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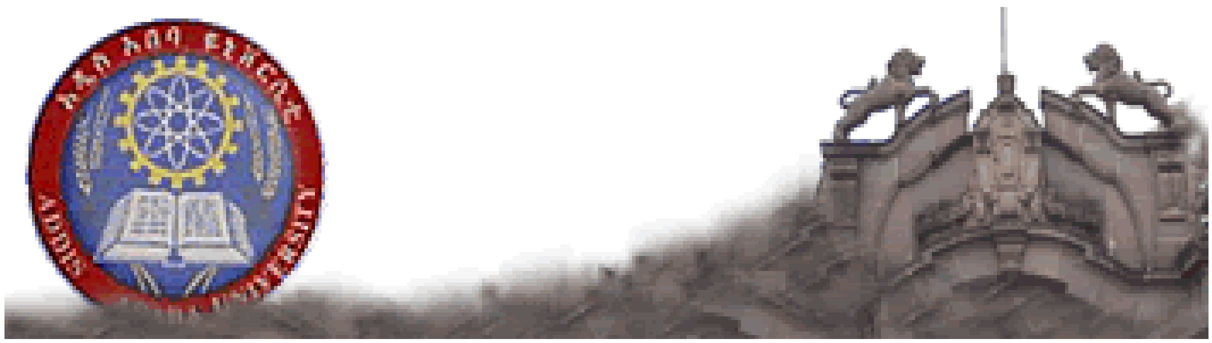

Modeling and Forecasting the Volatility of the Export Price of Sesame in Ethiopia

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This is to certify that the thesis prepared by Sebsib Muanenda, entitled: *modeling and forecasting the volatility of the export price of sesame in Ethiopia* and submitted in partial fulfillment of the requirements for the Degree of Master of Science complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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Acronyms

ACF	Autocorrelation function
ADF	Augmented Dickey-Fuller
AIC	Akaike information criterion
AICc	Bias correlated Akaike information criterion
AR	Autoregressive
ARCH	Autoregressive conditional heteroscedastic
ARCH LM	Autoregressive conditional heteroscedastic Langrage multiplier
ARDL	Autoregressive distributed lag
ARIMA	Autoregressive integrated moving average
ARMA	Autoregressive moving average
BIC	Bayesian information criterion
CSA	Central statistical agency
DAX	Deutscher Aktienindex
ECX	Ethiopia commodity exchange
EGARCH	Exponential generalized autoregressive conditionally heteroscedastic
ERCA	Ethiopian revenue and custom authority
FAIR	Federal Agriculture Improvement and Reform
FAO	Food and agricultural organization
GARCH	Generalized autoregressive conditionally heteroscedastic
GED	Generalized error distribution

JB	Jarque-Bera
LM	Lagrange multiplier
MA	Moving average
MAE	Mean absolute error
MAPE	Mean absolute percentage error
ML	Maximum likelihood
MPAE	Mean percentage absolute error
MSFE	Mean square forecast error
NBE	National bank of Ethiopia
OLS	Ordinary least square
PACF	Partial autocorrelation function
PP	Phillips-Perron
RMSE	Root mean square error
SARIMA	Seasonal autoregressive integrated moving average
SV	Stochastic volatility

Abstract

Sesame (Sesamum indicum L.) is one of the world's oldest spice and oilseed crops grown mainly for its seeds that contain approximately 50% oil and 25% protein. Ethiopian oilseeds and pulses are mostly organically produced, and are known for their flavor and nutritional value. Sesame is the second-largest export crop in Ethiopia, after coffee, and accounts for over 90% of the value of oil seeds exports. The aim of this study is to model the export price of sesame as well as its volatility in Ethiopia using ARIMA and GARCH family models. The data used are monthly observations of the export price of sesame, food price index, fuel oil price and exchange rate from January 1998 to June 2013.

Unit root tests of the series under study reveal that all the series are non-stationary at level and stationary after first difference. ARIMA and GARCH models were employed to analyze the monthly export price of sesame data. It was found that ARIMA(0,1,1) and ARMA(2,2)-GARCH(2,1) with normal distributional assumption for the residuals were adequate models for the data considered in this study. Among the exogenous variable that are considered in this study, food price index had an impact on the volatility of the export price of sesame in Ethiopia.

Finally, various forecast accuracy statistics indicate that the estimated ARIMA model is good enough to describe the export price of sesame. Moreover, the out-of-sample forecasts indicate that the export price of sesame has an increasing trend. The in-sample forecast using the best-fit GARCH model indicates that the export price volatility of sesame steadily increased at the beginning of the study period, remained at almost a constant level till 2007 and then exhibited a downward trend around the end of the study period.

Key words: *Sesame, ARIMA, GARCH, Forecasting, Ethiopia*

1. Introduction

1.1. Background of the study

Sesame (*Sesamum indicum* L.) is one of the world's oldest spice and oilseed crops grown mainly for its seeds that contain approximately 50% oil and 25% protein (Burden, 2005). The presence of some antioxidants (sesamum, sesamolin and sesamol) makes the oil to be one of the most stable vegetable oils in the world. The world production is estimated at 3.66 million tons with Asia and Africa producing 2.55 and 0.95 million tons, respectively (Anonymous, 2008). The continent of Africa is naturally endowed with favorable weather conditions that can support sesame production. The crop requires only 500-650 mm of rainfall per annum. Unfortunately, average world yield of sesame is still low at 0.46 ton ha^{-1} (FAO, 2004). Low yield had been attributed to cultivation of low yielding dehiscent varieties with low harvest index values, significant yield loss during threshing and lack of agricultural inputs such as improved varieties, fertilizers and other agro-chemicals (Ashri, 1994; Weiss, 2000; Uzun and Cagirgam, 2006). However, non- dehiscent sesame varieties with yield potential of over 1 ton ha^{-1} and suitable for mechanical combine harvest have been developed by Sesame Coordinators (SESACO) in USA (SESACO, 2007).

The original area of domestication of sesame is obscure but it seems likely to have first been brought into cultivation in Asia or India. The plant is usually 60 to 120cm tall and the fruit is a dehiscent capsule held close to the stem. When ripe, the capsule shatters to release a number of small seeds. The seeds are protected by a fibrous ‘hull’ or skin, which may be whitish to brown or black depending on the variety. 1000 seeds weigh some 4-8g (FAO, 2011).

The plant is deep rooting and well adapted to withstand dry conditions. It will grow on relatively poor soils in climates generally unsuitable for other crops, and so it is widely valued for its nutritional and financial yield from otherwise inclement areas. It is well suited to smallholder farming with a relatively short harvest cycle of 90 –140 days allowing other crops to be grown in the field. It is often intercropped with other grains (Chemonics, 2002).

Foreign trade payments have a significance role in the development plans of many developing countries. For most of these countries, export represents an important share of the total development. Trades in the foreign sales are important in forecasting of overall growth of a country. For some developing countries, export trade is such an important factor that an estimate of the foreign exchange earnings represents a first step in the formulation of development plans. As the export of a given country continue to expand, new profit opportunities develop not only to provide inputs for export sector but also to take the advantage of external economic benefits (Hultman, 1967).

Ethiopian oilseeds and pulses are mostly organically produced, and are known for their flavor and nutritional value. The Ethiopian white sesame seed is used as a reference for grading in international markets. Ethiopia's major oilseeds and pulses exports include the following: sesame seeds, Niger seeds, linseeds, sunflower seeds, groundnuts, rape seeds, castor oil seeds, pumpkin seeds, haricot beans, pea-beans, horse beans, chick peas, beans and lentils. The European Union, Asia, the Gulf States and neighboring African countries comprise the major markets for Ethiopia's oilseeds and pulses exports (ECX, 2012).

Sesame is the second-largest export crop in Ethiopia, after coffee, and accounts for over 90% of the value of oil seeds exports. Different reports indicate that Ethiopia is among the top-five sesame producing countries in the world, ranked at fourth place in 2011/2012 (FAOSTAT, 2012). And it is the third largest exporter of sesame seed after India and Sudan (Alemu and Meijerink, 2010). Sesame is grown mainly for export markets and little value is added in Ethiopia (Wijnands et al., 2009). It is mainly grown by small-scale farmers in four regions of Ethiopia (Tigray, Amhara, Oromia and Beneshangul Gumuz). In the past decade, the area under production has grown 8-fold to 316 thousand ha, or 2% of Ethiopia's arable land (FAOSTAT, 2012).

The Ethiopian Commodity Exchange (ECX) opened up for sesame trade in 2009, and became the mandatory channel for sesame exports in late 2010 (The Council of Ministers Regulation No. 178/2010). Not surprisingly, in light of the compulsory status, volumes of sesame and other crops traded via the ECX have grown rapidly. In its fourth year of operation (2011/12), the total traded volume reached 601,000 tones, consisting of coffee (50%), sesame (39%), and pea beans (11%) (ECX, 2012).

Oilseeds and wheat grains have witnessed unprecedented volatilities and price fluctuations in the recent past. Extreme volatility in commodity prices, particularly of food commodities, affects producers, consumers, traders, exporters and food procurement agencies of the central and state government. Sesame seed is a high value cash crop. Sesame prices have ranged between US 800 to 1700s per metric ton between 2008 and 2010. Sesame exports sell across a wide price range. Quality perception, particularly how the seed looks, is a major pricing factor.

Financial time series are often considered as heteroscedastic time series where the conditional variance given the past is no longer constant. In financial analysis, forecast of future volatility of a series under consideration is often of interest to assess the risk associated with certain assets. In that case, variance forecasts are of direct interest (Lütkepohl, 2005).

Engle (1982) developed the Autoregressive Conditional Hetroscedasticity (ARCH) model asserting that although many financial time series, such as stock returns and exchange rates, are unpredictable, there is apparent clustering in the variability or volatility. This is often referred to as conditional heteroscedasticity since it is assumed that overall the series could be stationary but the conditional expected value of the squared error term with respect to the information set is time dependent.

Later, Bollerslev (1986) modified Engle's ARCH model into a more generalized model called GARCH model. GARCH model may contains fewer parameters as compared to an ARCH model, and thus a GARCH model may be preferred to an ARCH model. GARCH is a mechanism that includes past variances in the explanation of future variances. More specifically, GARCH is a time series modeling technique that uses past variances and past squared error terms to forecast future volatility. Whenever a time series is said to have GARCH effects, the series is heteroscedastic, i.e. its variance varies with time.

1.2. Statement of the problem

Sesame production and marketing in Ethiopia has an effect on its export price due to low productivity and quality, poor market infrastructure, and long and traditional marketing channels, among others. These cause heavy and sudden fluctuation in export price of sesame and create serious problems in national income. The price volatility in response to shocks due to different factors is of crucial importance for a developing economy like Ethiopia whose policy is oriented towards price stability. Several studies have been conducted on sesame in Ethiopia (Aysheshm (2007), Amare (2009) and Geremew (2012)). Most of these researches were focused on the production related problem of the crop, ignoring the export price of the crop and its volatility. Thus, this study tries to utilize time series analysis using GARCH family models to analyze and forecast the volatility of export price of sesame in Ethiopia.

1.3. Objective of the Study

The general objective of this study is to model the export price of sesame as well as its volatility in Ethiopia using GARCH family models.

The specific objectives are:

- To fit appropriate ARIMA and GARCH family models for data on export price of sesame.
- To identify and analyze the key macroeconomic factors that have significant impact on the volatility of the export price of sesame in Ethiopia.
- To make in-sample forecast of the volatility of the export price of sesame.

1.4. Significance of the Study

- The result of this study helps to understand sesame export market volatility.
- The results could be of interest to further studies related to export price volatility.
- This research would be useful to cooperative societies, researchers, and governmental and non-governmental organizations for policy formulation, planning and development of agricultural marketing for both sesame exporters and producers in Ethiopia.

1.5. Organization of the study

This work is organized into the following chapters. Chapter one is introduction of the thesis. This chapter briefly addresses the thesis objectives, significance, and background of the study. Chapter 2 reviews the literature with emphasis on the statistical tools relevant to modeling the export price and volatility of sesame. Chapter 3 explains the methodology applied in building ARIMA and GARCH models and estimating their parameters. Chapter 4 presents the results of analysis. Chapter 5 conclusion and recommendation.

2. Literature review

2.1. Sesame Marketing in Ethiopia

Sesame marketing is highly linked with the international market and highly volatile following changes in the supply and demand at international markets. The major actors in the Ethiopian sesame market are exporters, wholesalers, brokers/agents, local traders (assemblers), primary cooperatives and their unions, commercial farms and small-scale farmers (Alemu, 2009). Understanding the scattered and small-scale nature of the Ethiopian production system, the role of aggregation in improving the agricultural marketing system is given due emphasis in the national agricultural marketing strategy and this is sought to be achieved through cooperatives and their respective unions.

The Council of Ministers Regulation No.178/2010 passed on 22 May 2010 mandates that sesame seed trading in Ethiopia shall be conducted only at primary transaction centers and the Ethiopian Commodity Exchange (ECX). Article 18 (2) of the Regulation reserves the right for any producer to export sesame seed directly, individually or through a cooperative in which he/she is a member. As a result of the enforcement of the mandatory trading provisions of the Regulation, nearly all of the country’s sesame has been traded through the Ethiopian Commodity Exchange (ECX, 2012).

Sesame in Ethiopia is grown mainly for the export market (Aysheshm, 2007; Alemu and Meijerink, 2010). According to Aysheshm (2007), only about 5% is believed to be consumed locally. Ethiopia is a major sesame seed exporter in the world market. For example, in 2005/06 Ethiopia exported 237, 565 tons of sesame seed, accounting for roughly 94% of the total export earnings from oilseeds and 19% of total national export earnings (ECX, 2012). In addition, reports suggest that there is a considerable international market demand for Ethiopian sesame seed, and it is expected to continue increasing in the future (Sorsa, 2009). According to the same author, this increasing international market demand for the crop is not only evident in the rise of export volume but also in new buyers coming to the market. China is the largest import market for Ethiopia’s sesame followed by Israel, Turkey and Jordan in 2011 (ERCA, 2012).

2.2. Theoretical and empirical studies on volatility

There are a number of different volatility estimating and forecasting models such as GARCH-family and stochastic volatility (SV) models. Among those models, the family of GARCH-type models introduced by Engle (1982), Bollerslev (1986) and Taylor (1986) and then developed by many other researchers is the most common approach and proved to be sufficient to capture the properties of return series.

Jin and Kim (2012), using real prices on rice, red pepper, onion and sesame for South Korea, test multiple structural change or regime switching techniques. They suggest a new type of measurement using a model which incorporates multiple structural changes in the unconditional mean to overcome the problem of amplified variance. They prove that multiple structural method performs better than others when the regime switches are given a form of parallel mean shift. However, if the series are more dominated by trends than by mean shifts this method is not suitable.

Udoh and Egwaikhide (2012) used multivariate GARCH methods to compare the performance of models by considering the normal and t-distributional assumptions for the error terms to capture volatility in food crop prices in Nigeria. They use monthly prices for wheat, rice, sugar, beef, coffee, sesame and groundnut and conclude that the t-distributional assumption is superior to the normal distribution one. This implies that the normality assumption of the residuals may lead to unreliable volatility results. Results of the multivariate GARCH model shows sugar price volatility transmission was better than that of wheat market.

Abule (2012) employed Autoregressive Distributed Lag (ARDL) model on yearly data for the period 1992-2010 to examine the impact of exchange rate variability on oilseeds in Ethiopia. The result shows that terms of trade is positive and significant implying that export of oilseeds is in favor of terms of trades. This is the result of an increasing demand for oilseeds all over the world. For instance, export of sesame has grown in double digits each year from 1998 to 2006: 50,000 tons in 1998 and more than 100,000 tons in 2006 (CSA, 2006; ECA, 2009).

Geremew (2012) used a truncated regression technique to analyze factors that are expected to influence and explain the income smallholder farmers are earning from sesame sale in the Giga district, East Wollega Zone of Oromia Regional state. According to the results, quantity of sesame marketed, sesame market price, selling to local cooperatives and selling channels were found to be have positive and statistically significant influence on the quantity of sesame production. On the other hand, travelling time to the nearest market center and sesame selling time to one, two and three months after harvest (these are categorical variables that allow us to understand the role of time in which farmers sold their sesame produce in explaining the price they received and, hence the income they earned) influence the quantity of sesame production inversely and significantly.

Engle (1982) studied on ARCH and GARCH models, and revealed that these models were designed to deal with the assumption of heteroscedasticity found in real life financial data. ARCH and GARCH models have been applied to a wide range of time series analyses, but applications in finance have been particularly successful. Financial decisions are generally based upon the tradeoff between risk and return; the econometric analysis of risk is therefore an integral part of asset pricing, portfolio optimization, option pricing and risk management. The analysis of ARCH and GARCH models and their many extensions provide a statistical approach on which many theories of asset pricing and portfolio analysis can be tested. The goal of such models is to provide a volatility measure like a standard deviation that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing.

According to Chatfield (2000), the idea behind a GARCH model is similar to that behind ARMA model in the sense that a higher order AR or MA model may often be approximated by a mixed ARMA model, with fewer parameters, using a rational polynomial approximation. Thus a GARCH model can be thought of as an approximation to a higher-order ARCH model. The GARCH(1,1) model has become the standard model for describing changing variance for no obvious reason other than relative simplicity.

Brooks (2008) studied stochastic volatility models and found that most time series models such as GARCH will have forecasts that tend towards the unconditional variance of the series as the prediction horizon increases. This is a good property for a volatility forecasting model to have, since it is well known that volatility series are mean-reverting. This implies that if they are at a lower level relative to their historic average, they will have a tendency to rise back towards the average. This feature is accounted for in GARCH volatility forecasting models.

Claessen and Mittnik (2002) investigated a range of alternative strategies for predicting volatility in financial markets applying GARCH model to daily returns on the DAX index, the major German stock index. The work of these two scholars investigated the suitability of several forecasting techniques for the volatility of the returns of the German DAX index and examined whether or not the German DAX–index options market is informational efficient. By combining both sources of information, the in–sample fitting and out–of–sample forecasting results give strong support for the hypothesis that historic returns contain no information beyond the market's volatility expectation that is reflected in DAX–index option prices. In summary, they concluded that implied volatility is a biased but highly informative predictor for future volatility. Moreover, implied volatilities are informational efficient relative to other historic volatility information sources.

Valadkhani, Layton and Karunaratne (2005), in their study of export price volatility in Australia, tried to show the extent to which Australia's export prices relate to world prices using quarterly time-series data spanning the period 1969 quarter 4-2002 quarter 3. Using a parsimonious GARCH (1,1), they showed that Australia's export prices are relatively more volatile in both the pre-1975 and post-1975 periods compared to that of other countries.

Song et al. (1998) employed GARCH models to analyze the volatility on the two main official stock exchange markets, Shanghai and Shenzhen, in China using monthly data from May 1992 to February 1996. The result showed that GARCH-M (1,1) model was a better-fit model among the GARCH Mean (M) models of stock return series of both Shanghai and Shenzhen markets. Moreover, the one month forecast suggests a similar pattern for the conditional variances of the two markets' return.

Amos (2009) studied the inflation rate in South Africa using data spanning from January 1994 to December 2008. In the study, the seasonal autoregressive integrated moving average (SARIMA) model and the generalized autoregressive conditional heteroscedasticity (GARCH) model were fitted to the data. A best fitting model for each family of models offering an optimal balance between goodness of fit and parsimony was selected. SARIMA(1,1,0)x(0,1,1) and GARCH(1,1) models were chosen to be the best fitting models for determining the two years forecasts of inflation rate of South Africa. According to the results, GARCH(1,1) was observed to be superior in producing future forecasts because of its ability to capture volatility in the data.

Zheng et al. (2008) apply GARCH model to examine whether unexpected news affects food price volatility. They use monthly prices for 45 foodstuffs in the US. The results confirm that the amplifying effect of the news is present only in one third of the products. They argue that the increasing concentration of the distribution and retailing of food on large firms is absorbing the price volatility.

Hayo et al. (2012) assessed using GARCH model the impact of the US monetary policy on the price volatility of different commodities (agricultural, livestock, energy and metals). They arrive at the conclusion that expected target interest rate changes and communications do decrease volatility, whereas unexpected interest rate movements and innovative measures do increase it.

Ghosh et al. (2010) used GARCH(1,1), the best fit model among GARCH, GARCH-dummy, EGARCH-M models and OLS estimation method, to examine the price volatility and supply response for rice, jowar, bajra, maize, groundnut and cotton in India. Using annual prices, they check whether trade liberalization indeed exacerbates volatility of agricultural products. The results show that the prices of major agricultural products become more unstable in India after the signing of the WTO agreement. Swaray (2007) in the same country used E-GARCH(2,1), the better fit model among an Exponential GARCH and a Threshold GARCH model, to assess the impact of the suspension of international commodity agreements on the asymmetry and persistency of volatility in monthly prices for cocoa, coffee, rubber, sugar and tin. The results demonstrate a rise in the asymmetry but a decrease in the persistence of the shocks.

Perniagaan et al. (2004) explored the varying volatility dynamics of inflation rate in Malaysia for the period from August 1980 to December 2004. The generalized autoregressive conditional heteroscedasticity (GARCH) models and the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) models were used to capture the stochastic variations and asymmetries in the series. An in-sample evaluation of the sub-periods volatility was done. Based on the results, both GARCH(1,1) model and EGARCH(1,1) produced good estimates of sub-periods volatility. But due to the nature of the data of having highly irregular fluctuation, EGARCH model was selected to be the best.

Yang et al. (2001) investigate the effects of the market-oriented US FAIR act in 1996 on agricultural price volatility using GARCH models for corn, oat, soybeans, wheat and cotton daily futures prices. The results show that agricultural liberalization policy causes: an increase in price volatility for the three major commodities (corn, soybean and wheat); a little change for oats; and a decrease for cotton.

Apergis and Rezitis (2003) employ a multivariate GARCH model of relative food price index in Greece. Macroeconomic factors such as money supply, income per capita, real exchange rate, budget deficit/surplus during 1985–1999 are used in order to investigate the volatility spillover effects between food and macroeconomic fundamentals. The results from a GARCH-X(1,1) model showed that a significant and positive effect is imposed by those derivations on the volatility of relative food prices. Moreover, the inclusion of macroeconomic shocks gets them closer to permanency.

Kontonikas (2004) analyzed the relationship between inflation and inflation uncertainty in the United Kingdom from 1973 to 2003 with monthly and quarterly data. Different types of GARCH Mean (M)-Level (L) models that allow for simultaneous feedback between the conditional mean and variance of inflation were used to test the relationship. They found that there was a positive relationship between past inflation and uncertainty about future inflation.

Assis et al. (2006) compared the forecasting performance of different time series methods for forecasting cocoa bean prices at Bagan Datoh cocoa bean in Malaysia. The monthly average of Bagan Datoh cocoa bean prices graded for the period of January 1992 to December 2006 was used. Four different types of univariate time series models were compared; namely the exponential smoothing, autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroscedasticity (GARCH) and the mixed ARIMA/GARCH models. Root mean squared error (RMSE), mean absolute percentage error (MAPE), mean absolute error (MAE) and Theil`s inequality coefficient (U-statistics) were used as selection criteria to determine the best forecasting model. The study revealed that the series was influenced by a positive linear trend factor while the regression test results showed the non-existence of seasonal factors. Moreover, the autocorrelation function (ACF) and the Augmented Dickey-Fuller (ADF) test showed that the time series was not stationary in levels but became stationary after the first differentiating process was carried out. Based on the results of the ex-post forecasting (starting from January until December 2006) GARCH model outperformed the exponential smoothing, ARIMA, and mixed ARIMA/GARCH models for forecasting Bagan Datoh bean prices.

Chong et al. (2002) used seven years of daily observed Sterling exchange rate in Malaysia. The GARCH model including the family of GARCH Mean model were used to explain the commonly observed characteristics of the unconditional distribution of daily rate of returns series. The results indicated that the hypotheses of constant variance model could be rejected, at least within sample, since almost all the parameter estimates of the ARCH and GARCH models were significant at five percent level of significant. The Box-Ljung statistics (Q- statistics) and the Lagrange Multiplier test revealed that the use of the long memory GARCH model was preferable to the short memory and high order ARCH model. The results from various goodness of fit statistics were not consistent for Sterling exchange rates. It appeared that the BIC and AIC test proposed GARCH models to be the best for within sample modeling while the mean square forecast error (MSFE) suggested the GARCH mean model to be the best for modeling the variance of daily exchange rates.

Igogo (2010) did a study on the effect of real exchange rate volatility on trade flows in Tanzania for the period from 1968 to 2007. The study employed recent ARCH family models to measure volatility. Firstly, the GARCH(1,1) model was employed and found to violate the non-negativity condition. Consequently, the study employed EGARCH(1,1) model proposed by Nelson (1991) to resolve the problem. The adequacy of the EGARCH(1,1) model to measure the real exchange rate volatility was confirmed by testing for ARCH effect after running the model. Furthermore, the study reveals that it is the real exchange rate rather than its volatility that is found to have a significant effect on trade flows although the effect is larger on exports than on imports. Therefore, in the short-run imports are mainly affected by domestic income while exports are mainly affected by the real exchange rate.

Shiferaw (2012) employed ARCH/GARCH models to capture the log-return price volatility of price returns of crops in Ethiopia. He found that GARCH(1,1), GARCH(1,2) and GARCH(2,1) models were good fit for the log-returns price of cereal, pulse and oil crops, respectively. The sum of the coefficients of the ARCH and GARCH terms is close to one in each crop. These results indicate that the volatility shocks are permanent or persistent in each of the three agricultural crops. Regarding the forecasting capability, after obtaining the three satisfactory

GARCH models, the forecast process was analyzed based on root mean square error (RMSE). The study concludes that the above GARCH models are suitable to forecast the long-return volatility of crops because the value of RMSE calculated using these models were small. Findings of this study could give good insight to forecast volatility of price of agricultural products in Ethiopia.

Amare (2009) studied the volatility of export prices in Ethiopia using data from January 1997 to May 2008 on two main export items, namely, coffee and oilseeds export prices. In the study, the autoregressive moving average (ARMA) and autoregressive conditional hetroscedasticity (GARCH) models were fitted to the data. A best fitting model for each family of model based on AIC and BIC criteria selection procedure was used. ARMA(1,1) is found as the most appropriate model for the conditional mean of coffee and oilseeds export prices. The study also found GARCH(2,1) and GARCH(2,2) for modeling volatility of coffee and oilseeds export prices as best fitting models respectively. Moreover, he also study on total export prices, the result showed that ARMA(1,1) were the appropriate model for the conditional mean of total export prices. And also GARCH(2,1) was the better fit model for modeling the volatility of the total export prices. In addition, the result suggest that the export price volatility is persistent in all the three cases indicating that past volatility is important in predicting (forecasting) future volatility.

3. Data and Methodology

3.1. Data and variable of the study

The purpose of this study is to model and forecast the volatility of the export price of sesame in Ethiopia. In order to propose suitable specifications, it is reasonable to investigate the nature of the data set. The characteristics of the data and their descriptive statistics partly indicate the appropriate models which should be employed.

Data for the study were obtained from National bank of Ethiopia (NBE) and Central statistical agency (CSA). Monthly time series data on export price of sesame, food price index, fuel oil price and exchange rate for the period from January 1998 to June 2013 were used for estimation process. The export price of sesame, exchange rate and fuel oil price data are obtained from NBE, and food price index data are obtained from CSA.

The variables of interest in this study are export price of sesame which is to be used as a dependent variable, and fuel oil price, exchange rate and food price index are exogenous variables used to model and forecast the volatility of the export price of sesame in Ethiopia.

3.2. Methodology

The methodology adopted in this study follows ARIMA and GARCH modelling approach. Before fitting time series models, the series should undergo stationary test so as statistical properties such as mean, variance, autocorrelation and auto-covariance function are all constant over time. Most statistical forecasting methods are based on the assumption that the time series can be rendered approximately stationary through the use of differencing. Thus, the first step would be conducting unit root tests such as the Augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979) and Phillips-Perron (1988) tests. And then we estimate the autoregressive integrated moving average (ARIMA) and the generalized autoregressive conditionally heteroscedastic (GARCH) model parameters and apply diagnostic tests of the models. Finally, the appropriate time series models will be used to forecast the volatility of the export price of sesame in Ethiopia.

3.2.1. Stationarity and Unit root test

The foundation of time series analysis is stationarity. A time series $\{Y_t\}$ is said to be strictly stationary if the joint distribution of $\{Y_1, Y_2, \dots, Y_t\}$ is identical to that of $\{Y_{1+h}, Y_{2+h}, \dots, Y_{t+h}\}$ where h is an arbitrary positive integers. In other words, strict stationarity requires that the joint distribution of $\{Y_1, Y_2, \dots, Y_t\}$ is invariant under time shift. This is a very strong condition that is hard to verify empirically. A weaker version of stationarity is often assumed. A time series $\{Y_t\}$ is weakly stationary if both the mean of Y_t and the covariance between Y_t and Y_{t-h} are time-invariant, where h is an arbitrary integer (Tsay, 2010).

More specifically, $\{ Y_t \}$ is weakly stationary if (a) $E(Y_t) = \mu$ (constant mean), and (b) $cov(Y_{t-h}, Y_t) = \gamma_h$, which only depends on h. In practice, suppose that we have observed T data points $\{Y_{t/t=1,\dots,T}\}$. Weak stationarity implies that the time plot of the data would show that the T values fluctuate with constant variation around a fixed level.

In the finance literature, it is common to assume that an asset return series is weakly stationary (Tsay, 2010). This assumption can be checked empirically provided that a sufficient number of historical returns are available. For example, one can divide the data into subsamples and check the consistency of the results obtained across the subsamples.

3.2.1.1. Unit Root Test

The assumption of stationarity is somewhat an unrealistic situation in most macroeconomic variables. Trivially, a non-stationary process arises when one of the conditions for stationarity does not hold. For non- stationary series, the effect of a shock never dies away. Moreover, a regression involving non-stationary variables leads to spurious regression; that is, one can regress completely unrelated series and find high R^2 (indicating how good one term is at predicting another) and the standard tests are not valid (Granger and Newbold, 1986). Thus, the first step for an appropriate analysis is to determine whether the series is stationary or not. The widely used unit-root tests are the Augmented Dickey Fuller (ADF) and Phillips Perron (PP) tests. The following discussion outlines the basic features of unit root tests (Hamilton, 1994).

Consider an AR (1) process:

$Y_t = \rho Y_{t-1} + \varepsilon_t$ .... [1]

where ρ is a parameter to be estimated and ε_t is assumed to be white noise. If $|\rho| \geq 1$, $\{Y_t\}$ is a non-stationary series and the variance of Y increases with time and approaches infinity. On the other hand, if $|\rho| < 1$, $\{Y_t\}$ is a stationary series. Thus, the hypothesis of (trend) stationarity can be evaluated by testing whether the value of ρ is strictly less than one. The hypotheses are:

$H_0: \rho = 1$ (the series is not stationary)

$H_1: \rho < 1$ (the series is stationary)

3.2.1.1.1. Augmented Dickey–Fuller(ADF) Unit Root Test

The standard Dickey-Fuller test is conducted in the following manner: from equation (1) we have:

$$Y_t - Y_{t-1} = (\rho - 1)Y_{t-1} + \varepsilon_t.$$

This can be rewritten as:

$\Delta Y_t = \pi Y_{t-1} + \varepsilon_t$ ....[2]

where $\pi = \rho - 1$. The null and alternative hypothesis may be rewritten as:

$H_0: \pi = 0$

$H_1: \pi < 0$ .... [3]

The test statistic is the conventional t-ratio for π:

$t_\pi = \hat{\pi} / se(\hat{\pi})$ ....[4]

where $\hat{\pi}$ is the OLS estimate of π and $se(\hat{\pi})$ is the standard error of $\hat{\pi}$.

Dickey and Fuller (1979) showed that, under the null hypothesis of a unit root, this statistic does not follow the conventional Student's t-distribution, and they derived asymptotic results and simulated critical values for various tests and sample sizes. MacKinnon (1996) implemented a much larger set of simulations than those tabulated by Dickey and Fuller. In addition, MacKinnon estimated response surfaces for the simulation results, permitting the calculation of Dickey-Fuller critical values and p-values for arbitrary sample sizes.

The simple Dickey-Fuller unit root test described above is valid only if the series is an AR (1) process. If the series is correlated at higher order lags, the assumption of white noise disturbances ε_t is violated. The ADF test constructs a parametric correction for higher-order correlation by assuming that the series follows an AR (p) process and adding lagged difference terms of the dependent variable Y_t to the right-hand side of the test regression:

$$\Delta Y_t = \pi Y_{t-1} + B_1 \Delta Y_{t-1} + B_2 \Delta Y_{t-2} + \cdots + B_p \Delta Y_{t-p} + U_t \qquad [5]$$

This augmented specification is then used to test for unit root using the t-ratio (equation 4). An important result obtained by Dickey-Fuller (1979) is that the asymptotic distribution of the t-statistic is independent of the number of lagged first differences included in the ADF regression. Moreover, while the assumption that Y_t follows an AR process may seem restrictive, Said and Dickey (1984) demonstrate that the ADF test is asymptotically valid in the presence of a moving average component, provided that sufficient lagged difference terms are included in the test regression.

3.2.1.1.2. Phillips-Perron (pp) Unit Root Test

Phillips and Perron (1988) developed a number of unit-root tests that have become popular in the analysis of financial time series. The Phillips-Perron (PP) unit-root tests differ from the ADF tests mainly in how they deal with serial correlation and heteroskedasticity in the errors. In particular, where the ADF tests use a parametric auto-regression to approximate the ARMA structure of the errors in the test regression, the PP tests ignore any serial correlation in the test regression. The test regression for the PP tests is:

$$\Delta Y_t = a_0 + a_2 t + \pi Y_{t-1} + \varepsilon_t \qquad [6]$$

where ε_t is stationary and may be heteroskedastic. The PP tests correct for any serial correlation and heteroskedasticity in the errors ε_t of the test regression by directly modifying the test statistics $t_{\pi=0}$ and $T_{\hat{\pi}}$ where

$$t_{\pi=0} = {}^{\hat{\rho}}/_{se(\hat{\rho})} \ and \ T_{\hat{\pi}} = {}^{\hat{\pi}}/_{se(\hat{\pi})}$$

where $\hat{\rho}$ and $\hat{\pi}$ are the OLS estimates of ρ and π, and $se(\hat{\rho})$ and $se(\hat{\pi})$ are the standard errors of $\hat{\rho}$ and $\hat{\pi}$, respectively.

These modified statistics, denoted Z_t and Z_π, are given by

$$Z_t = (\frac{\hat{\sigma}^2}{\hat{\tau}^2})^{1/2}. t_{\pi=0} - \frac{1}{2}\frac{(\hat{\tau}^2 - \hat{\sigma}^2)}{\hat{\tau}^2}.\frac{T.se(\hat{\pi})}{\hat{\sigma}^2}$$

$$Z_\pi = T_{\hat{\pi}} - \frac{1}{2}\left(\frac{T^2.se(\hat{\pi})}{\hat{\sigma}^2}\right)(\hat{\tau}^2 - \hat{\sigma}^2)$$

The terms $\hat{\tau}^2$ and $\hat{\sigma}^2$are consistent estimates of the variance parameters:

$$\sigma^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E[{\varepsilon_t}^2]$$

$$\tau^2 = \lim_{T\to\infty} \sum_{t=1}^{T} E[T^{-1}{S_T}^2]$$

where $S_T = \sum_{t=1}^{T} \varepsilon_t$. The sample variance of the least square residual $\hat{\varepsilon}_t$ is a consistent estimate of σ^2, and the Newey-West (1987) long-run variance estimate of ε_t is a consistent estimate of τ^2.

Under the null hypothesis that $\pi = 0$, the PP Z_t and Z_π statistics have the same asymptotic distributions as the ADF t-statistic. One advantage of the PP tests over the ADF tests is that the PP tests are robust to general forms of heteroskedasticity in the error term ε_t. Another advantage is that the user does not have to specify a lag length for the test regression.

3.2.2. Time domain approach and ARIMA model

The time domain approach is generally motivated by the presumption that correlation between adjacent points in time is best explained in terms of a dependence of the current value on past values. The time domain approach focuses on modeling some future value of a time series as a parametric function of the current and past values. One approach of time domain analysis is the Box-Jenkins (1970; see also Box et al., 1994) method which develops a systematic class of models called autoregressive integrated moving average (ARIMA) models. Conversely, the frequency domain approach assumes that the primary characteristics of interest in time series analyses are related to periodic or systematic sinusoidal variations found naturally in most data (Chatfield, 2000).

3.2.2.1. Autoregressive (AR) Process

Autoregressive models are based on the idea that the current value of a series, Y_t, can be explained as a function of the past values of itself. An autoregressive model of order p, abbreviated AR(p), is of the form:

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-p} + \varepsilon_t \quad [7]$$

where ε_t is a white noise series with mean zero and finite variance $\sigma^2{}_\varepsilon$. Using the backward shift operator B (defined as $B^p Y_t = Y_{t-p}$), equation (7) may be written as:

$$\emptyset(B) Y_t = \varepsilon_t \quad [8]$$

where $\emptyset(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p$ is a polynomial in B of order p. It can be shown that equation (8) has a unique causal stationary solution provided that the roots of $\emptyset(B) = 0$ lie outside the unit circle. If this is so, the solution can be expressed in the form

$$Y_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j} \quad [9]$$

for some constants φ_j such that $\sum_{j=0}^{\infty} \varphi_j{}^2 < \infty$. Typically an AR process is stationary provided that the roots of $\emptyset(B) = 0$ lie outside the unit circle (Chatfield, 2000).

A useful property of an AR(p) process is that the partial ACF is zero at all lags greater than p. This means that the sample partial ACF can be used to determine the order of an AR process (assuming the order is unknown as is usually the case) by looking for the lag value at which the sample partial ACF "cuts off" (meaning that it should be approximately zero, or at least not significantly different from zero, for higher lags) (Chatfield, 2000).

3.2.2.2. Moving average (MA) processes

A time series $\{Y_t\}$ is said to be a moving average process of order q (abbreviated $MA(q)$) if it is a weighted linear sum of the last q random shocks, that is,

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \quad [10]$$

where $\theta_1, \theta_2, \dots, \theta_q$ $(\theta_q \neq 0)$ are parameters and $\{\varepsilon_t\}$ is assumed to be white noise. Equation (10) may alternatively be written using the backward shift operator as:

$Y_t = \theta(B)\varepsilon_t$ .... [11]

where $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q$ is a polynomial in B of order q. If all roots of $\theta(B) = 0$ lie outside the unit circle, the MA process has an autoregressive representation of generally infinite order $\varepsilon_t = \sum_{j=0}^{\infty} \pi_j Y_{t-j}$ $with$ $\sum_{j=0}^{\infty} |\pi_j| < \infty$.

The order of such a model can be determined by analysis of the autocorrelation function, ACF, which cuts off after q lags and partial ACF that decays exponentially fast. Unlike the autoregressive process, the moving average process is stationary for any values of the parameters $\theta_1, \theta_2, \dots, \theta_q$ (Chatfield, 2000).

3.2.2.3. Autoregressive Moving Average (ARMA) Processes

In some applications, the AR or MA models become cumbersome because one may need a higher-order model with many parameters to adequately describe the dynamic structure of the series. To overcome this difficulty, the autoregressive moving-average (ARMA) models are introduced (Box, *et al* (1994)). Basically, an ARMA model combines the ideas of AR and MA models into compact form so that the number of parameters used is kept small. A mixed autoregressive moving average model with *p* autoregressive terms and *q* moving average terms (abbreviated $ARMA(p, q)$) can be written as:

$\emptyset(B)Y_t = \theta(B)\varepsilon_t$ ....[12]

where $\phi(B)$ and $\theta(B)$ are polynomials in B of finite order *p* and *q*, respectively. Equation (12) has a unique causal stationary solution provided that the roots of $\phi(B) = 0$ lie outside the unit circle. The process is invertible provided that the roots of $\theta(B) = 0$ lie outside the unit circle.

The importance of $ARMA$ process is that many real world time series may be approximated in a more parsimonious way by a mixed $ARMA$ model rather than by an AR or MA process.

3.2.2.4. Autoregressive Integrated Moving Average (ARIMA) Processes

In practice, many time series are non-stationary and we cannot apply stationary AR, MA or ARMA processes directly. One possible way of handling non-stationary series is to apply differencing so as to make them stationary. Differencing of a series can transform a non-stationary series to a stationary series. One advantage of differencing over de-trending to remove trend is that no parameters are estimated in the differencing operation. One disadvantage, however, is that differencing does not yield an estimate of the stationary process y_t . If an estimate of y_t is essential, then de-trending may be more appropriate. If the goal is to make the data stationary, then differencing may be more appropriate.

The first difference operator ∇ is defined by:

$\nabla y_t = y_t - y_{t-1} \quad => \quad \nabla y_t = (1-B)y_t$

so that ∇ can be expressed in terms of the backward shift operator B. In general, higher order differencing can be expressed as:

$$\nabla^n y_t = (1-B)^n y_t$$

A series that is stationary without any differencing is said to be integrated of order zero (denoted by I (0)), and a series which is stationary after being differenced d times is said to be integrated of order d (denoted by I (d)).

If the original data series is differenced d times before fitting an $ARMA(p,q)$ process, then the model for the original undifferenced series is said to be an $ARIMA(p,d,q)$ process where the letter 'I' in the acronym stands for integrated and d denotes the number of differences taken.

Mathematically, equation (12) is expressed in the following way:

$\emptyset(B)\nabla^d Y_t = \theta(B)\varepsilon_t$ ....[13]

where $\nabla = 1 - B$ and d is the order of integration (Wei, 2006).

3.2.3. Building ARIMA Models

There are a few basic steps to fitting ARIMA models to time series data. These steps involve plotting the data; possibly transform the data, identifying the appropriate model, parameter estimation, diagnostics and forecasting. The original Box-Jenkins modeling procedure involves an iterative three-stage process of model selection, parameter estimation and diagnostic checking. But, further explanations of the process by Makridakis et al. (1998) often add a preliminary stage of data preparation and a final stage of model application (forecasting and evaluation of the forecast).

3.2.3.1. Model Identification

The first step in ARIMA modeling is identification of the appropriate model. ARIMA procedures are mainly based on the analysis of the autocorrelation function (ACF) and the partial autocorrelation function (PACF), which are essential diagnostic instruments to identify the dependence structure of the series.

The autocorrelation of a series $\{Y_t\}$ at lag m is estimated by:

$$\hat{\boldsymbol{\rho}}(\boldsymbol{m}) = \frac{\sum_{t=m+1}^{T}(Y_t-\bar{Y})(Y_{t-m}-\bar{Y})}{\sum_{t=1}^{T}(Y_t-\bar{Y})^2} \quad \text{......[14]}$$

where $\bar{Y}$ is the sample mean of Y_t. This is the correlation coefficient for values of the series m periods apart.

The partial autocorrelation function at lag m is the correlation between Y_t and Y_{t-m} after removing the effect of the intervening variables $Y_{t-1}, Y_{t-2}, \dots, Y_{t-m+1}$. The partial autocorrelation function, donated by π_m, is calculated by iteration method (Shumway and Stoffer, 2010) as:

$$\pi_0 = 1$$

$$\pi_1 = \rho_1$$

$$\pi_m = \frac{\rho_m - \sum_{j=1}^{m-1}\pi_{m-1,j}\,\rho_{m-j}}{1-\sum_{j=1}^{m-1}\pi_{m-1,j}\,\rho_{m-j}} \qquad m > 1 \quad \text{......[15]}$$

where ρ_m is the autocorrelation at lag m and $\pi_{m,j}$ is given by the recursive equation:

$$\pi_{m,j} = \pi_{m-1,j} - \pi_m \pi_{m-1,m-j} \quad , \quad j = 1, 2, \dots, m-1$$

Table 3.1. Behavior of the ACF and PACF for ARMA Models

Model	ACF	PACF
$AR(p)$ $Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-p} + \varepsilon_t$	Infinite: damps out	Finite: cut off after lag p
$MA(q)$ $Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$	Finite: cut off after lag q	Infinite: damps out
$ARMA(p,q)$ $Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$	Infinite: damps out	Infinite: damps out

Model selection criteria

The idea here is to fit all $ARMA(p,q)$ models with orders $p \leq p_{max}$ and $q \leq q_{max}$ and choose the values of p and q which minimize some model selection criteria.

Akaike's Information Criterion (AIC)

Akaike's Information Criterion (AIC) is defined as:

$AIC = -2 \log L_k + 2k$ ....[16]

where L_k is the maximized log-likelihood and k = 1+p+q, is the number of parameters in the model. The best model is the one which minimizes the AIC, and there is no requirement for the models to be nested.

Bayesian Information Criterion (BIC)

The BIC was introduced by Schwarz (1978), and is defined as:

$BIC = -2 \log L_k + k \log T$ .... [17]

Various simulation studies have tended to verify that BIC does well at getting the correct order in large samples, whereas AICc tends to be superior in smaller samples where the relative number of parameters is large (McQuarrie and Tsai (1998)) as pointed out by Shumway and Stoffer (2010).

3.2.3.2. Parameter Estimation

Once the degree of differencing has been determined, we proceed to select the autoregressive and moving average orders by examining the sample autocorrelations and sample partial autocorrelations. After choosing the most appropriate model, the model parameters are estimated using several estimation procedures. In general, nonlinear estimation procedure is used to estimate ARIMA model parameters to maximize the likelihood function with respect to the parameters. That is, the maximum likelihood estimation method is used to test the parameters of the ARIMA model.

3.2.3.3. Diagnostic Test

Once a model has been identified and the parameters estimated, diagnostic checks are then applied to the fitted model. These include tests of serial correlation and normality of the residuals.

Ljung-Box test of serial correlation

Before an estimated model is used for statistical inference, the residuals must be examined for the presence of serial correlation. The Q-statistic is often used as a test of whether the series is white noise. The Q-statistic at lag m is a test statistic for the null hypothesis that there is no autocorrelation up to order m and is computed as:

$$Q = T(T+2)\sum_{j=1}^{m}\frac{{\hat{\rho}_j}^2}{T-j} \quad \text{[18]}$$

where $\hat{\rho}_j$ is the j-th lag autocorrelation and T is the number of observations. If the series is not based upon results of ARIMA estimation, then under the null hypothesis, Q is asymptotically distributed as chi-square with degrees of freedom equal to $(m-k)$, where k denotes the number of parameters. If the series represents the residuals from ARIMA estimation, the appropriate degrees of freedom should be adjusted to represent the number of autocorrelations less the number of AR and MA terms.

Jarque-Bera test of normality

One of the most commonly applied tests for normality is the Jarque-Bera (JB) test. JB uses the property of a normally distributed random variable that the entire distribution is characterized by the first two moments - the mean and the variance. The standardized third and fourth moments of a distribution are known as its skewness and kurtosis. Skewness measures the extent to which a distribution is not symmetric about its mean value, and kurtosis measures the peakedness or flatness of the distribution of the series.

Jarque and Bera (1981) formalize these ideas by testing whether the coefficients of skewness and excess kurtosis are jointly zero. If $\hat{\mu}_3$ and $\hat{\mu}_4$ are estimates of the third and the fourth central moments, respectively, $\overline{y}$ is the sample mean and $\hat{\sigma}^2$ is an estimate of the second central moment, the sample skewness and kurtosis coefficients are computed as:

$$\hat{b}_1 = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^3}{\{\frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^2\}^{3/2}} \quad \text{and} \quad \hat{b}_2 = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^4}{\{\frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^2\}^2}$$

The JB test statistic is defined as:

$$JB = T\,[\frac{\hat{b}_1}{6} + \frac{\hat{b}_2}{24}] \quad \text{[19]}$$

where T is the sample size. Under the null hypothesis this test statistic asymptotically follows a chi-square distribution with two degrees of freedom. Thus, large values of this test statistic relative to the quantiles from the chi-square distribution with two degrees of freedom lead to rejection of the null hypothesis (Brooks, 2002).

3.2.3.4. Forecasting

The last step in ARIMA modeling is forecasting. One of the reasons for the popularity of the ARIMA modeling is its success in forecasting.

Minimum mean square error forecasts

The general $ARIMA(p, d, q)$ model can be written as:

$$\emptyset(B)(1-B)^d Y_t = \theta(B)\varepsilon_t \quad \text{[20]}$$

where $\emptyset(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p$ is a stationary AR operator and $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$ is an invertible MA operator. Suppose at time t we have the observations $Y_t, Y_{t-1}, \ldots$ and wish to forecast the h-step ahead future value Y_{t+h} as a linear combination of the observations $Y_t, Y_{t-1}, \ldots$. (denoted by $\hat{Y}_t(h),\ h \geq 1$). Let F_t be the information set at time t. The minimum mean square error forecast $\hat{Y}_t(h)$ of Y_{t+h} is given by its conditional expectation. That is,

$\hat{Y}_t(h) = E(Y_{t+h}|F_t),\ h \geq 1$ ....[21]

For t is replaced by $t + h$, $h \geq 1$ we have

$$Y_{t+h} = \emptyset_1 Y_{t+h-1} + \emptyset_2 Y_{t+h-2} + \cdots + \emptyset_{p+d} Y_{t+h-p-d} + \varepsilon_{t+h} + \theta_1 \varepsilon_{t+h-1} + \cdots + \theta_q \varepsilon_{t+h-q}$$

Then, taking the conditional expectation given the information set at time t, we obtain

$$\hat{Y}_t(h) = \emptyset_1 \hat{Y}_t(h-1) + \emptyset_2 \hat{Y}_t(h-2) + \cdots + \emptyset_{p+d} \hat{Y}_t(h-p-d) + \hat{\varepsilon}_t(h) + \theta_1 \hat{\varepsilon}_{t-1}(h-1) + \cdots + \theta_q \hat{\varepsilon}_{t-q}(h-q).$$

This is the minimum mean square error forecast value of Y_{t+h}.

3.2.3.4.1. Forecasting accuracy measures

In most forecasting situations, accuracy is treated as the overriding criterion for selecting a forecasting method. In many instances, the word “accuracy” refers to the goodness of fit, which intern refers to how well the forecasting model is able to reproduce the data that are already known. To the consumer of forecasts, it is the accuracy of the future forecast that is most important.

If Y_{t+h}, h=1, 2, . . ., k, is the actual observation for the period $(t + h)$ and $\hat{Y}_t(h)$ is the forecast of Y_{t+h}, then the forecast error is defined as:

$\hat{\varepsilon}_t(h) = Y_{t+h} - \hat{Y}_t(h)$ .... [22]

If there are observations and forecasts for T time periods, then there will be T error terms, and the following standard statistical measures can be defined:

$$Mean\ Error\ (ME) = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t(h) \quad \text{.......[23]}$$

$$Mean\ Absolute\ Error\ (MAE) = \frac{1}{T}\sum_{t=1}^{T} |\hat{\varepsilon}_t(h)| \quad \text{.......[24]}$$

$$Mean\ Squared\ Error\ (MSE) = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t^{\ 2}(h) \quad \text{.......[25]}$$

To make comparisons we need to work with relative or percentage error measures. First let us define a relative or percentage error as

$$PE_t = \left(\frac{Y_{t+h} - \hat{Y}_t(h)}{Y_{t+h}}\right) * \ 100 \quad \text{.......[26]}$$

Then the following two relative measures are frequently used:

$$Mean\ Percentage\ Error\ (MPE) = \frac{1}{T}\sum_{t=1}^{T} PE_t \quad \text{.......[27]}$$

$$Mean\ percentage\ Absolute\ Error\ (MPAE) = \frac{1}{T}\sum_{t=1}^{T} |PE_t| \quad \text{.......[28]}$$

Equation [26] can be used to compute the percentage error for any time period. These can be averaged as in equation [27] to give the mean percentage error. However, as with the ME, the MPE is likely to be small since positive and negative PEs tend to offset one another. Hence the MAPE is defined using absolute values of PE in equation [28].

Alternatively, Theil's U statistic can be used as a measure of forecasting accuracy. Like MAPE statistic, high values suggest poor performance in the forecast. Theil's U can be estimated as:

$$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_{t+h} - \hat{Y}_t(h))^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\hat{Y}_t^{\ 2}(h)} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}Y_{t+h}^{\ 2}}} \quad \text{.......[29]}$$

The scaling of U is such that it will always lie between 0 and 1. If U = 0, $Y_{t+h} = \hat{Y}_t(h)$ for all forecasts and there is a perfect fit; if U = 1 the predictive performance is not good.

3.2.4. Introduction to ARCH and GARCH model

The analysis of financial data has received considerable attention in the literature over the last many years. Several models have been suggested for capturing special features of financial data, and most of these models have the property that the conditional variance (or the conditional scaling) depends on the past. Among the best well known and most often used are the autoregressive conditionally heteroscedastic (ARCH) and generalized autoregressive conditionally heteroscedastic (GARCH) models.

Most problems in finance have motivated the study of volatility, or variability, of a time series. Unlike ARMA models which assume a constant variance, models such as the ARCH model, first introduced by Engle (1982), were developed to model changes in volatility. These models were later extended to generalized ARCH (GARCH) models by Bollerslev (1986). In GARCH models, the variance of a time series is considered as variant (i.e., the error term have zero mean and non-constant variance). Here the error term is assumed to be serially uncorrelated and can be modeled by an autoregressive (AR) process.

If y_t is the export price of sesame at time t, then the return or relative gain, r_t, of the export price of sesame at time t is:

$$r_t = \frac{y_t - y_{t-1}}{y_{t-1}} \quad \text{.......[30]}$$

The basic idea behind volatility study is that the series $\{r_t\}$ is either serially uncorrelated or has lower order serial correlations, but is a dependent series. It is the study of r_t that is the focus of ARCH, GARCH, and other volatility models (Taylor, 1986). Typically for financial series, the return r_t does not have a constant variance, and highly volatile periods tend to be clustered together.

To put the volatility models in proper perspective, it is informative to consider the conditional mean and variance of r_t $given$ the information set available at time $(t-1)$, F_{t-1}:

$$\mu_t = E(r_t|F_{t-1}), \qquad var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}] = {\sigma_t}^2 \quad \text{.......[31]}$$

The equation for μ_t (equation 31) should be simple, and we assume that r_t follows a simple time series model such as a stationary ARMA(p, q) model with some explanatory variables. In other words, we entertain the model

$$r_t = \mu_t + \varepsilon_t, \qquad \mu_t = \emptyset_0 + \sum_{i=1}^{k} \beta_i x_{it} + \sum_{i=1}^{p} \emptyset_i r_{t-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \quad \text{.........................} [32]$$

where k, p, and q are non-negative integers, x_{it} are explanatory variables and $\emptyset_i,\ \theta_i\ and\ \beta_i$ are coefficients of AR term, MA term and explanatory variables, respectively. The model for μ_t in equation (32) is referred to as the *mean* equation for r_t.

3.2.4.1. ARCH Model

One of the simplest volatility models is the Autoregressive Conditional Heteroscedasticity (ARCH) model suggested by Engle (1982). ARCH model is the first attempt to capture the characteristics of financial time series without the assumption of constant variance. It is unlikely in financial time series that the variance of the error terms will be constant over time. Therefore, allowing for conditional heteroscedasticity in return series analysis is reasonable.

The $ARCH\ (m)$ model proposed by Engle (1982) formulates volatility as follows:

$$\varepsilon_t = \sigma_t v_t \quad \text{..} [33]$$

$$\sigma_t{}^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j \varepsilon^2{}_{t-j} \quad \text{..} [34]$$

we impose the non-negativity constraints $\alpha_0 > 0$ and $\alpha_j \geq 0, j = 1, 2, \dots, m$. The time varying volatility is captured by allowing volatility to depend on the lagged values of the squared innovation terms $\varepsilon^2{}_{t-j}$ and m is chosen such that the residuals of the variance equation are white noise. That is, $\{v_t\}$ is a sequence of independently and identically distributed random variables each with zero mean and unit variance.

While the ARCH model resembles a regression model, the fact that the conditional variance is not directly observable (and hence is called a latent variable) introduces some subtlety in the use of ARCH models in data analysis. Consider the ARCH(1) model:

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon^2{}_{t-1} \quad \text{..} [35]$$

Define π_t as: $\pi_t = {\varepsilon_t}^2 - {\sigma_t}^2$. It can be verified that $\{\pi_t\}$ is a serially uncorrelated series with zero mean. Moreover, π_t is uncorrelated with past returns. Equation (35) can be re-written as:

$${\varepsilon_t}^2 = \alpha_0 + \alpha_1 {\varepsilon_{t-1}}^2 + \pi_t \quad \ldots\ldots\ldots\ldots \text{[36]}$$

Thus, the squared return series satisfies an AR(1) model under the assumption of an ARCH(1) model for the return series. Based on this useful observation, an ARCH (1) model may be specified as an AR (1) specification for the squared returns (Cryer and Chan, 2008).

Besides its value in terms of data analysis, the deduced AR (1) model for the squared returns can be exploited to gain theoretical insights on the parameterization of the ARCH model. For example, because the squared returns must be nonnegative, it makes sense to always restrict the parameters α_0 and α_1 to be nonnegative. In addition, if the return series is stationary with variance σ^2 , then taking unconditional expectation on both sides of Equation (36) yields:

$$E({\varepsilon^2}_t) = E(\alpha_0 + \alpha_1 {\varepsilon_{t-1}}^2 + \pi_t)$$

$$\sigma^2 = \alpha_0 + \alpha_1 \sigma^2$$

$$\sigma^2 = \frac{\alpha_0}{1-\alpha_1} \quad \ldots\ldots\ldots\ldots \text{[37]}$$

That is, $\sigma^2 = \frac{\alpha_0}{1-\alpha_1}$, and hence, $0 \leq \alpha_1 < 1$. Indeed, it can be shown (Ling and McAleer, 2002) that the condition $0 \leq \alpha_1 < 1$ is necessary and sufficient for the (weak) stationarity of the ARCH (1) model.

Recall that weak stationarity of a process requires that the mean of the process is constant and the covariance of the process at any two epochs is finite and identical whenever the lags of the two epochs are the same. In particular, the variance is constant for a weakly stationary process. It is interesting to observe that weak stationarity does not preclude the possibility of a non-constant conditional variance process, as is the case for the ARCH(1) model. It can be checked that the ARCH (1) process is white noise. Hence, it is an example of a white noise that admits a non-constant conditional variance process.

A stationary ARCH(1) model need not have finite fourth moments. The existence of finite higher moments will further restrict the parameter range - a feature also shared by higher order analogues of the ARCH model and its variants. In addition the excess kurtosis of a stationary ARCH (1) process is greater than zero. This verifies our earlier statement that an ARCH(1) process has fat tails even with normal innovations. In other words, the fat tail is a result of the volatility clustering.

The ARCH model is simple. However, many parameters are required to estimate the volatility of price returns. The problem of parsimony and the violation of non-negativity constraints led to a more general framework proposed by Bollerslev (1986) and Taylor (1986).

3.2.4.2. $GARCH(m, r)$ model

Bollerslev (1986) extended Engle's original work by developing a technique that allows the conditional variance to be an ARMA process. Specifically the $GARCH(m, r)$ process is defined as:

$$\varepsilon_t = v_t \sigma_t$$

$$\sigma_t{}^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j \varepsilon^2{}_{t-j} + \sum_{j=1}^{r} \beta_j \sigma^2{}_{t-j} \quad \text{[38]}$$

where the restrictions $\alpha_0 > 0, \alpha_j \geq 0, \beta_j \geq 0$ ensure that the variance is always greater than zero and the restriction $(\sum_{j=1}^{m} \alpha_j + \sum_{j=1}^{r} \beta_j < 1)$ is necessary and sufficient condition for the stability of the conditional variance equation (Cryer and Chan, 2008). $\{v_t\}$ is i.i.d. random variable that is independent of past realization ε_{t-j}, and the conditional and unconditional means of ε_t are equal to zero.

The parameter α_0 is generally interpreted as long-term volatility to which the system converges. On the other hand, the ARCH term $\alpha_j \varepsilon^2{}_{t-j}$ reflects the effect of lagged shocks on the volatility at time t. Moreover, the GARCH term $\beta_j \sigma^2{}_{t-j}$ measures the effect of past-expected variance on the current volatility.

The $GARCH\ (m, r)$ allows both autoregressive and moving average components in the heteroscedastic variance equation. The benefits of the GARCH model should be clear; the GARCH model is a more parsimonious model of the conditional variance than a high-order ARCH model that is much easier to identify and estimate. Moreover, to ensure that the conditional variance is finite and stationary, $\sum_{j=1}^{m} \alpha_j + \sum_{j=1}^{r} \beta_j < 1$. Clearly, the more parsimonious model will entail fewer coefficient restrictions.

GARCH (1,1) Model

Although GARCH models with higher order allow for more complex autocorrelation structure, GARCH(1,1) is more commonly used because of its simplicity. The $GARCH(1,1)$ model specification is:

$$y_t = \mu + \varepsilon_t$$

$$\varepsilon_t = v_t \sigma_t$$

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1} \quad \text{...[39]}$$

where $v_t^{'}s$ are i.i.d. random variables with $E(v_t) = 0 \; and \; var(v_t) = 1$. We impose the restriction $\alpha_0 > 0, \alpha_1 \geq 0 \; and \; \beta_1 \geq 0$. Provided that $\alpha_1 + \beta_1 < 1$, conditional variance is stationary.

All autocorrelations of squared returns in GARCH(1,1) model are positive with an exponential decay. If $\alpha_1 + \beta_1$ is close to one, the decay is slow. Thus, $\alpha_1 + \beta_1$ can be labeled as the "persistence" parameter of the GARCH(1,1) model. The closer the persistence parameter is to one, the longer time the periods of volatility clustering will last. In addition, the larger α_1 relative to β_1, is the higher the immediate impact of lagged squared returns on volatility.

3.2.4.3. Exponential GARCH (EGARCH) model

To overcome some weaknesses of the GARCH model in handling financial time series, Nelson (1991) proposes the exponential GARCH (EGARCH) model. In particular, it allows for asymmetric effects between positive and negative returns.

An EGARCH(*m, r*) model can be written as:

$$\varepsilon_t = v_t \sigma_t$$

$$log\sigma_t{}^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j \left| v_{t-j} \right| + \sum_{j=1}^{m} \tau_j v_{t-j} + \sum_{i=1}^{r} \beta_i log\sigma^2{}_{t-i} \quad \text{...[40]}$$

The parameters α_0, α_j, β_i $and\, \tau_j$ can be estimated by using the maximum likelihood method. Note that the parameters τ_j indicates the leverage effect.

In contrast to the GARCH models, the EGARCH models do not have any restrictions on the parameters in the model. The EGARCH model always produces a positive conditional variance independently of the signs of the estimated parameters in the model and no restrictions are needed. This is preferable when the restrictions in the GARCH model sometimes create problems when estimated parameters violate the constraints.

3.2.4.4. Conditional Variance with Exogenous Variables model

In this section, we present a model that captures the influence of any exogenous variable on conditional variances. The GARCH-X model proposed by Hwang and Satchell (2005) for modeling aggregate stock market return volatility includes a measure of the lagged cross-sectional return variation as an explanatory variable in the GARCH conditional variance equation. To better model and forecast the volatility of economic and financial time series, empirical researchers and practitioners often include exogenous regressors in the specification of volatility dynamics. One particularly popular model within this setting is the so-called GARCH-X model where the basic GARCH specification of Bollerslev (1986) is augmented by adding exogenous regressors to the volatility equation:

$$y_t = \sigma_t(\vartheta)\varepsilon_t \quad \text{[41]}$$

where ε_t is the error process while ${\sigma_t}^2(\vartheta)$ is the volatility process given by:

$$ {\sigma_t}^2(\vartheta) = \alpha_0 + \alpha {y_{t-1}}^2 + \beta {\sigma_{t-1}}^2 + \gamma {x^2}_{t-1} \quad \text{[42]}$$

for some observed covariate x_t which is squared to ensure that ${\sigma_t}^2(\vartheta) > 0$ and where $\vartheta = (\alpha_0, \theta')'$, with $\theta = (\alpha, \beta, \gamma)'$, is a vector of parameters. The inclusion of the additional regressors x_t often helps explaining the volatilities of financial time series and tends to lead to better in-sample fit and out-of sample forecasting performance. Choices of covariates found in empirical studies using the GARCH with exogenous variable model span a wide range of various economic and financial indicators.

Depending on the formulation of Equation (42), different parameter restrictions are imposed to ensure that the non-negativity condition of variance is always satisfied (Hwang and Satchell, 2005). Alternatively, the model can be estimated by constrained maximum likelihood, restricting the parameter space so that the conditional volatility process is always positive (De Goeij and Marquering, 2004).

3.2.4.5. Testing for ARCH effects

Before estimating a full ARCH model for financial time series, it is usually a good practice to test for the presence of ARCH effect in the residuals. If there are no ARCH effects in the residuals, then the ARCH model is unnecessary and misspecified.

The Box-Jenkins (1976) approach is based on the assumption that the residuals are homoscedastic (remain constant over time) for ARMA or ARIMA models. Thus, the presence of ARCH effect (whether or not volatility varies over time) has to be tested in series through the squared residuals of the series (Tsay, 2010).

Lagrange multiplier (LM) test

This test was suggested by Engle (1982) and used to test significance of serial correlation in the squared residuals for the first q lags. This particular heteroskedasticity specification was motivated by the observation that in many financial time series, the magnitude of residuals appeared to be related to the magnitude of recent residuals. ARCH in itself does not invalidate standard least square inference. However, ignoring ARCH effects may result in loss of efficiency.

The null hypothesis of the test is that there is no serial correlation in the residuals up to a specified order, say q. The test statistic is $LM = T * R^2$, where R^2 is obtained from the regression of $\hat{\varepsilon}^2{}_t$ on $\hat{\varepsilon}^2{}_{t-i}, i = 1, \dots, q$ and T is the number of observations. This test statistic under the null hypothesis follows the chi-square distribution with q degree of freedom.

3.2.4.6. Parameter Estimation

Under the presence of ARCH effects, OLS estimation is not efficient since the conditional variance of the error is not constant and volatility models used in financial econometrics are non-linear in conditional variance. In addition, OLS involves minimizing the sum of squares of residuals which depends on the mean equation not on the conditional variance. Therefore, the maximum likelihood estimation method can be employed to estimate the parameters of GARCH family models. In maximum likelihood estimation, the distributional assumption on the residuals is the core point. In this study, normal, student-t and the generalized error distribution (GED) were considered to estimate the parameters. The appropriate distribution for the residuals can be identified based on in-sample forecast error statistics under specified error distributions and the final analysis may proceed based on the selected distribution for residuals in the mean equation.

Maximum likelihood estimation can be employed to find parameter values for both linear and non-linear models. Suppose that values of $\{y_t\}$ are drawn from a normal distribution having a mean of zero and conditional variance ${\sigma_t}^2$. The likelihood function of ε_t is given by:

$$l_t = \left(\frac{1}{\sqrt{2\pi{\sigma_t}^2}}\right) e^{-\frac{{\varepsilon_t}^2}{2{\sigma_t}^2}}$$

Since the realization $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T)$ of ε_t are independent, the likelihood of the joint realizations of $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T$ is the product of the individual likelihoods. Hence, if all have the same variance, the likelihood of the joint realization is:

$$L = \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi{\sigma_t}^2}}\right) e^{-\frac{{\varepsilon_t}^2}{2{\sigma_t}^2}}$$

The log likelihood function is:

$$ln\, L = -\frac{T}{2}\, ln\,(2\pi) - \frac{1}{2}\sum_{t=1}^{T} ln\,({\sigma_t}^2) - \frac{1}{2}\sum_{t=1}^{T} \frac{{\varepsilon_t}^2}{{\sigma_t}^2}$$

Now, suppose that $\varepsilon_t = y_t - \mu$ and, for GARCH(1,1), the conditional variance is ${\sigma_t}^2 = \alpha_0 + \alpha_1 {\varepsilon^2}_{t-1} + \beta_1 {\sigma^2}_{t-1}$. Substituting for ${\sigma_t}^2$ and y_t yields:

$$\ln L = -\frac{T-1}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=2}^{T}\ln[\alpha_0 + \alpha_1 {\varepsilon^2}_{t-1} + \beta_1 {\sigma^2}_{t-1}] - \frac{1}{2}\sum_{t=2}^{T}\frac{(y_t-\mu)^2}{\alpha_0+\alpha_1 {\varepsilon^2}_{t-1}+\beta_1 {\sigma^2}_{t-1}}.$$

Once we substitute $(y_{t-1} - \mu)^2$ for $\varepsilon^2{}_{t-1}$, it is possible to maximize $ln\, L$ with respect to μ, α_0, α_1 and β_1. However, there are no closed-form solutions to the first order conditions for a maximum.

Non-normal distributional case

We can assume that ε_t follows a student's t-distribution or a Generalized Error Distribution (GED), both of which can have fat tails. For example, the density function for the GED is given by:

$$f(\varepsilon_t) = \frac{v \exp\left(-\frac{1}{2}\left|\frac{\varepsilon_t}{\tau\sigma_t}\right|^v\right) \cdot \frac{1}{\sigma_t}}{2^{(1+\frac{1}{v})}\Gamma(1/v)\tau}, \qquad v > 0$$

where

$$\tau = \left[\frac{2^{-(2/v)}\Gamma(1/v)}{\Gamma(3/v)}\right]^{1/2}$$

and $\Gamma(.)$ is the Gamma function. The parameter v is a measure of fatness of the tails. The Gaussian distribution is a special case of the GED when v = 2, and when v < 2, the distribution has fatter tails than a Gaussian distribution.

The likelihood of the joint realization is the product of the individual likelihoods of $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T$ since the realization of ε_t are independent. That is;

$$L = \prod_{t=1}^{T} \frac{v \exp\left(-\frac{1}{2}\left|\frac{\varepsilon_t}{\tau\sigma_t}\right|^v\right) \cdot \frac{1}{\sigma_t}}{2^{(1+\frac{1}{v})}\Gamma(1/v)\tau}$$

The log-likelihood function becomes:

$$ln\, L = T\left(ln\left(\frac{v}{\tau}\right) - \left(1 + \frac{1}{v}\right) ln(2) - ln\left(\Gamma\left(\frac{1}{v}\right)\right)\right) - \frac{1}{2}\sum_{t=1}^{T} ln\sigma_t{}^2 - \frac{1}{2}\sum_{t=1}^{T}\left|\frac{\varepsilon_t}{\tau\sigma_t}\right|^v$$

For the case of t-distribution the density function is given by:

$$f(\varepsilon_t) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\,\Gamma(^{v}/_{2})} \cdot \left(1 + \frac{{\varepsilon_t}^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}$$

where v is the number of degrees of freedom (v > 2). Like the normal distribution, the t-distribution is symmetric. The likelihood function of the joint realization is:

$$L = \prod_{t=1}^{n} \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\,\Gamma(^{v}/_{2})(1 + \frac{{\varepsilon_t}^2}{v})^{-(\frac{v+1}{2})}}$$

The log-likelihood function becomes

$$\ln L = n[\ln\left(\Gamma\left(\frac{v+1}{2}\right)\right) - \ln(\sqrt{v\pi}) - \ln\left(\Gamma(^{v}/_{2})\right)] + \left(\frac{v+1}{2}\right)\ln(\sum_{t=1}^{T}(1 + \frac{{\varepsilon_t}^2}{v}))$$

Now, suppose that $\varepsilon_t = y_t - \mu$ and for GARCH(1,1) the conditional variance is ${\sigma_t}^2 = \alpha_0 + \alpha_1{\varepsilon_{t-1}}^2 + \beta_1{\sigma_{t-1}}^2$. Then, substituting for ${\sigma_t}^2$ and y_t, it is possible to maximize $ln\, L$ with respect to μ, α_0, α_1 and β_1.

3.2.4.7. Diagnostic checking

When a model has been fitted to a time series, it is advisable to check that the model really does provide an adequate description of the data. As with most statistical models, the goodness of fit of the ARCH-GARCH model is based on residuals and more specifically on the standardized residuals.

The followings are the model adequacy checking methods that are used for this study:

- ❖ The ACF and PACF of the standardized residuals: If the model is adequate the ACF and PACF of squared standardized residuals should be indicative of a white noise process.
- ❖ The standardized residuals should be identically and independently distributed standard normal even if student-t and generalized exponential distribution (GED) are assumed (Tsay, 2010). This can be checked through the Jarque-Bera test.

- The Ljung-Box test is one of the most widely used lack-of-fit tests, that is, a test for the appropriateness of the fitted model. It was developed by Box and Pierce (1970) and modified by McLeod and Li (1983). The Ljung-Box test statistic uses the Q-statistic to test whether there is a group of significant k autocorrelations, to test whether the mean model is appropriately specified and to test for remaining ARCH effects under the null hypothesis that there is no autocorrelation among k lags of standardized residuals and squared standardized residuals for mean and GARCH specification, respectively. Thus, if the statistic Q at all lags is non-significant, it indicates the absence of autocorrelation in the residuals and squared residuals, and this is evidence that the model selected fits the data well.

3.2.4.8. Forecasting by GARCH Family Model

Conditional variance forecasts from GARCH family models are obtained with similar approach to forecasts from ARMA models by iterating with the conditional expectations operator. In other words, when the estimation of the unknown parameters is done, estimates of the standard deviation series can be calculated recursively via the definition of the conditional variance for the $GARCHX(m, r)$ family process.

Suppose we are at time index t and interested in forecasting ${\sigma^2}_{t+l}$, where $l \geq 1$. Let ${\sigma_t}^2(l)$ be the forecast of ${\sigma^2}_{t+l}$ and F_t be the collection of information available at the forecast origin t. Then, the forecast ${\sigma_t}^2(l)$ is chosen such that:

$E\{[{\sigma^2}_{t+l} - {\sigma_t}^2(l)\,]^2|F_t\} \leq \min_g E[({\sigma^2}_{t+l} - g)^2|F_t]$ ....[43]

where g is a function of the information available at time t (inclusive), that is, a function of F_t. We refer to ${\sigma_t}^2(l)$ as the l-step-ahead-forecast of ${\sigma_t}^2$ at the forecast origin t.

For illustration consider the GARCH(1,1) model of equation (43) and assume the forecast origin is t. For 1-step-ahead-forecast, we have

$$ {\sigma_t}^2(1) = \alpha_0 + \alpha_1 {\varepsilon_t}^2 + \beta_1 {\sigma_t}^2 $$

The 2-step-ahead-forecast at the forecast origin t is:

$$\sigma_t^{\,2}(2) = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^{\,2}(1)$$

In general, we have

$$\sigma_t^{\,2}(l) = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^{\,2}(l-1), l > 1$$

3.2.4.8.1. Forecasting evaluation and Accuracy Criteria

The accuracy of a forecasting model depends on how close the forecast values ($\sigma_t^{\,2}(l), for\ l \geq 1$) are to the actual values ($\sigma^2{}_{t+l}$). In practice, the difference between the actual and the forecast values is defined as the forecast error,

$$e_t(l) = \sigma^2{}_{t+l} - \sigma_t^{\,2}(l)$$

If the model is doing a good job in forecasting the actual data, the forecast error will be relatively small. Because of the mathematical entity in the definition of forecast error, forecasters usually measure a model's accuracy by looking at various quantitative measures. Most frequently, these measures employ the absolute values of the forecast errors $|e_t(l)|$ or the square of the forecast errors $e^2{}_t(l)$. As a general rule, the smaller the sum of the absolute forecast errors $\sum_{t=1}^{T}|e_t(l)|$ or the sum of the squared forecast errors $\sum_{t=1}^{T} e^2{}_t(l)$, the more accurate the fit of the model.

The following statistical summary measures of a model's forecast accuracy are defined using the absolute errors:

- ❖ The mean absolute error

$$MAE = \frac{\sum_{t=1}^{T}|e_t(l)|}{T}$$

- ❖ The mean absolute percentage error

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\frac{|e_t(l)|}{\sigma^2{}_{t+l}} * 100$$

The following statistical summary measures of a model's forecast accuracy are defined using the squared errors:

- The mean square error

$$MSE = \frac{\sum_{t=1}^{T} e^2{}_t(l)}{T}$$

- The root mean square error

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T} e^2{}_t(l)}{T}}$$

Theil's inequality coefficient is another statistical measure of forecast accuracy. Theil's, U can be estimated as:

$$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(e_t(l))^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma_t{}^2(l))^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma^2{}_{t+l})^2}}$$

If U=0, $\sigma_t{}^2(l) = \sigma^2{}_{t+l}$ for all forecasts and there is a perfect fit. On the other hand, if U=1 the performance is not good.

4. Results and Discussion

4.1. Descriptive Analysis

The data in this study consist of monthly export price of sesame (in birr per metric ton), monthly food consumer price index, monthly fuel oil price (in US dollar) and monthly exchange rate (in birr against US dollar) in Ethiopia for the period spanning from January 1998 to June 2013.

Summary statistics of the export price of sesame and selected macro-economic variables are presented in Table 4.1. From the result, the mean value of the export price of sesame, food price index, fuel oil price and exchange rate over the study period were 8275.182, 40.331, 57.647 and 10.456, respectively. Moreover, the maximum and minimum value of the export price of sesame were 25070.650 and 2510.336, respectively.

Table 4.1: Descriptive statistics for macroeconomic variables

	SESAME	FOOD	OIL	EXCHANGE
Mean	8275.182	40.331	57.647	10.456
Median	5284.207	28.600	57.538	8.919
Maximum	25070.650	102.100	123.259	18.887
Minimum	2510.336	17.300	9.824	6.709
Std. Dev.	5720.587	23.741	34.434	3.308

4.2. Time plot and Unit Root Properties of Individual Series

The time plot of the series under consideration are shown in Figure 4.1 and Figure A2-A4 (Appendix). From the figures we can observe that the export price of sesame, food price index exchange rate show an increasing trend. Moreover, fuel oil price exhibits an upward trend up to 2007, tends to decrease from 2007 to 2008, and increases thereafter.

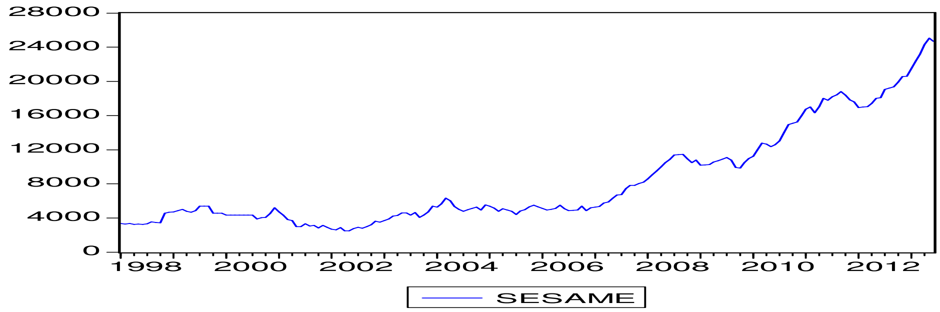

Figure 4.1 Time plot of export price of sesame at level

The time series under consideration should be checked for stationarity before one attempts to fit a suitable model. That is, variables have to be tested for the presence of unit root(s) and the order of integration of each series has to be determined. The stationarity of each series can be tested using the Augmented Dickey-Fuller test and the Phillips and Perron test. The null hypothesis of the tests is that there is a unit-root problem in the series.

The results of ADF and PP tests with intercept but no trend and with intercept and trend both at level and first difference for each series are presented in Tables 4.2 and 4.3. The critical values used for the tests are the McKinnon (1991) critical values. Test results presented in Table 4.2 indicate that the null hypothesis that the series in levels contain unit root could not be rejected for all of the four series.

Table 4.2: Unit root test results (At level)

Series	Include test equation	Test statistic		Prob.*		Test critical value		
		ADF	PP	ADF	PP	1% level	5% level	10% level
SESAME	with intercept	2.612	3.042	0.999	0.999	-3.466	-2.877	-2.575
	With trend and intercept	0.227	0.380	0.998	0.999			
FOOD	with intercept	5.108	6.755	0.999	0.999			
	With trend and intercept	1.011	1.351	0.999	0.998			
OIL	with intercept	-1.006	-0.971	0.751	0.763			
	With trend and intercept	-2.948	-2.974	0.150	0.143			
EXCHANGE	with intercept	3.642	3.828	0.999	0.999			
	With trend and intercept	1.103	0.999	0.999	0.999			

*MacKinnon (1996) one-sided p-values

Since the null hypothesis cannot be rejected, the same tests were applied to their first differences in order to determine the order of integration of the non stationary time series. The results in Table 4.3 indicate that the null hypothesis of unit root is rejected for the first differences of the four series under consideration. This implies that the four time series are integrated of order one (I (1)). Therefore, the ADF and PP tests show that all series are non-stationary in levels but stationary in first differences.

Table 4.3: Unit root test results (after first difference)

Series	Include test equation	Test statistic		Prob.*		Test critical value		
		ADF	PP	ADF	PP	1% level	5% level	10% level
SESAME	with intercept	-10.139	-10.351	0.000	0.000	-3.466	-2.877	-2.575
	With trend and intercept	-10.752	-10.809	0.000	0.000			
FOOD	with intercept	-5.711	-7.795	0.000	0.000			
	With trend and intercept	-9.842	-9.850	0.000	0.000			
OIL	with intercept	-9.749	-9.732	0.000	0.000			
	With trend and intercept	-9.722	-9.705	0.000	0.000			
EXCHANGE	with intercept	-5.313	-8.047	0.000	0.000			
	With trend and intercept	-8.491	-8.900	0.000	0.000			

*MacKinnon (1996) one-sided p-value

4.3. ARIMA Model

Box-Jenkins modeling of a stationary time series involves the following four steps: model identification, model estimation, diagnostic checking and forecasting. We have seen that the export price of sesame becomes a stationary time series after first order differencing. Now, the model that we are looking at is ARIMA(p,1,q). We have to identify a good-fit model, estimate its parameters, apply diagnostic checking for the residuals and finally achieve our objective of in-sample and out-of-sample forecasting of the export price of sesame.

4.3.1. Model Identification

Our first step would be to identify preliminary values of the autoregressive order p and the moving average order q. For this purpose, we compute the sample ACF and PACF of the stationary series. These are given in Figure A5 (in the Appendix). We observe from the ACF and PACF that there are insignificant spikes for most of the lags. In most applications, lower order ARIMA models are often considered. In this study, AR(0-3) and MA(0-3) orders are considered. Computing different model selection statistics in order to suggest the best fit model of all the ARIMA alternatives at the in-sample stage is useful. Here, two statistics, Akaike Information Criterion (AIC) and Bayesian Information criterion (BIC) are used. Table 4.4 shows the varies ARIMA models with their corresponding AIC and BIC.

Table 4.4: Model summery for varies orders of ARIMA models

Models	AIC	BIC	Serial correlation
ARIMA(0,1,1)	14.617	14.656	No
ARIMA(0,1,2)	14.629	14.687	No
ARIMA(0,1,3)	14.642	14.718	No
ARIMA(1,1,0)	14.623	14.661	No
ARIMA(1,1,1)	14.634	14.692	No
ARIMA(1,1,2)	14.634	14.711	No
ARIMA(1,1,3)	14.647	14.743	No
ARIMA(2,1,0)	14.641	14.699	No
ARIMA(2,1,1)	14.642	14.719	No
ARIMA(2,1,2)	14.633	14.729	No
ARIMA(2,1,3)	14.645	14.760	No
ARIMA(3,1,0)	14.656	14.733	No
ARIMA(3,1,1)	14.656	14.753	No
ARIMA(3,1,2)	14.667	14.783	No
ARIMA(3,1,3)	14.594	14.730	No

As discussed in the methodology part, a model with small AIC and BIC is preferable. Based on these selection criteria, ARIMA(0,1,1) is found to be the best fit model for the export price of sesame in Ethiopia.

4.3.2. Parameter Estimation

Estimating the parameters for Box-Jenkins models follows a non-linear estimation procedure and parameter estimates are usually obtained by maximum likelihood method which is asymptotically correct for any time series (Brockwell and Davis, 1996). Hence, we use maximum likelihood estimation method for monthly export price of sesame to estimate the parameters. The results are summarized in Table 4.5 below.

Table 4.5: Parameter estimation of ARIMA(0,1,1) model

Variables	Coefficients	Std. error	Statistic	P-value
Constant	88.867	34.258	2.594	0.010
MA(1)	0.212	0.078	2.708	0.008

From Table 4.5, the estimated parameters of the moving average term and the constant term are statistically significant at 5% level of significant. So the equation of the fitted ARIMA model is given by:

$$Y_t = 88.867 + 0.212\varepsilon_{t-1} + \varepsilon_t$$

4.3.3. Diagnostic checking

Before we consider the fitted model as a good-fit and interpret its findings, it is essential to check whether the model is correctly specified, that is, whether the model assumptions are supported by the data. If some key model assumptions seem to be violated, then a new model should be specified until it provides an adequate fit to the data.

4.3.3.1.Test for Serial Correlation

If a model fits well, the standardized residuals should behave as an i.i.d. sequence with mean zero and unit variance. The Ljung-Box (Q-statistic) in Figure A6 (in the Appendix) indicate that there is no significant serial correlation for lags up to 21 for the export price of sesame. An alternative test to the Q-statistic for testing serial correlation is the Breusch-Godfrey LM test. Unlike the Durbin-Watson test, the Breusch-Godfrey LM test is used to test for higher order serial correlation in the residuals. The null hypothesis of the LM test is that there is no serial correlation up to lag order *p*. The results are given in Table A1 (in the Appendix). The F-statistic and Breusch-Godfrey LM test statistic are 1.244 and 14.840, with their corresponding p-values 0.259 and 0.250, respectively. Thus, the tests do not reject the null hypothesis of no serial correlation up to lag order 12.

4.3.3.2.Test for normality

To investigate whether or not the residuals of the fitted model are normally distributed, the Jarque-Bera test has been applied. The result shows that the residuals are normally distributed at the 5% level of significance (Figure A7 in the Appendix). Moreover, the skewness and kurtosis coefficients are -0.045 and 3.076, respectively.

4.3.4. Forecasting of ARIMA(0,1,1) model

One of the fundamental applications of time series analysis or developing a time series model is forecasting. The previous discussion has shown that the ARIMA(0,1,1) model is a good-fit model to describe the export price of sesame. In this section we examine the forecasting accuracy of the fitted model and then make in-sample and out-of-sample forecast.

Evaluation of in-sample forecast

The forecasting performance of a model can be examined by the standardized statistical tools such as root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and Theil's inequality (U). But as discussed in the methodology part, MAPE and U are unit-less and preferable. Due to this, we consider the values of MAPE and U. The in-sample forecast accuracy measures are given in Table 4.6 below.

Table 4.6: In-sample forecast measures for ARIMA(0,1,1)

Accuracy measures	Export price of sesame
RMSE	356.830
MAE	277.489
MAPE	4.930
U	0.024
Bias proportion	0.000
Variance proportion	0.044
Covariance proportion	0.956

The mean absolute percentage error and Theil's inequality for ARIMA (0,1,1) model are 4.930 and 0.024, respectively. The value of Theil's inequality coefficient is close to zero. Besides that the bias and the variance proportion are also close to zero. This indicate that the forecasting performance of the model is good.

Out-of-sample forecasting

The out-of-sample forecast is performed from January 2011 to June 2013 generating 24 observations. The out-of-sample forecast graph is displayed in Figure 4.2 below. We can see that there is an increasing trend in the export price of sesame.

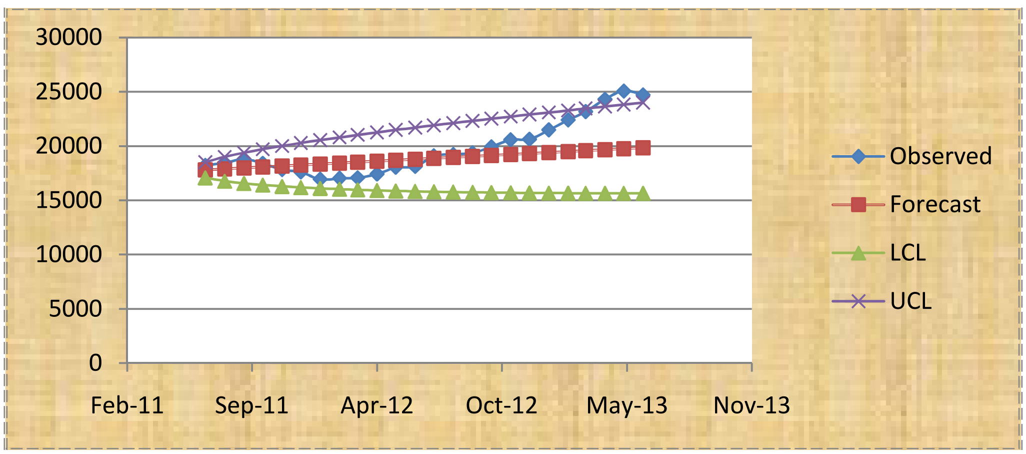

Figure 4.2: out-of-sample forecast graph of ARIMA(0,1,1) model for the export price of sesame

4.4. Volatility Modeling

4.4.1. Test of stationarity of the return series of the export price of sesame

If a time series data is non-stationary, it is necessary to look for possible transformations that might induce stationarity. In practice, researchers usually transform financial data series into return forms. Table 4.7 summarizes the unit root test of the return series for the export price of sesame. The table shows that the null hypothesis of unit root would be rejected. Hence the return series of the export price of sesame are stationary.

Table 4.7: Stationarity Test Statistics for the return series of the variables

Series Name	Include test equation	Test statistic		Prob.*		Test critical value		
		ADF	PP	ADF	PP	1% level	5% level	10% level
Return series of sesame	with intercept	-12.730	-12.759	0.000	0.000	-3.466	-2.877	-2.575
	With trend and intercept	-12.881	-12.884	0.000	0.000	-4.009	-3.434	-3.141

4.4.2. Specification of the mean equation

In order to model the volatility of the return series, we need first to specify their conditional mean equation. The return for current time will depend on returns in previous periods (autoregressive component) and the error terms in current and previous periods (moving average component). To specify the conditional mean equation for the series, comparison of various $AR(p)$, $MA\ (q)$ and $ARMA\ (p, q)$ models are performed and the one with smallest information criteria is selected.

In the specification of the mean equation, the sample ACF and PACF plots of the stationary series can be used to identify the order of autoregressive terms and moving average terms. In this study, AR (0-3) and MA (0-3) were considered since the return series show insignificant spikes for all of the lags (see Figure A8 in the Appendix). In order to select the best fit model of all the ARMA alternatives, Akaike Information Criterion (AIC) and Bayesian Information or Schwarz Criterion (BIC) are used. Table 4.8 gives the various ARMA models together with their AIC and BIC statistics.

Table 4.8: ARMA models of various orders for the return series of the export price of sesame

Model	AIC	BIC	Serial correlation
ARMA(0,1)	-2.852	-2.707	No
ARMA(0,2)	-2.863	-2.701	No
ARMA(0,3)	-2.845	-2.755	No
ARMA(1,0)	-2.859	-2.704	No
ARMA(1,1)	-2.860	-2.717	No
ARMA(1,2)	-2.847	-2.747	No
ARMA(1,3)	-2.858	-2.771	No
ARMA(2,0)	-2.865	-2.763	No
ARMA(2,1)	-2.860	-2.739	No
ARMA(2,2)	-2.866	-2.778	No
ARMA(2,3)	-2.886	-2.761	Yes
ARMA(3,0)	-2.854	-2.773	No
ARMA(3,1)	-2.845	-2.745	No
ARMA(3,2)	-2.853	-2.747	No
ARMA(3,3)	-2.841	-2.746	No

Among the various ARMA models considered, ARMA(2,2) model possesses minimum AIC and BIC and exhibits no serial correlation. Therefore, ARMA(2,2) model is the best-fit model for the conditional mean equation for the return series of the export price of sesame.

4.4.2.1. Parameter Estimation for the ARMA(2,2) Model

We use maximum likelihood estimation method for monthly return series of export price of sesame to estimate the parameters. The results are summarized in Table 4.9 below.

Table 4.9: Parameter estimate of ARMA(2,2) model

Parameters	Coefficients	Std. error	t-statistic	p-value
Constant	0.011	0.005	2.446	0.015**
AR(1)	-1.078	0.223	-4.832	0.000*
AR(2)	-0.513	0.177	-2.898	0.004*
MA(1)	1.149	0.195	5.907	0.000*
MA(2)	0.647	0.153	4.234	0.000*

*Note: * and ** indicates significant at 1% and 5% level, respectively*

4.4.2.2. Model adequacy checking

In this section, we will assess how well the selected model, ARMA(2,2), fits the return series of the export price of sesame. For this purpose, we have conducted diagnostic tests for the presence of serial correlation and normality of the residuals.

The presence of serial correlation in the residuals of the conditional mean equation was tested using the Lagrange Multiplier (LM) and Ljung-Box tests. The null hypothesis asserts that there is no serial correlation in the residual series up to lag order 3. The Breusch–Godfrey serial correlation LM test results in Table A3 (in Appendix) provide evidence that there is no serial correlation in the residuals of the mean equation. Moreover, the Ljung-Box test (Figure A9 in Appendix) indicates that there is no significant serial correlation for lags up to 21. Hence, there is no significant serial correlation in the residuals.

To investigate whether the residuals of the fitted model (mean equation) are normally distributed, the Jarque-Bera test has been applied. The results are reported in Figures A10 (in Appendix). The Jarque-Bera statistic is insignificant, and hence, there is no evidence to reject the null hypothesis of normality. This indicates that the residuals of the fitted model are normally distributed.

4.4.2.3. Test for ARCH effects

Based on the residuals from the mean equation it is possible to test for the existence of ARCH effects which will allow analysis using GARCH family models. The ARCH LM test helps to test the hypothesis that there is no ARCH effect up to lag p. If the test provides significant results, then it is possible to conclude that there is an ARCH effect up to lag p. The results of ARCH LM test for the residuals of our ARMA(2,2) model are displayed in Table 4.10. The results indicate that the null hypothesis of no ARCH effect in the first three lags of residuals from the mean equation for the return series of the export price of sesame is rejected. This implies that the conditional variance of the monthly return series of the export price of sesame is not time invariant.

Table 4.10: ARCH-LM test for the squared residuals of ARMA(2,2) model

ARCH Test:

F-statistic	8.056081	Probability	0.000046
Obs*R-squared	21.73314	Probability	0.000074

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 06/07/14 Time: 16:25
Sample (adjusted): 1998M07 2013M06
Included observations: 180 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001656	0.000493	3.360615	0.0010
RESID^2(-1)	0.029166	0.072830	0.400469	0.0487
RESID^2(-2)	0.195089	0.071393	2.732595	0.0069
RESID^2(-3)	0.257480	0.072832	3.535268	0.0005

4.4.3. Specification of Volatility Model

If an ARCH effect is found to be significant, the PACF of the squared residuals is helpful to determine the order of ARCH model. This is due to the expectation that ${\varepsilon_t}^2$ is linearly related to ${\varepsilon^2}_{t-1}, {\varepsilon^2}_{t-2}, \dots, {\varepsilon^2}_{t-q}$ in a manner similar to that of an $AR(p)$ model. In practice, a large lag is often needed for ARCH modeling and this requires estimating a large number of parameters. A GARCH model with low lags, on the other hand, results in a more parsimonious representation of the conditional variance process (Anderson, 2009).

The modeling procedure of ARCH models can also be used to build a GARCH model. However, specifying the order of a GARCH model is not easy. Often lower order GARCH models are used in most applications. Computing different statistics such as Akaike Information Criterion (AIC) and Bayesian information or Schwarz Criterion (BIC) in order to choose the best fit model is helpful. Note that the AIC and BIC of the GARCH models are obtained by estimating the mean and variance equations simultaneously.

In our model selection procedure we first fit different GARCH family models of different orders of m and r. The results are displayed in Table A4 (Appendix). GARCH(2,1) and EGARCH(1,1) models under normal distributional assumption for residuals, GARCH(1,1) model under both GED and student's t-distributional assumption for residuals, and EGARCH(2,1) and EGARCH(1,1) models under GED and student's t-distributional assumptions for residuals, respectively, were selected as candidate models for the export price volatility of sesame since they possess minimum AIC and/or BIC.

To select the appropriate conditional volatility model, we consider the forecasting performance of the selected GARCH family models. The forecast performance of the fitted GARCH family models are evaluated by RMSE, MAE, MAPE and Theil inequality coefficients . The model with the smallest statistics is considered to be better fit for modeling the conditional volatility of the export price of sesame in Ethiopia. The summary results are displayed in Table 4.11 below.

Table 4.11: Forecast accuracy statistics for residuals from the variance equation of the export price of sesame

Model	Error distribution	Forecast accuracy measures				Asymmetric effect
		RMSE	MAE	MAPE	Theil	
GARCH(2,1)	Normal	0.056995	0.043515	2758.162	0.809679	-
GARCH(1,1)	t-distribution	0.057060	0.043500	3079.246	0.793866	-
GARCH(1,1)	GED	0.057002	0.043515	2769.448	0.809174	-
EGARCH(1,1)	Normal	0.056917	0.043375	2625.691	0.815293	Non-significant
EGARCH(1,1)	t-distribution	0.056920	0.043369	2646.200	0.814062	Non-significant
EGARCH(2,1)	GED	0.056922	0.043414	2426.306	0.825612	Non-significant

The forecast accuracy measures indicate that EGARCH models with different error distributional assumption possess smallest accuracy measures in the majority of the statistics. However, the asymmetric effects are insignificant in all these models. Therefore, the best-fit model is selected from the symmetric GARCH models. Among the symmetric models, GARCH(2,1) model with normal distributional assumption for the residuals possesses the smallest forecast accuracy measures in most of the statistics. Thus, this model is a best-fit model to describe the volatility of the return series of the export price of sesame.

4.4.5. Parameter Estimation

Once the ARMA(2,2)-GARCH(2,1) model with normal distributional assumption for residuals was selected as a best-fit model, then the next step is to perform analysis of the monthly export price volatility of sesame. The parameters in the mean and variance equation are estimated by using maximum likelihood (ML) method. The results are shown in Table 4. 12 below.

Table 4.12: ML parameter estimates of ARMA(2,2)-GARCH(2,1) model under normal distributional assumption of residuals for the return series of export price of sesame

Parameters	Variables	Coefficients	Std. error	Statistic	P-value
Mean equation	Constant	0.012	0.003	3.973	0.000*
	AR(1)	-0.941	0.305	-3.087	0.002*
	AR(2)	-0.396	0.212	-1.873	0.061***
	MA(1)	1.013	0.296	3.419	0.001*
	MA(2)	0.476	0.206	2.311	0.021**
Variance equation	Constant	1.73E-04	7.67E-05	2.260	0.024**
	ARCH(-1)	-0.087	0.019	-4.607	0.000*
	ARCH(-2)	0.126	0.034	3.704	0.000*
	GARCH(-1)	0.926	0.030	31.322	0.000*
	Food price index	-8.68E-05	2.05E-05	-4.247	0.000*
	Fuel oil	-5.26E-06	1.47E-05	-0.359	0.720
	Exchange rate	-2.48E-04	5.90E-04	-0.420	0.674

*Note: *, ** and *** indicates significant at 1%, 5% and 10% level, respectively*

The coefficient estimate of food price index is negative and statistically significant at the 1% level, that is, food price index has a significant influence on the export price volatility of sesame. This indicates that an increase in food price index leads to a decrease in the monthly export price volatility of sesame. This result is consistent with the findings of Sorsa (2009) and Zheng et al. (2008).

As can be seen in Table 4.12, the coefficient of fuel oil price is statistically insignificant. This shows that fuel oil price has no significant effect on the export price volatility of sesame. This result is consistent with Khin (2010).

Among the explanatory variables which are considered in this study, the coefficient of exchange rate (in birr against US dollar) was negative and statistically insignificant. This results is inconsistent with the findings of Abule (2012) and Serge (2006).

The results from the variance equation show that the coefficient of the ARCH(-1) and ARCH(-2) term were statistically significant at the 1% level. This shows that the current month export price volatility of sesame was affected by its 1-month and 2-month lagged shocks. Similarly, the GARCH(-1) term is statistically significant at the1% level. This indicates that the current month export price volatility of sesame was affected by its 1-month lagged price volatility.

4.4.6. Diagnostic checking of the fitted model

For diagnostic checking of the presence of remaining ARCH effect in the residuals, ARCH LM test and Ljung-Box Q-test are used. The results are presented in Table A5 and Figure A11 (in Appendix), respectively. The p-values of both tests are greater than 5%. These results imply that we do not have enough evidence to reject the null hypothesis that there is no ARCH left in the residuals.

Additionally, the Jarque-Bera statistic is used to test the normality of the residuals in the fitted model. The results are presented in Figure A12 (Appendix). The result shows that the normality of the residuals in the fitted model is not rejected. Therefore, we conclude that the residuals of the fitted model are normally distributed.

4.5. Forecasting price volatility

One of the fundamental applications of developing GARCH family model is forecasting. In this section we make in-sample forecasts based on the fitted ARMA(2,2)-GARCH(2,1) model with normal distributional assumption for residuals. The plot of the dynamic in-sample forecast is presented in Figure 4.3.

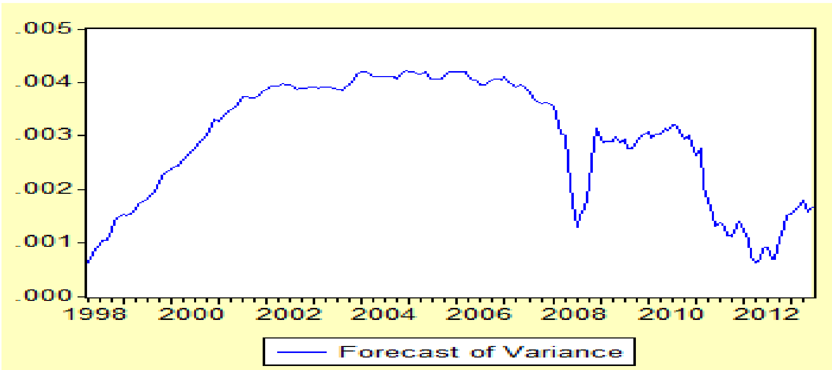

Figure 4.3: In-sample forecast of monthly export price volatility of sesame

As we can see from Figure 4.3, the export price volatility of sesame steadily increased from the years 1998 to around 2002, remained at almost a constant level till 2007 and then drops sharply. Moreover, low export price volatility was observed from around the year 2010 up to the end of the study period.

5. Conclusion and Recommendation

5.1. Conclusion

In this paper, we have examined ARIMA and GARCH family models for the export price of sesame. The conditional mean equation was estimated using ARMA model and the conditional variance equation using GARCH family models with different error distributions.

In the case of ARIMA model, after examining different competitive models, we came to the conclusion that ARIMA(0,1,1) model provides the best-fit model for the export price of sesame. On the other hand, ARMA(2,2)-GARCH(2,1) model with normal distributional assumption for the residuals provides the best-fit model for the export price volatility of sesame in Ethiopia.

Among the exogenous variable that are considered in this study, food price index had a significant effect on the volatility of the export price of sesame in Ethiopia. Moreover, the ARCH and GARCH terms were found to be statistically significant. These show that the current month export price volatility of sesame is affected by the recent past shocks and volatility.

Finally, forecasts are made using ARIMA and GARCH models. The in-sample forecast accuracy using root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and Theil's U statistics indicates that the estimated ARIMA model is good enough to describe the export price of sesame. Moreover, out-of-sample forecasts are made from July 2011 to June 2013. The result indicates that the export price of sesame has an increasing trend. The in-sample forecast using the best-fit GARCH model indicates a steady increase at the beginning of the study period followed by a period where volatility remained at almost a constant level. Moreover, low volatility was observed around the end of the study period.

5.2. Recommendation

The focus of this study was estimating and forecasting the export price volatility of sesame in Ethiopia. Further studies may employ multivariate models such as seasonal or dynamic conditional correlation multivariate model to analyze the time varying correlation of export price of sesame with other variables. Additionally, there are also other variable that might affect the volatility of the export price of sesame in Ethiopia.

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Appendix

Figure A1: Time plot of monthly export price of sesame at first difference

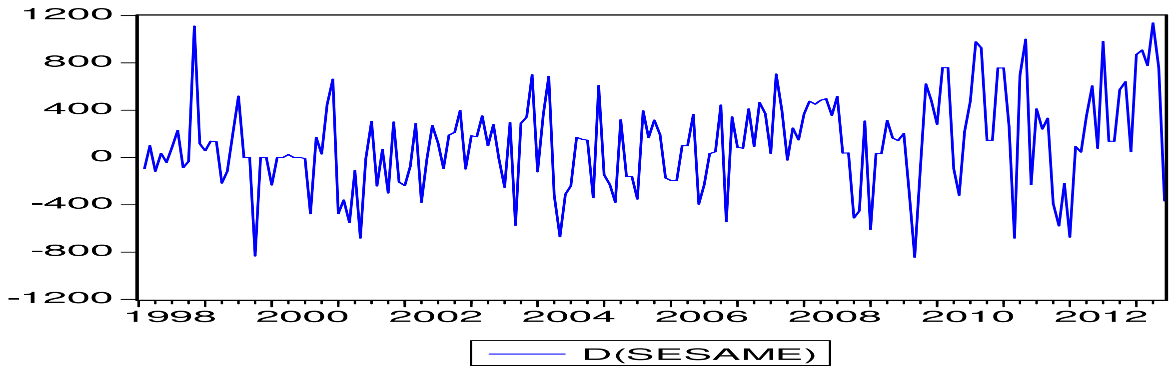


Figure A2: Time plot of monthly food price index at level and first difference

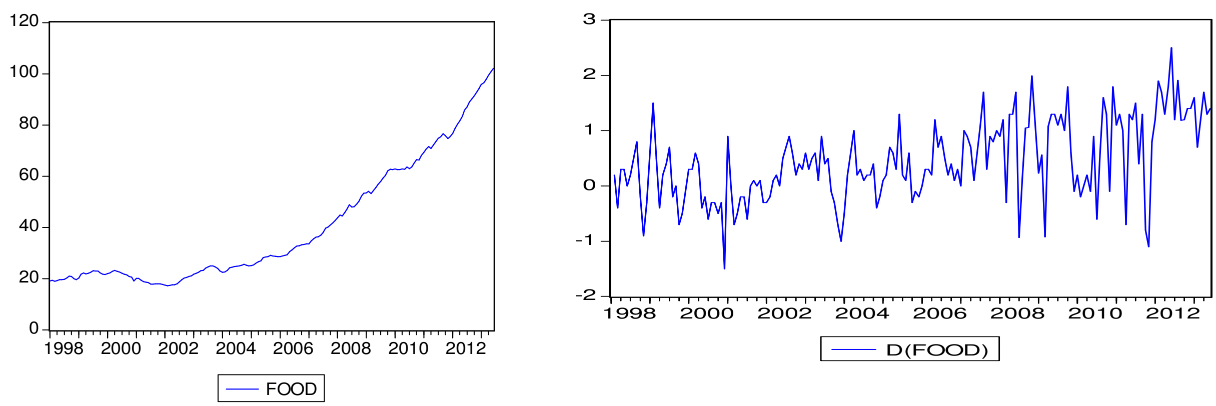


Figure A3: Time plot of monthly fuel oil price at level and first difference

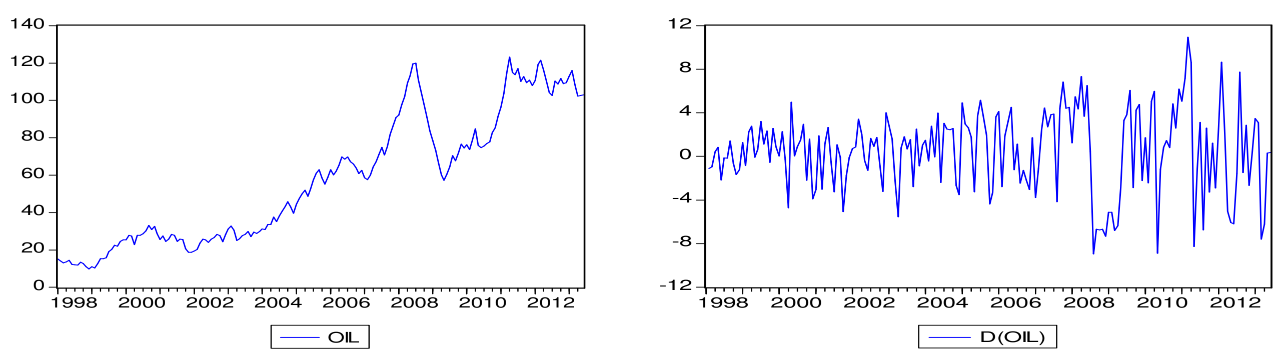


Figure A4: Time plot of monthly exchange rate at level and first difference

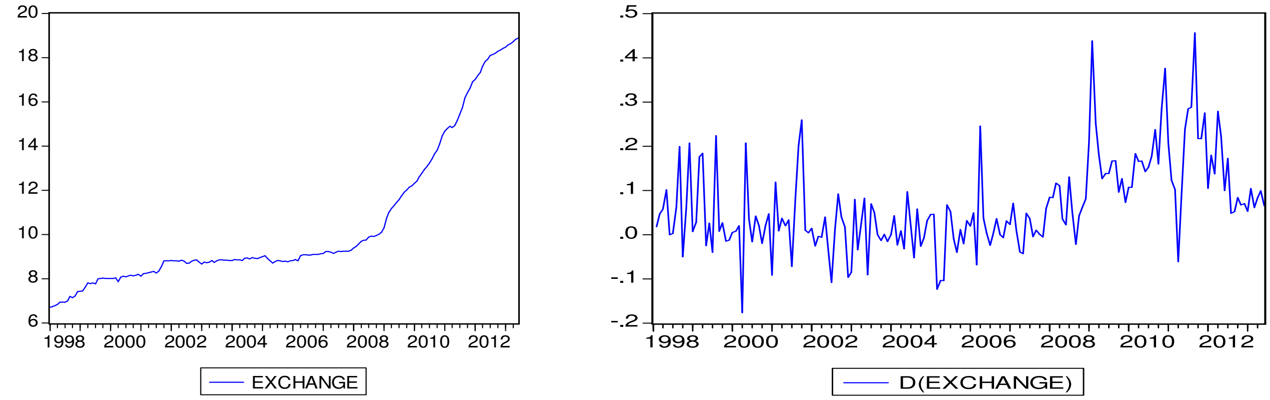


Figure A5: Correlogram of monthly export price of sesame at first difference up to 21 lags

Date: 07/03/14 Time: 09:39
Sample: 1998M01 2011M06
Included observations: 161

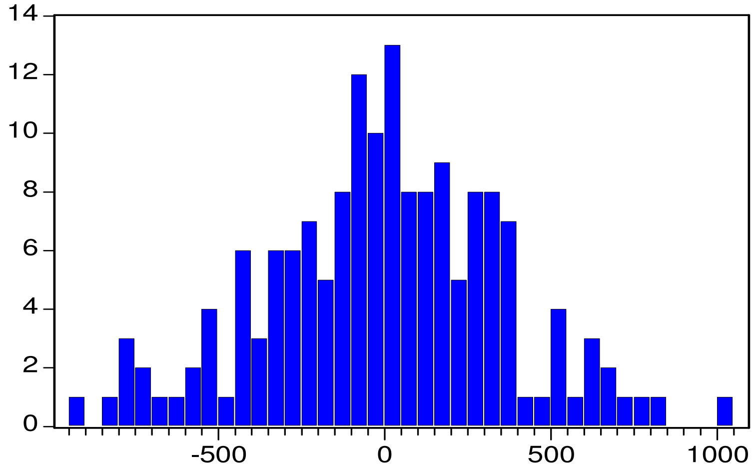
	AC	PAC	Q-Stat	Prob
1	0.221	0.221	7.9899	0.005
2	0.067	0.019	8.7276	0.013
3	0.009	-0.010	8.7411	0.033
4	0.075	0.078	9.6809	0.046
5	0.008	-0.025	9.6920	0.084
6	-0.036	-0.042	9.9164	0.128
7	0.128	0.156	12.697	0.080
8	0.152	0.096	16.639	0.034
9	0.089	0.025	18.018	0.035
10	0.060	0.042	18.647	0.045
11	0.025	-0.017	18.757	0.066
12	0.049	0.027	19.172	0.084
13	-0.045	-0.054	19.523	0.108
14	-0.101	-0.102	21.355	0.093
15	-0.087	-0.072	22.713	0.090
16	0.018	0.031	22.772	0.120
17	-0.001	-0.028	22.772	0.157
18	-0.025	-0.023	22.884	0.195
19	-0.048	-0.050	23.304	0.224
20	-0.025	-0.022	23.420	0.269
21	0.019	0.063	23.492	0.318

Figure A6: Sample ACF and PACF of residuals for ARIMA(0,1,1) up to 21 lags

Date: 07/03/14 Time: 10:17
Sample: 1998M02 2011M06
Included observations: 161
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.020	0.020	0.0673	
2	0.063	0.063	0.7243	0.395
3	-0.023	-0.025	0.8100	0.667
4	0.073	0.070	1.6964	0.638
5	0.006	0.006	1.7033	0.790
6	-0.052	-0.062	2.1629	0.826
7	0.115	0.122	4.4185	0.620
8	0.117	0.117	6.7800	0.452
9	0.050	0.027	7.2172	0.513
10	0.063	0.065	7.9136	0.543
11	-0.003	-0.019	7.9150	0.637
12	0.056	0.031	8.4648	0.671
13	-0.040	-0.029	8.7519	0.724
14	-0.080	-0.099	9.9036	0.702
15	-0.080	-0.096	11.047	0.682
16	0.034	0.024	11.259	0.734
17	-0.009	-0.028	11.274	0.792
18	-0.015	-0.021	11.316	0.840
19	-0.039	-0.041	11.602	0.867
20	-0.022	-0.043	11.693	0.898
21	0.030	0.058	11.865	0.921

Figure A7: Jarque-Bera normality test for residuals of ARIMA(0,1,1) model



Series: Residuals	
Sample 1998M02 2011M06	
Observations 161	
Mean	0.201177
Median	8.193199
Maximum	1039.350
Minimum	-908.7531
Std. Dev.	357.9431
Skewness	-0.044767
Kurtosis	3.075716
Jarque-Bera	0.092234
Probability	0.954930

Figure A8: Correlogram of return series of export price of sesame

Date: 07/04/14 Time: 04:32
Sample: 1998M01 2013M06
Included observations: 185

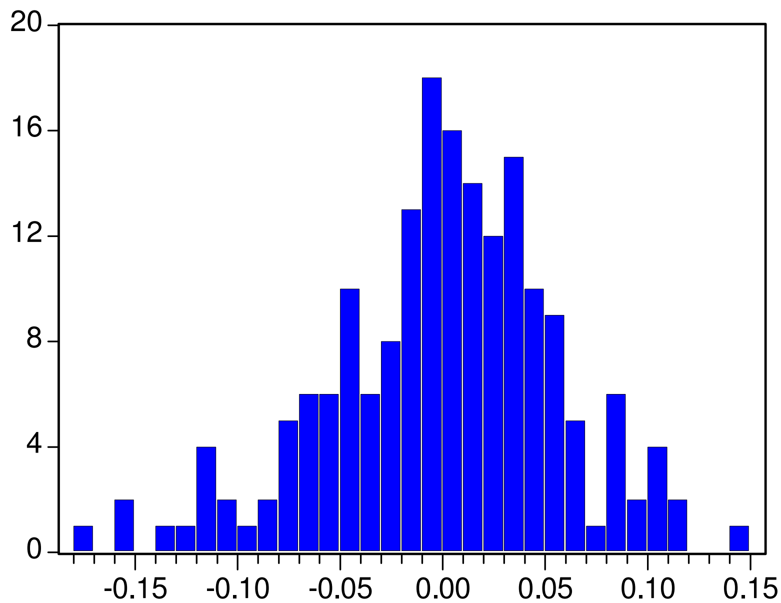
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.060	0.060	0.6662	0.414
		2	0.051	0.048	1.1613	0.560
		3	-0.030	-0.036	1.3376	0.720
		4	0.085	0.087	2.7319	0.604
		5	-0.060	-0.068	3.4329	0.634
		6	-0.116	-0.120	6.0507	0.418
		7	0.127	0.159	9.1629	0.241
		8	0.102	0.087	11.199	0.191
		9	0.074	0.049	12.268	0.199
		10	0.038	0.052	12.557	0.250
		11	0.058	0.012	13.217	0.279
		12	0.078	0.062	14.428	0.274
		13	-0.097	-0.077	16.333	0.232
		14	-0.059	-0.055	17.041	0.254
		15	-0.134	-0.133	20.711	0.146
		16	0.032	0.023	20.921	0.182
		17	0.017	0.040	20.979	0.227
		18	-0.028	-0.055	21.144	0.272
		19	-0.086	-0.126	22.669	0.252
		20	-0.041	-0.059	23.027	0.287
		21	0.072	0.083	24.114	0.288

Figure A9: Ljung-Box test of serial correlation the residuals of ARMA(2,2) model

Date: 06/07/14 Time: 16:17
Sample: 1998M04 2013M06
Included observations: 183
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.006	0.006	0.0058	
		2	-0.009	-0.009	0.0209	
		3	0.046	0.046	0.4215	
		4	0.032	0.032	0.6201	
		5	-0.035	-0.035	0.8578	0.354
		6	-0.109	-0.111	3.1483	0.207
		7	0.101	0.100	5.1117	0.164
		8	0.115	0.118	7.6794	0.104
		9	0.050	0.064	8.1668	0.147
		10	0.029	0.025	8.3331	0.215
		11	0.067	0.043	9.2206	0.237
		12	0.063	0.048	10.012	0.264
		13	-0.092	-0.070	11.690	0.231
		14	-0.048	-0.039	12.154	0.275
		15	-0.124	-0.149	15.252	0.171
		16	0.031	0.020	15.441	0.218
		17	0.034	0.043	15.672	0.267
		18	-0.043	-0.042	16.046	0.311
		19	-0.077	-0.130	17.272	0.303
		20	-0.050	-0.088	17.797	0.336
		21	0.079	0.081	19.093	0.323

Figure A10: Jarque-Bera normality test for the residuals of ARMA(2,2) model



Series: Residuals
Sample 1998M04 2013M06
Observations 183

Mean	0.000155
Median	0.002853
Maximum	0.141068
Minimum	-0.176907
Std. Dev.	0.056325
Skewness	-0.425545
Kurtosis	3.476640
Jarque-Bera	7.255489
Probability	0.056576

Figure A11: Correlogram of standard residuals of ARMA(2,2)-GARCH(2,1) with normal distributional assumption for residuals

Date: 06/08/14 Time: 04:08
Sample: 1998M04 2013M06
Included observations: 183
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.056	0.056	0.5749	
		2	0.041	0.038	0.8819	
		3	-0.006	-0.010	0.8879	
		4	0.006	0.005	0.8940	
		5	-0.001	-0.001	0.8942	0.344
		6	-0.136	-0.137	4.4381	0.109
		7	0.109	0.127	6.7254	0.081
		8	0.046	0.044	7.1392	0.129
		9	0.032	0.015	7.3427	0.196
		10	0.044	0.043	7.7176	0.260
		11	0.041	0.034	8.0541	0.328
		12	0.062	0.035	8.8218	0.358
		13	-0.097	-0.076	10.687	0.298
		14	-0.080	-0.078	11.960	0.288
		15	-0.097	-0.090	13.872	0.240
		16	0.033	0.054	14.093	0.295
		17	0.035	0.039	14.336	0.351
		18	-0.001	-0.007	14.336	0.425
		19	-0.078	-0.124	15.590	0.410
		20	-0.082	-0.087	16.978	0.387
		21	0.079	0.099	18.267	0.372

Figure A12: Normality test of the residuals of ARMA(2,2)-GARCH(2,1) model with normal distributional assumption for residuals

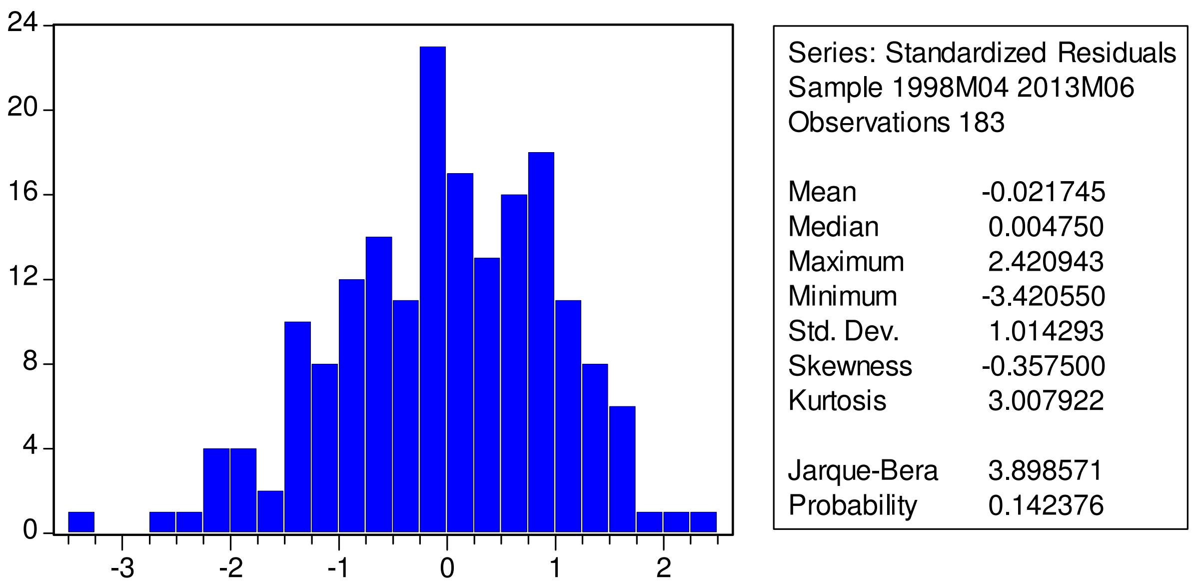


Table A1: Serial correlation Breush-Godfrey LM test for residuals of ARIMA(0,1,1) model

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.243746	Probability	0.258780
Obs*R-squared	14.83965	Probability	0.250323

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 07/03/14 Time: 10:25
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.707447	34.25730	0.137414	0.8909
MA(1)	2.174157	4.034798	0.538852	0.5908
RESID(-1)	-2.198875	4.035050	-0.544944	0.5866
RESID(-2)	0.476213	0.857973	0.555044	0.5797
RESID(-3)	-0.094273	0.198984	-0.473772	0.6364
RESID(-4)	0.162475	0.093286	1.741681	0.0837
RESID(-5)	0.015932	0.085585	0.186156	0.8526
RESID(-6)	-0.134610	0.085138	-1.581087	0.1160
RESID(-7)	0.105253	0.085914	1.225101	0.2225
RESID(-8)	0.158266	0.086354	1.832771	0.0689
RESID(-9)	0.083554	0.087263	0.957488	0.3399
RESID(-10)	0.159159	0.088043	1.807749	0.0727
RESID(-11)	-0.045375	0.090948	-0.498907	0.6186
RESID(-12)	-0.071216	0.091183	-0.781028	0.4360

Table A2: Out-of-sample forecast values of ARIMA(0,1,1) model

Month	Out of sample observed value	Forecast value	Month	Out of sample observed value	Forecast value
Jul-11	18210.27	17788.50	Jul-12	19081.95	18854.22
Aug-11	18451.54	17877.31	Aug-12	19219.06	18943.03
Sep-11	18784.83	17966.12	Sep-12	19356.17	19031.84
Oct-11	18395.08	18054.93	Oct-12	19928.19	19120.65
Nov-11	17817.10	18143.74	Nov-12	20567.77	19209.46
Dec-11	17597.94	18232.55	Dec-12	20617.29	19298.27
Jan-12	16925.96	18321.36	Jan-13	21486.71	19387.08
Feb-12	17018.48	18410.17	Feb-13	22394.71	19475.89

Mar-12	17064.83	18498.98	Mar-13	23172.14	19564.70
Apr-12	17416.91	18587.79	Apr-13	24309.63	19653.51
May-12	18022.34	18676.60	May-13	25070.65	19742.32
Jun-12	18102.15	18765.41	Jun-13	24704.51	19831.13

Table A3: Breusch-Godfrey Serial Correlation LM Test for residuals of ARMA(2,2) model

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.700844	Probability	0.552732
Obs*R-squared	2.171174	Probability	0.537649

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 06/07/14 Time: 16:19
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.77E-05	0.004561	0.021421	0.9829
AR(1)	-0.158526	0.392645	-0.403739	0.6869
AR(2)	0.034051	0.242898	0.140185	0.8887
MA(1)	-0.002692	0.230421	-0.011685	0.9907
MA(2)	-0.005143	0.159298	-0.032284	0.9743
RESID(-1)	0.168425	0.240095	0.701495	0.4839
RESID(-2)	-0.213049	0.182497	-1.167410	0.2446
RESID(-3)	0.182746	0.132043	1.383991	0.1681

Table A4: Comparison of GARCH family models of the residuals of the mean equation for the return series of the export price of sesame

GARCH Model with distribution	AIC	BIC	Serial corr.	EGARCH model with distribution	AIC	BIC	Serial corr.
GARCH(1,1) normal	-3.050	-2.847	No	EGARCH(1,1) normal	-3.105	-2.895	No
GARCH(1,2) normal	-3.033	-2.823	No	EGARCH(1,2) normal	-3.064	-2.836	No
GARCH(1,3)normal	-3.019	-2.792	No	EGARCH(1,3)normal	-3.076	-2.831	No
GARCH(2,1)normal	-3.060	-2.849	No	EGARCH(2,1)normal	-3.062	-2.834	No
GARCH(2,2)normal	-3.023	-2.795	No	EGARCH(2,2)normal	-3.086	-2.841	No

GARCH(2,3)normal	-3.043	-2.798	No	EGARCH(2,3)normal	-3.066	-2.803	No
GARCH(3,1)normal	-3.024	-2.796	No	EGARCH(3,1)normal	-3.085	-2.839	No
GARCH(3,2)normal	-3.035	-2.789	No	EGARCH(3,2)normal	-3.064	-2.801	No
GARCH(3,3)normal	-3.016	-2.753	No	EGARCH(3,3)normal	-3.027	-2.746	No
GARCH(1,1)t-dist.	-3.040	-2.829	No	EGARCH(1,1)t-dist.	-3.093	-2.865	No
GARCH(1,2)t-dist.	-3.020	-2.792	No	EGARCH(1,2)t-dist.	-3.064	-2.819	No
GARCH(1,3)t-dist.	-3.004	-2.759	No	EGARCH(1,3)t-dist.	-3.061	-2.798	No
GARCH(2,1)t-dist.	-3.031	-2.813	No	EGARCH(2,1)t-dist.	-3.066	-2.820	No
GARCH(2,2)t-dist.	-3.011	-2.765	No	EGARCH(2,2)t-dist.	-3.063	-2.800	No
GARCH(2,3)t-dist.	-3.016	-2.753	No	EGARCH(2,3)t-dist.	-3.039	-2.759	No
GARCH(3,1)t-dist.	-3.011	-2.765	No	EGARCH(3,1)t-dist.	-3.072	-2.809	No
GARCH(3,2)t-dist.	-3.018	-2.755	No	EGARCH(3,2)t-dist.	-3.029	-2.749	No
GARCH(3,3)t-dist.	-2.970	-2.690	No	EGARCH(3,3)t-dist.	-3.027	-2.729	No
GARCH(1,1) GED	-3.039	-2.829	No	EGARCH(1,1) GED	-3.066	-2.838	No
GARCH(1,2) GED	-3.018	-2.790	No	EGARCH(1,2) GED	-3.069	-2.824	No
GARCH(1,3) GED	-3.013	-2.767	No	EGARCH(1,3) GED	-3.078	-2.815	No
GARCH(2,1) GED	-3.033	-2.825	No	EGARCH(2,1) GED	-3.103	-2.858	No
GARCH(2,2) GED	-3.011	-2.765	No	EGARCH(2,2) GED	-3.077	-2.813	No
GARCH(2,3) GED	-2.997	-2.734	No	EGARCH(2,3) GED	-3.071	-2.790	No
GARCH(3,1) GED	-3.014	-2.768	No	EGARCH(3,1) GED	-3.089	-2.826	No
GARCH(3,2) GED	-2.957	-2.694	No	EGARCH(3,2) GED	-3.066	-2.786	No
GARCH(3,3) GED	-2.957	-2.677	No	GARCH(3,3) GED	-3.056	-2.758	No

Table A5: Remaining ARCH in residuals test for ARMA(2,2)-GARCH(2,1) model with normal distributional assumptions for residuals.

ARCH Test:

F-statistic	1.762337	Probability	0.156118
Obs*R-squared	5.249478	Probability	0.154414

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares
Date: 06/08/14 Time: 04:10
Sample (adjusted): 1998M07 2013M06
Included observations: 180 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
STD_RESID^2(-1)	0.050383	0.074386	0.677319	0.4991
STD_RESID^2(-2)	0.004005	0.074527	0.053742	0.9572
STD_RESID^2(-3)	0.161916	0.074332	2.178280	0.0607

DECLARATION

I, the undersigned, declare that this thesis is my original work and has not been presented for a degree in any other university, and that all sources of materials used for the thesis have been duly acknowledged.

Declared by:

Name: ________________________________

Signature: _____________________________

Date: _________________________________

Confirmed by Advisor:

Name: ________________________________

Signature: _____________________________

Date: _________________________________