NEUTRINO EMISSION AND THERMAL HISTORY OF NEUTRON STARS

By

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Table of Contents

List of Figures

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Abstract

In this work we have examined the formation of neutron stars (NS_s) in the core of a supernova explosion during the death of massive stars, neutrino emission mechanisms in the interior of NS_s under different assumptions on their internal structure, and the thermal history of NS_s by determining neutrino luminosities and the temperature as a function of time. The emphasis is made on new results such as modified Urca process in a NS core with spherical nuclear structures since both momentum and energy are conserved in these reactions. Neutrino emission from NS cores containing exotic phases of matter (pion or kaon condensates,quarks) is also outlined. The effects of neutrino emission on thermal histories of NS_s , particularly cooling isolated stars, are examined.

Chapter 1

Introduction

Neutron stars are compact objects with radius about 10 –16 kilometers and a typical mass of 1.4 to 3 times the mass of the Sun, consisting of mostly neutrons and supported by degenerate neutron pressure. NS_s are the collapsed cores of stars more massive than the sun, produced in the final evolutionary stages of these stars and associated with supernova explosions at a very high temperature. Rotating NS_s are observed as radio pulsars, and are also thought to be components of some X-ray binary stars.

The determination of surface temperatures of NS_s by detecting thermal blackbody radiation can in principle yield significant information about the interior hadronic matter and NS structure. For example, the current upper limit of $2 \times 10^6\ K$ for the surface temperature of the Crab pulsar imposes a severe constraint on the thermal history of a NS for the first 930 years after its formation. Knowing the thermal evolution of a NS also yields information on such temperature-sensitive properties as transport coefficients, transition to superfluid states, crust solidification, internal pulsar heating mechanisms such as frictional dissipation at the crust-superfluid interfaces, and so on.

It is generally believed that NS_s are formed at very high interior temperatures ($T \gtrsim 10^{11}\ K$) in the core of a supernova explosion. The predominant cooling mechanism immediately after formation is neutrino emission, with an initial cooling timescale of seconds. After about a day, the internal temperature drops to 10^9–$10^{10}\ K$. Photon emission overtakes neutrinos only when the internal temperature falls to $\sim 10^8\ K$, with a corresponding surface temperature roughly two orders of magnitude smaller. Neutrino cooling dominates for at least the first 10^5 years, and typically for much longer, in all standard cooling calculations performed recently. These theoretical calculations [1][2][3] provide curves of the neutron surface temperature as a function of time, which in principle are subject to observational verification.

Thermal evolution calculations are sensitive to the adopted nuclear equation of state, the NS mass, the assumed magnetic field strength, the possible existence of superfluidity, pion condensation, quark matter, and so on. A wide range of thermal histories result when all the possibilities are considered.

Typically, one finds that surface temperatures fall to several times $10^6\ K$ for objects approximately 300 years old, and remain in the vicinity of $(0.5–2) \times 10^6\ K$ for at least 10^4 yr. Such temperatures imply potentially detectable photon emission in the soft X-ray band, 0.2–3 keV. Inspired by this prospect, we discuss the physics of NS cooling to describe the neutrino emission and thermal history of NS_s in greater detail in this thesis.

Objectives

After studying this thesis, the readers are expected to:

- Briefly define or describe the NS_s.
- Explaine the degenerate neutron gases and the pressure supports the NS_s aginest inward pressure.
- Describe the physical properties of the NS_s.
- Briefly define the neutrino emission and the thermal history of neutrons in different reactions which take place in the core of NS_s.
- Discuse the temperatures of NS_s as a function of time.

Chapter 2

Neutron stars

2.1 Neutron Stars Formation

NS_s are one of the possible end states for a massive star. They result from massive stars which have mass greater than 6–8 times that of our Sun. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion. This explosion blows off the outer layers of a star into a beautiful supernova remnant. The central region of the star collapses under gravity. It collapses so much that protons and electrons combine to form neutrons.

2.2 Degenerate Neutron Gases

At about $10^{13} kg/m^3$, the inward pressure forces **inverse beta decay** to occur (Fig. 2.1). In this process, an electron (sometimes called a *beta particle*) and a proton join together to form a neutron and neutrino. This happens both to free protons and to protons in the nucleus of a heavy element. At around $10^{15} kg/m^3$, the neutrons are no longer bound to the nuclei and begin to form a separate gas. At $10^{17} kg/m^3$, the nuclei suddenly fall apart into a gas with 80 percent neutrons. A spoonful of this material on the earth would weigh about a thousand million tons.

At this density, the neutrons become degenerate and provide a degenerate gas pressure, called the *neutron degeneracy pressure*, and so balance the inward pull of gravity. This pressure allows the formation of a stable **neutron star**, a star composed mainly of neutrons. In an ordinary gas, the pressure depends directly on the temperature, but in a degenerate gas, the pressure is not related to the temperature; it depends only on density. The neutron star radius will be about 10 to 16 km, depending on its mass. (The greater the mass of a neutron star, the smaller its radius, because it is made of a degenerate gas.)

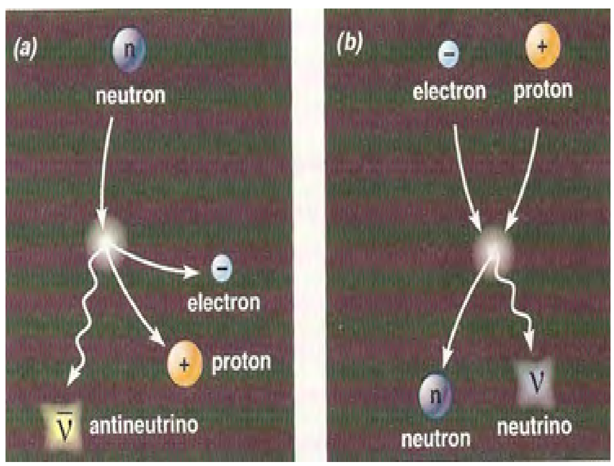

Figure 2.1: Beta decay and its reverse process, inverse beta decay. (An electron is sometimes called a beta particle for historic reasons.) (a) In beta decay, a neutron decomposes into a proton, an electron, and an antineutrino (an antimatter neutrino). (b) In inverse beta decay, a proton plus an electron transforms into a neutron and a neutrino(Astronomy: the evolving universe / Michael Zeilik –9th ed.).

2.3 Physical Properties of Neutron Stars

A neutron star is an eerie beast compared to an ordinary star. The exact structure of its model depends on assumptions made about the behavior of matter at very high densities. In one model for a 1.4-solar-mass neutron star, the radius is about 16 km(Fig.2.2). The inner 11 km is a fluid core, mostly of neutrons. The next 4 km out makes an inner crust, a neutron-rich fluid or perhaps a solid lattice. The outer crust, about 1 km thick, is a

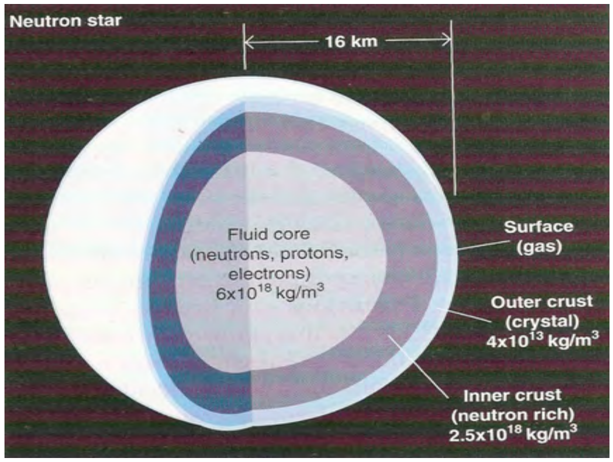

Figure 2.2: Theoretical model for the cross section of a 1.4–solar–mass neutron star. Note that the model shows three distinct interior layers. The exact details of any model depend on the known and assumed properties of the behavior of matter at high densities. (Adapted from a diagram by F.K. Lamb.)(Astronomy: the evolving universe / Michael Zeilik –9th ed.)

crystalline solid. In the outer few meters, where the density falls quickly, the neutron star has an atmosphere of atoms, electrons, and protons. The atoms are mostly iron.

Because a NS is so dense, it has an enormous surface gravity. For example, a 1-solar-mass NS with a radius of 12 km has a surface gravity 10^{11} times greater than that of the earth's surface gravity. The enormous pull means that there could be no mountains on a NS − mini-mountains reach a few centimeters at most. This intense gravitational field also results in a huge radial escape velocity, as much as about 80 percent of the speed of light. Objects falling onto a neutron star from a great distance have at least the escape speed when they hit the NS surface. That means that even a small mass carries a fantastic amount of kinetic energy. For example, a marshmallow dropped onto a neutron star from a few AU will clobber the surface with megatons of kinetic energy.

An ordinary star, having at the end of its evolution a mass equal to or greater than 1.4 solar masses, probably ends up as a NS. Theoretically, a stable NS with a mass of less than 1.4 solar masses can also form. These low-mass NS could be made in the pile-driver compression of a supernova explosion in which the interior of the star is crushed to the very high densities of neutron stars. The lowest mass possible is about 0.6 solar mass.

A crucial point arrives when the mass of the NS reaches a certain limit - not known exactly, but estimated at about 2 to 3 solar masses. Such a star has the highest density and smallest radius possible. Add a bit more mass and the gravitational forces overwhelm the degenerate neutron gas pressure. The star collapses and it cannot stable. This amount of mass, 2 to 3 solar mass, is called the **Oppenheimer-Volkoff limit** and signals the point at which gravitational collapse begins.

In general, a star with a mass between 1.4 and (roughly) 3 solar masses at the time of its death naturally forms into a degenerate neutron gas; hence the name "NS." NS_s lack any fusion reactions.

Chapter 3

Neutrino Emission

Neutrino is an elementary particle with no (or very little) mass and no electric charge that travels at the speed of light and carries energy away during certain types of nuclear reactions. In this chapter we consider neutrino emission from the core of neutron star during the predominant cooling mechanism immediately after its formation. We shall discuss neutrino emission through different process in the core of a neutron star. The emphasis is made on modified URCA process because powerful neutrino energy losses are produced by this process.

3.1 Neutrino Reactions in Neutron Stars ($T \lesssim 10^9\,K$)

We shall be interested in the thermal history of a NS after it has already cooled to an interior temperature below a few times $10^9\,K$. We shall discuss the relatively short epoch during which the temperature drops from $\sim 10^{11}\,K$ to $\sim 10^9\,K$. This earlier epoch, initiated by stellar core collapse and a supernova explosion, is significantly shorter (8 hrs) than the extended of th NS life time.

For internal temperatures below a few times $10^9\,K$, any neutrinos emitted during the cooling process escape freely from the NS, without further interacting with the NS matter. We shall verify this statement in section 4.1. This fact, which distinguishes the low-temperature NS cooling epoch from the earlier high-temperature NS formation epoch, greatly simplifies the determination of the late thermal evolution.

At the very high temperatures ($T \gtrsim 10^9$ K) found in the cores of evolved, massive stars, the dominant mode of energy loss via neutrinos is from the so-called URCA reactions:

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad e^- + p \rightarrow n + \nu_e \tag{3.1.1}$$

These reactions also dominate during core collapse. In both cases the nucleons in the hot interior are nondegenerate. However, when the nucleons become degenerate as in a NS that has cooled below 10^9 K, these reactions are highly suppressed. We now demonstrate this important result.

Matter in the degenerate interior satisfies the *β-equilibrium* condition

$$\mu_n = \mu_p + \mu_e, \tag{3.1.2}$$

where to good approximation $[\vartheta(kT/\mu_n)^2]$ the chemical potentials are just the Fermi energies. Thus

$$E_F(n) = E_F(p) + E_F(e), \tag{3.1.3}$$

where at nuclear densities

$$\begin{aligned} E_F(n) &\simeq m_n c^2 + \frac{p_F^2(n)}{2m_n}, \\ E_F(p) &\simeq m_p c^2 + \frac{p_F^2(p)}{2m_p}, \\ E_F(e) &\simeq p_F(e)c. \end{aligned} \tag{3.1.4}$$

Charge neutrality requires that

$$p_F(p) = p_F(e), \tag{3.1.5}$$

so Eq. (3.1.3) becomes

$$\frac{p_F^2(n)}{2m_n} \simeq p_F(e)c\left(1 + \frac{p_F(p)}{2m_p c}\right) - Q \tag{3.1.6}$$

where $Q = (m_n - m_p)c^2 = 1.293\, MeV$ is small in comparison to the other tems in Eq. (3.1.6)[cf. Eqs. (3.1.18) and (3.1.19)]. From Eq. (3.1.6) we see that the neutron Fermi energy (minus the rest mass energy) is very nearly equal to the electron Fermi energy:

$$E_F'(n) \equiv \frac{p_F^2(n)}{2m_n} \simeq p_F(e)c = E_F(e), \tag{3.1.7}$$

and thus

$$p_F(e) = p_F(p) \ll p_F(n), \tag{3.1.8}$$

$$E_F'(p) \ll E_F'(n). \tag{3.1.9}$$

Now consider the possibility of a reaction such as neutron decay, Eq. (3.1.1). The only neutrons capable of decaying lie within $\sim kT$ of the Fermi surface, $E_F'(n)$. Hence, by energy conservation, the final proton and electron must also be within $\sim kT$ of their Fermi surfaces; the energy of escaping neutrino must also be $\sim kT$. Now, according to inequality (3.1.8), the proton and electron must have small momenta compared to the neutron. But this is impossible: the decay cannot conserve momentum if it conserves energy.

In order for the process to work, a bystander particle must be present to absorb momentum. Chiu and Salpeter (1964) therefore proposed that "modified" URCA reactions

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e, \tag{3.1.10}$$

$$n + p + e^- \rightarrow n + n + \nu_e, \tag{3.1.11}$$

would be important for NS cooling. Accompanying these reactions are the muon-neutrino emitting reactions

$$n + n \rightarrow n + p + \mu^- + \bar{\nu}_\mu, \tag{3.1.12}$$

$$n + p + \mu^- \rightarrow n + n + \nu_\mu, \tag{3.1.13}$$

which occur whenever $\mu_e > m_\mu c^2 (\rho \gtrsim 8 \times 10^{14}\ g\ cm^{-3})$. Corresponding reactions involving τ-*neutrinos* do not occur at typical NS interior densities, since $m_\tau c^2 = 1784\ MeV \gg \mu_e$.

In the ideal Fermi gas approximation (cf. Appendix D), the following relations hold approximately for $\rho \leq 2\rho_{nuc}$ (i.e., nonrelativistic nucleons):

$$n_n = 1.7 \times 10^{38}\left(\frac{\rho}{\rho_{nuc}}\right) cm^{-3}, \tag{3.1.14}$$

$$n_e = n_p = 9.6 \times 10^{35}\left(\frac{\rho}{\rho_{nuc}}\right)^2 cm^{-3}, \tag{3.1.15}$$

$$E'_F(n) = E_F(e) = 60\left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} MeV, \tag{3.1.16}$$

$$E'_F(p) = 1.9\left(\frac{\rho}{\rho_{nuc}}\right)^{4/3} MeV, \tag{3.1.17}$$

$$p_F(n) = 340\left(\frac{\rho}{\rho_{nuc}}\right)^{1/3} MeV/c, \tag{3.1.18}$$

$$p_F(e) = p_F(p) = 60\left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} MeV/c. \tag{3.1.19}$$

Here $\rho_{nuc} = 2.8 \times 10^{14} g\ cm^{-3}$ is the standard nuclear matter density.

In the succeeding sections we shall calculate the cooling rate due to these modified URCA reactions.

3.2 Free Neutron Decay

The modified URCA reactions involve both strong and weak interactions. For example, pions are exchanged between the colliding neutrons in reaction (3.1.10) before one of the neutrons decays into a proton. Before calculating the rate of these processes, we therefore consider a simpler reaction: pure neutron decay in vacuum, reaction (3.1.1), where strong interaction effects are very small. Much of the discussion in this section also applies to β-*decay* in nuclei.

The energy released in a typical β-*decay* is small ($Q \sim MeV$) compared to the rest energy of the nucleons. Accordingly, we can employ the Golden Rule of time-dependent perturbation theory in the nonrelativistic limit to obtain the decay rate:

$$d\Gamma = \frac{2\pi}{\hbar}\left(\frac{1}{2}\sum_{spins}|H_{fi}|^2\right)\rho_e\rho_{\bar{\nu}}dE_e dE_{\bar{\nu}}\delta(E_{\bar{\nu}} + E_e - Q). \tag{3.2.1}$$

Here ρ_e and $\rho_{\bar{\nu}}$ are the densities of the final states of the e^- and $\bar{\nu}$, respectively, per unit energy interval, while E_e is the electron energy. The quantity H_{fi} is the weak interaction matrix element in the nonrelativestic limit,

$$H_{fi} = G_F \int \Psi_f^*\Psi_i dV = G_F \int \psi_p^*\psi_e^*\psi_{\bar{\nu}}^*\psi_n dV, \tag{3.2.2}$$

where $G_F, \psi_p, \psi_e, \psi_{\bar{\nu}}$, and ψ_n are the Universal Fermi coupling constant [cf. Appendix B Eq. (5.0.39)], proton, electron, anti–neutrino, and neutron wave functions, respectively. We imagine that the decaying neutron is in a box of unit volume and normalize all wave functions accordingly.

Note that the integral in Eq. (3.2.2) is the probability amplitude to find all four particles at the same point in space. Accordingly, the weak interaction in this low-energy domain (cf. Appendix B) is describable by a "contact potential":

$$H_{fi} = \int \Psi_f^*(\mathbf{r})V_W(\mathbf{r},\mathbf{r}')\Psi_i(\mathbf{r}')\, dVdV^{'}, \tag{3.2.3}$$

where

$$V_W(\mathbf{r},\mathbf{r}') = G_F\delta(\mathbf{r}-\mathbf{r}'). \tag{3.2.4}$$

If we neglect the coulomb distortion on the electron due to the proton, we can write ψ_e and $\psi_{\bar{\nu}}$ as plane-wave states. In addition, we note that the wavelengths of these states are much larger than the nuclear radius:

$$\lambda = \tfrac{1}{k} = \tfrac{\hbar}{p} \sim \tfrac{\hbar c}{E} \sim 10^{-11}\ cm$$

for $E \sim MeV$. We can thus expand the spatial part of lepton wave functions as

$$\psi_e^*\psi_{\bar{\nu}}^* = \exp[-i(\mathbf{k}_e + \mathbf{k}_{\bar{\nu}}) \bullet \mathbf{r}] = 1 - i(\mathbf{k}_e + \mathbf{k}_{\bar{\nu}}) \bullet \mathbf{r} + ... \tag{3.2.5}$$

and retain only the first term of the expansion (3.2.5) in the integeral (3.2.2). Provided the remaining nucleon overlap integral does not vanish, it is said to describe an *allowed transition*; higher-order terms in the expansion (3.2.5) give rise to forbidden transitions. Clearly, no orbital angular momentum is carried off by an allowed transition.

The spin part of the combined lepton wave function can be either a singlet state (total spin zero) or a triplet state (total spin unity). In singlet transitions, the nucleon wave function does not change its spin or its total angular momentum $\boldsymbol{J}$, so $\triangle\boldsymbol{J} = 0$. Triplet transitions require a spin flip, and total angular momentum can be conserved if $\triangle\boldsymbol{J} = \pm 1$ or 0. The singlet transition selection rule, $\triangle\boldsymbol{J} = 0$, is called the *Fermi selection rule*, while triplet transitions obey the *Gamow-Teller selection rule*. The spin flip in triplet transitions is effected by the pseudovector, or axial, part of the weak interaction, while singlet transitions occur via the pure vector part.

We can now write

$$\frac{1}{2}\sum_{spins}|H_{fi}|^2 = G_F^2\big[C_V^2|M_V|^2 + 3C_A^2|M_A|^2\big]. \tag{3.2.6}$$

Here M_V and M_A are the vector and axial vector nuclear matrix elements, which are determined by the overlap integrals of the initial and final nuclear states; C_V and C_A are coupling constants that would both be unity if strong interactions had no effect on the interaction, and the factor of 3 arises from the statistical weight of the triplet transition. Experimentally, it is found that

$$\begin{aligned} |C_V| &= 0.9737 \pm 0.0025 \\ \left|\frac{C_A}{C_V}\right| &\equiv a = 1.253 \pm 0.007. \end{aligned} \tag{3.2.7}$$

For neutron decay, it is a good approximation to take

$$M_V \simeq M_A \simeq 1, \tag{3.2.8}$$

since the neutron and proton wave functions are very similar (isotopic spin symmetry). Thus Eq. (3.2.6) becomes

$$\frac{1}{2}\sum_{spins}|H_{fi}|^2 = G_F^2 C_V^2(1+3a^2). \tag{3.2.9}$$

On integrating over $dE_{\bar{\nu}}$ using the δ-$function$, Eq. (3.2.1) becomes

$$d\Gamma = \frac{2\pi}{\hbar}G_F^2 C_V^2(1+3a^2)\rho_e\rho_{\bar{\nu}}dE_e. \tag{3.2.10}$$

For an electron with a definite spin orientation, we have

$$\rho_e = \frac{4\pi p^2}{h^3}\frac{dp}{dE_e} = \frac{4\pi p E_e}{c^2 h^3}, \tag{3.2.11}$$

where

$$p = \frac{(E_e^2 - m_e^2 c^4)^{1/2}}{c}. \tag{3.2.12}$$

Since the neutrino is massless, we have by energy conservation

$$E_{\bar{\nu}} = Q - E_e = p_{\bar{\nu}}c, \tag{3.2.13}$$

and so

$$\rho_{\bar{\nu}} = \frac{(Q-E_e)^2}{2\pi^2\hbar^3 c^3}. \tag{3.2.14}$$

Thus

$$d\Gamma = \frac{G_F^2 C_V^2(1+3a^2)}{2\pi^3\hbar^7 c^6}(E_e^2 - m_e^2c^4)^{1/2}E_e(Q-E_e)^2 dE_e. \tag{3.2.15}$$

Defining dimensionless energies

$$\varepsilon \equiv \frac{E_e}{m_e c^2}, \qquad \varepsilon_0 \equiv \frac{Q}{m_e c^2} = 2.5312, \tag{3.2.16}$$

we get

$$d\Gamma = \frac{m_e^5 c^4 G_F^2 C_V^2(1+3a^2)}{2\pi^3\hbar^7}df, \tag{3.2.17}$$

where the energy spectrum of the decay electron arises purely from phase-space factors:

$$df = (\varepsilon^2-1)^{1/2}\varepsilon(\varepsilon_0-\varepsilon)^2 d\varepsilon. \tag{3.2.18}$$

Integrating over the allowed range of ε from 1 to ε_0 gives

$$\begin{aligned}\int df \quad &= \int_1^{\varepsilon_0} (\varepsilon^2 - 1)^{1/2} \varepsilon (\varepsilon_0 - \varepsilon)^2 d\varepsilon \\ f &= \frac{1}{60}(\varepsilon_0^2 - 1)^{1/2}(2\varepsilon_0^4 - 9\varepsilon_0^2 - 8) + \frac{1}{4}\varepsilon_0 \ln[\varepsilon_0 + (\varepsilon_0^2 - 1)^{1/2}] \\ &= 1.6369, \end{aligned} \tag{3.2.19}$$

and so the neutron decay rate is

$$\Gamma = \frac{m_e^5 c^4 G_F^2 C_V^2 (1 + 3a^2)}{2\pi^3 \hbar^7} f = \frac{1}{972} s. \tag{3.2.20}$$

The observed rate is $1/(925 \pm 11)$ s; discrepancy arises because we ignored Coulomb effects in the calculation of f (f increases to about 1.70), and there are also quantum electrodynamic effects ("radiative corrections") of another 2 percent.

3.3 The Modified URCA Rate

Having evaluated the weak interaction matrix element for free neutron decay, we are now in a position to calculate the rate of the modified URCA reaction (3.1.10). We shall follow the original approach of Bahcall and Wolf (1965).

Let the subscripts 1, 2, 1', p, e, and $\bar{\nu}$ denote the two initial neutrons, the final neutron, the proton, the electron, and the antineutrino, respectively. The reaction rate for given initial states 1 and 2 is

$$d\Gamma = \frac{2\pi}{\hbar} \delta(E_f - E_i) |H_{fi}|^2 \rho_p \rho_e \rho_{\bar{\nu}} \, dE_p \, dE_e \, dE_{\bar{\nu}}, \tag{3.3.1}$$

where $|H_{fi}|^2$ must be summed over final spins and averaged over initial spins. We shall explicitly retain the normalization volume V in all phase-space factors and wave functions, so that in empty space for each species j we would have

$$\rho_j \, dE_j \equiv d^3 n_j = V \frac{d^3 p_j}{h^3}. \tag{3.3.2}$$

However, the reaction takes place in a dense gas and most of the low-energy cells in phase space are occupied. Hence each factor of d^3n_j must be multiplied by $1 - f_j$, where

$$f_j = \frac{1}{\exp[(E_j - \mu_j)/kT] + 1} \tag{3.3.3}$$

is the fraction of phase space occupied at energy E_j (Fermi-Dirac distribution). Factors of $(1 - f_j)$ reduce the reaction rate and are called *blocking factors*.

Note that in the Golden Rule as used in Eq. (3.3.1), the number of phase-space factors is one less than the number of final particles because of momentum conservation. We can obtain a more symmertrical expression by adding a factor

$$\begin{aligned} \delta^3(\mathbf{p}_f - \mathbf{p}_i)\, d^3\mathbf{p}_{1'} &= \delta^3(\mathbf{p}_f - \mathbf{p}_i)\, d^3n_{1'} \tfrac{h^3}{V} \\ &= \delta^3(\mathbf{k}_f - \mathbf{k}_i)\, d^3n_{1'} \tfrac{(2\pi)^3}{V}. \end{aligned}$$

On integrating over $\mathbf{p}_{1'}$, this factor gives unity.

The total antineutrino luminosity in a volume V is obtained by integrating the rate (3.3.1) over all initial states, multiplied by $E_{\bar\nu}$:

$$L_{\bar\nu} = \frac{(2\pi)^4}{\hbar V} \int d^3n_1\, d^3n_2\, d^3n_{1'}\, d^3n_p\, d^3n_e\, d^3n_{\bar\nu} \delta(E_f - E_i)\delta^3(\mathbf{k}_f - \mathbf{k}_i) S |H_{fi}|^2 E_{\bar\nu}. \tag{3.3.4}$$

Here

$$S = f_1 f_2 (1 - f_{1'})(1 - f_p)(1 - f_e), \tag{3.3.5}$$

the factors f_1 and f_2 accounting for the distribution of initial states.

The all-important matrix element for the interaction can be written as

$$H_{fi} = \langle n, p, e, \bar\nu | V_W | n, n \rangle, \tag{3.3.6}$$

where V_W is the weak interaction "contact" Hamiltonian given previously for free-neutron decay [Eq. (3.2.4)]. Now it is again appropriate to represent the leptons as free-particle state in Eq. (3.3.6). However, it is not at all valid to do so for the neutrons. The reason is that the total nucleon Hamiltonian is

$$H_{nuc} = H_{free} + H_s + V_W \equiv H_0 + V_W, \tag{3.3.7}$$

where H_{free} is the free-particle contribution (kinetic energy of each particle) and H_s is the strong interaction Hamiltonian. Thus Eq. (3.3.6) gives the lowest-order transition rate only if the nucleon wave functions appearing there are already eigenfunctions of H_0. The solution to the many-body Schrödinger equation including H_s is highly nontrivial and by no means resolved theoretically as yet. We shall simply follow Bahcall and Wolf and attempt to estimate the effect of H_s on the two-body nucleon wave function.

We therefore write

$$\begin{aligned} H_{fi} &= \int dV dV' \psi^*_{np}(\mathbf{r})\psi^*_e(\mathbf{r})\psi^*_{\bar{\nu}}(\mathbf{r})V_W(\mathbf{r},\mathbf{r'})\psi_{nn}(\mathbf{r'}) \\ &= \frac{G_F}{V}\int dV \psi^*_{np}(\mathbf{r})\psi_{nn}(\mathbf{r}). \end{aligned} \tag{3.3.8}$$

Here we have used Eq. (3.2.4)

$$\psi_e(\mathbf{r}) = \frac{1}{V^{1/2}}e^{i\mathbf{k}_e \bullet \mathbf{r}} \to \frac{1}{V^{1/2}} \tag{3.3.9}$$

and similary for $\psi_{\bar{\nu}}(\mathbf{r})$ as Eq. (3.2.5).

We assume that the interaction in the initial n-n state is dominated by s-wave scattering (*i.e.*, $L = 0$). Then the n-n state must have total spin $S = 0$. The V part of the weak interaction couples this state to an $S = 0$ n-p state, while the A part couples it to an $S = 1$ n-p. Thus [cf. Eq. (3.2.6)]

$$\sum_{spins}|H_{fi}|^2 = \frac{4G_f^2}{V^2}\left(C_V^2|M_V|^2 + 3C_A^2|M_A|^2\right), \tag{3.3.10}$$

where

$$\begin{aligned} M_V &= \int dV (\psi^0_{np})^*\psi^0_{nn}, \\ M_A &= \int dV (\psi^1_{np})^*\psi^0_{nn}. \end{aligned} \tag{3.3.11}$$

The factor of 4 occurs in Eq. (3.3.10) because either neutron in the n-n pair can become a proton, giving a factor of 2 in amplitude and 4 in probability.

Since the rang of the strong force is of order $\lambda_\pi = \hbar/m_\pi c$, we expect that the relative wave functions in Eq. (3.3.11) will overlap in a relatively small volume of order λ_π^3. Thus we expect $M \sim \lambda_\pi^3/V$. Defining nondimensional matrix elements

$$\tilde{M}_V = \frac{VM_V}{\lambda_\pi^3}, \qquad \tilde{M}_A = \frac{VM_A}{\lambda_\pi^3}, \tag{3.3.12}$$

we get for Eq. (3.3.4)

$$L_{\bar{\nu}} = 64\pi^4 V G_F^2 \hbar^{-1} \lambda_\pi^{-9} \big(C_V^2 |\tilde{M}_V|^2 + 3C_A^2 |\tilde{M}_A|^2\big) P, \tag{3.3.13}$$

where the dimensionless phase-space factor P is

$$P = V^{-6} \lambda_\pi^{15} \int \prod_{j=1}^{6} d^3 n_j S E_{\bar{\nu}} \delta^3(\mathbf{k}_f - \mathbf{k}_i) \delta(E_f - E_{i)}. \tag{3.3.14}$$

Note that since each factor $d^3 n_j$ is proportional to V, P is independent of V and so $L_{\bar{\nu}}$ is proportional to V. Numerically, we have for the antineutrino emissivity $\varepsilon_{\bar{\nu}}$

$$\varepsilon_{\bar{\nu}} \equiv \frac{L_{\bar{\nu}}}{V} = \big(5.1 \times 10^{48} erg\ cm^{-3} s^{-1}\big) P \big(|\tilde{M}_V|^2 + 4.7 |\tilde{M}_A|^2\big). \tag{3.3.15}$$

The phase-space factor is evaluated in Appendix A, where it is shown that

$$P \simeq 2.1 \times 10^{-30} \left(\frac{\rho}{\rho_{nuc}}\right)^2 T_9^8, \tag{3.3.16}$$

where $\rho_{nuc} = 2.8 \times 10^{14} g\ cm^{-3}$ and T_9 is the temperature in units of $10^9 K$. Note that all of the temperature dependence in $\varepsilon_{\bar{\nu}}$ is in the phase-space factor, a result that is generally true of neutrino cooling reactions. The 8 powers of T originate as follows: for each degenerate species, only a fraction $\sim kT/E_F$ can effectively contribute to the cooling rate. There are two such initial species and three such final species. The antineutrino phase space is proportional to $E_{\bar{\nu}}^2$, and the energy loss rate gives another factor $E_{\bar{\nu}}$. Since $E_{\bar{\nu}} \sim kT$, we have altogether 8 powers of T.

Evaluation of the nondimensional matrix elements is not so straightforward. we can define a fundamental length scale from the Fermi momentum of the dominant neutrons by

$$l = \frac{\hbar}{p_F(n)} \sim 0.4\lambda_\pi \left(\frac{\rho}{\rho_{nuc}}\right)^{-1/3}. \tag{3.3.17}$$

Bahcall and Wolf suggests that $\tilde{M}_V$ and $\tilde{M}_A$ might be expected to vary as $(l/\lambda_\pi)^3$ –that is, to be of order unity at $\rho \sim \rho_{nuc}$ and to decrease slowly as $1/\rho$. In any case they use some results of a nuclear matter calculation to formally calculate

$$|\tilde{M}_V|^2 = |\tilde{M}_A|^2 = 1.0\left(\frac{\rho_{nuc}}{\rho}\right)^{4/3}. \tag{3.3.18}$$

Thus Eq. (3.3.15) becomes

$$\varepsilon_{\bar{\nu}} = (6.1 \times 10^{19} erg\ cm^{-3} s^{-1})\left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} T_9^8. \tag{3.3.19}$$

To the expression (3.3.19) we must now add the rate of *neutrino* energy loss from the "inverse" reaction (3.1.11). By time–reversal invariance, the matrix elements M_A and M_V for reaction (3.1.11) are the complex conjugates of M_A and M_V for reaction (3.1.10). Since the phase–space factors are the same for both reactions (with the approximations we have made of neglecting all lepton momenta), the two reactions give the same energy loss rate.

The muon–neutrino reactions (3.1.12) and (3.2.13) must be considered when $\mu_e > m_\mu c^2$ (i.e., $\rho > 2.9\rho_{nuc}$ in the free–particle model). The ν_μ reactions differ from the ν_e reactions only in the phase–space factor P, where a factor $p_\mu^2 dp_\mu$ appears instead of $p_e^2 dp_e$. Thus the ratio of ν_μ to ν_e energy loss rate is

$$F = \frac{p_\mu^2 dp_\mu}{p_e^2 dp_e} \tag{3.3.20}$$

Each term in Eq. (3.3.20) is to be evaluated at the Fermi surface, since only that region of phase space contributes to the reaction rate.

By using equilibrium relation $E_F(\mu) = E_F(e)$, we have

$$\frac{dp_F(\mu)}{dE_F(\mu)} = \frac{E_F(\mu)}{p_F(\mu)c^2} \Rightarrow dE_F(\mu) = \frac{dp_F(\mu)}{E_F(\mu)} p_F(\mu)c^2 \tag{3.3.21}$$

where

$p_F(\mu) = \sqrt{\frac{E_F^2(\mu) - m_\mu^2 c^4}{c^2}}$.

Similarly, we have

$$\frac{dp_F(e)}{dE_F(e)} = \frac{E_F(e)}{p_F(e)c^2} \Rightarrow dE_F(e) = \frac{dp_F(e)}{E_F(e)} p_F(e)c^2 \tag{3.3.22}$$

where

$p_F(e) = \sqrt{\frac{E_F^2(e) - m_e^2 c^4}{c^2}}$.

But fron the above equilibrium relation, we have

$$dE_F(\mu) = dE_F(e). \tag{3.3.23}$$

By putting Eqs. (3.3.21) and (3.3.22) into Eq. (3.3.23), we get

$$p_F(\mu) dp_F(\mu) = p_F dp_F(e) \tag{3.3.24}$$

at the respective Fermi surfaces, so that

$$\begin{aligned} F &= 0, & \rho &\leq 2.9\rho_{nuc} \\ F &= \left[1 - \left(\frac{m_\mu c^2}{E_F(e)}\right)\right]^{1/2}, & \rho &\geq 2.9\rho_{nuc}. \end{aligned} \tag{3.3.25}$$

Multiplying Eq. (3.3.19) by $2(1+F)$ gives finally the total energy loss rate by modified URCA reactions:[6]

$$\begin{aligned} \varepsilon_\nu^{URCA} &= 2(1+F)\varepsilon_{\bar{\nu}} \\ &= 2(1+F)(6.1 \times 10^{19} erg\ cm^{-3} s^{-1}) \left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} T_9^8 \\ &= (1.2 \times 10^{20} erg\ cm^{-3} s^{-1}) \left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} T_9^8 (1+F). \end{aligned} \tag{3.3.26}$$

For a neutron satr of mass M and uniform density ρ, this gives a luminosity $L_\nu^{URCA} = \varepsilon_\nu^{URCA} V$, where $V = \frac{M}{\rho}$ is volume of neutron star.

So

$$L_\nu^{URCA} = (1.2 \times 10^{20} erg\ cm^{-3} s^{-1}) \left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} T_9^8 (1+F) \left(\frac{M}{\rho}\right) \tag{3.3.27}$$

If we multiply Eq. (3.3.27) by $(M_\odot/M_\odot)$ and (ρ_{nuc}/ρ_{nuc}) from the right and rearange it, we get

$$\begin{aligned} L_\nu^{URCA} &= (1.2\times 10^{20} erg\ cm^{-3} s^{-1})\left(\frac{\rho}{\rho_{nuc}}\right)^{2/3} T_9^8(1+F)(\frac{M}{\rho})\left(\frac{M_\odot}{M_\odot}\right)\left(\frac{\rho_{nuc}}{\rho_{nuc}}\right) \\ &= (8.5\times 10^{38} erg\ s^{-1})\frac{M}{M_\odot}\left(\frac{\rho_{nuc}}{\rho}\right)^{1/3} T_9^8(1+F) \end{aligned} \tag{3.3.28}$$

where $M_\odot = 1.989\times 10^{30} kg$ is the mass of the sun and $\rho_{nuc} = 2.8\times 10^{14} g\ cm^{-3}$ is the standard nuclear matter density.

Recently, Friman and Maxwell (1979) have repeated the above calculations using a more realistic expression for the strong NN interaction. They obtain the same density dependence, but a numerical coefficient of 7.4×10^{20} in Eq. (3.3.26), nearly an order of magnitude larger. For a uniform density star with no muons, this gives

$$L_\nu^{URCA} = (5.3\times 10^{39} erg\ s^{-1})\frac{M}{M_\odot}\left(\frac{\rho_{nuc}}{\rho}\right)^{1/3} T_9^8. \tag{3.3.29}$$

3.4 Other Reaction Rates

Having sketched the derivation of Eq. (3.3.26), we now discuss briefly the rates of other possible cooling reactions.

3.4.1 Nucleon Pair Bremsstrahlung

The most significant new cooling mechanism that becomes possible when neutral currents are considered is nucleon pair bremsstrahlung:

$$n+n\to n+n+\nu+\bar\nu, \quad n+p\to n+p+\nu+\bar\nu. \tag{3.4.1}$$

These reactions have been studied by Flowers et al. (1975) and later by Friman and Maxwell (1979), who found that while the rate also varies as T^8, it is smaller than the modified URCA rate by a factor of 30.

3.4.2 Neutrino Pair Bremsstrahlung

If the neutrons are "locked" in a superfluid state, the rates for all the reactions discussed so far are cut down by a factor $\sim \exp(-\triangle/kT)$, where $\triangle$ is the superfluid energy gap. In this case, cooling from neutrino pair bremsstrahlung from nuclei in the crust can be important. The rate for the process

$$e^- + (Z, A) \rightarrow e^- + (Z, A) + \nu + \bar{\nu} \tag{3.4.2}$$

is estimated to be

$$L_\nu^{brem} \sim (5 \times 10^{39} erg\ s^{-1}) \left(\frac{M_{cr}}{M_\odot}\right) T_9^6, \tag{3.4.3}$$

where M_{cr} is the mass of the crust[19]. Since this process goes as T_9^6, it decreases less rapidly than reaction(3.3.26) as the star cools.

3.4.3 Pionic Reaction

As Bahcall and Wolf originally pointed out, pion condensation can dramatically increase the cooling rate in NS interiors. If pion condensates exist , then "*quasi-particle*″ *β-decay* can occur via

$$N \rightarrow N' + e^- + \bar{\nu}_e \tag{3.4.4}$$

and its inverse. Here the *quasi-particles* N and N' are linear combinations of neutron and proton states in the pion sea. The pion condensate allows both energy and momentum to be conserved in the reaction, which is the analogue of the ordinary URCA process, Eq. (3.1.1).

Bahcall and Wolf considered a simplified version of reaction (3.4.4)–that is, cooling via the decay of free pions:

$$\pi^- + n \rightarrow n + e^- + \bar{\nu}_e, \tag{3.4.5}$$

$$\pi^- + n \rightarrow n + \mu^- + \bar{\nu}_\mu \tag{3.4.6}$$

and the "inverse" processes

$$n + e^- \rightarrow n + \pi^- + \nu_e, \tag{3.4.7}$$

$$n + \mu^- \rightarrow n + \pi^- + \nu_\mu. \tag{3.4.8}$$

As in the modified URCA reactions, the total rate for all four processes from Eq. (3.4.5) to (3.4.8) is essentially four times the rate of reaction (3.4.5) alone. (Recall that muons are already present when and if pions appear.)

We can make a rough estimate of the reaction rate as follows: Since there are two fewer fermions participating in the reactions than in the modified URCA reactions, the *phase-space* factor varies as T^6 rather than T^8. We therefore expect the total rate to be

$$L_\nu^\pi \sim L_\nu^{URCA} \times \left(\frac{E_f'(n)}{kT}\right)^2 \frac{n_\pi}{n_n}, \tag{3.4.9}$$

where n_π/n_n is the ratio of the number densities of pions and neutrons. Since $E_F'(n) \sim 60(\rho/\rho_{nuc})^{2/3} MeV$, Eqs. (3.3.28) and (3.4.9) give

$$L_\nu^\pi \sim (8 \times 10^{44} erg\ s^-) \left(\frac{M}{M_\odot}\right) \left(\frac{\rho}{\rho_{nuc}}\right) T_9^6 \frac{n_\pi}{n_n}. \tag{3.4.10}$$

Bahcall and Wolf actually obtained a numerical factor 1×10^{46} and no ρ dependence. The more recent calculation of Maxwall et al. (1977) gives

$$L_\nu^\pi = (1.5 \times 10^{46} erg\ s^-) \theta^2 \left(\frac{M}{M_\odot}\right) \left(\frac{\rho_{nuc}}{\rho}\right) T_9^6, \tag{3.4.11}$$

where $\theta \sim 0.3$ is an angle measuring the degree of pion condensation (i.e., it replaces the factor n_π/n_n). Since L_ν^π is larger than L_ν^{URCA} by a hefty factor, it will dominate the cooling rate at all temperatures of interest, provided pion condensation occurs.

3.4.4 Quark Beta Decay

If the core of a neutron star consists largely of quark matter, then there is the possibility of significant neutrino emission via the *β-decay* of degenerate, relativistic quarks [11][13].

Unlike ordinary NS matter in which the simple beta decay (i.e., URCA) processes described by Eq. (3.1.1) are suppressed, the corresponding processes can occur for quarks.

Recall Appendix C of three-component (u,d, and s) quark matter in beta equilibrium. There we showed that if we ignored the masses of the quars and their mutual interactions, the equilibrium composition was given by

$$n_u = n_d = n_s = n, \quad n_e = n_\mu = 0, \tag{3.4.12}$$

where $n = (n_u + n_d + n_s)/3$ is the baryon number density. According to Eq. (3.4.12), each quark species has the same Fermi momentum, given by

$$p_f(q) = 235\left(\frac{n}{n_{nuc}}\right)^{1/3}\frac{MeV}{c}, \tag{3.4.13}$$

where $n_{nuc} \equiv \rho_{nuc}/m_B = 0.17fm^{-3}$.

The existence of finite quark masses and quark-quark interactions is likely to alter the above composition. For example, the s quark is thought to be rather heavy ($m_s \sim$ 100–300 MeV) and, if present in a neutron star, it is not likely to be relativistic. On the other hand, the u and d quarks are probably very light ($m_u \sim m_d \sim$ 5–10 MeV), so they will be highly relativistic and their masses can still be ignored. Resulting modifications to the equilibrium composition include the presence of leptons to carry some of the negative charge while preserving overall charge neutrality. In fact, since characteristic quark Fermi energies greatly exceed $m_e c^2$, any small difference between μ_d and μ_u(or between μ_s and μ_u) will invariably result in electrons with relativistic Fermi energy[cf. Appendix C Eqs. (5.0.43) and (5.0.44)].

The simplest neutrino processes occurring in quark matter are the β-*decay* reactions involving the relativistic quarks,

$$d \rightarrow u + e^- + \bar{\nu}_e, \tag{3.4.14}$$

$$u + e^- \rightarrow d + \nu_e. \tag{3.4.15}$$

A detailed analysis of these reactions is given by Iwamoto (1980). He points out, for example, that if particle masses and *quark*-*quark* interactions are ignored, then the conservation of energy and momentum requires that the momentum of all the particles must be collinear for reaction (3.4.14) to occur at all. However, in that case, the WSG matrix element for the reaction turns out to vanish identically [13]. Iwamoto then shows that if one takes into account quark–quark interactions, the particles need not be collinear to satisfy energy and momentum conservation and the matrix element is nonzero. Specifically, he notes that the participating quarks must reside close to their Fermi surfaces. To lowest order in the strong interaction (QCD) coupling constant α_s, the reaction between quark chemical potential and Fermi momentum is [14]

$$\mu_{u,d} = \left(1 + \frac{8\alpha_s}{3\pi}\right) p_F(u,d)c. \tag{3.4.16}$$

It is easy to verify that this modification to the usual ideal gas relationship $\mu = p_F c$ for a relativistic fermion permits energy and momentum conservation for a *finite* angle between the momenta of the interacting particles in Eq. (3.4.14).

When interactions are included (but the small neutrino momentum ignored), the emissivities due to reactions (3.4.14) and (3.4.15) are equal and Iwamoto calculates their sum to be

$$\varepsilon_\nu^{quark} \approx (8.8 \times 10^{26} erg\ s^{-1} cm^{-3}) \alpha_s \frac{n}{n_{nuc}} Y_e^{1/3} T_9^6, \tag{3.4.17}$$

where $Y_e = n_e/n$ is the number of electrons per baryon. The value of α_s is momentum dependent and not very well determined experimentally. From Appendix C, the M.I.T. bag model suggests the value $\alpha_s \approx 0.55$. However, the analysis of charmonium decay gives $\alpha_s \approx 0.065$. Iwamato adopts the value $\alpha_s = 0.1$ in Eq. (3.4.17). He also sets $Y_e = 0.01$, a value typical of *normal* neutron star matter [20]. The resulting emissivity is

$$\varepsilon_\nu^{quark} \approx (1.9 \times 10^{25} erg\ s^{-1} cm^{-3}) \frac{n}{n_{nuc}} T_9^6. \tag{3.4.18}$$

The corresponding luminosity for a uniform density ρ and mass M star composed of quark matter is

$$L_\nu^{quark} = \varepsilon_\nu^{quark} V \tag{3.4.19}$$

Here $V = M/\rho$ is the volume of the star composed of quark matter.
By substituting Eq. (3.4.18) into Eq. (3.4.19), we get

$$L_\nu^{quark} \approx (1.9 \times 10^{25} erg\ s^{-1} cm^{-3}) \frac{n}{n_{nuc}} \Big(\frac{M}{\rho}\Big) T_9^6 \tag{3.4.20}$$

But $n_{nuc} = \rho_{nuc}/m_B$ and the baryon number density $n = \rho/m_B$. Thus

$$L_\nu^{quark} \approx (1.9 \times 10^{25} erg\ s^{-1} cm^{-3}) \Big(\frac{M}{\rho_{nuc}}\Big) T_9^6 \tag{3.4.21}$$

If we multiply Eq. (3.4.21) by $(M_\odot/M_\odot)$ from the right and put the values of $M_\odot$ and ρ_{nuc} in it, we get

$$L_\nu^{quark} \approx (1.3 \times 10^{44} erg\ s^{-1}) \frac{M}{M\odot} T_9^6. \tag{3.4.22}$$

The presence s quarks will lead to additional neutrino emission via β-*decay* reactions of the type

$$s \rightarrow u + e^- + \bar{\nu}_e, \tag{3.4.23}$$

$$u + e^- \rightarrow s + \nu_e. \tag{3.4.24}$$

Since the s quark is massive the momenta of the interacting particles can differ significantly from collinearity, thereby ensuring nonzero matrix elements. However, weak interactions coupling s and u quarks [e.g., reactions (3.4.23) and (3.4.24)] are “Cabibbo suppressed” in WSG theory relative to interactions coupling d and u quarks [e.g., reactions (3.4.14) and (3.4.15)]. More precisely, the former are proportional to the factor $sin^2\theta_C$ while the latter are proportional to $cos^2\theta_C$, where θ_C is called the Cabbibo angle. Experimentally θ_C is found to satisfy $cos^2\theta_C \approx 0.974$. Iwamoto thus concludes that s-quark neutrino processes will not alter the total emissivity (3.4.18) appreciably.

The question of whether quark matter exists in neutron star cores is by no means resolved. However, a comparison of Eqs. (3.3.29), (3.4.11), and (3.4.22) shows that a star with quark matter would cool at a rate much faster than an ordinary neutron star and comparable to that of a star with a pion-condensed core.

Chapter 4

Thermal History of Neutron Stars

The thermal history of NS_s can be described during the cooling process of NS_s. As we have been discussed in chapter two, when the reaction takes place in the core of NS the thermal energy loss through the neutrino emission in different ways. This thermal energy can be expressed in the following section by describing the temperature as a function of time in different reactions which take place during the cooling process of neutron stars not only in its interior but also on the surface. Base on this discussion the theoretical results are compared with observations of thermal radiation from neutron stars.

4.1 Neutrino Transparency

Once the neutrino luminosities are known, we can calculate cooling timescales. Fundamental to the discussion that follows is the assumption that neutrinos, once created, escape from neutron stars without undergoing further interactions or energy loss. We now show that for $T \lesssim 10^9 K$, this assumption is valid.

Before the discovery of neutral currents in 1974, the argument went as follows [6]: Reactions of the type

$$\begin{aligned} \nu_e + n &\rightarrow p + e^-, \\ \bar{\nu}_e + p &\rightarrow n + e^+, \\ \bar{\nu}_e + p + n &\rightarrow n + n + e^+, \end{aligned} \tag{4.1.1}$$

and so on are all forbidden in neutron star interiors by conservation of energy and momentum. The most important interaction for neutrino energy loss would then be inelastic scattering off electrons (for ν_e and $\bar{\nu}_e$) and off muons (for ν_μ and $\bar{\nu}_\mu$). In a degenerate gas, the cross section for $\nu_e - e^-$ scattering, assuming only charged current interactions, is [15]

$$\sigma_e \simeq \chi\sigma_0 \left(\frac{E_\nu}{m_e c^2}\right)^2 \frac{E_\nu}{E_F(e)}, \tag{4.1.2}$$

where

$$\sigma_0 \equiv \frac{4}{\pi}\left(\frac{\hbar}{m_e c}\right)^{-4}\left(\frac{G_F}{m_e c^2}\right)^2 = 1.76 \times 10^{-44} cm^2, \tag{4.1.3}$$

and where

$\chi \approx 0.1$ (V − A theory: only charged-current interactions),

≈ 0.06 (WSG theory: charged- and neutral-current interactions).

The cross sections for the other three scattering processes are comparable.

The mean free path of an electron neutrino is then

$$\lambda_e = (\sigma_e n_e)^{-1}. \tag{4.1.4}$$

Using the results of Equations (3.1.15),(3.1.16) and the old V − A value for σ_e, we get

$$\lambda_e \sim (9 \times 10^7 km)\left(\frac{\rho_{nuc}}{\rho}\right)^{4/3}\left(\frac{100\ keV}{E_\nu}\right)^3. \tag{4.1.5}$$

Since $\lambda_e \gg 10\ km$, the star is transparent to neutrinos.

With the possibility of neutral currents, the effective absorption mean free path is significantly reduced because of n-ν elastic scattering reactions, such as

$$\begin{aligned} n + \nu_e &\rightarrow n + \nu_e, \\ n + \nu_\mu &\rightarrow n + \nu_\mu, \end{aligned} \tag{4.1.6}$$

which only occur via neutral currents. Although elastic scattering does not degrade the neutrino energy, it does prevent the neutrino from escaping directly after emission. The neutrino scatters many times off neutrons in the interior, until it finally encounters an electron (or muon) and scatters inelastically, or else it random–walks its way to the surface and escapes with its energy unchanged.

The cross section for elastic scattering off neutrons is [15]

$$\sigma_n = \frac{1}{4}\sigma_0\left(\frac{E_\nu}{m_e c^2}\right)^2, \tag{4.1.7}$$

and the associated mean free path is

$$\begin{aligned} \lambda_n &= \frac{1}{\sigma_n n_n} \\ &\simeq 300\ km\ \frac{\rho_{nuc}}{\rho}\left(\frac{100\ keV}{E_\nu}\right)^2. \end{aligned} \tag{4.1.8}$$

Here we have used Eq. (3.1.14) for n_n. The *effective* mean free path for inelastic scattering off electrons, taking into account the increased probability of encountering an electron in the interior due to the zig-zag path of the neutrino, is

$$\lambda_{eff} \sim (\lambda_n \lambda_e)^{1/2} \tag{4.1.9}$$

$$\sim 2 \times 10^5\ km\left(\frac{\rho_{nuc}}{\rho}\right)^{7/6}\left(\frac{100\ keV}{E_\nu}\right)^{5/2}, \tag{4.1.10}$$

where we have used the WSG value for σ_e. So although λ_{eff} is significantly less than λ_e, it is still much larger than the radius of a neutron star. Hence, the "old" conclusion regarding the transparency of a neutron star to low-energy neutrinos ($T \lesssim 10^9\ K$, $kT \lesssim 100\ keV$) is still valid.

4.2 Cooling Curves

The temperature of a NS can now be calculated as a function of time. The thermal energy of the star resides almost exclusively in degenerate fermions (neutrons or quarks). Neglecting interactions, the heat capacity of N such particles of mass m and relativity parameter $x = p_f/mc$ is

$$C_\nu = Nc_\nu \equiv \left.\frac{dU}{dT}\right|_{N,V} = \frac{\pi^2(x^2+1)^{1/2}}{x^2}Nk\left(\frac{kT}{mc^2}\right), \tag{4.2.1}$$

where c_ν is the specific heat per particle.

The total thermal energy U for N particles can be calculated by integrating Eq. (4.2.1), which gives

$$U = \frac{\pi^2(x^2+1)^{1/2}}{2x^2}\left(\frac{Nk^2}{mc^2}\right)T^2 \tag{4.2.2}$$

Here the quantity x remains constant to lowest order.

The total thermal energy U_n for a normal neutron star of mass M, density ρ and temperature T is

$$U_n = \frac{\pi^2(x^2+1)^{1/2}}{2x^2}\left(\frac{Nk^2}{m_nc^2}\right)T^2. \tag{4.2.3}$$

Here we assume $x \ll 1$, we get

$$U_n = \frac{\pi^2}{2x^2}\left(\frac{Nk^2}{m_nc^2}\right)T^2. \tag{4.2.4}$$

But $N = n_nV$, where $V = M/\rho$ is the volume of the NS and use equations (3.1.14) and (3.1.18),thus

$$\begin{aligned} U_n &= \frac{\pi^2 n_n m_n k^2}{2p_F^2(n)}\left(\frac{M}{\rho}\right)T^2 \\ &= \frac{\pi^2\big[1.7\times10^{38}\big(\frac{\rho}{\rho_{nuc}}\big)cm^{-3}\big]m_nk^2\big(\frac{M}{\rho}\big)}{2\big[340\big(\frac{\rho}{\rho_{nuc}}\big)^{1/3}MeV/c\big]^2}T^2 \end{aligned} \tag{4.2.5}$$

where

$m_n = 1.6749\times10^{-27}kg$ is the neutron mass,

$c = 2.9979\times10^8 m/s$ is the speed of light in a vacuum and

$k = 1.3806\times10^{-23}J/K$ is Boltzmann's constant.

By substituting the values which describe above into Eq. (4.2.5) and multiplies it by $(M_\odot/M_\odot)$ and (ρ_{nuc}/ρ_{nuc}) from the right and rearange it, we get

$$U_n \simeq 6\times10^{47}erg\left(\frac{M}{M_\odot}\right)\left(\frac{\rho}{\rho_{nuc}}\right)^{-2/3}T_9^2. \tag{4.2.6}$$

Similarly, we can follow the same procedure to calculate the total thermal energy U_q for relativistic quark matter with $n_u = n_d = n_s = n$ and use Eq. (3.4.13), we get

$$U_q \simeq 9 \times 10^{47} erg\Big(\frac{M}{M_\odot}\Big)\Big(\frac{n}{n_{nuc}}\Big)^{-1/3} T_9^2. \tag{4.2.7}$$

The temperature T appearing in Eqs. (4.2.1)–(4.2.7) is the interior temperature. NS interiors are to good approximation isothermal, because of the high thermal conductivity of the degenerate electron gas. It is only in the low–density, nondegenerate, outermost layer that an appreciable temperature gradient exists [3].

The cooling equation is

$$\frac{dU}{dt} = C_\nu \frac{dT}{dt} = -(L_\nu + L_\gamma), \tag{4.2.8}$$

where L_ν is the total neutrino luminosity and L_γ is the photon luminosity. Assuming blackbody photon emission from the surface at an effective surface temperature T_e, we have

$$L_\gamma = 4\pi R^2 \sigma T_e^4 = 7 \times 10^{36} erg\ s^{-1} \left(\frac{R}{10km}\right)^2 T_{e,7}^4, \tag{4.2.9}$$

where σ is the Stefan–Boltzmann constant and $T_{e,7}$ is the temperature in units of $10^7 K$.

Inserting the appropriate luminosities into Eq. (4.2.8) and integrating gives the time for the star to cool from an initial interior temperature T(i) to a final temperature T(f).

The cooling time during neutrino emission in the modified URCA process is calculated from

$$\frac{dU_n}{dt} = -L_\nu^{(URCA)}, \tag{4.2.10}$$

where $L_\nu^{(URCA)} = (5.3 \times 10^{39} ergs^{-1}) \frac{M}{M_\odot} \left(\frac{\rho_{nuc}}{\rho}\right)^{1/3} T_9^8$ and U_n is given by Eq. (4.2.6).

Differentiating U_n with respect to temperature(T) we get
$\frac{dU_n}{dT} = (1.2 \times 10^{48} erg)(\frac{M}{M_\odot})\left(\frac{\rho_{nuc}}{\rho}\right)^{2/3} T_9$ from which follows

$$dU_n = (1.2 \times 10^{48} erg)(\frac{M}{M_\odot})\left(\frac{\rho_{nuc}}{\rho}\right)^{2/3} T_9 dT. \tag{4.2.11}$$

Subistitute this Eq. (4.2.11) into Eq. (4.2.10), then
$dt = -\frac{(1.2\times 10^{48} erg)(\frac{M}{M_\odot})\left(\frac{\rho_{nuc}}{\rho}\right)^{2/3} T_9 dT}{(5.3\times 10^{39} erg\ s^{-1})(\frac{M}{M_\odot})\left(\frac{\rho_{nuc}}{\rho}\right)^{1/3} T_9^8}.$
After we splify this equation and integrate it as follow,

$$\int_{t(i)}^{t(f)} dt = -2.26415094 \times 10^8 sec.\left(\frac{\rho_{nuc}}{\rho}\right)^{1/3} \int_{T(i)}^{T(f)} \frac{dT}{T_9^7}$$

$$\triangle t(URCA) = 37.74\ Msec.\left(\frac{\rho_{nuc}}{\rho}\right)^{1/3}\left(\frac{1}{T_9^6(f)} - \frac{1}{T_9^6(i)}\right)$$

$$\triangle t(URCA) \simeq 1.2\ yr\left(\frac{\rho_{nuc}}{\rho}\right)^{1/3}\left(\frac{1}{T_9^6(f)} - \frac{1}{T_9^6(i)}\right) \tag{4.2.12}$$

From Eq. (4.2.12) the temperature of a neutron star as a function of time for modified URCA reaction as:

$$T_f^{URCA}(t) = 10^9 K\left[\frac{t}{1.2yr}\left(\frac{\rho}{\rho_{nuc}}\right)^{1/3} + \frac{1}{10^{12}}\right]^{-1/6} \tag{4.2.13}$$

By using Eqs. (3.4.22) and (4.2.7) the cooling time of NS during neutrino emission in quark beta decay is calculated as follow:-
$\frac{dU_q}{dt} = -L_\nu^{(quark)}.$

$$dt = -\frac{dU_q}{L_\nu^{(quark)}} \tag{4.2.14}$$

Let us derivate U_q with respect to temperature(T) and we get
$\frac{dU_q}{dT} = (1.8 \times 10^{48} erg)(\frac{M}{M_\odot})\left(\frac{n_{nuc}}{n}\right)^{1/3} T_9.$
This implies that

$$dU_q = (1.8 \times 10^{48} erg)(\frac{M}{M_\odot})\left(\frac{n_{nuc}}{n}\right)^{1/3} T_9 dT. \tag{4.2.15}$$

Subistitute Eq. (4.2.15) into Eq. (4.2.14), then
$dt = -\frac{(1.8\times10^{48} erg)(\frac{M}{M_\odot})\left(\frac{n_{nuc}}{n}\right)^{1/3} T_9 dT}{(1.3\times10^{44} erg\ s^{-1})(\frac{M}{M_\odot})T_9^6}$.
After we splify this equation and integrate it as follow,

$$\int_{t(i)}^{t(f)} dt = -1.384615385\times10^4 sec.\left(\frac{n_{nuc}}{n}\right)^{1/3}\int_{T(i)}^{T(f)}\frac{dT}{T_9^5}$$
$$\triangle t(quark) = 3.46\times10^3 Sec.\left(\frac{n_{nuc}}{n}\right)^{1/3}\left(\frac{1}{T_9^4(f)}-\frac{1}{T_9^4(i)}\right)$$
$$\triangle t(quark) \simeq 1hr\left(\frac{n_{nuc}}{n}\right)^{1/3}\left(\frac{1}{T_9^4(f)}-\frac{1}{T_9^4(i)}\right) \tag{4.2.16}$$

From Eq. (4.2.16) the temperature of a NS as a function of time for quark beta decay can be expressed

$$T_f^{quark}(t) = 10^9 K\left[\frac{t}{1hr}\left(\frac{n}{n_{nuc}}\right)^{1/3}+\frac{1}{10^8}\right]^{-1/4} \tag{4.2.17}$$

Detailed calculations by Tsuruta (1974, 1979) and Malone (1974) indicate that in general the surface and interior temperatures are related by

$$\frac{T_e}{T} \sim 10^{-2}\alpha, \qquad 0.1 \lesssim \alpha \lesssim 1. \tag{4.2.18}$$

Now if photon emission is ever the dominant energy loss mechanism, then Eqs. (4.2.6), (4.2.9), and (4.2.18) give

$$\triangle t(photons) = 1.9\times10^3 yr\alpha^{-2}\left(\frac{M}{M_\odot}\right)^{1/3}\left\{\frac{1}{T_{e,7}^2(f)}-\frac{1}{T_{e,7}^2(i)}\right\}, \tag{4.2.19}$$

where the relation $M = 4\pi\rho R^3/3$ has been used to eliminate R, and it is assumed that $\alpha \simeq constant$ for short time intervals. From Eq. (4.2.19) the temperature of a neutron star as a function of time for photon emission can be expressed

$$T_{e,f}^{photon}(t) = 10^7 K\left[\frac{t\alpha^2}{1.9\times10^3\ yr}\left(\frac{M_\odot}{M}\right)^{1/3}+\frac{1}{10^4\alpha^2}\right]^{-1/2} \tag{4.2.20}$$

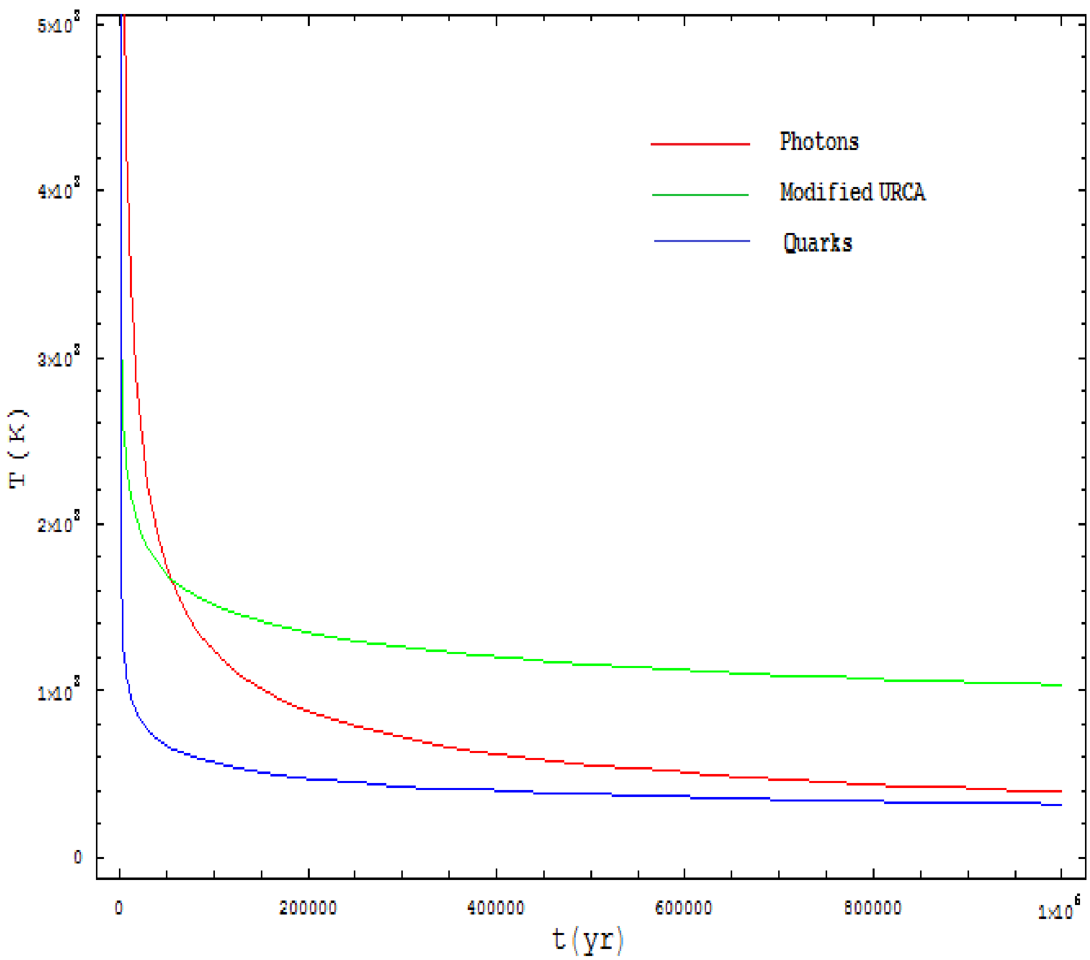

Figure 4.1: NS cooling curves of interior temperature versus time for various processes.

The above results are summarized in Figure 4.1, where the relative importance of the various cooling processes is plotted as a function of time for a neutron star with $M = M_{\odot}$, $\rho = \rho_{nuc}$ and α is very close to 0.1 . Each curve in the figure gives T(t) for each process separately assuming the others to be absent. The most effective cooling process at any time will be that with the lowest T(t); the interior temperature will be approximately T at that epoch.

These cooling timescales were determined assuming that the neutrons and protons comprised a normal fluid. Superfluidity would modify the results in two different ways.

The specific heat capacity increases discontinuously as the temperature falls below the transition temperature, and then decreases exponentially at lower temperatures. So immediately above the transition temperature the cooling timescale increases, while at lower temperatures it decreases. Second, neutrino production processes are suppressed in a superfluid, thus increasing the cooling timescale.

We have also neglected the role of a magnetic field in reducing the photon opacity of a NS near the surface. For a given value of T, a decrease in opacity increases T_e, and hence the photon luminosity.

Chapter 5

Conclusion

In this thesis as we have shown in chapter two a NS is formed at a very high temperature in the core of a supernova explosion caused by the death of a massive star that ran out its nuclear fuel. In the process inward pressure due to gravity forces inverse beta decay ($p+e \rightarrow n+\nu_e$) to occur at higher densities $\geq 10^{13} kg/m^3$. At densities $\geq 10^{17} kg/m^3$, the nuclei suddenly fall apart into a gas with 80 percent neutrons. The neutrons so formed are degenerate and provide the degenerate gas pressure required to balance the inward pull of gravity and form a NS. From this point of view, this pressure allows the formation of a stable NS which is composed mainly of neutrons. Neutron degenerate pressure is not related to temperature but rather on density. The diameter of NS_s is about 10 km to 20 km and it is mass dependent.

The emission of neutrinos take place in the core of NS_s during the predominant cooling stage immediately after its formation. Even though there are various neutrino emitting reaction, we have emphasis to the modified URCA reaction. The cooling from such a reaction is strongest during the first 10^5 years. If we have also seen the neutrino lumimosities in the other reactions and competed with each other in Eqs. (3.3.29),(3.4.11) and (3.4.22), the star with quark matter would cool at a rate much faster than an ordinary NS and comparable to that of a star with a pion-condensed core.

From chapter four we have seen that when the NS cools thermal energy losses through the emission of neutrino in the core and photon on the surface of NS. As we have seen from Figure 4.1 and Eqs. (4.2.13), (4.2.17) and (4.2.20) the most effective cooling process at $t \lesssim 10^5$ years will be the neutrino emission during the modified URCA reaction and the photon emission overtakes the neutrino on the NS when $t \gtrsim 10^5$ years . That means, after the formation of NS the the surface temperature of NS is decrease rapidly than interior temperature during different reactions take place in the core of NS.

Appendices

Appendix A

The Phase-space Factor, Eq. (3.3.16)

In this section we follow Bahcall and wolf (1965) in evaluating the phase–space factor P of Eq. (3.3.14). Such integrals appear frequently in neutrino and condensed matter *physics*.[1] We make use of the approximation that kT is negligible compared with all the fermi kinetic energies involved.

Using Eq. (3.3.2), rewrite P in the form

$$P = B \int \prod_{j=1}^{6} p_j^2 dp_j S E_{\bar{\nu}} \delta(E_f - E_i) A, \tag{5.0.1}$$

where

$$B = (m_\pi c)^{-15} (2\pi)^{-18}, \tag{5.0.2}$$

$$A = \hbar^{-3} \int \prod_{j=1}^{6} d\Omega_j \delta^3(\mathbf{k}_f - \mathbf{k}_i). \tag{5.0.3}$$

In the angular integral A, it is sufficient to consider only that region of phase space in which particle energies are within a few kT of the Fermi energies. The corresponding Fermi momentum of the neutron is large compared with the Fermi momenta of the proton and the electron (Eq. (3.1.19)); the neutrino momentum kT/c is completely negligible. Recalling that

$$\mathbf{k}_i = \mathbf{k}_1 + \mathbf{k}_2, \qquad \mathbf{k}_f = \mathbf{k}_{1'} + \mathbf{k}_s, \tag{5.0.4}$$

where

$$\mathbf{k}_s = \mathbf{k}_p + \mathbf{k}_e + \mathbf{k}_{\bar{\nu}}, \tag{5.0.5}$$

we have

$$\hbar^3 A = \int d\Omega_1 d\Omega_2 d\Omega_{1'} d\Omega_p d\Omega_e d\Omega_{\bar{\nu}} \delta^3(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_{1'} - \mathbf{k}_s). \tag{5.0.6}$$

Do the $d\Omega_{1'}$ integral first, writing the $\delta-$ function as

$$\delta(\mathbf{k}_{1'} - |\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_s|)\frac{\delta(\Omega_{1'} - \Omega_{1+2-s})}{k_{1'}^2}. \tag{5.0.7}$$

The $d\Omega_{1'}$ integral gives unity, leeving

$$\hbar^3 A = \int d\Omega_1 d\Omega_2 d\Omega_p d\Omega_e d\Omega_{\bar{\nu}} \frac{\delta(\mathbf{k}_{1'} - |\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_s|)}{k_{1'}^2}. \tag{5.0.8}$$

Rewrite the remaining $\delta-$ function as

$$\begin{aligned} &\delta\big[k_{1'} - (k_1^2 + |\mathbf{k}_2 - \mathbf{k}_s|^2 - 2k_1|\mathbf{k}_2 - \mathbf{k}_s|\cos\theta_1)^{1/2}\big] \\ &= \frac{\delta\big[\cos\theta_1 - (k_{1'}^2 - k_1^2 - |\mathbf{k}_2 - \mathbf{k}_s|^2)/(2k_1|\mathbf{k}_2 - \mathbf{k}_s|)\big]}{k_1|\mathbf{k}_2 - \mathbf{k}_s|/k_{1'}}. \end{aligned} \tag{5.0.9}$$

Here we have chosen the z axis for $\mathbf{k}_1$ along $\mathbf{k}_2 - \mathbf{k}_s$, and we have used the identity $\delta[f(x)] = \delta(x-a)/|f'(a)|$, where $f(a) = 0$. Doing the $d\Omega_1$ integral, we get

$$\hbar^3 A = \frac{2\pi}{k_{1'}k_1} \int \frac{d\Omega_2 d\Omega_p d\Omega_e d\Omega_{\bar{\nu}}}{|\mathbf{k}_2 - \mathbf{k}_s|}. \tag{5.0.10}$$

Recalling that $k_2 \gg k_s$, we find

$$\hbar^3 A = \frac{2\pi(4\pi)^4}{k_{1'}k_1k_2}, \tag{5.0.11}$$

or

$$A = (4\pi)^5(2p_1p_2p_{1'})^{-1}. \tag{5.0.12}$$

The momentum differentials in Eq. (5.0.1) may be simplified by using

$$\begin{aligned} p_j dp_j &= E_j \frac{dE_j}{c^2} \simeq m_j dE_j, \quad j = n, p, \\ dp_e &\simeq \frac{dE_e}{c}. \end{aligned} \tag{5.0.13}$$

Then all the p_j (except $p_{\bar{\nu}}$) can be set equal to $p_F(j)$ and removed from the integral. This gives

$$p = 2^9\pi^5c^{-4}Bm_n^3m_pp_F(p)p_F^2(e)\int\prod_{j=1}^{6}dE_jE_{\bar{\nu}}^3S\delta(E_f - E_i). \tag{5.0.14}$$

Now Eq. (3.3.5) for the statistical factor S can be written as

$$S = \prod_{j=1}^{5}(1 + e^x)^{-1}, \tag{5.0.15}$$

where the nondimensional energies x_j are defined by

$$\begin{aligned} x_1 &= \beta(E_1 - \mu_n), \\ x_2 &= \beta(E_2 - \mu_n), \\ x_3 &= -\beta(E_e - \mu_e), \\ x_4 &= -\beta(E_{1'} - \mu_n), \\ x_5 &= -\beta(E_p - \mu_p), \\ \beta &\equiv \frac{1}{kT}. \end{aligned} \tag{5.0.16}$$

Defining

$$y = \frac{E\bar{\nu}}{kT}, \tag{5.0.17}$$

we get for Eq. (5.0.14)

$$p = 2^9\pi^5c^{-4}Bm_n^3m_pp_F(p)p_F^2(e)(kT)^8I, \tag{5.0.18}$$

where

$$I \equiv \int_0^{\inf} dyy^3J, \tag{5.0.19}$$

$$J \equiv \int\prod_{j=1}^{5}dx_j(1 + e^{x_j})^{-1}\delta\Big[\sum_{j=1}^{5}x_j - y\Big]. \tag{5.0.20}$$

The integral J does not extend over energies less than mc^2. However, we make errors of at most $exp[-\beta E_F^{'}(p)]$ by extending the range of integration from $-\infty$ to $+\infty$ for each

x_j.

To evaluate J, start by introducing the representation

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{izx} dz. \tag{5.0.21}$$

Then

$$J = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dz\, e^{ixy} [f(z)]^5, \tag{5.0.22}$$

where

$$f(z) \equiv \int_{-\infty}^{+\infty} dx\, e^{izx} (e^x + 1)^{-1}. \tag{5.0.23}$$

In Eq. (5.0.23), z must have a small negative imaginary part for the integral to converge.

Evaluate f(z) by integrating

$$K \equiv \oint dw\, e^{izw} (e^w + 1)^{-1}, \tag{5.0.24}$$

around the contour shown in Figure 5.1. The vertical segments give no contribution in the limit $R \to \infty$. The integral along the real axis gives f(z), while the contribution along the line $Im(w) = 2\pi$ is -$exp(-2\pi z) f(z)$. The only pole of the integrand in Eq. (5.0.24) enclosed by the contour is at $w = i\pi$; the residue there is -$exp(-\pi z)$. Thus

$$K = f(z) - e^{-2\pi z} f(z) = -2\pi i e^{-\pi z}, \tag{5.0.25}$$

or

$$f(z) = \frac{\pi}{i\, sinh\, \pi z}. \tag{5.0.26}$$

Returning to Eq. (5.0.22), we have

$$J = \frac{1}{2\pi i} \int_{-\infty - ie}^{\infty - ie} dz\, e^{-izy} \left(\frac{\pi}{sinh\, \pi z} \right)^5. \tag{5.0.27}$$

Here we have inserted $-i\varepsilon$ in the limits of integration as a reminder that z has a small negative imaginary part. We would like to evaluate J by finding a suitable closed contour. If we make the substitution

$$z = z' - i \tag{5.0.28}$$

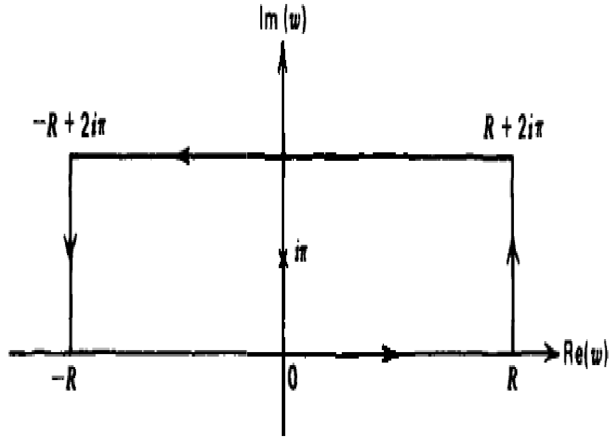

Figure 5.1: Contour for Eq. (5.0.24), with $R \to \infty$(Shapiro, S. L. and Teukolsky, S. A. 1983. Black Holes, White Dwarfs and Neutron stars. (Wiley-Interscience, New York)).

in Eq. (5.0.27), we find

$$J = \frac{-e^{-y}}{2\pi i} \int_{-\infty - ie + i\varepsilon}^{\infty - ie + i\varepsilon} dz' \, e^{-iz' y} \left(\frac{\pi}{sinh \, \pi z} \right)^5. \tag{5.0.29}$$

Equations (5.0.27) and (5.0.29) give

$$\begin{aligned} (1+e^y)J \quad &= \frac{1}{2\pi i} \left[\int_{-\infty - ie}^{\infty - ie} + \int_{-\infty - ie + i\varepsilon}^{\infty - ie + i\varepsilon} \right] dz \, e^{-izy} \left(\frac{\pi}{sinh \, \pi z} \right)^5 \\ &= \frac{1}{2\pi i} \oint dz \, e^{-izy} \left(\frac{\pi}{sinh \, \pi z} \right)^5 \end{aligned} \tag{5.0.30}$$

Here we have closed the contour as shown in Figure 5.2, the vertical segments not contributing. The only pole enclosed is at z = 0. Thus

$$(1+e^y)J = Residue \; at \; z = 0 \; of \left[e^{-izy} \left(\frac{\pi}{sinh \pi z} \right) \right]. \tag{5.0.31}$$

The residue is found by making series expansions of the exponential and sinh about Z = 0, and reading off the coefficient of 1/z. This gives

$$(1+e^y)J = \frac{3\pi^4}{8} + \frac{5\pi^2}{12} y^2 + \frac{1}{24} y^4. \tag{5.0.32}$$

Thus Eq. (5.0.19) gives

$$I = \int_0^\infty dy \left(\frac{3\pi^4}{8} y^3 + \frac{5\pi^2}{12} y^5 + \frac{1}{24} y^7 \right) (e^y + 1)^{-1}. \tag{5.0.33}$$

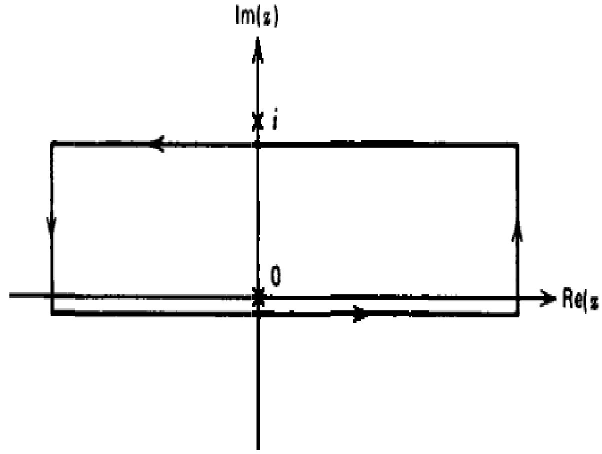

Figure 5.2: Contour for Eq. (5.0.30)(Shapiro, S. L. and Teukolsky, S. A. 1983. Black Holes, White Dwarfs and Neutron stars. (Wiley-Interscience, New York)).

The integrand of Eq. (5.0.33) gives the energy spectrum of the antineutrinos. The integral can be found in Gradshteyn and Ryzhik (1965), Eq. (3.4114). The result is

$$I = \frac{11513\pi^8}{120960}. \tag{5.0.34}$$

Using the results of Eq. (3.1.19) in Eq. (5.0.18), we finally obtain

$$P \simeq 2.1 \times 10^{-30} \left(\frac{\rho}{\rho_{nuc}} \right)^2 T_9^8, \tag{5.0.35}$$

the result used in Eq. (3.3.16).

Note that throughout our treatment we have naively employed free particle masses m_j for the nucleons, rather than effective masses $m*_j \lesssim m_j$. Effective masses take into account N–N interactions of the two–body nucleon system with the ambient many–body nucleon sea. Given the uncertainties in the effective masses, we have ignored this correction.

Appendix B

Weak Interaction Theory

Our current understanding of weak interactions such as *β–decay* is provided by the *Weinberg–Salam–Glashow theory*(WSG) [16]. In this theory, the weak force between fermions is mediated by the exchange of massive vector bosons, much as electromagnetic interactions are mediated by the exchange of photons (massless vector bosons).

In the standard WSG model, there are two charged "intermediate" bosons $W^+ and W^-$, and one neutral intermediate boson, the Z^0. They are predicted to have masses given by

$$m_W c^2 = \frac{37.3GeV}{sin\theta_W} = 78.1 \pm 1.7GeV,$$
$$m_Z c^2 = \frac{m_W c^2}{cos\theta_W} = 88.9 \pm 1.4GeV, \tag{5.0.36}$$

where the *Weinberg angle* θ_W is experimentally determined to be

$$sin^2\theta\, W0.228 \pm 0.010. \tag{5.0.37}$$

At the low–interaction energies,$E^2/c^4 \ll m_W^2, m_Z^2$, which characterize all processes occurring in neutron star interiors, the WSG Hamiltonian encompasses the old "V –A" Hamiltonian proposed by Feynman and Gell–Mann in 1958. Here V stands for the vector part of the interaction and A the axial vector part. The effective Hamiltonian density has the form in both theories

$$H_{WK} = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu, \tag{5.0.38}$$

where J_μ is the 4–current density of the interacting fermions. The universal Fermi coupling constant G_F appearing in the effective low–energy Hamiltonian is related to WSG parameters by ($\hbar = c = 1$)

$$\begin{aligned} G_F &= \frac{1}{4\sqrt{2}} \frac{\alpha}{m_W^2 sin^2\theta_W} \\ &= 1.16632 \pm 0.00004 \times 10^{-5} GeV^{-2} \\ &= 1.43582 \pm 0.00005 \times 10^{-49} ergcm^3 \\ &\simeq \frac{1.0 \times 10^{-5}}{m_p^2}. \end{aligned} \tag{5.0.39}$$

Here $\alpha = e^2/\hbar c \simeq \frac{1}{137}$ is the fine–structure constant.

An important difference between the old and new theories at low energy is that in the old theory only charged current reactions were allowed. These are reactions that, in the language of the new theory, are mediated by the charged vector bosons, $W^+ or W^-$. In the WSG model, additional neutral current reactions, mediated by the Z^0, can exist. In fact, the experimental confirmation of neutral current reactions in 1974 in part led to the award of a Nobel prize to Weinberg, Salam, and Glashow in 1979.

Reactions (3.1.1) proceed by a charged current interaction (the neutron changes into a charged proton), while the scattering processes

$$\nu + n \rightarrow \nu + n, \nu + p \rightarrow \nu + p \tag{5.0.40}$$

proceed via a neutral current. Some processes, such as *neutrino–lepton* scattering

$$\nu_e + e^- \rightarrow \nu_e + e^-, \nu_\mu + \mu^- \rightarrow \nu_\mu + \mu^-, \tag{5.0.41}$$

occur via both charged and neutral current (see Fig. 5.3).

Cross sections in the old V–A theory typically go as $G_F^2 E^2$, where E is the center–of–mass energy in a two–body interaction. Thus as $E \rightarrow \inf$, the cross section divereges rapidly. In WSG theory cross sections converge faster than $\sigma \propto (\ln E)^2$, where $n \leq 2$. The WSG model, like QED, is renormalizable: divergent integrals arising in

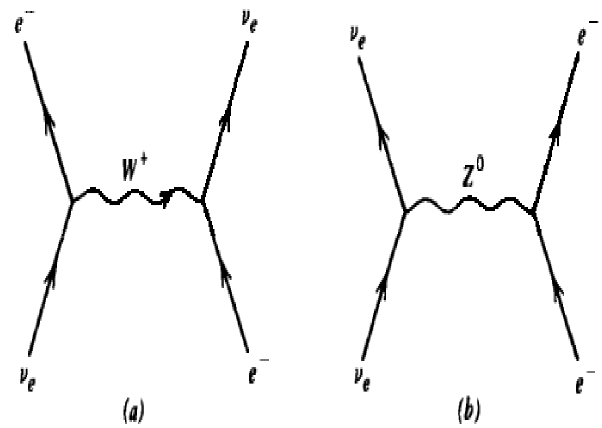

Figure 5.3: Feynman diagrams for electron–neutrino scattering $e^- + \nu_e \rightarrow e^- + \nu_e$: (a) charged current reaction and (b) neutral current reaction(Shapiro, S. L. and Teukolsky, S. A. 1983. Black Holes, White Dwarfs and Neutron stars. (Wiley-Interscience, New York)).

higher–order terms in perturbation expansions in the coupling constant can be removed in a well–defined manner [17].

Finally, we note that the WSG model unifies, in a single Lagrangian, both the weak and electromagnetic interactions. More precisely, the field equations obtained from the Lagrangian relate electromagnetic to weak fields just as Maxwell's equations relate **E** to **B** fields. This triumph has led to new attempts at a "Grand Unification" of all the known forces into a single theory. A discussion of this would take us too far afield, however.

Appendix C

Quark matter

In this section we have seen there is growing evidence the fundamental building blocks of all strongly interacting particles ($e.g, N, \triangle, \pi, \rho, ...$) are quarks. If this is true, any fundamental description of nuclear matter at high density mus involve quarks. Nuclei begin to "touch" at baryon density $\sim (4\pi r_n^3/3)^{-1} \sim few\, \rho_{nuc}$; here $r_n \sim 1\, fm$, characteristic nucleon radius. Just above this density, one might speculate that matter will undergo a phase transition at which quarks would begin to "drip" out of the nucleons. The result would be quark matter, a degenerate Fermi liquid.

Since on quarks have been observed in the free state in any experiment, they are believed to be permanently confined to the interior of hadrons by a force that increases as one tries to separate the quarks. However, the current theory of quark–quark interactions ("quantum chromodynamics") suggests that quark interactions become arbitrarily weak as the quarks are squeezed closer together ("asymptotic freedom"). Collins and Perry(1975) have therefore suggested that at sufficiently high densities, quark matter may be treated to first approximation as an ideal, relativistic Fermi gas. What would the asymptotic form of the equation of state be if this were true? We address this question immediately below.

Quarks can be transformed into other kinds of quarks (new "flavors") by weak interactions. In neutron stars, one is likely to be above threshold only for the three lightest quarks, u, d, and s. one has

$$d \rightarrow u + l + \bar{\nu},$$
$$s \rightarrow u + l + \bar{\nu}. \tag{5.0.42}$$

Here l denotes $e^{-} or \mu^{-}$, and $\bar{\nu}$ is the associated antineutrino. Assuming β*–equilibrium* and neglecting the neutrinos as usual, we have

$$\mu_d = \mu_u + \mu_l, \tag{5.0.43}$$

$$\mu_s = \mu_u + \mu_l \tag{5.0.44}$$

Now for an extremely relativistic degenerate Fermi gas,

$$n_i \propto g_l \mu_l^3, \tag{5.0.45}$$

where

$$g_i = 2, \quad i = l,$$
$$g_i = 6, \quad i = u, d, or\ s. \tag{5.0.46}$$

The 6 for quarks is the product of two spin states and three "color" states for each kind of quark. Thus Eqs, (5.0.43) – (5.0.45) give

$$\mu_d = \mu_s \quad and \quad n_d = n_s. \tag{5.0.47}$$

Equilibrium between μ^{-} and e^{-} gives

$$\mu_\mu = \mu_e = \mu_l.$$
$$n_\mu = n_e = n_l. \tag{5.0.48}$$

Charge neutrality implies

$$\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_\mu - n_e = 0, \tag{5.0.49}$$

which reduces to

$$n_u - n_s - 3n_l = 0. \tag{5.0.50}$$

Now, using Eqs. (5.0.44) and (5.0.45),

$$\left(\frac{n_s}{6}\right)^{1/3} = \left(\frac{n_u}{6}\right)^{1/3} + \left(\frac{n_l}{2}\right)^{1/3}. \tag{5.0.51}$$

Defing

$$x = \frac{n_e}{n_s}, \quad , y = \frac{n_u}{n_s}, \tag{5.0.52}$$

we find from Eqs. (5.0.50) and (5.0.51)

$$y = 1 - 3x, \tag{5.0.53}$$

$$1 = y^{1/3} + (3x)^{1/3}. \tag{5.0.54}$$

The only real solution to Eqs. (5.0.53) and (5.0.54) is y = 1, x= 0. Thus asymptotically we have

$$n_u = n_s = n_d, \quad n_e = n_\mu = 0. \tag{5.0.55}$$

The key feature of this model is that at high density the extreme relativistic free particle result applies; that is

$$P \rightarrow \frac{1}{3}\rho c^2, \quad \rho \rightarrow \infty. \tag{5.0.56}$$

This is a relatively soft equation of state.

For finite densities, quark interactions must be taken into account. At moderately high densities, one could use a perturbation expansion in the strong interaction coupling constant α_s since the quarks are asymptotically free. At lower densities, however, confinement occurs. A popular phenomenological model is the MIT "bag" model of Chodos et al. (1974). Here quarks in the nucleon are confined to a finite region of space or "bag" whose volume is held finite by a confining pressure $B > 0$ called the "bag constant" (i.e., B is the energy density needed to "inflate" tha bage). Reasonable fits to observed masses of hadrons are obtained in the model for $m_u = m_d = 0$, $B = 55 MeV fm^{-3}$, and $\alpha_s = g_s^2/16\pi\hbar c \simeq 0.55$, where g_s is the *quark–gluon* coupling constant. The energy density of quark matter is then given by the noninteracting Fermi contribution ($\propto n^{4/3}$) plus B.

Calculations of the pure neutron to quark phase transition in the bag model have been performed by a number of groups. For all the neutron equations of state considered, the phase transition occurs above the maximum density for a stable neutron star. However, the bag model is only phenomenological, and our understanding of strong interactions in is at present only tentative. The existence of stable "quark stars" is still an unresolved issue.

Appendix D

Results from Kinetic Theory

In kinetic theory, the number density in phase space for each species of particle, $d\wp/d^3x\, d^3p$, provides a complete description of the system. Equivalently, one can specify the dimensionless *distribution function* in phase space, $f(\mathbf{x},\mathbf{p},\mathrm{t})$, defined by

$$\frac{d\wp}{d^3x\, d^3p} = \frac{g}{h^3} f. \tag{5.0.57}$$

Here h^3 is the volume of a cell in phase space (h = Planck's constant), and g is the statistical weight–that is, the number of states of a particle with a given value of momentum **p**. For massive particles, g = 2S + 1 (S = spin), for photons g = 2, for neutrinos g = 1. The function f gives the average occupation number of a cell in phase space.

The number density of each species of particle is given by

$$n = \int \frac{d\wp}{d^3x d^3p} d^3p, \tag{5.0.58}$$

where the integral extends over all momenta.

For an ideal gas in equilibrium, f has the simple form

$$f(E) = \frac{1}{exp[(E-\mu)/kT] \pm 1}, \tag{5.0.59}$$

where the upper sign refers to fermions (*Fermi–Dirac* statistics) and the lower sign to bosons (*Bose–Einstein* statistics). Here k is Boltzmann's constant and μ is the chemical potential.

For sufficiently low particle densities and high temperatures, f(E) reduces to

$$f(E) \approx exp\left(\frac{\mu - E}{kT}\right), \tag{5.0.60}$$

the Maxwell–Boltzmann distribution. In this $f(E) \ll 1$.

For completely degenerate fermions ($T \to 0$, i.e, $\mu/kT \to \infty$), μ is called the *Fermi energy,* E_F, and

$$f(E) = \begin{cases} 1, & E \leq E_F \\ 0, & E > E_F. \end{cases} \tag{5.0.61}$$

Equation of State of a Completely Degenerate, Ideal Fermi Gas

An isolated neutron star ultimately cools to zero temperature, and it is the pressure associated with matter at T = 0 that supports these stars against gravitational collapse. The simplest cold, degenerate equation of state is that due to a single species of ideal (noninteracting) fermions.

For definiteness, let us take the gas to be one of neutrons at zero temperature. The gas can be treated as ideal if we ignore all electrostatic interactions.

If we define the *Fermi momentum of neutron* $p_F(n)$ by

$$E_F(n) \equiv \left(p_F^2(n)c^2 + m_n^2c^4\right)^{1/2}, \tag{5.0.62}$$

then Eqs. (5.0.58) and (5.0.61) give

$$\begin{aligned} n_n &= \frac{2}{h^3}\int d^3p \\ &= \frac{2}{h^3}\int_0^{p_F(n)}\left[\int_0^{\pi}\left(\int_0^{2\pi} d\Phi\right) sin\theta d\theta\right]p^2dp \\ &= \frac{2}{h^3}\int_0^{p_F(n)}(4\pi)p^2dp \\ &= \frac{8\pi}{3h^3}p_F^3(n). \end{aligned} \tag{5.0.63}$$

From Eq. (5.0.63) Fermi momentum of neutron is

$$p_F(n) = \left(\frac{3h^3}{8\pi}n_n\right)^{1/3}. \tag{5.0.64}$$

Since the neutron star mostly consists of neutrons, the number density of neutron can be expressed as

$$n_n = \frac{\rho}{m_n} \tag{5.0.65}$$

where $m_n = 1.6749 \times 10^{-27}$kg is the mass of the neutron.

If we multiply Eq. (5.0.65) by (ρ_{nuc}/ρ_{nuc}) and rearange it, we get

$$n_n = 1.7 \times 10^{38} \left(\frac{\rho}{\rho_{nuc}} \right) cm^{-3} \tag{5.0.66}$$

where $\rho_{nuc} = 2.8 \times 10^{17} kg\ m^{-3}$ is the standard nuclear matter density.

Inorder to get the Fermi momentum of neutron, we substitute Eq. (5.0.66) into Eq. (5.0.64). Thus

$$p_F(n) = h \left[\frac{3}{8\pi} \left(1.7 \times 10^{38} \right) \right]^{1/3} \left(\frac{\rho}{\rho_{nuc}} \right)^{1/3} cm^{-1} \tag{5.0.67}$$

where $h = 6.62608 \times 10^{-34} J\ s$.

If we multiply Eq. (5.0.67) by (c/c) ($c = 2.9978 \times 10^8 m/s$ is the speed of light) and rearange it, we get

$$p_F(n) = 340 \left(\frac{\rho}{\rho_{nuc}} \right)^{1/3} MeV/c. \tag{5.0.68}$$

Bibliography

[1] Glen, G., and Sutherland, P. G., 1980, Ap. J., **239**, 671G.

[2] Van Riper, K. A. and Lamb, D. Q., 1981, Ap. J. (Letters), **244**L, 13V.

[3] Nomoto, K. and Tsuruta, S., 1981, Ap. J. (Letters), **250**L, 19N.

[4] Chiu, H. Y., 1964, Ann. Phys., **26**, 364.

[5] Chiu, H. Y. and Salpeter, E. E., 1964, Phys. Rev. Letters, **12**, 413C.

[6] Bahcall, J.N. And Wolf, R.A., 1965, Phys. Rev. , **140**, 1452B.

[7] Tsuruta, S. and Cameron, A. G. W., 1965, Nature, **207**, 364.

[8] Hefland, D. J., 1981, in IAU symposium 95.

[9] Shapiro, S. L. and Teukolsky, S. A. 1983. Black Holes, White Dwarfs and Neutron stars. (Wiley-Interscience, New York)

[10] Tsuruta, S., 1974, IAU Symposium **53**, 209T.

[11] Iwamato, N., 1980, Phys. Rev. Lett., **44**, 1637.

[12] Astronomy: the evolving universe / Michael Zeilik –9th ed.

[13] Burrows, A., 1980, Phys. Rev. Lett., **44**, 1640.

[14] Baym, G. and chin, S. A., 1976 Ph. L.B., **62**, 241B

[15] Tubbs, D. L. and Schramm, D. N., 1975, Ap. J., **201**, 467T.

[16] Coleman, S., 1979, Scince **206**, 1290.

[17] t'Hooft, G., 1971, Nucl. Phys. B., **35**, 167T.

[18] Yakovlev D.G., Levenfish K.P., 1995, A and A **297**, 717Y.

[19] Maxwell O.V., 1979, ApJ **231**, 201.

[20] Duncan, M. J., 1983, Nu. Ph. B., **221**, 285D

Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been duly acknowledged.

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