THE CONTRIBUTION OF DEGENERATE ELECTRON PRESSURE TO THE STABILITY OF THIN KEPLERIAN ACCRETION DISKS AROUND A NEUTRON STAR WITH AXISYMMETRIC MAGNETIC DIPOLE

By

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Table of Contents

List of Figures

Abstract

The stability analysis of a geometrically thin, gas-pressure dominated accretion disk around a neutron star is presented.We have also derived the electron degenerate pressures and gas pressure in the middle and outer regions of the disk as a function of r using radial dependence of the central temperature and the density. The latter was obtained using the basic equations for thin accretion in non-relativistic case. The stability is analyzed at a small temperature, that is temperature approaching zero and at finite temperature. The contribution of both fully and partially degenerate electrons for the stability of the disk in its middle and outer regions are investigated. We find that the disk is stable in these regions, where the gas pressure is more dominant than radiation pressure.

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Introduction

An accretion disk is a structure (often a circumstellar disk) formed by diffuse material in orbital motion around a central body. The central body is typically a younger star, a protostar, a white dwarf, a neutron star, or a black hole. The accretion disk is likely to be formed when the compact star is a member of a close binary system and matter transferred from a giant type star onto its compact companion has high angular momentum. Shakura and Sunyaev(1973) initiated this discussion considering a very simplistic but effective standard model of a geometrically thin, optically thick accretion disk. According to the standard theory of accretion disks Shakura and Sunyaev (1973), the middle and outer parts of the disk are dominated by the gas pressure. Those regions have been found to be stable to the thermal and viscous modes but pulsationary unstable to the acoustic modes Blumenthal, G.R., Yang, and Lin: (1984) Shakura and Sunyaev(1973) formulated the standard model of a geometrically thin, optically thick accretion disk. They were able to obtain an analytical solution of the height integrated hydrodynamic equations, after having introduced the α-prescription for the turbulent stress, which transports the angular momentum outwards through the disk; since a disk in Keplerian rotation is stable according to Rayleighs criterion.Balbue , Hawley(1991) instead showed that it is unstable if there is a weak magnetic field in the disk. Subsequent numerical simulations confirmed that this instability generates turbulence and that the resulting turbulent stresses transport angular momentum outwards.

The stability of geometrically thin accretion disk has been studied extensively and it has been found that the disk is thermally and viscously unstable if it is optically thick and radiation pressure dominated Pringle and Pacholczyk,(1973) . There is also a possible mode of pulsational over stability.Kato (1978) considered the evolution of all three components of the fluid velocity. He found that the disk exits pulsation instability besides the viscous instability and thermal instability.Chen and Tamm (1995), pointed out the galactic black hole candidates may be due to pulsational over stability. Some early analysis about the stability of gas pressure dominated disk have incoperated azimuthal perturbation. However, the radial perturbation were neglected in all those studies. Mckee, (1991) has investigated the contribution of gas pressure to the stability of a standard disk. He found that the disk is stable when $\beta \leq 0.6$ (where β is the ratio of radiation pressure to total pressure). This implies that gas pressure dominated disk is more stable.

The structure of this thesis is as follows, The first chapter is a devoted to the discussion of electron degeneracy,in which we calculate electron degeneracy pressure.In chapter two the basic equations of thin accretion disk ,such as the equation of state, conservation of mass, angular and radial momentum, energy and equation for the vertical structure are briefly discussed.Structure equation and steady keplerian disks are fully dealt with in chapter three.The stability analysis of the thin accretion disk and estimating of the radius of instability of the middle and outer regions is given in chapter four.Finally we made a conclusion and discussion.

Chapter 1

Electron Degenerate Pressure

According to Paulis Exclusion Principle, no two fermions (particles with spin of one half) can occupy the same quantum state. The pressure exerted by fermions squeezed into a small box is what keeps cold stars from collapsing. A White dwarf is a remnant of a normal star which has exhausted its fuel fusing light elements into heavier ones. As the star cools, it shrinks in size until it is arrested by the degeneracy pressure of the electrons. If the white dwarf acquires more mass, the Fermi energy rises until electrons and protons abruptly combine to form neutrons and neutrinos, an event known as a supernova. The neutron star left behind is supported by degeneracy pressure of the neutrons. We can compute the degeneracy pressure from analyzing the dependence of the energy on volume for a fixed number of particles (fermions). The pressure generated by electrons that are forced in to higher momentum states as their density increases is called electron degeneracy pressure. A state of complete degeneracy is obtained when all the available momentum states are occupied up to a maximum momentum value. Such an ideal situation can only be achived at absolute zero temperature, but it consistes a good approximation to states of high degeneracy and has the advantage of enabling a straight forward calculation of the pressure. Therefore, although the transition from a Maxwellian to acompletely degenerate momentum distribution occurs gradually. The force provided by this pressure sets a limit on the extent to which matter can be squeezed together without collapsing into a neutron star. When electrons are squeezed too close together, the exclusion principle requires them to have different energy levels.To add another electron to a given volume requires raising an electron's energy level to make room, and this requirement for energy to compress the material appears as a pressure. The density of electrons is described by Fermi-Dirac statistics since an electron has half integral spin. For an electron with momentum p the density in the range (dp) can be described by J. McDougall and E.C.Stoner (1939)

$$n_e(p)dp = \frac{8\pi}{h^3}p^2dp\left[exp\left(\alpha + \frac{E}{K_BTc}\right) + 1\right]^{-1} \tag{1.0.1}$$

where $n_e(p)dp$ is the number density of electron momentum between p and p+dp. From Pauli exclusion principle, two identical electrons can't occupy the same state, that is

$$P_{(p)} = \left[exp\left(\alpha + \frac{E}{K_BTc}\right) + 1\right]^{-1} \tag{1.0.2}$$

Where k_B is Boltzmann's constant and T_C central temperature.
can't be larger than 1. If the density of electrons increases, the electrons will be forced to higher momentum states. The high momentum electrons contribute to the pressure, and the restriction on number density at each state is the souce of degeneracy pressure.

1.1 Non-relativistic Complete Degeneracy

We first consider a simple case which is T$\rightarrow$ 0. When the density is high enough, all the electron states with energy less than a maximum energy are filled. Applying the Heisenberg and Pauil principle to a completely degenerate isotropic electron gas yields the distribution; the number of electrons with momenta in the interval of $(p, p + dp)$ per unit volume:

$$n_e(p) = \frac{2}{\Delta V} \qquad\qquad p \leq p_0 \tag{1.1.1}$$

where δV is change of volume For completely non relativistic

$$P_0 \ll mc^2 \tag{1.1.2}$$

If the particle's location is known Δv and its momentum is with in an element $\Delta^3 p$ in three dimensional momentum space, then Δv and $\Delta^3 p$ are constrained by the condition:

$$\Delta v \Delta^3 P \geq h^3 \tag{1.1.3}$$

$$n_e(p) = \frac{2}{\Delta V}.\frac{\Delta^3 p}{\Delta^3 p} = \frac{2\Delta^3 p}{\Delta v \Delta^3 p}$$

Here: $\Delta v \Delta^3 p = h^3$ from equation (1.1.3)

and $\Delta^3 \rightarrow d^3p$=$4\pi p^2 dp$ in momentum space

$$n_e(p) = \frac{2}{h^3} 4\pi p^2 dp \tag{1.1.4}$$

The maximam momentum p_0 is obtained as $n_e = \int_0^{p_0} n_e(p)dp$

$n_e = \int_0^{p_0} \frac{8\pi p^2}{h^3} dp = \frac{8\pi p_0^3}{3h^3}$ solving for p_0 we can have:

$$P_0 = \left[\frac{3h^3 n_e}{8\pi}\right]^{1/3} \tag{1.1.5}$$

Using the pressure integral :

$$P = \frac{1}{3}\int_0^{\infty} \nu p n(p)dp \tag{1.1.6}$$

Where $\nu = \frac{p}{m}$ and $n(p) = \frac{2}{h^3}4\pi p^2 dp$, and integrating up to p_0 equation (1.1.6) becomes

$$P = \frac{1}{3}\int_0^{p_0} \frac{p^2}{m}.\frac{8\pi p^2}{h^3}dp = \frac{1}{3}\int \frac{8\pi p^4}{mh^3}dp \tag{1.1.7}$$

$$P = \frac{8\pi p_0^5}{15mh^3} \tag{1.1.8}$$

Substituting the value of p_0 from equation (1.1.5)

$$P = \frac{8\pi}{15mh^3}\left[\left(\frac{3h^3 n_0}{8\pi}\right)^{1/3}\right] = P_e \tag{1.1.9}$$

$$P_e = \frac{h^2}{20mm_p^{5/3}}\left(\frac{3}{\pi}\right)\left[\frac{\rho_c}{\mu_e}\right]^{5/3} \tag{1.1.10}$$

Where m is mass of electron,μ_e is the ratio of electron number to proton number, h is planks constant,m_p is mass of proton and ρ is density. The ideal electron pressure becomes

$$P_e = 9.77 \times 10^6\left[\frac{\rho_c}{\mu_e}\right]^{5/3} \tag{1.1.11}$$

We note that the degeneracy pressure is inversely proportional to the particle (electron) mass. Even at absolute zero, the ideal degenerate Fermi gas pressure does not disappear since only one electron is able to have zero momentum according to the the Pauli exclusion principle, i.e.

$$P = \frac{2U}{3V} = \frac{2E}{5V}\left[1 + \frac{5\pi^2}{12}\left(\frac{k_B T_c^2}{E}\right) + ...\right] \tag{1.1.12}$$

Because of this zero point pressure, all the other electrons have some finite momentum. Although the argument presented for electrons could be equally applied to proton and neutrons, as they are nearly 2000 times more massive than electrons, quantum effects become important in their case under much more extreme conditions (much higher densities for a given temperature, and much more lower temperature for a given density), and are usually ignored. We also note that, in spite of the high densities characteristic of degenerate matter, the particles may still be considered free, since the particle energy of the order of $\frac{P_o^2}{2m}$ is still higher than the Coulomb energy. If the electron density is increased further, the maximal momentum (Fermi momentum) in a completely degenerate electron gas grows larger.

1.2 Non relativistic Partial Degeneracy

In the real situation, the temperature is finite, and therefore the system is not in a complete degenerate state . Now we consider partial degeneracy of non-relativistic electrons. That means $v = \frac{p}{m}$ as in the non relativistic complete degenerate case. The energy of the electron particle can be evaluated as $\frac{p^2}{2m}$ Since the electrons are not in a complete degenerate state, there is no strict upper limit of energy level. We have to integrate the density as well as the pressure over all energy levels by extending the momentum upper limit to infinity [9].

$$n_e = \frac{8\pi}{h^3} \int_0^\infty \frac{p^2}{exp(\alpha + \frac{p^2}{2mK_BT_C} + 1)} dp \tag{1.2.1}$$

Where n_e is number density and α is a parameter which depends on density and temperature. It approaches negative infinity in completely degenerate gas.

$$P_e = \frac{8\pi}{h^3 m} \int_0^\infty \frac{p^4}{exp(\alpha + \frac{p^2}{2mK_BT_C} + 1)} dp \tag{1.2.2}$$

where P_e is electron degenerate pressure in momentum space for a partially degenerate system. Defining $u = \frac{p^2}{2mK_BT_C}$ and separating the integrals in equation (1.2.1) and (1.2.2), we get

$$n_e = \frac{4\pi}{h^3}(2mK_BT_C)^{3/2}F_{\frac{1}{2}}(\alpha) \tag{1.2.3}$$

and,

$$p_e = \frac{8\pi}{3h^3m}(2mK_BT_C)^{3/2}F_{\frac{3}{2}}(\alpha) \tag{1.2.4}$$

Where $F_{\frac{1}{2}}$ are given by

$$F_{\frac{1}{2}}(\alpha) = \int_0^\infty \frac{u^{\frac{1}{2}}}{exp(\alpha+u)+1}du \tag{1.2.5}$$

and'

$$F_{\frac{3}{2}}(\alpha) = \int_0^\infty \frac{u^{\frac{3}{2}}}{exp(\alpha+u)+1}du \tag{1.2.6}$$

respectively, Examining $F_{\frac{1}{2}}$ and $F_{\frac{3}{2}}$ at different α, we can measure the extent of degeneracy at different radial location. Combining P_e and n_e we find

$$P_e = n_eK_BT_C\left(\frac{2F_{\frac{3}{2}}(\alpha)}{3F_{\frac{1}{2}}(\alpha)}\right) \tag{1.2.7}$$

The quantity in the bracket the extent to which degenerate electron pressure differs from classical gas pressure.

Using the identity:

$\int_0^\infty x^{z-1}exp^{-x}dx = \Gamma(z)$

the ratio in the equation (1.2.7) can be simplified to give $\frac{F_{\frac{3}{2}}(\alpha)}{F_{\frac{1}{2}}(\alpha)} = \frac{3}{2}$ Therefore,the equation for electron degeneracy pressure becomes

$$P_e = n_eK_BT_C \tag{1.2.8}$$

The value of $F_{\frac{1}{2}}(\alpha)$ is approximated using the series representation of

$$F_{\frac{1}{2}}(\alpha) = \frac{2}{3}(-\alpha)^{3/2}\left[1+\frac{\pi^2}{8\alpha^2}+\frac{7\pi^4}{640\alpha^4}+...\right]$$

Approximating up to only the first term in the bracket $F_{\frac{1}{2}}(\alpha)$ becomes

$$F_{\frac{1}{2}}(\alpha) = \frac{2}{3}(-\alpha)^{\frac{3}{2}} \tag{1.2.9}$$

Combining equation (1.2.3) and (1.2.9) we have

$$n_e = \frac{8\pi}{3h^3}(2mK_BT_C)^{3/2} \mid \alpha \mid^{3/2} \tag{1.2.10}$$

Chapter 2

Basic Equations For Thin Accretion Disks

We study a steady, thin axisymmetric Keplerian disk around a neutron star with a magnetic dipole field. The basic equations describing the structure of the thin accretion disk can be derived from the equations of hydrodynamics.

2.1 Continuity Equation (Mass Conservation)

Any continuity equation can be expressed in an integral form (in terms of a flux integral), which applies to any finite region, or a differential form (in terms of a divergence operator) which applies to a point. To express the condition that a flow must conserve mass, we consider a volume V containing a parcel of fluid given by the volume integral:

$$m = \int_{v} \rho dV \tag{2.1.1}$$

where m is the mass and ρ is the density of the fluid flow. The mass of the fluid flowing at velocity v in unit time through element ds is equal to the area of surface element, and its direction is outward along the normal to the surface, which implies the mass flux is positive for fluid flow out of the volume. The total rate of flow out of the volume V is given by the surface integral $\int_A \rho v.ds$, where A is the area of the surface bounding V. Then the rate of decrease of mass in V is given by:

$$\frac{\partial}{\partial t}\int_V \rho dV = -\int_A \rho v.ds \tag{2.1.2}$$

The surface integral of the mass flux density v can then be written as a volume integral of its divergence

$$\int_A \rho v.ds = \int_V (\nabla.\rho v)dV \tag{2.1.3}$$

This may be rewritten as

$$\int_V \left[\frac{\partial \rho}{\partial t} + \nabla.\rho v\right] dV = 0 \tag{2.1.4}$$

This equation holds true for any volume V, and hence the equation of continuity in differential form becomes:

$$\frac{\partial \rho}{\partial t} + \nabla.\rho v = 0 \tag{2.1.5}$$

Where $v = (v_r, v_\phi, v_z)$,is the fluid velocity with radial, azimuthal and vertical components respectively.

Applying the divergence property of a vector for a thin cylinderically symmetric disk, using cylindrical coordinates, $\nabla.(\rho v) = 0$,can be expressed as:

$$\nabla.(\rho v) = \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial}{\partial \Phi}(\rho v_\Phi) + \frac{\partial}{\partial z}(\rho v_z) \tag{2.1.6}$$

Due to cylindrical symmetry the fluid variables are independent of Φ . Then it directly follows from the conservation of mass for steady state of axisymmetric disk.

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{2.1.7}$$

Neglecting a vertical outflow from the disk, the gas will have a radial velocity component only. Then for steady flow, the continuity equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) = 0 \tag{2.1.8}$$

The surface density function is defined as

$$\Sigma(r) = \int_{-H}^{H} \rho(r, z) dz = 2\rho H \tag{2.1.9}$$

Where ρ is an average density and H is the half-thickness of the disk. Then equation can be written as:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\Sigma v_r) = 0 \tag{2.1.10}$$

This can be vertically integrated to yield the constant mass influx (accretion) condition:

$$\dot{M} = -2\pi r \int_{-H}^{H} \rho v_r dz$$

$$\dot{M} = -2\pi r v_r \Sigma \tag{2.1.11}$$

Thus we also have

$$\frac{\partial \dot{M}}{\partial r} = 2\pi \frac{\partial \Sigma}{\partial t}$$

For steady state

$$\frac{\partial \dot{M}}{\partial r} = 0$$

Which means $\dot{M} = constant$ (independent of radius)

Provided ρv_r is negligible at $Z = \pm H$ a valid assumption except for the case of strong winds or magnetically driven outflows and $\dot{M}$ is the constant mass inflow rate (accretion rate).

2.2 Momentum Equation(Euler Equation)

For real fluid the transfer of momentum occurs in part by the transport of fluid volumes having different velocities, which is expressed by the advective term in Eulers equation. From Newtons second low, conservation of momentum being written for an arbitrary control volume as:

$$\frac{dv}{dt} = \frac{-1}{\rho}\nabla p - \nabla \Phi_G \tag{2.2.1}$$

where v is the flow velocity, ρ is the fluid density, and P is the pressure and $\Phi_G = -GM[r^2 + z^2]^{-1/2}$ is the gravitational potential energy. If dissipative terms representing the action of viscous forces are included on the right hand side of equation (2.2.1), it becomes Navier-Stokes equation. Using mathematical property ($\frac{d}{dt} = \frac{\partial}{\partial t} + v.\nabla$) the general form of the Navier-Stokes equation for the fluid motion:

$$\rho\frac{\partial v}{\partial t} + \rho v.\nabla v = -\rho\nabla\Phi_G - \nabla p + f \tag{2.2.2}$$

Where $\Phi_G = -GM[r^2 + z^2]^{-1/2}$ is the gravitational potential energy and f is the viscous force per unit volume responsible for momentum transport along velocity gradients by random (thermal and turbulent) motions. Accretion disks are differentially rotating flows, radially varying azimuthal, so we can only consider kinematic viscosity , which depends on shearing motions and the viscous force per unit volume is given by

$$f = \nabla.(2\rho\nu\xi) \tag{2.2.3}$$

where ν is the coefficient of kinematic viscosity and ξ is the fluid shear tensor. Using the continuity equation we can also rewrite the momentum equation as

$$\frac{\partial(\rho v)}{\partial t} + \nabla.(\rho v v) = -\rho\nabla\Phi_G - \nabla p + \nabla.(2\rho\nu\xi) \tag{2.2.4}$$

But it is possible to write the shear stress tensor as N. Straumann (1984)

$$\xi = \frac{1}{2}\left[\nabla v - \frac{2}{3}(\nabla.\nu)\right] \tag{2.2.5}$$

substituting this in to equation (2.2.4) we obtain

$$\frac{\partial(\rho v)}{\partial t} + \nabla.(\rho v v) = -\rho\nabla\Phi_G - \nabla p + \nabla.\left(\rho\nu(\nabla v - \frac{2}{3}(\nabla.\nu))\right) \tag{2.2.6}$$

From equation (2.2.6) the radial component of Navier-Stoke equation can be written as

$$\rho\left[v_r\frac{\partial v_r}{\partial r} - \frac{v_\Phi^2}{r}\right] = -\frac{\partial p}{\partial r} - \frac{\rho GMr}{\left(r^2 + z^2\right)^{3/2}} \tag{2.2.7}$$

and after integrating over z

$$\Sigma\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}\right)=\Sigma\left(\frac{v_\phi^2}{r}-\frac{GM}{r^2}\right)-\frac{\partial W}{\partial r} \tag{2.2.8}$$

where

$$W=\int pdz \tag{2.2.9}$$

Similarly, the vertical component of the momentum equation for the steady flow is

$$\rho\left[v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right]=-\frac{\partial p}{\partial z}-\frac{\rho GM}{r^2} \tag{2.2.10}$$

Neglecting vertical outflows and assuming the magnetic field to be weak the equation reduces to the equation hydrostatic equilibrium:

$$\frac{1}{\rho}\frac{\partial p}{\partial z}=\frac{GM}{r^2}\frac{z}{r} \tag{2.2.11}$$

Using H as half thickness of the disk the pressure at the mid plane of the disk is

$$P=\int_0^H\rho\frac{GMz}{r^3}dz=\frac{1}{2}(\rho H)\frac{GM}{r^3}H$$

$$P=\frac{1}{2}H\Sigma\frac{GM}{r^3} \tag{2.2.12}$$

From equation (2.2.11) making a slight approxmation:

$$\frac{1}{\rho}\left(\frac{\partial P}{\partial z}\right)\simeq\frac{1}{\rho}\frac{P}{z}$$

$$\frac{1}{\rho}\frac{P}{H}=\frac{GMz}{r^3}$$

Replacing z by half-thickness H it becomes

$$\frac{1}{\rho}\frac{P}{H}=\frac{GMH}{r^3} \tag{2.2.13}$$

Using keplerian velocity, $v_k^2=\frac{GM}{r}$ and $C_s^2=\frac{P}{\rho}\simeq\frac{dP}{d\rho}$, we can write the hydrostatic equilibrium equation as

$$C_s^2=v_k^2\left(\frac{H}{r}\right)^2$$

$$\left(\frac{H}{r}\right)=\frac{C_s}{v_k} \tag{2.2.14}$$

Where C_s^2 is the speed of sound.

The azimuthal component,the Φ component of momentum will be

$$\rho\left(v_r\frac{\partial v_\Phi}{\partial r}+\frac{v_\Phi}{r}+\frac{\partial v_\Phi}{\partial \Phi}+v_z\frac{\partial v_\Phi}{\partial v_z}+\frac{v_\Phi v_r}{r}\right)=\frac{1}{r}\frac{\partial P}{\partial \Phi}+\frac{\rho}{r}v_\Phi\Phi+\frac{1}{r}\frac{\partial}{\partial r}(rf_{\Phi r})\frac{\partial}{\partial z}(f_{\Phi z})+\frac{1}{r}f_{r\Phi}$$

Let

$$C=v_r\frac{\partial}{\partial r}+v_z\frac{\partial}{\partial z}$$

and since by symmetry the fluid variables are independent of Φ thus,

$$\rho\left(Cv_\Phi+\frac{v_\Phi v_r}{r}\right)=\frac{1}{r}\frac{\partial}{\partial r}(rf_{\Phi r})+\frac{\partial}{\partial z}(f_{\Phi z})+\frac{1}{r}f_{r\Phi}$$

Multiplying this equation with r then for Φ independent functions we obtain

$$\rho C(rv_\Phi)=\frac{1}{r}\frac{\partial}{\partial r}(r^2 f_{r\Phi})+\frac{\partial}{\partial z}(rf_{r\Phi}) \qquad (2.2.15)$$

Now we multiply the continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r)+\frac{\partial}{\partial z}(\rho v_z)=0$$

with rv_Φ and add the resulting equation to (2.2.15) and obtain

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r\rho rv_\Phi)+\frac{\partial}{\partial z}(v_z v_\Phi r\rho)=\frac{1}{r}\frac{\partial}{\partial r}(r^2 f_{r\Phi})+\frac{\partial(rf_{z\Phi})}{\partial z}$$

Integrating over z gives

$$\frac{1}{r}\frac{\partial}{\partial r}\int(r^2 v_r\rho rv_\Phi)dz=\frac{1}{r}\frac{\partial}{\partial r}(r^2 W_{r\Phi}) \qquad (2.2.16)$$

Where

$$W_{r\Phi}=\int f_{r\Phi}dz$$

where $W_{r\Phi}$ grvitational energy of $r\Phi$ component for thin disk Equation (2.2.16) is approximately

$$\frac{1}{r}\frac{\partial}{\partial r}(v_r\Sigma r^2 v_\Phi)=\frac{1}{r}\frac{\partial}{\partial r}(r^2 W_{r\Phi})$$

since for steady state

$$\frac{1}{r}\frac{\partial}{\partial r}(v_r\Sigma r)=0$$

Thus have

$$\Sigma v_r \frac{\partial}{\partial r}(r v_\Phi) = \frac{1}{r}\frac{\partial}{\partial r}(r^2 W_{r\Phi}) \tag{2.2.17}$$

where $r v_\Phi$ is the specific angular momentum and the component $f_{r\Phi}$ has the form $f_{r\Phi} = \eta r \frac{\partial}{\partial r}(\frac{v_\Phi}{r})$, where η is the dynamic vicosity.

Considering the radial component of momentum equation (2.2.7) the dominant terms are v_Φ^2 and gravitational terms, so ignoring the other gradient terms yields

$$\frac{v_\Phi^2}{r} \simeq \frac{GM\rho}{r^2}$$

$$v_\Phi^2 \simeq \frac{GM}{r} \equiv v_k^2 \tag{2.2.18}$$

where v_ϕ is the azimuthal velocity, G the gravitational constant and M is the mass of the accretor. This shows that the disk rotates in a Keplerian fashion.

so radial moment is conserved when keplerian rotation prevails, i.e. Keplerian rotational motion provides support against gravity. This is the differential rotation: each infinitesimal disk annulus rotates with a different $v_\Phi(r)$, which increases towards small r. This means there is a velocity shear between each annulus in the fluid flow.The Keplerian angular velocity is defined as

$$\Omega_k = \frac{v_k}{r} = \left(\frac{GM}{r^3}\right)^{1/2} \tag{2.2.19}$$

2.3 Conservation of Energy

An element of gas has two forms of energy: an amount $\frac{1}{2}\rho v^2$ of kinetic energy per unit volume, and internal or thermal energy $\rho\varepsilon$ per unit volume, ε the internal energy per unit mass, depends on the temperature T of the gas. According to the equipartition theorem of elementary kinetic theory, each degree of freedom of each gas particle is assigned a mean energy $\frac{1}{2}KT$. For monatomic gas the degrees of freedom are the three orthogonal directions of translational motion and

$$\varepsilon = \frac{3KT}{\mu m_p} \tag{2.3.1}$$

Molecular gases have additional internal degrees of freedom of vibration or rotation. In reality, cosmic gases are not quite monatomic and the effective number of degrees of freedom is not quite there; but in practice equation(2.3.1) is usually is a good approximation. The energy equation for the gas is

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\varepsilon\right) + \nabla.\left[\left(\frac{1}{2}\rho v^2 + \rho\varepsilon + P\right)v\right] = f.v - \nabla.F_{rad} - \nabla.q \tag{2.3.2}$$

The left hand side shows a family resemblance to the continuity equation(2.1.5), with the expected difference that the conserved quantity ρ is replaced by $(\frac{1}{2}\rho v^2 + \rho\varepsilon)$. The last term in the square brackets represents the so called pressure work. The new quantities appear on the right hand side: first, radiative flux vector

$$F_{rad} = \int dv \int d\Omega I_\nu(n, r) \tag{2.3.3}$$

where I_ν is the specific intensity of radiation at the point r in the direction n and the integrals are over frequency ν and solid angle Ω. The term $-\nabla.F_{rad}$ gives the rate at which radiant energy is being lost by emission, or gained by absorption; by unit volume of the gas. In general, specific intensity governed by further equation, the conservation of energy equation for the radiation field. The second new quantity in the energy equation(2.3.2) is the conductive flux of heat, q. This measures the rate at which random motions, chiefly those of electrons, transport thermal energy in the gas and thus act to smooth out temperature difference. For a slow inflow of matter through an optically thick disk, the local viscous dissipation $v.f$ is balanced by the radiative losses $\nabla.F_{rad}$ This is written as

$$\frac{9GM}{8r^3}\nu\Sigma = \frac{4\sigma T_c^4}{3\tau} \tag{2.3.4}$$

where T_c is the central temperature of the accretion disk, σ is the Stefan-Boltzmann constant and τ is the optical depth of the disk. The optical depth from the mid plane to the surface of the accretion disk, τ_c is defined in terms of the Rossel and mean opacity $K_R(\rho_c, T_c)$ by

$$\tau_c = \frac{\Sigma K_R}{2} = \rho_c h K_R \tag{2.3.5}$$

where ρ_c is the central density defined as $\rho_c = \frac{\Sigma}{2H}$

2.4 Viscous Spreading

The shear flow between neighboring Kepler orbits in the disk causes friction due to viscosity. The frictional torque is equivalent to exchange of angular momentum between these orbitals. But since the orbits are close to Keplerian, a change in angular momentum of a ring gas also means it must change its distance from the central mass. If the angular momentum transport (more precisely a non zero divergence of the angular momentum flux) there fore automatically implies redistribution of mass in the disk.A. Odrzywolek If at t=0 this ring is released to evolve under the viscous torques, one find that it first spreads in to an asymmetric hump with a long tail to large distances. As $t \rightarrow \infty$ the hump flattens in such a way that almost all the mass of the ring is accreted on to the center, while a vanishingly small fraction of the gas carries almost all the angular momentum to infinity. As a result of this asymmetric behavior essentially all the mass of a disk can accrete, even if there is no external torque to remove the angular momentum.

2.4.1 Viscous torque

The differential rotation of the gas means that elements on neighboring streamlines will slide past each other. Because of thermal and/or turbulent motions, viscous stresses are generated. The angular momentum is transported by shear viscosity.We now consider local mechanisms of angular momentum transport. First consider a uniform gas whose streaming motion is in the x-direction with velocity u(z).

Consider the x-momentum transport across an arbitrary plane $z = z_0$ due to the exchange of fluid elements (or molecules) between the levels $z_0 \pm \lambda/2$, with the typical turbulent scale λ (or mean free path) and speed v.There is no net mass motion across z_0, so the average upward and downward mass fluxes are the same. During the motion there is no force acting on the turbulent elements, so the linear momentum (and angular momentum) are

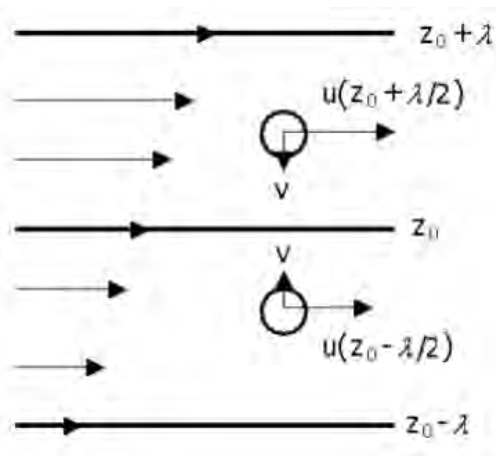

Figure 2.1: Uniform gas whose streaming motion is in the x-direction with velocity u(z).

conserved. The upward (downward) moving elements carry with them the x-momentum from a first level $\sim z_0\lambda/2(z_0 + \lambda/2)$ and the second level $\sim z_0\lambda/2(z_0 - \lambda/2)$. The net up ward x-momentum flux density is

$$\sim \rho v \left[u\left(z_0 - \frac{\lambda}{2} \right) - u\left(z_0 + \frac{\lambda}{2} \right) \right] \approx -r\nu\lambda u'(z_0)$$

The viscous stress exerted by the gas below on the gas above is given by

$$f_{xz} = -\eta \frac{\partial u}{\partial z} \sim \rho\nu\lambda u'(z_0) \tag{2.4.1}$$

Where $u' = \frac{\partial u}{\partial z}$ and η is the dynamical viscosity

The kinematic viscosity ν is defined as $\nu = \frac{\eta}{\rho}$ so $\nu = \lambda v$

In the case of molecular transport, λ and v are the mean free path and thermal speed of the molecules respectively. In the case of turbulent motions, λ is the characteristic spatial scale of the turbulence and v is the typical velocity of the eddies.

Now consider the similar process in a thin, differentially rotating disk in polar coordinates (R, Φ). Here the problem is which should be conserved, the angular momentum or linear momentum, when gas elements are constantly exchanged across the surface R = constant.

If the elements are not interacting with the streaming fluid, and subject only to external forces (e.g. gravity), the appropriate assumption is that angular momentum is conserved. If the effects of surrounding fluid (e.g. pressure gradients) must be included, the external

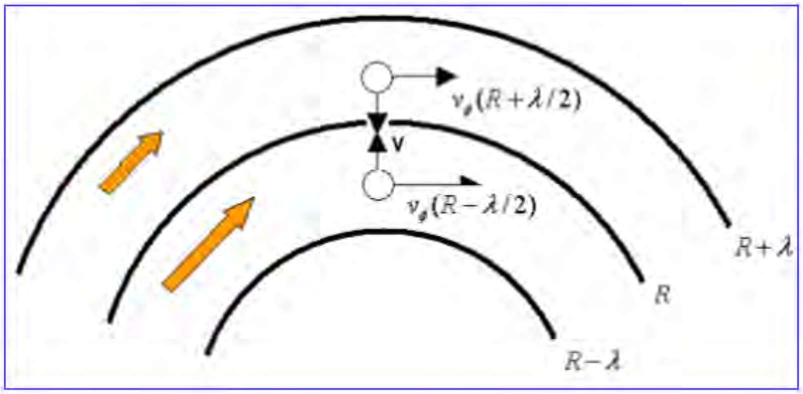

Figure 2.2: Differential viscous torque

forces are canceled by bulk rotation and pressure gradients in a steady state, so stream-wise momentum is conserved. The example of the first case is the ring of Saturn, while the later situation may be applied to accretion disks. Following the same method, the net upward ϕ-momentum flux density is

$$\sim \rho\tilde{v}\left[\left(R+\frac{\lambda}{2}\right)v_\Phi\left(R-\frac{\lambda}{2}\right)-\left(R-\frac{\lambda}{2}\right)v_\Phi\left(R+\frac{\lambda}{2}\right)\right] \approx -\rho\tilde{v}\lambda R^2\Omega' \tag{2.4.2}$$

The viscous stress in the Φ-direction per unit area is

$$f_{R\Phi} = -\eta\left(\frac{\partial v_\Phi}{\partial R}-\frac{v_\Phi}{R}\right) = -\eta R\Omega' \pm -\rho\tilde{v}\lambda R\Omega' \tag{2.4.3}$$

yielding a kinematic viscosity $\nu = \lambda v$

2.4.2 The magnitude of viscosity

The momentum transfer in the disk means that there is a force acting in the Φ -direction on the volume element due to shear viscosity,

$$f_{visc,shear} = \rho\lambda v\frac{\partial^2 v_\Phi}{\partial R^2} \sim \rho\lambda v\frac{v_\Phi}{R^2} \tag{2.4.4}$$

Comparing this with the inertia terms $\rho(\vec{v}.\nabla)\vec{v}$ in the Euler equation leads to the Reynolds number

$$Re = \frac{inertia}{viscousity} \sim \frac{v_\Phi^2/R}{\lambda v v_\Phi/R^2} = \frac{Rv_\Phi}{\lambda v} \tag{2.4.5}$$

If $Re << 1$, viscous force dominates the flow; if $Re >> 1$, the viscosity is dynamically unimportant.

A number of hypotheses have been proposed to explain the much larger effective viscosity in accretion disks. The most important of these are:

(1) A turbulent viscosity resulting from random small-scale turbulent fluid motions in the disk, generated by the strong shear in the differentially rotating disk.

(2) A magnetic viscosity associated with the magnetic Lorentz force in a disk containing magnetic fields.

We will discuss those, Since little is known about turbulence, the most we can do is to place plausible limits on λ_{turb} and v_{turb}. First, the typical size of the largest eddies cannot exceed the disk thickness H, so $\lambda_{turb} \leq H$. Second, it is unlikely that the turn over velocity v_{turb} is supersonic; otherwise the turbulent motions would be thermalized by shocks, so $v_{turb} \leq c_s$.

Hence we can write the viscosity as $v = \alpha c_s H$ with $\alpha \leq 1$. This is the famous α-prescription Shakura and Sunyaev(1973). Note that with this semi-empirical approach all our ignorance about viscosity mechanism has been isolated in α, which depends on other parameters and should not be taken as a constant. The shear stress in a thin, Keplerian disk is

$$\begin{aligned} f_{R\Phi} &= -\rho v R\Omega' = \frac{3}{2}\alpha\rho c_s H\Omega_k = \frac{3}{2}\alpha\rho c_s^2 \\ &= \frac{3}{2}\alpha P \end{aligned} \tag{2.4.6}$$

which means that the viscous stress shouldn't exceed the gas pressure in the disk.

Chapter 3

Structure Equation

Reducing those equations discussed above for the radial structure of the accretion disk and first assuming that the equation of state an ideal gas:

$$P_g = \frac{\rho_c K_B T_c}{m_p \mu} \tag{3.0.1}$$

where K_B is the Boltzmann constant,μ is the mean molecular weight, and m_p is the mass of proton. The equation of star pressure with the effect of electron degenerate pressure can be written using equations (1.1.11) and (3.0.1) as:

$$P = \frac{\rho_c K_B T_c}{m_p \mu} + 9.77 \times 10^6 \left(\frac{\rho_c}{\mu_e}\right)^{5/3} \tag{3.0.2}$$

where the first term stands for gas pressure and the second term stands for an ideal electron degenerate pressure on the right hand side of the equation.

3.1 Thickness of Accretion Disk

The thickness of an accretion disk is set by the balancing of pressure within the disk against the gravitational tidal force. As in the case of star, this structure depends on how the temperature of the gas varies with altitude above the plane of the disk. This in turn depends on the viscosity mechanisms that heat the disk and the radiative process and convection that transport the energy to the surface of the disk. For an axisymmetric magnetized disk around a neutron star having a dipole moment aligned with its rotation axis in steady state,the pressure can be expressed in terms of hydrostatic equilibrium as, A. Odrzywolek

$$P = P_g + P_e \tag{3.1.1}$$

Where P is the total pressure

$$\frac{GM}{2r^3} H\Sigma = \frac{\rho_c K_B T_C}{m_p \mu} + 9.77 \times 10^6 \left(\frac{\rho_c}{\mu_e} \right)^{5/3} \tag{3.1.2}$$

Using equation (3.1.) this can be written as

$$\frac{H^2 \rho_c GM}{r^3} = \frac{\rho_c K_B T_C}{m_p \mu} + 9.77 \times 10^6 \left(\frac{\rho_c}{\mu_e} \right)^{5/3} \tag{3.1.3}$$

From equation (3.1.3) the half-thickness of the accretion disk can be calculated as:

$$H = \left(GM \right)^{-1/2} \left[\frac{K_B T_C}{m_p \mu} + 9.77 \times 10^6 \left(\frac{\rho_c^2}{\mu_e^5} \right)^{1/3} \right]^{1/2} r^{3/2} \tag{3.1.4}$$

3.2 Regions of Accretion Disk

A disk around neutron star has three regions; inner, middle and outer regions. In this paper we do not consider the inner part of the disk, rather we study about stability of the disk in its middle and outer regions. The disk around a neutron star differs from the disk around a black hole in that it should eventually release the gravitational binding energy of accreted matter (which is converted to internal energy of the disk and the rotational kinetic energy) more efficiently.

The energy equation of the disk is N. Straumann(1984)

$$\Sigma \nu_r T \frac{ds}{dr} = Q^+ - Q^- \tag{3.2.1}$$

where s is the entropy per unit mass, r is the radius of a certain position in the disk, and Q^+ and Q^- are energy input (heating) and energy loss (cooling) rate in the disk,respectively. In the outer disk, the heat energy could be advected in ward, and we take $Q_{adv} = \frac{1}{2} \Sigma T \nu_r \frac{ds}{dr}$ to be the quantity of the energy advection rate, where $\frac{1}{2}$ is added because only the vertically integrated disk over a half-thickness H is considered thus the energy-conservation equation (3.2.1) is written as

$$Q^{+} = Q^{-}_{rad} + Q^{-}_{adv} + Q^{-}_{\nu} \tag{3.2.2}$$

where Q^{+} is viscous heat energy generation rate per unit surface area. according to the standard viscosity disk model, we have

$$Q^{+} = \frac{3GM\dot{M}}{8\pi r^3} f \tag{3.2.3}$$

Where $f = 1 - \left(\frac{r_0}{r}\right)^{1/2}$ The quantity Q_{rad} in equation (3.2.2) is the photon cooling rate per unit surface area of the disk. Since the disk is extremely dense, thus we can take $Q_{rad} = 0$ as a good approximation. If the accretion rate is not very high, most of the energy generated in the disk is advected in ward, and we call the disk as an advection dominated disk. As the accretion rate increases, the density and temperature of the disk also increase, and neutron cooling becomes the dominant cooling mechanism in some regions of the disk. Thus, we say that this region become neutrino-dominated. When the accretion rate is sufficiently large, the disk may become entirely neutrino-dominated. In addition to the accretion rate, there are some other factors, such as the central mass of neutron star, the electron-nucleon ratio, that can influence the structure.

3.3 Vertical Structure

For the vertical structure, we have the following equations

$$\frac{dp}{dz} = -\rho \frac{GM}{r^2} \frac{z}{r} \tag{3.3.1}$$

$$\frac{\rho_c K_B T_C}{m_p \mu} + 9.77 \times 10^6 \left(\frac{\rho_c}{\mu_e}\right)^{5/3} \tag{3.3.2}$$

We can determine with this equations the local structure of the disk. We derive analytic expressions we assume, a polytropic equation of state for fixed r,

$$p(z) = k\rho(z)^{1+\frac{1}{N}} \tag{3.3.3}$$

equation (3.3.1) can then immediately be solved with the result

$$k(1+N)\rho^{\frac{1}{N}} = \frac{1}{2}\frac{GM}{r}\left[\left(\frac{H}{r}\right)^2 - \left(\frac{z}{r}\right)^2\right] \tag{3.3.4}$$

For the values in the central plane(indexed by c) we obtained

$$\frac{p_c}{\rho_c} = \frac{1}{1+N}\frac{1}{2}\frac{GM}{r}\left(\frac{H}{r}\right)^2 \tag{3.3.5}$$

Further more

$$\Sigma = 2\rho_c H I(N)$$

$$W = 2p_c H I(N+1) \tag{3.3.6}$$

where $I(N) = \frac{(2^N N!)^2}{(2N+1)}$! Hence Shakura and Sunyaev (1973) propose

$$f_{r\Phi} = -\alpha p \tag{3.3.7}$$

from this equation we have generally

$$W_{r\Phi} = -\alpha W \tag{3.3.8}$$

3.4 Radial Structure

The vertical structure described depends on the parameters H and ρ_c which are functions of the radius.

The radial equations

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{3.4.1}$$

$W_{r\Phi} = -\alpha W$

From equation (2.2.17) and (3.3.8) we have

$$W(r) = \frac{\dot{M}}{2\pi r^2 \alpha}[l(r) - l(r_0)] \tag{3.4.2}$$

with $l(r) = \sqrt{GMr}$

on the left hand side of equation (3.4.2), we insert equation (3.3.6) to obtain

$$p_c = \frac{1}{I(N+1)}\frac{\dot{M}}{4\pi r^2 \alpha}\left(\frac{H}{r}\right)^{-1}\left(\frac{GM}{r}\right)^{1/2}\left[1 - \left(\frac{r_0}{r}\right)^{1/2}\right] \tag{3.4.3}$$

This equation contains still the parameter H using the relation between ρ_c and p_c gives

$$\rho_c(r) = \frac{2(N+1)}{I(N+1)} \frac{\dot{M}}{4\pi r^2 \alpha} \left(\frac{H}{r}\right)^{-3} \left(\frac{GM}{r}\right)^{-1/2} \left[1 - \left(\frac{r_0}{r}\right)^{1/2}\right] \quad (3.4.4)$$

3.5 Central Temprature and Central Density in The Middle Region of Accretion Disk

The central temperature in different regions of the disk should be high enough for the complete ionization of electron in the regions. The value $K_B T_C$ should be greater or equal to the critical value 0.5Mev in both middle and outer regions of the accretion disk, where the gas pressure is more dominant than radiation pressure, for complete ionization. Here, K_B is Boltzmann constant and T_c is the central temperature in both regions. It is possible to find the temperature of the accretion disk as a function of radius of the disk, N. Straumann(1984).

The central temperature in the middle part of the accretion disk can be calculated using a formula:N. Straumann(1984)

$$T_c = \left(6.5 \times 10^7 K\right) \tilde{I}^{-1/5} \alpha^{-1/5} \left(2\mu\right)^{1/5} \left(\frac{3r_g}{r_0}\right)^{9/10} \times \left(\frac{M}{M_\odot}\right)^{-1/5} \left(\dot{M}_{14}\right)^{2/5} \left(\frac{r}{r_0}\right)^{-9/10} f^{2/5} \quad (3.5.1)$$

Moreover,

$$I = \frac{(2^N N!)^2}{(2N+1)!}, \quad \tilde{I} = \frac{3}{2} I(N+1), \quad r_g = \frac{2GM}{c^2}, \quad f = 1 - \left(\frac{r_0}{r}\right)^{1/2} \quad (3.5.2)$$

in which c is the speed of light, $r_0 = r_A$ is Alfvens radius and N=3, for electron scattering. Some basic constants with their corresponding values used in this thesis.

Substituting all the constants and wrtting the central temperature in terms of the raduis of the accretion disk,r gives;

$$T_c = 2.22 \times 10^{9.3} r^{-11/10} \left[r^{1/2} - 10^3\right]^{2/5} K \quad (3.5.3)$$

The more simplified central density as a function of radius, r in the middle region of the accretion disk where gas-pressure is dominant is given by the equation Shakura and

Sunyaev (1973),

$$\rho_c = \left(1.4\right) \frac{(N+1)^{-1/2}}{\tilde{I}^{7/10}} \alpha^{-7/10} \left(2\mu\right)^{6/5} \left(\frac{3r_g}{r_0}\right)^{33/20} \times \left(\frac{M}{M_\odot}\right)^{-11/10} \left(\dot{M}_{14}\right)^{2/5} \left(\frac{r}{r_0}\right)^{-33/20} f^{2/5} \tag{3.5.4}$$

Using equation (3.5.2) to find the constants, the central density in the middle region becomes

$$\rho_c = 1.89 \times 10^{8.6} r^{-37/20} \left[r^{1/2} - 10^3\right]^{2/5} kg/m^3 \tag{3.5.5}$$

3.6 Central Temprature and Central Density in The Outer Region of Accretion Disk

The central temperature in the outer part of the accretion disk can be calculated using a formula:Shakura and Sunyaev (1973)

$$T_c = \left(2.7\times10^7 K\right)\left(N+1\right)^{-1/20}\tilde{I}^{-1/5}\alpha^{-1/5}\left(2\mu\right)^{1/5}\left(\frac{3r_g}{r_0}\right)^{3/4}\times\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\dot{M}_{14}\right)^{3/10}\left(\frac{r}{r_0}\right)^{-3/10}f^{3/10} \tag{3.6.1}$$

Using the above constants given and other constants the central temperature in terms of radius, r is given as:

$$T_c = 5.57\times10^{8.45}r^{-9/10}\left[r^{1/2}-10^3\right]^{3/10}K \tag{3.6.2}$$

Similar to equation (3.5.4) the equation of central density in the outer part of the accretion disk is given as:

$$\rho_c = \left(5.3\right)\frac{(N+1)^{-17/40}}{\tilde{I}^{7/10}}\alpha^{-7/10}\left(2\mu\right)^{9/8}\left(\frac{3r_g}{r_0}\right)^{15/8}\times\left(\frac{M}{M_\odot}\right)^{-5/4}\left(\dot{M}_{14}\right)^{11/20}\left(\frac{r}{r_0}\right)^{-15/8}f^{11/20} \tag{3.6.3}$$

Substituting the constants in to the equation (3.6.3), it is simplified to be

$$\rho_c = 7.21\times10^{8.7}r^{-43/20}\left[r^{1/2}-10^3\right]^{11/20}kg/m^3 \tag{3.6.4}$$

Chapter 4

Stability of the Accretion Disk

An equilibrium solution is physically relevant only if the disk is stable. When investigating the stability of an equilibrium condition we restrict ourselves to a linear analysis. One thus considers time-dependent perturbations and expands all equations which the system satisfies about the equilibrium condition, keeping only the linear terms. The system is stable provided the frequency of the corresponding normal modes all have negative imag inary parts. General thermos show that this result is unchanged by nonlinearities. On the other hand, if the imaginary part of the frequency of some normal mode is positive this mode grows exponentially, and one expects that the system is unstable. For cold spherically symmetric configuration one consider as the first step only radial perturbations, which are also adiabatic . Since hydrodynamic time scales are usually much shorter than the characteristic times for energy transports, this is reasonable. In this case one obtains an eigenvalue problem for ω^2 since the equations for adiabatic perturbations are time reversal invariant (there is no dissipation). The equilibrium is stable provided $\omega_0^2 > 0$ for the lowest mode frequency ω_0 . One can often find sufficient criteria for instability by making use of the Rayleigh-Ritz variational principle and simple trial functions sometimes the equilibrium solutions, whether or not it is stable against radial, adiabatic pulsations. There is a possibility that accretion disks around a neutron star or a black hole show non-linear oscillation caused by thermal instability, similar to dwarf nova outbursts. For a disk with high accretion rate, the heat generated by viscosity close to the transonic

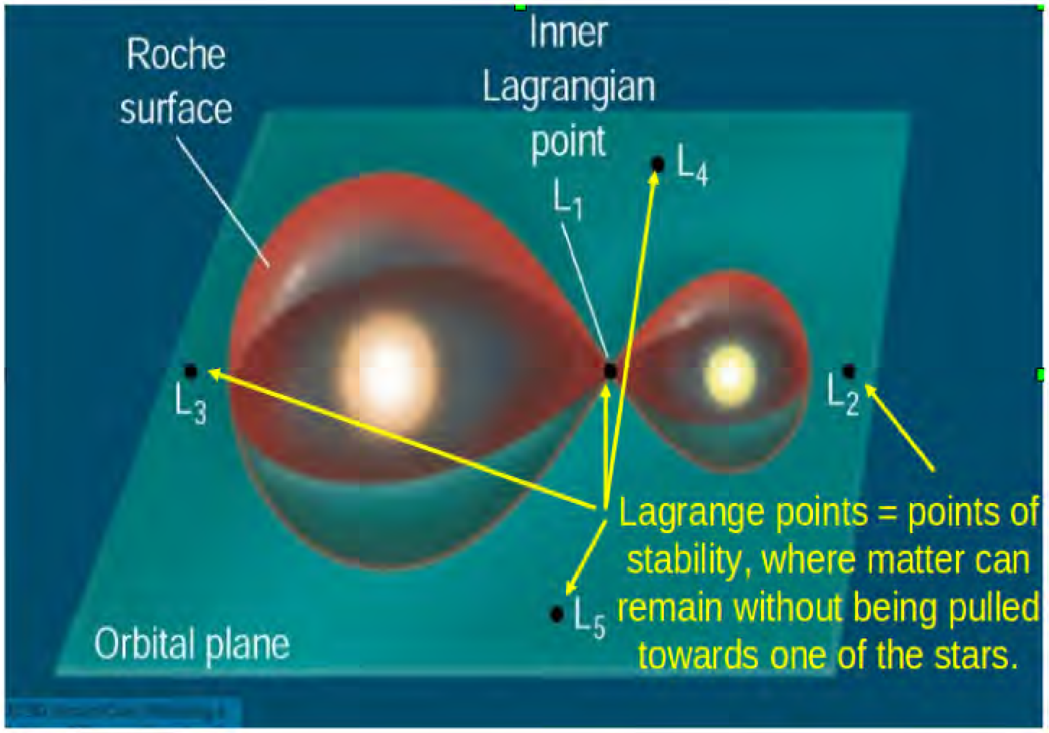

Figure 4.1: Lagrangian points of stability in binary stars.

part of the flow not only it locally radiant way, as standard Shakura and Sunyaev (1973) model, but also it is very efficient transported by advection. The size of the Roche lobes depends on the mass of stars and on the distance between the stars. If the stars are far apart, the lobes are very large, and the stars easily control their own mass. If the stars are close together, however the lobes are small and can interfere with the evolution of the stars.Matter inside each star's Roche lobe is gravitationally bound to the star, but matter that leaves a star's Roche lobe can fall into the star or leave the binary system completely.

The Lagrange points are places in the orbital plane of a binary star system where a bit of matter can reach stability.For astronomers, the most important of these points is the inner lagrange point where the two Roche lobes meet figure(4.1) If matter can leave a star and reach the inner lagrange point is the connection through which the stars can transfer matter.

The most development model for out bursts is the disk instability model. In this model the instability that gives rise to cyclic accretion is due to a temperature dependence of the resulting α-parameter will be a function of the main dimensionless parameter of the disk, the aspect ratio H/r. If this is sufficiently rapidly increasing function ,such that α is large in hot disks and low in cool disks, an instability results by the following mechanism. Suppose we start the disk in stationary state at the mean accretion rate. If this state is perturbed by small temperature increase, α goes up, and by the increased viscous stress the mass flux $\dot{M}$ increase. This increase the disk temperature further, resulting in a runaway to a hot state. Since $\dot{M}$ is larger than the average, the disk empties partly, reducing the surface density and the central temperature. A cooling front then transforms the disk to a cool state with an accretion rate below the mean. The disk in this model switches back and forth between hot and cool states. The contribution of electron degenerate pressure to the stability of accretion disk in different region of the disk is calculated from stability relation, N. Straumann (1984)

$$\beta = \frac{P_g}{P_g + P_e} \tag{4.0.1}$$

Where, P_g is gas pressure and P_e is electron degenerate pressure.

For gas-pressure dominated regions the disk is stable if

$$\bar{\beta} < \frac{3}{5} \tag{4.0.2}$$

Where $\bar{\beta} = 1 - \beta$

4.1 Stability in the Middle Region

It is possible to find P_g from equation (3.0.1) :

$$P_g = \left(\frac{K_B}{m_p \mu}\right) \rho_c T_c$$

Where;ρ_c and T_c are the central density and temperature in the middle region.

$$P_g = 5.72 \times 10^{21.9} r^{-59/20} \left[r^{1/2} - 10^3 \right]^{4/5} \tag{4.1.1}$$

Firstly, let us check the stability of the accretion disk due to a complete degeneracy electron pressure:

P_e for complete degeneracy pressure can be calculated from equation (1.1.11)

$$P_e = 9.77 \times 10^6 \left(\frac{\rho_c}{\mu_e}\right)^{5/3}$$

$$P_e = 2.8 \times 10^{21.2} r^{-3.08} \left[r^{1/2} - 10^3\right]^{2/3} \qquad (4.1.2)$$

The stability of accretion disk in the middle or outer region due to electron degeneracy and gas pressure is

$$\beta < 2/5$$

$$\frac{P_g}{P_g + P_e} < \frac{2}{5} \qquad (4.1.3)$$

substituting eqations (4.1.1) and (4.1.2) in to equation (4.1.3) we can have

$$3.35 \times 10^{21.9} r^{-59/20} \left[r^{1/2} - 10^3\right]^{4/5} - 9.12 \times 10^{21.2} r^{-37/12} \left[r^{1/2} - 10^3\right]^{2/3} < 0 \qquad (4.1.4)$$

Then the value of r for which the accretion disk is stable in the middle region is calculated to be New Jersy, (1995)

$1 \times 10^6 m < r < 5.33 \times 10^7 m$

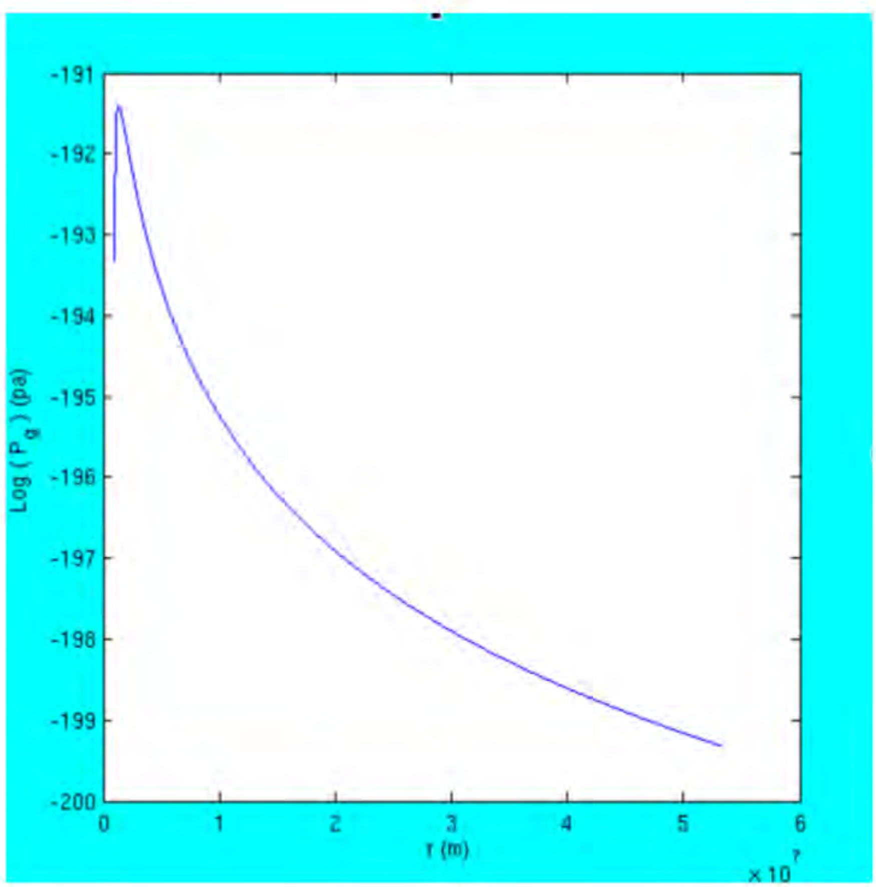

Figure 4.2: The relation between gas pressure and radius of the disk in the middle region.

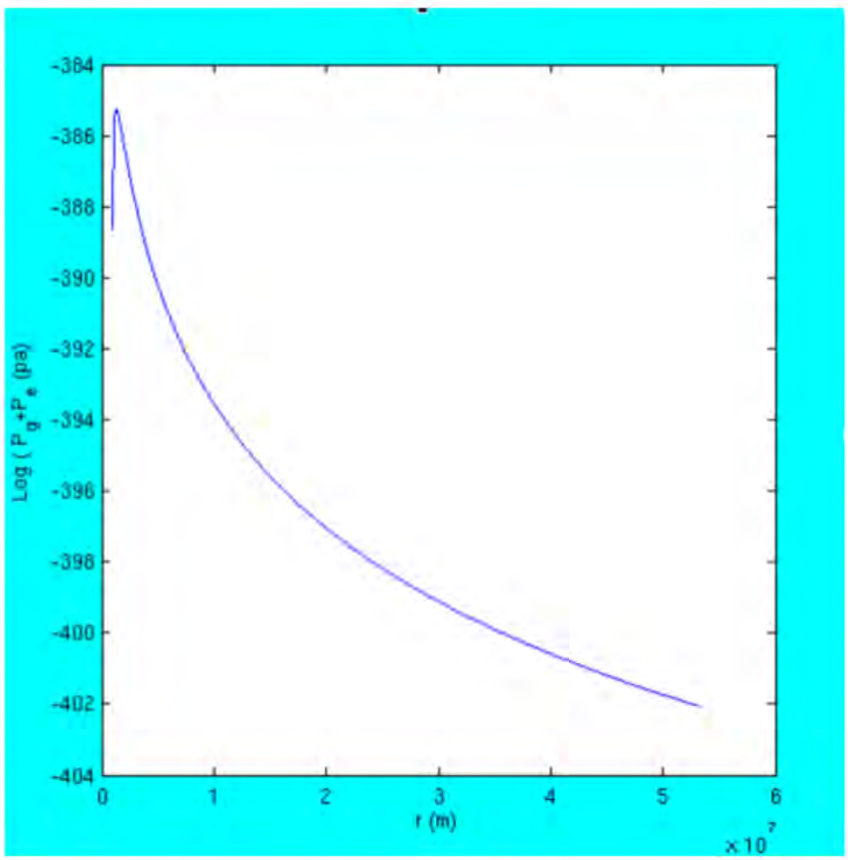

Figure 4.3: The relation between total pressure and radius of the disk for a complete degenerate electron pressure in the middle region.

Similarly, we can check the stability of the accretion disk due to a partial electron degenerate pressure. Firstly, we can write the relation for electron and proton number density as

$$n_e = n_p = \frac{\rho_c}{m_p} = 1.13 \times 10^{18.4} r^{-37/20} \left[r^{1/2} - 10^3 \right]^{2/5} \tag{4.1.5}$$

Where m_p is mass of proton and ρ_c is the central density in the middle region. Combining equations (1.2.10) and (4.1.6) we can have

$$|\alpha|^{3/2} = 9.4 \times 10^{-18.55} r^{-1/5} \left[r^{1/2} - 10^3 \right]^{-1/5} \tag{4.1.6}$$

Combining equations (1.2.8),(1.2.10) and (4.1.7) the relation for partial degeneracy electron pressure is given by

$$P_e = 3.48 \times 10^{4.73} r^{-2.95} \left[r^{1/2} - 10^3 \right]^{4/5} \tag{4.1.7}$$

When we consider the total pressure in the disk, we should include both the contribution from electron degeneracy pressure and also from ordinary particles, which is nuclear gas. So, the total pressure is given as

$$P = 5.58 \times 10^{21.9} r^{-59/20} \left[r^{1/2} - 10^3 \right]^{4/5} + 3.48 \times 10^{4.73} r^{-2.95} \left[r^{1/2} - 10^3 \right]^{4/5} \tag{4.1.8}$$

Where P is the sum of P_g and P_e.

The region of stability of the disk is checked by equation (4.1.3).That is

$$\frac{P_g}{P_g + P_e} < \frac{2}{5}$$

$$3.35 \times 10^{21.9} r^{-59/20} \left[r^{1/2} - 10^3 \right]^{4/5} - 1.39 \times 10^{4.73} r^{-6.1} \left[r^{1/2} - 10^3 \right]^{4/5} < 0 \tag{4.1.9}$$

Solving this for the radius of accretion disk, it is approximately equal to what we have found in the case of complete degenerate electron pressure

$1 \times 10^6 m < r < 5.33 \times 10^7 m$

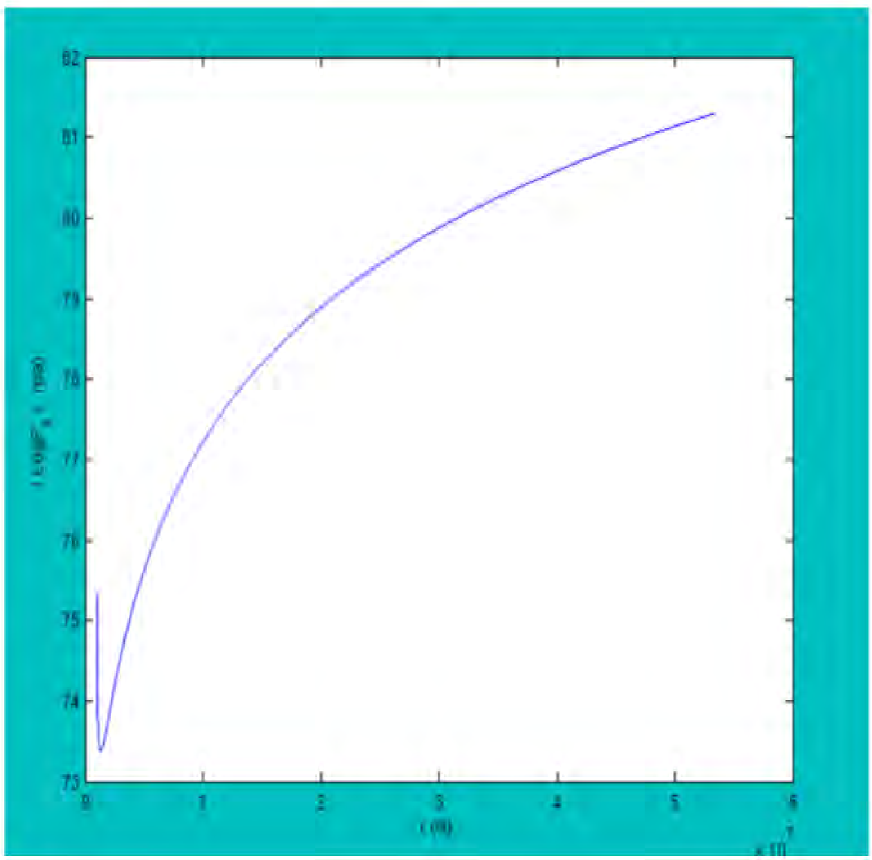

Figure 4.4: The relation between partial electron degeneracy pressure and radius of the disk in the middle region.

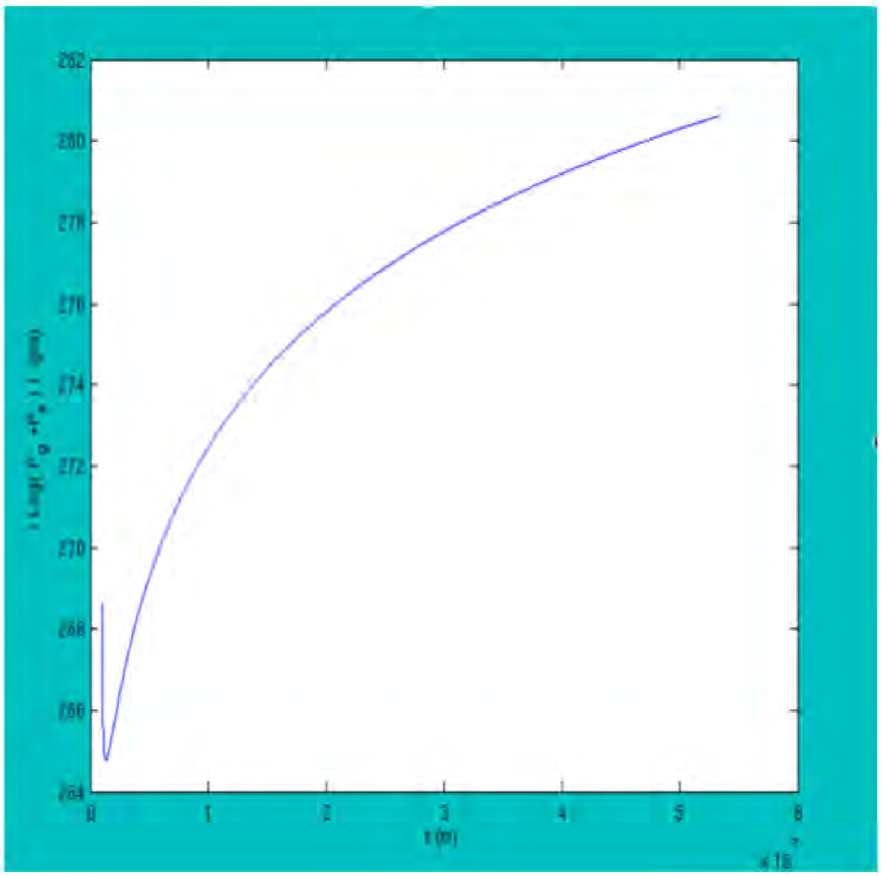

Figure 4.5: The relation between total pressure and radius of the disk for partial electron degeneracy pressure case in the middle region.

4.2 Stability in the Outer Region

We can use equation (3.0.1) to find the gas pressure in this region

$$P_g = \left(\frac{K_B}{m_p\mu}\right)\rho_c T_c \tag{4.2.1}$$

Where ρ_c and T_c are the central density and temperature in the outer region. Substituting equations (3.6.2) and (3.6.4) in to equation (4.2.1) we have:

$$P_g = 5.34 \times 10^{22.15} r^{-61/20} \left[r^{1/2} - 10^3\right]^{17/20} \tag{4.2.2}$$

A Complete electron degenerate pressure is derived from equations (1.1.11) and (3.6.4):

$$P_e = 2.63 \times 10^{22.15} r^{-3.58} \left[r^{1/2} - 10^3\right]^{11/12} \tag{4.2.3}$$

The stability of accretion disk in the outer region due to electron degeneracy and gas pressure is obtained by substituting equation (4.2.2) and (4.2.3) in to equation (4.1.3) to calculate for the radius of the accretion disk in the outer region where the disk is stable. That is

$$3.2 \times 10^{22.15} r^{-3.05} \left[r^{1/2} - 10^3\right]^{17/20} - 1.05 \times 10^{22.5} r^{-3.58} \left[10^{1/2} - 10^3\right]^{11/20} < 0 \tag{4.2.4}$$

Solving the inequality, the value of r for which the disk is stable in the outer region is New Jersy (1995) $5.33 \times 10^7 m \leq r \leq 1 \times 10^8 m$

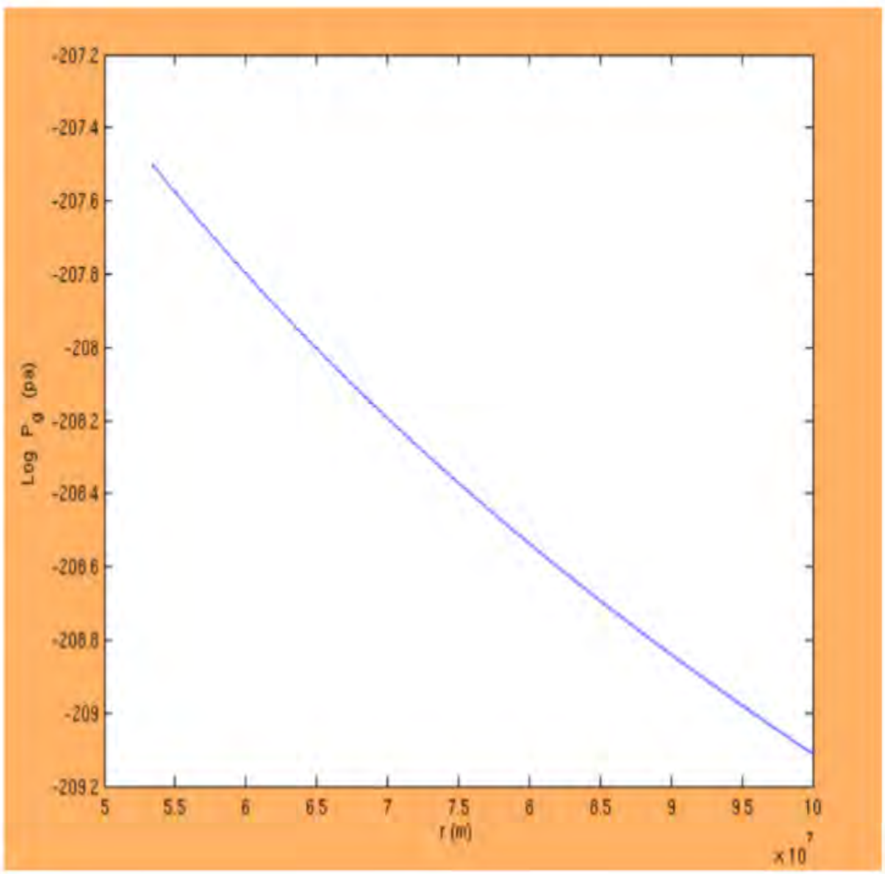

Figure 4.6: The relation between gas pressure and radius of the disk in the outer region.

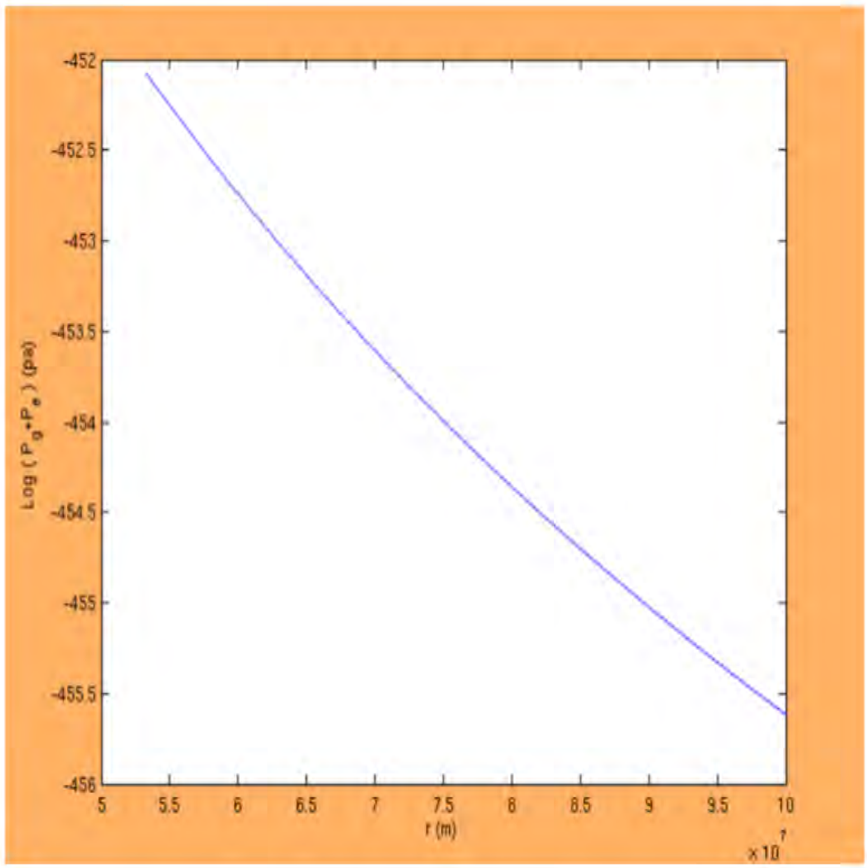

Figure 4.7: The relation between total pressure and radius of the disk for complete electron degeneracy pressure case in the outer region.

On the other hand, we can check the stability of the disk due to a partial degeneracy electron pressure in the outer region of the disk. The electron number density is given as

$$n_e = n_p = \frac{\rho_c}{m_p} = 4.32 \times 10^{18.3} r^{-43/20} \left[r^{1/2} - 10^3 \right]^{11/20} \quad (4.2.5)$$

Where ρ_c is the central density in the outer region. Combining equations (1.2.10) and (4.2.5) we can have

$$|\alpha|^{3/2} = 8.9 \times 10^{-17.38} r^{-16/20} \left[r^{1/2} - 10^3 \right]^{-7/20} \quad (4.2.6)$$

Similar to that of the middle region we combine equations (1.2.8) ,(1.2.10) and (4.2.6) to get an equation for partial degeneracy electron pressure in the outer region .

$$P_e = 3.29 \times 10^{3.95} r^{-3.05} \left[r^{1/2} - 10^3 \right]^{1/20} \quad (4.2.7)$$

So, the total pressure becomes

$$P = 5.34 \times 10^{22.15} r^{-61/20} \left[r^{1/2} - 10^3 \right]^{17/20} + 3.29 \times 10^{3.95} r^{-3.05} \left[r^{1/2} - 10^3 \right]^{1/20} \quad (4.2.8)$$

We use equation (4.1.3) to check the stability in this region

$$\frac{P_g}{P_g + P_e} < \frac{2}{5}$$

$$3.2 \times 10^{22.15} r^{-61/20} \left[r^{1/2} - 10^3 \right]^{0.85} - 1.32 \times 10^{-3.95} r^{-6.1} \left[r^{1/2} - 10^3 \right]^{0.05} < 0 \quad (4.2.9)$$

Solving this for the radius of accretion disk, it is approximately equal to what we have found in the case of complete degenerate electron pressure $5.33 \times 10^7 m \leq r \leq 1 \times 10^8 m$

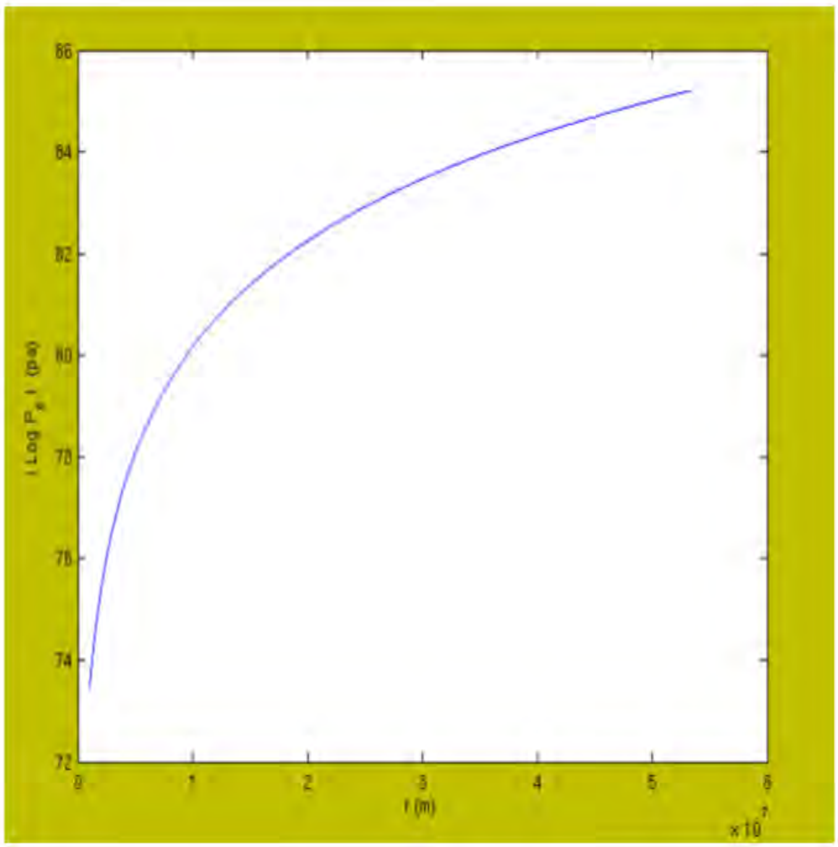

Figure 4.8: The relation between partial degenerate electron pressure and radius of the disk in the outer region.

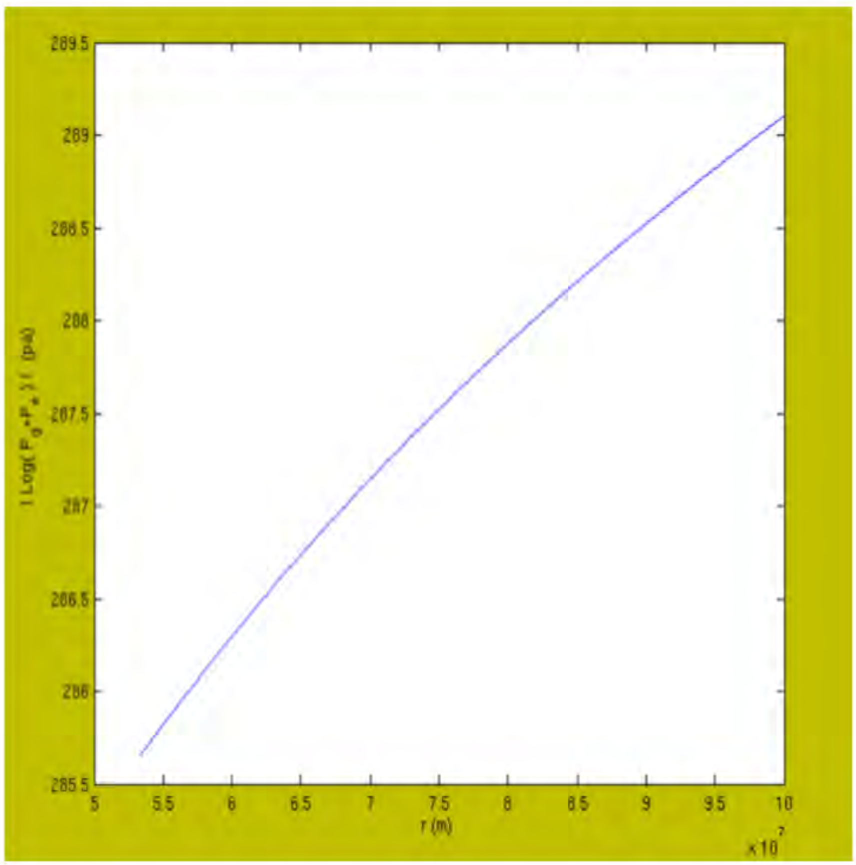

Figure 4.9: The relation between total pressure and radius of the disk for partial electron degeneracy pressure case in the outer region.

Chapter 5

Discussion

In this thesis we have studied the contribution of degenerate electron pressure to the stability of thin Keplerian accretion disks around neutron star with axisymmetric magnetic dipole, in which we ignored the situation when the particles are relativistic, so we took $v = \frac{p}{m}$ as a non-relativistic degenerate gas. For our fiducial model We have taken a neutron star of $M = 1.4M_{\odot}$. We included the complete non relativistic degeneracy electron pressure and partial non relativistic degeneracy electron pressure. Firstly, the central temperature and density of the disk is derived as a function of radius of accretion disk, which is the basis for the pressure derivation as a function of radius both in middle and outer regions of the disk. The degeneracy electron pressure is density dependent for complete degeneracy and temperature dependent for partial degeneracy.

Contribution of Degeneracy Electron Pressure in the Middle Region of Accretion Disk

This study indicates that the middle region of the disk is stable even though there is an additional pressure called degenerate electron pressure to a gas pressure. As it is discussed in chapter four, calculating the values of degenerate electron pressure in a complete and partial degenerate cases and adding to a gas pressure we have found that the disk is still stable even after the degenerate pressure is added. This implies that the contribution of degenerate pressure is insignificant for the stability of the disk in the middle region. For completely degenerate electron pressure contribution, which is on the basis that the central temperature approaches to zero, the disk is stable for the value of r in between $1 \times 10^6 m$ and $5.33 \times 10^7 m$. From fig. 4.2, we see that initially the pressure increases when the radius of the disk increases. Similarly in fig. (4.3) the pressure increases as the radius increases but in this case more rapidly.Similar phenomenon is observed in fig.(4.4) and (4.5). This is because of the additional term degeneracy electron pressure. Although, the magnitude of the pressure increases as the radius of the disk increases, the stability of the disk holds in this region. we have seen a similar situation in the case of partial degeneracy pressure. That is the contribution of partial degeneracy electron pressure is insignificant for the instability of the accretion disk in the middle region, so the disk is stable.

Contribution of Degeneracy Electron Pressure in the Outer Region of Accretion Disk

The contribution of electron degeneracy pressure for the outer region of the accretion disk is also insignificant. We have derived the relation for the temperature and density in this region, which are less than the values obtained in the middle region. Then we have found that the pressure in complete and partial degenerate electron pressure cases which are the inputs for the stability calculation. Using the stability relation, the contribution of both complete and partial degeneracy electron pressure is calculated. For the complete degenerate electron pressure case, we have gotten the interval for which the disk is stable in the outer region in terms of the radius of the disk, that is $5.33 \times 10^7 m \leq r \leq 1 \times 10^8 m$. In similar manner for the partial degenerate electron pressure contribution, the radius in which the disk is stable is calculated to be the same as the complete degenerate case. This has be illustrated by fig. (4.6) and (4.7). In both figures the pressure increases as the the radius increases but not linearly The increase in pressure in fig 4.7 with the existence of contribution of complete degeneracy electron pressure is greater pressure. Similar phenomenon is observed in fig. 4.8 and 4.9, for the contribution of partial degeneracy electron pressure. Even though, the pressures increase in the outer region with the contribution of electron pressure, it does not affect the stability of the disk. Generally, the middle and outer regions, where the radiation pressure is dominated by the gas pressure, are stable with the presence of a complete and partial degenerate electron pressure.

Chapter 6

Conclusion

In this study, our investigation is the stability in the case of gas-pressure dominated region of the accretion disk. According to the standard theory of accretion disk Shakura and Sunyaev (1973) , the middle and outer parts of a disk are dominated by the gas pressure. We have found that the temperature and density of the disk in the gas-pressure dominated region decreases as the radius of the disk increases. We have derived the relation for degeneracy electron pressure both in the case of complete and partial degeneracy. A complete degenerate electron pressure is calculated assuming that the temperature of the disk approaches to zero which means the temperature of the disk is less than Fermi temperature and a partial degenerate pressure is calculated at a finite temperature of the disk which is more practical. Comparing the gas pressure with the total pressure, we have observed that the total pressure rapidly increases when the radius of the disk increases.

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List of Symbols

Latin Symbols

c:	Speed of light in vacuum
ds:	Interval of space-time
dV,dv:	Element of a volume
E:	Specific energy of matter
F_{rad}:	Radiative energy flux density
$f_{r\Phi}$:	rΦ component of the viscosity stress tensor
G:	Gravitational constant
H:	Thickness of a layer; half-thickness of a disk
$\hbar$:	Planck constant divided by 2π
h:	Planck constant
I:	Nuclear spin
I_ν:	Spectral intensity of radiation
k_B:	Boltzmanns constant
M:	Stellar mass
$M_\odot$:	Mass of the Sun
$m = M/M_\odot$:	Non-dimensional stellar mass
m_e:	Electron mass
m_p:	Proton mass
n_e:	Number density of electrons
n_p:	Number density of protons
P:	Pressure of matter
P_e:	Electron pressure
P_g:	Gas pressure
p_z:	z Component of electron momentum
Q:	Energy release per nuclear reaction
r:	Radius of the star
$r_g = 2GM/C^2$:	Stellar gravitational radius
S:	Specific entropy of matter

T_C:	Central temperature of a star
u:	Velocity
W:	Gravitational energy of a star

Greek Symbols

α:	Coefficient connecting the mixing length of a convective element and the pressure scale height
β:	Non-dimensional chemical potential of electrons
η:	Coefficient of (dynamic) viscosity
λ:	Wavelength of a photon
μ:	Molecular weight $\equiv$ number of nucleons per particle
ν:	Photon frequency(Coefficient of kinematic viscosity)
ρ:	Matter density
ρ_c:	Central density of a star
σ:	The Stefan-Boltzmann constant
Σ:	Surface matter density of a disk
τ:	Optical depth of the disk
Φ_G:	Gravitational potential
Ω:	Angular velocity of matter
Ω_K:	Keplerian angular velocity

Physical Constants

α_{ss}	0.01
μ	0.62
$\frac{M}{M_\odot}$	1.4
$r_0 = r_A$	$1{\times}10^6 m$
μ	1
$M_\odot$	$1.99 \times 10^{30} Kg$
K_B	$1.38{\times}10^{-23} m^2 Kg s^{-2} K^{-1}$

h	$6.63 \times 10^{-34} m^2 Kg s^{-1}$
m_e	$9.11 \times 10^{-31} Kg$
m_p	$1.63 \times 10^{-23} Kg$
π	3.14

Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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Place and time of submission: Addis Ababa University, April 2013

This thesis has been submitted for examination with my approval as University advisor.

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