



ADDIS ABABA UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

A GRADUATE SEMINAR REPORT
ON
CONFORMAL MAPPINGS AND THE RIEMANN MAPPING THEOREM

COMPILED BY:
KEWANI WELAY

ADVISOR:
Dr. SEID MOHAMMED

ADDIS ABABA
JUNE 2010

ACKNOWLEDGEMENT

I would like to thank my advisor Dr. Seid Mohammed for his unreserved support of materials and advice in compiling this seminar paper with out him which was difficult. Finally, my thanks goes to my friend Ato Teklay G/here for his support of necessary devises in writing this seminar paper and all my friends in the department of mathematics.

Kewani Welay

TABLE OF CONTENTS

Contents	Page
Acknowledgement	II
Table Of contents	III
Preface	IV
Chapter One: BASIC PROPERTIES OF HOLOMORPHIC FUNCTIONS	1
1.1. Definitions and Theorems on Holomorphic Functions	1
1.2. Angle Preserving Properties of Holomorphic Functions	7
1.3. The Mobius Transformation	13
1.4. The Special Cases of Bilinear Transformation	18
1.5. The Implicit Formula	20
Chapter Two: The Schwarz-Christoffel Transformation	22
Chapter three: The Riemann mapping Theorem	29
3.1. Normal Families	29
3.2. The Riemann Mapping Theorem	35
3.3. Consequences of The Riemann mapping Theorem	39
References	41

Preface

It is easy to point out that any function of a complex-variable can be considered as a mapping from one complex plane into another. A great deal of attention is devoted to the study of holomorphic functions. The reason for this is that many problems from the theory of holomorphic functions can be solved according to the following procedures;

1. Solve the problem for the simplest possible type of domain;
2. Express the desired solution in terms of the one already found with the aid of a mapping.

A non-constant holomorphic function $w = f(z)$ maps a domain Ω of the z -plane onto another domain $f(\Omega)$ of the w -plane. At points where $f'(z) \neq 0$ such a map has the remarkable property that it is conformal. This means that any two smooth curves intersecting in Ω map into curves which intersect at the same angle in $f(\Omega)$. By means of conformal mapping, problems of fluid flow, electrostatics and other fields can be mapped into simpler problems of the same general sort in $f(\Omega)$. Solution of the problem in $f(\Omega)$ then solves the original problem in Ω . Conformal mapping also gives geometrical insight into analytic (holomorphic) questions. This seminar paper includes definitions and basic properties of holomorphic functions, conformal mappings (the bilinear transformations, The Schwarz-Christoffel transformation), Normal Families, the Riemann Mapping Theorem, which characterizes those domains that can be mapped conformally onto the unit open disk and concludes with one of the consequences of the Riemann mapping theorem.