A SIMPLE THERMALLY DRIVEN BROWNIAN MOTOR

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"Thanks be unto God for his unspeakable gift"

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Table of Contents

Abstract

In this work we study the directed motion (or current) of Brownian particles in an inhomogeneous medium due to spatially periodic temperature background, and a ratchet potential with or without load.

The nature of the current in our model system is discussed in detail. In particular we analyze the behavior of the current as a function of the parameters specifying our model system such as temperature, barrier height, trap potential, load and length of hot region.

Chapter 1

Introduction

The problem of Brownian motion has played a central role in the development of both foundations of thermodynamics and the dynamical interpretation of statistical physics. For the first time, Albert Einstein in 1905 proposed the solution for the above stated matter under a theory of "Brownian Motion" which is based on the molecular kinetic theory of heat [1]. His proposal provided the link between an elementary underlying microscopically dynamic and macroscopically observable phenomena. Latter, Smoluchowski proposed that Brownian motion is caused due to

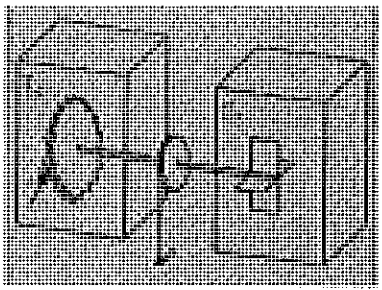

Figure 1.1: Feynman's ratchet and pawl system

rectification of thermal fluctuation [2]. Based on Smoluchowski's idea Feynman presented a model of a thermal fluctuation rectifier which is usually called ratchet-pawl system [3].

The ratchet-pawl system consists of an axle with peddles (vanes) surrounded by a gas in a box at one end at temperature T_1, and a ratchet and pawl at the other end surrounded by a gas in a box at temperature T_2. A ratchet is a disc with asymmetric saw tooth and the pawl serves as a rectifier of random motion as it prevents the ratchet from backward rotation. Figure (1.1) is an illustration of Feynman's ratchet-pawl system.

According to Feynman if thermal equilibrium is broken with $T_1 \neq T_2$, the random kick of gas molecules against vanes provide thermal fluctuation that are rectified by the ratchet and pawl. A rectification (or conversion) of random fluctuation into useful work is termed as ratchet effect. The ratchet effect is generated when the equilibrium conditions and symmetry are broken. The symmetry is broken either by the system characteristics, such as asymmetry of the potential, or by the dynamics itself that may break the symmetry in the time domain [4].

When thermal fluctuations are present, the equilibrium configuration on an energy potential is independent of initial conditions. A particle placed in a local energy minimum will not stay there when lower energy configurations are present. This principle allows us to always cause flow of particles by changing energy potentials.

There are essentially three ways that can cause a flow of particles in ratchets:

- change the level of energy barriers,

- change the level of energy minima

and

- add new particles at a rate faster than the system can keep up with [5].

When a machine becomes small enough, thermal fluctuations become large compared to the energies that drive the motor. A motor designed that actually incorporates thermal fluctuations into its functions is called Brownian motor. Brownian motor is defined by a set of characteristics:

-thermal noise plays central role,

-symmetry is broken,

-all forces average to zero except for the load,

-detailed balance is broken, that is, the system is kept away from the equilibrium, and

-periodicity of spatial or temporal is typical.

All are required for a clear definition of what constitutes "Transport" or "Work" in a Brownian motor [6].

Up to now, several qualitatively different models of Brownian motors have been proposed, namely, flashing ratchets, rocking ratchets, diffusion ratchets and frictional ratchets. Below we will briefly describe the first two.

Flashing ratchet is a simple model by which the periodic potential is allowed to fluctuate with finite time correlation between two states characterized by different barrier heights. The overdamped Brownian particles are subjected to two potential states periodically, i.e. a potential with barrier height V_1 for a time interval τ_1 and a potential with barrier height V_2 for a time interval τ_2. In general, the potential fluctuates between V_1 and V_2 states [7]. When one of these potential is zero, we have the so-called On and Off ratchet.

Another type of ratchet corresponds to rocking ratchet. In this ratchet, on an asymmetric potential one applies a random time varying force with zero mean. Due to the anisotropy of the potential when a force having the same magnitude but different signs +F and -F are applied, the motion of the particle on the average will be along positive and negative directions respectively. In rocking ratchet, this rocking or changing of slope can be done either periodically or randomly in time. This system acts as a nonlinear rectifier in the presence of zero average periodic or random force.

Unlike the case of flashing ratchets, the direction of current for the rocking ratchet is in the direction of the steeper slope and this mechanism of rocking is equivalent to generating dc current in semiconductor pn junctions under an applied ac bias [7].

In the case of a ratchet without load, particles moving in a periodic potential system end up with the same potential minimum. In such ratchet no useful work can be performed, no extra energy is stored in the Brownian particles which can be usefully expended when needed. Thus for a ratchet to perform work a small external force, which is called load, has to be added in a direction opposite to the induced motion. In this case, the ratchet still induces a motion against the force.

In this thesis we study the one dimensional motion of Brownian particles in inhomogeneous medium. Our model consists of an asymmetric sawtooth, so called ratchet, potential. The medium is subjected to non-homogeneous temperature background which has the same spatial periodicity between hot and cold as that of the ratchet potential. In addition the medium has equally spaced traps which are uniformly distributed and strong enough to trap the particles.

When Brownian particles are subjected to a spatially periodic, asymmetric potential in this medium, they begin to diffuse from one trap to another by absorbing heat from hot region and deliver it to cold region and then get trapped. Due to the rectification of thermal fluctuation by the ratchet and the spatially periodic temperature background, the Brownian particles will be induced to have directed motion. The Brownian particles are assumed to diffuse in a highly viscous medium such that inertial effects are negligible. Hence the dynamics of the Brownian particles is governed by an overdamped Langevin equation or equally by the associated Fokker-Plank equation. The thermal fluctuation we took is Gaussian white noise with vanishing meau which satisfies Einstein's fluctuation-dissipation relation [4]. Note that the above two stated ratchet models and ours are restricted to non-interacting Brownian particles.

The rest of this work is organized as follows. In chapter two, we present the diffusion equation for a Brownian particle that diffuses in an inhomogenous medium having traps. We then derive the general expression for the steady state probability current of the Brownian particle diffusing in the inhomogenous medium where temperature and external potential vary periodically. Such system is usually called thermally driven Brownian motor. In chapter three, we take a specific model of this thermally driven Brownian motor with or without load and find an exact expression for the steady state probability current. In chapter four, we present the results of our work and discuss them by plotting graphs of steady state probability current versus the different parameters describing our model. In chapter five, we deal with summary and conclusion.

Chapter 2

Directed motion in a particular non-homogeneous medium

Diffusion of Brownian particles in homogeneous medium is studied widely although there are still many more systems yet to be explored. Now a days, interest in studying diffusion of Brownian particles in inhomogeneous media has increased. Transport properties of Brownian particles in such media are essential in designing tiny smart devices of the order of few micrometers size. The devices could be used to extract energy available at tiny scales for useful work. In this work we consider a particular type of medium coupled to heat reservoirs and external potential. As the Brownian particles move through the medium they experience random as well as deterministic forces at the same time. For certain arrangements of the system, the Brownian particles attain unidirectional motion and act as thermally driven Brownian motors. In this chapter we introduce the characteristic of our system.

In the first part of this chapter, we describe the medium through which the Brownian particles diffuse and present the dynamic equation describing the particles' motion. The second part of this chapter considers a general thermally driven Brownian motor and derive the corresponding steady state probability current density for the Brownian particles.

2.1 Diffusion equation in a medium with traps

Consider a single electron moving in a one-dimensional lattice medium having equal spacing. Suppose the internal potential exerted on the electron by the medium gives rise to a trap potential at each lattice site. The trap potential is taken to be strong enough such that the time the electron spends within a trap before escaping to the next trap is much larger than that of the time it takes to move from one trap to the next. Within the period of its stay in a lattice, the electron loses all its memory of the past, its future state depending only on the present state and not on how it has arrived at the present state. This kind of motion of the electron can be well approximated by Markov process. Here it is important to note that the electron moves from one lattice to the next by hopping or jumping.

Approximating the sample path of the electron by a continuous time random walk, van Kampen derived the Fokker-Planck equation governing the electron's motion in such a medium [8]. When the medium's temperature and the external potential experienced by the electron vary with potion, the Fokker-Planck equation which van Kampen derived takes the form

$$\frac{\partial P(x,t)}{\partial t} = D_0 \frac{\partial}{\partial x}\left\{ \frac{exp[\frac{-\phi}{T(x)}]}{T(x)} V'(x)P(x,t) + \frac{\partial}{\partial x} exp[\frac{-\phi}{T(x)}]P(x,t) \right\}. \tag{2.1}$$

Here $P(x,t)$ is the probability density of getting the electron at position x at time t, $V(x)$ is the applied external potential, ϕ is trap potential, $T(x)$ is position dependent temperature, D_0 is a diffusion property of the medium and Boltzmann's constant, k_B, is taken as unity.

Now, we want to write Eq.(2.1) in the form of the usual diffusion equation ,

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}[\mu(x)V'(x)P(x,t) + \frac{\partial}{\partial x}(D(x)P(x,t))], \tag{2.2}$$

where $\mu(x)$ is mobility and D(x) is diffusion coefficient. Eq.(2.2) is the familiar Smoluchowski equation where the position dependent mobility is given by

$$\mu(x) = D_0 \frac{exp[\frac{-\phi}{T(x)}]}{T(x)}, \tag{2.3}$$

and the position dependent diffusion coefficient is given by

$$D(x) = D_0 exp[\frac{-\phi}{T(x)}]. \tag{2.4}$$

Note that the Einstein's relation $D(x) = \mu(x)T(x)$ is satisfied.

In the next section the general expression for the steady state probability current density will be derived for a thermally driven Brownian motor.

2.2 Derivation of steady state probability current

In a periodic ratchet potential there is no directed motion of Brownian particles, if the temperature of the medium is uniform and there is no external force. If the temperature of the medium is non-homogeneous, directed motion of Brownian particles may take place. In this section, we want to derive the probability current density for Brownian particles moving in the considered medium having non-homogeneous temperature background and experiencing external potential.

Consider a system where Brownian particles are subjected to a spatially periodic, asymmetric potential in the medium whose temperature also changes periodically having the same period as that of the potential. Eq.(2.1) can be written as a continuity equation.

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}, \tag{2.5}$$

where the probability current density, J(x,t) is given by

$$-J(x,t) = D_0 \left\{ \frac{V'(x)}{T(x)} exp[\frac{-\phi}{T(x)}] P(x,t) + \frac{\partial}{\partial x} exp[\frac{-\phi}{T(x)}] P(x,t) \right\}. \tag{2.6}$$

For infinitely extended system or a system that closes on itself like a ring, the system attains steady state. In such a situation the steady state probability current density is constant and hence Eq.(2.6) can be written as

$$-J = D_0\left\{\frac{V'(x)}{T(x)}exp[\frac{-\phi}{T(x)}]P_s(x) + \frac{d}{dx}exp[\frac{-\phi}{T(x)}]P_s(x)\right\}, \tag{2.7}$$

where $P_s(x)$ is the steady state probability density. By setting the probability current density, Eq.(2.7), equal to zero, the homogeneous solution for the steady state probability density is given by

$$P_s(x) = \frac{N}{\psi(x)}exp[\frac{\phi}{T(x)}], \tag{2.8}$$

where

$$\psi(x) = exp[\int_0^x \frac{V'(x')}{T(x')}dx'], \tag{2.9}$$

and N is a constant of integration which can be determined from the normalization condition. The probability for each trap to be occupied is given by the factor

$$exp[\frac{\phi}{T(x)}], \tag{2.10}$$

and the effect of the external potential, $V(x)$, on the stationary distribution is fully accounted by the inverse of $\psi(x)$.

Then the solution for $P_s(x)$ of the non-homogeneous differential equation Eq.(2.7), can be expressed,

$$P_s(x) = \frac{N(x)}{\psi(x)}exp[\frac{\phi}{T(x)}]. \tag{2.11}$$

Inserting the solution of $P_s(x)$, Eq.(2.11) in Eq.(2.7), finally, the solution for the steady state probability density becomes

$$P_s(x) = \left\{N(0) - \frac{J}{D_0}\int_0^x \psi(x')dx'\right\}exp[\frac{\phi}{T(x)}]\psi^{-1}(x). \tag{2.12}$$

Applying periodic boundary condition on the steady probability density, the potential and temperature, i.e.

$$P_s(x+\lambda) = P_s(x), \tag{2.13}$$

$$V(x+\lambda) = V(x), \tag{2.14}$$

and

$$T(x+\lambda) = T(x) \tag{2.15}$$

we obtain

$$P_s(x+\lambda) = \left\{N(0) - \frac{J}{D_0}\int_x^{x+\lambda}\psi(x')dx'\right\}exp[\frac{\phi}{T(x+\lambda)}]\psi^{-1}(x+\lambda). \tag{2.16}$$

Note that length of the period of both the potential and temperature profiles is taken to be λ.

Then equating Eq.(2.12) and Eq.(2.16), and using Eq.(2.13), Eq.(2.14) and Eq.(2.15) the value for the integration constant N(0) becomes

$$N(0) = \frac{J}{D_0}\frac{\{\int_0^x \psi(x')dx' - exp[-\int_x^{x+\lambda}\frac{V'(x')}{T(x')}dx']\int_0^\lambda \psi(x')dx'\}}{\{1 - exp[-\int_x^{x+\lambda}\frac{V'(x')}{T(x')}dx']\}}. \tag{2.17}$$

The normalization condition permits to normalize Eq.(2.12) in the spatial period λ:

$$\int_0^\lambda P_s(x)dx = 1. \tag{2.18}$$

After substituting Eq(2.12) in Eq.(2.18)we obtain,

$$\int_0^\lambda \left\{N(0) - \frac{J}{D_0}\int_0^x \psi(x')dx'\right\}exp[\frac{\phi}{T(x)}\psi(x)^{-1}]dx = 1. \tag{2.19}$$

After inserting the integration constant Eq.(2.17) in Eq.(2.19), the probability current density can be extracted and is given by

$$J = D_0 \left\{ \int_0^{\lambda} exp[\frac{\phi}{T(x)}]\{\frac{\int_x^{x+\lambda} \psi(x')dx'}{\psi(x+\lambda)-\psi(x)}\}dx \right\}^{-1}. \tag{2.20}$$

In the next chapter taking a specific ratchet potential with alternating hot and cold temperature background in a medium that has equally distributed traps, we get analytic expression for steady state probability current in terms of parameters specifying the model.

Chapter 3

Current of a simple thermally driven Brownian motor

In this chapter we will use the method developed in the previous chapter to get closed form expression for the probability current density of a specific model of a thermally driven Brownian motor with or without load.

3.1 The model

The model consists of a Brownian particle moving in a sawtooth potential with an external load where the particular medium we are considering is alternately in contact with hot and cold heat reservoirs along the space coordinate. The shape of a single sawtooth potential, $V_s(x)$, located in the interval $(0, \lambda)$ is described by

$$V_s(x) = \begin{cases} \frac{Qx}{\lambda_1} & \text{if } 0 \leq x < \lambda_1, \\ \frac{-Q(x-\lambda)}{\lambda_2} & \text{if } \lambda_1 \leq x < \lambda. \end{cases} \tag{3.1}$$

The potential corresponding to the external load is linear, fx, where f is the load. The temperature profile, $T(x)$, in the interval $(0, \lambda)$ is described by

$$T(x) = \begin{cases} T_H, & \text{if } 0 \le x < \lambda_1, \\ T_C, & \text{if } \lambda_1 \le x < \lambda. \end{cases} \tag{3.2}$$

Both $V_s(x)$ and $T(x)$ are taken to have the same period such that $V(x+\lambda) = V(x)$ and $T(x+\lambda) = T(x)$ with $\lambda = \lambda_1 + \lambda_2$. Note that the left side of each sawtooth from its barrier top overlaps with the hot region of the medium while the right side overlaps with the cold region. The sawtooth potential with the load, $V(x) = V_s(x) + fx$, and the temperature profile, $T(x)$ are shown in Fig.(3.1).

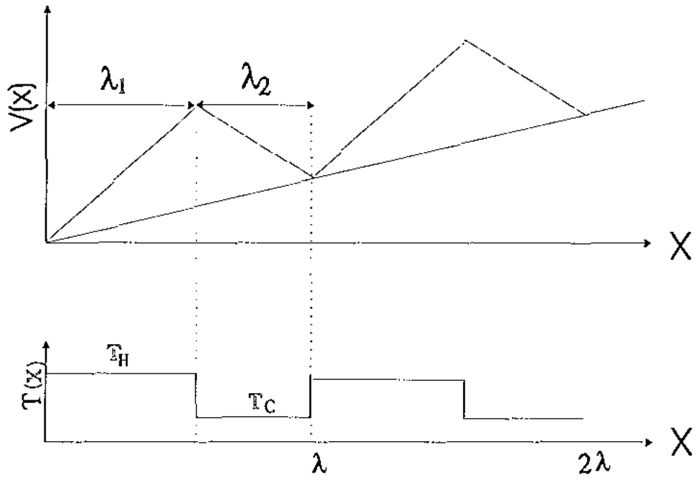

Figure 3.1: Profiles of the sawtooth potential with load and of temperature

3.2 Steady state probability current of the model

In this section we will derive an expression for the steady state probability current density of the model shown in Fig.(3.1). The quantities that specify the model are the barrier height Q, the shape of the sawtooth λ_1, λ, the load f, the temperature of the heat reservoirs T_C and T_H and the trap depth ϕ. We scale Q, λ_1, f, T_H and ϕ such that

$$q = \frac{Q}{T_C}, \tag{3.3}$$

$$\Phi = \frac{\phi}{T_C}, \tag{3.4}$$

$$F = \frac{f\lambda}{T_C}, \tag{3.5}$$

$$T_H = T_C(s+1) \tag{3.6}$$

and

$$\beta_1 = \frac{\lambda_1}{\lambda}. \tag{3.7}$$

Here s is a parameter quantifying the strength of the hot region in excess of the cold region, q is a parameter characterizing the barrier height, Φ is a parameter characterizing trap potential, F is a parameter characterizing the external force (load) and β_1 characterizes asymmetry of the potential.

Thus, we have five parameters q, s, F, β_1 and Φ characterizing the model for a given T_C and λ. Taking the steady state probability current density equation Eq.(2.20) we derived in chapter two for our model and after integration and certain re-arrangements, the steady state probability current turns out to be given by

$$J = \frac{J_0 J_1}{[J_2 + J_3 + J_4 + J_5 + J_6 + J_7 + J_8 + J_9]}, \tag{3.8}$$

where

$$J_0 = \frac{D_0}{\lambda^2}.$$

Converting the current density into a dimensionless quantity, $j = \frac{J}{J_0}$ then the above equation can be rewritten as

$$j = \frac{J}{J_0} = \frac{J_1}{[J_2 + J_3 + J_4 + J_5 + J_6 + J_7 + J_8 + J_9]}. \tag{3.9}$$

Here the expressions for J_1, J_2, J_3, J_4, J_5, J_6, J_7, J_8 and J_9 are given below

$$J_1 = -sinh[\frac{F(\beta_2 Y_1 + \beta_1) - qs}{2Y_1}]exp[\frac{F(\beta_2 Y_1 + \beta_1) - qs}{2Y_1}],$$

$$J_2 = \frac{\beta_1^2 Y_1^2}{Y_2^2} sinh[\frac{Y_2}{2Y_1}]exp[\frac{Y_2}{2Y_1}]exp[\frac{\Phi}{Y_1}],$$

$$J_3 = \frac{2\beta_1\beta_2 Y_1}{Y_1 Y_2} sinh[\frac{Y_3}{2}]sinh[\frac{Y_2}{2Y_1}]exp[\frac{F(\beta_2 Y_1 + \beta_1) - qs}{2Y_1}]exp[\frac{\Phi}{Y_1}],$$

$$J_4 = -\frac{\beta_1^2 Y_1^2}{Y_2^2} sinh[\frac{Y_2}{2Y_1}]exp[\frac{[F(2\beta_2 Y_1 + \beta_1) - q(2s+1)]}{2Y_1}]exp[\frac{\Phi}{Y_1}],$$

$$J_5 = -J_1 \frac{\beta_1^2 Y_1}{Y_2} exp[\frac{\Phi}{Y_1}],$$

$$J_6 = \frac{\beta_2^2}{Y_3^2} sinh[\frac{Y_3}{2}]exp[\frac{Y_3}{2}]exp[\Phi],$$

$$J_7 = \frac{2\beta_1\beta_2 Y_1}{Y_1 Y_2} sinh[\frac{Y_3}{2}]sinh[\frac{Y_2}{2Y_1}]exp[\frac{F(\beta_2 Y_1 + \beta_1) - qs}{2Y_1}]exp[\Phi],$$

$$J_8 = -\frac{\beta_2^2}{Y_3^2} sinh[\frac{Y_3}{2}]exp[\frac{[F(\beta_2(s+1) + 2\beta_1) - q(s-1)]}{2Y_1)}]exp[\Phi],$$

$$J_9 = -J_1 \frac{\beta_2^2}{Y_3} exp[\Phi]$$

where

$Y_1 = s+1$, $Y_2 = F\beta_1 + q$, $Y_3 = F\beta_2 - q$ and $\beta_2 = 1 - \beta_1$. The steady state probability current j that we found is a function of Φ, F, q, s and β_1.

In chapter four we will discuss about the behavior of the steady state probability current, j, as a function of our model parameters Φ, F, q, s and β_1.

Chapter 4

Result and discussion

In this chapter, we will present and discuss the behavior of the steady state probability current as a function of the parameters defining our model. In section one, we study how current is generated without the presence of external load and only due to the presence of thermal variation along the ratchet model. Section two deals with the behavior of current generated in the presence of load and thermal variation along the ratchet. The analytic expression for the steady state current is given by Eq.(3.9).

4.1 Steady state probability current of zero load

In this section, we will discuss the steady state probability current of our model in the absence of load. We discuss based on the plots of current j, versus Φ , q, s and β_1.

Fig.(4.1) shows plot of current, j, versus the hot temperature parameter, s. The current increases as the temperature increases and saturates for higher values of s. This is because first, as the temperature increases, the amount of heat absorbed by the Brownian particles increases and this enables the Brownian particles to escape faster across the potential barrier from hot region to the cold region, and consequently the current increases. The motion of the Brownian particles from hot region to cold region. At higher value of s, when the Brownian Particles (electrons) absorb sufficient amount of heat that enable them to cross the ratchet potential and escape out of the trap and thus the current gets saturated.

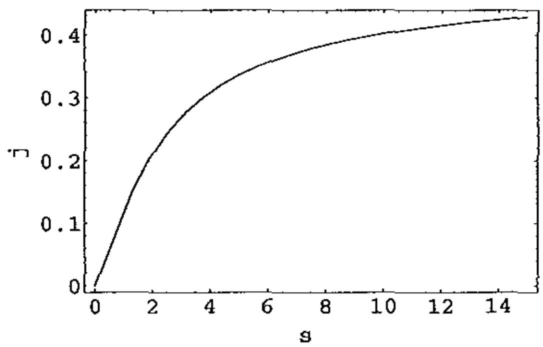

Figure 4.1: Plot of j versus s taking Φ=3, q=3, and β_1 = .75

Fig.(4.2) is a plot of j versus q. It shows that the current goes to zero as the barrier height goes either to zero or to infinity. As q goes to zero the particles are affected ouly by the periodic temperature profile and the trap potential which cannot cause net flow of Brownian particles. The current increases as the barrier height increases until it reaches an optimum value at a finite value of q. After attaiuing optimum value, the current decreases with the increment of q and gradually goes to zero as q goes to infinity. This is because the amount of heat absorbed by Brownian particles is not sufficient for them to cross a very high poteutial barrier. That is why the current tends to zero as the potential barrier becomes very large.

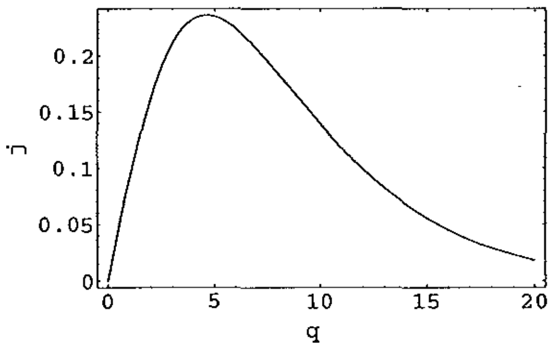

Figure 4.2: Plot of j versus q taking Φ=3, s=2, and β_1 = .75

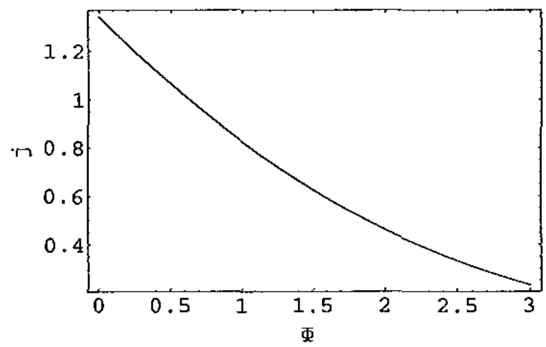

Figure 4.3: Plot of j versus Φ taking q=4, s=2, and β_1 = .75

The plot of j versus trap potential Φ, $Fig.$(4.3), shows that the current decreases monotonically and goes to zero as the trap potential increases. When the trap potential Φ is zero, the Brownian particles are affected only by the periodic temperature profile and the ratchet potential. This enables the Brownian particles to have maximum current. On the other hand, the increment of trap potential makes it difficult for the particles to escape out of the trap making the current to decrease and then vanish for high trap potential.

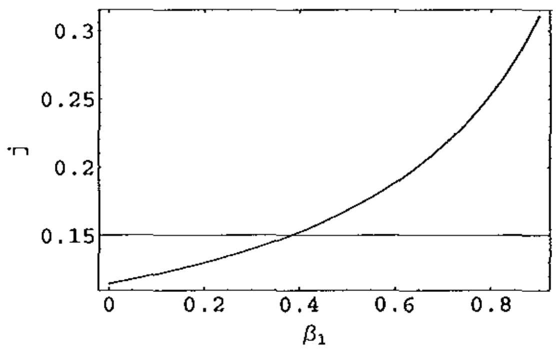

Figure 4.4: Plot of j versus β_1 taking Φ=3, q=4, and $s = 2$

The plot of j versus the asymmetry of the potential, β_1, $Fig.$(4.4), shows that the current increases monotonically as the asymmetry potential increases between zero and one. This is because as β_1 increase the Brownian particles stay longer in the hot region and absorb more heat from the heat reservoir. This makes Brownian particles to escape faster across the potential barrier from the hot region to the cold region, and consequently the current increases monotonically with the asymmetry potential.

4.2 Steady state probability current with load

In this section, we will discuss the behavior of steady state probability current of our ratchet model as a function of the different parameters including load.

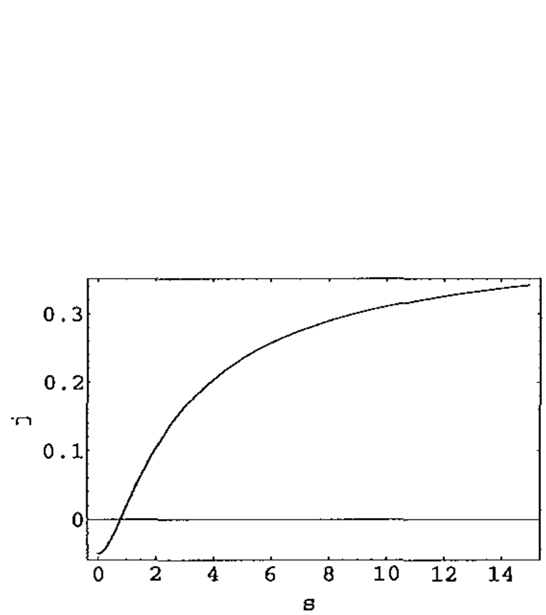

Figure 4.5: Plot of j versus s taking Φ=3, q=3, F=2 and β_1 = .75

The plot of j versus the hot temperature parameter, s, $Fig.$(4.5), shows that in the presence of load up to certain values of s the current is negative, positive for other values of s and saturates for very high value of s. The negative current shows that there is drift of Brownian particles from cold to hot region due to the presence of load and our model works as refrigerator. In this situation where the load, as it climbs down, does work on the engine forcing heat to flow from the cold to the hot reservoir and the current being reversed. Positive current indicates that there is a drift of Brownian particles from hot to cold region and our model (Brownian motor) works as heat engine. This positive current increases as s increases and saturates for large value of s. This is because as the hot temperature parameter increases the amount of heat absorbed by the Brownian particles increases and this enable them to escape faster out of the trap and cross the potential barrier from hot region to the cold region. Due to this current increases with the increment of s. At higher values of s, when the Brownian particles absorb sufficient amount of heat that enables them to cross the ratchet potential and escape out of the trap and thus the current gets saturated. In general we can say that the model in the presence of load acts as a heat engine for certain value of s and as a refrigerator for other values of s [9]. The presence of load causes current to decrease compared to the current in Fig.(4.1).

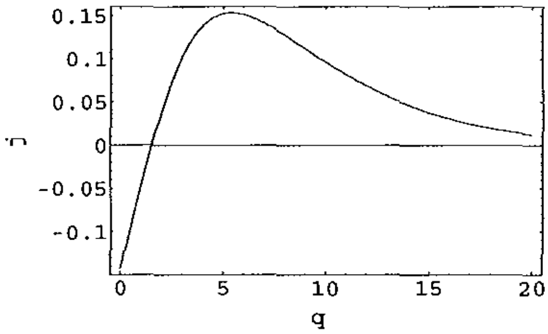

Figure 4.6: Plot of j versus q taking Φ=3, F=2, s=2, and β_1 = .75

The plot of j versus barrier height, q, *Fig.*(4.6), in the presence of load shows that up to a certain value of q the current is negative, positive for other value of q and tends towards zero for large q. Negative current shows that there is net drift of Brownian particles from cold region to hot region due to the presence of load. In this situation the engine works as a refrigerator. In this situation where the load, as it climbs down, does work on the engine forcing heat to flow from the cold to the hot reservoir and the current being reversed. Positive current indicates that there is drift of Brownian particles from hot region to cold region and the engine works as heat engine. The positive current increases as q increases until it reaches an optimum value at finite value of q. Once current attain optimum value it decreases with the increment of q and gradually tends towards zero for large value of q. This is because as q increases the amount of heat needed by the Brownian particles to escape out of the trap and cross such very high barrier height increases. Due to this the current decreases with the increment of q and gradually goes to zero.

Our model, in the presence of load, acts as a heat engine for certain values of

q and as a refrigerator for other values of q. When it works as heat engine, there is a finite value of q at which the current becomes optimum (peak value) and this point corresponds to the maximum rate of energy transfer [9]. The presence of load minimizes the magnitude of current compared to Fig.(4.2).

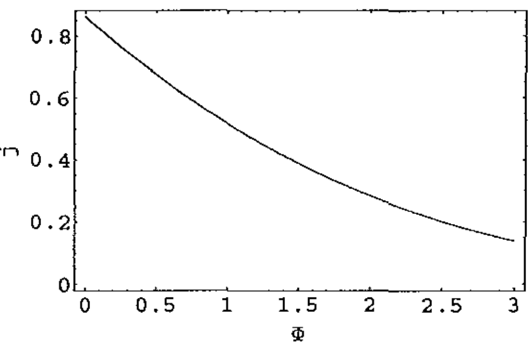

Figure 4.7: Plot of j versus Φ taking q=4, F=2, s=2, and β_1 = .75

The plot of j versus trap potential, Φ, *Fig.*(4.7) shows that the current decreases monotonically and goes to zero for large value of trap potential. When trap potential is zero current is maximum and with increase in trap potential the current monotonically decreases. This is because when Φ is zero, the heat absorbed by the Brownian particles is fully used to cross the potential barrier, but as the trap potential increases part of the heat absorbed by the particles is used to escape out of the trap and hence the current decreases. The presence of load minimizes magnitude of current and it slowly goes to zero compared to Fig.(4.3).

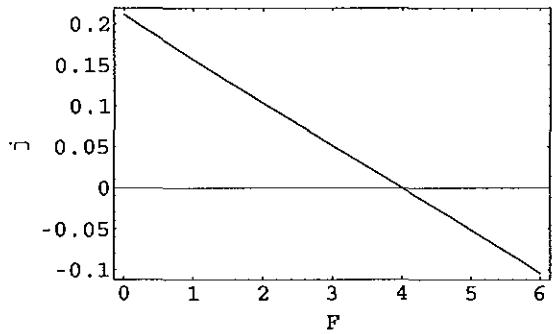

Figure 4.8: Plot of j versus F taking $\Phi = 3$, q=3, s=2, and $\beta_1 = .75$

The plot of j versus load F, $Fig.$(4.8), shows that current decreases linearly as the load increases. This is because as the load increases the number of Brownian particles that are induced opposite to the direction of the net drift increases. This causes a decrement for the net flow of Brownian particles for the increment of load. The plot of j versus the asymmetry of the potential, β_1, $Fig.$(4.9), shows that current increases monotonically as asymmetry of the potential, β_1, increases from a finite value. Because as β_1 increases the duration of Brownian particles in the hot region is longer and this enables the Brownian particles to absorb more heat. Due to this the Brownian particles escape faster across the potential barrier and out of the trap and thus current increases. The current decreases compared to Fig.(4.4) due to the presence of load.

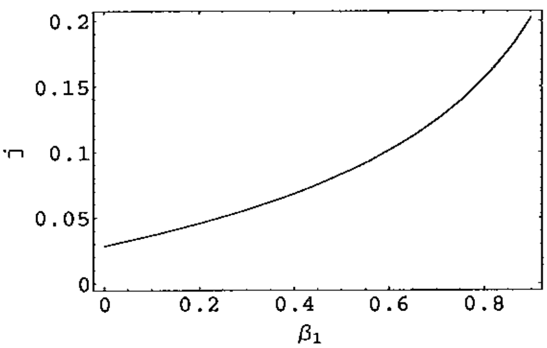

Figure 4.9: Plot of j versus β_1 taking Φ=3, q=4, $F = 2$, and $s = 2$

Chapter 5

Summary and conclusion

In this work we considered a model known as the hopping model to study the diffusion of Brownian particles (electrons) in non-homogeneous medium with ratchet potential and load. We designed our model to transport Brownian particles (electrons) in a piecewise linear asymmetric periodic potential ratchet driven by the thermal kicks the Brownian particles get from periodically placed hot and cold reservoirs along the path in a medium that has uniformly distributed traps. Based on this, we studied the dynamics of the Brownian particles (electrons) in such a system and then derived the expression for steady state probability current.

In the load-free case, the net drift of Brownian particles through the medium was from the hot region to the cold region. By keeping other parameters constant for particular finite value of q, current takes an optimum value. In addition, we have explored how the current of the model vary as a function of the parameters specifying the model.

In non-zero load case, the net drift of Brownian particles through the medium was from the cold region to the hot region up to certain value of q and from the hot region to the cold region for other value of q and the model works as refrigerator as well as heat engine. The load as it climbs down, does work on the engine forcing heat to flow from the cold to the hot reservoir and the current being reversed. In addition, we have explored how the current of the model vary as a function of the parameters specifying the model with the presence of load.

In conclusion, we believe that our work has, for the first time, quantitatively explored the quantities that determine the magnitude of current of the Brownian particles in this ratchet system. These theoretical results propose the extents and limits to be considered in the design of actual thermally driven Brownian motor acting either as a refrigerator as a heat engine especially for application in nano-technology. For the future, quantities such as power, efficiency and coefficient of performance of our model is open for further studies.

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