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ADDIS ABABA UNIVERSITY  
SCHOOL OF GRADUATE

FACULTY OF TECHNOLOGY  
DEPARTMENT OF CIVIL ENGINEERING

A COMPUTER PROGRAM FOR PRODUCING INTERACTION  
CHARTS OF T-COLUMN

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A thesis submitted to the school of Graduate Studies of Addis Ababa University in Partial fulfillment of the Requirements for the Degree of Masters of Science in Civil Engineering (Structures)

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# SCHOOL OF GRADUATE STUDIES

## A COMPUTER PROGRAM FOR PRODUCING DESIGN

### CHARTS OF T-COLUMN

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## **DECLARATION**

I, the undersigned, declare that this thesis is my original work and has not been presented for a degree in any other university, and that all sources of material used for the thesis have been duly acknowledged.

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## Table of Contents

|   |           |
|---|-----------|
| Table of Content.....   | ii        |
| List of Illustrations.....  | iv        |
| List of Tables.....   | v         |
| Notations.....  | iv        |
| <br>  |           |
| <b>1.Introduction .....</b>   | <b>1</b>  |
| 1.1. Back ground of the problem.....                                | 1         |
| 1.2. Objective of the Study.....                                    | 3         |
| 1.3. Methodology.....   | 4         |
| 1.4. Study Content.....   | 5         |
| <br>  |           |
| <b>2. Brief Review of column design .....</b>                       | <b>6</b>  |
| 2.1. Column interaction diagram.....                                | 6         |
| 2.2. Exact Method .....   | 8         |
| 2.2.1.Compression Plus Uniaxial Bending.....                        | 8         |
| 2.2.2.Compression Plus Biaxial Bending .....                        | 11        |
| 2.3. Approximate Methods.....                                       | 14        |
| 2.3.1 Load Contour Method.....                                      | 15        |
| 2.3.2 Reciprocal Load Method.....                                   | 16        |
| <br>  |           |
| <b>3. Design Charts and Equation Formulation .....</b>              | <b>19</b> |
| 3.1. General.....   | 19        |
| 3.2. Assumptions and Material Properties.....                       | 20        |
| 3.3. Stress Resultants.....   | 23        |
| 3.3.1. Axis designation and sign convention.....                    | 23        |
| 3.3.2. Stress resultants on concrete.....                           | 23        |
| 3.3.3. Stress resultants on reinforcement steel.....                | 25        |
| 3.3.4. Total Stress resultants about the global axes .....          | 25        |
| 3.4. Green’s Theorem and Gauss Quadrature Principle .....           | 27        |
| 3.4.1. Green’s Theorem and its application in biaxial bending ..... | 27        |

|   |           |
|---|-----------|
| 3.4.2. Gauss Quadrature applications.....   | 27        |
| 3.5. Computation of Compressive Stress and Stress Resultants on Concrete.....       | 28        |
| 3.6. Determination of Functions Q and P.....  | 29        |
| 3.7. Line Integration.....  | 30        |
| 3.8. The steel contribution.....  | 31        |
| 3.9. Cross-section analysis subjected to compression with bending(s).....           | 32        |
| 3.9.1. General .....  | 32        |
| 3.9.2. Computation of angle ' $\theta$ ' to choose design charts .....              | 32        |
| 3.9.3. Computation of X-Y coordinates of Salient points and Reinforcement bars..... | 36        |
| 3.9.4 Determination of Stress resultants: N, $M_x$ and $M_y$ .....                  | 38        |
| <b>4. Results and Discussion.....</b>   | <b>40</b> |
| 4.1. Discussion.....  | 40        |
| 4.2. Users guide.....   | 41        |
| 4.3. Design example.....  | 42        |
| <b>5. Conclusion, Recommendations and Limitations.....</b>                          | <b>48</b> |
| 5.1. Conclusion .....   | 48        |
| 5.2. Recommendations.....   | 49        |
| 5.3. Limitations.....   | 49        |
| <b>Appendix-A Interaction Charts .....</b>  | <b>50</b> |
| Chart for rectangular columns .....   | 51-52     |
| Charts for T-columns .....  | 53-55     |
| Charts for L-columns .....  | 56        |
| <b>Appendix-B Flow Charts and Code of the program.....</b>                          | <b>57</b> |
| Flow Chart of the program.....  | 58-64     |
| Code of the program .....   | 65-76     |
| <b>References: .....</b>  | <b>77</b> |

## List of Illustrations

|   |       |
|---|-------|
| Figure 2.1 Equivalent load eccentricity.....                                  | 9     |
| Figure 2.2 Compression member under eccentric force.....                      | 10    |
| Figure 2.3 Interaction diagram for compression plus uniaxial bending.....     | 11    |
| Figure 2.4 Interaction diagram for compression plus biaxial bending.....      | 10    |
| Figure 2.5 Interaction Contours at Constant $P_n$ varying eccentricities..... | 16    |
| Figure 2.6 Interaction surface for reciprocal load method.....                | 18    |
| Figure 3.1 Typical T-section.....   | 19    |
| Figure 3.2a Stress strain diagram for concrete in compression.....            | 21    |
| Figure 3.2b Stress strain diagram for reinforcing steel.....                  | 22    |
| Figure 3.2c Strain diagram in ultimate limit state.....                       | 22    |
| Figure 3.3 Bending moment about the local and global axes.....                | 23    |
| Figure 3.4 T-section stress and strain diagram.....                           | 24    |
| Figure 3.5 Six cases taken for analysis.....                                  | 33-35 |

**List of Tables**

|  |    |
|--|----|
| Table 2.1 $P_u/P_{uz}$ versus $\alpha$ .....   | 16 |
| Table 3.1 Range of angle ' $\theta$ ' for most compressed corners and highly tensile reinforcement<br>. bars ..... | 35 |
| Table 3.2 Global X and Y coordinates of vertexes of cross section.....   | 36 |
| Table 3.3 Global X and Y coordinates of reinforcement bars.....  | 37 |

### Notations

$P_u$  - Ultimate axial load

$P_d$  - Design compressive axial load

$M_u$  - Ultimate bending moment

$P_n$  – Nominal maximum compressive axial force capacity of a section

$M_{nx}$ ,  $M_{ny}$ , - Nominal maximum bending capacity of a section about x and y-axes respectively

$M_{xd}$ ,  $M_{yd}$ , - Design bending moment about global x and y-axis respectively

$M_{cx}$ ,  $M_{cy}$ , - Ultimate bending capacity of *concrete* in the section about x and y-axes respectively

$M_{sx}$ ,  $M_{sy}$ , - Ultimate bending capacity of *steel bars* in the section about x and y-axes respectively

$N_c$  - Ultimate Axial force capacity of concrete in the section.

$N_s$  - Ultimate Axial force capacity of reinforcement bars in the section

$f_{ck}$  - Characteristic cylinder compressive strength of concrete

$f_{cd}$  - Compressive Design strength of concrete

$f_c(y)$  - Compressive stress of concrete at distance ,y, from the local x-axis.

$f_{sk}$  - Characteristic strength of reinforcement

$f_{yd}$  - Design strength of reinforcement

$E_s$  - Elastic modulus of reinforcement steel.

$A$  - Partial area of a cross-section over which compressive stress acts.

$A_{sj}$  - area of steel bar 'j' at which tensile stress acts on it.

$A'_{sj}$  - area of steel bar 'j' at which compressive stress acts on it.

$f_{sj}$  - tensile stress acting on steel bar 'j'

$f_{csj}$  - compressive stress acting on steel bar 'j'

$a$  - inner face to face length of the cross-section

$b$  - inner face to face width of the cross-section

$t_f$  - thickness of the flange

$t_w$  - thickness of the flange

$l$  – left side length of the flange door

$r$  – right side length of the flange door

$\theta$  - angle of inclination of the neutral axis with the global X- axis towards the left.

$\varepsilon_b$  - Strain in the least compressive fiber

$\varepsilon_c(y)$  - Strain in the concrete at y-distance in the y-axis.

$\varepsilon_{cu}$  - limit compressive strain of concrete.

$\rho$  - reinforcement steel ratio ( $A_{s,tot}/A_c$ )

$A_{s,tot}$  - total steel area

$A_c$  - gross area of concrete section

## **Abstract**

In this study an algorithm for the computation of Interaction Surface of reinforced concrete T-shaped column subjected to axial load and biaxial bending is developed. The method is exact and it is based on the transformation of the double equilibrium integrals into line-integrals along the compressive perimeter of the concrete section using Green's Theorem. For up to third degree polynomial stress-strain relations, Gauss integration with only three sample integration points yields exact results.

By inserting the necessary cross-sectional dimensions and material properties on the user interface, the program locates the position of the plastic centroid of the cross section. The coordinates of corner points and reinforcement bars will be calculated using the cross sectional dimensions and position of the plastic centroid.

The interaction charts produced are for reinforcement ratios of 0 to 0.8 with an increment of 0.1. Parabolic-rectangular stress distribution is used for the analysis of the concrete section as is primarily recommended by EBCS-2,1995. The capacity of the section is checked for different positions of neutral axis. Green's theorem transforms the double integration over the compression area into a line-integration along the closed line that encloses the area. The line integration along the perimeter of the compression area of the section is numerically evaluated using Gauss integration.

The calculation of the stress and force carried by the steel is straightforward. From the strain we calculate the corresponding stress and finally the force on each bar. Note that from the stress of each compressive bar we subtract the stress of the concrete at the location of that bar in order to avoid double counting the compressive concrete area displaced by the bar.

The presented method for the construction of Interaction Surfaces of concrete sections is computationally efficient because the double integration is reduced to numerical evaluation of a few line integrals. The number of line integrations is equal to the number of the sides of the section in the compressive zone. Each of these integrals involves typically three sample points thus the number of computer operations is very small.

Although the initial objective of the study was to develop interaction chart for T columns; in the process of solving the problem other possibilities are also achieved. The program developed can also produce interaction surface of L and rectangular columns. Six of these interaction surfaces are shown in appendix A. It is clear that the computation time would allow the interactive use of the method as a design tool. The program is written in visual basic programming language.

## **CHAPTER ONE**

### **1. Introduction**

#### **1.1 Back ground of the problem**

When a column in a building structure is located at the end framing beams in three directions, it may assume a T-shape in cross-section. Moreover, when the width in the respective direction is to be contained within the wall thickness and subjected to axial force and bending moment in both orthogonal directions, the T-column analysis becomes mandatory for designing such columns. However, the design and analysis of such columns having such geometric cross-section is more complex than the commonly known rectangular or circular columns. In the light of this situation this study attempts to address the design problem and provide a simplified means of design solution.

Numerous structural analysis and design soft wares that simplify cumbersome manual analysis and design works have been developed and released to users. Nevertheless, to reduce the computational effort in the nearly exact methods in some complex structural design problem of solving using soft wares are replaced with design charts and Tables prepared by the practicing professional taking in to account the particular of a country. These enable end users to apply the charts & tables in solving design problems within short time with proper due regard to economy and safety.

For instance in our country various applicable design aids for different structural elements especially for columns have been prepared by different academicians and practitioners which are referde by prfetional practitioners and acadamicians such as referd in [3,9 & 11]

In addition to the works mentioned above on columns; other design aids like *Development of Computer Program and Preparation of Design Aids for Commonly Used Sections of Reinforced Concrete Shear Wall Systems* [6] and *Development of design aids for selected single channel flanged reinforced concrete core wall* [10] have been developed.

Researchers in different institutions have still continued to generate simple and user friendly programs that solve complicated civil engineering problems. So far no well defined and user friendly design aids for reinforced concrete T-shaped columns based on local building design code are available for designers.

Therefore, it is with these underlying facts that this study has been selected for the study. The study attempts to produce a program for generating design charts of T-shaped short reinforced concrete column subjected to axial load and bending moments. The fundamental principles of equilibrium of forces, compatibility of strains and known stress-strain relationships are used to set up the necessary equations for design chart generation based on EBCS 2, 1995.

In the process of this study, the research studies carried by academicians and practitioners referred have contributed to the findings and solution of this work. The program is developed using Visual Basic programming language and Microsoft excel on the basis of which design charts are generated.

## ***1.2 Objective of the study***

The initial objective of the research was to produce a program that can generate design charts of symmetrical T-shaped reinforced concrete columns subjected to axial load and bending moments on both axes.

This initial objective is widened from the suggestions given during thesis title defense. The suggestions were to make the section non symmetrical about both axis (i.e. bi-axially non symmetrical section). By doing so the program to be developed can produce design charts for T-sections of any dimensions. In addition to this; by adjusting the dimensions in appropriate way we can get design charts for L and rectangular columns of any dimension.

This research is mainly aimed at producing a program for generating design charts of T-shaped reinforced concrete columns with any side ratio, length of web and flange and with any location of web; subjected to axial load and bending moments in accordance with the principles of design laid in EBCS-2, 1995.

The final output of the research will be a program with attractive and easy user interface that can generate design charts for T, L and rectangular columns by just clicking a button after the user enters the necessary dimensions and material properties in the user interface.

The program will be used as a design aid for design institutions, a reference material for colleges and universities and an input for further research and development in the field. It can even be developed to a higher level program to handle other problems such as stability. This will definitely fill some gap that structural engineers face to design T, L and rectangular shaped reinforced concrete columns subjected to bending(s) in either one principal axis only or about the *two* orthogonal principal directions and axial compressive forces to facilitate the routine design practices based on the Ethiopian Building Code Standard: EBCS-2, 1995.

### **1.3 Methodology**

In order to achieve the study objectives review of literature on the state of the art has been carried where most are referred in this works. Accordingly, maximum flexural strength is attained when the extreme concrete compression fiber reaches the ultimate compression strain  $\epsilon_{cu}$ , [12]. Force equilibrium equations about the plastic centroid of the section, moment equilibrium about the plastic centroid of the section and a strain compatibility relationship enable the ideal flexural strength to be determined.

Exact method of cross section analysis is used. The method used for the determination of the ultimate strength of reinforced concrete members subjected to axial compression in combination with bending are based on *limiting the maximum compressive strain* (or stress) in concrete to some prescribed value and varying the position and orientation of the *neutral axis* by assuming different strain values of the less compressed steel in the section from  $+0.002$  to  $-0.01$ .

For each position of the neutral axis, the linear strain distribution across the section and actual stresses in both concrete and steel are considered. For the assumed strain and stresses distribution the corresponding stress resultants;  $N_u$ ,  $M_{ux}$  and  $M_{uy}$  that can create the assumed strain and stress distribution and values on the section on both concrete and steel are computed.

A computer program is developed using *VISUAL BASICS* for the analysis of T-shaped concrete columns and using the stress resultants obtained from the program output, Interaction Charts are generated in accordance with *Ethiopian Building Code of Standards Two* (EBCS-2,1995).

According to (Nilson A.H, 2004) there are two common methods of compression and biaxial bending moment's interaction chart presentation. "The failure surface can be described either by a set of curves defined by radial planes passing through the  $P_n$  axis, or by a set of curves defined by horizontal plane intersections, each for a constant  $P_n$ , defining load contours". In this thesis radial planes method is adopted to plot the chart

using normalized axial compression force ( $\nu$ ) and resultant moment ( $\mu$ ) of the bending moments about the principal X and Y axes.

### **1.4 Study Content**

The study has been organized in such a way that it can systematically convey the research works undertaken in a clear and consistent manner. In this regard the chapters are arranged as follows.

**Chapter one** is introduction where the background of the study, objectives and method statement are addressed.

**Chapter two** deals with review of state of the art where different sources selected to be of vital importance are assessed to fully understand the work and that of T-column and common methods of cross section analysis. In this view, exact and approximate methods are presented in detail.

**Chapter three** addresses the formulation of the design problem using different mathematical approach which includes integration, Gaussian Quadrature and Green's theorem. The double integral equations will be converted to line integral for simplification purpose in programming. The underlying philosophy as to the requirements, scope and limitation of the formulations are given in this chapter. The detail calculation procedure for position of geometric centroid, location of reinforcement bars and position of corner points are also addressed in this chapter.

**Chapter four** discloses the results of the study works with discussions of design examples and program guidelines for users using different cross sectional dimensions i.e. for symmetric T, for non-symmetric T, for L and rectangular columns.

**Chapter five** is devoted conclusion and recommendations are presented based on the results obtained in chapter four.

**Appendix**, finally design charts for selected section properties, flow charts for the program developed and the coding used to write the program are attached as an appendix.

## **CHAPTER TWO**

### **2. BRIEF REVIEW OF COLUMN DESIGN**

#### **2.1 Column Interaction Diagram**

The cross sectional dimensions of a column are generally considerably less than its height. Columns support vertical loads from floors and roofs and transmit those loads to the foundations. If a column is loaded to failure the reinforcement is likely to reach its yield strength before the concrete fails in compression. If high strength reinforcement is used the concrete may reach its maximum stress before the reinforcement yields. [10]

When a symmetrical column is subjected to a concentric axial load, longitudinal strain develops uniformly across the section. Because the steel and the concrete are bonded together, the strain in the concrete and steel are equal. For any given strain, it is possible to compute the stresses in the concrete and steel using the stress-strain curves for the two materials. [13]

Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to load not being centered on the column, or may result from columns resisting a portion of the unbalanced moment at the ends of the beams supported by the columns.

Structural column failure is of major significance in terms of economic as well as human loss. Thus extreme care needs to be taken in column design, with a higher reserved strength than in the case of beams and other horizontal structural elements, particularly since compression failure provides little visual warning. The analysis of reinforced concrete sections are characterized by material non-linearity arising from the non-linear stress-strain relationships and the cracking of the cross-sections. As a result, the determination of strain distribution for given internal forces necessitates the application of numerical methods accompanied by iterations.

The section strain distribution for the general case of bi-axial bending and axial load can be determined by the direction angle of the resultant curvature and the strains at two characteristic fibers of the cross-section, the greatest compressive and tensile strain for cracked section or the greatest and smallest compressive strain for uncracked sections. The design of a reinforced concrete column is essentially one of trial and error. Design consists of finding a cross section that will support satisfactorily both an axial load and bending moment in addition to the serviceability requirements. However, since the capacity of a section to carry axial load is dependent on the magnitude of the moment that is acting, there are no closed form solutions for determining a section uniquely. The problem will be compounded when the cross section is unsymmetrical.

An *interaction diagram* is a plot of the failure load and failure moment for a given column for the full range of eccentricities from zero to maximum point as shown in Figure 2.3. Failure of column could occur as a result of material failure by initial yielding of the tension steel at the tension face or initial crushing of the concrete at the compression face, or by buckling.

If the column fails due to initial material failure, it is classified as a short or non-slender column. As the length of the column increases, the probability that failure will occur by buckling also increases. In the case of design, where the aim is to determine the required area of reinforcement design chart will be used. Strength interaction diagrams for a given cross-section, full ranges of eccentricities from the smallest value to infinity are considered. For any eccentricity, there is a unique pair of values:  $P_n$  and  $M_n$  (for compression plus uniaxial bending) and unique triple values:  $P_n$ ,  $M_{nx}$  and  $M_{ny}$  (for compression plus biaxial bending) that will produce the state of failure [1].

There are two commonly used methods for the section analysis of a compression member with bending whose trial *cross-section dimension* and *reinforcement area* are known. [1]

The methods are referred as:-

1. Exact Method.
2. Approximate Method.

## **2.2 Exact Method**

Limiting the maximum compressive strain of concrete depending on the position of the neutral axis, the method assumes a strain distribution governed by *strain compatibility* equation i.e. the steel strain at any location are the same as the strains in the adjacent concrete. Corresponding stresses, compressive *parabolic-rectangular* on concrete and tensile and compressive on steel governed by Hooke's law till it yields are assumed. Applying strain compatibility and the two equilibrium equations, the stress resultants; compression and bending(s) that will give the assumed strain values and patterns are determined. In the construction of strength interaction diagrams for a given cross-section, full ranges of eccentricities from the smallest value to infinity are considered. For any eccentricity, there is a unique pair of values:  $P_n$  and  $M_n$  (for compression plus uniaxial bending) and unique triple values:  $P_n$ ,  $M_{nx}$  and  $M_{ny}$  (for compression plus biaxial bending) that will produce the state of incipient failure [1].

### **2.2.1 Compression plus uniaxial bending**

Members concentrically compressed occur rarely if ever in buildings and other structures. Columns are members which mainly carry vertical compression loads, so compression load is always present in columns. Bending moments are caused by continuity, i.e. by the fact that building columns are parts of monolithic frames in which the axial and bending loads of the roof and floor slabs are mainly resisted by the adjoining columns, by transverse loads such as wind and seismic actions, by loads carried eccentrically of columns.

Although the most frequent design loads of columns are compression with uniaxial bending, sometimes it can be designed for compression plus biaxial bending, neglecting the bending moment in the direction of the small eccentricity, especially if the ratio of the *smaller and larger moments* is less than 0.2. In general, biaxial bending should be taken into account when the estimated eccentricity ratio approaches or exceeds 0.2. So developing strength interaction diagrams for compression plus uniaxial bending is acceptable [1].

When a member is subjected to combined axial compression, P & moment, M such as in *Figure 2.1*, it is usually convenient to replace the axial load & moment with an equal force, P and applied eccentricity,  $e = M/P$ . The two loadings are statically equivalent. Columns having relatively small 'e' are generally characterized by compression over the entire section & if over loaded it will fail by *crushing* of concrete accompanied by yielding of steel in compression on the more heavily loaded side. Compression members with large eccentricity are subjected to *tension* over at least a part of the section, and if over loaded, may fail due to yielding of the steel on the side farthest from the load [1]

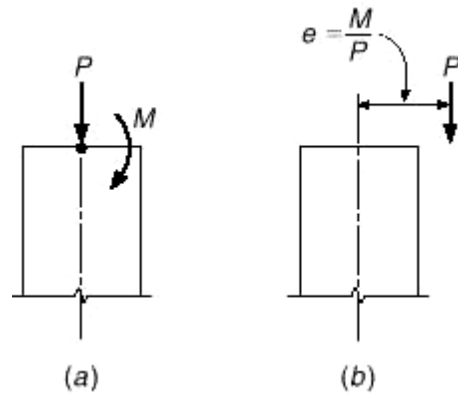


Figure 2.1: Equivalent load eccentricity

Figure 2.2a shows a member loaded parallel to its axis by a compressive force  $P_n$  at an eccentricity 'e' measured from the centre line. The distribution of strains at a section 'a-a' along its length, at incipient failure, is shown in Figure 2.2b. The corresponding stresses and forces are shown in Figure 2.2c. Equilibrium between external and internal axial forces shown in Figure 2.2c requires that:

$$P_n = \iint_A f_c(y) dA + \sum A'_{sj} [f_{csj} - f_c(y)] + \sum A_{sj} f_{sj} \text{-----} (2.1)$$

Also the moment about the center line of the section of the internal stresses and forces must be equal and opposite to the moment of the external force  $P_n$ , so that:

$$M_n = P_n e = \iint_A y * f_c(y) dA + \sum y * A'_{sj} [f_{csj} - f_c(y)] + \sum y * A_{sj} * f_{sj} \text{-----} (2.2)$$

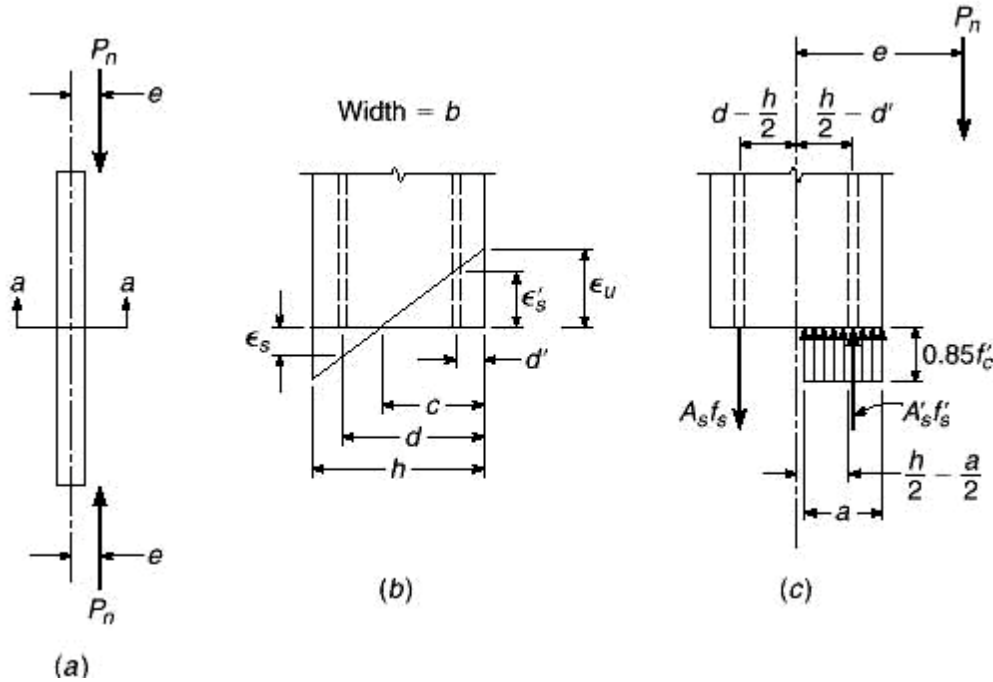
Where: y- is the coordinate of the axis orthogonal to the moment axis, where the resultant force act.

A- Partial area of the cross-section over which the compressive stress acts.

$f_c(y)$  - the exact compressive stress on concrete at  $y$ .

$A_{sj}$  and  $A'_{sj}$  - areas of tensile and compressive steel bars respectively at  $y$ .

$f_{sj}$  and  $f_{csj}$  - tensile and compressive stresses of steel bars respectively at  $y$ .



a) Loaded column    b) strain distribution at section a-a    c) Stresses and forces at normal section

Figure 2.2: Compression member under eccentric force.

For any eccentricity, there are unique pair of values of  $P_n$  and  $M_n$  that will produce the state of incipient failure. That pair of values can be plotted as a point on a graph relating  $P_n$  and  $M_n$ , such as shown in *Figure 2.3*. A series of such calculations, each corresponding to a different eccentricity, will result in a curve having a shape typically as shown in *Figure 2.3*. On such a diagram, any radial line represents a particular eccentricity  $e = M/P$ .

For that eccentricity, gradually increasing the load will define a load path as shown in *Figure 2.3*, and when that load path reaches the limit curve, failure will result. For a given compression member, selected for trial, the interaction diagram is most easily constructed by selecting successive choices of neutral axis distance,  $C$ , from infinity

(axial load with eccentricity zero) to a very small value found by trial to give  $P_n = 0.0$  (pure bending).

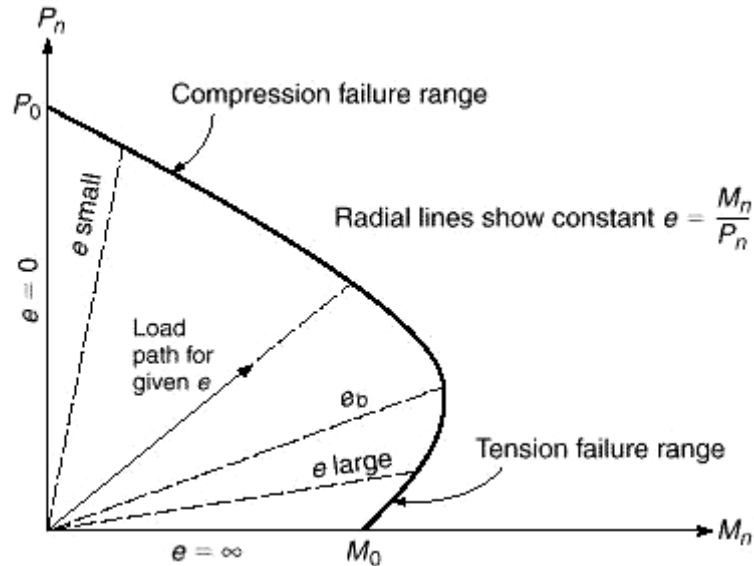


Figure 2.3: Interaction diagram for compression plus uniaxial bending.

### 2.2.2 Compression Plus Biaxial bending

The method discussed in the preceding section permits cross-section to be designed satisfying the criteria mentioned in section 2.2. But, there are situations that the ratio of smaller and larger eccentricities is greater or equal to 0.2. In this case the column should be designed using strength interaction diagrams prepared for compression with biaxial bending. The situation with respect to strength of biaxial loaded compression member is shown in Figure 2.4.

Let X and Y denote the directions of the principal (global) axes of the cross section. In *Figure 2.4a* a section is shown subject to bending about the Y-axis only, with load eccentricity  $e_x$  measured in the x-direction. The corresponding strength interaction curve is shown as *case (a)* in the three-dimensional sketch in *Figure 2.4d* and is drawn in the plane defined by the axes  $P_n$  and  $M_{ny}$ . Similarly, *Figure 2.4b* shows bending about the X-

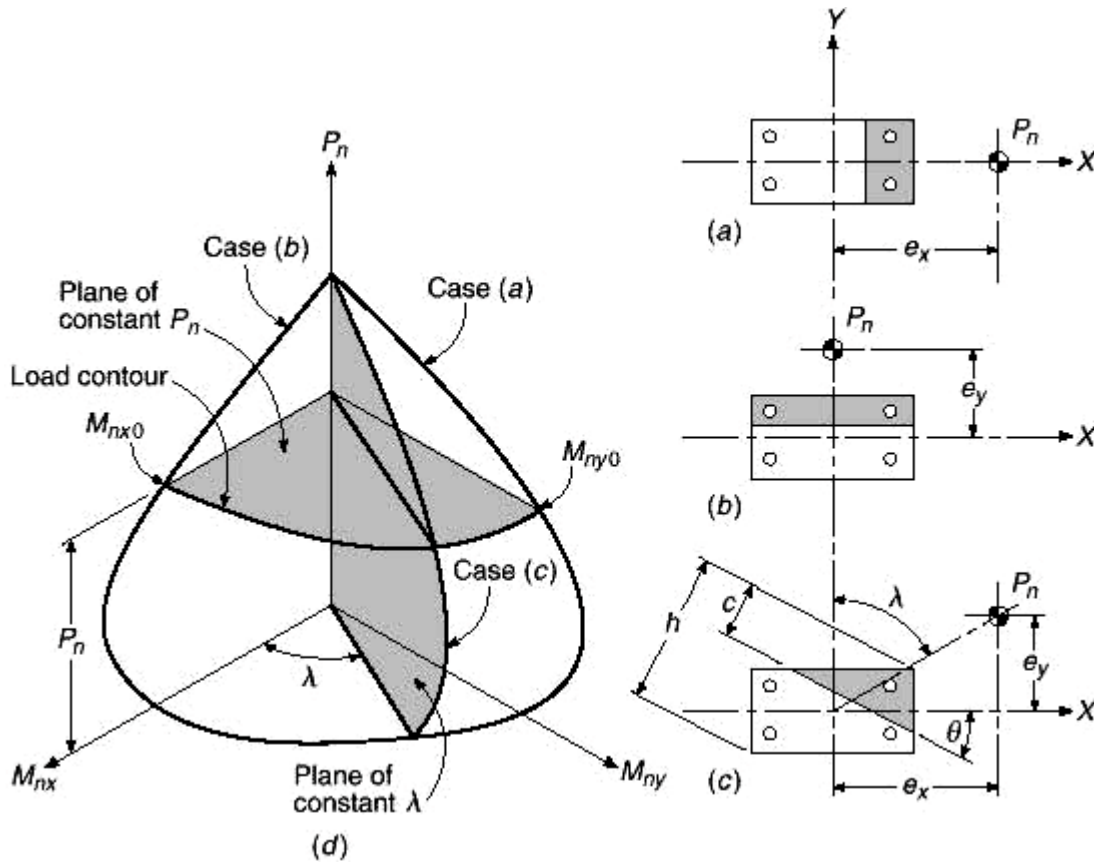
axis only, with eccentricity measured in the y direction. The corresponding interaction curve is shown as case (b) in the plane of  $P_n$  and  $M_{nx}$  in *Figure 2.4d*. For case (c), which combines X and Y axes bending, the orientation of the resultant eccentricity is defined by the angle  $\lambda$ .

$$\lambda = \arctan (e_x/e_y) = \arctan (M_{ny}/M_{nx}) \text{ ----- (2.3)}$$

Bending for this case is about an axis defined by the angle ' $\theta$ ' with respect to the X-axis. The angle  $\lambda$  in *Figure 2.4c* establishes a plane in *Figure 2.4d*, passing through the vertical  $P_n$  axis making an angle ' $\lambda$ ' with the ' $M_{nx}$ ' axis, as shown.

In that plane, x-section strength is defined by the interaction curve labeled *case (c)*. For other values of  $\lambda$ , similar curves are obtained to define a similar failure surface for axial load plus biaxial bending, such as shown in *Figure 2.4d*. Any combination of  $P_u$ ,  $M_{ux}$ , and  $M_{uy}$  falling inside the surface can be applied safely, but any point falling outside the surface would represent failure. [1].

Note that the failure surface can be described either by a set of curves defined by radial *planes* passing through the  $P_n$  axis, such as shown by case (c), or by a set of curves defined by *horizontal plane intersections*, each for a constant  $P_n$ , defining load contours.



- a) Uniaxial bending about the y-axis
- b) Uniaxial bending about the x-axis
- c) Biaxial bending about the diagonal axis
- d) Interaction surface

Figure 2.4 Interaction diagram for compression plus biaxial bending.

In Figure 2.4c, for selected value of  $\theta$ , successive choices of neutral axis distance “C” perpendicular distance between most compressed point and the neutral axis could be taken. For each, using strain compatibility and stress-strain relations to establish bar forces and the concrete compressive resultant, and then using the equilibrium equations to find  $P_n$ ,  $M_{nx}$ , and  $M_{ny}$ , one can determine a single point on the interaction surface.

Repetitive calculations, easily done by computer, then establish sufficient point to define the surface. The neutral axis will not, in general, be perpendicular to the resultant eccentricity, drawn from the *cross section center* to the load  $P_n$ . For each successive choice of neutral axis, there are unique values of  $P_n$ ,  $M_{nx}$ , and  $M_{ny}$  and only for special

cases will the ratio of  $M_{ny}/M_{nx}$  be such that the eccentricity is perpendicular to the neutral axis chosen for the calculation. The result is that, for successive choice of “C” for any given  $\theta$ , the value of  $\lambda$  in *Figure 2.4c and d* will vary. Points on the failure surface established in this way will *wander up* the failure surface for increasing  $P_n$ , not representing a plane intersection, as shown for case (c) in *Figure 2.4d*.

A compression member strength interaction curve is established for a trial cross-section, exactly analogous to the curve for axial load plus uniaxial bending. However, the curve is generated for the particular value of the eccentricity angle that applies, as determined by the ratio of  $M_{uy}/M_{ux}$  from the structural analysis see case (c) of *Figure 2.4d*. This is done by taking successive choices of neutral axis distance, measured from the most heavily compressed corner, from small eccentricity to large eccentricity, then calculating the axial force  $P_n$  and moments  $M_{nx}$  &  $M_{ny}$ . For each neutral axis distance, iteration is performed with successive values of the orientation angle  $\theta$ , *Figure 2.4c*, until  $\lambda = \arctan(M_{ny}/M_{nx})$  is in agreement with the value of  $\lambda = \arctan(M_{uy}/M_{ux})$  from the structural analysis or assumed value of  $\lambda$ . Thus, one point on the curve (c) of *Figure 2.6d* is established. The sequence of calculations is repeated: another choice of neutral axis distance is made; a value of  $\theta$  is interacted until  $\lambda$  is correct. Thus, the next point is established, and so on, until the computed strength interaction curve for the particular value of  $\lambda$  is completed [1].

### **2.3 Approximate Methods**

Approximate methods are frequently employed for both design and analysis purposes. The most commonly used approximate methods are based on the concept of failure surfaces introduced by Bresler. Bresler presented a load-contour and reciprocal load methods which lead to an interaction equation based on a failure surface,  $S(P_u, M_{ux}, M_{uy})$  and  $S_2(1/P_u, e_x, e_y)$  as shown in *Figure 2.5* and *2.6*, respectively, [1].

### 2.3.1 Load Contour Method.

The load contour method is based on representing the failure surface of *Figure 2.4d* by a family of curves corresponding to constant values of  $P_n$ . The general form of these curves can be approximated by a non-dimensional interaction equation.

$$\left[ \frac{M_{nx}}{M_{nxo}} \right]^{\alpha_1} + \left[ \frac{M_{ny}}{M_{nyo}} \right]^{\alpha_2} \leq 1.0 \text{-----} (2.4)$$

Where:  $M_{nx} = P_n e_y$

$M_{nxo} = M_{nx}$  when  $M_{ny} = 0$

$M_{ny} = P_n e_x$

$M_{nyo} = M_{ny}$  when  $M_{nx} = 0$

And  $\alpha_1$  and  $\alpha_2$  are exponents depending on *cross-section dimensions, amount and distribution of steel reinforcements, stress- strain characteristics of steel and concrete, amount of concrete cover, and arrangement and size of lateral ties or spiral [1]*.

In the British standards BS-8110, it has been suggested that a rectangular section of a short column subjected simultaneously to biaxial bending and axial compressive load can be designed in such a way that Eqn. (2.4) is satisfied. The values of ' $\alpha$ ' were related to the ratio of  $P_u/P_{uz}$  in which  $P_u$  is the ultimate axial load and  $P_{uz}$  is given as follows.

$$P_{uz} = 0.45 A_c f_{ck} + .75 f_{yk} A_{st} \text{-----} (2.5)$$

Where:  $A_c$ - concrete area of the section

$f_{ck}$ - characteristic strength of concrete

$f_{yk}$ - Characteristic strength of steel

$A_{st}$ - total steel area (tension and Compression reinforcement)

$P_{uz}$ - the capacity of cross-section under pure axial load.

For the given ratio of  $P_u/P_{uz}$ , the values of  $\alpha$  are given in Table 2.1 below.

Table 2.1:  $P_u/P_{uz}$  versus  $\alpha$

|              |      |      |      |      |
|--------------|------|------|------|------|
| $P_u/P_{uz}$ | 0.2  | .4   | 0.6  | 0.8  |
| $\alpha$     | 1.00 | 1.33 | 1.67 | 2.00 |

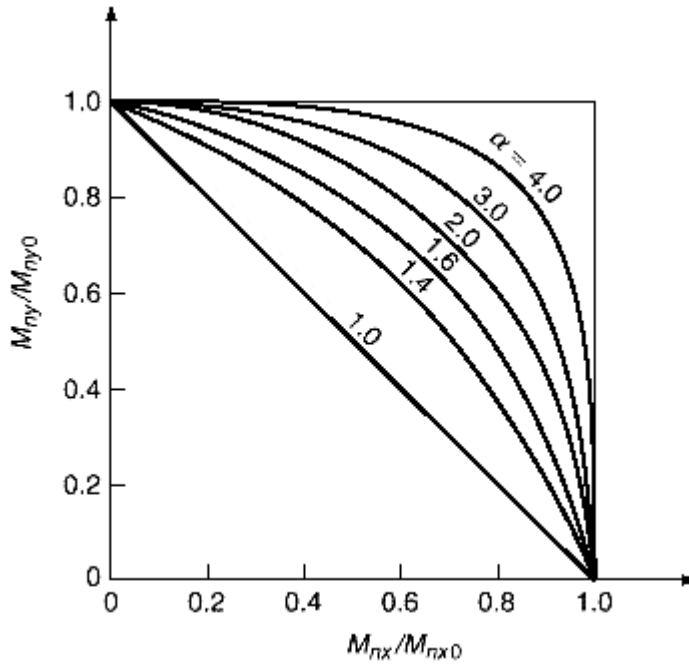


Figure.2.5: Interaction contours at constant  $P_n$  and varying eccentricities.

**2.3.2 Reciprocal Load Method.**

It is one of the approximate methods developed by Bresler. It is noted that the strength interaction surface in *Figure 2.4d* can alternatively, be plotted as a function of the axial load  $P_n$  and eccentricities  $e_x$  and  $e_y$  as shown in *Figure 2.4a*. The surface  $S_1$  of *Figure 2.6a* can be transformed into an equivalent failure surface  $S_2$ , as shown in *Figure 2.6b*. Where  $e_x$  and  $e_y$  are plotted against  $1/P_n$  rather than  $P_n$ . Thus,  $e_x = e_y = 0.0$  corresponds to the inverse of the capacity of the cross - section if it were concentrically loaded,  $P_0$ , and this is plotted as point C.

For  $e_y = 0.0$  and any given value of  $e_x$ , there is a load  $P_{nyo}$  (corresponding to moment  $M_{nyo}$ ) that would result in failure. The reciprocal of this load is plotted as point A. The

values of  $P_{nxo}$  and  $P_{nyo}$  are easily established, for known eccentricities of loading applied to a given cross-section, using uniaxial strength interaction charts.

An oblique plane  $S_2'$  is defined by the three points: A, B, and C. This plane is used as an approximation of the actual failure surface  $S_2$ . Note that, for any point on the surface  $S_2$  (i.e., for any given combination of  $e_x$  and  $e_y$ , there is a corresponding plane  $S_2'$ . Thus, the approximation of the true failure surface  $S_2$  involves an infinite number of planes  $S_2'$  determined by particular pairs of values of  $e_x$  and  $e_y$ , i.e., by particular points A, B, and C.

The vertical ordinate  $1/P_n$ , to the true failure surface will always be conservatively estimated by the distance  $1/P_n$ , approx to the oblique plane ABC, because of the concave upward egg shell of the true failure surface. In other words,  $1/P_n$ , approx is always greater than  $1/P_n$ , exact, which means that  $P_n$ , approx as always less than  $P_n$ , exact. Bresler's reciprocal load equation derives from the geometry of the approximating plane: It can be shown that.

$$\frac{1}{P_n} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_{no}} \text{-----} (2.6)$$

Where:  $P_n$ - approximate value of nominal load in biaxial bending with eccentricities  $e_x$  &  $e_y$ .

$P_{nyo}$  - nominal load when only eccentricity,  $e_x$ , is present ( $e_y = 0$ )

$P_{nxo}$ - nominal load when only eccentricity,  $e_y$ , is present ( $e_x = 0$ )

$P_{no}$ - nominal load for concentrically loaded compression member.

Eqn. (2.6) has been found to be acceptably accurate for design Purposes provided  $P_n \geq 0.1P_{no}$ . It is not reliable where biaxial bending is prevalent and accompanied by an axial force smaller than  $0.1P_{no}$ . In case of such strongly prevalent bending, failure is initiated by yielding of steel in tension and the situation corresponds to the lowest tenth of the interaction diagram of *Figure 2.4d*. In this range, it is conservative and accurate enough to neglect the axial force entirely and to calculate the section for biaxial bending only.

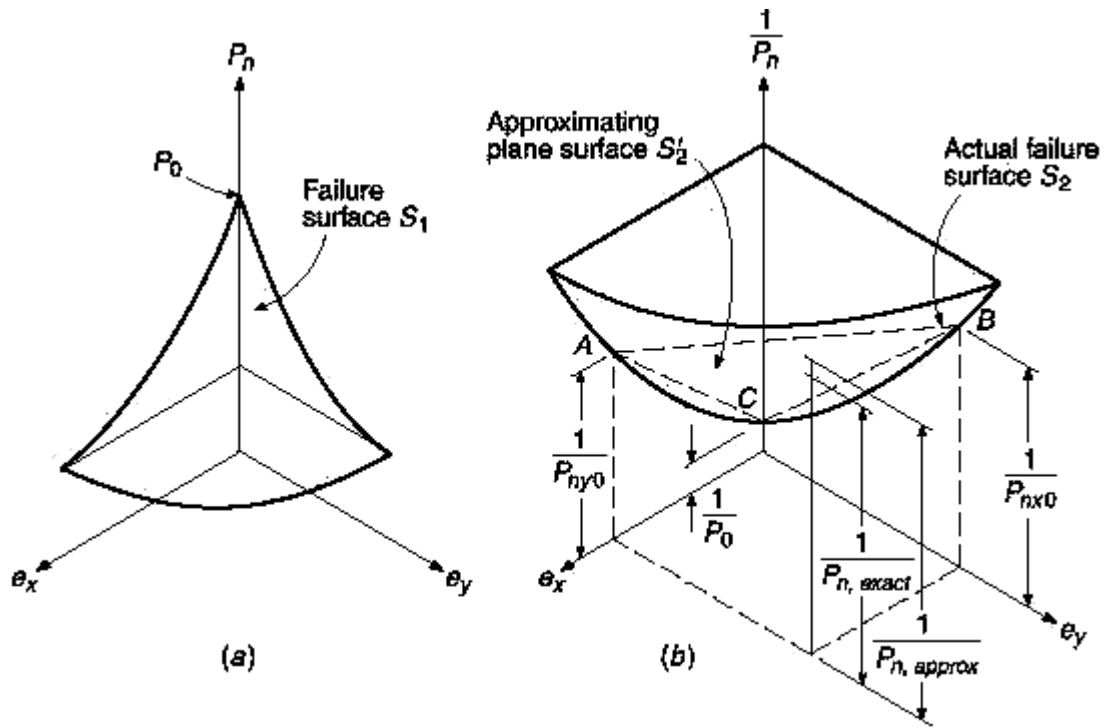


Figure 2.6: Interaction surface for reciprocal load method

## **CHAPTER THREE**

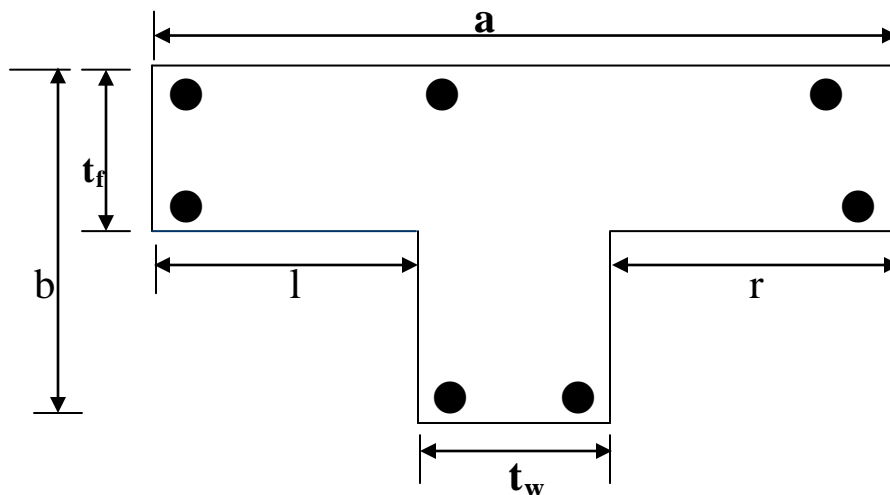
### **3. Design Charts and Equation Formulation**

#### **3.1 General**

In this thesis a T-shaped column with any given dimension such as shown in *Figure 3.1* is considered. By varying the dimensions we can develop design charts of T-shaped columns that are symmetrical or non-symmetrical. We can even have L-shaped and rectangular columns by giving the dimensions of the left or right sides (i.e.  $l$  &  $r$ ) a zero value.

Although the load contour and the reciprocal load methods are widely used in practice, each has serious short comings. With the load contour method, selection of the appropriate value of the exponent ' $\alpha$ ' is made difficult by a number of factors relating to cross-section shape, bar distribution etc. For many cases, the usual assumption that  $\alpha_1 = \alpha_2$  is a poor approximation. The reciprocal load method is very simple to use, but the representation of the curved failure surface by approximating plane is not reliable in the range of large eccentricities, where failure is initiated by steel yielding.

Even if it is laborious and time consuming and due to the significant short comings of the approximate method, Exact Method of Cross - Section Analysis is used for the development of strength interaction diagrams for the T-shaped concrete columns.



*Figure.3.1 Typical T-section*

Where: **a**-length of the flange  
**b**-depth of the section  
**t<sub>f</sub>** –thickness of the flange  
**t<sub>w</sub>** –thickness of the web

**l**-left wing length of the flange  
**r**-right wing length of the flange  
**d<sup>1</sup>**-cover to reinforcement

### **3.2 Assumptions and Material Properties**

In the cross-section analysis the following assumptions are made [8].

- a) Plane sections normal to the axis of member remain plane after bending and there is no bond-slip between reinforcement steel and concrete, i.e. strain compatibility is assumed.
- b) The tensile strength of concrete is ignored
- c) The relationship between stress-strain distribution in concrete is assumed to be *parabolic rectangular* as shown in *Figure 3.2a* (with maximum compressive stress equal to  $0.68f_{ck}/\gamma_c$ )
- d) The stresses in reinforcement are derived from the representative stress-strain curve for the type of steel used. Typical curves are shown in *Figure 3.2b*.
- e) The maximum compressive strain in concrete in axial uniform compression is taken 0.002.
- f) The maximum compression strain at the highly compressed extreme fiber in concrete subjected to axial compression and bending, but when there is no tension on the section, is taken as 0.0035 minus 0.75 times the strain at the least compressed extreme fiber *Figure 3.2c*.

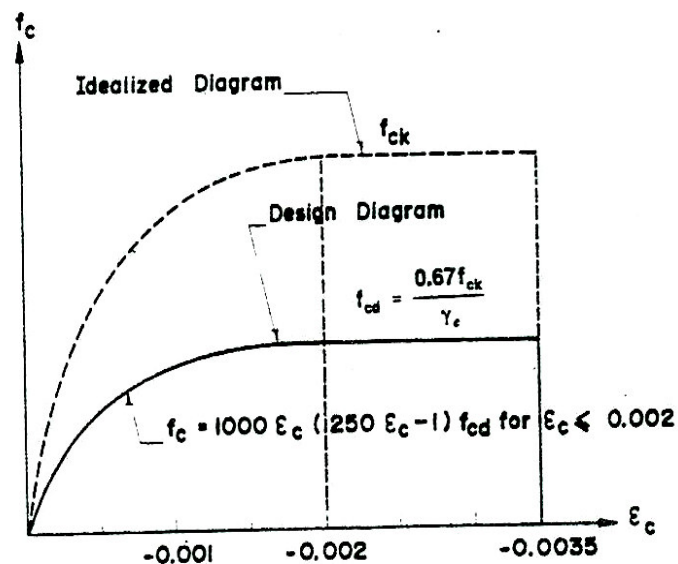
g) The maximum compressive strain at the highly compressed extreme fiber, when the neutral axis rests on the section in concrete subjected to axial compression and bending is taken as 0.0035, *Figure 3.2c*. In the limiting case, when the neutral axis lies along one edge of the section, the strain varies from 0.0035 at the highly compressed edge to zero at the opposite edge.

h) The maximum strain in the reinforcement steel is 0.01 as shown in *Figure 3.2b*.

i) For convenience of the programming the diameters of reinforcement bars used are the same.

j) For the section equal diameter of reinforcement bars are used. Reinforcement bars are provided at corner points and in the middle of the outer face of the flange.

k) Equal depth of concrete cover is provided for each reinforcement bar from the respective extreme face of the section.



**Figure 4.2 Parabolic-Rectangular Stress-Strain Diagram for Concrete in Compr**

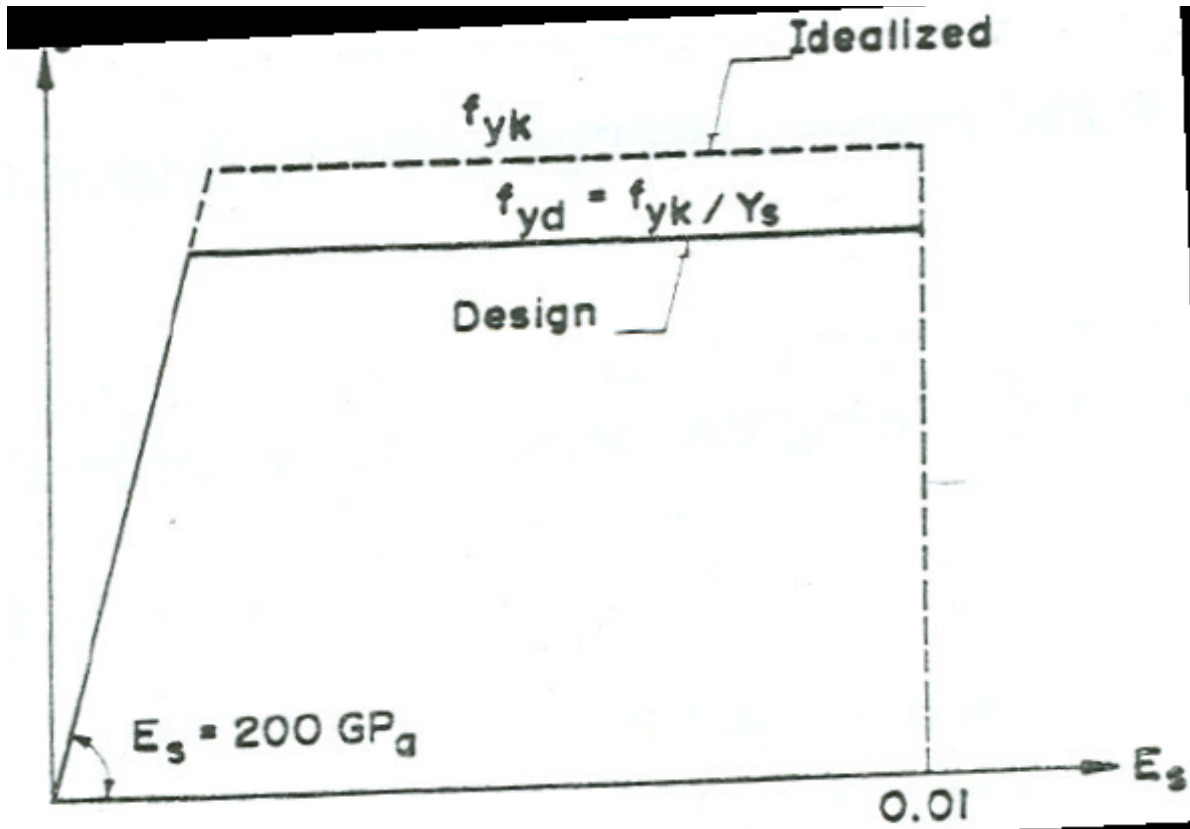


Figure 3.2b: Stress-strain diagram for Reinforcing Steel

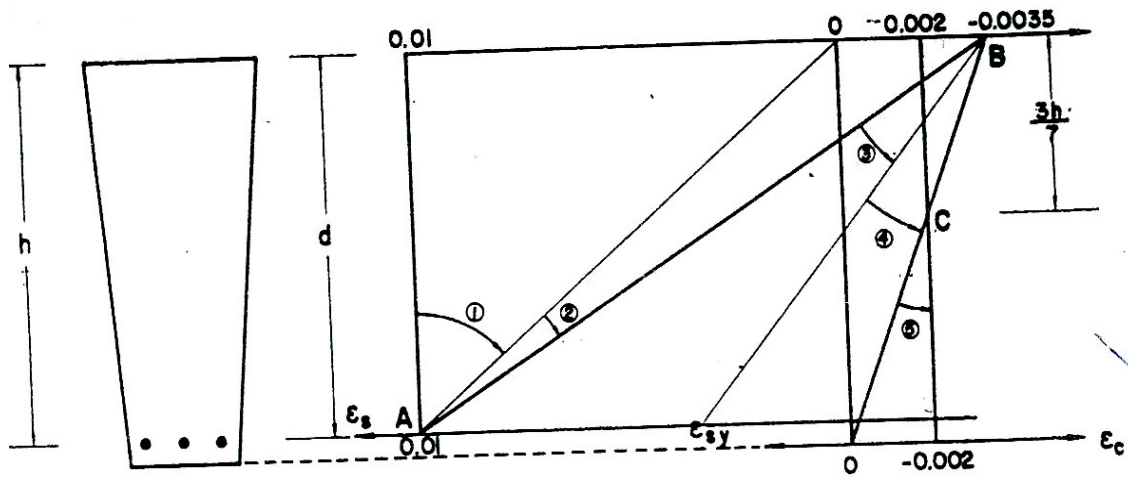


Figure 3.2c: Strain diagram in ultimate limit state

### **3.3 Stress Resultants**

#### **3.3.1 Axis Designation and Sign Convention**

- ❖ Local y-axis: is perpendicular to the assumed neutral axis and along it both strain and stress varies.
- ❖ Local x-axis: is parallel to the neutral axis and along it both stress and strain are constant.
- ❖ Global X and Y-axes are the principal centroidal axes of the section.
- ❖ The origin, O, of the coordinate systems is the plastic centroid of the cross-section.
- ❖ Compressive strains and stresses are taken positive; and tensile strains and stresses are taken negative.
- ❖ Bending moments whose arrow head orients towards the positive axes are positive otherwise negative.

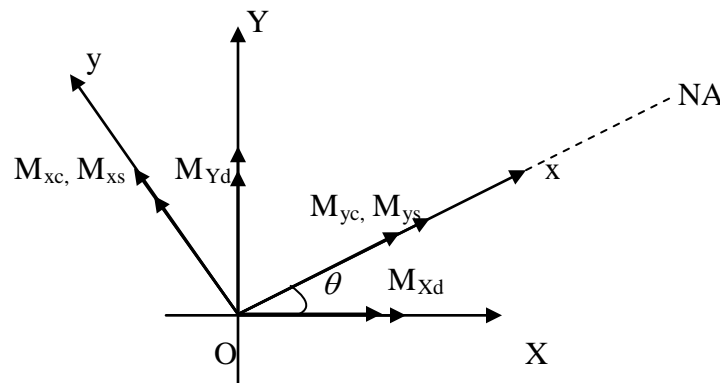


Figure 3.3: Bending moments about the local and global axes

#### **3.3.2 Stress resultants ( $N_c$ , $M_{xc}$ and $M_{yc}$ ) on concrete.**

Infinitesimal area, 'dA', in fig. 3.3 at y perpendicular distance from the x-axis in the compression zone of the section is considered. The stress resultants on the 'dA' and on the whole compression area are determined as follows.

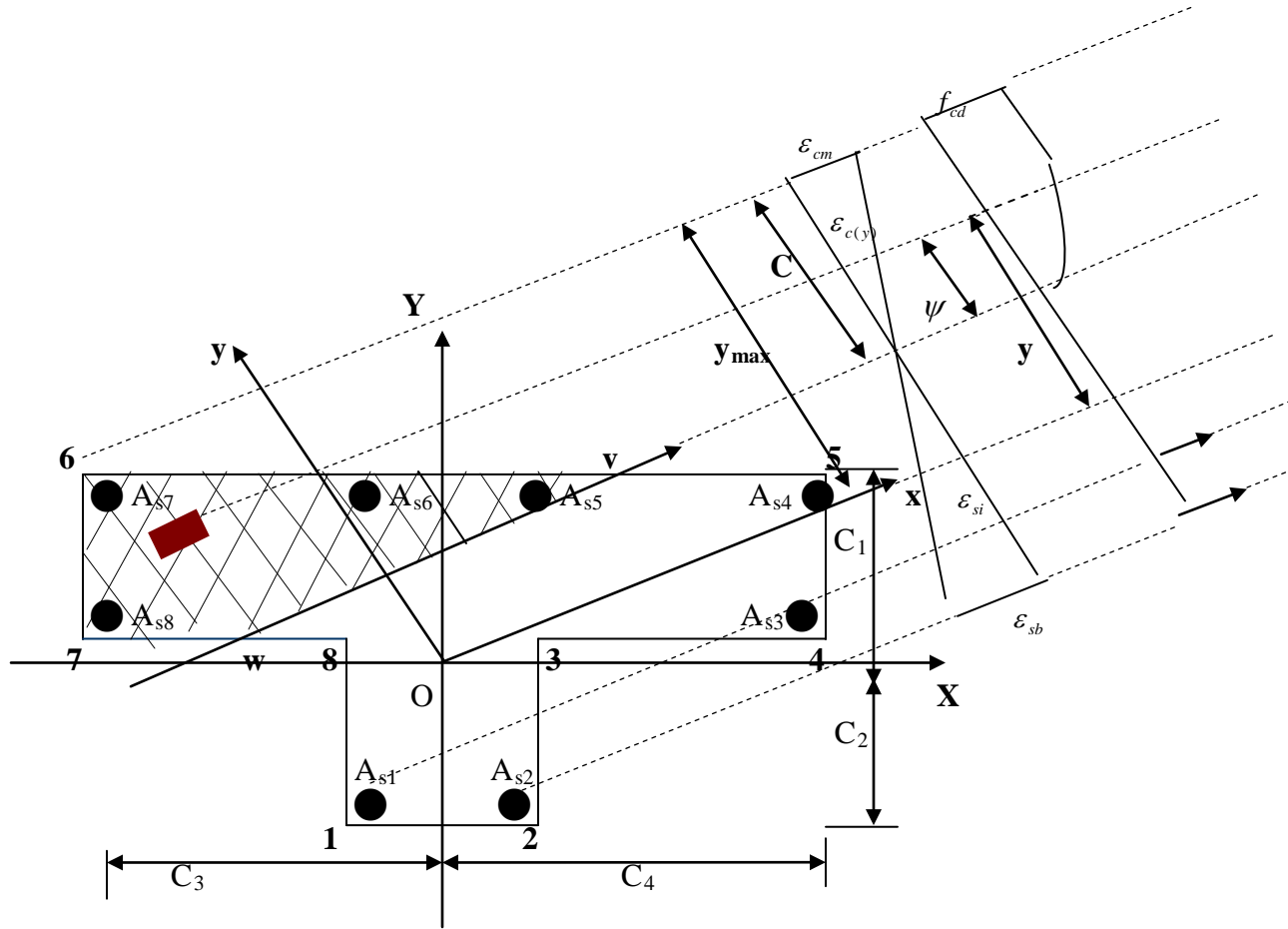


Figure 3.4 T-section stress and strain diagram

Stress resultants on concrete

$$\diamond dN_c = f_c(y)dA \quad \longrightarrow \quad N_c = \iint_A f_c(y)dA \quad (3.1)$$

$$\diamond dM_{xc} = -y f_c(y)dA \quad \longrightarrow \quad M_{xc} = -\iint_A y * f_c(y)dA \quad (3.2)$$

$$\diamond dM_{yc} = x f_c(y)dA \quad \longrightarrow \quad M_{yc} = \iint_A x * f_c(y)dA \quad (3.3)$$

### **3.3.3 Stress resultants ( $N_s$ , $M_{xs}$ and $M_{ys}$ ) on reinforcement bar.**

$$\diamond N_s = \sum_{j=1}^N A_{sj} * f_{sj} \quad (3.4)$$

$$\diamond M_{xs} = - \sum_{j=1}^N y_{sj} * A_{sj} * f_{sj} \quad (3.5)$$

$$\diamond M_{ys} = \sum_{j=1}^N x_{sj} * A_{sj} * f_{sj} \quad (3.6)$$

To account the ignored area of reinforcement in the compression zone in the computation of stress resultants resisted by concrete,  $f_c(y)$  should be deducted from  $f_{sj}$ , if it is compressive stress.

### **3.3.4 Total stress resultants about the global axes (X and Y)**

$$P_n = N_c + N_s \quad (3.7)$$

$$M_{xn} = (M_{xc} + M_{xs}) \cos(-\theta) + (M_{yc} + M_{ys}) \sin(-\theta) \quad (3.8)$$

$$M_{yn} = -(M_{xc} + M_{xs}) \sin(-\theta) + (M_{yc} + M_{ys}) \cos(-\theta) \quad (3.9)$$

Where:  $f_c(y)$  is the compressive stress on concrete

x and y – coordinates with respect to the local axes.

X and Y - coordinates with respect to the global axes.

$A_{sj}$ -is the area of reinforcement bar j.

$f_{sj}$ -is the stress on reinforcement bar j.

- $M_{xc}$  and  $M_{yc}$  are moment stress resultants of concrete about the local centroidal axis.
- $M_{xs}$  and  $M_{ys}$  are moment stress resultants of steel about the local centroidal axis.
- $M_{xn}$  and  $M_{yn}$  are moment capacities of the considered section about the centroidal global axes.

$\epsilon_{cu}$  –the limiting compressive strain of concrete

$\epsilon_{yd}$  –the yield strain of reinforcement steel

$\epsilon_b$  –strain value of the least compressed steel

$\epsilon_c(y)$  –the strain at ‘y’ distance from the x-axis

$y_{max}$  –the perpendicular distance between the local x-axis and the most compressed fiber.

$c$  –the perpendicular distance between the neutral axis (NA) and the most compressed fiber.

$$k_x = c/d$$

As shown in *Figure 3.2a* the limiting compressive strain of concrete can be expressed as shown below, depending on the value of  $k_x$ , which denotes the position of the neutral axis.

$$\varepsilon_{cm} = 0.01 / \left( \frac{d}{c} - 1 \right) \text{ ----- } k_x < 35/135$$

$$\varepsilon_{cm} = \varepsilon_{cu} = \begin{cases} 0.0035 \text{ ----- } (35/135) \leq k_x \leq 1 \\ 0.002 / \left( 1 - \frac{3 * d}{7 * c} \right) \text{ ----- } k_x > 1 \\ 0.002 \text{ ----- } k_x = \infty \end{cases}$$

$$\varepsilon_s(y) = \varepsilon_c(y) = \varepsilon_{cu} (c + y - y_{max})/c$$

$$f_c(y) = \begin{cases} 1000 \varepsilon_c(y) [250 * \varepsilon_c(y) - 1] f_{cd} \text{ ----- } \varepsilon_c(y) < 0.002 \\ f_{cd} \text{ ----- } 0.002 < \varepsilon_c(y) < 0.0035 \end{cases}$$

$$f_s(y) = \begin{cases} E_s \varepsilon_s(y) \text{ ----- } |\varepsilon_s(y)| < \varepsilon_{yd} \\ f_{cd} \text{ ----- } |\varepsilon_s(y)| < \varepsilon_{yd} \end{cases}$$

Where:  $f_{cd} = 0.68 \frac{f_{ck}}{\gamma_c} = 0.68 \frac{f_{ck}}{1.5} = 0.453 f_{ck}$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{f_{yk}}{1.15} = 0.87 f_{yk}$$

➤  $\varepsilon_{cm}$  is the maximum strain on concrete.

In the computation of stress resultants resisted by concrete, to simplify the computation, the area of reinforcing steel was not deducted from the area of concrete and it is accounted in the determination of stress resultants carried by reinforcing steel by subtracting  $f_c(y)$  from  $f(y)$  for reinforcements within the compression area of the section. So the stresses on reinforcing steel became:

$$f_{sc}(y) = \begin{cases} Es * \varepsilon_s(y) - fc(y) & \varepsilon_s(y) < \varepsilon_{yd} \\ fyd - fc(y) & \varepsilon_{yd} < \varepsilon_s(y) < 0.002 \\ fyd - fcd & 0.002 < \varepsilon_s(y) < 0.0035 \end{cases}$$

**3.4 Green’s Theorem and Gauss Quadrature Principle.**

**3.4.1 Green’s Theorem and its application in biaxial bending**

The double integral over an area, A, can be transformed in to a line integral along a closed curve, L, that encloses the area, A, as follows; [5]

$$\iint_A \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_L P dx + \oint_L Q dy \tag{3.10}$$

Where: P and Q are two functions of x and y, ‘A’ is the area of integration and ‘L’ is the closed curve that encloses the area, A. The theorem can be applied for both uni-axial and bi-axial problems, especially if parabolic–rectangular compressive stress distribution is assumed on concrete.

**3.4.2 Gauss Quadrature Application**

It is a numerical integration method and mostly used when the integrand is difficult to integrate analytically. Using Gauss quadrature principle a line integral can be approximated as follows; [5]

$$S_L = \int_L G_L(y) dy = \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i G_L(y_i) \tag{3.11}$$

Where: N<sub>G</sub>-is the order of Gauss quadrature.

w<sub>i</sub> - is the i<sup>th</sup> weight

y<sub>i</sub> -is the sample point at which the function G<sub>L</sub>(y<sub>i</sub>) is evaluated

### **3.5 Computation of Compressive Stress and Stress Resultants on Concrete**

As shown in figure 3.4 the stress-strain relationship of concrete is of the form:

$$\sigma_c(\psi) = f(\varepsilon_\psi) \text{ where; } \varepsilon_\psi - \text{ is the normal strain}$$

Because of the Bernoulli's assumption the strain  $\varepsilon_\psi$  can be expressed in terms of  $\psi$ , the distance from the neutral axis: [5]

$$\varepsilon_\psi = \varepsilon_{cu} * \psi / C \quad (3.12)$$

Where:  $\psi$  is the perpendicular distance from the neutral axis and a point on the compression zone.

From Figure 3.4 it is seen that:

$$\psi = y + c - y_{\max}$$

Therefore the stress can be expressed as:

$$\sigma_c(\psi) = f(\psi) = f(y + c - y_{\max}) = f_c(y)$$

i.e. the compressive stress of concrete at 'y' distance from the x-axis.

Taking differential area,  $dA$ , in the compression zone at 'y' distance from the x-axis, the stress resultants can be computed as follows:

$$\diamond N_c = \iint_A f_c(y) dA \quad (3.13)$$

$$\diamond M_{xc} = - \iint_A y * f_c(y) dA \quad (3.14)$$

$$\diamond M_{yc} = \iint_A x * f_c(y) dA \quad (3.15)$$

Where: 'A' is the compressive area of the section enclosed by the perimeter  $6, 7, w$  &  $v$  as shown in Figure 3.4. The coordinate system 'oxy' is a centroidal system with 'x' parallel to the neutral axis as shown in Figure 3.4. This is so because the design action effects are determined with respect to the centroidal axis system .

The strain and stress distributions shown in Figure 3.4 are the result of the external axial force  $P_d$  and the bending moments  $M_{Xd}$  and  $M_{Yd}$ . The equilibrium of those external forces with the internal force resultants  $N_c, M_{xc}, M_{yc}, M_{xs}, M_{ys}$  and  $N_s$  gives:

$$P_d = N_c + N_s \quad (3.16)$$

$$M_{xd} = (M_{Xc} + M_{Xs}) \cos(-\theta) + (M_{Yc} + M_{Ys}) \sin(-\theta) \quad (3.17)$$

$$M_{yd} = -(M_{Xc} + M_{Xs}) \sin(-\theta) + (M_{Yc} + M_{Ys}) \cos(-\theta) \quad (3.18)$$

Where:  $N_c$ ,  $M_{xc}$  and  $M_{yc}$  are the stress resultants of the concrete stress.

$N_s$ ,  $M_{xs}$  and  $M_{ys}$  are the stress resultants of the steel stress.

$\theta$ , is the angle that the neutral axis makes with positive X-axis (global) in the counter clock wise direction, see *Figure 3.4*

Using Green's Theorem and Gauss Quadrature principle the stress resultants;  $N_c$ ,  $M_{xc}$  and  $M_{yc}$  can be evaluated as follows.

$$\iint_A \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_L P dx + \oint_L Q dy \quad (3.19)$$

### **3.6 Determination of functions Q and P**

To simplify the computation the value of P is taken as zero. Function, Q in (3.19) should satisfy all the expressions in Eqns. 3.13, 3.14 and 3.15. Since it is differentiated with respect to 'x' it should be the product of  $f_c(y)$ ,  $x^{t+1}$  and  $y^s$  so that all Eqns. 3.13, 3.14 and 3.15 are satisfied. In addition,  $1/(t+1)$ , should be its multiplicative factor so that it cancels  $(t+1)$  obtained in differentiating Q with x. The expression that satisfies the above criteria for Q is as follows.

$$P = 0 \quad (3.20)$$

$$Q = \frac{1}{t+1} x^{t+1} y^s f_c(y) \quad (3.21)$$

Where: t and s are non negative integers.

$$\iint_A \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \iint_A x^t y^s f_c(y) dx dy = \frac{1}{t+1} \oint_L x^{t+1} y^s f_c(y) dy \quad (3.22)$$

Stress resultant

$$R = \frac{1}{t+1} \oint_L x^{t+1} y^s f_c(y) dy \quad (3.23)$$

This is the basic equation of the present formulation of the uniaxial and biaxial problems. Depending on the values of r and s, the left hand side of Eqn. (3.23) represents.

- a) The axial force  $N_c$  (Eq.19) for  $t = 0$  and  $s = 0$
- b) The bending moment  $M_{xc}$  (Eq.20) for  $t = 0$  and  $s = 1$
- c) The bending moment  $M_{yc}$  (Eq.21) for  $t = 1$  and  $s = 0$

If the function  $f_c(y) = 1$  then the left hand side of Eq. (29) represents:

- a) The area  $A$  for  $t = 0$  and  $s = 0$
- b) The area moment  $S_x$  for  $t = 0$  and  $s = 1$
- c) The area moment  $S_y$  for  $t = 1$  and  $s = 0$
- d) The moment of inertia,  $I_x$  for  $t = 2$  and  $s = 0$
- e) The moment of inertia  $I_y$  for  $t = 0$  and  $s = 2$
- f) The product of inertia  $I_{xy}$  for  $t = 1$  and  $s = 1$

The right hand side of Eqn.3.23 is the line integration along the sides of integration area  $A$  and can be written as:

$$\frac{1}{t+1} \oint_L x^{t+1} y^s f_c(y) dy = \frac{1}{t+1} \sum_L S_L \quad (3.24)$$

$$\text{Where: } S_L = \oint_L x^{t+1} y^s f_c(y) dy \quad (3.25)$$

is the integral along the 'L' side of the closed polygon that encloses the compressive zone of the section.

### **3.7 Line Integration**

The line integral of Eqn. (3.25) is evaluated along the sides of the compressive zone. The sides are defined by the xy-coordinates of the end points as shown in *Figure 3.4*

The equation of side 'L' is:

$$x = \alpha_L + \beta_L y \quad (3.26)$$

$$\beta_L = (x_{L+1} - x_L) / (y_{L+1} - y_L) \quad (3.27)$$

$$\alpha_L = x_L - \beta_L y_L \quad (3.28)$$

Where:  $(x_L, y_L)$  and  $(x_{L+1}, y_{L+1})$  are the starting and ending points of side 'L' respectively.

With Eqn. (29), the line integral, in Eqn. (28), becomes:

$$S_L = \int_L (\alpha_L + \beta y)^{t+1} y^s f_c(y) dy = \int_L G_L(y) dy \quad (3.29)$$

$$\text{Where: } G_L(y) = (\alpha_L + \beta_L y)^{t+1} y^s f_c(y) \quad (3.30)$$

$$S_L = \int_L G_L(y) dy \cong \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i G_L(y_i) \quad (3.31)$$

Where:  $N_G$  is the order of the Gauss Quadrature,  $w_i$  is the  $i^{\text{th}}$  weight, and ' $y_i$ ' is the sample point at which the function  $G_L(y_i)$  is evaluated. Numerical values for  $w_i$  and  $y_i$  can be found in numerical Analysis books and for this study  $N_G$  is three.

### **3.8) The steel contribution**

The contribution of the steel does not present computational difficulties. The steel bars are assumed discrete points with area  $A_{sj}$ , coordinates  $x_{sj}$ ,  $y_{sj}$  and stress  $f_{sj}$ . The total steel axial force and bending moment resultants are:

$$\diamond N_s = \sum_{j=1}^N A_{sj} * f_{sj} \quad (3.32)$$

$$\diamond M_{xs} = - \sum_{j=1}^N y_{sj} * A_{sj} * f_{sj} \quad (3.33)$$

$$\diamond M_{ys} = \sum_{j=1}^N x_{sj} * A_{sj} * f_{sj} \quad (3.34)$$

To avoid double counting of the concrete area displaced by the steel bars, the force  $A_{sj} f_c(y)$  is deducted from the bar force  $A_{sj} * f_{sj}$ ,  $f_c(y)$  is the concrete compressive stress at the centroid of the bar.

### **3.9 Cross-section Analysis subjected to Compression with bending(s)**

#### **3.9.1 General**

The section may have any dimension to meet the need of the software user. Thus the section might become bi-symmetric, symmetric along one axis or non-symmetrical along both axes; depending on the dimensions given to  $a$ ,  $b$ ,  $t_f$ ,  $t_w$  and especially for dimensions  $l$  and  $r$ .

Eight equal diameter reinforcement bars are to be placed as shown in *Figure 3.1* and *3.4*. For the assumed distribution of reinforcement the geometric and plastic centroids don't coincide, thus unlike bisymmetric sections axial force applied at the geometric centroid creates bending. For this study the geometric centroid is taken as moment center, due to this design moments should be obtained from structural analysis results done by considering the geometric centroid of the cross section as central axis.

#### **3.9.2 Computation of angle $\theta$ to choose design charts**

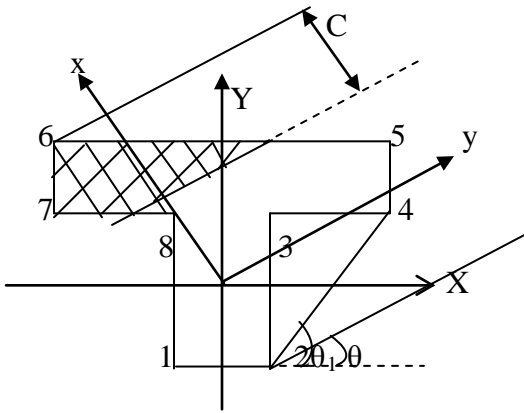
Due to biaxial non-symmetry (i.e. the section might be biaxial non symmetrical depending on the dimensions given to  $l$  &  $r$ ); the section is checked for ranges of angles that results different corners to be compressed and different reinforcement bars to be stressed (in tension and compression).

When varying the angle range in different intervals the distance between the most compressed corner of concrete and the highly tensiled reinforcement will also vary. This also changes the value of the distance between the most compressed corner and the highly tensile reinforcement bar.

By varying the angle from 0 to 360 we get six different cases that need special consideration. The cases are as shown below.

Where  $\ast \theta = \arctan\left(\frac{M_{yd}}{M_{xd}}\right)$        $\ast \theta_1 = \arctan\left(\frac{b-t_f}{r}\right)$   
 $\ast \theta_2 = \arctan\left(\frac{b-t_f}{l}\right)$

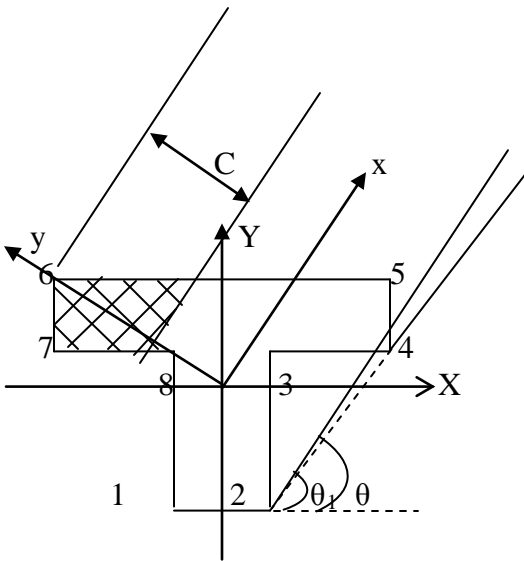
**Case 1.** For  $0 \leq \theta \leq \theta_1$



- ✦ Most compressed corner = 6
- ✦ Highly tensiled reinforcement bar =  $A_{s2}$

$$d = | -X_6 \cdot \sin\theta + Y_6 \cdot \cos\theta | + | -X_{As2} \cdot \sin\theta + Y_{As2} \cdot \cos\theta |$$

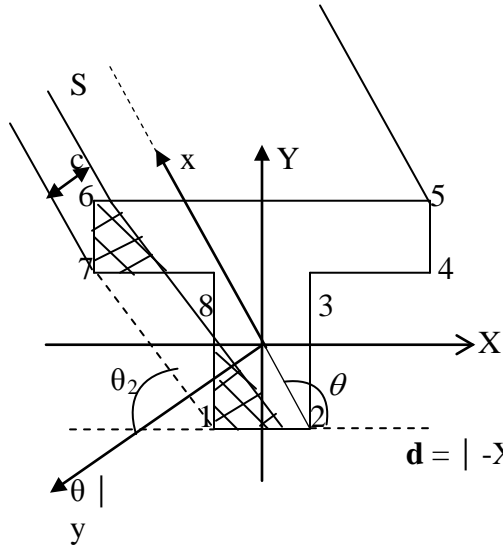
**Case 2.** For  $\theta_1 \leq \theta \leq 90$



- ✦ Most compressed corner = 6
- ✦ Highly tensiled reinforcement bar =  $A_{s4}$

$$d = | -X_6 \cdot \sin\theta + Y_6 \cdot \cos\theta | + | -X_{As4} \cdot \sin\theta + Y_{As4} \cdot \cos\theta |$$

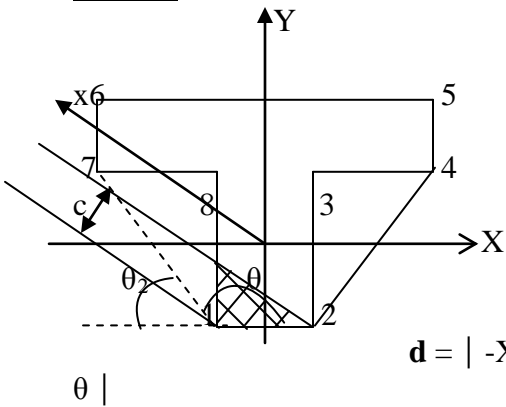
**Case 3.** For  $90 \leq \theta \leq 180 - \theta_2$



- ✦ Most compressed corner = 7
- ✦ Highly tensiled reinforcement bar =  $A_{s5}$

$$d = | -X_7 * \sin\theta + Y_7 * \cos\theta | + | -X_{As5} * \sin\theta + Y_{As5} * \cos\theta$$

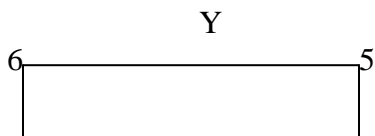
**Case 4.** For  $180 - \theta_2 \leq \theta \leq 180$



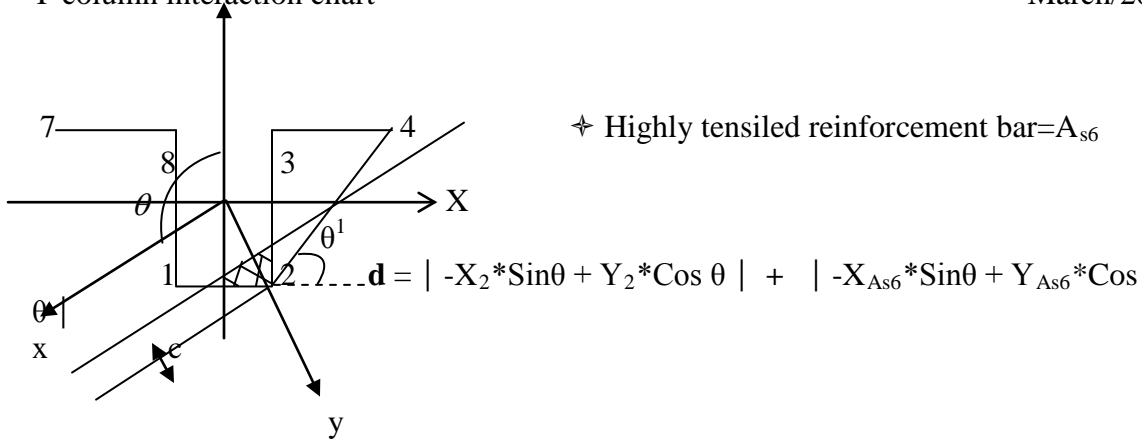
- ✦ Most compressed corner = 1
- ✦ Highly tensiled reinforcement bar =  $A_{s5}$

$$d = | -X_1 * \sin\theta + Y_1 * \cos\theta | + | -X_{As5} * \sin\theta + Y_{As5} * \cos\theta$$

**Case 5.** For  $180 \leq \theta \leq 180 + \theta_1$

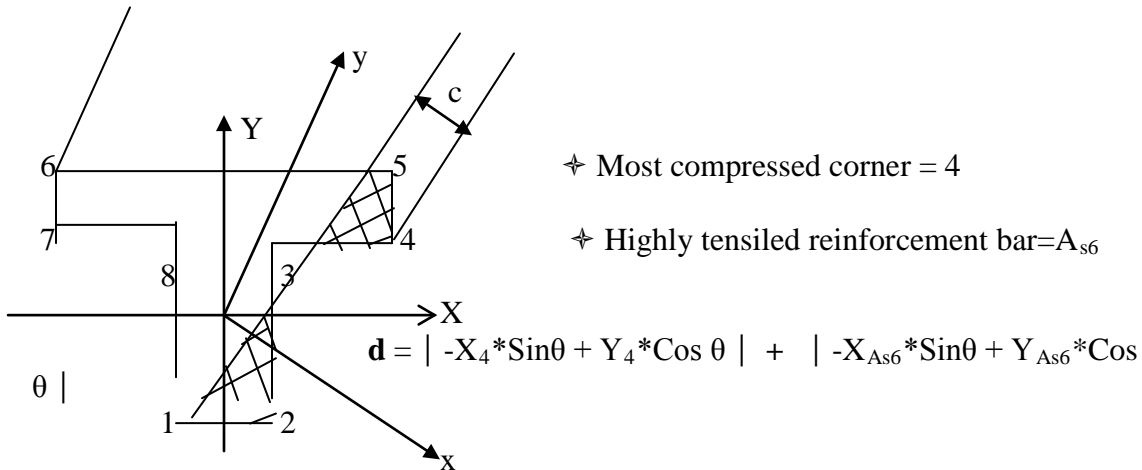


- ✦ Most compressed corner = 2



✦ Highly tensiled reinforcement bar= $A_{s6}$

**Case 6.** For  $180 + \theta_1 \leq \theta \leq 360$



✦ Most compressed corner = 4

✦ Highly tensiled reinforcement bar= $A_{s6}$

Where :-  $\theta_1 = \tan^{-1}(b-t_f/r)$  ,  $\theta_2 = \tan^{-1}(b-t_f/l)$

- d is the distance between the most compressed corner and highly tensiled reinforcement bar.

$d = | \text{local } y \text{ coordinate of the most compressed corner} | + | \text{local } y \text{ coordinate of the most tensile reinforcement bar} |$

Case 1.  $d = | -X_6 * \sin\theta + Y_6 * \cos\theta | + | -X_{As2} * \sin\theta + Y_{As2} * \cos\theta |$

Case 2.  $d = | -X_6 * \sin\theta + Y_6 * \cos\theta | + | -X_{As4} * \sin\theta + Y_{As4} * \cos\theta |$

Case 3.  $d = | -X_7 * \sin\theta + Y_7 * \cos\theta | + | -X_{As5} * \sin\theta + Y_{As5} * \cos\theta |$

Case 4.  $d = | -X_1 * \sin\theta + Y_1 * \cos\theta | + | -X_{As5} * \sin\theta + Y_{As5} * \cos\theta |$

Case 5.  $d = | -X_2 * \sin\theta + Y_2 * \cos\theta | + | -X_{As6} * \sin\theta + Y_{As6} * \cos\theta |$

Case 6.  $d = | -X_4 * \sin\theta + Y_4 * \cos\theta | + | -X_{As6} * \sin\theta + Y_{As6} * \cos\theta |$

Table 3.1 Range of angle 'θ' for most compressed corners and highly tensiled reinforcement bars

| Case | Range of angle $\theta$               | Most compressed corner point | Most tensile Reinforcement |
|------|---------------------------------------|------------------------------|----------------------------|
| 1    | $0 \leq \theta \leq \theta_1$         | 6                            | $A_{S2}$                   |
| 2    | $\theta_1 \leq \theta \leq 90$        | 6                            | $A_{S4}$                   |
| 3    | $90 \leq \theta \leq 180 - \theta_2$  | 7                            | $A_{S5}$                   |
| 4    | $180 - \theta_2 \leq \theta \leq 180$ | 1                            | $A_{S5}$                   |
| 5    | $180 \leq \theta \leq 180 + \theta_1$ | 2                            | $A_{S6}$                   |
| 6    | $180 + \theta_1 \leq \theta \leq 360$ | 4                            | $A_{S6}$                   |

### **3.9.3 Computation of X-Y coordinates of Salient points and Reinforcement bars.**

The x-y coordinates of the salient points with respect to both the global and local axes are computed as follows using T-section properties. Salient points stand for corner points of the section assigned from 1 to 8 as shown in fig 3.4.

$$C_{1=} \frac{[0.5 * F_{cd}(a * t_f^2 + t_w(b^2 - t_f^2)) + F_{ycmod} * 4 * A_s * d + F_{ycmod} * 2 * A_s * (t_f - d) + F_{ycmod} * 2 * A_s * (b - d)]}{[F_{cd}(a * t_f + t_w(b - t_f)) + F_{ycmod} * 8 * A_s]}$$

$$C_{2=} \frac{0.5 * F_{cd} * t_w(b - t_f)^2 + (F_{cd} * a * t_f * (b - 0.5 * t_f)) + F_{ycmod} * 2 * A_s * d + F_{ycmod} * 2 * A_s * (b - t_f + d) + F_{ycmod} * 4 * A_s * (b - d)}{[F_{cd}(a * t_f + t_w(b - t_f)) + F_{ycmod} * 8 * A_s]}$$

$$C_{3=} \frac{[0.5 * F_{cd} * t_f * a^2 + (F_{cd} * (1 + 0.5 * t_w) * (b - t_f)) + F_{ycmod} * 2 * A_s * d + F_{ycmod} * 2 * A_s * (1 + d) + F_{ycmod} * 2 * A_s * (1 + t_w - d) + F_{ycmod} * 2 * A_s * (a - d)]}{[F_{cd}(a * t_f + t_w(b - t_f)) + F_{ycmod} * 8 * A_s]}$$

$$C_{4=} \frac{[0.5 * F_{cd} * t_f * a^2 + (F_{cd} * (r + 0.5 * t_w) * (b - t_f)) + F_{ycmod} * 2 * A_s * d + F_{ycmod} * 2 * A_s * (r + d) + F_{ycmod} * 2 * A_s * (r + t_w - d) + F_{ycmod} * 2 * A_s * (a - d)]}{[F_{cd}(a * t_f + t_w(b - t_f)) + F_{ycmod} * 8 * A_s]}$$

- $C_1$  and  $C_2$  indicates the position of the plastic centroid of the section from the top and bottom faces of the cross section respectively as shown in *figure3.4*
- $C_3$  and  $C_4$  indicates the position of the plastic centroid of the section from the left and right faces of the flange respectively as shown in *figure3.4*

**Table 3.2 Global X and Y coordinates of vertexes of cross section**

| Salient points | Coordinates |           |
|----------------|-------------|-----------|
|                | X           | Y         |
| 1              | $-(C_3-l)$  | $-C_2$    |
| 2              | $C_4-r$     | $-C_2$    |
| 3              | $C_4-r$     | $C_1-t_f$ |
| 4              | $C_4$       | $C_1-t_f$ |
| 5              | $C_4$       | $C_1$     |
| 6              | $-C_3$      | $C_1$     |
| 7              | $-C_3$      | $C_1-t_f$ |
| 8              | $-(C_3-l)$  | $C_1-t_f$ |

Similarly global X and Y coordinates of each reinforcement bars are determined using the section properties of the core wall and sufficient reinforcement bar cover  $d^1$ .

Table 3.3 Global X and Y coordinates of reinforcement bars

| Reinforcement Bar | Coordinates    |               |
|-------------------|----------------|---------------|
|                   | X              | Y             |
| $A_{S1}$          | $-(C_3-l-d^1)$ | $-(C_2-d^1)$  |
| $A_{S2}$          | $(C_4-r-d^1)$  | $-(C_2-d^1)$  |
| $A_{S3}$          | $C_4-d^1$      | $C_1-t_f+d^1$ |
| $A_{S4}$          | $C_4-d^1$      | $C_1-d^1$     |
| $A_{S5}$          | $-(C_4-r-d^1)$ | $C_1-d^1$     |
| $A_{S6}$          | $-(C_3-l-d^1)$ | $C_1-d^1$     |
| $A_{S7}$          | $-(C_3-d^1)$   | $C_1-d^1$     |
| $A_{S8}$          | $-(C_3-d^1)$   | $C_1-t_f+d^1$ |

To computation of local stress resultants for ' $\theta$ ' orientation of the neutral axis, x and y coordinates of each necessary point about the local axis are determined using the transformation formula below.

$$x = X \cos(\theta) + Y \sin(\theta)$$

$$y = -X \sin(\theta) + Y \cos(\theta)$$

### **3.9.4 Determination of Stress resultants: $N$ , $M_x$ and $M_y$**

The stress resultants;  $N$ ,  $M_x$  and  $M_y$  are obtained by superposing the contribution of both concrete and reinforcement bars for the considered position and orientation,  $\theta$ , of the neutral axis.

**a) Computation of local stress resultants ( $N_c$ ,  $M_{xc}$  and  $M_{yc}$ ) of concrete contribution.**

From Eq.20, the general expression of stress resultants ( $R$ ) is:

$$R = \frac{1}{t+1} \int_L^{t+1} x^{t+1} y^s f_c(y) dy = \frac{1}{t+1} \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i G_L(y_i)$$

Where:  $G_L(y) = (\alpha_L + \beta_L y)^{t+1} y^s f_c(y)$ ,

$n$  is number of lines that enclose the compression area of the section.

By varying the values of  $t$  and  $s$  the following result is obtained from  $R$

➤  $N_c$ , when  $t = s = 0$  then  $R = N_c$

$$N_c = \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i G_L(y_i) = \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i (\alpha_L + \beta_L y_i) * f_c(y_i)$$

➤  $M_{xc}$ , when  $t = 0$  and  $s = 1$  then  $R = M_{xc}$

$$M_{xc} = - \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i G_L(y_i) = - \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i (\alpha_L + \beta_L y_i) * y_i * f_c(y_i)$$

➤  $M_{yc}$ , when  $t = 1$  and  $s = 0$  then  $R = M_{yc}$

$$M_{yc} = \frac{1}{2} \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i G_L(y_i) = \frac{1}{2} \sum_n \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{N_G} w_i (\alpha_L + \beta_L y_i)^2 * f_c(y_i)$$

**b) Computation of local stress resultants ( $N_s$ ,  $M_{xs}$  &  $M_{ys}$ ) of reinforcement bar contribution.**

$$N_s = \sum_{j=1}^N A_{sj} * f_{sj}$$

$$M_{xs} = - \sum_{j=1}^N y_{sj} * A_{sj} * f_{sj}$$

$$M_{ys} = \sum_{j=1}^N x_{sj} * A_{sj} * f_{sj}$$

The bending moment stress resultants obtained above are determined about the local centroidal x and y axes.

Using the local stress resultants, the Global ones ( $M_x$  and  $M_y$ ) which are with the design bending moments are determined as follows.

$$N = N_d = N_c + N_s$$

$$M_x = M_{xd} = (M_{xc} + M_{xs}) \cos(-\theta) + (M_{yc} + M_{ys}) \sin(-\theta)$$

$$M_y = M_{yd} = - (M_{xc} + M_{xs}) \sin(-\theta) + (M_{yc} + M_{ys}) \cos(-\theta)$$

Where  $N$ ,  $M_x$  and  $M_y$  represent a point on the interaction curve.

## **Chapter Four**

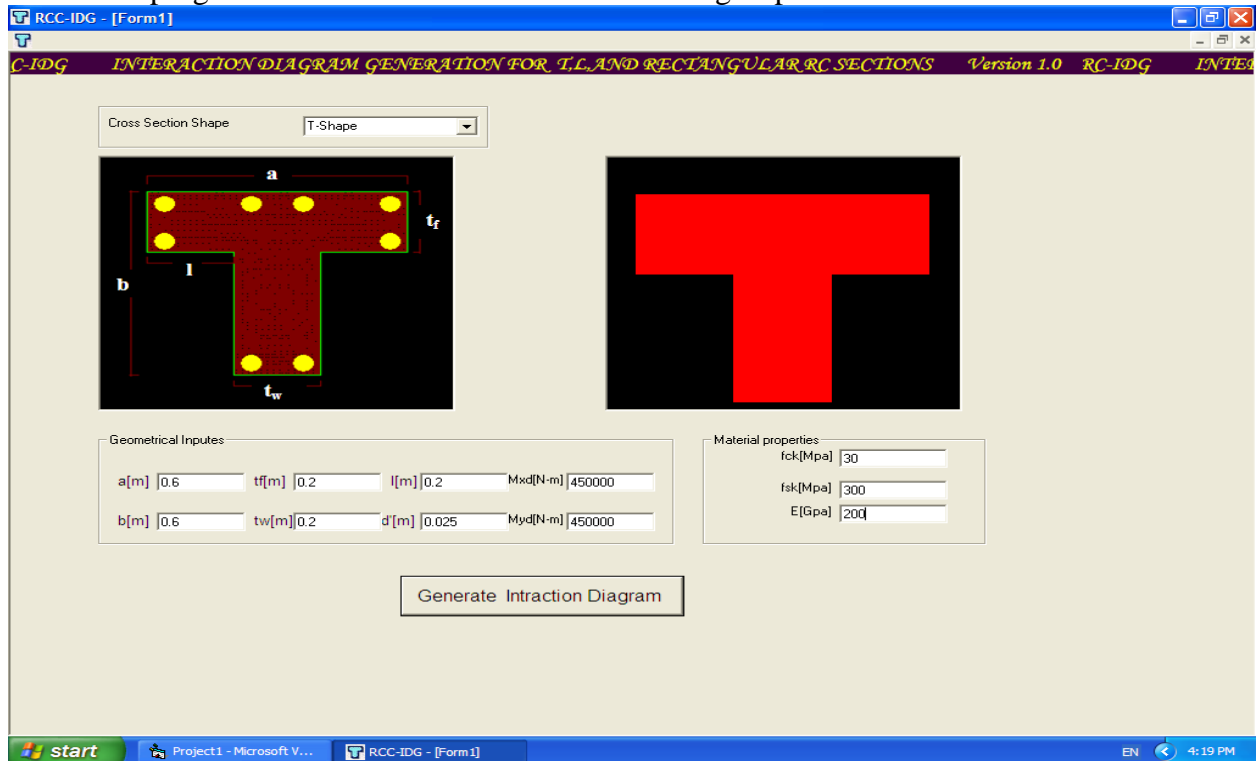
### **4. Results and Discussion**

#### **4.1 Discussion**

1. As the cross section is not symmetrical about both axis the geometric and plastic centroids don't coincide. In this study the plastic centroid is taken as the moment center.
2. Due to the rational in (1) above in addition to the compressive force (N), moment about the X-axis  $M_x$  and Y-axis  $M_y$  exist for uniform limiting compressive strain i.e.  $\epsilon_{cu} = 0.002$  on the section.
3. The capacity of the section depends on both the position and orientation of the neutral axis. Hence, the behaviors of the charts around the maximum compressive forces are different for each orientation of the neutral axis.
4. The charts prepared by the program are for different dimensions of T, L and rectangular section; and different orientation of web.
5. By changing the material properties used for concrete and reinforcement bar the strength of a section with the same cross-sectional area are observed.
6. Different reinforcement ratios are taken starting from 0 to 0.08 with an increment of 0.01. At lower reinforcement ratio the balanced point is expected to occur at higher axial load point than at higher reinforcement ratio.
7. The shape of the design chart looks like the design chart of rectangular and circular reinforced concrete column cross section with equally spaced failure surfaces for the given cross section and reinforcement arrangement.

## 4.2 User's Guide

To use the program the user should follow the following steps:



- The user should first choose shape of the section: i.e. T, L or rectangular.
- The user should next choose the concrete and reinforcement grades to be used for the chosen reinforced concrete column.
- The user should also input dimensions of the reinforced concrete cross-section that is going to be used for the design.
- The user should input reinforcement bar concrete cover.
- The inclination angle of the neutral axis from the positive global X-axis will be put as an input by giving the moment values obtained from analysis.

- Then run the program to get the interaction chart by simply kicking the ‘ Generate Interaction Diagram’ icon.
- The interaction chart will be produced on Excel sheet showing the shape of the section, position of reinforcement bars and inclination angle of the neutral axis from the positive global X-axis.

### 4.3 Design Examples

1. Design a rectangular reinforced concrete column subjected to biaxial bending solved in EBCS 2-1995: part 2 example 3.5 with the following design loads

$$[N_{sd} = 1305\text{KN}, M_{xd} = 100\text{KN-m and } M_{yd} = 200\text{KN-m}].$$

Material data: Concrete C30  
Steel S460

**Solution:**

- Assume column size ,b/h=400/400 mm and cover ratio h'/h=b'/b=0.1
- The same amount of reinforcement bars are positioned as shown in the appendix A.
- Design loads are  $N_{sd}=1305\text{KN}$ ,  $M_{xd}= 100\text{KN-m}$  and  $M_{yd}= 200\text{KN-m}$
- Determine angle  $\theta$  from the given moment,  $\theta = \tan^{-1}\left(\frac{M_{yd}}{M_{xd}}\right) = 63^\circ$
- Generate the design chart for the assumed section, reinforcement location and material properties following the steps given in the user guideline using the program provid interaction chart as shown in appendix A.
- Calculate  $\mu$  and  $\nu$  using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart

$$\nu = \frac{N_d}{f_{cd} * A_c} = \frac{1305 * 10^3}{(400 * 400) * 13.6} = 0.6$$

$$\mu_x = \frac{M_{xd}}{f_{cd} * A_c * b} = \frac{100 * 10^6}{(400 * 400) * 13.6 * 400} = 0.1149$$

$$\mu_y = \frac{M_{yd}}{f_{cd} * A_c * a} = \frac{200 * 10^6}{(400 * 400) * 13.6 * 400} = 0.229$$

$$\mu = \sqrt{(\mu_x^2 + \mu_y^2)} = \sqrt{(0.1149^2 + 0.229^2)} = 0.256$$

- $\rho$  when read from the graph will be approximately 0.55
- Therefore  $A_s = \rho * A_c * f_{cd} / f_{yd} = 0.55 * 400 * 400 * 13.6 / 400 = 2,992 \text{mm}^2$
- As the number of reinforcement bars are four and each bar has the same area,  $A_s$  will be

$$A_s = \frac{2992}{4} = 748 \text{mm}^2$$

- Therefore provide  $748 \text{mm}^2$  at each corner. This result is almost identical to the one done in example 3.5 of EBCS-2 part 2 which is  $734 \text{mm}^2$  at each corner.

2. Design a rectangular reinforced concrete column with the following design loads from analysis [ $N_{sd} = 500 \text{KN}$ ,  $M_{xd} = 175 \text{KN-m}$  and  $M_{yd} = 0 \text{KN-m}$ ].

**Solution:**

- Assume concrete grade C-30 and reinforcing steel grade S-460
- Assume a column section with  $a = b = 400 \text{mm}$  and cover to reinforcement = 20mm.
- The same amount and location of reinforcement bars are positioned in fig 3.1 is assumed.
- Design loads are  $N_{sd} = 500 \text{KN}$ ,  $M_{xd} = 175 \text{KN-m}$  and  $M_{yd} = 0 \text{KN-m}$
- Determine angle  $\theta$  from the given moment,  $\theta = \tan^{-1} \left( \frac{M_{yd}}{M_{xd}} \right) = 0^\circ$
- Generate the design chart for the assumed section, reinforcement location and material properties following the steps given in the user guideline using the program is shown in appendix A.
- Calculate  $\mu$  and  $\nu$  using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart

$$\nu = \frac{N_d}{f_{cd} * A_c} = \frac{500 * 10^3}{(400 * 400) * 13.6} = 0.2297$$

$$\mu_x = \frac{M_{xd}}{f_{cd} * A_c * b} = \frac{175 * 10^6}{(400 * 400) * 13.6 * 400} = 0.201$$

$$\mu_y = \frac{M_{yd}}{f_{cd} * A_c * a} = 0$$

$$\mu = \sqrt{(\mu_x^2 + \mu_y^2)} = 0.201$$

- $\rho$  when read from the graph will be approximately 0.017
- Therefore  $A_s = \rho \times A_c = 0.017 \times (400 \times 400) = 2,720 \text{mm}^2$
- As the number of reinforcement bars are four and each bar has the same area,  $A_s$  will be

$$A_s = \frac{2720}{4} = 680 \text{mm}^2$$

3. Design a T-shaped reinforced concrete column with the following design loads from analysis [ $N_{sd} = 650 \text{KN}$ ,  $M_{xd} = 175 \text{KN-m}$  and  $M_{yd} = 175 \text{KN-m}$ ].

**Solution:**

- Assume concrete grade C-30 and reinforcing steel grade S-460
- Since the section should not have edges in the interior of the room, the maximum thickness shall not exceed the thickness of the HCB wall i.e. 200mm therefore lets also assume a column section with  $a = b = 600 \text{mm}$  and  $l = 200 \text{mm}$ ,  $t_w = 200 \text{mm}$ ,  $t_f = 200 \text{mm}$   $d' = 25 \text{mm}$  based on the designation in Fig. (3.1).
- The same amount and location of reinforcement bars are positioned in fig 3.1 is assumed.
- Design loads are  $N_{sd} = 650 \text{KN}$ ,  $M_{xd} = 175 \text{KN-m}$  and  $M_{yd} = 175 \text{KN-m}$
- Determine angle  $\theta$  from the given moment,  $\theta = \tan^{-1} \left( \frac{M_{yd}}{M_{xd}} \right) = 45^\circ$
- Generate the design chart for the assumed section, reinforcement location and material properties following the steps given in the user guideline using the program is shown in appendix A.
- Calculate  $\mu$  and  $\nu$  using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart

$$\nu = \frac{N_d}{f_{cd} * A_c} = \frac{650 * 10^3}{(200 * 600 + 200 * 400) * 13.6} = 0.239$$

$$\mu_x = \frac{M_{xd}}{f_{cd} * A_c * b} = \frac{175 * 10^6}{(200 * 600 + 200 * 400) * 13.6 * 600} = 0.10723$$

$$\mu_y = \frac{M_{yd}}{f_{cd} * A_c * a} = \frac{175 * 10^6}{(200 * 600 + 200 * 400) * 13.6 * 600} = 0.10723$$

$$\mu = \sqrt{(\mu_x^2 + \mu_y^2)} = \sqrt{(0.10723^2 + 0.10723^2)} = 0.15164$$

- $\rho$  when read from the graph will be approximately 0.4
- Therefore  $A_s = \rho * A_c * f_{cd} / f_{yd} = 0.4 * 200,000 * 13.6 / 400 = 2,720 \text{mm}^2$
- As the number of reinforcement bars are eight and each bar has the same area,  $A_s$  will be

$$A_s = \frac{2720}{8} = 340 \text{mm}^2$$

- Therefore 8 $\Phi$ 20mm bars at the locations indicated.

4. Design the above T-shaped reinforced concrete column with the following design loads from analysis [ $N_{sd} = 650 \text{KN}$ ,  $M_{xd} = 243 \text{KN-m}$  and  $M_{yd} = 420 \text{KN-m}$ ].

**Solution:**

- Design loads are  $N_{sd} = 650 \text{KN}$ ,  $M_{xd} = 243 \text{KN-m}$  and  $M_{yd} = 420 \text{KN-m}$
- Determine angle  $\theta$  from the given moment,  $\theta = \tan^{-1} \left( \frac{M_{yd}}{M_{xd}} \right) = 60^\circ$
- Generate the design chart for the assumed section, reinforcement location and material properties following the steps given in the user guideline using the program is shown in appendix A.
- Calculate  $\mu$  and  $\nu$  using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart

$$\nu = \frac{N_d}{f_{cd} * A_c} = \frac{650 * 10^3}{(200 * 600 + 200 * 400) * 13.6} = 0.239$$

$$\mu_x = \frac{M_{xd}}{f_{cd} * A_c * b} = \frac{243 * 10^6}{(200 * 600 + 200 * 400) * 13.6 * 600} = 0.1489$$

$$\mu_y = \frac{M_{yd}}{f_{cd} * A_c * a} = \frac{420 * 10^6}{(200 * 600 + 200 * 400) * 13.6 * 600} = 0.257$$

$$\mu = \sqrt{(\mu_x^2 + \mu_y^2)} = \sqrt{(0.1489^2 + 0.257^2)} = 0.297$$

- $\rho$  when read from the graph will be approximately 0.8

- Therefore  $A_s = \rho * A_c * f_{cd} / f_{yd} = 0.8 * 200,000 * 13.6 / 400 = 5,440 \text{mm}^2$
- As the number of reinforcement bars are eight and each bar has the same area,  $A_s$  will be

$$A_s = \frac{5440}{8} = 680 \text{mm}^2$$

- Therefore provide  $680 \text{mm}^2$  reinforcement bars at the locations indicated.
5. Design the T-shaped reinforced concrete column in example 3 with the following design loads from analysis [ $N_{sd} = 500 \text{KN}$ ,  $M_{xd} = 175 \text{KN-m}$  and  $M_{yd} = 0 \text{KN-m}$ ].

**Solution:**

- Design loads are  $N_{sd} = 650 \text{KN}$ ,  $M_{xd} = 175 \text{KN-m}$  and  $M_{yd} = 0 \text{KN-m}$
- Determine angle  $\theta$  from the given moment,  $\theta = \tan^{-1} \left( \frac{M_{yd}}{M_{xd}} \right) = 0^\circ$
- Generate the design chart for the assumed section, reinforcement location and material properties following the steps given in the user guideline using the program is shown in appendix A.
- Calculate  $\mu$  and  $\nu$  using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart

$$\nu = \frac{N_d}{f_{cd} * A_c} = \frac{650 * 10^3}{(200 * 600 + 200 * 400) * 13.6} = 0.239$$

$$\mu_x = \frac{M_{xd}}{f_{cd} * A_c * b} = \frac{175 * 10^6}{(200 * 600 + 200 * 400) * 13.6 * 600} = 0.10723$$

$$\mu_y = \frac{M_{yd}}{f_{cd} * A_c * a} = 0$$

$$\mu = \sqrt{(\mu_x^2 + \mu_y^2)} = 0.10723$$

- $\rho$  when read from the graph will be approximately 0.2
- Therefore  $A_s = \rho * A_c * f_{cd} / f_{yd} = 0.2 * 200,000 * 13.6 / 400 = 1,360 \text{mm}^2$
- As the number of reinforcement bars are eight and each bar has the same area,  $A_s$  will be

$$A_s = \frac{1360}{8} = 170 \text{mm}^2$$

In this way any design problem with regard to T-shaped columns can be solved in short period of time.

6. Design an L-shaped reinforced concrete column with the following design loads from analysis [ $N_{sd} = 425\text{KN}$ ,  $M_{xd} = 140\text{KN-m}$  and  $M_{yd} = 140\text{KN-m}$ ].

**Solution:**

- Assume concrete grade C-25 and reinforcing steel grade S-460
- Since the section should not have edges in the interior of the room, the maximum thickness shall not exceed the thickness of the HCB wall i.e. 200mm therefore lets also assume a column section with  $a = b = 600\text{mm}$  and  $t_w=200\text{mm}, t_f=200\text{mm}$ ,  $d' = 30\text{mm}$ .
- The same amount and location of reinforcement bars are positioned in fig 3.1 is assumed.
- Design loads are  $N_{sd}=425\text{KN}$ ,  $M_{xd}= 140\text{KN-m}$  and  $M_{yd}= 140\text{KN-m}$
- Determine angle  $\theta$  from the given moment,  $\theta = \tan^{-1}\left(\frac{M_{yd}}{M_{xd}}\right) = 45^\circ$
- Generate the design chart for the assumed section, reinforcement location and material properties following the steps given in the user guideline using the program is shown in appendix A.
- Calculate  $\mu$  and  $\nu$  using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart.

$$\nu = \frac{N_d}{f_{cd} * A_c} = \frac{425 * 10^3}{(600 * 600 - 400 * 400) * 11.33} = 0.1875$$

$$\mu_x = \frac{M_{xd}}{f_{cd} * A_c * b} = \frac{140 * 10^6}{(600 * 600 - 400 * 400) * 11.33 * 600} = 0.1029$$

$$\mu_y = \frac{M_{yd}}{f_{cd} * A_c * a} = \frac{140 * 10^6}{(600 * 600 - 400 * 400) * 11.33 * 600} = 0.1029$$

$$\mu = \sqrt{(\mu_x^2 + \mu_y^2)} = \sqrt{(0.1029^2 + 0.1029^2)} = 0.1455$$

- $\rho$  when read from the graph will be approximately 0.45
- Therefore  $A_s = \rho * A_c * f_{cd} / f_{yd} = 0.45 * 200,000 * 11.33 / 400 = 2,550\text{mm}^2$

- As the number of reinforcement bars are eight and each bar has the same area,  $A_s$  will be

$$A_s = \frac{2550}{8} = 318.6mm^2$$

- Therefore 8Φ20mm bars at the locations indicated.

In this way any design problem with regard to T, L or rectangular shaped, columns can be solved in short period of time.

## **Chapter Five**

### **5. Conclusion , Recommendations and limitations**

#### **5.1 Conclusion**

- ❖ The width, depth and thickness of the cross-section, the grade of steel reinforcement and class of the concrete, the location of the reinforcement bars and concrete cover dimensions can be varied as the program can be modified to allow such changes to be incorporated.
- ❖ The program can develop design charts for T-section if the dimensions of left side of flange and width of the web are given different value from the dimension of the upper most surface (i.e.  $a$ ). It is also necessary that  $a \geq l + t_w$
- ❖ The program can develop design charts for L-section if the dimensions of left side of flange is given a zero or  $(a-t_w)$  value.
- ❖ The program can develop design charts for rectangular-section if the dimensions of left side of flange and width of the web are given equal value to the dimension of the upper most surface(i.e.  $a$ )
- ❖ Readily available any shaped reinforced concrete column design charts can be generated using the very same program with some modifications.
- ❖ The program is developed for normal grade of concrete and steel specified in the Ethiopian building code standard EBCS 2-1995.

### **5.2 Limitations of the program**

- ❖ There are some discrepancies above the balanced point in the results of the program output when compared with charts developed using other softwares.
- ❖ In pure axial case i.e the intersection of the chart with the Y axis the program gives less results than expected.
- ❖ The reason for the above two limitations might be the closeness of the data above the balanced point.

### **5.3 Recommendation**

- ❖ The program developed is user friendly and easily operable, by providing few necessary data and get the required interaction charts immediately.
- ❖ The program is made in such a way that it is flexible to provide interaction charts for reinforced concrete column cross sections to serve as an input for further research and development in the field of columns design.
- ❖ The program can be extended further to include other shapes and reinforcement bar arrangements.
- ❖ If the limitations of the program are solved it can be used for generating interaction charts of different shapes.
- ❖ The program methodology and mathematical approach used in this study can be used in different areas where double integration is required.

## **Appendix-A**

### **Interaction Charts**

## **Appendix-B**

### **Flow Charts & Code of the Program**



### Interaction Chart for Example 1

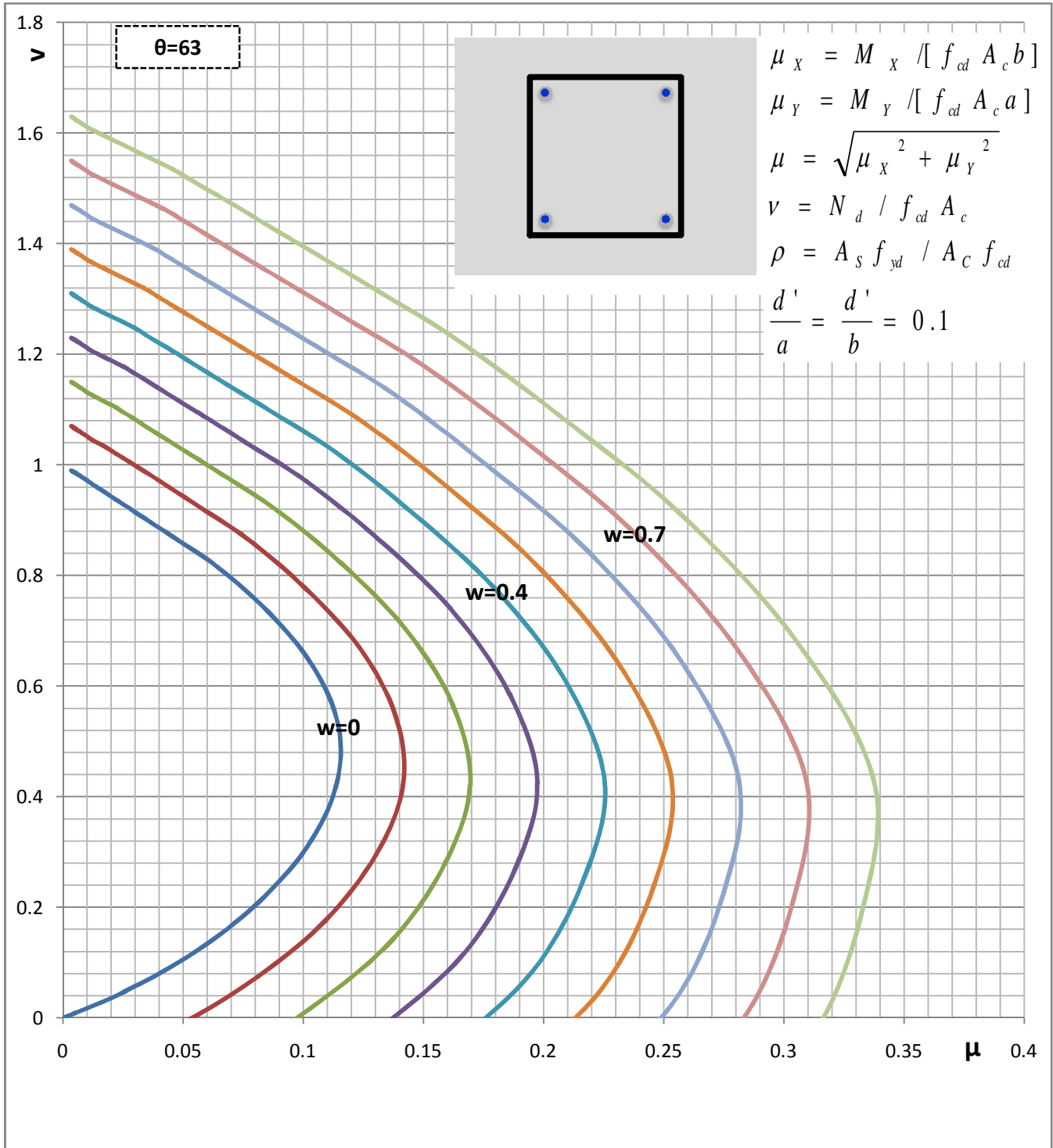


Chart for rectangular reinforced concrete column with dimensions  $a=b=400\text{mm}$   
 $M_{xd}=100\text{KNm}, M_{yd}=200\text{KNm}, C30, S460$

## Interaction Chart for Example 2

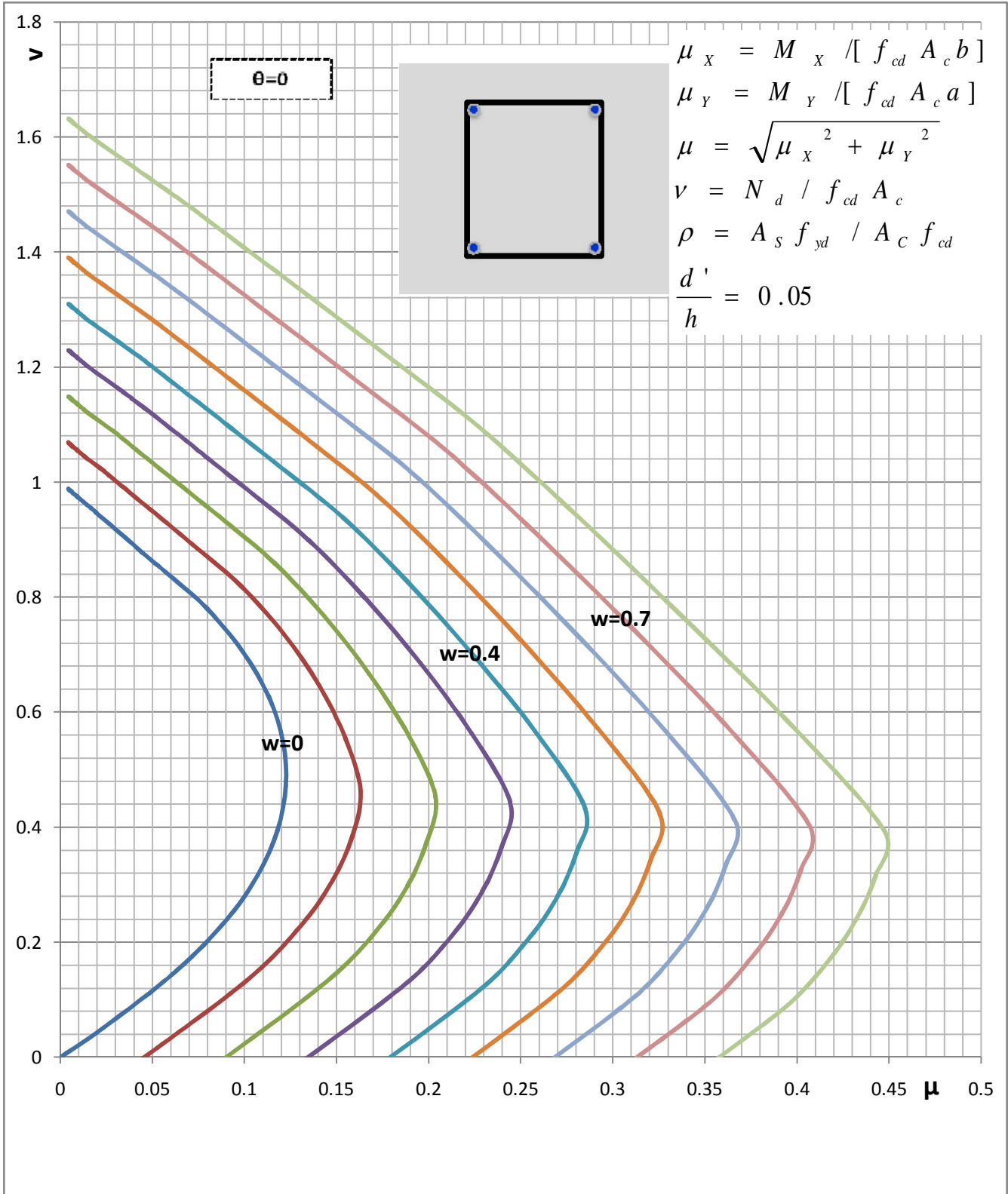


Chart for rectangular reinforced concrete column with dimensions  $a=b=400\text{mm}$   
 $M_{xd}=175\text{KNm}$ ,  $M_{yd}=0\text{KNm}$ , C25, S460

### Interaction Chart for Example 3

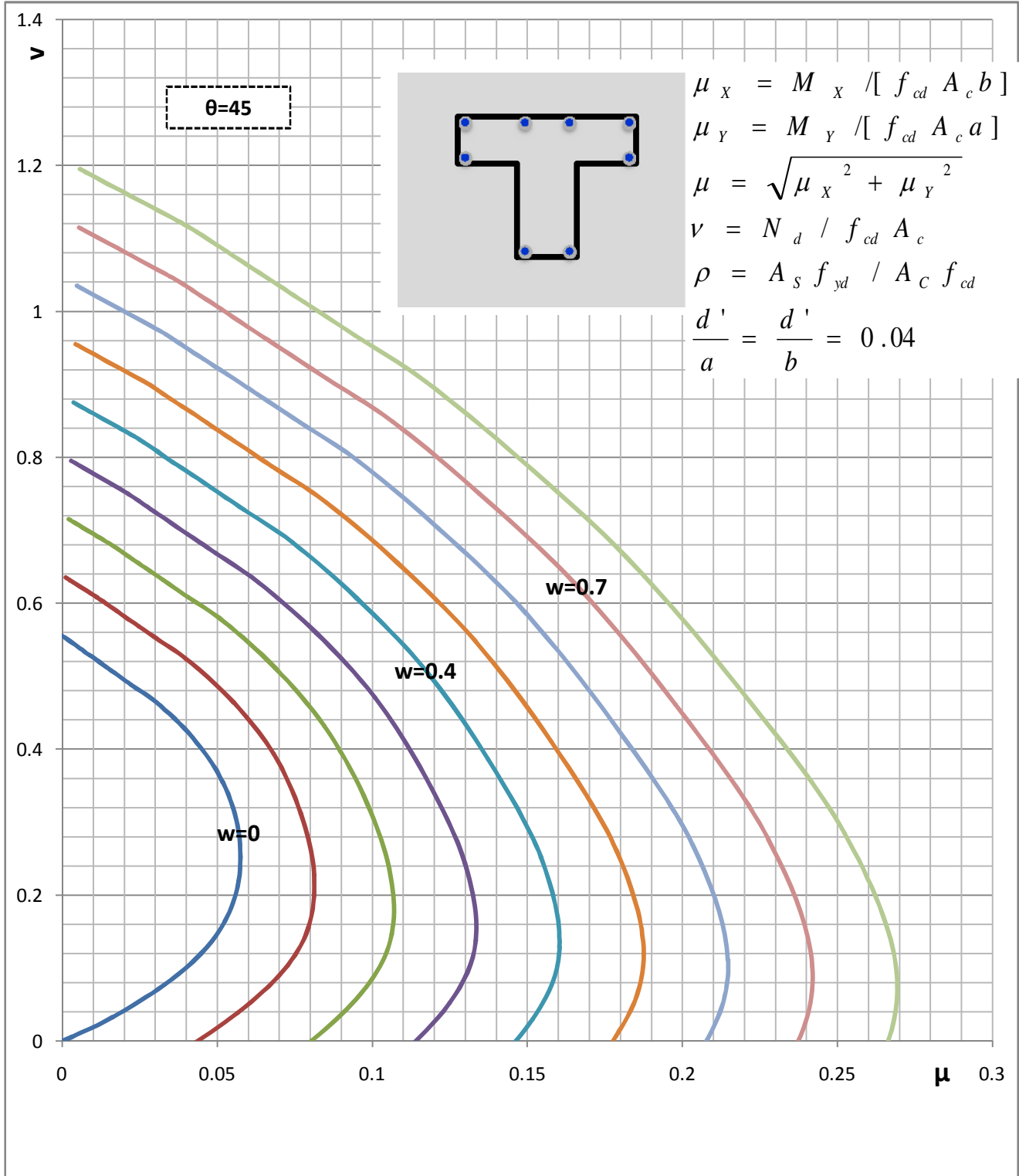


Chart for T-shaped reinforced concrete column with dimensions  $a=b=600\text{mm}$ ,  $l=tw=tf=200\text{mm}$   
 $M_{xd}=175\text{KNm}$ ,  $M_{yd}=175\text{KNm}$  C30,S460

### Interaction Chart for Example 4

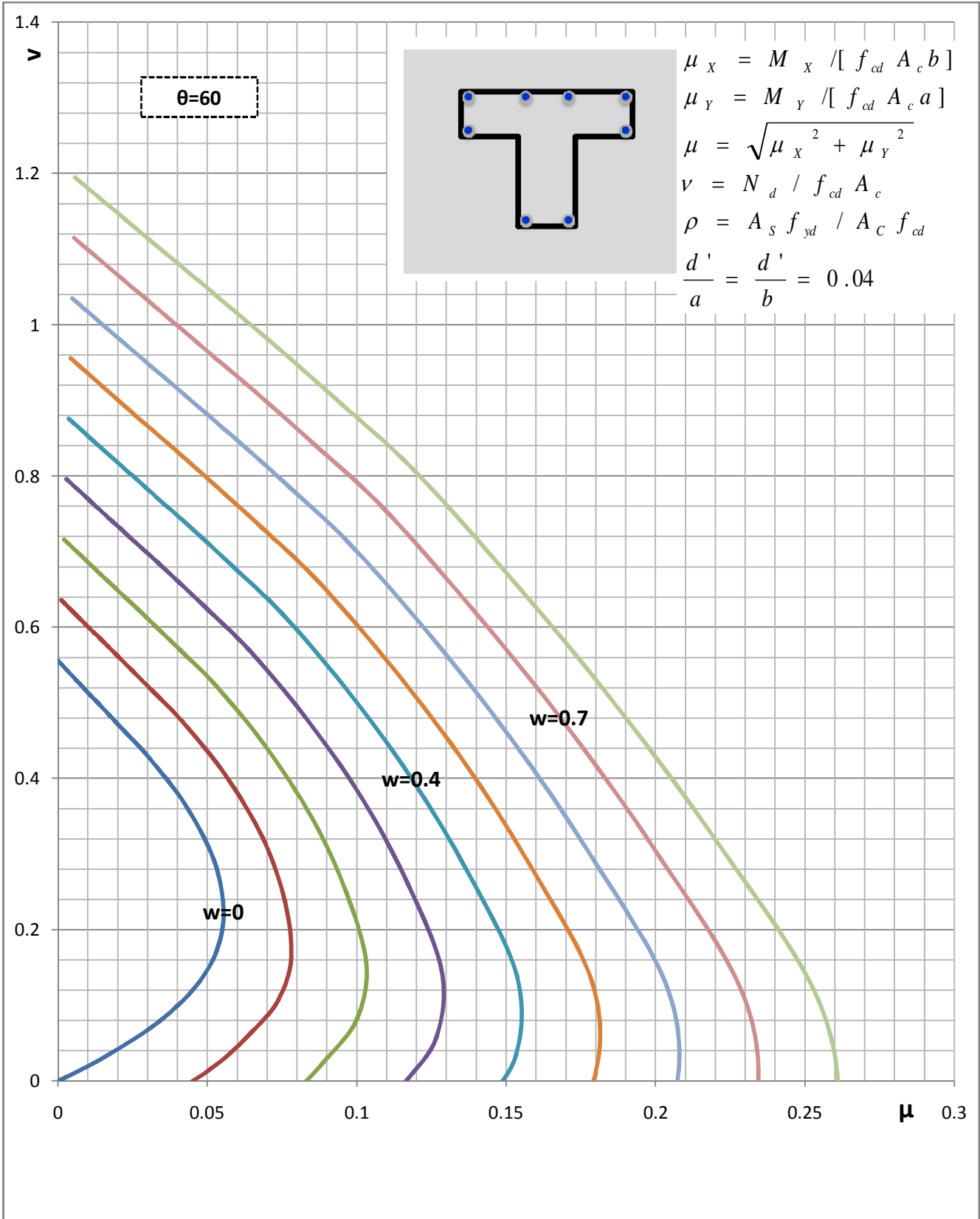


Chart for T-shaped reinforced concrete column with dimensions  $a=b=600\text{mm}$ ,  $l=tw=tf=200\text{mm}$   
 $M_{xd}=243\text{KNm}$ ,  $M_{yd}=420\text{KNm}$  C30,S460

### Interaction Chart for Example 5

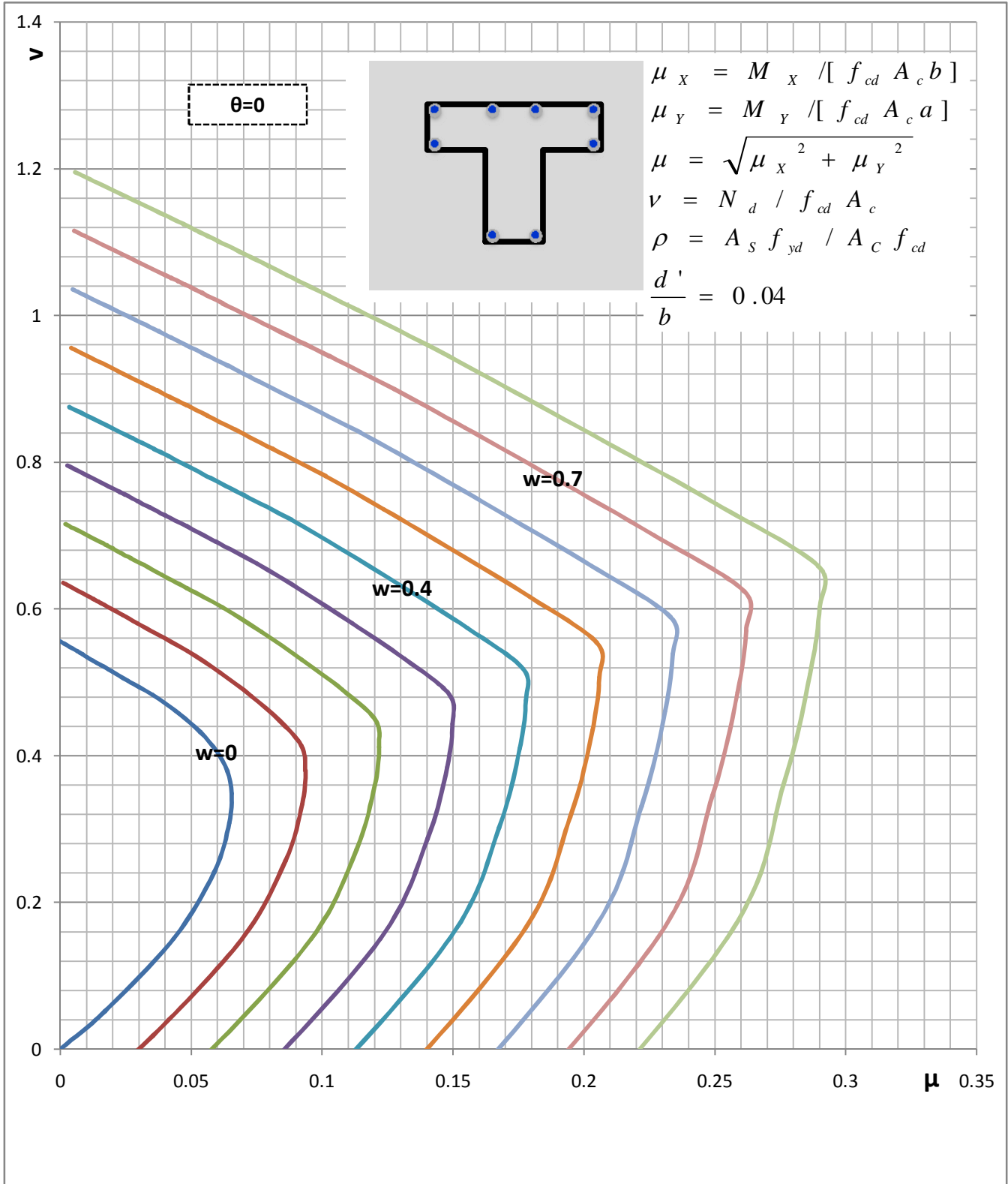


Chart for T-shaped reinforced concrete column with dimensions  $a=b=600\text{mm}$ ,  $l=tw=tf=200\text{mm}$   
 $M_{xd}=175\text{KNm}$ ,  $M_{yd}=0\text{KNm}$  C30,S460

### Interaction Chart for Example 6

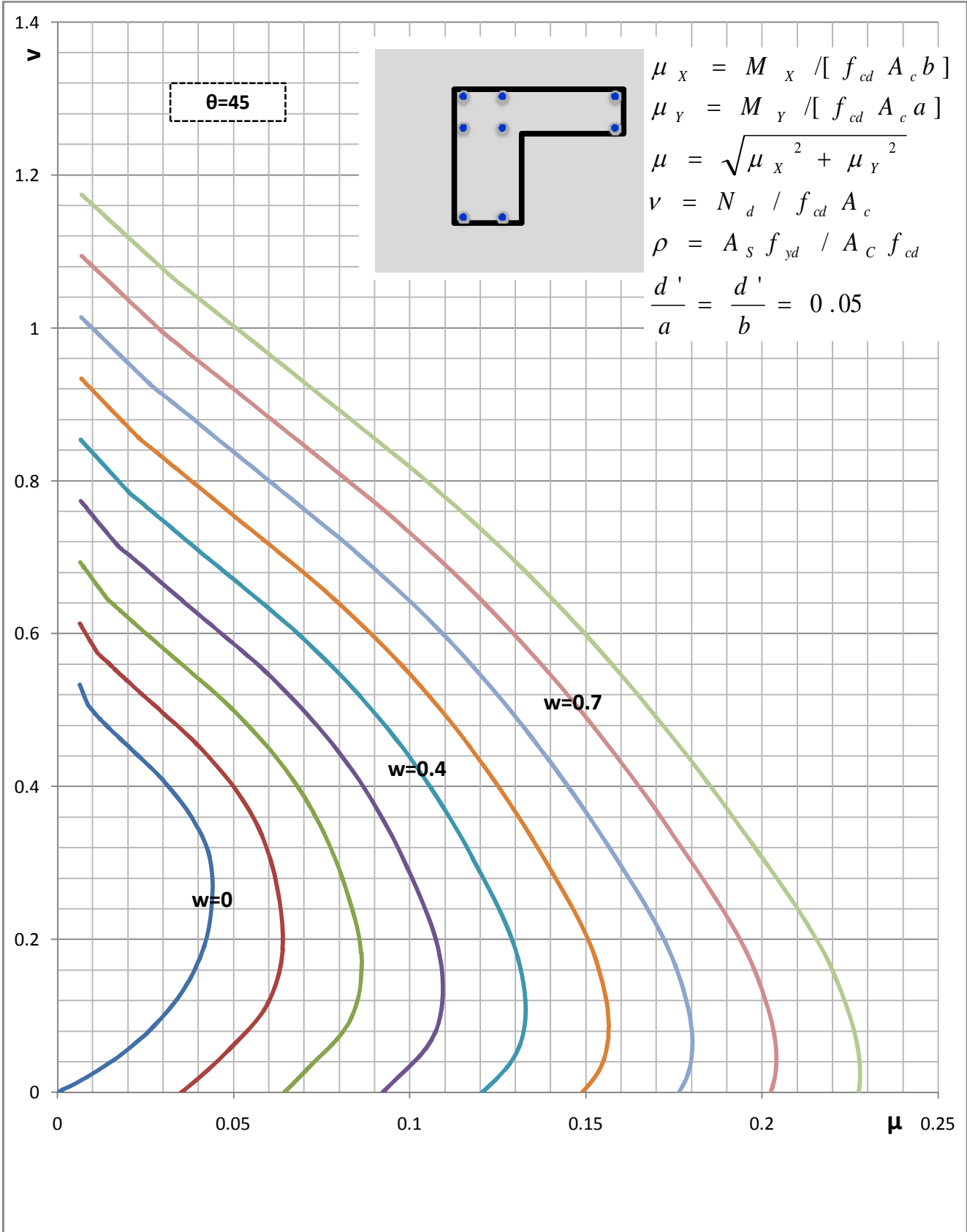
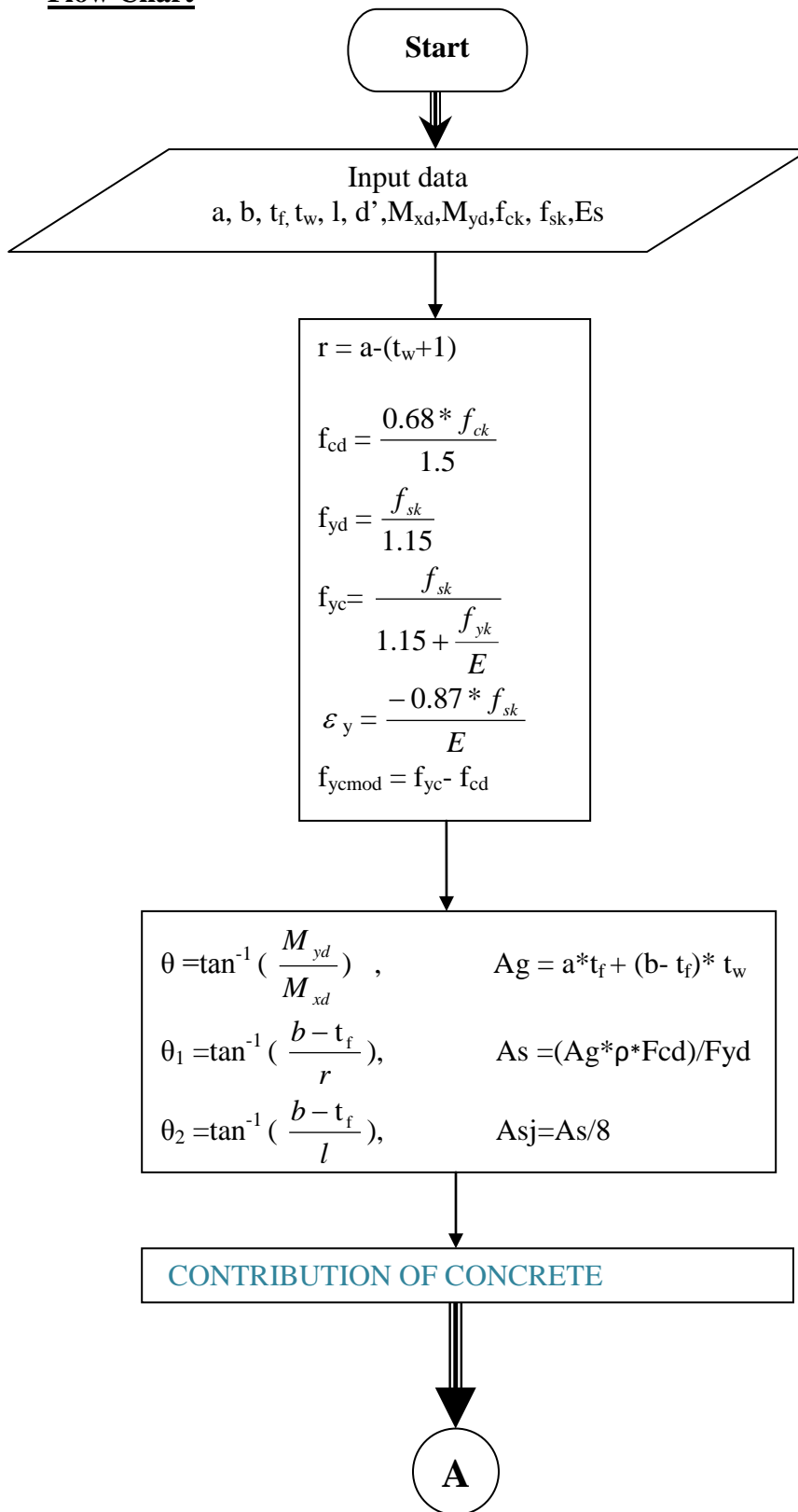
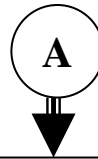


Chart for L-shaped reinforced concrete column with dimensions  $a=b=600\text{mm}$ ,  $t_w=t_f=200\text{mm}$   
 $M_{xd}=140\text{KNm}$ ,  $M_{yd}=140\text{KNm}$ , C25, S460

**Flow Chart**



-Position of plastic centroid

$$C_1 = \frac{[0.5 * f_{cd} * (a * t_f^2 + t_w (b^2 - t_f^2)) + f_{ycmod} * 4 * A_s * d + f_{ycmod} * 2 * A_s * (t_f - d) + f_{ycmod} * 2 * A_s * (b - d)]}{[f_{cd} (a * t_f + t_w (b - t_f)) + f_{ycmod} * 8 * A_s]}$$

$$C_2 = \frac{0.5 * f_{cd} * t_w (b - t_f)^2 + (f_{cd} * a * t_f * (b - 0.5 * t_f)) + f_{ycmod} * 2 * A_s * d + f_{ycmod} * 2 * A_s * (b - t_f + d) + f_{ycmod} * 4 * A_s * (b - d)]}{[f_{cd} (a * t_f + t_w (b - t_f)) + f_{ycmod} * 8 * A_s]}$$

$$C_3 = \frac{[0.5 * f_{cd} * t_f * a^2 + (f_{cd} * (l + 0.5 * t_w) * (b - t_f)) + f_{ycmod} * 2 * A_s * d + f_{ycmod} * 2 * A_s * (l + d) + f_{ycmod} * 2 * A_s * (l + t_w - d) + f_{ycmod} * 2 * A_s * (a - d)]}{[f_{cd} (a * t_f + t_w (b - t_f)) + f_{ycmod} * 8 * A_s]}$$

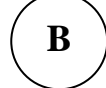
$$C_4 = \frac{[0.5 * f_{cd} * t_f * a^2 + (f_{cd} * (r + 0.5 * t_w) * (b - t_f)) + f_{ycmod} * 2 * A_s * d + f_{ycmod} * 2 * A_s * (r + d) + f_{ycmod} * 2 * A_s * (r + t_w - d) + f_{ycmod} * 2 * A_s * (a - d)]}{[f_{cd} (a * t_f + t_w (b - t_f)) + f_{ycmod} * 8 * A_s]}$$

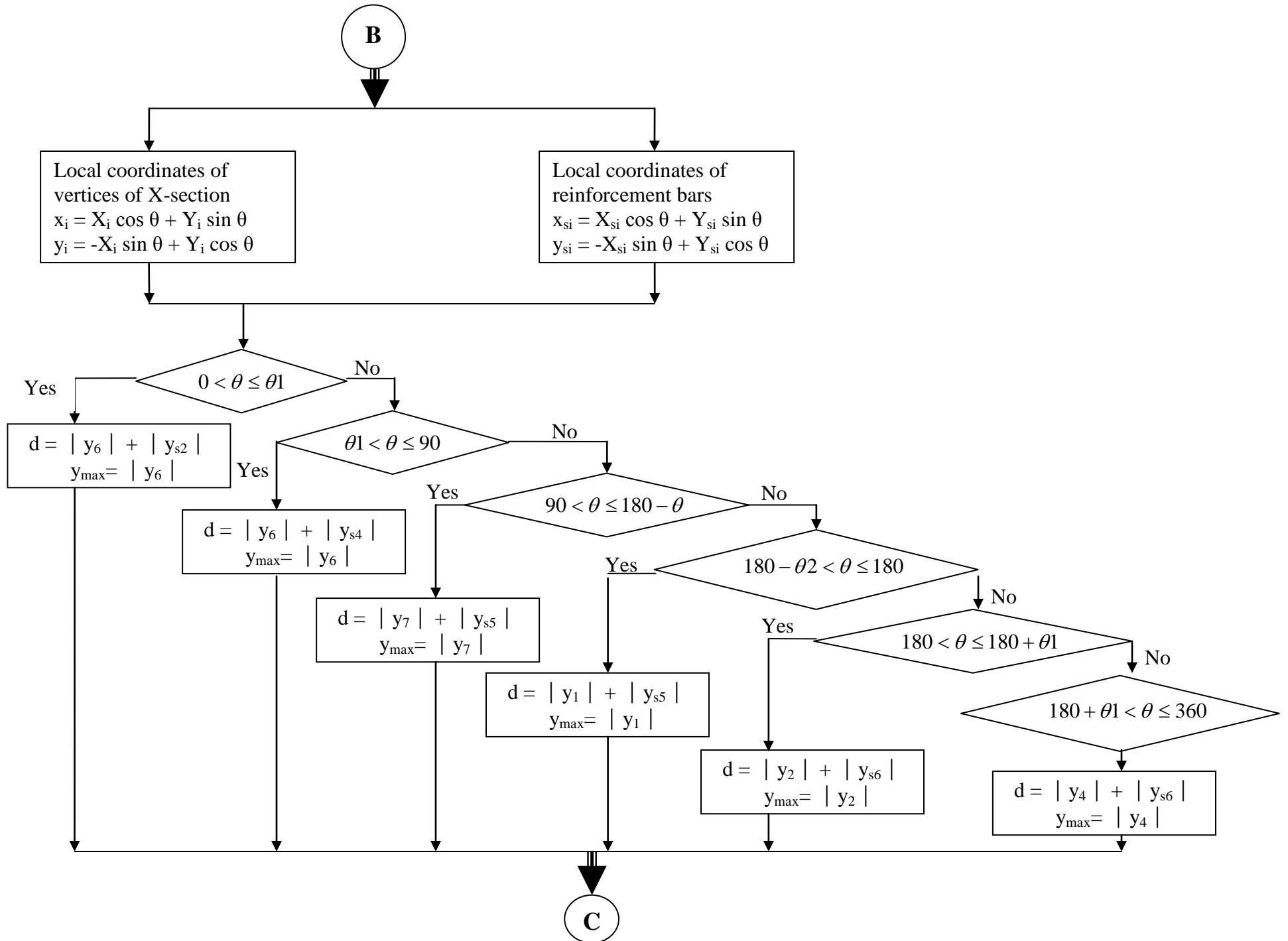
Global coordinates of vertices of X-section

|                     |   |                     |
|---------------------|---|---------------------|
| $X_1 = - (C_3 - l)$ | , | $Y_1 = -C_2$        |
| $X_2 = (C_4 - r)$   | , | $Y_2 = -C_2$        |
| $X_3 = (C_4 - r)$   | , | $Y_3 = (C_1 - t_f)$ |
| $X_4 = C_4$         | , | $Y_4 = (C_1 - t_f)$ |
| $X_5 = C_4$         | , | $Y_5 = C_1$         |
| $X_6 = -C_3$        | , | $Y_6 = C_1$         |
| $X_7 = -C_3$        | , | $Y_7 = (C_1 - t_f)$ |
| $X_8 = -(C_3 - l)$  | , | $Y_8 = (C_1 - t_f)$ |
| $X_9 = -(C_3 - l)$  | , | $Y_9 = -C_2$        |

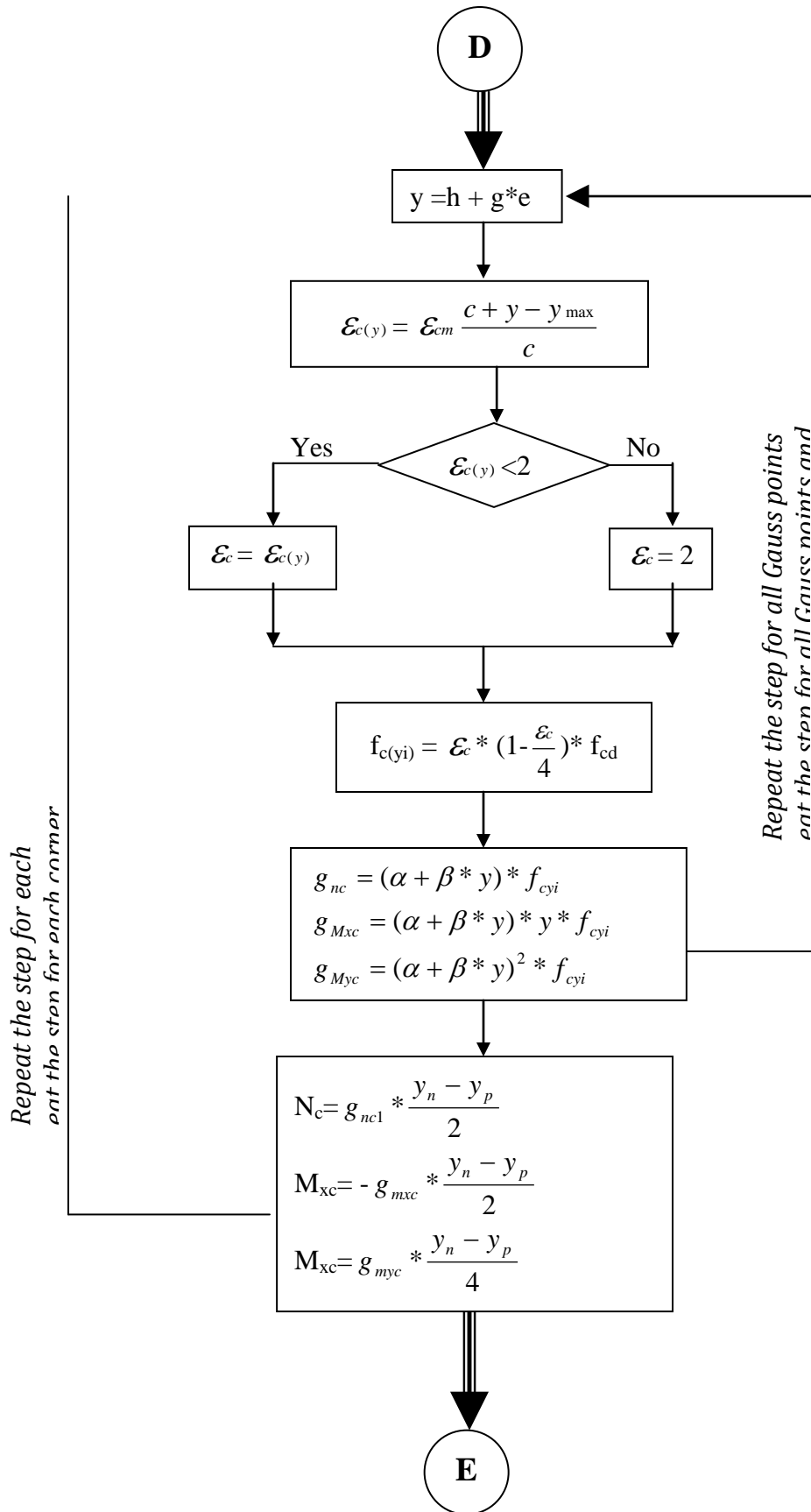
Global coordinates of reinforcement bars

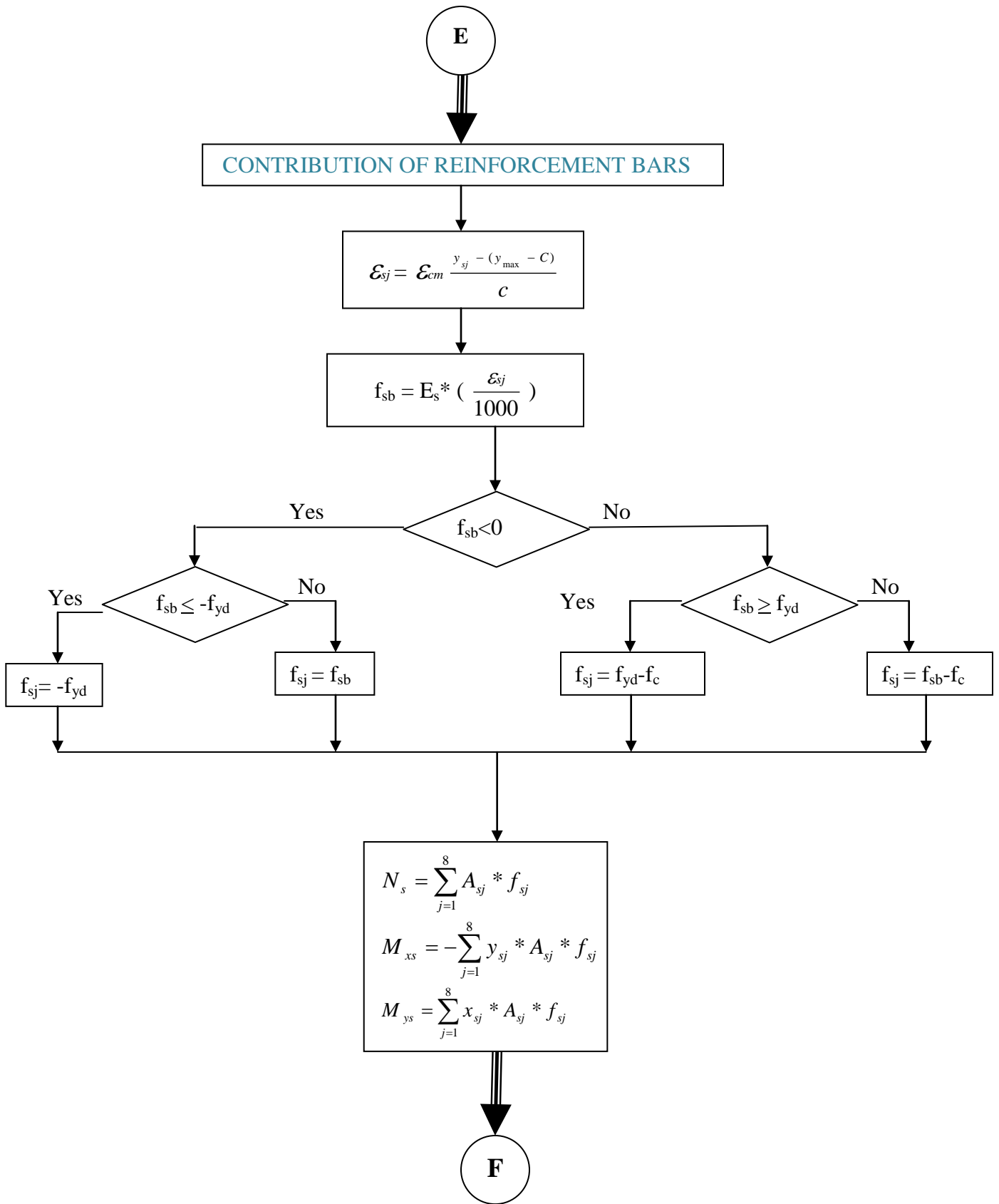
|                              |   |                              |
|------------------------------|---|------------------------------|
| $X_{s1} = - (C_3 - l - d^1)$ | , | $Y_{s1} = -(C_2 - d^1)$      |
| $X_{s2} = (C_4 - r - d^1)$   | , | $Y_{s2} = -(C_2 - d^1)$      |
| $X_{s3} = (C_4 - d^1)$       | , | $Y_{s3} = (C_1 - t_f + d^1)$ |
| $X_{s4} = (C_4 - d^1)$       | , | $Y_{s4} = (C_1 - d^1)$       |
| $X_{s5} = (C_4 - r - d^1)$   | , | $Y_{s5} = (C_1 - d^1)$       |
| $X_{s6} = -(C_3 - l - d^1)$  | , | $Y_{s6} = (C_1 - d^1)$       |
| $X_{s7} = -(C_3 - d^1)$      | , | $Y_{s7} = (C_1 - d^1)$       |
| $X_{s8} = -(C_3 - d^1)$      | , | $Y_{s8} = (C_1 - t_f + d^1)$ |

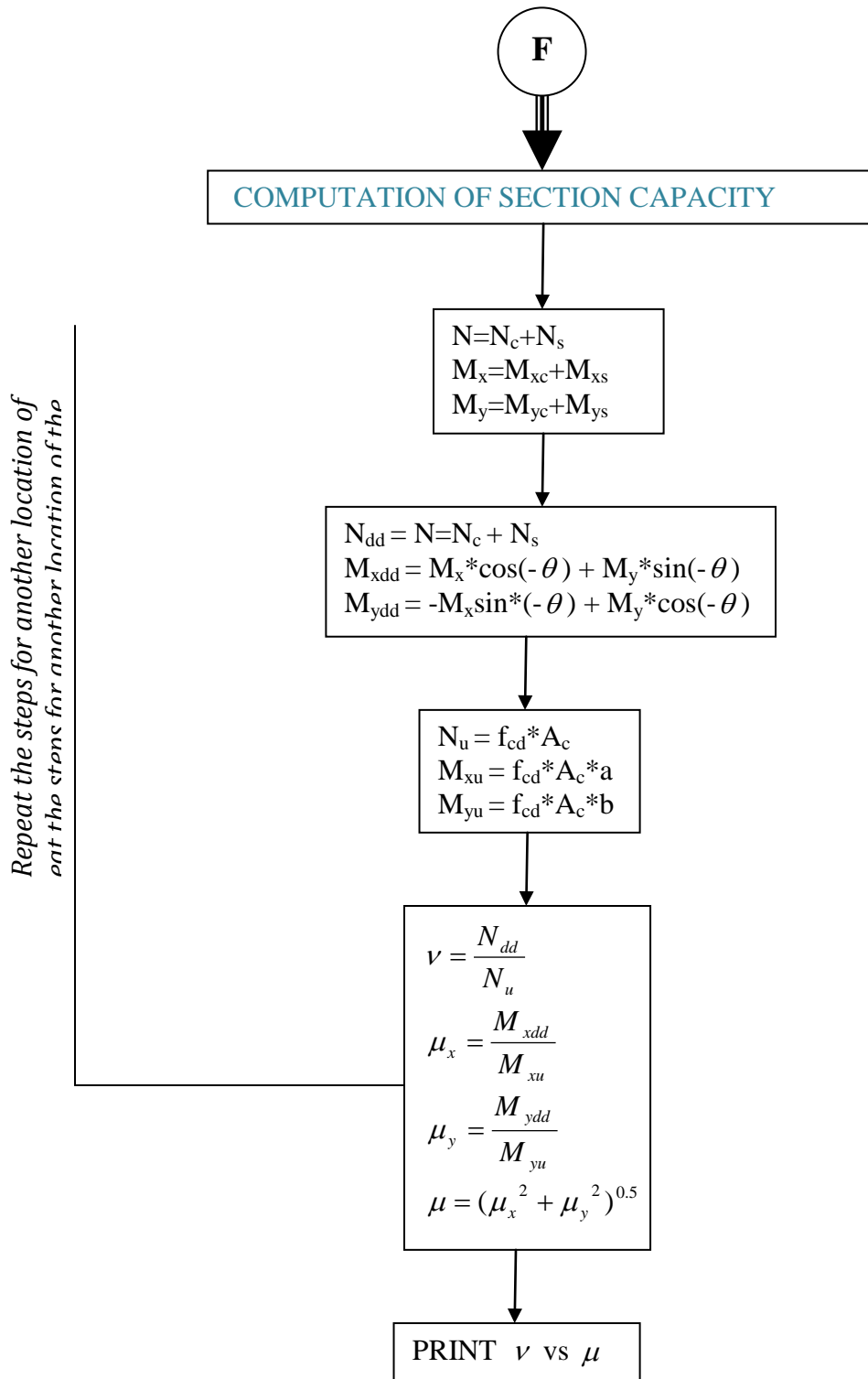














**Coding of the problem**

DefDbl I-N

DefDbl A-H, O-Z

Dim aa As Double, bb As Double, tff As Double, tww As Double, ll As Double, rr As Double, dd As Double

Dim fckk As Double, fskk As Double, teta As Double

Dim ComStrain, ComStrainyy, Y As Double, ModE As Double, Ymax As Double, Fcd As Double, Fyd As Double, fcc As Double

Dim roow As Double, Asj, Mxd As Double, Myd As Double

Dim teta11 As Double, teta22 As Double, dmax As Double, C As Double, ComStrainmax As Double

Dim CC1 As Double, CC2 As Double, CC3 As Double, CC4 As Double, Aas As Double, Ag As Double

Dim C1 As Double, C2 As Double, C3 As Double, C4 As Double

Dim Xloc(10), Yloc(10), GUC(3) As Double, GUW(3) As Double, XSS(9) As Double, YSS(9) As Double

Dim XXX1 As Double, XXX2 As Double, XXX3 As Double, XXX4 As Double, XXX5 As Double, XXX6 As Double, XXX7 As Double, XXX8 As Double, XXX9 As Double

Dim YYY1 As Double, YYY2 As Double, YYY3 As Double, YYY4 As Double, YYY5 As Double, YYY6 As Double, YYY7 As Double, YYY8 As Double, YYY9 As Double

Dim XXXSS1 As Double, XXXSS2 As Double, XXXSS3 As Double, XXXSS4 As Double, XXXSS5 As Double, XXXSS6 As Double, XXXSS7 As Double, XXXSS8 As Double, XXXSS9 As Double, XXXSS10 As Double

Dim YYYSS1 As Double, YYYSS2 As Double, YYYSS3 As Double, YYYSS4 As Double, YYYSS5 As Double, YYYSS6 As Double, YYYSS7 As Double, YYYSS8 As Double, YYYSS9 As Double, YYYSS10 As Double

Dim XSS1 As Double, XSS2 As Double, XSS3 As Double, XSS4 As Double, XSS5 As Double, XSS6 As Double, XSS7 As Double, XSS8 As Double, XSS9 As Double, XSS10 As Double

Dim YSS1 As Double, YSS2 As Double, YSS3 As Double, YSS4 As Double, YSS5 As Double, YSS6 As Double, YSS7 As Double, YSS8 As Double, YSS9 As Double, YSS10 As Double

Dim kx As Double

Dim Yn As Double, Yp As Double, Xn As Double, Xp As Double, Alpha As Double, Beta As Double, hh As Double, gg As Double

Dim gnc1 As Double, gmxc1 As Double, gmyc1 As Double, gnc2 As Double, gmxc2 As Double, gmyc2 As Double

Dim Ncf As Double, Mxcf As Double, Myc(9) As Double, Nc(9) As Double, Mxc(9) As Double, Mycc As Double

Dim Ey As Double, Fyc As Double, Eyc As Double, Fycmod As Double

Dim ew As Double, p, z, w, o, j

Dim TensStrainY(9) As Double, TensileStress(9) As Double, Fssj(9) As Double

Dim Ns As Double, Mxs As Double, Mys As Double

Dim N As Double, Mx As Double, My As Double, Ndd As Double, Mxdd As Double, Mydd As Double, Nu As Double, Mxu As Double, Myu As Double

Dim Axial As Double, MomentX As Double, MomentY As Double, Moment As Double, pi As Double

Private Sub Command1\_Click()

### **'MAKING THE CHART DRAWING PIC.BOX VISIBLE**

Picture2.Visible = True

Picture2.Cls

pi = CDBl(3.141592654)

### **'READING DATAS FROM TEXT BOX**

aa = CDBl(Val(txta))

bb = CDBl(Val(txtb))

tff = CDBl(Val(txttf))

tww = CDBl(Val(txttw))

ll = CDBl(Val(txll))

dd = CDBl(Val(txtd))

```
ModE = CDBl(Val(txtModules))
```

```
fckk = CDBl(Val(txtfck))
```

```
fskk = CDBl(Val(txtfsk))
```

```
Mxd = CDBl(Val(txtMomentMxd))
```

```
Myd = CDBl(Val(txtMomentMyd))
```

```
Dim xlTmp As Excel.Application
```

```
Set xlTmp = New Excel.Application
```

```
xlTmp.Workbooks.Open "c:\Book2.xlsx"
```

```
xlTmp.Visible = True
```

```
xlTmp.Sheets("Sheet1").Select
```

```
xlTmp.Range("A1:p100").Select
```

```
Selection.ClearContents
```

```
For roow = 0 To 0.81 Step 0.1
```

```
Ncf = CDBl(0): Nc(1) = CDBl(0): Nc(2) = CDBl(0): Nc(3) = CDBl(0): Nc(4) = CDBl(0): Nc(5) =  
CDBl(0): Nc(6) = CDBl(0): Nc(7) = CDBl(0): Nc(8) = CDBl(0)
```

```
Mxcf = CDBl(0): Mxc(1) = CDBl(0): Mxc(2) = CDBl(0): Mxc(3) = CDBl(0): Mxc(4) = CDBl(0):  
Mxc(5) = CDBl(0): Mxc(6) = CDBl(0): Mxc(7) = CDBl(0): Mxc(8) = CDBl(0)
```

```
Myc = CDBl(0): Myc(1) = CDBl(0): Myc(2) = CDBl(0): Myc(3) = CDBl(0): Myc(4) = CDBl(0):  
Myc(5) = CDBl(0): Myc(6) = CDBl(0): Myc(7) = CDBl(0): Myc(8) = CDBl(0)
```

**'MATERIAL DESIGN STRENGTH FROM GIVEN MATERIAL PROPERTY**

$$Fcd = (0.68 * fckk) / CDbl(1.5)$$

$$Fyd = (fskk) / CDbl(1.15)$$

$$Ey = CDbl(-0.87) * fskk / ModE$$

$$Fyc = fskk / (CDbl(1.15) + (fskk / CDbl(2000)))$$

$$Eyc = Fyc / ModE$$

$$Fycmod = Fyc - Fcd$$

**'ANGLE VALUES FROM INPUT DATA AND GEOMETRIC ORIENTATIONS**

$$rr = aa - (tww + ll)$$

$$teta = Atn((Myd / Mxd))$$

$$teta11 = Atn(((bb - tff) / rr))$$

$$teta22 = Atn(((bb - tff) / ll))$$

$$Ag = ((aa * tff) + ((bb - tff) * tww))$$

$$Aas = (Ag * roow * Fcd) / Fyd$$

$$Asj = Aas / 8$$

**'PLASTIC CENTROID for non symmetrical section**

$$C1 = (0.5 * Fcd * (aa * tff ^ 2 + tww * (bb ^ 2 - tff ^ 2)) + Fycmod * 4 * Asj * dd + Fycmod * 2 * Asj * (tff - dd) + Fycmod * 2 * Asj * (bb - dd)) / (Fcd * (aa * tff + tww * (bb - tff)) + Fycmod * 8 * Asj)$$

$$C2 = ((0.5 * Fcd * tww * (bb - tff) ^ 2) + (Fcd * aa * tff * (bb - 0.5 * tff)) + Fycmod * 2 * Asj * dd + Fycmod * 2 * Asj * (bb - tff + dd) + Fycmod * 4 * Asj * (bb - dd)) / (Fcd * (aa * tff + tww * (bb - tff)) + Fycmod * 8 * Asj)$$

$$C3 = ((0.5 * Fcd * tff * aa ^ 2) + (Fcd * tww * (ll + 0.5 * tww) * (bb - tff)) + Fycmod * 2 * Asj * dd + Fycmod * 2 * Asj * (ll + dd) + Fycmod * 2 * Asj * (ll + tww - dd) + Fycmod * 2 * Asj * (aa - dd)) / (Fcd * (aa * tff + tww * (bb - tff)) + Fycmod * 8 * Asj)$$

$$C4 = ((0.5 * Fcd * tff * aa ^ 2) + (Fcd * tww * (rr + 0.5 * tww) * (bb - tff)) + Fycmod * 2 * Asj * dd + Fycmod * 2 * Asj * (rr + dd) + Fycmod * 2 * Asj * (rr + tww - dd) + Fycmod * 2 * Asj * (aa - dd)) / (Fcd * (aa * tff + tww * (bb - tff)) + Fycmod * 8 * Asj)$$

**'GLOBAL COORDINATES OF VERTICES OF X-SECTION**

$$\text{XXX1} = -(C3 - ll); \quad \text{YYY1} = -C2$$

$$\text{XXX2} = C4 - rr; \quad \text{YYY2} = -C2$$

$$\text{XXX3} = C4 - rr; \quad \text{YYY3} = C1 - tff$$

$$\text{XXX4} = C4; \quad \text{YYY4} = C1 - tff$$

$$\text{XXX5} = C4; \quad \text{YYY5} = C1$$

$$\text{XXX6} = -C3; \quad \text{YYY6} = C1$$

$$\text{XXX7} = -C3; \quad \text{YYY7} = C1 - tff$$

$$\text{XXX8} = -(C3 - ll); \quad \text{YYY8} = C1 - tff$$

$$\text{XXX9} = -(C3 - ll); \quad \text{YYY9} = -C2$$

**'LOCAL COORIDINATES OF VESTICES OF X-SECTION**

$$\text{Xloc}(1) = \text{XXX1} * \text{Cos}(\text{teta}) + \text{YYY1} * \text{Sin}(\text{teta}); \text{Yloc}(1) = -\text{XXX1} * \text{Sin}(\text{teta}) + \text{YYY1} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(2) = \text{XXX2} * \text{Cos}(\text{teta}) + \text{YYY2} * \text{Sin}(\text{teta}); \text{Yloc}(2) = -\text{XXX2} * \text{Sin}(\text{teta}) + \text{YYY2} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(3) = \text{XXX3} * \text{Cos}(\text{teta}) + \text{YYY3} * \text{Sin}(\text{teta}); \text{Yloc}(3) = -\text{XXX3} * \text{Sin}(\text{teta}) + \text{YYY3} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(4) = \text{XXX4} * \text{Cos}(\text{teta}) + \text{YYY4} * \text{Sin}(\text{teta}); \text{Yloc}(4) = -\text{XXX4} * \text{Sin}(\text{teta}) + \text{YYY4} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(5) = \text{XXX5} * \text{Cos}(\text{teta}) + \text{YYY5} * \text{Sin}(\text{teta}); \text{Yloc}(5) = -\text{XXX5} * \text{Sin}(\text{teta}) + \text{YYY5} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(6) = \text{XXX6} * \text{Cos}(\text{teta}) + \text{YYY6} * \text{Sin}(\text{teta}); \text{Yloc}(6) = -\text{XXX6} * \text{Sin}(\text{teta}) + \text{YYY6} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(7) = \text{XXX7} * \text{Cos}(\text{teta}) + \text{YYY7} * \text{Sin}(\text{teta}); \text{Yloc}(7) = -\text{XXX7} * \text{Sin}(\text{teta}) + \text{YYY7} * \text{Cos}(\text{teta})$$

$$\text{Xloc}(8) = \text{XXX8} * \text{Cos}(\text{teta}) + \text{YYY8} * \text{Sin}(\text{teta}); \text{Yloc}(8) = -\text{XXX8} * \text{Sin}(\text{teta}) + \text{YYY8} * \text{Cos}(\text{teta})$$

$$X_{loc}(9) = XXX9 * \cos(\text{teta}) + YYY9 * \sin(\text{teta}); Y_{loc}(9) = -XXX9 * \sin(\text{teta}) + YYY9 * \cos(\text{teta})$$

### 'GLOBAL COORDINATES OF REINFORCEMENT BARS

$$XXXSS1 = -(C3 - ll - dd); \quad YYYSS1 = -(C2 - dd)$$

$$XXXSS2 = C4 - rr - dd; \quad YYYSS2 = -(C2 - dd)$$

$$XXXSS3 = C4 - dd; \quad YYYSS3 = C1 - tff + dd$$

$$XXXSS4 = C4 - dd; \quad YYYSS4 = C1 - dd$$

$$XXXSS5 = C4 - rr - dd; \quad YYYSS5 = C1 - dd$$

$$XXXSS6 = -(C3 - ll - dd); \quad YYYSS6 = (C1 - dd)$$

$$XXXSS7 = -(C3 - dd); \quad YYYSS7 = C1 - dd$$

$$XXXSS8 = -(C3 - dd); \quad YYYSS8 = C1 - tff + dd$$

### 'LOCAL COORDINATES OF REINFORCEMENT BARS

$$XSS(1) = XXXSS1 * \cos(\text{teta}) + YYYSS1 * \sin(\text{teta}); YSS(1) = -XXXSS1 * \sin(\text{teta}) + YYYSS1 * \cos(\text{teta})$$

$$XSS(2) = XXXSS2 * \cos(\text{teta}) + YYYSS2 * \sin(\text{teta}); YSS(2) = -XXXSS2 * \sin(\text{teta}) + YYYSS2 * \cos(\text{teta})$$

$$XSS(3) = XXXSS3 * \cos(\text{teta}) + YYYSS3 * \sin(\text{teta}); YSS(3) = -XXXSS3 * \sin(\text{teta}) + YYYSS3 * \cos(\text{teta})$$

$$XSS(4) = XXXSS4 * \cos(\text{teta}) + YYYSS4 * \sin(\text{teta}); YSS(4) = -XXXSS4 * \sin(\text{teta}) + YYYSS4 * \cos(\text{teta})$$

$$XSS(5) = XXXSS5 * \cos(\text{teta}) + YYYSS5 * \sin(\text{teta}); YSS(5) = -XXXSS5 * \sin(\text{teta}) + YYYSS5 * \cos(\text{teta})$$

$$XSS(6) = XXXSS6 * \cos(\text{teta}) + YYYSS6 * \sin(\text{teta}); YSS(6) = -XXXSS6 * \sin(\text{teta}) + YYYSS6 * \cos(\text{teta})$$

$$XSS(7) = XXXSS7 * \cos(\text{teta}) + YYYSS7 * \sin(\text{teta}); YSS(7) = -XXXSS7 * \sin(\text{teta}) + YYYSS7 * \cos(\text{teta})$$

$$XSS(8) = XXXSS8 * \cos(\text{teta}) + YYYSS8 * \sin(\text{teta}); YSS(8) = -XXXSS8 * \sin(\text{teta}) + YYYSS8 * \cos(\text{teta})$$

**'COMPUTATION OF d AND Ymax FOR DIFFERENT VALUES OF ANGLE**

If ( $0 < \text{teta} \leq \text{teta11}$ ) Then

$$d_{\max} = \text{Abs}(Y_{\text{loc}}(6)) + \text{Abs}(Y_{\text{SS}}(2))$$

$$Y_{\max} = \text{Abs}(Y_{\text{loc}}(6))$$

ElseIf ( $\text{teta11} < \text{teta} \leq (\pi / 2)$ ) Then

$$d_{\max} = \text{Abs}(Y_{\text{loc}}(6)) + \text{Abs}(Y_{\text{SS}}(4))$$

$$Y_{\max} = \text{Abs}(Y_{\text{loc}}(6))$$

ElseIf ( $\pi / \text{CDbl}(2) < \text{teta} \leq (\pi - \text{teta22})$ ) Then

$$d_{\max} = \text{Abs}(Y_{\text{loc}}(7)) + \text{Abs}(Y_{\text{SS}}(5))$$

$$Y_{\max} = \text{Abs}(Y_{\text{loc}}(7))$$

ElseIf ( $(\pi - \text{teta22}) < \text{teta} \leq \pi$ ) Then

$$d_{\max} = \text{Abs}(Y_{\text{loc}}(1)) + \text{Abs}(Y_{\text{SS}}(5))$$

$$Y_{\max} = \text{Abs}(Y_{\text{loc}}(1))$$

ElseIf ( $\pi < \text{teta} \leq (\pi + \text{teta11})$ ) Then

$$d_{\max} = \text{Abs}(Y_{\text{loc}}(2)) + \text{Abs}(Y_{\text{SS}}(6))$$

$$Y_{\max} = \text{Abs}(Y_{\text{loc}}(2))$$

ElseIf ( $(\pi + \text{teta11}) < \text{teta} \leq \text{CDbl}(2) * \pi$ ) Then

$$d_{\max} = \text{Abs}(Y_{\text{loc}}(4)) + \text{Abs}(Y_{\text{SS}}(6))$$

$$Y_{\max} = \text{Abs}(Y_{\text{loc}}(4))$$

Else: MsgBox "Incorrect angle", vbDefaultButton1, "Interaction"

End If

xlTmp.Cells(1, 2 \* (row \* 10) - 1).Value = "Moment"

xlTmp.Cells(1, 2 \* (row \* 10)).Value = " Axial"

**'POSITION OF NEUTRAL AXIS FROM THE MOST COMPRESSED CORNER**

For  $k_x = 0.01$  To 3 Step 0.1

$$C = k_x * d_{max}$$

If  $CDbl(0.01) \leq k_x$  And  $k_x < CDbl(35 / 135)$  Then

$$ComStrainmax = (CDbl(0.01 * C) / (d_{max} - C))$$

ElseIf  $CDbl(35 / 135) < k_x$  And  $k_x \leq CDbl(1)$  Then

$$ComStrainmax = CDbl(0.0035)$$

ElseIf  $CDbl(1) < k_x$  And  $k_x \leq CDbl(10)$  Then

$$ComStrainmax = CDbl(0.002 / (1 - (CDbl(3 * d_{max}) / CDbl(7 * C))))$$

Else:  $ComStrainmax = 0.002$

End

**'INTEGRATION STEPS**

For  $i = 1$  To 8

If  $(Y_{loc}(i) < (Y_{max} - C)$  And  $(Y_{max} - C) < Y_{loc}(i + 1))$  Then

$$Y_n = Y_{loc}(i + 1)$$

$$Y_p = Y_{max} - C$$

$$X_n = X_{loc}(i + 1)$$

$$X_p = X_{loc}(i + 1) - (((X_{loc}(i + 1) - X_{loc}(i)) * (Y_{loc}(i + 1) - (Y_{max} - C))) / (Y_{loc}(i + 1) - Y_{loc}(i)))$$

ElseIf  $(Y_{loc}(i + 1) > (Y_{max} - C)$  And  $Y_{loc}(i) > (Y_{max} - C))$  Then

$$Y_n = Y_{loc}(i + 1)$$

$$Y_p = Y_{loc}(i)$$

$$X_n = X_{loc}(i + 1)$$

$$X_p = X_{loc}(i)$$

ElseIf  $(Y_{loc}(i + 1) < (Y_{max} - C)$  And  $(Y_{max} - C) < Y_{loc}(i))$  Then

$$Y_n = Y_{\max} - C$$

$$Y_p = Y_{\text{loc}}(i)$$

$$X_n = X_{\text{loc}}(i) + ((X_{\text{loc}}(i + 1) - X_{\text{loc}}(i)) * ((Y_{\max} - C) - Y_{\text{loc}}(i))) / (Y_{\text{loc}}(i + 1) - Y_{\text{loc}}(i))$$

$$X_p = X_{\text{loc}}(i)$$

ElseIf ((Y<sub>max</sub> - C) > Y<sub>loc</sub>(i) And (Y<sub>max</sub> - C) > Y<sub>loc</sub>(i + 1)) Then GoTo 122

End

### **'INTEGRATION VARIABLES**

If Y<sub>n</sub> = Y<sub>p</sub> Then

$$\text{Beta} = \text{Cdbl}(0)$$

$$\text{Alpha} = (X_p - \text{Beta} * Y_p)$$

$$\text{hh} = (Y_n + Y_p) / \text{Cdbl}(2)$$

$$\text{gg} = (Y_n - Y_p) / \text{Cdbl}(2)$$

Else:

$$\text{Beta} = (X_n - X_p) / (Y_n - Y_p)$$

$$\text{Alpha} = (X_p - \text{Beta} * Y_p)$$

$$\text{hh} = (Y_n + Y_p) / \text{Cdbl}(2)$$

$$\text{gg} = (Y_n - Y_p) / \text{Cdbl}(2)$$

End If

### **'VALUES OF GAUSSIAN POINTS AND WEIGHTS**

$$\text{GUC}(1) = \text{Cdbl}(-0.774596669)$$

$$\text{GUC}(2) = \text{Cdbl}(0)$$

$$\text{GUC}(3) = \text{Cdbl}(0.7745966692)$$

$$\text{GUW}(1) = \text{Cdbl}(0.5555556)$$

$$\text{GUW}(2) = \text{Cdbl}(0.8888889)$$

$$\text{GUW}(3) = \text{Cdbl}(0.5555556)$$

**'DISTANCE FROM THE NEUTRAL AXIS**

For j = 1 To 3 Step 1

$$Y = hh + gg * GUC(j)$$

**'COMPRESIVE STRAIN AT Y DISTANCE**

$$\text{ComStrainyy} = \text{ComStrainmax} * ((C + Y - Y_{\text{max}}) / C)$$

If ComStrainyy < CDbl(0.002) Then

$$\text{fcc} = \text{ComStrainyy} * \text{CDbl}(1000) * (\text{CDbl}(1) - (\text{CDbl}(250) * \text{ComStrainyy})) * \text{Fcd}$$

Else: fcc = Fcd

End If

$$\text{gnc1} = \text{gnc1} + (\text{GUW}(j) * (\text{Alpha} + (\text{Beta} * Y)) * \text{fcc})$$

$$\text{gmxc1} = \text{gmxc1} + (\text{GUW}(j) * (\text{Alpha} + (\text{Beta} * Y)) * Y * \text{fcc})$$

$$\text{gmyc1} = \text{gmyc1} + (\text{GUW}(j) * \text{fcc} * (\text{Alpha} + (\text{Beta} * Y)) ^ \text{CDbl}(2))$$

Next j

$$\text{Nc}(i) = ((Y_n - Y_p) / \text{CDbl}(2)) * \text{gnc1}$$

$$\text{Mxc}(i) = ((Y_n - Y_p) / \text{CDbl}(2)) * \text{gmxc1}$$

$$\text{Myc}(i) = ((Y_n - Y_p) / 4) * \text{gmyc1}$$

122:

Next i

$$\text{Ncf} = \text{Nc}(1) + \text{Nc}(2) + \text{Nc}(3) + \text{Nc}(4) + \text{Nc}(5) + \text{Nc}(6) + \text{Nc}(7) + \text{Nc}(8)$$

$$\text{Mxcf} = \text{Mxc}(1) + \text{Mxc}(2) + \text{Mxc}(3) + \text{Mxc}(4) + \text{Mxc}(5) + \text{Mxc}(6) + \text{Mxc}(7) + \text{Mxc}(8)$$

$$\text{Mycf} = \text{CDbl}(0.5) * (\text{Myc}(1) + \text{Myc}(2) + \text{Myc}(3) + \text{Myc}(4) + \text{Myc}(5) + \text{Myc}(6) + \text{Myc}(7) +$$

$$\text{Myc}(8))$$

**'STEEL CONTRIBUTION**

For l = 1 To 8 Step 1

TensStrainY(l) = 0

Fssj(l) = 0

TensStrainY(l) = ComStrainmax \* (YSS(l) - (Ymax - C)) / C

If TensStrainY(l) <= 0 Then

    If TensStrainY(l) <= Ey Then

        Fssj(l) = (-Fyd)

        Else

        Fssj(l) = ModE \* TensStrainY(l)

    End If

Else

    If TensStrainY(l) >= Eyc Then

        Fssj(l) = Fycmod

        Else

        Fssj(l) = ModE \* TensStrainY(l) - Fcd

    End If

End If

**'STRESS DUE TO REINFORCEMENT BARS**

Ns = Ns + Asj \* Fssj(l)

Mxs = (Mxs + ((YSS(l) \* Asj \* Fssj(l))))

Mys = (Mys + ((XSS(l) \* Asj \* Fssj(l))))

**'COMBINED STRESS IN LOCAL AXIS**

N = Ncf + Ns

Mx = Mxcf + Mxs

$$M_y = M_{ycf} + M_{ys}$$

### 'COMBINED STRESS WITH RESPECT TO THE GLOBAL AXIS

$$N_{dd} = N$$

$$M_{xdd} = M_x * \cos(-\text{teta}) + M_y * \sin(-\text{teta})$$

$$M_{ydd} = -M_x * \sin(-\text{teta}) + M_y * \cos(-\text{teta})$$

### 'NORMALIZING STRESS

$$N_u = F_{cd} * A_g$$

$$M_{xu} = F_{cd} * A_g * b_b$$

$$M_{yu} = F_{cd} * A_g * a_a$$

### 'NORMALIZED STRESS

$$\text{Axial} = N_{dd} / N_u$$

$$\text{MomentX} = M_{xdd} / M_{xu}$$

$$\text{MomentY} = M_{ydd} / M_{yu}$$

$$\text{Moment} = (\text{MomentX}^2 + \text{MomentY}^2)^{0.5}$$

### 'PLOTING THE INTERACTION DIAGRAM

Picture2.Scale (-1, 4.5)-(1.5, -1)

Picture2.Line (0, 0)-(1, 0), vbBlue

Picture2.Line (0, 0)-(0, 3.5), vbBlue

Picture2.PSet (Moment, Axial)

xlTmp.Cells((kx \* 10 + 1.9), 1 + 2 \* ((roow+0.1) \* 10) - 2).Value = Moment

xlTmp.Cells((kx \* 10 + 1.9), 2 + 2 \* ((roow+0.1) \* 10) - 2).Value = Axial

Next kx

Next roow

End Sub

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