



THEORETICAL ESTIMATION OF GEOMETRY
DEPENDENT PHOTON DETECTION EFFICIENCY
OF NAI (TL)CRYSTAL OF DIFFERENT
SIZE FOR VARIOUS SOURCES TO
DETECTOR DISTANCES

By

Dinberu Faji Desalegn

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DEPARTMENT OF
PHYSICS

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Advisor:

Prof. A.K. Chaubey

Examiners:

Dr. Tilahun Tesfaye (Ass.professor)

ADDIS ABABA UNIVERSITY

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Author: **Dinberu Faji Desalegn**

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Signature of Author

Dedicated To
My Mother Simalo Setta Birra
and My Father Faji Desalegn Huluka

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Abstract

In this project the variation of the intrinsic efficiency of the Na I (Tl) detector against the source detector distance has been calculated for different gamma ray energy of 0.1, 0.2, 0.4, 0.6 1, 5, 7 and 10MeV and taking in to account various source to detector distance, ranging from 0 to 10 cm. The intrinsic efficiency depends not only on energy of photons but also on the geometry configuration of source and detector and source to detector distances. The intrinsic efficiency variation can be analyzed by the mean chord path length of the photon in the detector and the linear attenuation coefficient of photon in the detector.

INTRODUCTION

Nuclear radiation is conveniently categorized in to energetic charged particle radiation and uncharged radiation. Charged particles are fast electrons, heavy charged particle and uncharged radiations are electromagnetic radiation and neutrons.

Nuclear radiation is detected only through their interaction with matter. These radiations lose energy while traversing matter in qualitatively different ways. The energetic charged particles (electron, mesons, proton, and alpha-particle and heavy ion) interact with matter primarily by coulomb interaction with the electrons of the medium. The uncharged particle can not interact with Coulomb force; therefore, their detection is made possible by first liberation of an energetic charged particle then detection of the particle.

Radiation detector (scintillation crystal + PM tube) are an instrument used to detect or measure radiation. Radiation detectors depends basically on the interaction of incident radiations with the detector materials, which produces detectable out put signals. For each types of radiation, there is one or more suitable type of detectors and detection system. Radiation detectors are classified as detection of free charge carriers, light sensing, detection of free charge carriers and light sensing and track visualization.[3]

This project introduces the detection efficiency of scintillation detectors of sodium iodide activated with Thallium. Detection efficiency is the ratio of the number of particle or photon recorded by the detector to the number of particle or photon emitted

by the source, and this is called *absolute efficiency*. And the ratio of the number of particle or photon recorded by the detector to the number of particle or photon striking the detector is called *Intrinsic Efficiency* (ε_{intr}). The intrinsic efficiency is dependent on the solid angle (Ω) of the source-detector geometry arrangement. The main focus of this project is to determine theoretically the intrinsic efficiency of NaI(Tl) right cylindrical scintillator with different size and various sources to detector distances of different gamma ray energy. The geometry of the detector (length, diameter and shapes) can vary the intrinsic efficiency of the detectors. This variation in intrinsic efficiency can be analyzed by the mean chord length (l_m) which depends on the solid angle that the point source can have from the detector surface.

In Chapter one of this work, different types of radioactivity such as alpha, beta and gamma decays of radioactive process is discussed. In addition to this different radiation source; Auger electron and bremsstrahlung is explained.

In chapter two, the interaction of gamma radiation with matter mainly photoelectric absorption, Compton Effect and pair production of the main interaction of photon is discussed in details.

The scintillation of sodium iodide activated with Thallium detector of right cylindrical shape, mechanism of scintillation, photomultiplier have been explained in chapter three. The mean chord length of interaction photon in detector is calculated by considering the solid angle formed by the source from detector surfaces. The limiting case as the source goes to infinity and put on the detector surface is also evaluated.

Finally, I have calculated the intrinsic efficiency using linear attenuation coefficient and mean chord length of different solid angle of different size NaI(Tl) detectors in, and the gamma ray energies 0.1, 0.2, 0.4, 0.6, 1, 5, 7, and 10 MeV taking in to account with various source to detector distance from 0-10cm.

Chapter 1

RADIOACTIVITY

1.1 INTRODUCTION

Different combination of proton and neutron form a nucleus. Only certain combinations of this nucleon produce stable nuclei. Where others do not stable. Unstable nuclei decay by emitting alpha (helium nucleus), beta (electron or positron), and gamma ray radiation. The emission of these radiations by unstable nuclei in their decay process is called **Radioactivity**. The decay process is called *radioactive decay*. The Nuclei, which emit these radiations, are called *radioactive nuclei*. If the nuclei which emit these radiations are found in nature then their decay is called Natural radioactivity; but if they are produced in the laboratory, then their decay is called artificial radioactivity.[2]

Not all the three radiations (alpha, beta, and gamma) are emitted by the same nucleus in Radioactivity. Usually, a radioactive nucleus emits alpha-particle and gamma rays or beta-particle and gamma -rays.

Radiation is a process of emission of energy or particles. Various forms of radiations may be distinguished, depending on the type of the emitted energy matter, the types of the emission source, properties and purpose of the emission.

Radiations; heat and light are radiations that we can feel or see directly. But there are other kinds of radiations such as gamma ray, x-ray and neutrons that human can not recognize or feel it directly.[3]

Radiation are mainly classified in to *four* groups

- *Heavy charged* particles including all particles with mass equal to or heavier then one atomic mass unit (amu) such as alpha particles, proton and fission products.
- *Charged particles* including beta particles (negative electrons),positron (positive electron),internal conversion electron and auger electron.
- *Electromagnetic radiation* including gamma rays (following beta particle decay or nuclear reaction), characteristic x- ray, Annihilation radiation and Bremsstrahlung.
- *Neutrons* including fast neutrons, intermediate neutrons,epithermal neutron, thermal neutron and cold neutrons.

Radioactivity decay is statistical in nature i.e. it is impossible to predict when any given atom will disintegrate, and this hypothesis leads directly to the exponential law with great accuracy. When a radioactive sample containing N_0 nuclei at time $t = 0$ decays, the nuclei N after time t is given by

$$N = N_0 e^{-\lambda t} \quad (1.1.1)$$

Where λ is called the radioactive decay constant, which is just the probability of decay per unit interval of time. The time interval during which half of the given sample of radioactive substance has been decay ($N = \frac{1}{2}N_0$), is called half-life ($t_{\frac{1}{2}}$) of the substance which calculated as

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \quad (1.1.2)$$

By taking logarithms of equation (1.1.2) and solving for $t_{\frac{1}{2}}$.

$$-\ln 2 = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{0.693}{\lambda} \quad (1.1.3)$$

The average life (τ) expectancy of a particular radioactive nuclei is found by the sum of the times of existence of all the nuclei divided by their initial number.

$$\begin{aligned} \tau &= -\frac{1}{N_o} \int_{t=0}^{t=\infty} t dN \\ \tau &= \frac{1}{\lambda} \end{aligned} \quad (1.1.4)$$

1.2 ALPHA DECAY

The alpha-particle (α) is made up of two neutrons and two protons. Alpha particle emission is the characteristic of heavy nuclei with atomic number greater than 82 with few exceptions. Alpha particles a two neutron and two proton system, shortly Alpha particle is a helium nucleus 4_2He . Alpha particle is a large particle compared to beta particle or gamma rays. There fore, a nucleus able to emit alpha particle must it self be large.

A given nuclide often emits alpha particles with a number of different energies. This particles emitted in a particular decay are mono energetic. The energy spectra of alpha particles consist of a series of sharply defined lines corresponding to decay of the initial nucleus to various energy levels of the product nucleus.

Alpha particle cause intense initiation when passing through substance and they are easily observed. They can be absorbed by a sheet of paper or by an aluminum foil of about 0.04 mm thick or several centimeter of air.

There is a very strong correlation between alpha particle energy and half-life of the parent isotope. An alpha active element with shorter half life emits alpha particle of higher energy. And an alpha active nucleus with large half-life emits alpha particles of lower energy.

The alpha particles, emitted by an atom, can only come from the nucleus and the nucleus must therefore lose two positive charge and some mass equal to that of the helium nucleus. If Z is atomic number and A is atomic mass of initial nucleus, then the emission of alpha particles gives $Z-2$ atomic number and $A-4$ atomic mass of final nucleus. The decay process is written schematically as follow.[4]



Where x and y are the initial and final nuclear species respectively and Q_α is the amount of energy released in the processes. This energy is shared between the alpha particle emitted and residual nucleus in accordance with the laws of conservation of energy and linear momentum. Because alpha particles light particle compared to the residual nucleus y , it carries most of the energy.

1.3 BETA DECAY

Beta decay is a process where an electron (Beta-minus) or a positron (Beta -plus) is ejected from the nucleus or an orbital electron is captured by the nucleus. Basically the nucleus contains only neutrons and protons. But in beta decay, it is imagined that the electron or positron are created at the time of emission.

The emission of positrons or electron constitutes the main method for restoring nuclide to the stability of nuclei by beta decay. This decay changes the atomic number Z by one up or down. For positron and electron capture emission the atomic number of the residual nucleus decreases by one and for electron β^- emission the atomic number of the residual nucleus increase by one. But the mass number almost unchanged ($p \cong n$). This kind of transformation in which the mass number remains unchanged and the atomic number changed is called isobaric.[1]

The electron (β^-) involved in beta decay has been shown to be equivalent with atomic electron, but electron or positrons are emitted with a continuous spectrum of energies, whilst the atomic electron has discrete energy spectrum. The continuous spectrum of beta particles implies emission of a further mass less, changeless particle called *neutrino* (ν). This particle has got some typical properties. A neutrino is assumed to have a very small rest mass compared to the rest mass of electron; it is also fermions like-spin, no charge and stable particle moving with velocity of light. The existence of neutrino in beta decay solves the problem of conservation of energy, angular momentum, and linear momentum.

The decay process is written schematically within the nucleus as follows

$$n \rightarrow p + e^- + \bar{\nu} + Q_{\beta^-} \quad (1.3.1)$$

$$p \rightarrow n + e^+ + \nu + Q_{\beta^+} \quad (1.3.2)$$

$$p + e^- \rightarrow n + \nu + Q_{\beta} \quad (1.3.3)$$

When a nucleus has an excess of neutrons that is if its neutron: proton ratio is more than the value for the stable nucleus, then it will be unstable because of the deficiency of proton such a nucleus will achieve stability by changing its neutron into proton. It is represented by equation 1.3.1.

When a nucleus has more protons than permitted by the neutron: proton ratios for stable it will be unstable such a nucleus will achieve stability by converting its proton into neutron. It is represented by equation 1.3.2.

Nucleus, which has deficiency in neutron but has sufficient proton, decays either by β^+ -decay or electron capture. In electron capture a nucleus captures one of its own orbital electrons and emits a neutrino. Because a k-electron is captured in about 90 percent of the events, this process is often called k-capture. In some cases where

sufficient energy (1.022MeV) is not available for the creation of β^+ , electron capture is the only process of decay. Both these processes (electron capture and β^+ -decay) usually occurs in the same nucleus. The electron capture process is represented by equation 1.3.3.

Each of specific beta decay is characterized by a fixed energy or Q-value because the energy of the recoil nucleus is virtually zero; this energy is shared between the beta particle and the neutrino and /or anti-neutrino $\bar{\nu}$. The Beta particles thus appears with an energy that varies from decay to decay and can range from zero to the maximum (beta end point energy).

1.4 GAMMA DECAY

Gamma radiation is emitted by excited nuclei in their transition to lower-lying nuclear level (a nucleus may be produced in excited state by ($\beta^{+/-}$ or alpha-decay). Gamma rays are the most penetrating compared to alpha particle or beta particle. The energy spectrums of Gamma ray are discrete (quanta) energy.

Nucleus that disintegrates by gamma emission does not change in charge number and mass number since both the mass number and charge of a gamma ray is zero. When a nucleus decays by alpha emission or beta emission or as a consequence of nuclear reaction it is left in and excited state. De-excitation takes places through the emission of a gamma-ray whose energy is essentially equal to the difference in energy between the initial and final nuclear state.

The Gamma decay process is written as schematically as follow.

When the electromagnetic radiations, gamma rays and bremsstrahlung radiation, travel with the velocity of light, they are considered as particle and called photons. Gamma rays and x-rays have well defines energies, mono energetic, and have different

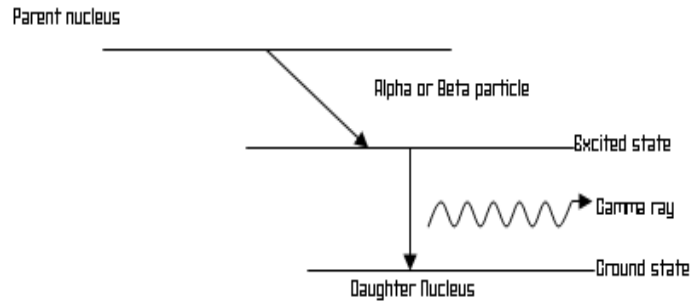


Figure 1.1: Schematically diagram of Gamma decay

origins. Gamma rays originated from the nuclei, while x-rays have originated from the atoms.

The relationship between energy (E), frequency ν , and wave length λ of gamma ray (photons) is given as

$$E = h\nu = h\frac{c}{\lambda} \quad (1.4.1)$$

Where h is Planck's constant.

1.5 DECAY SCHEMES

Decay scheme of a radioisotope is representation of its decay energy levels, properties of energy levels, life time, and branching ratio, energy of different radiations, parity

and spin of different levels. Radioactive decay may occur by one of the decay type as Alpha, Beta (β^- , β^+ , electron capture), and Gamma-decay or by a combination of several decays. The decays scheme of nuclei which decay by single mode is simple, but the decay schemes of nuclei which decay by a combination of several processes are complex. Transformation of beta and alpha are usually accompanied by gamma rays, and the presence of the gamma ray means that the product nucleus is formed in an excited state and passes to its ground state by emitting one or more gamma rays. The energy of gamma-ray is equal to the difference in energy between the initial and final nuclear states.

For example Cesium-137 ($^{137}_{55}\text{Cs}$) has a half life of 30 years and decays by beta emission to two states of barium, a Meta stable $^{137}_{56}\text{Ba}$ and $^{137}_{56}\text{Ba}$ in the ground state. Most frequently, 93.5percent of the time, the Cesium decays to barium meta stable states by emission of a low energy 514KeV beta particle. The Meta stable state is short lived (half life 2.55 minutes) and subsequently emits a 661.6Kev gamma ray in a transition to its ground state. Cesium is thought as a gamma emitter.[11] Examples widely used as gamma-ray calibration sources are illustrated as figure 1.2. The decay schemes of ^{22}Na , ^{57}Co , ^{60}Co and ^{137}Cs shown as follow.[3]

1.6 GAMMA SOURCES

Gamma radiation is emitted by excited nuclei in their transition to lower-lying nuclear levels. The excited nuclear states are created in the decay of apparent radionuclide, the emission of beta or alpha particle. Table 1.3 list some sources of known nuclide emitting gamma rays.[6]

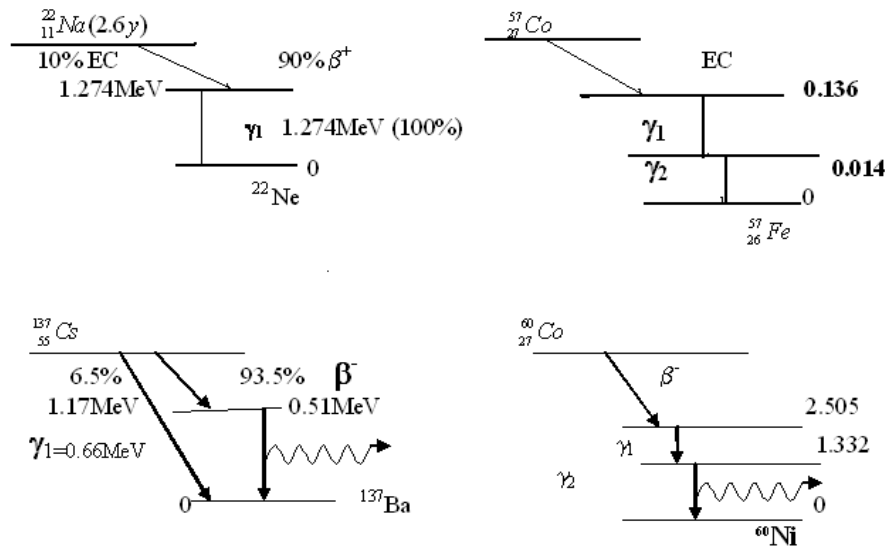


Figure 1.2: Decay schemes for some common Gamma references

1.7 BREMSSTRAHLUNG, CHARACTERISTIC X-RAY AND AUGER ELECTRON

BREMSSTRAHLUNG

Bremsstrahlung (Brems=breaking; Strahlung=radiation) is the process when fast electrons interact in matter, part of their energy is converted into electromagnetic radiation. The fraction of the electron energy converted into bremsstrahlung increases with increasing electron energy and is largest for absorbing materials of high atomic number.

For mono energetic electron that slow down and stop in a given material, the bremsstrahlung energy spectrum is a continuum with photon energies that extend as high as the electron energy itself. The emission of low energy photons predominates, and the average photon energy is a small fraction of the incident electron energy.

Source (Element)	Gamma Ray Energy in KeV
^{60}Co	14.359, 0.123, 0.14
^{203}Hg	70.830, 77.871, 82.572, 84.918
^{137}I	80.164, 254.307, 364.493
^{89}Sr	513.65
^{227}Bi	569.62, 1063.573
^{225}Th	583.139, 261.447
^{137}Cs	661.632
^{24}Mn	834.861
^{90}Y	893.01, 1836.11
^{60}Co	1173.226, 1332.483
^{22}Na	510.979, 1274.552
^{24}Na	1368.526, 2753.92

Figure 1.3: Gamma radiation sources

Because these spectrums are continua, they cannot be applied directly to the energy calibration of radiation detectors.

Bremsstrahlung is also produced by other sources of fast electrons, including beta particles. Therefore, some bremsstrahlung photons are generated by any beta active isotope encapsulated to stop the beta particles.[3]

CHARACTERISTIC X-RAYS

If the orbital electrons in an atom are disrupted from their normal configuration by some excitation process, the atom may exist in an excited state for a short period of time. There is a natural tendency for the electrons to rearrange themselves to return the atom to its lowest energy or ground state within a time that is characteristically a nanosecond or less in a solid material. The energy liberated in the transition from the excited to the ground states takes the form of a characteristic x-ray photon whose energy is given by the energy difference between the initial and final states. If

the vacancy is temporarily created in the K shell of an atom, then a characteristic K x-ray is emitted when that vacancy is subsequently filled. If the electron comes from the L shell, then the photon produced whose energy is equal to the difference in binding energies between the K and L shells. In general the excitation of the atom arises due to excitation by radioactivity decay and excitation by external radiation.

AUGER ELECTRON

An Auger electron is produced when an excitation of the atom is transferred directly to one of the outer electrons, causing it to be ejected from the atom, and the energy of the Auger electron is the difference between the original atomic excitation energy and the binding energy of the shell from which the electron was ejected. The energy of an Auger electron is a discrete spectrum; such a process is also observed when electron capture takes place, which may leave the atom with a vacancy in a normally complete electron shell. This vacancy is filled by an electron from one of the outer shells of the atom with the emission of a characteristic x-ray photon.[3]

Chapter 2

INTERACTION OF GAMMA RAY WITH MATTER

2.1 GENERAL INTRODUCTION OF INTERACTION

Gamma ray can transfer all or parts of its energy to electrons within the medium or matter with which it interacts. The resulting interaction is secondary electrons. Devices designed to detect gamma rays are failure to promote such interactions and to fully stop the resulting secondary electrons so that their energy may contribute to the output signal. If the interaction does not occur within the detector the gamma ray can pass completely through the detector active volume without revealing the slightest hint that they were ever there. There are a large number of possible interaction mechanisms known for gamma rays. But only three major types play an important role in radiation measurement. They are *photoelectric absorption*, *Compton scattering*, and *pair production*. All these processes lead to the partial or complete transfer of the gamma-ray photon energy to electron energy. They result in sudden and abrupt changes in the gamma-ray photon history, in that the photon either disappears entirely or is scattered through significant angle.

2.2 PHOTOELECTRIC ABSORPTION

In the photoelectric absorption process, a photon undergoes an interaction with absorber atom in which the photon completely disappears. In its place an energetic photon electron is ejected by the atom from one of its bound shells. The interaction is with the atom as a whole and can not take place with free electron. For gamma rays of sufficient energy, the most probable origin of the photo electron is the most tightly bound or K-shell electron of the atom. The photo electron appears with an energy given by

$$E_{e^-} = h\nu - E_b \quad (2.2.1)$$

Where E_{e^-} =photo electron kinetic energy

E_b =Binding energy of photoelectron in its original

h =Planck's constant. ν =Frequency of gamma ray

Schematically diagram for photoelectric absorption is given by figure 2.1.

Because of the interaction also creates an ionized absorber atom with a vacancy

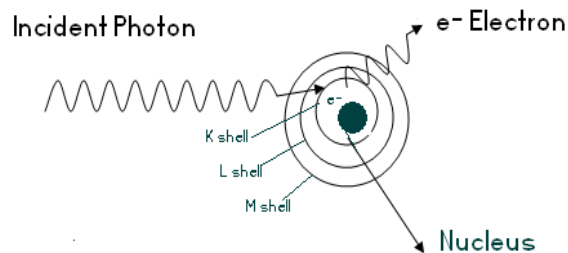


Figure 2.1: Schematic diagram of photoelectric absorption

in one of its bound shells, this vacancy is quickly filled through capture of a free electron

from the medium and/or rearrangement of electron from either shells of the atom. Therefore, one or more characteristic x-ray photon may also be generated. Although in most cases these x-ray are reabsorbed close to the original site through photoelectric absorption involving less tightly bound shells, their migration and possible escape from radiation detector and can influence their response.

Photoelectric coefficient (τ), the probability of photoelectric absorption per unit length, depends on the photon energy E_γ and the absorber atomic number (Z). The photoelectric process is the predominant of interaction for gamma rays (x-ray) of relatively low energy (100KeV-500KeV). No single analytical expression is valid for the probability of photoelectric absorption per unit length over energy ranges of E_γ and Z , but rough approximations is

$$\tau(m^{-1}) \cong constant \times \frac{Z^n}{E_\gamma^m} \quad (2.2.2)$$

Where: n and m are constant values varies between 3 and 5.

2.3 COMPTON SCATTERING

The scattering of an incident photon by an electron of the target materials resulting in the transfer of only a fraction of photon energy and momentum to the electron (assumed to be initially at rest) is known as *Compton scattering*. In Compton scattering, the incoming gamma-ray photon is deflected through an angle with respect to its original direction. Because all angles of scattering are possible, the energy transferred to the electron can vary from zero to a large fraction of the photon energy.

The expression that relate the energy transfer and the scattering angle for any given interaction can simply be derived by writing simultaneous equation for the conservation of energy and momentum. Therefore, the energy of the scattered photon is

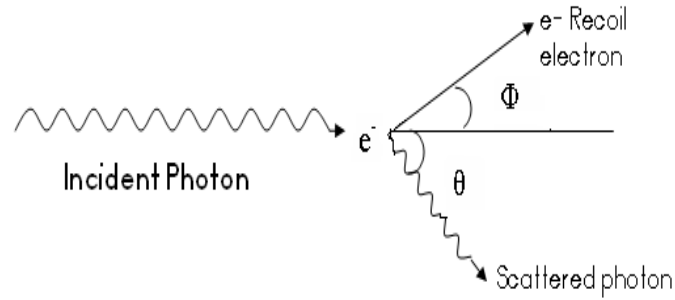


Figure 2.2: Schematic diagram illustrate Compton scattering.

given by

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2} \times (1 - \cos\theta)} \quad (2.3.1)$$

Where m_0c^2 is the rest mass energy of the electron(0.511MeV).

ν' is the frequency of scattered photon.

The energy of the scattered electron is given by

$$\begin{aligned} T &= h\nu - h\nu' \\ &= E_\gamma \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)} \end{aligned} \quad (2.3.2)$$

Where $\alpha = \frac{h\nu}{m_0c^2}$. For small scattering angle θ , very little energy is transferred. Some of the original energy is always retained by the incident photon. The maximum energy transfer T_{max} corresponding to $\theta = 180^\circ$ and is given by

$$T_{max} = \frac{h\nu}{1 + \frac{1}{2\alpha}} \quad (2.3.3)$$

The probability of Compton scattering per atom of the absorber depends on the number of electron available as scattering targets and there fore increases linearly with Z.[8]

The differential scattering cross-section for unpolarized photon is given by the Klein-Nishina formula as follow

$$\frac{d\sigma_c}{d\Omega} = Zr_o \left(\frac{1}{1 + \alpha(1 - \cos\theta)} \right)^2 \cdot \left(\frac{1 + \cos^2\theta}{2} \right) \cdot \left(1 + \frac{\alpha^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)[1 + (1 - \cos\theta)]} \right) \quad (2.3.4)$$

Here r_o is the classical electron radius. The corresponding average collision cross-section representing the integrated probability that some scattering event will occur per target electron given by

$$\sigma_c = 2\pi r_o^2 \left[\frac{1 + \alpha}{\alpha^2} \cdot \left(\frac{2(1 + \alpha)}{1 + 2\alpha} \right) - \frac{1}{\alpha} \ln(1 + \alpha) + \frac{1}{2\alpha} \ln(1 + 2\alpha) - \frac{1 + 3\alpha}{(1 + 2\alpha)^2} \right] \quad (2.3.5)$$

For $h\nu \gg m_o c^2$, the cross-section for Compton scattering is given approximation as

$$\sigma_c = \frac{\pi r_o^2 Z}{2h\nu} \left(\ln 4h\nu + \frac{1}{2} \right) \quad (2.3.6)$$

This show the cross-section is proportional to the atomic number Z and varies inversely as the photon energy. Compton Effect predominant in the energy ranges 100KeV to 1.0MeV.

Compton scattering coefficient (σ) is the probability of occurrence per unit length .and therefore given approximated by

$$\sigma(m^{-1}) = NZf(E_\gamma) \quad (2.3.7)$$

Where, $f(E_\gamma)$ is a function of E_γ .

2.4 PAIR PRODUCTION

When the incident photon energy exceeds twice the rest mass energy of an electron ($2m_0c^2$), the process of pair production is energetically possible. The probability of this interaction remains very low until the gamma-ray energy approaches several MeV and therefore pair production is predominantly confined to high energy gamma rays. In the interaction (which must take place in the coulomb field of nucleus), the gamma-ray (photon) disappears and is replaced by an *electron-positron pair*. All the excess energy carried in by the photon above the 1.02MeV required to create the pair goes in to kinetic energy shared by the positron and electron. The schematic representation of the pair production process in the field of a nucleus is shown in the figure 2.3.

If T_- and T_+ are the kinetic energies of the emitted electron and positron, it

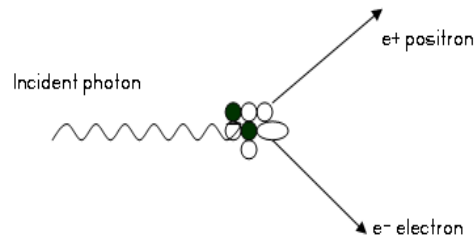


Figure 2.3: Schematic diagram of illustrate pair production.

follows from energy conservation that

$$T_- + T_+ = h\nu - 2m_0c^2 \quad (2.4.1)$$

Because the positron will subsequently annihilate after slowing down in the absorbing medium, two annihilation photons are normally produced as secondary products of interaction. The subsequent take of this radiation has an important effect on the response of gamma-ray detector.

For typical energies, both the electron and positron travel a few millimeters at most before losing all their kinetic energy to absorbing medium.

The relative importance of the three processes described above for different absorber materials and gamma ray energies is conveniently illustrated in the figure 2.4.

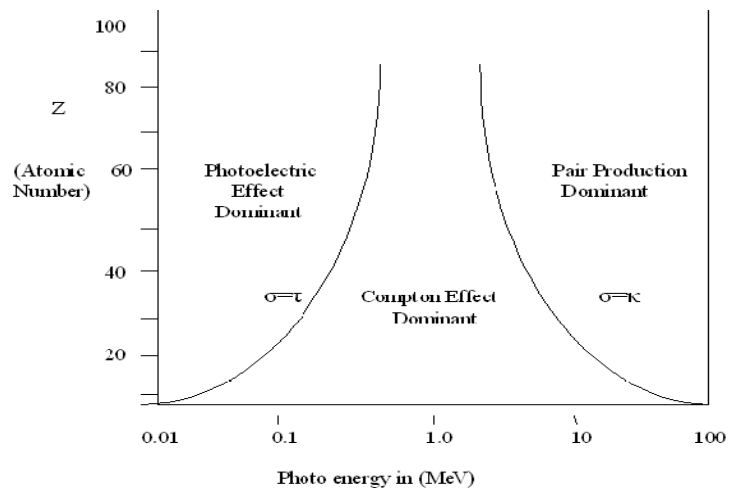


Figure 2.4: Relative importance of the three major types of gamma ray interaction (The graph is not drawn to scale)[6]

2.5 GAMMA RAY ATTENUATION

When x- and γ -ray impinges upon a materials, there are three possible out comes.

The photon:[6]

- 1 .Be absorbed (i.e.transfer its energy to atom of the target materials) during one or more interaction.
2. Be scattered during one or more interaction or
3. Traverse the material with out interaction. If the photon is absorbed or scattered it is said to be have been *attenuated*.

Attenuation of gamma-ray is proportional to the thickness of absorber and also

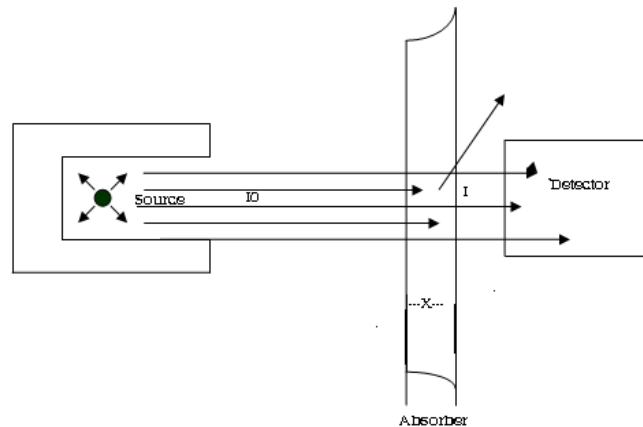


Figure 2.5: Geometrical representation of attenuation process collimated point source.

intensity I of gamma-ray.

$$dI \sim I dx \quad (2.5.1)$$

$$dI = -\mu I dx$$

Where, μ is linear attenuation coefficient.

$$\frac{dI}{I} = -\mu dx$$

$$\ln(I) = -\mu x + c$$

Where c is some integration constant and Negative is because $I_0 > I$

$$I = e^{-\mu x + c}$$

The number of transmitted gamma ray photons (I) through an absorber of thickness x , as shown in figure, from the incident gamma ray photons I_0 is given by

$$I = I_0 e^{-\mu_l x} \quad (2.5.2)$$

The exponent of e must possess no units. Therefore the unit for μ_l are $1/\text{cm}$ if the thickness x is expressed in centimeter, $1/\text{in}$ if X is expressed in inches and so on. An attenuation coefficient with units of $1/\text{length}$ is called a *linear attenuation coefficient* (μ_l). If ρ is an absorber density and we can put

$$\mu_l x = \mu_m \rho X$$

Where $\mu_m = \frac{\mu_l}{\rho}$ mass attenuation coefficient then equation 2.5.2 is can be re written as

$$I = I_0 e^{-\mu_m \rho x} \quad (2.5.3)$$

The half value thickness $X_{\frac{1}{2}}$, which is the value of the absorber that will reduce the intensity by a factor of 2. When $X = X_{\frac{1}{2}}$, $I = \frac{1}{2} I_0$ and following relationships lead to equation

$$I(x_{1/2}) = \frac{1}{2} I_0 = I_0 e^{-\mu_l x_{1/2}} \quad (2.5.4)$$

$$2^{-1} = e^{-\mu_l x_{1/2}}$$

$$\begin{aligned}
\ln(2^{-1}) &= \ln(e^{-\mu_l x_{1/2}}) \\
-\ln 2 &= -\mu_l x_{1/2} \\
\mu_l &= \frac{\ln 2}{x_{1/2}} = \frac{0.693}{x_{1/2}} \tag{2.5.5}
\end{aligned}$$

the main free path λ is defined as the average distance traveled in the absorber before an interaction takes places , and related to linear attenuation coefficient and half-value thickness $x_{1/2}$ by

$$\lambda = \frac{\int_0^\infty x e^{-\mu_l x} dx}{\int_0^\infty e^{-\mu_l x} dx} = \frac{1}{\mu_l} = 1.414 x_{1/2} \tag{2.5.6}$$

In macroscopic level, the incident photons may interact with the absorber material and their number decrease with increasing the thickness of the absorber, known as photon attenuation. photon attenuation is due to the main interaction mechanism of photon (photoelectric absorbtion, Compton effect and pair productions).that is photon completely absorbing and/or scattering. The linear attenuation coefficient (μ_l) is the probability per unit length that the photon is interacted and removed from the beam. The linear attenuation coefficient is the sum of the probabilities of the three main interaction mechanisms (photoelectric absorption, Compton scattering and pair-production) and given by

$$\mu_l = \tau(\text{potoelectric}) + \sigma(\text{Compton}) + \kappa(\text{pairproduction}) \tag{2.5.7}$$

Chapter 3

DETECTION OF GAMMA RAY

3.1 INTRODUCTION

Interaction of nuclear radiation with matter results in deposition of energy in the medium traversed by the radiation. Radiation detection relies on the sensing of this energy deposition through some suitable means. The collection of the free charges released in the medium resulting in a current or voltage pulse and the detection of the photon emitted during the de-excitation of excited or ionized atoms and molecules along the path of the incident radiation are two most convenient ways to sense the energy deposition. The neutral radiation (neutron, x-ray and gamma ray) which are not subject to the Coulomb force, deposit their energy in a medium in a two step process. The photons interact with the atom medium by any one of the well known process, namely photoelectric absorption, Compton scattering, and pair production. When the energetic photons impinge on a detector medium and interact by any of these process, energetic electrons are liberated which, being charged particles, can be detected through the resulting ionization or excitation of the atom of the medium.[3]

Ionization chamber, proportional counters, Geiger Muller counters and solid state detectors are devices in which the electric charge of the ion pair is collected and

amplified electronically. In scintillation counters the passages of the radiation through the scintillator produces excited states which decay by the emission of light in or near the visible part of the spectrum. And this light is converted in to electrical pulses by means of which the incident particles or radiation are detected.

3.2 SCINTILLATION DETECTOR

Certain materials scintillate on the passage of ionizing radiations through them. These materials (substance) are called *scintillator*. The interaction of different radiations with scintillator will be ionizing and excite its atoms and molecules. A large number of the absorbed energy will be transferred to heat, while after a short time; small percentage of the absorbed energy will be released due to scintillator atom de-excitation that produces *fluorescence light*, visible light pulses, known as *scintillation*. The light pulse (scintillation) is converted to *photoelectrons* that are magnified through the photomultiplier tube to electric signal. The prompt emission of visible light from a scintillator following its excitation due to energy absorption is known as fluorescence process. Delayed fluorescence has the same emission spectrum as the prompt fluorescence but with much longer emission of longer wave length of visible light than that of the fluorescence and generally with much slower emission time.[8]

The quality and suitability of scintillator as radiation detector depends on its ability to convert as large a fraction as possible of the incident radiation energy to prompt fluorescence and to minimize the delayed fluorescence and phosphorescence processes.

The quality of the scintillator as radiation detector is depends on the following properties.[3]

- It should convert the kinetic energy of charged particles into detectable light with a high scintillation efficiency.
- This conversion should be linear -the light yield should be proportional to deposited energy over as wide range as possible.
- The medium should be transparent to the wave length of its own emission for good light collection.
- The decay time of the induced luminescence should be short so that fast materials should be of good optical quality and subject to manufacture in sizes large enough to be of interest as particular detector.
- Its index of refraction should be near the glass (1.5) to permit efficient coupling of the scintillation light to a photomultiplier tube or other light sensor.

No material simultaneously meets all these criteria, and the choice of particular scintillation is always a compromise among these and other factors. One chooses a scintillator which is most suitable for the specific application.

Scintillators are divided in two classes, based on their physical structure; Organic and Inorganic. In general, organic scintillators are widely used for beta and alpha measurement, and are available in liquid and solid forms. They have faster response than the inorganics, but produce less light, and yield less photoelectric events due to the low atomic number Z of the materials. Inorganic materials are crystalline, transparent, and doped with an impurity. The most widely used is sodium iodide; doped with Thallium NaI (Tl). It is linear in its response to various energy gamma.

Anthracene, (C₁₄H₁₀), Trans-stilbene (C₁₂H₁₄), Toluene (C₆H₅.CH₃), P-xylene (C₆H₄.(CH₃)₂), p-terphenyl (C₁₈H₁₄) are some commonly used organic scintillator compounds and NaI(Tl), CsI (Tl), CsF, LiI (Eu), CaF₂(Eu), Bi₄Ge₃O₂ are some commonly used Inorganic scintillator.

3.3 DETECTION OF GAMMA-RAY USING NaI (Tl) SCINTILLATION COUNTER

A scintillation counter is composed of four main components; a scintillator, a light guide, a photomultiplier tube and the electronics in the photomultiplier and the recorder of the signal.

3.3.1 NaI (Tl) SCINTILLATOR

Sodium iodide activated with thallium is a water clear, hygroscopic crystal having the general properties at room temperature as the table below.[10]

The NaI (Tl) scintillator is a single crystal of alkali alides. Single crystal is needed

Scintillate matter	Wave length max. emission (nm)	Decay time (ns)	Index of refraction at wave length of max. emission	Density (g/cm ³)	Relative scintillation eff.
NaI (Tl)	410	230	1.85	3.67	100
NaI	303	60 (at 80 ⁰ K)	1.85	3.67	200(at 80 ⁰ K)

Figure 3.1: Properties of NaI and NaI (Tl).

to obtain transparency. A polycrystalline is result in reflection and absorption at crystal faces.

The interaction of the atom in a crystal cause the discrete energy level smearth out into a series of energy bands. The two highest bands are the valence band and conduction band. In insulating materials such as NaI, the valence band state is generally full and the conduction band states are empty. An electron may receive sufficient energy to be lifted from its normal position in the valence band across the forbidden band gap in to the conduction band, leaving a hole in the normally filled valence band.

If the electron has sufficient energy to be also lifted out of its lattice site (ionization process), both the electron and the hole are free to migrate independently through out the crystal. In an alternate possibility, the energy imparted to the electron is not sufficient to detach it from its lattice site but it is bound electro statically to the hole in the valence band. This process is referred to as excitation and the resulting loosely bound electron-hole pair is known as an *exaction*. An exaction can also migrate through the crystal lattice. In alkali halide, the forbidden band gap is about 6-8eV. In pure crystal (NaI), the return of the electron to the valence band with the emission of photon is an efficient process and the energy of the photons emitted is also too high to lie in the visible range.

To increase the probability for photon emission and to reduce self-absorption of light, small amount of impurities called activators are added to the crystal. A commonly used activator is *thallium* (Tl), and so these detectors are indicated as, NaI (Tl). The function of the thallium is to shift the wave length of the photon emitted by the excited molecules to a value which is not absorbed by the crystal. Typical Tl atom concentrations are about 0.1percent. The activator provides states in the energy gap and the light emission takes places between the activator states, as shown in the right side of figure 3.2. As a result, there will be states created with

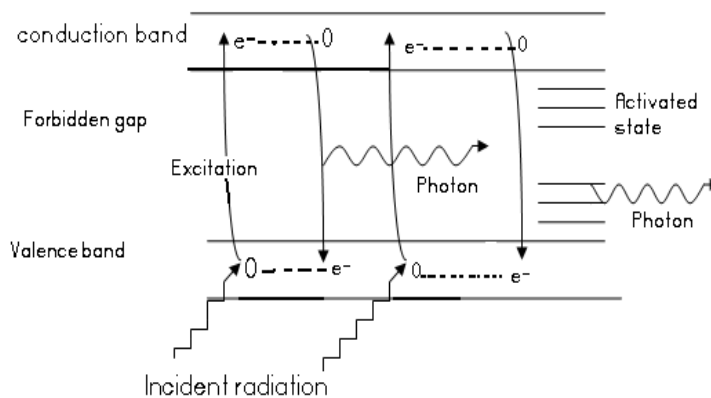


Figure 3.2: Energy band in the crystal

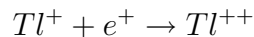
in the forbidden gap through which the electron can de-excited back to the valence band. Because the transition energy is less than that of the forbidden gap, these transitions give a change in wave length from ultraviolet region to the visible light (photon) region.[8]

MECHANISM OF SCINTILLATION IN NaI (Tl) SCINTILLATOR

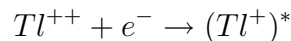
It is estimated that in the case of NaI(Tl) scintillator, the light out put is very close to one photon per electron-hole pair originally formed. The emission spectrum of pure sodium iodide which has a maximum at a wave length of approximately 303nm overlaps appreciably with its absorption spectrum. This results in a considerable self absorption of the scintillation photons in a pure sodium iodide crystal. The addition of small amount of thallium has the effect of shifting the scintillation spectrum to longer wave length corresponding to a maximum in the spectrum at about 410nm. An activated NaI (Tl) crystal is transparent to its scintillation light.

In a thallium activated crystal, the activator sited is normally present as Tl^+ ion in the lattice are substitutional. The holes created by the passing charged particle

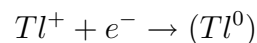
quickly drift to the activator sites and ionize them forming Tl^{++} because the ionization energy of the activator impurity atom is less than that of the normal lattice atom.



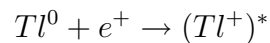
The free electron also migrates through the crystal and on encountering an ionized activator, drop in to the activator site forming an excited configuration $(Tl^+)^*$.



An other process leading to the formation of excited activator sites is one in which a free electron first diffuses to an activator site Tl^+ forming a Tl^o configuration.



Then a hole recombines with the Tl^o to give excited $(Tl^+)^*$. (* is stand for excitation, e^+ for Hole)



If $(Tl^+)^*$ has an allowed transition to the ground state, its de- excitation will occur quickly with a high probability for visible photon emission. Typical half lives of such excited states are of the order of 10^{-7} second. Because the hole and the electron migration lives are much shorter, the decay time of the excited activator states determines the rate of emission of the scintillation light.[8]

In case the transition to the ground state from the $(Tl^+)^*$ configuration is forbidden, the de- excitation can occur only through an additional increment of energy such as thermal excitation. The resulting slow component of light, called *phosphorescence* is a significant source of back ground light in inorganic scintillators.

3.3.2 PHOTOMULTIPLIER TUBES

Photomultiplier tube is an outer envelope (usually glass) serves as a pressure boundary to sustain vacuum condition inside the tube that are required so that low energy electrons can be accelerate efficiently by internal electric field. The two major components inside the tube are a photosensitive layer, called *photocathode* coupled to an *electron multiplier* structure. The photocathode serves to converts as many of the incident light photons as possible into low energy electrons. The electron multiplier section in PM tube provides efficient collection geometry for the photoelectrons as well as serving as a near ideal amplifier to greatly increase their number. After amplifications through the multiplier structure, a typical scintillation pulse will give rise to $10^7 - 10^{10}$ electrons, sufficient to serve as the charge signal for the original scintillation events. This charge is conventionally collected at the anode or out put stage of the multiplier structure.[7]

The photocathode

The photocathode is made of materials with exhibits photoelectric effect, that is, it emits electron when bombarded by light quanta. The process of photo emission can be thought of as occurring in three sequential steps:[3]

1. The absorption of the incident photon and transfer of energy to an electron with in the photo emissive materials.
2. The migration of that electron to the surface and
3. The escape of the electron from the surface of the photocathode.

The energy that can be transferred from the photon to an electron in the first step is given by the quantum energy of the photon . In step 2, some of the energy lost through electron-electron collision in the migration process. In step 3, there must be

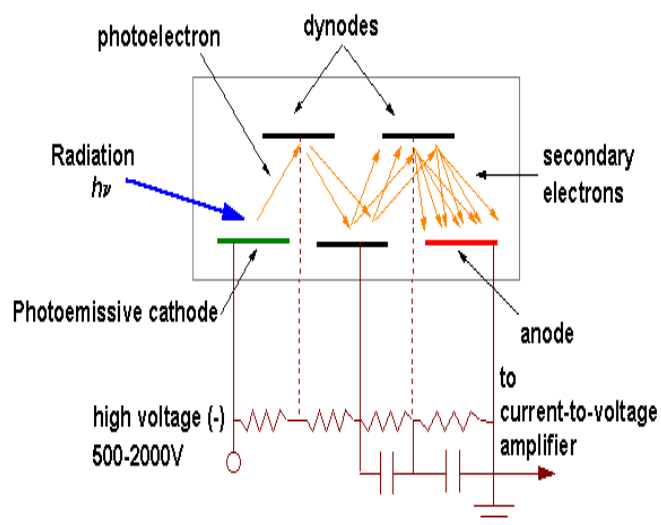


Figure 3.3: Schematic diagram illustrating the photomultiplier tube operation.

sufficient energy left for the electron to overcome the inherent potential barrier that always exists at any interface between materials and vacuum. This potential barrier (often called the work function) is normally greater than 3-4eV for most metals but can be as low as 1.5-2eV of suitably prepared semiconductor. These electrons travel towards an electrode that is maintained at a positive potential with respect to the photocathode. The fluctuation in the current produced by the impact of light quanta upon the photocathode are so weak, that a high degree of amplification is needed before use can be made of them and this is where the multiplier tubes come in. The photomultiplier tube operates on the principle of secondary emission. Certain materials when struck by an electron emit one or more electrons and in certain cases, a great many. This process is put to work a number of times in successive in the multiplier tubes. A single electron ejected from the photocathode by an incident light quanta in turn liberates a number of electrons from the secondary emission electrode (the first

dynode) and this repeated ten times or more, results in an avalanche of electrons.

The amplitude of the pulse appearing at the out put of a photomultiplier tube is proportional to the energy of the incident nuclear radiation. The pulse height V at the out put will be related to the amount of charge collected at the out put through the relation.

$$V = \frac{Q}{C} \quad (3.3.1)$$

Where C is the capacity of the out put point and Q is the charge.

If the incident nuclear radiation has energy E_n , a fraction f_n of this energy will be dissipated in the scintillator. If this energy $f_n E_n$ is converted in to light energy by the scintillator with an efficiency ε_l then $\varepsilon_l f_n E_n$ is the amount of light energy produced in the scintillator. if f_p is the fraction of the photon which reach photo cathode then an amount $f_p \varepsilon_l f_n E_n$ of light energy reaches the photocathode. Let f_m denotes the number of photoelectrons emitted by the photocathode when a 1.0eV of light energy strikes it and let f_d be the fraction of photoelectrons collected by the dynode. If the electron multiplication factor is M , then the total number of electron reaching the final stage will be

$$N_{e^-} = M f_n f_m f_p \varepsilon_l f_n E_n$$

and

$$Q = N_{e^-} e = e M f_n f_m f_p \varepsilon_l f_n E_n \quad (3.3.2)$$

This implies that

$$V \sim E_n$$

The final pulse height in a scintillation detector is proportional to the initial energy of the incident particle.[7]

3.4 DETECTION EFFICIENCY

All radiation detectors will in principle, give rise to an out put pulse for each quantum radiation that interacts with its active volume. For primary charged radiation such as alpha and beta particles, interaction in the form of ionization or excitation will takes places immediately upon enter of the particles in to the active volume. After traveling small fraction of its range, a typical particle will form enough ion pair along its path to ensure that the resulting pulse is large enough to be recorded. Under this condition, the detector is said to have a counting efficiency of 100 percent.[3]

For uncharged radiation such as gamma rays or neutrons must first undergo a significant interaction in the detector before detection is possible. Because this radiation can travel large distance between interactions, detectors are often less than 100 percent efficiency. The over all detection efficiency of specific detector set up depends on the solid angle subtended by the radiation source at the scintillator, and the intrinsic detection efficiency of the scintillator. The intrinsic efficiency (some times called Geometric efficiency) of detector usually depends primarily on the detector materials, the radiation energy, the physical thickness of the detector in the direction of the incident radiation and slightly depends on the distance between the source and detector.[3]

The size and shape of the scintillation crystal have a strong influence on the counting efficiency. So the geometry and shape of the scintillator crystal is commonly given in the form of *diameter* x *height*. The result of the efficiency, therefore specific to the detector geometry and gamma ray energy assumed and can not be generalized further. In the following we discuss only the *intrinsic efficiency* of solid right circular cylindrical crystal of NaI (Tl) scintillator.

3.5 DEFINITION OF EFFICIENCY

The detection efficiency of a scintillator crystal must be known if any quantitative measurement is desired. The efficiency of scintillator detector is determined by the following factor.[6]

1. The energy of gamma rays
2. The types of scintillator crystal
3. The types of geometry used. This includes the Size of the crystal, the dimension of the source, the relative position of source and the crystal.

The efficiency usually defined in two types, *total* and *intrinsic* efficiency.

The total efficiency of the spectrometry is defined as:[3]

$$\varepsilon_{total} = \frac{\text{Number of counted photons}}{\text{Number of photons emitted by the source}} \quad (3.5.1)$$

And the intrinsic efficiency of the spectrometry is defined as

$$\varepsilon_{intr} = \frac{\text{Number of counted photons}}{\text{Number of photons entered to detector}} \quad (3.5.2)$$

The schematically diagram of NaI (Tl) scintillator and source shown as figure 3.4. From the definition of intrinsic efficiency given above if I is the number of photon pass across the NaI (Tl) scintillator and I_o is the number of photon entered the NaI (Tl) scintillator. The number of photon attenuated I_{at} in a medium depends on the number transverse the medium. if all the photons posses the same energy(i.e. the photon is monoenergetic) and if the photons are attenuated under condition of good geometry(i.e. the beam is narrow and the transmitted beam contains no scattered photons),then the number I_{at} of photon attenuated (absorbed or scattered) from the beam is

$$I_{at} = I_0 - I$$

$$I_{at} = I_0 - I_0 e^{-\mu_l l}$$

$$I_{at} = I_0(1 - e^{-\mu_l l}) \quad (3.5.3)$$

If $I_0 - I$ is the number of photon detected by the detector or absorbed by NaI (Tl) scintillator then the intrinsic efficiency is calculated as follow.

$$\varepsilon_{intr} = \frac{I_{at}}{I_0} \quad (3.5.4)$$

Using the definition of I of equation 3.5.2

$$= \frac{I_0(1 - e^{-\mu_l(E)l})}{I_0}$$

$$\varepsilon_{intr} = 1 - e^{-\mu_l(E)l} \quad (3.5.5)$$

Where $\mu_l(E)$ is linear attenuation coefficient of NaI (Tl) at specific energy of gamma ray and l is the chord length of the photon travel in the detector.

From figure 3.4 we have the following trigonometric relation of the geometry of the detector.

$$\tan\theta_1 = \frac{R}{d+L}, \cos\theta_1 = \frac{d+L}{\sqrt{R^2 + (d+L)^2}}, \sin\theta_1 = \frac{R}{\sqrt{R^2 + (d+L)^2}} \quad (3.5.6)$$

$$\tan\theta_0 = \frac{R}{d}, \cos\theta_0 = \frac{d}{\sqrt{R^2 + (d)^2}}, \sin\theta_0 = \frac{R}{\sqrt{R^2 + (d)^2}} \quad (3.5.7)$$

For $\theta_1 \leq \theta < \theta_0$

$$\cos\theta = \frac{d+L}{l+s} \Rightarrow l+s = \frac{d+L}{\cos\theta}, \text{ But, } \cos\theta = \frac{d}{s}, \Rightarrow s = \frac{d}{\cos\theta}, l = \frac{d+L}{\cos\theta} - s = \frac{d+L}{\cos\theta} - \frac{d}{\cos\theta} = \frac{L}{\cos\theta}$$

For $0 \leq \theta \leq \theta_1$

$$\sin\theta = \frac{R}{s+l} \Rightarrow l = \frac{R}{\sin\theta} - s = \left(\frac{R}{\sin\theta} - \frac{d}{\cos\theta}\right) \frac{\sin\theta}{\sin\theta} = \frac{R - d\tan\theta}{\sin\theta}$$

Therefore the chord length is given as

$$\text{For } \theta_1 \leq \theta < \theta_0 \quad l = \frac{L}{\cos\theta} \quad \text{For } 0 \leq \theta \leq \theta_1 \quad l = \frac{R - d\tan\theta}{\sin\theta} \quad (3.5.8)$$

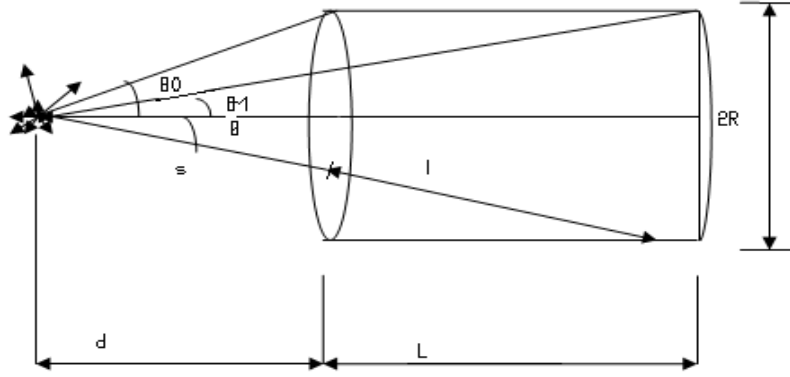


Figure 3.4: Geometry of source and NaI (Tl) detector.

The mean chord length of photon is defined as analytical average distance the photon can move inside the detector.

The mean chord length l_m over all path length taken by those gamma rays that strike the detector can be calculated by analytical method for simple geometrical right cylindrical as follows.

$$l_m = \frac{\int_0^{2\pi} \int_0^{\theta_0} l d\Omega}{\int_0^{2\pi} \int_0^{\theta_0} d\Omega} \quad (3.5.9)$$

Where $d\Omega$ is the solid angle $d\Omega = \sin\theta d\theta d\phi$ and $d\phi$ is azimuthal angle element.

$$\begin{aligned} l_m &= \frac{\int_0^{2\pi} \int_0^{\theta_0} l \sin\theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\theta_0} \sin\theta d\theta d\phi} \\ &= \frac{\int_0^{2\pi} [\int_0^{\theta_1} l \sin\theta d\theta + \int_{\theta_1}^{\theta_0} l \sin\theta d\theta] d\phi}{\int_0^{2\pi} [\int_0^{\theta_1} \sin\theta d\theta + \int_{\theta_1}^{\theta_0} \sin\theta d\theta] d\phi} \end{aligned}$$

From equation 3.5.8 substituting for l yields

$$= \frac{\int_0^{2\pi} [\int_0^{\theta_1} (\frac{L}{\cos\theta}) \sin\theta d\theta + \int_{\theta_1}^{\theta_0} (\frac{R-d\tan\theta}{\sin\theta}) \sin\theta d\theta] d\phi}{\int_0^{2\pi} [\int_0^{\theta_1} \sin\theta d\theta + \int_{\theta_1}^{\theta_0} \sin\theta d\theta] d\phi}$$

$$\begin{aligned}
&= \frac{\int_0^{2\Pi} [L \int_0^{\theta_1} \tan\theta d\theta + \int_{\theta_1}^{\theta_0} (R - d\tan\theta) d\theta] d\phi}{2\Pi[-\cos\theta|_0^{\theta_1} - \cos\theta|_{\theta_1}^{\theta_0}]} \\
&= \frac{2\Pi[L \int_0^{\theta_1} \tan\theta d\theta + \int_{\theta_1}^{\theta_0} (R - d\tan\theta) d\theta]}{2\Pi[-\cos\theta_1 + 1 - \cos\theta_0 + \cos\theta_1]} \\
&= \frac{[L \int_0^{\theta_1} \tan\theta d\theta + \int_{\theta_1}^{\theta_0} (R - d\tan\theta) d\theta]}{[1 - \cos\theta_0]} \\
&= \frac{[L \int_0^{\theta_1} \tan\theta d\theta + \int_{\theta_1}^{\theta_0} R d\theta - d \int_{\theta_1}^{\theta_0} \tan\theta d\theta]}{1 - \cos\theta_0} \\
&= \frac{-L\ln\cos\theta|_0^{\theta_1} + R(\theta)|_{\theta_1}^{\theta_0} + d\ln\cos\theta|_{\theta_1}^{\theta_0}}{[1 - \cos\theta_0]} \\
&= \frac{[-L\ln(\frac{\cos\theta}{1}) + R(\theta_0 - \theta_1) + d\ln(\frac{\cos\theta_0}{\cos\theta_1})]}{[1 - \cos\theta_0]} \\
l_m &= \frac{[L\ln(\frac{1}{\cos\theta_1}) + R(\theta_0 - \theta_1) - d\ln(\frac{\cos\theta_1}{\cos\theta_0})]}{[1 - \cos\theta_0]} \tag{3.5.10}
\end{aligned}$$

The limiting case of the mean chord length as d is approaching the surface of the detector and moving away from the surface on the axis of the cylindrical detector is calculated as follow. Now l_m is the function of d , L , R of the geometrical of the detector so we can write like $l_m = l(R, L, d)$.

$$\lim_{d \rightarrow \infty} l_m = \lim_{d \rightarrow \infty} \left[\frac{[L\ln(\frac{1}{\cos\theta_1}) + R(\theta_0 - \theta_1) - d\ln(\frac{\cos\theta_1}{\cos\theta_0})]}{(1 - \cos\theta_0)} \right]$$

By substituting $\cos\theta_0$, θ_0 , θ_1 , $\cos\theta_1$, in terms of the geometry as given in equation

$$\begin{aligned}
\lim_{d \rightarrow \infty} l_m &= \lim_{d \rightarrow \infty} \left[\frac{1}{1 - \frac{d}{\sqrt{R^2 + d^2}}} \left[L\ln\left(\frac{\sqrt{R^2 + (d+L)^2}}{d+L}\right) + R\left(\tan^{-1}\frac{R}{d} - \tan^{-1}\frac{R}{d+L}\right) \right] \right. \\
&\quad \left. - d\ln\left[\left(\frac{d+R}{d}\right)\left(\frac{\sqrt{R^2 + d^2}}{\sqrt{R^2 + (d+L)^2}}\right)\right] \right] \tag{3.5.11}
\end{aligned}$$

The second and third terms of the above equation canceled out because their limit as $d \rightarrow \infty$ approaches to zero.

$$\lim_{d \rightarrow \infty} l_m = \lim_{d \rightarrow \infty} \left[\frac{1}{1 - \left(\frac{d}{\sqrt{R^2 + d^2}}\right)} L\ln\left(\frac{\sqrt{R^2 + (d+L)^2}}{d+L}\right) \right]$$

$$\begin{aligned}
&= \lim_{d \rightarrow \infty} \left[\frac{\sqrt{R^2 + d^2}}{\sqrt{R^2 + d^2} - d} Lln\left(\frac{\sqrt{R^2 + (d+L)^2}}{d+L}\right) \right] \\
&= \lim_{d \rightarrow \infty} \left[Lln\left(\sqrt{1 + \frac{R^2}{(d+L)^2}}\right)^{\frac{\sqrt{R^2 + d^2}}{\sqrt{R^2 + d^2} - d}} \right] \\
&= Lln \lim_{d \rightarrow \infty} \left[\left(\sqrt{1 + \frac{R^2}{(d+L)^2}}\right)^{\frac{\sqrt{R^2 + d^2}}{\sqrt{R^2 + d^2} - d}} \right] \tag{3.5.12}
\end{aligned}$$

The limit in the logarithms function is similar to the definition of the e. i.e.

$$\lim_{x \rightarrow \infty} (1 + x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^x = e \tag{3.5.13}$$

Therefore equation 3.5.12 become

$$\lim_{d \rightarrow \infty} l_m = Lln e = L \tag{3.5.14}$$

This means as the mono energetic isotropic point sources is far from the detector surface the mean chord length of interaction is approach to or equal to the length (height) of the cylindrical detector, and

$$\begin{aligned}
\lim_{d \rightarrow 0} l_m &= \lim \frac{1}{1 - \frac{d}{\sqrt{R^2 + d^2}}} \left[Lln\left(\frac{\sqrt{R^2 + (d+L)^2}}{d+L}\right) + R\left(\tan^{-1} \frac{R}{d} - \tan^{-1} \frac{R}{d+L}\right) \right] \\
&\quad - dln\left[\left(\frac{d+R}{d}\right)\left(\frac{\sqrt{R^2 + d^2}}{\sqrt{R^2 + (d+L)^2}}\right)\right] \tag{3.5.15}
\end{aligned}$$

The third term is vanished because of $ln1 = 0$ and

$$\lim_{d \rightarrow 0} l_m = \left[Lln\left(1 + \frac{R^2}{L^2}\right)^{1/2} + R\left(\frac{\Pi}{2} - \tan^{-1} \frac{R}{L}\right) \right] \tag{3.5.16}$$

$$\lim_{d \rightarrow 0} l_m = \frac{1}{2} Lln\left(1 + \frac{R^2}{L^2}\right) + R\left(\frac{\Pi}{2} - \tan^{-1} \frac{R}{L}\right) \tag{3.5.17}$$

Where $\frac{\Pi}{2}$ and $\tan^{-1} \frac{R}{L}$ are in radian.

Chapter 4

4 RESULTS AND DISCUSSION

4.1 CALCULATION OF INTRINSIC EFFICIENCY

To calculate the intrinsic efficiency of the right cylindrical sodium iodide activated with thallium by using equation 3.5.5 theoretically (analytically) we have to have total linear attenuation coefficient of the scintillator or NaI (Tl) crystal. Here in this work I calculated the linear attenuation coefficient from the reference [3] from the table given on page (50). And the linear attenuation coefficient and mass attenuation coefficient for various gamma ray energies calculated given as table below. The energy is taken randomly. The mean chord length of interaction for photon in the crystal arranged according to figure 3.1 is calculated by putting the source (Gamma source) at different distance on the axis passes through the central cylinder using equation 3.5.10, for the source on the central surface of the detector ($d=0$) using eqn 3.5.17.

In this project a crystal of 1cmx1cm, 3cmx3cm, 5cmx5cm, 10cmx10cm, and 15cmx15cm are considered and the following mean chord length is calculated by using pocket calculator or FORTRAN program.

Finally, using table 4.1 and table 4.2 substituting in equation 3.5.5 the intrinsic efficiency for different size crystal is calculated as follow.

Gamma energy $E_\gamma(\text{Mev})$	Mass attenuation coefficient $\mu_m(\text{cm}^2/\text{gm})$	Linear attenuation coefficient $\mu_l(\text{cm}^{-1})$
0.1	1.4	5.138
0.2	0.30	1.101
0.4	0.11	0.4037
0.6	0.08	0.2936
0.8	0.065	0.23855
1	0.056	0.2055
2	0.043	0.15781
5	0.035	0.12845
6	0.0353	0.12910
7	0.360	0.13212
8	0.037	0.13579
10	0.038	0.13946
20	0.048	0.17616
50	0.056	0.20919

Figure 4.1: total linear and mass attenuation coefficient of NaI(Tl) for some gamma ray energy

1cmx1cm	
d(cm)	l_m(cm)
0.0	0.66515
0.1	0.51473
0.4	0.45917
0.5	0.46964
1	0.55258
2	0.68219
4	0.83580
6	0.85266
8	0.89138
9	0.89956
10	0.91106

A

3cmx3cm	
d(cm)	l_m(cm)
0.0	1.99543
0.1	1.75193
1	1.36953
2	1.48598
3	1.65940
4	1.81296
5	1.94305
6	2.04908
8	2.20882
9	2.26906
10	2.38427

B

5cmx5cm	
d(cm)	l_m(cm)
0.0	3.32571
0.1	3.03448
1	2.35197
2	2.29615
3	2.42170
4	2.59270
5	2.76534
6	2.92500
8	3.19635
9	3.30720
10	3.41489

C

10cmx10cm	
d(cm)	l_m(cm)
0.0	8.96968
0.1	7.02714
1	5.14726
2	4.70401
3	4.57057
4	4.59236
5	4.69631
6	4.83928
8	5.18525
9	5.36090
10	5.53067

D

15cmx15cm	
d(cm)	l_m(cm)
0.0	9.97719
0.1	9.58410
1	8.14044
2	7.42219
3	7.05600
4	6.88890
5	6.84760
6	6.88850
8	7.11362
9	7.26411
10	7.43053

E

Figure 4.2: The mean chord length calculated for various distances and detector size.

Energy E_γ (MeV)	Intrinsic Efficiency ϵ_{int} 1cmx1cm crystal				
	d=0cm	d=0.1cm	d=0.4cm	d=6cm	d=10cm
0.1	0.96716	0.92897	0.9055	0.9875	0.9907
0.2	0.52155	0.43261	0.3968	0.61133	0.63569
0.4	0.23549	0.18763	0.1692	0.29123	0.30774
0.6	0.1774	0.14027	0.12612	0.22147	0.23211
1	0.12776	0.10038	0.09005	0.16073	0.17048
5	0.08126	0.06398	0.05727	0.10295	0.1096
7	0.08426	0.06575	0.05886	0.10659	0.11346
10	0.08859	0.06926	0.06203	0.11211	0.11932

Figure 4.3: Intrinsic efficiency calculated for 1cmx1cm NaI(Tl) at various gamma energies.

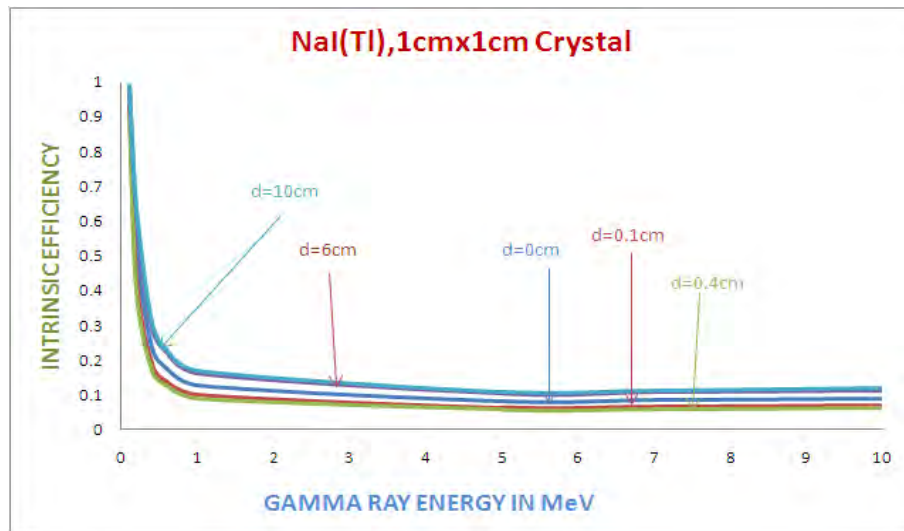


Figure 4.4: Intrinsic efficiency for NaI (Tl) detector with 1cmx1cm size Vs Gamma ray energies.

Energy E_γ (MeV)	Intrinsic Efficiency ε_{int} 3cmx3cm crystal				
	d=0cm	d=0.1cm	d=1cm	d=8cm	d=10cm
0.1	0.9999	0.99987	0.9991	0.9999	0.9999
0.2	0.89048	0.85469	0.78083	0.91354	0.92882
0.4	0.5848	0.507	0.45297	0.62213	0.63259
0.6	0.44337	0.40212	0.44337	0.47718	0.50343
1	0.33639	0.30236	0.2453	0.36486	0.38735
5	0.22451	0.2015	0.16012	0.24536	0.26199
7	0.23185	0.20663	0.1656	0.25322	0.27034
10	0.24292	0.21677	0.17386	0.26517	0.28289

Figure 4.5: Intrinsic efficiency calculated for 3cmx3cm NaI(Tl) at various gamma energy.

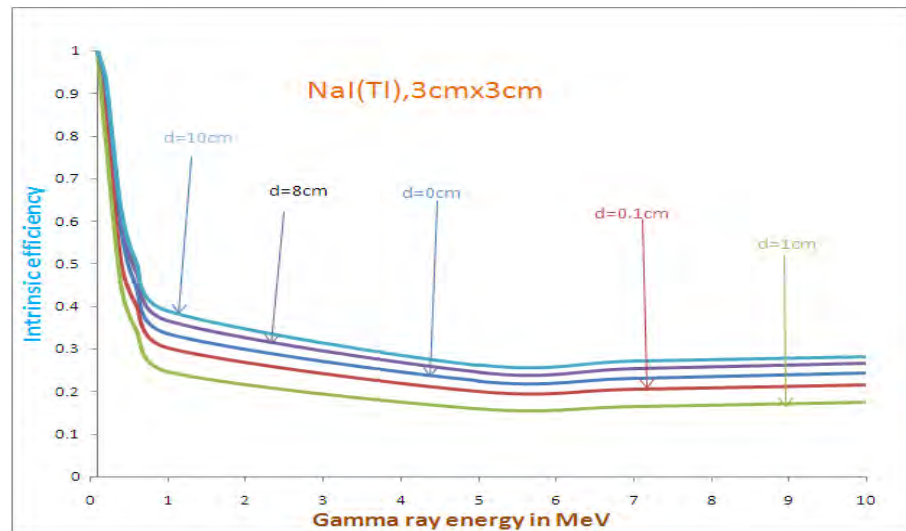


Figure 4.6: Intrinsic efficiency for NaI (Tl) detector with 3cmx3cm size Vs Gamma ray energies.

Energy E_γ (MeV)	Intrinsic Efficiency ϵ_{int} 5cmx5cm crystal			
	d=4cm	d=6cm	d=8cm	d=10cm
0.1	0.9996	0.9997	0.9998	0.9999
0.2	0.9435	0.9609	0.97106	0.97729
0.4	0.6489	0.69297	0.71036	0.74807
0.6	0.5319	0.57632	0.60877	0.63308
1	0.41304	0.45178	0.48152	0.50123
5	0.28134	0.31113	0.33455	0.35282
7	0.29017	0.32067	0.34461	0.36327
10	0.30542	0.33497	0.35966	0.37886

Figure 4.7: Intrinsic efficiency calculated for 5cmx5cm NaI(Tl) at various gamma energy.

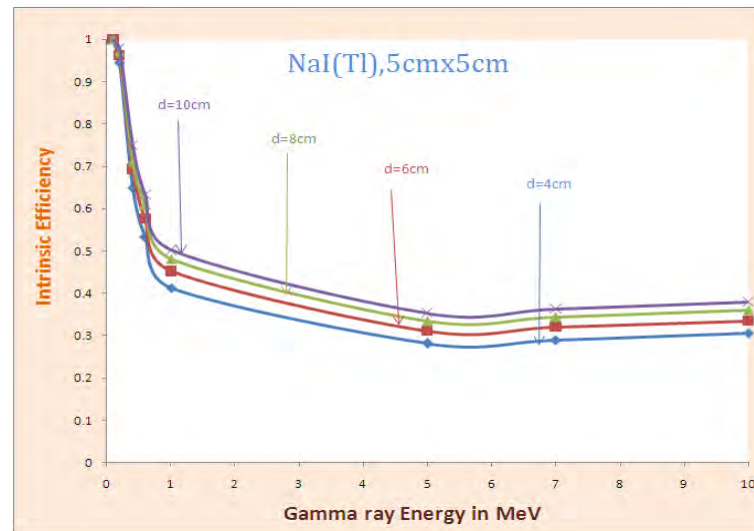


Figure 4.8: Intrinsic efficiency for NaI (Tl) detector with 5cmx5cm size Vs Gamma ray energies.

Energy E_γ (MeV)	Intrinsic Efficiency ε_{int} 10cmx10cm crystal					
	d=0cm	d=0.1cm	d=4cm	d=5cm	d=8cm	d=10cm
0.1	1	1	1	1	1	1
0.2	0.9999	0.9998	0.9936	0.9943	0.99668	0.99773
0.4	0.97325	0.94139	0.84338	0.84979	0.8767	0.89276
0.6	0.928	0.87295	0.74032	0.74809	0.7818	0.80285
1	0.8417	0.76407	0.61086	0.61905	0.6555	0.67910
5	0.68405	0.5949	0.44561	0.45032	0.48626	0.50856
7	0.69428	0.60482	0.45488	0.46228	0.49595	0.51843
10	0.71376	0.62469	0.47295	0.4805	0.51477	0.53759

Figure 4.9: Intrinsic efficiency calculated for 10cmx10cm NaI(Tl) at various gamma energy.

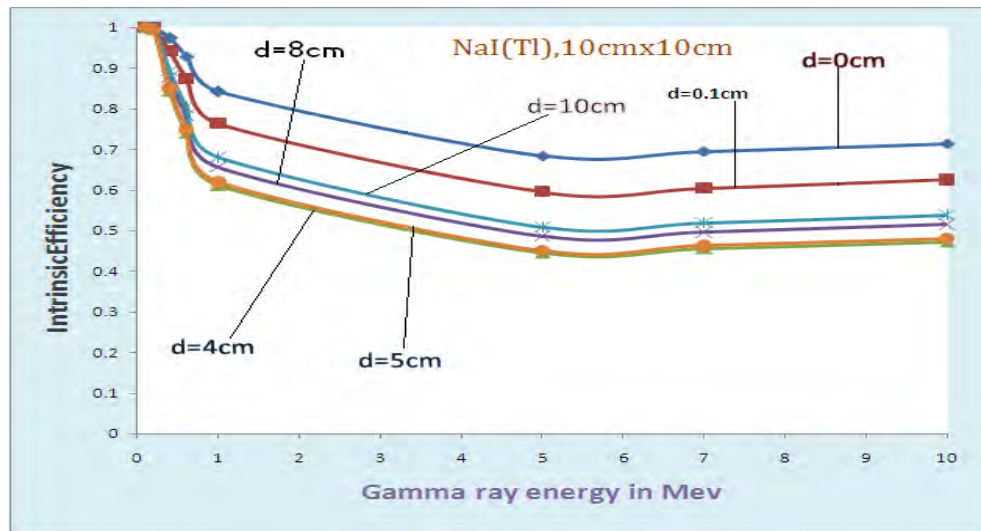


Figure 4.10: Intrinsic efficiency for NaI (Tl) detector with 10cmx10cm size Vs Gamma ray energies.

Energy E_γ (MeV)	Intrinsic Efficiency ϵ_{nt} 15cmx15cm crystal						
	d=0cm	d=0.1cm	d=2cm	d=6cm	d=7.5cm	d=8cm	d=10cm
0.1	1	1	1	1	1	1	1
0.2	0.999	0.999	0.9994	0.99946	0.99949	0.9996	0.9997
0.4	0.9822	0.979	0.95	0.938	0.94178	0.9434	0.9502
0.6	0.9466	0.94	0.88686	0.86767	0.87357	0.8761	0.8871
1	0.8713	0.8605	0.78247	0.75725	0.76488	0.7682	0.7828
5	0.7224	0.708	0.6145	0.587	0.59537	0.59898	0.61497
7	0.7324	0.718	0.6249	0.5975	0.6057	0.6093	0.62533
10	0.75128	0.73726	0.6448	0.61736	0.62557	0.6292	0.64522

Figure 4.11: Intrinsic efficiency calculated for 15cmx15cm NaI(Tl) at various gamma energy.

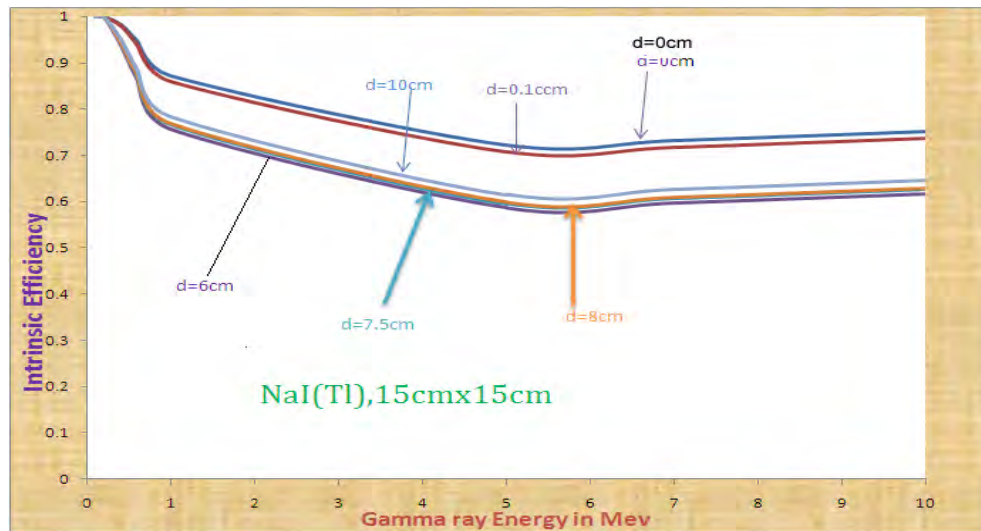


Figure 4.12: Intrinsic efficiency for NaI (Tl) detector with 15cmx15cm size Vs Gamma ray energies.

4.2 Conclusion

One advantage of scintillation counter is its efficiency to detect gamma ray radiation. Here from section 4.1 we can observe the following points for the sodium iodide activated by thallium.

- ♣ *As Energy increase , efficiency falls down and becomes almost constant and slightly increase for high energy ($E_\gamma > 5\text{MeV}$).*
- ♣ *As the size (volume) of the detector is increase, the intrinsic efficiency of NaI(Tl) detector for various energy of gamma ray, source to detector distances, as we can observe from graph 4.2.4, 4.2.6, 4.1.8, 4.1.10, and 4.1.12, the intrinsic efficiency is increase.*
- ♣ *For small detector size such as $1\text{cm} \times 1\text{cm}$, $3\text{cm} \times 3\text{cm}$ and $5\text{cm} \times 5\text{cm}$, the intrinsic efficiency of sodium iodide activated with Thallium is small for small source to detector distances ($d < R$).*
- ♣ *For large size detector, such as $10\text{cm} \times 10\text{cm}$ and $15\text{cm} \times 15\text{cm}$ the intrinsic efficiency of NaI (Tl) is small for large source to detector distances ($d \geq R$).*

Bibliography

- [1] Ashak Das Thomas Ferbel, 1994, "*Introduction to Nuclear and particle Physics*" print in USA.
- [2] B.R.Martin, 2006, "Nuclear and particle Physics, " John Wiley and son Ltd, England.
- [3] G.F.Knoll, 2000,"*Radiation Detection and measurement*, "Third Editio, Wiley Inc, Newyork.
- [4] J.M.Reid, 1972,"The Atomic Nucleus," William Clawes and sons Ltd.London.
- [5] Robley D.Evans, 1955,"*The Atomic Nucleus*," Mc Graw .Hill Book Company.Inc.
- [6] R.M.Singru, 1972,"*Introduction to Experimental nuclear Physics*," Wiley Eastern pvl,New Delhi.
- [7] Mukhin Konstantin Mikifovin, 1987, "Experimental Nuclear Physics," Mukhin,Moscow.
- [8] S.S.Kapoor and Ramamurthy, 1986,"*Nuclear Radiation Detectors*," Wiley East-ern pvt,New Delhi.
- [9] W.E.Burham, 1979,"*Elements of Nuclear Physics*," Longman Inc. Newyork.
- [10] Ernst BLemler and George J.Goldsmith, 1959, "*Experimental physics*," First Edn.Rinehart company Inc. Newyork

- [11] Ralph T.Overman,1960, "Radioisotope Technic," Mc GRAW. Hill Book company.Inc.