SCIENCE LIBRARY

CONTRIBUTION OF GRAVITATIONAL AND MAGNETIC DIPOLE RADIATIONS TO THE ELLIPTICITY OF PULSARS

By

Netsanet Kassa

A THESIS
SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES OF
ADDIS ABABA UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

SCHOOL OF GRADUATE STUDIES
ADDIS ABABA UNIVERSITY
ADDIS ABABA , ETHIOPIA
JUNE 2005

ADDIS ABABA UNIVERSITY

Date: **June 2005**

Author: **Netsanet Kassa**

Title: **Contribution of Gravitational and Magnetic Dipole Radiations to the Ellipticity of Pulsars**

Department: **Physics**

Degree: **M.Sc.** Convocation: **June** Year: **2005**

Permission is herewith granted to Addis Ababa University to circulate and to have copied for non-commercial purposes, at its discretion, the above title upon the request of individuals or institutions.

Signature of Author

THE AUTHOR RESERVES OTHER PUBLICATION RIGHTS, AND NEITHER THE THESIS NOR EXTENSIVE EXTRACTS FROM IT MAY BE PRINTED OR OTHERWISE REPRODUCED WITHOUT THE AUTHOR'S WRITTEN PERMISSION.

THE AUTHOR ATTESTS THAT PERMISSION HAS BEEN OBTAINED FOR THE USE OF ANY COPYRIGHTED MATERIAL APPEARING IN THIS THESIS (OTHER THAN BRIEF EXCERPTS REQUIRING ONLY PROPER ACKNOWLEDGEMENT IN SCHOLARLY WRITING) AND THAT ALL SUCH USE IS CLEARLY ACKNOWLEDGED.

To

CUD

Table of Contents

Table of Contents

Abstract

Acknowledgements

1 Basic formulas of power radiated by gravitational and dipole radiation 4
- 1.1 Introduction 4
- 1.2 Energy loss rate from gravitational radiation 4
- 1.3 Energy loss rate from magnetic dipole radiation 8

2 Estimation of Maximum Rotational Frequency of Pulsars 12
- 2.1 Introduction 12
- 2.2 Newtonian Approximation 12
- 2.3 Estimating Ω_{max} 14

3 Time Evolutions Of Ω Due To Gravitational And Magnetic Dipole Radiations 19
- 3.1 Introduction 19
- 3.2 Time evolution of Ω due to gravitational radiation 19
- 3.3 Time evolution of Ω due to magnetic dipole radiation 21

4 Ellipticity of Pulsars 24
- 4.1 Introduction 24
- 4.2 Definition of ellipticity ϵ 25
- 4.3 Time evolution of the parameter ellipticity ϵ 26
- 4.4 Results 28
- 4.5 Discussion of Results 32

Abstract

Accurate and reliable calculation of the ellipticity parameter is very important in order to theoretically study the gravitational waves emitted by pulsars which is a hot research area in the field astrophysics. Ellipticity is a result of lack in symmetry of spherical bodies. In the case of pulsars rotation and magnetic fields are the causes of deformation. According to relativistic plasma diffusion model for neutron star magnetic fields, the surface magnetic field of neutron stars depends on their rotation frequency. Hence it can be deduced that the parameter ellipticity depends on the rotational frequency.

But gravitational and magnetic dipole radiations affect the rotational frequency of pulsars which in turn will also affect the parameter ellipticity. In this work we will estimate the contributions of gravitational and dipole radiations to the parameter ellipticity.

SCIENCE LIBRARY

Acknowledgements

I would like to thank Legesse Wetro, my advisor, for his continues guidance and valuable comments in each and every part of my work, without him this work would never have come into completeness. I would like to acknowledge the financial support of the Region-14 Education Bearu and the physics department of AAU for providing the necessary materials to carry out my thesis work. I am indebted to my friends around me for I have benefited a lot from them. I would like also to thank W/r. Tsilat, secretary of physics department, for being cooperative and kindness she showed me during my stay in the department.

Finally I like to express my deepest gratitude to my families for they are the ***beauty*** and a ***reason*** of my life.

Introduction

Our universe is populated by massive objects like galaxies, stars, and so on. Clouds of interstellar gas consisting mostly of hydrogen and a little dust are the incubators of stars. Important factors in stellar formation are gravity, dust, gas pressure, rotation, magnetic fields, winds and radiation from near by young stars, and radiative shock waves. The dust in the molecular clouds originates on the cool stellar surfaces of redgiants (massive stars in a late stage of stellar evolution). Dust shields cloud interiors from UV star light, enabling their centers to cool. With lower thermal pressure, gravitational collapse of the denser regions becomes inevitable. A clump of gas then begins to fall towards its mass center under the attraction of gravity. Gravitational energy is converted to heat by compression.

The opacity of the gas increases as its density and establishes a temperature and thermal pressure gradient which approximately balances gravity until the core temperature raises to the ignition point for fusing hydrogen into helium. There after fusion becomes the important energy source and the thermal and radiation pressure will now nearly balance gravity for millions to billions of years. The protostar has now joined the main sequence of stars.(the evolutionary path way of stars first proposed by H. N. Russel). The star will spend most of its luminous life in this state of suspended collapse as it burns its large store of hydrogen, slowly radiating energy from its surface.Thermonuclear fusion provides the energy for the great furnace, in the phase of Eddington [1] which evolves stars through the various stages of combustion helium, carbon, neon, oxygen, magnesium, and silicon. Fusion will end when iron the most bound nuclear species is reached. Beyond iron, fusion is no more exothermic. The cessation of nuclear fusion signals the end of luminous stage of of the star. The

constant and estimate wrong values. According to the new model [3], however pulsar magnetic fields decay due to mainly the braking mechanisms involving quadrapole gravitational and magnetic dipole radiations but also due to neutrino and photon emissions. Based on this model we will estimate the time gravitational radiation will remain the dominant cause for pulsars ellipticity, die down and at the same time calculate the time evolution of the same parameter.

This thesis consists four chapters. In the first chapter we will derive the basic formulas for the rate at which energy is carried away from pulsars due to gravitational and magnetic dipole radiations. In the second chapter a mechanism, based on the physical conservation laws, will be developed out of which the initial angular frequency of pulsars can be estimated.

In chapter 3, with the help of the basic formulas derived in chapter 1, we will find expressions for the time evolutions of angular velocity for gravitational and dipole radiations. Finally in chapter 4, based on the expressions for the time evolutions of angular velocity found in chapter 3, we will derive the time evolution of the ellipticity due to gravitational and dipole radiations.

Let us next express D_{ij} as a sum of Fourier components

$$D_{ij}(t) = \sum_{\omega} e^{(-i\omega t)} D_{ij}(\omega) \tag{1.2.18}$$

Neutron stars emit gravitational waves mainly at frequencies twice the rotational one. For this particular frequency $\omega = 2\Omega$, the non vanishing Fourier coefficients can be calculated using

$$D_{ij}(t) = D_{ij}(2\Omega)e^{(-2i\Omega t)} \tag{1.2.19}$$

Hence from the above equation we get

$$D_{11}(t) = D_{11}(2\Omega)e^{(-2i\Omega t)} \tag{1.2.20}$$

or

$$D_{11}(2\Omega) = D_{11}(t)e^{(2i\Omega t)} \tag{1.2.21}$$

Substituting this into equation 1.2.14 we have

$$D_{11}(2\Omega) = \frac{1}{2}(I_{11} + I_{22})e^{(2i\omega t)} + \frac{1}{4}(I_{11} - I_{22})e^{(4i\Omega t)} + \frac{1}{4}(I_{11} - I_{22}) \tag{1.2.22}$$

When averaged over time the first two terms of the above two equation vanish.Therefore $D_{11}(2\Omega)$ becomes

$$D_{11}(2\Omega) = \frac{1}{4}(I_{11} - I_{22}) \tag{1.2.23}$$

In a similar procedure one conld arrive at the following results for D_{22} and D_{12}.

$$D_{22}(2\Omega) = \frac{1}{4}(I_{22} - I_{11}) \tag{1.2.24}$$

$$D_{12}(2\Omega) = \frac{i}{4}(I_{11} - I_{22}) \tag{1.2.25}$$

The total power radiated by gravitational radiation is given by[11]

$$P = \frac{2G\omega^6}{5c^5}\left\{ D_{ij}(\omega)^* D_{ij}(\omega) - \frac{1}{3} \mid D_{ii}(\omega) \mid^2 \right\} \tag{1.2.26}$$

Chapter 3

Time Evolutions Of Ω Due To Gravitational And Magnetic Dipole Radiations

3.1 Introduction

We know that pulsars lose energy due to radiations, as a result of which the rotational frequency of pulsars change with time. The rate at which the angular frequency changes can be formulated from the energy loss rate equations. In this chapter we will attempt to write the time evolutions of rotational frequency due to gravitational and magnetic dipole radiations. In both cases we will use the expression for moment of inertia and its time rate of change which will be given in the next chapter.

3.2 Time evolution of Ω due to gravitational radiation

As shown in the chapter 1, the time rate of change of rotational energy loss due gravitational radiation is given by

$$\dot{E}_{gr} = -\frac{32G}{5c^5} I^2 \epsilon^2 \Omega^6 \tag{3.2.1}$$

But

$$\dot{E}_{gr} = \frac{d}{dt}(\frac{1}{2}I\Omega^2) = I\Omega\dot{\Omega} + \frac{1}{2}\dot{I}\Omega^2 \tag{3.2.2}$$

In the next chapter it will be shown that the moment of inertia I of a pulsar can be expressed as (equation 4.3.8)

$$I = I_0(1 + K\Omega^2)$$

where K is a constant($\simeq 3.8 \times 10^{-9}$).

From which we can get its time derivative to be

$$\dot{I} = 2I_0K\Omega\dot{\Omega}$$

We can define the parameter ellipticity ϵ as

$$\epsilon = \frac{I - I_0}{I} \tag{3.2.3}$$

where I_0 is the initial moment of inertia. When the above three relations are substituted into equations 3.2.1 and 3.2.2 we find

$$(1 + 2\Omega^2K)\Omega\dot{\Omega} = -\frac{32G}{5c^5}I_0K^2\Omega^{10} \tag{3.2.4}$$

which can be rewritten as

$$\int_{\Omega_0}^{\Omega} \frac{(1 + 2\Omega^2K)}{\Omega^9}d\Omega = -\frac{32G}{5c^5}I_0K^2\int_0^t dt \tag{3.2.5}$$

Upon integration the above equation yields

$$\frac{1}{8\Omega^8} + \frac{K}{3\Omega^6} = \frac{1}{8\Omega_0^8} + \frac{K}{3\Omega_0^6} + \frac{32G}{5c^5}I_0K^2t \tag{3.2.6}$$

where Ω_0 is the initial rotational frequency. If we define the constants

$$\frac{1}{8\Omega_0^8} + \frac{K}{3\Omega_0^6} = c_1$$

and

$$\frac{32G}{5c^5}I_0K^2 = c_2$$

Substituting the values of I and $\dot{I}$ given in the preceding section into equation 3.3.3 we have

$$I_0(1+K\Omega^2)\Omega\dot{\Omega} = -\frac{2Q^2R^4\Omega^6}{27I_0c^5} \tag{3.3.4}$$

which can be rewritten as

$$\int_{\Omega_0}^{\Omega}\frac{d\Omega}{\Omega^5} + 2K\int_{\Omega_0}^{\Omega}\frac{d\Omega}{\Omega^3} = -\frac{Q^2R^4}{27I_0c^5}\int_0^t dt \tag{3.3.5}$$

which upon integration yields

$$\frac{1}{4\Omega^4} + \frac{K}{2\Omega^2} = \frac{1}{4\Omega_0^4} + \frac{K}{2\Omega_0^2} + \frac{Q^2R^4}{27I_0c^5}t \tag{3.3.6}$$

If we define the constants

$$\frac{Q^2R^4}{27I_0c^5} = b_2$$

and

$$\frac{1}{4\Omega_0^4} + \frac{K}{2\Omega_0^2} = b_1$$

we can rewrite the above equation as

$$4\Omega^4(b_1+b_2t) - 4K\Omega^2 - 1 = 0 \tag{3.3.7}$$

The physically meaningful root of the above equation is

$$\Omega(t) = \left(\frac{K+(K^2+b_1+b_2t)^{\frac{1}{2}}}{2(b_1+b_2t)}\right)^{\frac{1}{2}} \tag{3.3.8}$$

which is the required equation for the time evolution of angular frequency due to magnetic dipole radiation. If we assume the initial angular frequency to be in the order of $\sim 10^3 s^{-1}$ the above equation can be simplified with the help of the following approximation.

$$K^2 \sim 10^{-18}$$

and

$$b_2 \sim 10^{-12}$$

Thus $\Omega(t)$can be simplified to

$$\Omega(t) = \left[4(b_1 + b_2 t)\right]^{-\frac{1}{4}} \tag{3.3.9}$$

Chapter 4

Ellipticity of Pulsars

4.1 Introduction

The main objective of this thesis is to estimate the ellipticity of pulsars. Ellipticity arises due to lack in symmetry in shape of spherical objects which is zero for perfect spherical objects. Factors that may affect ellipticity of pulsars are rotation and magnetic field. Strong magnetic fields can induce large deformation in the star which is a possible source of gravitational radiation. As it will be shown in this unit, the ellipticity can be expressed as a function of angular velocity Ω Hence a change in the angular velocity will also change the parameter ellipticity. But we have already seen in the preceding unit that the angular velocity of pulsars varies with time. Thus we will try to estimate the time change of the ellipticity based on the time evolution of angular velocity which comes as a result of gravitational and dipole radiations. The former is strong in the early stage of pulsar life time during which gravitational radiation will be the main factor in defining the ellipticity. Eventually gravitational radiation will grow weak just to be replaced by magnetic dipole radiation. We will study the time evolution of the parameter for the two cases.

4.2 Definition of ellipticity ϵ

As already pointed out in chapter 3 the ellipticity ϵ of a rotating spheroid can be defined as

$$\epsilon = \frac{I - I_o}{I} \tag{4.2.1}$$

But for the star we are dealing with in addition to being a spheroid, the star is also magnetized. Therefore we will here adopt the expression for moment of inertia I as used by Macy [14]:

$$I = I_0\Big[1 + \frac{25 I_0 \Omega^2 R}{12GM^2} + \frac{25R^4}{24GM^2}(3B_p^2 - < B_t{}^2 >)(\cos^2\chi - 1/3)\Big] \tag{4.2.2}$$

where I_o is given by [15]

$$I_0 = \frac{1}{3}A + \frac{2}{3}C \tag{4.2.3}$$

where A and C are the equatorial and polar moments of inertia given by

$$A = \frac{2}{5}MR^2(1 - \frac{5\Omega_o^2}{16\pi G\rho}) \tag{4.2.4}$$

and

$$C = \frac{2}{5}MR^2(1 + \frac{5\Omega_0^2}{8\pi G\rho}) \tag{4.2.5}$$

ρ being the average density of the neutron star($\sim 10^{17}\frac{Kg}{m^3}$), M is the average mass of the star($\sim 1.4M_{\odot}$),and R is the radius of the star($\sim 10km$).

B_p, $< B_t^2 >$, and χ in equation 4.2.2 above are the poloidal magnetic field, the square of volume weighted average toroidal internal field and the angle between the dipole moment and symmetry axis of rotation respectively.

Basically there are two causes for the deformation of the star. These are the rotation of the star and the magnetic fields. In equation 4.2.2 the second term in the bracket represents the contribution of the rotational bulge to the ellipticity whereas the third term represents the contribution of the magnetic bulge to the ellipticity. Notice then

that studying the time evolution of ϵ means finding finding the time evolution of B and Ω.

4.3 Time evolution of the parameter ellipticity ϵ

In chapter 2 we have shown that the time evolution of angular frequencies from gravitational and dipole radiation are respectively given by

$$\Omega_{gr} = \left[8(c_1 + c_2 t)\right]^{-\frac{1}{8}} \tag{4.3.1}$$

and

$$\Omega_{dr} = \left[\frac{K + (K^2 + b_1 + b_2 t)^{\frac{1}{2}}}{2(b_1 + b_2 t)}\right]^{\frac{1}{2}} \tag{4.3.2}$$

We now derive the expressions for the quantities, B_p, and $< B_t^2 >$

From Kebede (2002) [3] the magnetic field at the surface of a pulsar is given by

$$B_p = -\left(\frac{2Q}{3cR}\right)\Omega \tag{4.3.3}$$

where Q is the magnitude of separated charge. Squaring the above equation we then get

$$B_p^2 = \left(\frac{2Q}{3cR}\right)^2 \Omega^2 \tag{4.3.4}$$

To get $< B_t^2 >$ we need to take the contribution to the external field of the positive charges at the core and the internal field of the separated negative charges at the crust. But the field of the separated positive charges in the core is small as compared to the field of the negative charges on the crust. Thus $< B_t^2 >$ will be approximately equals to the square of volume weighted average internal field from the crust only and according to Kebede (2002) it will become

$$< B_t^2 >= \left(\frac{2Q}{3cR}\right)^2 \Omega^2 \tag{4.3.5}$$

In this work we will restrict ourselves to the condition that the angle between the magnetic dipole moment and the symmetry axis of rotation to remain constant and

we will take its value to be 90^0, representing an orthogonal rotator. Therefore substituting equations 4.3.4, 4.3.5, and the value of χ into equation 4.2.2 we get

$$I = I_0\Big[1 + \frac{25I_0\Omega^2 R}{12GM^2} + \frac{25R^4}{24GM^2}\Big(\frac{2Q}{3cR}\Big)^2\Omega^2\Big(\frac{4}{3}\Big)\Big] \tag{4.3.6}$$

$$= I_0\Big[1 + \Omega^2\Big\{\frac{25I_0R}{12GM^2} + \frac{25R^4}{18GM^2}\Big(\frac{2Q}{3cR}\Big)^2\Big\}\Big] \tag{4.3.7}$$

$$= I_0\Big[1 + \Omega^2 K\Big] \tag{4.3.8}$$

where K is a constant given by

$$K = \frac{25I_0R}{12GM^2} + \frac{25R^4}{18GM^2}\Big(\frac{2Q}{3cR}\Big)^2$$

$$\simeq 3.8036 \times 10^{-9}s^2$$

The magnitude of the separated charges, Q is taken to be $10^{-27}esu$ [3]. Now if we substitute equation 4.3.10 into equation 4.2.1 we find that

$$\epsilon = \frac{I_0(1 + K\Omega^2) - I_0}{I_0(1 + K\Omega^2)} \tag{4.3.9}$$

which reduce to

$$\epsilon = \frac{K\Omega^2}{1 + K\Omega^2} \tag{4.3.10}$$

The last equation gives us the time evolution of ϵ if we express Ω as function of time. Using the expressions for the time evolutions of Ω due to gravitational radiation (eq. 3.2.7) and dipole radiation(eq. 3.3.9) we will now quantitatively study the time evolution of ϵ. In the early stage of their life, due to distortion induced by strong magnetic field, a large amount of gravitational wave is emitted [16] from pulsars. Thus during this period gravitational emission is the main mechanism for spin down of pulsars. Hence the ellipticity from gravitational braking will be

$$\epsilon = \frac{K\Omega_{gr}^2}{1 + K\Omega_{gr}^2} \tag{4.3.11}$$

For a typical pulsar with initial period $1.7ms$ or $\Omega_0 = 3969s^{-1}$ [13] our results for ellipticity are shown in Table-1 and the graph of ellipticity versus time due to gravitational radiation is shown in fig.1

Basically the dipole radiation becomes the dominant braking mechanism after the gravitational radiation gets weak and hence its contribution to ellipticity will be significant during this time. But in general we can express its contribution at any time as

$$\epsilon = \frac{K\Omega_{dr}^2}{1 + K\Omega_{dr}^2} \tag{4.3.12}$$

Therefore using the above equation we get the results shown in Table-2. The graph of ellipticity for dipole radiation contribution versus time is plotted in fig.2

4.4 Results

Based on the calculations made in the preceding section we now give quantitative estimates to the ellipticity of pulsars. As stated in the previous section gravitational radiation is dominant for some time in the early time of pulsars life time and therefore its contribution to ellipticity is dominant during this time. For the pulsar with $\Omega_0 = 3696s^{-1}$ the values of ellipticity at different times is shown in the table below and the graph of ellipticity versus time is plotted in fig 4.1.

From the fig above it can be clearly seen that the ellipticity decreases fast up to a certain time after which it changes very slowly with time.

On the other hand the value of ellipticity at different times from dipole radiation is shown in Table-2 below and the graph of ellipticity versus time for dipole radiation is plotted in fig 4.2. from which it can be seen that the ellipticity due to the dipole radiation changes at a relatively slower rate.

4.5 Discussion of Results

It is possible to roughly estimate the time during which gravitational radiation plays the major role for pulsars spin down. We can do this with the help of energy loss rate equations shown in unit one. i.e

$$\dot{E}_{gr} = \dot{E}_{dr} \tag{4.5.1}$$

where $\dot{E}_{gr}$ and $\dot{E}_{dr}$ are power radiated from gravitational and dipole radiations respectively. These are shown to be

$$\dot{E}gr = I_0(1 + 2\Omega_{gr}^2 K)\Omega_{gr}\dot{\Omega}_{gr}$$

and

$$\dot{E}_{dr} = I_0(1 + 2\Omega_{dr}^2 K)\Omega_{dr}\dot{\Omega}_{dr}$$

If we substitute Ω_{gr} and $\dot{\Omega}_{gr}$ into $\dot{E}_{gr}$ we find

$$\dot{E}_{gr} = -I_0 c_2\Big([8(c_1 + c_2 t)]^{-\frac{5}{4}} + 2K[8(c_1 + c_2 t)]^{-\frac{3}{2}}\Big) \tag{4.5.2}$$

Similarly for $\dot{E}_{dr}$ we get

$$\dot{E}_{dr} = -I_0 b_2\Big([4(b_1 + b_2 t)]^{-\frac{1}{2}} + 2K[4(b_1 + b_2 t)]^{-2}\Big) \tag{4.5.3}$$

Substituting the last two equations into equation 4.4.1 we get a numerical value for the required time $t \cong 9.04hrs.$ for a typical pulsar with initial rotational frequency $3696s^{-1}$. As stated earlier the dipole radiation becomes important braking mechanism after this time. If we take the initial angular velocity to be $1000s^{-1}$, we get the time during which the gravitational radiation plays the major role for the baking mechanism to be $t \simeq 6072hrs.$ From the above results it can be concluded that the faster a pulsar rotates initially, the shorter will be the time for which gravitational radiation becomes a dominant braking mechanism.

Next we will compare what the standard calculation show for the time evolution

of ϵ with our work. Palomba starting from the spin down rate equation due to gravitational radiation:

$$\dot{\Omega}_{gw} = -\frac{32G}{5c^5} I \epsilon^2 \Omega^5 \tag{4.5.4}$$

showed the ellipticity to be [17]

$$\epsilon = 1.9 \times [P\dot{P}_{gw}]^{\frac{1}{2}} \tag{4.5.5}$$

where P is the period and $\dot{P}_{gw}$ its time rate due to gravitational radiation.The above equation can be rewritten as a function of the observed pulsar period and its time derivative, and the ratio between the gravitational and electromagnetic spin down rates:

$$\epsilon = 1.9 \times 10^5 \left\{ P^3 \dot{P} \frac{Y}{Y+1} \right\} \tag{4.5.6}$$

where

$$Y = \frac{\dot{\Omega}_{gw}}{\dot{\Omega}_{em}}$$

$\dot{\Omega}_{gw}$ and $\dot{\Omega}_{em}$ being gravitational and electromagnetic spin down rates respectively. Using the above equation Palomba calculated the limiting value of ellipticity for three pulsars with measured braking index. The result is shown in the table bellow.

Table-3

pulsar	$\Omega(s^{-1})$	ϵ_{max}
Crab	365.3	3×10^{-4}
Vela	390.3	3.3×10^{-4}
PSR 0540	174.5	2.4×10^{-4}

According to our model the limiting values of ellipticity for the above three pulsars is shown in the table below.

[12] Stars and Relativity

Ya. B. Zel'dovich and I. D. Novikov, 1971

[13] Goussard, J. O., Haensel, P., Zdunik, J. L. (1998), AA, 330, 1005

[14] William W. Macy Jr. 1974 apj

[15] Goldreich P. 1970 APJ 160L 11G

[16] C.Palomba AA 367, 525-531(2001)

[17] C. Palomba Astron. Astrophys, 354, 163-168(2000)