STRONGLY MAGNETIZED TWO DIMENSIONAL ELECTRON GAS WITH ACCOUNT OF CONFINEMENT AND PAIR INTERACTION OF ELECTRONS

By

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SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE IN PHYSICS
AT
ADDIS ABABA UNIVERSITY
ADDIS ABABA, ETHIOPIA
JUNE 2011

ADDIS ABABA UNIVERSITY
COLLEGE OF NATURAL SCIENCE
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Department: **Physics**

Degree: **M.Sc.** Convocation: **JUNE** Year: **2011**

Signature of Author

For those family and friends of mine who has been helped me during my severe pain, which made me half-death. Also it extends to my advisor professor ***Vadim. N. Mal'nev*** *for his valuable encouragement and advise during that moment.*

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Acknowledgements

Above all, my innocent heavenly father the God for his graceful help to cope with my challenges.

I would like to thank my advisor Professor Mal'nev Vadium for his guidance, support, and encouragement. I am always impressed by his enthusiasm for science and his unlimited energy for research. I am very grateful that I have been able to study in his group under a stimulating atmosphere.

Special thanks go to Mr. Demake Datiko, Shewangtaw Hamelo and for all of my friends for their encoragement.

Finally, I hope this thesis will reward my family a little bit for their incessant support and love.

Addis Ababa University Zelalem Abebe

June, 2011

Abstract

During the past decade two-dimensional electron systems at low temperatuer and high magnetic field have repeatedly surprised with exotic electron correlation phenomena. The quantum liquids of the fractional quantum Hall effect (FQHE), the still enigmatic electron crystal at very low filling factors, and vanishing and reappearance of certain quantum Hall states in double layer electron systems all reflect the dominance of electron-electron interaction at very high magnetic fields. Recently the nature of the electronic interaction at the half-filled Landau levels has received much attention. The magnetization of two-dimensional electron gases at low temperatures and in a strong magnetic field B oriented normal to the plane of the electron motion was calculated along the same lines as in the work of Landau. With in two-dimensional electron system new surprises arise year after year, exotic states of electronic matter, new materials with fascinating quantum Hall properties keep triggering an intense activity in the field. Two-dimensional electron gases with crossing energy levels can be realized in a variety of systems with different relative sizes of orbital and spin effects, Coulomb energy, and different coupling between the components. Their study has lead to the discovery of many correlated Quantum Hall states and much effort is put into unravelling the energy-level structure of these systems. In this work we have four parts. The first part contens Two-Dimensional Electron gas, Quantum Confinement and Density of States, Potential wells in Semiconductors, Confinement of Electrons, Effect of Quantum Confinement on Electrons, Semiconductors Nanostructures with Confined Electrons and Properties and Applications of Confined Electron Gases. The second part includes Landau Levels and Landau Level degeneracy. The third part is about Classical and Quantum Hall effect. Finally the forth part presents two charged particles in a magnetic field and Parabolic confinement in strong magnetic field. Here we see charged particle in weak magnetic field and how the particle is confined.

INTRODUCTION

Confinement is important as it raises the energies of the surface state electrons, resulting in a depopulation of the surface state band and a concomitant modification of surface properties associated with the surface state electrons [1]. Historically, 1960s was a decade of intense research efforts to confine electrons to thin layers primarily in metals, superconductors, and metal-oxide-semiconductor devices [2, 3]. Relevant to this work are the 2D electron systems confined to the interface of a semiconductor-semiconductor heterojunction discovered in the early 1970s. Other much studied examples are electrons confined to move along the surface of a material (e.g. the first studied 2DEG on liquid Helium) or in a single sheet of graphene, the two-dimensional honeycomb crystal lattice of carbon atoms that has gained much attention in recent years [4]. After the discovery of the Integer Quantum Hall Effect (IQHE, 1980)[5], and the fractional one (FQHE, 1983)[6], two-dimensional electron systems submitted to a perpendicular magnetic field remain a very active field of research, be it experimentally or on the theory level [7]. The discovery of the quantum Hall Effects, in particular that of the FQHE, has taken a large part in a qualitative advance of condensed matter physics regarding electronic fluids in conducting materials. In large band metallic systems, the role of interactions was successfully taken into account until the sixties by the Landau liquid theory [8], which is a perturbation approach: interactions between electrons alter adiabatically the properties of the free electron model, so that the Drude-Sommerfeld model keeps its validity with renormalized coefficients. The theoretical tools corresponding to this physics have involved sophisticated diagrammatic techniques such as Feynman diagrams, which are all based on the

existence of a well controlled limit of zero interaction Greens function. If the dimensions of semiconductor structures shrink to only a few tenths of nanometers, very interesting phenomena in the electron transport can be observed. Most of these phenomena rely on the wave nature of the electrons and the quantization of energy levels. This implies that mostly low temperatures are required for their observation [9]. A two-dimensional electron gas can be realized at the surface of a semiconductor like silicon or gallium arsenide where the surface is usually in contact with a material which acts as an insulator (SiO_2 for silicon field effect transistors and, e.g. $Al_xGa_{1-x}As$ for heterostructures). Electrons are confined close to the surface of the semiconductor by an electrostatic field F_z normal to the interface, originating from positive charges, which causes a drop in the electron potential towards the surface [10]. Since the discovery of the quantum Hall effect the properties of a disordered two-dimensional electron gas (2DEG) subjected to a perpendicular magnetic field have been extensively studied. The nature of the density of states (DOS) of the 2DEG is a problem of vital importance for the understanding of many quantum phenomena observed in the system, e.g., cyclotron resonance, specific heat, magnetization, magnetocapacitance and magnetotransport. The problem has attracted a lot of attention from both experimentalists and theorists. Nevertheless, over the years of research the common conclusion has seemed to be only that the DOS in question is composed of disorder-broadened Landau levels with a significant number of electronic states in between. To date, there has been little or no consensus regarding the exact form of the Landau level DOS of the 2DEG and the magnetic-field dependence of the linewidth. This conclusion belongs not only to the calculated DOS but the measured DOS as well. Indeed, some experimentalists have reported a Gaussian lineshape while others claim that it is Lorentzian. Some report a broadening which is independent of the applied magnetic field while others report a broadening which is a square root or oscillating function of the magnetic field. In addition, in order to explain their experimental results several authors have invoked a constant background DOS whose origin is unclear.

The first calculation of the Landau level DOS was performed within a self-consistent Born approximation (SCBA) [11]. Magnetization measurements on a two dimensional electron system (2DES) subjected to strong perpendicular magnetic fields have revealed exciting new physics recently. Equilibrium magnetization measurements, manifested by the well known de Haas van Alphen effect (dHvA) have uncovered information on the broadening of Landau levels, the functional form of the density of states and gaps in the energy spectrum. This effect has been extensively studied in many 2DES's [12, 13, 14]. Compared to other 2D electronic system, this 2DEG offers a larger Fermi wavelength that is comparable to the smallest device size achievable and a much higher mobility. These advantages facilitate the studies of quantum transports in nanostructures. The goal of this work was to characterize the electron-electron interaction in comparatively low-magnetic field. Additionally, the effect of crossed magnetic fields on the 2DEG properties were studied.

Chapter 1

TWO-DIMENSIONAL ELECTRON GASES AND CONFINEMENT

1.1 Two-dimensional electron gases

An important system where quantum effects were observed is two-dimensional electron gas (2DEG). There are two basic systems where 2DEG has been studied. One of them is Si MOSFETs (metal-oxide-semiconductor field-effect transistors). A typical device is shown in (Fig. 1.1) A (100)Si surface serves as a substrate while SiO_2 layer behaves as an insulator. 2DEG is induced electrostatically by application a positive voltage V_g. The sheet density of 2DEG can be described as

$$n_s = \frac{\epsilon_{ox}}{e\, d_{ox}}(V_g - V_t)$$

where V_t is the threshold voltage for the barriers creation.

Another important systems with 2DEG involve modulation-doped GaAs-AlGaAs heterostructures. The bandgap in AlGaAs is wider than in GaAs. By variation of doping it is possible to move the Fermi level inside the forbidden gap. When the materials are put together, a unified level of chemical potential is established, and an inversion layer is formed at the interface. The 2DEG created by a modulation doping can be squeezed into narrow channels by selective depletion in spatially separated regions [15].

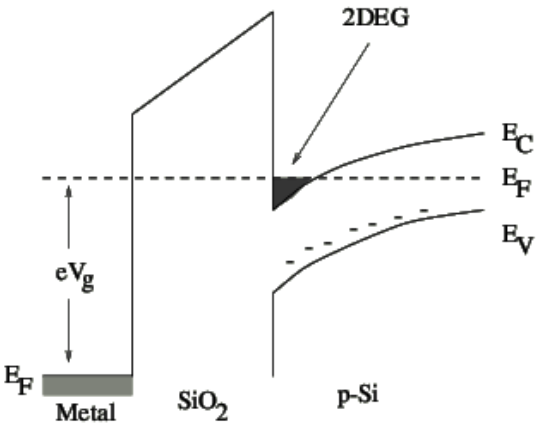

Figure 1.1: Band diagram showing conductance band E_C, valence band E_V and quasi-Fermi level E_F. A 2DEG is formed at the interface between the oxide (SiO_2) and p-type silicon substrate as a consequence of the gate voltage V_g [16].

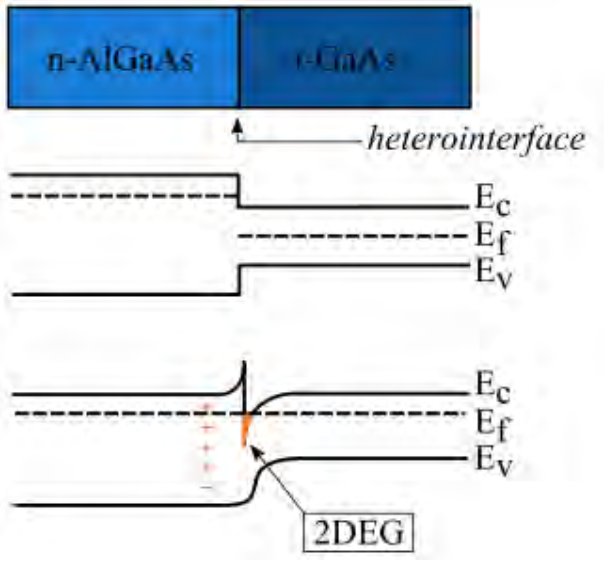

Figure 1.2: Band structure of the interface between n-AlGa As and intrinsic GaAs, (a)before and (b) after the charge transfer[15].

1.2 Quantum Confinement and Density of States

"Confinement" and "quantization" are two closely interrelated definitions: if a particle is "Confined" then its energy is "Quantized", and vice versa. According to the dictionary, to "Confine" means to"restrict within limits", "forced to stay in a closed space", and even to "imprison". A typical example, illustrating the relation between confinement and quantization, is the restriction of the motion of a particle by an infinite potential well of size L_A. Due to the presence of an infinite potential U(x) for $x < 0$ and $x > L_A$, the wave function $\psi(x)$ describing the particle is zero outside the well:$\psi = 0$ for $x < 0$ and $x > L_A$ and, in the region with $U(x) = 0\,(0 \leqslant x \leqslant L_A)$ [17]: Every physics student is familiar with

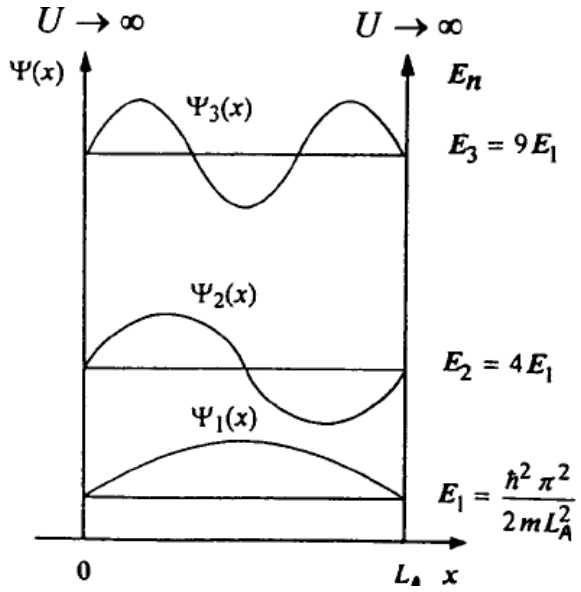

Figure 1.3: Confinement and quantization of the motion of a particle by an infinite potential well size L_A for n=1,2 and 3 [17]

the particle in a-box example from quantum dynamics. In it, a single particle is confined to a certain area by an external potential. So one-dimensional Schrödinger equation is:

$$\hat{H}\psi_n(x) = E_n\psi_n(x) \tag{1.2.1}$$

where $\hat{H}$ is Hamiltonian operator is given by

$$\hat{H} = -\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + U(x) \tag{1.2.2}$$

Where

E_n- is the (eigen) Energy of the system

$\hbar$- is Plank's constant

m^*-is the mass of the particle

U(x)- An infinite external potential

ψ_n is the electron wave function

$$\left\{ -\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + U(x) \right\}\psi_n(x) = E_n\psi_n(x) \tag{1.2.3}$$

From the boundary value problem U(x) is zero.

$$\begin{aligned} -\frac{\hbar^2}{2m^*}\frac{d^2\psi_n(x)}{dx^2} &= E_n\psi_n(x) \\ \frac{d^2\psi_n(x)}{dx^2} &= -\frac{2m^*}{\hbar^2}E_n\psi_n(x) \\ \frac{d^2\psi_n(x)}{dx^2} + \frac{2m^*}{\hbar^2}E_n\psi_n(x) &= 0 \\ \left\{ \frac{d^2}{dx^2} + k_{zn}^2 \right\}\psi_n(x) &= 0. \end{aligned} \tag{1.2.4}$$

Where $k_{zn}^2 = \frac{2m^*}{\hbar^2}E_n$

so the solution of this equation is

$$\psi_n(x) = A\, exp[i(k_{zn}x - wt)] \tag{1.2.5}$$

Thus the solutions of the one-dimensional Schrödinger equation correspond to standing waves with an integer number n of half wavelengths λ along this increase in energy is referred to as the confinement energy of the particle. It is a consequence of the uncertainty principle in quantum mechanics. When the particle is confined within a distance L_A in space (along the z direction in this case) the uncertainty in the z component of its momentum increases by an amount of the order of $\frac{\hbar}{L_A}$. $L_A : n\frac{\lambda_n}{2} = L_A$. This simple constraint results in the well-known quantized energy spectrum

$$E_n = \frac{\hbar^2 k_{zn}^2}{2m^*} = \frac{\hbar^2\pi^2}{2m^*L_A^2}n^2 \tag{1.2.6}$$

where n=1,2,3,.....

$$E_n = \frac{n^2 h^2}{8m^* L_A^2} \tag{1.2.7}$$

for n=1,2 and 3 the confinment, where $n-$ is Quantum number

$$\begin{aligned} E_1 &= \frac{\hbar^2 \pi^2}{2m^* L_A^2} \\ E_2 &= 4\frac{\hbar^2 \pi^2}{2m^* L_A^2} = 4E_1 \\ E_3 &= 9\frac{\hbar^2 \pi^2}{2m^* L_A^2} = 9E_1 \end{aligned} \tag{1.2.8}$$

Here k_{zn} is the wavevector and m^* is the free electron mass. This simple one-dimensional picture can be extended to three dimensions quite easily, but in this thesis the focus is on two-dimensional layers which are confined in only one direction. In such layers the particles are confined perpendicular to the layer, but are free to move within the layer. In that case, any additional quantum properties are well described by the one-dimensional case. To have an idea about the characterstics energy scales involved and their dependence upon the confinement length L_A. We have calculate (see table below) the energies E_1 for electrons confined by an infinite potential well the size, $1\mathring{A}$, 1nm and 1μm. The corresponding increase in the particles kinetic energy is then given by (1.1.6).

Table 1.1: Confinement by the infinite potential well

Confinement length L_A	EnergyE_1	Temperature T
$1\mathring{A}$	40 eV	$4 * 10^5$k
1nm	0.4 eV	$4 * 10^3$k
1μm	0.4 μeV	4 mk

Hence this effect is known also as quantum confinement. In addition to increasing the minimum energy of the particle, confinement also causes its excited state energies to become quantized. In addition to changing the energies of excitations, confinement also modifies their density of states (DOS). In general, reducing the dimensionality enhances the singularity in the DOS at a critical point. For instance, on reducing the dimension from three in bulk samples to two in a QW, the electronic DOS at the band gap E_g

changes from a threshold depending on photon energy as $\sqrt{\hbar w - E_g}$ to a step function [18].

1.3 Potential wells in Semiconductors

So far quantum theory has shown that a single particle in a potential well has distinct, numbered energy levels. To realize such a 2DEG system in an experimental device, three conditions need to be met:

(i). There has to be a potential well;

(ii). There have to be particles, here electrons, within the potential well;

(iii). The electrons within the well should not interact significantly.

Again, the interplay between theory and experiments forged the way to such devices. The application of quantum mechanics to solid state science gave rise to band structure theory. There, the (electrical) properties of materials are defined by the appearance of collections of electron levels named bands.If the Fermi level, the maximum energy level electrons can reach. If the Fermi levels lie within a band, then the small perturbations (e.g. an electric field) can cause an imbalance between the velocities and electrons may then contribute to conduction and the system is in its ground state. This is known as a metal (Conductor)(See Figure 1.24(c). If the Fermi level is within a gap between two

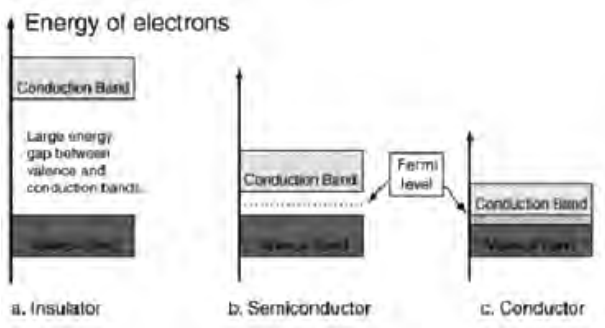

Figure 1.4: The simple band picture of insulators (a), semiconductors (b) and conductors (c). Figure taken from [19].

bands and Fermi energy within a small band gap (small compared with a few times k_BT), thermal fluctuations do not excite some electrons to the upper, conducting band, the material is an insulator (See Figure 1.4)(a). However, if some electrons are excited to the conduction band and both bands may contribute to conductions, it is a semiconductor (Figure 1.4(b)). The band gap is unique for each semiconductor material. Table 1.2 shows some band gaps for representative semiconductors and oxides.

Table 1.2: Important parameters for some representative semiconductors and oxides at room temperature[20−24].

material	band gap$E_g(eV)$	effective mass $m^*(-)$	electron mobility $\mu(\frac{cm^2}{Vs})$	electron density n
Si	1.12	1.08	1300	2×10^{16}
GaAs	1.42	0.067	6000	3×10^{15}
AlAs	2.168	0.146	200	2×10^{17}
Nb:SrTiO_3	1.8	6	3.2	1.4×10^{17}
LaAlO_3	5.6	−	−	−

From Table 1.2 it can be seen that the band gap varies substantially. By layering different semiconductors a potential well can be created. An example of such a potential well is shown in (Figure 1.5). The next question is how to get electrons into the potential well.

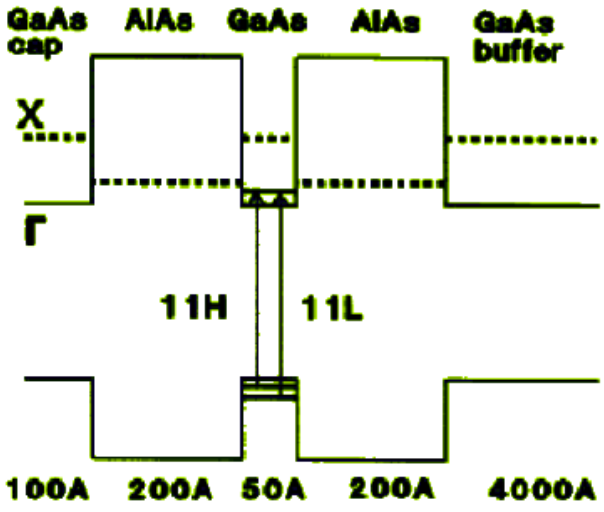

Figure 1.5: Band diagram of a simple semiconductor potential well [25].

The standard technique is to dope the semiconductor with atoms of an element that is more valent. Silicium is four-valent. Doping with phosphorus, which is five-valent would

result in an excess of electrons after covalent bonding. These excess electrons are now doped into the conduction band and can freely move throughout the potential well. Such doped atoms, however, also form scattering centers which inhibit the electron mobility. As the electrons should be disturbed as little as possible within the potential well, the active layer (the central GaAs layer in Figure 1.5) should not be doped. Here an additional bonus of the potential well appears. If the layers to the side of the actual well are doped, the free electrons will search out the energy minimum within the well as their ground state. This way the number of electrons within the well can be controlled, without disturbing the crystal perfection of the well itself. This control also gives the opportunity to make sure the electrons are independent from one another. As long as the mean free path is much smaller than the average distance between electrons, the electron-electron interaction will be negligible. Thus, by controlling the density of electrons within the potential well the independent electron regime can be maintained [25].

1.4 Confinement of Electrons

The most basic effect of such confinement would be the splitting of the energy bands into distinct states with energies as approximated by Equation (1.1.7). However, if the potential well is so large that the energy levels are very close together, thermal fluctuations blend the transitions and the energy levels are not distinct. If all levels are to be resolved, it can be stated that:

$$E_2 - E_1 > K_B T \tag{1.4.1}$$

where K_B is the Boltzmann constant and T the absolute temperature. After inserting Equation (1.1.6) this gives an upper limit to the potential well size:

$$L_{thermal} < \sqrt{\frac{3h^2}{8m^* K_B T}} \tag{1.4.2}$$

This is an upper limit to the size of the potential well to have distinct energy levels, or, to be a quantum well. Another important criterium is whether the electronic state of

the electron is larger than the potential well. Only in that case the external potential well forms the true potential well controlling the electron properties instead of the atomic potential well. This length scale is conveniently described by the Bohr radius:

$$r_{Bohr} = \frac{4\pi\epsilon\hbar}{m^* e^2} \tag{1.4.3}$$

If, however, these conditions are met and a quantum well is created, the density of states splits up. The lower the dimensions of the quantum well, the sharper the density of states is (see Figure 1.6). Note that even though for a 2D quantum well the density of states is a step function, (Figure 1.6) only gives the ground states. The transitions between levels is still well defined. Even if the electrons are confined within a potential well, but not to the quantum limit, the two-dimensional nature of the electron layer can influence its properties. Under both an applied magnetic and electric field, the Lorentz force will force the electrons in a material to undergo a cyclotron oscillation[27, 28] with a fundamental frequency of:

$$\omega_c = \frac{eB}{m^*} \tag{1.4.4}$$

where B is the applied magnetic field. ω_c can be written as $\omega_c = \frac{v_F}{r_c}$ where v_F is the Fermi velocity of the electron and r_c the radius of the cyclotron oscillation. Basic solid state physics gives then for the cyclotron radius:

$$r_c = \frac{m^* v_F}{eB} = \frac{\hbar K_F}{eB} \tag{1.4.5}$$

where K_F is the Fermi wave vector. This is a basic quantity in solid state physics and can, in approximation, easily be derived from the electron density. Depending on whether the system is two- or three-dimensional either $K_{F,3D} = \sqrt[3]{2\pi^2 n_{3D}}$ or $K_{F,2D} = \sqrt{2\pi n_{2D}}$ is used respectively. For interface and thin layer samples it is often the electron sheet density n_{2D} that can be measured instead of the full electron volume density n_{3D}. Under the assumption that $n_{3D} = \frac{n_{2D}}{L}$ where L is the average thickness of the electron layer two criteria for a two-dimensional electron layer can be derived. The first is whether the

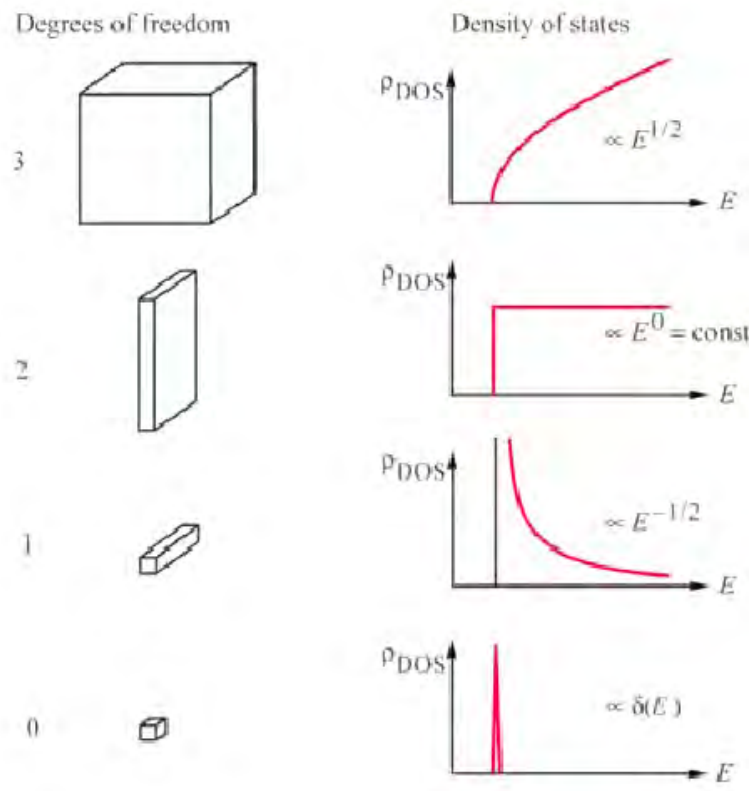

Figure 1.6: Density of states for different dimensionalities of quantum wells, taken from [26].

Fermi wavelength is larger than the electron layer thickness. This indicates that no Bloch waves can develop perpendicular to the layer, so the properties of the electron gas are two-dimensional. Combining the equations for the 3D Fermi wave vector $K_{F,3D}$ and the 3D electron density n_{3D} yields:

$$\lambda_F = \sqrt[3]{\frac{8\pi L}{3n_{2D}}} \tag{1.4.6}$$

From Equation (1.3.6) it follows that $\lambda_F > L$ if $L < \sqrt{\frac{8\pi}{3n_{2D}}}$. For an electron sheet density of $n_{2D} = 2 \times 10^{13} cm^{-2}$ this criterion can be evaluated to be $L < 6.5\,nm$. The second is whether the cyclotron radius is larger than the electron layer thickness. If the electron layer has a thickness less than the cyclotron radius no full cyclotron movement can be carried out by the electrons in that direction. As the cyclotron movement is always perpendicular to the magnetic field, this would be most pronounced if the magnetic field is parallel to the electron layer. Related electrical properties such as the Hall effect and

magnetoresistance would disappear for such magnetic field orientations. Indeed, this has been observed in STO field-effect transistor (FET) structures[29]. In a magnetic field of 3 T and with the same electron sheet density as above $r_c > L$ would be fulfilled for $L < 89\,nm$. For such systems the Fermi wavelength respectively the cyclotron radius form an upper bound to the electron layer thickness. These systems are not quantum confined, but do still exhibit a clear two-dimensional nature [25].

1.5 Effect of Quantum Confinement on Electrons

The effect of quantum confinement on electrons became important in the semiconductor area after the development of sophisticated growth techniques such as molecular beam epitaxy (MBE) and metal-organic chemical vapor deposition (MOCVD), since they allow to grow low-dimensional structures. The confinement effect should be taken into account when any dimension of a system is comparable to the particle or quasi-particle wavelength, defined by de Broglie as:

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{1.5.1}$$

where h is Planck's constant, m is the particle mass, v is the particle velocity, and c is the speed of light in vacuum. Typical examples of structures showing quantum confinement are two-dimensional structures referred as quantum wells (QWs), one-dimensional as nanowires, and zero- dimensional as quantum dots (QDs). A typical and very useful example of confinement is given by a particle with effective mass m confined in a one-dimensional system by infinite potential barriers in the z direction. The allowed wave vectors and energy of the Bloch wave functions are given by equation $K_{zn} = \frac{2\pi}{\lambda} = \frac{n\pi}{L_A}$ and energy in eqn.(1.1.6):where n = 1, 2, 3, ... are the indexes of each energy level and L_A is the thickness of the potential well (see Fig. 1.5) . Thinking of a real quantum well sample, in general the thickness L_A is subjected to fluctuations due to intrinsic inaccuracies of the growth mechanism, leading to the inhomogeneous broadening of the confined levels

(see Fig. 1.7). The energy dispersion δE as a function of the potential well thickness Fluctuations δL_A can be calculated from Eq.(1.1.6) as:

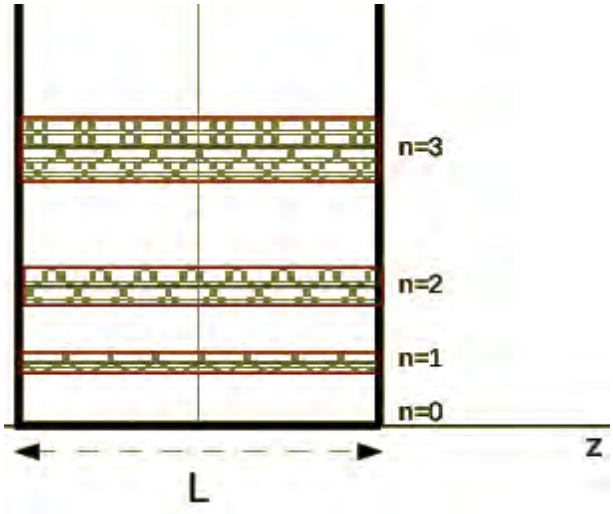

Figure 1.7: Scheme of a one-dimensional potential well with a barrier V . Four levels are shown denoted as n=0,1,2,3. The horizontal band around each level corresponds to changes in the energy of each level due to fluctuations in the thickness (L) of the potential well [30].

$$\delta E = \left| \frac{\hbar^2 \pi^2 n^2}{m^* L_A^2} \right| \delta L_A = 2E_n \frac{\delta L_A}{L_A} \tag{1.5.2}$$

This result shows that this broadening increases quadratically with the level number. In addition, if we consider a fixed value for the level number n this broadening increase as the well thickness decreases. This is a purely quantum effect and should not be confused with the inhomogeneous broadening observed, for example, in a distribution of QDs (or QWs) of different size [30].

1.6 Semiconductors Nanostructures with Confined Electrons

As mentioned above a 2DEG is a gas of electrons with typical densities of $10^{11} cm^{-2}$ that are free to move in two dimensions, but tightly confined in the third. This tight

confinement leads to quantized energy levels for motion in that direction. Thus the electrons appear to be a 2D sheet embedded in a 3D world. Such a system is currently realized in artificial (man-made) semiconductor structures based on different types of materials. The best material in terms of the mobility properties of the electrons is based on gallium arsenide GaAs and aluminum gallium arsenide AlGaAs. The diagram in (Fig.1.8) left panel shows a cross-section through a wafer consisting of layers of GaAs and AlGaAs. The wafer is grown by Molecular Beam Epitaxy (MBE) [31], which produces near-perfect crystalline layers of extreme purity, with nearly atomically sharp transitions between layers. To the left is shown the corresponding band diagram, i.e. the energy of the conduction band (the lowest kinetic energy that mobile electrons can have). The dashed line is the Fermi energy (roughly defined as the highest energy that electrons have in equilibrium). The conduction bands of GaAs and AlGaAs are offset from each other, and this allows electron to collect in the GaAs but not in the AlGaAs. To provide the electrons, silicon doping is included in the middle of the AlGaAs region. These donors become positively ionized and provide electrons, which collect in the neighboring GaAs, though they cannot go too far away because they are attracted back to the positive ions. Thus the resulting electric field created between the positive ions and the negative electrons distort the conduction band into the shape shown, where there is a triangular "well" at the interface, and this goes slightly below the Fermi energy so that electrons can collect there. This well is so narrow that all the electrons there behave as quantum-mechanical waves, with the same wavefunction in the vertical direction. Thus the only degrees of freedom for the electrons are in the plane of the interface, and so they are effectively in a two-dimensional world. It must be stressed that mobile electrons in the crystal environment are characterized by a charge e and by an effective mass which is only a fraction of the electron mass in vacuum. The effective mass is an elegant way to incorporate the complex pattern of interactions of the electrons with the crystal into a single parameter. When this is possible, our gas of electrons moving in the complex environment of a semiconductor

crystal can be mapped into a gas of virtually free electrons with the effective mass and a renormalized Coulomb interaction. Si donor atoms (+ sign in the figure (1.8)) are

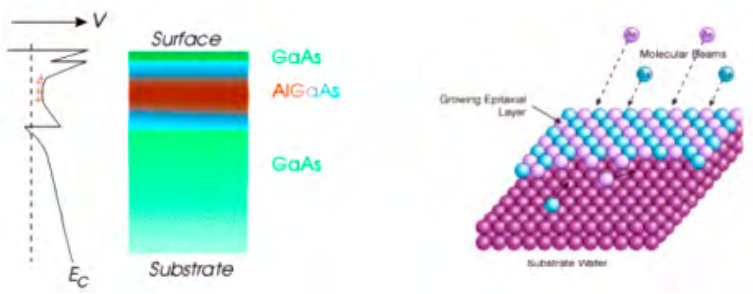

Figure 1.8: Left: Sequence of semiconductor layers along the growth direction in a typical semiconductor heterostructure, Right: schematics of the MBE growth process of GaAs [31].

inserted in the AlGaAs layers and provide electrons to the semiconductor. Electrons fall in the minimum of the energy profile (with triangular shape) and remain trapped in this minimum. They are free to move in the perpendicular plane. Further confinement in the plane can be achieved by modern techniques of e-beam nanolithograpy and chemical etching (the so-called top-down nanotechnology approaches) that are able to manipulate the matter up to a resolution of $10 - 20$ nm [31].

1.7 Properties and Applications of Confined Electron Gases

Probably the most common application of a confined electron gas is as the active layer in semiconductor transistors. Because of the confined nature in (at least) one direction, the absolute number of electrons that have to be manipulated at similar electron densities is small compared to bulk channels. In addition, because the electron donors are outside the actual conduction layer, very high mobilities can be achieved. Together this leads to high speed devices[32]. When the potential well is small enough to achieve quantum confinement, the distinct energy levels lead to interesting new possibilities. One of

the most obvious properties is the appearance of new optical features due to transitions between these levels. A familiar example is shown in (Figure 1.9). These flasks all contain cadmium selenide quantum dots, the zero-dimensional variety of a quantum well. The only difference between them is their respective size. But the light they give off by fluorescence is markedly different. Such dots (0D quantum wells) have a very distinctive

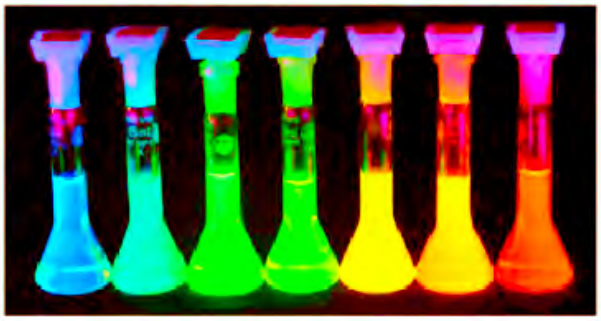

Figure 1.9: Cadmium selenide quantum dots fluorescence at different wavelengths, taken from [33].

spectrum, making them excellent choices for tracers in biological systems, their brightness and chemical stability making them superior to organic dies [33, 35]. This same sensitivity to light also makes them interesting for applications in photovoltaic cells. At the right (tunable) wavelength, the transition of electrons to higher states is easily facilitated. Again the reverse is also interesting: because the light emission of the quantum dots can be tuned, better blue, or even white, light emitting diodes may be created. Aside from active channels in semiconductors transistors, 2D quantum wells are commonly used as the active layer in diode lasers. Either the quantum well is the lasing medium itself or, as in a cascade quantum well laser, the lasing transitions are between levels in different quantum wells [36]. In the first case, the advantage is that the step in the density of states (see Figure 1.6) concentrates the electrons at that energy. This concentration of electrons at a specific energy increases the efficiency of the whole structure, but the transition energies themselves are still mostly determined by the material. The advantage of the second type of quantum well laser is that the lasing transition are between the

different levels in different quantum wells. Because these levels can be determined during the growth of the laser structure, the transition and thus the wavelength of the cascade laser can be tuned from device to device. One final, possible application of quantum wells that has to be mentioned is their use in quantum computing. Because quantum wells, especially quantum dots, are host to well-defined artificial (as in, non-atomic) electron levels they can be used to store quantum information. Coupled quantum dots, together with a method to read and manipulate those states, could form the basis for a quantum computer [37].

Chapter 2

LANDAU LEVELS AND LANDAU LEVEL DEGENERACY

2.1 Landau Levels

Landau quantization in quantum mechanics is the quantization of the cyclotron orbits of charged particles in magnetic fields. As a result, the charged particles can only occupy orbits with discrete energy values, called Landau levels. The Landau levels are degenerate, with the number of electrons per level directly proportional to the strength of the applied magnetic field. Landau quantization is directly responsible for oscillations in electronic properties of materials as a function of the applied magnetic field. The electron is in a stationary magnetic field in the direction of Z axis. The canonical momentum is, as a consequence:

$$\vec{p} = m\vec{v} - e\vec{A} = \hat{p} - e\hat{A} \tag{2.1.1}$$

Due to the fact that the electron has a negative charge whose value is $e \approx -1.6 \times 10^{-19}C$.The Hamiltonian of a nonrelativistic electron in an external magnetic field is:

$$\hat{H} = \frac{1}{2m^*}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 \tag{2.1.2}$$

Where e is positive, (i.e. the charge of the electron is $-e$). We consider a uniform external magnetic field H, $\vec{B} = H\vec{u_z}$, pointing along the z-axis. So we can use the equation $\vec{B} = \nabla \times \vec{A}$ in order to obtain the expression for $\vec{A}$:

$$\begin{pmatrix} 0 \\ 0 \\ H \end{pmatrix} = \nabla \times \vec{A} = \begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix}$$

Then choose the vectore potential $A_x = -Hy$

$$A_z = A_y = 0$$

That is to say,

$$\vec{A} = \begin{pmatrix} -Hy \\ 0 \\ 0 \end{pmatrix} \tag{2.1.3}$$

This is called "Landau gauge."

We can now continue with our Hamiltonian:

$$\hat{H} = \frac{1}{2m^*}\left(\hat{p} - \frac{e}{c}\hat{A}\right)^2 = \frac{1}{2m^*}\left\{\left[\hat{p}_x + \frac{e}{c}Hy\right]^2 + \hat{p}_y^2 + \hat{p}_z^2\right\} \tag{2.1.4}$$

So the Schrödinger equation is:

$$\begin{aligned} &\hat{H}\psi = E\psi \\ &\left\{\frac{1}{2m^*}\left(\left[\hat{p}_x + \frac{e}{c}Hy\right]^2 + \hat{p}_y^2 + \hat{p}_z^2\right)\right\}\psi = E\psi \end{aligned} \tag{2.1.5}$$

We solve the Schrödinger equation by assuming a wave function of the form:

$$\psi(x, y, z) = e^{i(k_x x + k_z z)}\phi(y) \tag{2.1.6}$$

Then $\phi(y)$ satisfies the equation for harmonic oscillator:

$$\begin{aligned} &\left[\frac{1}{2m^*}\hat{p}_y^2 + \frac{1}{2}m^*\omega_o^2(y - y_o)^2\right]\phi(y) = E'\phi(y) \\ &\omega_o = \frac{e}{m^* c}H \qquad , \qquad y_o = \left(\frac{\hbar c}{eH}\right)k_x \end{aligned} \tag{2.1.7}$$

Where $E' = E - \frac{\hbar k_z}{2m^*}$. The natural frequency of the harmonic oscillator ω_o is the "Cyclotron frequancy," that of a classical charge moving in a circular orbit normal to a uniform magnetic field. the energy eigenvalues are thus:

$$E_{n\,ap_z} = \frac{p_z^2}{2m^*} + \hbar\omega_o\left(n + \frac{1}{2}\right) \qquad (n = 0, 1, 2, 3, ...) \tag{2.1.8}$$

Where $p_z = \hbar k_z$. The above equation (2.0.8) are called Landau Energy Levels. Since they are independant of k_x, they have a degneracy equal to the number of allowed values of k_x, such that y_o lies within the container of the system. and the corresponding eigne state is:

$$\psi_{n\,p_x\,p_z} = \left(\frac{m\omega_c}{\pi\hbar}\right)^{1/4} (\exp\frac{i}{\hbar}(p_x x + p_z z))(\exp\frac{m_e\omega_c}{2\hbar}(y-y_0)^2)H_n(\sqrt{\frac{m\omega_c}{\hbar}}(y-y_0)) \quad (2.1.9)$$

2.2 Landau Level Degneracy

We have seen in the previous section that the Landau levels are degenerated, which means that different states can have the same energy eigenvalue. This raises the question how many states per unit area can be occupied in a certain Landau level at a particular magnetic field. The question can be answered if we start from a two-dimensional electron gas at zero magnetic field. If we apply a magnetic field we immediately see that the electrons in an energy interval $[\hbar\omega_c n, \hbar\omega_c(n+1)]$, with n = 0, 1, 2, ..., have to condensate into the n^{th} Landau level energetically located at the center of this interval $\hbar\omega_c(n+1/2)$. Let us put the system in a large cube of size L, and impose periodic boundary conditions. The allowed values of k_x are of the form $2\pi\frac{n_x}{L}$ where $n_x = 0, \pm1, \pm2, \pm3, ...$ For y_o to lie between 0 and L , the values of n_x must be positive and bounded by

$$g = \left(\frac{eH}{hc}\right)L^2 \quad (2.2.1)$$

which is the degeneracy of a Landau Level. The proportionality to L^2 reflects the fact that the projection of the electron orbit onto the xy plane can be centered anywhere in the plane without changing the energy. Thus, when the external field is turned on, the energy spectrum associated with the motion in the xy plane changes from a continuous spectrum to a discrete one, and the level spacing and degeneracy increases with the external field. Obviously, the degeneracy of a Landau level increases proportional to the magnetic field. For a given area L^2 the total number of states in a Landau level g_L is given by

$$g_L = n_L S = 2\frac{HS}{\phi_o} = 2\frac{\phi}{\phi_o} \quad (2.2.2)$$

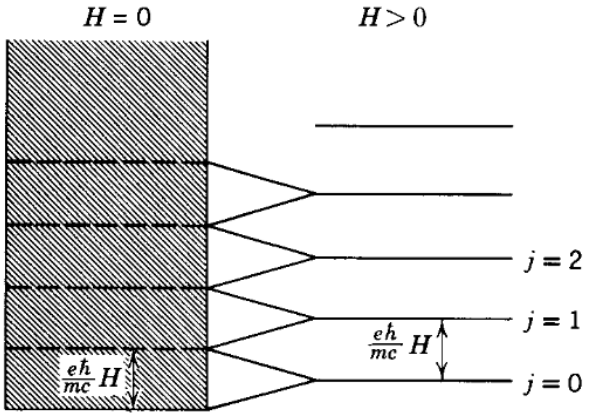

Figure 2.1: Comparison of energy spectra of a charged particle with and without magnetic field [39].

where, $\phi = HS$ is the magnetic flux penetrating the area $L^2 = S$. Thus, the number of states for a particular spin orientation, up or down, is simply given by the total flux ϕ divided by the flux quantum ϕ_o. In other words each electron state in a particular Landau level is connected to a magnetic flux equal to a flux quantum. In a classical picture, at a given magnetic field each of these states occupy an area

$$S_o = \frac{h}{2eH} \tag{2.2.3}$$

If we assume a circular area of $S_o = \pi r^2$ for each single electron, we directly obtain a radius corresponding to the magnetic length $r = \ell_m$.The filling factor, which is defined as the number of Landau levels occupied with electrons can be derived, which relates the number of electron in an area S, $g_{2D} = n_{2D}S$, to the number of flux quanta $g_\phi = \frac{\phi}{\phi_o}$ penetrating the same area. The relevance of flux quantization to the Landau levels lies in the fact that the degeneracy (2.1.1) is just the total magnetic flux measured in units of the flux quantum:

$$g = \frac{\phi}{\phi_o} \tag{2.2.4}$$

where Φ is the maximum magnetic flux, $\phi_0 = hc/e$ is the quanta of elementary magnetic flux. Filling ratio, ν is the number of filled Landau Levels

$$\nu = n_s/(eB/hc) \tag{2.2.5}$$

Chapter 3

CLASSICAL AND QUANTUM HALL EFFECT

3.1 Classical Hall Effect

The Hall effect has been important for many reasons. For example, in semiconductors it can be used for determining the sign and the concentration of charge carriers. The fractional quantum Hall effect, in terms of basic physics ideas, may be the most important discovery in solid-state physics in the last quarter of a century. To start, we first reconsider the classical quantum Hall effect for electrons only.

Let electrons move in the (x,y)-plane with a magnetic field in the z direction and an electric field also in the (x,y)-plane. In MKS units and standard notation ($e > 0$)

$$F_x = -eE_x - ev_yB - m\frac{v_x}{\tau} \tag{3.1.1}$$

$$F_y = -eE_y + ev_xB - m\frac{v_y}{\tau} \tag{3.1.2}$$

where the term involving the relaxation time τ is due to scattering. The current density is given by

$$j_x = -nev_x \tag{3.1.3}$$

$$j_y = -nev_y \tag{3.1.4}$$

where n-is the number of electrons per unit volume. Letting the dc conductivity be

$$\sigma_o = \frac{ne^2}{m}\tau \tag{3.1.5}$$

we can write (in the steady state when $F_x, F_y = 0$)

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_o} \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix} \tag{3.1.6}$$

where $\omega_c = \frac{eB}{m}$ is the cyclotron frequency and we can show (by (3.1.5))

$$\frac{B}{ne} = \frac{\omega_c\tau}{\sigma_o} \tag{3.1.7}$$

The inverse to (3.1.6) can be written

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{\sigma_o}{1+(\omega_c\tau)^2} \begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \tag{3.1.8}$$

We will use the geometry as shown in Fig. 3.1. We rederive the Hall coefficient. Setting $j_y = 0$, then

$$E_y = |v \times B| = -\frac{\omega_c\tau}{\sigma_o} j_x = -\frac{B}{ne} j_x \tag{3.1.9}$$

where $v = v_x = \frac{j_x}{ne}$ from (3.1.3). The Hall coefficient is defined as

$$R_H = \frac{E_y}{j_x B_z} = -\frac{1}{ne} \tag{3.1.10}$$

as usual. The Hall voltage over the length w would then be

$$V_H = -E_y w = B\frac{j_x w}{ne} \tag{3.1.11}$$

The current through the segment of area tw is

$$I_x = j_x t w \tag{3.1.12}$$

So

$$V_H = \frac{BI_x}{nte} \tag{3.1.13}$$

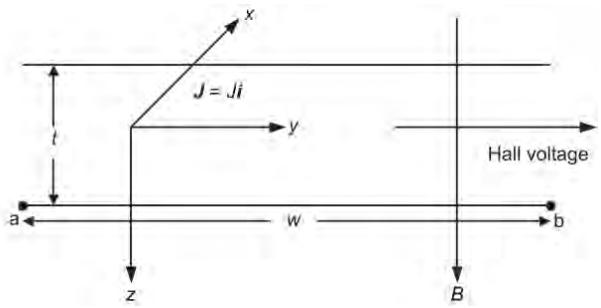

Figure 3.1: Schematic diagram of classical Hall effect [44].

Define n_a as the number of electrons per unit area (projected into the (x,y)-plane) so the Hall voltage can be written

$$V_H = \frac{I_x B}{n_a e} \tag{3.1.14}$$

The Hall conductance $\frac{1}{R_{xy}}$ is

$$\frac{1}{R_{xy}} = \frac{I_x}{V_H} = \frac{n_a e}{B} \tag{3.1.15}$$

Longitudinally over a length L, the voltage change is

$$V_L = E_x L = \frac{j_x L}{\sigma_o} = \frac{I_x L}{tw\sigma_o} \tag{3.1.16}$$

which we find to be independent of B. This is the usual Drude result. However, this result is based on the assumption that all electrons are moving with the same velocity. If we allow the electrons to have a distribution of velocities by doing a proper Boltzmann equation calculation, we find there is a magnetoresistance effect [44].

$$\sigma = \frac{\sigma_o}{1 + (\sigma_o R_H)^2 \frac{|j \times B|^2}{|j|^2}} \tag{3.1.17}$$

In addition, when band-structure effects are taken into account one finds there also may be a magnetoresistance even when $j \times B = 0$. Classically then we predict behavior for the Hall effect (with I_x =constant) as shown schematically in Fig. 3.2.

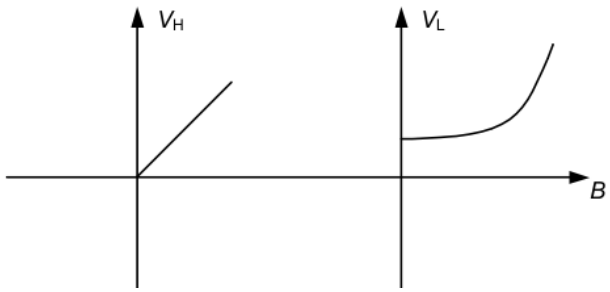

Figure 3.2: Schematic diagram of classical Hall effect behavior. See (3.1.14) for V_H and (3.1.16) for V_L [44].

3.2 Quantum Hall Effect

If an electric current flows along the x direction and a magnetic field is applied in the z direction, then an electric field is produced in the y direction. This phenomenon is called the Hall effect. More than one century ago, Hall showed that when the electron gas in a metal is placed in a magnetic field H, an electrical (transverse) resistivity develops for the electron motion in the direction perpendicular to the applied voltage (or electric field, see Fig.3.3). The essence of this effect is due to the fact that electrons while moving along the applied electric field start making circular orbits perpendicular to the magnetic field (the reason is the existence of the Lorentz force that is directed perpendicular to both the magnetic field and electron velocity). This produces a displacement of electrons perpendicular to the direction of the applied electric field. This transverse or Hall resistance displays a linear behavior in the values of H with a proportionality constant given by the inverse of the electron charge times the density n and the velocity of light c: $R_H = \frac{H}{nec}$. The Hall effect is nowadays used to measure the electron density in metals and semiconductors with great accuracy. Also the Hall effect refers to the measurable voltage that appears across a conductive material, for example silicon (Si), when an electric current flowing through the conductor is influenced by a magnetic field (Fig. 3.3). Under these conditions a transverse voltage is generated perpendicular to the applied current due to the balancing of the Lorentz and Electromagnetic forces [42]. Crossed magnetic and electric

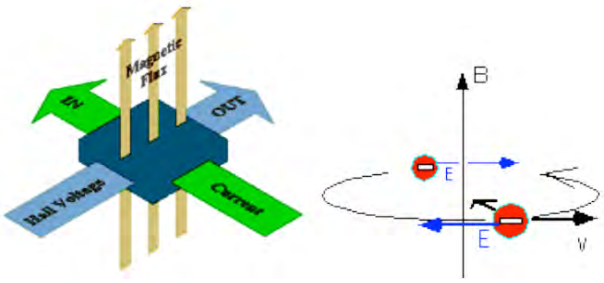

Figure 3.3: The classical Hall effect.Left:when the electrons moved in one direction under the action of an applied electric field, an electric field in the perpendicular direction (the Hall voltage) develops in the presence of a magnetic field. This effect is due to the circular motion of the electrons in the plane perpendicular to the magnetic field induced by the Lorentz force (right panel) [43].

fields, denoted by H and E respectively, as velocity filters to free charges letting through only whose those velocity v is such that

$$\begin{aligned} F &= E + \frac{v}{c} \times H = 0 \\ E &+ \frac{v}{c} H = 0 \\ \frac{v}{c} &= \frac{E}{H} \end{aligned} \tag{3.2.1}$$

For free charge carriers in a metal, the current density is:

$$j = qnv \tag{3.2.2}$$

where q-is the charge, and n-is the density. The hall resistivity ρ_{xy} is defined as the ratio of the electric field(in the y-direction) to the Hall current density(in the x-direction):

$$\begin{aligned} j_x &= \frac{E_y}{\rho_{xy}} \Rightarrow qnv_x = \frac{E_y}{\rho_{xy}} \\ qn &\frac{E}{H} c = \frac{E}{\rho_{xy}} \\ \rho_{xy} &= \frac{H}{qnc} \end{aligned} \tag{3.2.3}$$

Measurements of the Hall resistivity in various metals has yielded charge carrier densities and provided the first demonstrations that there are not only negative carriers(electrons), but also positive ones(holes). In the transistor technology the two-dimensional electron

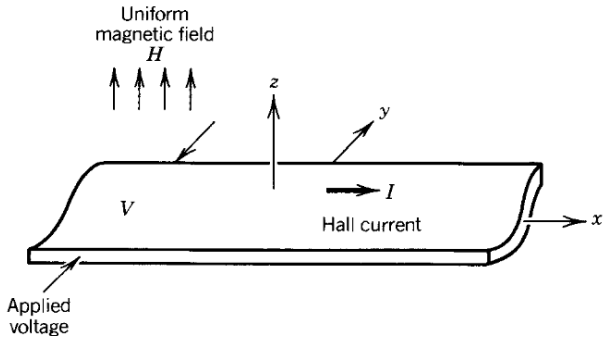

Figure 3.4: The Hall effect.A current I flows in a direction orthogonal to crossed electric and magnetic fields. The Hall resistivity is defined as $\rho_{xy} = \frac{V}{I}$. The conventional resistivity ρ_{xx} can be obtained by measuring the voltage drop along the direction of the current [39].

system used as a model. It can be made by injecting electrons into the interface of an alloy sandwich, which confines the electrons in a thin film about $500\mathring{A}$ thick. The Hall experiment has been performed on such two-dimensional electron systems at every low temperatures, and the direct resistivities ρ_{xx} and the Hall resistivities ρ_{xy} have been measured, as indicated in Fig. 3.4. The expermental results are quite dramatic, as shown in Fig. 3.5. As the magnetic field H increases the degeneracy of the Landau levels increases. Since the electron density does not depend on the field, the filling fraction ν of the lowest Landau level decreases:

$$\nu = \frac{hcn}{eH} \tag{3.2.4}$$

The Hall resistivity exhibits plateaus at $\nu = 1, \frac{2}{3}, \frac{1}{3}$, with values to $\frac{1}{\nu}$, in units of $\frac{h}{e^2}$. At the same time, the conventional resistivity ρ_{xx} drops to very low values. This indicates that in the neighborhood of these special filling fractions the two-dimensional electron fluid flows with almost no resistance. The value at $\nu = 1$, called the Integer Quantized Hall effect. It was first observed in MOSFIT(metal-oxide semiconductor Filed-effect transistor) at T=1.5k. The Hall resistivity was found to be quantized with a precision of one part in 10^5. The fractional values were found soon after [39]. The quantum Hall effect (or integer quantum Hall effect) is a quantum-mechanical version of the Hall effect, observed in two-dimensional electron systems subjected to low temperatures and strong magnetic fields,

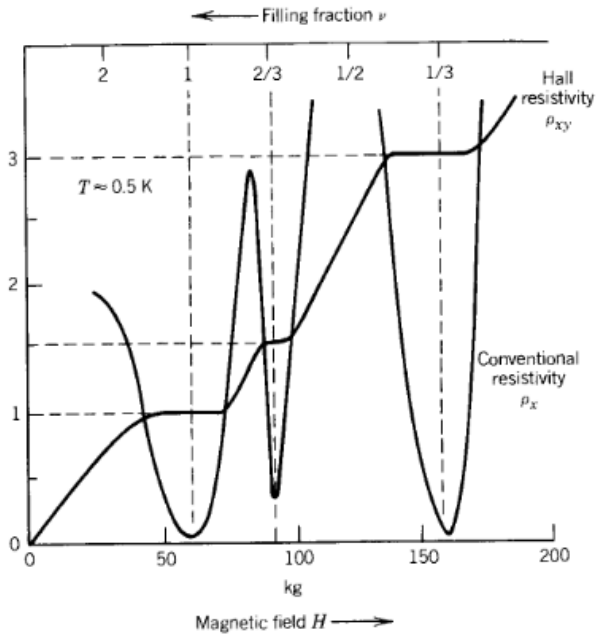

Figure 3.5: Quantized Hall effect: Schematic representation of expermental data. The filling fraction ν is the fraction of degenerate states in the lowest Landau levels occupied by electrons. The Hall resistivity exhibits plateau of value $\frac{1}{\nu}$, at $\nu = 1, \frac{2}{3}, \frac{1}{3} (inunitsof \frac{h}{e^2}.)$ The conventional resistivity becomes very small at these values. The quantization is accurate to at least one part in 10^4 [39]

in which the Hall conductivity σ takes on the quantized values . The Hall conductance, which is a measure of the conductance across the sample is :

$$\sigma_{xy} = \frac{j_x}{E_y} = \frac{n_{e\ell} e v_x}{v_x B} = n_{e\ell} \frac{e}{B} \tag{3.2.5}$$

Where $n_{e\ell}$ is the number density. Therefore, classically, we would expect the Hall conductance to vary linearly with $n_{e\ell} \frac{e}{B}$. where e is the elementary charge. The prefactor ν is known as the "filling factor", and can take on either integer ($\nu = 1, 2, 3, ...$) or rational fraction ($\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{2}{3}, \frac{3}{5}, \frac{1}{5}, \frac{2}{9}, \frac{3}{13}, \frac{5}{2}, \frac{12}{5}, ...$) values. The quantum Hall effect is referred to as the integer or fractional quantum Hall effect depending on whether is an integer or fraction respectively. The integer quantum Hall effect is very well understood, and can be simply explained in terms of single particle orbitals of an electron in a magnetic field. The fractional quantum Hall effect, is more complicated, as its existence relies fundamentally on electron-electron interactions. Interestingly, it is also very well understood as

an integer quantum Hall effect, not of electrons but of charge-flux composites known as composite fermions [43].

3.2.1 Integer Quantum Hall Effect (IQHE)

The IQHE was discovered by Klaus von Klitzing in 1980 and for this he was awarded the Nobel prize in 1985. About two years later, Stormer and Tsui discovered the FQHE and they along with Laughlin (for theory) were awarded the 1998 Nobel prize for this effect. Qualitatively, the IQHE can be fairly simply explained. As each Landau level is filled there is a gap to the next Landau level. The gap is filled by localized nonconducting states, and as the Fermi level moves through this gap, no change in current is observed. The Landau levels themselves are conducting. For the IQHE the electron-electron interactions effects are really not important, but the disorder that causes the localized states in the gap is crucial. The fractional quantum Hall effect occurs for partially filled Landau levels and electron-electron interaction effects are crucial. Potential fluctuations cause localized states and plateau formation. We give an elementary picture of the IQHE. We start with four results.

- The Landau degeneracy per spin is $\frac{eB}{h}$. (This follows because the number of states per unit area in k-space ($\triangle\ A$) and in real space is $\frac{(\triangle A)}{(2\pi)^2}$. Then, $(\triangle\ A) = (2\pi)^2(\frac{eB}{h})$. Thus, the number of states per unit area in real space is $n_B \equiv \frac{eB}{h}$).

- The drift velocity perpendicular to E and H field is $v = \frac{E}{H}c$.

- Flux quanta have the value $\phi_o = \frac{h}{e}$.

- The number of filled Landau levels $\nu = \frac{N}{N_\phi}$, where N is the number of electrons and N_ϕ is the number of flux quanta. This follows from $\nu = \frac{N}{eBLw/h} = \frac{N}{(\phi/\phi_o)}$.

Then

$$I_x = j_x wt = nevwt \tag{3.2.6}$$

where n- the number of electrons per unit volume

e-charge of an electron

v-velocity of an electron.

t-thikness of the sample.

w- width of the sample.

$$n = \frac{N}{wtL} = \nu\frac{eB}{h}(Lw)\frac{1}{wtL} = \nu\frac{eB}{ht} \tag{3.2.7}$$

So since $v = \frac{E}{B}c$,

$$I_x = \nu\frac{eB}{ht}e\frac{E}{B}wt = \nu e^2\frac{Ew}{h} = \nu e^2\frac{V_H}{h} \tag{3.2.8}$$

or

$$\frac{I}{V_H} = \frac{1}{R_{xy}} = theHallconductance = \nu\frac{e^2}{h} \tag{3.2.9}$$

If B changes, as long as the Fermi level stays in the gap, the Landau levels are filled or empty and the current over the voltage remains on a plateau of fixed n. (It can be shown that the total current carried by a full Landau level remains constant even as the number of electrons that fill it varies with the Landau degeneracy.) Incidentally, when $\frac{1}{R_{xy}} = \nu\frac{e^2}{h}$ then $\frac{1}{R_{xx}} = \frac{I}{V_L} \to \infty$ or $R_{xx} \to 0$. This is because the electrons in conducting states have no available energy states into which they can scatter[44].

Experiments have produced many surprises since the discovery of the Integer Quantized Hall Effect, in 1980. Understanding has required concepts and mathematical techniques from all corners of theoretical physics, including some completely new ideas. The goal of this talk is to review a few aspects of the quantized Hall effects, and show how our understanding of these phenomena stems from the work of L. D. Landau [46]. In two dimensions, when classical electrons are subjected to a magnetic field they follow circular cyclotron orbits. When the system is treated quantum mechanically, these orbits are

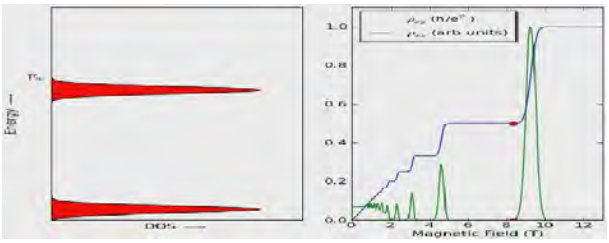

Figure 3.6: Integer Quantum Hall effect [46].

quantized. The energy levels of these quantized orbitals take on discrete values:

$$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right), \qquad n \geq 0 \tag{3.2.10}$$

Where $\omega_c = \frac{eB}{m^*}-$ is the cyclotron frequency. These orbitals are known as Landau levels, and at weak magnetic fields, their existence gives rise to many interesting "quantum oscillations" such as the Shubnikov-de Haas oscillations and the de Haas-van Alphen effect (which is often used to map the Fermi surface of metals) (Fig. 3.6). For strong magnetic fields, each Landau level is highly degenerate (i.e. there are many single particle states which have the same energy E_n). Specifically, for a sample of area A, in magnetic field B, the degeneracy of each Landau level is $N = g_s\frac{BA}{\phi_o}$ (where g_s represents a factor of 2 for spin degeneracy, and ϕ_o is the magnetic flux quantum). For sufficiently strong B-fields, each Landau level may have so many states that all of the free electrons in the system sit in only a few Landau levels; it is in this regime where one observes the quantum Hall effect [46].

3.2.2 Fractional Quantum Hall Effect (FQHE)

One needs to think about the FQHE both by thinking about the Laughlin wave functions and by thinking of their physical interpretation. For example, for the $\nu = \frac{1}{3}$ case with m=3, the wave function is

$$\psi(z_1, ..., z_N) = \Pi_{j<k}^{N}(z_j - z_k)^m exp\left(-\frac{1}{4\ell_\mu^2}\sum_{j=1}^{N}|z_j|^2\right) \tag{3.2.11}$$

where $z_j = x_j + iy_j$, x_j and y_j cordinates of electron in 2D. Positive and negative excitations at $z = z_o$, where $z_o = x_o + iy_o$and it can be given by:

$$\psi^+ = exp\left(-\frac{1}{4\ell_\mu^2}\sum_{j=1}^{N}|z_j|^2\right)\Pi_j^N(z_j - z_o)\Pi_{j<k}^N(z_j - z_k)^m \tag{3.2.12}$$

$$\psi^+ = exp\left(-\frac{1}{4\ell_\mu^2}\sum_{j=1}^{N}|z_j|^2\right)\Pi_j^N\left(2\ell_\mu^2\frac{\partial}{\partial z_j} - z_o^*\right)\Pi_{j<k}^N(z_j - z_k)^m \tag{3.2.13}$$

For m=3, these excitations have effective charges of magnitude $\frac{e}{3}$. The ground state of the FQHE is considered to be like an incompressible fluid as the density is determined by the magnetic field and is fairly rigidly locked. The papers by Laughlin should be consulted for full details. These wave functions have led to the idea of composite particles (CPs). Rather than considering electrons in 2D in a large magnetic field, it turns out to be possible to consider an equivalent system of electrons plus attached field vortices. The attached vortices account for most of the magnetic field and the new particles can be viewed as weakly interacting because the vortices minimize the electronelectron interactions.

That the composite particles may behave as either bosons or fermions according to the number of attached flux quanta. Electrons plus an odd number of surrounding flux quanta are Bose CPs and electrons with an even number of attached quanta are Fermi CPs. The $\nu = \frac{1}{3}, m = 3$ case involves electrons with three attached quanta and hence these CPs are bosons that can undergo a Bose Einstein-like condensation, produce an energy gap, and have a FQHE with plateaus. For the $\nu = \frac{1}{2}$ case, there are two attached quanta, the systems behaves as a collection of fermions, there is no BoseEinstein condensation and no FQHE. In general, when the magnetic field increases, electrons can "absorb" some field and become "anyons." These can be shown to obey fractional statistics and seem to be intermediate between fermions and bosons [44].

In samples of very high quality, in very strong magnetic fields, one finds additional plateaus, where

$$R_H^{-1} = \nu\frac{e^2}{h} \tag{3.2.14}$$

But ν is a simple rational fraction, usually with odd denominator.

Originally $\nu = \frac{1}{3}, \frac{2}{3}$. More recently include

$\nu = \frac{4}{3}, \frac{5}{3}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{3}{7}, \frac{4}{7}, \frac{4}{9}, \frac{5}{9}$, others [46].

Chapter 4

TWO CHARGED PARTICLES IN A MAGNETIC FIELD

The energy spectrum of a pure two-dimensional electron gas in a strong perpendicular magnetic field is completely determined by the Coulomb interactions, making it the paradigm for all "strongly interacting" systems, (for which standard many body perturbation theory is inapplicable). A two particle system is the simplest system that can be used to understand the physics of introducing the Chern-Simons(CS) flux. The two particles have masses m_1 and m_2 and charges q_1 and q_2 and they are moving in the x-y plane. A dc uniform magnetic field $\vec{B} = B\hat{z}$ is applied perpendicular to the plane. We work in the symmetric gauge where the vector potential is $\vec{A} = \frac{1}{2}Br\hat{\phi}$, where $\hat{\phi}$ is the unit vector in the direction of increasing the angular coordinate ϕ. The Hamiltonian contains the kinetic terms and the Coulomb term [47]. Solving schrodinger equation for two electrons in a uniform magnetic field. This studies appeared important because they led to discovery of a new state, the incompressible electron liquid. The principal aim of this work is by using a variational approach to obtain the ground state energy of a two-electron system. The Hamiltonian of two electrons in an interaction potential is given by:

$$\hat{H} = \frac{1}{2m_1}\left[\vec{p}_1 - \frac{q_1}{c}\vec{A}(\vec{r}_1)\right]^2 + \frac{1}{2m_2}\left[\vec{p}_2 - \frac{q_2}{c}\vec{A}(\vec{r}_2)\right]^2 + \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|} \tag{4.0.1}$$

Now, we want to reduce the two-body problem into a single-body one. Further we consider the case, $\quad m_1 = m_2 = m_e, \quad q_1 = q_2 = e$

Where m_e and e are the mass and charge of electron.

$\vec{r} = \vec{r}_1 - \vec{r}_2$- relative coordinate

$\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$- center of mass coordinate

$\mu = \frac{m_e}{2}$- reduced mass

$q = \frac{e}{2}$- reduced charge

And the vector potentials are

$$\vec{A}_1 = \frac{1}{2}\vec{B} \times \left(\vec{R} + \frac{\vec{r}}{2}\right) = \vec{A}_R + \frac{1}{2}\vec{A}_r \tag{4.0.2}$$

$$\vec{A}_2 = \frac{1}{2}\vec{B} \times \left(\vec{R} - \frac{\vec{r}}{2}\right) = \vec{A}_R - \frac{1}{2}\vec{A}_r \tag{4.0.3}$$

where $\vec{A}_R = \frac{1}{2}(\vec{A}_1 + \vec{A}_2) = \vec{B} \times \vec{R}$ and $\vec{A}_r = \vec{A}_1 - \vec{A}_2 = \frac{1}{2}\vec{B} \times \vec{r}$.

After appropriate substitution:

$$\hat{H} = \frac{1}{4m_e}P^2 + \frac{1}{2(m_e/2)}p^2 + \frac{e^2B^2}{4m_ec^2}\left[R^2 + \frac{r^2}{4}\right] - \frac{eB}{m_ec}[\vec{R} \times \vec{P} + \vec{r} \times \vec{p}] + \frac{e^2}{r} \tag{4.0.4}$$

where $\vec{P} = \vec{p}_1 + \vec{p}_2$ and $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$- are the center of mass and relative momentum of the electrons respectively. Schrödinger equation for the two electrons in the new coordinates is given by:

$$\hat{\mathbf{H}}\Psi(R, r) = \mathbf{E}_n\Psi(R, r) \tag{4.0.5}$$

For the case of interest here where the mass and charge of the two particles are equal, the center-of-mass and relative motion separate. The wave function depends on the on the center of mass position and relative position of the two electrons.

Our Hamiltonian takes the form:

$$\hat{H} = \hat{H}_{RCM} + \hat{H}_r \tag{4.0.6}$$

To decouple the Schrodinger equation (4.0.5), we use the variable separation method, the Hamiltonian can be separated into motion of center of mass (R_{CM}) and relative (r)

motion.

$$\hat{H}_{R_{CM}} = \frac{1}{2M}P^2 + \frac{M\omega_c^2}{8}R^2 - \frac{\omega_c}{2}\hat{L}_z \tag{4.0.7}$$

$$\hat{H}_r = \frac{1}{2\mu}p^2 + \frac{\mu\omega_c^2}{8}r^2 - \frac{\omega_c}{2}\hat{\ell}_z + \frac{e^2}{r} \tag{4.0.8}$$

Where $\hat{L}_z = \vec{R} \times \vec{P}$ and $\hat{\ell}_z = \vec{r} \times \vec{p}$ are the operators of angular momentum of center of mass and relative position respectively.

To decouple the Schrodinger equation, we use the variable technique separation method. To do this, we assume that the wave function can be separated, i.e.

$$\Psi(R, r) = \psi(R)\psi(r, \varphi) \tag{4.0.9}$$

Using equation (4.0.7) the Schrödinger equation of center of mass (R_{CM}) is:

$$\begin{aligned} &\hat{H}_{R_{CM}}\psi(R) = E_R\psi(R) \\ &\left\{\frac{1}{2M}P^2 + \frac{M\omega_c^2}{8}R^2 - \frac{\omega_c}{2}\hat{L}_z\right\}\psi_{n_R}(R) = E_{n_R}\psi_{n_R}(R) \end{aligned} \tag{4.0.10}$$

One particle Hamiltonian or the Landau Hamiltonian is given by:

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{m\omega_c^2}{8}r^2 - \frac{\omega_c}{2}\hat{\ell}_z \tag{4.0.11}$$

Its energy levels and wave functions are known. So equation (4.0.10) is one particle system, energy levels are then given by:

$$E_{n_R M} = \hbar\omega_c\left[n_R + \frac{1}{2}\left(|M| + M\right) + \frac{1}{2}\right] \tag{4.0.12}$$

Where $M = 0, \pm1, \pm2, \pm3, \pm4, ...$ and $n_R = 0, 1, 2, 3, ...$ One can see that these levels degenerated for negative M. Finally, we have:

$$E_{n_R} = \hbar\omega_c\left(n_R + \frac{1}{2}\right) \tag{4.0.13}$$

Where $n_R = 0, 1, 2, 3, ...$

The ground state energy $n_R = 0$ is:

$$E_o = \frac{1}{2}\hbar\omega_c \tag{4.0.14}$$

In view of the importance of the physics of the two-dimensional (2D) electron gas, the purpose of the present paper is to compute the effects of the coulomb interaction between electrons. We shall, moreover, go beyond the zero-field limit considered by Aslamazov and Larkin and compute the magnetization for arbitrary classically weak magnetic fields. We find that the interaction-induced magnetization generally dominates over the Landau and Pauli contributions at sufficiently low temperatures.

The relevant length scales of the problem are:

$$
\begin{aligned}
r &= \sqrt{\frac{\hbar}{\mu\omega_c}}\rho \\
\frac{e^2}{r} &\to \frac{\lambda}{\rho} \\
\lambda &= \frac{e^2}{\hbar\omega_c\sqrt{\hbar/\mu\omega_c}} = \frac{e^2}{\sqrt{\hbar^3\omega_c/\mu}} = \sqrt{\frac{e^4\mu}{\hbar^3\omega_c}} \\
\lambda &= \sqrt{\frac{\mu^2e^4}{\hbar^4}\frac{\hbar}{\mu\omega_c}} \\
\lambda &= \sqrt{\frac{a_H^2}{a_B^2}} = \frac{a_H}{a_B}
\end{aligned}
\tag{4.0.15}
$$

where $a_H = \sqrt{\frac{\hbar}{\mu\omega_c}} = \sqrt{\frac{\hbar c}{eB}}$-magnetic length for the relative motion.

$a_B = \sqrt{\frac{\hbar^4}{\mu^2e^4}}$- is Bohr radius.

$\omega_c = \frac{eB}{\mu c}$-Cyclotron frequency. and

λ- which characterizes the intensity of magnetic force and coulomb force.

The problem can be divided in three cases.

Case I: At high magnetic field regimes means $a_H << a_B$ *and* $\lambda << 1$ "**Quantum Hall Effect**". In this case the Coulomb interaction is small and can be taken into account by the perturbation theory [47].

Case II: At Low magnetic field regime means $a_H >> a_B$ *and* $\lambda >> 1$.

In this case the Coulomb interacion dominates and this is the usual 2DE gas.

Case III In this work we consider $a_H \sim a_B$ *and* $\lambda \sim 1$, when the perturbation theory

cannot be used.

Consider the relative motion, Schrödinger equation of the relative motion part (r) is:

$$\hat{H}_r\psi_{n_r}(r,\varphi) = \varepsilon\psi_{n_r}(r,\varphi) \tag{4.0.16}$$

where $\psi_{n_r}(r,\varphi)$ defin as:

$$\psi_{n_r}(r,\varphi) = e^{im\varphi}\phi(r) \tag{4.0.17}$$

Substituting equation (4.0.8) and (4.0.17) in to (4.0.16) we get:

$$\begin{aligned}
&\left\{\frac{1}{2\mu}p^2 + \frac{\mu\omega_c^2}{8}r^2 - \frac{\omega_C}{2}\hat{\ell}_z + \frac{e^2}{r}\right\}e^{im\varphi}\phi(r) = \varepsilon e^{im\varphi}\phi(r)\\
&\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{\omega_c\hbar}{2i}\frac{\partial}{\partial\varphi} + \frac{\mu\omega_c^2}{8}r^2 + \frac{e^2}{r}\right]e^{im\varphi}\phi(r) = \varepsilon e^{im\varphi}\phi(r)
\end{aligned} \tag{4.0.18}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$, thus our Hamiltonian become:

$$\begin{aligned}
&\left\{-\frac{\hbar^2}{2\mu}\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right\} + \frac{\mu\omega_c^2}{8}r^2 - \frac{\omega_c}{2i}\hbar\frac{\partial}{\partial\varphi} + \frac{e^2}{r}\right\}e^{im\varphi}\phi(r) = \varepsilon e^{im\varphi}\phi(r)\\
&\left\{-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{m^2}{r^2}\right) - \frac{\omega_c}{2}\hbar m + \frac{\mu\omega_c^2}{8}r^2 + \frac{e^2}{r}\right\}e^{im\varphi}\phi(r) = \varepsilon e^{im\varphi}\phi(r)
\end{aligned} \tag{4.0.19}$$

Further, we will be convenient to use dimensionless variable:

$$\rho = \sqrt{\frac{\mu\omega_c}{\hbar}}\,r \tag{4.0.20}$$

One express the Hamiltonian corresponding to the Schrödinger equation (4.0.20):

$$\hat{H}_r = -\frac{1}{2}\left\{\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{m^2}{\rho^2}\right\} + m + \frac{\rho^2}{8} + \frac{\lambda}{\rho} \tag{4.0.21}$$

Where $\lambda = \frac{a_H}{a_B}$.

The Lowest energy corresponds to m=0.

$$\hat{H}_r = -\frac{1}{2}\left\{\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\right\} + \frac{\rho^2}{8} + \frac{\lambda}{\rho} \tag{4.0.22}$$

For the case of $\lambda = 1$ and at m=0 the potential energy $U(\rho) = \frac{\rho^2}{8} + \frac{\lambda}{\rho}$ are shown in Fig. (4.1).

But in this case the problem can be solved numerically or with the help of variational

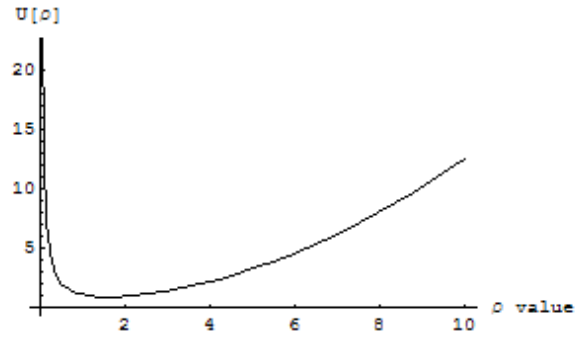

Figure 4.1: The potential energy, $U(\rho) = \frac{\rho^2}{8} + \frac{\lambda}{\rho}$, of the relative motion at m=0 and $\lambda = 1$.

method. To find the ground state of the Hamiltonian (4.0.22) we use the variational method. Let us take the normalized trial wave function in the form:

$$\phi_o(\rho) = \sqrt{\alpha}\, exp\Big(- \frac{\alpha\rho^2}{4} \Big) \tag{4.0.23}$$

where α- is a variational parameter. Usually, it is convenient to introduce parameters in the trial wave function in order to perform more efficiently the minimization of the energy. The Schrödinger equation with Hamiltonian (4.0.22) and wave function (4.0.23) is:

$$E_o(\alpha) = \int_0^\infty \phi(\rho) \Big[- \frac{1}{2} \Big\{ \frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} \Big\} + \frac{\rho^2}{8} + \frac{\lambda}{\rho} \Big] \phi(\rho)\rho d\rho$$

$$E_o(\alpha) = \alpha \int_0^\infty e^{-\alpha\rho^2/4} \Big[- \frac{1}{2} \Big\{ \frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} \Big\} + \frac{\rho^2}{8} + \frac{\lambda}{\rho} \Big] e^{-\alpha\rho^2/4} \rho d\rho$$

$$E_o(\alpha) = \alpha \int_0^\infty e^{-\alpha\rho^2/4} \Big[\Big[- \frac{1}{2} \Big\{ \frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} \Big\} + \frac{\rho^2}{8} + \frac{\lambda}{\rho} \Big] e^{-\alpha\rho^2/4} \Big] \rho d\rho$$

$$E_o(\alpha) = \alpha \int_0^\infty e^{-\alpha\rho^2/4} \Big[- \frac{1}{2} \Big(\frac{\alpha^2\rho^2}{4} - \frac{\alpha}{2} - \frac{\alpha}{2} \Big) + \frac{\rho^2}{8} + \frac{\lambda}{\rho} \Big] e^{-\alpha\rho^2/4} \rho d\rho$$

this integral gives

$$E_o(\alpha) = \alpha \Big[\frac{1}{4} + \frac{1}{4\alpha^2} + \lambda\sqrt{\frac{\pi}{2\alpha}} \Big] \tag{4.0.24}$$

This is the ground state energy of the system.

Where $E_o(\alpha) = \frac{\varepsilon(\alpha)}{\hbar\omega_c}$, that equation (4.0.24) gives:

$$\varepsilon(\alpha) = \hbar\omega_c \Big[\frac{1}{4} \Big(\alpha + \frac{1}{\alpha} \Big) + \lambda\sqrt{\frac{\pi\alpha}{2}} \Big] \tag{4.0.25}$$

To find the varetional parameters α , we minimize the function $E_o(\alpha)$,

this are $\frac{dE_o(\alpha)}{d\alpha}\Big|_{\alpha_{min}} = 0$, we obtain:

$$\begin{aligned}
&\frac{dE_o(\alpha)}{d\alpha}\Big|_{\alpha_{min}} = 0 \\
&\frac{1}{4}\left[1 - \frac{1}{\alpha^2}\right] + \lambda\sqrt{\frac{\pi}{2}}\frac{1}{2}\frac{1}{\sqrt{\alpha}} = 0 \\
&\alpha^2 - 1 + \frac{4\lambda}{2}\sqrt{\frac{\pi}{2}}\frac{\alpha^2}{\sqrt{\alpha}} = 0 \\
&\alpha^2 + \lambda\sqrt{2\pi}\alpha^{3/2} - 1 = 0
\end{aligned} \tag{4.0.26}$$

This equation is solved by using of Mathematica method, this gives us, for different values of λ, we get different value of α and $E_o(\lambda)$, so this shown as in the table below. From

Table 4.1: The relation of different vales of λ and $E_o(\lambda)$ value.

λ	α	$E_o(\lambda)$
0	1	$\frac{1}{2}$
0.25	0.763075	0.792096
0.5	0.621376	1.05165
0.75	0.52798	1.28851
1	0.461832	1.50851
1.25	0.412431	1.71538
1.5	0.374034	1.91166
1.75	0.343256	2.09915
2	0.317974	2.27919
2.25	0.296793	2.45281
2.5	0.278757	2.62082
2.75	0.263189	2.78387
3	0.249595	2.94247

this table we draw tha graph of $E_o(\lambda)$ versus λ are as shown below:

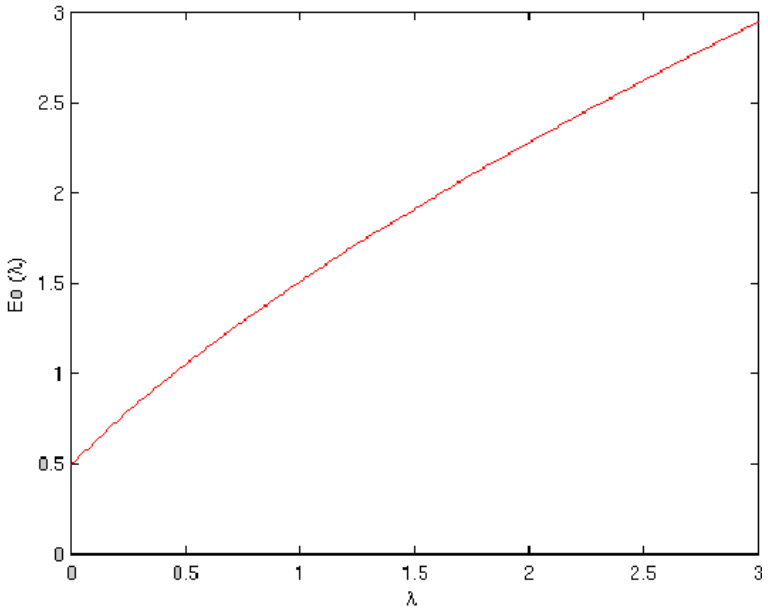

Figure 4.2: The graph of $E_o(\lambda)$ value at different given values of λ.

The corresponding results for the ground state energy E_o of the system in the presence of magnetic field are obtained and presented in figure 4.2.

To estimate the accuracy of variational method, we compare our results in the absence of Coulomb interaction with the exact solution.

4.1 Parabolic Confinement in Strong Magnetic Field

Confinement of the motion into two dimensions helps to strengthen many-body interactions as well. Since its first experimental observation on the surface of liquid helium two-dimensional electron gas (2DEG) has profited interest in wide variety of materials and types of quantum confinement. Especially its confinement into a semiconductor heterojunction in silicon inversion layer [2] has attracted attention at the beginning of its discovery and later on many investigations have been performed in such devices like Field Effect Transistors (FET) and Metal Oxide Semiconductor FET (MOSFET). Let's consider confinement of the electron motion in the z-direction described by the confinement potential v_{conf}. Electron motion is usually described here in the envelope function approximation (EFA), neglecting the part of the Bloch wave function periodic with the lattice constant of host material. Hamiltonian in EFA and effective mass approximation:

$$\hat{H} = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + \frac{m\omega^2}{2}x^2 \tag{4.1.1}$$

where $v_{conf} = \frac{m\omega^2}{2}x^2$- is external field which removes the Landau degeneracy

And the Landau gauge is given by:

$$\vec{A} = \begin{pmatrix} 0 \\ -Bx \\ 0 \end{pmatrix} \tag{4.1.2}$$

so the above Hamiltonian is written as follows:

$$\hat{H} = \frac{1}{2m}\hat{p}_x^2 + \frac{1}{2m}\left(\hat{p}_y + \frac{eB}{c}x\right)^2 + \frac{1}{2m}\hat{p}_z^2 + \frac{m\omega^2}{2}x^2 \tag{4.1.3}$$

$$\hat{H} = \frac{1}{2m}\left[\hat{p}_y^2 + \hat{p}_z^2 + \hat{p}_x^2 + \frac{2eBp_y}{c}x + \left(\frac{eB}{c}\right)^2 x^2\right] + \frac{m\omega^2}{2}x^2 \tag{4.1.4}$$

The Schrödinger equation is:

$$\begin{aligned} &\hat{H}\psi = E\psi \\ &\left\{\frac{1}{2m}\hat{p}_x^2 + \frac{1}{2m}\left(\hat{p}_y + \frac{eB}{c}x\right)^2 + \frac{1}{2m}\hat{p}_z^2 + \frac{m\omega^2}{2}x^2\right\}\psi = E\psi \end{aligned} \tag{4.1.5}$$

We solve the Schrödinger equation by assuming a wave function of the form:

$$\psi(x,y,z) \sim exp[i(k_y y + k_z z)]\varphi(x) \tag{4.1.6}$$

Where p_y and p_z are now constant numbers (these are eigenvalues). Substitution of (4.1.4) and (4.1.6)we get the schrödinger equation:

$$\hat{H}\psi = \left\{ \frac{1}{2m}\left[\hat{p}_y^2 + \hat{p}_z^2 + \hat{p}_x^2 + \frac{2eBp_y}{c}x + \left(\frac{eB}{c}\right)^2 x^2 \right] + \frac{1}{2}m\omega^2 x^2 \right\}\psi(x,y,z) = E\psi(x,y,z) \tag{4.1.7}$$

Note that in (4.1.7), p_y and p_z are constant numbers and only p_x and x are operators. Let us denote $\frac{1}{2m}(\hat{p}_z^2 + \hat{p}_y^2) = a$; then (4.1.7) can be written as:

$$\left[\frac{1}{2m}\hat{p}_x^2 + \left(\frac{eBp_y}{mc}\right)x + \frac{1}{2m}\left(\frac{eB}{c}\right)^2 x^2 + \frac{1}{2}m\omega^2 x^2\right]\psi(x,y,z) = (E-a)\psi(x,y,z)$$

$$\left[\frac{1}{2m}\hat{p}_x^2 + \left(\frac{eBp_y}{mc}\right)x + \frac{1}{2m}\left(\frac{eB}{c}\right)^2 x^2 + \frac{1}{2}m\omega^2 x^2\right]\psi(x,y,z) = E'\psi(x,y,z) \tag{4.1.8}$$

Where $E' = E - a$ and $p_y = \hbar k_y$, $\omega_c = \frac{eB}{mc}$- is cyclotrone frequency.

Substituting equation (4.1.6) in to the above equation we obtain:

$$\varphi'' + \frac{2m}{\hbar^2}\left[E' - \frac{m\omega_c^2}{2}\left(1+\left(\frac{\omega}{\omega_c}\right)^2\right)\left\{\left(x - \frac{x_o}{1+(\omega/\omega_c)^2}\right)^2 + \frac{x_o^2}{1+\left(\frac{\omega}{\omega_c}\right)^2} - \frac{x_o^2}{\left(1+\left(\frac{\omega}{\omega_c}\right)^2\right)^2}\right\}\right]\varphi = 0 \tag{4.1.9}$$

Where $x_o = \frac{\hbar k_y}{m\omega_c}$

we can rewrite equation (4.1.9) as follows:

$$\varphi'' + \frac{2m}{\hbar^2}\left[E' - \frac{m\Omega^2}{2}\left\{[x-\tilde{x}_o]^2 + \frac{x_o^2\omega^2}{\omega_c^2(1+(\omega/\omega_c)^2)^2}\right\}\right]\varphi = 0$$

$$\varphi'' + \frac{2m}{\hbar^2}\left[E' - \frac{m\Omega^2}{2}\left\{[x-\tilde{x}_o]^2 + \frac{x_o^2\omega^2}{\omega_c^2(\omega^2+\omega_c^2)^2\frac{1}{\omega_c^4}}\right\}\right]\varphi = 0$$

$$\varphi'' + \frac{2m}{\hbar^2}\left[E' - \frac{m\Omega^2}{2}\left\{[x-\tilde{x}_o]^2 + \frac{x_o^2\omega_c^2\omega^2}{\Omega^4}\right\}\right]\varphi = 0 \tag{4.1.10}$$

Here $\Omega^2 = \omega_c^2 + \omega^2$

$\tilde{x}_o = \frac{x_o}{1+(\omega/\omega_c)^2} = \frac{x_o\omega_c^2}{\Omega^2}$.

The solution of equation (4.1.10) reads:

$$E_{nk_zk_y} = \hbar\omega_c\sqrt{1+(\omega/\omega_c)^2}\left(n+\frac{1}{2}\right) - \frac{\hbar^2k_y^2}{2m^*} + \frac{\hbar^2k_z^2}{2m} \tag{4.1.11}$$

where $x_o = \frac{\hbar k_y}{m\omega_c}$ and

$m^* = m\frac{\Omega^2}{\omega^2} = m\frac{\omega_c^2}{\omega^2}\left(1 + \frac{\omega^2}{\omega_c^2}\right)$ is the new effective mass.

Case I: For weak confinement $\omega << \omega_c$

$$E_{nk_zk_y} = \hbar\omega_c\left(n + \frac{1}{2}\right) - \frac{\hbar^2 k_y^2}{2m\omega_c^2}\omega^2 + \frac{\hbar^2 k_z^2}{2m} \tag{4.1.12}$$

$m^* = m\frac{\omega_c^2}{\omega^2}$ is very large and the velocity along the y-axis is small, $v_y = \frac{\hbar k_y}{m^*}$. At $\omega \to 0,\ v_y \to 0$ and we have the Landau case.

Case II: For Strong Confinement $\omega \geq \omega_c$:

$$E_{nk_zk_y} = \hbar\omega_c\sqrt{2}(n + 1/2) - \frac{\hbar^2 k_y^2}{4m} + \frac{\hbar^2 k_z^2}{2m} \tag{4.1.13}$$

The term with k_y gives considerable contribution to the current:

$$j_y = e\frac{\hbar k_y}{2m}n \tag{4.1.14}$$

Where n-is the number of charge carriers.

Chapter 5

CONCLUSION

In this work we consider 2D electron gas in strong and weak magnetic field. When charged particles move in a non uniform magnetic field, the motion is complex.

- In strong magnetic field $a_H << a_B$ and $\lambda << 1$, it shows Quantum Hall Effect. This Hall effect arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic force they experience.

- Hall Effect gives us an information regarding the sign of the charge carriers and their density; it can also be used to measure the magnitude of magnetic fields.

- In the case of $\lambda << 1$, Coulomb interaction is small and to consider the contribution of coulomb on the system, we can taken into account by the perturbation theory.

- For $a_H \sim a_B$, particle get minimum potential energy from sketch (4.1), it shows Harmonic Oscillator. So particle is confined on some region of r.

- For higher values of the confinement energy the effect of the magnetic field is even smaller.

- Variational method has better accuracy when the system is far from the critical point.

- The type of Variational wave functions used involved an anharmonicity parameter.

- The presence of the magnetic field leads to the additional increase of the correlation energy of the system.

- Finally variational method was developed for the investigation of Coulomb correlation between two equally charged particles in systems with radial confinement in an external homogeneous magnetic field.

Bibliography

[1]. L. Jiutao, W.D. Schneider , R. Berndt and S. Crampin , Phys. Rev. Lett. **80**, 15 (1998).

[2]. A. B. Fowler, F. F. Fang, W. E. Howard, and P. J. Stiles, Phys. Rev. Lett **16**, 901 (1966).

[3]. R. Dingle, W. Wiegmann, and C. H. Henry, Phys. Rev. B **33**, 827 (1974).

[4]. E. Tolo ,"*Nanostructures in the Quantum Hall Regime*", PhD. Dissertation ,Aalto University of Science and Technology,Finland, Espoo (2010).

[5]. K. v. Klitzing, G. Dorda and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[6]. D. C. Tsui, H. Stormer and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[7]. S. Das Sarma and A. Pinczuk, eds., "*Perspectives in Quantum Hall Effects*",Wiley, New York (1997).

[8]. L. D. Landau et E. M. Lifschitz, "*Theoretical physicsts, T. V Stastical Physics*" , Editions Mir,Moscou (1973).

[9]. T. Heinzel, "*Mesoscopic Electronics in Solid State Nanostructures*". Wiley-VCH, New York (2006).

[10].K.v.Klitzing, Rev. Mod. Phys. **58**, 3, (1986).

[11].S. Glutsch, F. Bechstedt and D. N. Quang, "*Density of states of a two-dimensional electron gas in a perpendicular magnetic field and a random field of arbitrary correlation*", National Center for Science and Technology, Boho, Hanoi 10000, Vietnam (2003).

[12].S. A.J.Wiegers, M. Specht, L.P.Lévy, M.Y. Simmons, D. A. Ritchie, A. Cavanna, B.Etienne, G. Martinez, and P. Wyder, Phys. Rev. Lett . **79**, 3238 (1997).

[13].M.P. Scwarz, M.A. Wilde, S. Groth, D. Grundler, CH. Heyn, D. Heitmann, Phys. Rev. B **65**,245315 (2002).

[14]. M. Zhu, A. Usher, A.J. Matthews, A. Potts, M. Elliot, W.G. Herrenden-Harker, D.A. Ritchie and M.Y. Simmons, Phys. Rev. B **67**, 155329 (2003).

[15].Y.M.Galperin, "*Quantum Transport*", Lund University, Unpublished, (1998).

[16].R. Wagner, "*G-factor, Effective Mass and Spin Susceptibility of a 2-Dimensional Electron Gas*", Masters thesis, University of Basel, Basel (2009).

[17]V. V. Moshchalkov, V. Bruyndoncx, E. Rosseel, L. Van Look, M. Baert, M. J. Van Bael,T. Puig , C. Strunk and Y. Bruynseraede , " *Confinement and Quantization Effects in Mesoscopic Superconducting Structures*", Katholieke Universiteit Leuven, Belgium (1998).

[18].Y. Yu . Peter, "*Fundamentals of Semiconductors* ",3^{rd} Revised and Enlarged Edition , Springer Berlin Heidelberg New York , USA (2005).

[19]."*Band theory of solids*", http://hyperphysics.phy-astr.gsu.edu/hbase/solids/band.html (2008).

[20].F. J. Morin and J. P. Maita, Phys. Rev, B **96**, 28-35 (1954).

[21].G. E. Stillman, C. M. Wolfe, and J. O. Dimmock, Journal of Physics and Chemistry of Solids, **31**, 1199-1204 (1970).

[22].O. N. Tufte and P. W. Chapman, Phys. Rev., B **155**, 796-802 (1967).

[23].E. F. Schubert, "*Room temperature properties of semiconductors: IIIV arsenides*", http://www.rpi.edu/ S̃chubert/Educational-resources/Materials-Semiconductors-III-V arsenides.pdf (2009).

[24]. U. R. O. Madelung and M. Schulz, eds., "*Group IV Elements, IV-IV and III-V Compounds. Part b - Electronic, Transport, Optical and Other Properties*", vol. 41A1b of Landolt-Brnstein - Group III Condensed Matter, ch. Aluminum arsenide (AlAs),electrical and thermal conductivity, carrier concentration, pp. 12. Springer-Verlag, (2002).

[25].G.W.J. Hassink, "*Two-dimensional electron layers in perovskite oxides*", PhD thesis, University of Twente (2009).

[26].N. W. Ashcroft and N. D. Mermin, "*Solid state physics*". Philadelphia: Saunders College Publishing, (1976).

[27].C. Kittel, "*Introduction to solid state physics*", NJ: Wiley, 8^{th}ed., (2005).

[28].E. F. Schubert, "*Quantum mechanics applied to semiconductor devices - Ch 16 High doping effects*", http://www.ecse.rpi.edu/ schubert/Course-ECSE-6968 Quantum mechanics/Ch16 High doping effects. (2004).

[29].H. Nakamura, H. Tomita, H. Akimoto, R. Matsumura, I. H. Inoue, T. Hasegawa, K. Kono, Y. Tokura, and H. Takagi, Journal of the Physical Society of Japan, **78**, 083713 (2009).

[30].J. S. Reparaz , "*Optical Properties of Low-Dimensional Semiconductor Nanostructures under High Pressure* ",Doctoral Tesis ,Universidad Aut n oma de Barcelona ICMAB, (2008).

[31]."*A good introduction to the principles of operation of MBE can be found at*": http://projects.ece.ute

[32]. "*High electron mobility transistor*", http://en.wikipedia.org/wiki/HEMT (2008).

[33].P. Hadley, "*The nanoscale and quantum mechanics*", http://lamp.tu-graz.ac.at/ hadley/nanoscienc (2008).

[34].M. Dahan, S. Levi, C. Luccardini, P. Rostaing, B. Riveau, and A. Triller, Science Today, **302**, 442445, (2003).

[35].B. Ballou, C. Lagerholm, L. A. Ernst, M. P. Bruchez, and A. S. Waggoner, Bioconjugate Chemistry, **15**, 7986, (2004).

[36].J. Faist, F. Capasso, D. L. Sivco, C. Sirtori, A. L. Hutchinson, and A. Y. Cho, Science Today, **264**, 553-556, (1994).

[37]." *Quantum dot.*" http://en.wikipedia.org/wiki/Quantum dot (2008).

[38].G.Giuseppe and P.P.Giuseppe, "*Solid State Physics*",Antny Rowe Ltd, Eastbourne, New York (2005).

[39].K. Huang, "*Stastical Mechanics*", John willey and Sons, 2^{nd}ed, Canada, (1987).

[40].T.L. Chow, "*Mathematical Methods for Physicists:A Concise introduction*", California State University, Cambrig University Press, New York (2003).

[41].D. Yoshioka, "*The Quantum Hall Effect*", Springer (2002)

[42]. M. Hopkins, "*The State of the Art in Hall Effect Technology and Its Implications for Appliance Design and Development*", Allegro MicroSystems technical paper (2009).

[43]. T. Chakraborty and P. Pietilainen."*The Quantum Hall Effects Integral and Fractional*", 2^{nd}edt., Springer-Verlag, Heidelberg (1995).

[44].J.D. Patterson and B.C. Bailey, "*Solid-State Physics Introduction to the Theory*", Springer-Verlaga Berlin Heidelberg, New York, (2007).

[45].http::/www.Wikipedia, the free encyclopedia/Landau_quantization.

[46].T. Chakraborty and P. Pietilainen,"*The Quantum Hall Effects Integral and Fractional*", 2nd edition, Springer-Verlag, Heidelberg (1995).

[47]. G.E. Simion and J.J. Quinn, "Excitations of the $\nu = 5/2$ Fractional Quantum Hall State and the Generalized Composite Fermion Picture", Departement of physics and Astronomy, University of Tennessee, arXiv:0910,0828v1, (2009).

Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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