of the refractive index is responsible for the Kerr-effect. Depending upon the type of input signal, the Kerr-nonlinearity manifests itself in three different effects such as Self-Phase Modulation, Cross-Phase Modulation and Four-Wave Mixing. At high power level, the inelastic scattering phenomenon can induce stimulated effects such as Stimulated Brillouin-Scattering and Stimulated Raman-Scattering. The intensity of scattered light grows exponentially if the incident power exceeds a certain threshold value. The difference between Brillouin and Raman scattering is that the Brillouin generated phonons(acoustic) are coherent and give rise to a macroscopic acoustic wave in the fiber, while in Raman scattering the phonons (optical) are incoherent and no macroscopic wave is generated. Except for self phase modulation and cross phase modulation, all nonlinear effects provide gains to some channel at the expense of depleting power from other channels[10]. Self phase modulation and cross phase modulation affect only the phase of signals and can cause spectral broadening, which leads to increased dispersion[11]. Third-order nonlinear optical interactions (i.e., those described by a $\chi^{(3)}$ susceptibility) can occur for both centrosymmetric and noncentrosymmetric media. Fourth($\chi^{(4)}$) and higher order even harmonics were previously generated at solid surfaces only by irradiation far above the damage threshold, where the higher harmonic properties are determined by high intensity optical interaction with azimuthally isotropic solid density plasma[11].

The most usual procedure for describing nonlinear optical phenomena is based on expressing the polarization P(t) in terms of the applied electric field strength E(t). The reason why the polarization plays a key role in the description of nonlinear optical phenomena is that a time-varying polarization can act as the source of new components of the electromagnetic field. As mentioned electric field strength in an intense beam Laser approaches the internal electric field strength that exists in the medium. To estimate the magnitude of crystal internal field, it is considered the field acting on electron at the site of positive ion in a crystal.

1.1 Objectives

The objective of thesis report is to present basic characteristics of light intensity in optical fibers that are important from the point view of their effects in optical fiber communication system. Nonlinear optical effects in optical fibers are important subject of study from both fundamental and application point of view. They can offer for example novel function such as fiber Lasers, amplifiers, logic device, multiplexer, demultiplexer, generation of short pulse, techniques for format, wave length conversion for improved or added performances of fiber communication system.

1.2 General Objective

To study nonlinear polarization in optical fiber

1.3 Specific Objective

1 To study self phase modulation and evaluate the chirping frequencydescribes how frequency is chirped in nonlinear optical fibers

2 To evaluate the third harmonic component of polarization

1.4 Arrangements Of The work

This thesis report is organized as:

Introduction in chapter one, Propagation of light wave along dielectric medium in chapter two, Under this chapter there is section like, physics of optical fibers and Loss mechanism, group velocity and chirping phenomenon, Group velocity, Nonlinear optical process, Kerr-nonlinear effect like Self phase modulation, Cross phase modulation and Four wave mixing, Second order nonlinear optical process.

Mathematical formalism in chapter three, Under this chapter there is Third order nonlinear polarization basic of nonlinearity in optical fibers, Fourth and Fifth order harmonic polarization.

Result, Discusion and finally conclusion in chapter four.

Chapter 2

Propagation of light waves along dielectric medium

The four Maxwell equations may be written in general form as

$$\overrightarrow{\nabla}.\overrightarrow{E} = \frac{\rho}{\epsilon_o} \tag{2.0.1}$$

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial}{\partial t}\overrightarrow{B} \tag{2.0.2}$$

$$\overrightarrow{\nabla}.\overrightarrow{B} = o \tag{2.0.3}$$

$$c^2\overrightarrow{\nabla} \times \overrightarrow{E} = \frac{\partial}{\partial x}\overrightarrow{E} + \frac{\overrightarrow{J}}{\epsilon_o} \tag{2.0.4}$$

In this equations ρ is a charge density which in general includes both free charge density ρ_f and bound charge density ρ_b, so that $J = J_f + J_b$. In dielectric,however, $\rho_f = 0$. The bound charge density related polarization is $\rho_b = -\overrightarrow{\nabla}.\overrightarrow{P}$. The quantity $\overrightarrow{J}$ similarly represents the current density and can arise from both free and bound charges, as indicated earlier. In dielecric we have $\rho_f = J_f = 0$. Further more it can be shown that $\overrightarrow{J_b} = \frac{\partial}{\partial t}\overrightarrow{P}$. With these constraints, the four Maxwell equations for a dielectric can be

$$\overrightarrow{\nabla}.\overrightarrow{E} = -\frac{\nabla.\overrightarrow{P}}{\epsilon_o} \tag{2.0.5}$$

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial}{\partial t}\overrightarrow{B} \tag{2.0.6}$$

$$\overrightarrow{\bigtriangledown}.\overrightarrow{B} = o \tag{2.0.7}$$

$$c^2\overrightarrow{\bigtriangledown} \times \overrightarrow{B} = \frac{\partial}{\partial t}\overrightarrow{E} + \frac{1}{\epsilon_o}\frac{\partial}{\partial t}\overrightarrow{P} \tag{2.0.8}$$

Now we take curl of both side of equation (2.0.6) and making simplification using mathematical identity gives

$$\overrightarrow{\bigtriangledown}^2\overrightarrow{E} = \frac{\partial}{\partial t}(\bigtriangledown \times \overrightarrow{B}) \tag{2.0.9}$$

$$\overrightarrow{\bigtriangledown}^2\overrightarrow{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\overrightarrow{E} + \frac{1}{c^2}\frac{1}{\epsilon_o}\frac{\partial^2}{\partial t^2}\overrightarrow{P} \tag{2.0.10}$$

where $\frac{1}{c^2} = \mu_o\epsilon_o$ According to Newton-Lorentz equation a material medium is thought to consists of electron oscillators. When electromagnetic wave is incident on dielectric medium, electron experiences the following forces:

1. The external electric force e$\overrightarrow{E}$ which tries to align the dipole in the direction of$\overrightarrow{E}$.

2. Due to displacement $\overrightarrow{X}$ the spring force $K_s.\overrightarrow{X}$, which tries to restore the position of the dipole. For our case, when an intense electromagnetic energy is radiated through dielectric silica fiber(SiO_2) two oxygen atoms connected with silicon atom with ideal spring oscillates an harmonically. Due to this spring generated, displacement X is acted as electron displacement.

3. Newton force i.e.$m\frac{d^2\overrightarrow{X}}{dt^2}$

4. frictional force:$[F_{fric}] = b\frac{d\overrightarrow{X}}{dt}$. The friction force here represents different loss due to geometry of the fibers and others. It opposes the direction of velocity of light along the fiber[10].

Thus, the Newton-Lorentz equation can be written as,

$$K_s.\overrightarrow{X} + m\frac{d^2\overrightarrow{X}}{dt^2} + b\frac{d\overrightarrow{X}}{dt} = [e\overrightarrow{E}] \tag{2.0.11}$$

and electromagnetic field is given by:

$$\overrightarrow{E} = E_o \exp[i(\omega t - kz)] \tag{2.0.12}$$

The displacement $\overrightarrow{X}$ of the dipole will be in the direction of $\overrightarrow{E}$ and given by

$$\overrightarrow{X} = b_o \exp[i(\omega t - kz)] \tag{2.0.13}$$

$$\frac{d\overrightarrow{X}}{dt} = j\omega_o b_o \exp[i(\omega t - kz)] = i\omega$$

$$\frac{d^2\overrightarrow{X}}{dt^2} = i^2\omega^2 X = -\omega^2\overrightarrow{X} \tag{2.0.14}$$

where $\overrightarrow{X}$ is displacement of oscillators. By combining and rearranging above equations we have,

$$-\omega^2\overrightarrow{X} + 2\beta i\omega\overrightarrow{X} + \omega_o^2\overrightarrow{X} = \frac{e\overrightarrow{E}}{m}. \tag{2.0.15}$$

But using equations (2.0.13) and (2.0.15) we have

$$b_o = \frac{eE_o}{m}\frac{1}{-\omega^2 + 2\beta\omega i + \omega_o^2}. \tag{2.0.16}$$

By combining equations and make rearrangement we have

$$\overrightarrow{X} = \left(\frac{eE_o}{m}\frac{\exp[-i(\omega t - kz)]}{-\omega^2 + 2\beta\omega i + \omega_o^2}\right). \tag{2.0.17}$$

The perivious equation contains both imaginary and real part

$$\overrightarrow{X} = \textbf{real}\left(\frac{eE_o}{m}\frac{1}{-\omega^2 + 2\beta\omega i + \omega_o^2}\right)\exp[-i(\omega t - kz)] \tag{2.0.18}$$

$$\overrightarrow{X} = \frac{eE_o}{m}\left(\frac{\omega_o^2 - \omega^2}{[\omega_o^2 - \omega^2]^2 + 4\beta^2\omega^2}\right). \tag{2.0.19}$$

The dipole moment associated with displacement of the dipole ($\overrightarrow{X}$) is given by:

$$\overrightarrow{p} = e.\overrightarrow{X} = \frac{e^2E_o}{m}\left(\frac{\omega_o^2 - \omega^2}{[\omega_o^2 - \omega^2]^2 + 4\beta^2\omega^2}\right)\exp[i(\omega t - kz)] \tag{2.0.20}$$

where polarizeablity ($\overrightarrow{p}$) is given by

$$\overrightarrow{p} = \alpha(\omega) = \frac{eE_o}{m}\left[\frac{\omega_o^2 - \omega^2}{[\omega_o^2 - \omega^2]^2 + 4\beta^2\omega^2}\right] - 2\beta i\omega\frac{e^2}{m}\left[\frac{1}{\omega_o^2 - \omega^2 + 2\beta^2\omega^2}\right] \tag{2.0.21}$$

where is $\alpha(\omega)$ polarizability.

Polarization ($\overrightarrow{P}$) defined as

$$\overrightarrow{P} = N\alpha(\omega)\overrightarrow{E}$$

$$\overrightarrow{P} = N\frac{e^2}{m}\left[\frac{\omega_o^2 - \omega^2}{(\omega_o^2 - \omega^2)^2 + 4\beta^2\omega^2}\right] - 2\beta i\omega\frac{e^2}{m}\left[\frac{1}{\omega_o^2 - \omega^2 + 2\beta^2\omega^2}\right]\overrightarrow{E}_o\exp[i(\omega t - kz)] \tag{2.0.22}$$

This show linearly polarized light induced due to electromagnetic wave interaction within dielectric medium, where N is the density of dipole. By using equation (2.0.22) we have

$$\bigtriangledown^2\overrightarrow{E} - \frac{1}{c^2}\frac{\partial^2\overrightarrow{E}}{\partial t^2} - \frac{1}{c^2\epsilon_o}\frac{\partial^2\overrightarrow{P}}{\partial t^2} = 0 \tag{2.0.23}$$

$$\overrightarrow{\bigtriangledown}^2 E = \frac{\partial}{\partial t}[\mu\sigma E + \mu\epsilon_o E + \mu P] \tag{2.0.24}$$

From this equation dispersion relation between wave vector and frequency of electromagnetic field derived by using Maxwell's equation. Where $\rho = 0$ because no free charge in the medium, $\sigma = 0$ because medium is perfectly dielectric. $\mu_o\epsilon_o = \frac{1}{c^2}$, $\mu_o = \frac{1}{\epsilon_o c^2}$ for non-ferromagnetic. Using equations (2.0.23) we have

$$(\bigtriangledown^2 + \frac{\omega^2}{c^2})\overrightarrow{E} = \frac{-\omega^2}{c^2}N\alpha(\omega)\overrightarrow{E} \tag{2.0.25}$$

where $\overrightarrow{E}$ is harmonically varying fields, for $\overrightarrow{K}^2 = \bigtriangledown^2$,

$$\overrightarrow{K}^2 = \frac{\omega^2}{c^2}[1 + \frac{1}{\epsilon}N\alpha(\omega)]. \tag{2.0.26}$$

Equation (2.0.19) is the dispersion relation between wave vector ($\overrightarrow{K}$) and frequency (ω) in the dielectric medium. The diagram in (Figure 2.1) shows the pattern by which light is propagate along the fibers and guided mode.

2.1 Physics of Optical Fibers and Loss mechanism

An optical waveguide is a structure that can guide a light beam from one place to another. The most extensively used optical wave guide is the step index optical fiber that consists of cylindrical central dielectric core. Figure 2.2 shows the optical fibers. The cladding is dielectric material of slightly lower refractive index distribution of refractive index in transverse direction given by:

$$\begin{aligned} n_r &= n_1 \ , \ 0 < r < a, \quad \text{core} \\ &= n_2 \ , \ r > a, \quad \text{cladding} \end{aligned} \tag{2.1.1}$$

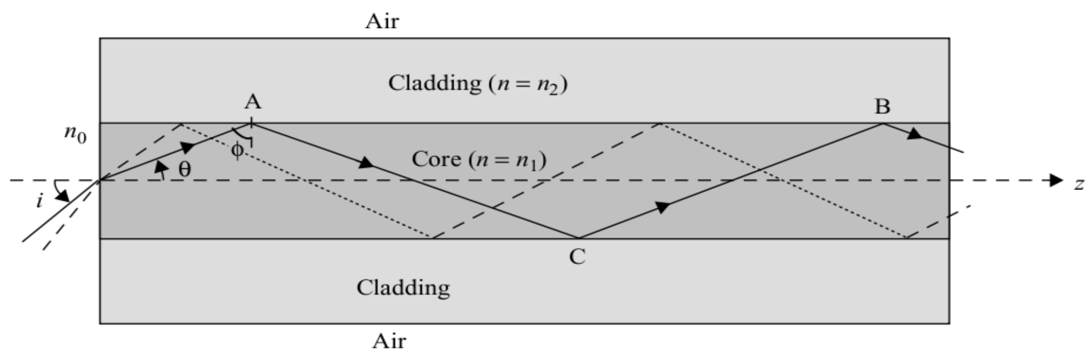

Figure 2.1: Light rays incident on the corecladding interface at an angle greater than the critical angle are trapped inside the fiber core.

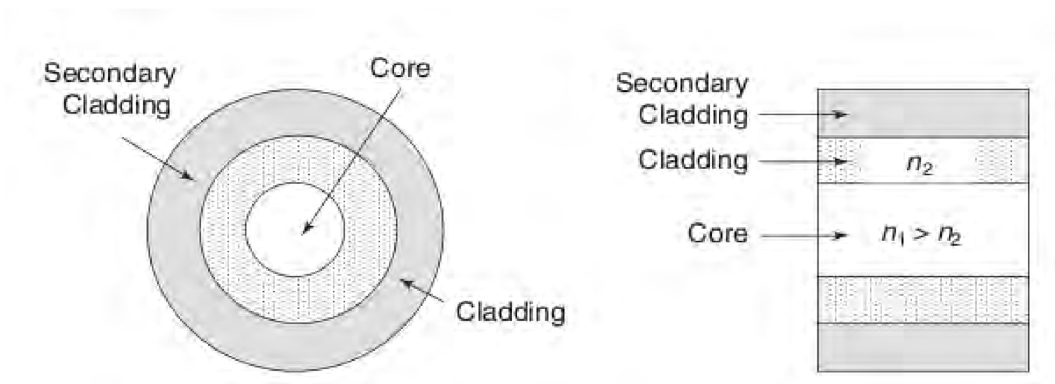

Figure 2.2: Cross section and longtudinal cross section of optical fibers

Where **r** is cylindrical radial coordinates, **a** is radius of the core. Actually, the core extends only finite distance b. However, for practical purpose we will assume that the cladding to extended upto finite. Typically for step index (multimode) silica fiber, $n_1 \approx 1.48, \quad n_2 \approx 1.46, \quad a \simeq 25\mu m, \quad b \simeq 62.5\mu m$. The ray guidance in optical fiber is shown in (Fig 2.1). If the angle of incidence at core-cladding interface (ϕ) is greater than the critical angle-that is $\phi_c = sin^{-1}(\frac{n_2}{n_1})$, then the rays undergo total internal reflection at that interface[9]. The basics of light propagation can be discussed with the use of geometric optics. The basic law of light guidance is Snells law (Figure 2.3). Consider two dielectric media with different refractive indices and with $n_1 > n_2$ and that are in perfect contact, as shown in Figure 2.2. At the interface between the two dielectrics, the incident and refracted rays satisfy Snells law of refraction-that is,

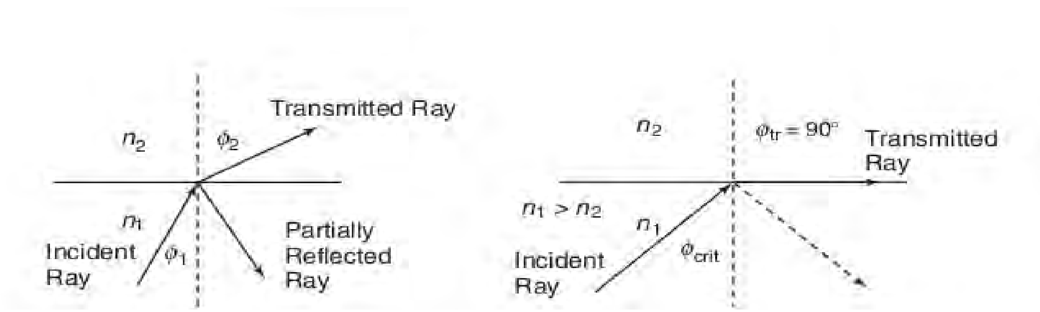

Figure 2.3: Snell's laws

$$n_1 sin\phi_1 = n_2 Sin\phi_2$$
$$\frac{Sin\phi_1}{Sin\phi_2} = \frac{n_2}{n_1} \tag{2.1.2}$$

On the other hand, A ray of light, incident upon the interface between two transparent optical materials having different indices of refraction, will be totally internally reflected (rather than refracted) if

(1). The ray is incident upon the interface from the direction of the more dense material. and

(2). The angle made by the ray with the normal to the interface is greater than some critical angle, the latter being dependent only on the indices of refraction of the media. It worth while to mention here that, a guided ray (also bound ray or trapped ray) is a ray of light in a multi-mode optical fiber, which is confined by the core. For step index fiber, light entering the fiber will be guided if it falls within the acceptance cone of the fiber-that is if it makes an angle with the fiber axis that is less than the acceptance angle:

$$Sin\theta \leq \sqrt{n_1^2 - n_2^2} \tag{2.1.3}$$

where θ is the angle the ray makes with the fiber axis, before entering the fiber, n_o is the refractive index along the central axis of the fiber, and n_c is the refractive index of the cladding. Rays that fall within this angular range are reflected from the core-cladding boundary by total internal reflection, and so are confined by the core. The confinement of light by the fiber can also be described in terms of bound modes or guided modes. This treatment is necessary when considering single mode fiber, since the ray mode does not accurately describe the propagation of light in this type of fiber. In more general cases, when total internal reflection occurs, the refracted wave penetrated into the cladding, but decay very fast[10].

2.1.1 Numerical Aperture

In analyze of Snell's law at the entrance of fibers defined by considering rays that is incident on entrance aperture of the fiber making an angle (i) with axis and the refracted ray make an angle θ with fiber axis. Assumed the outside medium to have refractive index n_0, If the ray has to suffer total internal reflection at core cladding interface

$$Sin\phi = \frac{n_2}{n_1}. \tag{2.1.4}$$

Thus, $sin\theta = [1 - \frac{n_2}{n_1}]^{\frac{1}{2}}$ hence we might have

$$Sini = \left[\frac{n_1^2 - n_2^2}{n_0^2}\right]^{\frac{1}{2}}. \tag{2.1.5}$$

If cone of light is incident in one end of the fiber, it will be guided the fiber provided semi angle of the cone is less than (i). This angle is measured of light gathering power of the fiber and defined as numerical aperture (NA)

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} \tag{2.1.6}$$

where assumed $n_1^2 < n_2^2 + 1$ true for practical purpose. It is used for efficiency of coupling.

2.1.2 Attenuation in optical fibers

The attenuation and dispersion represents two most important characteristics of an optical fiber that determines spacing the fiber optical communication system. The attenuation of optical fiber beam is usually measured in decibel(dB). If input power(p_1) results in an output power(p_2).

$$\alpha = 10log_{10}\frac{p_1}{P_2} \tag{2.1.7}$$

where α is attenuation coefficient.

2.1.3 Pulse dispersion in step index fibers

Pulse dispersion represents one of most important characteristics of an optical fiber that determine the information carrying capacity of fiber optic communication system. When a pulse of light is sent into a fiber, it broadens in the time as it propagates along the fiber. This phenomenon is known as pulse dispersion. This happens due to

- Different rays take different times to propagate through a given length of the fiber called intermodal dispersion.

- Any given source emits light over range of wavelengths. Due to different in wavelengths the amount of time taken to propagate along fiber is intrinsic property along the same path. This is wavelength take different amount of time to propagate along the same path called material dispersion [9].

Material dispersion and wave guide dispersion

For wave guide dispersion:-

There are only one guide mode that propagate through the fiber. Due to the presence of only single mode fiber such fiber free from inter modal dispersion. Pulse in such fibers is due to two mechanism. Material dispersion and waveguide dispersion. Normalized propagation constant is given by:

$$b = \frac{\frac{\beta^2}{k_o^2} - n_2^2}{n_1^2 - n_2^2} \tag{2.1.8}$$

where β propagation constant and k_o wave number. For guided modes $\left[\frac{\beta}{k_o}\right]$ lies between n_1 and n_2, propagation constant (**b**) lies between 0 and 1. For step index fibers **b** depends only on the $\mathbf{V_{number}}$ values of the fibers.

$$b = \left[\frac{(\frac{\beta}{k_o} - n_2)(\frac{\beta}{k_o} + n_2)}{(n_1 - n_2)(n_1 + n_2)}\right], \quad b \simeq \frac{\frac{\beta}{k_o} - n_2}{n_1 - n_2} \tag{2.1.9}$$

for $n_1 \simeq n_2$

$$\beta = \frac{\omega}{c}\left[n_2 + (n_1 - n_2)b(V)\right], \ \mathbf{b} \text{ depends on } \mathbf{V_{number}} \text{ value.} \tag{2.1.10}$$

Since n_1 and n_2 independent of wavelength(no material dispersion), $\left[\frac{d\beta}{d\omega}\right]$ depends on (ω) and due explicit dependence of $\mathbf{b}$ on $\mathbf{V_{number}}$ and hence on (ω).

$$\frac{1}{V_g} = \frac{d\beta}{d\omega} = \frac{1}{c}\left[n_2 + (n_1 - n_2)b(V)\right] + \frac{\omega}{c}(n_1 - n_2)\frac{db}{dV}.\frac{dV}{d\omega} \tag{2.1.11}$$

for the $\mathbf{V_{number}}$ value is written as:

$$V_{number} = \frac{\omega}{c}a\sqrt{n_1^2 - n_2^2} \tag{2.1.12}$$

$$\frac{1}{V_g} = \frac{n_2}{c} + \frac{n_1 - n_2}{c}[\frac{d(bV)}{dv}]. \tag{2.1.13}$$

Time spread of pulse is given by

$$\tau = \frac{L}{V_g} = \frac{L}{c}n_2\left[1 + \triangle\frac{d(bV)}{dv}\right] \tag{2.1.14}$$

where

$$\triangle = \frac{n_1 - n_2}{n_2}. \tag{2.1.15}$$

Now for source having spectral width $\triangle\lambda_o$ the corresponding wave guide dispersion is given by:

$$\triangle\tau \simeq -\frac{n_2\triangle}{3\lambda_o} \times 10^7\left[V\frac{d^2(bV)}{dV^2}\right]\frac{ps}{km.nm}. \tag{2.1.16}$$

From above equation and defined parameter waveguide dispersion of pulse is written as:

$$D_w = -\frac{2\pi c}{\lambda_o}\frac{d^2\beta}{d\omega^2} \simeq -\frac{n_2\triangle}{c\lambda_o}\left[V\frac{d^2(bV)}{dV^2}\right] \tag{2.1.17}$$

Material dispersion:

When temporal pulse propagates through a homogeneous medium it propagates with group velocity calculated as

$$V_g = \frac{1}{\frac{dk}{d\omega}} \tag{2.1.18}$$

where $k(\omega) = \frac{\omega}{c}n(\omega)$ represent propagation constant and $n(\omega)$ frequency dependent refractive index.

$$\frac{1}{V_g} = \frac{1}{c}\left[n(\omega) + \omega\frac{dn}{d\omega}\right]. \tag{2.1.19}$$

If it is expressed in terms of free space wavelength $\lambda_o = \frac{2\pi c}{\omega}$

$$\omega\frac{dn(\omega)}{d\omega} = \frac{2\pi c}{\lambda_o}\left[\frac{dn}{d\lambda_o}(\frac{-2\pi c}{\omega^2})\right] = -\lambda_o\frac{dn}{d\lambda_o} \tag{2.1.20}$$

$$V_g = \frac{1}{c}\left[n(\lambda) - \lambda_o\frac{dn}{d\lambda_o}\right]^{-1} \tag{2.1.21}$$

where λ_o is wavelength of source. The time spread of the pulse is

$$\tau = \tau(\lambda_o) = \frac{L}{V_g} = \frac{L}{c}\left[n(\lambda_o) - \lambda_o\frac{dn}{d\lambda_o}\right] \tag{2.1.22}$$

$$N(\lambda_o) = n(\lambda_o) - \lambda_o\frac{dn}{d\lambda_o} \tag{2.1.23}$$

called group refractive index, $\left[\frac{c}{N(\lambda_o)}\right]$ gives velocity. If the sources are characterized by spectral width $\triangle\lambda_o$ then each wavelength component transverse with different group velocity results in temporal broadening of the pulse given by

$$\triangle\tau = -\frac{L}{c}\left[\lambda_o^2\frac{d^2n}{d\lambda_o^2}\right]\frac{\triangle\lambda_o}{\lambda_o} = \frac{d\tau}{d\lambda_o}\Delta\lambda_o. \tag{2.1.24}$$

This broadening is called material dispersion and is given by:

$$D_{\lambda_o m} = \frac{\triangle\tau}{L\triangle\lambda_o} \tag{2.1.25}$$

for step index fibers with $n_1 = 1.45,\ \ n_2 = 1.445,\ \ a = 4.1\mu m$. If these value correspond to a typical step index fiber operating at 1300 nm, assume that refractive indices $n_1\ \ and\ \ n_2$ independent of wavelength, V$_{number}$ is given by

$$V_{number} = \frac{2\pi}{\lambda_o}a\sqrt{n_1^2 - n_2^2} \tag{2.1.26}$$

Thus, total dispersion is calculated as

$$D_{total} = D_w + D_m \tag{2.1.27}$$

$$D_{total} = -\frac{1}{c\lambda_o}\left[\lambda_o^2 \frac{d^2n}{d\lambda_o^2}\right] - \frac{n_2\triangle}{c\lambda_o}\left[V\frac{d^2}{dV^2}(bV)\right] \tag{2.1.28}$$

2.2 Group velocity and Chirping phenomena

By considering monochromatic wave in homogeneous medium in equation (2.0.12) When we consider a pulse envelope in which large number of oscillations exist within the pulse is the total sum of harmonic wave as

$$E(t) = \int_{-\infty}^{+\infty} C(\omega)\exp[i\omega t]d\omega \tag{2.2.1}$$

by using inverse Fourier transform

$$C(\omega) = \frac{1}{2\pi}\int E(t)\exp[-i\omega t]dt \tag{2.2.2}$$

the total field at z would be

$$E(z,t) = \int_{-\infty}^{+\infty} C(\omega)\exp\ i[\omega t - kz]d\omega \tag{2.2.3}$$

the equation shows the frequency component propagation

$$C(\omega) = |C(\omega)|\exp[i\phi(\omega)]. \tag{2.2.4}$$

Thus,

$$E(z,t) = \int_{-\infty}^{+\infty} |C(\omega)|\exp\ i[\omega t - kz + \phi(\omega)]d\omega. \tag{2.2.5}$$

When integration extended only over the domain of frequencies

$$\omega_o - \frac{1}{2}\Delta\omega \le \omega \le \omega_o + \frac{1}{2}\Delta\omega \tag{2.2.6}$$

Outside the domain $|C(\omega)|$ is assumed to be negligible.

Inside the domain $n(\omega)$ and k(ω) is to be smoothly varying.

So that the Taylor series expansion k(ω) around $\omega = \omega_o$

$$k(\omega) = k(\omega_o)+\left(\frac{\partial k}{\partial \omega}\right)(\omega-\omega_o)+\frac{1}{2}\left(\frac{\partial^2 k}{\partial \omega^2}\right)(\omega-\omega_o)^2+\frac{1}{3}\left(\frac{\partial^3 k}{\partial \omega^3}\right)(\omega-\omega_o)^3+\frac{1}{4}\left(\frac{\partial^4 k}{\partial \omega^4}\right)(\omega-\omega_o)^4+... \tag{2.2.7}$$

in similar way

$$\phi(\omega) = \phi(\omega_o)+\left(\frac{\partial \phi}{\partial \omega}\right)(\omega-\omega_o)+\frac{1}{2}\left(\frac{\partial^2 \phi}{\partial \omega^2}\right)(\omega-\omega_o)^2+\frac{1}{3}\left(\frac{\partial^3 \phi}{\partial \omega^3}\right)(\omega-\omega_o)^3+\frac{1}{4}\left(\frac{\partial^4 \phi}{\partial \omega^4}\right)(\omega-\omega_o)^4+... \tag{2.2.8}$$

by substituting equation 2.2.10 into equation 2.3.7 and assuming that in the domain of integration ($\Delta\omega$), k(ω) and $\phi(\omega)$ do not varying appreciable to $(\omega-\omega_o)^2$ to higher power of $(\omega-\omega_o)$. Under this approximation

$$E(z,t) \simeq \exp\ i[\omega_o t - k_o z + \phi]u(z,t) \tag{2.2.9}$$

where

$$u(z,t) = \int_{\Delta\omega} |C(\omega)| \exp[i(\omega-\omega_o)(t - \frac{dk}{d\omega}z + \frac{d\phi}{d\omega})]d\omega. \tag{2.2.10}$$

This equation represents the envelope wave packet. Thus,

$$u(z,t) = \int_{\Delta\omega} |C(\omega)| \exp[-i\frac{(\omega-\omega_o)}{v_g}(z - z_o - v_g t)]d\omega. \tag{2.2.11}$$

and shows distortion-less propagation of wave packet with group velocity(v_g), which occurs when (k) is strictly related to (ω). Where

$$V_g = \left[\frac{dk}{d\omega}\right]^{-1}, \quad z = v_g\left[\frac{d\phi}{d\omega}\right] \tag{2.2.12}$$

it may notes that

$$u(z,t=0) = \int_{\Delta\omega} |C(\omega)| \exp[-i\frac{(\omega-\omega_o)}{v_g}(z - z_o)]d\omega. \tag{2.2.13}$$

At z=z_o the integrand is everywhere positive and value of integral is maximum.

For $|z-z_0| \geq \frac{V_g}{\Delta\omega}$ the exponential function oscillates rapidly in the domain of integration and the value of integration is very small. This leads to uncertainty relation-that is,

$$\Delta\omega\frac{\Delta z}{v_g} \geq 1 \tag{2.2.14}$$

where Δz is spatial extent of pulse. When we introduce new function f(z,t) depends on z and t only through the combination $z - v_g t$, implies that the wave packet propagates without any distortion with velocity(V_g). This velocity is known as group velocity of the pulse.

At t=o the pulse is sharply peaked.

At z= z_o and as t increases the center of the pulse $z_c(t)$,

$$z_c(t) = z_o + V_g t \tag{2.2.15}$$

Since C(ω) real, $\phi(\omega)$ =0, if we substitute now for $C(\omega)$ and $K(\omega)$ from equation (2.2.6)

$$E(z,t) = D\exp[(\omega_o t - k_o z)]U(z,t) \tag{2.2.16}$$

where

$$\begin{aligned} U(z,t) &= \int C(\omega)\exp[i(\omega-\omega_o)(t-\frac{z}{V_g}) - \frac{1}{2}\alpha(\omega-\omega_o)^2 z]d\omega \\ &\approx \frac{\tau_o}{2\sqrt{\pi}}\int \exp[\Gamma^2(\frac{\tau_o^2}{4}+\frac{i}{2}\alpha(\omega-\omega_o)) + i\Gamma(t-\frac{z}{v_g})]d\omega \end{aligned} \tag{2.2.17}$$

where $\Gamma = \omega - \omega_o, \quad \alpha = \frac{d^2k}{d\omega^2}$.

Thus,

$$U(z,t) = \left(1 + i\frac{2\alpha z}{\tau_o^2}\right)^{-\frac{1}{2}} \tag{2.2.18}$$

and

$$E(z,t) = \frac{D}{(|\tau(z)|\,\tau_o)^{\frac{1}{2}}}\exp\left[-\frac{(t-\frac{z}{v_g})^2}{\tau^2(t)}\exp[i(\Phi(z,t)-k_o z]\right] \tag{2.2.19}$$

$$\Phi = \omega_o t + k(t-\frac{z}{v_g})^2 - \frac{1}{2}tan^{-1}\left(\frac{2\alpha z}{\tau_o^2}\right) \tag{2.2.20}$$

where

$$k = \frac{2\alpha z}{z^4}\left(1+\frac{4\alpha^2 z^2}{\tau_o^4}\right)^{-1} \tag{2.2.21}$$

$$\tau^2(z) = \tau_0^2\left(1+\frac{4\alpha^2 z^2}{\tau_o^4}\right). \tag{2.2.22}$$

The second term of equation(2.2.22) leads to nonlinear phenomenon Called chirping. Thus, the corresponding intensity distribution is given by

$$I(z,t) = \frac{I_o}{\frac{\tau_o(z)}{\tau_o}} \exp\left[-2\frac{(t-\frac{z}{v_g})^2}{\tau^2(z)}\right] \tag{2.2.23}$$

the pulse width from this equation

$$\tau(z) = \tau_o\left[1+\frac{4z^2}{\tau_o^4}(\frac{d^2k}{d\omega^2})^2\right]^{\frac{1}{2}}. \tag{2.2.24}$$

Equation(2.2.23) discussed as the corresponding intensity distribution of output pulse and so that the packet of the pulse broaden due to finite value of dispersion term (α) called group velocity dispersion [9]. Hence, the pulse broadening

$$\Delta\tau = [\tau^2(z)-\tau_o^2]^{\frac{1}{2}} = \frac{2z}{\tau_o}\left|\frac{d^2k}{d\omega^2}\right| \tag{2.2.25}$$

$$\frac{d^2k}{d\omega^2} = \frac{d}{d\omega}\left[\frac{1}{c}(n-\lambda_o\frac{dn}{d\lambda_o})\right] \tag{2.2.26}$$

$$= -\frac{\lambda_o^2}{2\pi}\frac{d}{d\lambda_o}\left[\frac{1}{c}(n-\lambda_o\frac{dn}{d\lambda_o})\right] \tag{2.2.27}$$

$$= \frac{\lambda_o^3}{2\pi c^2}\frac{d^2n}{d\lambda_o^2} \tag{2.2.28}$$

but, $\Delta\omega \simeq \frac{2}{\tau_o}$. Since $\omega = \frac{2\pi c}{\lambda_o}$, the corresponding wavelength will be $\Delta\lambda_o \simeq \frac{\lambda_o^2}{\pi c\tau_o}$.

Thus,

$$|\Delta\tau| \simeq \frac{z}{\lambda_o c}\left|\lambda_o^2\frac{d^2n}{d\lambda_o^2}\right|\Delta\lambda_o \tag{2.2.29}$$

When we see dispersion and nonlinear phenomena called chirping

Gaussian input pulse is

$$\overrightarrow{E}(t) = E_o\exp[-\frac{t^2}{\tau_o^2}]\exp[i\omega_o t] \tag{2.2.30}$$

where τ_o is initial spread of pulse. But after it propagate z distance within the fibers

$$\overrightarrow{E}(z,t) = \left(\frac{E_o}{(1+\sigma^2)^{\frac{1}{4}}}\right)\exp\left[\frac{-(t-\frac{z}{t})}{\tau^2(z)}\right]\exp\left[i(\Phi(z,t)-\beta(\omega_o)z)\right] \tag{2.2.31}$$

where

$$\Phi(z,t) = \omega_o t + k(t-\frac{z}{v_g})^2 - \frac{1}{2}tan^{-1}(\sigma) \tag{2.2.32}$$

$\sigma = \frac{2\pi z}{\tau_o^2}$, $k = \frac{\sigma}{(1+\sigma^2)\tau_o^2}$, $\tau^2(z) = \tau_o^2(1+\sigma^2)$, $\alpha = \frac{d^2\beta}{d\omega^2}$

Consider we have

$$\omega(t) = c\exp i[g(t)] \tag{2.2.33}$$

Then

$$\omega_{in}(t) = \frac{dg(t)}{dt} \tag{2.2.34}$$

hence, the instantaneous frequency within the pulse packet is

$$\omega_{in} = \frac{d\phi}{dt} = \omega_o + 2k(t-\frac{z}{v_g}). \tag{2.2.35}$$

Thus, the instantaneous frequency within the pulse envelope changes with time and is called chirped pulse. But if we see this chirping pulse without dispersion phenomenon for electric field $\overrightarrow{E}$ propagating along the fiber

$$\overrightarrow{E}(z,t) = E_o\exp i[\Phi(z,t) - k(\omega_o)z] \tag{2.2.36}$$

2.3 Description of nonlinear optical processes and their effects

2.3.1 Elecrostriction Effect

All dielectric materials undergo strain when subjected to an applied electric field. It results in slight change in shape. This change in length is termed electrostriction. It causes density changes in the core of cylindrical fiber that excite the transverse acoustic vibrational eigen modes of the fibers, generating refractive index variation by elastoptic effect. The electrostriction contribution (n_{n_l}) to the refractive index is due to intensity dependence of variation of material density associated with intense electric field from the source.

2.3.2 Kerr- nonlinear effect

When the optical medium is isotropic, as in the case of liquids and gases, the Pockels effect is absent and the polarization is modified by the third order nonlinear susceptibility as:

$$\overrightarrow{P} = \epsilon_o \chi^{(1)} \overrightarrow{E} + \epsilon_o \chi^{(2)} \overrightarrow{E}^2 + \epsilon_o \chi^{(3)} \overrightarrow{E}^3 + ... \tag{2.3.1}$$

in the expansion of polarization of medium electro-optic effect is better known as the Kerr effect. This effect like all third order effects occurs, whether or not material posses inversion symmetry. The Kerr nonlinearity(n_{n_l}) originates from third order nonresonant electronic susceptibility and Raman susceptibility. The Kerr and electrostrictive effects are strongly interrelated to each other, and both are associated with change in material density with high electromagnetic field interaction. However, the elecrostrictive effect is observed a long pulses and if the higher order susceptibilities are significant then the Kerr effect also contributes to the change of nonlinear refractive index. Electrostriction has the unique characteristics of being explicitly dependent on transverse optical intensity profile. Quadratic electrostriction is considered to be one of the effects that can influence the power damage threshold of materials used with high-power laser application [11].

Self phase modulation

The higher intensity portions of an optical pulse encounter a higher refractive index of the fiber compared with the lower intensity portions while it travels through the fiber. In fact time varying signal intensity produces a time varying refractive index in a medium that has an intensity dependent refractive index. The leading edge will experience a positive

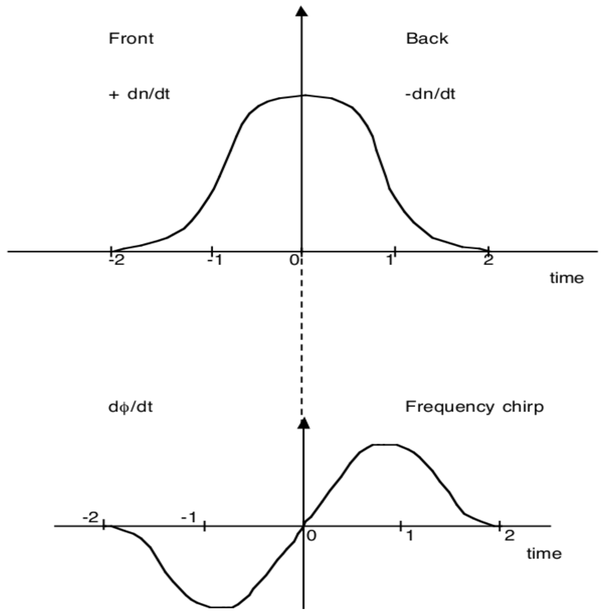

Figure 2.4: Phenomenological description of spectral broadening of pulse due to self phase modulation

refractive index gradient (dn/dt) and trailing edge a negative refractive index gradient ($-dn/dt$). This temporally varying index change results in a temporally varying phase.

The optical phase changes with time in exactly the same way as the optical signal. Since, this nonlinear phase modulation is self-induced the nonlinear phenomenon responsible for it is called as self-phase modulation. Different parts of the pulse undergo different phase shift because of intensity dependence of phase fluctuations. This results in frequency chirping. The rising edge of the pulse finds frequency shift in upper side whereas the trailing edge experiences shift in lower side. Hence primary effect of self phase modulation is to broaden the spectrum of the pulse, keeping the temporal shape unaltered. The self phase modulation effects are more pronounced in systems with high-transmitted power because the chirping effect is proportional to transmitted signal power[10]. The phase (ϕ) introduced by a field E over a fiber length L is given by

$$\phi = \frac{2\pi}{\lambda} nL, \quad \phi = k(\omega_o)z, \quad k = \frac{\omega}{c}n, \quad z = L \tag{2.3.2}$$

where λ is wavelength of optical pulse propagating in fiber of refractive index n, and nL is known as optical path length. For a fiber containing high-transmitted power n and L

can be replaced by n_{eff} and L_{eff} respectively i.e.,

$$\phi = \frac{2\pi}{\lambda} n_{eff} L_{eff} \quad \text{or}$$

$$\phi = \frac{2\pi}{\lambda} [n_l + n_{n_l} I] L_{eff}. \tag{2.3.3}$$

The first term on right hand side refers to linear portion of phase constant (ϕ_l) and second term provides nonlinear phase conistant (ϕ_{n_l}). If intensity is time dependent i.e., the wave is temporally modulated then phase (ϕ) will also depend on time [10]. This variation in phase with time is responsible for change in frequency spectrum, which is given by

$$\omega = \frac{d\phi}{dt}. \tag{2.3.4}$$

In a dispersive medium a change in the spectrum of temporally varying pulse will change the nature of the variation. To observe this, consider a Gaussian pulse, which modulates an optical carrier frequency (ω) and the new instantaneous frequency becomes,

$$\omega_{in} = \omega_o + \frac{d\phi}{dt}. \tag{2.3.5}$$

The sign of the phase shift due to self phase modulation is negative because of the minus sign in the expression for phase, ($\omega t - kz$) i.e.,

$$\phi = \frac{2\pi}{\lambda} [n_l + n_{n_l} I] L_{eff}. \tag{2.3.6}$$

And therefore ω_{in} becomes,

$$\omega_{in} = \omega_o - \frac{2\pi}{\lambda} n_{n_l} L_{eff} \frac{dI}{dt}. \tag{2.3.7}$$

At leading edge of the pulse $\frac{dI}{dt} > 0 \;\; \omega_{in} = \omega_o - \omega(t)$

At trailing edge of the pulse $\frac{dI}{dt} < 0 \;\; \omega_{in} = \omega_o + \omega(t)$

where

$$\omega(t) = \frac{2\pi}{\lambda} L_{eff} n_{eff} \frac{dI}{dt}. \tag{2.3.8}$$

This shows that the pulse is chirped i.e., frequency varies across the pulse. This chirping phenomenon is generated due to self phase modulation, which leads to the spectral broadening of the pulse. For single mode in which 10mW input power supplied at spectral wavelength of 1550nm along fiber of $n_l = 1.46$, $n_{n_l} = 3.2 \times 10^{-20} \frac{m^2}{W}$ and $A_{eff} = 50\mu m^2$ Using some above assumptions

$$\phi = \frac{2\pi}{\lambda} n_l L_{eff} + k_{n_l} p_{in} L_{eff} \tag{2.3.9}$$

$$\phi = \phi_l + \phi_{n_l} \tag{2.3.10}$$

where $K_{n_l} = \frac{2\pi}{\lambda} \frac{n_{n_l}}{A_{eff}} = 1.296 \times \frac{10^{-6}}{mW}$. Since, $\phi_{n_l} \ll 1$ equation 2.3.9 becomes

$$\phi = \phi_l = \frac{2\pi}{\lambda} L_{eff} \tag{2.3.11}$$

but,

$$L_{eff} = \frac{\left[1 - \exp(-\alpha z)\right]}{\alpha} \tag{2.3.12}$$

$$L_{eff} \backsimeq \frac{1}{\alpha} \tag{2.3.13}$$

where $\alpha = 0.2\frac{dB}{km}$. For single mode fiber of spectrum range of wavelength $1550nm$ dispersion is negligible

$$L_{eff} = \frac{1}{0.2\frac{dB}{km}} = 5 \times 10^3 \frac{m}{dB}. \tag{2.3.14}$$

Thus, equation 2.3.11 becomes

$$\phi = 11.994776 \times 10^7 \ \ \text{radians}. \tag{2.3.15}$$

Intensity is calculated as

$$I_o = \frac{P_{in}}{A_{eff}} = 2 \times 10^2 \frac{W}{m^2} \tag{2.3.16}$$

hence equation 3.0.32 becomes

$$\omega(t) = \frac{2\pi}{\lambda} L_{eff} n_{eff} \frac{dI}{dt} = \frac{4.5844 \times 10^4}{\lambda} \frac{dI(t)}{dt}. \tag{2.3.17}$$

$$\frac{dI(t)}{dt} = 0.12cos[4.05(3 \times 10^8 t - 1.46)] \tag{2.3.18}$$

Hence,

$$\omega(t) = 3.48 \times 10^{-9} cos[4.05(3 \times 10^8 t - 1.46)] \tag{2.3.19}$$

Threshold value of self phase modulation

Self phase modulation arises due to intensity dependence of refractive index. Fluctuation in signal intensity causes change in phase of the signal. This change in phase induces additional chirp, which leads to dispersion penalty. This penalty will be small if input power is less than certain threshold value[8]. The appropriate chirping of the input pulses can also be beneficial for reducing the self phase modulation effects. The power dependence of nonlinear phase constant (ϕ_{n_l}) is responsible for self phase modulation impact on communication systems [8]. To reduce this impact, it is necessary to have $\phi_{n_l} \ll 1$. Nonlinear phase constant $\phi_{n_l} = k_{n_l} P_{in} L_{eff}$ where nonlinear propagation constant $K_{n_l} = \frac{2\pi}{\lambda}\frac{n_{n_l}}{A_{eff}}$. So, with $L_{eff} \approx \frac{1}{\alpha}$; one may obtain $p_{in} \ll \frac{\alpha}{k_{n_l}}$. Therefore, to have $\phi_{n_l} \ll 1$ equivalent to $p_{in} \ll \frac{\alpha}{k_{n_l}}$. Typically, $\alpha = 0.2\frac{dB}{km}$ at $\lambda = 1550nm$ and $k_{n_l}= 2.35{\times}10^{-3}\frac{1}{mW}$ the input power should be kept below 19.6 mW. The move to increase the span between in-line optical amplifiers, more power must be launched into each fiber. This increased power increases self phase modulation effect on light wave systems which results in pulse spreading. The use of large-effective area fibers reduces intensity inside the fiber and hence self phase modulation impact on the system. The chirp produced by self phase modulation, which causes broadening depends on the input pulse shape and the instantaneous power level within the pulse[10]. For Gaussian shaped pulse, the chirp is even and gradual, and for a pulse that involves an abrupt change in power level (e.g.,square pulse) the amount of chirp is greater. Therefore a suitable input pulse shape may be able to reduce the chirp and hence self phase modulation induced broadening. In general, all nonlinear effects are weak and depend on long interaction length to build up. So any mechanism that reduces interaction length decreases the effect of nonlinearity. The damage due to self phase modulation-induced pulse broadening on system performance depends on the power transmitted and length of the link.

Cross phase modulation

Self phase modulation is the major nonlinear limitation in a single channel system. The intensity dependence of refractive index leads to another nonlinear phenomenon known as cross-phase modulation (CPM)[11]. When two or more optical pulses propagate simultaneously, the cross-phase modulation is always accompanied by Self phase modulation and occurs because the nonlinear refractive index seen by an optical beam depends not only on the intensity of that beam but also on the intensity of the other co-propagating beams. In fact cross phase modulation converts power fluctuations in a particular wavelength channel to phase fluctuations in other co-propagating channels. The result of cross phase modulation may be asymmetric spectral broadening and distortion of the pulse shape.

The effective refractive index of a nonlinear medium can be expressed in terms of the input power (p) and effective core area as:

$$n_{eff} = n_l + n_{n_l} \frac{p}{A_{eff}} \tag{2.3.20}$$

The nonlinear effects depend on ratio of light power to the cross-sectional area of the fiber. If the first-order perturbation theory is applied to investigate how fiber modes are affected by the nonlinear refractive index, it is found that the mode shape does not change but the propagation constant becomes power dependent.

$$k_{eff} = k_l + k_{n_l} p \tag{2.3.21}$$

where k_l is the linear portion of the propagation constant and k_{n_l} is nonlinear propagation constant. The phase shift caused by nonlinear propagation constant in traveling a distance L inside fiber is given as:

$$\phi_{n_l} = \int_o^L (k_{eff} - k_l) dz \tag{2.3.22}$$

$$\phi_{n_l} = k_{n_l} p_{in} L_{eff} \tag{2.3.23}$$

When several optical pulses propagate simultaneously the nonlinear phase shift of first channel (ϕ_{n_l}) depends not only on the power of that channel but also on signal power of other channels. For two channels, ϕ_{nl} can be given as,

$$\phi_{n_l}^{(1)} = k_{eff} L_{eff} (p_1 + 2p_2) \tag{2.3.24}$$

But for N-channel transmission:

$$\phi_{n_l}^{(i)} = k_{eff} L_{eff} \left(p_1 + 2 \sum_{n}^{N} p_n \right) \tag{2.3.25}$$

The factor 2 in above equation has its origin in the form of nonlinear susceptibility and indicates that cross phase modulation is twice as effective as self phase modulation for the same amount of power. The first term in above equation represents the contribution of

Self phase modulation and second term that of cross phase modulation. It can be observed that cross phase modulation is effective only when the interacting signals superimpose in time. Cross phase modulation can also occur only when two pulses overlap in time. Due to this overlapping, the intensity-dependent phase shift and consequent chirping is enhanced. Therefore the pulse broadening is also enhanced, which limits the performance of light wave systems. The effects of cross phase modulation can be reduced by increasing the wavelength spacing between individual channels. For increased wavelength spacing, pulse overlaps for such a short time that Cross phase modulation effects are virtually negligible. In fact, owing to fiber dispersion, the propagation constants of these channels become sufficiently different so that the pulses corresponding to individual channels walk away from each other. Due to this pulse walk-off phenomenon the pulses, which were initially temporally coincident, cease to be so after propagating for some distance and cannot interact further. Thus, effect of cross phase modulation is reduced. Optical switching and pulse compression can be done through the cross phase modulation.

Four wave mixing

The origin of four wave mixing process lies in the nonlinear response of bound electrons of a material to an applied optical field. In fact, the polarization induced in the medium contains not only linear terms but also the nonlinear terms. The magnitude of these terms is governed by the nonlinear susceptibilities of different orders. The four wave mixing process originates from third order nonlinear susceptibility. If three optical fields with carrier frequencies $\omega_1, \omega_2, \omega_3$, co -propagate inside the fiber simultaneously, $\chi^{(3)}$ generates a fourth field with frequency ω_4 , which is related to other frequencies by a relation, $\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$. In quantum-mechanical context, four wave mixing occurs when photons from one or more waves are annihilated and new photons are created at different frequencies such that net energy and momentum are conserved during the interaction. Self phase modulation and Cross phase modulation are significant mainly for high bit rate systems, but the four wave mixing effect is independent of the bit rate and is critically dependent on the channel spacing and fiber dispersion[11]. Decreasing the channel spacing increases the four-wave mixing effect and so does decreasing the dispersion. In order to understand four wave mixing effect, consider wave division multiplexing signals, which is the sum of n monochromatic plane wave. The electric field of such signals can be written as :

$$E = \sum_{p=1}^{n} E_p cos(\omega_p t - k_p z) \tag{2.3.26}$$

Then nonlinear polarization is given by:

$$P_{n_l} = \epsilon_o \chi^{(3)} E^3 \tag{2.3.27}$$

For this case P_{n_l} takes the form as:

$$P_{n_l} = \epsilon_o \chi^{(3)} \sum_{p=1}^{n} \sum_{q=1}^{n} \sum_{r=1}^{n} E_p cos(\omega_p t - k_p z) E_q cos(\omega_q t - k_q z) E_r cos(\omega_r t - k_r z) \tag{2.3.28}$$

Expansion of above expression gives:

$$\begin{aligned}
P_{n_l} = \frac{3}{4}\epsilon_o \chi^{(3)} \sum_{p=1}^{n} E_p^2 + 2 \sum_{q \neq p} E_p E_q E_p cos(\omega_p t - k_p z) \\
+\frac{1}{4}\epsilon_o \chi^{(3)} \sum_{p=1}^{n} E_p^3 cos(3\omega_p t - 3k_p z) \\
+\frac{3}{4}\epsilon_o \chi^{(3)} \sum_{p=1}^{n} \sum_{q \neq p}^{n} E_p^2 E_q cos[(2\omega_p - \omega_q)t - (2k_p - k_q)z] \\
+\frac{3}{4}\epsilon_o \chi^{(3)} \sum_{p=1}^{n} \sum_{q \neq 1}^{n} E_p^2 E_q cos[(2\omega_p + \omega_q)t - (2k_p + k_q)z] \\
\frac{6}{4}\epsilon_o \chi^{(3)} \sum_{p=1}^{n} \sum_{q>p} \sum_{r>} E_p E_q E_r cos[(2\omega_p + \omega_q + \omega_r)t - (k_p + k_q + K_r)z] \\
+cos[(\omega_p + \omega_q + \omega_r) - (k_p + K_q + k_r)z] \\
+cos[(\omega_p - \omega_q + \omega_r) - (k_p - K_q + k_r)z] \\
+cos[(\omega_p - \omega_q - \omega_r) - (k_p - K_q - k_r)z]
\end{aligned} \tag{2.3.29}$$

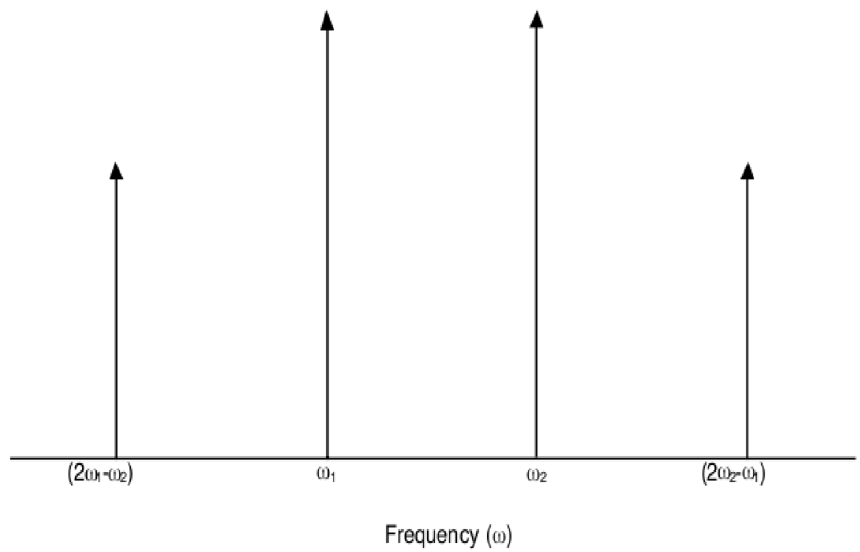

Figure 2.5: Showing mixing of two waves.

The first terms in above equation represents the effect of self phase modulation and cross phase modulation. Second, third and fourth terms can be neglected because of phase mismatch. The reason behind this phase mismatch is that, in real fibers $k(3\omega) = 3k(\omega)$ so any difference like $(3\omega,\ 3k)$ is called as phase mismatch. The phase mismatch can also be understood as the mismatch in phase between different signals traveling within the fiber at different group velocities. All these waves can be neglected because they contribute little. The last term represents phenomenon of four-wave mixing. It is this term, which tells that three electromagnetic waves propagating in a fiber generate new waves with frequencies $(\omega_p \pm \omega_q \pm \omega_r)$. Four-wave mixing is analogous to inter modulation distortion in electrical systems. The last term of polarization expression tells that four wave mixing comes from frequency combinations like $(\omega_p + \omega_q - \omega_r)$. In compact form all these combinations can be written as:

$$\omega_{pqr} = \omega_p + \omega_q - \omega_r \quad with \quad p,\ q\ \neq\ r \tag{2.3.30}$$

Fig shows a simple example of mixing of two waves at frequency ω_1 and ω_2. When these waves mixed up, they generate sidebands at $(2\omega_1 - \omega_2)$ *and* $(2\omega_2 - \omega_1)$. Similarly, three co propagating waves will create nine new optical side band. These sidebands travel along with original waves and will grow at the expense of signal-strength depletion. In general for N -wavelengths launched into fiber, the number of generated mixed products M is:

$$M = \frac{N^2}{2}.(N-1) \tag{2.3.31}$$

The efficiency of four wave mixing depends on fiber dispersion and the channel spacing. Since the dispersion varies with wavelength, the signal waves and the generated waves have different group velocities. This destroys the phase matching of interacting waves and lowers the efficiency of power transfer to newly generated frequencies. The higher the group velocity mismatch and wider the channel spacing, the lower the four-wave mixing. Four-wave mixing process results in power transfer from one channel to other. This phenomenon results in power depletion of the channel, which degrades the performance of that channel. Since, four wave mixing itself is inter channel crosstalk it induces interference of information from one channel with another channel. This interference again degrades the system performance. To reduce this degradation, channel spacing must be increased. This increases the group velocity mismatch between channels and hence four wave mixing penalty is reduced. The important applications are squeezing and wave length conversion.

2.3.3 Second harmonic generation

This nonlinear optical interaction of incident electric field-that is at $z = 0$

$$E(t) = E_o \exp[-i(\omega t)] + c.c \tag{2.3.32}$$

is incident upon dielectric crystals of fiber for which the second order susceptibility $\chi^{(2)}$ is zero. The nonlinear polarization that is created in such crystal is given according to:

$$P^{(2)} = \epsilon_o \chi^{(2)} E^2. \tag{2.3.33}$$

Explicitly this is

$$P^{(2)} = 2\epsilon_o \chi^{(2)} E_o E_o^* + \left[\epsilon_o \chi^{(2)} E_o^2 \exp[-i2\omega t] + c.c\right]. \tag{2.3.34}$$

Here we see that second order polarization contribution at zero frequency, the first term and the second term. Thus the second term leads to generation of electromagnetic radiation at twice original frequency (ω). Note that the first term contribution does not lead to the generation of electromagnetic radiation because the time derivation vanishes. It leads to process known as optical rectification which static Electric field create across the nonlinear crystals[12]. The process of second harmonic generation is more efficient in that nearly all of the power incident beam at frequency (ω) is converted to radiation at the second harmonic frequency 2ω. One use of second harmonic generation is to convert out put of fixed frequency Laser to the different spectral region.

Chapter 3

Mathematical formalism

3.1 Third order nonlinear polarization and basic of nonlinearity in optical fibers

For intense electromagnetic field any dielectric medium behaves like nonlinear medium. Fundamentally the origin of nonlinearity lies in the anharmonic motion of bound electrons under the influence of an applied field. Due to this harmonic motion the total polarization(P) induced by electric dipole is not linear. In linear medium the electric polarization is assumed to be a linear function of electric field.

$$\overrightarrow{P} = \epsilon_o \chi^{(1)} \overrightarrow{E} \tag{3.1.1}$$

The quantity $\chi^{(1)}$ is termed the linear dielectric susceptibilities. At high optical intensity (equivalently at high optical fields) the media behaves nonlinearly-that is the relation expressed is rewritten as:

$$P = \epsilon_o \chi^{(1)} \overrightarrow{E} + \epsilon_o \chi^{(2)} \overrightarrow{E}^2 + \epsilon_o \chi^{(3)} \overrightarrow{E}^3 + \epsilon_o \chi^{(4)} \overrightarrow{E}^4 + \epsilon_o \chi^{(5)} \overrightarrow{E}^5 + .. \tag{3.1.2}$$

If we consider plane optical wave with an electric field variation of the form:

$$\overrightarrow{E} = E_o cos(\omega t - kz) \tag{3.1.3}$$

where k is wave vector.

By using $\overrightarrow{E}$ into $\overrightarrow{P}$, equation for third order nonlinear polarization is written as:

$$\overrightarrow{P} = \epsilon_o\chi^{(1)}E_o cos(\omega t - kz) + \epsilon_o\chi^{(2)}E_o^2 cos^2(\omega t - kz) + \epsilon_o\chi^{(3)}E_o^3 cos^3(\omega t - kz) + ... \quad (3.1.4)$$

using some trigonometric relation that is:

$$2cos^2\theta = cos2\theta + 1 \quad (3.1.5)$$

By using relation 3.1.5 rewritten as:

$$\overrightarrow{P} = \epsilon_o\chi^{(1)}E_o cos(\omega t - kz) + \frac{1}{2}\epsilon_o\chi^{(2)}E_o^2[cos2(\omega t - kz) + 1]$$

$$\frac{1}{2}\epsilon_o\chi^{(3)}E_o^3 cos(\omega t - kz)[cos2(\omega t - kz) + 1] \quad (3.1.6)$$

using trigonometric relation: $\frac{1}{2}cos(A + B) + \frac{1}{2}cos(A - B) = cosAcosB$.

Hence, the equation for P written as:

$$\overrightarrow{P} = \frac{1}{2}\epsilon_o\chi^{(2)}E_o^2 + \epsilon_o E_o cos(\omega t - kz)[\chi^{(1)} + \frac{3}{4}\chi^{(3)}E_o^2] +$$

$$\frac{1}{2}\epsilon_o\chi^{(2)}E_o^2 cos2(\omega t - kz) + \frac{1}{4}\epsilon_o\chi^{(3)}E_o^3 cos3(\omega t - kz) \quad (3.1.7)$$

The first term of above equation gives the direct contribution to the polarization. It has a little practical importance. It gives dc fields across the medium. The second term of equation gives the direct contribution to the first or fundamental harmonic polarization(ω). The third term oscillating with frequency (2ω) is called second harmonic polarization. Similarly the fourth term with frequency (3ω) is called third harmonic polarization. Thus, the effect of nonlinearity gives rise to harmonic second, third and etc contribution to polarization. It is already mentioned that to generate harmonics $\overrightarrow{E}$ should be large, since χ^2, χ^3 are very small. This fact is just for homo-nuclear molecule such as $N = N$, $O = O$ and hetronuclear molecule such as $C = O$. But for molecules which lack inversion symmetry will have no second order susceptibility($\chi^{(2)}$). For fiber of fused silica $\chi^{(2)} = 0$, hence the equation 3.1.8 written as:

$$\overrightarrow{P} = \epsilon_o\left(\chi^{(1)} + \frac{3}{4}\chi^3 E_o^2\right)E_o cos(\omega t - kz) + \frac{1}{4}\epsilon_o\chi^{(3)}E_o^3 cos^3(\omega t - kz) \qquad (3.1.8)$$

The second term of this equation corresponding to third harmonic generation, which is negligible in optical fibers due to phase mismatch between frequencies ω and 3ω. The polarization at frequency ω is

$$\overrightarrow{P} = \epsilon_o\left(\chi^{(1)} + \frac{3}{4}\chi^{(3)}E_o^2\right)E_o cos(\omega t - kz) \qquad (3.1.9)$$

Now, this equation contain both linear (first term) and non linear(second term) polarization. For plane wave represented by equation (3.1.3), the intensity calculated as:

$$\overrightarrow{S} = \overrightarrow{E} \times \overrightarrow{H} \qquad (3.1.10)$$

where $\overrightarrow{H}$ is magnetic field, $\overrightarrow{E}$ is plane electric field, $\overrightarrow{S}$ is linearly polarization intensity of the light called poynting‘s vector. For time position depends on electric and magnetic field

$$\overrightarrow{E} = E_o cos(\omega t - kz)$$
$$\overrightarrow{B} = B_o cos(\omega t - kz) \qquad (3.1.11)$$

and both $\overrightarrow{E}$ and $\overrightarrow{B}$ are perpendicular to each others and direction of propagation. Moreover their amplitude are related according to $B_o = \frac{E_o}{2}$, time variation of poynting‘s vector

$$\overrightarrow{S}(t) = \frac{1}{\mu_o c}E_o^2 cos^2(\omega t - kz) \qquad (3.1.12)$$

where $c^2 = \frac{1}{\mu_o \epsilon_o}$. Time or space average of $cos^2(\omega t - kz)$ is $\frac{1}{2}$, it follows that

$$\langle S \rangle = \frac{1}{2z}E_o^2$$

where z is intrinsic impedance of the medium.

$$z = \sqrt{\frac{\mu}{\epsilon}} \qquad (3.1.13)$$

$$I = \frac{1}{2}E_o^2 \times \sqrt{\frac{\epsilon_r \times \epsilon_o}{\mu_o \times \epsilon_r}}, \ \sqrt{\epsilon_r} = n \tag{3.1.14}$$

$$I = \frac{1}{2}\epsilon_o E_o^2 n^2 c \quad \Rightarrow I = \frac{1}{2}c\epsilon_o E_o^2 n_l^2 \tag{3.1.15}$$

where c is speed of light n_l is linear refractive index of the medium at low field. Hence equation for P written as:

$$\overrightarrow{P} = \epsilon_o\left(\chi^{(1)} + \frac{3}{2}\frac{\chi^{(3)}}{c\epsilon_o n_l^2}I\right)E_o cos(\omega t - kz). \tag{3.1.16}$$

Then the general relationship between polarization and refractive index is given by

$$\overrightarrow{P} = \epsilon_o(n^2 - 1)E_o cos(\omega t - kz) \tag{3.1.17}$$

comparing equations (3.1.14 and 3.1.15) we see that nonlinear term containing $\chi^{(3)}$ leads to intensity dependent refractive index

$$n^2 = 1 + \chi + \frac{3}{2}\frac{\chi^{(3)}}{c\epsilon_o n_l}I. \tag{3.1.18}$$

Since the last term in above equation is usually very small even for intense light beams, we may approximate by a Taylor series expansion

$$n \simeq n_l + \frac{3}{4}\frac{\chi^{(3)}}{c\epsilon_o n_l^2}I = n_l + n_{n_l}I \tag{3.1.19}$$

where

$$n_l^2 = 1 + \chi^{(1)} \tag{3.1.20}$$

and

$$n_{n_l} = \frac{3}{4}\frac{\chi^{(3)}}{c\epsilon_o n_l^2} \tag{3.1.21}$$

is called nonlinear coefficient. Effective susceptibility will be:

$$\chi_{eff} = \frac{P}{\epsilon_o E}$$

$$\chi_{eff} = \chi^{(1)} + \frac{3}{2c\epsilon_o n_l^2}I \tag{3.1.22}$$

Effective refractive index will be

$$n_{eff} = n_l + n_{n_l} I \tag{3.1.23}$$

where n_{eff} is effective refractive index term. From this it is worst while to calculate $\chi^{(3)}$ which is nonlinear term-that is derived as:

$$n_{eff} \simeq n_l + \frac{3}{4} \frac{\chi^{(3)}\ I}{c\epsilon_o n_l^2} \tag{3.1.24}$$

$$\chi^{(3)} = \frac{4}{3} \Delta n [\frac{c\epsilon_o n_l^2}{I}] \tag{3.1.25}$$

where $\Delta n = n_{eff} - n_l$ and intensity(I) is

$$I = \frac{1}{2} c\epsilon_o E_o^2 \tag{3.1.26}$$

By inserting (3.1.24) into (3.1.23) we have

$$\chi^{(3)} = \frac{4}{3} \Delta n \frac{A_{eff}}{p} c\epsilon_o n_l^2 \tag{3.1.27}$$

where $I_0 = \frac{p_{in}}{A_{eff}} = 2 \times 10^2$, $c = 3 \times 10^8 \frac{m}{s}$, $\epsilon_o = 8.85 \times 10^{-12} \frac{N}{m}$,

$$\chi^{(3)} = 2.4 \times 10^{-22}. \tag{3.1.28}$$

Thus, polarization induced due to intense electromagnetic field of third order susceptibility is

$$\begin{aligned} \overrightarrow{P}(t) &= 1.7 \times 10^{-25} cos(\omega t - kz) \\ &= 1.7 \times 10^{-25} cos\left[\frac{18.84 \times 10^8}{\lambda} t - \frac{2\pi}{\lambda} z\right] \end{aligned} \tag{3.1.29}$$

3.2 Fourth and fifth harmonic nonlinear polarization

Equation 2.0.16 in nonlinear case written as

$$\bigtriangledown^2 \overrightarrow{E} - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \overrightarrow{E} = \frac{1}{\epsilon_o c} \frac{\partial^2}{\partial t^2} P^{(NL)} \tag{3.2.1}$$

where n is usual linear refractive index, c is speed of light in a vacuum, $\frac{\partial^2}{\partial t^2}P^{(NL)}$ measure of acceleration of charges that constitute the medium.

$$\overrightarrow{P} = \epsilon_o\chi^{(1)}\overrightarrow{E} + \epsilon_o\chi^{(2)}\overrightarrow{E}^2 + \epsilon_o\chi^{(3)}\overrightarrow{E}^3 + \epsilon_o\chi^{(4)}\overrightarrow{E}^4 + \epsilon_o\chi^{(5)}\overrightarrow{E}^5... \tag{3.2.2}$$

$$= \overrightarrow{P}^{(1)} + \overrightarrow{P}^{(2)} + \overrightarrow{P}^{(3)} + \overrightarrow{P}^{(4)} + \overrightarrow{P}^{(5)} + ... \tag{3.2.3}$$

But, as it is mentioned above, for plane wave propagated along the fibers

$$\overrightarrow{E}(z,t) = E_o cos(\omega t - kz) \tag{3.2.4}$$

$$\overrightarrow{P} = \epsilon_o\Big[\chi^{(1)}E_o cos(\omega t - kz) + \chi^{(2)}E_o^2 cos^2(\omega t - kz) +$$
$$\chi^{(3)}E_o^3 cos^3(\omega t - kz) + \chi^{(4)}E_o^4 cos^4(\omega t - kz) + \chi^{(3)}E_o^3 cos(\omega t - kz) + \chi^{(5)}E_o^5 cos^5(\omega t - kz)\Big] + ... \tag{3.2.5}$$

Using some trigonometric relations-that is

$$cos^3\theta = \frac{1}{4}cos3\theta + \frac{3}{4}cos\theta \tag{3.2.6}$$

$$cos^4\theta = \frac{1}{8}cos4\theta + cos^2\theta - \frac{1}{4} \tag{3.2.7}$$

$$cos^2\theta = \frac{1}{2}[cos2\theta + 1] \tag{3.2.8}$$

$$cos\theta cos\beta = \frac{1}{2cos(\theta - \beta)} + \frac{1}{2}cos(\theta + \beta) \tag{3.2.9}$$

$$cos^5\theta = \frac{1}{16}\left[cos3\theta + cos5\theta + \frac{1}{4}cos3\theta + \frac{1}{2}cos\theta\right] \tag{3.2.10}$$

by using these relations for equation 3.2.5 we have polarization:-

$$\overrightarrow{P} = \epsilon_o[\chi^{(1)}E_o cos(\omega t - kz)] + \frac{1}{2}\epsilon_o\chi^{(2)}E_o^2[cos2(\omega t - kz) + 1] + \epsilon_o\chi^{(3)}E_o^3[\frac{1}{4}cos3(\omega t - kz) + \frac{3}{4}cos(\omega t - kz)]$$
$$+\epsilon_o\chi^{(4)}E_o^4[\frac{1}{8}cos4(\omega t - kz) + \frac{1}{2}cos2(\omega t - kz) + \frac{1}{2} - \frac{1}{4}] + ... \tag{3.2.11}$$

In real fibers the term like $k(3\omega) \neq 3k(\omega)$, so that any difference like $(3\omega - 3k)$, $4\omega - 4k$ are called phase mismatches. This can understood as the mismatch in a phase between different signals traveling within the fiber at different group velocities and $\chi^{(2)}$ vanishes for the crystals displays inversion symmetry.

Thus equation 3.2.12 becomes

$$\overrightarrow{P} = \epsilon_o[\chi^{(1)} E_o cos(\omega t - kz)] + \frac{3}{4}\epsilon_o[\chi^{(3)} E_o^3 cos(\omega t - kz)] + \frac{1}{2}\epsilon_o[\chi^{(4)} E_o^4 cos2(\omega t - kz)] + \frac{1}{2}\epsilon_o \chi^{(4)} E_o^4 + ...$$

$$+\frac{1}{2}\epsilon_o[\chi^{(4)} E_o^4 cos2(\omega t - kz)] + \frac{1}{2}\epsilon_o \chi^{(4)} E_o^4 + ... \qquad (3.2.12)$$

In similar way that fifth harmonic nonlinear polarization is

$$\overrightarrow{P} = \epsilon_o \Big[\chi^{(1)} E_o cos(\omega t - kz) + \chi^{(2)} E_o^2 cos^2(\omega t - kz) +$$

$$\chi^{(3)} E_o^3 cos(\omega t - kz) + \chi^{(4)} E_o^4 cos(\omega t - kz) + \chi^{(5)} E_o^5 cos(\omega t - kz) \Big] + ... \qquad (3.2.13)$$

Using trigonometric relation of equations(3.2.6-3.2.10) and those phase mismatch, lack inversion symmetries canceled, the fifth harmonic polarization would be

$$\overrightarrow{P} = \epsilon_o(\chi^{(1)} + \frac{3}{4}\chi^{(3)} E_o^2) E_o cos(\omega t - kz) + \frac{1}{2}\epsilon_o \chi^{(5)} E_o^5 cos(\omega t - kz) +$$

$$\frac{1}{2}\epsilon_o \chi^{(4)} E_o^4 cos2(\omega t - kz) + \frac{1}{2}\epsilon_o \chi^{(4)} E^4 \qquad (3.2.14)$$

$$\overrightarrow{P} = \epsilon_o \chi^{(1)} E_o cos(\omega t - kz) + \frac{3}{4}\epsilon_o \chi^{(3)} E_o^3 cos(\omega t - kz)$$

$$+\frac{1}{2}\epsilon_o \chi^{(5)} E_o^5 cos(\omega t - kz) + \frac{1}{2}\epsilon_o \chi^{(4)} E_o^4 cos2(\omega t - kz) + \frac{1}{2}\epsilon_o \chi^{(4)} E_o^4 \qquad (3.2.15)$$

$$\overrightarrow{P} = \epsilon_o \chi^{(1)} E_o cos(\omega t - kz) + \frac{3}{4}\epsilon_o \chi^{(3)} E_o^3 cos(\omega t - kz)$$

$$+\frac{1}{2}\epsilon_o \chi^{(5)} E_o^5 cos(\omega t - kz) + \qquad (3.2.16)$$

$$\overrightarrow{P} = (\epsilon_o \chi^{(1)} E_o + \frac{3}{4}\epsilon_o \chi^{(3)} E_o^3 + \frac{1}{2}\epsilon_o \chi^{(5)} E_o^5) cos(\omega t - kz) \qquad (3.2.17)$$

$$\overrightarrow{P} = \epsilon_o \big(\chi^{(1)} E_o + \frac{3}{4}\chi^{(3)} E_o^3 + \frac{1}{2}\chi^{(5)} E_o^5\big) cos(\omega t - kz) \qquad (3.2.18)$$

The equation contain both third and fifth harmonic polarization.

Chapter 4

Results and Discusion

This chapter will presents the results of chirp frequency, time variation of third order nonlinear polarization with operating wavelength range.their variation with wavelength. From equations (2.3.17-2.3.19) one observes chirped pulse due to frequency variation across the pulse. Figure(4.1) shows frequency response of chirp system in dispersion less medium. In calculation I use single mode fibers that inversion symmetry at molecular level of fibers in which second order susceptibility vanishes and I obtained the numerical value of third order susceptibility. Using the value obtained the graph of third order time polarization vs time using thee operating wavelength of the source like 850nm,1550nm, 3100nm wavelength obtained. From graphs of (Figure 4.2-4.11) we observe that the light is polarized in the fibers of any length within various spread pulse time. Using operating wavelength of the fibers polarization is increased at 3100nm, because this is wave length at which dispersion through the fiber is zero. When we compare our results shown on (Fig 4.1-4.11) with composition of optical fibers at molecular level-that is (SiO_2), then two oxygen atoms are connected to the silicon atom via springs that undergoes into anharmonic vibration when high optical monochromatic Laser light is propagated through the fiber at given wavelength. We also used some parameter derived in sections (2.3.3 and 3.0.1) which are very important to generate data for nonlinear index of refraction, susceptibilities, input Laser power and etc as shown in table 4.1.

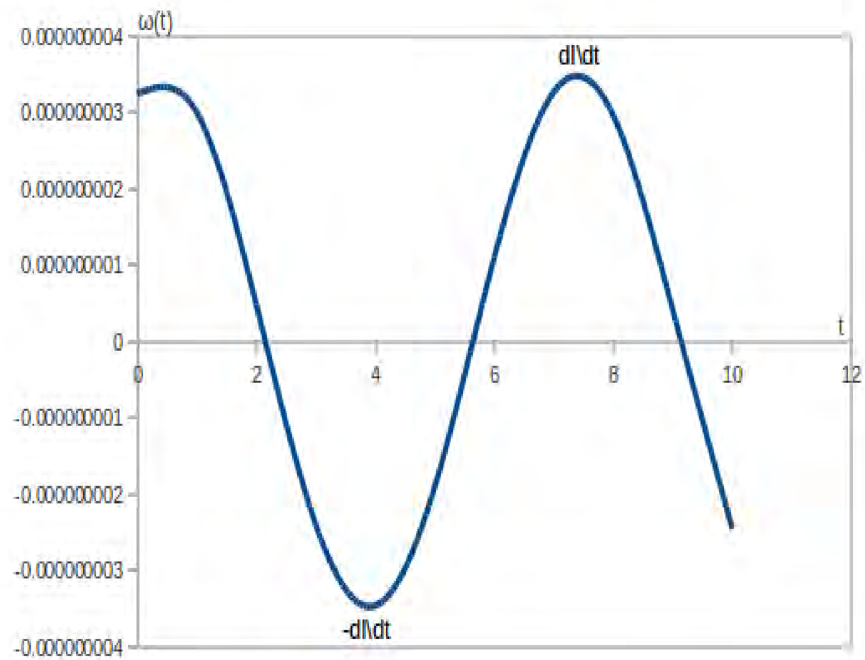

Figure 4.1: Frequency response of chirping phenomena

Table 4.1: Results for third order susceptibility, effective length, nonlinear phase constants and nonlinear phase number.

Parameters	Numerical values
Intensity(I_o)	$200[\frac{W}{m^2}]$
Speed of light(c)	$3 \times 10^8[\frac{m}{s}]$
Permitivty in free space(ϵ_o)	$8.85 \times 10^{-12}[\frac{N}{m}]$
Third order susceptibility($\chi^{(3)}$)	2.4×10^{-12}
Effective length(L_{eff})	$5 \times 10^3[\frac{m}{dB}]$
Attenuation coefficient(α)	$0.2[\frac{dB}{km}]$
Nonlinear phase number(k_{n_l})	$1.296{\times}10^{-6}[\frac{1}{mW}]$
Linear phase constant(ϕ_l)	$8.215610^7\ [rad]$
Number of cycle obtained	1.3×10^7
Input Laser power (p)	$10[mW]$
Effective Area (A_{eff})	$50[\mu m^2]$
Linear refractive index (n_l)	1.46
Nonlinear refractive index(n_{n_l})	$3.2 \times 10^{-20}[\frac{m^2}{W}]$
Wavelength at negligible dispersion (λ_o)	$1550[nm]$

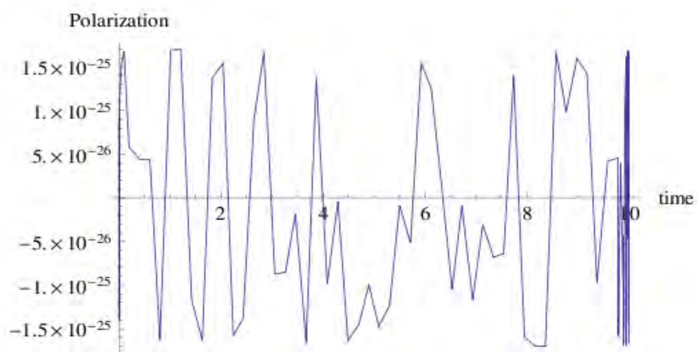

Figure 4.2: Variation of third order nonlinear polarization versus time. By using length of fibers L=($10^6 m$) at operating wavelength fibers communication ($850nm$).

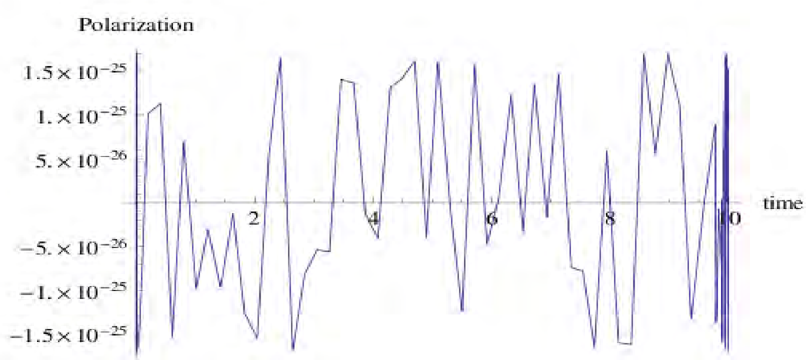

Figure 4.3: Variation of third order nonlinear polarization versus time. By using length of fibers L=($10^5 m$) operating wavelength of fibers communication ($850nm$).

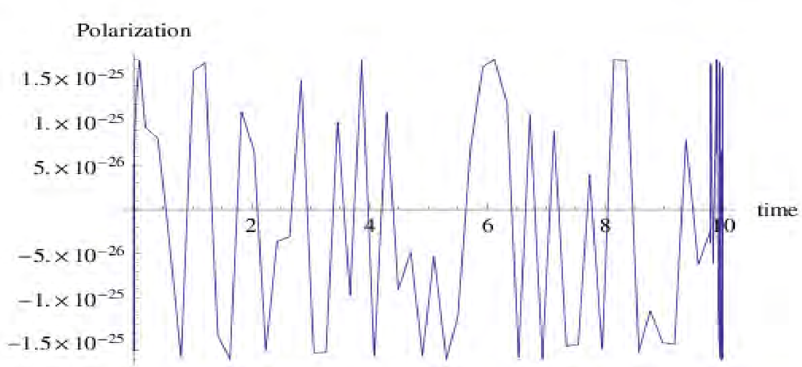

Figure 4.4: Variation of third order nonlinear polarization versus time. By using length of fibers (L=$10^3 m$) operating wavelength of fibers communication ($850nm$).

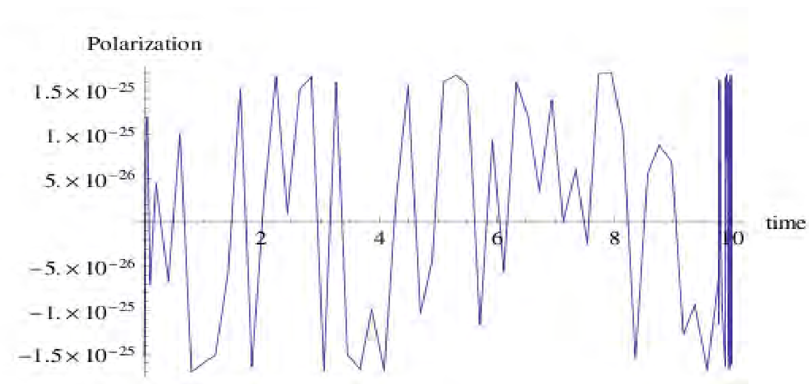

Figure 4.5: Variation of third order nonlinear polarization versus time. By using length of fibers (L=$10^3 m$) operating wavelength of fibers communication ($1550nm$).

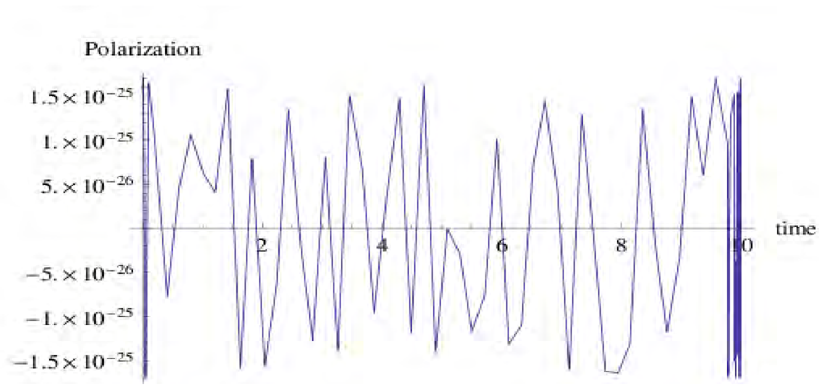

Figure 4.6: Variation of third order nonlinear polarization versus time. By using length of fibers (L=$10^5 m$) operating wavelength of fibers communication ($1550nm$).

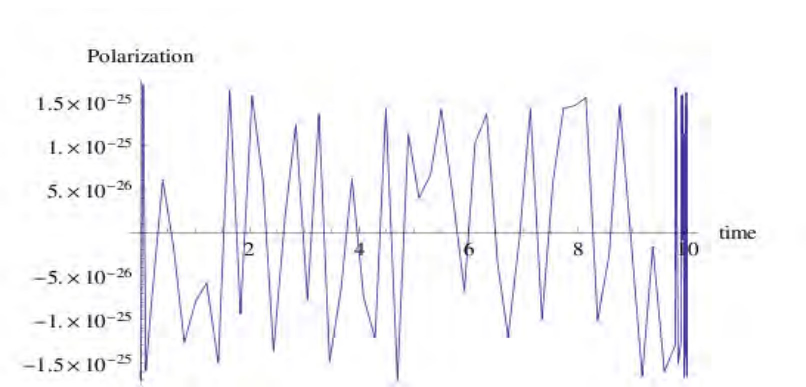

Figure 4.7: Variation of third order nonlinear polarization versus time. By using length of fibers (L=$10^6 m$) operating wavelength of fibers communication ($1550nm$).

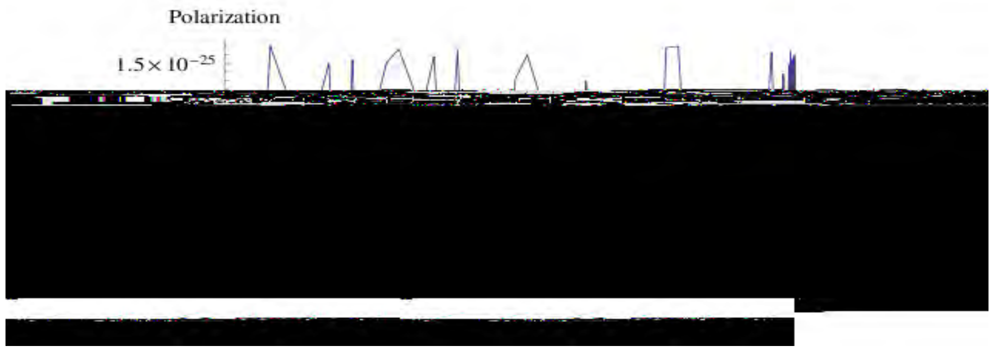

Figure 4.8: aa

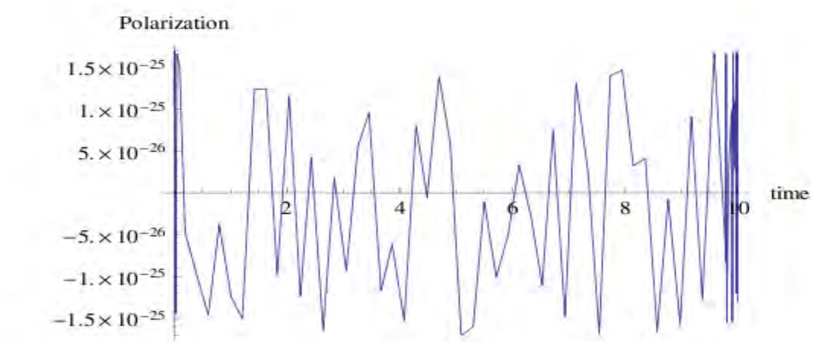

Figure 4.9: Variation of third order nonlinear polarization versus time. By using length of fibers (L=$10^5 m$) operating wavelength of fibers communication ($3100nm$).

Figure 4.10: Variation of third order nonlinear polarization versus time. By using length of fibers (L=$10^6 m$) operating wavelength of fibers communication ($3100nm$).

Conclusion

Self phase modulation by it self leads to chirping, regardless of pulse shape. If dispersion that is responsible for pulse broadening, the self phase modulation induced chirp modifies the pulse broadening effects of dispersion. chirp signal is ingenious way of handling practical problem in echo location system such as radar. Graphs are in general used to look at nonlinear response in pure silica core of single mode fibers. This is because, in single mode fibers dispersion is negligible and have good contribution for light to polarized. Nonlinearities in optical fibers and materials play crucial role in a long haul and high band with communication. Nonlinearities in optical fibers and materials play crucial role in a long haul and high band with communication. It has many loss mechanism such as attenuation, dispersion, distortion in signal propagation. As the size of the pulse become narrow due we use single mode fibers and Laser power increases polarization increases, nonlinearities become more predominant. In order to understand the origin and mechanism of such effects we develop simple mathematical derivation for silica core fiber as bound electron vibrate anharmonically to generate induced polarization of second, third, fourth and fifth order susceptibilities. We see that our calculation gives quite nice quantitative understanding of nonlinear effects contributed from dispersion-less medium polarization. In addition to this all we would like to clarify that the choice of pump source for generating monochromatic Laser light depends on the application prospectives. As in optical communication and medical applications, low power Laser are desirable. However, for applications in optical switches and optical limiters, the nonlinear effects are required. Therefore, high power Lasers are used. This is only drawback that the invention of optical switches is taking time for the commercial market.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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