QUANTUM STATISTICAL DESCRIPTION OF THERMAL RADIATION

By

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Abstract

In this project we have studied and presented a review of the quantum statistical description of thermal radiation that offers a comprehensive framework for understanding the emission and absorption of electromagnetic radiation from a microscopic perspective. This approach integrates principles of quantum mechanics and statistical mechanics to describe how radiation interacts with matter and how it is distributed across different frequencies or wavelengths. At the core of this description is the concept of photons quantized packets of energy whose distribution follows Bose-Einstein statistics due to their bosonic nature.

Plancks law was pivotal in the development of quantum theory, as it introduced the concept of quantized energy levels to explain the observed spectral distribution, resolving the "ultraviolet catastrophe" predicted by classical physics. Together, the Stefan-Boltzmann law and Plancks law form the foundational principles of thermal radiation.

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Chapter 1

1.1 INTRODUCTION

Thermal radiation is electromagnetic radiation emitted by the thermal motion of particles in matter. Thermal radiation transmits as an electromagnetic wave through both matter and vacuum. When matter absorbs thermal radiation its temperature will tend to rise. All matter with a temperature greater than absolute zero emits thermal radiation.[1]

Thermal radiation, the electromagnetic radiation emitted by a body in thermal equilibrium, is a fundamental concept in both classical and quantum physics.The quantum statistical description of thermal radiation provides a deeper insight into this phenomenon, moving beyond classical theories and offering a more accurate depiction grounded in quantum mechanics.[2]

Classically, thermal radiation was described by the Rayleigh Jeans law, which works well at low frequencies but fails at high frequencies, leading to the so-called ultraviolet catastrophe .Planck proposed that electromagnetic radiation is quantized, which means it can only be emitted or absorbed in discrete amounts called quanta or photons .Plancks law of black body radiation describes how the intensity of radiation emitted by a perfect black body (an idealized object that absorbs all incident radiation) varies with frequency at a given temperature. According to this law, the spectral distribution of radiation shifts to higher frequencies (shorter wavelengths) as the temperature increases, and the distribution shape is characteristic of the black body's temperature.[3]

The quantum statistical description of thermal radiation also involves understanding the occupation numbers of these photons[1].According to Bose-Einstein statistics, photons obey Bose-Einstein distribution, which accounts for the indistinguishability of photons and their tendency to occupy the same quantum state [3]. Instead, Plancks black body radiation can be implemented to correlate the temperature of an object with its emission spectrum and energy. For most applications, the strength of emission and absorption are expressed in terms of empirical properties of materials, their emissivity and absorptivity values. These parameters can also be derived starting from the fundamental properties of dielectric constant and magnetic susceptibility, which relate atomic level vibrations and electron flow in matter to emission and absorption of electromagnetic waves [4].

Thermal radiative heat transfer refers to the process of heat transfer between surfaces or bodies through thermal radiation. Unlike conduction or convection, which require a medium for heat transfer, radiation can occur through a vacuum. This type of heat transfer is governed by the principles of electromagnetic radiation and is crucial in many engineering and scientific applications[5] .

1.2 Background of the Study

Classically, thermal radiation was described by the Rayleigh-Jeans law, which works well at low frequencies but fails at high frequencies, leading to the so called ultraviolet catastrophe .The problem of black body radiation (that is, the electromagnetic radiation emitted by a hot surface), has been under active consideration since the middle of the nineteenth century. It was the inability of classical statistical mechanics to account for the black body spectrum (the emitted power at a given temperature as a function of frequency) that led Planck1 in 1900 to introduce the concept of the quantum. On the other hand, in 1884 Boltzmann used thermodynamic reasoning to correctly predict the temperature dependence of the total radiation from a hot body, and we begin by discussing the thermodynamics of radiation, leaving the statistical treatment till later [1].

Heat transfer can take place from one body to another in the form of radiation with no material contact between the two bodies[2].This form of heat was called heat radiation. When it was discovered that motion of charges produced electromagnetic radiation, the idea that heat radiation was a form of electromagnetic radiation was taken up, especially in the works of Gustav Kirchhoff (1824-1887),and Wilhelm Wien (1864-1928) and its thermodynamic consequences were investigated [3] .

1.3 Objective of the study

General Objective

To review quantum statistical description of thermal radiation

Specific Objective

Deriving the Planck law of black body radiation, which explains how the intensity of radiation varies with frequency at a given temperature.

Explaining the essential characteristics of thermal radiation and the fundamental parameters which describe thermal radiation properties.

1.4 Organization of the Study

This project is organized in to six chapters.Chapter one discuss about the introduction to quantum statistical description of thermal radiation,chapter two thermodynamics of black body radiation that includes radiant energy,black-body radiation and emission and absorption of thermal radiation.Chapter three the fundamentals law of black body radiation that includes photon statistics,black body law ,thermodynamics of black body radiation and application.Chapter four radiative transfer equation that includes Beer's law and Schwarszchilds equation.Chapter five thermal radiation heat transfer that include radiative heat transfer and finally in chapter six summary and conclusions of the study are included.

Chapter 2

THERMODYNAMICS OF BLACK BODY RADIATION

2.1 Introduction

Black body radiation is a fundamental concept in the study of thermodynamics and quantum mechanics. The study of black body radiation how an objects emits radiation as a function of temperature, which is crucial for understanding various phenomena ranging from stellar astrophysics to the design of efficient energy systems. This topic is particularly notable for its historical significance, as it led to the development of quantum theory [2,13].

2.2 Radiant Energy

The Sun as the Source of Radiation.The energy carried by electromagnet waves is called radiant energy. Radiant energy, also knows thermal radiation , is transfer of electromagnetic radiation which describes the exchanges of energy by photons [13]. The effective temperature T_e of a star or any other source of radiant energy is determined by:

$$F = \sigma T_e^4 \tag{2.2.1}$$

where σ is the Stefan-Boltzmann constant

F = flux of radiant energy from the object, the effective temperature of which is being determined.

2.3 Black Body Radiation

2.3.1 Black Body

A Black body is a defined as an ideal body that allows all the incident radiation pass into it (no reflect energy) and internally absorbs all incident radiation (no transmitted energy). This is true radiation for wavelength and for incidence. The concept of black is basic to study of radiative energy transfer [2]. The black body emits the maximum radiant energy and hence serves an ideal standard of comparison with real black body emitting radiation only a few materials, such as carbon black, carborundum, platinum black, gold black paints of absorbing substrates, approach the black body in their ability to absorbs radiant energy [4].

The amount of radiation energy emitted from a surface at a given wavelength depends on the material of the body and the condition of its surface as well as the surface temperature. Therefore, different bodies may emit different amounts of radiation per unit surface area, even when they are at the same temperature. Maximum possible emission occurs at all wavelengths and in all directions, thus the radiance is independent of direction, known as isotropic radiation [2].

2.3.2 Gray Body

If the radiative properties absorptivity, reflectivity, transmissivity of a body are assumed to be uniform over the entire wavelength then such a body is called gray body. Most real surfaces do not behave as black bodies. They offer up some resistance to

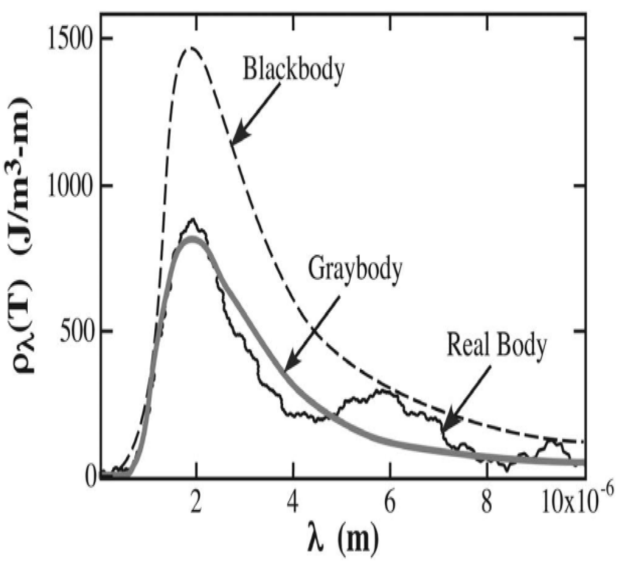

Figure 2.1: Comparison of ideal versus real radiators.[21]

absorbing or emitting radiation. Figure 2.3 shows the energy flux emitted by a black body at a given temperature and a corresponding real body at that temperature. The real body emits much less energy than the black body. It also has a much more complicated emission pattern since that pattern is not a smooth function of wavelength. The gray body uses the same type of energy spectrum as the black body but scales the black bodys output by a function termed the emissivity, ε [24].The total radiative power of the gray body can then be written as:

$$E = \varepsilon \sigma T^4 \tag{2.3.1}$$

2.3.3 Thermal Electromagnetic Radiation

Thermal electromagnetic radiation is the radiation emitted by objects as a result of their temperature.

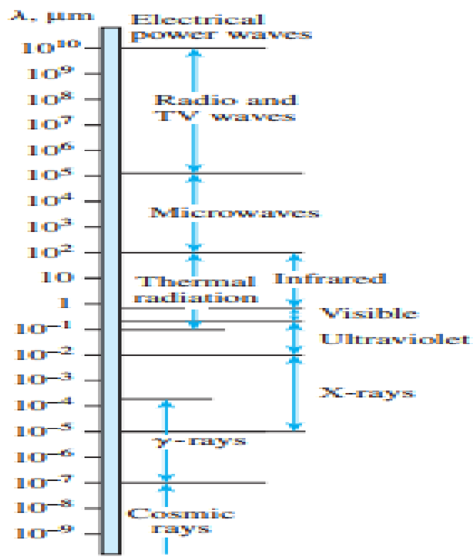

Figure 2.2: The electromagnetic wave spectrum [8].

It is a type of electromagnetic radiation that such as radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays,as shown in Fig 2.4 [2].

The higher the temperature of an object, the more thermal radiation it emits, and the higher the energy of the emitted radiation. The electromagnetic radiation encountered in practice covers a wide range of wavelengths, varying from less than $10^{-10}\mu m$ for cosmic rays to more than $10^{10}\mu m$ for electrical power waves.

2.3.4 Radiant Intensity

The intensity of radiation, denoted by I, is the main quantitative characteristic of a radiation field. The value of radiant intensity depends on the wavelength of radiation λ, the time t, the coordinates x, y, z of the point under consideration, P, and the direction r of the ray. The dependence of the radiant intensity upon all these values is usually denoted as $I_\lambda(t, P, r)$ [30].

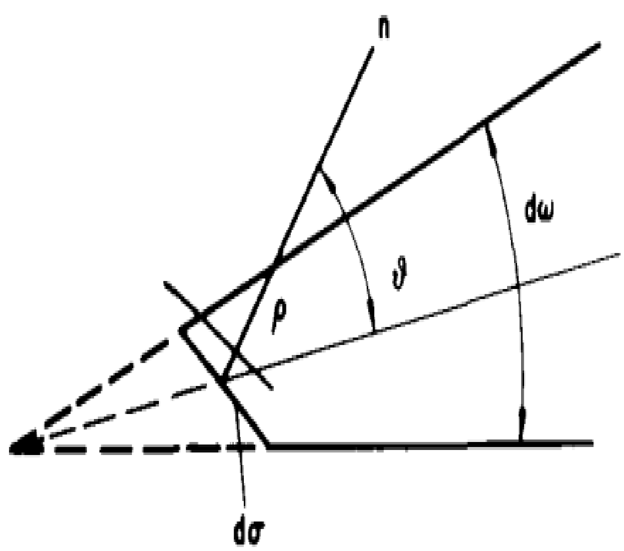

Figure 2.3: The determination of radiant intensity [30].

Consider only those beams that are grouped around some definite direction and find the quantity of radiation passing in this direction. Let n be the normal to $d\sigma$ at the point P, and r the line passing through the point P and constituting the angle θ with the direction of the normal n. Determine now the value of the intensity $I_\lambda(t, P, r)$ of the wavelength λ at the point P in the direction r, using the following relation:

$$dF_\lambda = I_\lambda(t, P, r) cos\theta d\sigma d\omega d\lambda dt, \tag{2.3.2}$$

Thus the radiant intensity $I_\lambda(t, P, r)$ is a quantity of energy in the unit wavelength interval, and in the unit solid angle per unit time per unit area of the surface perpendicular to the direction r of the beam of rays.
The total (integral) intensity of radiation can be obtained by integrating over all wavelengths and frequencies :

$$I = \int_0^\infty I_\lambda d\lambda = \int_0^\infty I_\nu d\nu. \tag{2.3.3}$$

2.3.5 Radiance

The radiance L(r, s) is the power per unit area, per unit solid angle at a point r in the direction of the unit vector s; in other words it is the integral of L_ν over frequency:

$$L(r, \mathbf{r}) = \int_0^\infty L_\nu(r, s) d\nu \tag{2.3.4}$$

2.3.6 Radiant flux

The flux of radiant energy of the wavelength across the surface, whose orientation is characterized by the direction of the normal n :

$$F_{\lambda,n} = \int I_\lambda(t, P, r) cos\theta d\omega, \tag{2.3.5}$$

All that we have said about the intensity of radiation also holds for the description of the energy distribution in the spectrum of radiant fluxes.The total (integral) radiant flux is determined [analogously to (2.3.3)], by the following integral :

$$F = F_\lambda d_\lambda = F_\nu d_\nu, \tag{2.3.6}$$

Introducing the spherical coordinates θ and φ and taking into consideration that

$$d\omega = sind\theta d\varphi, \tag{2.3.7}$$

we can use the formula (2.3.6) to derive the flux from a hemisphere

$$F_{\lambda,n} = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_\lambda(t, P, \theta, \varphi) cos\theta sin\theta d\theta, \tag{2.3.8}$$

The flux of radiant energy of the wavelength λ thus can be determined at a given moment and at a given point by integration over θ and φ, given in (2.3.9), provided the intensity I_λ as the function of the coordinates θ and φ is known. It I_λ does not depend on the direction, the radiation field is isotropic. In this case the integration is easily performed, and we obtain the following expression for the flux:

$$F_\lambda = \pi I_\lambda, \tag{2.3.9}$$

An analogous relation will evidently hold for the total flux:

$$F = \pi I. \tag{2.3.10}$$

2.3.7 Irradiance

The irradiance (or flux density) F(r, n) is the power per unit area at a point r through a surface of normal n, i.e. the integral of F_ν over frequency, and also the integral of the radiance L over a hemisphere [31].

$$F(r, n) = \int_0^\infty F_\nu(r, n) d\nu. \tag{2.3.11}$$

2.4 Emission and Absorption of Thermal Radiation

2.4.1 Emissivity

For a given wavelength λ, the emissivity (ε) is defined as the ratio of actual emitted radiance I_λ to the black body radiance B_λ:

$$\varepsilon_\lambda = \frac{I_\lambda}{B_\lambda} \tag{2.4.1}$$

The emissivity ranges from 0 to 1 for real objects and is a measure of how strongly a body radiates at a given wavelength [2].

2.4.2 Absorptivity

we can define the monochromatic adsorptively α_λ, as the fraction of the incident monochromatic intensity that a black body absorbs[2].

$$Absorptivity : a_\lambda = \frac{I_\lambda(absorbed)}{I_\lambda(incident)} \tag{2.4.2}$$

2.4.3 Transmissivity

Transmissivity (t_λ) is a measure of how much radiation passes through a material.The fraction of irradiation transmitted by the surface is called transmissivity. Transmittance of the surface of a material is its effectiveness in transmitting radiant energy.

$$Transmissivity : t_\lambda = \frac{I_\lambda(transmitted)}{I_\lambda(incident)} \tag{2.4.3}$$

2.4.4 Reflectivity

Reflectivity r_λ is a measure of how much incident radiation is reflected by a surface compared to the total incident radiation. It is given by

$$Reflectivity : r_\lambda = \frac{I_\lambda(reflected)}{I_\lambda(incident)}. \tag{2.4.4}$$

Chapter 3

THE FUNDAMENTAL LAW OF BLACK BODY RADIATION

3.1 Photon Statistics

We have assumed that the number of particles, N, contained in a given system is a fixed number. This is a reasonable assumption if the particles possess non-zero mass, because we are not generally considering relativistic systems (i.e., we are assuming that the particle energies are much less than their rest-mass energies)[1,8].

However, this assumption breaks down for the case of photons, which are zero-mass bosons. In fact, photons enclosed in a container of volume V, maintained at temperature T, can readily be absorbed or emitted by the walls. Thus, for the special case of a gas of photons, there is no requirement that limits the total number of particles .That photons obey a simplified form of Bose-Einstein statistics in which there is an unspecified total number of particles. This type of statistics is called photon statistics [1].

The mechanism of establishing equilibrium in a photon gas is absorption and emission of photons by matter that N can be found from the equilibrium condition: On other hand $\left(\frac{\partial F}{\partial N}\right)=\mu_{ph}=0$. However, we cannot use the usual expression for the chemical potential, because one cannot increase N (i.e., add photons to the system) at constant volume and at the same time keep the temperature constant: $\left(\frac{\partial F}{\partial N}\right)_{T,V}$ - does not exist for the photon gas.

3.1.1 Density of States for Photons

In order to calculate the average number of photons per small energy interval $d\varepsilon$, the average energy of photons per small energy interval $d\varepsilon$, as well as the total average number of photons in a photon gas and its total energy, we need to know the **density of states for photons** as a function of photon energy. The particle is confined to a

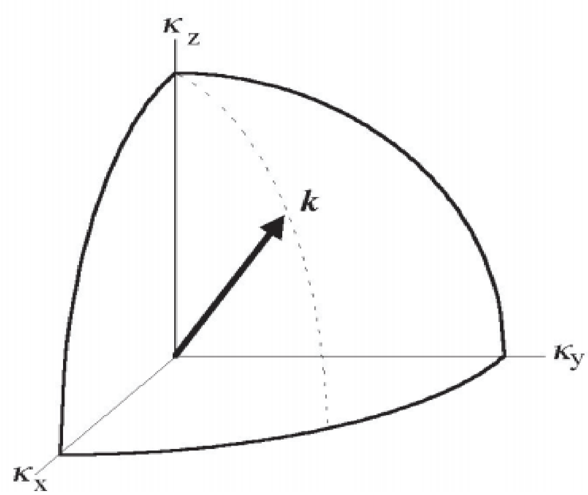

Figure 3.1: Reciprocal space for a three-dimensional system. The octant of the sphere of radius k encloses N(k) points, each point corresponding to one stationary state; [1].

rectangular box of sides $L_x L_y L_z$, with the boundary condition that ψ vanish on all six faces of the box. The normal modes now have the form

$$\psi(x, y, z, t) = A \sin k_x x \sin k_y y \sin k_z z e^{i\omega t} \tag{3.1.1}$$

where, to satisfy the boundary conditions, $k_x = \frac{n_x \pi}{L_x}$, $k_y = \frac{n_y \pi}{L_y}$, $k_z = \frac{n_z \pi}{L_z}$. Where n_x, n_y and n_z being positive non zero integers. $k^2 = k_x^2 + k_y^2 + k_z^2$. Again we start by calculating N(k). This is the number of states within the octant of a sphere of radius k in the three-dimensional k-space defined by the axes k_x, k_y,k_z (see Figure 3.1). The volume of this octant is $\frac{1}{8}\frac{4\pi k^3}{3} = \frac{\pi k^3}{6}$.Note that this is a volume in k-space, which must be clearly distinguished from the volume of the box in real space, which is $L_x L_y L_z$. The possible values of k_x, k_y, k_z form a three-dimensional lattice in k-space. Each point in this lattice is associated with a rectangular parallelepiped with sides $\frac{\pi}{L_x} \cdot \frac{\pi}{L_y} \cdot \frac{\pi}{L_z}$ so that the volume per point is $\frac{\pi}{L_x L_y L_z}$. The number of points within the octant of radius k is the volume of the octant divided by the volume per point, that is,

$$N(k) = \frac{L_x L_y L_z}{\pi^3}, \tag{3.1.2}$$

Since the volume of the box is V = $L_x L_y L_z$, the number of normal modes per unit volume of real space is

$$G(k) = \frac{N(k)}{L_x L_y L_z} = \frac{k^3}{6\pi^2}, \tag{3.1.3}$$

The dispersion relation (k) for a wave of constant phase velocity c is so that, from Equation (3.1.4)

$$\nu = \frac{ck}{2\pi}, \tag{3.1.4}$$

$$G(\nu) = 3\frac{1}{6\pi^2}\left(\frac{2\pi\nu}{c}\right)^3, \tag{3.1.5}$$

and

$$g(\nu) = \frac{dG}{d\nu} = \frac{12\pi\nu^2}{c^3} \tag{3.1.6}$$

The density of states per unit volume with respect to photon frequency, $g(\nu)$, can be obtained from equation (3.1.6). Since photons have two possible polarizations while phonons have three, we multiply by 2/3 to obtain.

$$g(\nu) = \frac{8\pi\nu^2}{c^3}. \tag{3.1.7}$$

3.2 Ideal Photon Gas

In the language of quantum mechanics, electromagnetic waves can be quantized as a set of particles, which are known as photons . According to the quantum theory of radiation, photons are massless bosons of spin 1 (in units $\hbar$),they move with the speed of light. The linearity of Maxwell equations implies that the photons do not interact with each other. (Non-linear optical phenomena are observed when a large-intensity radiation interacts with matter)[23].

$$E_{ph} = h\nu, \tag{3.2.1}$$

$$E_{ph} = cP_{ph}, \tag{3.2.2}$$

$$P_{ph} = \frac{E_{ph}}{c} = h\frac{\nu}{c}. \tag{3.2.3}$$

The mechanism of establishing equilibrium in a photon gas is absorption and emission of photons by matter. Presence of a small amount of matter is essential for establishing equilibrium in the photon gas. We will treat a system of photons as an ideal photon gas, and, in particular, will apply the BE statistics to this system [23].

3.3 Black Body Radiation Laws

The laws of blackbody radiation are basic to an understanding of the absorption and emission processes. A black body is a basic concept in physics and can be visualized by considering a cavity with a small entrance hole. Most of the radiant flux entering this hole from the outside will be trapped within the cavity, regardless of the material and surface characteristics of the wall. Repeated internal reflections occur until all the fluxes are absorbed by the wall. The probability that any of the entering flux will escape back through the hole is so small that the interior appears dark. The term blackbody is used for a configuration of material where absorption is complete, emission by a blackbody is the converse of absorption[17].

3.3.1 Planks Distribution Function

The Planck distribution function describes the distribution of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature [8].
P(n) for photons: Our current problem is slightly different from the problem of the electron spins, in that instead of just two energy levels, we now have an infinite number of energy levels.The probability $P(E_n)$ is given by a normalizing constant A times a Boltzmann factor.

$$P(E_n) = Ae^{-E_n/kT}, \tag{3.3.1}$$

To determine what A is, we again note that the sum of all the probabilities must be 1.

$$\sum_{n=0}^{\infty} P(E_n) = A\sum_{n=0}^{\infty} e^{-En/kT} = A\sum_{n=0}^{\infty} e^{-nh\nu/kT} = 1, \tag{3.3.2}$$

The above sum is of the form

$$\sum x^n = 1 + x + x^2...., \tag{3.3.3}$$

where x $\equiv e^{h\nu/kT}$ is a number that is less than 1. This series, which converges when x is less than 1, turns out to be exactly what you get for the binomial expansion of 1/(1 - x) . Its a power series expansion,hence

$$A = 1 - x = 1 - e^{-h\nu/kT}, \tag{3.3.4}$$

and

$$P(E_n) = Ae^{-E_n/kT} = \left(1 - e^{-h\nu/kT}\right) e^{-E_n/kT}, \tag{3.3.5}$$

$$P(n) = \left(1 - e^{-h\nu/kT}\right) e^{-nh\nu/kT}, \tag{3.3.6}$$

Now that we have an expression for the probability P (n) that there are n photons in a mode of frequency ν, we can determine $\bar{n}$, the mean (or average) number of photons in a mode when the temperature of the walls of our box is T.

$$\bar{n} = \sum_{n=0}^{\infty} nP(n) = \left(1 - e^{-h\nu/kT}\right) \sum_{n=0}^{\infty} ne^{-nh\nu/kT}, \tag{3.3.7}$$

To calculate this last sum, we notice that it can be written in the form

$$\sum_{n=0}^{\infty} ne^{-nh\nu/kT} = \sum ne^{-an}, \tag{3.3.8}$$

where $a \equiv h\nu/kT$

Thus we can write

$$\sum ne^{-an} = -\frac{\partial}{\partial a} \sum e^{-an} = -\frac{\partial}{\partial a}\left(\frac{1}{1 - e^{-a}}\right) = \frac{e^{-a}}{(1 - e^{-a})^2} \tag{3.3.9}$$

For chemical potential is zero,μ=0, the BE distribution reduces to the Planck's distribution:

$$\bar{n_{ph}} = f_{ph}(\varepsilon, T) = \frac{1}{exp\left(\frac{\varepsilon}{k_B T}\right) - 1} = \frac{1}{exp\left(\frac{h\nu}{k_B T}\right) - 1} \tag{3.3.10}$$

Planck's distribution provides the average number of photon in a single mode of frequency ν=ε/h

The average energy in the mode.

$$\bar{\varepsilon} = \bar{n}h\nu = \frac{h\nu}{exp(\frac{h\nu}{k_BT}) - 1}. \tag{3.3.11}$$

In the classical ($h\nu \ll k_BT$) limit.

In order to calculate the average number of photons per small energy interval $d\varepsilon$, the average energy of photons per small energy interval $d\varepsilon$, as well as the total average number of photons in a photon gas and its total energy, we need to know the density of states for photons as a function of photon energy.

3.3.2 Spectrum of Black-Body Radiation

Black-body radiation has a characteristic, continuous frequency spectrum that depends only on the body's temperature, called the Planck spectrum or Planck's law [20]. The spectrum is peaked at a characteristic frequency that shifts to higher frequencies with increasing temperature, and at room temperature most of the emission is in the infrared region of the electromagnetic spectrum [19]. The average energy of photons with frequency between $\nu$and $\nu + d\nu$ (per unit volume) is $g(\nu)(\bar{n_{ph}})d\nu$,since each photon carries energy E=$h\nu$,the spectral energy density is:

$$u(\nu, T) = h\nu g(\nu) f(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{exp\,(\beta h\nu) - 1}, \tag{3.3.12}$$

u as a function of the energy

$$u(\varepsilon, T) = \frac{8\pi}{(hc)^3} \frac{\varepsilon^3}{exp(\frac{\varepsilon}{k_BT}) - 1}, \tag{3.3.13}$$

$u(\varepsilon, T)$- the energy density per unit photon energy for a photon gas in equilibrium with a black body at temperature T.

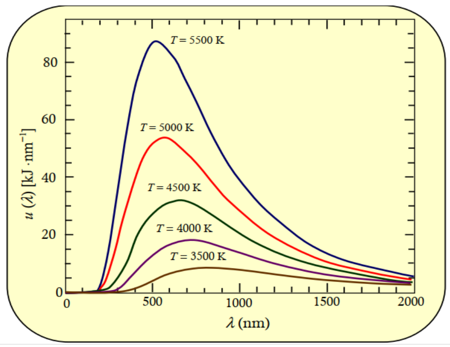

Figure 3.2: Black body isotherm curves [10].

In a graphical representation, the black-body spectrum is characterized by:

A peak in intensity that shifts to shorter wavelengths as temperature increases (according to Wien's Law).

A continuous spectrum that increases steeply with temperature.

The total area under the curve represents the total energy emitted per unit area, which increases rapidly with temperature (Stefan-Boltzmann Law).

3.3.3 Planks Law of Black-Body Radiation

Planck's Law describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T.

According to quantum statistics principles,the spectral volume density of radiation energy can be determined by calculating the equilibrium distribution of photons,for which the radiation field entropy is maximum,and taking in to consideration that the photon energy with frequency ν (angular frequency ω=$2\pi\nu$) equal to $h\nu$=$\hbar\omega$ .If radiation field is considered to be a gas obeying the Einstein-bose statistics ,then we obtain the planck formula for the volume of radiation.

$$u_s(\nu,T)d\nu = \frac{8\pi h\nu^3\pi}{c^3}\frac{1}{exp\,(h\nu/k_BT)-1)}, \tag{3.3.14}$$

In terms of angular frequency the spectral volume density of radiation is:

$$u(\omega) = \frac{\hbar\omega^2}{\pi^2c^3}\frac{1}{exp\,(\hbar\omega/k_BT)-1}, \tag{3.3.15}$$

where k_B is the Boltzmann constant.The planck law law for energy distribution in the black body spectrum maximum value of intensity of radiation that can be emitted by any body at the given temperature and wave length.

3.3.4 Rayleigh-Jeans Radiation Law

Rayleigh-Jeans Law describes the spectral density of black body radiation at low frequencies (long wavelengths). It was an earlier model that was later modified by Plancks Law. Rayleigh recognized that equipartition could only apply at low frequencies and assumed an arbitrary cut off at high frequencies [1].

A black body at thermodynamic equilibrium at low frequencies or high temperature, $e^{\frac{h\nu}{k_BT}} \approx 1+\frac{h\nu}{k_BT}$ and equation (3.3.12) becomes.

$$u(\nu,T) = \frac{8\pi\nu^2}{c^2}\times\frac{h\nu}{1+\frac{h\nu}{k_BT}-1} = \frac{8\pi\nu^2}{c^3}k_BT, \tag{3.3.16}$$

The temperature dependent term in Equation (3.3.16) is a purely classical result (note that it does not contain Plancks constant h) and can be obtained directly from equipartition. It is known as the Rayleigh-Jeans radiation law, u as a function of the wavelength:

$$u(\lambda,T)d\lambda = -u(\varepsilon,T)[\frac{d\varepsilon}{d\lambda} = \frac{-hc}{\lambda^2}], \tag{3.3.17}$$

$$u(\lambda, T) = \frac{8\pi}{(hc)^3} \frac{\left(h\frac{c}{\lambda}\right)^3}{e^{\left(\frac{hc}{\lambda k_B T}\right)} - 1} \left(\frac{hc}{\lambda^2}\right) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\left(\frac{hc}{\lambda k_B T}\right)} - 1}. \tag{3.3.18}$$

3.3.5 Ultraviolet Catastrophe

According to classical physics, the total energy density of electromagnetic radiation inside an enclosed cavity is infinite. This is clearly an absurd result, and was recognized as such in the latter half of the nineteenth century. In fact, this prediction is

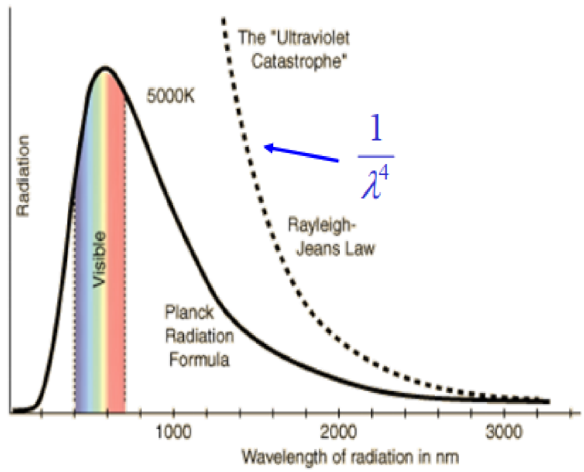

Figure 3.3: u as a function of the wave length [1].

known as the ultra-violet catastrophe, because the Rayleigh-Jeans law usually starts to diverge badly from experimental observations (by over-estimating the amount of radiation) in the ultra-violet region of the spectrum [14]. The equation 3.3.16 This equation predicts the so-called ultraviolet catastrophe an infinite amount of energy being radiated at high frequencies or short wavelengths.

3.3.6 Weins Radiation Formula

In 1896 proceeding from classical concepts ,Wein derived the law of energy distribution in the black-body spectrum (Wein radiation law). At high frequencies,when $h\nu \gg k_B T$ and equation (3.3.12) becomes

$$u_s(\nu, T) \approx h\nu^3 e^{-\frac{1}{K_B T} h\nu} \tag{3.3.19}$$

He give the spectral distribution of black body radiation for short wave length when the temperature T increases.
From equation (3.3.12) high frequencies, which is the so called Wien law.So spectral density decreases exponential when $\nu \to +\infty$

3.3.7 Wien's Displacement Law

Wien's Displacement Law states that the frequency, ν_{max} at which the emission of radiation is maximized is directly proportional to the temperature T of the black body.

$$h\nu_{max} T = 2.8 k_B T, \tag{3.3.20}$$

Where ν_{max} is thefrequency at which the emission is maximum
T is the absolute temperature of the blackbody in kelvin.

$$\frac{du}{d\nu} = const \times \frac{d}{d\left(\frac{h\nu}{k_B T}\right)} \left[\frac{\left(\frac{h\nu}{k_B T}\right)}{e^{(\frac{h\nu}{k_B T})} - 1} \right] = const \times \left[\frac{3x^2}{e^x - 1} - \frac{x^6 e^x}{(e^x - 1)^2} \right] = 0. \tag{3.3.21}$$

$$(3 - x)e^x = 3 \to x \approx 2.8. \tag{3.3.22}$$

3.3.8 Stefan Boltzmann Law

Stefan-Boltzmann Law describes the total energy radiated per unit surface area of a black body across all wavelengths per unit time. It is proportional to the fourth power of the black bodys absolute temperature, [13].Integrating equation (3.3.12) over the entire range of possible frequencies $\nu \in [0, +\infty]$ and multiplying by the volume ,V one obtains the total energy.The number of photons per unit volume:

$$\bar{n} = \frac{\bar{N}}{V} = \int_0^\infty \bar{n}(\varepsilon)g(\varepsilon)d\varepsilon = \left(\frac{k_BT}{h}\right)^3 \frac{8\pi}{c^3} \int_0^\infty \frac{\nu^2}{e^{\frac{h\nu}{k_BT}} - 1} d\nu = \frac{8\pi}{c^3} \left(\frac{k_BT}{h}\right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1} = \frac{8\pi}{c^3} \left(\frac{k_BT}{h}\right. \tag{3.3.23}$$

The total energy of photons per unit volume(the energy density of a photon gas:

$$u(T) \equiv \frac{U}{V} = \int_0^\infty \frac{\varepsilon \times g(\varepsilon)}{e^{(\beta\varepsilon)} - 1} d\varepsilon = \frac{8\pi^5 (k_BT)^4}{15(hc)^3}, \tag{3.3.24}$$

$\sigma = \frac{2\pi^5 {k_B}^4}{15h^3c^2}$ the **Stefan-Boltzmann constant**

$$u(T) = \frac{4\sigma T^4}{c}. \tag{3.3.25}$$

3.3.9 The Kirchhoff Law

A body in local thermodynamic equilibrium will emit the same amount of energy that it absorbs [8]. Therefore the body does not heat up or cool down. Consider a body which is able to absorb and emit radiation. If I_λ is the incident spectral radiance, then the emitted spectral radiance E_λ is

$$E_\lambda = \varepsilon_\lambda = a_\lambda I_\lambda, \tag{3.3.26}$$

where $a_\lambda I_\lambda$ is the absorbed spectral radiance and a_λ is the absorbance at a given wavelength. For thermal equilibrium the emitted and absorbed radiation is the same;

therefore, in the case of a blackbody:

$$I_\lambda = B_\lambda, \tag{3.3.27}$$

So,

$$E_\lambda = a_\lambda. \tag{3.3.28}$$

This is known as Kirchhoffs law, which means that at a given wavelength, weak absorbers are weak emitters, and conversely, strong absorbers are strong emitters. Kirchhoffs law is a general statement equating emission and absorption in heated objects, following from general considerations of thermodynamic equilibrium.

An object at some nonzero temperature radiates electromagnetic energy. If it is a perfect blackbody, absorbing all light that strikes it, it radiates energy according to the blackbody radiation formula. More generally, if it is a gray body that radiates with some emissivity multiplied by the blackbody formula, Kirchhoffs law states that at thermal equilibrium, the emissivity of a body equals its absorptivity [23].

3.4 Thermodynamic Function of Black-Body Radiation

3.4.1 Internal Energy of a Photon Gas

The walls of the cavity emit and absorb electromagnetic waves. The cavity is filled with radiation that is in equilibrium with the heat bath surrounding it, so that it can be assigned a temperature . The total energy of photons per unit volume(the energy density of a photon gas toward to Stefan-Boltzmann in equation (3.3.26)

3.4.2 Heat Capacity

Heat capacity is the amount of heat required to change the temperature of an object. For a black body, this concept can be applied to understand how the total energy emitted changes with temperature. The heat capacity of a photon gas at constant volume:Differtiating equation (3.3.26) in terms of temperature.

$$c_V \equiv [\frac{\partial u(T)}{\partial T}] = \frac{16\sigma}{c} V T^3 \tag{3.4.1}$$

3.4.3 Entropy

In black body radiation, entropy is a measure of the disorder or randomness associated with the energy distribution of the emitted radiation. The entropy of the cosmic black body radiation has not changed with time because the number of photons in each mode has not changed with time, although the frequency of each mode has decreased as the wavelength has increased with the expansion of the universe.[8] The equation (3.7.3) holds for all T (it agrees with the **Nernst theorem**), and we can integrate it to get the entropy of a photon gas.

$$S(T) = \int_0^T \frac{C_v(T')}{T'} dT' = \frac{16\sigma V}{c} \int_0^T T'^2 dT' = \frac{16\sigma}{3c} V T^3 \tag{3.4.2}$$

3.4.4 Free Energy

The Helmholtz free energy F is a thermodynamic potential that measures the useful work obtainable from a thermodynamic system at constant volume and temperature. All other quantities of interest can be computed from the free energy,

$$F = U - TS = \frac{4\sigma}{c} V T^4 - \frac{16\sigma}{3c} V T^4 = -\frac{4\sigma}{3c} V T^4 \tag{3.4.3}$$

The negative sign indicates that, as expected, black body radiation involves energy being released (emitted) as the temperature increases, which corresponds to a decrease in the Helmholtz free energy.

3.4.5 Gibbs Free Energy

Gibbs free energy is not commonly computed directly since blackbody radiation is usually studied in terms of energy density, temperature, and entropy. However, for a thermodynamic system in equilibrium.

$$G = U - TS, \tag{3.4.4}$$

where U is the internal energy,T is the temperature, and S is the entropy. For blackbody radiation in a cavity, the Gibbs free energy would be

$$G = \frac{4\sigma T^4}{c} - T \times \frac{4\sigma T^3 V}{c} = 0. \tag{3.4.5}$$

This result indicates that, for blackbody radiation in equilibrium, the Gibbs free energy is zero, reflecting that the system is in a state where no additional work can be extracted, consistent with the equilibrium condition.

3.4.6 Radiation pressure

Radiation pressure is the pressure exerted by electromagnetic radiation on a surface [22]. If photons hit a material surface a momentum transfer takes place, which implies that a stream of photons exerts pressure on the walls of a container, known as radiation pressure or photon pressure [21]. As a consequence, the photon gas pressure is a function of temperature alone.

$$P = -(\frac{\partial F}{\partial V})_{T,V} = \frac{4\sigma}{3c}T^4 = \frac{1}{3}u \tag{3.4.6}$$

3.5 Application

3.5.1 Human-body Emission

The human body can be approximated as a black body radiator in terms of thermal radiation. While the human body isn't a perfect black body, its emission can be approximated reasonably well using black body radiation principles.

The average surface temperature of the human body is approximately 37^0C (98.6^0F) [26]. According to Planck's law, the amount of radiation emitted by a black body increases with temperature. For the human body, this means that it primarily emits infrared radiation, which is invisible to the naked eye but can be detected with infrared cameras.

Because the human body temperature is much lower than that of a typical black body at high temperatures (like the Sun), the peak of its emission spectrum falls in the infrared region of the electromagnetic spectrum. The peak wavelength of this emission can be estimated using Wien's displacement law, which states that the wavelength at which the emission is maximum is inversely proportional to the absolute temperature of the black body. Wien's displacement law determine wavelength of emission human body:

$\lambda_{max}=\frac{b}{T}$,where b is Wiens displacement constant ($\approx 2898\ \mu mK$) and T is the temperature in Kelvin. For a body temperature of 37^0C (310 K):

$\lambda_{max}=\frac{2898\mu K}{310K} \approx 9.35\mu m$

This wavelength falls well into the infrared range.

The emission of infrared radiation is a significant way the human body loses heat. This process is crucial for regulating body temperature and can be affected by factors like clothing, environmental conditions, and metabolic rate.

The net power radiated is difference between the emitted and the absorbed.

$$p_{net} = p_{emit} - p_{absorb}, \tag{3.5.1}$$

Applying the Stefan-Boltzman law

$$p_{net} = A\delta\epsilon(T^4 - T_0^4), \tag{3.5.2}$$

where A and T are the body surface area and temperature , ϵ is emissivity ,σ is Stefan-Boltzman constant,and T_0 is ambient temperature.

The total radiated energy radiated in one day is about 8MJ,or 200k cal (food calories).The emissivity of skin is considerably less than 1, it emits sufficient infrared radiation to be easily detectable by modern techniques.

3.5.2 Temperature Relation Between a Planet and its Star

The black-body law may be used to estimate the temperature of a planet orbiting the Sun. Earths long wave thermal radiation intensity, from clouds, atmosphere and ground.The temperature of a planet depends on several factors: Incident radiation from its star emitted radiation of the planet (for example, Earths infrared glow).

The albedo effect causing a fraction of light to be reflected by the planet.The greenhouse effect for planets with an atmosphere Energy generated internally by a planet itself due to radioactive decay, tidal heating, and adiabatic contraction due to cooling. The analysis only considers the Suns heat for a planet in a Solar System. The Stefan-Boltzmann law gives the total power (energy/time) that the Sun emits:

$$P_{Semit} = 4\pi R_s^2 \sigma T_S^4, \tag{3.5.3}$$

where σ is the Stefan-Boltzman constant

T_S is is the effective temperature of the Sun, and

R_S is is the radius of the Sun

The Sun emits that power equally in all directions. Because of this, the planet is hit with only a tiny fraction of it. The power from the Sun that strikes the planet (at the top of the atmosphere) is:

$$P_{SE} = P_{Semit}(\frac{\pi R_E^2}{4\pi D^2}), \tag{3.5.4}$$

where R_E is is the radius of the planet, and D is is the distance between the Sun and the planet.

Because of its high temperature, the Sun emits to a large extent in the ultraviolet and visible (UV-Vis) frequency range. In this frequency range, the planet reflects a fraction α of this energy where α is the albedo or reflectance of the planet in the UV-V is range. In other words, the planet absorbs a fraction $1 - \alpha$ of the Sun's light, and reflects the rest. The power absorbed by the planet and its atmosphere is then:

$$P_{abs} = (1 - \alpha)P_{SE}, \tag{3.5.5}$$

Even though the planet only absorbs as a circular area πR^2 it emits in all directions; the spherical surface area being $4\pi R^2$. If the planet were a perfect black body, it would emit according to the StefanBoltzmann law

$$P_{embb} = 4\pi R_E^2 \sigma T_E^4, \tag{3.5.6}$$

Where T_E is the temperature of the planet. This temperature, calculated for the case of the planet acting as a black body by setting $P_{abs}= P_{embb}$, is known as the effective temperature. The actual temperature of the planet will likely be different, depending on its surface and atmospheric properties. Ignoring the atmosphere and greenhouse effect, the planet, since it is at a much lower temperature than the Sun, emits mostly in the infrared (IR) portion of the spectrum. In this frequency range, it emits $\bar{\epsilon}$ of the radiation that a black body would emit where $\bar{\epsilon}$ is the average emissivity in the IR range. The power emitted by the planet is then:

$$P_{emit} = \bar{\epsilon} P_{emitbb} \tag{3.5.7}$$

The power emitted by the planet is the then:For a body in radiative exchange equilibrium with its surroundings, the rate at which it emits radiant energy is equal to the rate at which it absorbs it:

$$P_{abs} = P_{emit}, \tag{3.5.8}$$

Substituting the expressions for solar and planet power in equations (3.5.3-3.5.7) and simplifying yields the estimated temperature of the planet, ignoring greenhouse effect, T_P

$$T_P = T_S \sqrt{R_S \sqrt{\frac{(1-\alpha)/\bar{\epsilon}}{2D}}}. \tag{3.5.9}$$

tice that a gray (flat spectrum) ball where $(1-\alpha)=\bar{\epsilon}$ comes to the same temperature as a black body no matter how dark or light gray.

3.5.3 Cosmology

The cosmic microwave background radiation observed today is the most perfect black body radiation ever observed in nature, with a temperature of about 2.7 K [27]. It is a "snapshot" of the radiation at the time of decoupling between matter and radiation in the early universe. Prior to this time, most matter in the universe was in the form of an ionized plasma in thermal, though not full thermodynamic, equilibrium with radiation.

According to Kondepudi and Prigogine, at very high temperatures (above 10^{10} K; such temperatures existed in the very early universe), where the thermal motion separates protons and neutrons in spite of the strong nuclear forces, electron-positron pairs appear and disappear spontaneously and are in thermal equilibrium with electromagnetic radiation. These particles form a part of the black body spectrum, in addition to the electromagnetic radiation.

3.5.4 Radiation in Universe

There are two sources of high energy particles entering the Earth's atmosphere from outer space: the sun and deep space. The sun continuously emits particles, primarily free protons, in the solar wind, and occasionally augments the flow hugely with coronal mass ejections (CME). The particles from deep space (inter- and extra-galactic) are much less frequent, but of much higher energies. These particles are also mostly protons, with much of the remainder consisting of helions (alpha particles).

A few completely ionized nuclei of heavier elements are present. The origin of these galactic cosmic rays is not yet well understood, but they seem to be remnants of supernovae and especially gamma-ray bursts (GRB), which feature magnetic fields capable of the huge accelerations measured from these particles.[31]

3.5.5 Cosmic Microwave Background

In 1978, Penzias and Wilson of Bell Labs, New Jersey, USA won the Nobel Prize for their serendipitous discovery (in 1963-1965) of seemingly uniform microwave emission coming from all directions in the sky,which has come to be known as the cosmic microwave background [20].

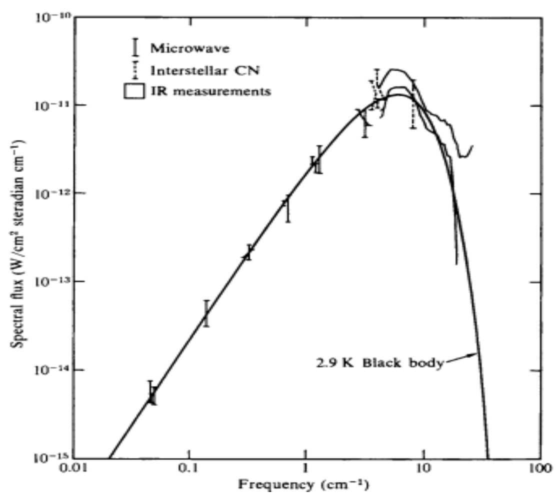

Figure 3.4: Experimental measurements of the spectrum of the cosmic black body radiation observations of the flux were made with microwave heterodyne receivers at frequencies below the peak, were deduced from optical measurements of the spectrum of interstellar CN molecules near the peak, and were measured with a balloon-borne infrared spectrometer at frequencies above the peak [20].

The cosmic microwave background (CMB or CMBR) is microwave radiation that fills all space in the observable universe. A major recent discovery is that the universe accessible to us is filled with radiation approximately like that of a black body at 2.9 K. The existence of this radiation (Figure 3.4) is important evidence for big bang cosmological models which assume that the universe is expanding and cooling with time. This radiation is left over from an early epoch when the universe was composed primarily of electrons and protons at a temperature of about 4000 K. The plasma of electrons and protons interacted strongly with electromagnetic radiation at all important frequencies, so that the matter and the black body radiation were in thermal equilibrium. By the time the universe had cooled to 3000 K, the matter was primarily in the form of atomic hydrogen. This interacts with black body radiation only at the frequencies of the hydrogen spectral lines.

The further expansion of the Universe can be considered as quasistatic adiabatic(isentropic) for the radiation:

S(T)$=\frac{16\sigma}{3c}VT^3$ =Constant

After the decoupling the evolution of matter into heavier atoms (which are organized into galaxies, stars, and dust clouds) was more complicated than before decoupling. Electromagnetic radiation, such as starlight, radiated by the matter since the decoupling is superimposed on the cosmic black body radiation [12].

3.5.6 Thermal Radiation Emission by planets

Planets emit thermal radiation primarily as black bodies, which means they absorb and re-radiate energy across a spectrum of wavelengths. The amount and spectrum of radiation emitted depends on the planets temperature.

Thermal radiation emission by planets an essential concepts in understanding how these celestial bodies interact with their environment, particularly with respect to their temperature and atmosphere.

The total power emitted by the entire planet, you need to consider the planets surface area. The surface area, A of a spherical planet is:

$$A = 4\pi R^2, \tag{3.5.10}$$

where R is the radius of the planet.

Thus, the total power, P radiated by the planet according to the Stefan-Boltzmann law :

$$P = 4\pi\sigma R^2 T^4. \tag{3.5.11}$$

A planet's thermal emission depends on its surface temperature. Hotter planets emit more radiation. For example, Earths average surface temperature is about 288 K ($15^0 C$), and it radiates in the infrared part of the spectrum.

3.5.7 The Greenhouse Effect

The absorptivity of any substance varies with frequency in a way characteristic of that substance. The study of such variation is called spectroscopy and is an enormous held in itself. An environmentally important consequence of this variation is the greenhouse effect.

In the earths atmosphere, carbon dioxide CO_2 is one of the greenhouse gases that play the role of the glass, being transparent at visible and near-infrared wavelengths but strongly absorbing in the 10 to $15\mu m$ range where the earths thermal radiation is concentrated [1].

The Greenhouse effect describes the warming of surface Earth caused by the interposition of an infrared absorbent layer of water,as vapor and in clouds,and carbon dioxide in the atmosphere between sun and Earth. The water may contribute as 90 percent of the warming effect.[8] We assume as an example that the atmosphere is a perfect greenhouse, defined as absorbent layer that transmits all visible radiation that fails on it from sun, but absorbs and re-emits all the radiation (which lies in the infrared),from the surface of the earth.

Absorption: Power in $=\alpha(\pi R_E^2)\sigma(T_{sun})^4(\frac{R_{sun}}{R_{orbit}})^2$,

Power out $=4\pi R_E^2\sigma T_E^4$,

$T_E=[\frac{\alpha}{4}(\frac{R_{sun}}{R_{orbit}})^2]^{1/4}T_{sun}$,

where R_{orbit}=$1.5\times10^{11}m$, $R_{sun}=7\times10^8m$

α=$1-T_{Earth}$=280K,

In reality α=$0.7-T_{Earth}$=256K.

To maintain a comfortable temperature on the Earth, we need the Greenhouse Effect! However, too much of the greenhouse effect leads to global warming.

3.5.8 Thermal Radiation Emission by stars

Thermal Radiation Emission by stars is a fundamental aspects of astrophysics, it describes how stars produce and emit energy.The emission is primarily the result of the process occurring in stars core, which nuclear fusion take place, and if affect various aspects of stellar behavior, including temperature, luminosity, and spectral characterizes.

Stars can be approximated as black bodies, which are idealized objects that absorbs all incident radiation. Thermal radiation emitted by a black body is described by planck's, which defines intensity of radiation at different wavelength for a given temperature. The surface of the earth, which warms up during the day as a result of the absorption of solar energy, cools down at night by radiating its energy into deep space as infrared radiation [2].

3.5.9 Sun's Mass Loss due to Radiation Emission

The Sun is the star closest to the earth, and its radiant energy is practically the only source of energy that influences atmospheric motions and our climate [14].

The Sun loses mass in two major ways.The first is through solar wind. The surface of the Sun is hot enough that electrons and protons boil off its surface and stream away from the Sun, generating a wind of ionized particles. [17]

The second way the Sun loses mass is through nuclear fusion.The Sun is powered by the conversion of hydrogen into helium in its core, through the process of nuclear fusion [16] producing its life-giving glow over billions of years . The production of helium transforms some of the hydrogens mass into energy, which radiates away from the Sun in the form of light and neutrinos. This reaction results in a decrease in the Suns mass, and in the release of energy through electromagnetic radiation and the by observing just how much energy the Sun radiates, and using Einsteins equation relating mass and energy, it was found that the Sun loses about 4.2 million tonnes of mass each second due to fusion.

3.5.10 Instruments for Measuring Atmospheric Thermal Emission

Measuring atmospheric thermal emission involves detecting the energy radiated by the Earths atmosphere and surface in the infrared spectrum. In spectral photometric measurements of the low temperature sources of thermal radiation (the atmosphere among them) the so called differential method is commonly used. This method consists essentially of measuring the difference between the investigated radiation and a certain referent flux of radiation [30]. The method is realized by making use of a double beam lighting system at the aperture of the spectrophotometer.

A spectrophotometer is a vital instrument for measuring atmospheric thermal emission.

3.5.11 Investigations of the Earths Net Radiation by Means of Satellites

The instruments used with the TIROS and NIMBUS meteorological satellites to measure the outgoing radiation in various spectral regions and the first results of these measurements. Satellites provide valuable data that help researchers understand the energy balance of the Earth, monitor climate change, and analyze weather patterns. Satellites use various sensors to measure solar radiation (incoming shortwave) and terrestrial radiation (outgoing longwave). These measurements are often made using radiometers that capture data in different spectral bands.

Satellite data are processed using radiative transfer models to correct for atmospheric effects, such as scattering and absorption by gases and aerosols.These models help estimate how much radiation reaches the Earth's surface versus what is emitted back into space. Satellites reveal that equatorial regions typically receive a surplus of net radiation, while polar areas experience deficits, confirming established climatic patterns. Seasonal variations in net radiation are also observed, with higher values in summer months for mid-latitude regions.

Chapter 4

Radiative Transfer Equation

4.1 Beer's Law

Beers law, also known as Bouguers law or Lamberts law, states that the monochromatic intensity I_λ decreases monotonically with path length as the radiation passes the layer [13,30]. The law can be demonstrated as follows. Consider a parallel beam of radiation, I_λ, passing through an infinitesimally thin layer of the atmosphere containing absorbing gases and aerosols along a specific path (see Fig. 4.1). After passing through the layer, the monochromatic intensity of radiation is decreased by

$$dI_\lambda = -I_\lambda \rho r k_\lambda \sigma ds, \tag{4.1.1}$$

where ρ is the density of air, r is the mass of the absorbing gas per unit mass of air, and k_λ is the mass absorption or scattering coefficient. As shown in Fig 4.1, the differential path length along the ray path of the incident radiation is

$$ds = sec\theta dz, \tag{4.1.2}$$

Substituting this value for ds in Eq. (4.1.1), we get

$$dI_\lambda = -I_\lambda \rho r k_\lambda sec\theta dz \tag{4.1.3}$$

Now let us consider the case of depletion of radiation due to absorption and scattering from the top of the atmosphere ($z = \infty$) down to any level (z). Integrating Eq. (4.1.3) gives

$$lndI_{\lambda_\infty} - lnI\lambda = sec\theta \int_z^\infty k_\lambda \rho r dz, \tag{4.1.4}$$

Taking the antilog on both sides, we ge

$$I_\lambda = I_{\lambda_\infty} exp^{(-\tau sec\theta)} = I_{\lambda_\infty} t. \tag{4.1.5}$$

where $\tau = k_\lambda \rho r dz$ is a dimensionless quantity referred to as the normal optical depth or optical thickness. It is a measure of the cumulative depletion that a beam of radiation directed straight downward with zero zenith angle would experience in passing through the layer, and t_λ $=exp^{(-\tau_\lambda sec\theta)}$ is the transmissivity of the layer. In the absence of scattering, the monochromatic absorptivity

$$a_\lambda = 1 - t_\lambda = 1 - exp^{(-\tau_\lambda sec\theta)} \tag{4.1.6}$$

approaches unity exponentially with increasing optical depth. In the above cases, we have dealt with the scattering and absorption of solar radiation in the atmosphere in the absence of emission. Now we will see the absorption and emission of infrared radiation in the absence of scattering.

4.2 Schwarszchilds Equation

Let us derive the equation that governs the transfer of the infrared radiation through a gaseous medium[13]. The rate of change of the monochromatic intensity of outgoing terrestrial radiation along the path length of ds = $sec\theta dz$, due to absorption within the layer, is written as

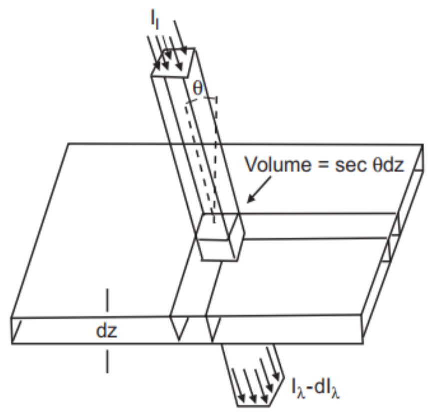

Figure 4.1: Depletion of incoming beam of parallel radiation on passing through a slab of absorbing material [13].

$$dI_\lambda(absorption) = -I_\lambda k_{a\lambda} r sec\theta \lambda dz = -I_\lambda a_\lambda, \tag{4.2.1}$$

where ka_λ is the mass absorption coefficient, and a_λ is the absorptivity of the layer. The corresponding rate of change of radiation due to emission is

$$dI_\lambda(emission) = -B_\lambda(T)\varepsilon_\lambda, \tag{4.2.2}$$

According to Kirchhoffs law,the emissivity ε_λ of a black body is equal to its absoptivity of the layer Eq. (4.2.2) can be written as

$$dI_\lambda(emission) = -B_\lambda(T)k_\lambda \rho r sec\theta dz, \tag{4.2.3}$$

Now subtract the absorption from the emission to obtain the net contribution of the layer to monochromatic intensity of the radiation passing through it.

$$dI_\lambda = -[I_\lambda a_\lambda - B_\lambda(T)]k_\lambda \rho r sec\theta dz. \tag{4.2.4}$$

his equation is known as Schwarzschilds equation. It states that as the radiation passes through an isothermal layer, its monochromatic intensity exponentially approaches that of a black body radiation corresponding to the temperature of the layer.

Chapter 5

Thermal Radiation Heat Transfer

5.1 Introduction

Thermal radiation heat transfer is a fundamental mode of energy transfer that occurs through electromagnetic waves. Thermal radiation can occur through a vacuum, making it crucial in a variety of scientific and engineering applications. Thermal radiation heat transfer is a vital process that impacts many aspects of modern technology and science. Its study encompasses the emission, absorption, and transmission of electromagnetic radiation and is fundamental to understanding energy interactions in both terrestrial and extraterrestrial [4].

5.2 Radiative Heat Transfer

All matter emits and absorbs radiation under all conditions, there is always radiative transfer of energy, even within an isothermal system. If two objects are at different temperatures, there will be net radiation energy transfer between them, even if there is no matter between the objects. An obvious example is radiation emitted by the sun, which travels through the vacuum of space and is partially absorbed and scattered upon entering the earths atmosphere [5]. The amount of emitted and absorbed radiation are functions of the physical and chemical properties of the material as well as its internal energy level, as quantified by its temperature [6].

5.2.1 Translation of Energy Deposits to Heating Rates

If we denote the flux I as units in Wm^{-2}, then the heating rate can be related directly to the rate of energy deposition.

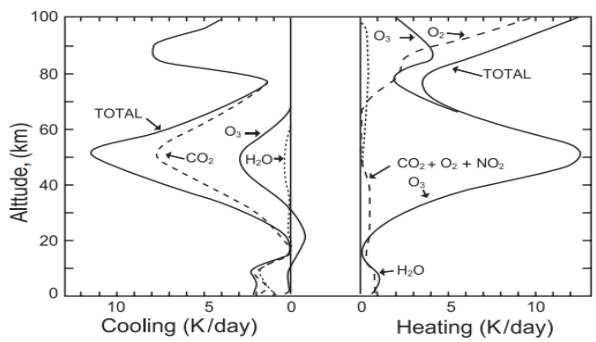

Figure 5.1: Vertical distribution of shortwave heating rates and long wave cooling rates [13].

The flux of radiation integrated over the entire range of wavelengths can be written as

$$I = I_\infty \int exp\left[-\sigma_{\alpha\lambda} n(z) sec\theta dz\right] d\lambda, \tag{5.2.1}$$

Now integrate Eq.(5.2.1) over a range of wavelengths. Only O_2 and O_3 really contribute to the heating:

$$I = \int I_\infty exp\left[-\sigma o_2[O_2] sec\theta dz - \int \sigma o_3 \lambda [0_3] sec\theta dz\right] d\lambda, \tag{5.2.2}$$

The rate of energy deposition is

$$r = \int_\lambda I\sigma o_2(\lambda)[O_2] + \sigma o_3(\lambda)[O_3] d\lambda, \tag{5.2.3}$$

where r has units of J $s^{-1}m^{-3}$. If all of this energy goes into heating, then the rate of energy deposition equals the rate of heating. By the First Law of Thermodynamics.

$$\frac{dU}{dt} = \dot{Q}, \tag{5.2.4}$$

which can be represented as

$$\rho c_p \frac{DT}{Dt} = r, \tag{5.2.5}$$

$$\frac{DT}{Dt} = \frac{1}{C_p} \int_\lambda I\{\delta(o_2)(\lambda)[o_2] + \delta o_3(\lambda)[o_3]\} d\lambda. \tag{5.2.6}$$

where ρ is the density, and C_p is the specific heat, or specific enthalpy (1,005 J $kg^{-1}K^{-1}$).

5.2.2 Infrared Heating and Cooling

Infrared heating plays a relatively small role in stratospheric heating. The main infrared heating is by absorption of the 2,700 nm and 4,300 nm bands of CO_2. The 9,600 nm band of O_3 provides some heating near the tropopause.

Infrared cooling is primarily due to the 15,000 nm band of CO_2. The second most important infrared cooling is in the 9,600 nm band of O_3, which is most important near the stratopause. We can use the cooling to space approximation to estimate the cooling rate. In this approximation, we ignore the downward flux of infrared and look only at the upward flux. This is not a bad approximation, as can be seen in the figure on upward and downward fluxes. The temperature change with time is given by the equation:

$$\frac{DT}{Dt} = CB_\nu(T), \tag{5.2.7}$$

where $B_\nu(T)$ is the Plancks Law, C is a constant that depends on the infrared lines, their shape and overlap, and their oscillator strengths. For this case, in which the absorption is strong, the expression becomes

$$\frac{DT}{Dt} = -\{5.4 \times 10^5(\theta CO_2/3.3 \times 10^-4)^{1/2}\}\{3.5 \times 10^{-4} exp(960/T)\}. \quad (5.2.8)$$

where the second half of the right-hand-side is an approximation of the Plancks function for the CO_2, in 15,000 nm band.

5.2.3 Radiative Heating due to Absorption

In order to determine the heating rate of the layer between altitude z and z+dz, the energy balance at each boundary of the layer must be identified. As the thickness dz approaches zero, the energy absorbed per unit volume is given by the net flux divergence DF/dz. Therefore, the variation of the temperature in the layer per unit time is given by

$$\frac{DT}{Dt} = -\frac{1}{\rho C_p}\frac{DF}{Dz}, \quad (5.2.9)$$

$$\frac{DT}{Dz} = \frac{g}{C_p}\frac{DF}{DP}. \quad (5.2.10)$$

where C_p is the specific heat at constant pressure,ρ is the total air density, g is the acceleration due to gravity, and p is the pressure.

5.2.4 Heat Waves

Infrared radiations are produced by hot bodies and molecules or due to vibrations of atoms and molecules or due to transition of electrons between two (closely spaced) energy levels in an atom, so infrared waves are called heat waves as they cause heating effect.

The term heat wave in black body radiation can be linked to the concept of thermal radiation and its effects at high temperatures. As the temperature of a black body increases, the peak wavelength of emitted radiation shifts to shorter wavelengths and the total emitted radiation increases significantly [29].

5.2.5 Equation for the Heat Inflow

The equation of heat inflow in the atmosphere can be generally written as

$$\varrho q + D = C_p \frac{dT}{dt} + p\varrho \frac{d}{dt}\left(\frac{1}{\varrho}\right) \tag{5.2.11}$$

where p is the pressure, ϱ is air density, T is the air temperature, q is the amount of heat received by unit mass of air in unit time, D is the dissipation of mechanical energy, C_p is the specific heat of air at constant volume, and t is time.

The heat inflow can be presented with sufficient accuracy as the sum of the following three components

$$\varrho q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \tag{5.2.12}$$

where ε_1 is the heat inflow due to turbulent conductivity, ε_1 is the heat in- flow due to radiative exchange, and ε_2 is the heat inflow due to phase transformations of water in the atmosphere. Taking account only of the vertical turbulent mixing, the heat influx is expressed as

$$\epsilon_1 = \frac{\partial}{\partial z}\left(\frac{\lambda \partial T}{\lambda \partial z}\right), \tag{5.2.13}$$

Where λ is the coefficient of turbulent thermal conductivity. Taking into account the dependence of I_ν, on direction, it is easy to see that the total absorption of radiant energy by an elementary layer of mass dw can be written as

$$dw \int_0^\infty k_\nu I_\nu \int (t, p, r) dw, \tag{5.2.14}$$

where the second integral is taken over all possible directions. In like manner we obtain for the total emission of the elementary layer

$$4\pi dw \int_0^\infty \eta_\lambda d\lambda = 4\pi\eta dw, \tag{5.2.15}$$

where the second integral is taken over all possible directions. In like manner we obtain for the total emission of the elementary layer: Using (5.2.14) and (5.2.15) and having in mind that the main absorbing and emitting component of the atmosphere is water vapor.Equation for the heat inflow radiative heat inflow per unit mass of the absorbing substance:

$$\frac{\varepsilon_2}{\varrho_w} = \int_0^\infty k_\lambda d\lambda \int I_\lambda dw - 4\pi\eta, \tag{5.2.16}$$

where ε_2, is the heat gain per unit volume. Let us now derive the expression for the radiative heat gain, using the accurate transfer equation. Denoting the directing cosines of the beam in the direction r by cos(r, x), cos(r, y), and cos(r, z), we have

$$\frac{\partial I_\lambda}{\partial S} = -\frac{\partial I_\lambda}{\partial x} cos(r,x) + \frac{\partial I_\lambda}{\partial y} cos(r,y) + \frac{\partial I_\lambda}{\partial z} cos(r,z), \tag{5.2.17}$$

Thus can be rewritten as

$$\frac{1}{\varrho}\left[\frac{\partial I_\lambda}{\partial x} cos(r,x) + \frac{\partial I_\lambda}{\partial y} cos(r,y) + \frac{\partial I_\lambda}{\partial z} cos(r,z)\right] = \eta_\lambda + \frac{\sigma_\lambda}{4\pi}\int I_\lambda(P,r')\gamma_\lambda(P,r',r)dw' - (K_\lambda + \sigma), \tag{5.2.18}$$

Integrating both sides of the last equation with respect to r (that is, over all possible directions), we find

$$\int \frac{I_\lambda}{\partial S} dw = \frac{\partial}{\partial x}\int I_\lambda cos(r,x)dw - \frac{\partial}{\partial y}\int I_\lambda cos(r,y)dw + \frac{\partial}{\partial z}\int I_\lambda cos(r,z)dw = \frac{\partial F_{\lambda,x}}{\partial x} + \frac{\partial F_{\lambda,y}}{\partial y} + \frac{\partial F_{\lambda,z}}{\partial z}, \tag{5.2.19}$$

The right-hand side of the transformed equation is now

$$\int\left[\int I_\lambda(P,r')\gamma_\lambda(P,r')dw'\right]dw, \int dw' I_\lambda(P,r')\int \gamma_\lambda(P,r',r)dw = \tag{5.2.20}$$

$$4\pi \int I_\lambda \lambda(P, r')dw' = 4\pi \int I_\lambda \lambda(P, r)dw, \tag{5.2.21}$$

thus we have

$$\frac{1}{\varrho} divF_\lambda = 4\pi\eta_\lambda + \sigma_\lambda \int I_\lambda(P, r)dw - (k_\lambda + \sigma_\lambda) \int I_\lambda(P, r)dw, \tag{5.2.22}$$

or, after simplification

$$\frac{1}{\varrho} divF_\lambda = 4\pi\eta_\lambda - k_\lambda \int I_\lambda(P, r)dw, \tag{5.2.23}$$

Integrating both sides of this equation with respect to all wavelengths within 0 to ∞, we finally have

$$\frac{1}{\varrho} divF_\lambda = 4\pi\eta_\lambda - \int_0^\infty k_\lambda \int I_\lambda(P, r)dw, \tag{5.2.24}$$

. Comparing this relation with (5.2.16) and having in mind that here, as in (5.2.16),ϱ should be replaced by ϱ_w, we find

$$\varepsilon_2 = -divF \tag{5.2.25}$$

$$C_p \varrho \frac{\partial T}{\partial t} = \varepsilon_2 = -divF. \tag{5.2.26}$$

we find that at div $F > 0$, radiative heat transfer causes a cooling of air temperature, while at div $F < 0$, its thermal effect is positive. The case div F = 0 appears to describe the state of radiative equilibrium at the given point (the heat inflow is zero).

$$\varepsilon_2 = \varrho w \sum_j k_j(G_j + U_j) - 2P_\lambda B, \tag{5.2.27}$$

This shows that

$$divF = -\varrho w \sum_j k_j(G_j + U_j) - 2P_\lambda B, \tag{5.2.28}$$

Since with an approximate treatment of radiative heat transfer, F = U - G and the fluxes U and G vary only in the vertical direction, we obtain

$$divF = \frac{\partial F}{\partial z} = \frac{\partial}{\partial z}\left(U - G\right). \tag{5.2.29}$$

Thus the sign of the temperature changes due to radiative heat transfer is determined by the sign of the effective radiant flux.

5.2.6 Results of Calculations of Radiative Flux Divergence

As was shown in Sec. 5.2.11 the temperature changes due to radiative heat exchange can be calculated from the following formula:

$$C_P \frac{\partial T}{\partial t} = -\frac{\partial F}{\partial t} \tag{5.2.30}$$

It is evident that an increase of the effective flux with height causes radiative cooling of the air.If, however,$\frac{\partial F}{\partial t}$ less than 0,then $\frac{\partial T}{\partial t}$ less than 0

5.2.7 Climatology of Net Radiation of the Earth

The net radiation is the defined as the difference between the radiant energy absorbed and that emitted by the underlying surface, by the atmosphere or the system earth atmosphere [30]. The equation for the net radiation of the atmosphere given us:

$$R_n = R_{in} - R_{out} \tag{5.2.31}$$

where R_n is net radiation, R_{in} is incoming radiation which is Primarily from the Sun, this is the energy received at the Earth's surface, which varies by location, season, and time of day. R_{out} is outgoing radiation which is the energy emitted by the Earth back into space as infrared radiation. It is affected by the surface temperature and emissivity of the Earths surface.

Net radiation varies significantly with latitude. Equatorial regions receive more incoming solar radiation than polar regions, leading to a net surplus.In contrast, polar regions often experience a net deficit due to long wave radiation losses, especially during winter months. Seasonal changes influence net radiation due to variations in solar angle and day length. For instance, during summer, net radiation increases in the Northern Hemisphere, while it decreases during winter. Topography, vegetation, and land cover can modify local net radiation. Urban areas often have higher net radiation due to heat absorption by buildings and infrastructure (urban heat island effect).

Chapter 6

SUMMARY AND CONCLUSION

In this project, we have presented a review of the the quantum statistical description of thermal radiation or long-wave radiation. Fundamentally, radiation arises from the principles of quantum mechanics and statistical physics. At its core, it explains how electromagnetic radiation is emitted and absorbed by matter in thermal equilibrium. The theoretical concept of a black body, an idealized physical body that absorbs all incident radiation, serves as a model for understanding thermal radiation.

Max Planck introduced the idea that electromagnetic radiation is quantized, meaning energy is emitted or absorbed in discrete packets (quanta) called photons. This quantization leads to the Planck distribution of energy states. According to the Planck distribution, the mean number of photons occupying a photon state of frequency ν is $\bar{n_{ph}} = \frac{1}{exp\left(\frac{h\nu}{k_B T}\right)-1}$.

The intensity and spectral distribution of thermal radiation depend on the temperature of the emitting body. As the temperature increases, the peak of the radiation spectrum shifts to shorter wavelengths (Wien's Displacement Law). The total energy radiated per unit surface area of a black body to the fourth power of its absolute temperature, highlighting the significant increase in emitted energy with temperature(Stefan-Boltzmann Law).

Radiative heat transfer is governed by the propagation of electromagnetic energy and the interaction of that energy with matter. In most instances, a complete description of radiative heat transfer starting from the basic equation governing the interaction of electromagnetic waves with matter is intractable.

In general, the quantum statistical description of thermal radiation provides a comprehensive framework for understanding the emission and absorption of radiation at a fundamental level. It resolves classical discrepancies, incorporates quantum mechanics, and accurately describes the behavior of thermal radiation across a wide range of conditions.

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