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Design Aid for High Strength Reinforced Concrete Members:

Beam Design Aids and Column Interaction Charts

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This is to certify that the thesis prepared by Misgun Samuel, entitled: Design Aid for High Strength Reinforced Concrete Members**:** Beam Design Aids and Column Interaction Charts and submitted in fulfilment of the requirement for degree of Degree of Masters of Sciences (Structural Engineering) complies with the regulation of the University and meets the accepted standards with respect to originality and quality.

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Abstract

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Applicability of high-strength concrete (HSC) in reinforced concrete structures has gained acceptability in the world as it allows efficient utilization of space and enhanced structural performance. The Ethiopian Building Code Standards provides thorough coverage for the design of reinforced concrete structures made of normal strength concrete (NSC) and also provides design aids for NSC. The design considerations and aids provided in the current code cannot be simply extrapolated for use in reinforced HSC. This is because the mechanical properties of HSC under compression, shear and tension are different from that of NSC. Thus a separate design consideration and aids are required for reinforced HSC structural members. The current building code which is under revision will include design consideration for reinforced HSC, however design aids for HSC are missing. This study focuses on constructing design charts and tables for solid rectangular beams and interaction charts for solid rectangular columns sections made of HSC by using an iterative numerical based approaches. Stress-strain relationship for concrete and reinforcing steel is adapted from Euro Code. The effect of strain hardening for reinforcement steel is incorporated in the design aids. Exact method of cross section analysis is used to estimate the ultimate capacity of structural members when developing the design aids. Multiple application of Simpsons 1/3 rule is used to estimate the total compressive force in the concrete when developing design table and charts for beams. The design tables and charts can be used for any class of work as there are no built-in safety factors. Uniaxial and biaxial interaction charts are constructed by applying Green's Theorem to transform double integrals to line integrals along the compressive perimeter of the concrete section. The line integral itself is evaluated by using Third Order Gauss integration to yield accurate results. Double counting of concrete area that is displaced by the steel bars are taken into account. The algorithms developed for preparation of these design aids are exact and computational efficient.

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Notations

Latin upper case letters

A Cross sectional area

A_c Cross sectional area of concrete

A_{s1} Cross sectional area of tensile reinforcement

A_{s2} Cross sectional area of compression reinforcement

A_{si} Cross sectional area of steel bar i

C_c Compressive force in the concrete

E_{cm} Secant modulus of elasticity of concrete

HSC High Strength concrete

LSD Limit state design

M_{sd} Design value of the applied bending moment

M_X Moment in the x direction

M_{xc} Resultant Moment component in the x direction carried by the concrete

M_{xs} Resultant moment component in the x direction carried by the steels

M_y Moment in the y direction

M_{yc} Resultant moment component in the y direction carried by the concrete

M_{ys} Resultant moment component in the y direction carried by the steels

N_c Resultant axial carried by the concrete

N_s Resultant axial carried by the steels

N_{Sd} Design value of the applied axial force

NSC Normal strength concrete

P_u Axial load

SLS Serviceability limit state

ULS Ultimate limit state

Latin lower case letters

b Overall width of a cross-section

b' Concrete cover to the center of reinforcement

c neutral axis depth

d Effective depth of a cross-section

f_c Compressive strength of concrete

f_{cd} Design value of concrete compressive strength

f_{ck} Characteristic compressive cylinder strength of concrete at 28 days

f_{cm} Mean value of concrete cylinder compressive strength

f_{ctm} Mean value of axial tensile strength of concrete

f_{si} Stress in steel bar i

f_y Yield strength of reinforcement

f_{yd} Design yield strength of reinforcement

f_{yk} Characteristic yield strength of reinforcement

h Overall depth of a cross-section

h' Concrete cover to the center of reinforcement

k Coefficient

k_x Relative neutral axis depth

k_z Relative lever arm

n Exponent

x Neutral axis depth

x_{si} Distance of steel bar *i* from the x axis

x,y,z Coordinates

y_i Distance from the neutral axis

y_{si} Distance of steel bar i from the y axis

z Lever arm of internal forces

Greek lower case letters

α_c Relative compressive force in concrete under compression

β_c Relative distance of point of application of compressive force from the outer most concrete fiber under compression

δ Redistribution ratio

ε_c Compressive strain in the concrete

ε_{c2} Peak compressive strain in the concrete

ε_{ci} Compressive strain at distance yi

ε_{cm} Compressive strain in the most compressed fiber

ε_{cu2} Ultimate compressive strain in the concrete

ε_{s1} Strain in the tensile reinforcement

ε_{s2} Strain in the compression reinforcement

ε_{ud} Design strain of reinforcement

ε_{uk} Characteristic strain of reinforcement steel at maximum load

ε_{yd} Design yield strain of reinforcement

θ Angle from the positive direction of the neutral axis to the + X axis

λ Angle between eccentricities in x and y direction

μ_{Sd} Relative moment resistance

$\mu_{Sd,lim}$ Limiting relative moment resistance

σ_c Compressive stress in the concrete

σ_{cu} Compressive stress in the concrete at the ultimate compressive strain ε_{cu}

$\sigma_{cd,s2}$ Concrete stress at the compression reinforcement level,

σ_{s2} Stress in the compression reinforcement

ψ Distance from the neutral axis

$\mu_{Sd,x}$ Normalized moment ratio in the x direction

$\mu_{Sd,y}$ Normalized moment ratio in the y direction

v_{sd} Normalized axial load ratio

ω Mechanical reinforcement ratio / Tensile reinforcement ratio

ω' Compression reinforcement ratio

ω_{lim} Limiting tensile reinforcement ratio

CHAPTER ONE

1. Introduction

1.1 Nature of the problem

In the developed world, the applicability of high-strength concrete (HSC) in structures is increasing as it allows efficient utilization of space and enhances structural performance. Structures made of HSC can put in service at much earlier age. In high-rise buildings, column size can be reduced significantly without affecting the lateral stability of the building to achieve a maximum usable floor area. HSC can also be used to build superstructures of long-span bridges when required to have enhanced durability, modulus of elasticity and flexural strength. The ingredients of HSC are readily available and can be produced by professionals on site or batching plants by mixing Portland cement, sand and aggregates at very low water-cement ratio with the aid of admixtures. The production of HSC has enabled man kind to construct structures which were taught to be impossible by using normal strength concrete (NSC). Recognizing these benefits, the demand for using HSC is also evolving in the Ethiopian construction industry as well.

According to the American Concrete Institute (ACI) state of the art report on HSC revised and reissued in 1997[1], HSC is defined as concrete with specified compressive strength over 41 MPa. The report also recognizes that the definition of high strength varies on a geographical basis. According to the state of the art report on HSC in Japan [2], concrete strength in excess of 60 MPa in the field of building engineering is defined as HSC. Currently, in our building code EBCS-2 1995: Part I [3], there is no clear demarcation between NSC and HSC. However, for this study, concrete strength in excess of 60 MPa is defined as HSC.

Several experiments done all over the world suggested that most engineering properties of concrete improve with increased strength but some characteristics need special attention when it comes to design HSC structures. The mechanical properties of HSC under compression, shear and tension are different from that of NSC. An analytical study by Ibrahim and MacGregor [4] showed that the ACI 318M-11 [5] rectangular stress block, which is very similar to the stress block given in EBCS-2 1995: Part I [3] overestimate moment capacity of HSC. They have also proposed a modified stress block parameter α_1 and β_1 for use in HSC. Other point worth mentioning is that Stress-Strain relationship for HSC and NSC are quite different. The ascending branch of the stress-strain relationship is steeper for HSC, indicating higher elastic modulus. It changes from approximately a second-order parabola for concretes within the normal strength range to almost a straight line. Thus, predicting the capacities of reinforced HSC members by mere extrapolation of the equations developed for NSC is unsafe. The abovementioned facts clearly indicate the urgency for revision and inclusion of stress blocks for HSC and the associated design aids in EBCS-2 1995: Part I [3] and EBCS-2 1995: Part II [6] respectively.

Therefore, taking into account these basic fundamental difference between NSC and HSC, most codes nowadays have incorporated design guides for reinforced HSC. To this regards, EBCS-2 1995: Part I [3], which is under revision allows the use of HSC for reinforced concrete members. Therefore design guides and design aids for HSC need to be developed and incorporated in the new code as well.

1.2 Objective and scope of the study

The main objective of the study focuses on preparing design aids for estimating the ultimate capacity of (solid rectangular) reinforced HSC section under flexure and compression based on mechanical properties of HSC given in the EN 1992-1-1:2004 [7]. The choice of the EN 1992-1-1:2004 [7] is justifiable because the current revision of the Ethiopian Building Code Standard bases this code as its foundation. Therefore, the current study aims to develop design tables and general design charts for NSC and HSC sections under flexure and uniaxial and biaxial interaction charts for columns made of NSC and HSC to conform to the revised building code standards.

The design aids developed for reinforced concrete beams can be used for single and double reinforced rectangular section. The design aids can be directly used for flanged section in which the neutral axis lies within the flange or in which the compression zone is rectangular. However, when the compression zone is not rectangular, the design aids cannot be used directly and require modifications. This type of problem is not covered in this study. The design aids are prepared for cover to reinforcement ratios of 0.05, 0.10 and 0.15 for compression reinforcement. The design aids can also be used for section analysis of rectangular sections.

The uniaxial and biaxial interaction charts for columns presented in this study cover only solid rectangular section. Also the methodology given in this study can be extended to cover rectangular hollow sections, circular, circular hollow section, L sections and any arbitrary shaped section with arbitrary arrangement of reinforcement, this study presents only interaction charts with uniformly distributed reinforcement with cover ratio of 0.10.

1.3 Methodology

Reinforced concrete structures are designed such that the ultimate load carrying capacity of the section is not exceeded when the structure is carrying the anticipated ultimate design load and does not have excessive crack and deflection while carrying service loads. This design philosophy which uses probability for load and materials property is known as limit state design (LSD). The current building codes thoroughly discuss design consideration and presents design aids for NSC. Unfortunately, design guides and design aids for HSC are scanty. To this end, the current code has completed its final stages of revision to include considerations to be taken when using HSC in structural members. However design aids for estimating the ultimate capacity of section made of HSC are not available.

Exact method of cross section analysis based on stress-strain compatibility and equilibrium is used for estimating the ultimate capacity of section subjected to pure flexure and compression accompanied by bending. Stress-strain relationship for concrete under compression are adopted from EN 1992-1-1:2004 [7] to cover classes from C12/15 – C90/105. Similar relationship for reinforcing steel with the effect of strain hardening is also adopted from the same code. Green's Theorem is used to transform double integrals to single line integrals along a closed and reducible boundaries. Iterative numerical procedures such as Multiple Application of Simpson's 1/3 Rule and Third Order Gauss Quadrature are used for evaluating line integrals. To systematical prepare the design aids and interaction charts, a computer program is compiled with C^{++} language.

CHAPTER TWO

2. Ultimate Limit State of Reinforced Concrete Sections

2.1 Introduction

Reinforced concrete members which are the fundamental building blocks of almost all reinforced concrete structures have unique load carrying capacity. If these load carrying capacities whether the structural member is slab, beam, column or wall is exceeded; failure to part of the structure or the whole structure is unavoidable. If either of the above case does occur, we say that the structure is unfit for use. In the study of reinforced concrete structures, structures are designed such that the ultimate load carrying capacity of the section is not exceeded when the structure is carrying the anticipated ultimate design load. This design philosophy which uses probability for load and materials property is known as limit state design philosophy. In limit state design philosophy, designer should make sure that ultimate load carrying capacity of the element is not exceeded at any time for the expected design loads and the structure does not have excessive crack and deflection while carrying service loads. The first philosophy is referred to as Ultimate Limit State (ULS) while the latter is known as Serviceability Limit State (SLS).

This chapter is dedicated to reviewing basic assumption taken when carrying out cross section analysis of beam or column in the ULS. The analysis procedure which takes into account the material non-linearity for concrete and reinforcement, the interaction between concrete and reinforcements as well as the behavior of concrete under tension is discussed. The assumption for the stress-strain diagrams for concrete and reinforcement bars which are taken from EN 1992-1-1:2004 [7] are presented. Although the main objective of this study aims at generating design aid for reinforced HSC section, reinforced NSC section are also discussed to include the additional strength gain due to strain hardening of the reinforcement steels.

2.2 Basic Assumption in the Ultimate Limit State

Whether analysis or design is carried out in the ULS, assumptions have to be taken so as to accurately capture the behaviors of the elasto-plastic material that results from the interaction of concrete and reinforcement bars due to bonding. These fundamental assumptions are the same whether the structural member is a slab, beam or column. The assumptions originate from flexural theory which are,

1. Section perpendicular to axis of bending which are plane remain plane after bending,
2. Strain in reinforcement bar is equal to strain in the concrete at the same level if relative slip between concrete and reinforcement bars is avoided by using standard ribs and
3. Stress in both concrete and reinforcement steel can be calculated from the idealized stress-strain curves.

The above three assumption can be sufficient to calculate section capacities. However, additional assumption can be taken to further simplify the analysis process without losing accuracy. These additional assumptions taken vary from code to code. The following additional assumptions taken for this study are in accordance to EN 1992-1-1:2004 [7] and are stated below.

1. Tensile strength of concrete is neglected,
2. Stress-strain curve for concrete under compression assumes parabolic-rectangle diagram for concrete under compression found in EN 1992-1-1:2004 [7] and is shown in Figure 2-1,
3. Stress-strain curves for reinforcing steel assumes design stress-strain curve for reinforcement bars with inclined top branch found in EN 1992-1-1:2004 [7] and is shown in Figure 2-2,
4. The maximum tensile strain in reinforcing steel is assumed to be 25‰ and
5. The strain diagrams are assumed to pass through either point A, B or C which are shown in Figure 2-3.

Failure of reinforced concrete member occurs when the strain in the extreme fiber reaches the ultimate compressive strain of concrete or when the strain in the reinforcement steel exceeds the fracturing strain. The total compression force in concrete can be calculated from strain distribution, referred to as stress blocks, that are derived from cylinder tested in uniaxial compression. EN 1992-1-1:2004 [7] gives a parabolic-rectangular stress block expressed by the following equation.

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right], \quad \text{for } 0 \le \varepsilon_c \le \varepsilon_{c2} \tag{2.1}$$

$$\sigma_c = f_{cd}, \quad \text{for } \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \tag{2.2}$$

where: n is the exponent according to Table 2-1

ε_{c2} is the peak strain

ε_{cu2} is the ultimate strain at peak stress

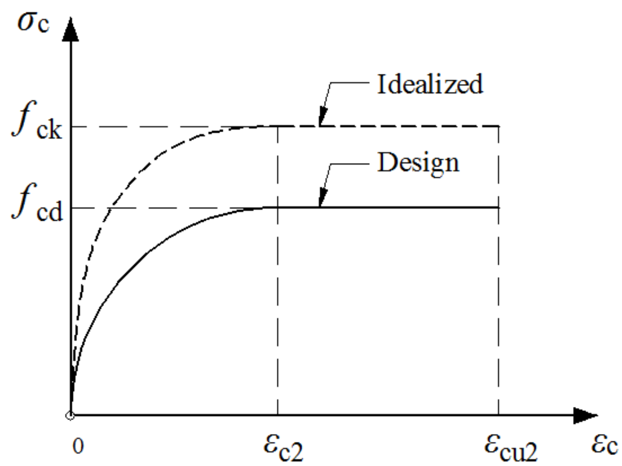


Figure 2-1 Parabolic-rectangle diagram for concrete under compression

The stress-strain diagram for reinforcing steel is also given in EN 1992-1-1:2004 [7]. The code gives two types of stress-strain diagram for section design which are shown in Figure 2-2. The first assumption which does not take into account the strength gain due to strain hardening, does not limit the strain in the reinforcing steel when it reaches ULS. However, the second assumption takes into account the strength gain due to strain hardening, by limiting the maximum strain to ε_{uk} for different ductility class of reinforcing steel.

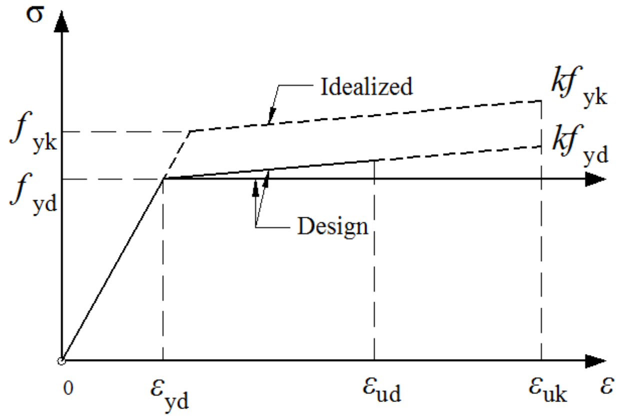


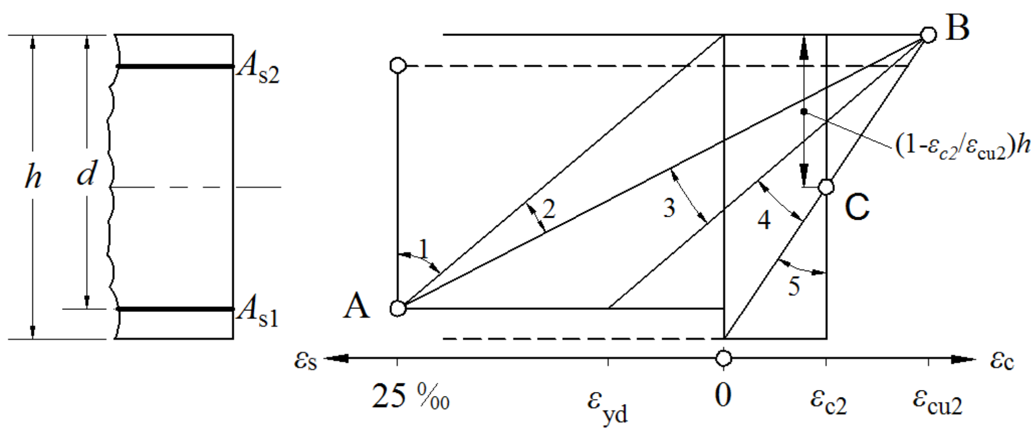
Figure 2-2 Idealized and design stress-strain diagram for reinforcing steel

EN 1992-1-1:2004 [7] gives minimum values of maximum strain ε_{uk} as 25‰, 50‰ and 75‰ corresponding to Class A, Class B and Class C reinforcing steel respectively. In this study, the ultimate strain is limited to 25‰ by assuming Class A reinforcing steel which has the lowest ductility. This upper limit of strain is selected because the author believes that higher classes of reinforcing steel (Class B and C) are very difficult to obtain in the Ethiopian market.

Table 2-1 Strength and deformation characteristics for concrete

Strength classes for concrete														
Class	**12/15**	**16/20**	**20/25**	**25/30**	**30/37**	**35/45**	**40/50**	**45/55**	**50/60**	**55/67**	**60/75**	**70/85**	**80/95**	**90/105**
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90
f_{cu} (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105
f_{ctm} (MPa)	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.8	4.1	4.2	4.4	4.6	4.8	5.0
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44
ε_{c2} (‰)	2.0									2.2	2.3	2.4	2.5	2.6
ε_{cu2} (‰)	3.5									3.1	2.9	2.7	2.6	2.6
n	2.0									1.75	1.6	1.45	1.4	1.4

After establishing stress-strain diagram for concrete and reinforcing steel it is possible to calculate force carried by each constitute elements by assuming proper strain distributions in the ULS. The strain distribution in the ULS shown in Figure 2-3 applies to beams, slabs and columns in which section remain plane before and after bending. In the figure, region (1) represents when the whole section is under tension and it can be assumed that all the tensile force is carried by reinforcing steel alone. Region (2), (3) and (4) represents when the section is under bending which the neutral axis lies within the section. Region (5) when the neutral axis is outside the section indicates that the section is under pure compression. Therefore, by varying the neutral axis depth and using possible ranges of strain distribution in the ULS, the analysis of any given section can be done by applying stress-strain compatibility and equilibrium principles.



A – Reinforcing steel tension strain limit

B – Concrete compression strain limit

C – Concrete pure compression strain limit

Figure 2-3 Possible strain distribution in the ultimate limit state

2.3 Analysis of Ultimate Limit Sate for Reinforced Concrete Section

Two basic requirements are satisfied throughout the analysis and design of reinforced concrete beams and columns [8]. They are,

1. Stress and strain compatibility and
2. Equilibrium.

The first point states that stress at any point should correspond to the strain at that point. This will enable as to calculate stress based on strain distribution. The second requirement states that internal forces should balance external load effects. Therefore, one can use equilibrium to calculate force carried by each constitute element.

When carrying out section analysis or design, for cross-section not fully under compression, the concrete is assumed to fail when the strain at extreme fiber reaches ε_{cu2} as given in Table 2-1 for different classes of concrete. However, for concrete under pure compression, the strain is limited to ε_{c2} as given in Table 2-1.

Setting strain limits for both constitute materials, force resultants are calculated based on the strain distributions assumed in the ULS. Equations can be developed for estimating relative compressive force in concrete under compression (α_c) and for calculating relative distance of point of application of compressive force from the outer most concrete fiber under compression (β_c) using stress-strain curve of concrete. Equations for estimating α_c and β_c are given in EBCS-2 1995: Part II [6] for reinforced NSC up to class C 50/60. However, for higher class of concrete, the equations for α_c and β_c on the code becomes useless. The reason for the equation not to work comes from the fact that these equations were derived for $n = 2$ in equation (2.1), i.e. for parabolic only. Therefore new design tables, design charts and interaction charts shall be developed for HSC.

In Chapter three, ultimate limit state of reinforced concrete beams is discussed briefly. Procedures for developing design table and general design chart are presented for reinforced NSC and HSC solid rectangular sections. Also, design aids have been prepared and presented in the same chapter by developing a program in Visual Studio C^{++}.

CHAPTER THREE

3. Design Aid for Reinforced Concrete Beams

3.1 Introduction

A beam is a structural member that supports applied loads and its own weight primarily by internal moments and shears [8]. Although much of the beams in concrete structures are subjected to moments and shears alone, under rare condition, reinforced concrete beams can also be subjected to significant amount of axial force in addition to moments. The fundamental principles stated in Chapter 2 still applies for analysis of this type of loading as well.

Reinforced concrete beam can reach ULS in three ways. If the extreme fibers in the concrete under compression reaches ε_{cu2} after the tensile reinforcing steel have yielded, the type of failure resulting is referred as tensile failure. This type of failure is ductile and shows visible warning before failure. The second type of failure occurs when the extreme fibers in the concrete under compression reaches ε_{cu2} before the tensile reinforcing steel yield. The failure resulting is called compression failure. This failure is sudden and brittle. The third type of failure which rarely occurs is called balanced failure. This failure results from yielding of the tensile reinforcement and by crushing of concrete simultaneously.

Failure of reinforced concrete beams by tension failure is preferred because large deflection and wide cracking occurs indicate impending failure. Therefore, reinforced concrete beams are designed for tension failure. EBCS-2 1995: Part I [3] and EN 1992-1-1:2004 [7] imposes tension failure by limits the ratio of neutral axis depth for different percent of plastic moment redistribution. Provisions stated in EN 1992-1-1:2004 [7] Section 5.5 were used for this study and presented below for convince.

$$\begin{aligned} &\delta \geq 0.44 + 1.25(\frac{x_u}{d}) \qquad \text{for } f_{ck} \leq 50\text{MPa} \\ &\delta \geq 0.54 + 1.25(0.6 + 0.0014/\varepsilon_{cu2})(\frac{x_u}{d}) \qquad \text{for } f_{ck} > 50\text{MPa} \end{aligned} \tag{3.1}$$

In clause 5.5(4) of EN 1992-1-1:2004 [7], the redistribution factor δ in equations (3.1) must be greater than or equal to 0.70 when Class B and Class C reinforcement are used. If Class A reinforcement are used the redistribution factor must be greater than or equal to 0.80. The above limits are employed in the preparation of the design tables and general design charts.

3.2 Formulation of Expressions for Force Resultants

When dealing with the ULS of reinforced concrete beams, developing new expression for α_c and β_c for reinforced HSC will be difficult because the values of ε_{c2}, ε_{cu2} and n changes as the compressive strength of the concrete increases. The solution for this problem will be employing numerical integration to obtain the total force in the concrete under compression. Therefore, an algorithm was developed to uses Newton-Cotes Closed Integration Formulas. Multiple Application of Simpson's 1/3 Rule with n = 1000 segments was used in particular to accurately estimate the compressive force in the concrete. Similar algorithm was also developed to calculate β_c.

Depending on the location of the neutral axis, two general cases can be observed.

Case 1: The neutral axis within the section

For the neutral axis inside the section, simple expressions for relating the strain in concrete with the strain in reinforcement steel can be derived from similarity of triangle. Referring to Figure 3-1, the strain at any point in concrete can be related to a known strain in reinforcement steel or concrete. Similarly, the strain in the reinforcement steel can be obtained from known value of strain in concrete.

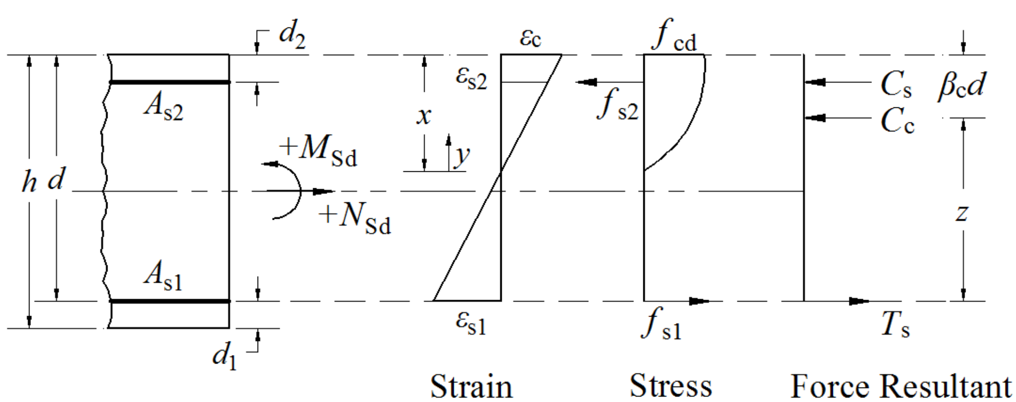


Figure 3-1 Stress and strain distribution for neutral Axis within the section

If we denotes ε_{ci} as the strain in concrete at distance y_i from the neutral axis and also ε_{cm} as the maximum strain in concrete at the most compressed fiber, the expression for strain at any level of the cross section for a known maximum strain of ε_{cm} becomes,

$$\varepsilon_{ci} = \frac{y_i}{x}\varepsilon_{cm} \tag{3.2}$$

Also the expression for strain at any level of the cross section for a known strain of the tension reinforcing steel ε_{s1} becomes,

$$\varepsilon_{ci} = \frac{y_i}{d-x}\varepsilon_{s1} \tag{3.3}$$

The stress at every point in the compressed concrete can be calculated from equation (2.1) or (2.2) by substituting the value of ε_{ci} from equations (3.2) or (3.3) as,

$$\sigma_c(\varepsilon_{ci}) = f_{cd}\left[1-\left(1-\frac{\varepsilon_{ci}}{\varepsilon_{c2}}\right)^n\right], \quad \text{for } 0 \le \varepsilon_{ci} \le \varepsilon_{c2} \tag{3.4}$$

$$\sigma_c(\varepsilon_{ci}) = f_{cd}, \quad \text{for } \varepsilon_{c2} \le \varepsilon_{ci} \le \varepsilon_{cu2} \tag{3.5}$$

After finding the strain at every point in the compressed zone, the total compressive force can be calculated by integrating equation (3.4) or (3.5) over the compressed area as,

$$C_c = \int_0^x \sigma_c(\varepsilon_{ci})dA \tag{3.6}$$

Numerically integrating equations (3.6) over the neutral axis depth x, the total compressive force in the concrete becomes,

$$C_c = x\frac{\sigma_c(\varepsilon_{c0}) + 4\sum_{i=1,3,5}^{n-1}\sigma_c(\varepsilon_{ci}) + 2\sum_{i=2,4,6}^{n-2}\sigma_c(\varepsilon_{ci}) + \sigma_c(\varepsilon_{cm})}{3n} \tag{3.7}$$

The relative compressive force α_c in the concrete under compression can be obtained from the equation (3.7) by simply dividing it by the term $f_{cd}bd$. Therefore, the relative compressive force in the concrete under compression become,

$$\alpha_c = \frac{C_c}{f_{cd}bd} \tag{3.8}$$

Similarly, the expression for β_c can be derived by applying fundamental principles of statics for locating point of application of the resultant force,

$$\beta_c = x - \frac{C_{cc}}{C_c} \tag{3.9}$$

Where the value C_{cc} is given by,

$$C_{cc} = \int_0^x \sigma_c y_i dA$$

$$C_{cc} = x \frac{\sigma_c(\varepsilon_{c0})y_0 + 4\sum_{i=1,3,5}^{n-1}\sigma_c(\varepsilon_{ci})y_i + 2\sum_{i=2,4,6}^{n-2}\sigma_c(\varepsilon_{ci})y_i + \sigma_c(\varepsilon_{cm})y_m}{3n} \tag{3.10}$$

Case 2: The neutral axis outside the section

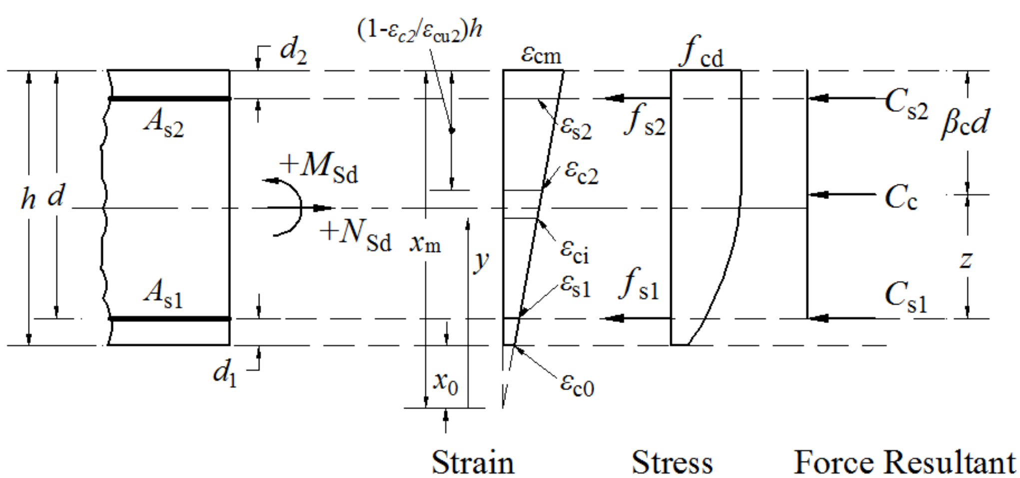


Figure 3-2 Stress and strain distribution for neutral axis outside the section

The concrete strain ε_{ci} at distance y_i from the neutral axis for a known peak strain, ε_{c2} becomes,

$$\varepsilon_{ci} = \frac{y_i}{x_m - (1 - \varepsilon_{c2}/\varepsilon_{cu2})h}\varepsilon_{c2} \tag{3.11}$$

Similarly, the strain in concrete for the least compressed fiber ε_{c0} can be expressed by,

$$\varepsilon_{c0} = \frac{x_m - h}{x_m - (1 - \varepsilon_{c2}/\varepsilon_{cu2})h}\varepsilon_{c2} \tag{3.12}$$

The total compressive force can be calculated by integrating equation (2.1) or (2.2) over the compressed area by substituting the values of ε_{ci} from equation (3.11) as,

$$C_c = \int_{x_0}^{x_m} \sigma_c(\varepsilon_{ci})dA$$

$$C_c = h\frac{\sigma_c(\varepsilon_{c0}) + 4\sum_{i=1,3,5}^{n-1}\sigma_c(\varepsilon_{ci}) + 2\sum_{i=2,4,6}^{n-2}\sigma_c(\varepsilon_{ci}) + \sigma_c(\varepsilon_{cm})}{3n} \tag{3.13}$$

The relative compressive force in the concrete under compression takes the form of equation (3.8). Similarly, the expression for β_c can be derived from equation (3.9) by modifying the term C_{cc} in equation (3.9). Therefore, the expression for C_{cc} becomes,

$$C_{cc} = \int_{x_o}^{x_m} \sigma_c y_i dA$$

$$C_{cc} = h\frac{\sigma_c(\varepsilon_{c0})y_0 + 4\sum_{i=1,3,5}^{n-1}\sigma_c(\varepsilon_{ci})y_i + 2\sum_{i=2,4,6}^{n-2}\sigma_c(\varepsilon_{ci})y_i + \sigma_c(\varepsilon_{cm})y_m}{3n} \tag{3.14}$$

Therefore by using the relationships developed for α_c and β_c, design tables and design charts can be formulated and presented in a systematic way.

3.3 General Design Chart for Reinforced Concrete Beams

The preparation of the general design chart is a very simple and straight forward problem. If a singly reinforced concrete beam subjected to moment and axial load is shown in Figure 3-3, expressions for the moment capacity of the section can be easily developed by applying stress and strain compatibility and equilibrium principles.

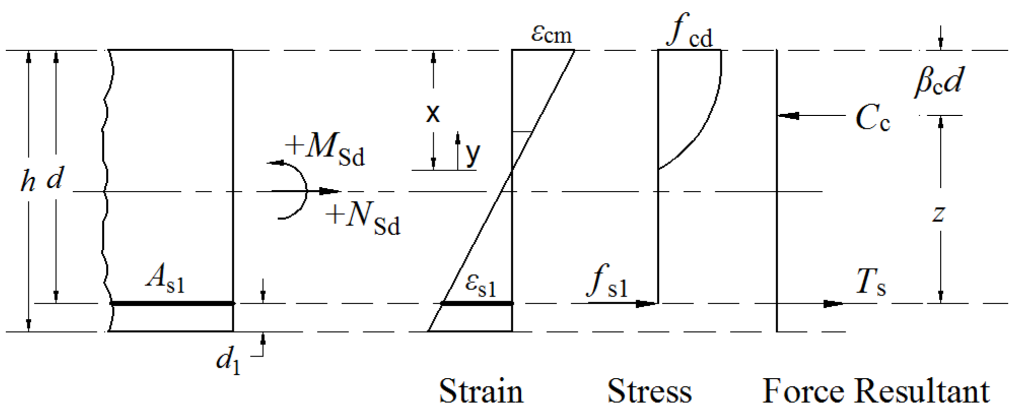


Figure 3-3 Singly reinforced beam subjected to moment

Referring to Figure 3-3, the moment resistance M_{Sd} of the cross-section about tensile reinforcement without compression reinforcement becomes,

$$M_{Sd} = C_c(d - \beta_c d) \quad (3.15)$$

$$M_{Sd} = \alpha_c f_{cd} b d^2 (1 - \beta_c) \quad (3.16)$$

In equation (3.16), if we rearrange such that the known terms for design are on the left side, hence giving a dimension less value on the right hand side, the above equation has a form,

$$\mu_{Sd} = \frac{M_{Sd}}{f_{cd} b d^2} = \alpha_c (1 - \beta_c) \quad (3.17)$$

The term μ_{Sd} is known as the relative moment resistance (without compression reinforcement). It was proposed for the first time by Rüsch [6].

In order to insure tension failure, upper limits of μ_{Sd} for 0%, 10% and 20% moment redistribution are included in the General Design Chart as $\mu_{Sd,lim.}$ If these limits shown on the chart by a vertical dashed line are exceeded, tension failure can be achieved by providing compression reinforcements.

A computer program was developed to calculate the values of strain in the most compressed concrete fiber (ε_c), strain in the tensile reinforcement (ε_{s1}), strain in the compression reinforcement (ε_{s2}), relative compressive force (α_c), relative lever arm (β_c), relative moment resistance (μ_{Sd}), limiting relative moment resistance ($\mu_{Sd,lim}$), relative neutral axis depth (k_x) and relative lever arm (k_z) by applying stress-strain compatibility and equilibrium. The program calculates α_c, β_c, μ_{Sd}, k_x and k_z by selecting different possible strain distribution in the ULS.

The general design chart can be applied directly to any design situation and class of works, as it contains no built-in safety factors. The chart generated by the program was verified to the General Design Chart given in EBCS-2 1995: Part II [6] for NSC. The verification is given in section 3.5. General Design Charts for HSC were generated using the same program by varying the concrete stress-strain diagram. A new General Design Chart for NSC was also developed by setting the maximum tensile strain in the tensile reinforcement to be 25‰.

The areas of reinforcement required are determined from the following equations:

If $\mu_{Sd} \leq \mu_{Sd,lim}$ then compression reinforcement is not required and

$$A_{s1} = \frac{1}{f_{yd}}\left(\frac{M_{Sd}}{z} + N_{Sd}\right) \tag{3.18}$$

If $\mu_{Sd} > \mu_{Sd,lim}$, then compression reinforcement is required and

$$A_{s2} = \frac{1}{\sigma_{s2}}\left(\frac{M_{Sd} - M_{Sd,lim}}{d - d_2}\right) \tag{3.19}$$

$$A_{s1} = \frac{1}{f_{yd}}\left(\frac{M_{Sd,lim}}{z} + \frac{M_{Sd} - M_{Sd,lim}}{d - d_2} + N_{Sd}\right) \tag{3.20}$$

When applying equation (3.19) in HSC, double counting of the area displaced by the compression reinforcement should be taken into account. This is because the concrete stress are significantly higher than NSC. This can be achieved by simply subtracting the concrete stress at the compression reinforcement level, $\sigma_{cd,s2,}$ from the stress in the compression reinforcement, σ_{s2}.

3.4 Design Table for Reinforced Concrete Beams

The program developed for generating points on the General Design Chart was used to develop design tables for both NSC and HSC. This method employs a dimensionless approach for developing the design table. It involves separating the known design terms with the unknown variables that comes from integration of the stress block. Here also factor of safety are not built-in the design table which makes it applicable to any design situation unlike the dimension bound design tables A and B in EBCS-2 1995: Part II [6]. The derivations for the design table can be summarized as follows.

Referring to section 3.3, if we use the same expression for μ_{Sd} as in equation (3.17), the area of reinforcement can be calculated for known values of μ_{Sd} and β_c with a very simple mathematical manipulations.

Referring to Figure 3-3, the moment resistance of the cross-section about centroid of the compressed zone without compression reinforcement becomes,

$$M_{Sd} = A_{s1} z f_{yd} \tag{3.21}$$

Rearranging and substituting M_{Sd} with expression from equation (3.17), the area of reinforcement required for equilibrium without compression reinforcement becomes,

$$A_{s1} = \frac{M_{Sd}}{z f_{yd}} = \frac{\mu_{Sd} f_{cd} b d^2}{f_{yd} d(1-\beta_c)} \tag{3.22}$$

If we collect the dimensionless values on the right side, if the resulting value is referred as ω,

$$\omega = \frac{A_{s1} f_{yd}}{f_{cd} b d} = \frac{\mu_{Sd}}{(1-\beta_c)} = \alpha_c \tag{3.23}$$

Therefore by tabulating the different values of μ_{Sd} and ω for different ratio of neutral axis depth, a design table shown in Table 3-1 without built-in factor of safety can be developed.

Table 3-1 Design Table for reinforced NSC

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd}bd^2}$	$\omega = \frac{A_{s1}f_{yd}}{f_{cd}bd}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	**Percentage Redistribution**
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.030	0.990	0.773	25.000	
0.020	0.020	0.044	0.985	1.146	25.000	
0.030	0.031	0.055	0.980	1.464	25.000	
0.040	0.041	0.066	0.976	1.763	25.000	
0.050	0.051	0.076	0.971	2.060	25.000	
0.060	0.062	0.086	0.967	2.365	25.000	
0.070	0.073	0.097	0.962	2.682	25.000	
0.080	0.084	0.107	0.956	3.009	25.000	
0.090	0.095	0.118	0.951	3.349	25.000	
0.100	0.106	0.131	0.946	3.500	23.294	
0.110	0.117	0.145	0.940	3.500	20.709	
0.120	0.128	0.159	0.934	3.500	18.552	
0.130	0.140	0.173	0.928	3.500	16.726	
0.140	0.152	0.188	0.922	3.500	15.159	
0.150	0.164	0.202	0.916	3.500	13.799	
0.160	0.176	0.217	0.910	3.500	12.608	
0.170	0.188	0.232	0.903	3.500	11.555	
0.180	0.201	0.248	0.897	3.500	10.618	
0.190	0.213	0.264	0.890	3.500	9.777	
0.200	0.226	0.280	0.884	3.500	9.019	
0.205	0.233	0.288	0.880	3.500	8.653	20%
0.210	0.239	0.296	0.877	3.500	8.332	
0.220	0.253	0.312	0.870	3.500	7.706	
0.230	0.266	0.329	0.863	3.500	7.132	
0.240	0.280	0.346	0.856	3.500	6.605	
0.250	0.295	0.364	0.849	3.500	6.118	
0.252	0.298	0.368	0.847	3.500	6.011	10%
0.260	0.309	0.382	0.841	3.500	5.667	
0.270	0.324	0.400	0.834	3.500	5.247	
0.280	0.339	0.419	0.826	3.500	4.856	
0.290	0.355	0.438	0.818	3.500	4.490	
0.295	0.363	0.448	0.814	3.500	4.313	0%

It should be noted that the design table is a manipulation of the General Design Chart and can be applied to any design situations. The shaded rows in the design table indicate the limit for tensile failure of a selected amount of redistribution. If the values of μ_{sd} above these limits are obtained, compression reinforcement shall be provided to make the mode of failure tensile. The moment capacity can then be calculated by assuming two superimposed section as shown in Figure 3-4.

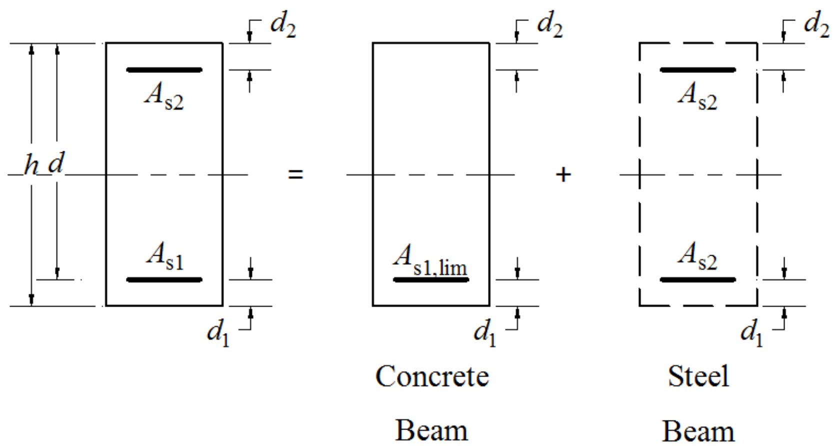


Figure 3-4 Moment Capacity of Doubly reinforced section

The concrete beam in Figure 3-4 is made to carry a maximum moment of a singly reinforced section. The moment over and above the capacity of singly reinforced section is to be carried by the additional compression reinforcement, A_{s2}. The capacity of the steel beam in Figure 3-4 can be obtained by taking moment about the tensile reinforcement,

$$M_{\mathrm{Sd}} - M_{\mathrm{lim}} = A_{s2}\sigma_{s2}(d - d_2) \tag{3.24}$$

$$\mu_{\mathrm{Sd}} f_{\mathrm{cd}} bd^2 - \mu_{\mathrm{Sd,lim}} f_{\mathrm{cd}} bd^2 = A_{s2}\sigma_{s2} d(1 - d_2/d) \tag{3.25}$$

$$\mu_{\mathrm{Sd}} - \mu_{\mathrm{Sd,lim}} = \frac{A_{s2}\sigma_{s2} d(1 - d_2/d)}{f_{\mathrm{cd}} bd^2} = \frac{A_{s2}\sigma_{s2}(1 - d_2/d)}{f_{\mathrm{cd}} bd} \tag{3.26}$$

Rearranging and defining ω' as,

$$\omega' = \frac{A_{s2}\sigma_{s2}}{f_{cd}bd} = \frac{\mu_{sd} - \mu_{sd,lim}}{(1 - d_2/d)} \quad (3.27)$$

Therefore, the area of steel required for the steel beam can be calculated from,

$$A_{s2} = \omega' bd \frac{f_{cd}}{\sigma_{s2}} \quad (3.28)$$

It should be noted that σ_{s2} in equation (3.28) should be taken as f_{yd} if the compression reinforcement has yielded. The strain in the compression reinforcement can be easily calculated from equation (3.29).

$$\varepsilon_{s2} = \left(1 - \frac{d_2/d}{k_x}\right)\varepsilon_c \quad (3.29)$$

When applying equation (3.28) in HSC, double counting of the area displaced by the compression reinforcement should be taken into account. This is because the concrete stress are significantly higher than NSC. This can be achieved by simply subtracting the concrete stress at the compression reinforcement level, $\sigma_{cd,s2}$, from the stress in the compression reinforcement, σ_{s2}.

Also the area of steel required for the concrete beam is given by,

$$A_{s1,lim} = \omega_{lim} bd \frac{f_{cd}}{f_{yd}} \quad (3.30)$$

Superimposing the concrete and steel beam, the area of tensile reinforcement required for the doubly reinforced beam becomes,

$$A_{s1} = (\omega_{lim} + \omega')bd \frac{f_{cd}}{f_{yd}} \quad (3.31)$$

The values of $\mu_{Sd,lim}$ and ω_{lim} for 0%, 10% and 20% plastic moment redistribution is also provided with the design table and is shown in Table 3-2 for reinforced NSC.

Table 3-2 Limiting values for singly reinforced sections

Percentage Redistribution %	δ	$\mu_{sd,lim}$	ω_{lim}	$k_{x,lim}$	$k_{z,lim}$
0	1.00	0.295	0.366	0.448	0.814
10	0.90	0.252	0.298	0.368	0.847
20	0.80	0.205	0.233	0.288	0.880

The step to be taken to when designing beams using design tables are summarized in Table 3-3.

Table 3-3 Steps to be taken when using Design Tables

- Calculate $\mu_{Sd} = \dfrac{M_{Sd}}{f_{cd} b d^2}$
- If $\mu_{Sd} \leq \mu_{Sd,lim}$, then compression reinforcement is not required and

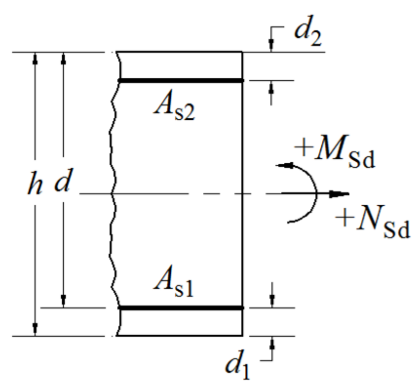
$$A_{s1} = \frac{1}{f_{yd}}\left(\omega \cdot b \cdot d \cdot f_{cd} + N_{Sd}\right)$$

- If $\mu_{Sd} > \mu_{Sd,lim}$, then compression reinforcement is required and

$$\omega' = \frac{\mu_{sd} - \mu_{sd,lim}}{(1 - d_2/d)}$$

$$A_{s2} = \omega' b d \frac{f_{cd}}{\sigma_{s2}}$$

$$A_{s1} = \frac{1}{f_{yd}}\left(\left(\omega_{lim} + \omega'\right) \cdot b \cdot d \cdot f_{cd} + N_{Sd}\right)$$



3.5 Verification

The accuracy of the algorithm was checked by comparing the values of α_c and β_c in this study with the values of α_c and β_c calculated from formulae given in EBCS-2 1995: Part

II [6]. The formulae are derived for NSC with ε_{c2}, ε_{cu2} and n of 2.0‰, 3.5‰ and 2 respectively and are stated below in equations (3.32 – 3.37).

(i) $\varepsilon_{cm} \leq \varepsilon_{c2}$ and neutral axis within section

$$\alpha_c = \frac{\varepsilon_{cm}}{12}\left(6 - \varepsilon_{cm}\right)k_x \tag{3.32}$$

$$\beta_c = \frac{8 - \varepsilon_{cm}}{4\left(6 - \varepsilon_{cm}\right)}\, k_x \tag{3.33}$$

(ii) $\varepsilon_{cm} > \varepsilon_{c2}$ and neutral axis within section

$$\alpha_c = \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}}\, k_x \tag{3.34}$$

$$\beta_c = \frac{\varepsilon_{cm}\left(3\varepsilon_{cm} - 4\right) + 2}{2\varepsilon_{cm}\left(3\varepsilon_{cm} - 2\right)}\, k_x \tag{3.35}$$

(iii) $\varepsilon_{cm} > \varepsilon_{c2}$ and neutral axis outside of section

$$\alpha_c = \frac{1}{189}\left(125 + 64\varepsilon_{cm} - 16\varepsilon_{cm}^2\right) \tag{3.36}$$

$$\beta_c = 0.5 - \frac{40}{7} \cdot \frac{\left(\varepsilon_{cm} - 2\right)^2}{125 + 64\varepsilon_{cm} - 16\varepsilon_{cm}^2} \tag{3.37}$$

The yield strength f_{yd} and the ultimate rupture strain ε_{s2} in the steel bars were taken to be 460 MPa and 10‰ respectively. As it can be seen from Table 3-4, the percentage difference is negligible. Therefore, this way of generating design aid is valid.

Table 3-4 Verification for the values α_c and β_c

k_x	ε_c [‰]	ε_{s1} [‰]	Zerayohannes [6]		This Study		% difference	
			[1] α_c	[2] β_c	[3] α_c	[4] β_c	$\left\lvert\frac{[1]-[3]}{[1]}\right\rvert\times 100$	$\left\lvert\frac{[2]-[4]}{[2]}\right\rvert\times 100$
0.00	0.00	10.00	0.000	0.000	0.000	0.000	0.0000	0.0000
0.04	0.42	10.00	0.008	0.014	0.008	0.014	0.0001	0.0001
0.08	0.87	10.00	0.030	0.028	0.030	0.028	0.0002	0.0000
0.12	1.36	10.00	0.063	0.043	0.063	0.043	0.0003	0.0000
0.16	1.90	10.00	0.104	0.060	0.104	0.060	0.0000	0.0000
0.20	2.50	10.00	0.147	0.078	0.147	0.078	0.0002	0.0000
0.24	3.16	10.00	0.189	0.098	0.189	0.098	0.0001	0.0000
0.28	3.50	9.00	0.227	0.116	0.227	0.116	0.0001	0.0004
0.32	3.50	7.44	0.259	0.133	0.259	0.133	0.0001	0.0002
0.36	3.50	6.22	0.291	0.150	0.291	0.150	0.0001	0.0001
0.40	3.50	5.25	0.324	0.166	0.324	0.166	0.0001	0.0003
0.44	3.50	4.45	0.356	0.183	0.356	0.183	0.0001	0.0001
0.48	3.50	3.79	0.389	0.200	0.389	0.200	0.0001	0.0001
0.52	3.50	3.23	0.421	0.216	0.421	0.216	0.0001	0.0002
0.56	3.50	2.75	0.453	0.233	0.453	0.233	0.0001	0.0001
0.60	3.50	2.33	0.486	0.250	0.486	0.250	0.0001	0.0001
0.64	3.50	1.97	0.518	0.266	0.518	0.266	0.0000	0.0002
0.68	3.50	1.65	0.550	0.283	0.550	0.283	0.0000	0.0001
0.72	3.50	1.36	0.583	0.299	0.583	0.299	0.0000	0.0001
0.76	3.50	1.11	0.615	0.316	0.615	0.316	0.0000	0.0001
0.80	3.50	0.88	0.648	0.333	0.648	0.333	0.0000	0.0000
0.84	3.50	0.67	0.680	0.349	0.680	0.349	0.0000	0.0001
0.88	3.50	0.48	0.712	0.366	0.712	0.366	0.0000	0.0001
0.92	3.50	0.30	0.745	0.383	0.745	0.383	0.0000	0.0000
0.96	3.50	0.15	0.777	0.399	0.777	0.399	0.0000	0.0001
1.00	3.50	0.00	0.810	0.416	0.810	0.416	0.0000	0.0001
1.04	3.50	-0.13	0.842	0.433	0.842	0.433	0.0000	0.0000
1.08	3.43	-0.25	0.869	0.447	0.869	0.447	0.0037	0.0050
1.12	3.34	-0.36	0.890	0.457	0.890	0.457	0.0078	0.0102
1.16	3.27	-0.45	0.907	0.466	0.907	0.466	0.0109	0.0137
1.20	3.20	-0.53	0.922	0.473	0.922	0.473	0.0137	0.0168

3.6 General Design Charts and Design Tables

General design charts and design tables prepared as presented in section 3.3 and section 3.4 are given in this section for concrete classes C 12/15 up to C 90/105.

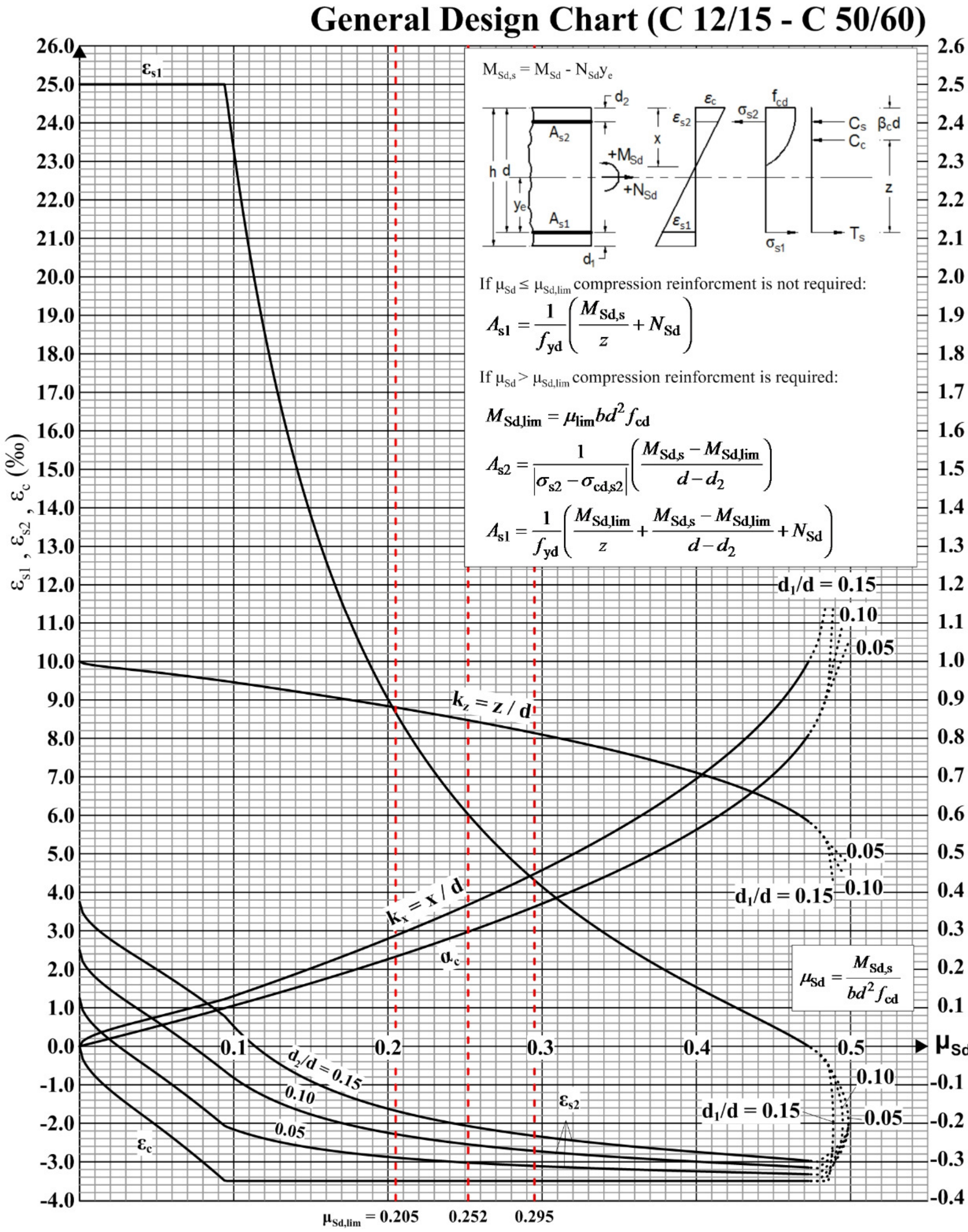


Figure 3-5 General Design Chart for C 12/15 – C 50/60

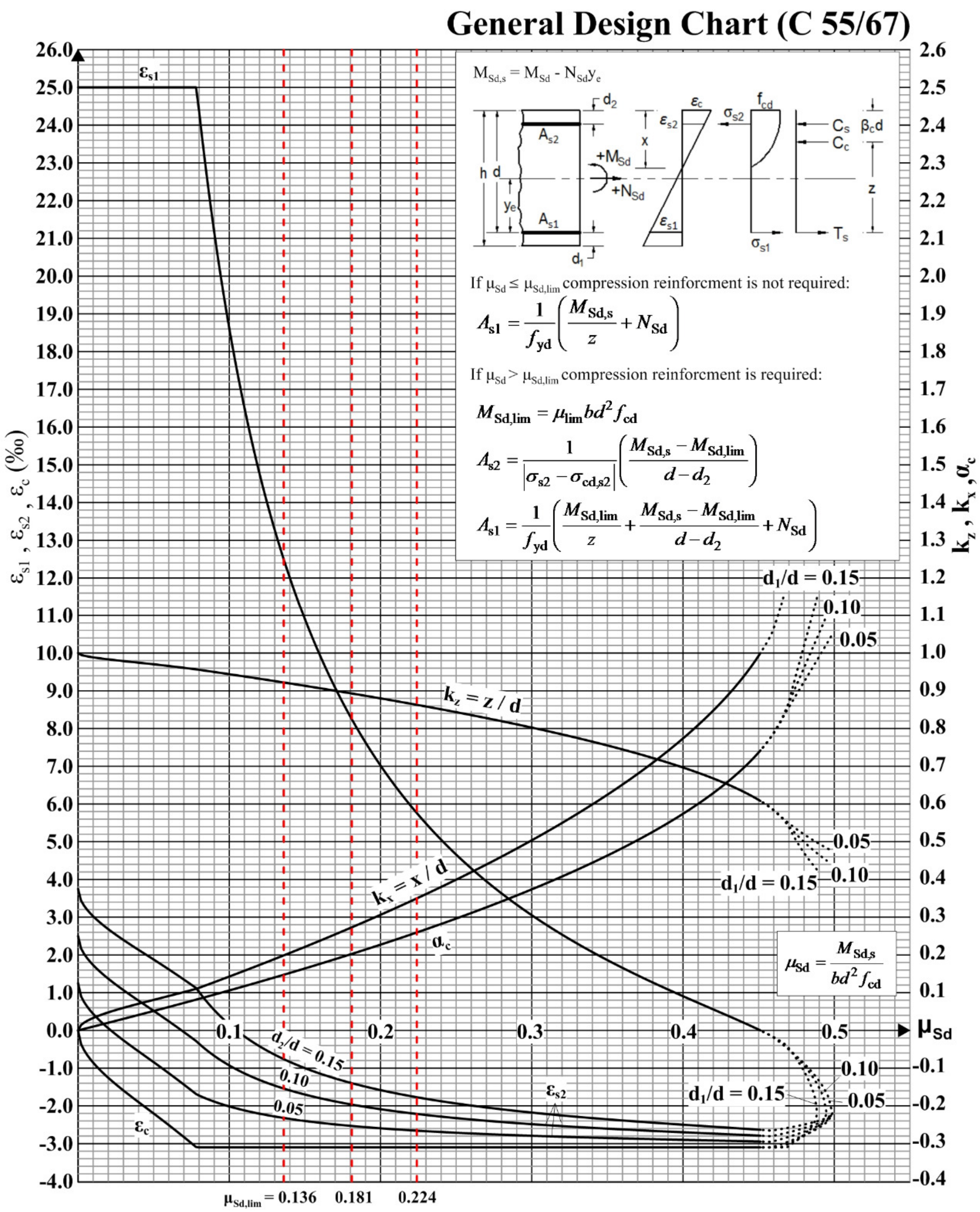


Figure 3-6 General Design Chart for C 55/67

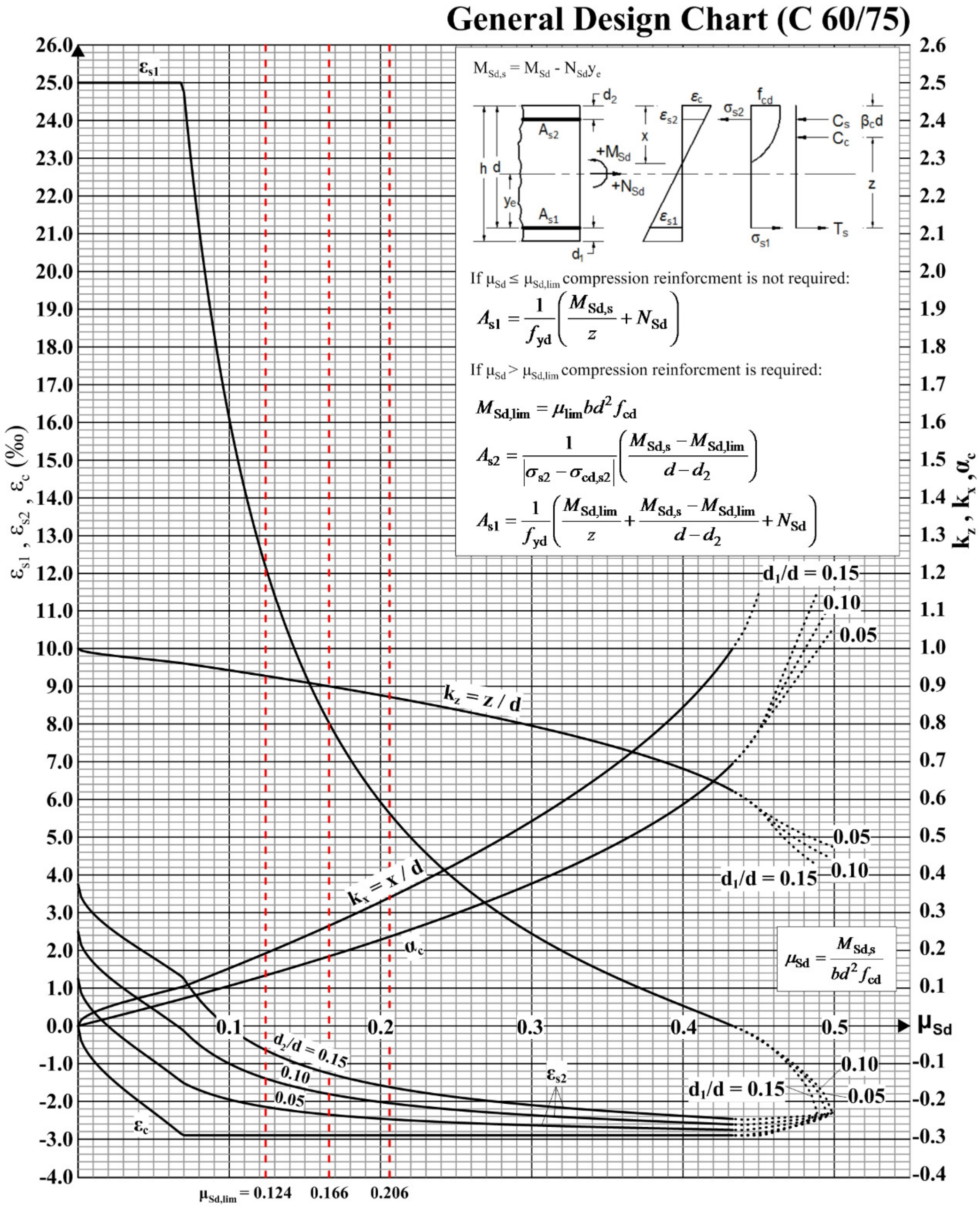


Figure 3-7 General Design Chart for C 60/75

General Design Chart (C 70/85)

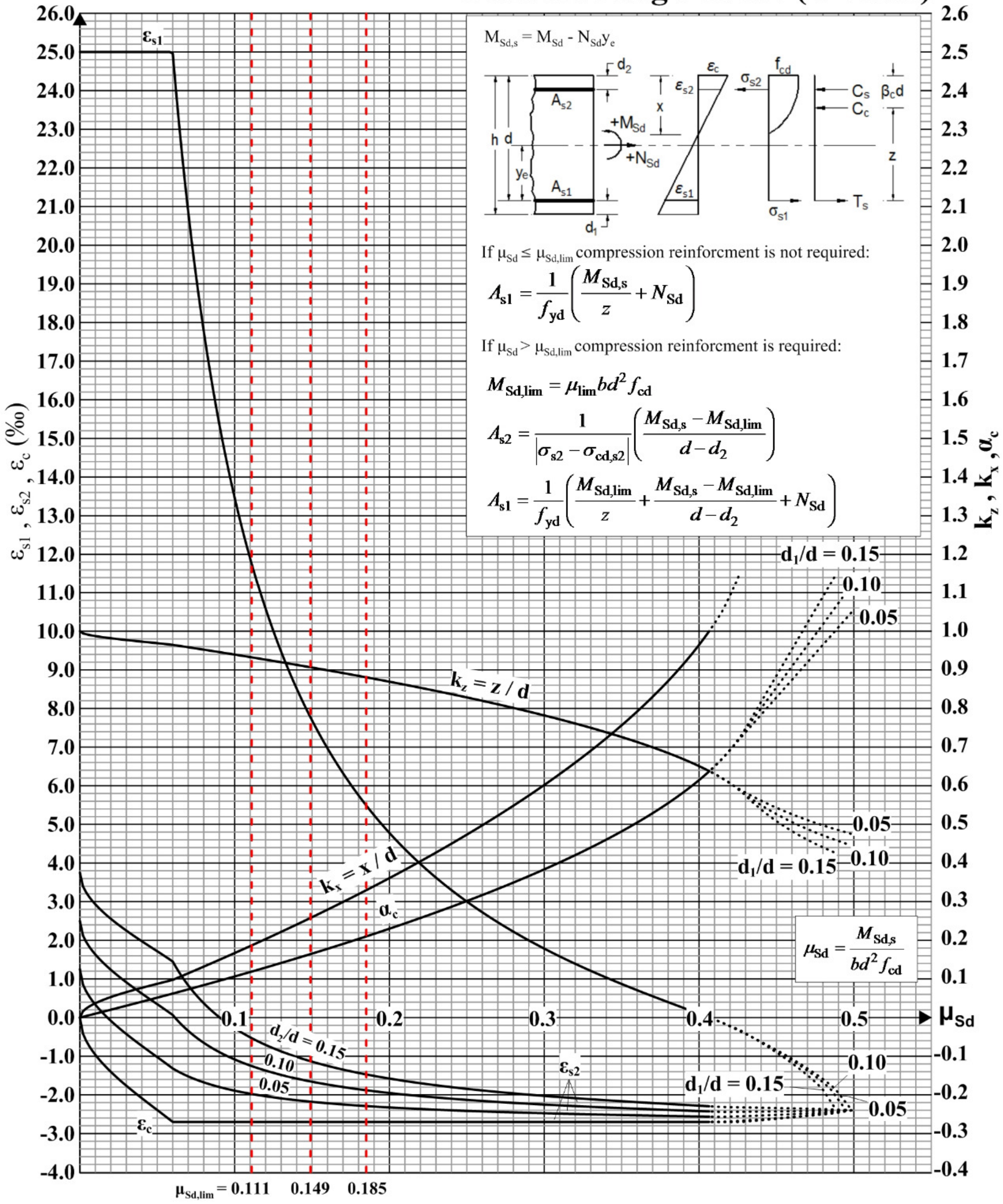


Figure 3-8 General Design Chart for C 70/85

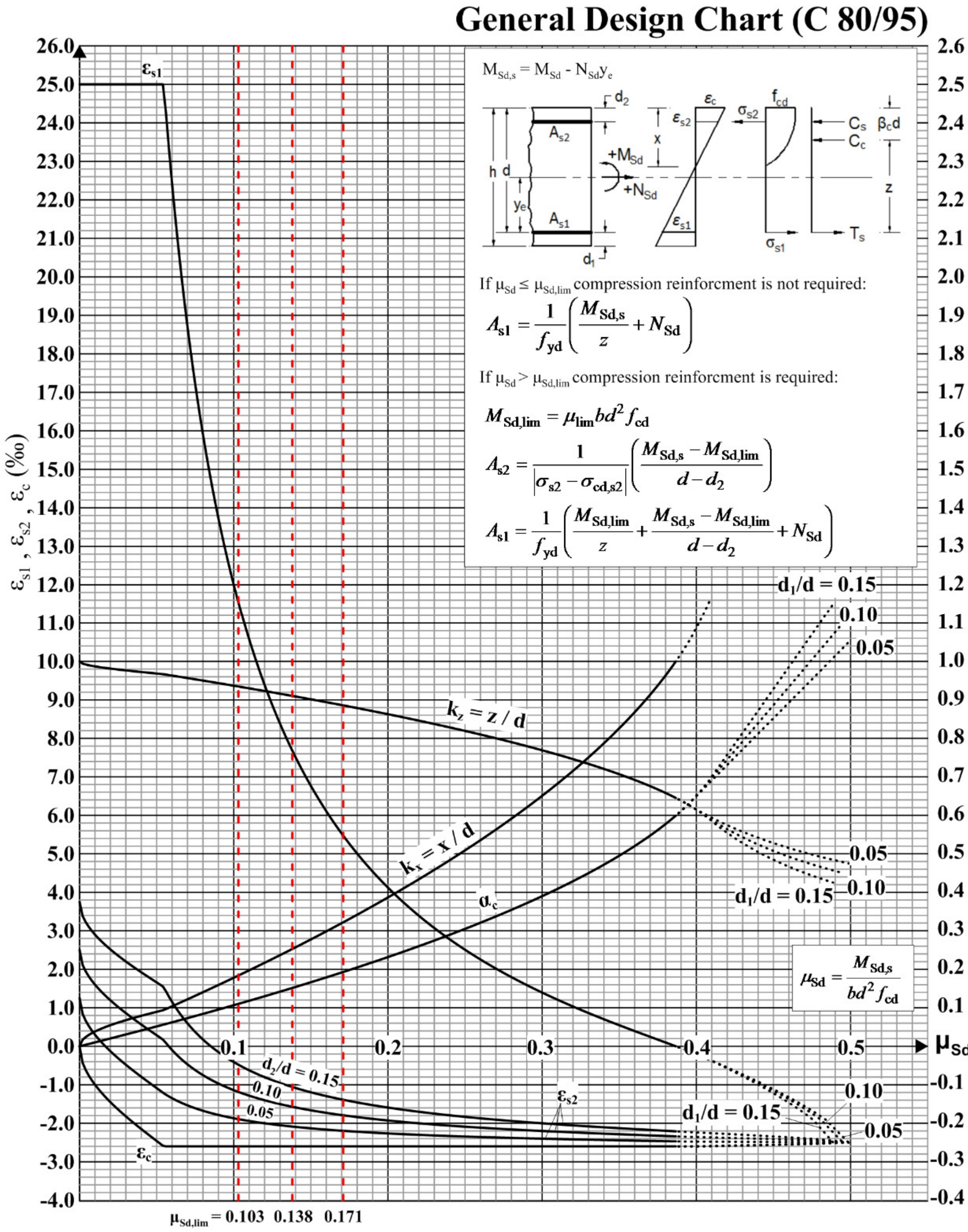


Figure 3-9 General Design Chart for C 80/95

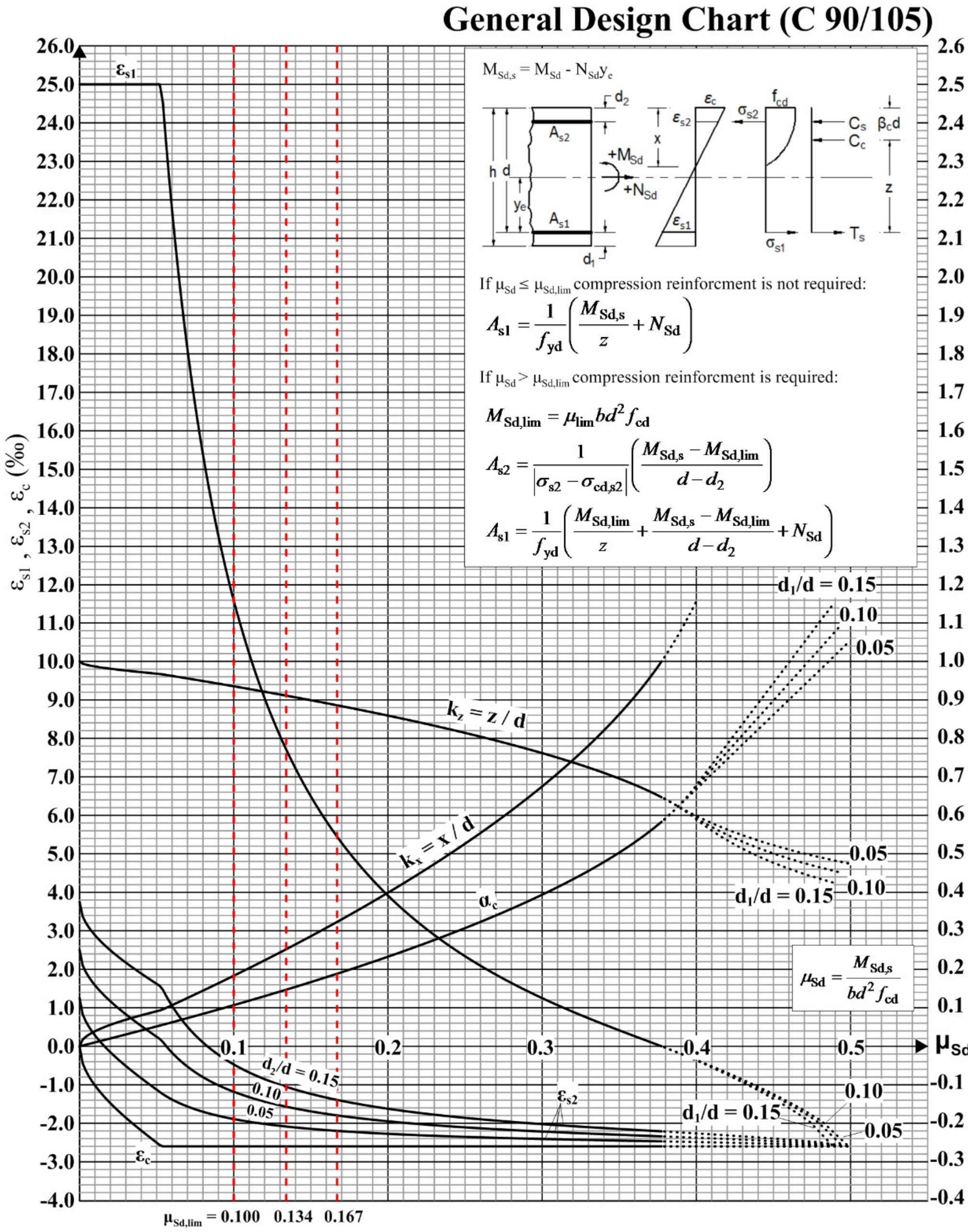


Figure 3-10 General Design Chart for C 90/105

Table 3-5 Design Table for C 12/15 – C 50/60

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd} b d^2}$	$\omega = \frac{A_{s1} f_{yd}}{f_{cd} b d}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistribution
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.030	0.990	0.773	25.000	
0.020	0.020	0.044	0.985	1.146	25.000	
0.030	0.031	0.055	0.980	1.464	25.000	
0.040	0.041	0.066	0.976	1.763	25.000	
0.050	0.051	0.076	0.971	2.060	25.000	
0.060	0.062	0.086	0.967	2.365	25.000	
0.070	0.073	0.097	0.962	2.682	25.000	
0.080	0.084	0.107	0.956	3.009	25.000	
0.090	0.095	0.118	0.951	3.349	25.000	
0.100	0.106	0.131	0.946	3.500	23.294	
0.110	0.117	0.145	0.940	3.500	20.709	
0.120	0.128	0.159	0.934	3.500	18.552	
0.130	0.140	0.173	0.928	3.500	16.726	
0.140	0.152	0.188	0.922	3.500	15.159	
0.150	0.164	0.202	0.916	3.500	13.799	
0.160	0.176	0.217	0.910	3.500	12.608	
0.170	0.188	0.232	0.903	3.500	11.555	
0.180	0.201	0.248	0.897	3.500	10.618	
0.190	0.213	0.264	0.890	3.500	9.777	
0.200	0.226	0.280	0.884	3.500	9.019	
0.205	0.233	0.288	0.880	3.500	8.653	20%
0.210	0.239	0.296	0.877	3.500	8.332	
0.220	0.253	0.312	0.870	3.500	7.706	
0.230	0.266	0.329	0.863	3.500	7.132	
0.240	0.280	0.346	0.856	3.500	6.605	
0.250	0.295	0.364	0.849	3.500	6.118	
0.252	0.298	0.368	0.847	3.500	6.011	10%
0.260	0.309	0.382	0.841	3.500	5.667	
0.270	0.324	0.400	0.834	3.500	5.247	
0.280	0.339	0.419	0.826	3.500	4.856	
0.290	0.355	0.438	0.818	3.500	4.490	
0.295	0.363	0.448	0.814	3.500	4.313	0%

Table 3-6 Design Table for C 55/67

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd}bd^2}$	$\omega = \frac{A_{s1}f_{yd}}{f_{cd}bd}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistribution
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.033	0.989	0.855	25.000	
0.020	0.020	0.048	0.983	1.256	25.000	
0.030	0.031	0.060	0.979	1.592	25.000	
0.040	0.041	0.071	0.975	1.901	25.000	
0.050	0.052	0.081	0.970	2.202	25.000	
0.060	0.062	0.091	0.966	2.509	25.000	
0.070	0.073	0.102	0.961	2.827	25.000	
0.080	0.084	0.113	0.956	3.100	24.379	
0.090	0.095	0.128	0.950	3.100	21.177	
0.100	0.106	0.143	0.944	3.100	18.613	
0.110	0.117	0.158	0.938	3.100	16.514	
0.120	0.129	0.174	0.932	3.100	14.763	
0.130	0.140	0.189	0.926	3.100	13.280	
0.136	0.147	0.198	0.922	3.100	12.557	20%
0.140	0.152	0.205	0.920	3.100	12.007	
0.150	0.164	0.221	0.913	3.100	10.903	
0.160	0.176	0.238	0.907	3.100	9.935	
0.170	0.189	0.255	0.900	3.100	9.080	
0.180	0.201	0.271	0.894	3.100	8.318	
0.181	0.203	0.274	0.893	3.100	8.214	10%
0.190	0.214	0.289	0.887	3.100	7.635	
0.200	0.227	0.306	0.880	3.100	7.019	
0.210	0.241	0.324	0.873	3.100	6.461	
0.220	0.254	0.342	0.866	3.100	5.951	
0.224	0.260	0.350	0.863	3.100	5.757	0%

Table 3-7 Design Table for C 60/75

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd}bd^2}$	$\omega = \frac{A_{s1}f_{yd}}{f_{cd}bd}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistribution
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.035	0.988	0.906	25.000	
0.020	0.020	0.050	0.983	1.326	25.000	
0.030	0.031	0.063	0.978	1.673	25.000	
0.040	0.041	0.074	0.974	1.988	25.000	
0.050	0.052	0.084	0.970	2.292	25.000	
0.060	0.062	0.094	0.965	2.600	25.000	
0.070	0.073	0.105	0.960	2.900	24.752	
0.080	0.084	0.121	0.955	2.900	21.146	
0.090	0.095	0.137	0.948	2.900	18.340	
0.100	0.106	0.153	0.942	2.900	16.093	
0.110	0.117	0.169	0.936	2.900	14.253	
0.120	0.129	0.186	0.930	2.900	12.719	
0.124	0.133	0.192	0.928	2.900	12.204	20%
0.130	0.141	0.203	0.924	2.900	11.419	
0.140	0.153	0.220	0.917	2.900	10.303	
0.150	0.165	0.237	0.911	2.900	9.334	
0.160	0.177	0.255	0.904	2.900	8.486	
0.166	0.185	0.266	0.900	2.900	8.002	10%
0.170	0.189	0.273	0.897	2.900	7.736	
0.180	0.202	0.291	0.890	2.900	7.068	
0.190	0.215	0.310	0.883	2.900	6.469	
0.200	0.228	0.328	0.876	2.900	5.928	
0.206	0.236	0.340	0.872	2.900	5.629	0%

Table 3-8 Design Table for C 70/85

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd}bd^2}$	$\omega = \frac{A_{s1}f_{yd}}{f_{cd}bd}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistributi-on
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.037	0.987	0.964	25.000	
0.020	0.020	0.053	0.982	1.404	25.000	
0.030	0.031	0.066	0.977	1.763	25.000	
0.040	0.041	0.077	0.973	2.086	25.000	
0.050	0.052	0.087	0.969	2.393	25.000	
0.060	0.062	0.098	0.965	2.700	24.960	
0.070	0.073	0.115	0.959	2.700	20.858	
0.080	0.084	0.132	0.952	2.700	17.779	
0.090	0.095	0.149	0.946	2.700	15.382	
0.100	0.106	0.167	0.940	2.700	13.464	
0.110	0.118	0.185	0.933	2.700	11.893	
0.111	0.119	0.186	0.933	2.700	11.816	20%
0.120	0.130	0.203	0.926	2.700	10.582	
0.130	0.141	0.222	0.920	2.700	9.471	
0.140	0.153	0.241	0.913	2.700	8.518	
0.149	0.164	0.257	0.907	2.700	7.806	10%
0.150	0.166	0.260	0.906	2.700	7.690	
0.160	0.178	0.279	0.899	2.700	6.965	
0.170	0.191	0.299	0.892	2.700	6.324	
0.180	0.204	0.319	0.884	2.700	5.753	
0.185	0.210	0.329	0.881	2.700	5.507	0%

Table 3-9 Design Table for C 80/95

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd} b d^2}$	$\omega = \frac{A_{s1} f_{yd}}{f_{cd} b d}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistribution
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.038	0.987	0.999	25.000	
0.020	0.020	0.055	0.981	1.451	25.000	
0.030	0.031	0.068	0.977	1.820	25.000	
0.040	0.041	0.079	0.972	2.149	25.000	
0.050	0.052	0.090	0.968	2.459	25.000	
0.060	0.062	0.104	0.963	2.600	22.414	
0.070	0.073	0.122	0.957	2.600	18.698	
0.080	0.084	0.140	0.950	2.600	15.908	
0.090	0.095	0.159	0.944	2.600	13.737	
0.100	0.107	0.178	0.937	2.600	11.999	
0.103	0.110	0.183	0.935	2.600	11.608	20%
0.110	0.118	0.197	0.930	2.600	10.575	
0.120	0.130	0.217	0.923	2.600	9.387	
0.130	0.142	0.237	0.916	2.600	8.380	
0.138	0.152	0.253	0.910	2.600	7.677	10%
0.140	0.154	0.257	0.909	2.600	7.516	
0.150	0.166	0.278	0.901	2.600	6.766	
0.160	0.179	0.299	0.894	2.600	6.108	
0.170	0.192	0.320	0.886	2.600	5.526	0%

Table 3-10 Design Table for C 90/105

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd} b d^2}$	$\omega = \frac{A_{s1} f_{yd}}{f_{cd} b d}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistribution
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.039	0.987	1.018	25.000	
0.020	0.020	0.056	0.981	1.480	25.000	
0.030	0.031	0.069	0.976	1.855	25.000	
0.040	0.041	0.081	0.972	2.189	25.000	
0.050	0.052	0.091	0.968	2.504	25.000	
0.060	0.062	0.107	0.962	2.600	21.724	
0.070	0.073	0.126	0.956	2.600	18.107	
0.080	0.084	0.145	0.949	2.600	15.391	
0.090	0.096	0.164	0.942	2.600	13.278	
0.099	0.107	0.183	0.935	2.600	11.608	20%
0.100	0.107	0.183	0.935	2.600	11.586	
0.110	0.118	0.203	0.928	2.600	10.199	
0.120	0.130	0.223	0.921	2.600	9.043	
0.130	0.142	0.244	0.914	2.600	8.063	
0.134	0.148	0.253	0.911	2.600	7.677	10%
0.140	0.154	0.265	0.907	2.600	7.221	
0.150	0.167	0.286	0.899	2.600	6.490	
0.160	0.179	0.308	0.891	2.600	5.850	
0.167	0.188	0.323	0.886	2.600	5.450	0%

3.7 Design Examples

3.7.1 Singly Reinforced Section without Normal Load

Given: Cross-section: b/h/d = 300/600/550 mm

Action Effects: Design moment = 350 kNm

Material Data: Concrete C 70/85 and Steel S 460

Required: Amount of tension reinforcement for Class I Works

Solution:

Using Design Table:

$$\mu_{\mathrm{Sd,lim}} = 0.185$$

$$\mu_{\mathrm{Sd,s}} = \frac{M_{\mathrm{Sd,s}}}{f_{\mathrm{cd}} b d^2} = \frac{350 \times 10^6}{46.67 \times 300 \times 550^2} = 0.083 \leq \mu_{\mathrm{Sd,lim}}$$

Since $\mu_{\mathrm{Sd,s}} \leq \mu_{\mathrm{Sd,lim}}$, the section is singly reinforced.

Reading ω from Table 3-8,

$$\rightarrow \omega = 0.0873$$

$$A_{s1} = \omega b d \frac{f_{\mathrm{cd}}}{f_{yd}} = 0.0873 \times 300 \times 550 \times \frac{46.67}{400} = 1680.65 \text{ mm}^2$$

Using General Design Chart:

$$\mu_{\mathrm{Sd,lim}} = 0.185$$

$$\mu_{\mathrm{Sd,s}} = \frac{M_{\mathrm{Sd,s}}}{f_{\mathrm{cd}} b d^2} = \frac{350 \times 10^6}{46.67 \times 300 \times 550^2} = 0.083 \leq \mu_{\mathrm{Sd,lim}}$$

Since $\mu_{\mathrm{Sd,s}} \leq \mu_{\mathrm{Sd,lim}}$, the section is singly reinforced.

Reading k_z from Figure 3-8,

$$\rightarrow k_z = 0.945$$

$$A_{s1} = \frac{M_{\text{Sd,s}}}{zf_{yd}} = \frac{350 \times 10^6}{0.945 \times 550 \times 400} = 1683.51\,\text{mm}^2$$

3.7.2 Singly Reinforced Section with Normal Load (Compressive)

Given: Cross-section: b/h/d = 300/600/550 mm

Action Effects: Design moment = 350 kNm & Normal Load = 500 kNm

Material Data: Concrete C 70/85 and Steel S 460

Required: Amount of tension reinforcement for Class I Works

Solution:

Using Design Table:

$$M_{\text{Sd,s}} = 350 + 500 \times (550 - 300) \times 10^{-3} = 475\,\text{kNm}$$

$$\mu_{\text{Sd,lim}} = 0.185$$

$$\mu_{\text{Sd,s}} = \frac{M_{\text{Sd,s}}}{f_{\text{cd}} bd^2} = \frac{475 \times 10^6}{46.67 \times 300 \times 550^2} = 0.112 \leq \mu_{\text{Sd,lim}}$$

Since $\mu_{\text{Sd,s}} \leq \mu_{\text{Sd,lim}}$, the section is singly reinforced.

Reading ω from Table 3-8,

$$\rightarrow \omega = 0.1204$$

$$A_{s1} = \omega bd \frac{f_{\text{cd}}}{f_{yd}} + \frac{N_{\text{Sd}}}{f_{\text{yd}}} = 0.1204 \times 300 \times 550 \times \frac{46.67}{400} - \frac{500 \times 10^3}{400} = 1067.87\ \text{mm}^2$$

Using General Design Chart:

$$M_{\text{Sd,s}} = 350 + 500 \times (550 - 300) \times 10^{-3} = 475\,\text{kNm}$$

$$\mu_{\text{Sd,lim}} = 0.185$$

$$\mu_{\text{Sd,s}} = \frac{M_{\text{Sd,s}}}{f_{\text{cd}} bd^2} = \frac{475 \times 10^6}{46.67 \times 300 \times 550^2} = 0.112 \leq \mu_{\text{Sd,lim}}$$

Since $\mu_{Sd,s} \leq \mu_{Sd,lim}$, the section is singly reinforced.

Reading k_z from Figure 3-8,

$$\rightarrow k_z = 0.925$$

$$A_{s1} = \frac{M_{Sd,s}}{zf_{yd}} + \frac{N_{Sd}}{f_{yd}} = \frac{475 \times 10^6}{0.925 \times 550 \times 400} - \frac{500 \times 10^3}{400} = 1084.15 \text{ mm}^2$$

3.7.3 Doubly Reinforced Section with Moment Redistribution

Given: Cross-section: $b/h/d/d_2$ = 300/600/550/55 mm

Action Effects: Design moment = 600 kNm after moment redistribution

Material Data: Concrete C 70/85 and Steel S 460

Percent Moment Redistribution: 20%

Required: Amount of tension and compression reinforcement for Class I Works

Solution:

Using Design Table:

$\mu_{Sd,lim}$ = 0.111 for 20% moment redistribution

$$\mu_{Sd,s} = \frac{M_{Sd,s}}{f_{cd} b d^2} = \frac{600 \times 10^6}{46.67 \times 300 \times 550^2} = 0.142 > \mu_{Sd,lim}$$

Since $\mu_{Sd,s} > \mu_{Sd,lim}$, the section is doubly reinforced.

Reading ω_{lim} from Table 3-8,

$$\rightarrow \omega_{lim} = 0.119$$

$$\omega' = \frac{\mu_{Sd,s} - \mu_{Sd,lim}}{(1 - d_2/d)} = \frac{0.142 - 0.111}{(1 - 55/550)} = 0.0344$$

Area of tension reinforcement,

$$A_{s1} = (\omega_{lim} + \omega^{'})bd\frac{f_{cd}}{f_{yd}} = (0.119 + 0.0344) \times 300 \times 550 \times \frac{46.67}{400} = 2953.16\ \text{mm}^2$$

Check whether the compression reinforcement has yielded,

$$\varepsilon_{s2} = \left(1 - \frac{d_2/d}{k_x}\right)\varepsilon_c = \left(1 - \frac{0.10}{0.186}\right) \times 2.7 = 1.23\ ‰$$

$$\rightarrow \varepsilon_{s2} = 1.23\ ‰ \leq \varepsilon_{yd} = 2.0‰ \rightarrow \sigma_{s2} = \varepsilon_{s2} E_s = 1.23 \times 200 = 246\ \text{MPa}$$

Calculate the stress in concrete at the level of compression reinforcement to avoid double counting of area,

$$\rightarrow \varepsilon_{cs2} = 1.23\ ‰ \leq \varepsilon_{c2} = 2.4‰$$

$$\sigma_{cd,s2} = f_{cd}\left[1 - \left(1 - \frac{\varepsilon_{cs2}}{\varepsilon_{c2}}\right)^n\right] = 46.67 \times \left[1 - \left(1 - \frac{1.23}{2.4}\right)^{1.45}\right] = 30.20\ \text{MPa}$$

Area of compression reinforcement,

$$A_{s2} = \omega^{'} bd\frac{f_{cd}}{|\sigma_{s2} - \sigma_{cd,s2}|} = 0.0344 \times 300 \times 550 \times \frac{46.67}{|246 - 30.20|} = 1227.52\ \text{mm}^2$$

Using General Design Chart:

$\mu_{Sd,lim} = 0.111$ for 20% moment redistribution

$$\mu_{Sd,s} = \frac{M_{Sd,s}}{f_{cd} bd^2} = \frac{600 \times 10^6}{46.67 \times 300 \times 550^2} = 0.142 > \mu_{Sd,lim}$$

Since $\mu_{Sd,s} > \mu_{Sd,lim}$, the section is doubly reinforced.

$$M_{Sd,lim} = \mu_{Sd,lim} bd^2 f_{cd} = 0.111 \times 300 \times 550^2 \times 46.67 \times 10^{-6} = 470.12\ \text{kNm}$$

Reading $k_{z,lim}$ from Figure 3-8,

$$\rightarrow k_{\mathrm{z,lim}} = 0.933$$

Area of tension reinforcement,

$$A_{\mathrm{s1}} = \frac{M_{\mathrm{Sd,lim}}}{z_{\mathrm{lim}} f_{yd}} + \frac{(M_{\mathrm{Sd}} - M_{\mathrm{Sd,lim}})}{(d - d_2) f_{yd}} = \frac{470.12 \times 10^6}{0.933 \times 550 \times 400} + \frac{(600 - 470.12) \times 10^6}{(550 - 50) \times 400}$$

$$A_{\mathrm{s1}} = 2939.76\,\mathrm{mm}^2$$

Check whether the compression reinforcement has yielded,

Reading $\varepsilon_{\mathrm{s2}}$ from Figure 3-8,

$$\frac{d_2}{d} = \frac{55}{550} = 0.10$$

$$\rightarrow \varepsilon_{\mathrm{s2}} = -1.25\,‰ \le \varepsilon_{yd} = -2.0‰ \rightarrow \sigma_{s2} = \varepsilon_{\mathrm{s2}} E_{\mathrm{s}} = 1.25 \times 200 = 250\ \mathrm{MPa}$$

Calculate the stress in concrete at the level of compression reinforcement to avoid double counting of area,

$$\rightarrow \varepsilon_{\mathrm{cs2}} = 1.25\,‰ \le \varepsilon_{\mathrm{c2}} = 2.4‰$$

$$\sigma_{\mathrm{cd,s2}} = f_{cd}\left[1 - \left(1 - \frac{\varepsilon_{\mathrm{cs2}}}{\varepsilon_{\mathrm{c2}}}\right)^n\right] = 46.67 \times \left[1 - \left(1 - \frac{1.25}{2.4}\right)^{1.45}\right] = 30.61\,\mathrm{MPa}$$

Area of compression reinforcement,

$$A_{\mathrm{s2}} = \frac{1}{|\sigma_{s2} - \sigma_{c2}|} \frac{(M_{\mathrm{Sd}} - M_{\mathrm{Sd,lim}})}{(d - d_2)\sigma_{s2}} = \frac{1}{|250 - 30.61|} \frac{(600 - 470.12) \times 10^6}{(550 - 50)}$$

$$A_{\mathrm{s2}} = 1184.01\,\mathrm{mm}^2$$

4. Interaction Chart for Reinforced Concrete Columns

4.1 Introduction

Columns are vertical structural members supporting axial compressive forces with or without moments. Almost all columns in concrete structures are subjected to moments in addition to axial loads [8]. These moments may originate from the positions of the column in a structure, from alignment of columns or from the nature of the loading. Column subjected to earthquake or wind loads are usually subjected to moment in both of their principal directions. Based on the moment that is acting on a column, a column may be termed as uniaxial column if an axial compression is accompanied by bending in only one of the orthogonal direction and biaxial if the axial compression is accompanied by bending in the two orthogonal directions.

In the past, analysis and design of columns ignored biaxial bending effects. The majority of columns found in buildings were designed by simplified procedures given in different codes. These simplified procedures allows biaxial column to be designed as uniaxial columns. Although the procedures are justifiable, they are most of the time conservative resulting in uneconomical design. The reason for avoiding biaxial bending in design is because it presents a great deal of difficulty and computational intensity in obtaining the ultimate capacity of cross-sections unlike uniaxial bending problems [9]. Therefore the objective of this chapter is to develop an algorithm for construction of uniaxial and biaxial interaction charts for any arbitrary shaped sections. The method offered uses exact method of cross section analysis and is computationally effective. Although the solution presented in this chapter can be used for any arbitrary shaped sections, interaction charts are constructed only for solid rectangular or square sections.

The procedures for generating interaction charts for uniaxial and biaxial bending are discussed in the following sections which sets its base on the work of Fafitis A. [9].

4.2 Formulation of Expression for Force Resultant

In reality, most of the column located in a structure are subjected to biaxial bending in addition to axial loads. Therefore, computationally effective and exact methods for cross section analysis should be formulated. Figure 4-1 shows an arbitrary shaped reinforced concrete section subjected to an axial load P_u and a bending moment components M_X and M_Y. The basic assumption taken in the analysis of this cross section remain the same as discussed in section 2.2 and section 2.3.

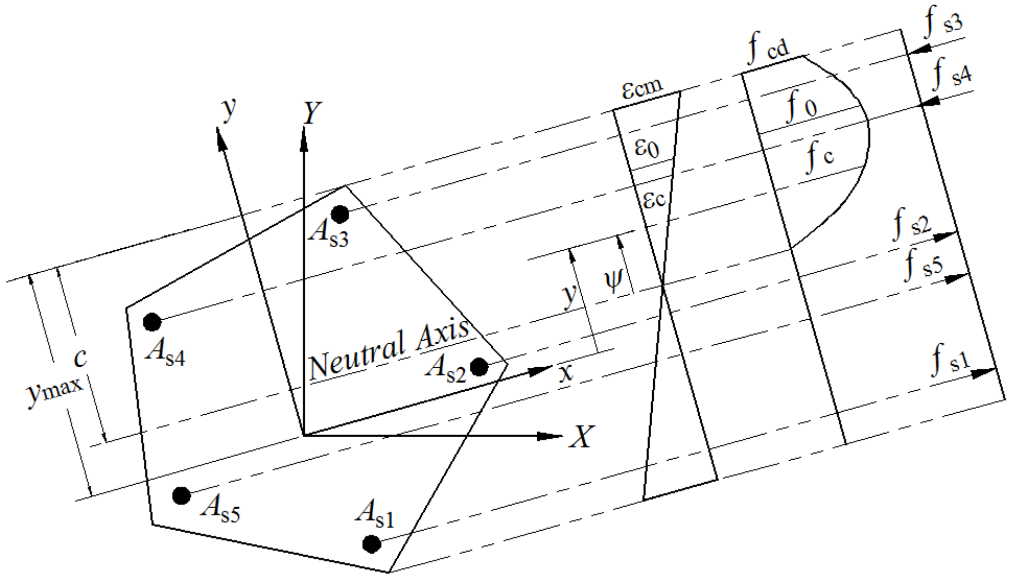


Figure 4-1 Arbitrary shaped reinforced concrete section

Referring to Figure 4-1, the strain in the concrete ε_c at a distance ψ from the neutral axis can be expressed in terms of the strain in the most compressed fiber ε_{cm} and depth from the most compressed fiber to the neutral axis as,

$$\varepsilon_c = \frac{\psi}{c}\varepsilon_{cm} \tag{4.1}$$

The distance ψ should be expressed in terms of the chosen coordinate system. In this study the coordinate system is chosen such that the origin of the coordinate system coincides with the geometrical center of the section. This is done because moments and forces obtained from structural analysis are referred to this axis. The x axis is made to be parallel to the neutral axis to increase computational efficiency. Thus the expression in equation (4.1) becomes,

$$\varepsilon_c = \frac{(y + c - y_{max})}{c}\varepsilon_{cm} \tag{4.2}$$

Therefore, the stress in the concrete at any position y from the centroid of the section becomes,

$$\sigma_c = f(\psi) = f(y + c - y_{max}) = g(y) \tag{4.3}$$

The total force and moment carried by the concrete under compression can then be calculated by integrating equation (4.3) over the compressed area *A*. Therefore, the force and moment resultants becomes,

$$\begin{aligned} N_c &= \iint_A g(y)dA \\ M_{xc} &= \iint_A yg(y)dA \\ M_{yc} &= -\iint_A xg(y)dA \end{aligned} \tag{4.4}$$

where: N_c is resultant axial carried by the concrete

M_{xc} is the moment component in the *x* direction carried by the concrete

M_{yc} is the moment component in the *y* direction carried by the concrete

The contributions of the reinforcement bars for force and moment capacity can be easily expressed as,

$$\begin{aligned} N_s &= \sum_{i=1}^{n} A_{si} f_{si} \\ M_{xs} &= \sum_{i=1}^{n} y_{si} A_{si} f_{si} \\ M_{ys} &= -\sum_{i=1}^{n} x_{si} A_{si} f_{si} \end{aligned} \tag{4.5}$$

where: N_s is resultant axial carried by the steels

M_{xs} is the resultant moment component in the *x* direction carried by the steels

M_{ys} is the resultant moment component in the *y* direction carried by the steels

x_{si} is the distance of steel bar *i* from the x axis

y_{si} is the distance of steel bar *i* from the y axis

A_{si} is the area of steel bar i

f_{si} is the stress in steel bar i

n is the number of steel bars

Therefore the total axial load and moment capacity of the section can be obtained by adding the contributions of the concrete and reinforcement. These capacities in the global axis X-Y are given as,

$$\begin{aligned} P_u &= N_c + N_s \\ M_X &= (M_{xc} + M_{xs})\cos\theta + (M_{yc} + M_{ys})\sin\theta \\ M_Y &= -(M_{xc} + M_{xs})\sin\theta + (M_{yc} + M_{ys})\cos\theta \end{aligned} \tag{4.6}$$

where: P_u is axial load carrying capacity of the section

M_X is the moment capacity of the section in the X direction

M_Y is the moment capacity of the section in the Y direction

θ is the angle from the positive direction of the neutral axis to the positive X axis

From the equation (4.4), its straight forward that evaluation double integrals over the compressive zone of the cross section is a must to obtain force and moment resultants. The evaluation of the double integral presents computational difficulties, especially when the shape of the compression zone becomes complicated. The evaluation of the double integral can be avoided by dividing the section either in strips parallel to the neutral axis or in small rectangular elements. This way of evaluating the double integral requires a complex computer code and requires a control over the loss of accuracy that comes from discretization [9].

The way to avoid computational difficulties that arise from evaluating double integrals is to apply elementary calculus i.e, by application of Green's Theorem. Green's Theorem is an exact integration method in which double integrals are transformed to single line integrals along a closed and reducible boundaries. The application of Green's Theorem in construction of interaction chart is presented in the next section.

4.3 Application of Green's Theorem in Construction of Interaction Charts

If the compression zone of a section can be accurately defined by a line that is closed and reducible, the double integration over the compressed area *A* can be transformed to line integration by applying Green's Theorem as,

$$\iint_A (\partial Q / \partial x - \partial P / \partial y) dxdy = \oint_l P dx + \oint_l Q dy \tag{4.7}$$

where: P and Q are two functions of x and y

A is area of integration

L is the curve that enclosed the area A

Therefore, applying Green's Theorem to biaxial bending problem by setting P and Q as follows,

$$P = 0$$
$$Q = \frac{1}{r+1}\oint_l x^{r+1} y^s g(y) dy \tag{4.8}$$

Substituting equation (4.8) in equation (4.7), the stress resultants are reduced to,

$$R = \iint_A x^r y^s g(y) dxdy = \frac{1}{r+1}\oint_l x^{r+1} y^s g(y) dy \tag{4.9}$$

Equation (4.9) depending on the values of r and s which are non- negative integers, equation (4.9) represents,

- The axial force N_c for $r = 0$ and $s = 0$,
- The bending moment M_{xc} for $r = 0$ and $s = 1$ and
- The bending moment $-M_{yc}$ for $r = 1$ and $s = 0$.

Furthermore, if $g(y)$ in equation (4.9) is set to unit,

- The area A, for $r = 0$ and $s = 0$,
- The area moment S_x, for $r = 0$ and $s = 1$,
- The area moment S_y, for $r = 1$ and $s = 0$,
- The moment of inertia I_x, for $r = 2$ and $s = 0$,
- The moment of inertia I_y, for $r = 0$ and $s = 2$ and
- The product of inertia I_{xy}, for $r = 1$ and $s = 1$.

Equation (4.9) can be written as,

$$R = \frac{1}{r+1}\oint_{l} x^{r+1} y^{s} g(y)dy = \frac{1}{r+1}\sum_{l} S_{l} \tag{4.10}$$

$$S_{l} = \oint_{l} x^{r+1} y^{s} g(y)dy \tag{4.11}$$

The function S_l for NSC is directly integrable because the function for the material constitutive law of concrete $g(y)$ is parabolic. However, for HSC the function $g(y)$ is no longer linear. Therefore, a numerical integration method is required. In this study, a numerical approach is chosen for both NSC and HSC. Third-order Gauss Quadrature formula was used to evaluate the line integral in equation (4.11).The line integral in equation (4.11) should be evaluated along the sides of the compressive zone. Therefore, equation for defining the sides are required and given below for straight lines.

Referring to Figure 4-2,

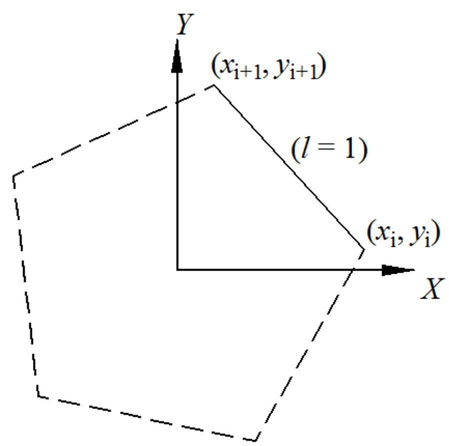


Figure 4-2 Definition of side of the compressed zone

$$\begin{aligned} x &= \alpha_i + \beta_i y \\ \alpha_i &= x_i - \beta_i y_i \\ \beta_i &= (x_{i+1} - x_i)/(y_{i+1} - y_i) \end{aligned} \tag{4.12}$$

where: (x_i, y_i) and (x_{i+1}, y_{i+1}) are starting and ending point of side i respectively.

Substituting the expression in equations (4.12) in equation (4.11) yields,

$$S_{l} = \oint_{l} (\alpha_i + \beta_i y)^{r+1} y^{s} g(y)dy \tag{4.13}$$

Applying third-order Gauss Quadrature method, equation (4.13) takes the form,

$$S_1 \cong (y_{i+1} - y_i)\sum_{i=1}^{n} w_i(\alpha_i + \beta_i y)^{r+1} y^s g(y) \tag{4.14}$$

where: n is the order of the Gauss Quadrature and taken as 3 for this study,

w_i is the weight of sample points and taken as (0.278,0.444,0.278) [9]

y_i is the sample point at which the function is evaluated.

Therefore, depending on the values of r and s in equation (4.14), the axial force, bending moments, area, area of moments, moment of inertias and product of inertia can be easily calculated by evaluating the integral along the sides of the compression zone.

A worked example showing the application of Green's Theorem on biaxial bending problem and for verifying the program written can be found on Fafits A. [9].

4.4 Preparation of Interaction Charts

If the solution discussed above for biaxial bending is further expanded, an interaction surface can be constructed for a given cross-section and material constitutive laws by simply varying the possible strain distribution in the ULS. This is done by incrementing θ from 0^0 to 360^0 and for each θ, the neutral axis depth c is also incremented from zero to infinity. The calculated axial load and moment capacities for the selected θ and c are then plotted in P_u-M_X-M_Y space resulting in an interaction surface shown in Figure 4-3.

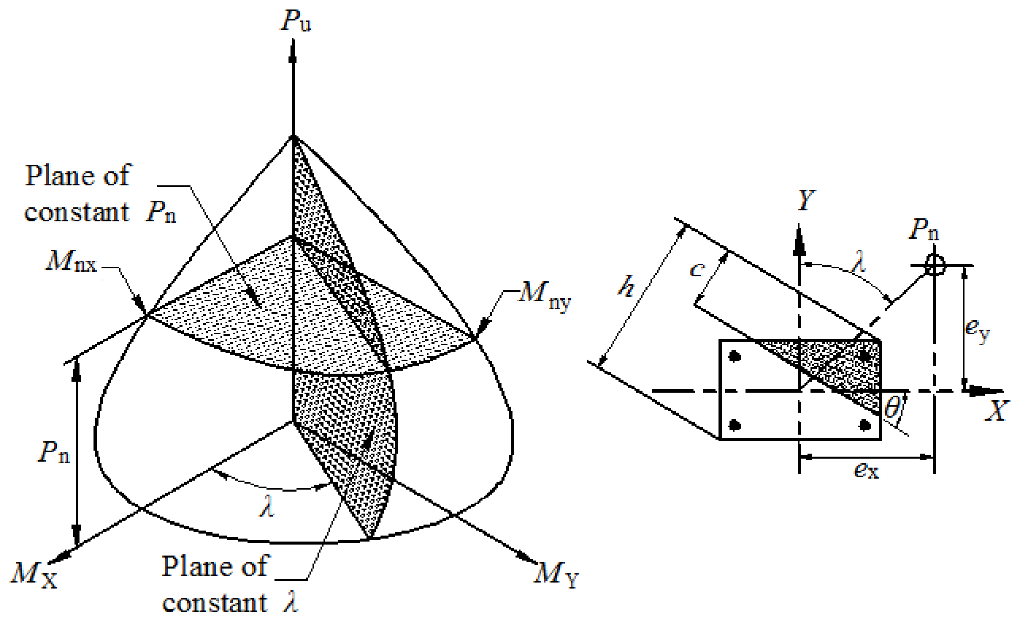


Figure 4-3 Interaction surface for column

Although this method of obtaining values on the interaction surface is acceptable, using this interaction surface for design purpose becomes difficult. Therefore, the interaction surface shown in Figure 4-3 should be plotted for plane of constant load P_n or plane of constant λ = 0^0 or 90^0 whether the required interaction chart is biaxial or uniaxial respectively. Furthermore, the interaction chart plotted in this way can be made more useful for design purpose by representing the interaction charts independent of column dimensions and concrete compressive strength. This can be achieved by dividing the factored axial-load resistances P_u, moment resistance M_X and M_Y by $f_{cd}A$, $f_{cd}Ah$ and $f_{cd}Ab$ respectively. The resulting fractions are non-dimensional and are called normalized ratios for axial loads and moments (v_{sd}, $\mu_{Sd,x}$ and $\mu_{Sd,y}$ respectively).

However, points obtained in this way may not have a constant spacing, i.e. points on the chart may be closely spaced in some portion while widely spaced in other portions. This will resulting in loss of accuracy when plotting the interaction charts. The solution of problem is employing iterative schemes which can calculate the possible strain profile, angle and neutral axis depth for a selected points. In preparation of uniaxial chart the normalized axial load v_{sd} is varied at an increment of 0.01 to obtain equally spaced points on the chart. Furthermore, when constructing biaxial interaction chart, the chart is plotted for a selected constant ν_{sd} of 0.0 up to 1.4 with an increment of 0.2. Here again, to obtain an equally spaced points on the chart, the chart is constructed for λ equal to 0^0 up to 90^0 incremented at 2^0 intervals.

4.5 Verification

The accuracy of the program was checked with work done by Zerayohannes [10].The co-ordinates of the interaction chart presented in Table 4-1 are of a rectangular solid section made of concrete strength class C100/115 with ε_{c2}, ε_{cu2} and n of 2.2‰, 2.2‰ and 1.55 respectively, 4 corner reinforcement with cover ratios $d_1/h = b_1/b = 0.10$, mechanical reinforcement ratio $\omega = 0.5$, and related normal force component $\nu_{sd} = -0.8$. The yield strength f_{yd} and the ultimate rupture strain ε_{s2} in the steel bars were taken to be 500 MPa and 25‰ respectively. As it can be seen from Table 4-1, the percentage difference is negligible. Therefore, this way of generating interaction charts is valid.

Table 4-1 Comparison between charts

ν_{sd}	λ	Zerayohannes [10]		This Study		% difference	
		[1] μ_{Edy}	[2] μ_{Edz}	[3] μ_{Edy}	[4] μ_{Edz}	$\left\lvert\frac{[1]-[3]}{[1]}\right\rvert \times 100$	$\left\lvert\frac{[2]-[4]}{[2]}\right\rvert \times 100$
-0.8	0	0.1635	0	0.1637	0	0.11	0.00
-0.8	2	0.1589	0.0055	0.1591	0.0056	0.12	1.01
-0.8	4	0.1545	0.0108	0.1547	0.0108	0.10	0.14
-0.8	6	0.1502	0.0158	0.1504	0.0158	0.12	0.03
-0.8	8	0.1461	0.0205	0.1462	0.0206	0.10	0.26
-0.8	10	0.1422	0.0251	0.1423	0.0251	0.04	0.07
-0.8	12	0.1383	0.0294	0.1384	0.0294	0.06	0.05
-0.8	14	0.1346	0.0336	0.1346	0.0336	0.03	0.09
-0.8	16	0.1310	0.0376	0.1310	0.0376	0.01	0.09
-0.8	18	0.1275	0.0414	0.1275	0.0414	0.02	0.05
-0.8	20	0.1240	0.0451	0.1240	0.0451	0.03	0.10
-0.8	22	0.1207	0.0487	0.1207	0.0488	0.02	0.11
-0.8	24	0.1174	0.0523	0.1174	0.0523	0.01	0.07
-0.8	26	0.1142	0.0557	0.1142	0.0557	0.04	0.04
-0.8	28	0.1110	0.0590	0.1110	0.0590	0.01	0.02
-0.8	30	0.1079	0.0623	0.1079	0.0623	0.03	0.04
-0.8	32	0.1048	0.0655	0.1048	0.0655	0.01	0.03
-0.8	34	0.1018	0.0686	0.1017	0.0686	0.06	0.04
-0.8	36	0.0988	0.0717	0.0987	0.0717	0.08	0.04
-0.8	38	0.0958	0.0748	0.0957	0.0748	0.08	0.02
-0.8	40	0.0928	0.0779	0.0927	0.0778	0.06	0.10
-0.8	42	0.0898	0.0809	0.0898	0.0808	0.04	0.09
-0.8	44	0,0868	0.0839	0.0868	0.0838	0.01	0.10
-0.8	46	0.0839	0.0868	0.0838	0.0868	0.10	0.01
-0.8	48	0.0809	0.0898	0.0808	0.0898	0.09	0.04

-0.8	50	0.0779	0.0928	0.0778	0.0927	0.10	0.06
-0.8	52	0.0748	0.0958	0.0748	0.0957	0.02	0.08
-0.8	54	0.0717	0.0988	0.0717	0.0987	0.04	0.08
-0.8	56	0.0686	0.1018	0.0686	0.1017	0.04	0.06
-0.8	58	0.0655	0.1048	0.0655	0.1048	0.03	0.01
-0.8	60	0.0623	0.1079	0.0623	0.1079	0.04	0.03
-0.8	62	0.0590	0.1110	0.0590	0.1110	0.02	0.01
-0.8	64	0.0557	0.1142	0.0557	0.1142	0.04	0.04
-0.8	66	0.0523	0.1174	0.0523	0.1174	0.07	0.01
-0.8	68	0,0487	0.1207	0.0488	0.1207	0.11	0.02
-0.8	70	0.0451	0.1240	0.0451	0.1240	0.10	0.03
-0.8	72	0.0414	0.1275	0.0414	0.1275	0.05	0.02
-0.8	74	0.0376	0.1310	0.0376	0.1310	0.09	0.01
-0.8	76	0.0336	0.1346	0.0336	0.1346	0.09	0.03
-0.8	78	0.0294	0.1383	0.0294	0.1384	0.05	0.06
-0.8	80	0.0251	0.1422	0.0251	0.1423	0.07	0.04
-0.8	82	0.0205	0.1461	0.0206	0.1462	0.26	0.10
-0.8	84	0.0158	0.1502	0.0158	0.1504	0.03	0.12
-0.8	86	0.0108	0.1545	0.0108	0.1547	0.14	0.10
-0.8	88	0.0056	0.1589	0.0056	0.1591	0.11	0.12
-0.8	90	0	0.1635	0.0000	0.1637	0.00	0.11

The points for the interaction chart presented in Table 4-1 where generated based on the net cross-section. i.e., to avoid double counting of the concrete area that is displaced by the steel bars. This can be done by subtracting $A_{si} f_{ci}$ from the steel bar force $A_{si} f_{si}$ where f_{ci} is the concrete compressive stress at the centroid of the bar. For the purpose of comparison the interaction chart in Table 4-1 is shown with the corresponding interaction chart in Figure 4-4 which is determined on the basis of the gross cross-section. In Figure 4-4, charts drawn based on both net and gross cross-section for NSC has also been plotted. As it can be seen from the chart, avoiding of double counting of the concrete area is significant for HSC whereas for NSC the effect is small compared to the former one. However, in the construction of all of the interaction charts, double counting of the concrete area is avoided.

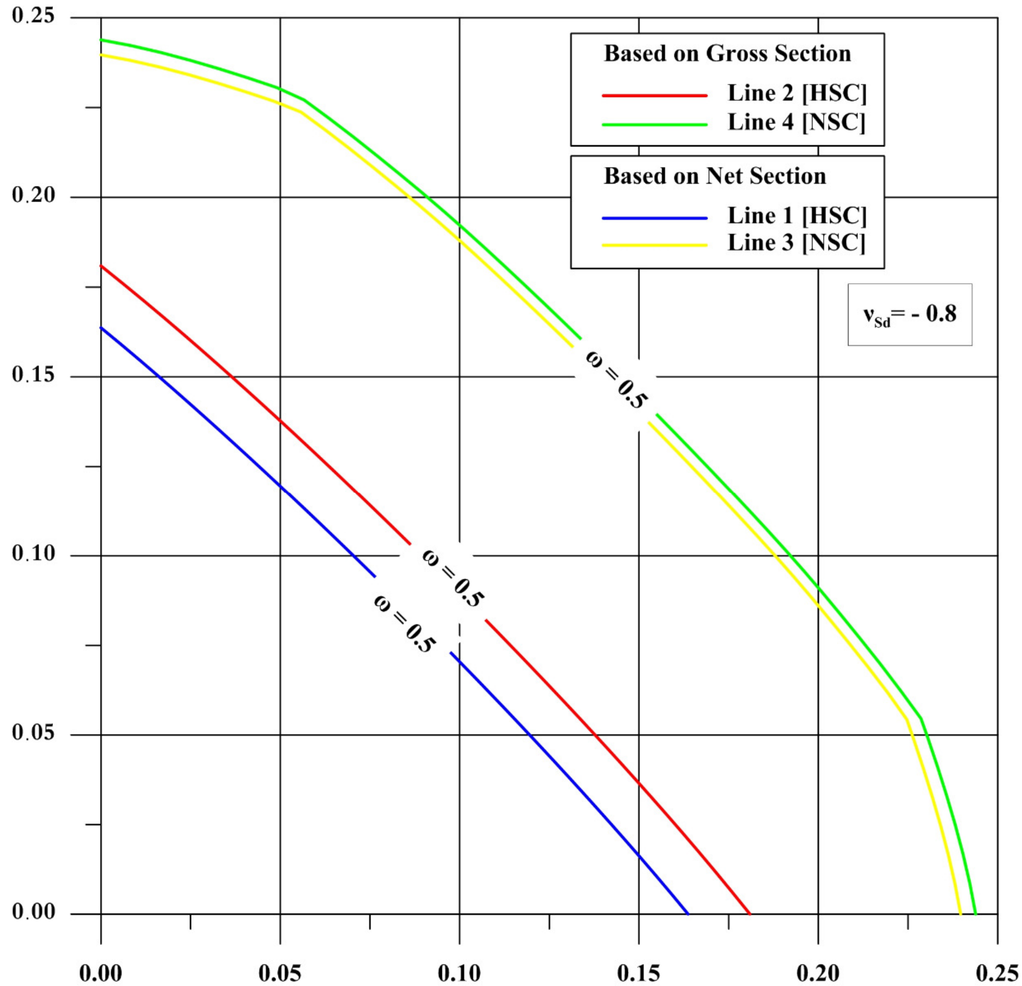


Figure 4-4 Comparison of interaction curves determined on the basis of the net and gross cross-sections

4.6 Uniaxial and Biaxial Interaction Charts

Uniaxial and biaxial interaction charts prepared as presented in section 4.4 are given in this section for concrete classes C 12/15 up to C 90/105.

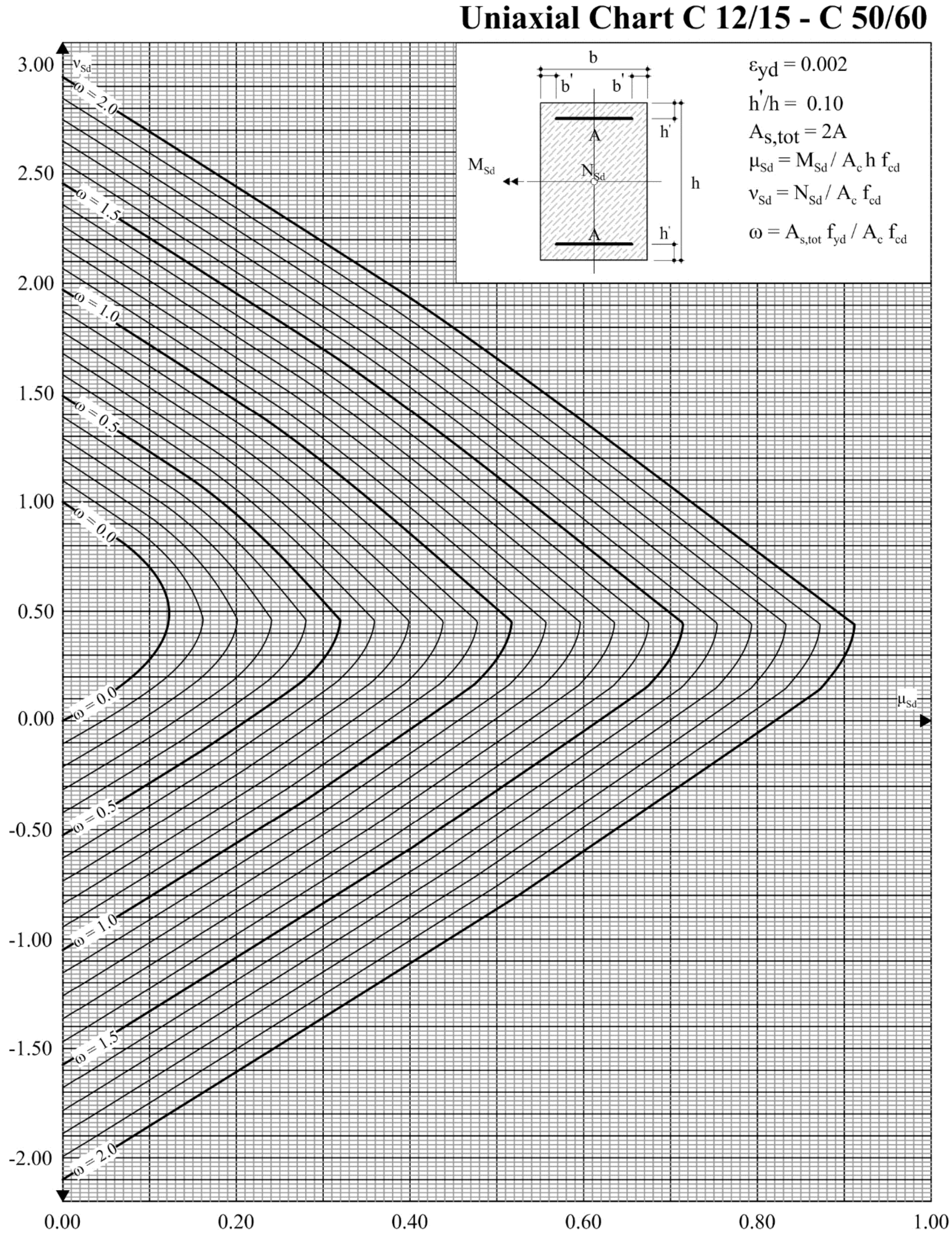


Figure 4-5 Uniaxial Chart for C 12/15 – C 50/60 ($A_{s,tot}$ = 2A)

Uniaxial Chart C 55/67

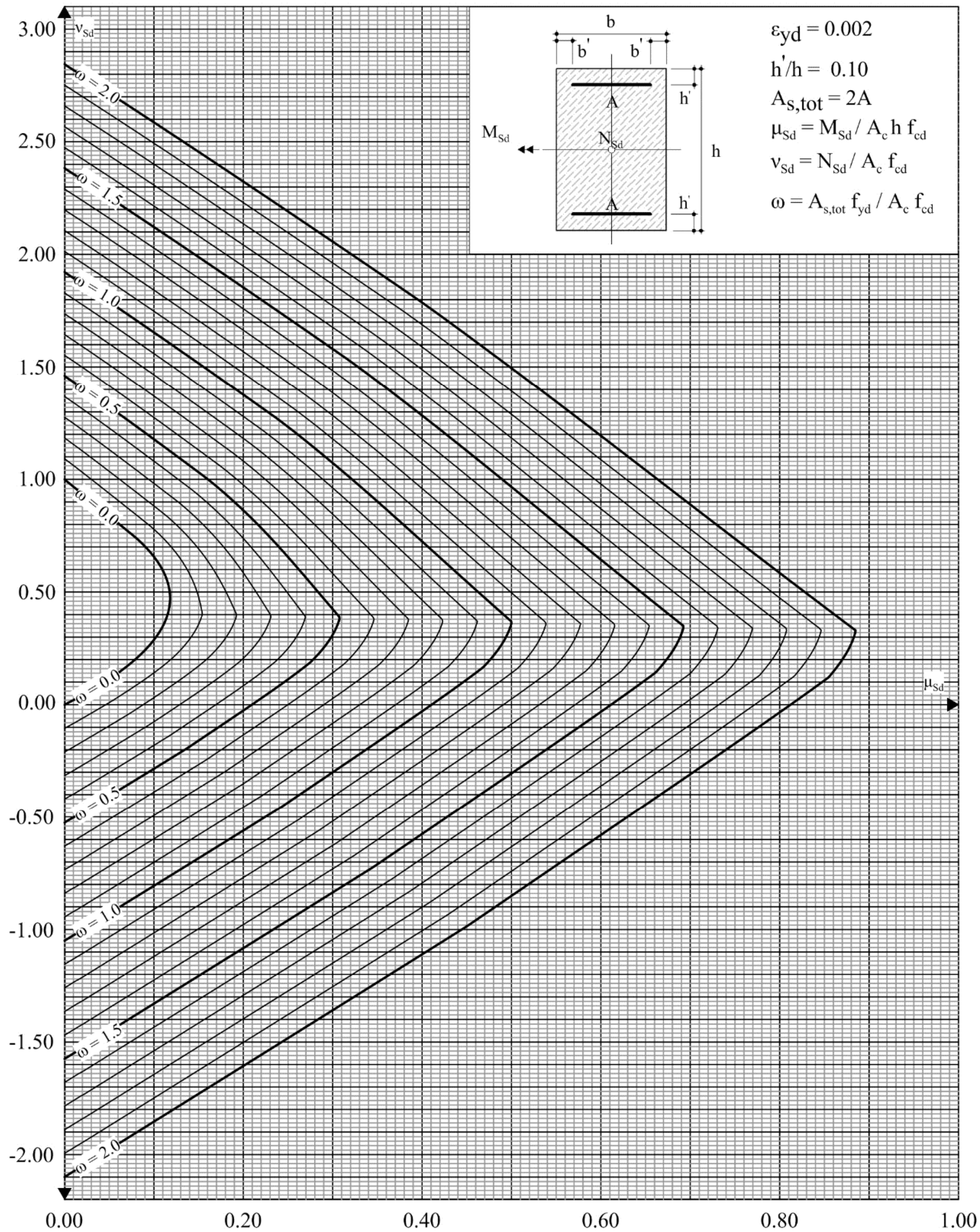


Figure 4-6 Uniaxial Chart for C 55/67 ($A_{s,tot}$ = 2A)

Uniaxial Chart C 60/75

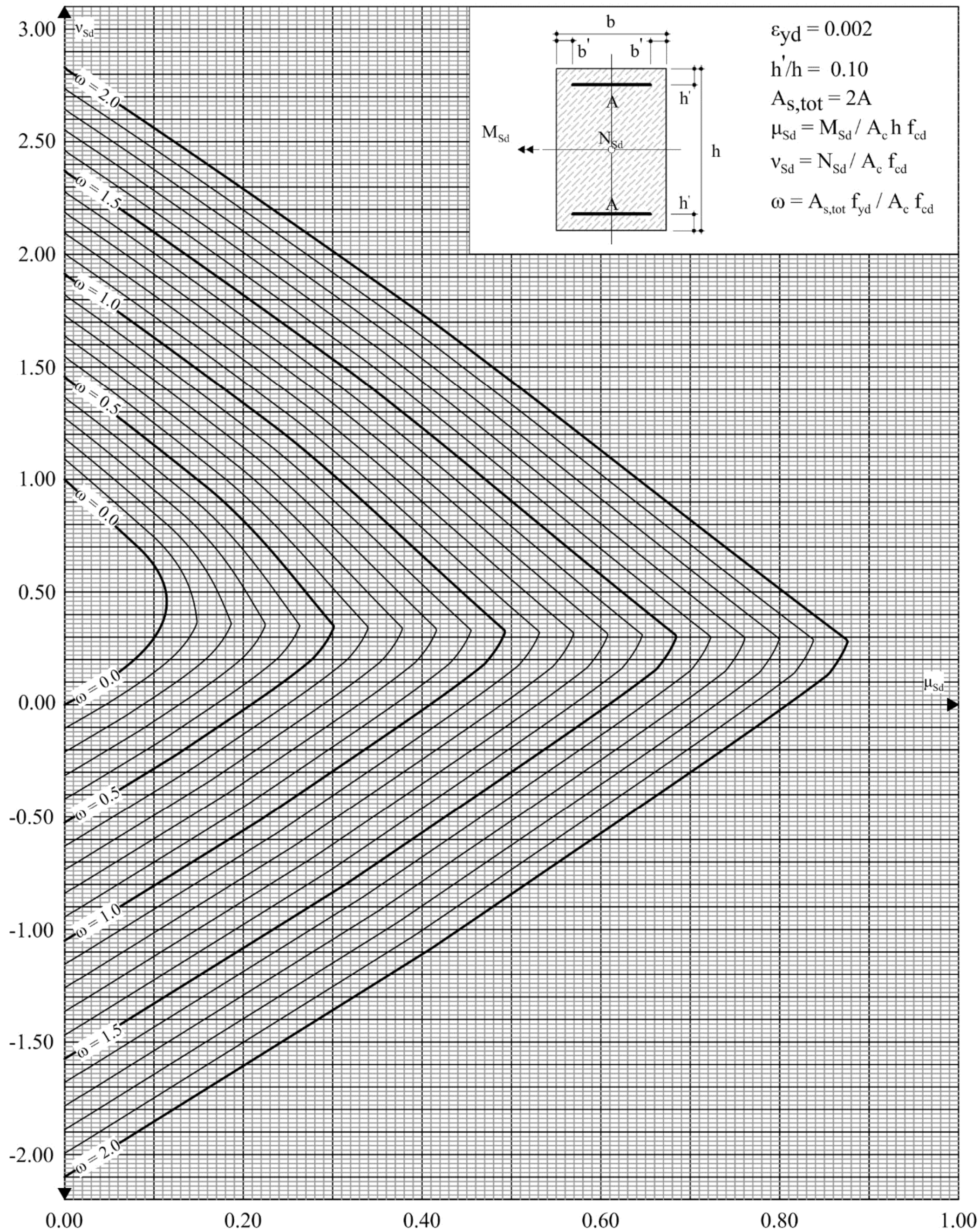


Figure 4-7 Uniaxial Chart for C 60/75 ($A_{s,tot}$ = 2A)

Uniaxial Chart C 70/85

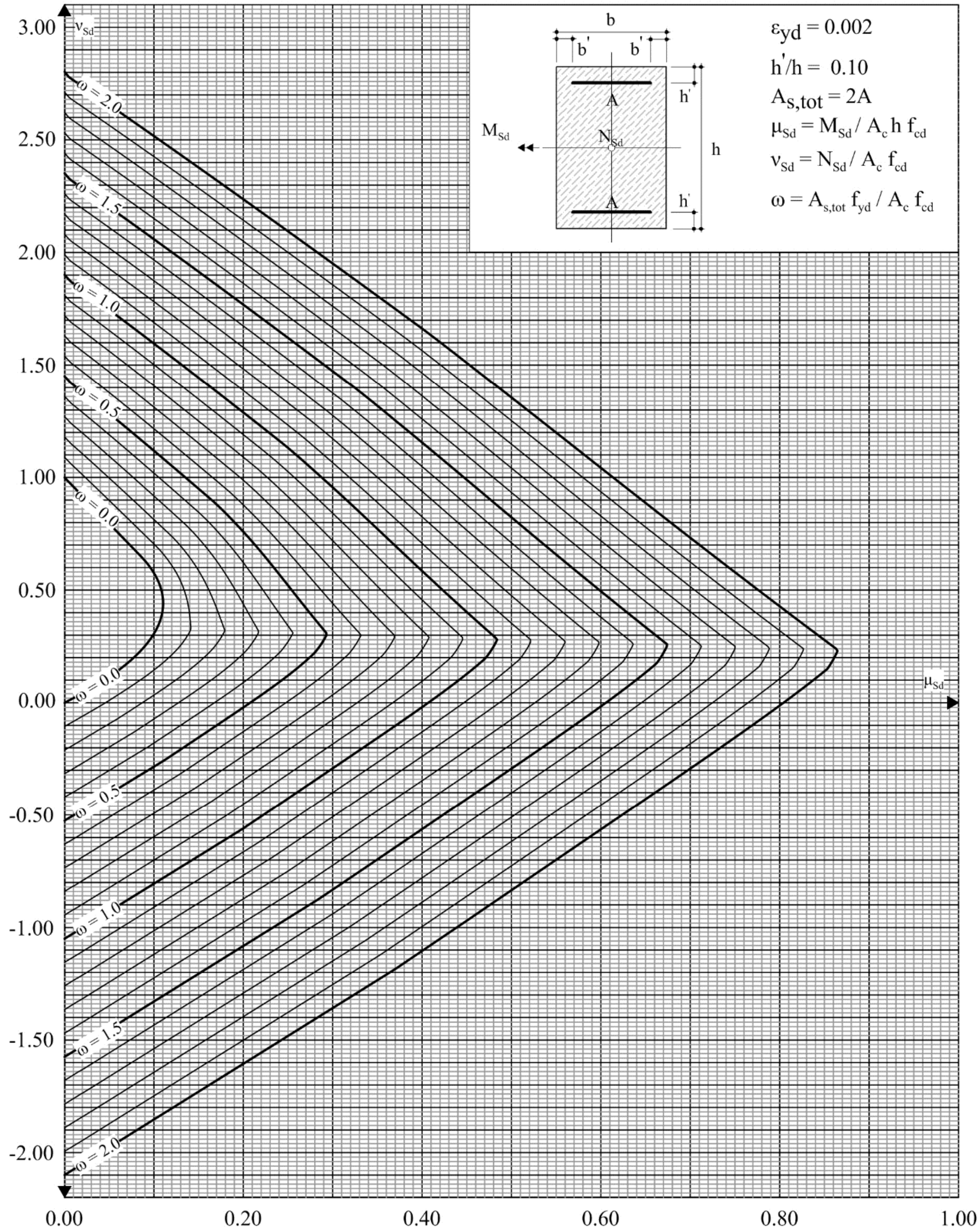


Figure 4-8 Uniaxial Chart for C 70/85 ($A_{s,tot}$ = 2A)

Uniaxial Chart C 80/95

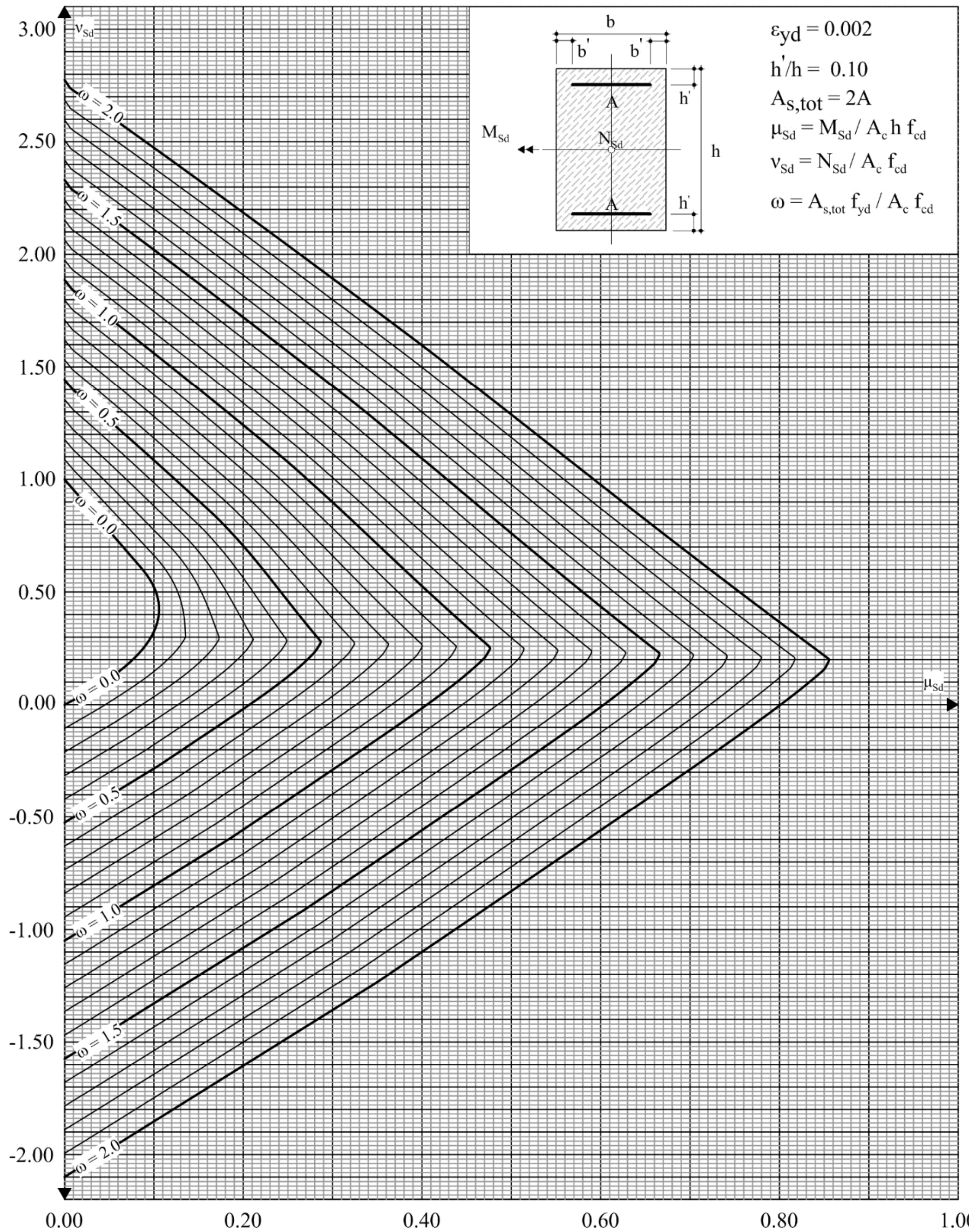


Figure 4-9 Uniaxial Chart for C 80/95 ($A_{s,tot} = 2A$)

Uniaxial Chart C 90/105

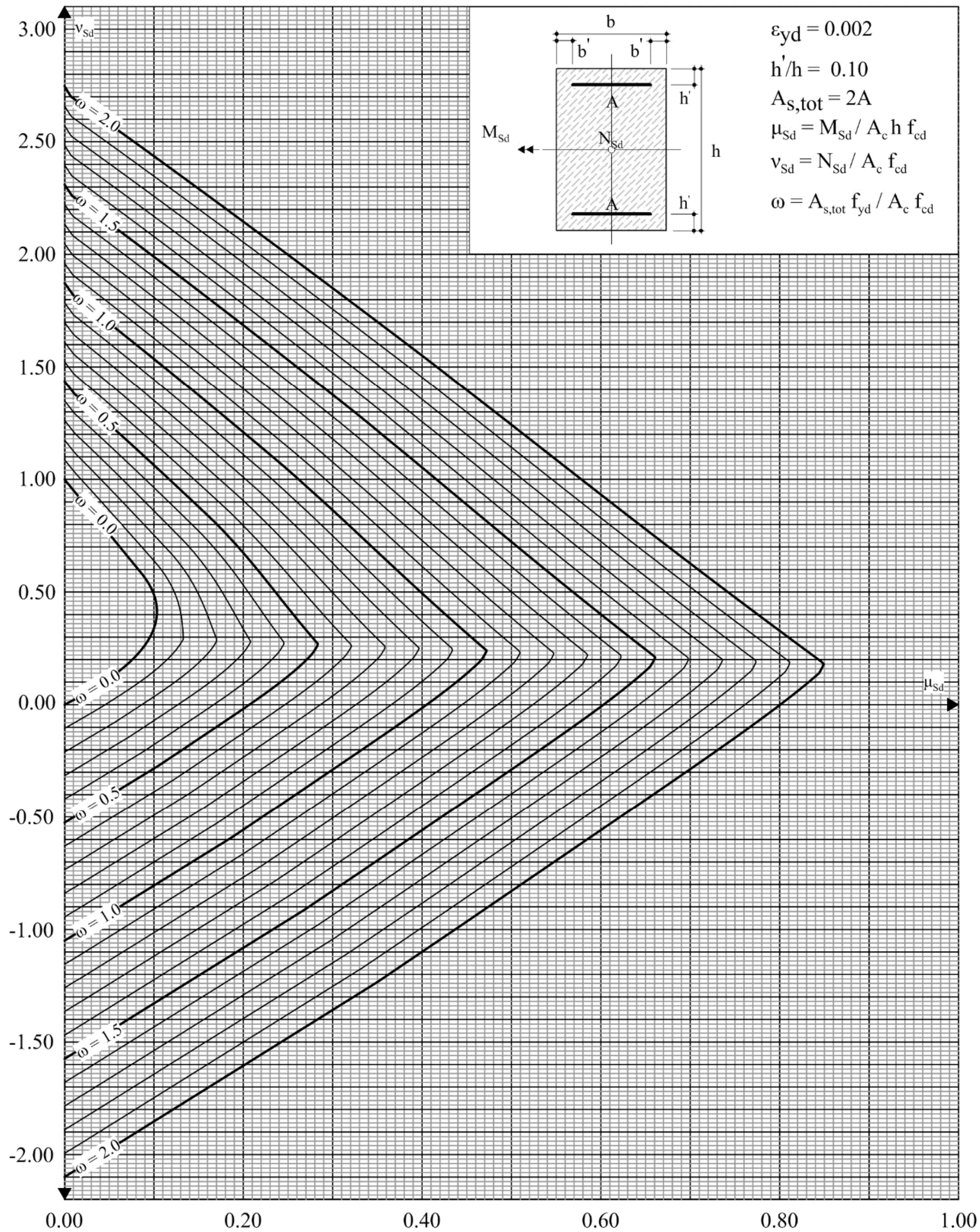


Figure 4-10 Uniaxial Chart for C 90/105 ($A_{s,tot}$ = 2A)

Uniaxial Chart C 12/15 - C 50/60

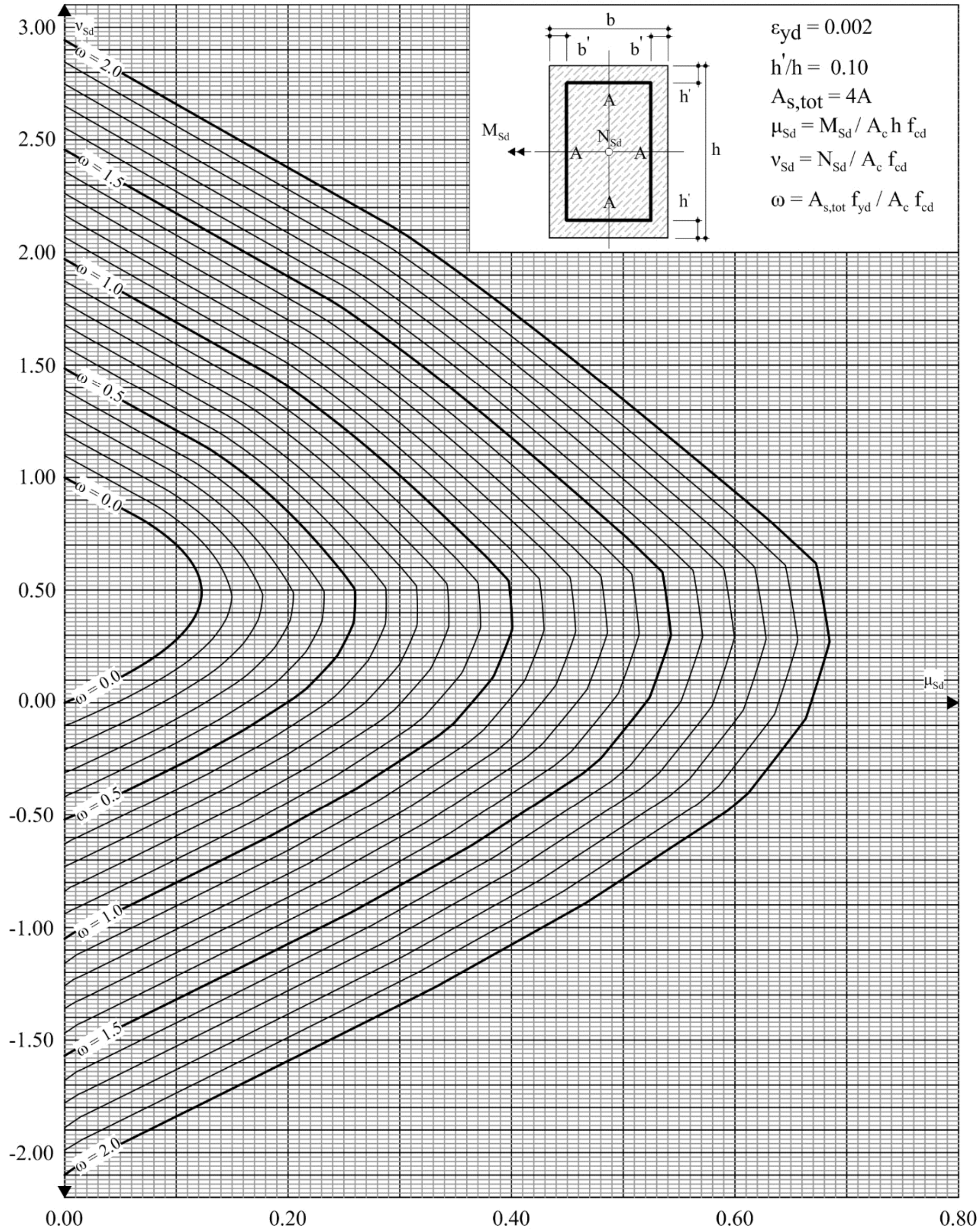


Figure 4-11 Uniaxial Chart for C 12/15 – C50/60 ($A_{s,tot}$ = 4A)

Uniaxial Chart C 55/67

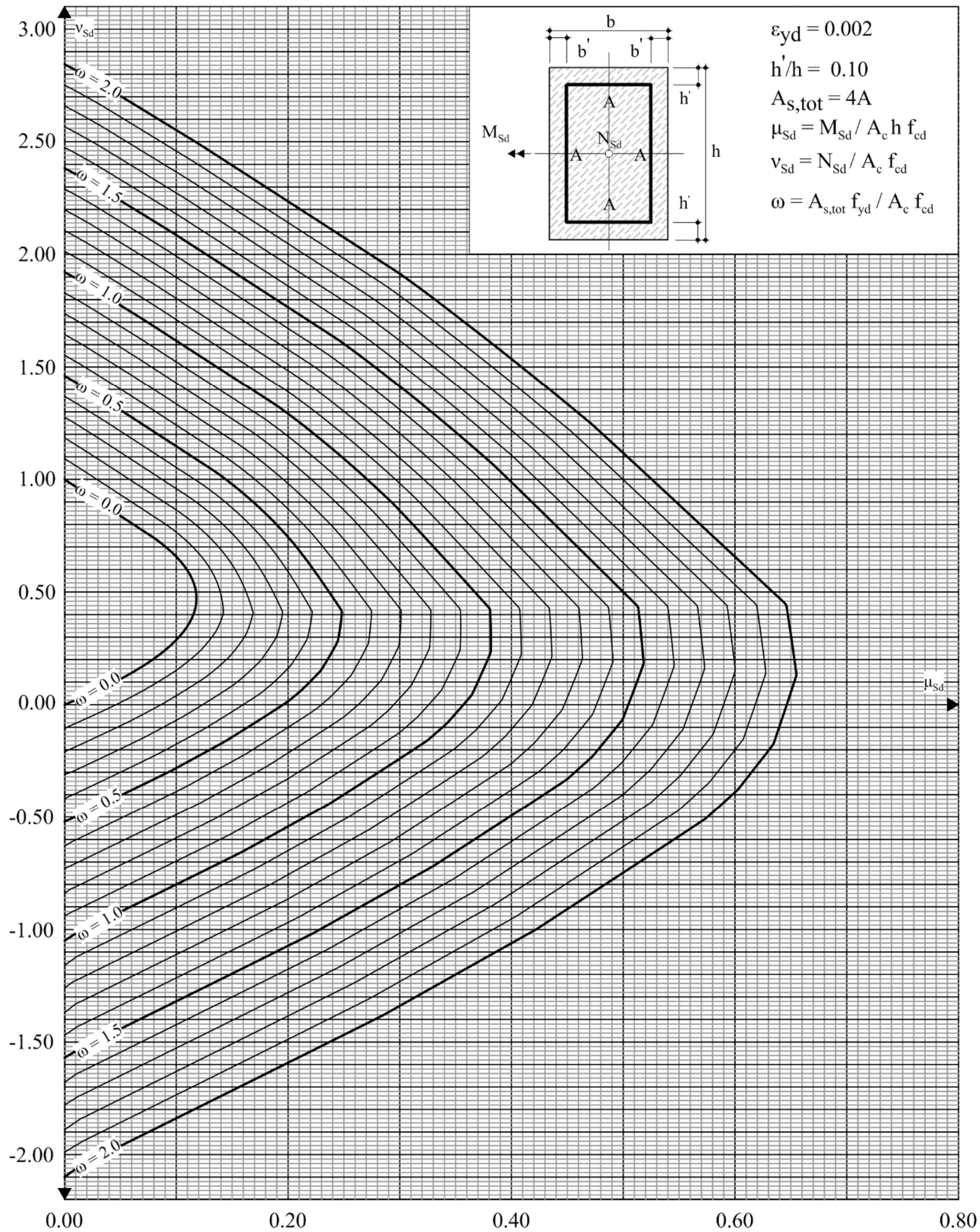


Figure 4-12 Uniaxial Chart for C 55/67 ($A_{s,tot}$ = 4A)

Uniaxial Chart C 60/75

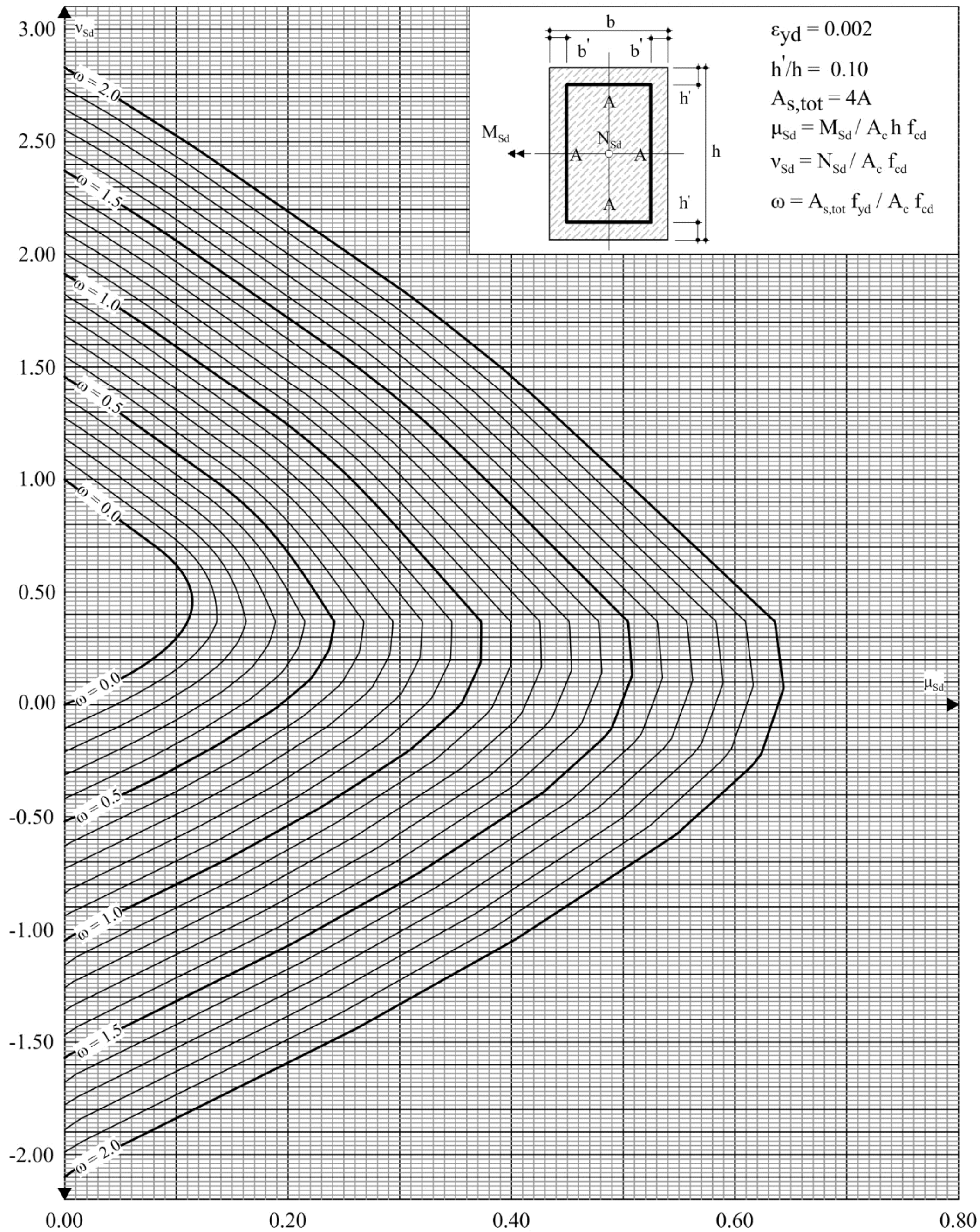


Figure 4-13 Uniaxial Chart for C 60/75 ($A_{s,tot}$ = 4A)

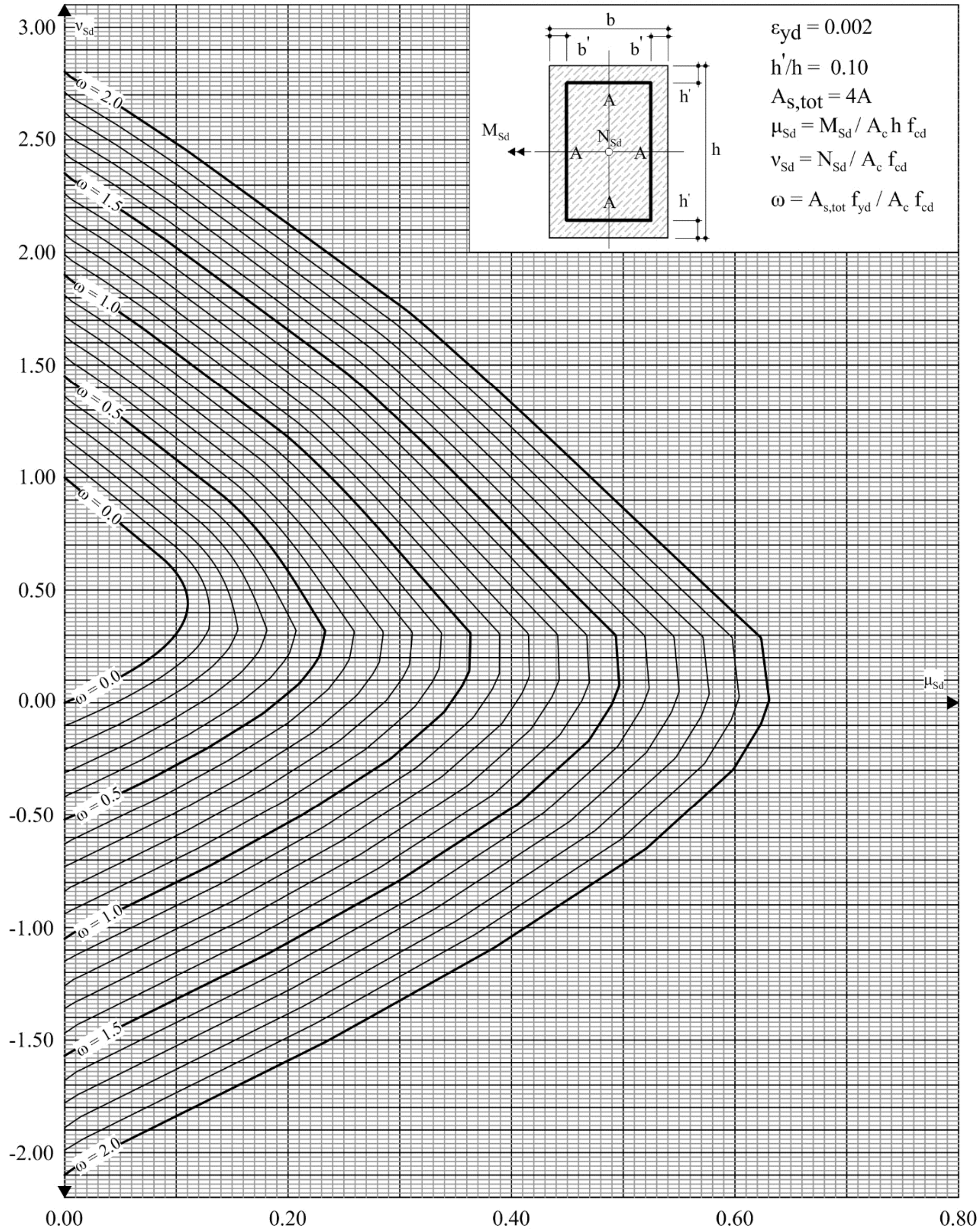


Figure 4-14 Uniaxial Chart for C 70/85 ($A_{s,tot}$ = 4A)

Uniaxial Chart C 80/95

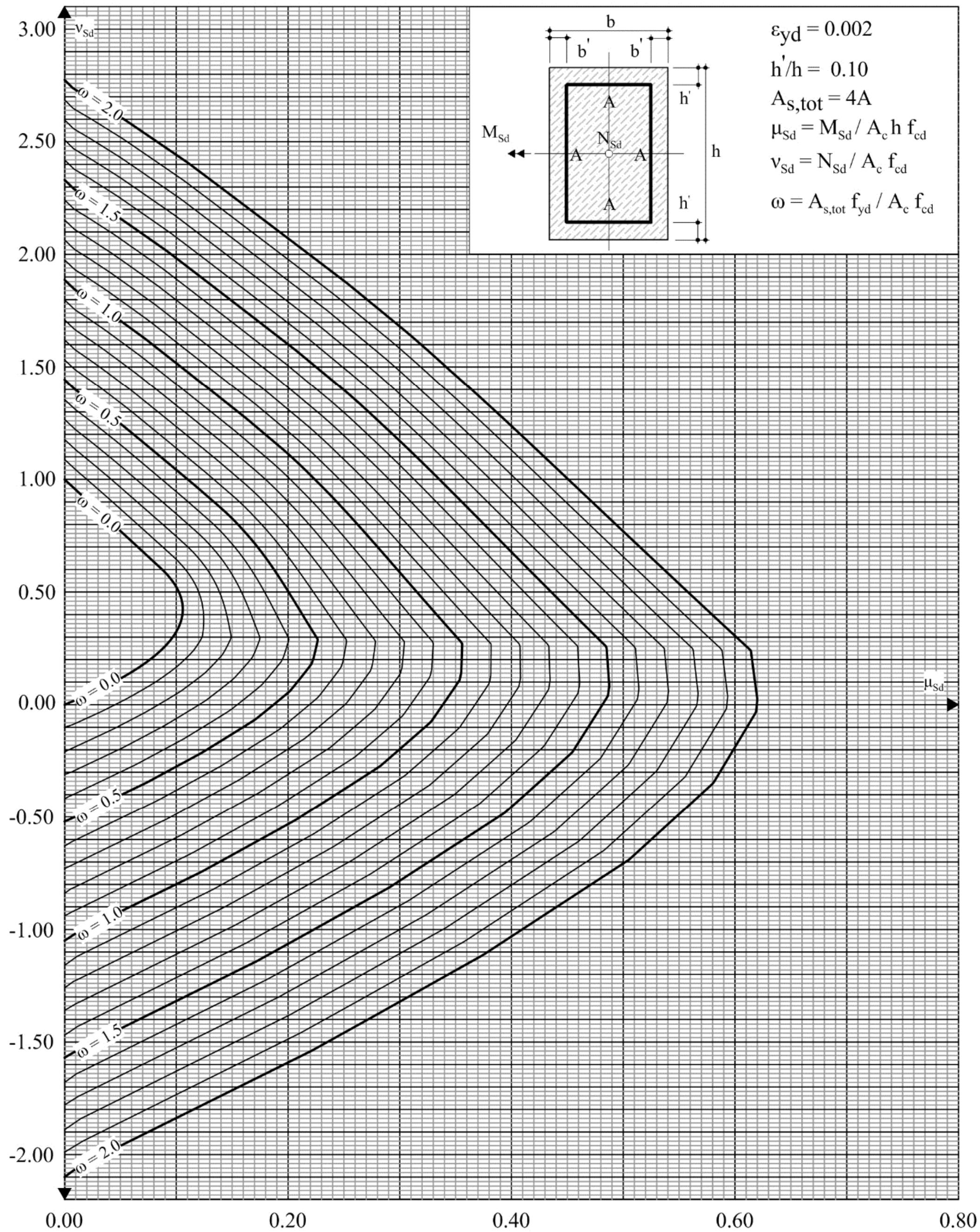


Figure 4-15 Uniaxial Chart for C 80/95 ($A_{s,tot}$ = 4A)

Uniaxial Chart C 90/105

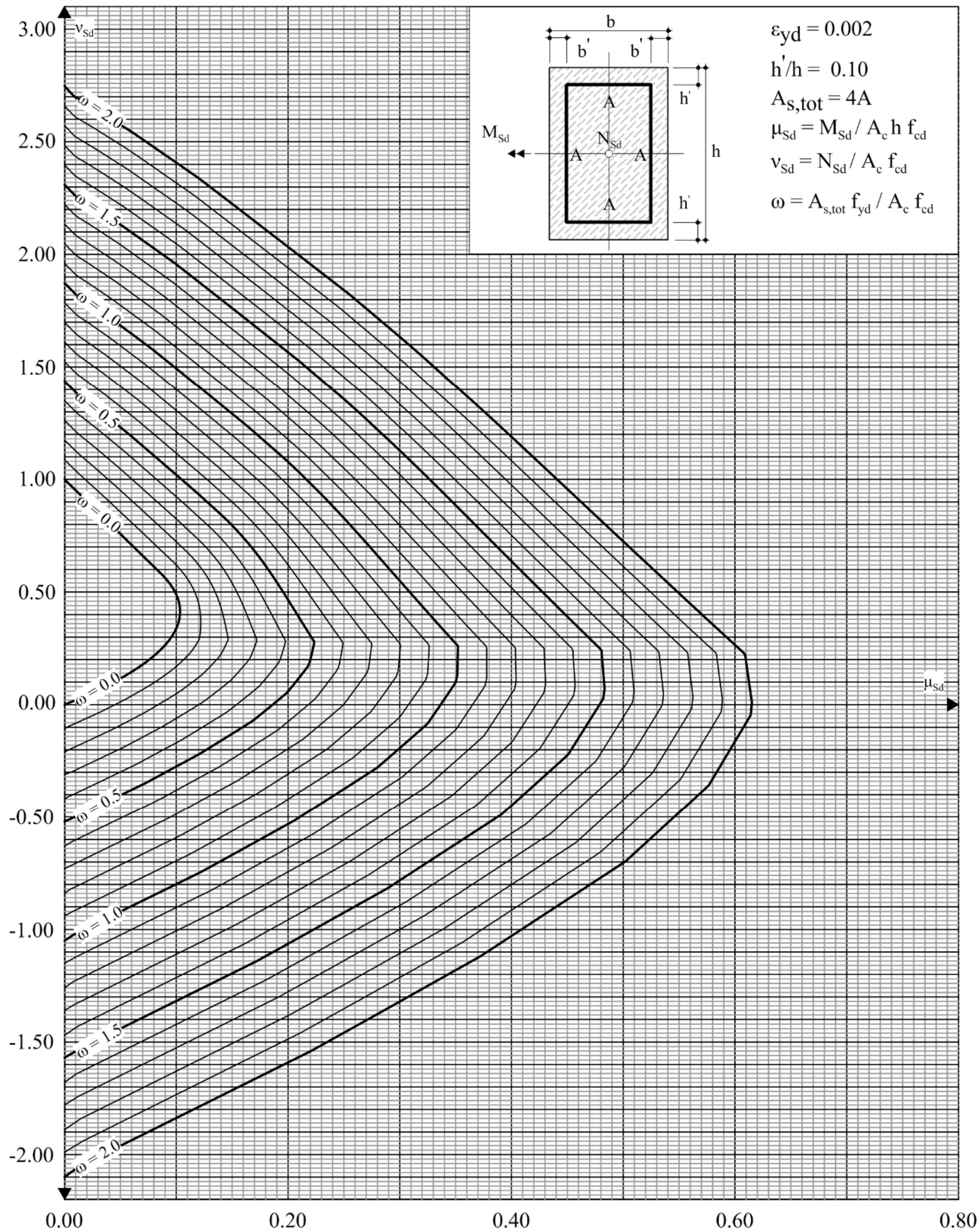
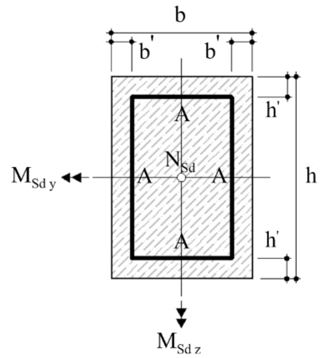


Figure 4-16 Uniaxial Chart C 90/105 ($A_{s,tot} = 4A$)

Biaxial Chart C 12/15 - C 50/60



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

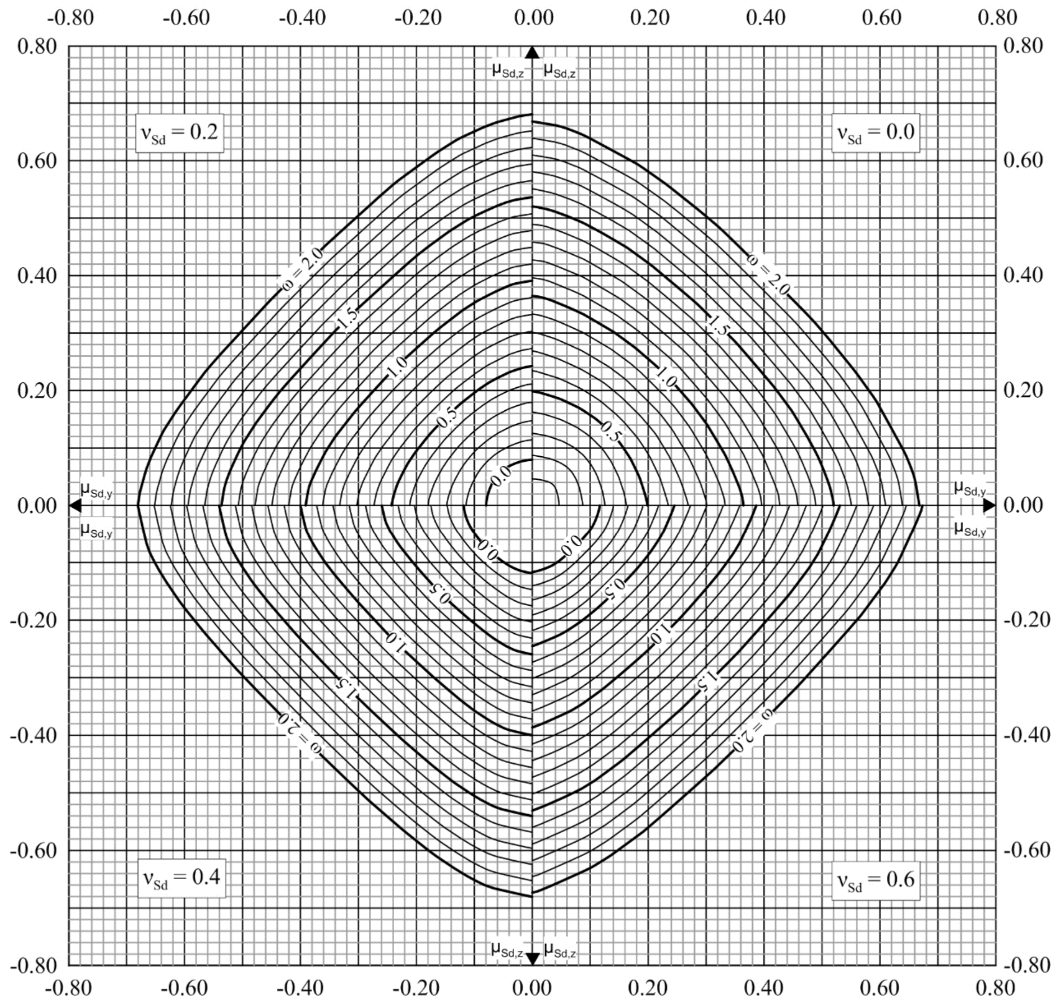
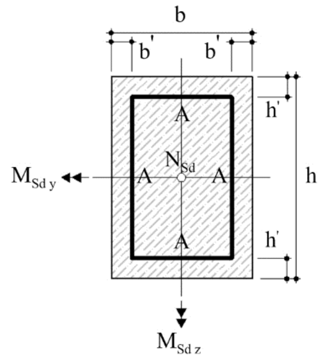


Figure 4-17 Biaxial Chart C 12/15 – C 50/60 I ($A_{s,tot} = 4A$)

Biaxial Chart C 12/15 - C 50/60



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

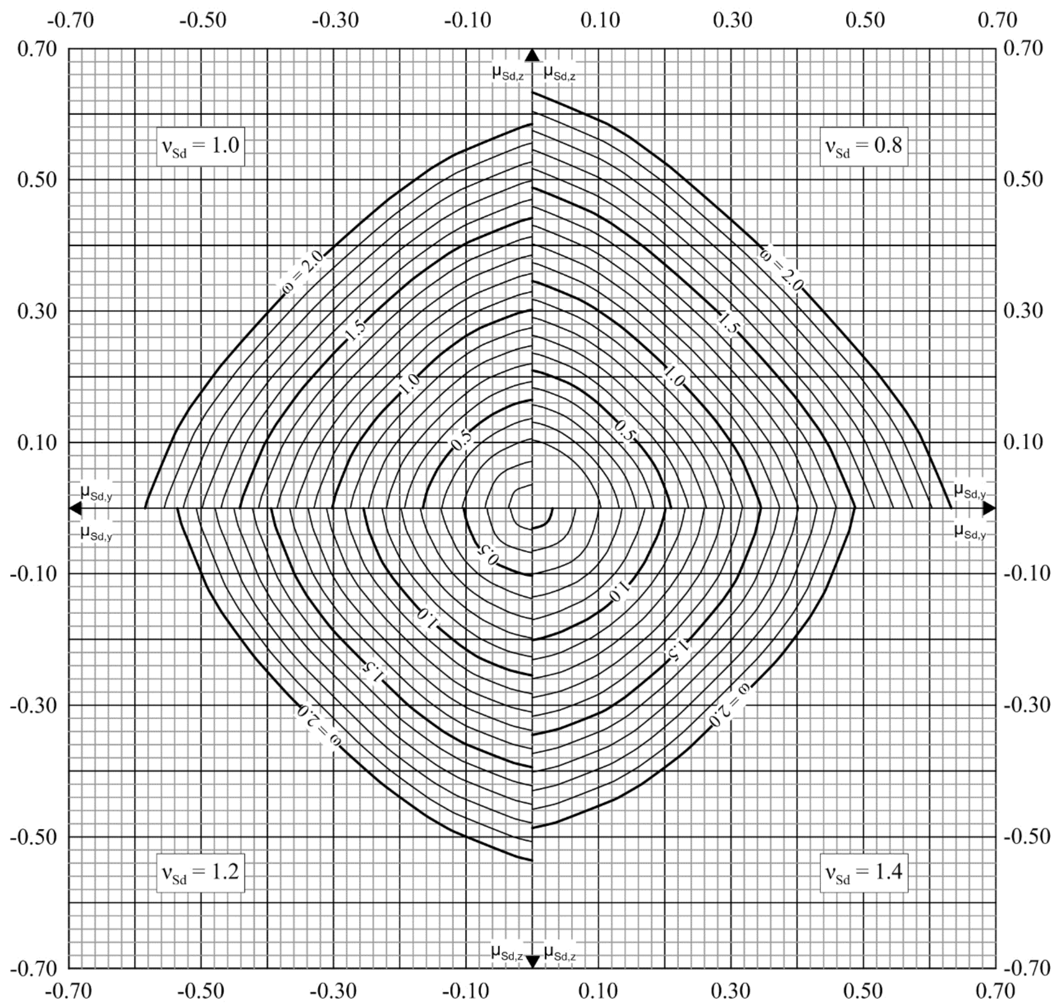
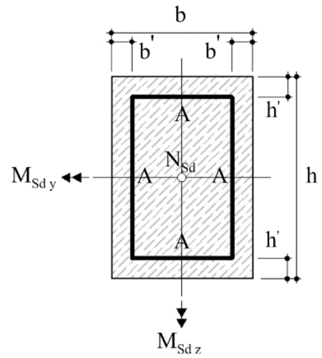


Figure 4-18 Biaxial Chart C 12/15 – C 50/60 II ($A_{s,tot} = 4A$)

Biaxial Chart C 55/67



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

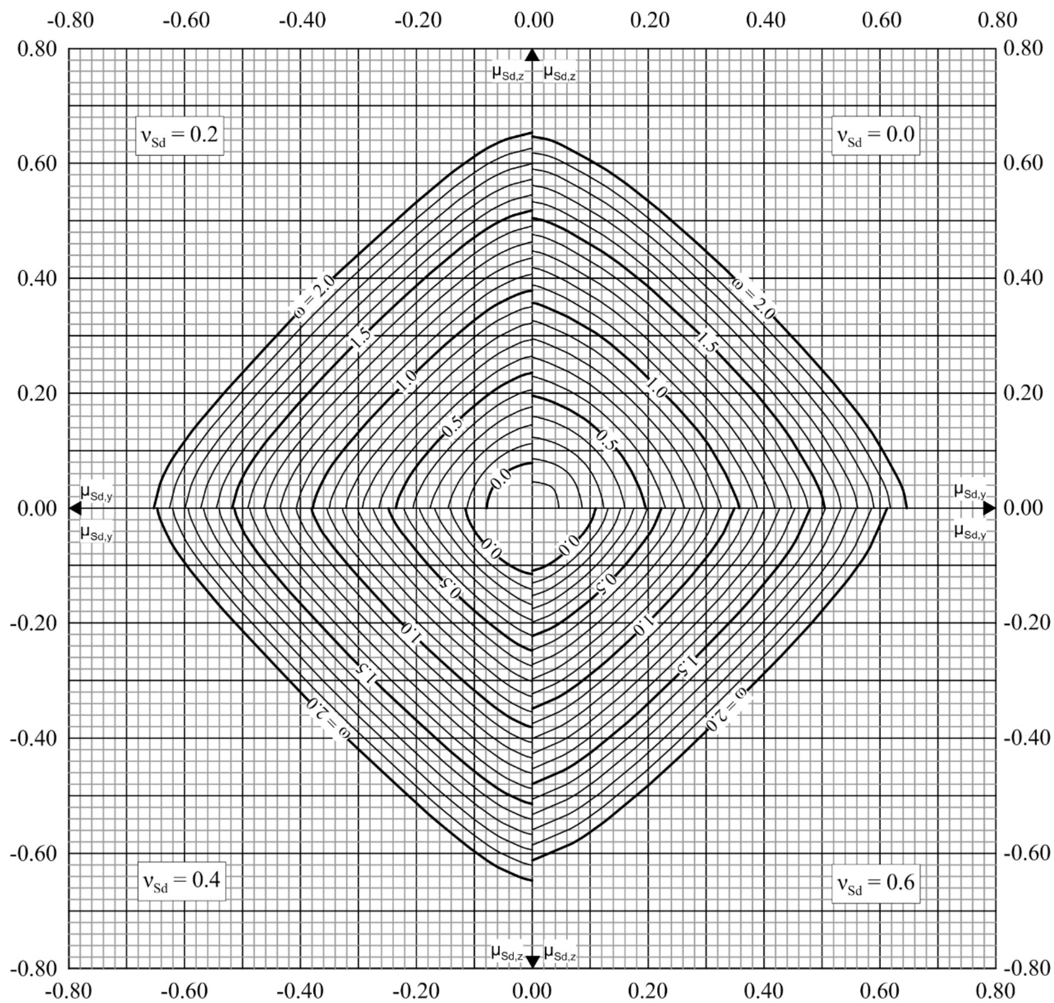
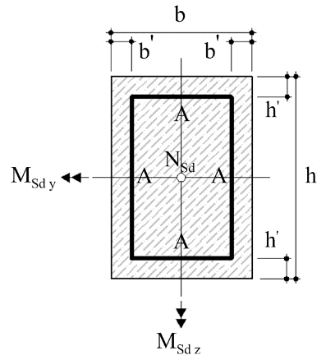


Figure 4-19 Biaxial Chart C 55/67 I ($A_{s,tot}$ = 4A)

Biaxial Chart C 55/67



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

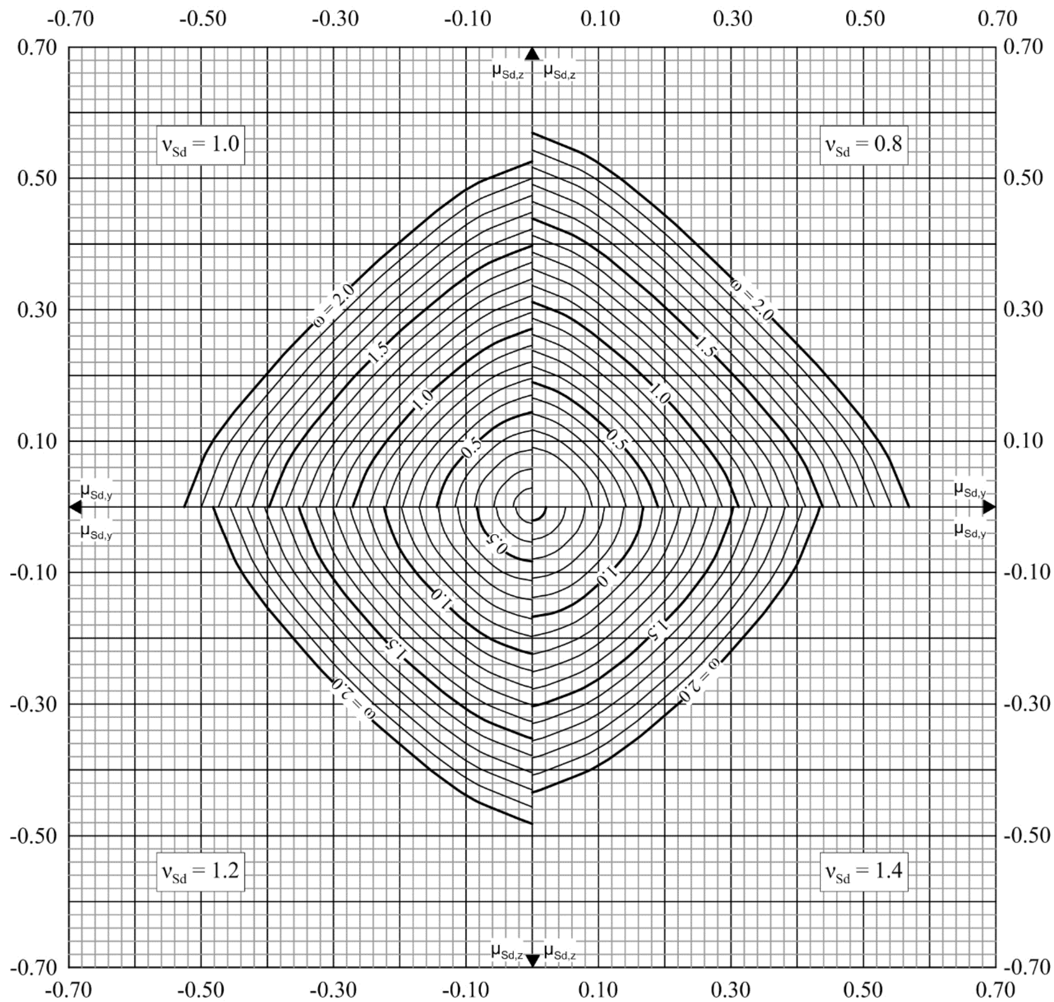


Figure 4-20 Biaxial Chart C 55/67 II ($A_{s,tot}$ = 4A)

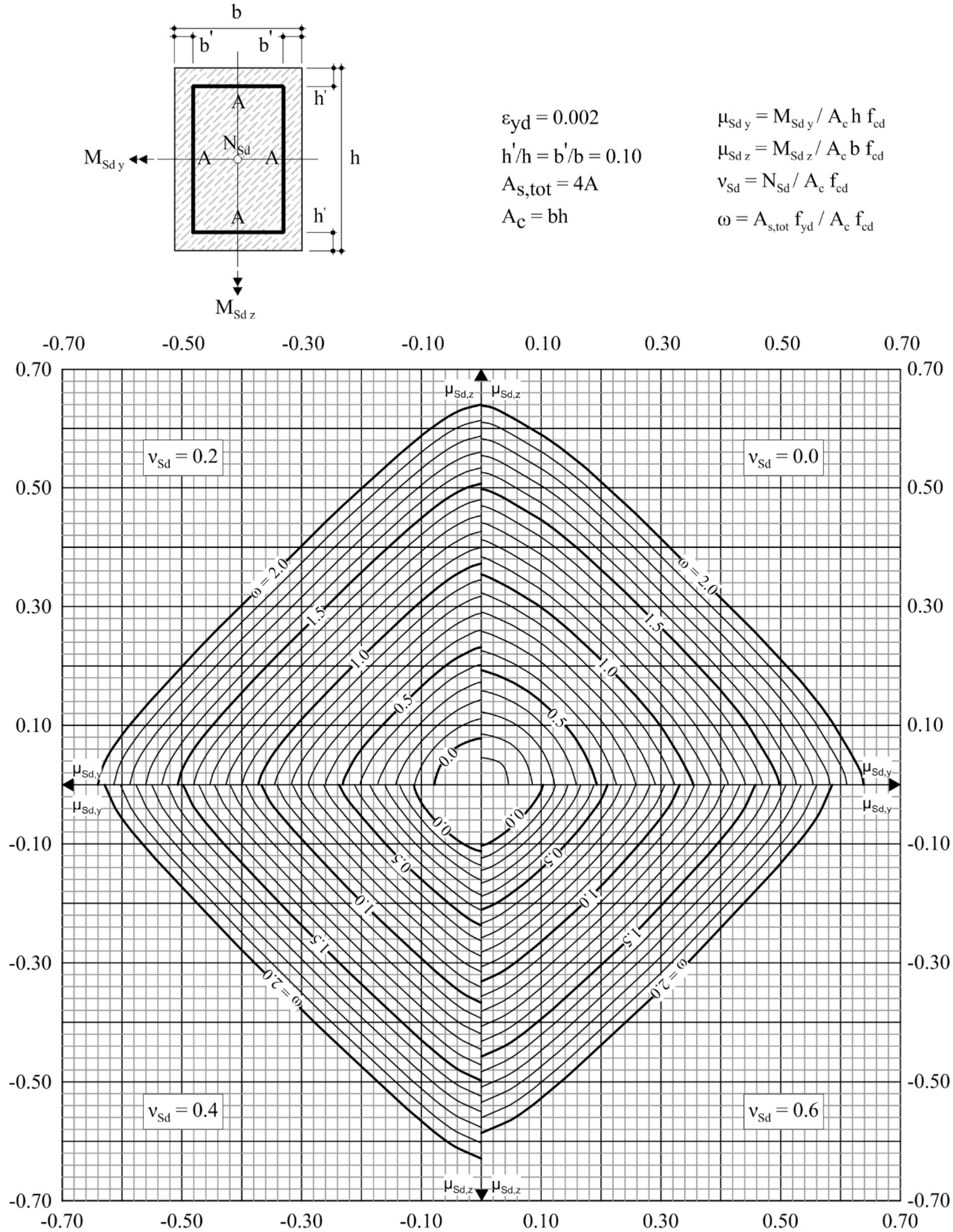
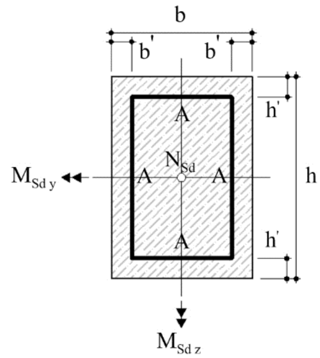


Figure 4-21 Biaxial Chart C 60/75 I ($A_{s,tot} = 4A$)

Biaxial Chart C 60/75



$\varepsilon_{yd} = 0.002$
$h'/h = b'/b = 0.10$
$A_{s,tot} = 4A$
$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$
$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$
$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$
$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

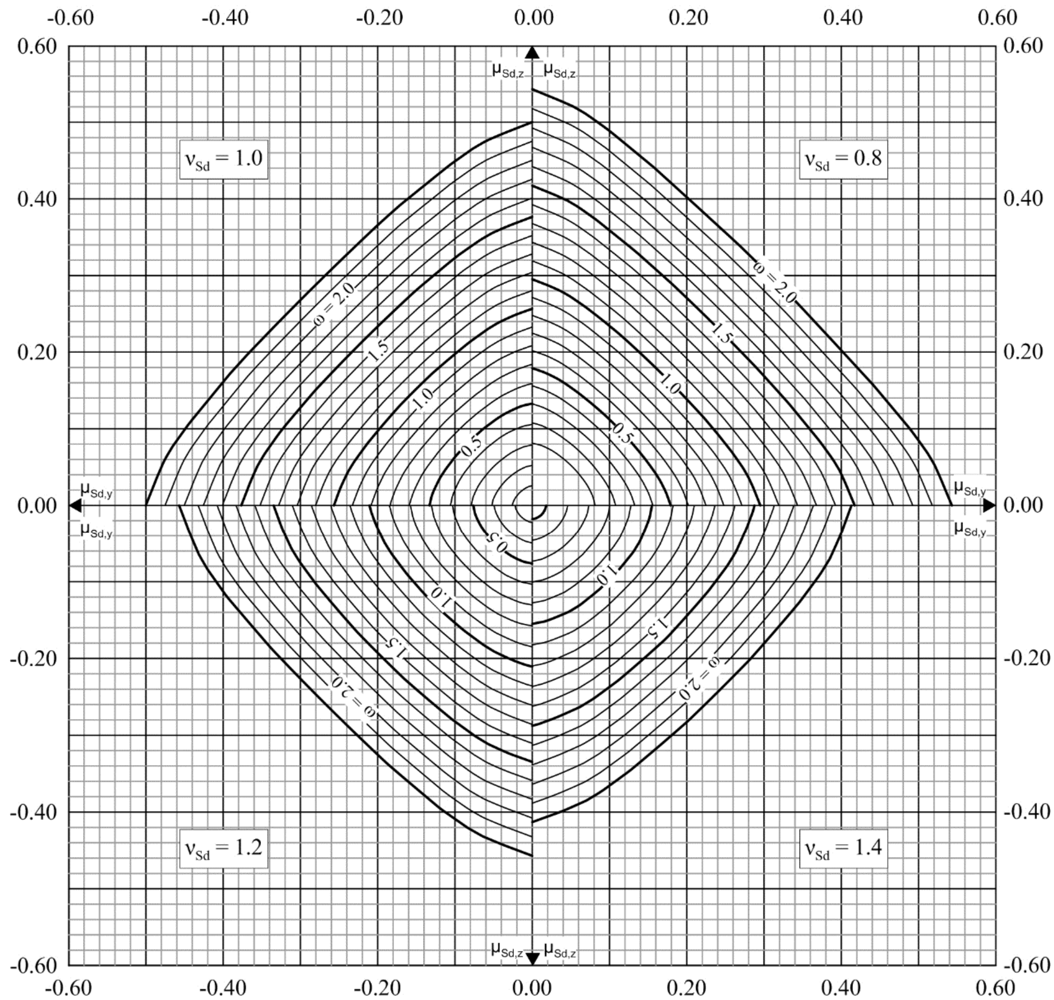


Figure 4-22 Biaxial Chart C 60/75 II ($A_{s,tot}$ = 4A)

Biaxial Chart C 70/85

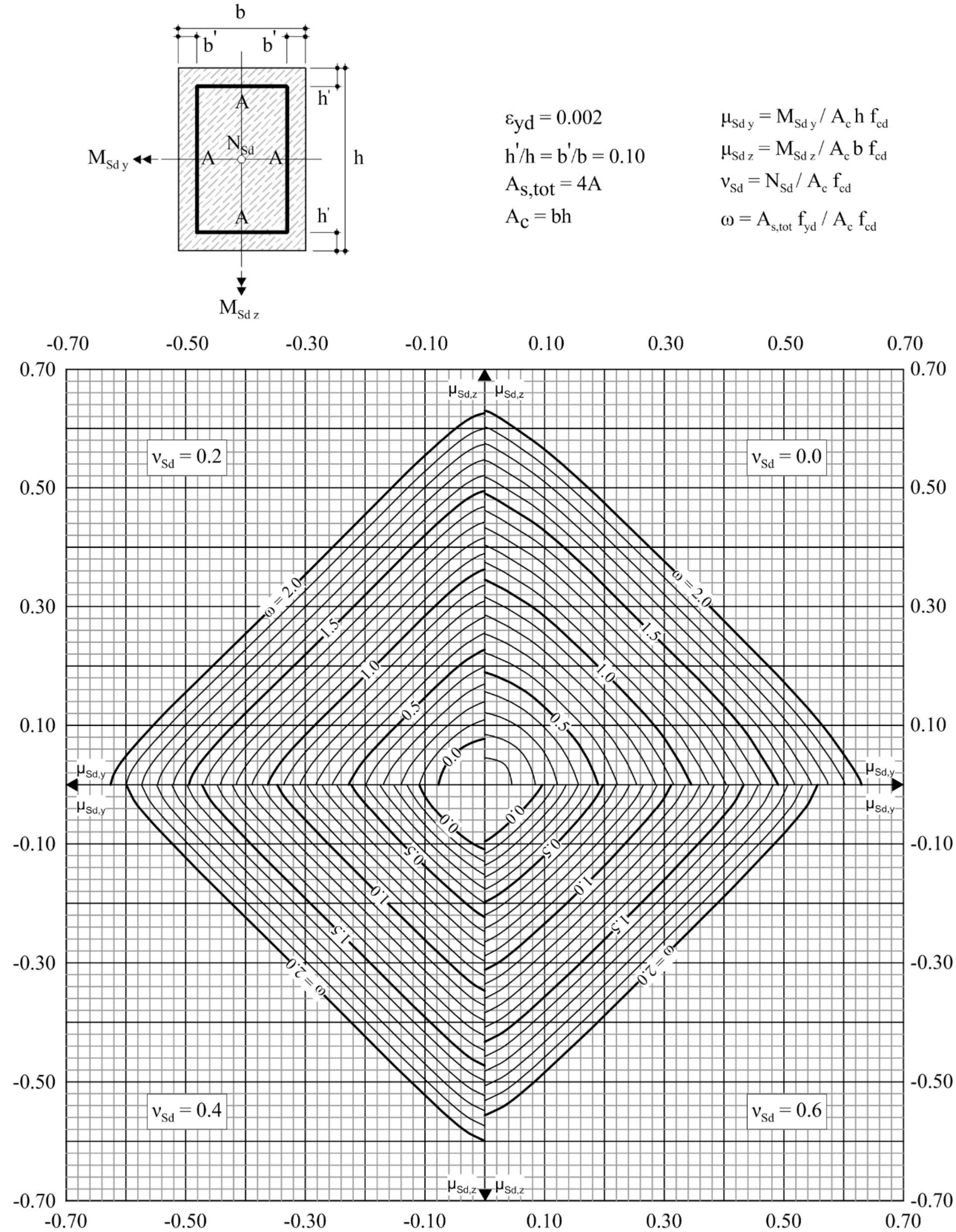
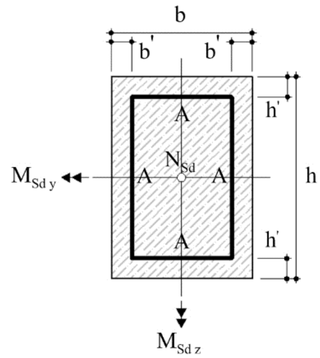


Figure 4-23 Biaxial Chart C 70/85 I ($A_{s,tot}$ = 4A)

Biaxial Chart C 70/85



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

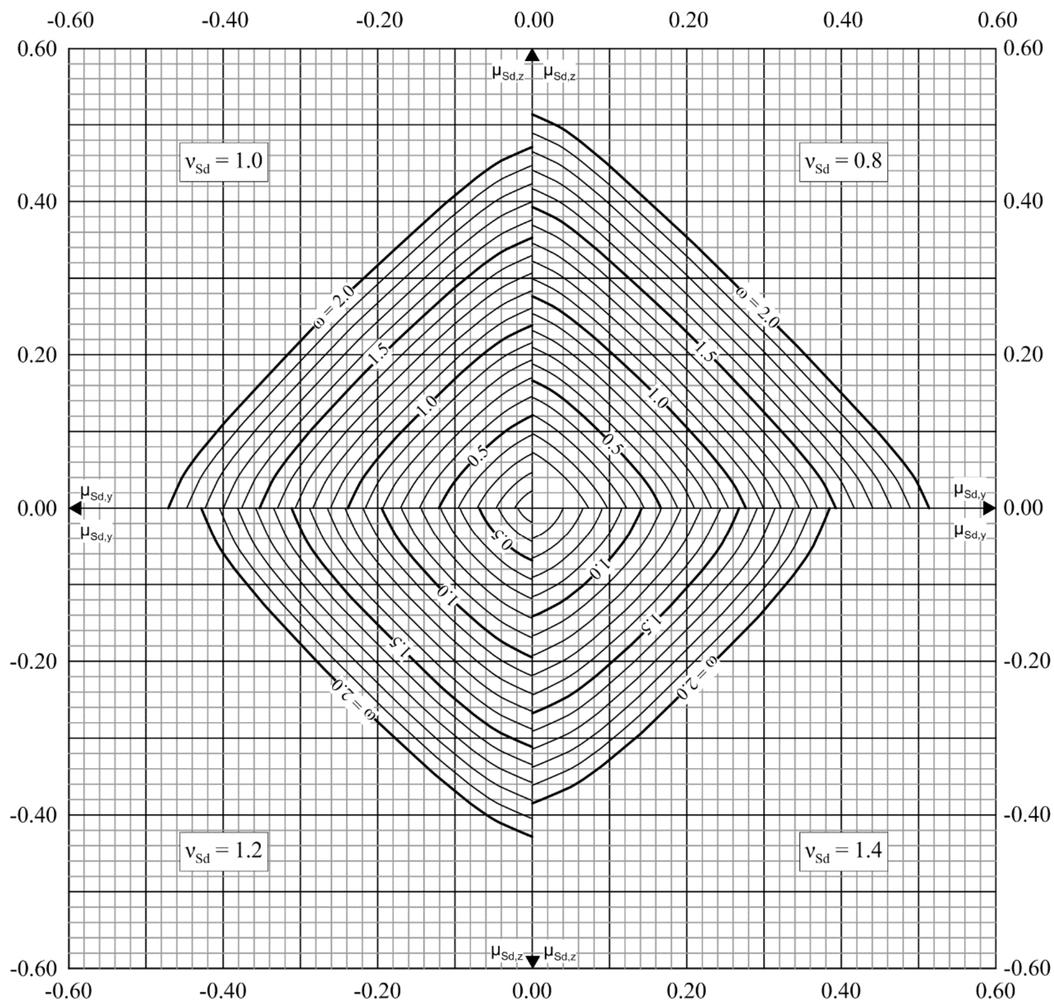
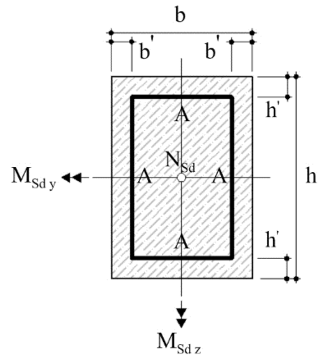


Figure 4-24 Biaxial Chart C 70/85 II ($A_{s,tot} = 4A$)

Biaxial Chart C 80/95



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

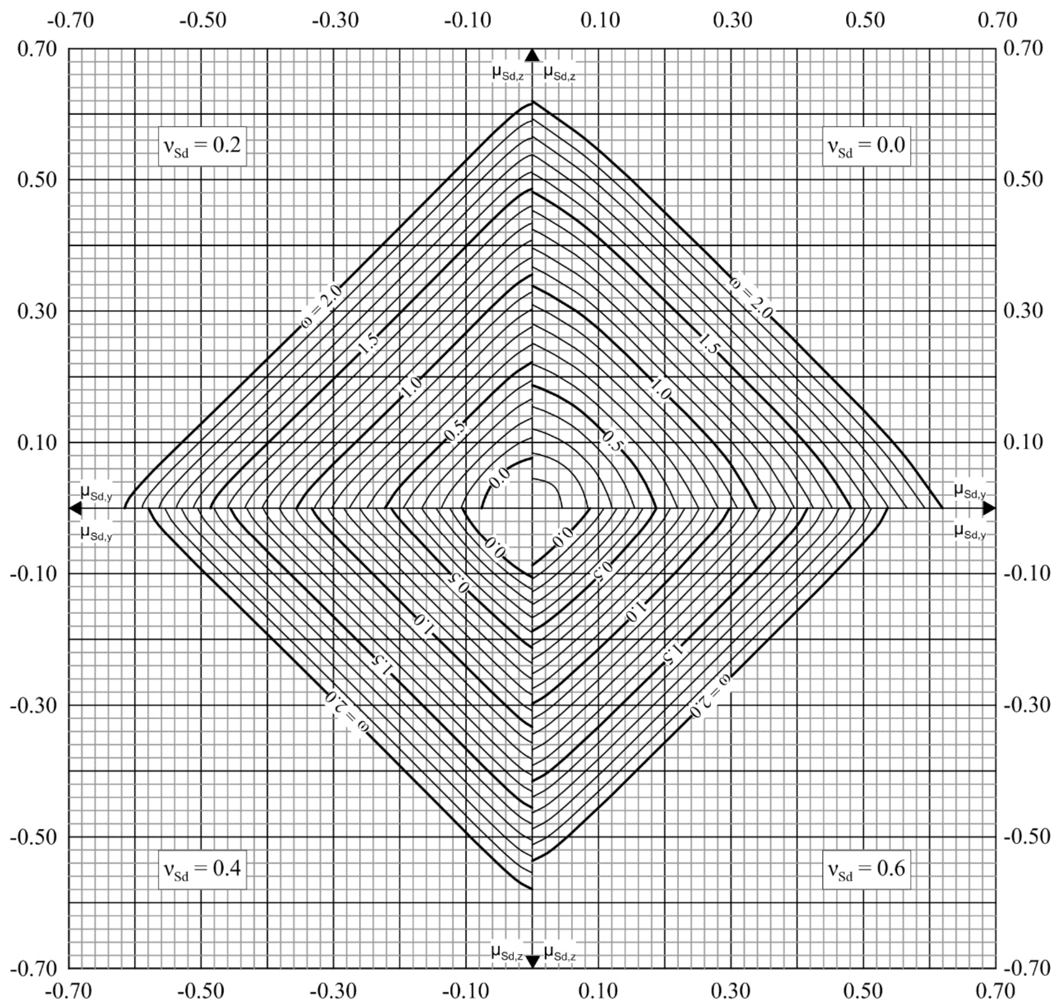
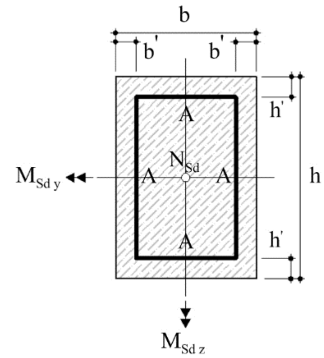


Figure 4-25 Biaxial Chart C 80/95 I ($A_{s,tot}$ = 4A)

Biaxial Chart C 80/95



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

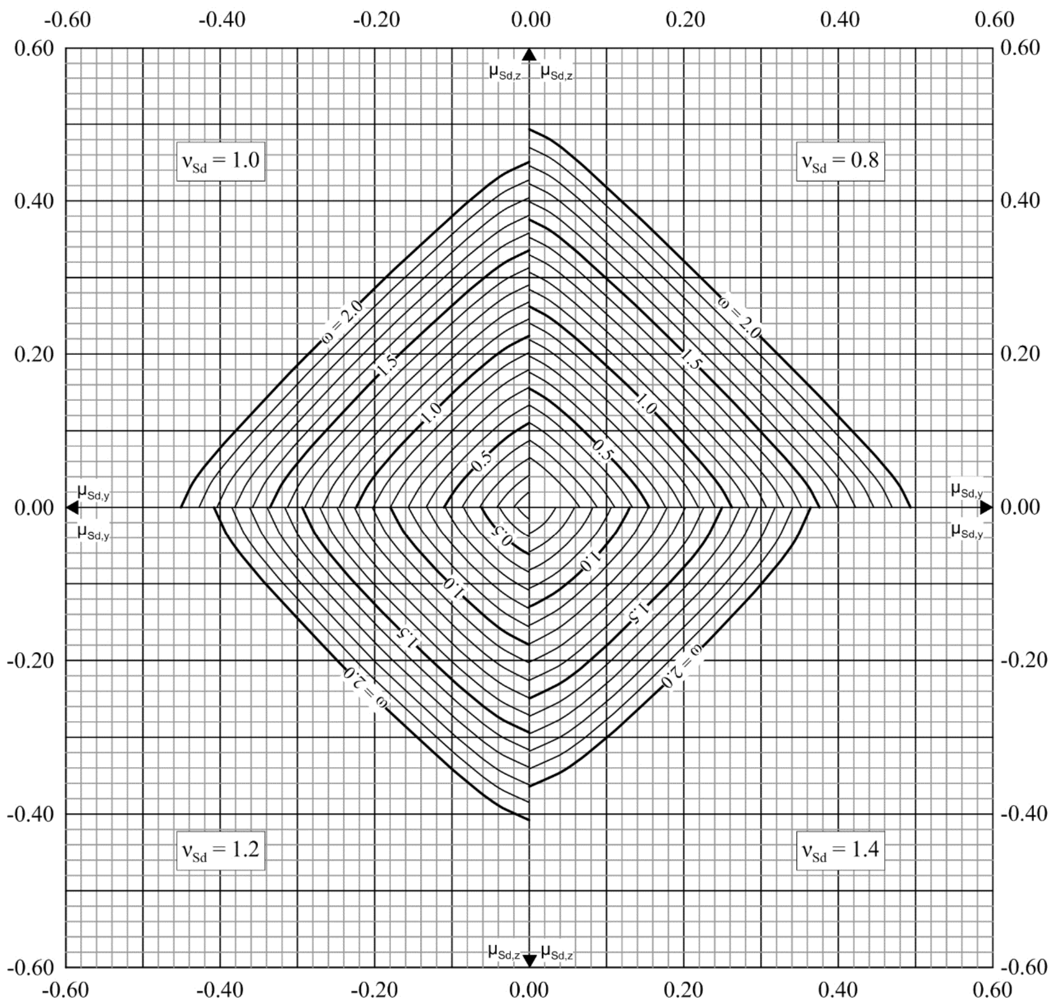
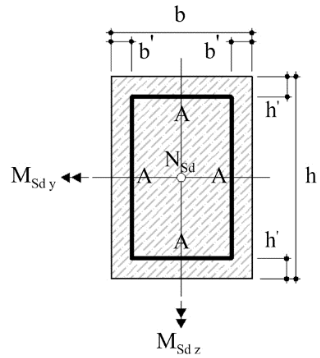


Figure 4-26 Biaxial Chart C 80/95 II ($A_{s,tot} = 4A$)



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

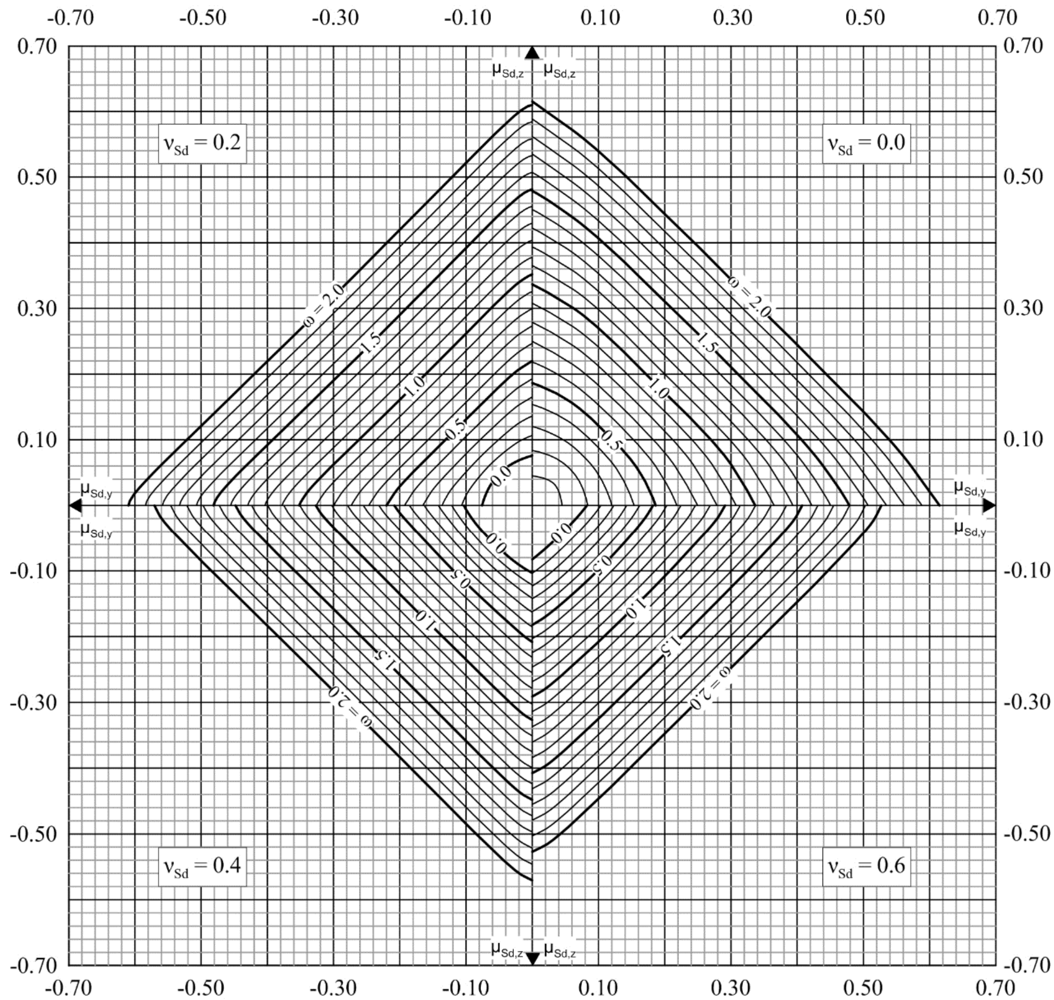
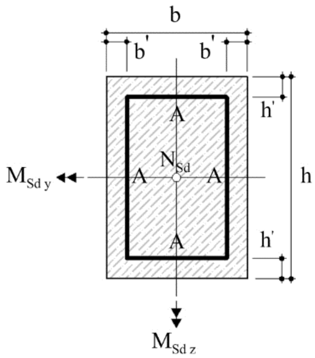


Figure 4-27 Biaxial Chart C 90/105 I ($A_{s,tot}$ = 4A)

Biaxial Chart C 90/105



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = M_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = M_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$

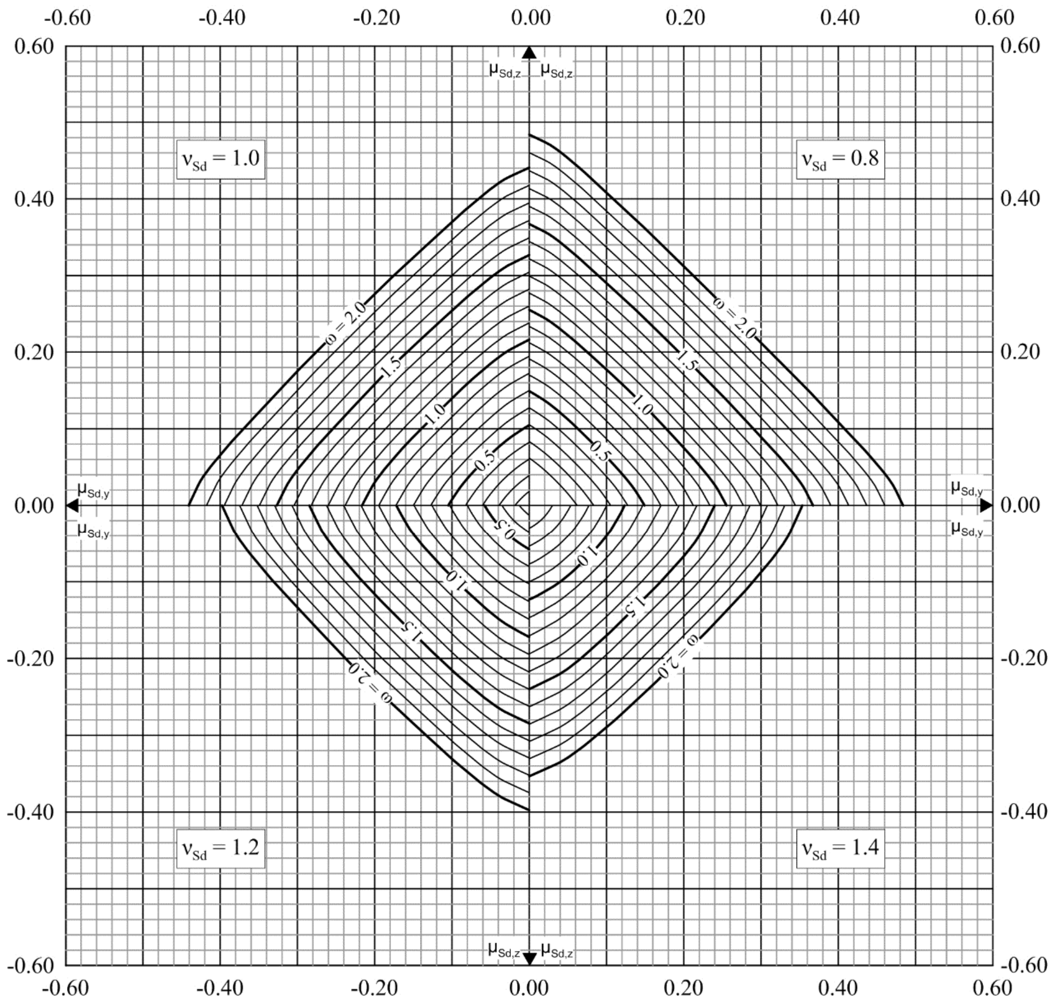


Figure 4-28 Biaxial Chart C 90/105 II ($A_{s,tot}$ = 4A)

4.7 Design Examples

4.7.1 Design of Column Subject to Uniaxial Bending

Given: Action Effects: factored action effect including slenderness effect,

N_{Sd} = 2500 kN and $M_{Sd,y}$ = 350 kNm

Material Data: Concrete C 70/85 and Steel S 460

Required: Quantity of reinforcement for two sides uniformly distributed reinforcement

Solution:

Assume column size, b/h = 300/400 mm and cover ratio $h'/h = b'/b = 0.10$

$$\nu_{Sd} = \frac{N_{Sd}}{A_c f_{cd}} = \frac{2500\times10^3}{300\times400\times46.67} = 0.45$$

$$\mu_{Sd,y} = \frac{M_{Sd,y}}{A_c h f_{cd}} = \frac{350\times10^6}{300\times400\times400\times46.67} = 0.16$$

Reading ω from Figure 4-8,

$$\omega = 0.20$$

$$A_{s,tot} = \frac{\omega A_c f_{cd}}{f_{yd}} = \frac{0.20\times300\times400\times46.67}{400} = 2801\, mm^2$$

Provide $A_{s,tot} / 2 = 1400.5\ mm^2$ uniformly distributed on the shorter sides

4.7.2 Design of Column Subject to Biaxial Bending

Given: Action Effects: factored action effect including slenderness effect,

N_{Sd} = 2500 kN, $M_{Sd,y}$ = 350 kNm and $M_{Sd,z}$ = 250 kNm

Material Data: Concrete C 70/85 and Steel S 460

Required: Quantity of reinforcement for four sides uniformly distributed reinforcement

Solution:

Assume column size, b/h = 400/400 mm and cover ratio $h'/h = b'/b = 0.10$

$$v_{Sd} = \frac{N_{Sd}}{A_c f_{cd}} = \frac{2500\times10^3}{400\times400\times46.67} = 0.335$$

$$\mu_{Sd,y} = \frac{M_{Sd,y}}{A_c h f_{cd}} = \frac{350\times10^6}{400\times400\times400\times46.67} = 0.12$$

$$\mu_{Sd,z} = \frac{M_{Sd,z}}{A_c b f_{cd}} = \frac{250\times10^6}{400\times400\times400\times46.67} = 0.084$$

Reading ω from Figure 4-23 and Figure 4-24,

$\omega = 0.35$ for $v_{Sd} = 0.2$ Figure 4-23

$\omega = 0.35$ for $v_{Sd} = 0.4$ Figure 4-24

Interpolating for $v_{Sd} = 0.335$, $\omega = 0.35$

$$A_{s,tot} = \frac{\omega A_c f_{cd}}{f_{yd}} = \frac{0.35\times400\times400\times46.67}{400} = 6534\ mm^2$$

Provide $A_{s,tot} / 4 = 1633.5\ \text{mm}^2$ uniformly distributed on four sides

5. Conclusions and Recommendations

1. An iterative numerical integration procedure such as Multiple application of Simpson's 1/3 rule with n = 1000 segments can be used to accurately estimate the relative compressive force in the concrete and relative distance of point of application of compressive force from the outer most concrete fiber under compression for NSC and HSC when developing design table and general design charts.
2. For sections entirely under compression, no accurate reading is possible when using general design charts. The designer must use uniaxial interaction chart to estimate the capacity of the section.
3. Evaluation of double integrals in the preparation of interaction charts for columns can be reduced to numerical integration of few line integrals by applying Green's Theorem. The line to be integrated should define the boundaries.
4. An iterative numerical integration procedure such as Third-Order Gauss Quadrature formulas can be used to accurately integrate the line integrals when applying Green's Theorem.
5. The algorithm discussed for preparation of interaction chart can be extended to cover arbitrary shaped solid and hollow section with arbitrary reinforcement arrangements.
6. Resistance of cross sections should be based on net cross section for HSC by avoiding double counting of concrete area that is displaced by the reinforcing steels.
7. Using iterative based numerical methods for preparation of deign aids insures computational efficient computer program.

6. References

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