43

GRADUATE SEMINAR REPORT

ON

TECHNIQUES FOR FINDING CRITICAL PATH OF A PROJECT NETWORK MODEL.

BY

TADESSE TILAHUN

ADVISOR

DR. MOBIN AHMAD

SCHOOL OF GRADUATE STUDIES
DEPARTMENT OF MATHEMATICS
ADDIS ABABA UNIVERSITY.

JUNE 2005.

Contents

Acknowledgment.

This seminar report deals with a specific topic of Operation Research **: Project Network.** Basically it has two parts : The first part deals with more general feature of Project Network; such as Basic terms, their representation ,Labeling etc. while the Second part deals with techniques for determining the critical path of a Project Network model. For clarity illustrative examples are incorporated with each concept.

A Project may range from simple day to day activities such as making household item to complicated projects that require considerable amount of resource, such as launching space shuttle.

Given a Project, it can be breakdown into different (but interdependent) activities each requiring some amount of time for completion.

The relationship (interdependence) between activities of the project can be represented using a Network Model.

It is natural that an activity may not be started or completed according to the plan for so many reasons. In such cases, the earliest time that we can start an activity or latest time to start the activity can be calculated.

It is interesting to know which activities really determine the completion of a project. i.e. Activities the delay of which can affect the total project completion.

This report presents ways of solving such kind of questions that may arise in any project management.

In the preparation of this seminar I would like to acknowledge those who have done their contribution to the successful completion of this report. The first and foremost goes to the almighty **GOD**, who gave me peace and strength from the beginning to the end of this report in particular and my study in general. Great thanks to my advisor **Dr. Mobin Ahmad** for his invaluable support by providing useful materials, constructive comments and suggestions.

I would like to thank also **Prof. Dr. R. Deumlich** for his comments and advises in writing this report.

Finally I would like to thank my family and working colleagues without whom support my study might have not be realized.

Tadesse Tilahun.

2. Historical background of Operations Research.

Operations research, sometimes abbreviated O.R. or often called management science, means a scientific approach to decision making, which seeks to determine what the best design is and how to operate a system. Operations research formally began in the early 1940's in the United States at around the same time it developed in Great Britain. O.R. grew up around military problems .

During the 1940's there were no mathematical methods of operations research. The only methods that were available were those belonging to individual scientists. One of the first operations research teams was the Blackett's Circus. This group was concerned with the study of military problems.

During World War II the British military leaders asked scientists and engineers to analyze the military's problems. These problems were the deployment of radar and the management of convoy, bombing, antisubmarine, and mining operations (Winston 1). The role of the Blackett's team was to provide scientific assistance in the coordination of British radar equipment at gun sites .This is the earliest operations research study. Most of these early studies included teams of civilian scientists working on military problems.

Once the war had ended, interest in operations research outside of military use began to grow.

This increase led to the development of educational programs. "The Massachusetts Institute of Technology, Case Institute of Technology, and John Hopkins University were early leaders in developing seminars, short courses, and conferences in operations research.

Operations research is concerned with decision problems. There are eight areas of decision problems that can be specified in operations research. They are inventory, allocation, waiting lines, scheduling, sequencing, competition, replacement, and search.

There are four scheduling techniques and three applications of scheduling. They are time, resource, production, and general scheduling. Of the techniques, the single most important dimension is time. "The time scheduling techniques are principally based upon network logical plans. The time scheduling techniques Critical Path Method (CPM), Program Evaluation and Review Techniques (PERT), and Precedence Diagrams all utilize logic nets." Over the years these techniques have become similar and have the same amount of effectiveness. The technique used is up to the person's preference. Other time-based scheduling techniques include close-order, time-motion, and short-interval scheduling.

3. Introduction

Networks arise in numerous settings and variety of guises. Transportation, electrical and communication networks pervade our daily lives. Network representation are also widely used for problems in such diverse areas as production, distribution, project planning, facilities location, resource allocation management and financial planning – to name just few examples.

In fact, a network representation provides such a powerful visual and conceptual aid for portraying the relationship between the components of systems that is used in virtually every field of scientific, social and economic endeavor.

One of the most exciting developments in Operations Research (OR) in recent years has been the unusually rapid advance in both the methodology and application of network optimization methods.

Network optimization is a special type of linear programming model. Network models have three main advantages over linear programming.

- They can be solved very quickly.
- They have naturally integer solutions
- They are intuitive. (i.e. Network Models provide a language for talking about problems that is much more intuitive than the " variables, objectives and constraints " language of linear and integer programming.

Some of the most important kinds of Network Problem are: -

Shortest Path Problem.

Minimum Spanning Tree Problem.

Maximum Flow Problem.

Minimum Cost Flow Problem.

An important economic application for Network system is the description of projects, where projects are set of finite sequences of activities.

Network model ranges from simple to complicated models that can be analyzed by dividing the original network into sub- graph.

The first part of this seminar report will scratch the surface of the current state of art of network methodology, while the second part deals with techniques for finding the critical path of network problem.

4. Basic Concepts, Terminologies and Definition.

A **Network** consists of a set of points and lines connecting certain pairs of points.

- The points are called **nodes (vertices).**
- The lines are called arcs **(links/branches),**

OR,

A **Network** is a flow of diagram consisting of **activities** and **events** connected logically and sequentially.

An **Activity** is the actual performance of a given task.

(It is task or item of work to be done that consume time, money, effort or other resources.)

An **Event** is the commencement of work to be done that does not consume time or resource.

Dummy activity is an activity, which only determines the dependency of one activity over the other but does not consume any time. It is represented by dotted line.

Notation :-

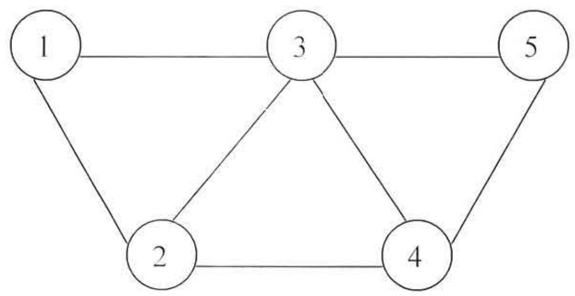
For Simplicity,

Nodes are represented by small circles labeled with letters/numbers.

Arcs are represented by straight lines (through the actual situation represented may not be straight.)

The notation for describing a network is **(N,A)** where **N** is the set of nodes and **A** is the set of arcs.

As an illustration, the Network,



is described as

$$N = \{1,2,3,4,5\}$$

$$A = \{(1,3),(1,2),(2,3),(2,4),(3,4),(3,5),(4,5)\}$$

The arcs may have a **flow** of some type through them. In a road Network, for example, we have automobile traffic flow, in pipeline networks the flow may be of oil products or fluids, and in airways we have a flow of aircrafts.

The direction of flow through an arc in a network model may be only in the direction or either direction.

If the flow through an arc is in one direction, the arc is used to be **directed arc**. The direction is indicated by adding an arrowhead at the end of the line representing the arc.

If the flow through an arc is in either direction, the arc is said to be **undirected arc.**

Although the flow through an undirected arc is allowed to be in either direction, we do assume that the flow will be one way in the direction of choice rather than having simultaneous flow in opposite directions.

However, in the process of making the decision on the flow through an undirected arc, it is possible to make a sequence of assignment of flows in opposite directions, but with the understanding that the actual flow will be the **net flow**.

A Network that has only directed arcs is called **directed Network**, and if all arcs are undirected, the Network is said to be **undirected Network**.

A Network with a mixture of directed and undirected arcs (or even all undirected arcs) can be converted to directed Network, if desired by replacing each undirected arc by a pair of directed arcs in opposite directions.

A path between two nodes is a sequence of distinct arcs connecting these nodes.

A **directed path** from node i to node j is a sequence of connecting arcs whose directions (if any) is toward node j so that flow node i to node j along this path is feasible.

An **undirected path** from node i to node j, is a sequence of connecting arcs whose direction (if any) can be toward or away from any node j. (It is obvious that a directed

path satisfies the definition of undirected path, but not vice-versa.) Surprisingly, undirected paths play a major role in the analysis of directed networks.

A path that begins and ends at the same node is called a **cycle**.

In a directed network, a cycle is either a directed or undirected cycle, depending on whether the path involved is directed or undirected.

Since a directed path is also undirected path, a directed cycle is undirected cycle, but not vice versa in general.

Two nodes are said to be **connected** if the network contains at least one undirected path between them.

A **connected Network** is a network where every pair of nodes is connected.

The maximum amount of flow (possibly infinite) that can be carried on directed arc is referred to as the **arc capacity**.

For nodes, a distinction is made among those that are net generators of flow, net absorbers of flow or neither.

A **supply node (or source node/ Source)** has the property that the flow out of the node exceeds the flow into the node. The reverse case is a **demand node (sink node/sink)** , where the flow into the node exceeds the flow out of the node. A **transshipment node (or intermediate node)** satisfies conservation of flow, so flow in equals the flow out.

A network without any directed cycle is called **acyclic network** while a network consisting of directed cycle is called **cyclic network**.

5. Network Model.

A **Model** in operations research is defined as idealized representation of the real life situation. The objective of the model is to provide a means for analyzing the behavior of the system for the purpose of improving performance.

Network Models are applicable in enormous variety of decision problems that can be modeled as Network Optimization problem and solved efficiently and effectively.

Some of these decision problems are really physical problems such as transportation of flow of commodities. However many problems are more of an abstract representation of processes or activities such as the critical path activity network management.

Network models have three important characteristics. First, many real world problems can be modeled by using networks. Problems relating to transportation and distribution are solved using network models. Other types of network models such as **CPM** (Critical Path Method) and **PERT** (Project Evaluation and Review Techniques) can be used to find the shortest or longest path in a network.

The second feature is that the model created of the network has a visual interpretation in addition to the mathematical formulation. The visual representation of a network reduces the problem of communication between the analysis and the organization. This is the most important feature of network models.

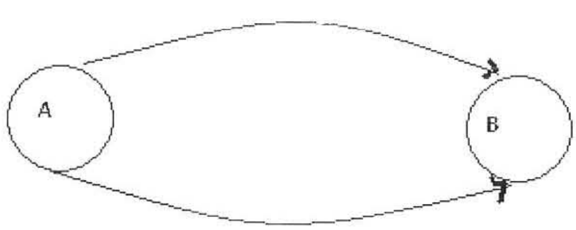
The last characteristic of the network is that the corresponding mathematical formulation has a special structure that allows extremely large problems to be solved very quickly.

Network scheduling models are used to plan, schedule, control, and evaluate complex projects and tasks. The basic idea is to analyze a project, identify the tasks and relate them in a network.

6. Rules for Network Construction of Project Network.

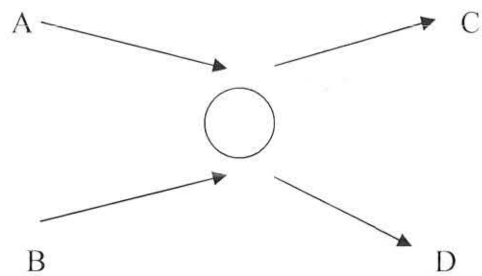
There are a number of ground lines in connection with the handling of events and activities of a project network that should be followed.

I- Each activity must be represented by one and only one arrow.



Such representations are not allowed in Project Network.

II- **No two activities should be represented by same end events.**



Here it is difficult to determine the relationship of A with C and D; and that of B with C and D.

III- **No event should occur until every activity preceding it has been completed.**

IV- **An activity succeeding another cannot be started until that event has occurred.**

V- **An event can not occur twice. i.e. path of activities cannot form a loop that returns to any event previously accomplished. Thus no event can depend for its completion of succeeding event.**

VI- **Each activity must start from and terminate in an event.**

VII- **Dummy activities should only be inserted if absolutely necessary.**

SUGGESTIONS :-

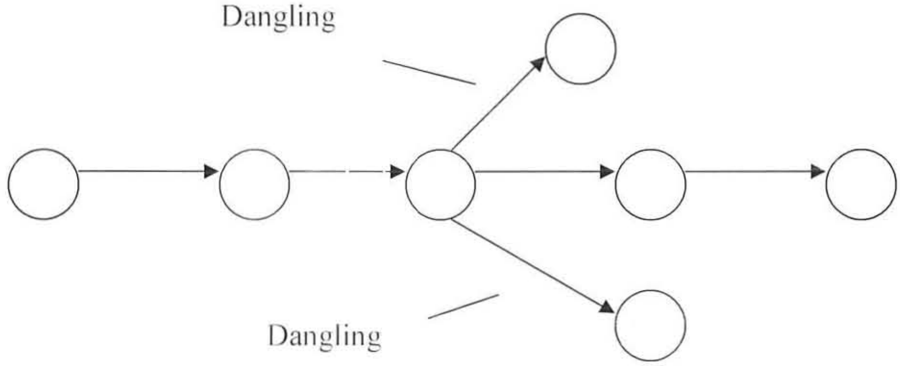
The following suggestions in Network Drawing to construct simple and easy to follow Network :-

- ⊙ Avoid arrows, which cross each other. (If possible).
- ⊙ Do not attempt to represent duration of an activity by its arrow length.
- ⊙ Time flows from left to right.
- ⊙ Network should have only one entry point (called **Start Node**) and one point of emergency (called **End event**).

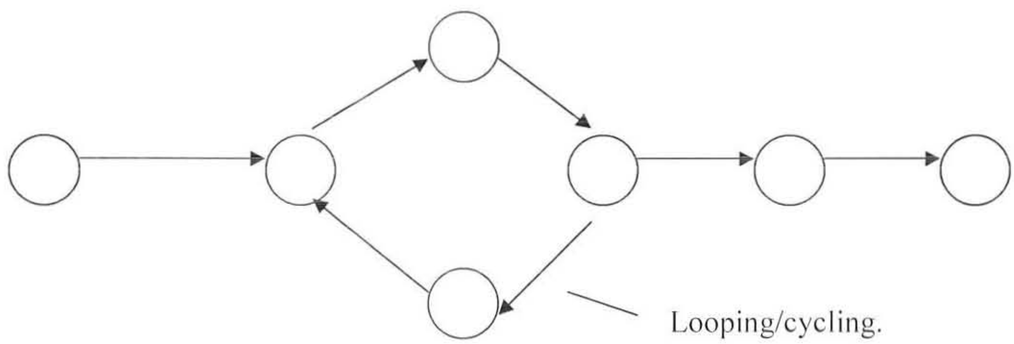
7. Common Errors in Network Drawing.

There are three types of errors, which are most common in Network Drawing.

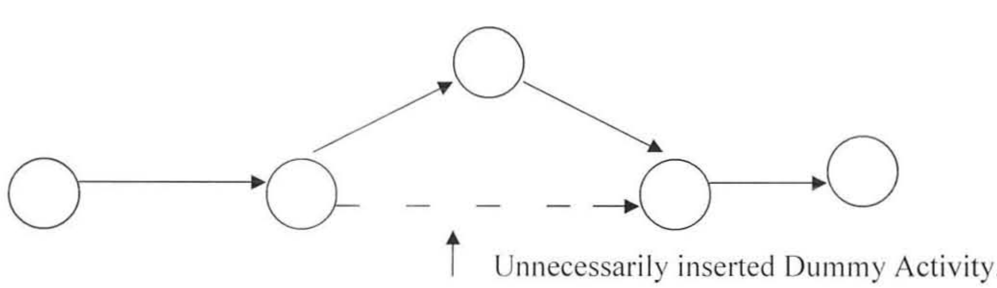
I- Dangling:- To disconnect an activity before the completion of all activities in a network diagram is known as **dangling**. No activity should end without being joined to end event.



II- Looping (Cycling) :- Looping error is known as **cycling** in a network diagram. Drawing an endless loop in a network is known as an error of looping as shown in the following figure.



III- Redundancy :- Unnecessarily inserting the dummy activity in a network logic is known as the **error of redundancy** as shown in the diagram. If the dummy activity is the only activity emanating from an event, it can be eliminated.



8. Fulkerson's " I – J " Rule For Labeling (Numbering) Events.

After the network is drawn in logical sequence, every event is assigned a number, which is placed inside the node circle.

The number sequence should be so as to reflect the flow of the network.

A standard procedure called " **I – J** " rule developed by **D.R. Fulkerson**, is most commonly used for this purpose.

The following steps are used to label a project network.

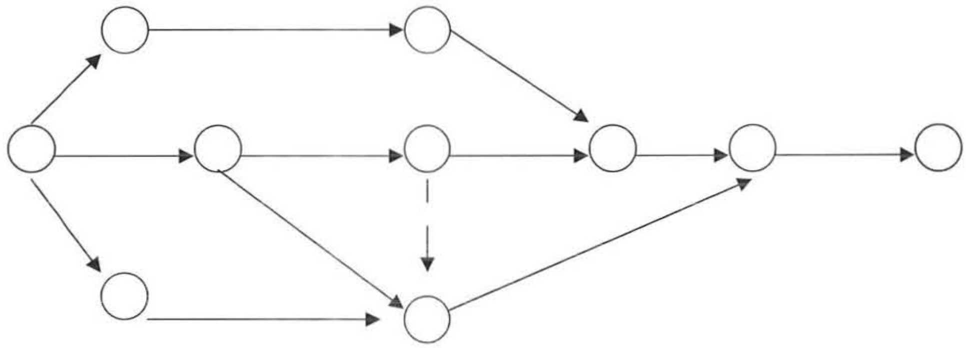
Step 1 :- A start event is the one which has arrows emerging from it but none entering into it. Label it as 1.

Step 2:- Delete all arrows from all numbered events.
This will create at least one new start event out of preceding events.

Step 3:- Number all new start events 2,3 and so on. (Preferably from top to bottom when there are more than one new start events.) Repeat Step 2 and 3 until all events are labeled.

Example :-

Consider the following Network diagram for Labeling Furkerson's Rule.

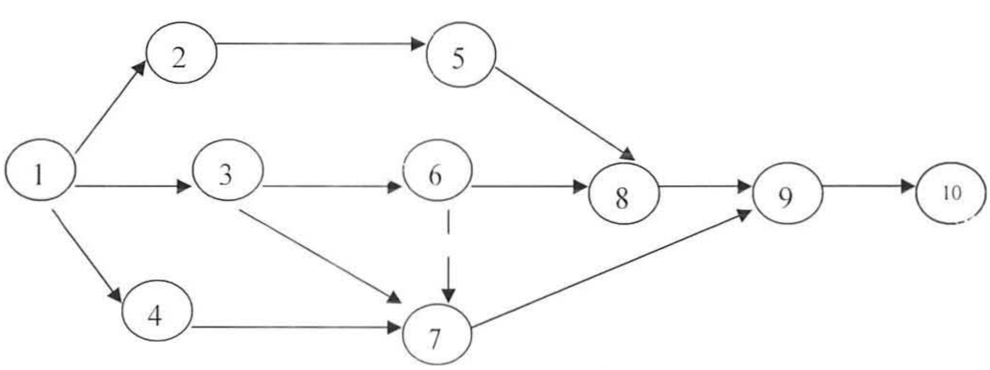


It is obvious to label the start event as 1.

Then by deleting all arrows from this event we will get three new start events.

We can label them as 2, 3 and 4 starting from top to bottom.

Continuing the process the complete labeled Network diagram becomes :-



9. Time Estimates and Critical Path in Network Analysis.

9.1 Critical Path Method (CPM).

Let us consider a project represented by a Network. Once a Network represents the project, the time analysis of the network becomes essential for various activities of the project.

An **activity time** is a forecast of the time an activity is expected to take from starting point to its completion, under normal conditions.

The main objective of time analysis is to prepare a planning schedule of the project.

The planning schedule should include the following factors: -

I - Total completion time of the project.

II - Earliest time when an activity can start.

III - Latest time when each activity can be started without delaying the total project.

IV - Float for each activity. i.e. Amount of time by which the completion of an activity can be delayed without delaying the total project completion.

V - Identification of Critical Activities and Critical paths.

9.1.1 Terms and Notations.

Notation: - We shall use the following Notations for basic Scheduling Computation.

(i,j)	-	Activity (i,j)
E_i	-	Earliest Occurrence time of event i.
L_j	-	Latest Occurrence time of event j.
D_{ij}	-	Estimated Completion time (time duration) of activity (i,j)
$(E_s)_{ij}$	-	Earliest starting time of activity (i,j)
$(E_f)_{ij}$	-	Earliest Finishing time of activity (i,j)
$(L_s)_{ij}$	-	Latest Starting time of activity (i,j)
$(L_f)_{ij}$	-	Latest Finishing time of activity (i,j)

Float - it is the amount of time by which the completion of an activity can be delayed without delaying the total project completion.

i.e. the difference b/n Latest starting time and Earliest starting time of an activity.

Slack - is the difference between the latest event and earliest event times.

Critical Event - an event with zero slack times are called critical events. In other words, event **i** is said to be critical if $E_i = L_j$

Critical Activity - since the difference b/n the latest start time and earliest start time of an activity is called a total float, the activities with zero total float are known as critical activities.

In other words, an activity is said to be critical if a delay in its start will cause a further delay in the completion date of the entire project.

Critical Path - the sequence of critical activities in a network is called Critical Path.

The critical path is the **longest path** in the network from the starting event to ending event and defines the **minimum time** required to complete the project.

9.1.2 Main Features of Critical path: -

The critical Path has two main features: -

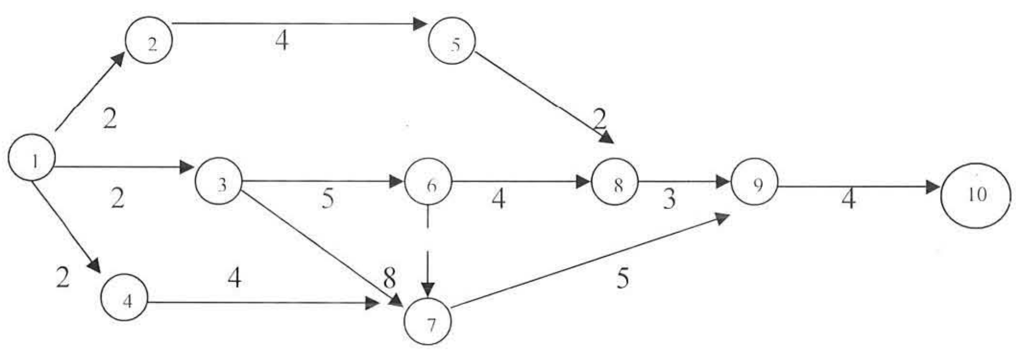
I - If the project has to be shortened, then some of the activities on that path must be Shortened. The application of additional resource on the other activities will not give the desired result unless that critical path is shortened first.

II - The variation in actual performance from the expected activity duration time will be completely reflected in one-to-one fashion in the anticipated completion of the whole project.

Now, lets consider a method to determine critical path of a network (and hence the minimum time required to complete the project) and jobs (activities) that are critical to the completion of the project in time.

Example: -

Consider a project represented by the following network, where nodes are numbered according to Fulkerson's Rule. Numbers along various activities represent the normal time (D_{ij}) required to finish that activity. (in days)



For this project, we are interested to find out

i) the time required to complete the project.

ii) Critical activities (jobs) and Critical Path.

<u>Solution :-</u> First it is necessary to find out the earliest and the latest completion time for each activity in the network.

This can be done by the methods called "**Forward Pass**" and "**Backward Pass**" Computation respectively.

To begin with the procedure, we define

E_i = the earliest expected occurrence time of event i

L_j = the latest allowable event occurrence time of event j

D_{ij} = the expected duration (time) to complete activity (i,j)

9.1.3 Determination of Earliest Time (E_j). (Forward Pass Computation.)

Now, let's start the solution by forward pass computation.

Step 1 :- Determination of Earliest Time (E_j). (Forward Pass Computation.)

The purpose of the forward pass computation is to find out earliest start times for all activities. For this, it is necessary to assign some initial value for the starting node 1.

Usually this value is taken to be zero so that the subsequent earliest time could be interpreted as the project duration up to that point in question.

Rule for Computation: -

Rule 1: - Initial event is supposed to occur at time equal to 0.

I.e. $E_1 = 0$

Rule 2: - Any activity can start immediately when all preceding activities are completed.

i.e. The earliest time E_j for node j is given by

$$E_j = \underset{i}{\text{Max}} \{ E_i + D_{ij} \}$$

Rule 3:- Repeat Step 2, for the next eligible activity until end node is reached.

Thus, using the rule we can have numerical calculation.

$E_1 = 0$

$E_2 = \underset{i}{\text{Max}} \{ E_i+D_{i2} \} = \text{Max}\{ E_1+D_{12} \} = E_1+D_{12} = 0+2=2$

$E_3 = \underset{i}{\text{Max}} \{ E_i+D_{i3} \} = \text{Max}\{ E_1+D_{13} \} = E_1+D_{13} = 0+2=2$

$E_4 = \underset{i}{\text{Max}} \{ E_i+D_{i4} \} = \text{Max}\{ E_1+D_{14} \} = E_1+D_{14} = 0+2=2$

$E_5 = \underset{i}{\text{Max}} \{ E_i+D_{i5} \} = \text{Max}\{ E_2+D_{25} \} = E_2+D_{25} = 2+4=6$

$E_6 = \underset{i}{\text{Max}} \{ E_i+D_{i6} \} = \text{Max}\{E_3+D_{36}\} = E_3+D_{36} = 2+5=7$

$E_7 = \underset{i}{\text{Max}} \{E_i + E_{i7}\} = \text{Max} \{ E_3+D_{37}, E_4+D_{47} \} = \text{Max}\{10,6\} = 10$

$E_8 = \underset{i}{\text{Max}}\{ E_i+D_{i8} \} = \text{Max}\{E_5+D_{58}, E_6+D_{68}\} = \text{Max}\{8,11\} = 11$

$E_9 = \underset{i}{\text{Max}}\{ E_i+D_{i9} \} = \text{Max}\{E_7+D_{79}, E_8+D_{89}\} = \text{Max}\{15,14\} = 15$

$E_{10} = \underset{i}{\text{Max}}\{E_i+D_{i10}\} = \text{Max} \{E_9+D_{(9)(10)}\} = E_9+D_{(9)(10)} = 19$

E_{10}= This implies that the earliest time for the occurrence of last event of the project is on the 19 th days.

Conlusion: - The project will take minimum 19 days to complete.

9.1.4 Determination of Latest time (L_i). (Backward Pass Computation)

Step 2: - Determination of Latest time (L_i). (Backward Pass Computation)

In forward computation, the earliest time when a particular activity will be considered is known. It is also seen that some activities are not critical to the completion of the project.

A question may arise: " Can their starting time be delayed so that total completion time is not affected?"

Such a question may arise while scheduling the resources such as manpower, equipment, finance and so on.

If delay is allowable, then what can be the maximum delay?

For this is the **latest time** for various activities which is desired.

Backward Pass Computation can be done by reversing the method of calculation used for earliest event times.

In forward pass computation, assignment of $E_1 = 0.$, was arbitrarily. Likewise for the backward pass computation, it is possible to assign the project terminal event the date on which the project should be over (latest allowable time determined in Forward pass computation.)

Rule 1:- Set $L_i = E_i$ or T_s

Where Ts is the schedule date for completion and E_i is the earliest terminal time.

Rule 2:- $L_i = \text{Min} \{ L_i - D_{ij} \}$

i.e. the latest time for activities is minimum of the latest time for all succeeding activities reducing their activity time.

Rule 3:- Repeat Rule 2 until starting activity is reached.

Latest time for activities of the network are calculated below:

By Rule 1, set $L_{19} = 19$. Applying Rule 2, until starting activity is reached.

$L_9 = \underset{j}{\text{Min}} \{ L_i - D_{9j} \} = 19 - 4 = 15$ for $j = 10$

$L_8 = \underset{j}{\text{Min}} \{ L_i - D_{8j} \} = L_9 - D_{8,9} = 15 - 3 = 12$ (j contain only node 9)

$L_7 = \underset{j}{\text{Min}} \{ L_i - D_{7,j} \} = L_9 - D_{7,9} = 15 - 5 = 10$ (j contain only node 9)

Now consider node 6, for this node, there are two succeeding activities, namely 6-8 and 6-7,

$$\text{Hence, } L_6 = \underset{j=7,8}{\text{Min}}\{L_j, D_{6,j}\} = \text{Min} \begin{vmatrix} L_7 - D_{6,7} \\ L_8 - D_{6,8} \end{vmatrix} = \text{Min} \begin{vmatrix} 12-4=8\ * \\ 10-0=10 \end{vmatrix} = 8$$

Similarly,

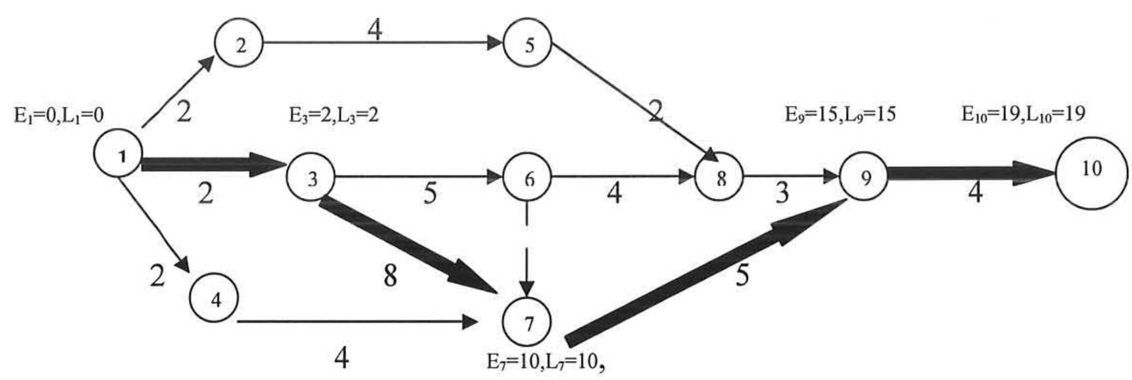
$L_5 = L_8 - D_{5,8} = 12-2 = 10$

$L_4 = L_7 - D_{4,7} = 10-4 = 6$

$$L_3 = \text{Min} \begin{vmatrix} L_6 - D_{3,6} \\ L_7 - D_{3,7} \end{vmatrix} = \text{Min} \begin{vmatrix} 8-5=3 \\ 10-8=2\ * \end{vmatrix} = 2$$

$L_2 = L_5 - D_{25} = 10-4 = 6$

$$L_1 = \underset{j=(2,3,4)}{\text{Min}}\{L_j - D_{1,j}\} = \text{Min} \begin{vmatrix} 7-5=2 \\ 2-2=0\ * \\ 6-2=4 \end{vmatrix} = 0$$



The minimum value of $L_1 = 0$ is not surprising result. Since, started with $L_i = E_i$, it is always possible to set $L_1 = 0$. If this is not so, it means that some error is made in calculations of forward pass or backward pass values.

One can easily observe that Earliest time (E_i) and Latest time (L_j) for activities 1-3, 3-7, 7-9, and 9-10 are equal. Along this path, the fact that the latest and earliest times are the same implies that any activity along this path cannot be delayed without affecting the duration of the project.

9.1.5 Computation of Floats

Step 3. Computation of Floats.

By definition, for the activity 8-9, the float is one day (L_8-E_8=12-11=1). This float represents the amount by which this particular activity can be delayed without influencing the duration of the project.

Also, by definition, free float if any, will exist only on the activities merge points. To illustrate the concept of free float, consider path 1-2-5-8-9, total float on activity 8-9 is one day and sine this is the last activity prior to merging two activities, this float is free float also. Similarly, consider the activity 8-9,. If the activity 5-8, is delayed up to three days, the early start time of no activity in the network will be affected. Therefore, the concept of free float clearly states that the use of free float time will not influence any succeeding activity float time.

If free float of any activity comes out to negative, it is taken zero. For example, independent float of (1,2) = free float of (1,2) – (L_1-E_1)=0-(0-0)=0.

In a similar manner, (total) float of each activity can be calculated by taking the difference of latest and earliest and latest start of each activity. The float of each activity is given on Table 9.1.

9.1.6. Identifying the Critical path.

Step 4. To Identify the Critical Path.

The earlier calculation shows that the path or paths, which have zero floats, are called critical ones. If this logic is extended little further, it would provide a guide rule determine the next critical path, and so on. Such information will be useful for managers in the control of projects. In this example, path 1-3-8-9-10 happens to next to critical; path because it has float of one many of its activities.

Activity (i-j)	Duration (D_{ij})	Start		Finish		Float		
		Earliest	Latest	Earliest	Latest	Total	Free	Independent
(1)	(2)	(3)E_i	(4)=(6)-(2)	(5)=(3)+(2)	(6)=L_j	(7)=(4)-(3)	(8)=E_j-E_i-D_{ij}	(9)=(8)-(L_i-E_i)
1-2	2	0	4	2	6	4	0	0
1-3	2	0	0	2	2	0*	0	0
1-4	2	0	4	2	6	4	0	0
2-5	4	2	6	6	10	4	0	0
3-6	5	2	3	7	8	1	0	0
3-7	8	2	2	10	10	0*	0	0
4-7	4	2	6	6	10	4	4	0
5-8	2	6	10	8	12	4	3	0
6-8	4	7	8	11	12	1	0	0
7-9	5	10	10	15	15	0*	0	0
8-9	3	11	12	14	15	1	1	0
9-10	4	15	15	19	19	0*	0	0

Table 9.1 (* - activities with zero total float. i.e. **critical activities.**)

Thus, we can see that the critical activities are 1-3, 3-7, 7-9 and 9-10. Hence the critical path of the given project network is 1-3-7-9-10. This path is indicated by dark double headed arrow.( ).

9.2 Project Evaluation And Review Technique (PERT).

9.2.1 Terms and Definitions.

The Network method discussed previously (i.e Critical Path Method – **CPM**) has dealt with deterministic models since estimated activity times are assumed to be the expected values.

CPM (Critical Path Method) is an effective for the project, planning and control when activity times are known with certainty. However certain projects like research and development work, it seems unrealistic to assume that we can know with certainty the time duration in which the activities can be completed.

Deterministic Network methods assume that the expected time is the actual time taken. Probabilistic on the other hand, assume the reverse, more realistic situation, where activity times are represented by probability distribution.

PERT is a technique that statistically presents knowledge about activity uncertainties.

There are two approaches that are used for the assessment of duration for activity completion.

I – CPM – deterministic approach in which we may assume that we know enough about each job or operation; so that a single estimate of their duration is sufficiently accurate to give reasonable results.

II – PERT – non – deterministic or probabilistic approach in which one may only be able to state limits within which it is virtually certain that activity duration will lie.

PERT was developed and has been used most frequently in research and development type projects, such as industry, aerospace industry, defense products industry etc.

PERT system is best suited for projects that involve a work that is never done before, or there is a little past history on which to base network construction and time estimates.

The main objective in the analysis through **PERT** is to find out the completion of a particular event with specified date. This approval takes into account the uncertainties.

Time is the most essential and basic variable in **PERT** system of planning and control.

In **PERT** exact estimation of times of completion for various activities is difficult. Estimate is made of not only the most probable time required to complete the activity, but some measure of uncertainty is incorporated in this estimate to consider two more time estimates.

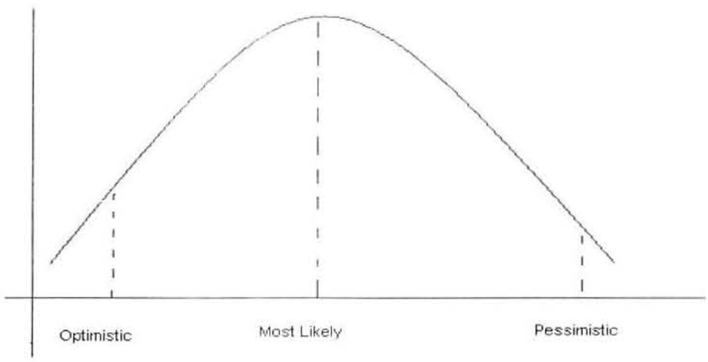
In this approach, three time values are associated with each activity: the **optimistic value**, the **pessimistic time** and the **most likely time**. Thus three time values provide a measure of uncertainty associated with that activity.

The Optimistic time: - is the shortest possible time in which an activity can be finished. It assumes that everything goes well. This is denoted by t_o.

The Most likely time: - is the estimate of the normal time the activity would take. This assumes normal delays. If a graph is plotted in the time of completion in that time period, then the most likely time will represent the highest frequency of occurrence. This is denoted by t_m.

The pessimistic time: - represents the longest time the activity could take if everything goes wrong. As in optimistic estimate, this value may be such that only one in hundred or one in twenty will take time longer than this value. This is denoted by t_p.

These three time values are shown below.



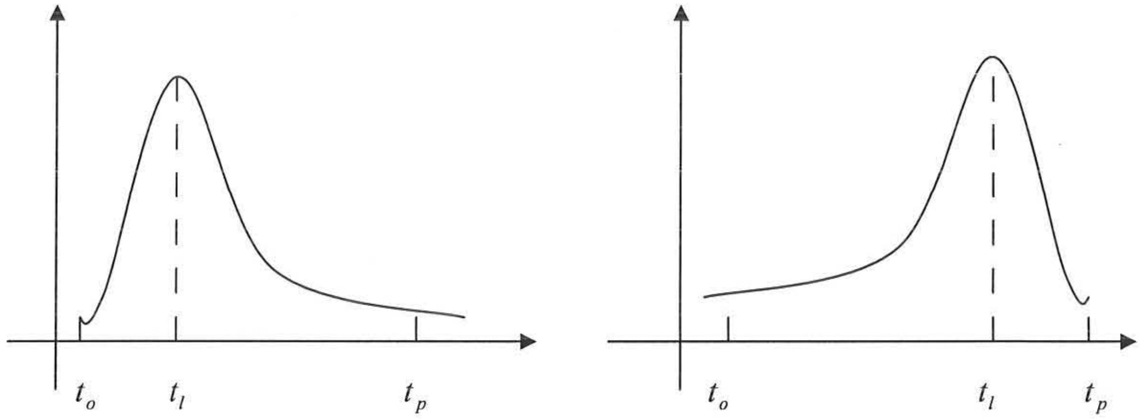
If a curve is plotted between the 'time' of completion and the number of jobs completed in that 'time', a frequency distribution curve will be obtained.

From the curve, it is clear that there are large number of cases of the activity that are completed in the most likely time.

In **PERT** calculation, all values are used to obtain the percent expected value.

Expected Time: - is the average time an activity will take if there were repeated on large number of times and is based on the assumption that an activity time follows beta distribution.*

Beta distribution – the Beta distribution is typical type of probability distribution, which fits well for PERT analysis. A beta distribution is not symmetrical about its apex. It may skew to **left (optimistic estimator)** or the **right (pessimistic estimator)**.



Beta Distribution is chosen in PERT analysis because:-

1. The distribution should have small probability of reaching the most optimistic time (shortest time).

2. The distribution should have a small probability of reaching the most pessimistic time (longest time).

3. The distribution should have one and only one most likely time (i.e. Unimodal) which would be free to move between the two extremes mentioned in 1 and 2 above.

4. The distribution should be such that the amount of uncertainty in the estimating can be measured easily.

The **Standard deviation** for the activity is then given by $\sigma = (t_p - t_o)/6$.

As, **as Variance** for the activity is then given by the formula $\sigma^2 = [(t_p - t_o)/6]^2$

However, one must combine the three time estimates (t_o, t_l, t_p) into a single time, the average time for the computation of the activity of job.

This average time is the so called expected time. (t_e).

If the exact shape of probability distribution curve is known, the average time expected could be accurately calculated. However since the precise curves are never available, we must use approximation. This is done by algebraically using weighed average derived by staticians.

In computing the expected time, a weightage of 1 is given to optimistic time t_o, weightage of 4 to the most likely time t_l and weightage of 1 to the pessimistic time t_p.

Thus the **expected time** is given by the formula:-

$$t_e = (t_o + 4t_m + t_p)/6$$

where t_o - optimistic time

t_p - pessimistic time

t_m – most likely time

t_e - expected time

The expected time (t_e), represents average value while the most likely time (t_l) represents mode of the B – distribution. The expected time represents a particular time on the probability curve, that has a 50-50 chance of being met or excdeed. There is same chance the actual time taken will be greater than te as there is it is less than te. Naturally the vertical line through te will divide the probability curve in to two equal areas.

The difference between **PERT** and **CPM** is that instead of activity duration, expected time, te for the activity is considered with each node, variance is associated. Thus, the duration of the project is the mean expected time with variance.

9.2.2 Calculating the Earliest Time.

As in the case of **CPM**, first we have to find earliest expected times E_i and latest expected times L_i for each activity.

Calculating the Earliest Expected Time :- The earliest expected time E_i, for each node can be obtained by taking the sum of expected times for all the activities leading to node i, when more than one activity leads to a node i, the maximum of E_i is selected.

9.2.3 Calculating Latest Time

Calculating the latest expected time: - To find the latest expected times we start with the latest time T_L for the last node as equal to E_i. (Earliest expected time for last node or schedule completion time of the project (if available).

Then moving backwards for each path, subtracting the expected time, t_e, for each activity link. If more than one succeeding activities occur, we take minimum of all possible values.

9.2.4 Calculating Slacks and Floats

Calculating Slacks and Floats:- As Slack of an event is the difference between the latest and earliest event time and Float is the amount of the by which completion an activity could be delayed beyond the expected completion time, without affecting the overall project duration time;

Slack of an Event = Latest Event time – Earliest Event time.

Float for an Activity = Latest start of Activity – Earliest Start of an Activity.

9.2.5 Determination of Critical of Path.

Since Critical Path is defined as sequence of critical activities (activities with zero Total Float), the critical path can be traced along zero total float activities.

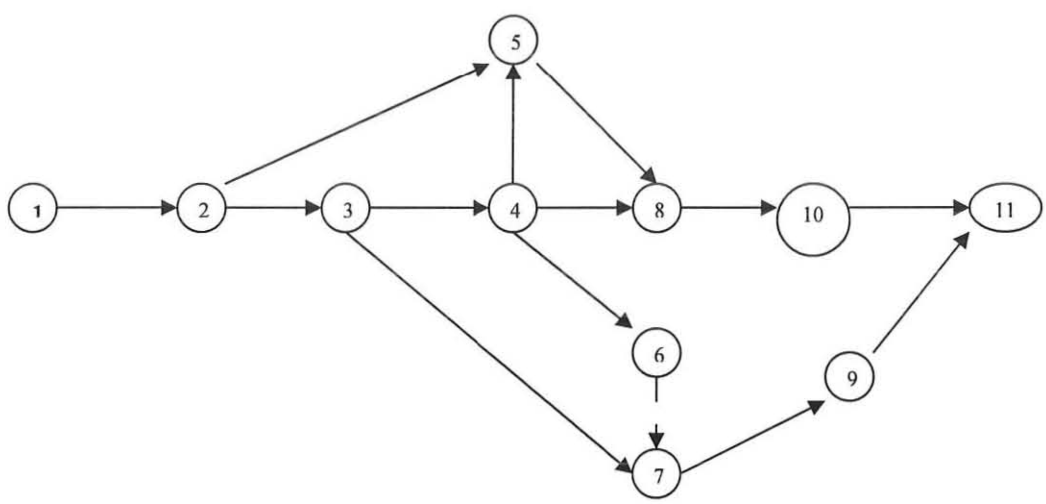
Illustrative Examples on PERT

Example: - For the Project represented by the following Network

i) Find the expected time for each event.

ii) Compute Earliest and Latest expected times.

iii) Find the Critical Path.



(t_o,t_m,t_p for each activity is given on Table 9.2)

Solution:-

a) The Expected time :- $t_e = \frac{t_o + 4t_m + t_p}{6}$

Activity	t_o	t_m	t_p	$T_e = \frac{t_o+4t_m+t_p}{6}$
1-2	3	11	13	10
2-3	1	2	3	2
2-5	4	5	12	6
3-4	5	10	27	12
3-7	6	8	16	9
4-5	4	7	16	8
4-6	3	4	11	5
4-8	4	11	12	10
5-8	2	3	10	4
6-7	0	0	0	0
7-9	4	5	18	7
8-10	3	4	11	5
9-11	6	7	14	8
10-11	7	8	21	10

b) The Earliest and latest Time for Each event can be calculated as :

$E_1 = 0$, $E_2 = 0 + 10 = 10$, $E_3 = 10 + 2 = 12$, $E_4 = 10 + 2 + 12 = 24$,

$E_5 = \text{Max}\{ 10+6, 24+ 8\} = 32$, $E_6 = 24 +5 = 29$, $E_7 = \text{Max} \{29, 12+9 \} = 29$

$E_8 = \text{Max} \{ 32+4, 24+10\}=36$, $E_9 = 29 + 7 = 36$, $E_{10} = 36 +5 = 41$

$E_{11}= \text{Max} \{ 41 +10, 36+8\} = 51$

and,

$L_{11} = E11 = 51$, $L_{10} = 51-10 = 41$, $L_9 = 51 - 8 = 43$, $L_8 = 41 - 5 = 36$,

$L_7 = 43-7= 36$, $L_6 = 36 - 0 = 36$, $L_5 = 63 -4 = 32$, $L_4 = \text{Min}\{ 36-10,32-8\}= 24$,

$L_3 = \text{Min}\{ 36-9, 24-12 \} =12$, $L_2 = \text{Min} \{12-2, 32-6\} = 10$

$L_1 = 10-10 = 0$

c) The Critical Path.

i) Computation of Total Float.

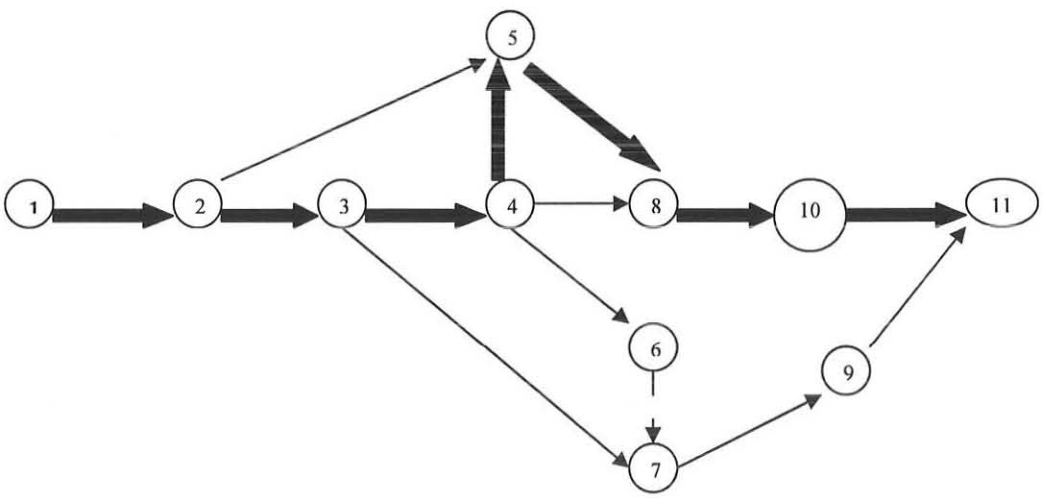
Activity	Duration	Start		Finish		Total Float
		Earliest	Latest	Earliest	Latest	
1-2	10	0	0	10	10	0 *
2-3	2	10	10	12	12	0 *
2-5	6	10	26	16	32	16
3-4	12	12	12	24	24	0 *
3-7	9	12	27	21	36	15
4-5	8	24	24	32	32	0 *
4-6	5	24	31	29	36	7
4-8	10	24	26	34	36	2
5-8	4	32	32	36	36	0 *
6-7	0	29	36	29	36	7
7-9	7	29	36	36	43	7
8-10	5	36	36	41	41	0 *
9-11	8	36	43	44	51	7
10-11	10	41	41	51	51	0 *

(* - activities with zero Total Float. i.e. critical activities)

ii) Thus by tracing the Critical activities on the Network Diagram, we can see the critical path to be:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 8 \rightarrow 10 \rightarrow 11$$

The Critical path is shown by Double Lines on the below network.



10. Differences between PERT and CPM

While **PERT** is mainly for planning and scheduling research programs, **CPM** is used for construction and business problems taking from space programs to marriage proposals to horse-shows.

PERT Network is a probabilistic model with uncertainty is activity duration, **CPM** is deterministic model with well known activity times based on past experience. It assumes that the expected time is actually the time taken.

PERT is an event oriented approach while **CPM** is an activity oriented system. Event represents a significant point on the time while the activity is a time consuming element.

The concept of crashing is applied mainly to **CPM** models. **CPM** deals with costs of project schedules and their minimization.

The above differences between the two techniques are only historical importance and today **PERT** and **CPM** are used as one technique of project scheduling.

10. Advantages of Network Techniques.

The twin techniques of **PERT** and **CPM** find wide range of engineering non-engineering applications.

The main advantages of these techniques as tools of management are:-

1. Techniques of **PERT** and **CPM** help the management in planning, scheduling and control. At the planning stage of various tasks to be performed and resources requirement for accomplishing these tasks are laid out. Network analysis helps at this stage, when alternative programmes are being considered. At the scheduling stage network calculates the time and resources needed at each stage of production or activity. Finally in the operational phase, the network is useful as a control device for measuring the actual against the planned performance.

2. Network shows in simple way, the inter-relationship of various activities constituting a project or a programme. This helps in bringing out clearly the technological interdependence of various activities and so in integrating the project planning.

3. Network points to the smallest critical areas and the exceptional problems so that corrective action may be taken in time.

4. **CPM** deals with time-cost trade off and determine the optimum schedule for a project. Crashing programmes assist in avoiding resource bottlenecks.

5. Cost control can be exercised on project basis. For this purpose, activity or work package is treated as a cost center.

6. Application of network techniques has resulted in better managerial control, improved utilization of resources, improved communication and progress reporting and better decision making.

12. Limitations Of Network Techniques.

Apart from their advantages, Network Techniques have some limitations associated to it.

Some of these are:-

i. In Network Techniques, time estimates are often guess work. Therefore there is difficulty in securing the realistic estimates.

ii. It has natural tendency to oppose changes. (i.e it lacks flexibility.)

iii. It requires trained personnel in the network methodology.

iv. Developing a clear logical network is troublesome.

v. Determination of level of network detail is also troublesome – as it varies from planner to planner.

References.

[1] **Operations Research.**
By A.P Verma, S.K. Kataria & Sons, 2004.

[2] **Introduction to Operations Research.**
By Hiller & Lieberman.

[3] **Optimization Methods (2nd Edition)**
By K.V Mital, Wiley Eastern Limited.1990.

[4] **Project Planning and Control with CPM and PERT.**
By Dr. B.C. Pummia, K.K Khandelwal.Laxmi Publications, 1988.

[5] **Operations Research for Management Decisions.**
By S.D. Sharma.(1999).

[6] **Optimization, Theory and Applications. (2nd Edition).**
By S.S Rao, Wiley Eastern Limited, 1984.

[7] **Problems in Operations Research, Methods and Solutions.**
By P.K. Gupta, Man Mohan, Sultan Chad and Sons, 1995.

[8] **Applied operations Research and Management Science.**
W.J. Fabrycky/ P.M. Ghare/ P.E. Torgensen; Prentice Hall of India, 1987.

[9] **Operations Research, Theory and Applications.**
By J.K. Sharma; Faculty of Management Studies, University of Delhi.